2.1: PROBLEM DEFINITION

<u>Find</u>:

How density differs from specific weight

PLAN

Consider their definitions (conceptual and mathematical)

SOLUTION

Density is a [mass]/[unit volume], and specific weight is a [weight]/[unit volume]. Therefore, they are related by the equation $\gamma = \rho g$, and density differs from specific weight $\boxed{\text{by the factor } g}$, the acceleration of gravity.

2.2: PROBLEM DEFINITION

Find:

Fluids for which we can (usually) assume density to be nearly constant

Fluids for which density should be calculated as a function of temperature and pressure?

SOLUTION

Density can usually be assumed to be nearly constant for liquids, such as water, mercury and oil. However, even the density of a liquid varies slightly as a function of either pressure or temperature. Slight changes in the volume occupied by a given mass of a liquid as a function of pressure can be calculated using the equation for elasticity.

One must know the temperature and the pressure to determine the density of a gas.

2.3: PROBLEM DEFINITION

Find:

Where in this text you can find density data for such fluids as oil and mercury.

SOLUTION

Table A.4 in the Appendix contains density data for such fluids as oil and mercury.

2.4: PROBLEM DEFINITION

Situation:

An engineer needs to know the local density for an experiment with a glider.

$z = 2500\,\text{ft}$.

Find:

Calculate density using local conditions.

Compare calculated density with the value from Table A.2, and make a recommendation.

Properties:

From Table A.2, $R_{\text{air}} = 287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$, $\rho = 1.22\,\text{kg/m}^3$.

Local temperature = $74.3\,^\circ\text{F} = 296.7\,\text{K}$.

Local pressure = 27.3 in.-Hg = 92.45 kPa.

PLAN

Apply the ideal gas law for local conditions.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{92,450\,\text{N/m}^2}{(287\,\text{kg/m}^3)(296.7\,\text{K})} \\ &= 1.086\ \text{kg/m}^3\end{aligned}$$

$$\boxed{\rho = 1.09\ \text{kg/m}^3\ \text{(local conditions)}}$$

Table value. From Table A.2

$$\boxed{\rho = 1.22\ \text{kg/m}^3\ \text{(table value)}}$$

The density difference (local conditions versus table value) is about 12%. Most of this difference is due to the effect of elevation on atmospheric pressure.

Recommendation–use the local value of density because the effects of elevation are significant.

REVIEW

Note: Always use absolute pressure when working with the ideal gas law.

2.5: PROBLEM DEFINITION

Situation:
Carbon dioxide.

Find:
Density and specific weight of CO_2.

Properties:
From Table A.2, $R_{CO_2} = 189$ J/kg·K.
$p = 300\,\text{kPa}$, $T = 60\,°\text{C}$.

PLAN

1. First, apply the ideal gas law to find density.
2. Then, calculate specific weight using $\gamma = \rho g$.

SOLUTION

1. Ideal gas law

$$
\begin{aligned}
\rho_{CO_2} &= \frac{P}{RT} \\
&= \frac{300,000\,\text{kPa}}{(189\,\text{J/kg K})\,(60+273)\,\text{K}}
\end{aligned}
$$

$$\boxed{\rho_{CO_2} = 4.767 \text{ kg/m}^3}$$

2. Specific weight

$$\gamma = \rho g$$

Thus

$$
\begin{aligned}
\gamma_{CO_2} &= \rho_{CO_2} \times g \\
&= 4.767\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2
\end{aligned}
$$

$$\boxed{\gamma_{CO_2} = 46.764 \text{ N/m}^3}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

2.6: PROBLEM DEFINITION

Situation:

Methane gas.

Find:

Density (kg/m^3).

Specific weight (N/m^3).

Properties:

From Table A.2, $R_{\text{Methane}} = 518\ \frac{J}{kg \cdot K}$.

$p = 300\,\text{kPa}$, $T = 60\,^\circ\text{C}$.

PLAN

1. Apply the ideal gas law to find density.
2. Calculate specific weight using $\gamma = \rho g$.

SOLUTION

1. Ideal gas law

$$\begin{aligned}\rho_{\text{Methane}} &= \frac{P}{RT} \\ &= \frac{300,000\frac{N}{m^2}}{518\frac{J}{kg\cdot K}(60 + 273\,K)}\end{aligned}$$

$$\boxed{\rho_{\text{Methane}} = 1.74\ kg/m^3}$$

2. Specific weight

$$\gamma = \rho g$$

Thus

$$\begin{aligned}\gamma_{\text{Methane}} &= \rho_{\text{Methane}} \times g \\ &= 1.74\,kg/m^3 \times 9.81\,m/s^2\end{aligned}$$

$$\boxed{\gamma_{\text{Methane}} = 17.1\ N/m^3}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

2.7: PROBLEM DEFINITION

Situation:
Natural gas is stored in a spherical tank.

Find:
Ratio of final mass to initial mass in the tank.

Properties:
$p_{atm} = 100\,\text{kPa}$, $p_1 = 100\,\text{kPa-gage}$.
$p_2 = 200\,\text{kPa-gage}$, $T = 10\,°\text{C}$.

PLAN

Use the ideal gas law to develop a formula for the ratio of final mass to initial mass.

SOLUTION

1. Mass in terms of density

$$M = \rho \forall \tag{1}$$

2. Ideal gas law

$$\rho = \frac{p}{RT} \tag{2}$$

3. Combine Eqs. (1) and (2)

$$\begin{aligned} M &= \rho \forall \\ &= (p/RT)\forall \end{aligned}$$

4. Volume and gas temperature are constant, so

$$\frac{M_2}{M_1} = \frac{p_2}{p_1}$$

and

$$\begin{aligned} \frac{M_2}{M_1} &= \frac{300 \text{ kPa}}{200 \text{ kPa}} \\ & \boxed{\frac{M_2}{M_1} = 1.5} \end{aligned}$$

2.8: PROBLEM DEFINITION

Situation:

Wind and water at 100 °C and 5 atm.

Find:

Ratio of density of water to density of air.

Properties:

Air, Table A.2: $R_{\text{air}} = 287$ J/kg·K.

Water (100°C), Table A.5: $\rho_{\text{water}} = 958$ kg/m^3.

PLAN

Apply the ideal gas law to air.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho_{\text{air}} &= \frac{p}{RT} \\ &= \frac{506,600\,\text{kPa}}{(287\,\text{J}/\,\text{kg}\,\text{K})\,(100+273)\,\text{K}} \\ &= 4.73\ \text{kg/m}^3\end{aligned}$$

For water

$$\rho_{\text{water}} = 958\ \text{kg/m}^3$$

Ratio

$$\frac{\rho_{\text{water}}}{\rho_{\text{air}}} = \frac{958\,\text{kg}/\,\text{m}^3}{4.73\,\text{kg}/\,\text{m}^3}$$

$$\boxed{\frac{\rho_{\text{water}}}{\rho_{\text{air}}} = 203}$$

REVIEW

Always use absolute pressures when working with the ideal gas law.

2.9: PROBLEM DEFINITION

Situation:

Oxygen fills a tank.

$V\!\!\!/_{\text{tank}} = 10\,\text{ft}^3$, $W_{\text{tank}} = 150\,\text{lbf}$.

Find:

Weight (tank plus oxygen).

Properties:

From Table A.2, $R_{O_2} = 1555$ ft·lbf/(slug ·^{o} R) .

$p = 500$ psia, $T = 70\,°\text{F}$.

PLAN

Apply the ideal gas law to find density of oxygen.
Find the weight of the oxygen using specific weight (γ) and add this to the weight of the tank.

SOLUTION

1. Ideal gas law

$$\begin{aligned} p_{\text{abs.}} &= 500 \text{ psia} \times 144 \text{ psf/psi} = 72,000 \text{ psf} \\ T &= 460 + 70 = 530^\circ R \\ \rho &= \frac{p}{RT} \\ &= \frac{72,000 \text{ psf}}{(1555\,\text{ft lbf/ slug}^o R)\,(530^o R)} \\ \rho &= 0.087 \text{ slugs/ft}^3 \end{aligned}$$

2. Specific weight

$$\begin{aligned} \gamma &= \rho g \\ &= 0.087\frac{\text{slug}}{\text{ft}^3} \times 32.2\frac{\text{ft}}{\text{s}^2} \\ \gamma &= 2.80 \text{ lbf/ft}^3 \end{aligned}$$

3. Weight of filled tank

$$\begin{aligned} W_{\text{oxygen}} &= 2.80 \text{ lbf/ft}^3 \times 10 \text{ ft}^3 \\ &= 28 \text{ lbf} \\ W_{\text{total}} &= W_{\text{oxygen}} + W_{\text{tank}} \\ &= 28.0\,\text{lbf} + 150\,\text{lbf} \\ &\boxed{W_{\text{total}} = 178 \text{ lbf}} \end{aligned}$$

REVIEW

1. For compressed gas in a tank, pressures are often very high and the ideal gas assumption is invalid. For this problem the pressure is about 34 atmospheres–it is a good idea to check a thermodynamics reference to analyze whether or not real gas effects are significant.
2. Always use absolute pressure when working with the ideal gas law.

2.10: PROBLEM DEFINITION

Situation:

Oxygen is released from a tank through a valve.
$\forall = 10\,\mathrm{m}^3$.

Find:

Mass of oxygen that has been released.

Properties:

$R_{O_2} = 260 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$.
$p_1 = 800\,\mathrm{kPa}$, $T_1 = 15\,^\circ\mathrm{C}$.
$p_2 = 600\,\mathrm{kPa}$, $T_2 = 20\,^\circ\mathrm{C}$.

PLAN

1. Use ideal gas law, expressed in terms of density and the gas-specific (not universal) gas constant.
2. Find the density for the case before the gas is released; and then mass from density, given the tank volume.
3. Find the density for the case after the gas is released, and the corresponding mass.
4. Calculate the mass difference, which is the mass released.

SOLUTION

1. Ideal gas law

$$\rho = \frac{p}{RT}$$

2. Density and mass for case 1

$$\begin{aligned}
\rho_1 &= \frac{800,000 \frac{\mathrm{N}}{\mathrm{m}^2}}{(260 \frac{\mathrm{N \cdot m}}{\mathrm{kg \cdot K}})(288\,\mathrm{K})} \\
\rho_1 &= 10.68 \frac{\mathrm{kg}}{\mathrm{m}^3}
\end{aligned}$$

$$\begin{aligned}
M_1 &= \rho_1 \forall \\
&= 10.68 \frac{\mathrm{kg}}{\mathrm{m}^3} \times 10\,\mathrm{m}^3 \\
M_1 &= 106.8\ \mathrm{kg}
\end{aligned}$$

3. Density and mass for case 2

$$\begin{aligned}
\rho_2 &= \frac{600,000 \frac{\mathrm{N}}{\mathrm{m}^2}}{(260 \frac{\mathrm{N \cdot m}}{\mathrm{kg \cdot K}})(288\,\mathrm{K})} \\
\rho_2 &= 8.01 \frac{\mathrm{kg}}{\mathrm{m}^3}
\end{aligned}$$

$$
\begin{aligned}
M_2 &= \rho_1 \not{V} \\
&= 8.01 \frac{\text{kg}}{\text{m}^3} \times 10\,\text{m}^3 \\
M_1 &= 80.1 \text{ kg}
\end{aligned}
$$

4. Mass released from tank

$$
\begin{aligned}
M_1 - M_2 &= 106.8 - 80.1 \\
& \boxed{M_1 - M_2 = 26.7 \text{ kg}}
\end{aligned}
$$

2.11: PROBLEM DEFINITION

Situation:

Properties of air.

Find:

Specific weight (N/m^3).
Density (kg/m^3).

Properties:

From Table A.2, $R = 287 \, \frac{J}{kg \cdot K}$.
$p = 600 \, kPa$, $T = 50 \, °C$.

PLAN

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

SOLUTION

1. Ideal gas law

$$\begin{aligned} \rho_{air} &= \frac{P}{RT} \\ &= \frac{600,000 \, kPa}{(287 \, J/\, kg \, K) \, (50 + 273) \, K} \\ & \boxed{\rho_{air} = 6.47 \ kg/m^3} \end{aligned}$$

2. Specific weight

$$\begin{aligned} \gamma_{air} &= \rho_{air} \times g \\ &= 6.47 \, kg/\, m^3 \times 9.81 \, m/\, s^2 \\ & \boxed{\gamma_{air} = 63.5 \ \ N/\, m^3} \end{aligned}$$

REVIEW

Always use absolute pressure when working with the ideal gas law.

2.12: PROBLEM DEFINITION

Situation:

Consider a mass of air in the atmosphere.

$V\!\!\!\!- = 1\,\text{mi}^3$.

Find:

Mass of air using units of slugs and kg.

Properties:

From Table A.2, $\rho_{\text{air}} = 0.00237$ slugs/ft^3.

Assumptions:

The density of air is the value at sea level for standard conditions.

SOLUTION

Units of slugs

$$M = \rho V\!\!\!\!-$$

$$M = 0.00237 \frac{\text{slug}}{\text{ft}^3} \times (5280)^3\,\text{ft}^3$$

$$\boxed{M = 3.49 \times 10^8 \quad \text{slugs}}$$

Units of kg

$$M = \left(3.49 \times 10^8\,\text{slug}\right) \times \left(14.59 \frac{\text{kg}}{\text{slug}}\right)$$

$$\boxed{M = 5.09 \times 10^9 \quad \text{kg}}$$

REVIEW

The mass will probably be somewhat less than this because density decreases with altitude.

2.13: PROBLEM DEFINITION

Situation:

For a cyclist, temperature changes affect air density, thereby affecting both aerodynamic drag and tire pressure.

Find:

a.) Plot air density versus temperature for a range of -10°C to 50°C.

b.) Plot tire pressure versus temperature for the same temperature range.

Properties:

From Table A.2, $R_{\text{air}} = 287$ J/kg/K.

Initial conditions for part b: $p = 450\,\text{kPa}$, $T = 20\,°\text{C}$.

Assumptions:

For part b, assume that the bike tire volume does not change.

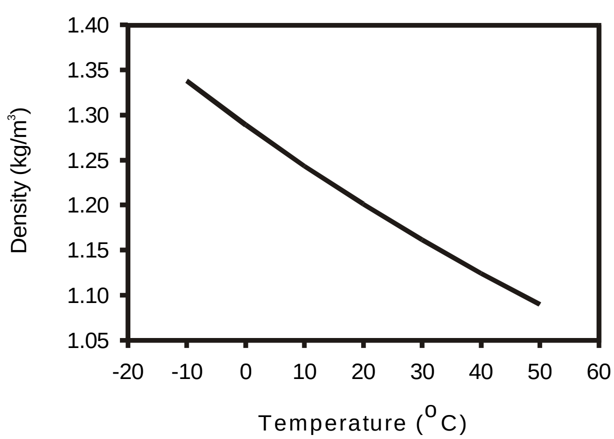
PLAN

Apply the ideal gas law.

SOLUTION

a.) Ideal gas law

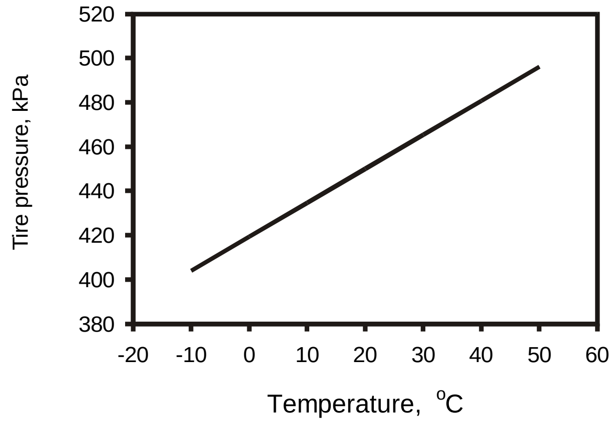
$$\rho = \frac{p}{RT} = \frac{101000\,\text{kPa}}{(287\,\text{J/kg K})\,(273+T)}$$



b.) If the volume is constant, since mass can't change, then density must be constant. Thus

$$\frac{p}{T} = \frac{p_o}{T_o}$$

$$p = 450\,\text{kPa}\left(\frac{T}{20\,°\text{C}}\right)$$



Tire pressure, kPa
520
500
480
460
440
420
400
380
-20
-10
0
10
20
30
40
50
60
Temperature, °C

2.14: PROBLEM DEFINITION

Situation:

Design of a CO_2 cartridge to inflate a rubber raft.

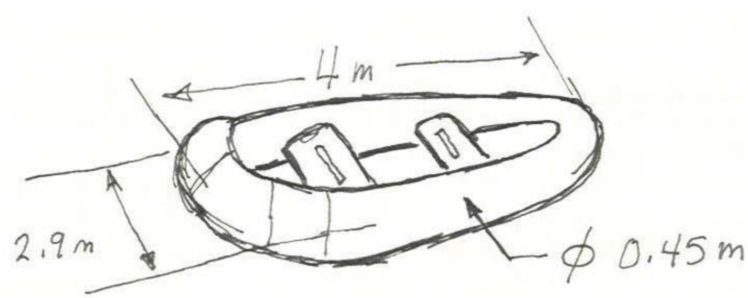
Inflation pressure = 3 psi above p_{atm} = 17.7 psia = 122 kPa abs.

Find:

Estimate the volume of the raft.

Calculate the mass of CO_2 (in grams) to inflate the raft.

Sketch:



Assumptions:

CO_2 in the raft is at 62 °F = 290 K.

Volume of the raft ≈ Volume of a cylinder with $D = 0.45\,\text{m}$ & $L = 16\,\text{m}$ (8 meters for the length of the sides and 8 meters for the lengths of the ends plus center tubes).

Properties:

CO_2, Table A.2, $R = 189$ J/kg·K.

PLAN

Since mass is related to volume by $m = \rho V$, the steps are:

1. Find volume using the formula for a cylinder.
2. Find density using the ideal gas law (IGL).
3. Calculate mass.

SOLUTION

1. Volume

$$\begin{aligned} V &= \frac{\pi D^2}{4} \times L \\ &= \left(\frac{\pi \times 0.45^2}{4} \times 16 \right) \text{m}^3 \end{aligned}$$

$$\boxed{V = 2.54 \ \text{m}^3}$$

2. Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{122,000\,\mathrm{N/\,m^2}}{(189\,\mathrm{J/\,kg\cdot\ K})\,(290\,\mathrm{K})} \\ &= 2.226\ \mathrm{kg/m^3} \end{aligned}$$

3. Mass of CO_2

$$\begin{aligned} m &= \rho \forall \\ &= \left(2.226\ \mathrm{kg/m^3}\right)\left(2.54\,\mathrm{m^3}\right) \\ & \boxed{m = 5660\,\mathrm{g}} \end{aligned}$$

REVIEW

The final mass ($5.66\,\mathrm{kg} = 12.5\,\mathrm{lbm}$) is large. This would require a large and potentially expensive CO_2 tank. Thus, this design idea may be impractical for a product that is driven by cost.

2.15: PROBLEM DEFINITION

Situation:

A helium filled balloon is being designed.
$r = 1.3\,\mathrm{m}$, $z = 80,000\,\mathrm{ft}$.

Find:

Weight of helium inside balloon.

Properties:

From Table A.2, $\mathrm{R_{He}} = 2077$ J/kg·K.
$p = 0.89\,\mathrm{bar} = 89\,\mathrm{kPa}$, $T = 22\,^{\circ}\mathrm{C} = 295.2\,\mathrm{K}$.

PLAN

Weight is given by $W = mg$. Mass is related to volume by $M = \rho * V\!\!\!/$. Density can be found using the ideal gas law.

SOLUTION

Volume in a sphere

$$
\begin{aligned}
V\!\!\!/ &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi\,(1.3\,\mathrm{m})^3 \\
&= 9.203\,\mathrm{m}^3
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{89,000\,\mathrm{N/\,m^2}}{(2077\,\mathrm{J/\,kg\cdot\,K})\,(295.2\,\mathrm{K})} \\
&= 0.145\ \mathrm{kg/m^3}
\end{aligned}
$$

Weight of helium

$$
\begin{aligned}
W &= \rho \times V\!\!\!/ \times g \\
&= \left(0.145\ \mathrm{kg/m^3}\right) \times \left(9.203\,\mathrm{m^3}\right) \times \left(9.81\,\mathrm{m/\,s^2}\right) \\
&= 13.10\,\mathrm{N}
\end{aligned}
$$

$$\boxed{\text{Weight} = 13.1\ \mathrm{N}}$$

2.16: PROBLEM DEFINITION

Situation:

Hydrometers are used to measure alcohol content of wine and beer by measuring specific weight at various stages of fermentation.

Fermentation is described by the following equation:

$$C_6H_{12}O_6 \rightarrow 2(CH_3CH_2OH) + 2(CO_2)$$

Find:

Final specific gravity of the wine.

Percent alcohol content by volume after fermentation.

Assumptions:

All of the sugar is converted to alcohol.

Initial liquid is only sugar and water.

Properties:

$S_{alcohol} = 0.80$, $S_s = 1.59$, $S_w = 1.08$.

PLAN

Imagine that the initial mixture is pure water plus saturated sugar solution and then use this visualization to find the mass of sugar that is initially present (per unit of volume). Next, apply conservation of mass to find the mass of alcohol that is produced (per unit of volume). Then, solve for the problem unknowns.

SOLUTION

The initial density of the mixture is

$$\rho_{mix} = \frac{\rho_w V_w + \rho_s V_s}{V_o}$$

where ρ_w and ρ_s are the densities of water and sugar solution (saturated), V_o is the initial volume of the mixture, and V_s is the volume of sugar solution. The total volume of the mixture is the volume of the pure water plus the volume of saturated solution

$$V_w + V_s = V_o$$

The specific gravity is initially 1.08. Thus

$$\begin{aligned} S_i &= \frac{\rho_{mix}}{\rho_w} = (1 - \frac{V_s}{V_o}) + \frac{\rho_s}{\rho_w}\frac{V_s}{V_o} \\ 1.08 &= (1 - \frac{V_s}{V_o}) + 1.59\frac{V_s}{V_o} \\ \frac{V_s}{V_o} &= 0.136 \end{aligned}$$

Thus, the mass of sugar per unit volume of mixture

$$\begin{aligned}\frac{M_s}{\mathcal{V}_o} &= 1.59 \times 0.136 \\ &= 0.216 \text{ kg/m}^3\end{aligned}$$

The molecular weight of glucose is 180 and ethyl alcohol 46. Thus 1 kg of glucose converts to 0.51 kg of alcohol so the final density of alcohol is

$$\begin{aligned}\frac{M_a}{\mathcal{V}_o} &= 0.216 \times 0.51 \\ &= 0.110 \text{ kg/m}^3\end{aligned}$$

The density of the final mixture based on the initial volume is

$$\begin{aligned}\frac{M_f}{\mathcal{V}_o} &= (1 - 0.136) + 0.110 \\ &= 0.974 \text{ kg/m}^3\end{aligned}$$

The final volume is altered because of conversion

$$\begin{aligned}\frac{\mathcal{V}_f}{\mathcal{V}_o} &= \frac{M_w}{\rho_w \mathcal{V}_o} + \frac{M_a}{\rho_a \mathcal{V}_o} \\ &= \frac{\mathcal{V}_w}{\mathcal{V}_o} + \frac{0.51 M_s}{\rho_a \mathcal{V}_o} \\ &= \frac{\mathcal{V}_w}{\mathcal{V}_o} + \frac{0.51 \rho_s}{\rho_a}\frac{\mathcal{V}_s}{\mathcal{V}_o} \\ &= 0.864 + \frac{0.51 \times 1.59}{0.8} \times 0.136 \\ &= 1.002\end{aligned}$$

The final density is

$$\begin{aligned}\frac{M_f}{\mathcal{V}_f} &= \frac{M_f}{\mathcal{V}_o} \times \frac{\mathcal{V}_o}{\mathcal{V}_f} \\ &= 0.974 \times \frac{1}{1.002} \\ &= 0.972 \text{ kg/m}^3\end{aligned}$$

The final specific gravity is

$$\boxed{S_f = 0.972}$$

The alcohol content by volume

$$\begin{aligned}\frac{\mathcal{V}_a}{\mathcal{V}_f} &= \frac{M_a}{\rho_a \mathcal{V}_f} \\ &= \frac{M_a}{\mathcal{V}_o}\frac{1}{\rho_a}\frac{\mathcal{V}_o}{\mathcal{V}_f} \\ &= 0.110 \times \frac{1}{0.8} \times \frac{1}{1.002} \\ &= 0.137\end{aligned}$$

Thus,

$$\boxed{\text{Percent alcohol by volume} = 13.7\%}$$

2.17: PROBLEM DEFINITION

Situation:

Several preview questions about viscosity are answered.

Find:

(a) The primary dimensions of viscosity and five common units of viscosity.
(b) The viscosity of motor oil (in traditional units).
(c) How and why viscosity of water varies with temperature?
(d) How and why viscosity of air varies with temperature?

SOLUTION

a) Primary dimensions of viscosity are $\boxed{[\frac{M}{LT}]}$.

Five common units are:

$\boxed{\text{i) } \frac{\text{N}\cdot\text{s}}{\text{m}^2}\text{; ii) } \frac{\text{dyn}\cdot\text{s}}{\text{cm}^2}\text{; iii) poise; iv) centipoise; and v) } \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}}$

(b) To find the viscosity of SAE 10W-30 motor oil at 115 °F, there are no tablular data in the text. Therefore, one should use Figure A.2. For traditional units (because the temperature is given in Fahrenheit) one uses the left-hand axis to report that $\boxed{\mu = 1.2 \times 10^{-3} \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}}$.

Note: one should be careful to identify the correct factor of 10 for the log cycle that contains the correct data point. For example, in this problem, the answer is between 1×10^{-3} and 1×10^{-2}. One should be able to determine that the answer is 1.2×10^{-3} and not 1×10^{-2}.

(c) The $\boxed{\text{viscosity of water decreases with increasing temperature}}$. This is true for all liquids, and is because the loose molecular lattice within liquids, which provides a given resistance to shear at a relatively cool temperature, has smaller energy barriers resisting movement at higher temperatures.

(d) The $\boxed{\text{viscosity of air increases with increasing temperature}}$. This is true for all gases, and is because gases do not have a loose molecular lattice. The only resistance to shear provided in gases is due to random collision between different layers. As the temperature increases, there are more likely to be more collisions, and therefore a higher viscosity.

2.18: PROBLEM DEFINITION

Situation:

Change in viscosity and density due to temperature.

$T_1 = 10\,°\text{C}$, $T_2 = 70\,°\text{C}$.

Find:

Change in viscosity and density of water.

Change in viscosity and density of air.

Properties:

$p = 101\,\text{kN/m}^2$.

PLAN

For water, use data from Table A.5. For air, use data from Table A.3

SOLUTION

Water

$$\mu_{70} = 4.04 \times 10^{-4} \text{ N·s/m}^2$$
$$\mu_{10} = 1.31 \times 10^{-3} \text{ N·s/m}^2$$
$$\boxed{\Delta\mu = -9.06 \times 10^{-4} \text{ N·s/m}^2}$$

$$\rho_{70} = 978 \text{ kg/m}^3$$
$$\rho_{10} = 1000 \text{ kg/m}^3$$
$$\boxed{\Delta\rho = -22 \text{ kg/m}^3}$$

Air

$$\mu_{70} = 2.04 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$$
$$\mu_{10} = 1.76 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$$
$$\boxed{\Delta\mu = 2.8 \times 10^{-6} \text{ N} \cdot \text{s/m}^2}$$

$$\rho_{70} = 1.03 \text{ kg/m}^3$$
$$\rho_{10} = 1.25 \text{ kg/m}^3$$
$$\boxed{\Delta\rho = -0.22 \text{ kg/m}^3}$$

2.19: PROBLEM DEFINITION

Situation:

Air at certain temperatures.
$T_1 = 10\,^\circ\mathrm{C}$, $T_2 = 70\,^\circ\mathrm{C}$.

Find:

Change in kinematic viscosity.

Properties:

From Table A.3, $\nu_{70} = 1.99 \times 10^{-5}$ m^2/s, $\nu_{10} = 1.41 \times 10^{-5}$ m^2/s.

PLAN

Use properties found in Table A.3.

SOLUTION

$$\begin{aligned} \Delta v_{\text{air},10\to70} &= (1.99 - 1.41) \times 10^{-5} \\ & \boxed{\Delta \mathrm{v}_{\text{air},10\to70} = 5.8{\times}10^{-6}\mathrm{m^2/s}} \end{aligned}$$

REVIEW

Sutherland's equation could also be used to solve this problem.

2.20: PROBLEM DEFINITION

Situation:

Viscosity of SAE 10W-30 oil, kerosene and water.

$T = 38\,^\circ\text{C} = 100\,^\circ\text{F}$.

Find:

Dynamic and kinematic viscosity of each fluid.

PLAN

Use property data found in Table A.4, Fig. A.2 and Table A.5.

SOLUTION

	Oil (SAE 10W-30)	kerosene	water
$\mu(\text{N}\cdot\text{s/m}^2)$	$\boxed{6.7\times10^{-2}}$	$\boxed{1.4\times10^{-3}}$ (Fig. A-2)	$\boxed{6.8\times10^{-4}}$
$\rho(\text{kg/m}^3)$	880	814	993
$\nu(\text{m}^2/\text{s})$	$\boxed{7.6\times10^{-5}}$	$\boxed{1.7\times10^{-6}}$ (Fig. A-2)	$\boxed{6.8\times10^{-7}}$

2.21: PROBLEM DEFINITION

Situation:

Dynamic and kinematic viscosity of air and water.
$T = 20\,^\circ\text{C}$.

Find:

Ratio of dynamic viscosity of air to that of water.
Ratio of kinematic viscosity of air to that of water.

Properties:

From Table A.3, $\mu_{\text{air},20^\circ C} = 1.81 \times 10^{-5}$ N·s/m^2; $\nu = 1.51 \times 10^{-5}$ m^2/s
From Table A.5, $\mu_{\text{water},20^\circ C} = 1.00 \times 10^{-3}$ N·s/m^2; $\nu = 1.00 \times 10^{-6}$ m^2/s

SOLUTION

Dynamic viscosity

$$\frac{\mu_{\text{air}}}{\mu_{\text{water}}} = \frac{1.81 \times 10^{-5}\,\text{N} \cdot \text{s/m}^2}{1.00 \times 10^{-3}\,\text{N} \cdot \text{s/m}^2}$$

$$\boxed{\frac{\mu_{\text{air}}}{\mu_{\text{water}}} = 1.81 \times 10^{-2}}$$

Kinematic viscosity

$$\frac{\nu_{\text{air}}}{\nu_{\text{water}}} = \frac{1.51 \times 10^{-5}\,\text{m}^2/\text{s}}{1.00 \times 10^{-6}\,\text{m}^2/\text{s}}$$

$$\boxed{\frac{\nu_{\text{air}}}{\nu_{\text{water}}} = 5.1}$$

2.22: PROBLEM DEFINITION

Situation:

Sutherland's equation and the ideal gas law describe behaviors of common gases.

Find:

Develop an expression for the kinematic viscosity ratio ν/ν_o, where ν is at temperature T and pressure p.

Assumptions:

Assume a gas is at temperature T_o and pressure p_o, where the subscript "o" defines the reference state.

PLAN

Combine the ideal gas law and Sutherland's equation.

SOLUTION

The ratio of kinematic viscosities is

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} = \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S}\frac{p_o}{p}\frac{T}{T_o}$$

$$\boxed{\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o+S}{T+S}}$$

2.23: PROBLEM DEFINITION

Situation:

The dynamic viscosity of air.
$\mu_o = 1.78 \times 10^{-5}$ N·s/m^2.
$T_o = 15\,^\circ$C, $T = 100\,^\circ$C.

Find:

Dynamic viscosity μ.

Properties:

From Table A.2, $S = 111K$.

SOLUTION

Sutherland's equation

$$\begin{aligned} \frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + S}{T + S} \\ &= \left(\frac{373\,\mathrm{K}}{288\,\mathrm{K}}\right)^{3/2} \frac{288\,\mathrm{K} + 111\,\mathrm{K}}{373\,\mathrm{K} + 111\,\mathrm{K}} \\ \frac{\mu}{\mu_o} &= 1.21 \end{aligned}$$

Thus

$$\begin{aligned} \mu &= 1.21\mu_o \\ &= 1.21 \times \left(1.78 \times 10^{-5}\,\mathrm{N \cdot s/\,m^2}\right) \end{aligned}$$

$$\boxed{\mu = 2.15 \times 10^{-5}\ \mathrm{N{\cdot}s/m^2}}$$

2.24: PROBLEM DEFINITION

Situation:
Methane gas.
$v_o = 1.59 \times 10^{-5}\,\mathrm{m^2/s}$.
$T_o = 15\,^\circ\mathrm{C}$, $T = 200\,^\circ\mathrm{C}$.
$p_o = 1\,\mathrm{atm}$, $p = 2\,\mathrm{atm}$.

Find:
Kinematic viscosity $(\mathrm{m^2/s})$.

Properties:
From Table A.2, $S = 198\,\mathrm{K}$.

PLAN

Apply the ideal gas law and Sutherland's equation.

SOLUTION

$$\begin{aligned} \nu &= \frac{\mu}{\rho} \\ \frac{\nu}{\nu_o} &= \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} \end{aligned}$$

Ideal-gas law

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{p_o}{p}\frac{T}{T_o}$$

Sutherland's equation

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

so

$$\begin{aligned} \frac{\nu}{\nu_o} &= \frac{1}{2}\left(\frac{473\,\mathrm{K}}{288\,\mathrm{K}}\right)^{5/2}\frac{288\,\mathrm{K} + 198\,\mathrm{K}}{473\,\mathrm{K} + 198\,\mathrm{K}} \\ &= 1.252 \end{aligned}$$

and

$$\begin{aligned} \nu &= 1.252 \times 1.59 \times 10^{-5}\ \mathrm{m^2/s} \\ \nu &= \boxed{1.99 \times 10^{-5}\,\mathrm{m^2/s}} \end{aligned}$$

2.25: PROBLEM DEFINITION

Situation:

Nitrogen gas.

$\mu_o = 3.59 \times 10^{-7}\,\text{lbf} \cdot \text{s/ft}^2$.

$T_o = 59\,^\circ\text{F}$, $T = 200\,^\circ\text{F}$.

Find:

μ using Sutherland's equation.

Properties:

From Table A.2, $S = 192^o\text{R}$.

SOLUTION

Sutherland's equation

$$\begin{aligned}
\frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S} \\
&= \left(\frac{660^o\text{R}}{519^o\text{R}}\right)^{3/2}\frac{519^o\text{R}+192^o\text{R}}{660^o\text{R}+192^o\text{R}} \\
&= 1.197 \\
\mu &= 1.197 \times \left(3.59 \times 10^{-7}\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \\
&= 4.297 \times 10^{-7}
\end{aligned}$$

$$\boxed{\mu = 4.30 \times 10^{-7}\ \text{lbf-s/ft}^2}$$

2.26: PROBLEM DEFINITION

Situation:

Helium gas.

$v_o = 1.22 \times 10^{-3}\,\text{ft}^2/\,\text{s}$.

$T_o = 59\,°\text{F}$, $T = 30\,°\text{F}$.

$p_o = 1\,\text{atm}$, $p = 1.5\,\text{atm}$.

Find:

Kinematic viscosity using Sutherland's equation.

Properties:

From Table A.2, S =143^{o}R.

PLAN

Combine the ideal gas law and Sutherland's equation.

SOLUTION

$$
\begin{aligned}
\frac{\nu}{\nu_o} &= \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o+S}{T+S} \\
&= \frac{1.5}{1}\left(\frac{490^o\text{R}}{519^o\text{R}}\right)^{5/2}\frac{519^o\text{R}+143^o\text{R}}{490^o\text{R}+143^o\text{R}} \\
&= 1.359 \\
\nu &= 1.359 \times \left(1.22 \times 10^{-3}\frac{\text{ft}^2}{\text{s}}\right) \\
&= 1.658 \times 10^{-3}\frac{\text{ft}^2}{\text{s}}
\end{aligned}
$$

$$\boxed{\nu = 1.66 \times 10^{-3}\ \text{ft}^2/\,\text{s}}$$

2.27: PROBLEM DEFINITION

Situation:

Absolute viscosity of propane.

$T_o = 100\,^\circ\text{C}$, $\mu_o = 1 \times 10^{-5}\,\text{N s/ m}^2$.

$T = 400\,^\circ\text{C}$, $\mu = 1.72 \times 10^{-5}\,\text{N s/ m}^2$.

Find:

Sutherland's constant.

SOLUTION

Sutherland's equation

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}}$$

Also

$$\begin{aligned} \frac{\mu}{\mu_o} &= 1.72 \\ \frac{T_o}{T} &= \frac{373\,\text{K}}{673\,\text{K}} \end{aligned}$$

Thus

$$\frac{S}{T_o} = 0.964$$

$$\boxed{S = 360\text{ K}}$$

2.28: PROBLEM DEFINITION

Situation:

Ammonia at room temperature.
$T_o = 68\,^\circ\mathrm{F}$, $\mu_o = 2.07 \times 10^{-7}\,\mathrm{lbf\,s/\,ft^2}$.
$T = 392\,^\circ\mathrm{F}$, $\mu = 3.46 \times 10^{-7}\,\mathrm{lbf\,s/\,ft^2}$.

Find:

Sutherland's constant.

SOLUTION

Sutherland's equation

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}} \tag{1}$$

Calculations

$$\frac{\mu}{\mu_o} = \frac{3.46 \times 10^{-7}\,\mathrm{lbf\,s/\,ft^2}}{2.07 \times 10^{-7}\,\mathrm{lbf\,s/\,ft^2}} = 1.671 \tag{a}$$

$$\frac{T_o}{T} = \frac{528\,^\circ\mathrm{R}}{852\,^\circ\mathrm{R}} = 0.6197 \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\frac{S}{T_o} = 1.71$$

$$\boxed{S = 903\ ^\circ\mathrm{R}}$$

2.29: PROBLEM DEFINITION

Situation:

SAE 10W30 motor oil.
$T_o = 38\,^\circ\text{C}$, $\mu_o = 0.067\,\text{N s/m}^2$.
$T = 99\,^\circ\text{C}$, $\mu = 0.011\,\text{N s/m}^2$.

Find:

The viscosity of motor oil, $\mu(60^o\text{C})$, using the equation $\mu = Ce^{b/T}$.

PLAN

Use algebra and known values of viscosity (μ) to solve for the constant b. Then, solve for the unknown value of viscosity.

SOLUTION

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$. Thus, evaluate μ at temperatures T and T_o and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[b\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$$

Take the logarithm and solve for b.

$$b = \frac{\ln(\mu/\mu_o)}{\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

Data

$$\begin{aligned} \mu/\mu_o &= \frac{0.011\,\text{N s/m}^2}{0.067\,\text{N s/m}^2} = 0.164 \\ T &= 372\,\text{K} \\ T_o &= 311\,\text{K} \end{aligned}$$

Solve for b

$$b = 3430\ (\text{K})$$

Viscosity ratio at 60°C

$$\begin{aligned} \frac{\mu}{\mu_o} &= \exp\left[3430\left(\frac{1}{333\,\text{K}} - \frac{1}{311\,\text{K}}\right)\right] \\ &= 0.4833 \\ \mu &= 0.4833 \times 0.067\,\text{N s/m}^2 \\ \mu &= \boxed{0.032\ \text{N}\cdot\text{s/m}^2} \end{aligned}$$

2.30: PROBLEM DEFINITION

Situation:

Viscosity of grade 100 aviation oil.
$T_o = 100\,°\text{F}$, $\mu_o = 4.43 \times 10^{-3}\,\text{lbf s/ ft}^2$.
$T = 210\,°\text{F}$, $\mu = 3.9 \times 10^{-4}\,\text{lbf s/ ft}^2$.

Find:

$\mu(150^o\text{F})$, using the equation $\mu = Ce^{b/T}$.

PLAN

Use algebra and known values of viscosity (μ) to solve for the constant b. Then, solve for the unknown value of viscosity.

SOLUTION

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$. Thus, evaluate μ at temperatures T and T_o and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Take the logarithm and solve for b

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Data

$$\begin{aligned}\frac{\mu}{\mu_o} &= \frac{0.39 \times 10^{-3}\,\text{lbf s/ ft}^2}{4.43 \times 10^{-3}\,\text{lbf s/ ft}^2} = 0.08804 \\ T &= 670^o\text{R} \\ T_o &= 560^o\text{R}\end{aligned}$$

Solve for b

$$b = 8293\ (^o\text{R})$$

Viscosity ratio at 150°F

$$\begin{aligned}\frac{\mu}{\mu_o} &= \exp\left[8293\left(\frac{1}{610^o\text{R}} - \frac{1}{560^o\text{R}}\right)\right] \\ &= 0.299 \\ \mu &= 0.299 \times \left(4.43 \times 10^{-3}\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right)\end{aligned}$$

$$\boxed{\mu = 1.32 \times 10^{-3}\,\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}}$$

2.31: PROBLEM DEFINITION

Situation:

Oil (SAE 10W30) fills the space between two plates.

$\Delta y = 1/8 = 0.125\,\text{in}$, $u = 25\,\text{ft/s}$.

Lower plate is at rest.

Find:

Shear stress in oil.

Properties:

Oil (SAE 10W30 @ 150 °F) from Figure A.2: $\mu = 5.2 \times 10^{-4}$ lbf·s/ft^2.

Assumptions:

1.) Assume oil is a Newtonian fluid.

2.) Assume Couette flow (linear velocity profile).

SOLUTION

Rate of strain

$$\begin{aligned} \frac{du}{dy} &= \frac{\Delta u}{\Delta y} \\ &= \frac{25\,\text{ft/s}}{(0.125/12)\text{ ft}} \\ \frac{du}{dy} &= 2400\,\text{s}^{-1} \end{aligned}$$

Newton's law of viscosity

$$\begin{aligned} \tau &= \mu \left(\frac{du}{dy} \right) \\ &= \left(5.2 \times 10^{-4} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} \right) \times \left(2400 \frac{1}{\text{s}} \right) \\ &= 1.248 \frac{\text{lbf}}{\text{ft}^2} \end{aligned}$$

$$\boxed{\tau = 1.25 \ \frac{\text{lbf}}{\text{ft}^2}}$$

2.32: PROBLEM DEFINITION

Situation:

Properties of air and water.

$T = 40\,°\mathrm{C}$, $p = 170\,\mathrm{kPa}$.

Find:

Kinematic and dynamic viscosities of air and water.

Properties:

Air data from Table A.3, $\mu_{\mathrm{air}} = 1.91 \times 10^{-5}$ N·s/m^2

Water data from Table A.5, $\mu_{\mathrm{water}} = 6.53 \times 10^{-4}$ N·s/m^2, $\rho_{\mathrm{water}} = 992$ kg/m^3.

PLAN

Apply the ideal gas law to find density. Find kinematic viscosity as the ratio of dynamic and absolute viscosity.

SOLUTION

A.) Air

Ideal gas law

$$\begin{aligned}\rho_{\mathrm{air}} &= \frac{p}{RT} \\ &= \frac{170,000\,\mathrm{kPa}}{(287\,\mathrm{J/\,kg\,K})\,(313.2\,\mathrm{K})} \\ &= 1.89\ \mathrm{kg/m}^3\end{aligned}$$

$$\boxed{\mu_{\mathrm{air}} = 1.91 \times 10^{-5}\ \frac{\mathrm{N{\cdot}s}}{\mathrm{m}^2}}$$

$$\begin{aligned}\nu &= \frac{\mu}{\rho} \\ &= \frac{1.91 \times 10^{-5}\,\mathrm{N\,s/\,m}^2}{1.89\,\mathrm{kg/\,m}^3}\end{aligned}$$

$$\boxed{\nu_{\mathrm{air}} = 10.1 \times 10^{-6}\,\mathrm{m}^2/\,\mathrm{s}}$$

B.) water

$$\boxed{\mu_{\mathrm{water}} = 6.53 \times 10^{-5}\ \mathrm{N{\cdot}s/m}^2}$$

$$\begin{aligned}\nu &= \frac{\mu}{\rho} \\ \nu &= \frac{6.53 \times 10^{-4}\,\mathrm{N\,s/\,m}^2}{992\,\mathrm{kg/\,m}^3}\end{aligned}$$

$$\boxed{\nu_{\mathrm{water}} = 6.58 \times 10^{-7}\ \mathrm{m}^2/\mathrm{s}}$$

2.33: PROBLEM DEFINITION

Situation:

Sliding plate viscometer is used to measure fluid viscosity.
$A = 50 \times 100\,\text{mm}$, $\Delta y = 1\,\text{mm}$.
$u = 10\,\text{m/s}$, $F = 3\,\text{N}$.

Find:

Viscosity of the fluid.

Assumptions:

Linear velocity distribution.

PLAN

1. The shear force τ is a force/area.
2. Use equation for viscosity to relate shear force to the velocity distribution.

SOLUTION

1. Calculate shear force

$$\begin{aligned} \tau &= \frac{Force}{Area} \\ \tau &= \frac{3\,\text{N}}{50\,\text{mm} \times 100\,\text{mm}} \\ \tau &= 600\,\text{N} \end{aligned}$$

2. Find viscosity

$$\begin{aligned} \mu &= \frac{\tau}{\left(\frac{du}{dy}\right)} \\ \mu &= \frac{600\,\text{N}}{[10\,\text{m/s}]\,/\,[1\,\text{mm}]} \end{aligned}$$

$$\boxed{\mu = 6 \times 10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

2.34: PROBLEM DEFINITION

Situation:

Water flows near a wall. The velocity distribution is

$$u(y) = a\left(\frac{y}{b}\right)^{1/6}$$

$a = 10\,\mathrm{m/s}$, $b = 2\,\mathrm{mm}$ and y is the distance (mm) from the wall.

Find:

Shear stress in the water at $y = 1$ mm.

Properties:

Table A.5 (water at 20 °C): $\mu = 1.00 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$.

SOLUTION

Rate of strain (algebraic equation)

$$\begin{aligned} \frac{du}{dy} &= \frac{d}{dy}\left[a\left(\frac{y}{b}\right)^{1/6}\right] \\ &= \frac{a}{b^{1/6}}\frac{1}{6y^{5/6}} \\ &= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6} \end{aligned}$$

Rate of strain (at $y = 1\,\mathrm{mm}$)

$$\begin{aligned} \frac{du}{dy} &= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6} \\ &= \frac{10\,\mathrm{m/s}}{6 \times 0.002\,\mathrm{m}}\left(\frac{2\,\mathrm{mm}}{1\,\mathrm{mm}}\right)^{5/6} \\ &= 1485\,\mathrm{s^{-1}} \end{aligned}$$

Shear Stress

$$\begin{aligned} \tau_{y=1\,\mathrm{mm}} &= \mu\frac{du}{dy} \\ &= \left(1.00 \times 10^{-3}\frac{\mathrm{N \cdot s}}{\mathrm{m^2}}\right)\left(1485\,\mathrm{s^{-1}}\right) \\ &= 1.485\,\mathrm{Pa} \end{aligned}$$

$$\boxed{\tau\,(y = 1\,\mathrm{mm}) = 1.49\,\mathrm{Pa}}$$

2.35: PROBLEM DEFINITION

Situation:

Velocity distribution of crude oil between two walls.
$\mu = 8 \times 10^{-5}\,\text{lbf s/ ft}^2$, $B = 0.1\,\text{ft}$.
$u = 100y(0.1 - y)\,\text{ft/ s}$, $T = 100\,°\text{F}$.

Find:

Shear stress at walls.

SOLUTION

Velocity distribution

$$u = 100y(0.1 - y) = 10y - 100y^2$$

Rate of strain

$$\begin{aligned} du/dy &= 10 - 200y \\ (du/dy)_{y=0} &= 10\text{ s}^{-2} \quad (du/dy)_{y=0.1} = -10\text{ s}^{-1} \end{aligned}$$

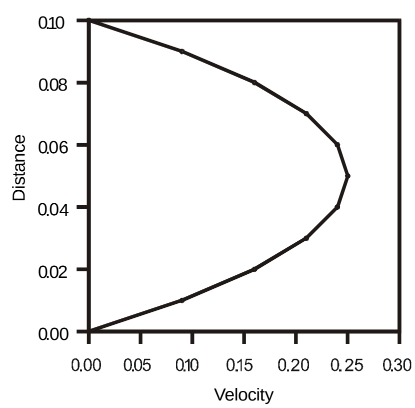
Shear stress

$$\tau_0 = \mu\frac{du}{dy} = (8 \times 10^{-5}) \times 10$$

$$\boxed{\tau_0 = 8{\times}10^{-4}\text{ lbf/ft}^2}$$

$$\boxed{\tau_{0.1} = 8{\times}10^{-4}\text{ lbf/ft}^2}$$

Plot



2.36: PROBLEM DEFINITION

Situation:

A liquid flows between parallel boundaries.

$y_0 = 0.0\,\text{mm}$, $V_0 = 0.0\,\text{m/s}$.

$y_1 = 1.0\,\text{mm}$, $V_1 = 1.0\,\text{m/s}$.

$y_2 = 2.0\,\text{mm}$, $V_2 = 1.99\,\text{m/s}$.

$y_3 = 3.0\,\text{mm}$, $V_3 = 2.98\,\text{m/s}$.

Find:

(a) Maximum shear stress.

(b) Location where minimum shear stress occurs.

SOLUTION

(a) Maximum shear stress

$$\begin{aligned} \tau &= \mu dV/dy \\ \tau_{\max} &\approx \mu(\Delta V/\Delta y) \text{ next to wall} \\ \tau_{\max} &= (10^{-3}\text{N}\cdot\text{s/m}^2)((1\text{ m/s})/0.001\text{ m}) \\ &\boxed{\tau_{\max} = 1.0\text{ N/m}^2} \end{aligned}$$

(b)The **minimum shear stress will occur midway between the two walls**. Its magnitude will be zero because the velocity gradient is zero at the midpoint.

2.37: PROBLEM DEFINITION

Situation:

Glycerin is flowing in between two stationary plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

$dp/dx = -1.6\,\text{kPa/m}$, $B = 5\,\text{cm}$.

Find:

Velocity and shear stress at a distance of 12 mm from wall (i.e. at $y = 12\,\text{mm}$).
Velocity and shear stress at the wall (i.e. at $y = 0\,\text{mm}$).

Properties:

Glycerin (20 °C), Table A.4: $\mu = 1.41\,\text{N}\cdot\text{s/m}^2$.

PLAN

Find velocity by direct substitution into the specified velocity distribution.
Find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative du/dy) is found by differentiating the velocity distribution.

SOLUTION

a.) Velocity (at $y = 12\,\text{mm}$)

$$\begin{aligned} u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\ &= -\frac{1}{2\,(1.41\,\text{N}\cdot\text{s/m}^2)}\left(-1600\,\text{N/m}^3\right)\left((0.05\,\text{m})\,(0.012\,\text{m}) - (0.012\,\text{m})^2\right) \\ &= 0.258\,7\frac{\text{m}}{\text{s}} \end{aligned}$$

$$\boxed{u\,(y = 12\,\text{mm}) = 0.259\,\text{m/s}}$$

Rate of strain (general expression)

$$\begin{aligned} \frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \end{aligned}$$

Rate of strain (at $y = 12\,\text{mm}$)

$$\begin{aligned} \frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\ &= \left(-\frac{1}{2\,(1.41\,\text{N}\cdot\text{s/m}^2)}\right)\left(-1600\frac{\text{N}}{\text{m}^3}\right)(0.05\,\text{m} - 2\times 0.012\,\text{m}) \\ &= 14.75\,\text{s}^{-1} \end{aligned}$$

Definition of viscosity

$$\begin{aligned}\tau &= \mu \frac{du}{dy} \\ &= \left(1.41 \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right)\left(14.75\,\mathrm{s}^{-1}\right) \\ &= 20.798\,\mathrm{Pa}\end{aligned}$$

$$\boxed{\tau\,(y = 12\,\mathrm{mm}) = 20.8\,\mathrm{Pa}}$$

b.) Velocity (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\ &= -\frac{1}{2\left(1.41\,\mathrm{N \cdot s/\,m^2}\right)}\left(-1600\,\mathrm{N/\,m^3}\right)\left((0.05\,\mathrm{m})(0\,\mathrm{m}) - (0\,\mathrm{m})^2\right) \\ &= 0.00\frac{\mathrm{m}}{\mathrm{s}}\end{aligned}$$

$$\boxed{u\,(y = 0\,\mathrm{mm}) = 0\,\mathrm{m/\,s}}$$

Rate of strain (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\ &= \left(-\frac{1}{2\left(1.41\,\mathrm{N \cdot s/\,m^2}\right)}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m}^3}\right)(0.05\,\mathrm{m} - 2 \times 0\,\mathrm{m}) \\ &= 28.37\,\mathrm{s}^{-1}\end{aligned}$$

Shear stress (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}\tau &= \mu \frac{du}{dy} \\ &= \left(1.41 \frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right)\left(28.37\,\mathrm{s}^{-1}\right) \\ &= 40.00\,\mathrm{Pa}\end{aligned}$$

$$\boxed{\tau\,(y = 0\,\mathrm{mm}) = 40.0\,\mathrm{Pa}}$$

REVIEW

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.
2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

2.38: PROBLEM DEFINITION

Situation:

Laminar flow occurs between two horizontal parallel plates. The velocity distribution is

$$u=-\frac{1}{2\mu}\frac{dp}{ds}\left(Hy-y^2\right)+u_t\frac{y}{H}$$

Pressure p decreases with distance s, and the speed of the upper plate is u_t. Note that u_t has a negative value to represent that the upper plate is moving to the left.

Moving plate: $y=H$.

Stationary plate: $y=0$.

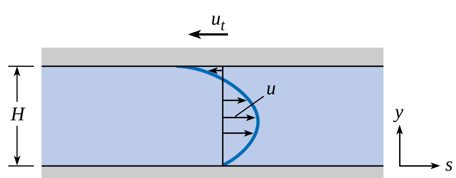
Find:

(a) Whether shear stress is greatest at the moving or stationary plate.

(b) Location of zero shear stress.

(c) Derive an expression for plate speed to make the shear stress zero at $y=0$.

Sketch:



PLAN

By inspection, the rate of strain (du/dy) or slope of the velocity profile is larger at the moving plate. Thus, we expect shear stress τ to be larger at $y=H$. To check this idea, find shear stress using the definition of viscosity: $\tau=\mu\,(du/dy)$. Evaluate and compare the shear stress at the locations $y=H$ and $y=0$.

SOLUTION

Part (a)

1. Shear stress, from definition of viscosity

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \mu\frac{d}{dy}\left[-\frac{1}{2\mu}\frac{dp}{ds}\left(Hy-y^2\right)+u_t\frac{y}{H}\right] \\
&= \mu\left[-\frac{H}{2\mu}\frac{dp}{ds}+\frac{y}{\mu}\frac{dp}{ds}+\frac{u_t}{H}\right] \\
\tau\left(y\right) &= -\frac{(H-2y)}{2}\frac{dp}{ds}+\frac{\mu u_t}{H}
\end{aligned}$$

Shear stress at $y = H$

$$\begin{aligned}\tau(y=H) &= -\frac{(H-2H)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\ &= \frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H} \qquad (1)\end{aligned}$$

2. Shear stress at $y = 0$

$$\begin{aligned}\tau(y=0) &= -\frac{(H-0)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\ &= -\frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H} \qquad (2)\end{aligned}$$

Since pressure decreases with distance, the pressure gradient dp/ds is negative. Since the upper wall moves to the left, u_t is negative. Thus, maximum shear stress occurs at $y = H$ because both terms in Eq. (1) have the same sign (they are both negative.) In other words,

$$|\tau(y=H)| > |\tau(y=0)|$$

.

$$\boxed{\text{Maximum shear stress occur at } y = H}.$$

Part (b)
Use definition of viscosity to find the location (y) of zero shear stress

$$\begin{aligned}\tau &= \mu\frac{du}{dy} \\ &= -\mu(1/2\mu)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H} \\ &= -(1/2)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H}\end{aligned}$$

Set $\tau = 0$ and solve for y

$$\begin{aligned}0 &= -(1/2)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H} \\ &\boxed{y = \frac{H}{2} - \frac{\mu u_t}{H\,dp/ds}}\end{aligned}$$

Part (c)

$$
\begin{aligned}
\tau &= \mu \frac{du}{dy} = 0 \text{ at } y = 0 \\
\frac{du}{dy} &= -(1/2\mu)\frac{dp}{ds}(H - 2y) + \frac{u_t}{H} \\
\text{Then, at } y &= 0 : du/dy = 0 = -(1/2\mu)\frac{dp}{ds}H + \frac{u_t}{H} \\
\text{Solve for } u_t &: \boxed{u_t = (1/2\mu)\frac{dp}{ds}H^2} \\
\textit{Note} &: \text{ because } \frac{dp}{ds} < 0, u_t < 0.
\end{aligned}
$$

2.39: PROBLEM DEFINITION

Situation:

Oxygen at 50 °F and 100 °F.

Find:

Ratio of viscosities: $\frac{\mu_{100}}{\mu_{50}}$.

SOLUTION

Because the viscosity of gases increases with temperature $\mu_{100}/\mu_{50} > 1$. Correct choice is $\boxed{\text{(c)}}$.

2.40: PROBLEM DEFINITION

Situation:

A cylinder falls inside a pipe filled with oil.
$d = 100\,\text{mm},\ D = 100.5\,\text{mm}.$
$\ell = 200\,\text{mm},\ W = 15\,\text{N}.$

Find:

Speed at which the cylinder slides down the pipe.

Properties:

SAE 20W oil (10°C) from Figure A.2: $\mu = 0.35\ \text{N}\cdot\text{s/m}^2$.

SOLUTION

$$\begin{aligned}
\tau &= \mu \frac{dV}{dy} \\
\frac{W}{\pi d \ell} &= \frac{\mu V_{\text{fall}}}{(D-d)/2} \\
V_{\text{fall}} &= \frac{W(D-d)}{2\pi d \ell \mu} \\
V_{\text{fall}} &= \frac{15\,\text{N}(0.5 \times 10^{-3}\,\text{m})}{(2\pi \times 0.1\,\text{m} \times 0.2\,\text{m} \times 3.5 \times 10^{-1}\,\text{N}\,\text{s}/\,\text{m}^2)} \\
&\boxed{V_{\text{fall}} = 0.17\text{m/s}}
\end{aligned}$$

2.41: PROBLEM DEFINITION

Situation:

A disk is rotated very close to a solid boundary with oil in between.
$\omega_a = 1\,\mathrm{rad/s}$, $r_2 = 2\,\mathrm{cm}$, $r_3 = 3\,\mathrm{cm}$.
$\omega_b = 2\,\mathrm{rad/s}$, $r_b = 3\,\mathrm{cm}$.
$H = 2\,\mathrm{mm}$, $\mu_c = 0.01\,\mathrm{N\,s/m^2}$.

Find:

(a) Ratio of shear stress at 2 cm to shear stress at 3 cm.
(b) Speed of oil at contact with disk surface.
(c) Shear stress at disk surface.

Assumptions:

Linear velocity distribution: $dV/dy = V/y = \omega r/y$.

SOLUTION

(a) Ratio of shear stresses

$$\begin{aligned} \tau &= \mu\frac{dV}{dy} = \frac{\mu\omega r}{y} \\ \frac{\tau_2}{\tau_3} &= \frac{\mu \times 1 \times 2/y}{\mu \times 1 \times 3/y} \\ &\boxed{\frac{\tau_2}{\tau_3} = \frac{2}{3}} \end{aligned}$$

(b) Speed of oil

$$\begin{aligned} V &= \omega r = 2 \times 0.03 \\ &\boxed{V = 0.06\mathrm{m/s}} \end{aligned}$$

(c) Shear stress at surface

$$\begin{aligned} \tau &= \mu\frac{dV}{dy} = 0.01\,\mathrm{N\,s/m^2} \times \frac{0.06\,\mathrm{m/s}}{0.002\,\mathrm{m}} \\ &\boxed{\tau = 0.30\mathrm{N/m^2}} \end{aligned}$$

2.42: PROBLEM DEFINITION

Situation:

A disk is rotated in a container of oil to damp the motion of an instrument.

Find:

Derive an equation for damping torque as a function of D, S, ω and μ.

PLAN

Apply the Newton's law of viscosity.

SOLUTION

Shear stress

$$\begin{aligned}\tau &= \mu \frac{dV}{dy} \\ &= \frac{\mu r \omega}{s}\end{aligned}$$

Find differential torque—on an elemental strip of area of radius r the differential shear force will be τdA or $\tau(2\pi r dr)$. The differential torque will be the product of the differential shear force and the radius r.

$$\begin{aligned}dT_{\text{one side}} &= r[\tau(2\pi r dr)] \\ &= r\left[\frac{\mu r \omega}{s}(2\pi r dr)\right] \\ &= \frac{2\pi\mu\omega}{s} r^3 dr \\ dT_{\text{both sides}} &= 4\left(\frac{r\pi\mu\omega}{s}\right) r^3 dr\end{aligned}$$

Integrate

$$\begin{aligned}T &= \int_0^{D/2} \frac{4\pi\mu\omega}{s} r^3 dr \\ T &= \frac{1}{16}\frac{\pi\mu\omega D^4}{s}\end{aligned}$$

2.43: PROBLEM DEFINITION

Situation:

One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder. $T_{\min} = 50\,^{\circ}\mathrm{F}$, $T_{\max} = 200\,^{\circ}\mathrm{F}$.

Find:

(a) Design a viscometer that can be used to measure the viscosity of motor oil.

Assumptions:

Motor oil is SAE 10W-30. Data from Fig A-2: μ will vary from about 2×10^{-4}lbf-s/ft^2 to 8×10^{-3}lbf-s/ft^2.

Assume the only significant shear stress develops between the rotating cylinder and the fixed cylinder.

Assume we want the maximum rate of rotation (ω) to be 3 rad/s.

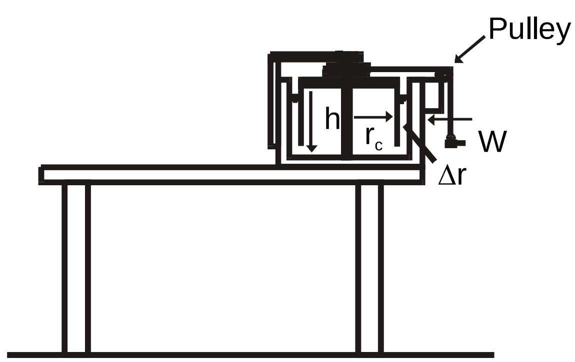
Maximum spacing is 0.05 in.

SOLUTION

One possible design solution is given below.
Design decisions:

1. Let $h = 4.0$ in. $= 0.333$ ft
2. Let I.D. of fixed cylinder $= 9.00$ in. $= 0.7500$ ft.
3. Let O.D. of rotating cylinder $= 8.900$ in. $= 0.7417$ ft.

Let the applied torque, which drives the rotating cylinder, be produced by a force from a thread or small diameter monofilament line acting at a radial distance r_s. Here r_s is the radius of a spool on which the thread of line is wound. The applied force is produced by a weight and pulley system shown in the sketch below.



The relationship between μ, r_s, ω, h, and W is now developed.

$$T = r_c F_s \tag{1}$$

where T = applied torque

r_c = outer radius of rotating cylinder
F_s = shearing force developed at the outer radius of the rotating cylinder but $F_s = \tau A_s$ where A_s = area in shear $= 2\pi r_c h$

$$\tau = \mu dV/dy \approx \mu \Delta V/\Delta r \text{ where } \Delta V = r_c\omega \text{ and } \Delta r = \text{ spacing}$$

Then $T = r_c(\mu\Delta V/\Delta r)(2\pi r_c h)$

$$= r_c\mu(\frac{r_c\omega}{\Delta r})(2\pi r_c h) \tag{2}$$

But the applied torque $T = Wr_s$ so Eq. (2) become

$$Wr_s = r_c^3\mu\omega(2\pi)\frac{h}{\Delta r}$$

Or

$$\mu = \frac{Wr_s\Delta r}{2\pi\omega h r_c^3} \tag{3}$$

The weight W will be arbitrarily chosen (say 2 or 3 oz.) and ω will be determined by measuring the time it takes the weight to travel a given distance. So $r_s\omega = V_{\text{fall}}$ or $\omega = V_{\text{fall}}/r_s$. Equation (3) then becomes

$$\mu = \left(\frac{W}{V_f}\right)\left(\frac{r_s^2}{r_c^3}\right)\left(\frac{\Delta r}{2\pi h}\right)$$

In our design let $r_s = 2$ in. $= 0.1667$ ft. Then

$$\begin{aligned}
\mu &= \left(\frac{W}{V_f}\right)\frac{(0.1667)^2}{(.3708)^3}\frac{0.004167}{(2\pi \times .3333)} \\
\mu &= \left(\frac{W}{V_f}\right)\left(\frac{0.02779}{0.05098}\right) \\
\mu &= \left(\frac{W}{V_f}\right)(1.085 \times 10^{-3}) \text{ lbf} \cdot \text{s/ft}^2
\end{aligned}$$

Example: If $W =$ 2oz. $=$ 0.125lb. and V_f is measured to be 0.24 ft/s then

$$\begin{aligned}
\mu &= \frac{0.125}{0.24}(1.085 \times 10^{-3}) \text{ lbf s/ ft}^2 \\
&= 0.564 \times 10^{-4} \text{ lbf} \cdot \text{s/ ft}^2
\end{aligned}$$

REVIEW

Other things that could be noted or considered in the design:

1. Specify dimensions of all parts of the instrument.
2. Neglect friction in bearings of pulley and on shaft of cylinder.
3. Neglect weight of thread or monofilament line.
4. Consider degree of accuracy.
5. Estimate cost of the instrument.

2.44: PROBLEM DEFINITION

Situation:

Elasticity of ethyl alcohol and water.
$E_{ethyl} = 1.06 \times 10^9$ Pa.
$E_{water} = 2.15 \times 10^9$ Pa.

Find:

Which substance is easier to compress, and why.

PLAN

Consider bulk density equation.

SOLUTION

The bulk modulus of elasticity is given by:

$$E = -\Delta p \frac{\forall}{\Delta \forall} = \frac{\Delta p}{d\rho/\rho}$$

This means that elasticity is inversely related to change in density, and to the negative change in volume.
Therefore, the liquid with the smaller elasticity is easier to compress.
Ethyl alcohol is easier to compress because it has the smaller elasticity, because elasticity is inversely related to change in density.

2.45: PROBLEM DEFINITION

Situation:

Pressure is applied to a mass of water.
$\forall = 2000\,\text{cm}^3$, $p = 2 \times 10^6\,\text{N/m}^2$.

Find:

Volume after pressure applied (cm^3).

Properties:

From Table A.5, $E = 2.2 \times 10^9$ Pa

PLAN

1. Use modulus of elasticity equation to calculate volume change resulting from pressure change.
2. Calculate final volume based on original volume and volume change.

SOLUTION

1. Elasticity equation

$$\begin{aligned} E &= -\Delta p \frac{\forall}{\Delta \forall} \\ \Delta \forall &= -\frac{\Delta p}{E} \forall \\ &= -\left[\frac{(2 \times 10^6)\ \text{Pa}}{(2.2 \times 10^9)\ \text{Pa}}\right] 2000\,\text{cm}^3 \\ &= -1.82\,\text{cm}^3 \end{aligned}$$

2. Final volume

$$\begin{aligned} \forall_{final} &= \forall + \Delta \forall \\ &= (2000 - 1.82)\ \text{cm}^3 \end{aligned}$$

$$\boxed{\forall_{final} = 1998\,\text{cm}^3}$$

2.46: PROBLEM DEFINITION

Situation:
Water is subjected to an increase in pressure.

Find:
Pressure increase needed to reduce volume by 2%.

Properties:
From Table A.5, $E = 2.2 \times 10^9$ Pa.

PLAN

Use modulus of elasticity equation to calculate pressure change required to achieve the desired volume change.

SOLUTION

Modulus of elasticity equation

$$\begin{aligned} E &= -\Delta p \frac{V}{\Delta V} \\ \Delta p &= E \frac{\Delta V}{V} \\ &= -\left(2.2 \times 10^9 \text{ Pa}\right)\left(\frac{-0.01 \times V}{V}\right) \\ &= \left(2.2 \times 10^9 \text{ Pa}\right)(0.02) \\ &= 4.4 \times 10^7 \text{ Pa} \end{aligned}$$

$$\boxed{\Delta p = 44 \text{ MPa}}$$

2.47: PROBLEM DEFINITION

Situation:

Open tank of water.

$T_{20} = 20\,^\circ\text{C}$, $T_{80} = 80\,^\circ\text{C}$.

$V\!\!\!/ = 400\,\text{l}$, $d = 3\,\text{m}$.

Hint: Volume change is due to temperature.

Find:

Percentage change in volume.

Water level rise for given diameter.

Properties:

From Table A.5: $\rho_{20} = 998\frac{\text{kg}}{\text{m}^3}$,and $\rho_{80} = 972\frac{\text{kg}}{\text{m}^3}$.

PLAN

This problem is NOT solved using the elasticity equation, because the volume change results from a change in temperature causing a density change, NOT a change in pressure. The tank is open, so the pressure at the surface of the tank is always atmospheric.

SOLUTION

a. Percentage change in volume must be calculated for this mass of water at two temperatures.
For the first temperature, the volume is given as $V\!\!\!/_{20} = 400$ L $= 0.4\,\text{m}^3$.Its density is $\rho_{20} = 998\frac{\text{kg}}{\text{m}^3}$. Therefore, the mass for both cases is given by.

$$\begin{aligned} m &= 998\frac{\text{kg}}{\text{m}^3} \times 0.4\,\text{m}^3 \\ &= 399.2\,\text{kg} \end{aligned}$$

For the second temperature, that mass takes up a larger volume:

$$\begin{aligned} V\!\!\!/_{80} &= \frac{m}{\rho} = \frac{399.2\,\text{kg}}{972\frac{\text{kg}}{\text{m}^3}} \\ &= 0.411\,\text{m}^3 \end{aligned}$$

Therefore, the percentage change in volume is

$$\begin{aligned} \frac{0.411\,\text{m}^3 - 0.4\,\text{m}^3}{0.4\,\text{m}^3} &= 0.0275 \\ \text{volume \% change} &= \boxed{= 2.8\%} \end{aligned}$$

b. If the tank has $D = 3\,\text{m}$, then $V\!\!\!/ = \pi r^2 h = 7.68h$.Therefore:

$$h_{20} = .052\,\mathrm{m}$$
$$h_{80} = .054\,\mathrm{m}$$

And water level rise is $0.054 - 0.52\,\mathrm{m} = 0.002\,\mathrm{m} = 2\,\mathrm{mm}$.

$$\boxed{\text{water level rise is} = 0.002\,\mathrm{m} = 2\,\mathrm{mm}}$$

REVIEW

Density changes can result from temperature changes, as well as pressure changes.

2.48: PROBLEM DEFINITION

Situation:

Surface tension is an energy/area.

Find:

Show that $\frac{\text{Energy}}{\text{Area}}$ equals $\frac{\text{Force}}{\text{Length}}$.

$$\begin{aligned}\frac{\text{Energy}}{\text{Area}} &= \frac{\text{force} \cdot \text{distance}}{\text{area}} \\ &= \left[\frac{M\frac{L}{T^2} \cdot L}{L^2}\right] \\ &= \left[\frac{M}{T^2}\right]\end{aligned}$$

$$\begin{aligned}\frac{\text{Force}}{\text{Length}} &= \left[\frac{M\frac{L}{T^2}}{L}\right] \\ &= \left[\frac{M}{T^2}\right]\end{aligned}$$

The primary dimensions for $\frac{\text{Energy}}{\text{Area}}$ and $\frac{\text{Force}}{\text{Length}}$ are both $\left[\frac{M}{T^2}\right]$, so they are equal.

2.49: PROBLEM DEFINITION

Situation:

Very small spherical droplet of water.

Find:

Pressure inside.

SOLUTION

Refer to Fig. 2-6(a). The surface tension force, $2\pi r\sigma$, will be resisted by the pressure force acting on the cut section of the spherical droplet or

$$p(\pi r^2) = 2\pi r\sigma$$
$$p = \frac{2\sigma}{r}$$
$$\boxed{p = \frac{4\sigma}{d}}$$

2.50: PROBLEM DEFINITION

Situation:

A spherical soap bubble.

Inside radius R, wall-thickness t, surface tension σ.

Special case: $R = 4\,\text{mm}$.

Find:

Derive a formula for the pressure difference across the bubble

Pressure difference for bubble with $R = 4\,\text{mm}$.

Assumptions:

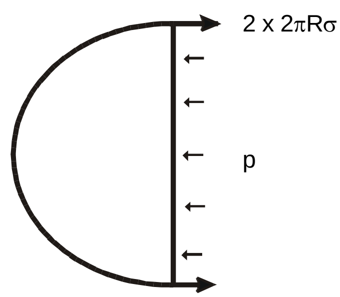
The effect of thickness is negligible, and the surface tension is that of pure water.

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION

Force balance



Surface tension force

$$\begin{aligned} \sum F &= 0 \\ \Delta p \pi R^2 - 2(2\pi R\sigma) &= 0 \end{aligned}$$

Formula for pressure difference

$$\Delta p = \frac{4\sigma}{R}$$

Pressure difference

$$\Delta p_{4\text{mm rad.}} = \frac{4 \times 7.3 \times 10^{-2}\ \text{N/m}}{0.004\ \text{m}}$$

$$\boxed{\Delta \text{p}_{4\text{mm rad.}} = 73.0\ \text{N/m}^2}$$

2.51: PROBLEM DEFINITION

Situation:

A water bug is balanced on the surface of a water pond.

$n = 6$ legs, $\ell = 5\,\text{mm/leg}$.

Find:

Maximum mass of bug to avoid sinking.

Properties:

Surface tension of water, from Table A.4, $\sigma = 0.073$ N/m.

PLAN

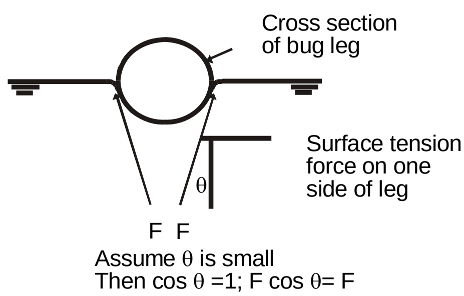
Apply equilibrium, then the surface tension force equation.

SOLUTION

Force equilibrium

$$\begin{aligned} \text{Upward force due to surface tension} &= \text{Weight of Bug} \\ F_T &= mg \end{aligned}$$

To find the force of surface tension (F_T), consider the cross section of one leg of the bug:



Surface tension force

$$\begin{aligned} F_T &= (2/\text{leg})(6 \text{ legs})\sigma\ell \\ &= 12\sigma\ell \\ &= 12(0.073 \text{ N/m})(0.005 \text{ m}) \\ &= 0.00438\,\text{N} \end{aligned}$$

Apply equilibrium

$$\begin{aligned} F_T - mg &= 0 \\ m &= \frac{F_T}{g} = \frac{0.00438\,\text{N}}{9.81\,\text{m}^2/\text{s}} \\ &= 0.4465 \times 10^{-3}\,\text{kg} \end{aligned}$$

$$\boxed{m = 0.447 \times 10^{-3}\,\text{kg}}$$

2.52: PROBLEM DEFINITION

Situation:

A water column in a glass tube is used to measure pressure.
$d_1 = 0.25\,\text{in}$, $d_2 = 1/8\,\text{in}$, $d_3 = 1/32\,\text{in}$.

Find:

Height of water column due to surface tension effects for all diameters.

Properties:

From Table A.4: surface tension of water is 0.005 lbf/ft.

SOLUTION

Surface tension force

$$\begin{aligned}
\Delta h &= \frac{4\sigma}{\gamma d} = \frac{4 \times 0.005\,\text{lbf/ ft}}{62.4\,\text{lbf/ ft}^3 \times d} = \frac{3.21 \times 10^{-4}}{d}\ \text{ft.} \\
d &= \frac{1}{4}\ \text{in.} = \frac{1}{48}\ \text{ft.};\ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}}{1/48} = 0.0154\ \text{ft.} = \boxed{0.185\ \text{in.}} \\
d &= \frac{1}{8}\ \text{in.} = \frac{1}{96}\ \text{ft.};\ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}}{1/96} = 0.0308\ \text{ft.} = \boxed{0.369\ \text{in.}} \\
d &= \frac{1}{32}\ \text{in.} = \frac{1}{384}\ \text{ft.};\ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}}{1/384} = 0.123\ \text{ft.} = \boxed{1.48\ \text{in.}}
\end{aligned}$$

2.53: PROBLEM DEFINITION

Situation:

Two vertical glass plates

$y = 1\,\text{mm}$

Find:

Capillary rise (h) between the plates.

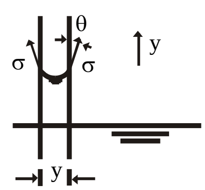
Properties:

From Table A.4, surface tension of water is 7.3×10^{-2} N/m.

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION



Equilibrium

$$\begin{aligned} \sum F_y &= 0 \\ \text{Force due to surface tension} &= \text{Weight of fluid that has been pulled upward} \\ (2\ell)\,\sigma &= (h\ell t)\,\gamma \end{aligned}$$

Solve for capillary rise (h)

$$\begin{aligned} 2\sigma\ell - h\ell t\gamma &= 0 \\ h &= \frac{2\sigma}{\gamma t} \\ h &= \frac{2 \times (7.3 \times 10^{-2}\,\text{N/m})}{9810\,\text{N/m}^3 \times 0.001\,\text{m}} \\ &= 0.0149\ \text{m} \\ h &= \boxed{14.9\ \text{mm}} \end{aligned}$$

2.54: PROBLEM DEFINITION

Situation:

A spherical water drop.

$d = 1\,\text{mm}$

Find:

Pressure inside the droplet (N/m^2)

Properties:

From Table A.4, surface tension of water is 7.3×10^{-2} N/m

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION

Equilibrium (half the water droplet)

$$\begin{aligned} \text{Force due to pressure} &= \text{Force due to surface tension} \\ pA &= \sigma L \\ \Delta p \pi R^2 &= 2\pi R\sigma \end{aligned}$$

Solve for pressure

$$\begin{aligned} \Delta p &= \frac{2\sigma}{R} \\ \Delta p &= \frac{2 \times 7.3 \times 10^{-2}\,\text{N/m}}{(0.5 \times 10^{-3}\,\text{m})} \end{aligned}$$

$$\boxed{\Delta p = 292\ \text{N/m}^2}$$

2.55: PROBLEM DEFINITION

Situation:

A tube employing capillary rise is used to measure temperature of water
$T_0 = 0\,^\circ\mathrm{C}$, $T_{100} = 100\,^\circ\mathrm{C}$
$\sigma_0 = 0.0756\,\mathrm{N/m}$, $\sigma_{100} = 0.0589\,\mathrm{N/m}$

Find:

Size the tube (this means specify diameter and length).

PLAN

Apply equilibrium and the surface tension force equation.

SOLUTION

The elevation in a column due to surface tension is

$$\Delta h = \frac{4\sigma}{\gamma d}$$

where γ is the specific weight and d is the tube diameter. For the change in surface tension due to temperature, the change in column elevation would be

$$\Delta h = \frac{4\Delta\sigma}{\gamma d} = \frac{4 \times 0.0167\,\mathrm{N/m}}{9810\,\mathrm{N/m^3} \times d} = \frac{6.8 \times 10^{-6}}{d}$$

The change in column elevation for a 1-mm diameter tube would be 6.8 mm. Special equipment, such the optical system from a microscope, would have to be used to measure such a small change in deflection It is unlikely that smaller tubes made of transparent material can be purchased to provide larger deflections.

2.56: PROBLEM DEFINITION

Situation:

A soap bubble and a droplet of water of equal diameter falling in air
$d = 2\,\text{mm}$, $\sigma_{bubble} = \sigma_{droplet}$

Find:

Which has the greater pressure inside.

SOLUTION

The soap bubble will have the greatest pressure because there are two surfaces (two surface tension forces) creating the pressure within the bubble. The correct choice is **a)**

2.57: PROBLEM DEFINITION

Situation:

A hemispherical drop of water is suspended under a surface

Find:

Diameter of droplet just before separation

Properties:

Table A.5 (20 °C): $\gamma = 9790\,\text{N/m}^3, \sigma = 0.073\,\text{N/m}$.

SOLUTION

Equilibrium

$$\begin{aligned}
\text{Weight of droplet} &= \text{Force due to surface tension} \\
\left(\frac{\pi D^3}{12}\right)\gamma &= (\pi D)\,\sigma
\end{aligned}$$

Solve for D

$$\begin{aligned}
D^2 &= \frac{12\sigma}{\gamma} \\
&= \frac{12 \times (0.073 \text{ N/m})}{9790 \text{ N/m}^3} = 8.948 \times 10^{-5}\,\text{m}^2 \\
D &= 9.459 \times 10^{-3}\,\text{m}
\end{aligned}$$

$$\boxed{D = 9.46\,\text{mm}}$$

2.58: PROBLEM DEFINITION

Situation:

Surface tension is being measured by suspending liquid from a ring
$D_i = 10\,\text{cm},\ D_o = 9.5\,\text{cm}$
$W = 10\,\text{g},\ F = 16\,\text{g}$

Find:

Surface tension

PLAN

1. Force equilibrium on the fluid suspended in the ring. For force due to surface tension, use the form of the equation provided in the text for the special case of a ring being pulled out of a liquid.
2. Solve for surface tension - all the other forces are known.

SOLUTION

1. Force equilibrium

$$\begin{aligned} \text{(Upward force)} &= \text{(Weight of fluid)} + \text{(Force due to surface tension)} \\ F &= W + \sigma(\pi D_i + \pi D_o) \end{aligned}$$

2. Solve for surface tension

$$\begin{aligned} \sigma &= \frac{F - W}{\pi(D_i + D_o)} \\ \sigma &= \frac{(0.016 - 0.010)\,\text{kg} \times 9.81\,\text{m/s}^2}{\pi(0.1 + 0.095)\,\text{m}} \\ &= 9.61 \times 10^{-2}\,\frac{\text{kg}}{\text{s}^2} \end{aligned}$$

$$\boxed{\sigma = 0.0961\ \text{N/m}}$$

2.59: PROBLEM DEFINITION

Situation:

A liquid reaches the vapor pressure

Find:

What happens to the liquid

SOLUTION

If a liquid reaches its vapor pressure for a given temperature, it boils.

2.60: PROBLEM DEFINITION

Find:

How does vapor pressure change with increasing temperature?

SOLUTION

The vapor pressure increases with increasing temperature. To get an everyday feel for this, note from the Appendix that the vapor pressure of water at 212 °F (100 °C) is 101 kPa (14.7 psia). To get water to boil at a lower temperature, you would have to exert a vacuum on the water. To keep it from boiling until a higher temperature, you would have to pressurize it.

2.61: PROBLEM DEFINITION

Situation:
Watar at 60 °F

Find:
The pressure that must be imposed for water to boil

Properties:
Water (60 °F), Table A.5: $P_v = 0.363$ psia

SOLUTION

The pressure to which the fluid must be exposed is **P = 0.363 psia.** This is lower than atmospheric pressure. Therefore, assuming atmospheric pressure is 14.7 psia gage, or 14.7 psig, the pressure needed could also be reported as **P = -14.34 psig**.

2.62: PROBLEM DEFINITION

Situation:
$T = 20\,^{\circ}\mathrm{C}$,fluid is water.

Find:
The pressure that must be imposed to cause boiling

Properties:
Water ($60\,^{\circ}\mathrm{F}$), Table A.5: $P_v = 2340$ Pa abs

SOLUTION

Bubbles will be noticed to be forming when $P = P_v$.

$$\boxed{P = 2340 \text{ Pa abs}}$$

2.63: PROBLEM DEFINITION

Situation:

Water in a closed tank

$T = 20\,°\text{C}$, $p = 10400\,\text{Pa}$

Find:

Whether water will bubble into the vapor phase (boil).

Properties:

From Table A.5, at $T = 20\,°\text{C}$, $P_v = 2340$ Pa abs

SOLUTION

The tank pressure is 10,400 Pa abs, and $P_v = 2340$ Pa abs. So the tank pressure is higher than the P_v. Therefore the **water will not boil**.

REVIEW

The water can be made to boil at this temperature only if the pressure is reduced to 2340 Pa abs. Or, the water can be made to boil at this pressure only if the temperature is raised to approximately $50\,°\text{C}$.

2.64: PROBLEM DEFINITION

Situation:

The boiling temperature of water decreases with increasing elevation $\frac{\Delta p}{\Delta T} = \frac{-3.1\,\text{kPa}}{^oC}$.

Find:

Boiling temperature at an altitude of 3000 m

Properties:

$T = 100^o\text{C}$, $p = 101\,\text{kN/m}^2$.

$z_{3000} = 3000\,\text{m}$, $p_{3000} = 69\,\text{kN/m}^2$.

Assumptions:

Assume that vapor pressure versus boiling temperature is a linear relationship.

PLAN

Develop a linear equation for boiling temperature as a function of elevation.

SOLUTION

Let BT = "Boiling Temperature." Then, BT as a function of elevation is

$$BT\ (3000\text{ m}) = \text{BT}\ (0\text{ m}) + \left(\frac{\Delta BT}{\Delta p}\right)\Delta p$$

Thus,

$$\begin{aligned} BT\ (3000\text{ m}) &= 100\,^\circ\text{C} + \left(\frac{-1.0\,^\circ\text{C}}{3.1\,\text{kPa}}\right)(101 - 69)\text{ kPa} \\ &= 89.677\,^\circ\text{C} \end{aligned}$$

$$\boxed{\text{Boiling Temperature (3000 m)} = 89.7\,^\circ\text{C}}$$