Problem 7.1

No solution provided

Problem 7.2

No solution provided

Problem 7.3

Answer the questions below.

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a. What are common forms of energy? Which of these are relevant to fluid mechanics?

- Mechanical Energy = Energy that matter has because of its motion (i.e. kinetic energy) or its position.(potential energy associated with a spring; gravitational potential energy). Highly relevant to fluid mechanics. Examples of KE include.
  - KE in a river that is flowing. KE in air that is being used to drive a wind turbine. KE in the moving air in the throat of a venturi.
  - Elastic potential energy associated with compression of a gas. Elastic potential energy associated with compression of water during water hammer.
  - Gravitational potential energy associated with water stored behind a dam. Gravitational potential energy associated with water in a stand pipe. Gravitational potential energy associated with water stored in a water tower for a municipal water supply.
- Other forms of energy include electrical energy, nuclear energy, thermal energy, and chemical energy. Each of these forms of energy can be related to fluid mechanics.

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b. What is work? Give three examples that are relevant to fluid mechanics.

- In mechanics, work is done when a force acts on a body as the body moves through a distance. Often, the magnitude of work is force multiplied by the distance moved ($W = Fd$).
  - When water in a river tumbles a rock, the water is exerting a force on the rock because the rock is moving through a distance.
  - When the heart contracts, the walls of the heart exert a force on the blood within the chambers of the heart, thereby doing work (force of wall x displacement of wall).
  - When water flows through a turbine in a dam, the water exerts a force on the turbine blade and the turbine blade rotates in response to this force.

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c. Common units of power: horsepower and watts.

---

d. Differences between power and energy

- power is a rate (amount per time) whereas energy in an amount.

- different primary dimensions: power (ML^2/T^3) and Energy (ML^2/T^2).
- different units: power has units of horsepower, watts, etc.; energy has units of Joules, ft-lbf, Btu, cal, etc.

Problem 7.4

Apply the grid method to each situation described below. Note: Unit cancellations are not shown in this solution.

a)

Situation:

A pump operates for 6 hours.
Cost $= C = \$0.15/\,\text{kW}\cdot\text{h}$.
$P = 1\,\text{hp}$, $t = 6\,\text{h}$.

Find:

Amount of energy used (joules).
Cost of this energy ($'s).

Solution:

$$\Delta E = P\Delta t = \left(\frac{1\,\text{hp}}{1.0}\right)\left(\frac{6\,\text{h}}{1.0}\right)\left(\frac{745.7\,\text{W}}{1.0\,\text{hp}}\right)\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right)$$

$$\boxed{\Delta E = 1.61\times 10^{7}\,\text{J}}$$

$$\text{Cost} = C\Delta E = \left(\frac{0.15}{\text{kW}\cdot\text{h}}\right)\left(\frac{1.611\times 10^{7}\,\text{J}}{1.0}\right)\left(\frac{\text{kW}\cdot\text{h}}{3.6\times 10^{6}\,\text{J}}\right)$$

$$\boxed{\text{Cost} = \$0.67}$$

b)

Situation:

A motor is turning the shaft of a centrifugal pump.
$T = 100\,\text{lbf}\cdot\text{in}$, $\omega = 850$ rpm.
$P = T\omega$.

Find:

Power in watts.

$$P = T\omega$$

$$= \left(\frac{100\,\text{lbf}\cdot\text{in}}{1.0}\right)\left(\frac{850\text{ rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1.0\text{ rev}}\right)\left(\frac{1.0\,\text{min}}{60\,\text{s}}\right)\left(\frac{1.0\,\text{m}}{39.37\,\text{in}}\right)\left(\frac{4.448\,\text{N}}{\text{lbf}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)$$

$$\boxed{P = 1010\,\text{W}}$$

c)

Situation:

A turbine produces power.
$P = 7500\,\text{ft lbf}/\,\text{s}$.

Find:

Covert the power to watts and hp.

Solution:

$$P = \left(\frac{7500\,\text{ft}\cdot\,\text{lbf}}{\text{s}}\right)\left(\frac{\text{s}\cdot\,\text{W}}{0.7376\,\text{ft}\cdot\,\text{lbf}}\right)$$

$$\boxed{P = 10.2\,\text{kW}}$$

$$P = \left(\frac{7500\,\text{ft}\cdot\,\text{lbf}}{\text{s}}\right)\left(\frac{\text{s}\cdot\,\text{hp}}{550\,\text{ft}\cdot\,\text{lbf}}\right)$$

$$\boxed{P = 13.6\,\text{hp}}$$

7.5: PROBLEM DEFINITION

Situation:

Water is being sprayed out a spray bottle.

Find:

Estimate the power (watts) required.

PLAN

1. Acquire data from a spray bottle.
2. Find power using $P = \Delta W / \Delta t$, where ΔW is the amount of work in the time interval Δt.

SOLUTION

1. Data:

- Ten sprays took approximately 15 seconds.
- Deflecting the trigger involves a motion of $\Delta x = 1.0\,\text{in} = 0.0254\,\text{m}$.
- The force to deflect the trigger was about $F = 3\,\text{lb}$ which is about $15\,\text{N}$.

2. Power:

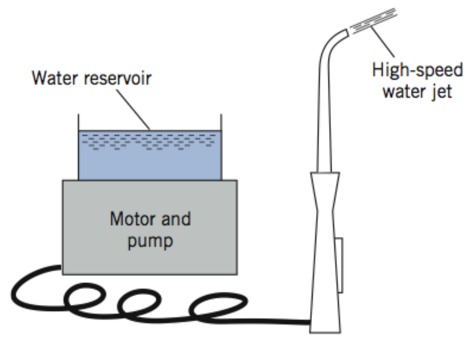
$$\begin{aligned} P &= \frac{\Delta W}{\Delta t} = \frac{F\,(N\Delta x)}{\Delta t} = \left(\frac{(15\,\text{N})\,(10\text{ sprays})\,(0.0254\,\text{m/spray})}{15\,\text{s}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right) \\ &= \boxed{0.25\,\text{W (estimate)}} \end{aligned}$$

7.6: PROBLEM DEFINITION

Situation:

A water pik is producing a high-speed jet.

$d = 3\,\text{mm}$, $V_2 = 25\,\text{m/s}$.



Find:

Minimum electrical power (watts).

Assumptions:

Neglect energy losses in the mechanical system–e.g. motor, gears, and pump.
Neglect all energy losses associated with viscosity.
Neglect potential energy changes because these are very small.

Properties:

Water $(10\,^\circ\text{C})$, Table A.5: $\rho = 1000\,\text{kg/m}^3$.

PLAN

Balance electrical power with the rate at which water carries kinetic energy out of the nozzle.

SOLUTION

$$\begin{aligned}\text{Power} &= \frac{\text{Amount of kinetic energy that leaves the nozzle}}{\text{Each interval of time}} \\ &= \frac{\Delta m \frac{V_2^2}{2}}{\Delta t}\end{aligned}$$

where Δm is the mass that has flowed out of the nozzle for each interval of time (Δt). Since the mass per time is mass flow rate: $(\Delta m/\Delta t = \dot{m} = \rho A_2 V_2)$

$$\begin{aligned}\text{Power} &= \frac{\dot{m} V_2^2}{2} \\ &= \frac{\rho A_2 V_2^3}{2}\end{aligned}$$

Exit area

$$
\begin{aligned}
A_2 &= \frac{\pi}{4} \times \left(3.0 \times 10^{-3}\,\mathrm{m}\right)^2 \\
&= 7.07 \times 10^{-6}\,\mathrm{m}^2
\end{aligned}
$$

Thus:

$$
\text{Power} = \frac{(1000\,\mathrm{kg/\,m^3})\,(7.07 \times 10^{-6}\,\mathrm{m}^2)\,(25\,\mathrm{m/\,s})^3}{2} = 55.2\,\mathrm{W}
$$

$$
\boxed{P = 55.2\,\mathrm{W}}
$$

REVIEW

Based on Ohm's law, the current on a U.S. household circuit is about: $I = P/V = 55.2\,\mathrm{W}/115\,\mathrm{V} = 0.48\,\mathrm{A}$.

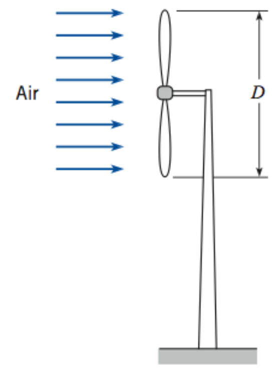
7.7: PROBLEM DEFINITION

Situation:

A small wind turbine is being developed.

$D = 1.25\,\mathrm{m}$, $V = 15$ mph $= 6.71\,\mathrm{m/s}$.

Turbine efficiency: η=20%.



Find:

Power (watts) produced by the turbine.

Properties:

Air (10 °C, 0.9 bar = 90 kPa), $R = 287\,\mathrm{J/kg \cdot K}$.

PLAN

Find the density of air using the idea gas law. Then, find the kinetic energy of the wind and use 20% of this value to find the power that is produced.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{90,000\,\mathrm{Pa}}{(287\,\mathrm{J/kg\cdot K})\,(10+273)\,\mathrm{K}} \\ &= 1.108\,\mathrm{kg/m^3}\end{aligned}$$

Kinetic energy of the wind

$$\begin{aligned}\text{Rate of KE} &= \frac{\text{Amount of kinetic energy}}{\text{Interval of time}} \\ &= \frac{\Delta m V^2/2}{\Delta t}\end{aligned}$$

where Δm is the mass of air that flows through a section of area $A = \pi D^2/4$ for each unit of time (Δt). Since the mass for each interval of time is mass flow rate:

$(\Delta m/\Delta t = \dot{m} = \rho A V)$

$$\begin{aligned} \text{Rate of KE} &= \frac{\dot{m}V^2}{2} \\ &= \frac{\rho A V^3}{2} \end{aligned}$$

The area is

$$\text{Rate of KE} = \frac{(1.108\,\text{kg/m}^3)\left(\pi\,(1.25\,\text{m})^2/4\right)(6.71\,\text{m/s})^3}{2}$$

$$\text{Rate of KE} = 205\,\text{W}$$

Since the output power is 20% of the input kinetic energy:

$$P = (0.2)\,(205\,\text{W})$$

$$\boxed{P = 41.0\,\text{W}}$$

REVIEW

The amount of energy in the wind is diffuse (i.e. spread out). For this situation, the wind turbine provides enough power for approximately one 40 watt light bulb.

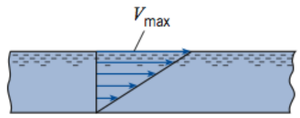
Problem 7.8

No solution provided.

7.9: PROBLEM DEFINITION

Situation:

There is a linear velocity distribution in a rectangular channel.



Find:

Kinetic energy correction factor: α

PLAN

1. Use the definition of α (Eq. 7.21 in EFM9e), and then do a term-by-term analysis.
2. Combine terms.

SOLUTION

1. Definition of α

$$\alpha = \frac{1}{A}\int_A \left(\frac{V(y)}{\bar{V}}\right)^3 dA \tag{1}$$

- $A = dw$ where d is the depth of the channel and w is the width.
- To find $V(y)$, use the equation of a straight line where y is distance from the channel floor.

$$V(y) = V_{\max}\frac{y}{d}$$

- Since the velocity profile is linear, the mean velocity is 1/2 of the maximum velocity

$$\bar{V} = V_{\max}/2$$

- $dA = wdy$

2. Substitute terms into Eq. (1)

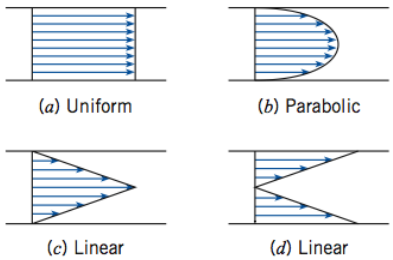
$$\begin{aligned}\alpha &= \frac{1}{wd}\int_{y=0}^{y=d}\left(\left(V_{\max}\frac{y}{d}\right)\left(\frac{2}{V_{\max}}\right)\right)^3 wdy \\ &= \frac{1}{d}\int_{y=0}^{y=d}\left(\frac{2y}{d}\right)^3 dy = \left(\frac{1}{d}\right)\left(\frac{8}{d^3}\right)\left(\frac{d^4}{4}\right) = 2\end{aligned}$$

$$\boxed{\alpha = 2}$$

7.10: PROBLEM DEFINITION

Situation:

Velocity distributions (a) through (d) are shown in the sketch.



Find:

Indicate whether α is less than, equal to, or less than unity.

SOLUTION

a) $\boxed{\alpha = 1.0}$

b) $\boxed{\alpha > 1.0}$

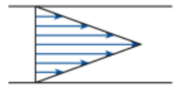
c) $\boxed{\alpha > 1.0}$

d) $\boxed{\alpha > 1.0}$

7.11: PROBLEM DEFINITION

Situation:

There is a linear velocity distribution in a round pipe.



Find:

Kinetic energy correction factor: α

SOLUTION

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\overline{V}}\right)^3 dA$$

Flow rate equation

$$\begin{aligned} V &= V_m - (\frac{r}{r_0})V_m \\ V &= V_m(1-(\frac{r}{r_0})) \\ Q &= \int V dA \\ &= \int_0^{r_0} V(2\pi r dr) \\ &= \int_0^{r_0} V_m(1-\frac{r}{r_0})2\pi r dr \\ &= 2\pi V_m \int_0^{r_0}[r-(\frac{r^2}{r_0})]dr \end{aligned}$$

Integrating yields

$$\begin{aligned} Q &= 2\pi V_m[(r^2/2)-(r^3/(3r_0))]_0^{r_0} \\ Q &= 2\pi V_m[(1/6)r_0^2] \\ Q &= (1/3)V_m A \end{aligned}$$

Thus

$$\overline{V} = \frac{Q}{A} = \frac{V_m}{3}$$

Kinetic energy correction factor

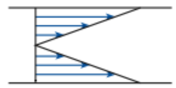
$$\begin{aligned} \alpha &= \frac{1}{A}\int_0^{r_0}\left[\frac{V_m(1-\frac{r}{r_0})}{(\frac{1}{3}V_m)}\right]^3 2\pi r dr \\ &= \frac{54\pi}{\pi r_0^2}\int_0^{r_0}(1-(\frac{r}{r_0}))^3 r dr \end{aligned}$$

$$\boxed{\alpha = 2.7}$$

7.12: PROBLEM DEFINITION

Situation:

There is a linear velocity distribution in a round pipe.



Find:

Kinetic energy correction factor: α

SOLUTION

Flow rate equation

$$\begin{aligned}
V &= kr \\
Q &= \int_0^{r_0} V(2\pi r dr) \\
&= \int_0^{r_0} 2\pi k r^2 dr \\
&= \frac{2\pi k r_0^3}{3} \\
\bar{V} &= \frac{Q}{A} \\
&= \frac{\frac{2}{3}k\pi r_0^3}{\pi r_0^2} \\
&= \frac{2}{3}kr_0
\end{aligned}$$

Kinetic energy correction factor

$$\begin{aligned}
\alpha &= \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \\
\alpha &= \frac{1}{A}\int_0^{r_0} \left(\frac{kr}{\frac{2}{3}kr_0}\right)^3 2\pi r dr \\
\alpha &= \frac{(3/2)^3 2\pi}{(\pi r_0^2)}\int_0^{r_0} \left(\frac{r}{r_0}\right)^3 r dr \\
\alpha &= \frac{27/4}{r_0^2}\left(\frac{r_0^5}{5r_0^3}\right)
\end{aligned}$$

$$\boxed{\alpha = \frac{27}{20}}$$

7.13: PROBLEM DEFINITION

Situation:

The velocity distribution in a pipe with turbulent flow is given by

$$\frac{V}{V_{\max}} = \left(\frac{y}{r_0}\right)^n$$

Find:

Derive a formula for α as a function of n.

Find α for $n = 1/7$.

SOLUTION

Flow rate equation

$$\begin{aligned}
\frac{V}{V_{\max}} &= \left(\frac{y}{r_0}\right)^n = \left(\frac{r_0 - r}{r_0}\right)^n = \left(1 - \frac{r}{r_0}\right)^n \\
Q &= \int_A V dA \\
&= \int_0^{r_0} V_{\max}\left(1 - \frac{r}{r_0}\right)^n 2\pi r dr \\
&= 2\pi V_{\max}\int_0^{r_0}\left(1 - \frac{r}{r_0}\right)^n r dr
\end{aligned}$$

Upon integration

$$Q = 2\pi V_{\max} r_0^2 \left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right]$$

Then

$$\begin{aligned}
\bar{V} &= Q/A = 2V_{\max}\left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right] \\
&= \frac{2V_{\max}}{(n+1)(n+2)}
\end{aligned}$$

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_0^{r_0}\left[\frac{V_{\max}\left(1 - \frac{r}{r_0}\right)^n}{\frac{2V_{\max}}{(n+1)(n+2)}}\right]^3 2\pi r dr$$

Upon integration one gets

$$\boxed{a = \frac{1}{4}\left[\frac{[(n+2)(n+1)]^3}{(3n+2)(3n+1)}\right]}$$

If $n = 1/7$, then

$$\alpha = \frac{1}{4}\left[\frac{\left[(\frac{1}{7}+2)(\frac{1}{7}+1)\right]^3}{\left(3\left(\frac{1}{7}\right)+2\right)\left(3\left(\frac{1}{7}\right)+1\right)}\right]$$

$$\boxed{\alpha = 1.06}$$

7.14: PROBLEM DEFINITION

Situation:

The velocity distribution in a rectangular channel with turbulent flow is given by

$$V/V_{\max} = (y/d)^n$$

Find:

Derive a formula for the kinetic energy correction factor.

Find α for $n = 1/7$.

SOLUTION

Solve for q first in terms of $u_{\max}$ and d

$$q = \int_0^d V\,dy = \int_0^d V_{\max}\left(\frac{y}{d}\right)^n dy = \frac{V_{\max}}{d^n}\int_0^d y^n dy$$

Integrating:

$$\begin{aligned} q &= \left(\frac{V_{\max}}{d^n}\right)\left[\frac{y^{n+1}}{n+1}\right]_0^d \\ &= \left(\frac{V_{\max} d^{n+1} d^{-n}}{n+1}\right) \\ &= \frac{V_{\max} d}{n+1} \end{aligned}$$

Then

$$\bar{V} = \frac{q}{d} = \frac{V_{\max}}{n+1}$$

Kinetic energy correction factor

$$\begin{aligned} \alpha &= \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \\ &= \frac{1}{d}\int_0^d \left[\frac{V_{\max}\left(\frac{y}{d}\right)^n}{V_{\max}/(n+1)}\right]^3 dy \\ &= \frac{(n+1)^3}{d^{3n+1}}\int_0^d y^{3n} dy \end{aligned}$$

Integrating

$$\alpha = \frac{(n+1)^3}{d^{3n+1}}\left[\frac{d^{3n+1}}{3n+1}\right]$$

$$\boxed{a = \frac{(n+1)^3}{3n+1}}$$

When $n = 1/7$

$$\alpha = \frac{(1+1/7)^3}{1+3/7}$$

$$\boxed{\alpha = 1.05}$$

7.15: PROBLEM DEFINITION

Situation:
Turbulent flow in a circular pipe.
$r = 3.5\,\text{cm}$.

Find:
Kinetic energy correction factor: α.

SOLUTION

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

The integral is evaluated using

$$\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \simeq \frac{1}{\bar{V}^3}\sum_i \pi(r_i^2 - r_{i-1}^2)\left(\frac{v_i + v_{i-1}}{2}\right)^3$$

Results. The mean velocity is 24.32 m/s and the kinetic energy correction factor is **1.19.**

Problem 7.16

Answer the questions below.

a. What is the conceptual meaning of the first law of thermodynamics for a system?

- The law can be written for
  - energy (amount) with units such as joule, cal, or BTU.
  - rate of energy (amount/time) with units such as watts, J/s, or BTU/s.
- Meaning (rate form):

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

$$\left\{ \begin{array}{c} \text{net rate of energy} \\ \text{entering system} \\ \text{at time } t \end{array} \right\} - \left\{ \begin{array}{c} \text{net rate of work} \\ \text{done by system} \\ \text{at time } t \end{array} \right\} = \left\{ \begin{array}{c} \text{net rate of change in energy} \\ \text{of matter in system} \\ \text{during time } t \end{array} \right\}$$

- Meaning (amount form):

$$Q - W = \Delta E$$

$$\left\{ \begin{array}{c} \text{net amount of energy} \\ \text{entering system} \\ \text{during time } \Delta t \end{array} \right\} - \left\{ \begin{array}{c} \text{net amount of work} \\ \text{done by system} \\ \text{during time } \Delta t \end{array} \right\} = \left\{ \begin{array}{c} \text{net change in energy} \\ \text{of matter in system} \\ \text{during time } \Delta t \end{array} \right\}$$

b. What is flow work? How is the equation for flow work derived?

- Flow work is work done by forces associated with pressure.
- Eq. for flow work is derived by using the definition of work: $W = F \cdot d$.

c. What is shaft work? How is shaft work different than flow work?

- Shaft work is any work that is not flow work.
- Shaft work differs from flow work by the physical origin of the work:
  - flow work is done by a force associated with pressure.
  - shaft work is done by any other force.

Problem 7.17

Answer the questions below.

---

a. What is head? How is head related to energy? To power?

- Head is a way of characterizing energy or power or work.
- Head is the ratio of (energy of a fluid particle)/(weight of a fluid particle) at a point in a fluid flow.
  - example: KE of a fluid particle divided by its weight gives "velocity head": velocity head = $(mV^2/2)\,/\,(mg) = V^2/(2g)$
- Head is related to power by

$$P = \dot{m}gh$$

---

b. What is head of a turbine?

- Head of a turbine is a way of describing the work (or power) that is done on the turbine blades by a flowing fluid:

$$P_{\text{turbine}} = \frac{\Delta W_{\text{turbine}}}{\Delta t} = \dot{m}gh_t$$

---

c. How is head of a pump related to power? To energy?

$$P_{\text{pump}} = \frac{\Delta W_{\text{pump}}}{\Delta t} = \dot{m}gh_p$$

---

d. What is head loss?

- Conversion of mechanical energy in a flowing fluid to thermal energy via the action of viscous stresses.
- Mechanical energy is generally useful. The thermal energy that is generated is generally not useful because it cannot be recovered and used productively.
- The thermal energy usually heats the fluid. This is analogous to frictional heating.

Problem 7.18

Answer the questions below.

---

a. What are the five main terms in the energy equation (7.29)? What does each term mean?

See the text and figure that follow Eq. (7.29).

---

b. How are terms in the energy equation related to energy? To power?

- Each term in the energy equation is a "head term" with units of length. To relate head, power, energy, and work use:

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t} = \dot{m}gh$$

---

c. What assumptions are required to use Eq. (7.29)?

- The flow is steady.
- The control volume has one inlet port and one exit port.
- The fluid density is the same at all spatial locations.

How is the energy equation similar to the Bernoulli equation? List three similarities. How is the energy equation different from the Bernoulli equation? List three differences.
Similarities:

- Both equations involve head terms.
- Both equations are applied from a "location 1" to a "location 2."
- There are some similar terms (all terms in the Bernoulli equation appear in the energy equation).

Differences:

- The energy equation applies to viscous or inviscid flow whereas the Bernoulli equation applies only to inviscid flow.
- The energy equation includes terms for work of a pump and turbine; these terms do not appear in the Bernoulli equation.
- The energy equation includes head loss, whereas the Bernoulli equation lacks any term to account for energy loss.

Problem 7.19

Situation:
Flow in a pipe.

Find:
Prove that fluid in a constant diameter pipe will flow from a location with high piezometric head to a location with low piezometric head.

Assumptions:
No pumps or turbines in the pipeline.
Steady flow.

SOLUTION

The energy equation (7.29).

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Term-by-term analysis:

- In a constant diameter pipe, KE terms cancel out.
- No machines means that $h_p = h_t = 0$.

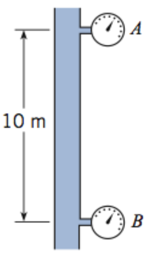
Energy equation (simplified form):

$$\begin{aligned} \left(\frac{p_1}{\gamma} + z_1\right) &= \left(\frac{p_2}{\gamma} + z_2\right) + h_L \\ h_1 &= h_2 + h_L \end{aligned}$$

Since section 1 is upstream of section 2, and head loss is always positive, we conclude that $h_1 > h_2$. This means that fluid flows from high to low piezometric head.

7.20: PROBLEM DEFINITION

Situation:
Water flows in a vertical pipe.
$L = 10\,\text{m}$, $p_A = 10\,\text{kPa}$.
$p_B = 98.1\,\text{kPa}$.



Find: Direction of flow in a pipe:
(a) Upward.
(b) Downward.
(c) No flow.

PLAN

1. Calculate the piezometric head h at A and B.,
2. To determine flow direction, compare the piezometric head values.
   Whichever location has the larger value of h is upstream.
   If the h values are the same, there is no flow.

SOLUTION

1. Piezometric head:

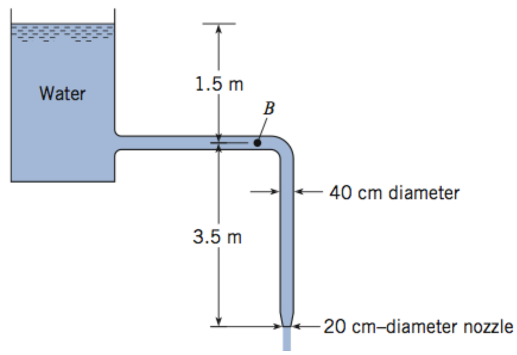
$$\begin{aligned} h_A &= \frac{p_A}{\gamma} + z_A = \left(\frac{10000\,\text{Pa}}{9810\,\text{N/m}^3}\right) + 10\,\text{m} = 11.02\,\text{m} \\ h_B &= \frac{p_B}{\gamma} + z_B = \left(\frac{98100\,\text{Pa}}{9810\,\text{N/m}^3}\right) + 0 = 10\,\text{m} \end{aligned}$$

2. Since $h_A > h_B$, the correct selection is $\boxed{\text{(b)}}$.

7.21: PROBLEM DEFINITION

Situation:

Water flowing from a tank into a pipe connected to a nozzle.
$\alpha = 1.0$, $D_B = 40\,\text{cm}$.
$D_0 = 20\,\text{cm}$, $z_0 = 0\,\text{m}$.
$z_B = 3.5\,\text{m}$, $z_r = 5\,\text{m}$.



Find:

(a) Discharge in pipe (m^3/s).
(b) Pressure at point B (kPa).

Assumptions:

$\gamma = 9810\,\text{N/m}^3$.

PLAN

1. Find velocity at nozzle by applying the energy equation.
2. Find discharge by applying $Q = A_o V_o$
3. Find the pressure by applying the energy equation.

SOLUTION

1. Energy equation (point 1 on reservoir surface, point 2 at outlet)

$$\begin{aligned}
\frac{p_{\text{reser.}}}{\gamma} + \frac{V_r^2}{2g} + z_r &= \frac{p_{\text{outlet}}}{\gamma} + \frac{V_0^2}{2g} + z_0 \\
0 + 0 + 5 &= 0 + \frac{V_0^2}{2g} \\
V_0 &= 9.90 \text{ m/s}
\end{aligned}$$

2. Flow rate equation

$$\begin{aligned}
Q &= V_0 A_0 \\
&= 9.90\,\text{m/s} \times (\frac{\pi}{4}) \times (0.20\,\text{m})^2
\end{aligned}$$

$$\boxed{Q = 0.311 \text{ m}^3/\text{s}}$$

3. Energy equation (point 1 on reservoir surface, point 2 at B)

$$0 + 0 + 5 = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 3.5$$

where

$$\begin{aligned} V_B &= \frac{Q}{V_B} = \frac{0.311\,\mathrm{m^3/\,s}}{(\pi/4) \times (0.4\,\mathrm{m})^2} = 2.48 \text{ m/s} \\ \frac{V_B^2}{2g} &= 0.312\,\mathrm{m} \end{aligned}$$

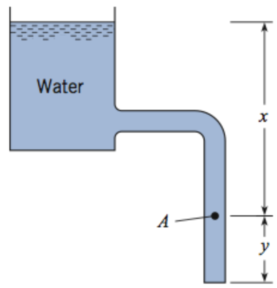
$$\begin{aligned} \frac{p_B}{\gamma} - 5 - 3.5 &= 0.312 \\ & \boxed{p_B = 86.4 \text{ kPa}} \end{aligned}$$

7.22: PROBLEM DEFINITION

Situation:

Water drains from a tank into a pipe.

$x = 10\,\text{ft}$, $y = 4\,\text{ft}$.



Find:

Pressure at point A (psf).

Velocity at exit (ft/s).

Assumptions:

$\alpha_2 = 1$

PLAN

To find pressure at point A, apply the energy equation between point A and the pipe exit. Then, then apply energy equation between top of tank and the exit.

SOLUTION

Energy equation (point A to pipe exit).

$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 =$ 0-gage; $(z_A - z_B) = y$; $h_p = 0$, $h_t = 0$, $h_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ &= -62.4\,\text{lbf}/\,\text{ft}^3 \times 4\,\text{ft} \end{aligned}$$

$$\boxed{p_A = -250 \text{ psf}}$$

Energy equation (1= top of tank; 2 = pipe exit)

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

$$z_1 = \frac{V_2^2}{2g} + z_2$$

Solve for velocity at exit

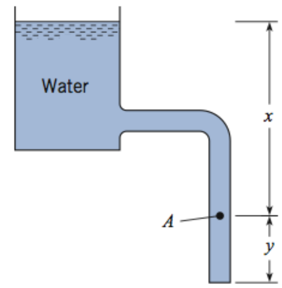
$$\begin{aligned} V_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2 \times 32.2\,\mathrm{ft/\,s^2} \times 14\,\mathrm{ft}} \end{aligned}$$

$$\boxed{V_2 = 30.0\,\mathrm{ft/\,s}}$$

7.23: PROBLEM DEFINITION

Situation:

Water drains from a tank into a pipe.
$x = 10\,\text{m}$, $y = 4\,\text{m}$.



Find:

Pressure at point A (kPa).
Velocity at exit (m/s).

Assumptions:

$\alpha_1 = 1$.

PLAN

1. Find pressure at point A by applying the energy equation between point A and the pipe exit.
2. Find velocity at the exit by applying the energy equation between top of tank and the exit.

SOLUTION

1. Energy equation (section A to exit plane):

$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 = 0$-gage; $(z_A - z_B) = y$; $h_p = 0$, $h_t = 0$, $\dot{h}_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ p_A &= -\left(9810\,\text{N}/\,\text{m}^3\right)(1.5\,\text{m}) \end{aligned}$$

$$\boxed{p_A = -14.7\,\text{kPa}}$$

2. Energy equation (top of tank to exit plane):

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
z_1 &= \frac{V_2^2}{2g} + z_2 \\
V_2 &= \sqrt{2g(z_1 - z_2)} \\
&= \sqrt{2 \times 9.81\,\mathrm{m/s^2} \times 11.5\,\mathrm{m}} \\
&\boxed{V_2 = 15.0\ \mathrm{m/s}}
\end{aligned}
$$

7.24: PROBLEM DEFINITION

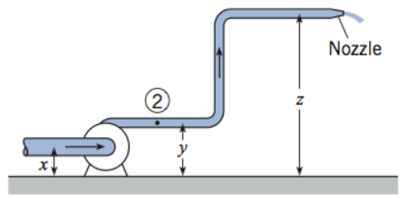
Situation:

Water is pushed out a nozzle by a pump.

$Q = 0.1\,\text{m}^3/\text{s}$, $D_2 = 30\,\text{cm}$.

$D_n = 10\,\text{cm}$, $z_n = 7\,\text{m}$.

$z_1 = 1\,\text{m}$, $z_2 = 2\,\text{m}$.



Find:

Pressure head at point 2.

SOLUTION

Flow rate equation to find V_n (velocity at nozzle)

$$V_n = \frac{Q}{A_n} = \frac{0.10\,\text{m}^3/\text{s}}{(\pi/4)\times(0.10\,\text{m})^2} = 12.73 \text{ m/s}$$

$$\frac{V_n^2}{2g} = 8.26 \text{ m}$$

Flow rate equation to find V_2

$$V_2 = \frac{Q}{A_2} = \frac{0.10\,\text{m}^3/\text{s}}{(\pi/4)\times(0.3\,\text{m})^2} = 1.41 \text{ m/s}$$

$$\frac{V_2^2}{2g} = 0.102 \text{ m}$$

Energy equation

$$\frac{p_2}{\gamma} + 0.102 + 2 = 0 + 8.26 + 7$$

$$\boxed{\frac{p_2}{\gamma} = 13.2 \text{ m}}$$

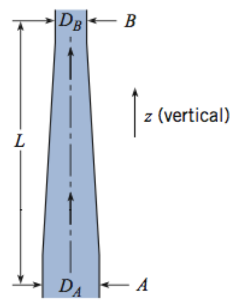
7.25: PROBLEM DEFINITION

Situation:

Oil moves through a narrowing section of pipe.

$D_A = 20\,\text{cm}$, $D_B = 12\,\text{cm}$.

$L = 1\,\text{m}$, $Q = 0.06\,\text{m}^3/\,\text{s}$.



Find:

Pressure difference between A and B.

Properties:

$S = 0.90$.

SOLUTION

Flow rate equation

$$\begin{aligned} V_A &= \frac{Q}{A_1} \\ &= 1.910\text{m/s} \\ V_B &= \left(\frac{20}{12}\right)^2 \times V_A \\ &= 5.31\ \text{m/s} \end{aligned}$$

Energy equation

$$\begin{aligned} p_A - p_B &= 1\gamma + (\rho/2)(V_B^2 - V_A^2) \\ p_A - p_B &= (1)\left(9810\,\text{N}/\,\text{m}^3\right)(0.9) + \left(\frac{900\,\text{kg}/\,\text{m}^3}{2}\right)\left((5.31\,\text{m}/\,\text{s})^2 - (1.91\,\text{m}/\,\text{s})^2\right) \end{aligned}$$

$$\boxed{p_A - p_B = 19.9\,\text{kPa}}$$

7.26: PROBLEM DEFINITION

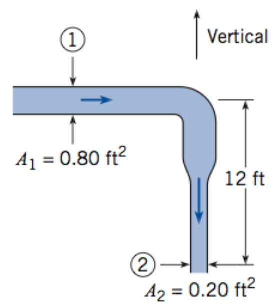
Situation:

Gasoline flows in a pipe that narrows into a smaller pipe.

$Q = 5\,\text{ft}^3/\,\text{s}$, $p_1 = 18$ psig.

$h_L = 6\,\text{ft}$, $\Delta z = 12\,\text{ft}$.

$A_1 = 0.8\,\text{ft}^2$, $A_2 = 0.2\,\text{ft}^2$.



Find:

Pressure at section 2 (psig).

Properties:

Gasoline, $S = 0.8$.

Water, Table A.5: $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

PLAN

Apply flow rate equation and then the energy equation.

SOLUTION

Flow rate equation:

$$V_1 = \frac{Q}{A_1} = \frac{5}{0.8} = 6.25\,\text{ft}/\,\text{s}$$

$$\frac{V_1^2}{2g} = \frac{(6.25\,\text{ft}/\,\text{s})^2}{2(32.2\,\text{ft}/\,\text{s}^2)} = 0.6066\,\text{ft}$$

$$V_2 = \frac{Q}{A_2} = \frac{5}{0.2} = 25\,\text{ft}/\,\text{s}$$

$$\frac{V_2^2}{2g} = \frac{(25\,\text{ft}/\,\text{s})^2}{2(32.2\,\text{ft}/\,\text{s}^2)} = 9.705\,\text{ft}$$

Energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + 6$$

$$\frac{18\,\text{lbf}/\,\text{in}^2 \times 144\,\text{in}^2/\,\text{ft}^2}{0.8 \times 62.4\,\text{lbf}/\,\text{ft}^3} + 0.6066\,\text{ft} + 12\,\text{ft} = \frac{p_2}{0.8 \times 62.4\,\text{lbf}/\,\text{ft}^3} + 9.705\,\text{ft} + 0\,\text{ft} + 6\,\text{ft}$$

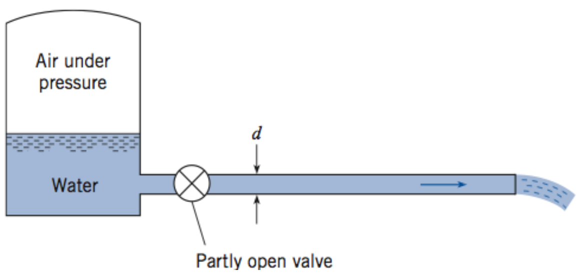
$$p_2 = 2437 \text{ psfg}$$

$$\boxed{p_2 = 16.9 \text{ psig}}$$

7.27: PROBLEM DEFINITION

Situation:

Water flows from a pressurized tank, through a valve and out a pipe.
$p_1 = 100\,\text{kPa},\ z_1 = 8\,\text{m}.$
$p_2 = 0\,\text{kPa},\ z_2 = 0\,\text{m}.$
$h_L = K_L \frac{V^2}{2g},\ V_2 = 10\,\text{m/s}.$



Find:

The minor loss coefficient (K_L).

Assumptions:

Steady flow.
Outlet flow is turbulent so that $\alpha_2 = 1.0$.
$V_1 \approx 0$.

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\text{N/m}^3$.

PLAN

Apply the energy equation and then solve the resulting equation to find the minor loss coefficient.

SOLUTION

Energy equation (section 1 on water surface in tank; section 2 at pipe outlet)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \tag{1}$$

Term by term analysis:

- At the inlet. $p_1 = 100\,\text{kPa},\ V_1 \approx 0,\ z_1 = 8\,\text{m}$
- At the exit , $p_2 = 0\,\text{kPa},\ V_2 = 10\,\text{m/s},\ \alpha_2 = 1.0.$
- Pumps and turbines. $h_p = h_t = 0$
- Head loss. $h_L = K_L \frac{V^2}{2g}$

Eq. (1) simplifies to

$$\begin{aligned}\frac{p_1}{\gamma}+z_1 &= \alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g}\\ \frac{(100,000\,\mathrm{Pa})}{(9800\,\mathrm{N/\,m^3})}+8\,\mathrm{m} &= \frac{(10\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})}+K_L\frac{(10\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})}\\ 18.2\,\mathrm{m} &= (5.097\,\mathrm{m})+K_L\,(5.097\,\mathrm{m})\end{aligned}$$

Thus

$$\boxed{K_L = 2.57}$$

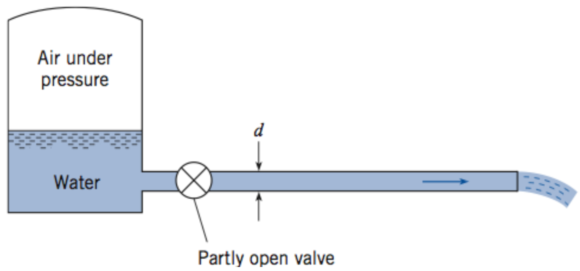
REVIEW

1. The minor loss coefficient ($K_L = 2.57$) is typical of a valve (this information is presented in Chapter 10).
2. The head at the inlet $\left(\frac{p_1}{\gamma}+z_1 = 22.2\,\mathrm{m}\right)$ represents available energy. Most of this energy goes to head loss $\left(K_L\frac{V_2^2}{2g} = 17.1\,\mathrm{m}\right)$. The remainder is carried as kinetic energy out of the pipe $\left(\alpha_2\frac{V_2^2}{2g} = 5.1\,\mathrm{m}\right)$.

7.28: PROBLEM DEFINITION

Situation:

Water flows from a pressurized tank, through a valve and out a pipe.
$Q = 0.1\,\text{ft}^3/\,\text{s}$, $\Delta z = 10\,\text{ft}$.
$p_2 = 0$ psig, $D = 1\,\text{in}$.
$h_L = K_L \frac{V^2}{2g}$.



Find:

Pressure in tank (psig).

PLAN

Apply the energy equation and then solve the resulting equation to give pressure in the tank.

SOLUTION

Energy equation (from the water surface in the tank to the outlet)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\begin{aligned}
\frac{p_1}{\gamma} &= \frac{V_2^2}{2g} + h_L - z_1 = \frac{6V_2^2}{2g} - 10 \\
V_2 &= \frac{Q}{A_2} = \frac{0.1\,\text{ft}^3/\,\text{s}}{(\pi/4)\left(\frac{1}{12}\,\text{ft}\right)^2} = 18.33 \text{ ft/s} \\
\frac{p_1}{\gamma} &= \frac{6\,(18.33\,\text{ft}/\,\text{s})^2}{64.4\,\text{ft}^2/\,\text{s}^2} - 10\,\text{ft} = 21.3 \text{ ft} \\
p_1 &= 62.4\,\text{lbf}/\,\text{ft}^3 \times 21.3\,\text{ft} = 1329 \text{ psfg}
\end{aligned}$$

$$\boxed{p_1 = 9.23 \text{ psig}}$$

7.29: PROBLEM DEFINITION

Situation:

Water flows from an open tank, through a valve and out a pipe.

$A = 9\,\text{cm}^2$, $\Delta z = 11\,\text{m}$.

$p_1 = p_2 = 0\,\text{kPa}$.

$h_L = 5\frac{V^2}{2g}$.

Find:

Discharge in pipe.

Assumptions:

$\alpha = 1$.

PLAN

Apply the energy equation from the water surface in the reservoir (pt. 1) to the outlet end of the pipe (pt. 2).

SOLUTION

Energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

Term by term analysis:

$$\begin{aligned} p_1 &= 0; \quad p_2 = 0 \\ z_2 &= 0; \quad V_1 \simeq 0 \end{aligned}$$

The energy equation becomes.

$$\begin{aligned} z_1 &= \frac{V_2^2}{2g} + h_L \\ 11\,\text{m} &= \frac{V_2^2}{2g} + 5\frac{V_2^2}{2g} = 6\frac{V_2^2}{2g} \\ V_2^2 &= \left(\frac{2g}{6}\right)(11) \\ V_2 &= \sqrt{\left(\frac{2 \times 9.81\,\text{m/s}^2}{6}\right)(11\,\text{m})} \\ V_2 &= 5.998\text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= (5.998\,\text{m/s})\left(9\,\text{cm}^2\right)\left(\frac{10^{-4}\,\text{m}^2}{\text{cm}^2}\right) \\ &= 5.398\,2 \times 10^{-3}\frac{\text{m}^3}{\text{s}} \end{aligned}$$

$$\boxed{Q = 5.40 \times 10^{-3}\ \text{m}^3/\text{s}}$$

7.30: PROBLEM DEFINITION

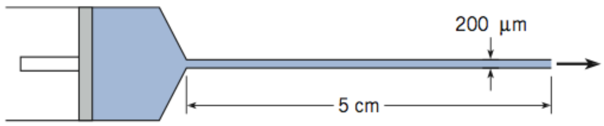
Situation:

A microchannel is designed to transfer fluid in a MEMS application.

$D = 200\,\mu\text{m}$, $L = 5\,\text{cm}$.

$Q = 0.1\ \mu\text{L/s}$.

$h_L = \frac{32\mu LV}{\gamma D^2}$.



Find:

Pressure in syringe pump (Pa).

Assumptions:

$\alpha = 2$.

Properties:

Table A.4: $\rho = 799\,\text{kg/m}^3$.

PLAN

Apply the energy equation and the flow rate equation.

SOLUTION

Energy equation (locate section 1 inside the pumping chamber; section 2 at the outlet of the channel)

$$\begin{aligned}\frac{p_1}{\gamma} &= h_L + \alpha_2\frac{V^2}{2g} \\ &= \frac{32\mu LV}{\gamma D^2} + 2\frac{V^2}{2g} \qquad (1)\end{aligned}$$

Flow rate

The cross-sectional area of the channel is 3.14×10^{-8} m^2. A flow rate of 0.1 μl/s is 10^{-7} l/s or 10^{-10} m^3/s. The flow velocity is

$$\begin{aligned}V &= \frac{Q}{A} \\ &= \frac{10^{-10}\,\text{m}^3/\text{s}}{3.14\times10^{-8}\,\text{m}^2} \\ &= 0.318\times10^{-2}\ \text{m/s} \\ &= 3.18\ \text{mm/s}\end{aligned}$$

Substituting the velocity and other parameters in Eq. (1) gives

$$\begin{aligned}
\frac{p_1}{\gamma} &= \frac{32 \times 1.2 \times 10^{-3} \times 0.05 \times 0.318 \times 10^{-2}}{7,850 \times 4 \times (10^{-4})^2} + 2 \times \frac{(0.318 \times 10^{-2})^2}{2 \times 9.81} \\
&= 0.0194 \text{ m}
\end{aligned}$$

The pressure is

$$p_1 = 799 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 0.0194 \text{ m}$$

$$\boxed{p_1 = 152 \text{ Pa}}$$

7.31: PROBLEM DEFINITION

Situation:

Water flows out of a fire nozzle.

$V = 40\,\mathrm{m/s}$, $\Delta z = 50\,\mathrm{m}$.

$A_e/A_{hose} = 1/4$.

$h_L = 10\frac{V^2}{2g}$.

Find:

Pressure at hydrant.

Assumptions:

Hydrant supply pipe is much larger than the firehose.

Properties:

Water, Table A.5, $\gamma = 9810\,\mathrm{N/m^3}$.

PLAN

Apply the energy equation.

SOLUTION

Energy equation

$$\frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + z_2 + h_L$$

where the kinetic energy of the fluid feeding the hydrant is neglected. Because of the contraction at the exit, the outlet velocity is 4 times the velocity in the pipe, so the energy equation becomes

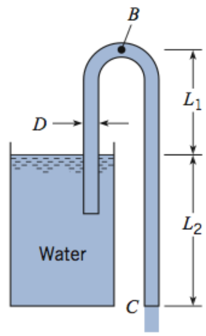
$$\begin{aligned}
\frac{p_1}{\gamma} &= \frac{V_2^2}{2g} + z_2 - z_1 + 10\frac{V^2}{16 \times 2g} \\
p_1 &= \left(\frac{1.625}{2g}V^2 + 50\right)\gamma \\
&= \left(\frac{1.625}{2 \times 9.81\,\mathrm{m/s^2}} \times (40\,\mathrm{m/s})^2 + 50\,\mathrm{m}\right) 9810\,\mathrm{N/m^3} \\
&= 1.791 \times 10^6\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{p_1 = 1790\,\mathrm{kPa}}$$

7.32: PROBLEM DEFINITION

Situation:

Water flows out of a syphon.
$L_1 = L_2 = 3\,\text{ft}$.
$Q = 2.8\,\text{ft}^3/\,\text{s}$, $D = 8\,\text{in}$.



Find:

Head loss between reservoir surface and point C.
Pressure at point B.

Properties:

Water (60 °F), Table A.5, $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

Assumptions:

$\alpha = 1$.
Three quarters of head loss is between reservoir surface and point B.

PLAN

To find head loss between reservoir surface and point C
1. Develop an equation for head loss by applying the energy equation from the reservoir surface to section C.
2. Find V using the flow rate equation.
3. Combine results of steps 1 and 2 and solve for the head loss.

To find the prssure at B.
4. Develop an equation for the pressure at B by applying the energy equation from the reservoir surface to section B

SOLUTION

1. Energy equation (from reservoir surface to section C)

$$
\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_c}{\gamma} + \frac{V_c^2}{2g} + z_c + h_t + h_L \\
0 + 0 + 3\,\text{ft} + 0 &= 0 + \frac{V_c^2}{2g} + 0 + 0 + h_L \\
3\,\text{ft} &= \frac{V_c^2}{2g} + h_L
\end{aligned}
$$

2. Flow rate equation

$$
\begin{aligned}
V_c &= \frac{Q}{A_2} \\
V_c &= \frac{2.8\,\text{ft}^3/\,\text{s}}{(\pi/4) \times \left(\frac{8}{12}\,\text{ft}\right)^2} = 8.02 \text{ ft/s}
\end{aligned}
$$

3. Combine results of step 1 and 2.

$$
\begin{aligned}
3\,\text{ft} &= \frac{V_c^2}{2g} + h_L \\
3\,\text{ft} &= \frac{(8.02\,\text{ft}/\,\text{s})^2}{2\,(32.2\,\text{ft}/\,\text{s}^2)} + h_L
\end{aligned}
$$

$$
\boxed{h_L = 2.00\,\text{ft}}
$$

4. Energy equation (from reservoir surface to section B).

$$
\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_t + h_L \\
0 + 0 + 3\,\text{ft} + 0 &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 6\,\text{ft} + 0 + (3/4) \times 2\,\text{ft} \\
3\,\text{ft} &= \frac{p_B}{62.4\,\text{lbf}/\,\text{ft}^3} + \frac{(8.02\,\text{ft}/\,\text{s})^2}{2\,(32.2\,\text{ft}/\,\text{s}^2)} + 6\,\text{ft} + (3/4) \times 2\,\text{ft}
\end{aligned}
$$

$$
\begin{aligned}
\frac{p_B}{\gamma} &= 3\,\text{ft} - 1\,\text{ft} - 6\,\text{ft} - 1.5\,\text{ft} = -5.5\,\text{ft} \\
p_B &= -5.5\,\text{ft} \times 62.4\,\text{lbf}/\,\text{ft}^3 \\
&= -343 \text{ psfg}
\end{aligned}
$$

$$
\boxed{p_B = -2.38 \text{ psig}}
$$

7.33: PROBLEM DEFINITION

Situation:

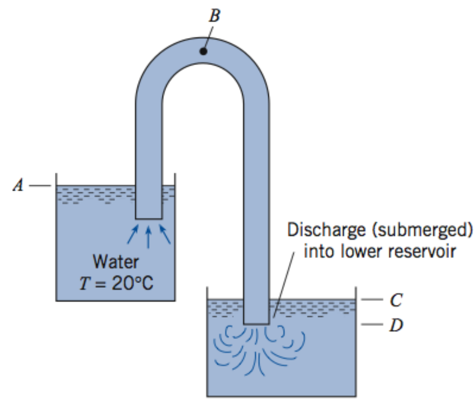
A siphon transports water from one reservoir to another.

$z_A = 30\,\text{m},\ z_B = 32\,\text{m}.$

$z_C = 27\,\text{m},\ z_D = 26\,\text{m}.$

$D = 25\,\text{cm}.$

$h_{\ell_{\text{pipe}}} = \frac{V_p^2}{2g}.$



Find:

Discharge.

Pressure at point B.

Assumptions:

$\alpha = 1.$

PLAN

Apply the energy equation from A to C, then from A to B.

SOLUTION

Head loss

$$h_{\ell_{\text{pipe}}} = \frac{V_p^2}{2g}$$

$$h_{\text{total}} = h_{\ell_{\text{pipe}}} + h_{\ell_{\text{outlet}}} = 2\frac{V_p^2}{2g}$$

Energy equation (from A to C)

$$0 + 0 + 30\,\text{m} = 0 + 0 + 27\,\text{m} + 2\frac{V_p^2}{2g}$$

$$V_p = 5.42\ \text{m/s}$$

Flow rate equation

$$
\begin{aligned}
Q &= V_p A_p \\
&= 5.42\,\mathrm{m/\,s} \times (\pi/4) \times (0.25\,\mathrm{m})^2 \\
& \boxed{Q = 0.266\ \mathrm{m^3/s}}
\end{aligned}
$$

Energy equation (from A to B)

$$
\begin{aligned}
30\,\mathrm{m} &= \frac{p_B}{\gamma} + \frac{V_p^2}{2g} + 32\,\mathrm{m} + 0.75\frac{V_p^2}{2g} \\
\frac{p_B}{\gamma} &= -2 - 1.75 \times 1.497\ \mathrm{m} \\
& \boxed{p_B = -45.3\ \mathrm{kPa,\ gage}}
\end{aligned}
$$

7.34: PROBLEM DEFINITION

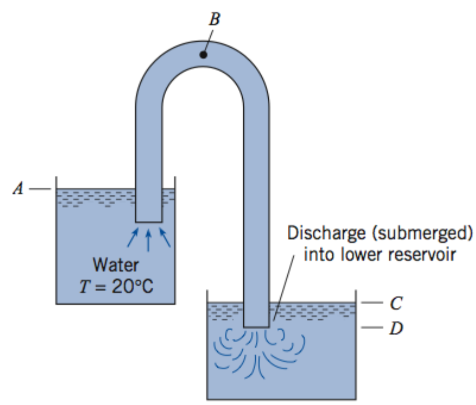
Situation:

A siphon transports water from one reservoir to another.

$p_v = 1.23\,\text{kPa}$, $p_{atm} = 100\,\text{kPa}$.

$Q = 8 \times 10^{-4}\,\text{m}^3/\text{s}$, $A = 10^{-4}\,\text{m}^2$.

$h_{L,A\to B} = 1.8V^2/2g$.



Find:

Depth of water in upper reservoir for incipient cavitation.

PLAN

Apply the energy equation from point A to point B.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{8 \times 10^{-4}\ \text{m}^3/\text{s}}{1 \times 10^{-4}\ \text{m}^2} \\ &= 8\,\text{m}/\text{s} \end{aligned}$$

Calculations

$$\begin{aligned} \frac{V^2}{2g} &= \frac{(8\,\text{m}/\text{s})^2}{2 \times 9.81\,\text{m}/\text{s}^2} = 3.262\,\text{m} \\ h_{L,A\to B} &= 1.8\frac{V^2}{2g} = 5.872\ \text{m} \end{aligned}$$

Energy equation (from A to B; let $z = 0$ at bottom of reservoir)

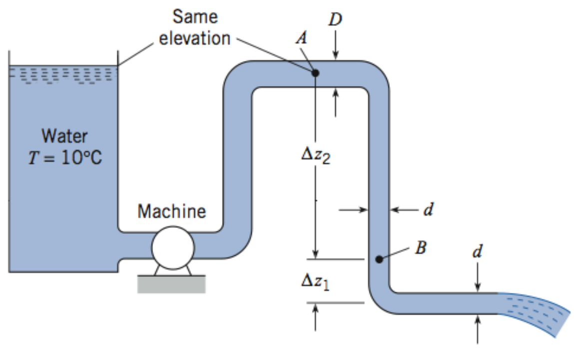
$$
\begin{aligned}
\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_L \\
\frac{100000\,\mathrm{Pa}}{9810\,\mathrm{N/m^3}} + 0 + z_A &= \frac{1230\,\mathrm{Pa}}{9810\,\mathrm{N/m^3}} + 3.262\,\mathrm{m} + 10\,\mathrm{m} + 5.872\,\mathrm{m}
\end{aligned}
$$

$$
\boxed{z_A = \text{depth} = 9.07\,\mathrm{m}}
$$

7.35: PROBLEM DEFINITION

Situation:

A reservoir discharges water into a pipe with a machine.
$d = 6\,\text{in},\ D = 12\,\text{in}.$
$\Delta z_1 = 6\,\text{ft},\ \Delta z_2 = 12\,\text{ft}.$
$Q = 10\,\text{ft}^3/\,\text{s}.$



Find:

Is the machine a pump or a turbine?
Pressures at points A and B (psig).

Assumptions:

Machine is a pump.
$\alpha = 1.0.$

PLAN

Apply the energy equation between the top of the tank and the exit, then between point B and the exit, finally between point A and the exit.

SOLUTION

Energy equation

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

Assuming the machine is a pump. If the machine is a turbine, then h_p will be negative. The velocity at the exit is

$$V_2 = \frac{Q}{A_2} = \frac{10\,\text{ft}^3/\,\text{s}}{\frac{\pi}{4}\,(0.5\,\text{ft})^2} = 50.93\ \text{ft/s}$$

Solving for h_p and taking the pipe exit as zero elevation we have

$$h_p = \frac{(50.93\,\text{ft}/\,\text{s})^2}{2 \times 32.2\,\text{ft}/\,\text{s}^2} - (6 + 12)\,\text{ft} = 22.3\ \text{ft}$$

Therefore the machine is a pump.
Applying the energy equation between point B and the exit gives

$$\frac{p_B}{\gamma} + z_B = z_2$$

Solving for p_B we have

$$\begin{aligned} p_B &= \gamma(z_2 - z_B) \\ p_B &= -6\,\text{ft} \times 62.4\,\text{lbf}/\,\text{ft}^3 = -374 \text{ psfg} \\ &\boxed{p_B = -2.6 \text{ psig}} \end{aligned}$$

Velocity at A

$$V_A = \left(\frac{6}{12}\right)^2 \times 50.93\,\text{ft}/\,\text{s} = 12.73 \text{ ft/s}$$

Applying the energy equation between point A and the exit gives

$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

so

$$\begin{aligned} p_A &= \gamma\left(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}\right) \\ &= 62.4\,\text{lbf}/\,\text{ft}^3 \times \left(\frac{(50.93\,\text{ft}/\,\text{s})^2 - (12.73\,\text{ft}/\,\text{s})^2}{2 \times 32.2\,\text{ft}/\,\text{s}^2} - 18\right) \\ &= 1233 \text{ psfg} \\ &\boxed{p_A = 8.56 \text{ psig}} \end{aligned}$$

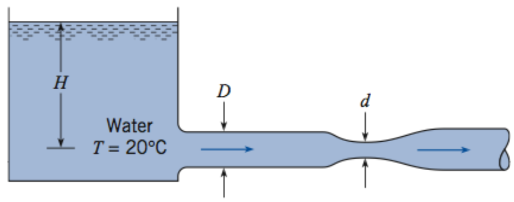
7.36: PROBLEM DEFINITION

Situation:

Water flows through a pipe with a venturi meter.

$D = 30\,\text{cm}$, $d = 15\,\text{cm}$.

$p_{atm} = 100\,\text{kPa}$, $H = 5\,\text{m}$.



Find:

Maximum allowable discharge before cavitation.

Assumptions:

$\alpha = 1.0$.

Properties:

Water $(10\,^\circ\text{C})$, Table A.5: $p_v = 2340$ Pa, abs.

SOLUTION

Energy equation (locate 1 on the surface of the tank; 2 at the throat of the venturi)

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \\
0 + 0 + 5 &= \frac{p_{2,\text{vapor}}}{\gamma} + \frac{V_2^2}{2g} + 0 \\
p_{2,\text{vapor}} &= 2340 - 100,000 = -97,660 \text{ Pa gage}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{V_2^2}{2g} &= 5\,\text{m} + \frac{97660\,\text{Pa}}{9790\,\text{N/m}^3} = 14.97 \text{ m} \\
V_2 &= 17.1 \text{ m/s}
\end{aligned}$$

Flow rate equation

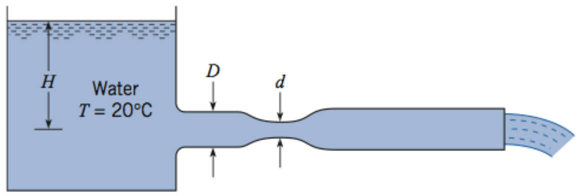
$$\begin{aligned}
Q &= V_2 A_2 \\
&= 17.1\,\text{m/s} \times \pi/4 \times (0.15\,\text{m})^2
\end{aligned}$$

$$\boxed{Q = 0.302 \text{ m}^3/\text{s}}$$

7.37: PROBLEM DEFINITION

Situation:

A reservoir discharges to a pipe with a venturi meter before draining to atmosphere.
$D = 40\,\text{cm}$, $d = 25\,\text{cm}$.
$p_{\text{atm}} = 100\,\text{kPa}$, $h_L = 0.9V_2^2/2g$.



Find:

Head at incipient cavitation (m).
Discharge at incipient cavitation (m^3/s).

Assumptions:

$\alpha = 1.0$.

Properties:

From Table A.5 $p_v = 2340$ Pa, abs.

PLAN

First apply the energy equation from the Venturi section to the end of the pipe. Then apply the energy equation from reservoir water surface to outlet:

SOLUTION

Energy equation from Venturi section to end of pipe:

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
\frac{p_{\text{vapor}}}{\gamma} + \frac{V_1^2}{2g} &= 0 + \frac{V_2^2}{2g} + 0.9\frac{V_2^2}{2g} \\
p_{\text{vapor}} &= 2,340 \text{ Pa abs.} = -97,660 \text{ Pa gage}
\end{aligned}$$

Continuity principle

$$\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_1 &= \frac{V_2 A_2}{A_1} \\
&= 2.56 V_2
\end{aligned}$$

Then

$$\frac{V_1^2}{2g} = 6.55\frac{V_2^2}{2g}$$

Substituting into energy equation

$$
\begin{aligned}
-97,660/9,790 + 6.55\frac{V_2^2}{2g} &= 1.9\frac{V_2^2}{2g} \\
V_2 &= 6.49 \text{ m/s}
\end{aligned}
$$

Flow rate equation

$$
\begin{aligned}
Q &= V_2 A_2 \\
&= 6.49\,\text{m/s} \times \pi/4 \times (0.4\,\text{m})^2 \\
&\boxed{Q = 0.815 \text{ m}^3/\text{s}}
\end{aligned}
$$

Energy equation from reservoir water surface to outlet:

$$
\begin{aligned}
z_1 &= \frac{V_2^2}{2g} + h_L \\
H &= 1.9\frac{V_2^2}{2g} \\
&\boxed{H = 4.08 \text{ m}}
\end{aligned}
$$

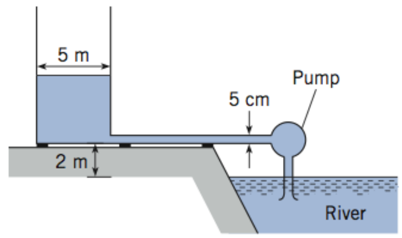
7.38: PROBLEM DEFINITION

Situation:

A pump fills a tank with water from a river.

$D_{\text{tank}} = 5\,\text{m}$, $D_{\text{pipe}} = 5\,\text{cm}$.

$h_L = 10V_2^2/2g$, $h_p = 20 - 4 \times 10^4 Q^2$.



Find:

Time required to fill tank to depth of 10 m.

Assumptions:

$\alpha = 1.0$.

SOLUTION

Energy equation (locate 1 on the surface of the river, locate 2 on the surface of the water in the tank).

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

but $p_1 = p_2 = 0$, $z_1 = 0$, $V_1 = 0$, $V_2 \simeq 0$. The energy equation reduces to

$$0 + 0 + 0 + h_p = 0 + 0 + (2\text{ m} + h) + h_L$$

where h =depth of water in the tank

$$20 - (4)(10^4)Q^2 = h + 2 + \frac{V^2}{2g} + 10\frac{V^2}{2g}$$

where $V^2/2g$ is the head loss due to the abrupt expansion. Then

$$18 = (4)(10^4)Q^2 + 11\frac{V^2}{2g} + h$$

$$\begin{aligned}
V &= \frac{Q}{A} \\
11\frac{V^2}{2g} &= \frac{11}{2g}\left(\frac{Q^2}{A^2}\right) = (1.45)(10^5)Q^2 \\
18 &= 1.85 \times 10^5 Q^2 + h \\
Q^2 &= \frac{18 - h}{1.85 \times 10^5} \\
Q &= \frac{(18 - h)^{0.5}}{430}
\end{aligned}$$

But $Q = A_T dh/dt$ where A_T = tank area, so

$$\therefore \quad \frac{dh}{dt} = \frac{(18-h)^{0.5}}{(430)(\pi/4)(5)^2} = \frac{(18-h)^{0.5}}{8,443}$$

$$dh/(18-h)^{0.5} = dt/8,443$$

Integrate:

$$-2(18-h)^{0.5} = \frac{t}{8,443} + \text{ const.}$$

But $t = 0$ when $h = 0$ so const. $= -2(18)^{0.5}$. Then

$$t = (18^{0.5} - (18-h)^{0.5})(16,886)$$

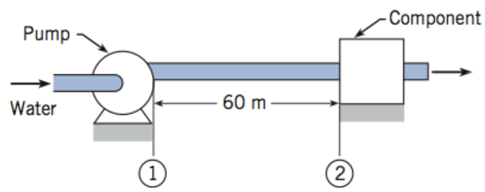
For $h = 10$ m

$$\begin{aligned} t &= (18^{0.5} - 8^{0.5})(16,886) \\ &= 23,880 \text{ s} \\ & \boxed{t = 6.63\,\text{h}} \end{aligned}$$

7.39: PROBLEM DEFINITION

Situation:

Water is flowing in a horizontal pipe.
$D = 0.15\,\text{m}$, $L = 60\,\text{m}$.
$V = 2\,\text{m/s}$. $h_L = 2\,\text{m}$.



Find:

Pressure drop (Pa).
Pumping power (W).

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Find pressure drop using the energy equation.
2. Find power using the power equation.

SOLUTION

1. Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- let $\Delta p = p_1 - p_2$
- KE terms cancel.
- Elevation terms cancel.
- $h_p = h_t = 0$

$$\begin{aligned} \Delta p &= \gamma h_L \\ &= \left(9800\,\text{N/m}^3\right)(2\,\text{m}) \\ & \boxed{\Delta p = 19.6 \text{ kPa}} \end{aligned}$$

2. Power equation:

$$\dot{W}_p = \gamma Q h_p = \dot{m} g h_p$$

- The head of the pump must equal the head loss.

- $Q = VA = (2\,\mathrm{m/s})\,(\pi/4)\,(0.15\,\mathrm{m})^2 = 0.0353\,\mathrm{m^3/s}$.

$$
\begin{aligned}
\dot{W}_p &= \gamma Q h_L \\
&= \left(\frac{9800\,\mathrm{N}}{\mathrm{m^3}}\right)\left(0.0353\,\mathrm{m^3/s}\right)(2\,\mathrm{m}) \\
&\boxed{\dot{W}_p = 692\,\mathrm{W}}
\end{aligned}
$$

REVIEW The pump would need to supply about 0.9 hp to the water.

7.40: PROBLEM DEFINITION

Situation:

A fan is moving air through a hair dryer.

$\Delta p = 6\,\text{mm-H20} = 58.8\,\text{Pa}$, $V = 10\,\text{m/s}$.

$D = 0.044\,\text{m}$, $\eta = 60\%$.

Find:

Electrical power (watts) to operate the fan.

Assumptions:

Air is at a constant temperature.

Constant diameter: $D_1 = D_2 = 0.044\,\text{m}$.

Properties:

Air (60 °C), Table A.3, $\gamma = 10.4\,\text{N/m}^3$.

PLAN

1. Find head loss using the energy equation.
2. Find power supplied to the air using the power equation.
3. Find electrical power using the efficiency equation.

SOLUTION

1. Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- locate section 1 just downstream of fan ($p_1 = 58.8\,\text{Pa}$); section 2 at exit plane ($p_2 = 0$).
- KE terms cancel.
- Elevation terms cancel.
- $h_p = h_t = 0$.

$$h_L = \frac{p_1}{\gamma} = \frac{58.8\,\text{Pa}}{10.4\,\text{N/m}^3} = 5.65\,\text{m}$$

2. Power equation:

$$\dot{W}_p = \gamma Q h_p = \dot{m} g h_p$$

- Assume that head supplied by the fan equals the head loss.
- $Q = VA = (10\,\text{m/s})\,(\pi/4)\,(0.044\,\text{m})^2 = 0.0152\,\text{m}^3/\text{s}$.

$$\begin{aligned} \dot{W}_{\text{fan}} &= \gamma Q h_L \\ &= \left(\frac{10.4\,\text{N}}{\text{m}^3}\right)\left(0.0152\,\text{m}^3/\text{s}\right)(5.65\,\text{m}) \\ &= 0.893\,\text{W} \end{aligned}$$

3. Efficiency equation:

$$P_{\text{electrical}} = \frac{P_{\text{fan}}}{\eta} = \frac{0.893\,\text{W}}{0.6}$$

$$\boxed{P_{\text{electrical}} = 1.49\,\text{W}}$$

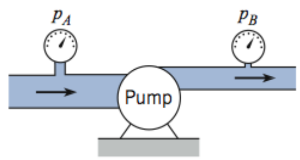
REVIEW

The electrical power to operate the fan ($\approx 1.5\,\text{W}$) is small compared to the electrical power to heat the air.

7.41: PROBLEM DEFINITION

Situation:

A pump supplies energy to a flowing fluid
$Q = 5\,\text{ft}^3/\text{s}$, $\alpha = 1.0$.
$D_A = 1.0\,\text{ft}$, $p_A = 5$ psig.
$D_B = 0.5\,\text{ft}$, $p_B = 90$ psig.



Find:

Horsepower delivered by pump (hp).

PLAN

Apply the flow rate equation, then the energy equation from A to B. Then apply the power equation.

SOLUTION

Flow rate equation:

$$\begin{aligned}
V_A &= \frac{Q}{A_A} = \frac{5\,\text{ft}^3/\text{s}}{(\pi/4)\times(1\,\text{ft})^2} = 6.366\,\text{ft/s} \\
\frac{V_A^2}{2g} &= \frac{(6.366\,\text{ft/s})^2}{2\times(32.2\,\text{ft/s}^2)} = 0.6293\,\text{ft} \\
V_B &= \frac{Q}{A_B} = \frac{5\,\text{ft}^3/\text{s}}{(\pi/4)\times(0.5\,\text{ft})^2} = 25.47\ \text{ft/s} \\
\frac{V_B^2}{2g} &= \frac{(25.47\,\text{ft/s})^2}{2\times(32.2\,\text{ft/s}^2)} = 10.07\,\text{ft}
\end{aligned}$$

Energy equation:

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_p = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$5\times\frac{144}{62.4} + (0.6293) + 0 + h_p = 90\times\frac{144}{62.4} + 10.07 + 0$$

$$h_p = 205.6\,\text{ft}$$

Power equation:

$$P(\text{hp}) = \frac{Q\gamma h_p}{550}$$

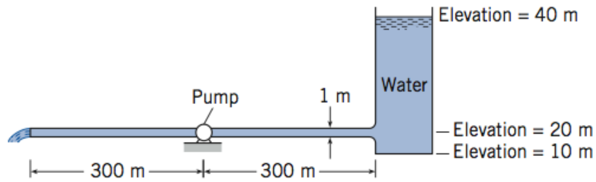
$$= \frac{5\,\text{ft}^3/\,\text{s} \times 62.4\,\text{lbf}/\,\text{ft}^3 \times 205.6\,\text{ft}}{550}$$

$$\boxed{P = 117\,\text{hp}}$$

7.42: PROBLEM DEFINITION

Situation:

A pump moves water from a tank through a pipe.
$Q = 8\,\mathrm{m^3/s}$, $h_L = 7V^2/2g$.
$D = 1\,\mathrm{m}$.



Find:

Power supplied to flow (MW).

Assumptions:

$\alpha = 1.0$.

PLAN

Find power using the power equation. The steps are
1. Find velocity in the pipe using the flow rate equation.
2. Find head of the pump using the energy equation.
3. Calculate power.

SOLUTION

1. Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{8}{\pi/4 \times (1\,\mathrm{m})^2} \\ &= 10.2 \quad \mathrm{m/s} \end{aligned}$$

2. Energy equation (locate 1 on the reservoir surface; locate 2 at the out of the pipe).

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + 40 + h_p &= 0 + \frac{V^2}{2g} + 20 + 7\frac{V^2}{2g} \\ \frac{V^2}{2g} &= \frac{(10.2\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}} = 5.30 \text{ m} \end{aligned}$$

Then

$$
\begin{aligned}
40 + h_p &= \frac{V^2}{2g} + 20 + 7\frac{V^2}{2g} \\
h_p &= 8 \times 5.30 + 20 - 40 \\
&= 22.4 \text{ m}
\end{aligned}
$$

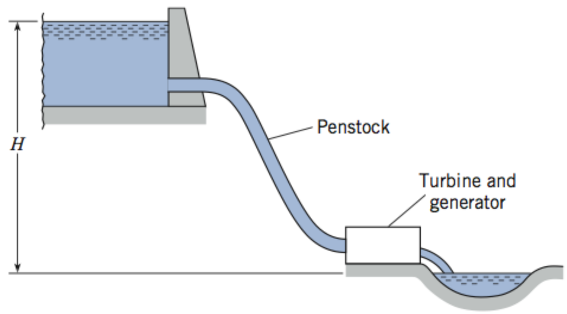
3. Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= 8\,\mathrm{ft}^3/\,\mathrm{s} \times 9810\,\mathrm{N}/\,\mathrm{m}^3 \times 22.4\,\mathrm{m} \\
&\boxed{P = 1.76\,\mathrm{MW}}
\end{aligned}
$$

7.43: PROBLEM DEFINITION

Situation:

An engineer is estimating the power that can be produced by a small stream.
$Q = 1.4$ cfs, $T = 40\,°\text{F}$, $H = 34\,\text{ft}$.



Find:
Estimate the maximum power that can be generated (kW) if:
$h_L = 0\,\text{ft}$, $\eta_t = 100\%$, $\eta_g = 100\%$.
$h_L = 5.5\,\text{ft}$, $\eta_t = 70\%$, $\eta_g = 90\%$.

PLAN To find the head of the turbine (h_t), apply the energy equation from the upper water surface (section 1) to the lower water surface (section 2). To calculation power, use $P = \eta(\dot{m}gh_t)$, where η accounts for the combined efficiency of the turbine and generator.

SOLUTION

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L \tag{1}$$

Term by term analysis

$$\begin{aligned} p_1 &= 0; \quad V_1 \approx 0 \\ p_2 &= 0; \quad V_2 \approx 0 \\ z_1 - z_2 &= H \end{aligned}$$

Eq. (1) becomes

$$\begin{aligned} H &= h_t + h_L \\ h_t &= H - h_L \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m}g &= \gamma Q \\ &= \left(62.4\,\mathrm{lbf/\,ft^3}\right)\left(1.4\,\mathrm{ft^3/\,s}\right) \\ &= 87.4\,\mathrm{lbf/\,s} \end{aligned}$$

Power (case a)

$$\begin{aligned} P &= \dot{m}gh_t \\ &= \dot{m}gH \\ &= (87.4\,\mathrm{lbf/\,s})\,(34\,\mathrm{ft})\,(1.356\,\mathrm{J/\,ft\cdot\,lbf}) \\ &= 4.02\,\mathrm{kW} \end{aligned}$$

Power (case b).

$$\begin{aligned} P &= \eta\dot{m}g\,(H - h_L) \\ &= (0.7)(0.9)\,(87.4\,\mathrm{lbf/\,s})\,(34\,\mathrm{ft} - 5.5\,\mathrm{ft})\,(1.356\,\mathrm{J/\,ft\cdot\,lbf}) \\ &= 2.128\,\mathrm{kW} \end{aligned}$$

$$\boxed{\text{Power (case a)} = 4.02\text{ kW}}$$

$$\boxed{\text{Power (case b)} = 2.13\text{ kW}}$$

REVIEW

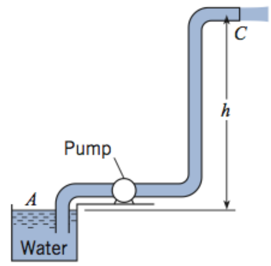
1. In the ideal case (case a), all of the elevation head is used to make power. When typical head losses and machine efficiencies are accounted for, the power production is cut by nearly 50%.

2. From Ohm's law, a power of 2.13 kW will produce a current of about 17.5 amps at a voltage of 120V. Thus, the turbine will provide enough power for about 1 typical household circuit. It is unlikely the turbine system will be practical (too expensive and not enough power for a homeowner).

7.44: PROBLEM DEFINITION

Situation:

A pump draws water out of a tank and moves this water to a higher elevation.
$D_A = 8\,\text{in},\ D_C = 4\,\text{in}.$
$V_C = 12\,\text{ft/s},\ P = 25\,\text{hp}.$
$\eta = 60\%,\ \ h_L = 2V_C^2/2g.$



Find:

Height (h)above water surface.

Assumptions:

$\alpha = 1.0.$

PLAN

Apply the energy equation from the reservoir water surface to the outlet.

SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \\ 0 + 0 + 0 + h_p &= 0 + \frac{V_c^2}{2g} + h + 2\frac{V_c^2}{2g} \\ h_p &= h + 3\frac{V_c^2}{2g} \qquad (1) \end{aligned}$$

Velocity head

$$\frac{V_c^2}{2g} = \frac{(12\,\text{ft/s})^2}{64.4\,\text{ft/s}^2} = 2.236 \text{ ft} \qquad (2)$$

Flow rate equation

$$\begin{aligned} Q &= V_C A_C \\ &= \left(\frac{12\,\text{ft}}{\text{s}}\right)\left(\frac{\pi\,(4/12\,\text{ft})^2}{4}\right) \\ &= 1.047\,\text{ft}^3/\text{s} \end{aligned}$$

Power equation

$$\begin{aligned} P\,(\text{hp}) &= \frac{Q\gamma h_p}{550\eta} \\ h_p &= \frac{P\,(550)\,\eta}{Q\gamma} \\ &= \frac{25\,\text{hp}\,(550)\,0.6}{1.047\,\text{ft}^3/\,\text{s} \times 62.4\,\text{lbf}/\,\text{ft}^3} \\ &= 126.3\,\text{ft} \end{aligned} \tag{3}$$

Substitute Eqs. (2) and (3) into Eq. (1)

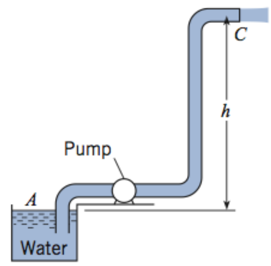
$$\begin{aligned} h_p &= h + 3\frac{V_c^2}{2g} \\ 126.3\,\text{ft} &= h + (3 \times 2.236)\ \text{ft} \\ h &= 119.6\,\text{ft} \end{aligned}$$

$$\boxed{h = 120\ \text{ft}}$$

7.45: PROBLEM DEFINITION

Situation:

A pump draws water out of a tank and moves this water to a higher elevation.
$D_A = 20\,\text{cm}$, $D_C = 10\,\text{cm}$.
$V_C = 3\,\text{m/s}$, $P = 35\,\text{kW}$.
$\eta = 60\%$, $h_L = 2V_C^2/2g$.



Find:

Height h in meters.

Assumptions:

$\alpha = 1.0$.

Properties:

Water (20 °C), Table A.5: $\gamma = 9790\,\text{N/m}^3$.

SOLUTION

Energy equation:

$$\begin{aligned}
\frac{p_A}{\gamma} + \alpha_A \frac{V_A^2}{2g} + z_A + h_p &= \frac{p_C}{\gamma} + \alpha_C \frac{V_C^2}{2g} + z_C + h_L \\
0 + 0 + 0 + h_p &= 0 + \frac{V_c^2}{2g} + h + 2\frac{V_c^2}{2g} \\
h_p &= h + 3\frac{V_c^2}{2g} \qquad (1)
\end{aligned}$$

Velocity head:

$$\frac{V_c^2}{2g} = \frac{(3\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} = 0.4587\,\text{m}$$

Flow rate equation:

$$\begin{aligned}
Q &= V_C\, A_C \\
&= (3\,\text{m/s}) \left(\frac{\pi\,(0.1\,\text{m})^2}{4} \right) \\
&= 0.02356\,\text{m}^3/\text{s}
\end{aligned}$$

Power equation:

$$
\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
h_p &= \frac{P\eta}{Q\gamma} = \frac{35000\,\text{W} \times 0.6}{(0.02356\,\text{m}^3/\,\text{s})\,(9790\,\text{N}/\,\text{m}^3)} = 91.05\,\text{m}
\end{aligned}
$$

Eq. (1):

$$
h = h_p - 3\frac{V_c^2}{2g} = (91.05\,\text{m}) - 3\,(0.4587\,\text{m})
$$

$$
\boxed{h = 89.7\,\text{m}}
$$

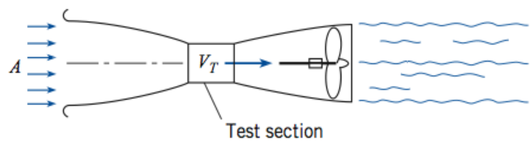
7.46: PROBLEM DEFINITION

Situation:

A subsonic wind tunnel is being designed.

$A = 4\,\text{m}^2$, $V = 60\,\text{m/s}$.

$h_L = 0.025\frac{V_T^2}{2g}$.



Find:

Power required (kW).

Assumptions:

$\alpha = 1.0$.

Properties:

$\rho = 1.2\,\text{kg/m}^3$.

PLAN

To find power, apply the power equation. The steps are

1. Find the head of the pump by applying the energy equation and the continuity equation together.
2. Calculate power.

SOLUTION

1. Finding head of the pump

- Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\ 0 + 0 + 0 + h_p &= 0 + \frac{V_2^2}{2g} + 0 + 0.025\frac{V_T^2}{2g} \end{aligned}$$

- Continuity principle

$$\begin{aligned} V_T A_T &= V_2 A_2 \\ V_2 &= \frac{V_T A_T}{A_2} \\ &= V_T \times 0.4 \\ \frac{V_2^2}{2g} &= 0.16\frac{V_T^2}{2g} \end{aligned}$$

- Combining previous two equations

$$\begin{aligned} h_p &= \frac{V_T^2}{2g}(0.185) \\ &= \frac{(60\,\mathrm{m/\,s})^2}{2 \times 9.81\,\mathrm{m/\,s^2}}(0.185) \\ h_p &= 33.95 \text{ m} \end{aligned}$$

2. Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= (VA)(\rho g)\, h_p \\ &= \left(60\,\mathrm{m/\,s} \times 4\,\mathrm{m^2}\right)\left(1.2\,\mathrm{kg/\,m^3} \times 9.81\,\mathrm{m/\,s^2}\right)(33.95\,\mathrm{m}) \end{aligned}$$

$$\boxed{P = 95.9 \text{ kW}}$$

7.47: PROBLEM DEFINITION

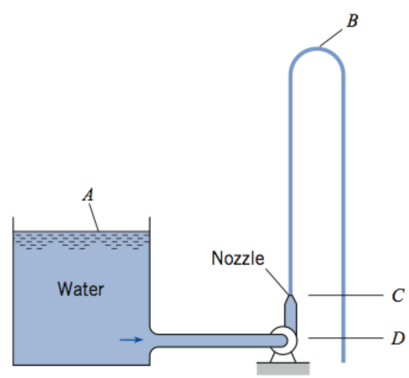
Situation:

A pumping system delivers water.

$z_A = 110\,\text{ft},\ z_B = 200\,\text{ft}.$

$z_C = 110\,\text{ft},\ z_D = 90\,\text{ft}.$

$A = 0.1\,\text{ft}^2.$



Find:

Power delivered by pump (hp).

SOLUTION

$$
\begin{aligned}
0 + 0 + 110 + h_p &= 0 + 0 + 200;\ h_p = 90 \text{ ft} \\
P(\text{hp}) &= \frac{Q\gamma h_p}{550} \\
Q &= V_j A_j = 0.10\, V_j \\
V_j &= \sqrt{2g \times (200 - 110)\,\text{ft}} = 76.13 \text{ ft/s} \\
Q &= 7.613 \text{ ft}^3/\text{s}
\end{aligned}
$$

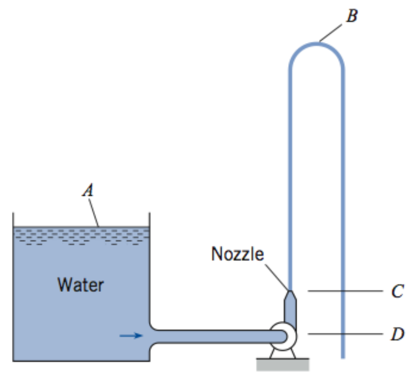
Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
P &= \frac{7.613\,\text{ft}^3/\,\text{s} \times 62.4\,\text{lbf}/\,\text{ft}^3 \times 90\,\text{ft}}{550} \\
& \boxed{P = 77.7 \text{ hp}}
\end{aligned}
$$

7.48: PROBLEM DEFINITION

Situation:

A pumping system delivers water.
$z_A = 40\,\text{m}$, $z_B = 65\,\text{m}$.
$z_C = 35\,\text{m}$, $z_D = 30\,\text{m}$.
$A = 25\,\text{cm}^2$.



Find:

Power delivered by pump (kW).

PLAN

Apply the energy equation from the reservoir water surface to point B. Then apply the power equation.

SOLUTION

Energy equation

$$\begin{aligned} \frac{p}{\gamma} + \frac{V^2}{2g} + z + h_p &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \\ 0 + 0 + 40 + h_p &= 0 + 0 + 65;\ h_p = 25 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_j A_j = 25 \times 10^{-4} \text{ m}^2 \times V_j \\ \text{where } V_j &= \sqrt{2g \times (65 - 35)} = 24.3 \text{ m/s} \\ Q &= 25 \times 10^{-4} \times 24.3 = 0.0607 \text{ m}^3/\text{s} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ P &= 0.0607\,\text{m}^3/\,\text{s} \times 9{,}810\,\text{N}/\,\text{m}^3 \times 25\,\text{m} \\ & \boxed{P = 14.9 \text{ kW}} \end{aligned}$$

7.49: PROBLEM DEFINITION

Situation:

A pumping system pumps oil.
$L = 1\,\text{mi}$, $D = 1\,\text{ft}$.
$Q = 3500$ gpm, $\Delta z = 200\,\text{ft}$.
$\Delta p = 60$ psi.

Find:

Power required for pump (hp).

Properties:

$\gamma = 0.53\,\text{lbf/ ft}^3$.

SOLUTION

Energy equation

$$h_p = z_2 - z_1 + h_L$$

Expressing this equation in terms of pressure

$$\gamma h_p = \gamma z_2 - \gamma z_1 + \Delta p_{loss}$$

Thus pressure rise across the pump is

$$\gamma h_p = 53 \text{ lbf/ft}^3 \times 200 \text{ ft} + 60 \times 144 \text{ lbf/ft}^2 = 19,240 \text{ psf}$$

Flow rate equation

$$\begin{aligned} Q &= V \times A \\ Q &= 3500 \text{ gpm} \times 0.002228 \frac{\text{ft}^3/\text{s}}{\text{gpm}} = 7.80 \text{ cfs} \end{aligned}$$

Power equation

$$\begin{aligned} \dot{W} &= Q\gamma h_p \\ &= 7.80 \text{ cfs} \times \frac{19,240 \text{ psf}}{550} \\ & \boxed{\dot{W} = 273 \text{ hp}} \end{aligned}$$

7.50: PROBLEM DEFINITION

Situation:

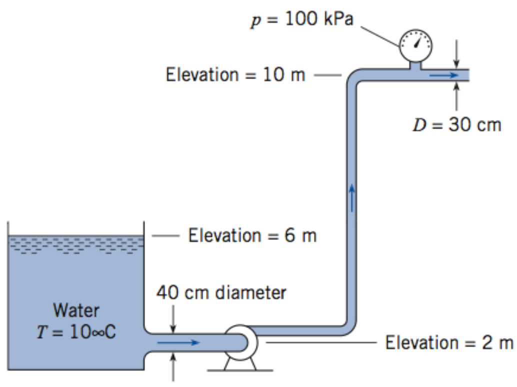
Water is pumped from a reservoir through a pipe.

$Q = 0.35\,\text{m}^3/\,\text{s}$, $h_L = 2\frac{V_2^2}{2g}$.

$z_1 = 6\,\text{m}$, $z_2 = 10\,\text{m}$.

$D_1 = 40\,\text{cm}$, $D_2 = 10\,\text{cm}$.

$p_2 = 100\,\text{kPa}$.



Find:

Power (kW) that the pump must supply.

Assumptions:

$\alpha = 1.0$.

Properties:

Water (10 °C), Table A.5: $\gamma = 9810\,\text{N}/\,\text{m}^3$.

PLAN

Apply the flow rate equation, then the energy equation from reservoir surface to the 10 m elevation. Then apply the power equation.

SOLUTION

Flow rate equation:

$$
\begin{aligned}
V &= \frac{Q}{A} \\
&= \frac{0.35}{(\pi/4) \times (0.3\,\text{m})^2} \\
&= 4.95 \text{ m/s} \\
\frac{V_2^2}{2g} &= 1.250 \text{ m}
\end{aligned}
$$

Energy equation (locate 1 on the reservoir surface; locate 2 at the pressure gage)

$$\begin{aligned} 0+0+6\,\mathrm{m}+h_p &= \frac{100000\,\mathrm{Pa}}{9810\,\mathrm{N/\,m^3}}+1.25\,\mathrm{m}+10\,\mathrm{m}+2.0\,(1.25\,\mathrm{m}) \\ h_p &= 17.94\,\mathrm{m} \end{aligned}$$

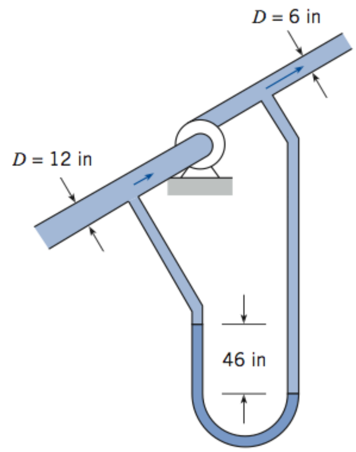
Power equation:

$$\begin{aligned} P &= Q\gamma h_p \\ &= \left(0.35\,\mathrm{m^3/\,s}\right)\left(9810\,\mathrm{N/\,m^3}\right)(17.94\,\mathrm{m}) \\ &\boxed{P = 61.6\,\mathrm{kW}} \end{aligned}$$

7.51: PROBLEM DEFINITION

Situation:

Oil is pumped through a pipe attached to a manometer filled with mercury.
$Q = 6\,\text{ft}^3/\,\text{s}$, $\Delta h = 46\,\text{in}$.
$D_1 = 12\,\text{in}$, $D_2 = 6\,\text{in}$.



Find:

Horsepower pump supplies (hp).

Assumptions:

$\alpha = 1.0$.

Properties:

Table A.5: $S_{\text{mercury}} = 13.55$.
$S_{\text{oil}} = 0.88$

PLAN

Apply the flow rate equation, then the energy equation. Then apply the power equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V_{12} &= \frac{Q}{A_{12}} = \frac{6\,\text{ft}^3/\,\text{s}}{(\pi/4)\times(1\,\text{ft})^2} = 7.64\,\text{ft}/\,\text{s} \\ V_{12}^2/2g &= 0.906\text{ ft} \\ V_6 &= 4V_{12} = 30.56\text{ ft/s} \\ V_6^2/2g &= 14.50\text{ ft} \end{aligned}$$

Energy equation

$$\begin{aligned}
\left(\frac{p_6}{\gamma}+z_6\right)-\left(\frac{p_{12}}{\gamma}+z_{12}\right) &= \frac{(13.55-0.88)\left(\frac{46}{12}\,\mathrm{ft}\right)}{0.88} \\
\left(\frac{p_{12}}{\gamma}+z_{12}\right)+\frac{V_{12}^2}{2g}+h_p &= \left(\frac{p_6}{\gamma}+z_6\right)+\frac{V_6^2}{2g} \\
h_p &= \left(\frac{13.55}{0.88}-1\right)\times 3.833\,\mathrm{ft}+14.50\,\mathrm{ft}-0.906\,\mathrm{ft} \\
h_p &= 68.8\ \mathrm{ft}
\end{aligned}$$

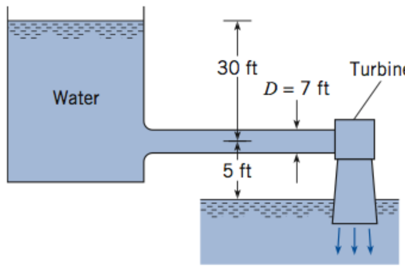
Power equation

$$\begin{aligned}
P(\mathrm{hp}) &= Q\gamma h_p/550 \\
P &= \frac{6\,\mathrm{ft}^3/\,\mathrm{s}\times\left(0.88\times 62.4\,\mathrm{lbf}/\,\mathrm{ft}^3\right)\times 68.8\,\mathrm{ft}}{550} \\
& \boxed{P = 41.2\ \mathrm{hp}}
\end{aligned}$$

7.52: PROBLEM DEFINITION

Situation:

$Q = 500$ cfs, $\eta = 90\%$.
$h_L = 1.5\frac{V^2}{2g}$, $D = 7\,\text{ft}$.
$z_1 = 35\,\text{ft}$, $z_2 = 0\,\text{ft}$.



Find:

Power output from turbine.

Assumptions:

$\alpha = 1.0$.

PLAN

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

SOLUTION

Energy equation:

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L + h_t \\
0 + 0 + 35 &= 0 + 0 + 0 + 1.5\frac{V^2}{2g} + h_t \\
V &= \frac{Q}{A} = \frac{500\,\text{ft}^3/\,\text{s}}{(\pi/4) \times (7\,\text{ft})^2} = 12.99\ \text{ft/s} \\
\frac{V^2}{2g} &= 2.621\ \text{ft} \\
h_t &= 35\,\text{ft} - 1.5\,(2.621\,\text{ft}) = 31.07\ \text{ft}
\end{aligned}$$

Power equation:

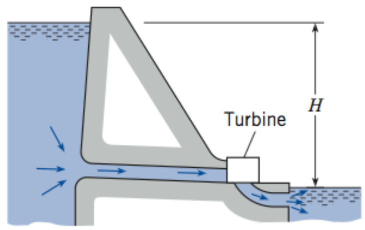
$$\begin{aligned}
P(\text{hp}) &= \frac{Q\gamma h_t \eta}{550} \\
&= \frac{(500\,\text{ft}^3/\,\text{s})(62.4\,\text{lbf}/\,\text{ft}^3)(31.07\,\text{ft} \times 0.9)}{550}
\end{aligned}$$

$$\boxed{P = 1590\,\text{hp} = 1.18\,\text{MW}}$$

7.53: PROBLEM DEFINITION

Situation:

A hydraulic power system consists of a dam with an inlet connected to a turbine.
$H = 15\,\text{m}$, $V_2 = 5\,\text{m/s}$.
$Q = 1\,\text{m}^3/\text{s}$.



Find:

Power produced by turbine (kW).

Assumptions:

All head loss is expansion loss.
100% efficiency.

PLAN

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L + h_t \\
0 + 0 + 15 \text{ m} &= 0 + 0 + 0 + h_t + \frac{V_2^2}{2g} \\
h_t &= 15 \text{ m} - \frac{(5\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} \\
&= 13.73 \text{ m}
\end{aligned}$$

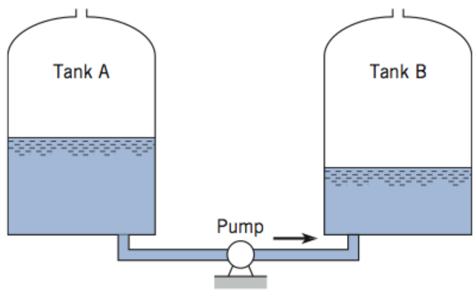
Power equation

$$\begin{aligned}
P &= Q\gamma h_t \\
&= (1 \text{ m}^3/\text{s})(9810 \text{ N/m}^3)(13.73 \text{ m}) \\
&\boxed{P = 135 \text{ kW}}
\end{aligned}$$

7.54: PROBLEM DEFINITION

Situation:

A pump transfers SAE-30 oil between two tanks.
$D_{\text{tank}} = 12\,\text{m}$, $D_{\text{pipe}} = 20\,\text{cm}$.
$h_L = 20\frac{V^2}{2g}$, $h_p = 60\,\text{m}$.
$z_A = 20\,\text{m}$, $z_B = 1\,\text{m}$.



Find:

Time required to transfer oil (h).

PLAN

Apply the energy equation between the top of the fluid in tank A to that in tank B.

SOLUTION

Energy equation

$$h_p + z_A = z_B + h_L$$

or

$$h_p + z_A = z_B + 20\frac{V^2}{2g} + \frac{V^2}{2g}$$

Solve for velocity

$$\begin{aligned} V^2 &= \frac{2g}{21}(h_p + z_A - z_B) \\ V^2 &= \frac{2 \times 9.81}{21}(60 + z_A - z_B) \\ V &= 0.9666\,(60 + z_A - z_B)^{1/2} \end{aligned}$$

The sum of the elevations of the liquid surfaces in the two tanks is

$$z_A + z_B = 21$$

So the energy equation becomes

$$V = 0.9666(81 - 2z_B)^{1/2}$$

Continuity equation

$$\begin{aligned} \frac{dz_B}{dt} &= V\frac{A_{\text{pipe}}}{A_{\text{tank}}} = V\frac{(0.2\,\text{m})^2}{(12\,\text{m})^2} \\ &= \left(2.778 \times 10^{-4}\right) V \\ &= \left(2.778 \times 10^{-4}\right) 0.9666(81 - 2z_B)^{1/2} \\ &= 2.685 \times 10^{-4}(81 - 2z_B)^{1/2} \end{aligned}$$

Separate variables

$$\frac{dz_B}{(81 - 2z_B)^{1/2}} = 2.685 \times 10^{-4} dt$$

Integrate

$$\begin{aligned} \int_1^{20\,\text{ft}} \frac{dz_B}{(81 - 2z_B)^{1/2}} &= \int_0^{\Delta t} 2.685 \times 10^{-4} dt \\ \left(-\sqrt{81 - 2z_B}\right)_{1\,\text{ft}}^{20\,\text{ft}} &= \left(2.685 \times 10^{-4}\right) \Delta t \\ \left(-\sqrt{81 - 2\,(20)} + \sqrt{81 - 2\,(1)}\right) &= \left(2.685 \times 10^{-4}\right) \Delta t \\ 2.\,485\,1 &= \left(2.685 \times 10^{-4}\right) \Delta t \\ \Delta t &= 9256\,\text{s} \end{aligned}$$

$$\boxed{t = 9260\,\text{s} = 2.57\,\text{h}}$$

7.55: PROBLEM DEFINITION

Situation:

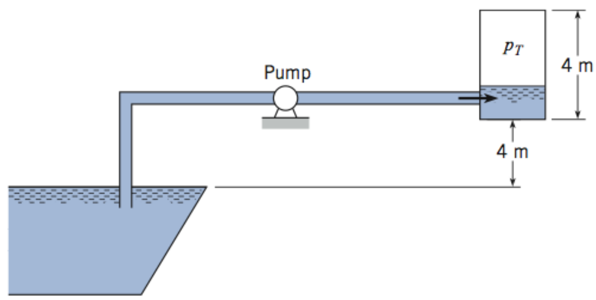
A pump is used to pressurize a tank.

$D_{\text{tank}} = 2\,\text{m}$, $D_{\text{pipe}} = 4\,\text{cm}$.

$h_L = 10\frac{V^2}{2g}$, $h_p = 50\,\text{m}$.

$z_A = 20\,\text{m}$, $z_B = 1\,\text{m}$.

$p_T = \frac{3}{4-z_t}p_0$, $p_0 = 0\,\text{kPa gage} = 100\,\text{kPa}$.



Find:

Write a computer program to show how the pressure varies with time.

Time to pressurize tank to 300 kPa (s).

PLAN

Apply the energy equation between the water surface at the intake and the water surface inside the tank.

SOLUTION

Energy equation

$$h_p + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

Expressing the head loss in terms of the velocity allows one to solve for the velocity in the form

$$V^2 = \frac{2g}{10}(h_p + z_1 - z_t - \frac{p_t}{\gamma})$$

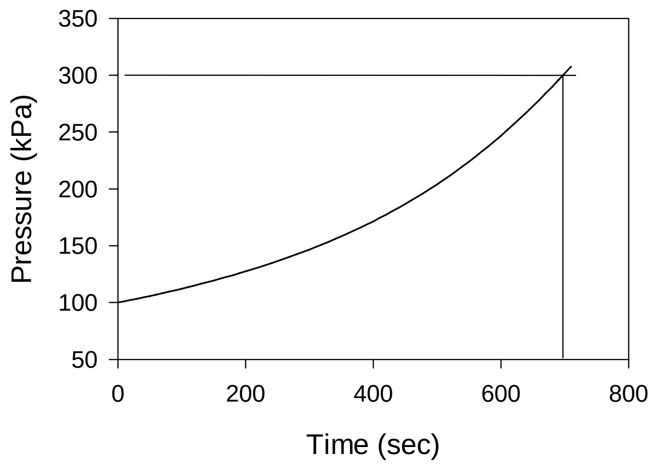
Substituting in values

$$V = 1.401(46 - z_t - 10.19\frac{3}{4 - z_t})^{1/2}$$

The equation for the water surface elevation in the tank is

$$\Delta z_t = V\frac{A_p}{A_t}\Delta t = \frac{V}{2500}\Delta t$$

A computer program can be written taking time intervals and finding the fluid level and pressure in the tank at each time step. The time to reach a pressure of 300 kPa abs in the tank is **698 seconds or 11.6 minutes.** A plot of how the pressure varies with time is provided.

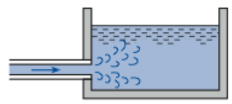


7.56: PROBLEM DEFINITION

Situation:

A pipe discharges water into a reservoir.

$Q = 10$ cfs, $D = 12\,\text{in}$.



Find:

Head loss at pipe outlet (ft).

PLAN

Apply the flow rate equation, then the sudden expansion head loss equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{10\,\text{ft}^3/\,\text{s}}{(\pi/4)\times(1\,\text{ft})^2} \\ &= 12.73\ \text{ft/s} \end{aligned}$$

Sudden expansion head loss equation

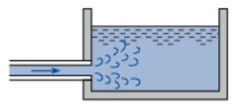
$$\begin{aligned} h_L &= V^2/2g \\ & \boxed{h_L = 2.52\ \text{ft}} \end{aligned}$$

7.57: PROBLEM DEFINITION

Situation:

A pipe discharges water into a reservoir.

$Q = 0.5\,\mathrm{m}^3/\,\mathrm{s},\ D = 50\,\mathrm{cm}.$



Find:

Head loss at pipe outlet (m).

PLAN

Apply the flow rate equation, then the sudden expansion head loss equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.5\,\mathrm{ft}^3/\,\mathrm{s}}{(\pi/4)\times(0.5\,\mathrm{ft})^2} \\ &= 2.546\ \mathrm{m/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= \frac{V^2}{2g} \\ &= \frac{(2.546\,\mathrm{m/\,s})^2}{2\times 9.81\,\mathrm{m/\,s^2}} \\ & \boxed{h_L = 0.330\ \mathrm{m}} \end{aligned}$$

7.58: PROBLEM DEFINITION

Situation:

Water flows through a sudden expansion.
$D_8 = 8\,\text{cm}$, $V_8 = 2\,\text{m/s}$.
$D_{15} = 15\,\text{cm}$.

Find:

Head loss caused by the sudden expansion (m).

PLAN

Apply the continuity principle, then the sudden expansion head loss equation.

SOLUTION

Continuity principle:

$$\begin{aligned} V_8 A_8 &= V_{15} A_{15} \\ V_{15} &= \frac{V_8 A_8}{A_{15}} = 2 \times \left(\frac{8}{15}\right)^2 = 0.569 \text{ m/s} \end{aligned}$$

Sudden expansion head loss:

$$\begin{aligned} h_L &= \frac{(V_8 - V_{15})^2}{(2g)} \\ h_L &= \frac{(2\,\text{m/s} - 0.569\,\text{m/s})^2}{(2 \times 9.81\,\text{m/s}^2)} \\ &\boxed{h_L = 0.104\,\text{m}} \end{aligned}$$

7.59: PROBLEM DEFINITION

Situation:

Water flows through a sudden expansion.
$D_6 = 6\,\text{in} = 0.5\,\text{ft}$.
$D_{12} = 12\,\text{in} = 1\,\text{ft}$.
$Q = 5$ cfs.

Find:

Head loss

PLAN

Apply the flow rate equation, then the sudden expansion head loss equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V_6 &= \frac{Q}{A_6} = \frac{5}{(\pi/4) \times (0.5\,\text{ft})^2} = 25.46 \text{ ft/s}; \\ V_{12} &= \frac{1}{4} V_6 = 6.37 \text{ ft/s} \end{aligned}$$

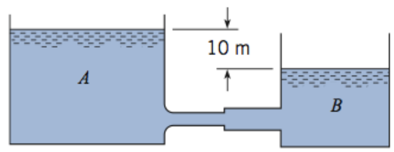
Sudden expansion head loss equation

$$\begin{aligned} h_L &= \frac{(V_6 - V_{12})^2}{2g} \\ &= \frac{(25.46\,\text{ft/s} - 6.37\,\text{ft/s})^2}{2 \times 32.2\,\text{ft/s}^2} \\ & \boxed{h_L = 5.66 \text{ ft}} \end{aligned}$$

7.60: PROBLEM DEFINITION

Situation:

Two tanks are connected by a pipe with a sudden expansion.
$\Delta z = 10\,\text{m},\ A_1 = 8\,\text{cm}^2.$
$A_2 = 25\,\text{cm}^2.$



Find:

Discharge between two tanks (m^3/s)

PLAN

Apply the energy equation from water surface in A to water surface in B.

SOLUTION

Energy equation (top of reservoir A to top of reservoir B)

$$\begin{aligned}\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + \Sigma h_L \\ 0 + 0 + 10\,\text{m} &= 0 + 0 + 0 + \Sigma h_L \end{aligned} \tag{1}$$

Let the pipe from A be called pipe 1. Let the pipe into B be called pipe 2
Then

$$\Sigma h_L = \frac{(V_1 - V_2)^2}{2g} + \frac{V_2^2}{2g} \tag{2}$$

Continuity principle:

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_1 &= \frac{V_2 A_2}{A_1} = V_2 \frac{(25\,\text{cm}^2)}{(8\,\text{cm}^2)} = 3.125 V_2 \end{aligned} \tag{3}$$

Combine Eq. (1), (2), and (3):

$$\begin{aligned} 10\,\text{m} &= \frac{(3.125V_2 - V_2)^2}{2\,(9.81\,\text{m}/\text{s}^2)} + \frac{{V_2}^2}{2\,(9.81\,\text{m}/\text{s}^2)} \\ V_2 &= 5.964\,\text{m}/\text{s} \end{aligned}$$

Flow rate equation:

$$\begin{aligned} Q &= V_2 A_2 \\ &= (5.964\,\text{m}/\text{s})\,(25\,\text{cm}^2)\left(\frac{1.0\,\text{m}}{100\,\text{cm}}\right)^2 \end{aligned}$$

$$\boxed{Q = 0.0149\,\text{m}^3/\text{s}}$$

7.61: PROBLEM DEFINITION

Situation:

A horizontal pipe with an abrupt expansion.
$D_{40} = 40\,\text{cm}$, $D_{60} =$ cm.
$Q = 1.0\,\text{m}^3/\text{s}$, $p_1 = 70\,\text{kPa}$ gage.

Find:

Horizontal force required to hold transition in place (kN).
Head loss (m).

Assumptions:

$\alpha = 1.0$.

Properties:

Water, $\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the flow rate equation, the sudden expansion head loss equation, the energy equation, and the momentum principle.

SOLUTION

Flow rate equation

$$\begin{aligned} V_{40} &= \frac{Q}{A_{40}} = \frac{1.0\,\text{m}^3/\text{s}}{(\pi/4) \times (0.40\,\text{m})^2} = 7.96 \text{ m/s} \\ \frac{V_{40}^2}{2g} &= 3.23 \text{ m} \\ V_{60} &= V_{40} \times \left(\frac{4}{6}\right)^2 = 3.54 \text{ m/s} \\ \frac{V_{60}^2}{2g} &= 0.639 \text{ m} \end{aligned}$$

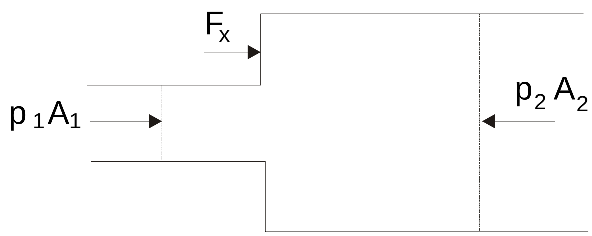
Sudden expansion head loss equation

$$\begin{aligned} h_L &= \frac{(V_{40} - V_{60})^2}{2g} \\ &= \boxed{0.996 \text{ m}} \end{aligned}$$

Energy equation

$$\begin{aligned} \frac{p_{40}}{\gamma} + \frac{V_{40}^2}{2g} &= \frac{p_{60}}{\gamma} + \frac{V_{60}^2}{2g} + h_L \\ p_{60} &= 70,000\,\text{Pa} + 9810\,\text{N/m}^3(3.23 - 0.639 - 0.996)\,\text{m} = 85,647 \text{ Pa} \end{aligned}$$

Momentum principle



$$\sum F_x = \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i}$$

Substituting values

$$\begin{aligned} & 70,000\,\mathrm{Pa} \times \pi/4 \times (0.4\,\mathrm{m})^2 - 85,647\,\mathrm{Pa} \times \pi/4 \times (0.6\,\mathrm{m})^2 + F_x \\ = \;& 1000\,\mathrm{kg/\,m^3} \times 1.0\,\mathrm{m^3/\,s} \times (3.54\,\mathrm{m/\,s} - 7.96\,\mathrm{m/\,s}) \end{aligned}$$

The result is

$$\begin{aligned} F_x &= -8796 + 24,216 - 4,420 \\ &= 11,000 \text{ N} \\ & \boxed{F_x = 11.0 \text{ kN}} \end{aligned}$$

7.62: PROBLEM DEFINITION

Situation:

Water flows in a horizontal pipe before discharging to atmosphere.

$A = 9\,\text{in}^2$, $V = 15\,\text{ft/s}$.

$h_L = 3\,\text{ft}$.

Find:

Force on pipe joint.

Assumptions:

$\alpha = 1.0$.

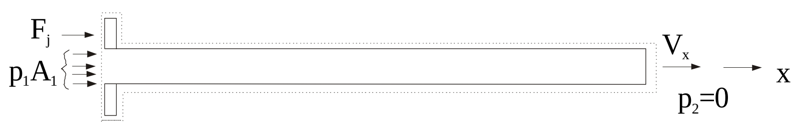
Properties:

$\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN

Apply the momentum principle, then the energy equation.

SOLUTION



Momentum Equation

$$\begin{aligned}\sum F_x &= \dot{m}V_{o,x} - \dot{m}V_{i,x} \\ F_j + p_1 A_1 &= -\rho V_x^2 A + \rho V_x^2 A \\ F_j &= -p_1 A_1\end{aligned}$$

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$p_1 - p_2 = \gamma h_L$$

$$p_1 = \gamma(3) = 187.2 \text{ psfg}$$

$$F_j = -187.2\,\text{lbf/ft}^2 \times \left(\frac{9\,\text{in}^2}{144\,\text{in}^2/\text{ft}^2}\right)$$

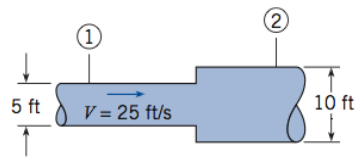
$$F_j = -11.7\,\text{lbf}$$

$$\boxed{F_j = 11.7\,\text{lbf acting to the left}}$$

7.63: PROBLEM DEFINITION

Situation:

An abrupt expansion dissipates high energy flow.
$D_1 = 5\,\text{ft}$, $p_1 = 5$ psig.
$V = 25\,\text{ft/s}$, $D_2 = 10\,\text{ft}$.



Find:

(a) Horsepower lost (hp).
(b) Pressure at section 2 (psig).
(c) Force needed to hold expansion (lbf).

Assumptions:

$\alpha = 1.0$.

Properties:

Water, $\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN

Find the head loss by applying the sudden expansion head loss equation, first solving for V_2 by applying the continuity principle. Then apply the power equation, the energy equation, and finally the momentum principle.

SOLUTION Continuity equation

$$\begin{aligned} V_2 &= V_1 \frac{A_1}{A_2} \\ &= 25\,\text{ft/s}\left(\frac{1}{4}\right) \\ &= 6.25\ \text{ft/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= (V_1 - V_2)^2/(2g) \\ h_L &= \frac{(25\,\text{ft/s} - 6.25\,\text{ft/s})^2}{64.4\,\text{ft/s}^2} \\ &= 5.46\ \text{ft} \end{aligned}$$

a) Power equation

$$\begin{aligned} P(\text{hp}) &= Q\gamma h/550 \\ Q &= VA = 25\,\text{ft/s}(\pi/4)\,(5\,\text{ft})^2 = 490.9\ \text{ft}^3/\text{s} \\ P &= (490.9\,\text{ft}^3/\text{s})(62.4\,\text{lbf/ft}^3)(5.46)/550 \\ & \boxed{P = 304\ \text{hp}} \end{aligned}$$

b) Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\ \frac{(5 \times 144)\,\text{lbf/ft}^2}{62.4\,\text{lbf/ft}^3} + \frac{(25\,\text{ft/s})^2}{64.4\,\text{ft/s}^2} &= \frac{p_2}{\gamma} + \frac{(6.25\,\text{ft/s})^2}{64.4\,\text{ft/s}^2} + 5.46\,\text{ft} \\ p_2/\gamma &= 15.18\ \text{ft} \\ p_2 &= 15.18\,\text{ft} \times 62.4\,\text{lbf/ft}^3 \\ &= 947\ \text{psfg} \end{aligned}$$

$$\boxed{p_2 = 6.58\ \text{psig}}$$

c) Momentum equation

$$\begin{aligned} \dot{m} &= 1.94\,\text{slug/ft}^3 \times (\pi/4) \times (5\,\text{ft})^2 \times 25\,\text{ft/s} \\ &= 952.3\ \text{kg/s} \end{aligned}$$

$$\sum F_x = \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i}$$

$$p_1 A_1 - p_2 A_2 + F_x = \dot{m}(V_2 - V_1)$$

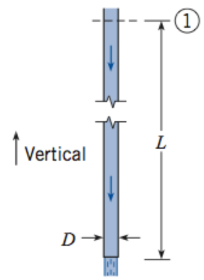
$$\begin{aligned} & (5\,\text{lbf/in}^2)(144\,\text{in}^2/\text{ft}^2)(\pi/4)\,(5\,\text{ft})^2 - (6.58\,\text{lbf/in}^2)(144\,\text{in}^2/\text{ft}^2)(\pi/4)\,(10\,\text{ft})^2 + F_x \\ = \ & 952.3\,\text{kg/s} \times (6.25\,\text{ft/s} - 25\,\text{ft/s}) \end{aligned}$$

$$\boxed{F_x = 42{,}400\ \text{lbf}}$$

7.64: PROBLEM DEFINITION

Situation:

Rough aluminum pipe discharges water.
$L = 50\,\text{ft},\ D = 6\,\text{in}.$
$W = (1.5\,\text{lbf})\,L,\ Q = 6$ cfs.
$h_L = 10\,\text{ft}.$



Find:

Longitudinal force transmitted through pipe wall (lbf).

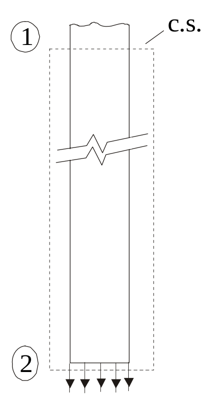
Properties:

Water, $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

PLAN

Apply the energy equation, then the momentum principle.

SOLUTION



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

but $V_1 = V_2$ and $p_2 = 0$. Therefore

$$\begin{aligned} p_1/\gamma &= -50\,\text{ft} + 10\,\text{ft} \\ p_1 &= -2496 \text{ lbf/ft}^2 \end{aligned}$$

Momentum principle

$$\sum F_y = \dot{m}V_{y,o} - \dot{m}V_{y,i} = \rho Q(V_{2y} - V_{1y})$$

$$-p_1 A_1 - \gamma AL - 2L + F_{\text{wall}} = 0$$

$$\begin{aligned} F_{\text{wall}} &= 1.5L + \gamma A_1 L - p_1 A_1 \\ &= 75\,\text{lbf} + \left[(\pi/4) \times (0.5\,\text{ft})^2 \,(62.4\,\text{lbf/ft}^3 \times 50\,\text{ft} - 2,496\,\text{lbf/ft}^3)\right] \\ &= 75\,\text{lbf} + 122.5\,\text{lbf} \end{aligned}$$

$$\boxed{F_{\text{wall}} = 198 \text{ lbf acting upward}}$$

7.65: PROBLEM DEFINITION

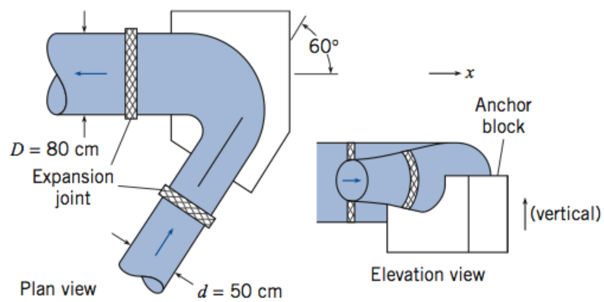
Situation:

Water flows in a bend.

$Q = 5\,\text{m}^3/\,\text{s}$, $p = 650\,\text{kPa}$.

$h_L = 10\,\text{m}$, $D = 80\,\text{cm}$.

$d = 50\,\text{cm}$.



Find:

Pressure at outlet of bend (kPa).

Force on anchor block in the x-direction (kN).

Assumptions:

$\alpha = 1.0$.

PLAN

Apply the energy equation, then the momentum principle.

SOLUTION

Energy equation

$$\frac{p_{50}}{\gamma} + \frac{V_{50}^2}{2g} + z_{50} = \frac{p_{80}}{\gamma} + \frac{V_{80}^2}{2g} + z_{80} + h_L$$

where $p_{50} = 650,000$ Pa and $z_{50} = z_{80}$

Flow rate equation

$$V_{80} = \frac{Q}{A_{80}} = \frac{5\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.8\,\text{m})^2} = 9.947 \text{ m/s}$$

$$V_{80}^2/2g = 5.04 \text{ m}$$

Continuity equation

$$V_{50} = V_{80} \times (8/5)^2 = 25.46 \text{ m/s}$$

$$\frac{V_{50}^2}{2g} = 33.04 \text{ m}$$

$$h_L = 10 \text{ m}$$

Then

$$
\begin{aligned}
\frac{p_{80}}{\gamma} &= \frac{650,000\,\mathrm{Pa}}{\gamma} + 33.04 - 5.04 - 10 \\
p_{80} &= 650,000\,\mathrm{Pa} + \left(9,810\,\mathrm{N/\,m^3}\right)(33.04 - 5.04 - 10)\,\mathrm{m} = 826,600\ \mathrm{Pa} \\
&\boxed{p_{80} = 827\ \mathrm{kPa}}
\end{aligned}
$$

Momentum principle

$$
\begin{aligned}
\sum F_x &= \dot{m}V_o - \dot{m}V_i = \rho Q(V_{80,x} - V_{50,x}) \\
p_{80}A_{80} + p_{50}A_{50} \times \cos 60^\circ + F_x &= 1,000\,\mathrm{kg/\,m^3} \times 5\,\mathrm{m^3/\,s}(-9.947\,\mathrm{m/\,s} - 0.5\,\mathrm{m} \times 25.46\,\mathrm{m/\,s}) \\
F_x &= -415,494 - 63,814 - 113,385 \\
&= -592,693\,\mathrm{N} \\
&\boxed{F_x = \text{-593 kN}}
\end{aligned}
$$

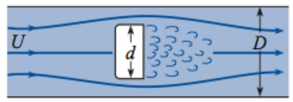
7.66: PROBLEM DEFINITION

Situation:

Fluid in a pipe flows around an accelerated disk.

$U = 10\,\mathrm{m/s}$, $D = 5\,\mathrm{cm}$.

$d = 4\,\mathrm{cm}$.



Find:

Develop an expression for the force required to hold the disk in place in terms of U, D, d, and ρ.

Force required under given conditions (N).

Assumptions:

$\alpha = 1.0$.

Properties:

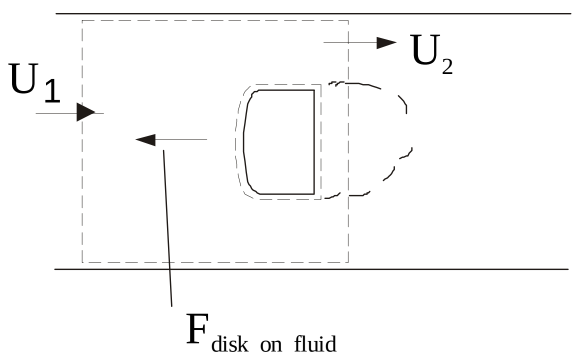
$\rho = 1.2\,\mathrm{kg/m^3}$.

PLAN

Apply the energy equation from section (1) to section (2), and apply the momentum principle.

SOLUTION

Control volume



Energy equation

$$
\begin{aligned}
p_1 + \frac{\rho U_1^2}{2} &= p_2 + \frac{\rho U_2^2}{2} \\
p_1 - p_2 &= \frac{\rho U_2^2}{2} - \frac{\rho U_1^2}{2}
\end{aligned}
$$

but

$$\begin{aligned} U_1 A_1 &= U_2(\pi/4)(D^2 - d^2) \\ U_2 &= \frac{U_1 D^2}{(D^2 - d^2)} \end{aligned} \tag{1}$$

Then

$$p_1 - p_2 = \left(\frac{\rho}{2}\right) U_1^2 \left[\frac{D^4}{(D^2 - d^2)^2} - 1\right] \tag{2}$$

Momentum principle for the C.V.

$$\begin{aligned} \sum F_x &= \dot{m}_o U_o - \dot{m}_i U_i = \rho Q(U_{2x} - U_{1x}) \\ p_1 A - p_2 A + F_{\text{disk on fluid}} &= \rho Q(U_2 - U_1) \\ F_{\text{fluid on disk}} &= F_d = \rho Q(U_1 - U_2) + (p_1 - p_2)A \end{aligned}$$

Eliminate $p_1 - p_2$ by Eq. (2), and U_2 by Eq. (1):

$$F_d = \rho U A \left(U_1 - \frac{U_1 D^2}{(D^2 - d^2)}\right) + \left(\frac{\rho U^2}{2}\right)\left[\frac{D^4}{(D^2 - d^2)^2} - 1\right] A$$

$$\boxed{F_d = \frac{\rho U^2 \pi D^2}{8}\left[\frac{1}{(D^2/d^2 - 1)^2}\right]}$$

When $U = 10$ m/s, $D = 5$ cm, $d = 4$ cm and $\rho = 1.2$ kg/m^3

$$F_d = \frac{1.2\,\text{kg/m}^3 \times (10\,\text{m/s})^2 \times \pi \times (0.05\,\text{m})^2}{8}\left[\frac{1}{((0.05\,\text{m}/0.04\,\text{m})^2 - 1)^2}\right]$$

$$\boxed{F_d = 0.372 \text{ N}}$$

Problem 7.67

No solution provided.

Problem 7.68

Answer the following questions.

---

a. What are three important reasons that engineers use the HGL and EGL?

- Identify where head loss is occurring.
- Identify potential sites of cavitation.
- Visualize how the energy equation is being satisfied.

---

b. What factors influence the magnitude of the HGL? What factors influence the magnitude of the EGL?

- Since the HGL $= (p/\gamma) + z$, the factors are:
  - Pressure head (p/γ), which is influenced by the pressure and the specific weight of the fluid.
  - Elevation head (z).
- Since the EGL $= (p/\gamma) + z + (V^2/(2g))$, the factors are:
  - Pressure head and elevation head (as described above).
  - Velocity head $(V^2/(2g))$, which is influenced by the flow rate and pipe section area.

---

c. How are the EGL and HGL related to the piezometer? To the stagnation tube?

- When liquid flows in a pipe, the HGL is coincident with the water level in a piezometer that is tapped into the pipe.
- When liquid flows in a pipe, the EGL is coincident with the water level in a stagnation tube that is tapped into the pipe.

---

d. How is the EGL related to the energy equation?

- The EGL involves three of the terms that appear in the energy equation.
- The terms represent transport of energy across the control surface (PE + KE) plus flow work at the control surface.

---

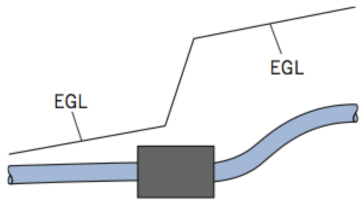
e. How can you use an EGL or an HGL to determine the direction of flow?

- In a pipe of constant diameter, the flow goes from locations of high EGL & HGL to locations of low HGL & EGL.
- In a pipe of variable diameter, flow goes from locations of high EGL to locations of low EGL.

7.69: PROBLEM DEFINITION

Situation:

A piping system with a black box shows a large EGL change at the box.



Find:

What the black box could be.

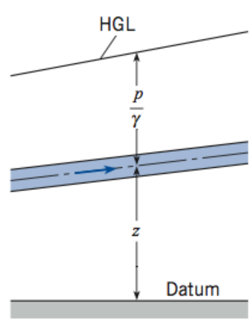
SOLUTION

- Because the EGL slopes downward to the left, the flow is from right to left.
- In the "black box" is a device that removes energy from the flowing fluid because the EGL drop.
- Thus, there could either be a turbine, an abrupt expansion or a partially closed valve. b, c, d.

7.70: PROBLEM DEFINITION

Situation:

A constant diameter pipe is shown with an HGL.



Find:

Whether this system is possible, and if so under what conditions.

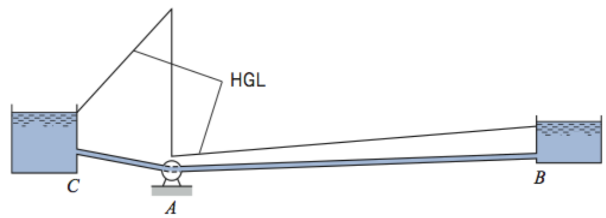
SOLUTION

Possible if the fluid is being accelerated to the left.

7.71: PROBLEM DEFINITION

Situation:

Two tanks are connected by a pipe with a machine.

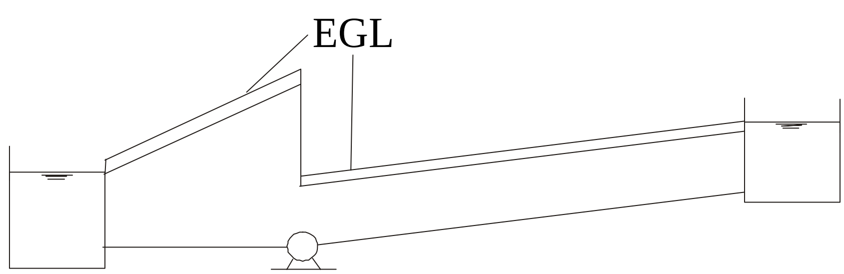


Find:

(a) Direction of flow.
(b) What kind of machine is at point A.
(c) Compare the diameter of pipe sections.
(d) Sketch the EGL.
(e) If there is a vacuum at anywhere, if so where it is.

SOLUTION

(a) Flow is from right to left.
(b) Machine is a pump.
(c) Pipe CA is smaller because of steeper HGL
(d)

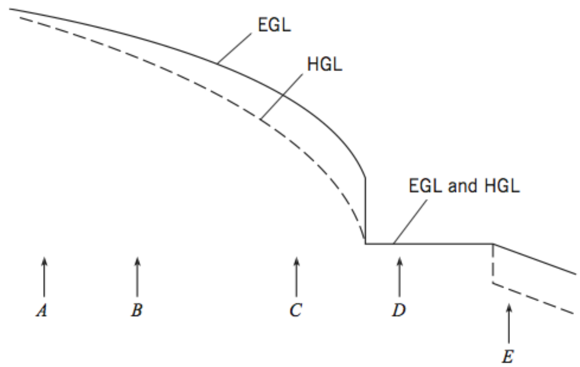


(e) No vacuum in the system.

7.72: PROBLEM DEFINITION

Situation:

An HGL and EGL are shown for a flow system.

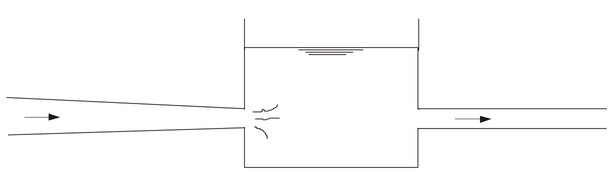


Find:

(a) Direction of flow.
(b) Whether there is a reservoir.
(c) Whether the diameter at E is uniform or variable.
(d) Whether there is a pump.
(e) Sketch a physical set up that could exist between C and D.
(f) Whether there is anything else revealed by the sketch.

SOLUTION

(a) Flow is from A to E because EGL slopes downward in that direction.
(b) Yes, at D, because EGL and HGL are coincident there.
(c) Uniform diameter because $V^2/2g$ is constant (EGL and HGL uniformly spaced).
(d) No, because EGL is always dropping (no energy added).
(e)

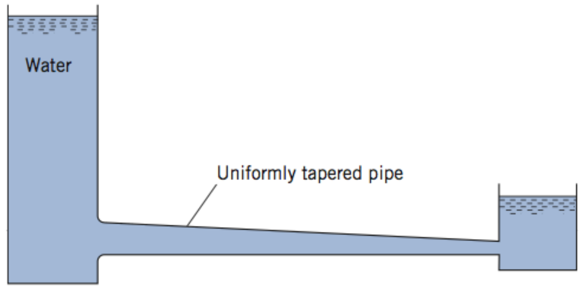


(f) Nothing else.

7.73: PROBLEM DEFINITION

Situation:

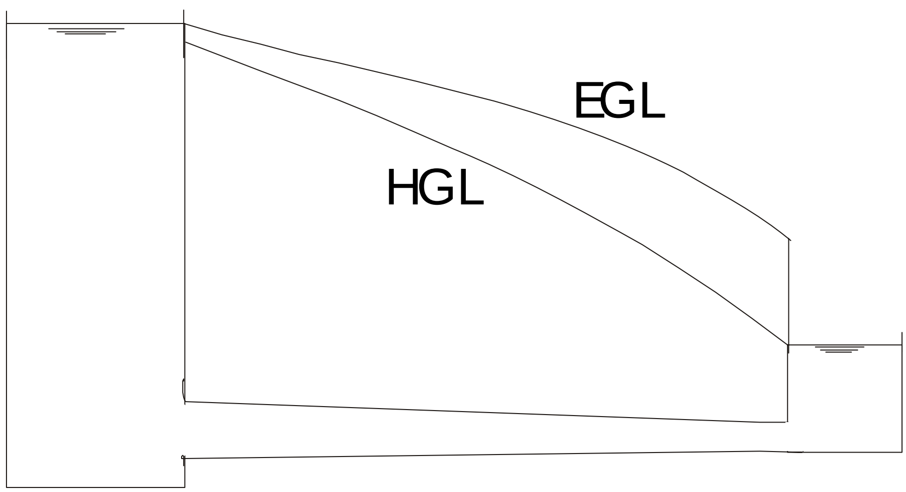
Two tanks are connected by a uniformly tapered pipe.



Find:

Draw the HGL and EGL.

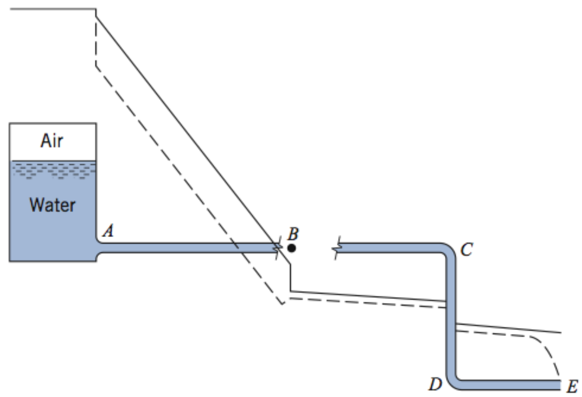
SOLUTION



7.74: PROBLEM DEFINITION

Situation:

A system with an HGL and EGL is described in the problem statement.



Find:

(a) Which line is HGL and which is EGL.
(b) If pipes are the same size and which is smallest.
(c) If and where pipe pressure falls below zero.
(d) Point of max pressure.
(e) Point of minimum pressure.
(f) What is at point E.
(g) Air pressure in the tank above or below atmospheric.
(h) What is at point B.

SOLUTION

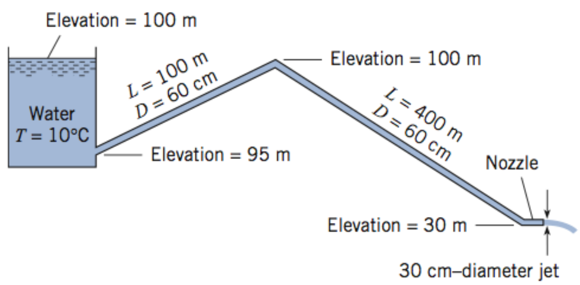
(a) Solid line is EGL, dashed line is HGL.
(b) No; AB is smallest.
(c) From B to C.
(d) p_{max} is at the bottom of the tank.
(e) p_{min} is at the bend C.
(f) A nozzle.
(g) above atmospheric pressure.
(h) abrupt expansion.

7.75: PROBLEM DEFINITION

Situation:

Water flows from a tank through a pipe system before discharging through a nozzle.
$z_1 = 100\,\text{m},\ z_2 = 30\,\text{m}.$
$L_1 = 100\,\text{m},\ L_2 = 400\,\text{m}.$
$D_1 = D_2 = 60\,\text{cm},\ D_{\text{jet}} = 30\,\text{cm}.$
Head loss in the pipe is given by

$$h_L = 0.014\frac{L}{D}\frac{V_p^2}{2g}$$



Find:

(a) Discharge.
(b) Draw HGL and EGL.
(c) location of maximum pressure.
(d) location of minimum pressure.
(e) values for maximum and minimum pressure.

Properties: Water (15 °C), Table A.5, $\gamma = 9800\,\text{N/m}^3$.

SOLUTION

Energy equation (locate 1 on the reservoir water surface; locate 2 at outlet of the nozzle).

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 100 &= 0 + \frac{V_2^2}{2g} + 30 + 0.014\left(\frac{L}{D}\right)\left(\frac{V_p^2}{2g}\right) \\
100 &= 0 + \frac{V_2^2}{2g} + 30 + 0.014\left(\frac{500}{0.6}\right)\frac{V_2^2}{2g}
\end{aligned}$$

Continuity equation

$$\begin{aligned} V_2 A_2 &= V_p A_p \\ V_2 &= V_p \frac{A_p}{A_L} \\ V_2 &= 4V_p \end{aligned}$$

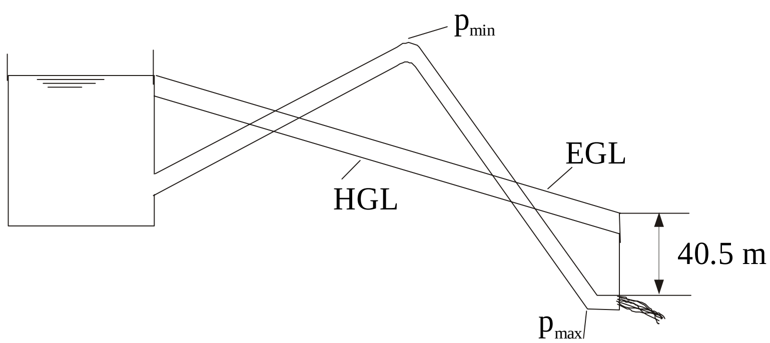
Then

$$\begin{aligned} \frac{V_p^2}{2g}(16 + 11.67) &= 70\,\mathrm{m} \\ V_p &= 7.045\,\mathrm{m/s} \\ \frac{V_p^2}{2g} &= 2.53\ \mathrm{m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_p A_p \\ &= 7.045\,\mathrm{m/s} \times (\pi/4) \times (0.60\,\mathrm{m})^2 \end{aligned}$$

$$\boxed{Q = 1.99\ \mathrm{m^3/s}}$$



Minimum pressure. Apply the Energy equation (point 1 on reservoir surface; point 2 in pipe at location of minimum pressure)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + z_1 + 0 &= \frac{p_{\min}}{\gamma} + \frac{V_p^2}{2g} + z_2 + 0 + 0.014 \frac{L_1}{D} \frac{V_p^2}{2g} \\ z_1 &= \frac{p_{\min}}{\gamma} + \frac{V_p^2}{2g} + z_2 + 0.014 \frac{L_1}{D} \frac{V_p^2}{2g} \\ 100\,\mathrm{m} &= \frac{p_{\min}}{9800\,\mathrm{N/m^3}} + \frac{(7.045\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} + 100\,\mathrm{m} + 0.014 \left(\frac{100\,\mathrm{m}}{0.6\,\mathrm{m}}\right) \frac{(7.045\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} \end{aligned}$$

$$\boxed{p_{\min} = \text{-82.6 kPa gage}}$$

Maximum pressure. Apply the Energy equation (point 1 on reservoir surface; point 2 in pipe at location of maximum pressure)

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
0 + 0 + z_1 + 0 &= \frac{p_{\max}}{\gamma} + \frac{V_p^2}{2g} + z_2 + 0 + 0.014 \frac{L}{D} \frac{V_p^2}{2g} \\
z_1 &= \frac{p_{\max}}{\gamma} + \frac{V_p^2}{2g} + z_2 + 0.014 \frac{L}{D} \frac{V_p^2}{2g} \\
100\,\mathrm{m} &= \frac{p_{\max}}{9800\,\mathrm{N/\,m^3}} + \frac{(7.045\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})} + 30\,\mathrm{m} + 0.014\left(\frac{500\,\mathrm{m}}{0.6\,\mathrm{m}}\right)\frac{(7.045\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})}
\end{aligned}
$$

$$
\boxed{p_{\max} = 373\,\mathrm{kPa\ gage}}
$$

7.76: PROBLEM DEFINITION

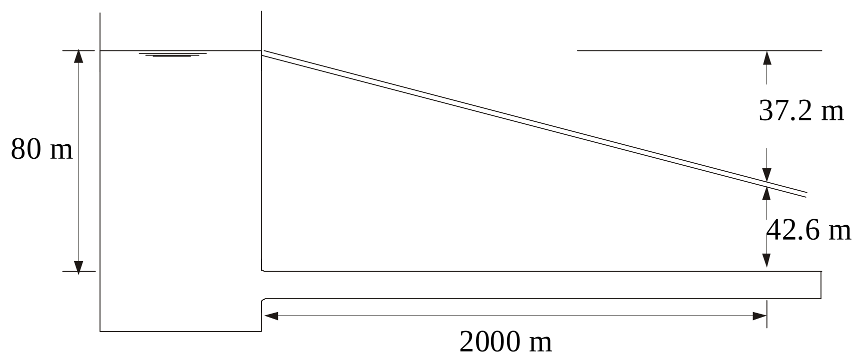
Situation:

A reservoir discharges into a pipe.

Find:

Draw the HGL and EGL.

SOLUTION



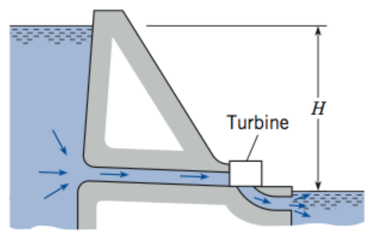
7.77: PROBLEM DEFINITION

Situation:

Water discharges through a turbine.

$Q = 1000$ cfs, $\eta = 85\%$.

$h_L = 4\,\text{ft}$, $H = 100\,\text{ft}$.



Find:

Power generated by turbine (hp).

Sketch the EGL and HGL.

PLAN

Apply the energy equation from the upper water surface to the lower water surface. Then apply the power equation.

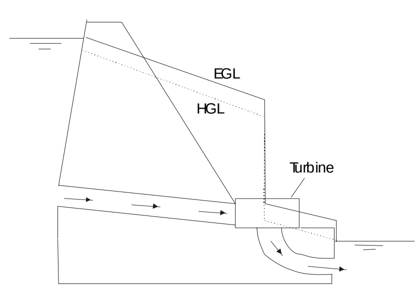
SOLUTION

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 &= \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\sum h_L+h_t\\ 0+0+100\text{ ft} &= 0+0+4\text{ ft }+h_t\\ h_t &= 96\text{ ft}\end{aligned}$$

Power equation

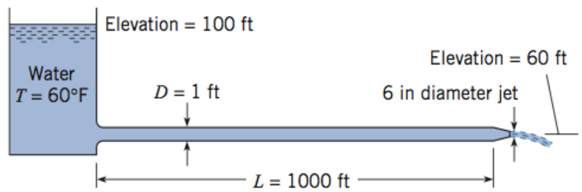
$$\begin{aligned}P &= (Q\gamma h_t)(\text{eff.})\\ P(\text{hp}) &= \frac{Q\gamma h_t\eta}{550}=\frac{1,000\,\text{ft}^3/\,\text{s}\times 62.4\,\text{lbf}/\,\text{ft}^3\times 96\,\text{ft}\times 0.85}{550}\\ &\boxed{P=9260\text{ hp}}\end{aligned}$$



7.78: PROBLEM DEFINITION

Situation:

Water discharges from a reservoir through a pipe and out a nozzle.
$h_L = 0.025\frac{L}{D}\frac{V^2}{2g}$, $D = 1\,\text{ft}$.
$L = 1000\,\text{ft}$, $D_{\text{jet}} = 6\,\text{in}$.
$z_1 = 100\,\text{ft}$, $z_2 = 60\,\text{ft}$.



Find:

(a) Discharge (cfs).
(b) Draw the HGL and EGL.

Assumptions:

$\alpha = 1.0$.

PLAN

Apply the energy equation from the reservoir surface to the exit plane of the jet.

SOLUTION

Energy equation. Let the velocity in the 6 inch pipe be V_6. Let the velocity in the 12 inch pipe be V_{12}.

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_6^2}{2g} + z_2 + h_L \\
0 + 0 + 100 &= 0 + \frac{V_6^2}{2g} + 60 + 0.025\left(\frac{1000}{1}\right)\frac{V_{12}^2}{2g}
\end{aligned}$$

Continuity principle

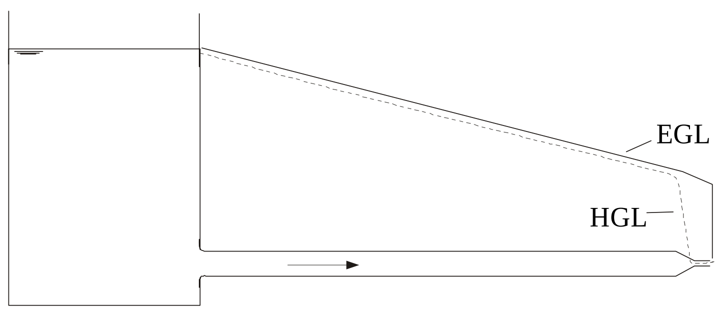
$$\begin{aligned}
V_6 A_6 &= V_{12} A_{12} \\
V_6 &= V_{12}\frac{A_{12}}{A_6} \\
V_6 &= V_{12}\frac{12^2}{6^2} = 4V_{12} \\
\frac{V_6^2}{2g} &= 16\frac{V_{12}^2}{2g}
\end{aligned}$$

Substituting into energy equation

$$
\begin{aligned}
40\,\text{ft} &= \left(\frac{V_{12}^2}{2g}\right)(16+25) \\
V_{12}^2 &= \left(\frac{40\,\text{ft}}{41}\right)\left(2\times 32.2\,\text{ft/s}^2\right) \\
V_{12} &= 7.927\ \text{ft/s}
\end{aligned}
$$

Flow rate equation

$$
\begin{aligned}
Q &= V_{12}A_{12} \\
&= (7.927\,\text{ft/s})(\pi/4)\,(1\,\text{ft})^2 \\
&\boxed{Q = 6.23\ \text{ft}^3/\text{s}}
\end{aligned}
$$



7.79: PROBLEM DEFINITION

Situation:

Water moves between two reservoirs through a contracting pipe.
$h_L = 0.02\frac{L}{D}\frac{V^2}{2g}$.
$D_d = 15\,\text{cm}$, $L_d = 100\,\text{m}$.
$L_u = 100\,\text{m}$, $D_u = 30\,\text{cm}$.
$z_1 = 100\,\text{m}$, $z_2 = 60\,\text{m}$.

Find:

Discharge of water in system (m^3/s).

PLAN

Apply energy equation from upper to lower reservoir.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 100\,\text{m} &= 0 + 0 + 70\,\text{m} + \sum h_L \\
\sum h_L &= 30\text{ m}
\end{aligned}$$

$$\begin{aligned}
h_L &= .02 \times (L/D)(\frac{V^2}{2g}) \\
30 &= 0.02 \times \left(\frac{200}{0.3}\right)\left(\frac{V_u^2}{2g}\right) + \left(0.02\left(\frac{100\,\text{m}}{0.15\,\text{m}}\right) + 1.0\right)\frac{V_d^2}{2g} \qquad (1)
\end{aligned}$$

Flow rate equation

$$V_u = \frac{Q}{A_u} = \frac{Q}{(\pi/4) \times (0.3\,\text{m})^2} \qquad (2)$$

$$V_d = \frac{Q}{A_d} = \frac{Q}{(\pi/4) \times (0.15\,\text{m})^2} \qquad (3)$$

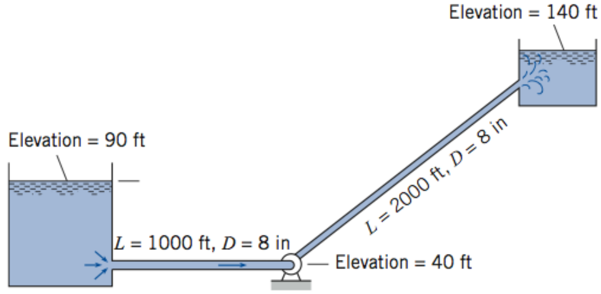
Substituting Eq. (2) and Eq. (3) into (1) and solving for Q yields:

$$\boxed{Q = 0.110\,\text{m}^3/\text{s}}$$

7.80: PROBLEM DEFINITION

Situation:

Water is pumped from a lower reservoir to an upper one.
$z_1 = 90\,\text{ft},\ z_2 = 140\,\text{ft}$.
$L_1 = 1000\,\text{ft},\ L_2 = 2000\,\text{ft}$.
$D_1 = 8\,\text{in},\ D_2 = 8\,\text{in}$.
$Q = 3$ cfs, $h_L = 0.018\frac{L}{D}\frac{V^2}{2g}$.



Find:

(a) Power supplied to the pump (hp).
(b) Sketch the HGL and EGL.

Properties:

Water (68 °F), Table A.5: $\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN

Apply the flow rate equation to find the velocity. Then calculate head loss. Next apply the energy equation from water surface to water surface to find the head the pump provides. Finally, apply the power equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{3.0\,\text{ft}^3/\text{s}}{(\pi/4)\times(8/12\,\text{ft})^2} \\ &= 8.594\ \text{ft/s} \end{aligned}$$

Head loss

$$\begin{aligned} h_L &= \left(0.018\frac{L}{D}\frac{V^2}{2g}\right) + \left(\frac{V^2}{2g}\right) \\ &= 0.018\left(\frac{3000\,\text{ft}}{8/12\,\text{ft}}\right)\frac{(8.594\,\text{ft/s})^2}{2\,(32.2\,\text{ft/s}^2)} + \frac{(8.594\,\text{ft/s})^2}{2\,(32.2\,\text{ft/s}^2)} \\ &= 94.04\,\text{ft} \end{aligned}$$

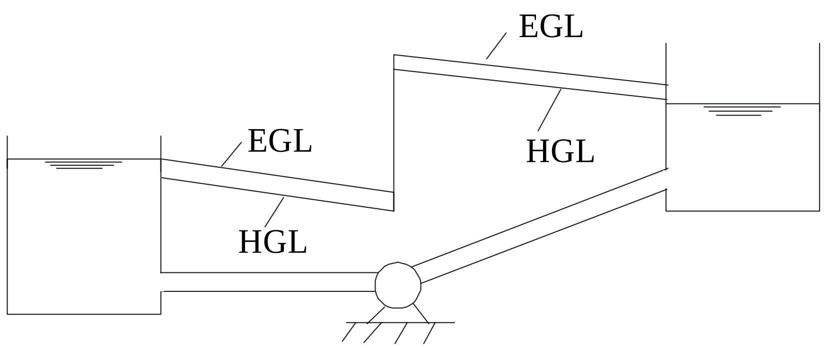
Energy equation

$$\begin{aligned}\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p &= \frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L \\ 0+0+90+h_p &= 0+0+140+94.04 \\ h_p &= 144.0 \text{ ft}\end{aligned}$$

Power equation

$$\begin{aligned}P &= Q\gamma h_p \\ &= 3.0\,\text{ft}^3/\,\text{s}\times 62.4\,\text{lbf}/\,\text{ft}^3\times 144\,\text{ft} \\ &= 26957\frac{\text{ft lbf}}{\text{s}}\left(\frac{\text{ft}\cdot\text{ lbf}}{550\,\text{hp}\cdot\text{ s}}\right)\end{aligned}$$

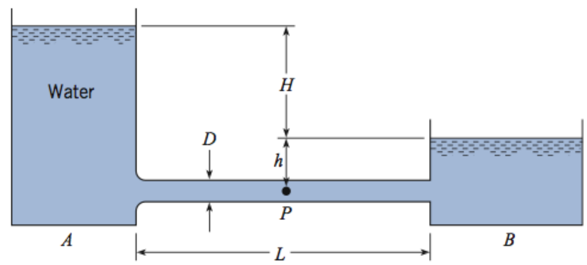
$$\boxed{P = 49.0\,\text{hp}}$$



7.81: PROBLEM DEFINITION

Situation:

Water flows between two reservoirs.
$D = 1\,\mathrm{m}$, $L = 300\,\mathrm{m}$.
$H = 16\,\mathrm{m}$, $h = 2\,\mathrm{m}$.
$h_L = 0.01\frac{L}{D}\frac{V^2}{2g}$.



Find:

(a) Discharge in pipe (m^3/s).
(b) Pressure halfway between two reservoirs (kPa).

PLAN

To find the discharge, apply the energy equation from water surface A to water surface in B. To find the pressure at location P, apply the energy equation from water surface A to location P.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + H &= 0 + 0 + 0 + 0.01 \times \left(\frac{300}{1}\right)\frac{V_p^2}{2g} + \frac{V_p^2}{2g} \\
16 &= 4V_p^2/2g \\
V_p &= \sqrt{4 \times 2 \times 9.81} = 8.86 \text{ m/s}
\end{aligned}$$

Flow rate equation

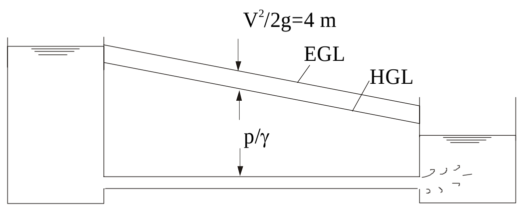
$$\begin{aligned}
Q &= VA \\
&= 8.86 \times (\pi/4) \times 1^2 \\
&\boxed{Q = 6.96 \text{ m}^3/\text{s}}
\end{aligned}$$

Energy equation between the water surface in A and point P:

$$
\begin{aligned}
0+0+H &= \frac{p_p}{\gamma}+\frac{V_p^2}{2g}-h+0.01\times\left(\frac{150\,\mathrm{m}}{1\,\mathrm{m}}\right)\frac{V_p^2}{2g} \\
16 &= \frac{p_p}{\gamma}-2+2.5\frac{V_p^2}{2g}
\end{aligned}
$$

where $V_p^2/2g = 4$ m. Then

$$
\begin{aligned}
p_p &= \left(9,810\,\mathrm{N/\,m^3}\right)(16+2-10)\,\mathrm{m} \\
& \boxed{p_p = 78.5\ \mathrm{kPa}}
\end{aligned}
$$

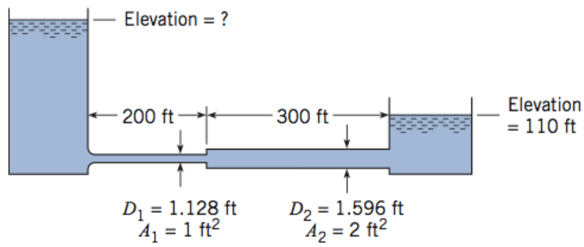


7.82: PROBLEM DEFINITION

Situation:

Two reservoirs are connected by a pipe with an abrupt expansion.

$h_L = 0.02\frac{L}{D}\frac{V^2}{2g}$, $Q = 16\,\text{ft}^3/\,\text{s}$.



Find:

Elevation in left reservoir.

Assumptions:

$\alpha = 1.0$.

PLAN

Apply the energy equation from the left reservoir to the right reservoir.

SOLUTION

Energy equation

$$\begin{aligned}\frac{p_L}{\gamma} + \frac{V_L^2}{2g} + z_L &= \frac{p_R}{\gamma} + \frac{V_R^2}{2g} + z_R + h_L \\ 0 + 0 + z_L &= 0 + 0 + 110 + 0.02\left(\frac{200}{1.128}\right)\frac{V_1^2}{2g} \\ &\quad +0.02\left(\frac{300}{1.596}\right)\frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{V_2^2}{2g}\end{aligned}$$

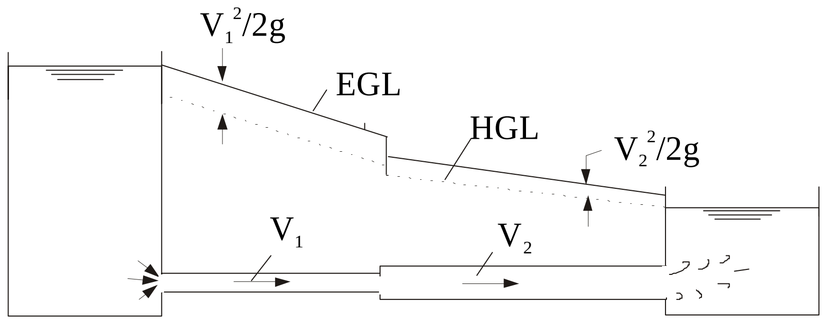
Flow rate equation

$$\begin{aligned}V_1 &= \frac{Q}{A_1} \\ &= \frac{16\,\text{ft}^3/\,\text{s}}{1\,\text{ft}^2} = 16 \text{ ft/s} \\ V_2 &= 8 \text{ ft/s}\end{aligned}$$

Substituting into the energy equation

$$\begin{aligned}z_L &= 110 + \left(\frac{0.02}{2 \times 32.2}\right)\left[\left(\frac{200}{1.238}\right)(16)^2 + \left(\frac{300}{1.596}\right)(8)^2\right] + \left(\frac{(16\,\text{ft}/\,\text{s} - 8\,\text{ft}/\,\text{s})^2}{64.4\,\text{ft}/\,\text{s}^2}\right) + \frac{(8\,\text{ft}/\,\text{s})^2}{64.4\,\text{ft}/\,\text{s}^2} \\ &= 110 + 16.58 + 0.99 + 0.99\end{aligned}$$

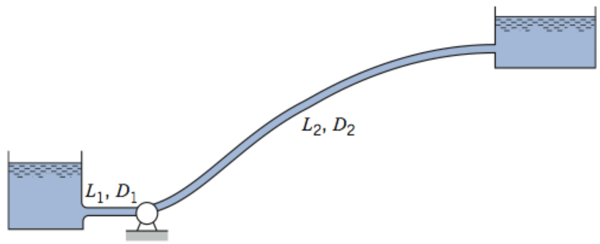
$$\boxed{z_L = 129 \text{ ft}}$$



7.83: PROBLEM DEFINITION

Situation:

Water is pumped from a lower to an upper reservoir.
$L_1 = 100\,\text{m}$, $L_2 = 1000\,\text{m}$.
$D_1 = 1\,\text{m}$, $D_2 = 50\,\text{cm}$.
$\eta = 74\%$, $Q = 3\,\text{m}^3/\,\text{s}$.
$z_1 = 150\,\text{m}$, $z_2 = 250\,\text{m}$.



Find:

(a) Pump power (MW).
(b) Sketch the HGL and EGL.

Assumptions:

$\alpha = 1.0$.

PLAN

Apply the energy equation from the upper reservoir surface to the lower reservoir surface.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 150 + h_p &= 0 + 0 + 250 + \sum 0.018 \left(\frac{L}{D}\right) \frac{V^2}{2g} + \frac{V^2}{2g}
\end{aligned}$$

Flow rate equation

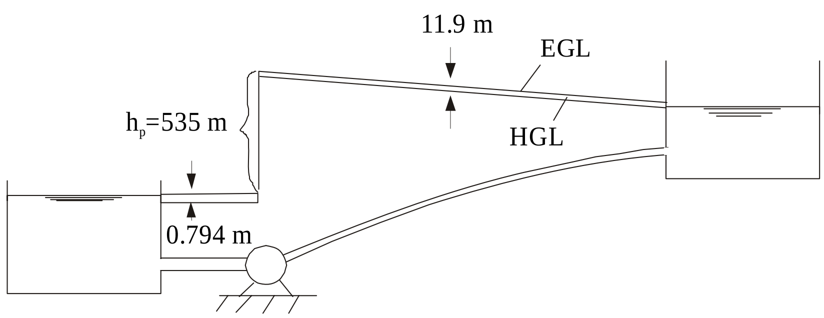
$$\begin{aligned}
V_1 &= \frac{Q}{A_1} = \frac{3\,\text{m}^3/\,\text{s}}{(\pi/4) \times (1\,\text{m})^2} = 3.82 \text{ m/s} \\
\frac{V_1^2}{2g} &= 0.744 \text{ m} \\
V_2 &= Q/A_2 = 4V_1 = 15.28 \text{ m/s} \\
\frac{V_2^2}{2g} &= 11.9 \text{ m}
\end{aligned}$$

Substituting into the energy equation

$$
\begin{aligned}
h_p &= 250\,\text{m} - 150\,\text{m} + 0.018\left[\left(\frac{100\,\text{m}}{1\,\text{m}}\right)\times 0.744\,\text{m} + \left(\frac{1000\,\text{m}}{5\,\text{cm}}\right)\times 11.9\,\text{m}\right] + 11.9\,\text{m} \\
&= 541.6\ \text{m}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
&= \frac{3\,\text{m}^3/\,\text{s}\times 9,810\,\text{N}/\,\text{m}^3\times 541.6\,\text{m}}{0.74} \\
&\boxed{P = 21.5\ \text{MW}}
\end{aligned}
$$



7.84: PROBLEM DEFINITION

Situation:

Water flows out of a reservoir into a pipe that discharges to atmosphere.
$h_L = 0.018 \frac{L}{D} \frac{V^2}{2g}$, $z_1 = 200\,\text{m}$.
$z_2 = 185\,\text{m}$, $z_{\text{pipe}} = 200\,\text{m}$.
$D = 30\,\text{cm}$, $L = 200\,\text{m}$.

Find:

(a) Water discharge in pipe (m^3/s).
(b) Pressure at highest point in pipe (kPa).

Assumptions:

$\alpha = 1.0$.

PLAN

First apply energy equation from reservoir water surface to end of pipe to find the V to calculate the flow rate. Then to solve for the pressure midway along pipe, apply the energy equation to the midpoint:

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 200 &= 0 + \frac{V^2}{2g} + 185 + 0.02(200/0.30)\frac{V^2}{2g} \\
14.33\frac{V^2}{2g} &= 15 \\
\frac{V^2}{2g} &= 1.047 \\
V &= 4.53 \text{ m/s}
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
Q &= VA \\
&= 4.53\,\text{m/s} \times (\pi/4) \times (0.30\,\text{m})^2
\end{aligned}$$

$$\boxed{Q = 0.320 \text{ m}^3/\text{s}}$$

Energy equation to the midpoint:

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_m}{\gamma} + \frac{V_m^2}{2g} + z_m + h_L \\
0 + 0 + 200 &= \frac{p_m}{\gamma} + \frac{V_m^2}{2g} + 200 + 0.02\left(\frac{100}{0.3}\right)\frac{V^2}{2g} \\
p_m/\gamma &= -\left(\frac{V^2}{2g}\right)(1 + 6.667) \\
&= (-1.047)(7.667) = -8.027 \text{ m} \\
p_m &= -8.027\gamma \\
&= -78,745 \text{ Pa}
\end{aligned}$$

$$\boxed{p_m = -78.7 \text{ kPa}}$$