10.1: PROBLEM DEFINITION

Situation:

Kerosene flows in a pipe.

$Q = 0.04\,\text{m}^3/\,\text{s},\ D = 25\,\text{cm}.$

Find:

Is the flow laminar or turbulent?

Entrance length (meters).

Properties:

Kerosene (20 °C),Table A.4, $\nu = 2.37 \times 10^{-6}\,\text{m}^2/\,\text{s}.$

SOLUTION

Reynolds number:.

$$\text{Re} = \frac{4Q}{\pi D \nu} = \frac{4\,(0.04\,\text{m}^3/\,\text{s})}{\pi\,(0.25\,\text{m})\,(2.37 \times 10^{-6}\,\text{m}^2/\,\text{s})} = 85957$$

$$\boxed{\text{Re} > 3000 \qquad \text{Flow is turbulent}}$$

Entrance length:

$$\begin{aligned} \frac{L_e}{D} &= 50 \\ L_e &= 50D = 50\,(0.25\,\text{m}) = \boxed{12.5\,\text{m}} \end{aligned}$$

10.2: PROBLEM DEFINITION

Situation:

A compressor is drawing ambient air through a duct.
$Q = 0.3\,\mathrm{m^3/s}$, $D = 0.15\,\mathrm{m}$.
$L = 10\,\mathrm{m}$, $T = 10\,^\circ\mathrm{C}$.

Find:

Determine if the flow is laminar or turbulent.
Entrance length (meters).

Assumptions:

Smooth inlet, so L_e correlations apply.
Pressure in duct is 1.0 atm.

Properties:

Air ($10\,^\circ$C), Table A.3, $\nu = 14.1 \times 10^{-6}\,\mathrm{m^2/s}$.

SOLUTION

Reynolds number:.

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} = \frac{4\,(0.3\,\mathrm{m^3/s})}{\pi\,(0.15\,\mathrm{m})\,(14.1 \times 10^{-6}\,\mathrm{m^2/s})} = 180601$$

Since Re > 3000, the flow is turbulent.

$$\boxed{\text{Flow is turbulent}}$$

Entrance length:

$$\begin{aligned} \frac{L_e}{D} &= 50 \\ L_e &= 50D = 50\,(0.15\,\mathrm{m}) = 7.5\,\mathrm{m} \end{aligned}$$

$$\boxed{L_e = 7.5\,\mathrm{m}}$$

10.3: PROBLEM DEFINITION

Situation:

A tube carries SAE 10W-30 oil.
$Q = 0.2$ L/s $= 0.0002\,\mathrm{m^3/s}$, $T = 38\,^\circ\mathrm{C}$.
Flow must be laminar and fully developed.

Find:

Specify a tube length (millimeters).
Specify a tube diameter (millimeters).

Assumptions:

Smooth inlet, so L_e correlations apply.

Properties:

SAE 10W-30 oil (38 °C), Table A.4, $\nu = 7.6 \times 10^{-5}\,\mathrm{m^2/s}$.

PLAN

1. Find diameter D by specifying a Reynolds number of 1500 (laminar).
2. Find length L by specifying a length so $L > 0.05D\,\mathrm{Re}$.

SOLUTION

Reynolds number:.

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} = \frac{4\,(0.0002\,\mathrm{m^3/s})}{\pi D\,(7.6 \times 10^{-5}\,\mathrm{m^2/s})} = 1500$$

$$\boxed{D = 2.23\,\mathrm{mm}}$$

Entrance length:

$$\begin{aligned} L &> 0.05D\,\mathrm{Re} \\ L &> 0.05\,(0.00223\,\mathrm{m})\,(1500) = 0.167\,\mathrm{m} \end{aligned}$$

Select

$$\boxed{L = 200\,\mathrm{mm}}$$

REVIEW Notice that the answer will change if a different value of Re is selected.

Problem 10.4

Answer the questions below.

---

a.) What is pipe head loss? How is pipe head loss related to total head loss?

- Pipe head loss ==> head loss associated with fully developed flow in straight sections of pipe.
- Total head loss = (Pipe head loss) + (Component head loss).

---

b.) What is the friction factor f? How is f related to wall shear stress?

- f is a π-group
- f is defined as the ratio of wall shear stress to kinetic pressure (with an extra constant of 0.25 included)

$$f \equiv \frac{\tau_{\text{wall}}}{0.25 \times \rho V^2/2}$$

- f can be thought of as a dimensionless wall shear stress

---

c.) What assumptions need to be satisfied to apply the Darcy Weisbach equation?

- the conduit needs to be completely full of the flowing fluid
- fully developed flow
- steady flow

Problem 10.5

Apply the grid method to each situation described below. Apply the DW equation. Note: Unit cancellations are not shown in this solution.

Situation: (a)

Water is flowing in a pipe.
$Q = 20$ gpm, $V = 180\,\text{ft}/\,\text{min}$.
$L = 200\,\text{ft}, \quad f = 0.02$.

Find:

Head loss (ft).

Solution:

Flow rate eqn:

$$
\begin{aligned}
Q &= \left(\frac{\pi D^2}{4}\right) V \\
D &= \sqrt{\frac{4Q}{\pi V}} = \sqrt{\frac{4}{\pi}\left(\frac{20\,\text{gal}}{\text{min}}\right)\left(\frac{\text{min}}{180\,\text{ft}}\right)\left(\frac{1.0\,\text{ft}^3}{7.481\,\text{gal}}\right)} = 0.1375\,\text{ft}
\end{aligned}
$$

Darcy Weisbach eqn:

$$
h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.02\left(\frac{200\,\text{ft}}{0.1375\,\text{ft}}\right)\left(\frac{180\,\text{ft}}{\text{min}}\right)^2\left(\frac{1.0}{2\,(32.2\,\text{ft}/\,\text{s}^2)}\right)\left(\frac{1.0\,\text{min}}{60\,\text{s}}\right)^2
$$

$$
\boxed{h_f = 4.07\,\text{ft}}
$$

Situation: (b)

Flow in a PVC pipe.
$h_f = 0.8\,\text{m}, \quad f = 0.012, \quad L = 15\,\text{m} \quad Q = 2\,\text{ft}^3/\,\text{s}$..

Find:

Pipe diameter (meters).

Solution:

$$
\begin{aligned}
h_f &= f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{(Q/A)^2}{2g} = f\frac{L}{D}\frac{16Q^2}{2g\pi^2 D^4} \\
D &= \sqrt[5]{f \quad \left(\frac{L}{h_f}\right)\frac{16Q^2}{2g\pi^2}} = \sqrt[5]{0.012 \quad \left(\frac{15\,\text{m}}{0.8\,\text{m}}\right)\frac{16}{2\,(9.81\,\text{m}/\,\text{s}^2)\,\pi^2}\left(\frac{1\,\text{ft}^3}{\text{s}}\right)^2\left(\frac{\text{m}^3}{35.31\,\text{ft}^3}\right)^2}
\end{aligned}
$$

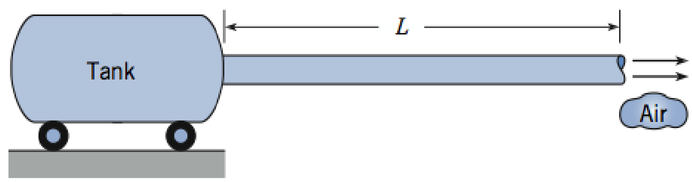
$$
\boxed{D = 0.108\,\text{m}}
$$

10.6: PROBLEM DEFINITION

Situation:

Air is flowing from a large tank to ambient through a horizontal pipe.
Pipe is 1" Schedule 40. $D = 1.049\,\text{in} = 0.0266\,\text{m}$.
$V = 10\,\text{m/s}$. $f = 0.015$, $L = 50\,\text{m}$.

Sketch:



Find:

Pressure in the tank (Pa).

Assumptions:

Air has constant density (look up properties at 1 atm).
KE correction factor is $a_2 = 1.0$.

Properties:

Air (20 °C, 1 atm), Table A.3, $\rho = 1.2\,\text{kg/m}^3$.

PLAN

1. Relate pressure in tank to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

SOLUTION

1. Energy eqn. (location 1 inside the tank, location 2 at the pipe exit)

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + 0 + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \qquad (1)\end{aligned}$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

3. Combine Eqs. (1) and (2).

$$\begin{aligned}p_1 &= \gamma \frac{V_2^2}{2g}\left(1 + f\frac{L}{D}\right) = \frac{\rho V_2^2}{2}\left(1 + f\frac{L}{D}\right) \\ &= \frac{(1.2\,\text{kg/m}^3)\,(10\,\text{m/s})^2}{2}\left(1 + (0.015)\,\frac{(50\,\text{m})}{(0.0266\,\text{m})}\right) \\ &= 1.75\,\text{kPa-gage}\end{aligned}$$

$$\boxed{p_{\text{tank}} = 1.75\,\text{kPa gage}}$$

REVIEW The constant density assumption is valid because the pressure in the tank is less than 2% of atmospheric pressure.

10.7: PROBLEM DEFINITION

Situation:

Water is flowing through a horizontal pipe (garden hose).
$D = 0.018\,\text{m}$, $L = 20\,\text{m}$.
$f = 0.012$, $V = 1.5\,\text{m/s}$.

Find:

Pressure drop (Pa) for 20 m of hose.

Properties:

Water (15 °C), Table A.5, $\rho = 999\,\text{kg/m}^3$.

PLAN

1. Relate pressure drop to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

SOLUTION

1. Energy eqn. (location 1 upstream; location 2 is 20 m downstream)

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + 0 + 0 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + 0 + h_L \\
\text{since } V_1 &= V_2, \text{ KE terms cancel} \\
\Delta p &= \gamma h_L \qquad (1)
\end{aligned}$$

2. Darcy-Weisbach eqn.:

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

3. Combine Eqs. (1) and (2):

$$\begin{aligned}
\Delta p &= \gamma \left(f \frac{L}{D} \frac{V^2}{2g} \right) = \frac{\rho V^2}{2} \left(f \frac{L}{D} \right) \\
&= \frac{(999\,\text{kg/m}^3)\,(1.5\,\text{m/s})^2}{2} \left(0.012 \times \frac{20\,\text{m}}{0.018\,\text{m}} \right)
\end{aligned}$$

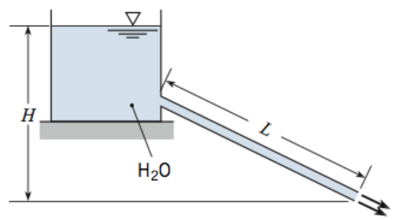
$$\boxed{\Delta p = 15.0\,\text{kPa}}$$

10.8: PROBLEM DEFINITION

Situation:

Water is flowing from a tank through a tube & then discharging to ambient.
$D = 0.008\,\text{m},\ L = 6\,\text{m}.$
$H = 3\,\text{m},\ \ f = 0.015.$

Sketch:



Find:

Exit velocity (m/s).
Discharge (L/s).
Sketch the HGL & EGL.

Assumptions:

The only head loss is in the tube.
Turbulent flow so $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5, $\rho = 999\,\text{kg/m}^3 \quad \nu = 1.14 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Relate H to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Find V by combining steps 1 & 2.
4. Find Q by using the flow rate equation.

SOLUTION

1. Energy eqn. (location 1 at the free surface, location 2 at the pipe exit)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + H + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_L \qquad (1) \end{aligned}$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

3. Combine Eqs. (1) and (2):

$$
\begin{aligned}
H &= \frac{V_2^2}{2g}\left(1 + f\frac{L}{D}\right) \\
V_2 &= \sqrt{\frac{2gH}{1 + f\frac{L}{D}}} \\
&= \sqrt{\frac{2\,(9.81\,\mathrm{m/s^2})\,(3\,\mathrm{m})}{1 + 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}}}
\end{aligned}
$$

$$\boxed{V = 2.19\,\mathrm{m/s}}$$

4. Flow rate equation:

$$Q = VA = V\frac{\pi D^2}{4} = (2.192\,\mathrm{m/s})\,\frac{\pi\,(0.008\,\mathrm{m})^2}{4}$$

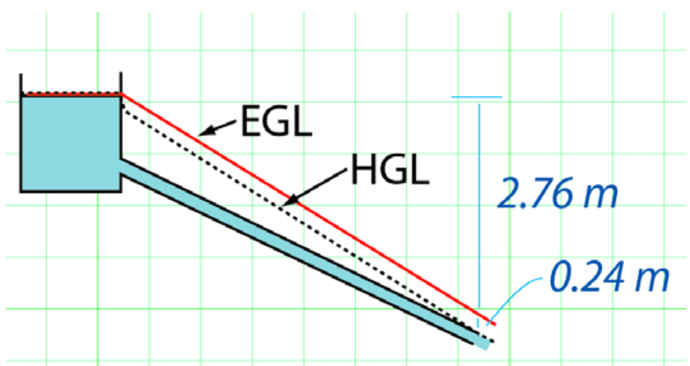
$$\boxed{Q = 0.110\ \mathrm{L/s}}$$

5. Sketch HGL & EGL

- Locate the EGL & HGL on free surface of tank.
- Velocity head and head loss:

$$
\begin{aligned}
\frac{V^2}{2g} &= \frac{(2.192\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} = 0.24\,\mathrm{m} \\
h_f &= f\frac{L}{D}\frac{V^2}{2g} = 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}\frac{(2.192\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} = 2.76\,\mathrm{m}
\end{aligned}
$$

- Locate the EGL and HGL at the end of the pipe. Sketch lines.



REVIEW Check the turbulent flow assumption.

$$
\begin{aligned}
\mathrm{Re} &= \frac{VD}{\nu} = \frac{(2.192\,\mathrm{m/s})\,(0.008\,\mathrm{m})}{(1.14 \times 10^{-6}\,\mathrm{m^2/s})} \\
\mathrm{Re} &= 15400 > 3000
\end{aligned}
$$

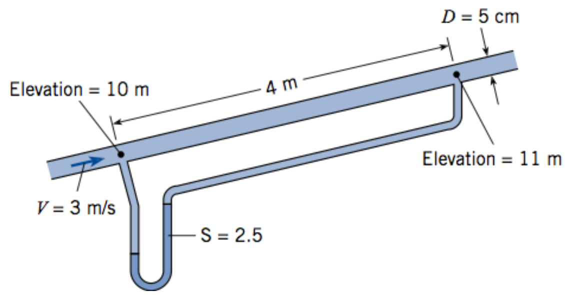
Thus, the assumption of turbulent flow is valid.

10.9: PROBLEM DEFINITION

Situation:

Water flows through a pipe.
$V = 3\,\text{m/s}$, $z_1 = 10\,\text{m}$.
$z_2 = 11\,\text{m}$, $D = 5\,\text{cm}$

Sketch:



Find: Resistance coefficient, f.

SOLUTION

Manometer equation

$$h_f = \Delta h_{\text{manometer}} \left(\gamma_m / \gamma_{\text{H2O}} - 1\right)$$
$$h_f = (0.90\,\text{m})\,(2.5 - 1) = 1.35 \text{ m of water}$$

Darcy Weisbach

$$h_f = f(L/D)V^2/2g$$

Solve for f

$$f = h_f \left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right)$$
$$= (1.35\,\text{m})\left(\frac{0.05\,\text{m}}{4\,\text{m}}\right)\left(\frac{2\,(9.81\,\text{m/s}^2)}{(3\,\text{m/s})^2}\right)$$

$$\boxed{f = 0.037}$$

Problem 10.10

Answer the questions below.

---

a. What are the main characteristics of laminar flow?

- Flow in layers
- Smooth flow; visualize flow of honey.
- Low rates of mixing.
- Low values of Reynolds number.

---

b. What is the meaning of each variable that appears in Eq. (10.27)?

- $\overline{V}$ is the area-averaged velocity in the pipe; also called the mean velocity
- r_o is the pipe radius.
- μ is the dynamic viscosity of the fluid.
- γ is the specific weight of the fluid.
- Δh is the change in piezometric head from location 1 to location 2.
- ΔL is the length between location 1 to location 2.

---

c. In Eq. (10.33), what is the meaning of h_f?

- h_f is the head loss associated with flow in a conduit.

10.11: PROBLEM DEFINITION

Situation:

Fluid flowing in a pipe.

$V = 0.04\,\text{m/s}$, $D = 0.1\,\text{m}$.

Find:

(a) Reynolds number.

(b) Maximum velocity in the pipe.

(c) Friction factor f.

(d) Shear stress at the wall.

(e) Shear stress 25 mm from pipe center.

Properties: $\mu = 10^{-2}\,\text{Pa}\cdot\text{s}$, $\rho = 800\,\text{kg/m}^3$

SOLUTION Reynolds number:

$$\begin{aligned}\text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.04 \times 0.1 \times 800}{0.01}\end{aligned}$$

$$\boxed{\text{Re} = 320}$$

Therefore, the flow is laminar:

$$\begin{aligned}V_{\max} &= 2V = \boxed{8\text{ cm/s}} \\ f &= 64/\text{Re} \\ &= 64/320 \\ &= \boxed{0.20}\end{aligned}$$

Wall shear stress (from definition of f) :

$$\begin{aligned}f &= \frac{4\tau_o}{\rho V^2/2} \\ \tau_o &= \left(\frac{f}{4}\right)\left(\frac{\rho V^2}{2}\right) = \frac{0.2}{4}\left(\frac{(800\,\text{kg/m}^3)(0.04\,\text{m/s})^2}{2}\right) = 0.032\,\text{Pa}\end{aligned}$$

$$\boxed{\tau_o = 0.032\,\text{Pa}}$$

Get $\tau_{r=0.025}$ by using proportions. Rationale: shear stress varies linearly from $\tau = 0$ at $r = 0$ to $\tau = \tau_o$ at $r = r_o$.

$$
\begin{aligned}
0.025/0.05 &= \tau/\tau_0 \\
\tau &= 0.5\tau_0 \\
\tau &= 0.5\,(0.032\,\text{Pa}) \\
&= \boxed{0.016\ \text{N/m}^2}
\end{aligned}
$$

10.12: PROBLEM DEFINITION

Situation:

Water is flowing in a pipe. Re $= 1000$, $T = 15\,^{\circ}\mathrm{C}$.

Nominal diameter = 1/2" Schedule 40. $D = 0.622\,\mathrm{in} = 0.0158\,\mathrm{m}$.

Find:

(a) Mass flow rate (kg/s).

(b) Friction factor f.

(c) Head loss per meter of pipe length.

(d) Pressure drop per meter of head length.

Properties: Water (15 °C), Table A.5, $\rho = 999\,\mathrm{kg/m^3}$, $\gamma = 9800\,\mathrm{N/m^3}$, $\nu = 1.14 \times 10^{-6}\,\mathrm{m^2/s}$.

PLAN

1. Find V by using known Re.
2 Find $\dot{m}$ by using $\dot{m} = \rho A V$.
3. Find f by using 64/Re.
4. Find h_L by using Darcy-Weisbach eqn.
5. Find Δp by using the energy eqn.

SOLUTION

1. Reynolds Number:

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ V &= \frac{\nu\,\mathrm{Re}}{D} = \frac{(1.14 \times 10^{-6}\,\mathrm{m^2/s})\,(1000)}{(0.0158\,\mathrm{m})} \end{aligned}$$

$$V = 0.0722\,\mathrm{m/s}$$

2. Mass flow rate:

$$\begin{aligned} \dot{m} &= \rho A V \\ &= \left(999\,\mathrm{kg/m^3}\right)\left(\frac{\pi\,(0.0158\,\mathrm{m})^2}{4}\right)(0.0722\,\mathrm{m/s}) \\ &= 0.0141\,\mathrm{kg/s} \end{aligned}$$

$$\boxed{\dot{m} = 0.0141\,\mathrm{kg/s}}$$

3. Friction factor:

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1000}$$

$$\boxed{f = 0.064}$$

4. Darcy-Weisbach Eqn.:

$$\frac{h_f}{L} = \frac{f}{D}\frac{V^2}{2g} = \frac{(0.064)}{(0.0158\,\mathrm{m})}\left(\frac{(0.0722\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})}\right)$$

$$\boxed{\frac{h_f}{L} = 0.00108\,\mathrm{m} \quad \text{per m of pipe length}}$$

5. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- KE terms cancel.
- Assume horizontal pipe.
- $h_p = h_t = 0$. $h_L = h_f$

$$\frac{p_1}{\gamma} + 0 + 0 + 0 = \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f$$

$$\Delta p = \gamma h_f = \left(9800\,\mathrm{N/m^3}\right)(0.00108)$$

$$\boxed{\frac{\Delta p}{L} = 10.6\,\mathrm{Pa} \quad \text{per m of pipe length}}$$

10.13: PROBLEM DEFINITION

Situation:

Liquid flows through a smooth pipe.

$V = 1\,\mathrm{m/s}$. $h_f = 2\,\mathrm{m}$ per m, $D = 0.03\,\mathrm{m}$.

Find:

Friction factor.

Reynolds number.

Prove that doubling the flow will double the head loss.

Assumptions:

Laminar flow.

Fully developed flow.

PLAN

1. Find f using the Darcy-Weisbach eqn.
2. Find Re using $f = 64/\operatorname{Re}$.
3. Determine the effect of doubling Q by logical reasoning with the head loss eqn.

SOLUTION

1. Darcy-Weisbach:

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ f &= \frac{h_f}{L}\frac{2gD}{V^2} \\ &= \left(\frac{2\,\mathrm{m}}{1\,\mathrm{m}}\right)\frac{2\,(9.81\,\mathrm{m/s^2})\,(0.03\,\mathrm{m})}{(1\,\mathrm{m/s})^2} = 1.177 \end{aligned}$$

2. Assume laminar flow:

$$\begin{aligned} f &= \frac{64}{\operatorname{Re}} \\ \operatorname{Re} &= \frac{64}{f} = \frac{64}{1.177} = 54.4 \end{aligned}$$

Since Re << 2000, we conclude that the flow is laminar.

3. Head loss in laminar flow

$$h_f = \frac{32\mu LV}{\gamma D^2}$$

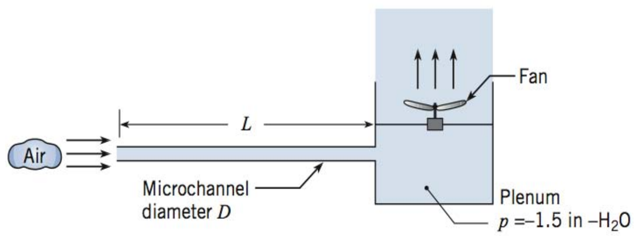
is linear with V. Thus, doubling Q will increase V by a factor of 2 and increase h_f by a factor of 2.

10.14: PROBLEM DEFINITION

Situation:

Air flows through a round tube.
$D = 0.0005\,\text{m}, \quad L = 0.75\,\text{m}.$
$p_2 = -1.5$ inch H20 = -373 Pa gage.

Sketch:



Find:

Velocity in the tube (m/s).

Assumptions:

Fully developed flow.
Laminar flow.
Only source of head loss is flow in tube.

Properties:

Air (20 °C), Table A.3, $\mu = 1.81 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2$, $\nu = 15.1 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Relate velocity to pressure using the energy equation.
2. Find head loss.
3. Find velocity by combining steps 1 and 2.
4. Check laminar flow assumption by calculating Re.

SOLUTION

1. Energy eqn (point 1 back from tube inlet; point 2 at tube outlet):

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + 0 + 0 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + 0 + 0 + h_f \qquad (1) \end{aligned}$$

2. Head loss (laminar flow):

$$h_f = \frac{32\mu L V}{\gamma D^2} \qquad (2)$$

3. Combine Eq. (1) and (2):

$$
\begin{aligned}
-p_2 &= \alpha_2 \frac{\rho V_2^2}{2} + \frac{32\mu L V_2}{D^2} \\
(+373\,\mathrm{Pa}) &= 2\frac{(1.2\,\mathrm{kg/\,m^3})\,V_2^2}{2} + \frac{32\,(1.81\times10^{-5}\,\mathrm{N\cdot\ s/\,m^2})\,(0.75\,\mathrm{m})\,V_2}{(0.0005\,\mathrm{m})^2}
\end{aligned}
$$

Solve using quadratic equation:

$$
\boxed{V_2 = 0.215\,\mathrm{m/\,s}}
$$

4. Reynolds number:

$$
\mathrm{Re} = \frac{VD}{\nu} = \frac{(0.215\,\mathrm{m/\,s})\,(0.0005\,\mathrm{m})}{(15.1\times10^{-6}\,\mathrm{m^2/\,s})} = 7.1
$$

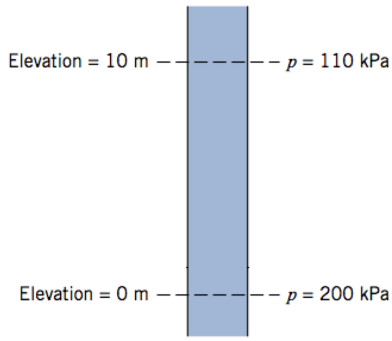
Since $\mathrm{Re} < 2000$, the laminar flow assumption is valid.

10.15: PROBLEM DEFINITION

Situation:

Liquid flows in a vertical pipe; flow direction is unknown.
$D = 0.008\,\text{m}$.

Sketch:



Find:

Direction of flow.
Velocity (m/s).

Properties:

$\gamma = 10\,\text{kN/m}^3, \quad \mu = 3.0 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2.$

PLAN

1. Find the flow direction by calculating the piezometric head at locations 1 and 2. Rationale: using the HGL shows that flow goes from high to low piezometric head in a constant diameter pipe.
2. Find head loss using the energy eqn.
3. Find velocity by using the equation for head loss in laminar flow.

SOLUTION

1. Piezometric Head (location 1 at $z = 10\,\text{m}$):

$$
\begin{aligned}
h_1 &= \frac{p_1}{\gamma} + z_1 = \frac{110\,\text{kPa}}{10\,\text{kN/m}^3} + 10\,\text{m} = 21\,\text{m} \\
h_2 &= \frac{p_2}{\gamma} + z_2 = \frac{200\,\text{kPa}}{10\,\text{kN/m}^3} + 0\,\text{m} = 20\,\text{m}
\end{aligned}
$$

Since $h_1 > h_2$, the direction of flow is downward.

2. Energy equation:

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
\frac{p_1}{\gamma} + 0 + z_1 + 0 &= \frac{p_2}{\gamma} + 0 + z_2 + h_f \\
21\,\mathrm{m} &= 20\,\mathrm{m} + h_f \\
h_f &= 1\,\mathrm{m}
\end{aligned}
$$

3. Head loss (laminar flow):

$$
\begin{aligned}
h_f &= \frac{32\mu L V}{\gamma D^2} \\
V &= \frac{h_f \gamma D^2}{32\mu L} \\
&= \frac{(1\,\mathrm{m})\,(10000\,\mathrm{N/\,m^3})\,(0.008\,\mathrm{m})^2}{32\,(3.0\times 10^{-3}\,\mathrm{N\cdot\ s/\,m^2})\,(10\,\mathrm{m})} \\
&= 0.667\,\mathrm{m/\,s}
\end{aligned}
$$

$$
\boxed{V = 0.667\,\mathrm{m/\,s}}
$$

10.16: PROBLEM DEFINITION

Situation:

Oil is pumped through a horizontal pipe. $Q = 0.004\,\text{ft}^3/\,\text{s}$.

Nominal diameter = 1 in. Schedule 80. $D = 0.957\,\text{in} = 0.0798\,\text{ft}$.

Find:

Head loss (ft) per 100 feet of pipe.

Properties:

Oil, $S = 0.97$, $\mu = 10^{-2}\,\text{lbf}\cdot\,\text{s}/\,\text{ft}^2$.

PLAN

1. Find V using the flow rate equation.
2. Find flow regime by calculating Re.
3. Find h_f.

SOLUTION

1. Flow rate equation:

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4\left(0.004\,\text{ft}^3/\,\text{s}\right)}{\pi\left(0.0798\,\text{ft}\right)^2} = 0.8\,\text{ft}/\,\text{s}$$

2. Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} = \frac{\left(0.8\,\text{ft}/\,\text{s}\right)\left(0.0798\,\text{ft}\right)\left(0.97\times 1.94\,\text{slug}/\,\text{ft}^3\right)}{\left(10^{-2}\,\text{lbf}\cdot\,\text{s}/\,\text{ft}^2\right)} \\ &= 12 \text{ (thus, flow is laminar)} \end{aligned}$$

3. Head loss (laminar flow):

$$\begin{aligned} h_f &= \frac{32\mu LV}{\gamma D^2} \\ &= \frac{32\left(10^{-2}\,\text{lbf}\cdot\,\text{s}/\,\text{ft}^2\right)\left(100\,\text{ft}\right)\left(0.8\,\text{ft}/\,\text{s}\right)}{\left(0.97\times 62.4\,\text{lbf}/\,\text{ft}^3\right)\left(0.0798\,\text{ft}\right)^2} \end{aligned}$$

$$\boxed{h_f = 66.4\,\text{ft} \text{ per 100 ft run of pipe}}$$

10.17: PROBLEM DEFINITION

Situation:

A liquid flows in a pipe.
$D = 0.1\,\text{m},\ V = 1.5\,\text{m/s}$.

Find:

Show that the flow is laminar.
Friction factor f.
Head loss per meter of pipe length. .

Properties:

$\rho = 1000\,\text{kg/m}^3,\quad \mu = 10^{-1}\ \text{N}\cdot\text{s/m}^2,\quad \nu = 10^{-4}\,\text{m}^2/\text{s}$.

SOLUTION

1. Reynolds number:

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{(1.5\,\text{m/s})\,(0.1\,\text{m})}{(10^{-4}\,\text{m}^2/\text{s})} \\ &= 1500\end{aligned}$$

Since Re < 2000, the **flow is laminar**.

2. Friction factor:

$$f = \frac{64}{\text{Re}} = \frac{64}{1500}$$

$$\boxed{f = 0.043}$$

3. Darcy Weisbach eqn.:

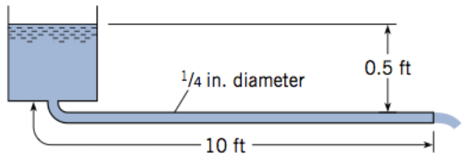
$$\frac{h_f}{L} = \frac{f}{D}\frac{V^2}{2g} = \frac{0.043}{(0.1\,\text{m})}\frac{(1.5\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)}$$

$$\boxed{\frac{h_f}{L} = 0.049\,\text{m}\quad \text{per m of pipe length}}$$

10.18: PROBLEM DEFINITION

Situation:

Kerosene flows out a tank and through a tube.
$D = 0.25\,\text{in}$, $L = 10\,\text{ft}$.
$z_1 = 0.5\,\text{ft}$.



Find:

Mean velocity in the tube.
Discharge.

Assumptions:

Laminar flow so $\alpha = 2$.
Only head loss is in the tube.

Properties:

Kerosene (68 °F): $S = 0.8$.

PLAN

Apply the energy equation from the surface of the reservoir to the pipe outlet.

SOLUTION

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + 2V^2/2g + z_2 + 32\mu LV/(\gamma D^2) \\ 0 + 0 + 0.50 &= 0 + V^2/g + 32\mu LV/(\gamma D^2) \end{aligned}$$

Thus

$$\begin{aligned} V^2/g + 32\mu LV/(\gamma D^2) - 0.50 &= 0 \\ V^2/32.2 + 32(4\times10^{-5})(10)V/(0.80\times62.4\times(1/48)^2) - 0.50 &= 0 \\ V^2 + 19.0V - 16.1 &= 0 \end{aligned}$$

Solving the above quadratic equation for V yields:

$$\boxed{V = 0.81 \text{ ft/s}}$$

Check Reynolds number to see if flow is laminar

$$\begin{aligned} \text{Re} &= VD\rho/\mu \\ &= 0.81\times(1/48)(1.94\times0.8)/(4\times10^{-5}) \\ \text{Re} &= 654.8 \text{ (laminar)} \\ Q &= VA \\ &= 0.81\times(\pi/4)(1/48)^2 = 2.76\times10^{-4} \text{ cfs} \end{aligned}$$

$$\boxed{Q = 2.76 \times 10^{-4} \text{ cfs}}$$

10.19: PROBLEM DEFINITION

Situation:

Oil is pumped through a horizontal pipe.

$D = 0.05\,\text{m}, \quad V = 0.5\,\text{m/s}.$

Find:

Head loss per 10 m of pipe.

Properties:

$S = 0.94, \ \mu = 0.048\,\text{N}\cdot\text{s/m}^2.$

PLAN

1. Determine flow regime (laminar or turbulent) by finding the Reynolds number.
2. Relate pressure drop to head loss using the energy eqn.
3. Apply eqn. for head loss in laminar flow.
4. Combine steps 2 and 3.

SOLUTION

1. Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{\rho V D}{\mu} = \frac{(0.94 \times 1000\,\text{kg/m}^3)\,(0.5\,\text{m/s})\,(0.05\,\text{m})}{(0.048\,\text{N}\cdot\text{s/m}^2)} \\ &= 490 \text{ (laminar flow)} \end{aligned}$$

2. Energy equation:

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \qquad (1) \end{aligned}$$

3. Head loss (laminar flow):

$$h_f = \frac{32 \mu L V}{\gamma D^2} \qquad (2)$$

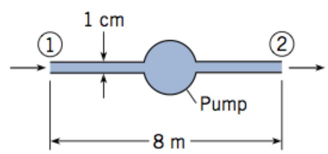
4. Combine Eqs. (1) and (2):

$$\Delta p = \gamma h_f = \frac{32 \mu L V}{D^2} = \frac{32\,(0.048\,\text{N}\cdot\text{s/m}^2)\,(10\,\text{m})\,(0.5\,\text{m/s})}{(0.05\,\text{m})^2}$$

$$\boxed{\Delta p = 3.07\,\text{kPa} \ \text{ per 10 m of pipe length}}$$

10.20: PROBLEM DEFINITION

Situation:
SAE 10-W oil is pumped through a tube.
$L = 8\,\text{m},\ D = .01\,\text{m}.$
$Q = 7.85 \times 10^{-4}\,\text{m}^3/\,\text{s},\ \eta = 1.0$
Pressures at points 1 and 2 are equal.



Find: Power to operate the pump.

Properties: SAE 10W-30 Oil, $\nu = 7.6 \times 10^{-5}\,\text{m}^2/\,\text{s},\ \ \gamma = 8630\,\text{N}/\,\text{m}^3.$

SOLUTION

Energy equation

$$p_1/\gamma + z_1 + \alpha_1 V_1^2/2g + h_p = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Simplify

$$h_p = h_L = f(L/D)(V^2/2g)$$

Flow rate equation

$$V = Q/A = 7.85 \times 10^{-4}/((\pi/4)(0.01)^2) = 10 \text{ m/s}$$

Reynolds number

$$\text{Re} = VD/\nu = (10)(0.01)/(7.6 \times 10^{-5}) = 1316 \text{ (laminar)}$$

Friction factor (f)

$$\begin{aligned} f &= \frac{64}{\text{Re}} \\ &= \frac{64}{1316} \\ &= 0.0486 \end{aligned}$$

Head of the pump

$$\begin{aligned} h_p &= f(L/D)(V^2/2g) \\ &= 0.0486(8/0.01)(10^2/((2)(9.81)) \\ &= 198 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= h_p \gamma Q \\ &= 198 \times 8630 \times \left(7.85 \cdot 10^{-4}\right) = 1341\,\mathrm{W} \end{aligned}$$

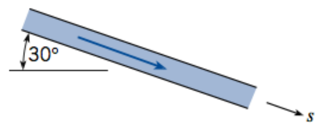
$$\boxed{P = 1341\,\mathrm{W}}$$

10.21: PROBLEM DEFINITION

Situation:

Oil flows downward in a pipe.
$D = 0.1\,\text{ft}$, $V = 2\,\text{ft/s}$, Slope of $30\,^\circ$.

Sketch:



Find:

Pressure gradient along the pipe (psf/ft).

Properties:

Oil, $S = 0.9$, $\mu = 10^{-2}\,\text{lbf s/ft}^2$, $v = 0.0057\,\text{ft}^2/\text{s}$.

SOLUTION

$$\begin{aligned}
\text{Re} &= \frac{VD}{v} \\
&= \frac{(2\,\text{ft/s})(0.10\,\text{ft})}{0.0057\,\text{ft}^2/\text{s}} \\
&= 35.1 \text{ (laminar)} \\
-\frac{d}{ds}(p+\gamma z) &= \frac{32\mu V}{D^2} \\
-\frac{dp}{ds} - \gamma\frac{dz}{ds} &= \frac{(32)(10^{-2}\,\text{lbf s/ft}^2)(2\,\text{ft/s})}{(0.1\,\text{ft})^2} \\
-\frac{dp}{ds} - \gamma(-0.5) &= 64\,\text{lbf/ft}^3 \\
\frac{dp}{ds} &= (0.5)(0.9)(62.4\,\text{lbf/ft}^3) - 64 \\
\frac{dp}{ds} &= 28.08 - 64
\end{aligned}$$

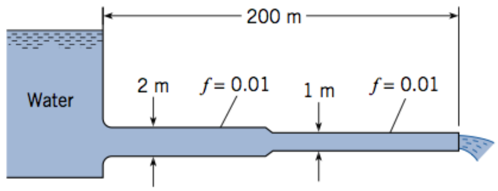
$$\boxed{\frac{dp}{ds} = -35.9 \text{ psf/ft}}$$

10.22: PROBLEM DEFINITION

Situation:

Fluid flows out of a tank through a pipe with an abrupt contraction.
$L_1 = L_2 = 100\,\text{m},\ f = 0.01.$
$D_1 = 2\,\text{m},\ D_2 = 1\,\text{m}.$

Sketch:



Find:

Ratio of head loss.

$$\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})}$$

SOLUTION

$$\begin{aligned}
h_L &= f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \\
\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})} &= \left(\frac{f_1 L_1 V_1^2/(D_1)}{f_2 L_2 V_2^2/(D_2)}\right) \\
&= \left(\frac{D_2}{D_1}\right)\left(\frac{V_1^2}{V_2^2}\right) \\
V_1 A_1 &= V_2 A_2 \\
\frac{V_1}{V_2} &= \frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 \\
\left(\frac{V_1}{V_2}\right)^2 &= \left(\frac{D_2}{D_1}\right)^4
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})} &= \left(\frac{D_2}{D_1}\right)\left(\frac{D_2}{D_1}\right)^4 \\
&= \left(\frac{D_2}{D_1}\right)^5 = 2^5 = 32
\end{aligned}$$

Correct choice is (d)

10.23: PROBLEM DEFINITION

Situation:
Glycerin flows in a pipe
$D = 0.5\,\text{ft}$, $\bar{V} = 2\,\text{ft/s}$.

Find:
Determine if the flow is laminar or turbulent.
Plot the velocity distribution.

Properties:
Glycerin at 68 °F from Table A.4:
$\mu = 0.03\,\text{lbf}\cdot\text{s/ft}^2$, $\nu = 1.22\times10^{-2}\,\text{ft}^2/\text{s}$.

SOLUTION

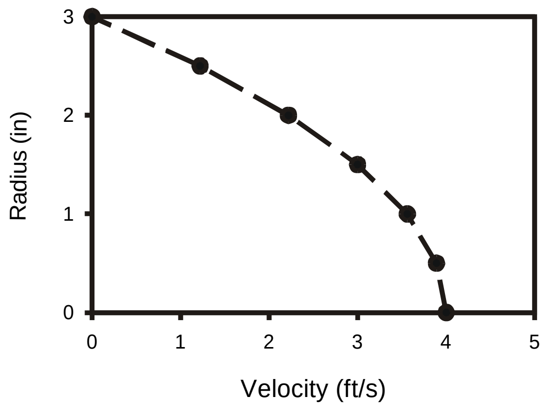
$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{2\times0.5}{1.22\times10^{-2}} \\ &= 81.97\ \boxed{\text{(laminar)}}\end{aligned}$$

To plot the velocity distribution, begin with Eq. (10.23) from EFM9e.

$$\begin{aligned}V(r) &= V_{\max}\left(1-\frac{r^2}{r_o^2}\right) \\ &= (4\,\text{ft/s})\left(1-\left(\frac{r}{r_o}\right)^2\right)\end{aligned}$$

Create a table of values and then plot

r (in)	r/r_0	$V(r)$ (ft/s)
0	0	4
0.5	1/6	3.89
1.0	1/3	3.56
1.5	1/2	3.00
2	2/3	2.22
2.5	5/6	1.22
3	1	0


Radius (in)
3
2
1
0
0
1
2
3
4
5
Velocity (ft/s)

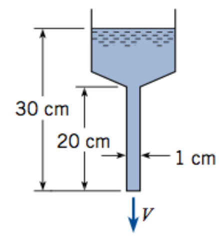
10.24: PROBLEM DEFINITION

Situation:

Glycerin flows through a funnel

$D = 1\,\mathrm{cm}$, $L = 20\,\mathrm{cm}$.

Sketch:



Find: Mean velocity (in m/s)at the exit.

Assumptions:

Laminar flow $(\alpha_2 = 2.0)$.

The only head loss is due to friction in tube.

Properties:

Glycerin $(20\,^\circ\mathrm{C})$,Table A.4:

$\rho = 1260\,\mathrm{kg/m^3}$, $\gamma = 12,300\,\mathrm{N/m^3}$.

$\mu = 1.41\,\mathrm{N\cdot s/m^2}$, $\nu = 1.12\times 10^{-3}\,\mathrm{m^2/s}$.

SOLUTION

Energy equation (Let section 1 be the surface of the liquid and section 2 be the exit plane of the funnel).

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

.

$$\begin{aligned} 0 + 0 + 0.30 &= 0 + 2.0\left(\frac{V_2^2}{2g}\right) + 0 + \frac{32\mu L V_2}{\gamma D^2} \\ 0.30 &= 2.0\left(\frac{V_2^2}{2\times 9.81}\right) + \left(\frac{32\times 1.41\,\mathrm{N\,s/m^3}\times 0.2\,\mathrm{m}\times V_2}{12300\,\mathrm{N/m^3}\times (0.01\,\mathrm{m})^2}\right) \end{aligned}$$

Solve quadratic equation.

$$\begin{aligned} V_2 &= -72.01\,\mathrm{m/s} \\ V_2 &= 4.087\times 10^{-2}\,\mathrm{m/s} \end{aligned}$$

Select the positive root

$$\boxed{V_2 = 0.0409\ \mathrm{m/s}}$$

Check the laminar flow assumption

$$\begin{aligned}\mathrm{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.0409\,\mathrm{m/\,s} \times 0.01\,\mathrm{m} \times 1260\,\mathrm{kg/\,m^3}}{1.41\,\mathrm{N\,s/\,m^2}} \\ &= 0.365\end{aligned}$$

Since $\mathrm{Re} \leq 2000$, the laminar flow assumption is valid.

10.25: PROBLEM DEFINITION

Situation:

Castor oil flows through a steel pipe.

$Q = 0.2\,\text{ft}^3/\,\text{s},\ L = 0.5\,\text{mi} = 2640\,\text{ft}.$

Allowable pressure drop is 10 psi.

Find:

Nominal diameter of pipe (ft).

Assumptions:

Laminar flow.

Horizontal pipe.

Properties:

Castor oil (90 °F): $\mu = 0.085\,\text{lbf}\cdot\text{s}/\,\text{ft}^2,\ \ S = 0.85.$

SOLUTION

1. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- KE terms cancel.
- $h_p = h_t = 0.\ \ h_L = h_f$

$$\begin{aligned}\frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \qquad (1)\end{aligned}$$

2. Head loss (laminar flow)

$$h_f = \frac{32\mu LV}{\gamma D^2} \qquad (2)$$

3. Combine Eq. (1) and (2)

$$\Delta p = \frac{32\mu LV}{D^2}$$

4.Let $V = Q/A$

$$\Delta p = \frac{32\mu LQ}{(\pi/4)\times D^4}$$

5. Solve for diameter.

$$\begin{aligned}D^4 &= \frac{128\mu LQ}{\pi\Delta p} \\ &= \frac{128\left(0.085\,\text{lbf}\cdot\text{s}/\,\text{ft}^2\right)(2640\,\text{ft})\left(0.2\,\text{ft}^3/\,\text{s}\right)}{\pi\left(1440\,\text{lbf}/\,\text{ft}^2\right)} \\ D &> 1.06\,\text{ft}\end{aligned}$$

6. Select a nominal pipe size.

$$D > 12.7\,\text{in}$$

Thus, $\boxed{\text{select a 14 inch nominal diameter NPS schedule 40 pipe } (ID = 13.1\,\text{in})}$

7. Check Laminar flow assumption:
Velocity:

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{\left(0.2\,\text{ft}^3/\,\text{s}\right)}{\pi/4\,(1.094\,\text{ft})^2} \\ &= 0.213\,\text{ft}/\,\text{s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.213\,\text{ft}/\,\text{s})\,(1.094\,\text{ft})\left(0.85 \times 1.94\,\text{slug}/\,\text{ft}^3\right)}{\left(0.085\,\text{lbf}\cdot\,\text{s}/\,\text{ft}^2\right)} \\ &= 4.52 \end{aligned}$$

Thus, the initial assumption of laminar flow is valid.

10.26: PROBLEM DEFINITION

Situation:

Velocity measurements are made in a pipe.
$D = 0.3\,\mathrm{m}$, $V = 1.5\,\mathrm{m/s}$.
$\Delta p = 1.9\,\mathrm{kPa}$ per $100\,\mathrm{m}$ of pipe.

Find:

Kinematic viscosity of fluid ($\mathrm{m^2/s}$).

Assumptions:

Laminar flow (since velocity profile is parabolic).
Horizontal pipe.

Properties:

$S = 0.8$

SOLUTION

1. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- KE terms cancel.
- Assume horizontal pipe.
- $h_p = h_t = 0.\;\; h_L = h_f$

$$\begin{aligned} \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \end{aligned} \quad (1)$$

2. Head loss (laminar flow)

$$h_f = \frac{32\mu LV}{\gamma D^2} \quad (2)$$

3. Combine Eq. (1) and (2)

$$\begin{aligned} \Delta p &= \frac{32\mu LV}{D^2} \\ \nu &= \frac{\mu}{\rho} = \frac{\Delta p D^2}{32\rho LV} \\ &= \frac{(1900\,\mathrm{Pa})\,(0.3\,\mathrm{m})^2}{32\,(0.8\times 1000\,\mathrm{kg/m^3})\,(100\,\mathrm{m})\,(0.75\,\mathrm{m/s})} = \end{aligned}$$

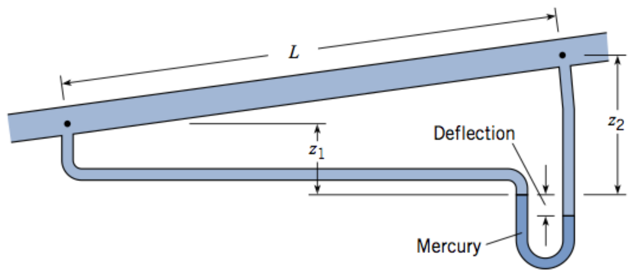
$$\boxed{\nu = 8.91\times 10^{-5}\,\mathrm{m^2/s}}$$

10.27: PROBLEM DEFINITION

Situation:

Oil flows through a smooth pipe.
$L = 12\,\text{m}$, $z_1 = 1\,\text{m}$, $z_2 = 2\,\text{m}$.
$V = 1.2\,\text{m/s}$, $D = 5\,\text{cm}$.

Sketch:



Find:

Flow direction.
Resistance coefficient.
Nature of flow (laminar or turbulent).
Viscosity of oil (N s/m^2).

Properties:

$S = 0.8$.

SOLUTION

Based on the deflection on the manometer, the piezometric head (and HGL) on the right side of the pipe is larger than that on the left side. Thus, the

flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_L$$

Assume $\alpha_1 V_1 = \alpha_2 V_2$. Let $z_2 - z_1 = 1\,\text{m}$. Also the head loss is given by the Darcy Weisbach equation: $h_f = f(L/D)V^2/(2g)$. The energy principle becomes

$$\frac{p_2 - p_1}{\gamma_{\text{oil}}} = (-1\,\text{m}) + f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2 + (2\,\text{m})\,\gamma_{\text{oil}} + (0.1\,\text{m})\,\gamma_{\text{oil}} - (0.1\,\text{m})\,\gamma_{\text{Hg}} - (1\,\text{m})\,\gamma_{\text{oil}} = p_1$$

Algebra gives

$$
\begin{aligned}
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= -(2\,\text{m}) - (0.1\,\text{m}) + (0.1\,\text{m})\frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}} + (1\,\text{m}) \\
&= -(1\,\text{m}) + (0.1\,\text{m})\left(\frac{S_{\text{Hg}}}{S_{\text{oil}}} - 1\right) \\
&= -(1\,\text{m}) + (0.1\,\text{m})\left(\frac{13.6}{0.8} - 1\right) \\
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= 0.6\,\text{m} \qquad (2)
\end{aligned}
$$

Substituting Eq. (2) into (1) gives

$$
\begin{aligned}
(0.6\,\text{m}) &= (-1\,\text{m}) + f\frac{L}{D}\frac{V^2}{2g} \\
&\text{or} \\
f &= 1.6\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 1.6\left(\frac{0.05\,\text{m}}{12\,\text{m}}\right)\left(\frac{2 \times 9.81\,\text{m/s}^2}{(1.2\,\text{m/s})^2}\right) \\
&\boxed{f = 0.0908}
\end{aligned}
$$

Since the resistance coefficient is now known, this value can be used to find viscosity. To perform this calculation, assume the flow is laminar.

$$
\begin{aligned}
f &= \frac{64}{\text{Re}} \\
0.0908 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.0908\rho V D}{64} \\
&= \frac{0.0908 \times (0.8 \times 1000) \times 1.2 \times 0.05}{64} \\
&\boxed{\mu = 0.068\,\text{N} \cdot \text{s/m}^2}
\end{aligned}
$$

Now, check Reynolds number to see if laminar flow assumption is valid

$$
\begin{aligned}
\text{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{1.2 \times 0.05 \times (0.8 \times 1000)}{0.068} \\
&= 706
\end{aligned}
$$

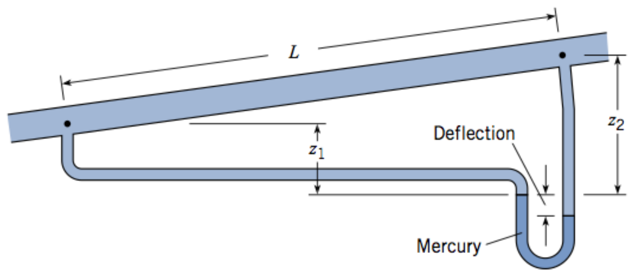
Thus, flow is $\boxed{\text{laminar.}}$

10.28: PROBLEM DEFINITION

Situation:

Oil flows through a smooth pipe.
$D = 2\,\text{in}, \quad V = 5\,\text{ft/s}.$
$L = 30\,\text{ft}, \quad z_1 = 2\,\text{ft}, \quad z_2 = 4\,\text{ft}.$

Sketch:



Find:

The direction of the flow.
Resistance coefficient.
Nature of the flow (laminar or turbulent).
Viscosity of oil $\left(\text{lbf s/ft}^2\right)$.

Properties:

Oil, $S = 0.8$.

SOLUTION

Based on the deflection on the manometer, the piezometric head (and HGL) on the right side of the pipe is larger than that on the left side. Thus, the flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_L$$

Term by term analysis

$$\alpha_1 V_1 = \alpha_2 V_2; \qquad z_2 - z_1 = 2\,\text{ft}$$

Darcy Weisbach equation

$$h_L = f(L/D)V^2/(2g)$$

Combine equations

$$\frac{p_2 - p_1}{\gamma_{\text{oil}}} = (-2\,\text{ft}) + f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2 + (4\,\text{ft})\,\gamma_{\text{oil}} + (0.33\,\text{ft})\,\gamma_{\text{oil}} - (0.33\,\text{ft})\,\gamma_{\text{Hg}} - (2\,\text{ft})\,\gamma_{\text{oil}} = p_1$$

Calculate values

$$\begin{aligned}
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= -(4\,\text{ft}) - (0.33\,\text{ft}) + (0.33\,\text{ft})\frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}} + (2\,\text{ft}) \\
&= -(2\,\text{ft}) + (0.33\,\text{ft})\left(\frac{S_{\text{Hg}}}{S_{\text{oil}}} - 1\right) \\
&= -(2\,\text{ft}) + (0.33\,\text{ft})\left(\frac{13.6}{0.8} - 1\right) \\
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= 3.28\,\text{ft} \qquad (2)
\end{aligned}$$

Substitute Eq. (2) into (1)

$$\begin{aligned}
(3.28\,\text{ft}) &= (-2\,\text{ft}) + f\frac{L}{D}\frac{V^2}{2g} \\
&\text{or} \\
f &= 5.28\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 5.28\left(\frac{1/6\,\text{ft}}{30\,\text{ft}}\right)\left(\frac{2 \times 32.2\,\text{ft/s}^2}{(5\,\text{ft/s})^2}\right) \\
&\boxed{f = 0.076}
\end{aligned}$$

Since the resistance coefficient (f) is now known, use this value to find viscosity.

Resistance coefficient (f) (assume laminar flow)

$$\begin{aligned}
f &= \frac{64}{\text{Re}} \\
0.076 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.076\rho V D}{64} \\
&= \frac{0.076 \times \left(0.8 \times 1.94\,\text{slug/ft}^3\right) \times 5\,\text{ft/s} \times (1/6\,\text{ft})}{64} \\
&\boxed{\mu = 0.00154\ \text{lbf} \cdot \text{s/ft}^2}
\end{aligned}$$

Check laminar flow assumption

$$\begin{aligned}
\text{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{5\,\text{ft/s} \times (1/6\,\text{ft}) \times \left(0.8 \times 1.94\,\text{slug/ft}^3\right)}{0.00154\,\text{lbf s/ft}^2} \\
&= 840
\end{aligned}$$

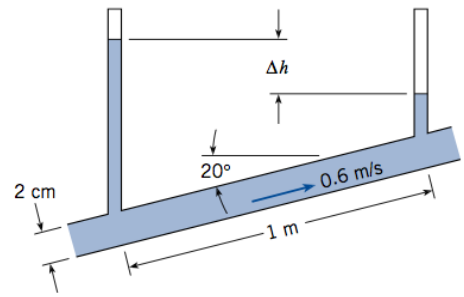
Answer ⇒ $\boxed{\text{Flow is laminar}}$.

10.29: PROBLEM DEFINITION

Situation:

Glycerin flows through a commercial steel pipe connected to two piezometers.
$D = 2\,\text{cm}$, $V = 0.6\,\text{m/s}$.

Sketch:



Find:

Height differential (in m).

Properties:

Glycerin (20 °C), Table A.4, $\mu = 1.41\,\text{N}\cdot\text{s/m}^2$, $\nu = 1.12\times10^{-3}\,\text{m}^2/\text{s}$.

SOLUTION

Energy equation (apply from one piezometer to the other)

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L \\
p_1/\gamma + z_1 &= p_2/\gamma + z_2 + h_L \\
((p_1/\gamma) + z_1)) - ((p_2/\gamma) + z_2) &= h_L \\
\Delta h &= h_L
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{(0.6)(0.02)}{1.12\times10^{-3}} \\
&= 10.71
\end{aligned}$$

Since Re < 2000, the flow is laminar. The head loss for laminar flow is

$$\begin{aligned}
h_L &= \frac{32\mu LV}{\gamma D^2} \\
&= \frac{(32)(1.41)(1)(0.6)}{12300\times0.02^2} \\
&= 5.502\,\text{m}
\end{aligned}$$

Energy equation

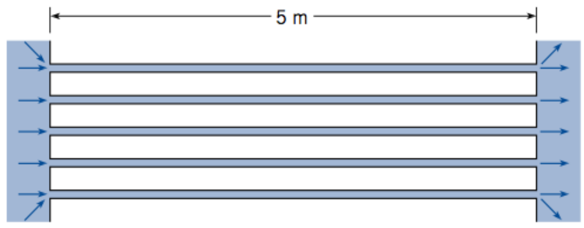
$$\Delta h = h_L$$
$$\boxed{\Delta h = 5.50 \text{ m}}$$

10.30: PROBLEM DEFINITION

Situation:

Water is pumped through tubes in a heat exchanger
$D = 5\,\text{mm}$, $L = 5\,\text{m}$, $V = 0.12\,\text{m/s}$.
$T_1 = 20\,°\text{C}$, $T_2 = 30\,°\text{C}$.

Sketch:



Find:

Pressure difference across heat exchanger.

Properties:

Water (20 °C), Table A.5: $\upsilon = 10^{-6}\,\text{m}^2/\text{s}$.

SOLUTION

Reynolds number (based on temperature at the inlet)

$$\text{Re}_{20°} = \frac{VD}{\nu} = \frac{0.12\,\text{m/s} \times 0.005\,\text{m}}{10^{-6}\,\text{m}^2/\text{s}} = 600$$

Since $\text{Re} \leq 2000$, the flow is laminar. Thus,

$$\Delta p = \frac{32\mu LV}{D^2}$$

Assume linear variation in μ and use the temperature at 25^oC. From Table A.5

$$\begin{aligned} \mu_{\text{avg.}} &= \mu_{25°} \\ &= 8.91 \times 10^{-4}\ \text{N} \cdot \text{s/m}^2 \end{aligned}$$

and

$$\begin{aligned} \Delta p &= \frac{32\mu LV}{D^2} \\ &= \frac{32 \times 8.91 \times 10^{-4}\,\text{N s/m}^2 \times 5\,\text{m} \times 0.12\,\text{m/s}}{(0.005\,\text{m})^2} = 684\,\text{Pa} \end{aligned}$$

$$\boxed{\Delta p = 684\,\text{Pa}}$$

10.31: PROBLEM DEFINITION

Situation:

Water flows through a PVC pipe.
4" Schedule 40. $D = 4.026\,\text{in} = 0.3355\,\text{ft}$.
$Q = 2\,\text{ft}^3/\text{s}$, $k_s = 0$.

Find:

Resistance coefficient f.

Properties:

Water (70 °F), Table A.5, $\nu = 1.06 \times 10^{-5}\,\text{ft}^2/\text{s}$.

SOLUTION

1. Reynolds number.

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} = \frac{4\left(2\,\text{ft}^3/\text{s}\right)}{\pi\,(0.3355\,\text{ft})\left(1.06 \times 10^{-5}\,\text{ft}^2/\text{s}\right)} \\ &= 7.16 \times 10^5 \quad \text{(turbulent flow)} \end{aligned}$$

2. Swamee and Jain eqn.

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{(7.16\times10^5)^{0.9}}\right)\right]^2} = 0.0123 \end{aligned}$$

$$\boxed{f = 0.012}$$

10.32: PROBLEM DEFINITION

Situation:

Water flows through a brass tube.
$k_s = 0,\ D = 3\,\text{cm}.$
$Q = 0.002\,\text{m}^3/\,\text{s}.$

Find:

Resistance coefficient, f.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.002\,\text{m}^3/\,\text{s}}{\pi/4 \times (0.03\,\text{m})^2} \\ &= 2.83\ \text{m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{v} \\ &= \frac{2.83\,\text{m/}\,\text{s} \times 0.03\,\text{m}}{10^{-6}\,\text{m}^2/\,\text{s}} \\ &= 8.49 \times 10^4 \end{aligned}$$

Friction factor (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{(8.49\times10^4)^{0.9}}\right)\right]^2} \\ &\boxed{f = 0.0185} \end{aligned}$$

10.33: PROBLEM DEFINITION

Situation:

Water flows through a smooth pipe.
$D = 0.25\,\text{m}$, $Q = 0.05\,\text{m}^3/\,\text{s}$.
$k_s = 0$.

Find:

Resistance coefficient, f.

Properties:

Water (10 °C), Table A.5, $\nu = 1.31 \times 10^{-6}\ \text{m}^2/\,\text{s}$.

SOLUTION

1. Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4\,(0.05\,\text{m}^3/\,\text{s})}{\pi\,(0.25\,\text{m})\,(1.31 \times 10^{-6}\ \text{m}^2/\,\text{s})} = 1.94 \times 10^5 \end{aligned}$$

2. Moody diagram:

$$\boxed{f = 0.016}$$

10.34: PROBLEM DEFINITION

Situation:
Water flows through a cast-iron pipe.
$D = 10\,\text{cm}$, $V = 4\,\text{m/s}$.

Find:
Calculate the resistance coefficient.
Plot the velocity distribution.

Properties:
Water (10 °C), Table A.5: $\nu = 1.31 \times 10^{-6}\ \text{m}^2/\text{s}$.

SOLUTION

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu}\\ &= \frac{4\,\text{m/s}(0.1\,\text{m})}{1.31\times10^{-6}\,\text{m}^2/\text{s}}\\ &= 3.053\times10^5\end{aligned}$$

Sand roughness height

$$\begin{aligned}\frac{k_s}{D} &= \frac{0.00026}{0.1}\\ &= 0.0026\end{aligned}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned}f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D}+\frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}\\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.0026}{3.7}+\frac{5.74}{(3.053\times10^5)^{0.9}}\right)\right]^2}\\ &= 0.0258\end{aligned}$$

$$\boxed{f = 0.0258}$$

Velocity profile (turbulent flow)

$$\frac{u}{u_*} = 5.75\ \log\left(\frac{y}{k_s}\right) + 8.5$$

Friction velocity(u_*)

$$u_* = \sqrt{\tau_0/\rho} \tag{1}$$

Resistance coefficient

$$\tau_o = \frac{f}{4}\left(\frac{\rho V^2}{2}\right) \tag{2}$$

Combine Eqs. (1) and (2)

$$\begin{aligned} u_* &= V\sqrt{\frac{f}{8}} \\ &= 4\sqrt{\frac{0.0258}{8}} \\ &= 0.227\,2\ \mathrm{m/\,s} \end{aligned}$$

Velocity profile

$$u = (0.227\,2\ \mathrm{m/\,s})\left[5.75\log\left(\frac{y}{0.00026}\right) + 8.5\right]$$

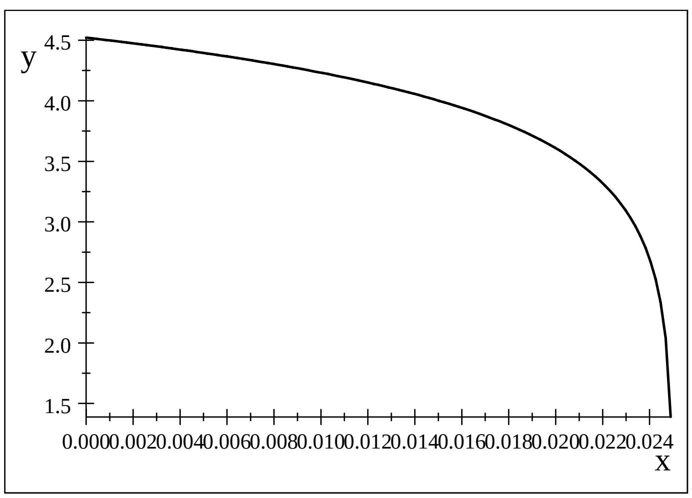
The distance from the wall (y) is related to pipe radius (R) and distance from the centerline (r) by

$$y = R - r$$

Velocity Profile

$$u(r) = (0.227\,2\ \mathrm{m/\,s})\left[5.75\log\left(\frac{0.025 - r}{0.00026}\right) + 8.5\right]$$

Plot



10.35: PROBLEM DEFINITION

Situation:

A fluid flows in a smooth pipe.

$D = 100\,\text{mm}$, $\bar{V} = 500\,\text{mm/s}$.

Find:

(a) Maximum velocity (m/s).

(b) Resistance coefficient.

(c) Shear velocity (m/s).

(d) Shear stress 25 mm from pipe center (N/m^2).

(e) Determine if the head loss will double if discharge is doubled.

Properties:

$\mu = 10^{-2}\,\text{N}\cdot\text{s/m}^2$, $\rho = 800\,\text{kg/m}^3$.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.5\,\text{m/s})(0.1\,\text{m})(800\,\text{kg/m}^3)}{10^{-2}} \\ &= 4000 \end{aligned}$$

Because Re > 2000, assume the flow is turbulent.

a) Table 10.2 relates mean and centerline velocity. From this table,

$$\begin{aligned} V_{\max} &= \frac{\bar{V}}{0.791} \\ &= \frac{0.50\,\text{m/s}}{0.791} \end{aligned}$$

$$\boxed{V_{\max} = 0.632 \text{ m/s}}$$

b) Resistance coefficient (from Moody diagram)

$$\boxed{f = 0.041}$$

c) Shear velocity is defined as

$$u_* = \sqrt{\frac{\tau_o}{\rho}} \tag{1}$$

Wall shear stress

$$\tau_o = \frac{f}{4}\frac{\rho V^2}{2}$$

Combine equations

$$\begin{aligned} u_* &= V\left(\frac{f}{8}\right)^{0.5} \\ &= (0.5\,\mathrm{m/s})\left(\frac{0.041}{8}\right)^{0.5} \end{aligned}$$

$$\sqrt{\frac{0.041 \times 0.5^2}{8}}$$

$$\boxed{u_* = 0.0358\,\mathrm{m/s}}$$

d) In a pipe flow, shear stress is linear with distance from the wall. The distance of 25 mm from the center of the pipe is half way between the wall and the centerline. Thus, the shear stress is 1/2 of the wall value:

$$\tau_{25\text{ mm}} = \frac{\tau_o}{2}$$

The shear stress at the wall is given by Eq. (1)

$$\begin{aligned} \tau_o &= \rho u_*^2 \\ &= 800\,\mathrm{kg/m^3} \times (0.0358\,\mathrm{m/s})^2 \\ &= 1.025\,\mathrm{N/m^2} \end{aligned}$$

Thus

$$\begin{aligned} \tau_{25\text{ mm}} &= \frac{\tau_o}{2} \\ &= \frac{1.025\,\mathrm{N/m^2}}{2} \end{aligned}$$

$$\boxed{\tau_{25\text{ mm}} = 0.513\ \mathrm{N/m^2}}$$

e) If flow rate (Q) is doubled, the velocity will also double. Thus, head loss will be given by

$$h_f = f_{\text{new}}\left(\frac{L}{D}\right)\frac{(2V)^2}{2g}$$

The increase in velocity will increase Reynolds number, thereby decreasing the friction factor so that $f_{\text{new}} < f_{\text{original}}$ Overall the head loss will increase by slightly less than a factor of 4.0.

$$\boxed{\text{No, the increase in head loss will be closer to a factor of 4.0}}$$

10.36: PROBLEM DEFINITION

Situation:

Water flows in a cast iron pipe.

$D = 0.15\,\text{m}$, $Q = 0.075\,\text{m}^3/\,\text{s}$.

$k_s = 0.26\,\text{mm}$.

Find:

Reynolds number.

Friction factor, f.

Shear stress at the wall (Pa).

Properties:

Water (20 °C), Table A.5: $\rho = 998\,\text{kg}/\,\text{m}^3$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

SOLUTION

1. Flow rate eqn.

$$
\begin{aligned}
V &= \frac{Q}{A} = \frac{(0.075\,\text{m}^3/\,\text{s})}{(\pi/4)\,(0.15\,\text{m})^2} \\
&= 4.244\,\text{m}/\,\text{s}
\end{aligned}
$$

2. Reynolds number

$$
\text{Re} = \frac{VD}{\nu} = \frac{(4.244\,\text{m}/\,\text{s})\,(0.15\,\text{m})}{(1.00 \times 10^{-6}\,\text{m}^2/\,\text{s})} = 6.366 \times 10^5
$$

$$
\boxed{\text{Re} = 6.37 \times 10^5}
$$

3. Relative roughness

$$
\frac{k_s}{D} = \frac{0.26\,\text{mm}}{150\,\text{mm}} = 1.733 \times 10^{-3}
$$

4. Swamee Jain eqn.

$$
\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{1.733\times10^{-3}}{3.7} + \frac{5.74}{(6.366\times10^5)^{0.9}}\right)\right]^2} = 0.023
\end{aligned}
$$

$$
\boxed{f = 0.023}
$$

5. Definition of f:

$$
\begin{aligned}
\tau_0 &= \frac{f\rho V^2}{8} = \frac{0.023\,(998\,\text{kg}/\,\text{m}^3)\,(4.244\,\text{m}/\,\text{s})^2}{8} \\
&= 51.68\,\text{Pa}
\end{aligned}
$$

$$
\boxed{\tau_0 = 51.7\,\text{Pa}}
$$

10.37: PROBLEM DEFINITION

Situation:

Water flows in a uncoated cast iron pipe.
$D = 4\,\text{in}$, $Q = 0.02\,\text{ft}^3/\,\text{s}$.

Find: Resistance coefficient f.

Properties:

From Table A.5 (60 °F): $\nu = 1.22 \times 10^{-5}$ ft^2/s.
From Table 10.4: $k_s = 0.01$ in.

SOLUTION

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4 \times 0.02\,\text{ft}^3/\,\text{s}}{\pi \times (4/12)\,\text{ft} \times (1.22 \times 10^{-5}\,\text{ft}^2/\,\text{s})} \\ &= 6.3 \times 10^3\end{aligned}$$

Sand roughness height

$$\begin{aligned}\frac{k_s}{D} &= \frac{0.01}{4} \\ &= 0.0025\end{aligned}$$

Friction factor (from Moody diagram)

$$\boxed{f = 0.038}$$

10.38: PROBLEM DEFINITION

Situation:

Fluid flows in a concrete pipe.
$D = 6\,\text{in}, \quad L = 900\,\text{ft}.$
$Q = 3$ cfs, $\quad k_s = 0.0002\,\text{ft}.$

Find:

Head loss (ft).

Properties:

$\rho = 1.5$ slug/ft^3, $\nu = 3.33 \times 10^{-3}\,\text{ft}^2/\,\text{s}.$

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4(3.0\,\text{ft}^3/\,\text{s})}{\pi(0.5\,\text{ft})3.33 \times 10^{-3}\,\text{ft}^2/\,\text{s}} \\ &= 2294 \text{ (laminar)} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{\pi D^2/4} \\ &= \frac{3\,\text{ft}^3/\,\text{s}}{\pi/4 \times (0.5\,\text{ft})^2} \\ &= 15.28 \text{ ft/s} \end{aligned}$$

Head loss (laminar flow)

$$\begin{aligned} h_f &= \frac{32\mu L V}{\gamma D^2} \\ &= \frac{32 \times (5 \times 10^{-3}\,\text{lbf s}/\,\text{ft}^2) \times 900\,\text{ft} \times 15.28\,\text{ft}/\,\text{s}}{1.5\,\text{slug}/\,\text{ft}^3 \times 32.2\,\text{ft}/\,\text{s}^2 \times (0.5\,\text{ft})^2} = 182.2\,\text{ft} \end{aligned}$$

$$\boxed{h_f = 182\,\text{ft}}$$

10.39: PROBLEM DEFINITION

Situation:

Crude oil flows through a steel pipe.
$D = 15\,\text{cm}$, $Q = 0.03\,\text{m}^3/\,\text{s}$.
$p_B = 300\,\text{kPa}$, $L = 1\,\text{km}$.

Find:

Pressure at point A (kPa).

Properties:

$S = 0.82$, $\mu = 10^{-2}\,\text{N}\,\text{s}/\,\text{m}^2$.
From Table 10.4: $k_s = 4.6 \times 10^{-5}$ m.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{v} \\ &= \frac{4Q}{\pi D\nu} \\ &= \frac{4 \times 0.03\,\text{m}^3/\,\text{s}}{\pi \times 0.15\,\text{m} \times (10^{-2}\,\text{N}\,\text{s}/\,\text{m}^2/0.82)} \\ & \quad 2.09 \times 10^4 \text{ (turbulent)} \end{aligned}$$

Sand roughness height

$$\begin{aligned} \frac{k_s}{D} &= \frac{4.6 \times 10^{-5}}{0.15} \\ &= 3.1 \times 10^{-4} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.03\,\text{m}^3/\,\text{s}}{\pi \times (0.15\,\text{m})^2/4} \\ &= 1.698 \text{ m/s} \end{aligned}$$

Friction factor (from Moody diagram)

$$f = 0.027$$

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.027\left(\frac{1000\,\text{m}}{0.15\,\text{m}}\right)\left(\frac{(1.698\,\text{m}/\,\text{s})^2}{2 \times 9.81\,\text{m}/\,\text{s}^2}\right) \\ &= 26.4 \text{ m} \end{aligned}$$

Energy equation

$$p_A/\gamma + \alpha_A V_A^2/2g + z_A = p_B/\gamma + \alpha_B V_B^2/2g + z_B + h_f$$

$$p_A = 0.82 \times 9810[(300000/(0.82 \times 9810)) + 20 + 26.41] = 673\,\text{kPa}$$

$$\boxed{p_A = 673\,\text{kPa}}$$

10.40: PROBLEM DEFINITION

Situation:

A pipe is being using to measure viscosity of a fluid.
$D = 1\,\text{cm}$, $L = 1\,\text{m}$.
$V = 3\,\text{m/s}$, $h_f = 50\,\text{cm}$.

Find:

Kinematic viscosity.

SOLUTION

$$\begin{aligned} h_f &= f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \\ 0.50 &= f\left(\frac{1\,\text{m}}{0.01\,\text{m}}\right)\left(\frac{(3\,\text{m/s})^2}{2\times 9.81\,\text{m/s}^2}\right) \\ f &= 0.0109 \end{aligned}$$

At this value of friction factor, Reynolds number can be found from the Moody diagram. The result is:

$$\text{Re} = 1.5\times 10^6$$

Thus

$$\begin{aligned} \nu &= \frac{VD}{\text{Re}} \\ &= \frac{(3\,\text{m/s})(0.01\,\text{m})}{1.5\times 10^6} = 2.0\times 10^{-8}\,\text{m}^2/\text{s} \end{aligned}$$

$$\boxed{\nu = 2.0\times 10^{-8}\,\text{m}^2/\text{s}}$$

10.41: PROBLEM DEFINITION

Situation:

For a selected pipe:
$f = 0.06$, $D = 40\,\text{cm}$.
$V = 3\,\text{m/s}$, $\nu = 10^{-5}\,\text{m}^2/\text{s}$.

Find:

Change in head loss per unit meter if the velocity were doubled.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{v} \\ &= \frac{3\,\text{m/s} \times 0.4\,\text{m}}{10^{-5}\,\text{m}^2/\text{s}} \\ &= 1.2 \times 10^5 \end{aligned}$$

Since Re > 3000, the flow is turbulent and obviously the conduit is very rough ($f = 0.06$); therefore, one would expect f to be virtually constant with increased velocity. Since $h_f = f(L/D)(V^2/2g)$, we expect, $h_f \sim V^2$, so if the velocity is doubled, the head loss will be **quadrupled.**

10.42: PROBLEM DEFINITION

Situation:

Water flows through a horizontal run of PVC pipe

$V = 5\,\text{ft/s}, \quad L = 100\,\text{ft}.$

Nominal diameter 2.5" Schedule 40 $D = 2.45\,\text{in.} = 0.204\,\text{ft}$ (look up on internet).

Find:

(a) Pressure drop in psi.

(b) Head loss in feet.

(c) Power in horsepower needed to overcome the head loss.

Assumptions:

Assume $k_s = 0$.

Assume $\alpha_1 = \alpha_2$, where subscripts 1 and 2 denote the inlet and exit of the pipe.

Properties:

Water $(50\,^\circ\text{F})$, Table A.5:

$\rho = 1.94\,\text{slug/ft}^3$, $\gamma = 62.4\,\text{lbf/ft}^3$, $\nu = 14.1 \times 10^{-6}\,\text{ft}^2/\text{s}$.

PLAN

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor (f) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using $P = \dot{m}gh_f$.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(5\,\text{ft/s})(0.204\,\text{ft})}{(14.1 \times 10^{-6}\,\text{ft}^2/\text{s})} \\ &= 72,400 \end{aligned}$$

Thus, flow is turbulent.

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{72,400^{0.9}}\right)\right]^2} \\ &= 0.0191 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0191\left(\frac{100\,\text{ft}}{0.204\,\text{ft}}\right)\frac{(5\,\text{ft/s})^2}{2\times 32.2\,\text{ft/s}^2} \\ &= 3.635\,\text{ft} \end{aligned}$$

$$\boxed{h_f = 3.64\,\text{ft}\quad\text{(part b)}}$$

Energy equation (section 2 located 100 ft downstream of section 1).

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L$$

Term by term analysis

$$\begin{aligned} \frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p &= \frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L \\ \frac{p_1}{\gamma}+\alpha\frac{V^2}{2g}+0+0 &= \frac{p_2}{\gamma}+\alpha\frac{V^2}{2g}+0+0+h_f \\ \frac{p_1}{\gamma} &= \frac{p_2}{\gamma}+h_f \\ \text{or}\quad \Delta p &= \gamma h_f \\ &= \left(62.4\,\text{lbf/ft}^3\right)(3.635\,\text{ft}) \\ &= 227\text{ psf} \\ &= 227\left(\frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{\text{ft}^2}{144\,\text{in}^2}\right) \end{aligned}$$

$$\boxed{\Delta p = 1.58\text{ psi}\quad\text{(part a)}}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho A V \\ &= \left(1.94\,\text{slug/ft}^3\right)\left(\frac{\pi\,(0.204\,\text{ft})^2}{4}\right)(5\,\text{ft/s}) \\ &= 0.317\,\text{slug/s} \end{aligned}$$

Power equation

$$\begin{aligned} \dot{W} &= \dot{m} g h_f \\ &= (0.317\,\text{slug/s})\left(32.2\,\text{ft/s}^2\right)(3.635\,\text{ft})\left(\frac{1.0\,\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}\right) \\ &= 0.06746\,\text{hp} \end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 0.0675\,\text{hp}\quad\text{(part c)}}$$

REVIEW

1. The pressure drop for a 100 ft run of pipe ($\Delta p = 227$ psf ≈ 1.6 psi)could be decreased by selecting a larger pipe diameter.

2. The power to overcome the frictional head loss is about 1/15 of a horsepower.

10.43: PROBLEM DEFINITION

Situation:

Water flows with a through a horizontal run of PVC pipe

$V = 2\,\mathrm{m/s}$, $L = 50\,\mathrm{m}$.

Nominal diameter 2.5" Schedule 40. $D = 2.45\,\mathrm{in.} = 0.0622\,\mathrm{m}$.

Find:

(a) Pressure drop in kPa.

(b) Head loss in meters.

(c) Power in watts needed to overcome the head loss.

Assumptions:

1.) Assume $k_s = 0$.

2.) Assume $\alpha_1 = \alpha_2$, where subscripts 1 and 2 denote the inlet and exit of the pipe.

Properties:

Water (10 °C), Table A.5:

$\rho = 1000\,\mathrm{kg/m^3}$, $\gamma = 9810\,\mathrm{N/m^3}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/s}$.

PLAN

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor (f) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using $P = \dot{m}gh_f$.

SOLUTION

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{(2\,\mathrm{m/s})\,(0.0622\,\mathrm{m})}{(1.31 \times 10^{-6}\,\mathrm{m^2/s})} \\ &= 94{,}960 \end{aligned}$$

Thus, flow is turbulent.

Friction factor (f) (Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{94{,}960^{0.9}}\right)\right]^2} \\ &= 0.0181 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f \frac{L}{D} \frac{V^2}{2g} \\ &= 0.0181 \left(\frac{50\,\text{m}}{0.0622\,\text{m}} \right) \frac{(2\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} \\ &= 2.966\,\text{m} \end{aligned}$$

$$\boxed{h_f = 2.97\,\text{m} \quad (\text{part b})}$$

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Select a control volume surrounding the pipe. After analysis of each term, the energy equation simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_f \\ \text{or} \quad \Delta p &= \gamma h_f \\ &= \left(9810\,\text{N/m}^3\right)(2.966\,\text{m}) \\ &= 29,096\,\text{kPa} \end{aligned}$$

$$\boxed{\Delta p = 29.1\,\text{kPa} \quad (\text{part a})}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho A V \\ &= \left(1000\,\text{kg/m}^3\right) \left(\frac{\pi\,(0.0622\,\text{m})^2}{4} \right) (2\,\text{m/s}) \\ &= 6.077\,\text{kg/s} \end{aligned}$$

Power equation

$$\begin{aligned} \dot{W} &= \dot{m} g h_f \\ &= (6.077\,\text{kg/s}) \left(9.81\,\text{m/s}^2\right)(2.966\,\text{m}) \\ &= 176.8\,\text{W} \end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 177\,\text{W} \quad (\text{part c})}$$

REVIEW

1. The pressure drop (29 kPa) is about 1/3 of an atmosphere This value could be decreased by increasing the pipe diameter to lower the speed of the water.
2. The power to overcome the frictional head loss is small, about 1/4 of a horsepower.

10.44: PROBLEM DEFINITION

Situation:

Air flows through a smooth tube.
$Q = 0.015\,\text{m}^3/\,\text{s},\ D = 3\,\text{cm}.$
$p = 110\,\text{kPa-absolute}.$

Find:

Pressure drop per meter of tube length.

Properties:

Air (20 °C) Table A.3: $\mu = 1.81 \times 10^{-5}$ N·s/m^2, $\rho = 1.2\,\text{kg}/\,\text{m}^3$.

Assumptions:

1. Pipe is horizontal.
2. Fully developed flow so V and α are constant.

PLAN

Solve the problem by applying the energy equation. The steps are:

1. Develop an equation for Δp by applying the energy equation.
2. Calculate V using the flow rate equation.
3. Calculate ρ using the ideal gas law
4. Calculate Reynolds number.
5. Look up f on the Moody diagram using the Re from step 4.
6. Calculate h_f using the Darcy-Wiesbach equation.
7. Combine results.

SOLUTION

1. Energy equation (cv surrounding a 1-m length of pipe)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Since the velocity head terms cancel, $z_1 = z_2$, and $h_p = h_t = 0$, the energy equation simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_L \\ \Delta p &= \gamma h_L = \rho g h_f \end{aligned}$$

2. Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.015\,\text{m}^3/\,\text{s}}{\pi/4 \times (0.03\,\text{m})^2} \\ &= 21.2 \text{ m/s} \end{aligned}$$

3. Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{110,000\,\text{Pa}}{287\,\text{J/kg K} \times 293\,\text{K}} \\
&= 1.31\ \text{kg/m}^3
\end{aligned}$$

4. Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{21.2\,\text{m/s} \times 0.03\,\text{m} \times 1.20\,\text{kg/m}^3}{1.81 \times 10^{-5}\,\text{N s/m}^2} \\
&= 42166
\end{aligned}$$

5. Friction factor (f) (Moody diagram)

$$f = 0.0212$$

6. Darcy Weisbach equation

$$\begin{aligned}
h_f &= f\frac{L}{D}\frac{V^2}{2g} \\
&= 0.0212\left(\frac{1\,\text{m}}{0.03\,\text{m}}\right)\left(\frac{(21.2\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2}\right) \\
&= 16.19\,\text{m for a 1.0 m length of pipe}
\end{aligned}$$

7. Combine results

$$\begin{aligned}
\Delta p &= h_f \rho g \\
&= (16.19\,\text{m})\left(1.31\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right) \\
&= 207.6\,\text{Pafor a 1.0 m length of pipe}
\end{aligned}$$

$$\boxed{\Delta p/L = 208\,\text{Pa/m}}$$

10.45: PROBLEM DEFINITION

Situation:

Water flows from point A to B in a cast iron pipe.
$L = 3\,\text{mi}$, $D = 24\,\text{in.}$
$\Delta p_{B-A} = 20$ psi, $\Delta h_{A-B} = 30\,\text{ft}$.

Find:

Direction and rate of flow $\left(\text{ft}^3/\text{s}\right)$.

Assumptions:

Flow is from A to B.

Properties:

Water (50 °F), Table A.5: $\nu = 1.41 \times 10^{-5}$ ft^2/s.
Pipe Roughness, Table 10.4 (EFM9e), $k_s = 0.01\,\text{in} = 0.000833$ ft.

SOLUTION

$$\begin{aligned}
h_f &= \Delta(p/\gamma + z) \\
&= (-20 \times 144/62.4) + 30 \\
&= -16.2 \text{ ft}
\end{aligned}$$

Therefore, flow is from B to A.

Parameters for the Moody diagram

$$\begin{aligned}
\text{Re}\, f^{1/2} &= (D^{3/2}/\nu)(2gh_f/L)^{1/2} \\
&= (2^{3/2}/(1.41 \times 10^{-5}) \times 64.4 \times 16.2/(3 \times 5,280))^{1/2} \\
&= 5.14 \times 10^4 \\
k_s/D &= 4.2 \times 10^{-4}
\end{aligned}$$

Resistance coefficient (from the Moody diagram,Fig. 10.8)

$$f = 0.0175$$

Darcy Weisbach equation

$$\begin{aligned}
V &= \sqrt{h_f 2gD/fL} \\
&= \sqrt{(16.2 \times 64.4 \times 2)/(0.0175 \times 3 \times 5,280)} \\
&= 2.74 \text{ ft/s}
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
q &= VA \\
&= 2.74 \times (\pi/4) \times 2^2 \\
&= \boxed{q = 8.60\text{cfs}}
\end{aligned}$$

10.46: PROBLEM DEFINITION

Situation:

Air flows through a smooth tube.

$D = 1\,\text{in}$, $Q = 30\,\text{ft}^3/\,\text{min}$.

$p = 15$ psia.

Find:

Pressure drop per foot of tube.

Properties:

Air (80°F) Table A.3: $\mu = 3.85 \times 10^{-7}$ lbf-s/ft^2.

SOLUTION

$$\begin{aligned}
V &= \frac{Q}{A} = \frac{30\,\text{ft}^3/\,\text{min} \times 4}{60\,\text{s}/\,\text{min} \times \pi \times \left(\frac{1}{12}\,\text{ft}\right)^2} = 91.67 \text{ ft/s} \\
\rho &= \frac{p}{RT} = \frac{15 \text{ psia} \times 144\,\text{in}^2/\,\text{ft}^2}{1716 \times 540} = 0.00233 \text{ slugs/ft}^3 \\
\text{Re} &= \frac{VD\rho}{\mu} = \frac{91.67\,\text{ft}/\,\text{s} \times (1/12)\,\text{ft} \times 0.00233\,\text{slug}/\,\text{ft}^3}{3.85 \times 10^{-7}\,\text{lbf}\,\text{s}/\,\text{ft}^2} \\
&= 4.623 \times 10^4
\end{aligned}$$

Resistance coefficient (f) (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{(4.623\times10^4)^{0.9}}\right)\right]^2} \\
&= 0.0211
\end{aligned}$$

Pressure drop

$$\begin{aligned}
\Delta p &= f\frac{L}{D}\left(\frac{\rho V^2}{2}\right) \\
&= 0.0211\left(\frac{1\,\text{ft}}{1/12\,\text{ft}}\right)\left(\frac{0.00233\,\text{slug}/\,\text{ft}^3 \times (91.67\,\text{ft}/\,\text{s})^2}{2}\right) \\
&= 2.479 \text{ psf/ft}
\end{aligned}$$

$$\boxed{\frac{\Delta p}{L} = 2.48 \text{ psf per foot of tube}}$$

10.47: PROBLEM DEFINITION

Situation:

Water is pumped through a vertical steel pipe to an elevated tank.
$D = 10\,\text{cm}$, $p_1 = 1.6\,\text{MPa}$.
$L = 80\,\text{m}$, $Q = 0.02\,\text{m}^3/\,\text{s}$.

Find:

Pressure at point 80 m above pump.

Properties:

Water (20 °C), Table A.5: $\gamma = 9790\,\text{N}/\,\text{m}^3$.

SOLUTION

$$\begin{aligned}
\text{Re} &= \frac{4Q}{\pi D\nu} \\
&= \frac{4 \times 0.02\,\text{m}^3/\,\text{s}}{\pi \times 0.10\,\text{m} \times 10^{-6}\,\text{m}^2/\,\text{s}} = 2.55 \times 10^5 \\
\frac{k_s}{D} &= \frac{4.6 \times 10^{-2}}{100} = 4.6 \times 10^{-4}
\end{aligned}$$

Resistance coefficient

$$f = 0.0185$$

Then

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

where

$$\begin{aligned}
V &= \frac{0.02\,\text{m}^3/\,\text{s}}{\pi/4) \times (0.1\,\text{m})^2} = 2.546\ \text{m/s} \\
h_f &= 0.0185 \times \frac{80\,\text{m}}{0.10\,\text{m}} \times \frac{(2.546\,\text{m}/\,\text{s})^2}{2 \times 9.81\,\text{m}/\,\text{s}^2} = 4.89\ \text{m}
\end{aligned}$$

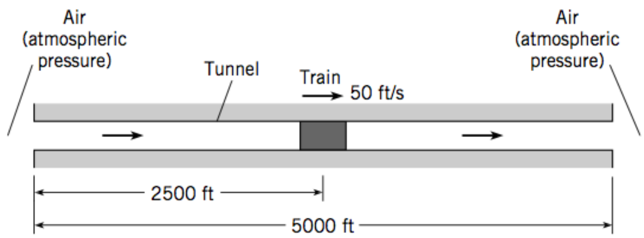
Energy equation (from pump to location 80 m higher)

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_f \\
\frac{1.6 \times 10^6\,\text{Pa}}{9,790\,\text{N}/\,\text{m}^3} + \frac{V_1^2}{2g} &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 80 + 4.89 \\
V_1 &= V_2 \\
& \boxed{p_2 = 769\,\text{kPa}}
\end{aligned}$$

10.48: PROBLEM DEFINITION

Situation:

A train travels through a tunnel.
$D = 10\,\text{ft}$, $k_s = 0.05\,\text{ft}$.
$V = 50\,\text{ft/s}$, $L = 2500\,\text{ft}$



Find:

(a) Change in pressure between the front and rear of the train.
(b) Power required to produce the air flow in the tunnel.
(c) Sketch an EGL and a HGL.

Properties:

Air (60 °F) Table A.3: $\gamma = 0.0764$ lbf/ft^3, $\nu = 1.58 \times 10^{-4}$ ft^2/s.

PLAN

Apply the energy equation from front of train to outlet of tunnel.

SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f \\ \frac{p_1}{\gamma} + \frac{V_1^2}{2g} &= 0 + 0 + 0 + \frac{V_2^2}{2g} + f \frac{L}{D} \frac{V_2^2}{2g} \\ \frac{p_1}{\gamma} &= f \frac{L}{D} \frac{V_2^2}{2g} \end{aligned}$$

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.05}{10} = 0.005 \\ \text{Re} &= \frac{VD}{v} = \frac{50\,\text{ft/s} \times 10\,\text{ft}}{1.58 \times 10^{-4}\,\text{ft}^2/\text{s}} = 3.2 \times 10^6 \end{aligned}$$

Resistance coefficient (from Moody diagram)

$$f = 0.030$$

Darcy Weisbach equation

$$\begin{aligned} p_1 &= \gamma f \frac{L}{D}\frac{V_2^2}{2g} \\ &= (0.0764\,\text{lbf/ft}^3)(0.03)\left(\frac{2500\,\text{ft}}{10}\right)\left(\frac{(50\,\text{ft/s})^2}{64.4\,\text{ft/s}^2}\right) \\ p_1 &= 22.24 \text{ psfg} \end{aligned}$$

Energy equation (from outside entrance to rear of train)

$$\begin{aligned} \frac{p_3}{\gamma} + \alpha_3\frac{V_3^2}{2g} + z_3 &= \frac{p_4}{\gamma} + \alpha_4\frac{V_4^2}{2g} + z_4 + \sum h_L \\ 0+0+0 &= \frac{p_4}{\gamma} + \frac{V_4^2}{2g} + 0 + K_e + f\frac{L}{D}\frac{V^2}{2g} \\ \frac{p_4}{\gamma} &= -\left(\frac{V^2}{2g}\right)\left(1.5 + f\frac{L}{D}\right) \\ &= -\frac{(50\,\text{ft/s})^2}{2g}\left(1.5 + 0.03\left(\frac{2500\,\text{ft}}{10\,\text{ft}}\right)\right) \\ p_4 &= -\gamma(349.4\,\text{ft}) = -26.69 \text{ psf} \end{aligned}$$

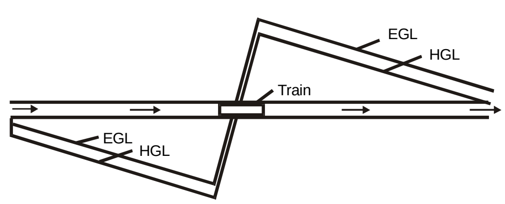
$$\begin{aligned} \Delta p &= p_1 - p_4 \\ &= 22.24 - (-26.69) = 48.93 \text{ psf} \end{aligned}$$

$$\boxed{\Delta p = 48.9 \text{ psf}}$$

Power equation

$$\begin{aligned} P &= FV \\ &= (\Delta p A)(50\,\text{ft/s}) \\ &= (48.93 \text{ psf} \times \pi/4 \times (10\,\text{ft})^2)(50\,\text{ft/s}) \\ &= 192,158 \text{ ft-lbf/s} = 349\,\text{hp} \end{aligned}$$

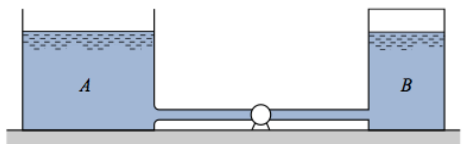
$$\boxed{P = 349\,\text{hp}}$$



10.49: PROBLEM DEFINITION

Situation:
Water is pumped from a reservoir to a tank.
$D = 4\,\text{in}$, $L = 300\,\text{ft}$.
$Q = 1\,\text{ft}^3/\,\text{s}$, $\eta = 0.9$.
$p_B = 10$ psig, $p_A = 0$ psig.

Sketch:



Find:
Power to operate the pump.

Assumptions:
Assume the entrance is smooth.

Properties:
Water (60 °F) Table A.5: $\nu = 1.22 \times 10^{-5}$ ft^2/s.
Pipe roughness, Table 10.4 (EFM9e), $k_s = 0.002\,\text{in} = 1.67 \times 10^{-5}\,\text{ft}$.
Loss Coefficients, Table 10.5 (EFM9e), $K_e = 0.03$, $K_E = 1$.

SOLUTION Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} = \frac{1.0\,\text{ft}^3/\,\text{s}}{(\pi/4)D^2} \\ &= \frac{1.0\,\text{ft}^3/\,\text{s}}{(\pi/4)(0.333\,\text{ft})^2} \\ &= 11.46\ \text{ft/s} \end{aligned}$$

Then

$$\begin{aligned} \text{Re} &= \frac{11.46\,\text{ft}/\,\text{s} \times (1/3)\,\text{ft}}{1.22 \times 10^{-5}\,\text{ft}^2/\,\text{s}} = 3.13 \times 10^5 \\ \frac{k_s}{D} &= 4.5 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from Moody diagram)

$$f = 0.0165$$

Then

$$f\frac{L}{D} = 0.0165\frac{300\,\text{ft}}{(1/3)\,\text{ft}} = 14.86$$

Energy equation (from water surface A to water surface B)

$$\begin{aligned}
\frac{p_A}{\gamma} + \alpha_A\frac{V_A^2}{2g} + z_A + h_p &= \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 0 + h_p &= \frac{10 \text{ psig} \times 144\,\text{in}^2/\,\text{ft}^2}{62.4\,\text{ft}/\,\text{s}^2} + 0 + \left(K_e + K_E + f\frac{L}{D}\right)\frac{V^2}{2g}
\end{aligned}$$

Thus

$$\begin{aligned}
h_p &= 23.08\,\text{ft} + (0.03 + 1 + 14.86)\,\text{ft}\frac{(11.46\,\text{ft}/\,\text{s})^2}{64.4\,\text{ft}/\,\text{s}^2} \\
&= 55.48 \text{ ft}
\end{aligned}$$

Power equation

$$\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
&= \frac{1.0\,\text{ft}^3/\,\text{s} \times 62.4\,\text{ft}/\,\text{s}^2 \times 55.48\,\text{ft}}{0.9} \\
&= 3847 \text{ ft} \cdot \text{lbf/s} \\
&= \boxed{P = 6.99 \text{ horsepower}}
\end{aligned}$$

Problem 10.50

Classify problems as case 1, 2, or 3.

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a. Problem 10.49.

- Classification: Case 1.
- Rationale for classification: Flow rate is specified. Solution path involves finding head loss and then finding pump power.

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b. Problem 10.52

- Classification: Case 3.
- Rationale for classification: Hose diameter is the goal. Flow rate is specified.

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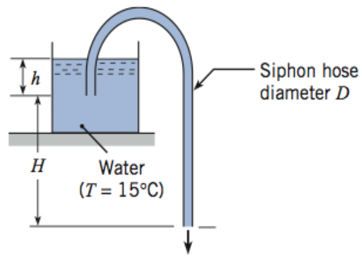
c. Problem 10.55

- Classification: Case 3.
- Rationale for classification: Pipe diameter is the goal. Flow rate is specified and head loss can be easily related to the specified pressure drop.

10.51: PROBLEM DEFINITION

Situation:

Water is flowing out a plastic siphon hose.
$D = 0.012\,\text{m}$, $H = 3\,\text{m}$.
$h = 1.0\,\text{m}$, $L = 5.5\,\text{m}$.
$k_s = 0$.



Find:

Velocity (assume the Bernoulli equation applies).
Velocity (include the head loss in the hose).

Assumptions:

Steady flow.
Neglect all head loss (part 1 of problem).
Neglect component head loss (part 2 of problem).
Turbulent flow. Also, $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5: $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

PLAN

1. Use the Bernoulli equation to find velocity
Classify this problem as case 2 (V is unknown), then
2. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
3. Solve the set of equations using a computer program (we used TK Solver).

SOLUTION

1. Bernoulli equation (point 1 on tank surface; point 2 on exit plane of hose):

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$
$$0 + (H + h) + 0 = 0 + 0 + \frac{V_2^2}{2g}$$

$$V = \sqrt{2g\,(H+h)} = \sqrt{2\,(9.81\,\text{m}/\,\text{s}^2)\,(3\,\text{m} + 1\,\text{m})}$$
$$\boxed{V = 8.86\,\text{m}/\,\text{s}}$$

2. Equations for finding velocity:

- Energy equation:

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\
0 + 0 + (H + h) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \qquad (1)
\end{aligned}
$$

- Darcy-Weisbach:

$$
h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)
$$

- Swamee-Jain:

$$
f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad (3)
$$

- Reynolds number:

$$
\mathrm{Re} = \frac{VD}{\nu} \qquad (4)
$$

3. Solution of Eqs. (1) to (4):

$$
\begin{aligned}
h_f &= 3.66\,\mathrm{m} \\
\mathrm{Re} &= 26930 \\
f &= 0.024
\end{aligned}
$$

$$
\boxed{V = 2.56\,\mathrm{m/s}}
$$

REVIEW

1. Notice that the turbulent flow assumption is valid because Re > 2300.
2. An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.

10.52: PROBLEM DEFINITION

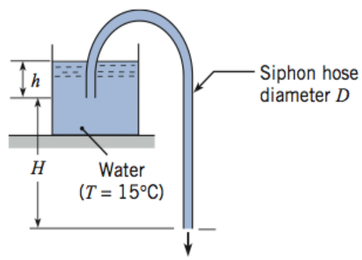
Situation:

Water is flowing out a plastic siphon hose.

$Q = 0.0015\,\text{m}^3/\,\text{s}, \quad H = 5\,\text{m}.$

$h = 0.5\,\text{m}, \quad L = 7\,\text{m}, \; k_s = 0.$

Sketch:



Find:

Diameter of hose (meters).

Assumptions:

Steady flow.

Component head loss is zero.

Turbulent flow. Also, $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5, $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}.$

PLAN Classify this problem as case 3 (D is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 5 equations with 5 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

SOLUTION

1. Governing equations:

- Flow rate equation:

$$Q = V\left(\frac{\pi D^2}{4}\right) \tag{1}$$

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + h) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \end{aligned} \tag{2}$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{3}$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{4}$$

- Reynolds number:

$$\mathrm{Re} = \frac{VD}{\nu} \tag{5}$$

3. Solution of Eqs. (1) to (5):

$$\begin{aligned} h_f &= 4.715\,\mathrm{m} \\ \mathrm{Re} &= 75900 \\ f &= 0.0189 \\ V &= 3.92\,\mathrm{m/s} \end{aligned}$$

$$\boxed{D = 0.022\,\mathrm{m}}$$

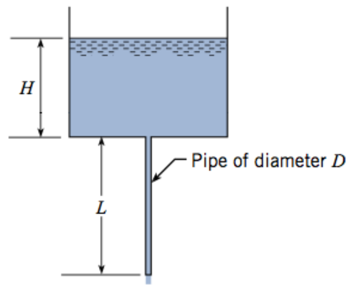
REVIEW

- Notice that the turbulent flow assumption is valid because $\mathrm{Re} > 2300$.
- An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.
- Notice that most of the elevation head $(5.5\,\mathrm{m})$ is converted to head loss $(4.72\,\mathrm{m})$.

10.53: PROBLEM DEFINITION

Situation:

Water is draining out of a tank through a galvanized iron pipe.
$k_s = 0.006\,\text{in} = 5 \times 10^{-4}\,\text{ft}$, $L = 10\,\text{ft}$, $H = 4\,\text{ft}$.
The pipe is 1-in schedule 40 NPS, $D = 1.049\,\text{in} = 0.08742\,\text{ft}$.



Find:

Velocity in the pipe (ft/s).
Flow rate (cfs)

Assumptions:

Steady flow.
Component head loss is zero.
Turbulent flow. Also, $\alpha_2 = 1.0$.

Properties:

Water (70 °F), Table A.5, $\rho = 1.94\,\text{slug/ft}^3$, $\gamma = 62.3\,\text{lbf/ft}^3$, $\nu = 1.06 \times 10^{-5}\,\text{ft}^2/\text{s}$.

PLAN Classify this problem as case 2 (V is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).
3. Find the flow rate with $Q = VA$.

SOLUTION

1. Governing equations:

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + (H + L) + 0 = 0 + \frac{V_2^2}{2g} + 0 + h_f \quad (1)$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{3}$$

- Reynolds number:

$$\mathrm{Re} = \frac{VD}{\nu} \tag{4}$$

2. Solution of Eqs. (1) to (4):

$$\begin{aligned} h_f &= 11.04\,\mathrm{ft} \\ \mathrm{Re} &= 113800 \\ f &= 0.0326 \end{aligned}$$

$$\boxed{V = 13.8\,\mathrm{ft/s}}$$

3. Flow rate equation:

$$\begin{aligned} Q &= \frac{\pi D^2}{4} V \\ &= \left(\frac{\pi\,(0.08742\,\mathrm{ft})^2}{4}\right)(13.8\,\mathrm{ft/s}) = 8.2831 \times 10^{-2}\,\mathrm{ft^3/s} \end{aligned}$$

$$\boxed{Q = 0.0828 \text{ cfs}}$$

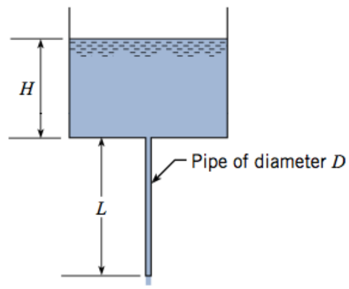
REVIEW

- Notice that the turbulent flow assumption is valid because Re > 2300.
- An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.
- Notice that most of the elevation head (14 ft) is converted to head loss (11 ft).

10.54: PROBLEM DEFINITION

Situation:

Water is draining out of a tank through a galvanized iron pipe.
$L = 2\,\mathrm{m}$, $H = 1\,\mathrm{m}$, $k_s = 0.15 \times 10^{-3}\,\mathrm{m}$
The pipe is 0.5-in schedule 40 NPS, $D = 0.622\,\mathrm{in} = 0.0158\,\mathrm{m}$.



Find:

Velocity in the pipe (ft/s).

Assumptions:

Steady flow.
Component head loss is zero.
Turbulent flow, so $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5, $\nu = 1.14 \times 10^{-6}\,\mathrm{m}^2/\,\mathrm{s}$.

PLAN

Classify this problem as case 2 (V is unknown), then
1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

SOLUTION

1. Governing equations:

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + L) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \qquad (1) \end{aligned}$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{3}$$

- Reynolds number:

$$\mathrm{Re} = \frac{VD}{\nu} \tag{4}$$

3. Solution of Eqs. (1) to (4):

$$\begin{aligned} h_f &= 2.50\,\mathrm{m} \\ \mathrm{Re} &= 43600 \\ f &= 0.039 \end{aligned}$$

$$\boxed{V = 3.15\,\mathrm{m/s}}$$

REVIEW

1. Notice that the turbulent flow assumption is valid because Re > 2300.
2. An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.

10.55: PROBLEM DEFINITION

Situation:

Air is flowing in a horizontal copper tube.
$L = 150\,\text{m}$, $Q = 0.1\,\text{m}^3/\,\text{s}$, $k_s = 1.5 \times 10^{-6}\,\text{m}$.
Pressure drop in the tube cannot exceed $\Delta p = 6$ in-H20 $= 1493\,\text{Pa}$.

Find:

Tube diameter (meters).

Assumptions:

Steady flow. Fully developed flow.
Component head loss is zero.
Turbulent flow.

Properties:

Air (40 °C, 1 atm), Table A.5, $\gamma = 11.1\,\text{N}/\,\text{m}^3$, $\nu = 1.69 \times 10^{-5}\,\text{m}^2/\,\text{s}$.

PLAN

Classify this problem as case 3 (D is unknown), then
1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 5 equations with 5 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

SOLUTION

1. Governing equations:

- Flow rate equation:

$$Q = V\left(\frac{\pi D^2}{4}\right) \tag{1}$$

- Energy equation (section 1 is located 150 m upstream from section 2).

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

  - Since the flow is fully developed, the KE terms cancel out.
  - let $p_1 - p_2 = \Delta p$
  - $z_1 = z_2 = h_p = h_t = 0$

$$\Delta p = \gamma h_f \tag{2}$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D}\frac{V^2}{2g} \tag{3}$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \tag{4}$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \tag{5}$$

3. Solution of Eqs. (1) to (5):

$$\begin{aligned} h_f &= 134.5\,\text{m} \\ \text{Re} &= 67000 \\ f &= 0.0195 \\ V &= 10.1\,\text{m/s} \end{aligned}$$

$$\boxed{D > 0.112\,\text{m}}$$

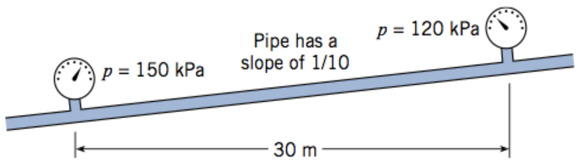
REVIEW. The turbulent flow assumption is valid because Re > 2300.

10.56: PROBLEM DEFINITION

Situation:

A fluid flows through a galvanized iron pipe.
$D = 8\,\text{cm}$.
Pipe slope is 1 Horizontal to 10 Vertical.

Sketch:



Find:

Flow rate.

Properties:

From Table 10.4 $k_s = 0.15$ mm.
$\rho = 800\,\text{kg/m}^3$, $\nu = 10^{-6}\,\text{m}^2/\text{s}$.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f \\
\frac{150000\,\text{Pa}}{800\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2} + \frac{V_1^2}{2g} + 0 &= \frac{120000\,\text{Pa}}{800\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2} + \frac{V_2^2}{2g} + 3\,\text{m} + h_f \\
h_f &= 0.823\,\text{m} \\
((D^{3/2})/(\nu)) \times (2gh_f/L)^{1/2} &= ((0.08)^{3/2}/10^{-6}) \times (2 \times 9.81 \times 0.823/30.14)^{1/2} \\
&= 1.66 \times 10^4
\end{aligned}$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.08} = 1.9 \times 10^{-3}$$

Resistance coefficient. From Fig. 10-8 $f = 0.025$. Then

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Solving for V

$$
\begin{aligned}
V &= \sqrt{\left(\frac{h_f}{f}\right)\left(\frac{D}{L}\right)2g} \\
&= \sqrt{\left(\frac{0.823\,\mathrm{m}}{0.025}\right)\left(\frac{0.08\,\mathrm{m}}{30.14\,\mathrm{m}}\right)\times 2\times 9.81\,\mathrm{m/\,s^2}} = 1.312\ \mathrm{m/s} \\
Q &= VA \\
&= 1.312\,\mathrm{m/\,s}\times(\pi/4)\times(0.08\,\mathrm{m})^2 = 6.59\times 10^{-3}\,\mathrm{m^3/\,s}
\end{aligned}
$$

$$
\boxed{Q = 6.59\times 10^{-3}\,\mathrm{m^3/\,s}}
$$

10.57: PROBLEM DEFINITION

Situation:

Commercial steel pipe will convey water.
$h_L = 1$ ft per 1000 ft of pipe length.
$Q = 300\,\mathrm{ft}^3/\,\mathrm{s}$.

Find:

Pipe diameter to produce specified head loss.

Assumptions:

The pipes are available in even inch sizes (e.g. 10 in., 12 in., 14 in., etc.)

Properties:

Water (60 °F), Table A.5: $\nu = 1.22 \times 10^{-5}$ ft^2/s.
From Table 10.4: $k_s = 0.002\,\mathrm{in} = 1.\,7 \times 10^{-4}\,\mathrm{ft}$.

SOLUTION

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= f\frac{L}{D}\frac{Q^2}{2gA^2} \\ &= f\frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Solve for diameter

$$D = \left(f\frac{8LQ^2}{g\pi^2 h_f}\right)^{1/5}$$

Assume $f = 0.015$

$$\begin{aligned} D &= \left(0.015\frac{8\,(1000\,\mathrm{ft})\,\left(300\,\mathrm{ft}^3/\,\mathrm{s}\right)^2}{32.2\,\mathrm{ft}/\,\mathrm{s}^2 \times \pi^2 \times 1\,\mathrm{ft}}\right)^{1/5} \\ &= 8.06\,\mathrm{ft} \end{aligned}$$

Now get a better estimate of f:

$$\begin{aligned} \mathrm{Re} &= \frac{4Q}{\pi D\nu} = 3.9 \times 10^6 \\ f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.002/12}{3.7\times 8.06} + \frac{5.74}{(3.9\times 10^6)^{0.9}}\right)\right]^2} \\ &= 0.0104 \end{aligned}$$

Compute D again:

$$\begin{aligned} D &= \left(0.0104 \frac{8\,(1000\,\text{ft})\left(300\,\text{ft}^3/\,\text{s}\right)^2}{32.2\,\text{ft}/\,\text{s}^2 \times \pi^2 \times 1\,\text{m}}\right)^{1/5} \\ &= 7.49\,\text{ft} \end{aligned}$$

Thus, specify a pipe with $\boxed{D = 90 \text{ in}}$

10.58: PROBLEM DEFINITION

Situation:

A steel pipe will carry crude oil.

$Q = 0.1\,\mathrm{m^3/\,s}$.

$h_L = 50\,\mathrm{m}$ per km of pipe length.

Find:

(a) Diameter of pipe for a head loss of 50 m.

(b) Pump power.

Assumptions:

Available pipe diameters are $D = 20,\ 22$, and $24\,\mathrm{cm}$.

Properties:

From Table 10.4: $k_s = 0.046$ mm.

$S = 0.93$, $\nu = 10^{-5}\,\mathrm{m^2/\,s}$.

SOLUTION

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= f\frac{L}{D}\frac{Q^2}{2gA^2} \\ &= f\frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Solve for diameter

$$D = \left(f\frac{8LQ^2}{g\pi^2 h_f}\right)^{1/5}$$

Assume $f = 0.015$

$$\begin{aligned} D &= \left(0.015\frac{8\,(1000\,\mathrm{m})\,(0.1\,\mathrm{m^3/\,s})^2}{9.81\,\mathrm{m/\,s^2} \times \pi^2 \times 50\,\mathrm{m}}\right)^{1/5} \\ &= 0.19\,\mathrm{m} \end{aligned}$$

Calculate a more accurate value of f

$$\begin{aligned}
\text{Re} &= \frac{4Q}{\pi D \nu} \\
&= \frac{4 \times 0.1\,\text{m}^3/\,\text{s}}{\pi \times 0.19\,\text{m} \times 10^{-5}\,\text{m}^2/\,\text{s}} \\
&= 6.7 \times 10^4 \\
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{0.046}{3.7\times 190} + \frac{5.74}{67000^{0.9}}\right)\right]^2} \\
&= 0.021
\end{aligned}$$

Recalculate diameter using new value of f

$$\begin{aligned}
D &= \left(\frac{0.021}{0.015}\right)^{1/5} \times 0.19 \\
&= 0.203 \text{ m} = 20.3 \text{ cm}
\end{aligned}$$

Use the next larger size of pipe; $\boxed{D = 22 \text{ cm}}$

Power equation (assume the head loss is remains at $h_L \approx 50$ m/1,000 m)

$$\begin{aligned}
P &= Q\gamma h_f \\
&= 0.1 \times (0.93 \times 9810) \times 50 = 45.6\,\text{kW}
\end{aligned}$$

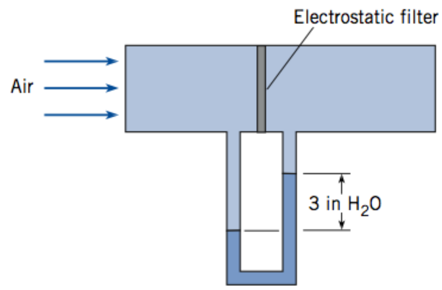
$$\boxed{P = 45.6\,\text{kW for each kilometer of pipe length}}$$

10.59: PROBLEM DEFINITION

Situation:

An electrostatic air filter is being tested.

Pressure drop is Δp =3 in.-$H_2$0. Air speed is $V = 10\,\mathrm{m/s}$.



Find: The minor loss coefficient (K) for the filter.

Properties:

Air (20 °C) Table A.3: $\nu = 15.1 \times 10^{-6}\,\mathrm{m^2/s}$.

$\rho = 1.2\,\mathrm{kg/m^3}$, $\gamma = 11.8\,\mathrm{N/m^3}$.

PLAN

Apply the energy equation to relate the pressure drop to head loss. Then, find the minor loss coefficient using $h_L = KV^2/2g$.

SOLUTION

Energy equation (select a control volume surrounding the filter)

$$\left(\frac{p}{\gamma}\right)_1 = \left(\frac{p}{\gamma}\right)_2 + h_L$$

Thus

$$\begin{aligned} h_L &= \frac{\Delta p}{\gamma_{\text{air}}} \\ &= \frac{(3\text{ in.-H}_2\text{O})\left(249.2\frac{\text{Pa}}{\text{in.-H}_2\text{O}}\right)}{11.8\,\mathrm{N/m^3}} \\ &= 63.36\,\mathrm{m} \end{aligned}$$

Head loss

$$
\begin{aligned}
h_L &= \frac{KV^2}{2g} \\
K &= \frac{2gh_L}{V^2} \qquad (1) \\
&= \frac{2\,(9.81\,\mathrm{m/\,s^2})\,(63.36\,\mathrm{m})}{(10\,\mathrm{m/\,s})^2} \\
&= 12.43 \\
&\boxed{K = 12.4} \qquad (2)
\end{aligned}
$$

REVIEW

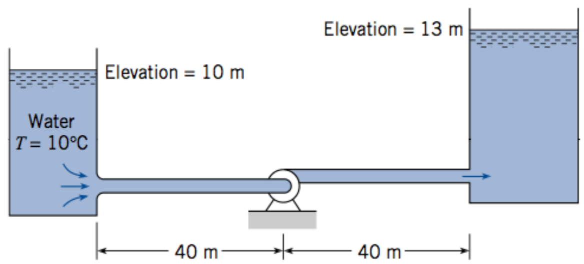
1.) This minor loss coefficient is larger than the coefficient for any components listed in Table 10.5.
2.) Combining Eqs. (1) and (2) gives $K = \Delta p/(\rho V^2/2)$. Thus, the pressure drop for the filter is about 12 times larger that the pressure change that results when the flow is brought to rest.

10.60: PROBLEM DEFINITION

Situation:

Water is pumped between reservoirs through a steel pipe.
$Q = 0.1\,\text{m}^3/\,\text{s}$, $D = 15\,\text{cm}$.

Sketch:



Find:

Power that is supplied to the system by the pump.

Properties:

From Table 10.4: $k_s = 0.046$ mm.
Water (10 °C), Table A.5: $v = 1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}$, $\gamma = 9810\,\text{N}/\,\text{m}^3$.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.1\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.15\,\text{m})^2} \\ &= 5.66\ \text{m/s} \\ \frac{V^2}{2g} &= 1.63\ \text{m} \\ \frac{k_s}{D} &= \frac{0.0046}{15} = 0.0003 \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{v} = \frac{5.66\,\text{m}/\,\text{s} \times 0.15\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}} \\ &= 6.4 \times 10^5 \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

$$f = 0.016$$

Energy equation (between the reservoir surfaces)

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
h_p &= z_2 - z_1 + \frac{V^2}{2g}(K_e + f\frac{L}{D} + K_{\mathrm{E}}) \\
&= 13\,\mathrm{m} - 10\,\mathrm{m} + 1.63\,\mathrm{m} \times (0.1 + 0.016 \times \frac{80\,\mathrm{m}}{0.15\,\mathrm{m}} + 1) \\
&= 3 + 15.7 = 18.7\ \mathrm{m}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= 0.10\,\mathrm{m^3/\,s} \times 9810\,\mathrm{N/\,m^3} \times 18.7\,\mathrm{m} \\
&= 18,345\ \mathrm{W}
\end{aligned}
$$

$$
\boxed{P = 18.3\,\mathrm{kW}}
$$

10.61: PROBLEM DEFINITION

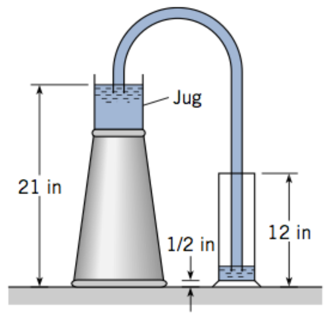
Situation:

A siphon tube is used to drain water from a jug into a graduated cylinder.

$D_{\text{tube}} = 3/16$ in. $= 0.01562$ ft.

$L_{\text{tube}} = 50$ in, $\forall = 500\,\text{ml}$.

Sketch:



Find:

Time to fill cylinder.

Assumptions:

$T \simeq 60^{\text{o}}\text{F}$ with $\nu = 1.2 \times 10^{-5}$ ft^2/s.

Neglect head loss associated with any bend in the Tygon tube.

SOLUTION

Energy equation (from the surface of the water in the jug to the surface in the graduated cylinder)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \quad (1)$$

Assume that the entrance loss coefficient is equal to 0.5. It could be larger than 0.5, but this should yield a reasonable approximation. Therefore

$$\sum h_L = (0.5 + f\frac{L}{D} + K_E)\frac{V^2}{2g}$$

The exit loss coefficient, K_E, is equal to 1.0. Therefore, Eq. 1 becomes

$$\begin{aligned} \Delta z &= z_j - z_c = \left(\frac{V_2^2}{2g}\right)\left(1.5 + f\frac{L}{D}\right) \\ \text{or } V &= \sqrt{\frac{2g\Delta z}{1.5 + f\frac{L}{D}}} \quad (1) \\ &= \sqrt{\frac{2g\Delta z}{1.5 + f \times 267}} \end{aligned}$$

Assume $f = 0.03$ and let $\Delta z = (21 - 2.5)/12 = 1.54$ ft. Then

$$\begin{aligned} V &= \sqrt{\frac{2g \times 1.54\,\text{ft}}{1.5 + 10.7}} \\ &= 2.85 \text{ ft/s} \\ \text{Re} &= \frac{VD}{\nu} \\ &= \frac{2.85\,\text{ft/ s} \times .01562\,\text{ft}}{1.2 \times 10^{-5}\,\text{ft}^2/\,\text{s}} \\ &= 3710 \end{aligned}$$

Resistance coefficient (recalculate)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{3710^{0.9}}\right)\right]^2} \\ &= 0.040 \end{aligned}$$

Repeat calculations with a new value of friction factor.

$$\begin{aligned} V &= \sqrt{\frac{2g \times 1.54}{1.5 + 10.68}} \\ &= 2.85 \text{ ft/s} \\ \text{Re} &= \frac{VD}{\nu} \\ &= 3710 \end{aligned}$$

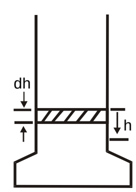


Fig. A

Use $f = 0.040$ for final solution. As a simplifying assumption assume that as the cylinder fills the level of water in the jug has negligible change. As the cylinder is being filled one can visualize (see figure) that in time dt a volume of water equal to Qdt will enter the cylinder and that volume in the cylinder can be expressed as $A_c dh$, that is

$$\begin{aligned} Qdt &= A_c dh \\ dt &= \frac{A_c}{Q} dh \end{aligned}$$

But

$$Q = V_t A_t \tag{3}$$

so

$$dt = \frac{A_c/A_t}{V} dh$$

Substitute V of Eq. (1) into Eq. (2):

$$\begin{aligned} dt &= \frac{A_c/A_t}{(2g\Delta z/(1.5+267f))^{1/2}} dh \\ \not{V}_c &= 0.500 \text{ liter } = 0.01766 \text{ ft}^3 \end{aligned}$$

or

$$\begin{aligned} 0.01766\,\text{ft}^3 &= A_c \times \left(\frac{11.5}{12}\right) \text{ ft} \\ A_c &= 0.01842 \text{ ft}^2 \\ A_{\text{tube}} &= (\pi/4)\left(\frac{3/16}{12}\text{ ft}\right)^2 = 0.0001917 \text{ ft}^2 \\ \frac{A_c}{A_t} &= 96.1 \end{aligned}$$

The differential equation becomes

$$dt = \frac{96.1}{(2g\Delta z/(1.5+10.9))^{1/2}} dh$$

Let h be measured from the level where the cylinder is 2 in full. Then

$$\begin{aligned} \Delta z &= \frac{21 \text{ in} - 2.5 \text{ in}}{12} - h \\ \Delta z &= 1.542 - h \end{aligned}$$

Now we have

$$\begin{aligned} dt &= \frac{96.1}{(2g(1.54-h)/12.2)^{1/2}} dh \\ dt &= \frac{42.2}{(1.54-h)^{1/2}} dh \\ dt &= -\frac{42.2}{(1.54-h)^{1/2}} (-dh) \end{aligned}$$

Integrate:

$$
\begin{aligned}
t &= -\frac{42.2(1.54-h)^{1/2}}{1/2}|_0^h \\
&= -84.4(1.54-h)^{1/2}|_0^{0.75} \\
&= -84.4[(0.79)^{1/2}-(1.54)^{1/2}] \\
&= -84.4(0.889-1.241) \\
& \boxed{t = 29.7 \text{ s}}
\end{aligned}
$$

REVIEW

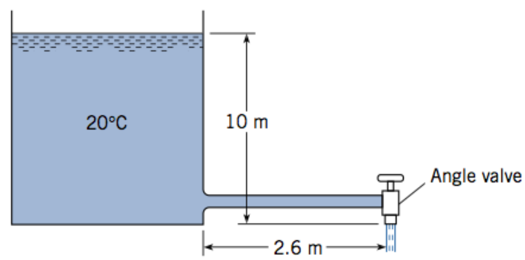
Possible problems with this solution: The Reynolds number is very close to the point where turbulent flow will occur and this would be an unstable condition. The flow might alternate between turbulent and laminar flow.

10.62: PROBLEM DEFINITION

Situation:

Water exits a tank through a short galvanize iron pipe.
$D_{\text{tank}} = 2\,\text{m}$, $D_{\text{pipe}} = 26\,\text{mm}$.
$L_{\text{pipe}} = 2.6\,\text{m}$, $z_1 = 10\,\text{m}$.
Fully open angle valve: $K_v = 5.0$.

Sketch:



Find:

Time required for the water level in tank to drop from 10 m to 2 m.

Assumptions:

The pipe entrance is smooth: $K_e \approx 0$
The kinetic energy correction factor in the pipe is $\alpha_2 = 1.0$

PLAN

Apply the energy equation from the top of the tank (location 1) to the exit of the angle valve (location 2).

SOLUTION

Energy equation

$$h = \alpha_2 \frac{V^2}{2g} + \frac{V^2}{2g}(K_e + K_v + f\frac{L}{D})$$

Term by term analysis

$$\begin{aligned} \alpha_2 &= 1.0 \\ K_e &\approx 0,\ K_v = 5.0 \\ L/D &= 2.6/0.026 = 100.0 \end{aligned}$$

Combine equation and express V in terms of h

$$V = \sqrt{\frac{2gh}{6 + 100 \times f}}$$

Sand roughness height

$$\frac{k_s}{D} = \frac{0.15}{26} = 5.8 \times 10^{-3}$$

Reynolds number

$$\text{Re} = \frac{V \times 0.026}{10^{-6}} = 2.6 \times 10^4 V$$

Rate of decrease of height

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{0.000531}{3.14} V = -0.000169V$$

A program was written to first find V iteratively for a given h using the Swamee-Jain equation for the friction factor. Then a new h was found by

$$h_n = h_{n-1} - 0.000169V\Delta t$$

where Δt is the time step. The result was 1424 s or 23.7 minutes.

$$\boxed{t = 23.7\,\text{min}}$$

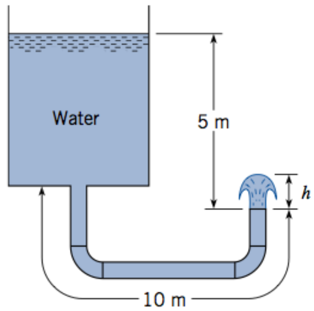
REVIEW

1. When valves are tested to evaluate K_{valve} the pressure taps are usually connected to pipes both upstream and downstream of the valve. Therefore, the head loss in this problem may not actually be $5V^2/2g$.
2. The velocity exiting the valve will probably be highly non-uniform; therefore, this solution should be considered as an approximation only.

10.63: PROBLEM DEFINITION

Situation:

Water drains from a tank, passes through a pipe and then jets upward.
$D = 1.5\,\text{cm}$, $L = 10\,\text{m}$, $\Delta z = 5\,\text{m}$.
Two 90° elbows in pipe.



Find:

(a) Exit velocity of water (m/s).
(b) Height of water jet (cm).

Assumptions:

The pipe is galvanized iron.
The water temperature is 20°C so $\nu = 10^{-6}$ m^2/s.
Relative roughness $k_s/D = .015/1.5 = 0.01$. Start iteration at $f = 0.035$.

Properties:

From Table 10.4 $k_s = 0.15$ mm $= 0.015$ cm.
From Table 10.5 $K_b = 0.9$ and $K_e = 0.5$.

PLAN

Apply the energy equation from the water surface in the tank to the pipe outlet.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 5 &= 0 + \alpha_2 \frac{V_2^2}{2g} + 0 + (K_e + 2K_b + f\frac{L}{D})\frac{V_2^2}{2g} \\
5 &= \left(\frac{V_2^2}{2g}\right)\left(1 + 0.5 + 2 \times 0.9 + .035 \times \frac{10\,\text{m}}{0.015\,\text{m}}\right) \\
5 &= \left(\frac{V_2^2}{2 \times 9.81}\right)(26.6) \\
V_2 &= 1.920 \text{ m/s}
\end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= 1.92 \times 0.015/10^{-6} \\ &= 2.88 \times 10^4. \end{aligned}$$

Resistance coefficient (new value)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.01}{3.7} + \frac{5.74}{28800^{0.9}}\right)\right]^2} \\ &= 0.040 \end{aligned}$$

Recalculate V_2 with this new value of f

$$\boxed{V_2 = 1.81 \text{ m/s}}$$

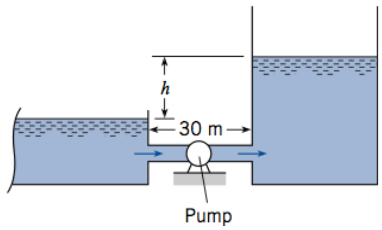
Energy equation (from the pipe outlet to the top of the water jet)

$$\begin{aligned} h &= \frac{V^2}{2g} \\ &= \frac{(1.81\,\text{m/ s})^2}{2 \times 9.81\,\text{m/ s}^2} \\ &= 0.1670 \text{ m} \\ &= \boxed{h = 16.7\text{cm}} \end{aligned}$$

10.64: PROBLEM DEFINITION

Situation:

A pump operates between a reservoir and a tank.
$h_p = h_o(1 - Q^2/Q_{\max}^2)$, $h_o = 50\,\mathrm{m}$.
$Q_{\max} = 2\,\mathrm{m}^3/\,\mathrm{s}$, $f = 0.18$.
$D = 90\,\mathrm{cm}$, $A_{\tan\mathrm{k}} = 100\,\mathrm{m}^2$.



Find:

Time to fill tank to 40 meters.

Properties:

From Table 10.5: $K_e = 0.5$ and $K_E = 1.0$.

PLAN

Apply the energy equation from the reservoir water surface to the tank water surface. The head losses will be due to entrance, pipe resistance, and exit.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + z_1 + h_p &= 0 + 0 + z_2 + (K_e + f\frac{L}{D} + K_E)\frac{V^2}{2g} \\
h_p &= (z_2 - z_1) + \left(0.5 + \left(0.018 \times \frac{30}{0.9}\right) + 1.0\right)\frac{V^2}{2g} \\
h_p &= h + (2.1)\frac{V^2}{2g}
\end{aligned}$$

But the head supplied by the pump is $h_o(1 - (Q^2/Q_{\max}^2))$ so

$$\begin{aligned}
h_o\left(1 - \frac{Q^2}{Q_{\max}^2}\right) &= h + 1.05\frac{V^2}{g} \\
50\left(1 - \frac{Q^2}{4}\right) &= h + 1.05\frac{Q^2}{gA^2} \\
50 - 12.5Q^2 &= h + 1.05\frac{Q^2}{gA^2}
\end{aligned}$$

Area

$$A = (\pi/4)D^2 = (\pi/4)(0.9^2) = 0.63\,\mathrm{m}^2$$

So

$$\begin{aligned} 50 - 12.5Q^2 &= h + 0.270Q^2 \\ 50 - h &= 127.77Q^2 \\ \sqrt{50-h} &= 3.57Q \end{aligned}$$

The discharge into the tank and the rate of water level increase is related by

$$Q = A_{\text{tank}}\frac{dh}{dt}$$

so

$$\sqrt{50-h} = 3.57A_{\text{tank}}\frac{dh}{dt}$$

or

$$dt = 3.57A_{\text{tank}}(50-h)^{-1/2}dh$$

Integrating

$$t = 2 \times 3.57A_{\text{tank}}(50-h)^{1/2} + C$$

when $t = 0$, $h = 0$ and $A_{\text{tank}} = 100$ m^2 so

$$t = 714(7.071 - (50-h)^{1/2})$$

When $h = 40$ m

$$t = 2791 \text{ s}$$

$$\boxed{t = 46.5 \text{ min}}$$

10.65: PROBLEM DEFINITION

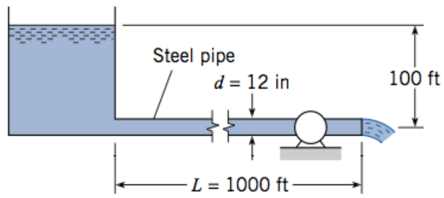
Situation:

Water flows out of reservoir, through a steel pipe and a turbine.

$Q = 5\,\text{ft}^3/\,\text{s}$, $\eta = 0.8$, $\Delta z = 100\,\text{ft}$.

$D = 12\,\text{in}$, $L = 1000\,\text{ft}$.

Sketch:



Find:

Power delivered by turbine.

Assumptions:

Turbulent flow, so $\alpha_2 \approx 1$.

Properties:

Water (70°F), Table A.5: $\nu = 1.06 \times 10^{-5}$ ft^2/s

PLAN

Apply the energy equation from the reservoir water surface to the jet at the end of the pipe.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \sum h_L \\
0 + 0 + z_1 &= 0 + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \left(K_e + f \frac{L}{D} \right) \frac{V^2}{2g} \\
z_1 - z_2 &= h_T + (1 + 0.5 + f \frac{L}{D}) \frac{V^2}{2g} \\
100 \text{ ft} &= h_T + \left(1.5 + f \frac{L}{D} \right) \frac{V^2}{2g}
\end{aligned}$$

But

$$
\begin{aligned}
V &= \frac{Q}{A} = \frac{5\,\mathrm{ft}^3/\,\mathrm{s}}{(\pi/4)\,(1\,\mathrm{ft})^2} = 6.37\ \mathrm{ft/s} \\
\frac{V^2}{2g} &= 0.6293\ \mathrm{ft} \\
\mathrm{Re} &= \frac{VD}{v} = 6.0\times 10^5
\end{aligned}
$$

From Fig. 10.8 $f = 0.015$ for $k_s/D = 0.000167$. Then

$$
\begin{aligned}
100\ \mathrm{ft} &= h_T + \left(1.5 + 0.0150\times\frac{1000\,\mathrm{ft}}{1\,\mathrm{ft}}\right)(0.6293\,\mathrm{ft}) \\
h_T &= (100 - 10.35)\,\mathrm{ft} = 89.65\,\mathrm{ft}
\end{aligned}
$$

Power equation

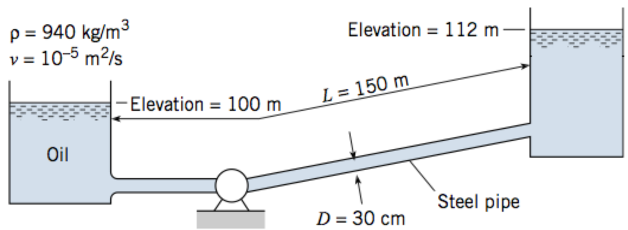
$$
\begin{aligned}
P &= Q\gamma h_T\eta \\
&= \left(5\,\mathrm{ft}^3/\,\mathrm{s}\right)\left(62.4\,\mathrm{lbf}/\,\mathrm{ft}^3\right)(89.6\,\mathrm{ft})\,(0.80) \\
&= 22,364\ \mathrm{ft}\cdot\mathrm{lbf/s} \\
& \boxed{P = 40.7\ \mathrm{horsepower}}
\end{aligned}
$$

10.66: PROBLEM DEFINITION

Situation:

Oil is pumped from a lower reservoir to an upper reservoir through a steel pipe.
$D = 30\,\text{cm}$, $Q = 0.20\,\text{m}^3/\,\text{s}$.
$z_1 = 100\,\text{m}$, $z_2 = 112\,\text{m}$, $L = 150\,\text{m}$.

Sketch:



Find:

(a) Pump power.
(b) Sketch an EGL and HGL.

Properties:

$\rho = 940\,\text{kg}/\,\text{m}^3$, $v = 10^{-5}\,\text{m}^2/\,\text{s}$.
From Table 10.4 $k_s = 0.046$ mm

PLAN

Apply the energy equation between reservoir surfaces .

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
100 + h_p &= 112 + \frac{V^2}{2g}(K_e + f\frac{L}{D} + K_E) \\
h_p &= 12 + \left(\frac{V^2}{2g}\right)\left(0.03 + f\frac{L}{D} + 1\right)
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
V &= \frac{Q}{A} \\
&= \frac{0.2\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.30\,\text{m})^2} \\
&= 2.83\ \text{m/s} \\
\frac{V^2}{2g} &= 0.408\,\text{m}
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{v} \\
&= \frac{2.83\,\text{m/s} \times 0.30\,\text{m}}{10^{-5}\,\text{m}^2/\text{s}} \\
&= 8.5 \times 10^4 \\
\frac{k_s}{D} &= \frac{4.6 \times 10^{-5}\,\text{m}}{0.3\,\text{m}} \\
&= 1.5 \times 10^{-4}
\end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

$$f = 0.019$$

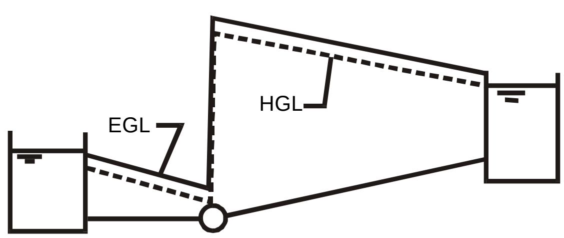
Then

$$\begin{aligned}
h_p &= 12\,\text{m} + 0.408\,\text{m}(0.03 + (0.019 \times \frac{150\,\text{m}}{0.3\,\text{m}}) + 1.0) \\
&= 16.3 \text{ m}
\end{aligned}$$

Power equation

$$\begin{aligned}
P &= Q\gamma h_p \\
&= 0.20\,\text{m}^3/\text{s} \times (940\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2) \times 16.3\,\text{m} = 30100\,\text{W}
\end{aligned}$$

$$\boxed{P = 30.1\,\text{kW}}$$



10.67: PROBLEM DEFINITION

Situation:

A cast iron pipe joins two reservoirs.

$D = 1.0\,\text{ft}$, $L = 200\,\text{ft}$.

$z_1 = 100\,\text{ft}$, $z_2 = 40\,\text{ft}$.

$z_{\text{pipe1}} = 70\,\text{ft}$, $z_{\text{pipe2}} = 30\,\text{ft}$.

Find:

(a) Calculate the discharge in the pipe.

(b) Sketch the EGL and HGL.

Properties:

From Table 10.4: $k_s = 0.01\,\text{in}$

Water (60°F), Table A.5:

$\nu = 1.22 \times 10^{-5}\ \text{ft}^2/\text{s}$, $\mu = 2.36 \times 10^{-5}\,\text{N} \cdot \text{s}/\text{m}^2$, $\rho = 1.94\,\text{slug}/\text{m}^3$.

PLAN

Apply the energy equation from the water surface in the upper reservoir to the water surface in the lower reservoir.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 100 &= 0 + 0 + 40 + \left(K_e + 2K_{\text{v}} + +K_E + f\frac{L}{D} \right) \frac{V^2}{2g} \\
100\,\text{ft} &= 40\,\text{ft} + \left(0.5 + 2 \times 0.2 + 1.0 + f \times \frac{200\,\text{ft}}{1\,\text{ft}} \right) \frac{V^2}{2g}
\end{aligned}$$

The equation for V becomes

$$\frac{V^2}{2g} = \frac{60}{1.9 + 200f} \tag{1}$$

Relative roughness

$$\begin{aligned}
\frac{k_s}{D} &= \frac{0.01\,\text{ft}}{12\,\text{ft}} \\
&= 8.3 \times 10^{-4}
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{V \times 1.0\,\text{ft}}{1.22 \times 10^{-5}\,\text{ft}^2/\text{s}} \\
&= \left(8.20 \times 10^4 \times V\right) \qquad (2)
\end{aligned}$$

Friction factor (Swamee-Jain correlation–Eq. 10.26)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{8.3\times10^{-4}}{3.7} + \frac{5.74}{(8.20\times10^{4}\times V)^{0.9}}\right)\right]^2} \tag{3}$$

Solve Eqs. (1) to (3) simultaneously (we applied a computer program, TK Solver)

$$\begin{aligned} V &= 26.0\,\mathrm{m/\,s} \\ \mathrm{Re} &= 2,130,000 \\ f &= 0.019 \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 26.0\,\mathrm{ft/\,s}(\pi/4 \times (1\,\mathrm{ft})^2) \\ &= \boxed{Q = 20.4 \text{ cfs}} \end{aligned}$$

10.68: PROBLEM DEFINITION

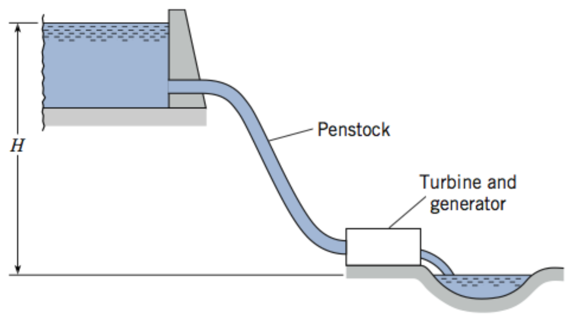
Situation:

A small stream fills a reservoir–water from this reservoir is used to create electrical power.

$Q = 2$ cfs, $H = 34\,\text{ft}$.

h_f =3 ft, $L = 87\,\text{ft}$.

Sketch:



Find:

Find the minimum diameter for the penstock pipe.

Assumptions:

Neglect minor losses associated with flow through the penstock.

Assume that pipes are available in even sizes–that is, 2 in., 4 in., 6 in., etc.

Assume a smooth, plastic pipe– $k_s = 0$.

Assume turbulent flow (check this after the calculation is done).

Properties:

Water (40 °F), Table A.5: $\nu = 1.66 \times 10^{-5}\,\text{ft}^2/\,\text{s}$.

PLAN

Apply the Darcy-Weisbach equation to relate head loss (h_f) to pipe diameter. Apply the Swamee-Jain correlation to relate friction factor (f) to flow velocity. Also, write equations for the Reynolds number and the flow rate. Solve these four equations simultaneously to give values of D, V, f, and Re.

SOLUTION

Darcy-Weisbach equation

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \tag{2}$$

Reynolds number

$$\mathrm{Re} = \frac{VD}{\nu} \tag{3}$$

Flow rate equation

$$Q = V\frac{\pi D^2}{4} \tag{4}$$

Solve Eqs. (1) to (4) simultaneously. The computer program TKSolver was used for our solution.

$$\begin{aligned} f &= 0.01448 \\ V &= 9.026\,\mathrm{ft/s} \\ D &= 6.374\,\mathrm{in} \\ \mathrm{Re} &= 289,000 \end{aligned}$$

Recommendation

Select a pipe with $D = 8\,\mathrm{in}$.

REVIEW

With an 8-inch-diameter pipe, the head loss associated with flow in the pipe will be less than 10% of the total available head (34 ft). If an engineer selects a pipe that is larger that 8 inches, then cost goes up.

10.69: PROBLEM DEFINITION

Situation:

A pipe runs from a reservoir to an open drain.

$z_{\text{reservoir}} = 120\,\text{ft}$, $z_{\text{pipe1}} = 100\,\text{ft}$, $z_{\text{pipe2}} = 70\,\text{ft}$.

$D = 6\,\text{in}$, $L = 100\,\text{ft}$, $p_1 = p_2 = 0$ psi.

Find:

Discharge $\left(\text{ft}^3/\,\text{s}\right)$.

Properties:

From Table 10.4: $k_s = 4 \times 10^{-4}$ ft.

Water (50 °F), Table A.5: $\nu = 1.41 \times 10^{-5}$ ft^2/s.

From Table 10.5: $K_e = 0.5$.

PLAN

Apply the energy equation from water surface in reservoir to the outlet.

SOLUTION

Energy equation

$$
\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 120\,\text{ft} &= 0 + \frac{V^2}{2g} + 70\,\text{ft} + (K_e + +K_E + f\frac{L}{D})\frac{V^2}{2g} \\
\frac{V^2}{2g}\left(1.5 + f\frac{L}{D}\right) &= 50 \text{ ft} \\
\frac{V^2}{2g} &= \frac{50\,\text{ft}}{1.5 + 200f} \qquad (1)
\end{aligned}
$$

Sand roughness height

$$\frac{k_s}{D} = \frac{4 \times 10^{-4}\,\text{ft}}{0.5\,\text{ft}} = 0.0008$$

Reynolds number

$$\text{Re} = 3.54 \times 10^4 \times V \qquad (2)$$

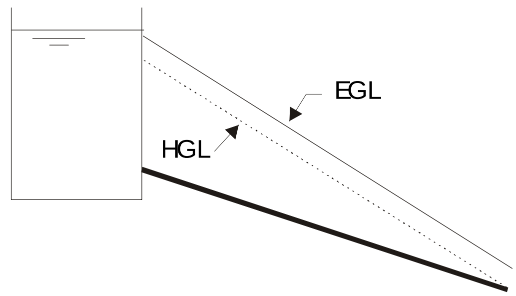
Solve a set of three simultaneous equations: Eq. (1) Eq. (2), and the Swamee-Jain correction. The result is

$$V = 24.6 \text{ ft/s}$$

Flow rate equation

$$
\begin{aligned}
Q &= VA \\
&= 24.6(\pi/4)(0.5^2)
\end{aligned}
$$

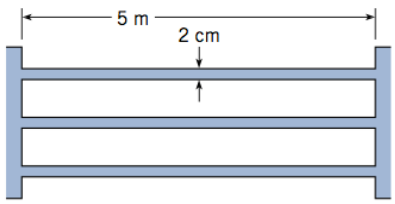
$$\boxed{Q = 4.83 \text{ cfs}}$$



10.70: PROBLEM DEFINITION

Situation:

A shell and tube heat exchanger is used in a geothermal power system
Clean fluid inside the tubes; brine outside of the tubes.
100 tubes total. Galvanized iron.
$D = 2\,\text{cm}$, $L = 5\,\text{m}$, $\dot{m} = 50\,\text{kg/s}$.
After continued used, 2 mm of build up, $k_s = .5\,\text{mm}$.



Find:

Power required to operate heat exchanger with:
(a) clean tubes.
(b) scaled tubes.

Properties:

Pipe roughness (galvanized iron), Table 10.4 (EFM9e), $k_s = 0.15$ mm.
Given fluid properties ($T = 200\,°\text{C}$) $\rho = 860\,\text{kg/m}^3$, $\mu = 1.35 \times 10^{-4}\,\text{N s/m}^2$.

SOLUTION

$$\dot{m}/\text{tube} = 0.50 \text{ kg/s}$$

$$Q/\text{tube} = \frac{0.50\,\text{kg/s}}{860\,\text{kg/m}^3} = 5.8139 \times 10^{-4} \text{ m}^3/\text{s}$$

$$V = \frac{Q}{A} = \frac{5.8139{\times}10^{-4}\,\text{m}^3/\text{s}}{(\pi/4) \times (0.02\,\text{m})^2} = 1.851 \text{ m/s}$$

$$\text{Re} = \frac{VD\rho}{\mu} = \frac{1.851\,\text{m/s} \times 0.02\,\text{m} \times 860\,\text{kg/m}^3}{1.35 \times 10^{-4}\,\text{N s/m}^2} = 2.35 \times 10^5$$

$$\frac{k_s}{D} = \frac{0.15\,\text{m}}{20\,\text{m}} \approx 0.007$$

From the Moody diagram, $f = 0.034$. Then

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.034 \times \frac{5\,\text{m}}{0.02\,\text{m}} \times \frac{(1.851\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} = 1.48 \text{ m}$$

a) $P = Q\gamma h_f = 5.8139 \times 10^{-4}\,\text{m}^3/\text{s} \times 860\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2 \times 1.48\,\text{m} \times 100 = 726\,\text{W}$

$$\boxed{P = 726\,\text{W (clean tubes)}}$$

Part (b)

$$\frac{k_s}{D} = \frac{0.5}{16}$$
$$= 0.031$$

From the Moody diagram, $f = 0.058$

$$P = 726\,\text{W} \times \frac{0.058}{0.034} \times \left(\frac{20}{16}\right)^4 = 3.03\,\text{kW}$$

$$\boxed{P = 3.03\,\text{kW (scaled tubes)}}$$

10.71: PROBLEM DEFINITION

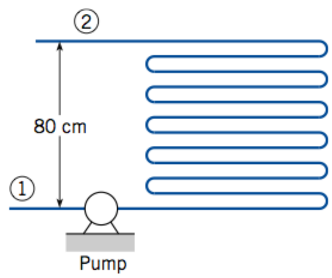
Situation:

Water flows through a heat exchanger.

$L = 20\,\text{m}$, $D = 2\,\text{cm}$, $Q = 3.4 \times 10^{-4}\,\text{m}^3/\,\text{s}$.

$T_1 = 20\,°\text{C}$, $T_2 = 80\,°\text{C}$, $\Delta z = 0.8\,\text{m}$, $p_1 = p_2$.

Sketch:



Find:

Pump power required.

Assumptions:

K =2 x K for smooth bends of 90°, $r/d \approx 1$, $K_b \approx 2 * 0.35 = 0.7$

Properties can be found at the average temperature in the heat exchanger.

Smooth tubes ($k_s = 0.0\,\text{m}$)

Properties:

Water (50 °C), Table A.5: $\nu = 5.53 \times 10^{-7}\ \text{m}^2/\text{s}$, $\rho = 998\,\text{kg}/\,\text{m}^3$, $\gamma = 9693\,\text{N}/\,\text{m}^3$.

SOLUTION

Energy equation (section 1 at inlet, section 2 at exit)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Since $V_1 = V_2$ and $p_1 = p_2$

$$h_p = h_L + (z_2 - z_1)$$

Velocity

$$V = \frac{Q}{A} = \frac{3 \times 10^{-4}\,\text{m}^3/\,\text{s}}{\pi/4 \times (0.02\,\text{m})^2} = 0.955\ \text{m/s}$$

Reynolds number and resistance coefficient

$$\begin{aligned} \text{Re} &= \frac{VD}{\upsilon} = \frac{0.955\,\text{m}/\,\text{s} \times (0.02\,\text{m})}{5.53 \times 10^{-7}\,\text{m}^2/\,\text{s}} = 34539 \\ f &= 0.022 \end{aligned}$$

Head loss

$$
\begin{aligned}
h_L &= \left(f\frac{L}{D} + 19K_b\right)\frac{V^2}{2g} = \left(0.022\frac{20\,\mathrm{m}}{0.02\,\mathrm{m}} + 19\times 0.7\right)\frac{(0.955\,\mathrm{m/\,s})^2}{2\times 9.81\,\mathrm{m/\,s^2}} \\
&= 1.64\ \mathrm{m}
\end{aligned}
$$

Final calculations

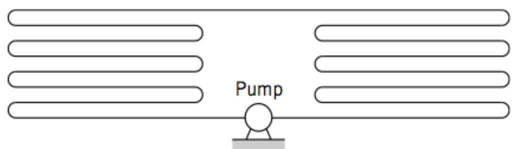
$$
\begin{aligned}
h_p &= z_2 - z_1 + h_L = 0.8 + 1.64 = 2.44\ \mathrm{m} \\
P &= \gamma h_p Q = 9693\,\mathrm{N/\,m^3}\times 2.44\,\mathrm{m}\times 3\times 10^{-4} \\
& \boxed{P = 7.10\ \mathrm{W}}
\end{aligned}
$$

10.72: PROBLEM DEFINITION

Situation:

A heat exchanger is described in the problem statement.
$V = 10\,\text{m/s}$, $D = 2\,\text{cm}$.
$\eta = 0.8$, $L = 10\,\text{m}$.
14 - 180° elbows, $K_L = 2.2$.

Sketch:



Find:

Power required to operate pump.

Properties:

Water (40 °C), Table A.5: $\nu = 6.58 \times 10^{-7}$ m²/s, $\gamma = 9732\,\text{N/m}^3$
From Table 10.4 $k_s = 0.0015$ mm.

SOLUTION

Reynolds number

$$\text{Re} = \frac{0.02 \times 10}{6.58 \times 10^{-7}} = 3.04 \times 10^5$$

Flow rate equation

$$Q = \frac{\pi}{4} \times (0.02\,\text{m})^2 \times 10\,\text{m/s} = 0.00314\ \text{m}^3/\text{s}$$

Relative roughness (copper tubing)

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3}\ \text{mm}}{20\ \text{mm}} = 7.5 \times 10^{-5}$$

Resistance coefficient (from Moody diagram)

$$f = 0.0155$$

Energy equation

$$\begin{aligned} h_p &= \frac{V^2}{2g}(f\frac{L}{D} + \sum K_L) \\ &= \frac{(10\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2}(0.0155 \times \frac{10\ \text{m}}{0.02\ \text{m}} + 14 \times 2.2) = 196\ \text{m} \end{aligned}$$

Power equation

$$
\begin{aligned}
P &= \frac{\gamma Q h_p}{\eta} \\
&= \frac{9732\,\mathrm{N/\,m^3} \times 0.00314\,\mathrm{m^3/\,s} \times 196\,\mathrm{m}}{0.8} \\
&= 7487\,\mathrm{W}
\end{aligned}
$$

$$
\boxed{P = 7.49\,\mathrm{kW}}
$$

10.73: PROBLEM DEFINITION

Situation:

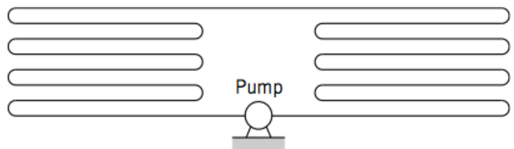
A heat exchanger is described in the problem statement.

$L = 15\,\mathrm{m},\ D = 15\,\mathrm{mm}.$

$Q_{\max} = 10^{-3}\,\mathrm{m}^3/\,\mathrm{s}.$

$h_p = h_{p0}\left[1 - (Q/Q_{\max})^3\right].$

14 - 180° elbows, $K_L = 2.2$.



Find:

System operating points for h_{p0} of 2 m, 10 m and 20 m.

Properties:

From Table 10.4: $k_s = 1.5 \times 10^{-3}$ mm.

SOLUTION

Energy equation

$$h_p = \frac{V^2}{2g}(\sum K_L + f\frac{L}{D})$$

Substitute in the values for loss coefficients, L/D and the equation for h_p

$$h_{p0}\left[1 - \left(\frac{Q}{Q_{\max}}\right)^3\right] = \frac{V^2}{2g}(14 \times 2.2 + f \times 1000)$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 1.767 \times 10^{-4} V \end{aligned}$$

Combine equations

$$h_{p0}\left[1 - \left(\frac{Q}{Q_{\max}}\right)^3\right] = 1.632 \times 10^6 Q^2 (30.8 + f \times 1000) \qquad (1)$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3}}{15} = 10^{-4}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{V \times 0.015}{6.58 \times 10^{-7}} = 2.28 \times 10^4 V = 1.29 \times 10^8 Q \end{aligned}$$

Eq. (1) becomes

$$F(Q) = h_{p0}\left[1 - \left(\frac{Q}{Q_{\max}}\right)^3\right] - 1.632 \times 10^6 Q^2 (30.8 + f \times 1000)$$

A program was written to evaluate $F(Q)$ by inputting a value for Q and trying different Q's until $F(Q) = 0$. The results are

h_{p0} (m)	Q (m^3/s)
2	$\boxed{0.000356}$
10	$\boxed{0.000629}$
20	$\boxed{0.000755}$

10.74: PROBLEM DEFINITION

Situation:

Gasoline being pumped from a gas tank.

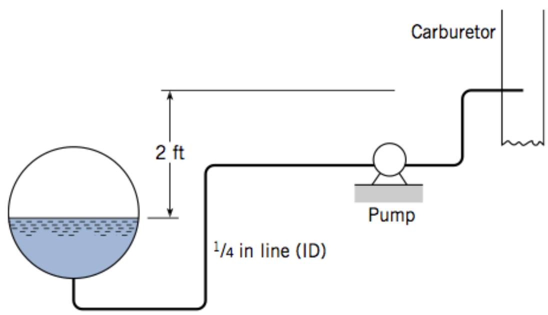
Pressure in the tank = 14.7 psia, pressure in the carburetor = 14.0 psia.

$D_{\text{tube}} = 0.25\,\text{in} = 0.0208\,\text{ft},\ D_{\text{jet}} = 1/32\,\text{in} = 0.00260\,\text{ft}.$

$L = 10\,\text{ft},$

$\eta = 0.80$, $Q = 0.12$ gpm $= 2.68 \times 10^{-4}$ cfs

Sketch:



Find:

Pump power.

Properties:

Gasoline Fig. A.2: $S = 0.68$, $\nu = 5.5 \times 10^{-6}$ ft^2/s, $\gamma = 62.4\,\text{lbf/ft}^3 \times 0.68 = 42.4$ lbf/ft^3

Loss coefficient, Table 10.5 (EFM9e), 90° smooth bends, $r/d = 6$, $K_b = 0.21$.

SOLUTION

Velocity values

$$V_{\text{tube}} = \frac{4Q}{\pi D_{\text{tube}}^2} = \frac{4\left(2.68 \times 10^{-4}\,\text{ft}^3/s\right)}{\pi\,(0.0208\,\text{ft})^2} = 0.789\,\text{ft/s}$$

$$V_{\text{jet}} = V_{\text{tube}}\left(\frac{.25\,\text{in}}{1/32\,\text{in}}\right)^2 = (0.789\,\text{ft/s})\left(\frac{.25\,\text{in}}{1/32\,\text{in}}\right)^2 = 50.5\,\text{ft/s}$$

$$\frac{V_{\text{tube}}^2}{2g} = 0.00959\ \text{ft}$$

$$\frac{V_{\text{jet}}^2}{2g} = 39.3\ \text{ft}$$

Reynolds number (fuel line)

$$\begin{aligned} \text{Re} &= \frac{VD}{v} \\ &= \frac{0.789\,\text{ft/ s} \times 0.0208\,\text{ft}}{5.5 \times 10^{-6}\,\text{ft}^2\text{/ s}} \\ &= 2972 \end{aligned}$$

From Moody diagram

$$f \approx 0.040$$

Energy equations

$$\begin{aligned} h_L &= \left(f\frac{L}{D} + 5K_b\right)\frac{V_1^2}{2g} \\ &= \left(0.040 \times \frac{10\,\text{ft}}{0.0208\,\text{ft}} + 5 \times 0.21\right) 0.00959\,\text{ft} = 0.194 \text{ ft} \end{aligned}$$

$$\begin{aligned} h_p &= \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2}{2g} + h_L \\ &= \frac{(14.0 \text{ psia} - 14.7 \text{ psia})\,144\,\text{in}^2\text{/ ft}^2}{42.4\,\text{lbf/ ft}^3} + 2 + 39.3 + 0.194 = 39.1 \text{ ft} \end{aligned}$$

Power equation

$$P = \frac{\gamma h_p Q}{550\eta} = \frac{42.4\,\text{lbf/ ft}^3(39.1\,\text{ft})0.000268\,\text{ft}^3\text{/ s}}{550 \times 0.8} = 10.1 \times 10^{-4}\,\text{hp}$$

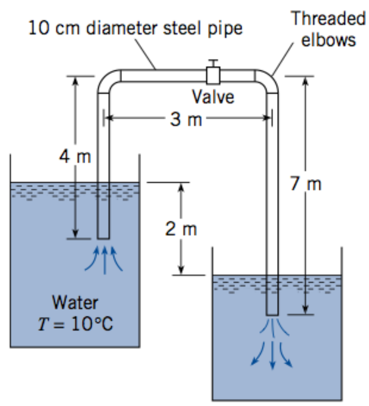
$$\boxed{P = 10.1 \times 10^{-4}\,\text{hp}}$$

10.75: PROBLEM DEFINITION

Situation:

A partially-closed valve on a steel pipeline between two reservoirs.
$D = 10\,\text{cm}$, $\Delta z = 2\,\text{m}$, $L = 14\,\text{m}$.

Sketch:



Find:

Loss coefficient for valve, K_v.

Properties:

From Table 10.4: $k_s = 0.046$ mm
Water (10 °C), Table A.5: $v = 1.31 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

PLAN

First find Q for valve wide open. Assume valve is a gate valve.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\
2 &= 0 + 0 + 0 + \frac{V^2}{2g}(0.5 + 0.9 + 0.2 + 0.9 + 1 + f\frac{L}{D}) \\
V^2 &= \frac{4g}{3.5 + f\frac{L}{D}}
\end{aligned}$$

Assume $f = 0.015$. Then

$$
\begin{aligned}
V &= \left(\frac{4 \times 9.81\,\mathrm{m/\,s^2}}{3.5 + 0.015 \times 14\,\mathrm{m}/0.1\,\mathrm{m}}\right)^{1/2} = 2.65 \text{ m/s} \\
\frac{k_s}{D} &\simeq 0.0005 \\
\mathrm{Re} &= \frac{2.65\,\mathrm{m/\,s} \times 0.10\,\mathrm{m}}{1.3 \times 10^{-6}\,\mathrm{m^2/\,s}} = 2.0 \times 10^5
\end{aligned}
$$

From the Moody diagram, $f = 0.019$. Then

$$
\begin{aligned}
V &= \left(\frac{4 \times 9.81\,\mathrm{m/\,s^2}}{3.5 + 0.019 \times 14\,\mathrm{m}/0.1\,\mathrm{m}}\right)^{1/2} = 2.52 \text{ m/s} \\
\mathrm{Re} &= 2.0 \times 10^5 \times \frac{2.52}{2.65} = 1.9 \times 10^5;\ \text{O.K.}
\end{aligned}
$$

This is close to $2.0{\times}10^5$ so no further iterations are necessary. For one-half the discharge

$$
\begin{aligned}
V &= 1.26 \text{ m/s} \\
\mathrm{Re} &= 9.5 \times 10^4
\end{aligned}
$$

and from the Moody diagram $f = 0.021$. So

$$
\begin{aligned}
V^2 &= 1.588 = \frac{4 \times 9.81\,\mathrm{m/\,s^2}}{3.3 + K_v + 0.021 \times 14\,\mathrm{m}/0.1\,\mathrm{m}} \\
3.3 + K_v + 2.94 &= 24.7 \\
& \boxed{K_v = 18.5}
\end{aligned}
$$

10.76: PROBLEM DEFINITION

Situation:

A galvanized steel pipe connects a water main to a factory.
$p_1 = 300\,\text{kPa}$, $Q = 0.025\,\text{m}^3/\,\text{s}$.
$L = 160\,\text{m}$, $z_2 = 10\,\text{m}$, $p_2 = 60\,\text{kPa}$.

Find:

The pipe size.

Properties:

From Table 10.4 $k_s = 0.15$ mm.
Water (10 °C), Table A.5: $\gamma = 9810\,\text{N}/\,\text{m}^3$, $\nu = 1.31 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f \\
\frac{300000\,\text{Pa}}{9810\,\text{N}/\,\text{m}^3} + 0 &= \frac{60000\,\text{Pa}}{9810\,\text{N}/\,\text{m}^3} + 10 + h_f \\
h_f &= 14.46\text{ m} \\
f\frac{L}{D}\frac{Q^2/A^2}{2g} &= 14.46 \\
f\frac{L}{D}\frac{Q^2}{\left(\pi/4 \times D^2\right)^2/2g} &= 14.46 \\
\frac{4^2 f L Q^2/\pi^2}{2gD^5} &= 14.46 \\
D &= \left[\frac{(8/14.46) f L Q^2}{\pi^2 g}\right]^{1/5}
\end{aligned}$$

Assume $f = 0.020$. Then

$$\begin{aligned}
D &= \left[\frac{(8\,\text{m}/14.46\,\text{m}) \times 0.02 \times 140\,\text{m} \times (0.025\,\text{m}^3/\,\text{s})^2}{\pi^2 \times 9.81\,\text{m}/\,\text{s}^2}\right]^{1/5} \\
&= 0.1027\text{ m}
\end{aligned}$$

Relative roughness

$$\begin{aligned}
\frac{k_s}{D} &= \frac{0.15}{103} \\
&= 0.00146
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{4Q}{\pi D \nu} \\
&= \frac{4 \times (0.025\,\text{m}^3/\,\text{s})}{\pi \times (0.1027\,\text{m}) \times (1.31 \times 10^{-6}\,\text{m}^2/\,\text{s})} \\
&= 2.266 \times 10^5
\end{aligned}$$

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{.00146}{3.7} + \frac{5.74}{(2.266\times 10^5)^{0.9}}\right)\right]^2} \\
&= 2.2717 \times 10^{-2}
\end{aligned}$$

Recalculate pipe diameter

$$D = 0.1027 \times \left(\frac{0.0227}{0.020}\right)^{1/5} = 0.105\,\text{m}$$

$$\boxed{\text{Specify a 12-cm pipe}}$$

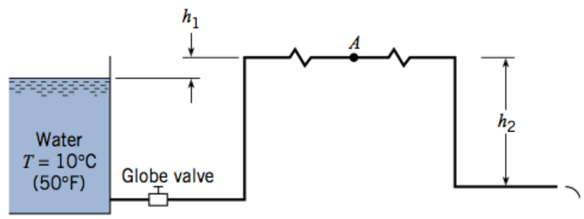
10.77: PROBLEM DEFINITION

Situation:

A steel pipe discharges into the atmosphere.

$D = 10\,\text{cm},\ L = 1000\,\text{m},\ z_1 = 12\,\text{m}.$

Sketch:



Find:

Discharge (m^3/s).

Pressure at point A.

Assumptions:

Water temperature is 10 °C.

Properties:

Water (10 °C), Table A.5: $\nu = 1.31 \times 10^{-6}\ \text{m}^2/\text{s}$.

From Table 10.5: $K_v = 10, K_b = 0.9,\ K_e = 0.5$.

SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 12 + 0 &= 0 + 0 + \frac{V_2^2}{2g}(1 + K_e + K_v + 4K_b + f \times \frac{L}{D}) \end{aligned}$$

Using a pipe diameter of 10 cm and assuming $f = 0.025$

$$\begin{aligned} 24g &= V^2(1 + 0.5 + 10 + 4(0.9) + 0.025 \times \frac{1000\,\text{m}}{0.1\,\text{m}}) \\ V^2 &= 24g/265.1 = 0.888\ \text{m}^2/\text{s}^2 \\ V &= 0.942\ \text{m/s} \\ Q &= VA \\ &= 0.942\,\text{m/s} \times \pi/4 \times (0.10\,\text{m})^2 \\ &\boxed{Q = 0.0074\text{m}^3/\text{s}} \\ \text{Re} &= 0.942 \times 0.1/1.31 \times 10^{-6} = 7 \times 10^4 \end{aligned}$$

From Fig. 10.8 $f \approx 0.025$

$$\begin{aligned}
\frac{p_A}{\gamma} + \frac{V^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V^2}{2g} + z_B + \sum h_L \\
\frac{p_A}{\gamma} + 15 &= \frac{V^2}{2g}(2K_b + f \times \frac{L}{D}) \\
\frac{p_A}{\gamma} &= \frac{0.888\,\mathrm{m}^2/\,\mathrm{s}^2}{2g}(2 \times 0.9 + 0.025 \times \frac{500\,\mathrm{m}}{0.1\,\mathrm{m}}) - 15\,\mathrm{m} = -9.26\ \mathrm{m} \\
p_A &= 9810\,\mathrm{N}/\,\mathrm{m}^3 \times (-9.26\,\mathrm{m}) \\
&= \boxed{p_A = -90.8\ \mathrm{kPa}}
\end{aligned}$$

Note that this is not a good installation because the pressure at A is near cavitation level.

10.78: PROBLEM DEFINITION

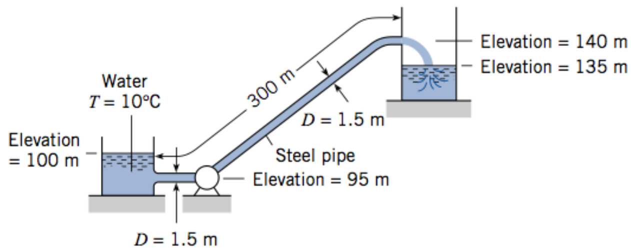
Situation:

Water is pumped from one reservoir to another reservoir.

$D = 1.5\,\text{m}$, $Q = 25\,\text{m}^3/\,\text{s}$.

$L = 300\,\text{m}$, $z_2 = 140\,\text{m}$, $z_1 = 100\,\text{m}$.

Sketch:



Find:

Pump power.

Properties:

From Table 10.4: $k_s = 0.046$ mm.

Water (10 °C), Table A.5 $\nu = 1.31 \times 10^{-6}$ mm.

SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 100 + h_p &= 0 + \frac{V_2^2}{2g} + 140 + \frac{V_2^2}{2g}\left(0.03 + f\frac{L}{D}\right) \end{aligned}$$

Flow rate

$$\begin{aligned} V_2 &= \frac{Q}{A_p} \\ &= \frac{25\,\text{m}^3/\,\text{s}}{(\pi/4) \times (1.5\,\text{m})^2} \\ &= 14.15\ \text{m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{14.15\,\text{m}/\,\text{s} \times 1.5\,\text{m}}{1.31 \times 10^{-6}\,\text{m}^2/\,\text{s}} \\ &= 1.620 \times 10^7 \\ \frac{k_s}{D} &= \frac{0.046}{1500} \\ &= 0.00003 \end{aligned}$$

Friction factor (Moody Diagram) or the Swamee-Jain correlation:

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{0.00003}{3.7} + \frac{5.74}{(1.620\times10^7)^{0.9}}\right)\right]^2} \\
&= 0.009995 \\
&\approx 0.01
\end{aligned}$$

Then

$$\begin{aligned}
h_p &= 140\,\mathrm{m} - 100\,\mathrm{m} + \frac{V_2^2}{2g}\left(1.03 + 0.010 \times \frac{300\,\mathrm{m}}{1.5\,\mathrm{m}}\right) \\
&= 140\,\mathrm{m} - 100\,\mathrm{m} + \frac{(14.15\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}}\left(1.03 + 0.010 \times \frac{300\,\mathrm{m}}{1.5\,\mathrm{m}}\right) \\
h_p &= 70.92\,\mathrm{m}
\end{aligned}$$

Power equation

$$\begin{aligned}
P &= Q\gamma h_p \\
&= \left(25\,\mathrm{m^3/s}\right) \times \left(9810\,\mathrm{N/m^3}\right) \times (70.92\,\mathrm{m}) = 17.4\,\mathrm{MW}
\end{aligned}$$

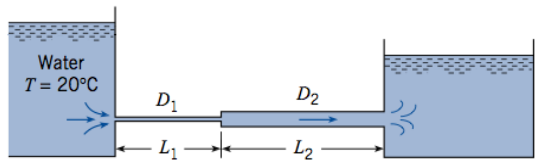
$$\boxed{P = 17.4\,\mathrm{MW}}$$

10.79: PROBLEM DEFINITION

Situation:

A system with two pipe sizes connects two reservoirs.
$k_s = 0.1\,\text{mm}$, $Q = 0.1\,\text{m}^3/\text{s}$.
$D_1 = 15\,\text{cm}$, $L_1 = 50\,\text{m}$.
$D_2 = 30\,\text{cm}$, $L_2 = 160\,\text{m}$.

Sketch:



Find:

Difference in water surface between two reservoirs.

Properties:

Water (20°C), Table A.5: $\nu = 10^{-6}\ \text{m}^2/\text{s}$.

SOLUTION

$$\begin{aligned}
\frac{k_s}{D_{15}} &= \frac{0.1}{150} = 0.00067 \\
\frac{k_s}{D_{30}} &= \frac{0.1}{300} = 0.00033 \\
V_{15} &= \frac{Q}{A_{15}} = \frac{0.1\,\text{m}^3/\text{s}}{\pi/4 \times (0.15\,\text{m})^2} = 5.659\ \text{m/s} \\
V_{30} &= 1.415\ \text{m/s} \\
\text{Re}_{15} &= \frac{VD}{v} = \frac{5.659\,\text{m/s} \times 0.15\,\text{m}}{10^{-6}\,\text{m}^2/\text{s}} = 8.49 \times 10^5 \\
\text{Re}_{30} &= \frac{1.415\,\text{m/s} \times 0.3\,\text{m}}{10^{-6}\,\text{m}^2/\text{s}} = 4.24 \times 10^5
\end{aligned}$$

Resistance Coefficient (from the Moody diagram, Fig. 10-8)

$$\begin{aligned}
f_{15} &= 0.0185 \\
f_{30} &= 0.0165
\end{aligned}$$

Energy equation

$$z_1 - z_2 = \sum h_L$$

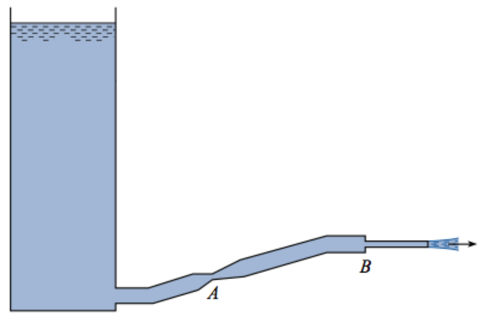
$$
\begin{aligned}
z_1 - z_2 &= \frac{V_{15}^2}{2g}(0.5 + 0.0185 \times \frac{50}{0.15}) \\
&\quad + \frac{V_{30}^2}{2g}\left(1 + 0.0165 \times \frac{160\,\mathrm{m}}{0.3\,\mathrm{m}}\right) + \frac{(V_{15} - V_{30})^2}{2g} \\
z_1 - z_2 &= \left(\frac{(5.659\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}}\right)(6.67) \\
&\quad + \left(\frac{(1.415\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}}\right)(9.80) + \frac{(5.659\,\mathrm{m/s} - 1.415\,\mathrm{m/s})^2}{2 \times 9.81} \\
z_1 - z_2 &= 1.632\,\mathrm{m}(6.67) + 1.00\,\mathrm{m} + 0.918\,\mathrm{m} \\
&\boxed{z_1 - z_2 = 12.80\ \mathrm{m}}
\end{aligned}
$$

10.80: PROBLEM DEFINITION

Situation:

A tank discharges to atmosphere through a piping system.

Sketch:

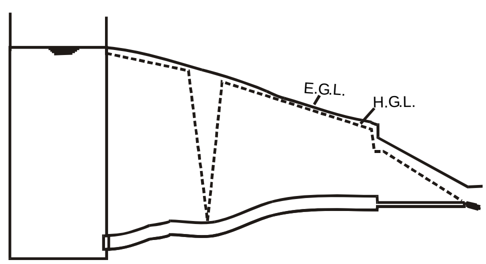


Find:

Sketch the EGL and HGL.

Identify where cavitation might occur.

SOLUTION



Answer ==> Cavitation could occur in the venturi throat section or just downstream of the abrupt contraction (where there will be a contraction of the flow area).

10.81: PROBLEM DEFINITION

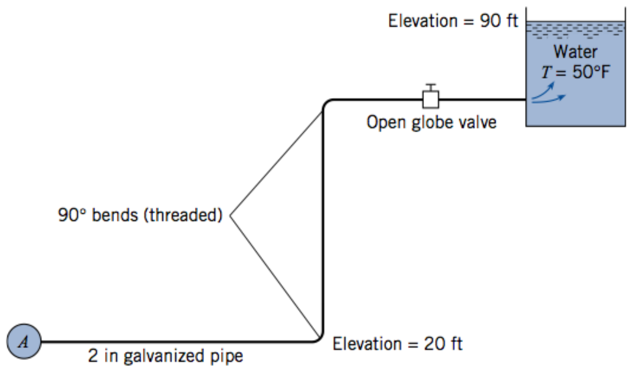
Situation:

A steel pipe carries water from a main pipe to a reservoir.

$z_1 = 20\,\text{ft}$, $z_2 = 90\,\text{ft}$.

$Q = 50$ gpm, $D = 2\,\text{in}$, $L = 240\,\text{ft}$.

Sketch:



Find:

Pressure at point A.

Properties:

From Table 10.5: $K_b = 0.9$, $K_v = 10$.

From Table 10.4: $k_s = 5 \times 10^{-4}$ ft.

Water (50 °F), Table A.5: $\nu = 1.41 \times 10^{-5}$ ft^2/s.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\
\frac{p_A}{\gamma} + 20\,\text{ft} + 0 &= 0 + 90\,\text{ft} + 0 + \frac{V^2}{2g}(0.5 + 2K_b + K_v + f\frac{L}{D} + 1) \\
V &= \frac{Q}{A} = \frac{50/449}{\pi/4 \times \left(\frac{2}{12}\,\text{ft}\right)^2} = 5.1\,\text{ft/s} \\
\frac{V^2}{2g} &= \frac{(5.1\,\text{ft/s})^2}{64.4\,\text{ft/s}^2} = 0.404\,\text{ft} \\
\text{Re} &= \frac{5.1\,\text{ft/s} \times \frac{2}{12}\,\text{ft}}{1.41 \times 10^{-5}\,\text{ft}^2/\text{s}} = 6 \times 10^4 \\
\frac{k_s}{D} &= \frac{5 \times 10^{-4} \times 12}{2} = 0.003
\end{aligned}$$

Resistance coefficient (from Moody diagram)

$$f = 0.028$$

Energy equation becomes

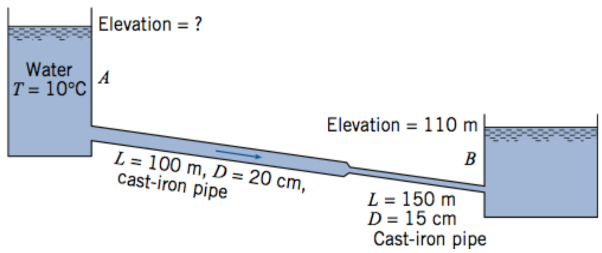
$$\begin{aligned} p_A &= \gamma \left[70\,\text{ft} + 0.404\,\text{ft} \left(0.5 + 2 \times 0.9 + 10 + \left(0.028 \times \frac{240}{2/12} \right) + 1.0 \right) \right] \\ &= 62.4\,\text{lbf/ft}^3 \times 91.7\,\text{ft} = 5722 \text{ psfg} \\ & \boxed{p_A = 39.7 \text{ psig}} \end{aligned}$$

10.82: PROBLEM DEFINITION

Situation:

Two reservoirs are connected by a cast-iron pipe of varying diameter.
$z_2 = 110\,\text{m}$, $Q = 0.3\,\text{m}^3/\,\text{s}$.
$D_1 = 20\,\text{cm}$, $L_1 = 100\,\text{m}$.
$D_2 = 15\,\text{cm}$, $L_2 = 150\,\text{m}$.

Sketch:



Find:

Water surface elevation in reservoir A.

Properties:

From Table 10.4: $k_s = 0.26$ mm.
Water (10 °C), Table A.5: $\nu = 1.3 \times 10^{-6}$ m^2/s.

SOLUTION

$$\frac{k_s}{D_{20}} = \frac{0.26}{200} = 0.0013$$

$$\frac{k_s}{D_{15}} = 0.0017$$

$$V_{20} = \frac{Q}{A_{20}} = \frac{0.03\,\text{m}^3/\,\text{s}}{\pi/4 \times (0.20\,\text{m})^2} = 0.955 \text{ m/s}$$

$$\frac{Q}{A_{15}} = 1.697 \text{ m/s}$$

$$\text{Re}_{20} = \frac{VD}{v} = \frac{0.955\,\text{m}/\,\text{s} \times 0.2\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}} = 1.5 \times 10^5$$

$$\text{Re}_{15} = \frac{1.697\,\text{m}/\,\text{s} \times 0.15\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}} = 1.9 \times 10^5$$

From Fig. 10-14: $f_{20} = 0.022$; $f_{15} = 0.024$

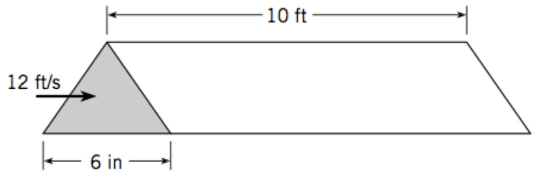
$$\begin{aligned}
z_1 &= z_2 + \sum h_L \\
z_1 &= 110 + \frac{V_{20}^2}{2g}(0.5 + 0.022 \times \frac{100\,\text{m}}{0.2\,\text{m}} + 0.19) \\
&\quad + \frac{V_{15}^2}{2g}\left[(0.024 \times \frac{150\,\text{m}}{0.15\,\text{m}}) + 1.0 + 0.19)\right] \\
&= 110\,\text{m} + 0.0465\,\text{m}(11.7) + 0.1468\,\text{m}(25.19) \\
&= 110 + 0.535 + 3.70 = 114.2\,\text{m}
\end{aligned}$$

$$\boxed{z_1 = 114\,\text{m}}$$

10.83: PROBLEM DEFINITION

Situation:

Air flowing through an equilateral triangle shaped horizontal duct.
$L = 100\,\text{ft}$, $V = 12\,\text{ft/s}$.
$k_s = 0.0005\,\text{ft}$, Triangle side = 6 in.



Find:

Pressure drop over 100 ft length.

Properties:

Air (60 °F), Table A.3: $\nu = 1.58 \times 10^{-4}$ ft^2/s and $\rho = 0.00237$ slug/ft^3.

SOLUTION

$$\begin{aligned}
h &= (6\text{ in})(\cos\ 30°) = 5.20\,\text{in} \\
A &= (6\,\text{in})(5.20\,\text{in})/2 = 15.6\text{ in}^2 = 0.108\text{ ft}^2 \\
R_h &= \frac{A}{P} = \frac{15.6\text{ in}^2}{3 \times 6\,\text{in}} = 0.867\text{ in.} \\
4R_h &= 3.47\text{ in.} = 0.289\text{ ft.} \\
\frac{k_s}{4R_h} &= \frac{0.0005}{0.289} = 0.00173 \\
\text{Re} &= \frac{(V)(4R_h)}{v} = \frac{(12\,\text{ft/s})(0.289\,\text{ft})}{1.58 \times 10^{-4}\,\text{ft}^2/\text{s}} = 2.2 \times 10^4
\end{aligned}$$

From the Moody diagram, $f = 0.030$ so the pressure drop is

$$\begin{aligned}
\Delta p_f &= f\left(\frac{L}{4R_h}\right)\frac{\rho V^2}{2} \\
\Delta p_f &= 0.030\left(\frac{100\,\text{ft}}{0.289\,\text{ft}}\right)\left(\frac{0.00237 \times (12\,\text{ft/s})^2}{2}\right)
\end{aligned}$$

$$\boxed{\Delta p_f = 1.77\text{ lbf/ft}^2}$$

10.84: PROBLEM DEFINITION

Situation:

Air moves through a galvanized iron cold-air duct.
$b = 100\,\text{cm}$, $A = 100\,\text{cm} \times 15\,\text{cm}$.
$Q = 6\,\text{m}^3/\,\text{s}$.

Find:

Power loss in duct.

Assumptions:

$k_s = .15$ mm$= 1.5 \times 10^{-4}$ m.

Properties:

Air $(15\,^\circ\text{C})$, Table A.3: $\nu = 1.46 \times 10^{-5}\,\text{m}^2/\,\text{s}$.
From Table A.2: $\rho = 1.22$ kg/m^3.

SOLUTION

Hydraulic radius

$$\begin{aligned} A &= 0.15 \text{ m}^2 \\ P &= 2.30 \text{ m} \\ R &= \frac{A}{P} = 0.0652 \text{ m} \\ 4R &= 0.261 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{6\,\text{m}^3/\,\text{s}}{0.15\,\text{m}^2} \\ &= 40 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= V \times \frac{4R}{v} \\ &= \frac{40\,\text{m}/\,\text{s} \times 0.261\,\text{m}}{1.46 \times 10^{-5}\,\text{m}^2/\,\text{s}} \\ &= 7.15 \times 10^5 \end{aligned}$$

Friction factor (f) (turbulent flow: Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{1.5\times10^{-4}}{3.7\times0.261} + \frac{5.74}{(7.15\times10^5)^{0.9}}\right)\right]^2} \\ &= 0.01797 \approx 0.018 \end{aligned}$$

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.018 \times \left(\frac{100\,\mathrm{m}}{0.261\,\mathrm{m}}\right)\left(\frac{(40\,\mathrm{m/\,s})^2}{2 \times 9.81\,\mathrm{m/\,s^2}}\right) \\ &= 562.4 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P_{\text{loss}} &= Q\gamma h_f \\ &= 6\,\mathrm{m^3/\,s} \times 1.22\,\mathrm{kg/\,m^3} \times 9.81\,\mathrm{m/\,s^2} \times 562.4\,\mathrm{m} = 40400\,\mathrm{W} \end{aligned}$$

$$\boxed{P_{\text{loss}} = 40.4\,\mathrm{kW}}$$

10.85: PROBLEM DEFINITION

Situation:

Air flows through a horizontal, rectangular, air-conditioning duct.

$L = 20\,\text{m}$, Section area is 4 by 10 inches.

V =10 m/s, $k_s = 0.004\,\text{mm}$.

Find:

(a) The pressure drop in inches of water.

(b) The power in watts needed to overcome head loss.

Assumptions:

Neglect all head loss associated with minor losses.

$\alpha_1 = \alpha_2$

Properties:

Air at 20 °C from Table A.3: $= 15.1 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

$\rho = 1.2\,\text{kg}/\,\text{m}^3$, $\gamma = 11.8\,\text{N}/\,\text{m}^3$.

PLAN

To account for the rectangular section, use hydraulic diameter. Calculate Reynolds number and then choose a suitable correlation for the friction factor (f). Apply the Darcy-Weisbach equation to find the head loss (h_f). Apply the energy equation to find the pressure drop, and calculate power using $P = \dot{m}gh_f$.

SOLUTION

Hydraulic diameter (D_H) (four times the hydraulic radius)

$$\begin{aligned} D_H &= \frac{4A}{P} \\ &= \frac{4\,(0.102\,\text{m})\,(0.254\,\text{m})}{(0.102\,\text{m} + 0.102\,\text{m} + 0.254\,\text{m} + 0.254\,\text{m})} \\ &= 0.1456\,\text{m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD_H}{\nu} \\ &= \frac{(10\,\text{m}/\,\text{s})\,(0.1456\,\text{m})}{(15.1 \times 10^{-6}\,\text{m}^2/\,\text{s})} \\ &= 96,390 \end{aligned}$$

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D_H} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{4\times10^{-6}\,\text{m}}{3.7\times(0.1456\,\text{m})} + \frac{5.74}{96{,}390^{0.9}}\right)\right]^2} \\ &= 0.0182 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0182\left(\frac{20\,\mathrm{m}}{0.1456\,\mathrm{m}}\right)\left(\frac{10^2\,\mathrm{m}^2/\,\mathrm{s}^2}{2\times 9.81\,\mathrm{m}/\,\mathrm{s}^2}\right) \\ &= 12.72\,\mathrm{m} \end{aligned}$$

Energy equation (section 1 and 2 are the inlet and exit of the duct)

$$\left(\frac{p}{\gamma}\right)_1 = \left(\frac{p}{\gamma}\right)_2 + h_L$$

Thus

$$\begin{aligned} \Delta p &= \gamma_{\mathrm{air}} h_f \\ &= \left(11.8\,\mathrm{N}/\,\mathrm{m}^3\right)(12.72\,\mathrm{m}) \\ &= 150\,\mathrm{Pa} \\ &= 150\,\mathrm{Pa}\left(\frac{1.0\ \text{in.-H}_2\text{O}}{248.8\,\mathrm{Pa}}\right) \end{aligned}$$

$$\Delta p = 0.6\,\text{in.-H}_2\text{O}$$

Power equation

$$\begin{aligned} P &= \gamma Q h_f \\ &= \Delta p A V \\ &= (150\,\mathrm{Pa})\,(0.102\,\mathrm{m}\times 0.254\,\mathrm{m})\,(10\,\mathrm{m}/\,\mathrm{s}) \\ & \boxed{P = 38.9\,\mathrm{W}} \end{aligned}$$

REVIEW

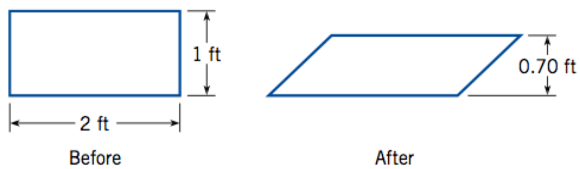
The power to overcome head loss is small (39 W)—this is equivalent to the power required to light a small light bulb.

10.86: PROBLEM DEFINITION

Situation:

A rectangular duct (initial state).
A trapezoidal duct (after being run over by a truck).
Rectangular area is 1 by 2 feet. Trapezoidal area is 0.7 by 2 feet.

Sketch:



Find:

Ratio of velocity in trapezoidal to rectangular duct.

SOLUTION

$$\Delta h_{\text{rect}} = \Delta h_{\text{trap}}$$

$$\therefore h_{f,\text{rect}} = h_{f,\text{trap}}$$

$$\left(\frac{f_b L}{4R_b}\right)\frac{V_b^2}{2g} = \left(\frac{f_a L}{4R_a}\right)\frac{V_a^2}{2g}$$

$$R_b = \frac{A_b}{P_b} = \frac{2\,\text{ft}^2}{6\,\text{ft}} = 0.333\text{ ft}$$

$$R_a = \frac{A_a}{P_a} = \frac{1.4\,\text{ft}^2}{6\,\text{ft}} = 0.233\text{ ft}$$

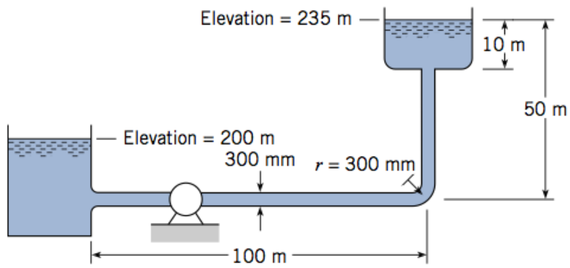
$$\frac{V_a^2}{V_b^2} = \frac{R_a}{R_b} = 0.70$$

$$\boxed{\frac{V_{\text{trap}}}{V_{\text{rect}}} = 0.84}$$

10.87: PROBLEM DEFINITION

Situation:

Water is pumped through a steel pipe from one tank to another.
$D = 300\,\text{mm}$, $L = 140\,\text{m}$, $Q = 0.314\,\text{m}^3/\text{s}$.
$z_1 = 200\,\text{m}$, $z_2 = 235\,\text{m}$, Elbow radius is 300 mm.



Find:

The pump power.

Assumptions:

Pipe entrance is well-rounded: $r/D > 0.2$.

Properties:

From Table 10.5: $K_e = 0.03; K_b = 0.35; K_E = 1.0$.
Water (20 °C), Table A.5: $\nu = 10^{-6}\ \text{m}^2/\text{s}$.
From Table 10.4: $k_s = 0.046$ mm.

PLAN

Apply the energy equation from the water surface in the lower reservoir to the water surface in the upper reservoir.

SOLUTION

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 200\ \text{m} + h_p &= 0 + 0 + 235\ \text{m}\ + \frac{V_2^2}{2g}(K_e + K_b + K_E + f\frac{L}{D})\end{aligned}$$

Flow rate equation

$$\begin{aligned}V &= \frac{Q}{A} \\ &= \frac{0.314\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.3\,\text{m})^2} \\ &= 4.44\ \text{m/s} \\ \frac{V^2}{2g} &= 1.01\ \text{m}\end{aligned}$$

Reynolds number

$$
\begin{aligned}
\text{Re} &= \frac{VD}{v} \\
&= \frac{4.44\,\text{m/s} \times 0.3\,\text{m}}{10^{-6}\,\text{m}^2/\text{s}} \\
&= 1.33 \times 10^6 \\
\frac{k_s}{D} &\approx 0.00015
\end{aligned}
$$

Resistance coefficient (from the Moody diagram)

$$f = 0.014$$

So

$$
\begin{aligned}
f\frac{L}{D} &= 0.014 \times \frac{140\,\text{m}}{0.3\,\text{m}} = 6.53 \\
h_p &= 235\,\text{m} - 200\,\text{m} + 1.01\,\text{m}(0.03 + 0.35 + 1 + 6.53) \\
&= 43.0\ \text{m}
\end{aligned}
$$

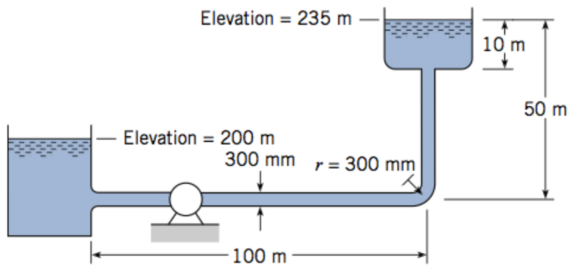
Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= 0.314\,\text{m}^3/\text{s} \times 9,790\,\text{N/m}^3 \times 43.0\,\text{m} \\
& \boxed{P = 132\ \text{kW}}
\end{aligned}
$$

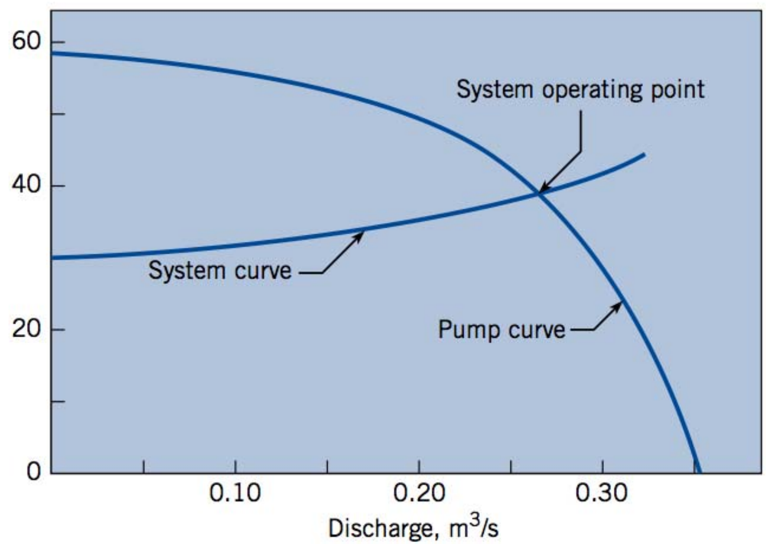
10.88: PROBLEM DEFINITION

Situation:

Water is pumped through the system sketched below.
$D = 300\,\text{mm}$, $L = 140\,\text{m}$.
$z_1 = 200\,\text{m}$, $z_2 = 235\,\text{m}$.
Elbow radius is 300 mm.



The pump curve is shown on the figure below.



Find:

Discharge.

PLAN

For the system curve, follow the solution for problem 10.87 in EFM9e. Then plot the system curve on the above diagram to find the operating point.

SOLUTION

The solution to Prob. 10.87 in EFM9e, gives the system curve

$$0 + 0 + 200\text{ m} + h_p = 0 + 0 + 235\text{ m } + \frac{V_2^2}{2g}(K_e + K_b + K_E + f\frac{L}{D})$$

$$h_p = 35\,\text{m} + \frac{V^2}{2g}\,(0.03 + 0.35 + 1 + 6.53)$$

$$h_p = 35\,\text{m} + 7.91\frac{V^2}{2g}$$

$$h_p = 35\,\text{m} + 7.91\left[\frac{Q^2/((\pi/4)\times 0.3^2)^2}{2g}\right] = 35 + 85.6Q^2$$

System data computed and shown below:

$Q(\text{m}^3\text{s})$	$\rightarrow$	0.05	0.10	0.15	0.20	.30
$h_p(\text{m})$	$\rightarrow$	35.2	35.8	36.9	38.4	42.7

Then, plotting the system curve on the pump performance curve of Fig.10.19b in EFM9e yields the operating point

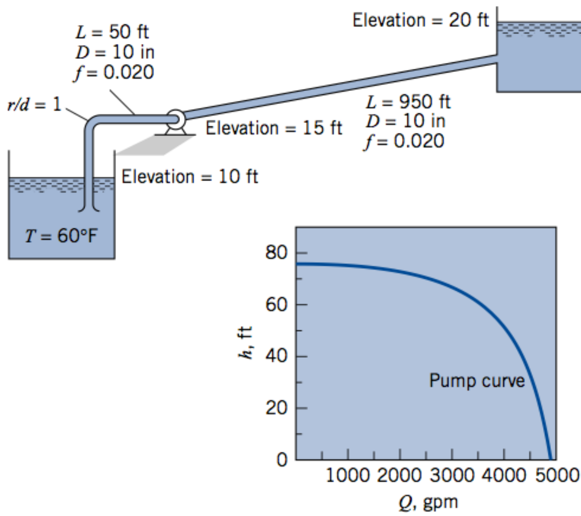
$$\boxed{Q = 0.25\ \text{m}^3/\text{s}}$$

10.89: PROBLEM DEFINITION

Situation:

Water is pumped from one tank to another.
$r/d = 1$, $D_1 = 10\,\text{in}$, $L_1 = 50\,\text{ft}$.
$f_2 = 0.020$, $z_1 = 10\,\text{ft}$.
$D_2 = 10\,\text{in}$, $L_2 = 950\,\text{ft}$, $f_2 = 0.020$.
$z_2 = 20\,\text{ft}$, $T = 60\,°\text{F}$.

Sketch:



Find:

Discharge $\left(\text{ft}^3/\,\text{s}\right)$.

Properties:

From Table 10.5: $K_e = 0.03$; $K_b = 0.35$; $K_E = 1.0$.

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 10 + h_p &= 0 + 0 + 20 + \frac{V_2^2}{2g}\left(K_e + f\frac{L}{D} + K_b + K_E\right) \\
h_p &= 10 + \frac{Q^2}{2gA^2}\left(0.03 + 0.02 \times \frac{1000\,\text{ft}}{\frac{10}{12}\,\text{ft}} + 0.35 + 1\right) \\
A &= (\pi/4) \times \left(\frac{10}{12}\,\text{ft}\right)^2 = 0.545\ \text{ft}^2
\end{aligned}$$

$$h_p = 10 + 1.31 Q_{\text{cfs}}^2$$
$$1 \text{ cfs} = 449 \text{ gpm}$$
$$h_p = 10 + \frac{1.31 Q_{\text{gpm}}^2}{(449)^2}$$
$$h_p = 10 + 6.51 \times 10^{-6} Q_{\text{gpm}}^2$$

$Q \rightarrow$	1,000	2,000	3,000
$h \rightarrow$	16.5	36.0	68.6

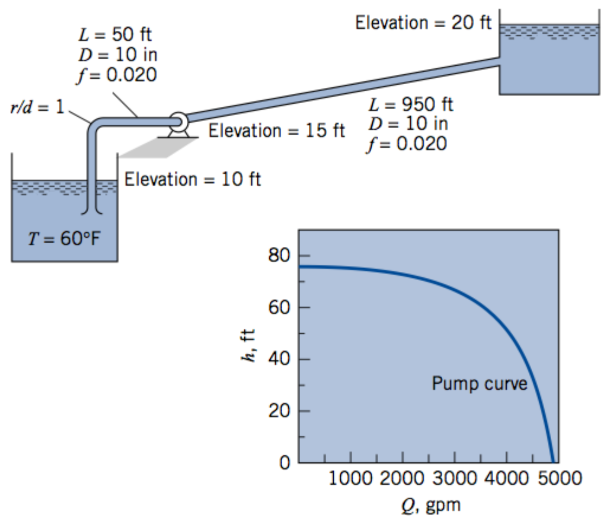
Plotting this on pump curve figure yields $\boxed{Q \approx 2,950 \text{ gpm}}$

10.90: PROBLEM DEFINITION

Situation:

A liquid is pumped through a pipe from one tank to another.
$r/d = 1$, $D_1 = 10\,\text{in}$, $L_1 = 50\,\text{ft}$.
$f_2 = 0.020$, $z_1 = 10\,\text{ft}$.
$D_2 = 10\,\text{in}$, $L_2 = 950\,\text{ft}$.
$f_2 = 0.020$, $z_2 = 20\,\text{ft}$.

Sketch:



Find:

Pumping rate (gpm).

Assumptions:

No head loss for this liquid.

SOLUTION $h_p = 20$ ft - 10 ft $= 10$ ft

Then from the pump curve for problem 10.89 one finds $Q = 4700$ gpm.

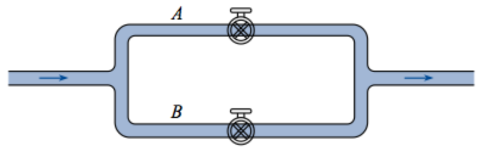
10.91: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.

$K_{vA} = 0.2$, $K_{vB} = 10$, $2 \times A_A = A_B$.

Sketch:



Find:

Ratio of discharge in line B to that in line A.

Assumptions:

Head loss due to valves overshadows losses due to junctions, elbows and friction.

SOLUTION

$$
\begin{aligned}
h_{LA} &= h_{LB} \\
0.2\frac{V_A^2}{2g} &= 10\frac{V_B^2}{2g} \qquad (1)\\
V_A &= \sqrt{50}V_B \\
\frac{Q_B}{Q_A} &= \frac{V_B A_B}{V_A A_A} \\
&= \frac{V_B A_B}{V_A \times \frac{1}{2}A_B} \qquad (2)\\
\frac{Q_B}{Q_A} &= \frac{2V_B}{V_A}
\end{aligned}
$$

Solve Eqs. (1) and (2) for Q_B/Q_A:

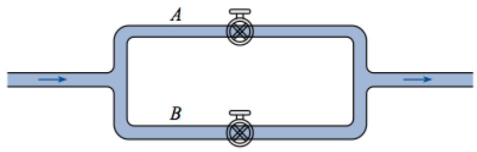
$$
\begin{aligned}
\frac{Q_B}{Q_A} &= \frac{2 \times V_B}{\sqrt{50}V_B} \\
& \boxed{\frac{Q_B}{Q_A} = 0.283}
\end{aligned}
$$

10.92: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.
Line A has a half open gate valve, line B a fully open globe valve.

Sketch:



Find:

Ratio of velocity in line A to B.

Assumptions:

Head loss due to friction is negligible.

Properties:

From Table 10.5: $K_{vA} = 5.6$, $K_{vB} = 10$, $K_b = 0.9$.

SOLUTION

$$\begin{aligned}
\sum h_{LB} &= \sum h_{LA} \\
h_{L,\text{globe}} + 2h_{L,\text{elbow}} &= h_{L,\text{gate}} + 2h_{L,\text{elbow}} \\
10\frac{V_B^2}{2g} + 2\left(0.9\frac{V_B^2}{2g}\right) &= 5.6\frac{V_A^2}{2g} + 2\left(0.9\frac{V_A^2}{2g}\right) \\
11.8\frac{V_B^2}{2g} &= 7.4\frac{V_A^2}{2g}
\end{aligned}$$

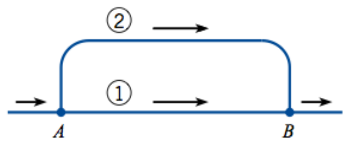
$$\boxed{\frac{V_A}{V_B} = 1.26}$$

10.93: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.
$L_1 = 1000\,\text{m}$, $D_1 = 50\,\text{cm}$, $L_2 = 1500\,\text{m}$.
$D_2 = 40\,\text{cm}$, $Q = 1.2\,\text{m}^3/\,\text{s}$.

Sketch:



Find:

Division of flow of water.

Assumptions:

Friction factor, f, is equal in both lines.

SOLUTION

$$\frac{V_1}{V_2} = \left[\left(\frac{f_2}{f_1}\right)\left(\frac{L_2}{L_1}\right)\left(\frac{D_1}{D_2}\right)\right]^{1/2}$$

Initially assume $f_1 = f_2$
Then

$$\begin{aligned}
\frac{V_1}{V_2} &= \left[\left(\frac{1500\,\text{m}}{1000\,\text{m}}\right)\left(\frac{0.5\,\text{m}}{0.4\,\text{m}}\right)\right]^{1/2} \\
&= 1.369 \\
V_1 &= 1.37V_2 \\
1.2 &= V_1A_1 + V_2A_2 \\
1.2 &= 1.37V_2 \times (\pi/4) \times (0.5\,\text{m})^2 + V_2 \times (\pi/4) \times (0.4\,\text{m})^2 \\
&\boxed{V_2 = 3.04 \text{ m/s}} \\
\text{Then } V_1 &= 1.37 \times 3.04 \\
&\boxed{V_1 = 4.16 \text{ m/s}} \\
Q_1 &= V_1A_1 \\
&= 4.16\,\text{m}/\,\text{s} \times (\pi/4) \times (0.5\,\text{m})^2 \\
&\boxed{Q_1 = 0.816 \text{ m}^3/\text{s}} \\
&\boxed{Q_2 = 0.382 \text{ m}^3/\text{s}}
\end{aligned}$$

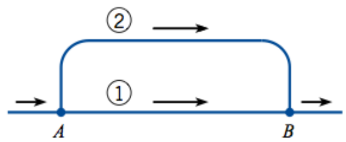
10.94: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.

$L_2 = 4L_1,\ D_1 = D_2,\ Q_2 = 1\,\text{ft}^3/\,\text{s}.$

Sketch:



Find:

Discharge in pipe 1.

Assumptions:

Friction factor, f, is equal in both lines.

SOLUTION

$$h_{f,1} = h_{f,2}$$

$$\begin{aligned} f\left(\frac{L}{D}\right)\frac{V_1^2}{2g} &= f\left(\frac{4L}{D}\right)\frac{V_2^2}{2g} \\ V_1^2 &= 4V_2^2 \\ V_1 &= 2V_2 \end{aligned}$$

Thus

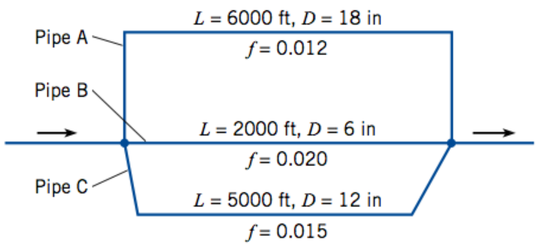
$$\begin{aligned} Q_1 &= 2Q_2 \\ & \boxed{Q_1 = 2 \text{ cfs}} \end{aligned}$$

10.95: PROBLEM DEFINITION

Situation:

Three pipes are connected in parallel
$L_A = 6000\,\text{ft},\ D_A = 18\,\text{in},\ f_A = 0.012.$
$L_B = 2000\,\text{ft},\ D_B = 6\,\text{in},\ f_B = 0.020.$
$L_C = 5000\,\text{ft},\ D_C = 12\,\text{in},\ f_C = 0.015.$

Sketch:



Find:

The pipe having the greatest velocity.

SOLUTION

$$
\begin{aligned}
h_{p,A} &= h_{f,B} = h_{f,C} \\
f\frac{L}{D}\left(\frac{V^2}{2g}\right)_A &= f\frac{L}{D}\left(\frac{V^2}{2g}\right)_B = f\frac{L}{D}\left(\frac{V^2}{2g}\right)_C \\
0.012\left(\frac{6000\,\text{ft}}{1.5\,\text{ft}}\right)V_A^2 &= 0.02\left(\frac{2000\,\text{ft}}{0.5\,\text{ft}}\right)V_B^2 = 0.015\left(\frac{5000\,\text{ft}}{1\,\text{ft}}\right)V_C^2 \\
48V_A^2 &= 80V_B^2 = 75V_C^2
\end{aligned}
$$

Therefore, V_A will have the greatest velocity. Correct choice is (a)

10.96: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.

$L_1 = 3L_2,\ D_1 = 2D_2.$

$f_1 = 0.010,\ f_2 = 0.014.$

Find:

Ratio of discharges in two pipes.

SOLUTION

$$\frac{V_1}{V_2} = \left[\left(\frac{f_2}{f_1}\right)\left(\frac{L_2}{L_1}\right)\left(\frac{D_1}{D_2}\right)\right]^{1/2}$$

Let pipe 1 be large pipe and pipe 2 be smaller pipe. Then

$$\frac{V_1}{V_2} = \left[\left(\frac{0.014}{0.01}\right)\left(\frac{L}{3L}\right)\left(\frac{2D}{D}\right)\right]^{1/2} = 0.966$$

$$\frac{Q_1}{Q_2} = \frac{V_1}{V_2}\frac{A_1}{A_2} = 0.966 \times \left(\frac{2D}{D}\right)^2 = 3.86$$

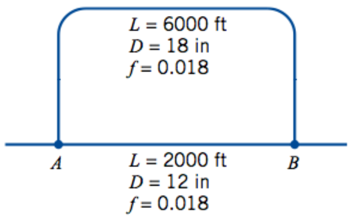
$$\boxed{(Q_{\text{large}}/Q_{\text{small}}) = 3.86}$$

10.97: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.
$Q = 14\,\text{ft}^3/\,\text{s},\ L_1 = 6000\,\text{ft}.$
$D_1 = 18\,\text{in},\ f_1 = 0.018.$
$L_1 = 2000\,\text{ft},\ D_1 = 12\,\text{in},\ f_1 = 0.018.$

Sketch:



Find:

Division of flow (cfs).
Head loss (ft).

SOLUTION

$$\begin{aligned} Q_{18}+Q_{12} &= 14\text{ cfs} \\ h_{L_{18}} &= h_{L_{12}} \\ f_{18}\frac{L_{18}}{D_{18}}\frac{V_{18}^2}{2g} &= f_{12}\frac{L_{12}}{D_{12}}\frac{V_{12}^2}{2g} \\ f_{18} &= 0.018 = f_{12} \end{aligned}$$

so

$$\begin{aligned} \frac{L_{18}Q_{18}^2}{D_{18}^5} &= \frac{L_{12}Q_{12}^2}{D_{12}^5} \\ Q_{18}^2 &= \left(\frac{D_{18}}{D_{12}}\right)^5\left(\frac{L_{12}}{L_{18}}\right)Q_{12}^2 \\ &= \left(\frac{18\text{ ft}}{12\text{ ft}}\right)^5\left(\frac{2000}{6000}\right)Q_{12}^2 \\ &= 2.53Q_{12}^2 \\ Q_{18} &= 1.59Q_{12} \end{aligned}$$

$$\begin{aligned} 1.59Q_{12}+Q_{12} &= 14 \\ 2.59Q_{12} &= 14 \\ & \boxed{Q_{12} = 5.4\text{ cfs}} \end{aligned}$$

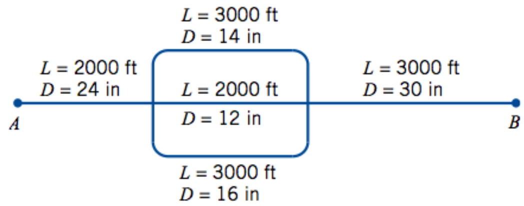
$$
\begin{aligned}
Q_{18} &= 1.59 Q_{12} \\
&= 1.59(5.4) \\
& \boxed{Q_{18} = 8.6 \text{ cfs}}
\end{aligned}
$$

$$
\begin{aligned}
V_{12} &= \frac{5.4\,\text{ft}^3/\text{s}}{\pi/4 \times (1\,\text{ft})^2} = 6.88\,\text{ft}/\text{s} \\
V_{18} &= \frac{8.6\,\text{ft}^3/\text{s}}{\pi/4 \times (1.5\,\text{ft})^2} = 4.87\,\text{ft}/\text{s} \\
h_{L_{12}} &= 0.018\left(\frac{2,000\,\text{ft}}{1\,\text{ft}}\right)\frac{(6.88\,\text{ft}/\text{s})^2}{64.4\,\text{ft}/\text{s}^2} = 26.5\,\text{ft} \\
h_{L_{18}} &= 0.018\left(\frac{6,000\,\text{ft}}{1.5\,\text{ft}}\right)\frac{(4.87\,\text{ft}/\text{s})^2}{64.4\,\text{ft}/\text{s}^2} = 26.5\,\text{ft} \\
& \text{Thus, } \boxed{h_{L_{A-B}} = 26.5 \text{ ft}}
\end{aligned}
$$

10.98: PROBLEM DEFINITION

Situation:

A concrete piping system is described in the problem statement.
$Q = 25\,\text{ft}^3/\text{s},\ f = 0.030,\ L_{24} = 2000\,\text{ft},\ D_{24} = 24\,\text{in}.$
$L_{30} = 3000\,\text{ft},\ D_{30} = 30\,\text{in},\ L_{14} = 3000\,\text{ft},\ D_{14} = 14\,\text{in}.$
$L_{12} = 2000\,\text{ft},\ D_{12} = 12\,\text{in},\ L_{16} = 3000\,\text{ft},\ D_{16} = 16\,\text{in}.$



Find:

Division of flow.
Head loss.

SOLUTION

Sketch:

$$\begin{aligned} Q &= Q_{14} + Q_{12} + Q_{16} \\ 25 &= V_{14} \times (\pi/4) \times \left(\frac{14}{12}\,\text{ft}\right)^2 + V_{12} \times (\pi/4) \times (1\,\text{ft})^2 + V_{16} \times (\pi/4) \times \left(\frac{16}{12}\,\text{ft}\right)^2 \quad (1) \end{aligned}$$

Also, $h_{f_{14}} = h_{f_{12}} = h_{f_{16}}$ and assuming $f = 0.03$ for all pipes

$$\left(\frac{3000\,\text{ft}}{14\,\text{in}}\right) V_{14}^2 = \left(\frac{2000\,\text{ft}}{12\,\text{in}}\right) V_{12}^2 = \left(\frac{3000\,\text{ft}}{16\,\text{in}}\right) V_{16}^2 \quad (2)$$

$$V_{14}^2 = 0.778 V_{12}^2 = 0.875 V_{16}^2$$

From Eq.(1)

$$\begin{aligned} 25 &= 1.069 V_{14} + 0.890 V_{14} + 1.49 V_{14} \\ V_{14} &= 7.25 \text{ ft/s} \end{aligned}$$

and $V_{12} = 8.22$, $V_{14} = 7.25$ ft/s; $V_{16} = 7.25$ ft/s. Calculate flow rate using $Q = VA$

Q (12 inch pipe) = 6.46 cfs

Q (14 inch pipe) = 7.75 cfs

Q (16 inch pipe) = 10.8 cfs

$$
\begin{aligned}
V_{24} &= \frac{Q}{A_{24}} = \frac{25}{\pi/4 \times (2\,\mathrm{ft})^2} = 7.96 \text{ ft/s;} \\
V_{30} &= 5.09 \text{ ft/s} \\
h_{L\mathrm{AB}} &= \frac{0.03}{64.4\,\mathrm{ft/\,s^2}}[\left(\frac{2000\,\mathrm{ft}}{2\,\mathrm{ft}}\right)(7.96\,\mathrm{ft/\,s})^2 + \left(\frac{2000\,\mathrm{ft}}{1\,\mathrm{ft}}\right) \times (8.21\,\mathrm{ft/\,s})^2 \\
&\quad + \left(\frac{3000\,\mathrm{ft}}{2.5\,\mathrm{ft}}\right) \times (5.09\,\mathrm{ft/\,s})^2]
\end{aligned}
$$

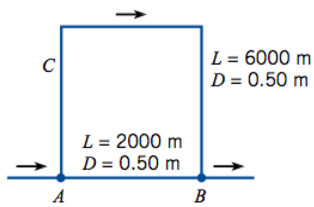
$$
\boxed{h_{L_{\mathrm{AB}}} = 107\,\mathrm{ft}}
$$

10.99: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel with the pump from figure 10.16 on line C.
$L_1 = 2000\,\mathrm{m}$, $D_1 = 0.50\,\mathrm{m}$, $L_2 = 6000\,\mathrm{m}$.
$D_2 = 0.50\,\mathrm{m}$, $Q = 0.60\,\mathrm{m}^3/\,\mathrm{s}$.

Sketch:



Find:

Division of flow between pipes ($\mathrm{m}^3/\,\mathrm{s}$).
Head loss (m).

Properties:

From Table 10.4 $k_s = 0.046$ mm.

SOLUTION

Call pipe A-B pipe and pipe ACB pipe 2. Then

$$
\begin{aligned}
h_{f,1} + h_p &= h_{f,2} \\
\frac{k_s}{D} &= \frac{0.046}{500} \simeq 0.0001
\end{aligned}
$$

Assume $f_1 = f_2 = 0.013$ (guess from Fig. 10-8)

$$
\begin{aligned}
f\frac{L_1}{D_1}\frac{V_1^2}{2g} + h_p &= f\frac{L_2}{D_2}\frac{V_2^2}{2g} \\
0.013\left(\frac{2000\,\mathrm{m}}{0.5\,\mathrm{m}}\right)\frac{V_1^2}{2g} + h_p &= 0.013\left(\frac{6000\,\mathrm{m}}{0.5\,\mathrm{m}}\right)\frac{V_2^2}{2g} \\
2.65V_1^2 + h_p &= 7.951V_2^2 \qquad (1)
\end{aligned}
$$

Continuity principle

$$
\begin{aligned}
(V_1 + V_2)A &= 0.60\ \mathrm{m}^3/\mathrm{s} \\
V_1 + V_2 &= \frac{0.6\,\mathrm{m}^3/\,\mathrm{s}}{A} = \frac{0.6\,\mathrm{m}^3/\,\mathrm{s}}{\pi/4 \times (0.5\,\mathrm{m})^2} = 3.0558 \\
V_1 &= 3.0558 - V_2 \qquad (2)
\end{aligned}
$$

By iteration (Eqs. (1), (2) and pump curve) one can solve for the division of flow:

$$\boxed{Q_1 = 0.27 \text{ m}^3/\text{s}}$$

$$\boxed{Q_2 = 0.33 \text{ m}^3/\text{s}}$$

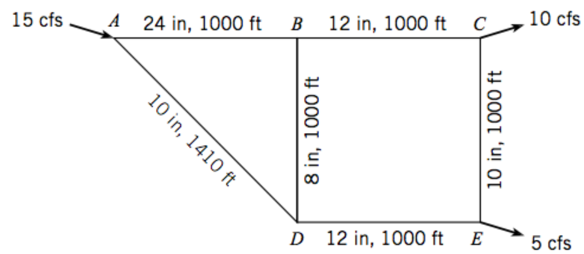
Head loss determined along pipe 1

$$\begin{aligned}
h_L &= f\frac{L}{D}\frac{V_1^2}{2g} \\
V_1 &= \frac{Q_1}{A} = \frac{0.27}{\pi/4 \times (0.5\,\text{m})^2} = 1.38 \text{ m/s} \\
h_l &= 0.013\left(\frac{2000\,\text{m}}{0.5\,\text{m}}\right)\frac{(1.38\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} \\
&\boxed{h_l = 5.05 \text{ m}}
\end{aligned}$$

10.100: PROBLEM DEFINITION

Situation:

A piping network with sources and loads is specified.
$f = 0.012$, $p_A = 60$ psi, $Q_A = 15\,\mathrm{ft}^3/\,\mathrm{s}$.
$Q_C = 10\,\mathrm{ft}^3/\,\mathrm{s}$, $Q_E = 5\,\mathrm{ft}^3/\,\mathrm{s}$.



Find:

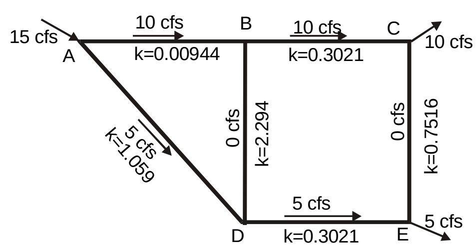
Load distribution and pressure at load points.

Properties:

Water, Table A.5: $\gamma = 62.4\,\mathrm{lbf}/\,\mathrm{ft}^3$.

SOLUTION

An assumption is made for the discharge in all pipes making certain that the continuity equation is satisfied at each junction. The following figure shows the network with assumed flows.



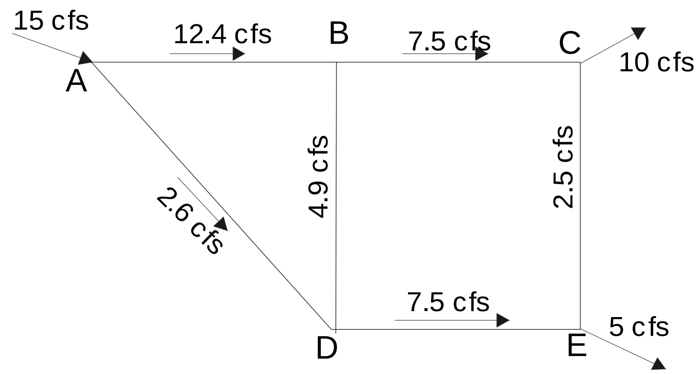
Darcy-Weisbach equation

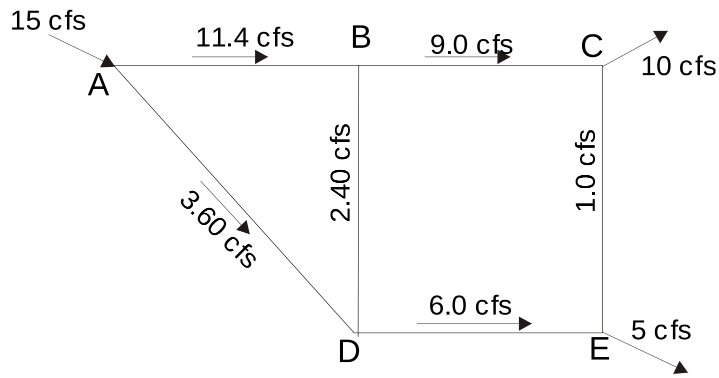
$$\begin{aligned} h_f &= f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \\ &= 8\left(\frac{fL}{gD^5\pi^2}\right)Q^2 \\ &= kQ^2. \end{aligned}$$

where $k = 8\left(\frac{fL}{gD^5\pi^2}\right)$. The loss coefficient, k, for each pipe is computed and shown in Fig. A. Next, the flow corrections for each loop are calculated as shown in the accompanying table. Since $n = 2$ (exponent on Q), $nkQ^{n-1} = 2kQ$. When the correction obtained in the table are applied to the two loops, we get the pipe discharges shown in Fig. B. Then with additional iterations, we get the final distribution of flow as shown in Fig. C. Finally, the pressures at the load points are calculated.

Loop ABC		
Pipe	$h_f = kQ^2$	$2kQ$
AB	+0.944	0.189
AD	-26.475	10.590
BD	0	0
$\sum kQ_c^2 - \sum kQ_{cc}^2$	-25.53	$\sum 2KQ = 10.78$
ΔQ=-22.66/9.062=2.50 cfs		

Loop $BCDE$		
Pipe	h_f	$2kQ$
BC	+30.21	6.042
BD	0	0
CE	0	0
DE	-7.55	3.02
	+22.66	9.062
ΔQ=-25.53/10.78=-2.40 cfs		





$$
\begin{aligned}
p_C &= p_A - \gamma(k_{AB}Q_{AB}^2 + k_{BC}Q_{BC}^2) \\
&= 60 \text{ psi } \times 144 \text{ psf/psi} - 62.4\,\text{lbf/ ft}^3(0.00944 \times 11.4^2 + 0.3021 \times 9.0^2) \\
&= 8640 \text{ psf } - 1603 \text{ psf} \\
&= 7037 \text{ psf} \\
&\boxed{p_C = 48.9 \text{ psi}} \\
p_E &= 8640 - \gamma(k_{AD}Q_{AD}^2 + k_{DE}Q_{DE}^2) \\
&= 8640 - 62.4(1.059 \times 3.5^2 + 0.3021 \times 6^2) \\
&= 7105 \text{ psf} \\
&\boxed{p_E = 49.3 \text{ psi}}
\end{aligned}
$$

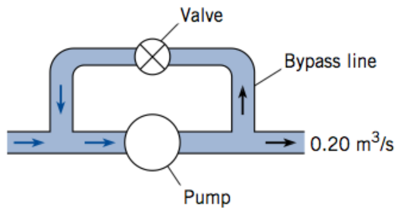
10.101: PROBLEM DEFINITION

Situation:

Two pipes are in parallel. One has a bypass valve and the other a pump.

$h_p = 100 - 100Q_p$, $Q_p = Q_v + 0.2\,\mathrm{m^3/s}$.

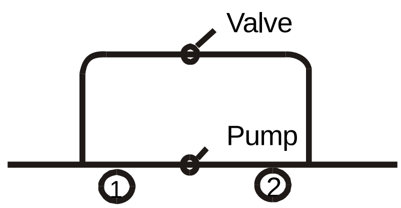
$D_v = 10\,\mathrm{cm}$, $K_v = 0.2$.



Find:

Discharge through pump and bypass line.

SOLUTION



$$
\begin{aligned}
Q_p &= Q_v + 0.2 \\
\frac{p_2 - p_1}{\gamma} &= h_p \\
A &= \pi/4 \times (0.1\,\mathrm{m})^2 \\
&= 0.00785\ \mathrm{m}^2 \\
\frac{K_v V_v^2}{2g} &= \frac{K_v Q_v^2}{2gA^2} = h_p \\
h_p &= 100 - 100(Q_v + 0.2) \\
\frac{(0.2)Q_v^2}{2 \times 9.81\,\mathrm{m/s^2} \times 0.00785\,\mathrm{m^2}} &= 100 - 100Q_v - 20 \\
165Q_v^2 + 100Q_v - 80 &= 0
\end{aligned}
$$

Solve by quadratic formula

$$
\boxed{Q_v = 0.456\mathrm{m^3/s}}
$$

$$
\begin{aligned}
Q_p &= 0.456 + 0.2 \\
&\boxed{Q_v = 0.656\ \mathrm{m^3/s}}
\end{aligned}
$$