Mech Family

تقدم

دفتر

FLUID

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No.

Fluid Mechanics I

18/2/2014

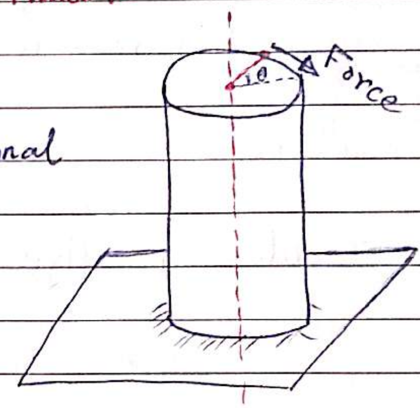
CH 1 # introduction

1.1 Fluid ???   [liquids + gases]

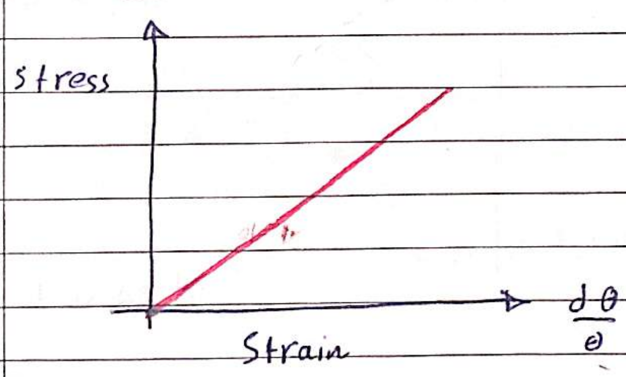
→ any substance that will continously deform under shear stress, no matter what the value of stress

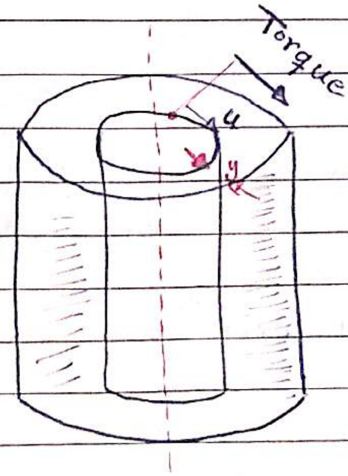
→ comparison Between Solids and fluids :-

* solids :-



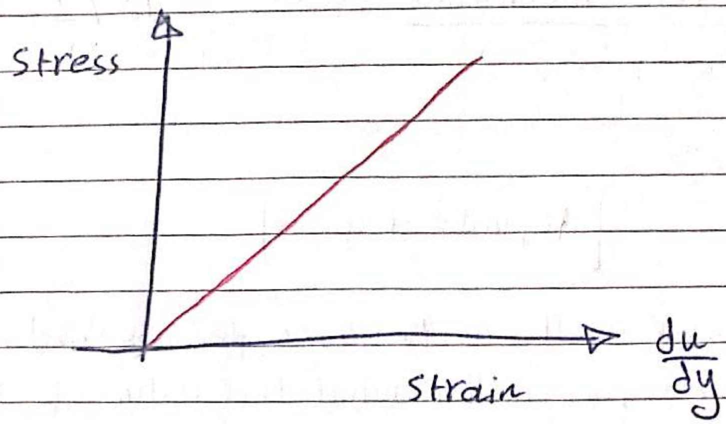
in solids :- the stress is Proportional to strain





* Fluids

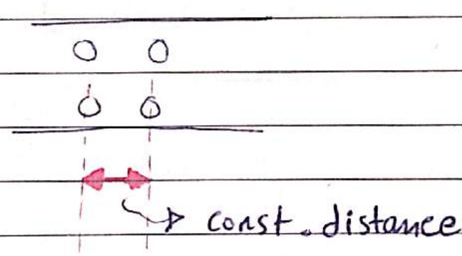
in fluids :- Stress is proportional with rate of Strain



another differince is Molecules

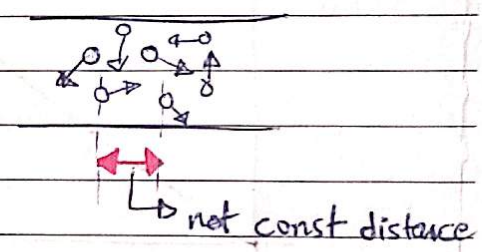
in solids :-

Molecules are fixed



in fluids :-

Molecules are not fixed



1.2 Continuum

* in some conditions [very low pressure] there will Be no Continuoty on the fluid

→ our course consists of Continuous fluids only.

No.

* Newtonian and non Newtonian fluids :-

* non Newtonian such as fresh concrete , Jelly

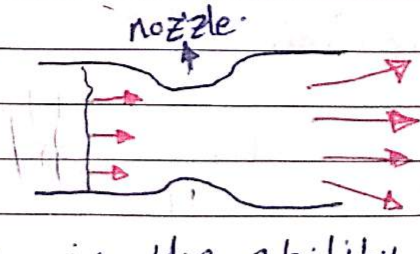
⟶ in our course ⟶ all fluids are Newonian .

* fluid classification :-

fluid Mechanics

Hydrodynamics	Gas dynamics
- no density change - Hydraulics :- liquids only - low-speed gas flows :- - such :- air cond in ducts	- where there is change in density

~~*nozzle~~



* speed has a role in the ability to compress the fluid

if < 0.3 speed of sound ⟶ $\boxed{\text{incompressible}}$

if > 0.3 " " " ⟶ $\boxed{\text{compressible}}$

No.

such that: 0.3 is $\boxed{\text{Mach Number}}$

also the speed must Be in terms of Mach No.

such: $\boxed{\text{Mach No} = \dfrac{\text{Speed measured}}{\text{speed of sound}}}$

1.3 Units:

1] Primary or Basic, such:

Mass M — Temp T
length L
time t

2] Secondary or derived, such
force F
pressure P
Velocity $\mathcal{V}$
acceleration a

	SI	British or	USCS
M	kg	lbm	slug = 32.2 lbm
L	m	ft	ft
t	s	s	s
T	$k = C° + 273.15$	R° Rankin	$R° = F° + 460$
F	N	lbf	lbf
P	N/m^2 or Pascal	lbf/m^2	lbf/m^2
$\mathcal{V}$	m/s	ft/s	ft/s
α	m/s^2	ft/s^2	ft/s^2

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* relation Between force and mass

Newton's 2^{nd} law

$F \propto ma \rightarrow F = \frac{ma}{g_c}$ g_c :- is constant

units of g_c

$m = 1\,kg$ $a = 1\,m/s^2$ $F = 1\,N$

SI units :- $F = 1\,N = \frac{kg \cdot m/s^2}{g_c}$ $[g_c] = 1 \frac{kg \cdot m/s^2}{N}$

But $N = kg \cdot m/s^2$ $[g_c] = 1$ dimentionless

British units :- $F = lb_f = \frac{1\,slug \cdot ft/s^2}{g_c}$ Type II

$$\boxed{[g_c] = 1 \frac{slug \cdot ft}{lbf \cdot s^2}}$$ which we used maity in this book

Type I $F = lb_f = 32.2\ lbm * 1\,ft/s^2$

$$[g_c] = 32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}$$

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chapter 2

fluid properties

[1] - Density (ρ) [kg/m^3] or [$slug/ft^3$]

for water :- $1000\ kg/m^3$ or $1.94\ slug/ft^3$

[2] - specific weight (γ) :- mass per unit Volume

$\gamma = \rho g$ [γ] = N/m^3 or lbf/ft^3

for water :- $9810\ N/m^3$ or $62.4\ lbf/ft^3$

* for liquids :- ρ is usually constant

→ in compressible.

* for gases :- ρ may Be varible, depending on velocity, such that

1) at low velocity :- ρ = const

2) " High " :- $\rho \neq$ const

No.

3 - Specific gravity (s) :- = $\frac{\text{Weight of certain Vol. of material}}{\text{Wieght of same Vol of water}}$

$$\boxed{s = \frac{\gamma}{\gamma_w} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w}}$$ dimentionless

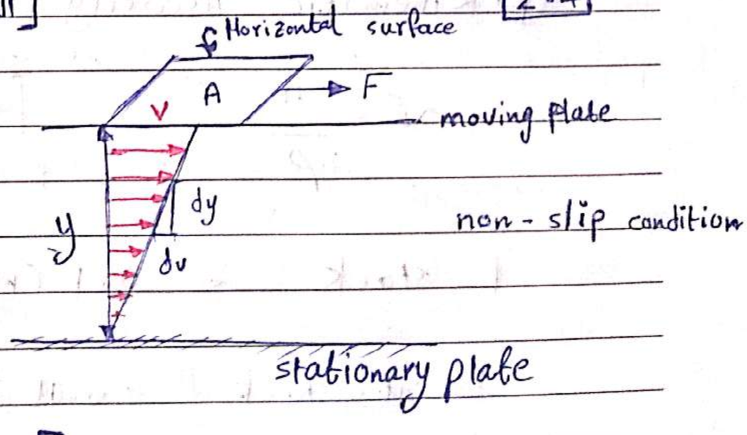
4 - Specific volume v :- $= \frac{1}{\rho} = \frac{V}{m} \equiv m^3/kg$

* eq of state for ideal gas :-

$$\boxed{\rho = \frac{P}{RT}} \equiv \frac{kPa}{kJ/kg.k \cdot K}$$

5 - Viscocity [اللزوجة] **2.4**

$$F \propto \frac{AV}{y}$$



$$\frac{F}{A} \propto \frac{V}{y}$$

But τ (shear stress) $= \frac{F}{A}$

$\frac{V}{y} = \frac{du}{dy}$ تشابه المثلثات $\Rightarrow$ $\boxed{\tau = \boxed{\mu} * \frac{du}{dy}}$ → constant

law of viscocity

μ :- is const → viscocity

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- the viscocity has two types:-

1) absolute (μ)   2) dynamic

units of viscocity

$$[\mu] = \frac{[\tau]}{\left[\frac{dv}{dy}\right]} = \frac{N/m^2}{(m/s)/m} = \frac{N.s}{m^2} = \frac{kg}{m.s}$$

$$1 \text{ poise} = 0.1 \frac{kg}{m.s}$$

1 centi poise = 0.01 poise = 0.001 kg/m.s

Kinematic viscocity (ν)

$$\nu = \frac{\mu}{\rho} \qquad [\nu] = m^2/s$$

1 stock = 0.1 cm^2/s = 0.0001 m^2/s

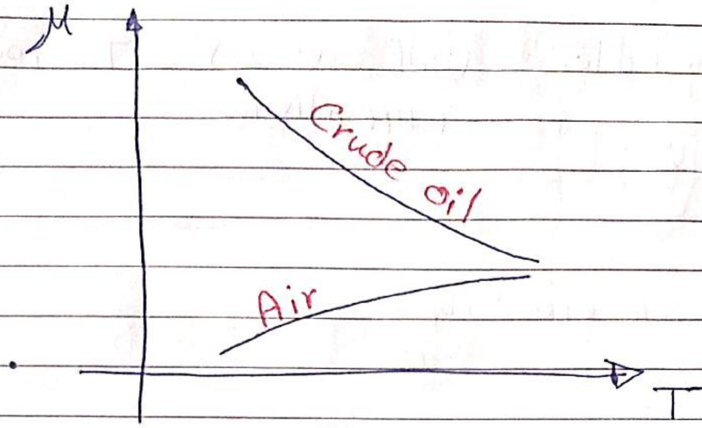
1 centi stock = 0.01 stock = $1*10^{-6}$ m^2/s

for water $\boxed{\nu = 1*10^{-6}\ m^2/s = 1 \text{ centi stoke}}$

see figure A.2 , A.3

Hw #1 with quiz 4/3/2014

10th ed 2. 31, 33, 55, 37
9th 35, 37, 39, 40
No. 8th 31, 33, ~~38~~, 39



Crudeoil :- زيوت المحركات والبريك

* variation of μ with Temprature :-

1) liquids :- μ goes down with T goes up

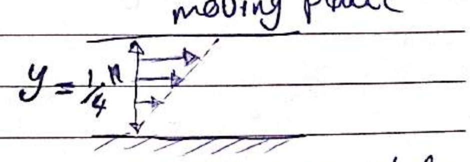
2) ~~gas~~ gases :- μ " up " T " down

* notice that $\mu_{liq} > \mu_{gas}$ and $\nu_{liq} < \nu_{gas}$

25/2/2014 example 2.34

shaft $V = 10\ ft/s$

100°F



oil (SAE 10 W - 30)

find Shear stress (τ) in the oil ???

SAE 10 W - 30)

society of automotive engineers (SAE)

W: winter

sol :-

$$\tau = \mu \frac{dU}{dy}$$

No.

μ :- from tables of (SAE 10W-30) at T = 100°F
(Appendix A)

$$\frac{dV}{dy} \approx \frac{V}{y}$$

$$\mu = 1.4 * 10^{-3} \ \frac{lb_f \cdot s}{ft^2}$$

$$\tau = 1.4 * 10^{-3} * \frac{10}{\underbrace{1/4/12}_{\text{inch} \to ft}} = 0.672 \ lb_f / ft^2$$

* extension :- find F

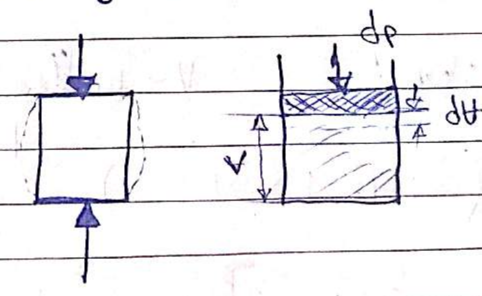
$$\tau = \frac{F}{A} \qquad A = 2\pi r * \text{depth}$$

(see figure)

$$F = \tau * A$$
$$= 0.672 * 2\pi r * 1 = - -$$

2.5 * elasticity and compressibility

$$dp \propto \frac{d\forall}{\forall}$$



$$dp = -E_v \frac{d\forall}{\forall}$$

حطيناها للحفاظ على خاصية

$$dp \propto \frac{1}{\forall}$$

E_v :- Moleculessof elasticity or (liquids)
or Compressebility factor (solid)

No.

example :- pipeline of 1m diameter and 5km long. water, at atm pressure, the working pressure is 40 Atm above atmospheric, the test pressure is 1.5 the working pressure, calculate the change in water volume under the test pressure, compare it to the working pressure ???

solution :- $d\forall = -\frac{\forall \, dP}{E_v} \Rightarrow \Delta\forall = -\forall \frac{\Delta P}{E_v}$

$$\Delta\forall = -\frac{\pi D^2}{4} * L \frac{\Delta P}{E_v} = -\frac{\pi}{4} 1^2 * 5*10^3 \left(\frac{1.5*40*101{,}325}{2200*10^6}\right)$$

$= -10.54 \ m^3$ ~~compared to the~~ tables

compared to the original volume : 0.3 % less

* ملاحظة صغيرة ومهمة على حساب [shear stress] :-

* أولاً :- السرعة عند الصفيحة الثابتة يجب أن تساوي $\boxed{\text{صفر}}$ أشي بديهي

* ثانياً :- كلما أبتعدنا عن الصفيحة كلما زاد الـ shear stress ومنها

1) أعلى قيمة لـ shear stress هي عندما تكون y أبعد ما يمكن عن الصفيحتين ((يعني بالنص))

2) أقل - - - - - - - - - - أقل ما يمكن ((صفر y))

No.

27/2/2017

CHAPTER 3

Fluid Statics

3.1 Pressure

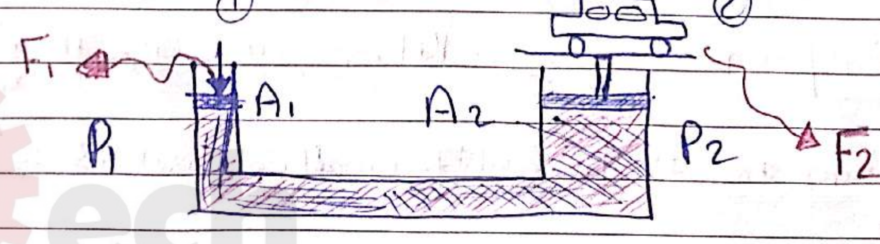
- $\boxed{P \equiv \frac{F}{A}}$ = force per unit Area

- pressure at a point is $\boxed{P = \frac{dF}{dA}}$

* such that, Pressure is a scalar quantity

* Pressure transmission ((Pascal's principle))

→ in a closed System, a pressure change is produced at one point in the system, will Be transmitted through out the whole system



$\boxed{P_1 = P_2} \rightsquigarrow \boxed{\frac{F_1}{A_1} = \frac{F_2}{A_2}}$

* positive displacement pump :- رداو

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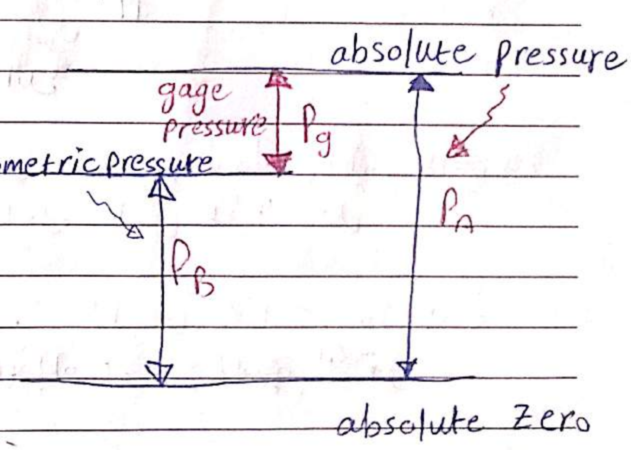
* All the HyDraulic systems depends on Pascal's Principle

* Pressure measurment

→ 3 types :-

1) ABSOLUTE :- measured from an absolute point which is outer space (($P=0$))



2) Barometric

* الضغط الجوي.

* ~~101.3 KPa~~

يساوي 101.3 KPa في الظروف الطبيعية.

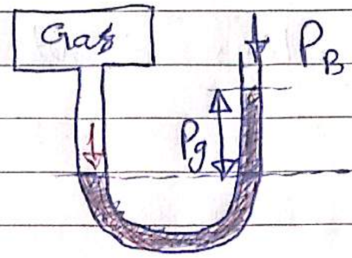
3) gage ((gauge))

$$\boxed{P_A = P_B + P_g}$$

* some types of gage pressure measuring equipments :-

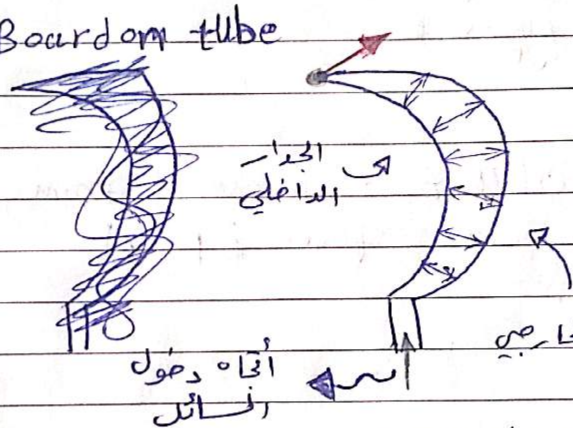
1) → manometer

No.



U-tube

2) ~~Bourdom~~ Boardom tube



* مبدأ عمل Bourdontube :-

1) يدخل السائل المراد قياس ضغطه إلى داخل أنبوب خاص رفيع ومكون من جدارين داخلي وخارجي محاني الشكل

2) بسبب الضغط يتوسع الجدار الخارجي للخارج مما يؤدي إلى تحرك الرأس المدبب بالاتجاه الموضح بالسهم الأحمر

⟵ شو الفايدة من هالحكي ؟؟؟!!!

⟵ أكثر من 99 % من ساعات قياس الضغط بالعالم تعتمد هذا المبدأ بالقياس ، حيث يتم تركيب سهم مدبب على النقطة المتحركة ويتم معايرتها لكي تعطي قراءات دقيقة للضغط .

See page 72 for a good picture

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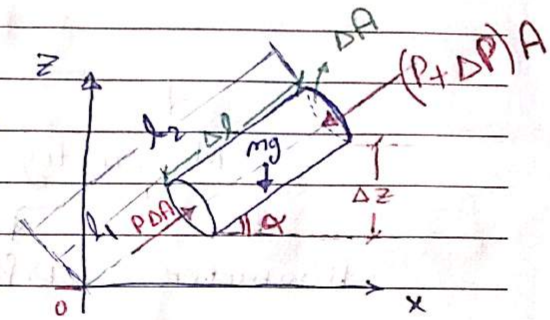
2/3/2014

3.2 * Pressure variation ~~and~~ with elevation

$\Sigma F_l = 0$



$P\Delta A - (P+\Delta P)\Delta A - mg\sin\alpha = 0$

$mg = \forall \rho g = \Delta A\, \Delta l\, \rho g$
$= \Delta A\, \Delta l\, \gamma$

$\dfrac{\Delta P}{\Delta l} = -\gamma \sin\alpha$

remember $\boxed{\gamma = \rho g}$

take limit when $\Delta l \to 0$ and Put $\boxed{\sin\alpha = \dfrac{\Delta Z}{\Delta l}}$ $\Rightarrow \dfrac{dZ}{dl}$

$\dfrac{dP}{dl} = -\gamma \dfrac{dZ}{\partial l}$

$\Rightarrow \dfrac{dP}{dZ} = -\gamma \Rightarrow \int_1^2 dp = \int_1^2 -\gamma\, dZ$

$$\boxed{P_2 - P_1 = -\gamma\,(\underbrace{Z_2 - Z_1}_{h}) = -\rho g h}$$ * قانون ثابت

Call Z :- head or ((geometrical head))

* Re arrange the eq :-

$\dfrac{P_2}{\gamma} + Z_2 = \dfrac{P_1}{\gamma} + Z_1 \Rightarrow \boxed{\dfrac{P}{\gamma} + Z} = \text{constant}$

Peisometric head

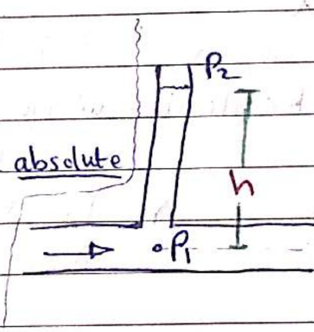
* such that P_1, P_2 has the same unit and the same type

No.

3.3 Pressure Measurment   complete

P. measurment
- Manometry
  - Piesometer
  - Differental manometer
- Bourdon tube
- Pressure transducer

1 Pieso meter



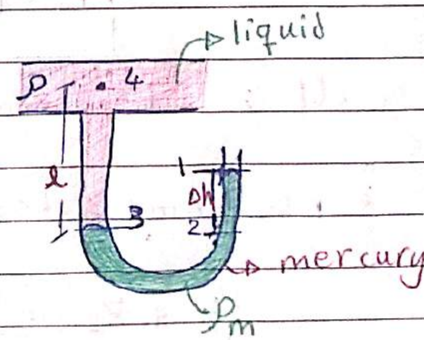
$P_1 - P_2 = \rho g h$. * if P_1, P_2 are absolute

* if P_1, P_2 ~~fa~~are gage

$P_1 = \rho g h$ $P_2 = P_{atm} = 0$ ((in gage measuring))

* can be used for small pressure measuring ~~≤~~ ≤ 10 kPa

2 Differental manometer



① there is an~~d~~ equibrium at the inter face between any two points.

$P_3 - P_4 = \rho g l$ --- ①

② Pressure is the same in the same liquid in same elevation $P_2 = P_3$ --- ②

No.

also $P_2 - P_1 = \rho_m g \Delta h$ ... ③

$$P_4 = P_3 + \rho g l = P_2 - \rho g l$$

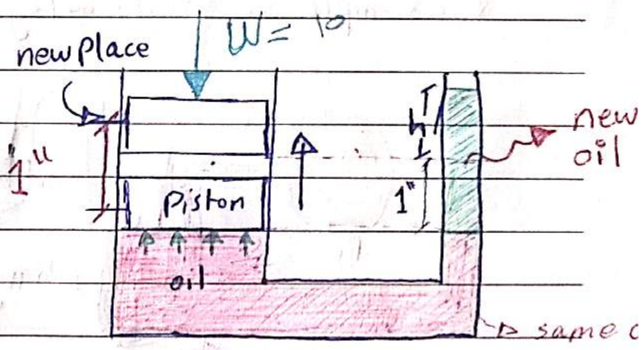
$$\boxed{P_4 = P_1 + \rho_m g \Delta h + \rho g l}$$ for absolute pressure

$$\boxed{P_4 = 0 + \rho_m g \Delta h - \rho g l}$$ for gage pressure

- example :- Problem 3.20 with British units

- $W = 10$ lbf
- piston dim $= 4"$

calculate the volume added to the system in order to raise the piston 1" ??



$\gamma_{oil} = 62.4$
$S = 0.85$ specific gravity

Remember

$$\boxed{S = \frac{\gamma_{oil}}{\gamma_w}}$$

solution :- Force Balance on the piston

$$P_p A_p = W = 10 \text{ lbf}$$

$$P_p = \frac{10}{A_p} = \frac{10}{\frac{\pi}{4} 4^2} = 0.796 \underbrace{\text{lbf/in}^2}_{\text{psi}} = 114.6 \text{ lbf/ft}^2$$

$$114.6 = \gamma_{oil} h = \underset{\text{specific gravity}}{S} \gamma_w h \rightarrow h = \frac{114.6}{0.85 * 62.4}$$

$$= 2.161 \text{ ft} = 25.93 \text{ in}$$

No.

→ volume added to the device :-

$$V_{add} = \frac{\pi}{4} 4^2 * 1'' + \frac{\pi}{4} * 1^2 * (25.93 + 1)$$

$$= 33.7 \text{ in}^3$$

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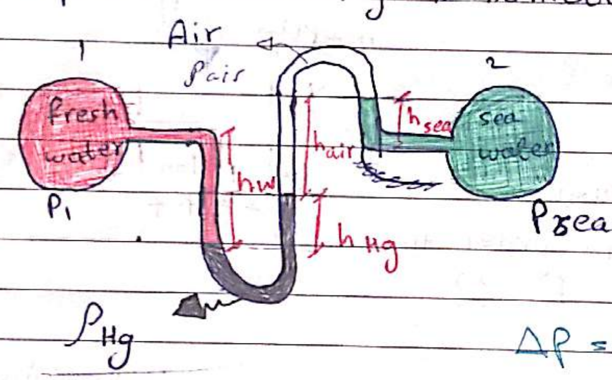
* example on multi-leg manometers :-



hw = 60 cm

h_{Hg} = 10 cm

hair = 70 cm

hsea = 40 cm

$\Delta P = ??$

$$P_1 + \rho_w g h_w - \rho_H g h_{Hg} - \rho_{air} g h_{air}$$

$$+ \rho_{sea} g h_{sea} = P_2$$

neglected ≈ 0

$$\Rightarrow P_1 - P_2 = g\left[-\rho_w h_w + \rho_{Hg} h_{Hg} + \cancel{\rho_{air} h_{air}} - \rho_{sea} h_{sea}\right]$$

$$P_1 - P_2 = 9.81\,(13600 * 0.1 - 1000 * 0.6 - 1035 * 0.4)\frac{1}{1000}$$

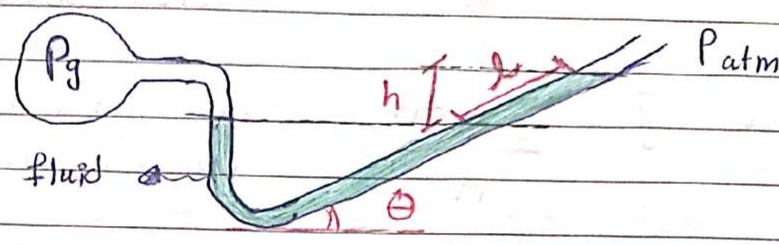
$$= \boxed{3.39 \text{ kPa}}$$

with quiz

9th 3.13, 33, 67, 78, 100

No.

* inclined tube mano meter



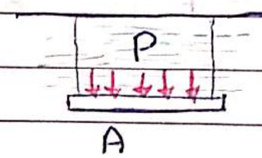
$$P_g = P_{atm} + \rho g h$$

$$h = l \sin\theta$$

3.4 Hydro static Forces on plane surfaces

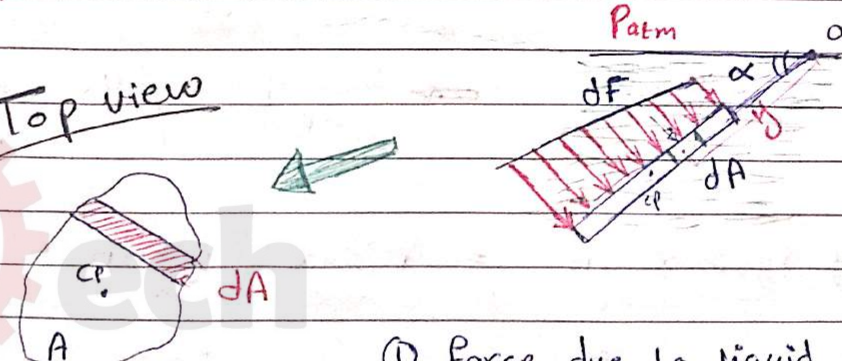
* Horizontal plane surface

$$F = \frac{P}{A}$$



* inclined surface

Top view



① Force due to liquid

$\boxed{P = \rho g h = \gamma h}$

$$dF = P\,dA = \gamma\, y \sin\alpha\, dA$$

$$F = \int_A P dA = \int \gamma\, y \sin\alpha\, dA = \gamma \sin\alpha \int_A y\, dA$$

No.

remember : $\int_A y\, dA$ is First moment of plate area about x- axis

also $\bar{y} = \frac{\int_A y\, dA}{A}$, the distance of center of gravity ((cintroid)) from OX.

→ $F = \underbrace{\gamma\, \bar{y}\, \sin\alpha}_{\text{Pressure at cintroind } \bar{P}}\, A$

$F = \bar{P}\, A$ such that P is in gage pressure

→ if P is absolute,

$$\boxed{F_{abs} = P_{atm} A + \bar{P} A}$$

② line of action

$F_{cp} = \bar{P} A$

also $y_{cp} = \frac{\int_F y\, dF}{F}$ → $y_{cp}\, F = \int_A y\, dPA$

$= \int_A y\, \gamma y \sin\alpha\, dA = \gamma \sin\alpha \boxed{\int y^2\, dA}$

2nd moment of area or [[moment of inertia]] I_o

No.

Put $I_o = \boxed{\bar{I}} + y^2 A$

moment of inertia around the centroid

$\Rightarrow$ $\boxed{\text{Tables. Fig A.1}}$

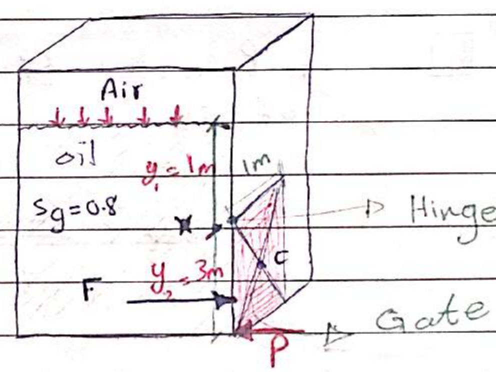
$\Rightarrow$ $y_{cp} F = \gamma \sin\alpha \, (\bar{I} + \bar{y}^2 A)$

$$\boxed{y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y} A}}$$

example :-

Find $\boxed{a}$ Force P if Air is at atmospheric pressure

$\boxed{b}$ Force P if air at 40 KPa



solution :

$\boxed{a}$ $y_{\text{from} \times \text{to } F} = y_{cp} - y_1$

$\Sigma F_x = 0$, $\Sigma F_y = 0$
, $\Sigma M = 0$

Take moments around Henge

$F(y_{cp} - y_1) - P(y_2)$ But $F = \gamma \cancel{\sin\alpha} \, \bar{y} \, dA$

$F = \gamma \bar{y} A$, also $\bar{y} = y_1 + \frac{1}{2} y_2$ / and $\gamma = Sg * \gamma_w$

No.

$\gamma = 0.8 * 9810$

$F = \underbrace{0.8 * 9810}_{\gamma} * \underbrace{2.5}_{\bar{y}} \underbrace{(1\times 3)}_{A} = 58860\ N$

$y_{cp} = \bar{y} + \dfrac{\bar{I}}{\bar{y}A}$

$\bar{I} = \dfrac{bh^3}{12} = 2.25\ m^4$

$2.5 + \dfrac{2.25}{2.5(3)} = 2.5\ m$

now $P = \dfrac{F\,(y_{cp} - y_1)}{y_2} = 35316$

$\boxed{b}$ if $P_{air} = 40$ KPa , add an equivelant depth of oil

$y_{eq} = \dfrac{P_{air}}{\gamma} = \dfrac{40000}{0.8 * 9810} = 5.1\ m$

$\bar{y} = \frac{1}{2}y_2 + y_1 + y_{eq} = 7.6\ m$

$F = 0.8 * 9810 * 7.6\,(3) = 178860\ N$

$P = 95312.4\ N.$

Notes:- for curved surfaces (section 3-5)

① أوجد المسافة العمودية للسنترويد (z_c))

② أوجد $\bar{P}$ ومنها F_{cp} $\quad F_{cp} = \bar{P}A = \gamma z_c A$

③ أوجد $\bar{y}$ وهي المسافة المائلة للسنترويد (إن كان هناك ميلان أصلاً))

④ أوجد المقدار $y_{cp} - \bar{y}$ وهو عبارة عن مقدار بعد نقطة تأثير الضغط الحقيقية عن السنترويد .

⑤ هيك صار عندك قوة واتجاه ومكان التأثير وبس

11/3/2014

another solution :- super position method

$$F = F_{air} + F_{Hydro}$$

$$F_{Air} = P_{air} * Area = 40000 * (3*1)$$

Area of the gate

$$F_{air} = 120 \text{ kN}$$

$$F_{Hydro} = 58,680 \text{ N}$$

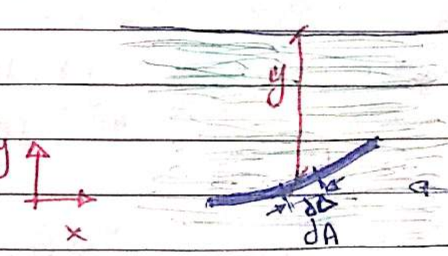
$$F = 120000 + 58680 = 178680 \text{ N}$$

now :- $\bar{P} = \frac{F}{A} = \frac{178860}{3*1} = 59620 \text{ Pa}$

$$\bar{y} = \frac{\bar{P}}{S_g \cdot \gamma_w} = \frac{59620}{0.8 * 9810} = \boxed{7.6 \text{ m}}$$ measured from oil surface

3.5 Hydrostatic forces on curved surfaces

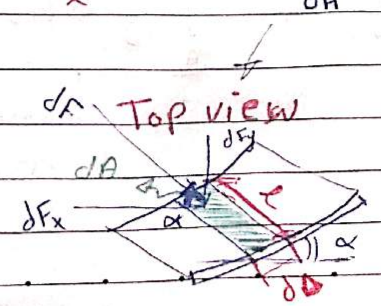
$$dF = P\,dA$$
$$= \gamma y\, dA$$



$$dF_x = \underbrace{\gamma y}_{P} \underbrace{l\, d\Delta \sin\alpha}_{dA_v}$$

$$F_x = \int \gamma y\, dA_v$$

$$\boxed{F_x = \gamma \int y\, dA_v}$$

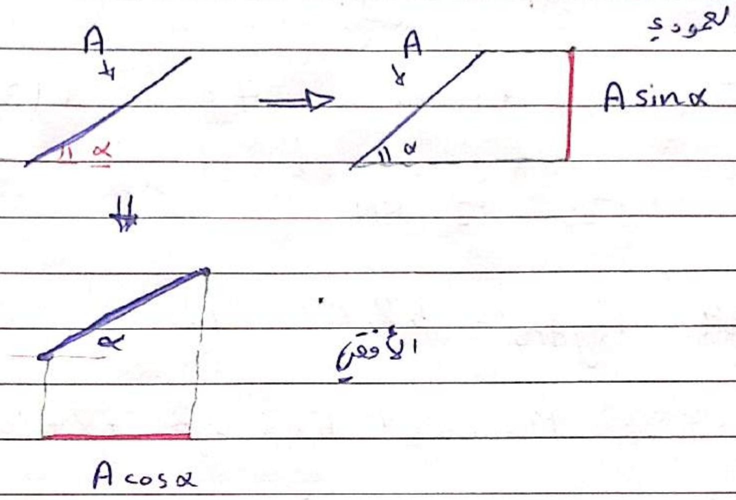


→ first moment plane projected Area on V. dirction

No.

$$\rightarrow \int_A y \, dA_v = \bar{y}_v \, A_v$$

* حيث $\bar{y}$ هي مركز الأسقاط العمودي للمساحة للتوضيح و A_v هي المساحة العمودية

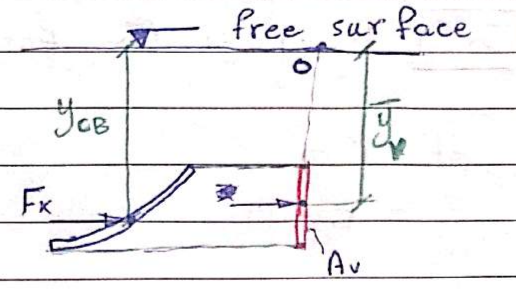


$$\therefore F_x = \gamma \, \bar{y}_v \, A_v \quad \text{or} \quad F_x = \bar{P} \cdot A_v$$

now :- line of action of the Horizontal direction

نقطة عشوائية

$$\sum M_o :- y_{cp} * F_x = \int y \, dF_x$$



$$\rightarrow y_{cp} * \underbrace{\gamma \, \bar{y}_v \, A_v}_{F_x} = \int_A y \, \underbrace{\gamma y \, dA_v}_{dF_x}$$

$$y_{cp} \, \bar{y}_v \, A_v = \underbrace{\int_A y^2 \, dA_v}_{I_{ov}}$$

I_{ov} :- moment of inertia of vertical projection about $\boxed{O}$

first exam 25/3/2014

ch 2, 3, 4 open book

No.

→ replace $I_{ov} = \bar{I}_v + \frac{\bar{I}}{\bar{y}_v \cdot A_v}$

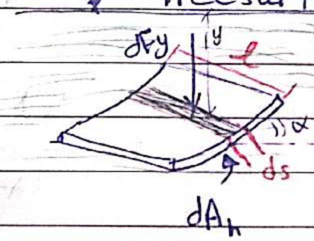
solve for $\boxed{y_{cp}}$

$$y_{cp} = \bar{y}_v + \frac{\bar{I}_v}{\bar{y}_v A_v}$$

12/3/2014

→ forces in the vertical direction

(1) $dF_y = P\, dA_h = \gamma \underbrace{y\, l\, ds \cos\alpha}_{d\forall}$

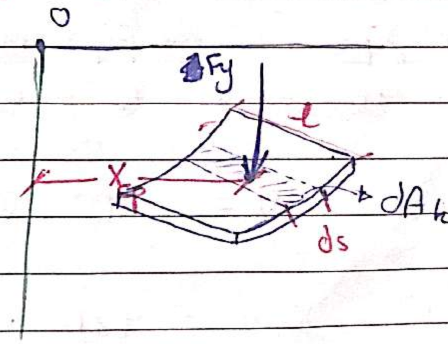


$$\int dF_y = \int \gamma\, d\forall$$

$$\boxed{F_y = \gamma \forall}$$

such that $\forall$ is the total $\forall$ above the curve $(W + F_y)$

(2) line of action



$$x_{cp} F_y = \int x \gamma\, d\forall$$

$$\boxed{x_{cp} = \frac{\int x\, d\forall}{\forall}}$$

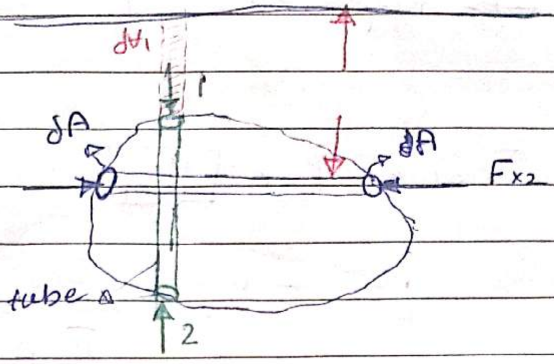
distance of centroid from ref. point

No.

3.6 Bouyancy

$F_{x_1} = F_{x_2}$

$\Sigma F_x = 0$



→ Force on $\boxed{1}$

$F_1 = \gamma \, d\forall_1$

$F_2 = \gamma \, d\forall_2$ → $\boxed{d\forall_2 = d\forall_1 + d\forall_{tube}}$

vertical net force on prizm : $dF_v = \gamma \left(d\forall_2 - d\forall_1 \right)$

$dF_v = \gamma \, d\forall_{tube}$ upward

$\boxed{F_v = \int \gamma \, d\forall_{tube} = \gamma \, \forall_{tube}}$ → Archimedis principle

example :- A pipe ; 0.8 m diameter and 200 m long is submerged in water, calculate bouyancy force on the pipe. if the pipe mass = 80000 kg, is the Bouyancy force enough to float it ???

solution :- $\forall = \frac{\pi}{4} D^2 * l = \frac{\pi}{4} (0.8)^2 * 200 = 100.53 \ m^3$

$F_v = \gamma \, \forall = 9810 * 100.53 = 986199.3 \ N$

Pipe wieght $= 80000 * 9.8 = 784800 \ N$

→ pipe will float

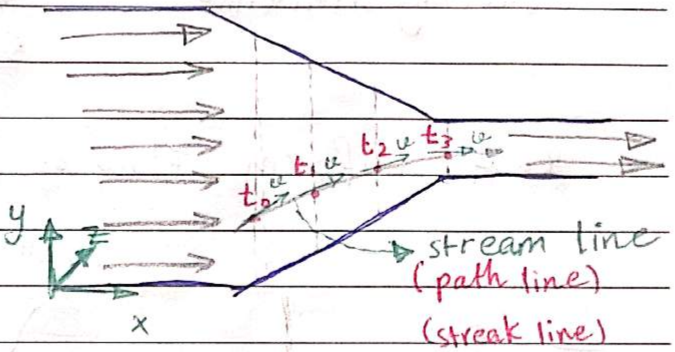
16/3/2014

* CHAPTER 4 *

* flowing fluids and Pressure Variation

[a] lagrangian approach :-

position vector



$\vec{r}(t) = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

→ velocity $\vec{U}(t)$

$$\vec{U}(t) = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

→ acceleration

$$\vec{a}(t) = \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

[b] eulerian approach

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

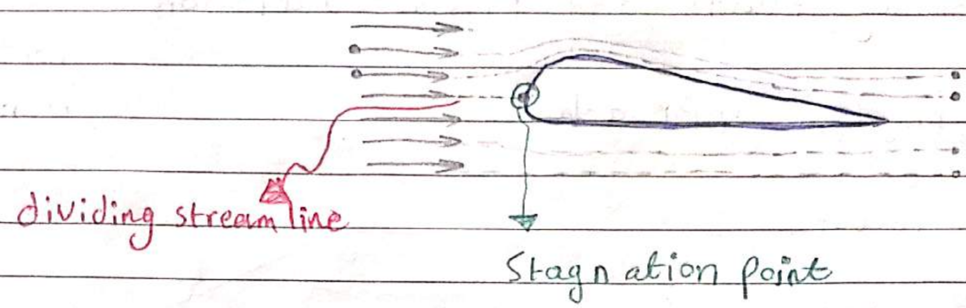
✱ stream line :- the line that the fluid Particles move at

→ in S. line Coordinates :-

$v \rightarrow \vec{U} = \vec{U}(s, t)$

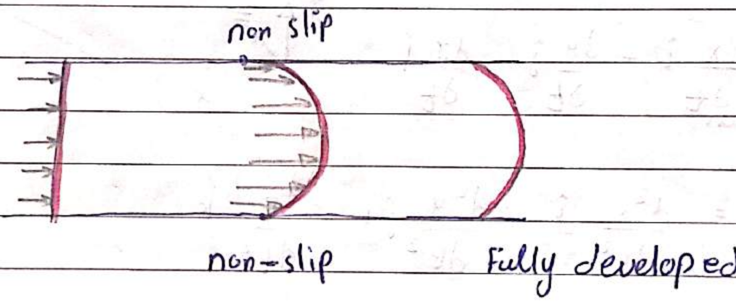
No.

* Flow about an air foil



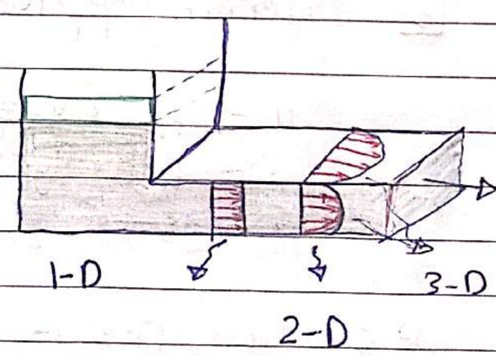
* A uniform flow :-

$\frac{dV}{dt} = 0$



18/3/2014 * steady flow : $\frac{d\vec{v}}{dt} = 0$

none steady flow → $\frac{d\vec{v}}{dt} \neq 0$

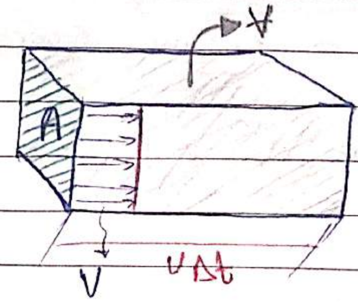


4.2 Rate of flow

① ~~Continuous~~ Constant Velocity
② Variable "
③ Average "

now : Constant flow rate :-

$\Delta \forall = V \, \Delta t \, A$



$\frac{\Delta \forall}{\Delta t} = A.V$

No.

$$\frac{\Delta \forall}{\Delta t} = Q \quad \text{((Volumetric flow rate))} \ m^3/s$$

or l/s

or $\boxed{gpm}$ or ft^3/s

↳ gallon per minute

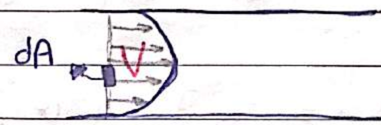
~~note:~~

Knowing that $\boxed{Q = A \cdot V}$

$\boxed{\text{and} \quad \dot{m} = \rho V A}$

* Variable velocity

$$dQ = V \, dA$$



integrating

$$\int dQ = \int_A V \cdot dA$$

mass flow rate

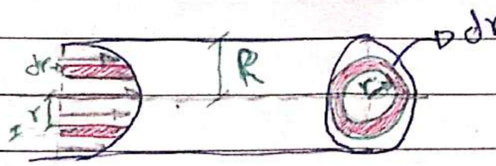
$$\dot{m} = \int_A \rho V \cdot dA$$

for constant $\boxed{\rho}$ incompressible fluid

$$\boxed{\dot{m} = \rho \int_A V \cdot dA}$$

* Average ~~flow~~ velocity:

given $V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$



$$\bar{V} = \frac{\int_0^R V_{max}\left(1 - \frac{r^2}{R^2}\right) 2\pi r \, dr}{\pi R^2}$$

$$\bar{V} = \frac{1}{2} V_{max}$$

No.

4.3 Acceleration

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

→ acceleration :-

$$a_x = \frac{du}{dt} \quad , \text{ and since } u = f(x, y, z, t)$$

$$\rightarrow \quad a_x = \frac{\partial u}{\partial x}\cdot \underbrace{\boxed{\frac{dx}{dt}}}_{u} + \frac{\partial u}{\partial y}\cdot \underbrace{\boxed{\frac{dy}{dt}}}_{v} + \frac{\partial u}{\partial z}\cdot \underbrace{\boxed{\frac{dz}{dt}}}_{w} + \frac{\partial u}{\partial t}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Convective acceleration

local or timperal acceleration

No.

20/3/2014

examples:- acceleration

Given $u = xt + 2y$, $v = xt^2 - 2y$, $w = 0$ } velocity of a fluid particle in flow field

what is the total acceleration w, at a point $x = 1\ m$, $y = 1\ m$, $t = 2\ s$

solution:-

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= (xt+2y)\,t + (xt^2-2y)\,2 + 0 + x$$

$$(2+2)(2) + (4-2)2 + 0 + 1 = \boxed{13\ m/s^2}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \rightarrow$$

$$= (xt+2y)t^2 + (xt^2-2y)(-2) + 0 + 2xt$$

$$4 * 4 + 2 * -2 + 0 + 4 = \boxed{16\ m/s^2}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$= (xt+2y)\,0 + 0 + 0 + 0 = \boxed{0\ m/s^2}$$

$$\vec{a} = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{13^2 + 16^2 + 0^2} = \_\_\ m/s^2$$

No.

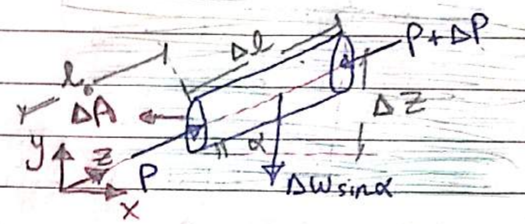
4.3 Euler's equation :

Pressure Variation due to weight and acceleration

$\Sigma F_l = M a_l$ ((z-axis))

$P \Delta A - (P+\Delta P)\Delta A - \Delta W \sin\alpha$

$= \rho \Delta l \Delta A a_l$



and since $\Delta W = \gamma \Delta l \Delta A$

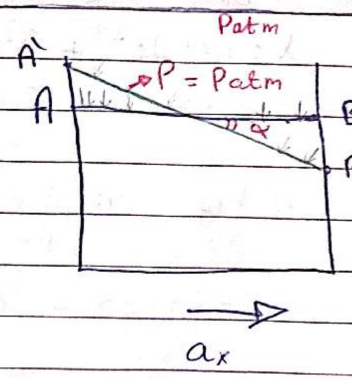
$$-\frac{\Delta P}{\Delta l} - \gamma \sin\alpha = \rho a_l$$

$$-\frac{\partial P}{\partial l} - \gamma \frac{\partial z}{\partial l} = \rho a_l$$

$$\boxed{-\frac{\partial}{\partial l}(P + \gamma z) = \rho a_l}$$ euler's eq for motion of fluids

---

* linear acceleration :



وعاء يتحرك لليمين

$P = P_{atm} = const$

$\frac{dP}{dl} = 0$ where l is the distance along sides

$a_l = a_x \cos\alpha$

Euler eq $\longrightarrow$ $-\frac{d}{dl}(\overset{0}{\cancel{P}} + \gamma z) = \rho a_x \cos\alpha$

No.

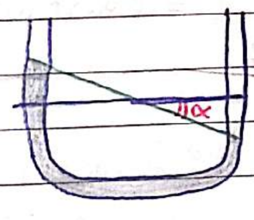
for constant γ :-

$$\gamma * -\frac{\partial}{\partial l}(z) = \rho\, a_x \cos x \qquad \text{since } \boxed{\frac{dz}{dl} = \sin\alpha}$$

$$-\frac{\partial z}{\partial l} = \frac{\rho\, a_x \cos x}{\gamma} \cancel{= \tan\alpha}$$

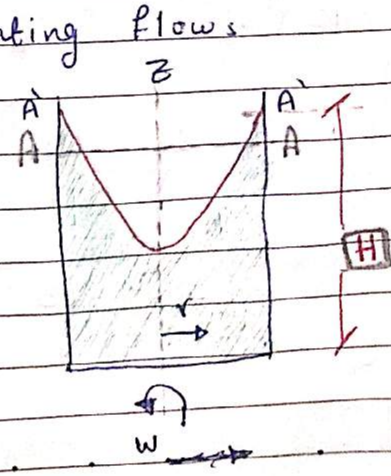
$$\boxed{\tan\alpha = \frac{a_x}{g}}$$

applecation :- accelerometer



23/3/2014

4.4 Pressure distribution in Rotating flows

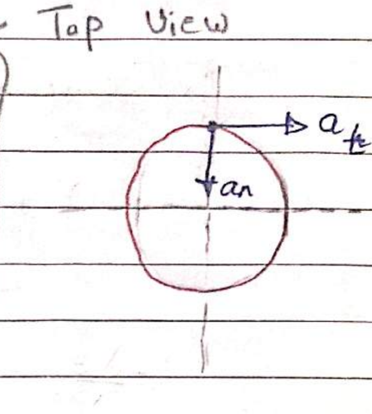


eq 4.8 in Cartisian coordinates

$$\frac{\partial}{\partial l}(P + \gamma z) = \rho\, a_l$$

→ convert to cylindrical

$$-\frac{\partial}{\partial r}\left(P+\gamma z\right)=\rho\left(\frac{-V^2}{r}\right)$$



$$\Rightarrow \frac{\partial}{\partial r}\left(P+\gamma z\right)=\rho\, r\omega^2$$

integrating

$$P+\gamma z=\rho\,\frac{\overset{V^2}{\boxed{r^2\omega^2}}}{2}+\text{Const}$$

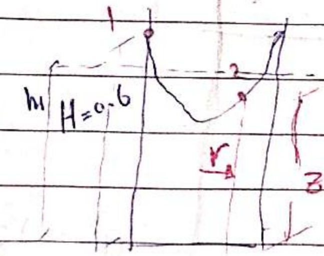
$$P+\gamma z-\rho\frac{V^2}{2}=\text{const} \qquad /\gamma \qquad \boxed{\gamma=\rho g}$$

$$\boxed{\frac{P}{\gamma}+z-\frac{V^2}{g*2}=\text{Constant}} \qquad 4.13\,a$$

* example :- find ω required for water to spill from the edge of the cylinder $H=0.6\,m$, $r=0.1\,m$

solution :- apply 4.13 a at the nip of cylinder

$P=P_{atm}$ $\quad r=R$ $\quad z=H$



$$\frac{P_a}{\gamma}+H-\frac{\omega^2R^2}{2g}=c \quad \text{---} \quad (1)$$

$$\frac{P_a}{\gamma}+z-\frac{\omega^2r^2}{2g}=c \quad \text{--} \quad (2)$$

No.

① - ② $\rightsquigarrow$ $$Z = H - \frac{w^2}{2g}\left(R^2 - r^2\right)$$

at any Point on the free surface water is spilling = initial volume

$$\int_0^R 2\pi r Z \, dr = \pi R^2 h_1$$

$$2\pi \int_0^R \left[H - \frac{w^2 (R^2 - r^2)}{2g} \right] r dr = \pi R^2 \left[H - \frac{w^2 R^2}{2g} \right]$$

$$= \pi R^2 h_1$$

$$w = \frac{\sqrt{4g(H-h_1)}}{R} = \frac{\sqrt{4 * 9.81\,(0.6-0.4)}}{0.1} = 28.01 \text{ rad/s}$$

No.

4.5 Bernoulli Equation :-

Back to euler eq :- (4.8)

$$-\frac{\partial}{\partial l}(P+\gamma z) = \rho a_l$$

put that in cartisian coordinates

$$\Rightarrow -\frac{\partial}{\partial x}(P+\gamma z) = \rho a_x$$ divide By $\boxed{\gamma}$

$$-\frac{\partial}{\partial x}\left(\frac{P}{\gamma}+z\right) = \frac{a_x}{g}$$

* it canbe shawn that

$$\frac{\partial}{\partial x}\left(h + \frac{V_s^2}{2g}\right) = 0$$

→ similarly in y - direction

$$\frac{\partial}{\partial y}\left(h + \frac{V_s^2}{2g}\right) = 0$$

∴ put $\boxed{h + \frac{V_s^2}{2g} = \text{constant}}$ but $\boxed{h = \frac{P}{\gamma} + z}$

No.

$$\boxed{\frac{P}{\gamma} + Z + \frac{V_s^2}{2g} = \text{Constant}}$$

at any points, suppose $\boxed{1}$, $\boxed{2}$

$$\boxed{\frac{P_1}{\gamma} + Z_1 + \frac{V_{s_1}^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_{s_2}^2}{2g}}$$

* Conditions:-

1. Steady State
2. Irrotational flow
3. non Viscous ((invescosid))
4. in compressible ((ρ = const))
5. on the same Stream line

note:- we cannot apply Bernoulli eq on Branches

→ according to point $\boxed{5}$

* such that:-

$\frac{P}{\gamma}$: Static head

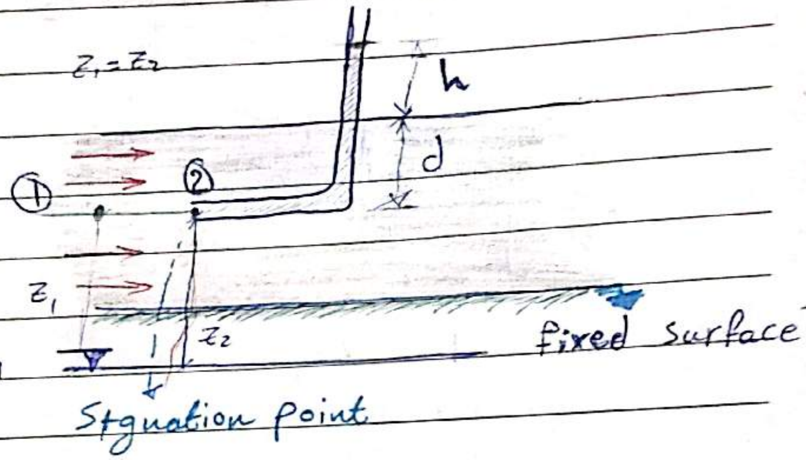
Z :- elevation head

$\frac{V_s^2}{2g}$:- velocity head

* applications to Bernoulli's eq:-

No.

* Stagnation tube :-



$$\frac{U_1^2}{2g} = \frac{P_2}{\gamma} - \frac{P_1}{\gamma}$$

$$= \frac{\cancel{\gamma}(h+d)}{\cancel{\gamma}} - \frac{\cancel{\gamma}d}{\cancel{\gamma}} = h$$

notes:-

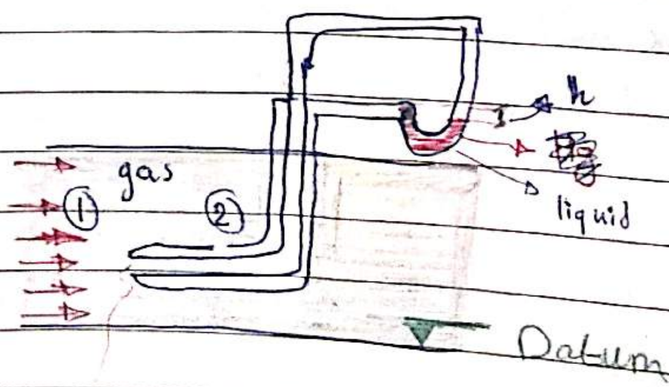
① $Z_1 = Z_2$ so will Be eleminated Both

② U_1 on ~~the Bat~~ ② $= 0$ so $\frac{U_2^2}{2g}$ will Be eleminated

$$\boxed{U_1 = \sqrt{2gh}}$$

notes:- we cannot use this device inside pipes? Because the flow inside it has ~~so~~ abig Pressure so we will need avery high tube.

* Pitot tube



$$\frac{P_1}{\gamma_1} + \cancel{Z_1} + \cancel{\frac{U_1^2}{2g}} = \frac{P_2}{\gamma} + \cancel{Z_2} + \frac{U_2^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{P_2}{\gamma} = \frac{U_2^2}{2g}$$

Hw # 3
4.62 , 64 , 68 , 71   9th

with quiz   6/4/2014

No.

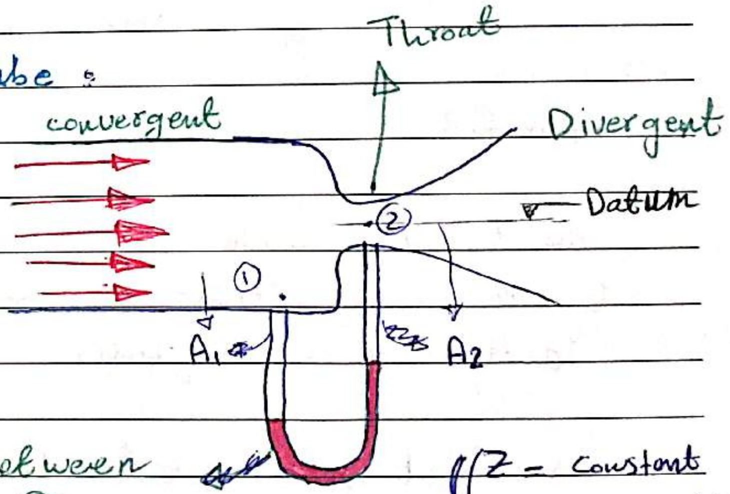
$$V_2 = \sqrt{\frac{2g\,(P_2 - P_1)}{\gamma}} \quad \text{and} \quad \boxed{P_2 - P_1 = \Delta P = \rho_L\, g\, h}$$

$$V_2 = \frac{2\,\Delta P}{\rho g}$$

$$\boxed{V_2 = \sqrt{2\,\frac{\rho_{liq}}{\rho_{gas}}\, g h}}$$

30/3/2014

* Application :- Venturi tube :



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \quad \text{--- } \boxed{1}$$

$V_2 > V_1$, $A_2 > A_1$

Bernoulli Between ① and ②

((Z = constant (same elevation)))

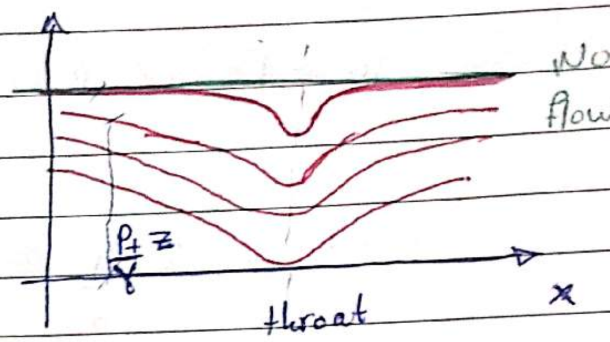
* continuouty eq :-

$$Q_1 = Q_2 \qquad A_1 V_1 = A_2 V_2 \quad \text{---} \ \boxed{2}$$

solve for V_1, V_2 ⟹ obtain Q_2

No.

* Cavitation :-



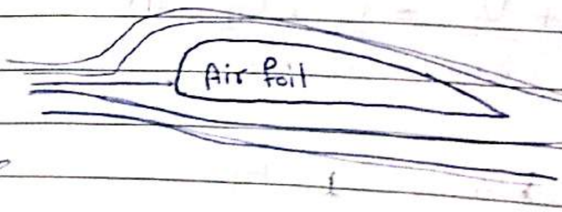
Pressure will decrease untill Becomes $\boxed{\text{Negative}}$.

which cause the liquid to vaporize and gas forms at the throttle.

→ this cause $\boxed{\text{cavitation}}$ التكهف.

another applications : Carborettor, Air foil orifice meter.

*Air foil :

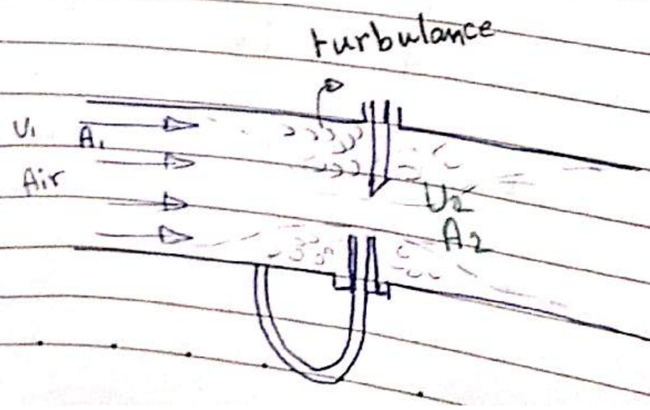


the destance made By the air particles on the above is grater

→ U_{above} is greater than U_{down}

→ this make a lift force that raise the plane above

* orifice meter :-

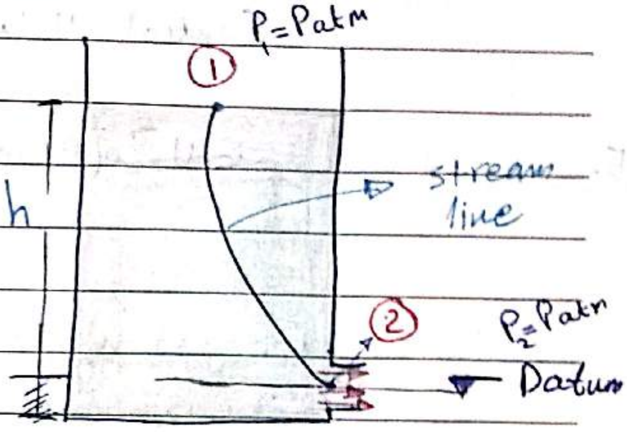


No.

1/4/2014

* Free flow out of a ~~manometer~~ reservoir :-

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{U_1^2}{2g}}^{0} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{U_2^2}{2g} + \cancel{Z_2}^{0}$$



$\boxed{P_1 = P_2}$

$\boxed{Z = h}$ $\quad \frac{U_2^2}{2g} = Z_1$

U = ??

$$\boxed{U_2 = \sqrt{2gh}}$$

* another case :- what if there was a tube in the outlet??

$P_2 \neq P_{atm}$

example :- $h = 15\ m$ $\quad U_2 = 8\ m/s$ ((reservoir with tube))

Problem (4.96) $\quad P_2 \neq P_{atm}$ $\quad$ find P_g at 2

$$\frac{P_1}{\gamma} + \cancel{\frac{U_1^2}{2g}}^{0} + Z_1 = \frac{P_2}{\gamma} + \frac{U_2^2}{2g} + \cancel{Z_2}^{0} \quad \text{Re arrange}$$

$$\underbrace{\frac{P_2 - P_{atm}}{\gamma}}_{P_g} = Z_1 - \frac{U_2^2}{2g}$$

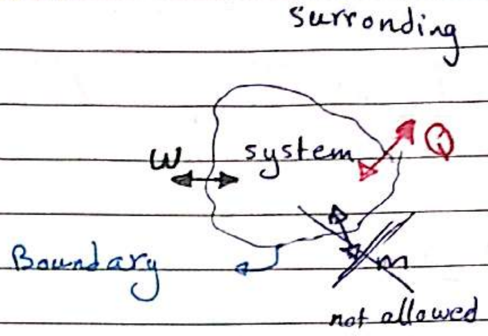
$$P_g = \gamma\left(Z_1 - \frac{U_2^2}{2g}\right) = 9810\left(15 - \frac{8^2}{2*9.81}\right)$$

$$\boxed{P_g = 115.15\ kPa}$$

No.

** CHAPTER 5 **

Control Volume Approach



5.2 Basic C.V approach

let B an extensive property
such as Mass M
Momentum $m\vec{V}$
Energy E

b is an intensive property

$\frac{M}{M} = 1$ for mass ((dimensionless))

$\frac{M\vec{V}}{M} = \vec{V}$ for momentum m/s

$\frac{E}{M} = e$ for Energy kJ/kg

so $\rightarrow$ $\boxed{\frac{B}{M} = b}$

* for a differential mass dm

$$\rightarrow B = \int_M b\, dm = \int_{\forall} \rho\, d\forall \quad \text{---} \quad \boxed{5.1d}$$

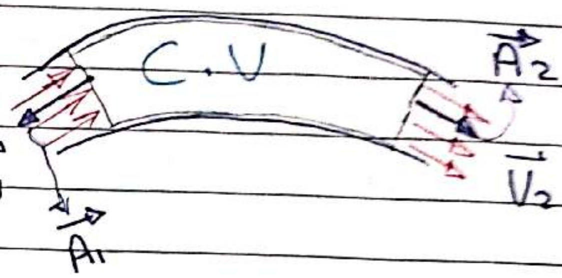
apply this on flow in and out of a C.V

No.

$Q = VA$ ((Vol. flow rate))

on vectorial basis

$$Q = \vec{V} \cdot \vec{A}$$



* apply to station ①

$$Q = V_1 A_1 \cos 180 = -V_1 A_1$$

* apply on station (2)

$$Q = V_2 A_2 \cos 0 = V_2 A_2$$

* Net rate of flow out of $\boxed{C.V = \sum_{C.S} \vec{V} \vec{A} = V_2 A_2 - V_1 A_1}$

* if the quantity $(V_2 A_2 - V_1 A_1) > 0$ ((positive))
  then the flow is $\boxed{up}$

* " " " $(V_2 A_2 - V_1 A_1) < 0$ (negative)
  " " " " $\boxed{Down}$

* " " " $(V_2 A_2 - V_1 A_1) = 0$
  then there is $\boxed{\text{no flow}}$

* flow on mass Basis

$$\boxed{\dot{m} = \sum_{C.S} \rho \vec{V} \cdot \vec{A} = \rho_2 V_2 A_2 - \rho_1 V_1 A_1}$$

No.

3/4/2014 $\dot{B}$ similarly, for any extensive property

$$\boxed{\dot{B} = \sum_{c.s} b \rho \vec{U} \cdot \vec{A}} \quad \dots \boxed{5.12}$$

cond :- ① 1-Dimension ② steady

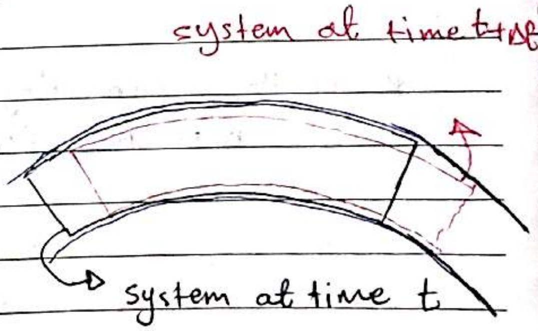
or, in integral form :-

$$\dot{B} = \int_{c.s} b \rho \vec{U} \cdot d\vec{A} \quad \dots \boxed{5.13}$$

cond :- ① 2 Dimension ② steady

now : non steady flow

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{t+\Delta t} - B_t}{\Delta t} \right]$$



* mass of the system at $t+\Delta t$

= mass within C.V at $(t + \Delta t)$ + mass moved out of c.v during $(t+\Delta t)$ − mass moved in to c.v during $(t+\Delta t)$

$$M_{sys,t+\Delta t} = M_{c.v,t+\Delta t} + \Delta M_{out} - \Delta M_{in}$$

No.

* similarly for any extensive property

$$B_{sys,\,t+\Delta t} = B_{c.v,\,t+\Delta t} + \Delta B_{out} - \Delta B_{in}$$

hence $$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0}\left[\frac{(B_{cv,\,t+\Delta t} + \Delta B_{out} - \Delta B_{in} - B_{cv,\,t})}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{B_{cv,\,t+\Delta t} - B_{cv,\,t}}{\Delta t}\right] + \lim_{\Delta t \to 0}\left[\frac{\Delta B_{out} - \Delta B_{in}}{\Delta t}\right]$$

$$= \frac{dB_{c.v}}{dt} + \dot{B}_{out} - \dot{B}_{in}$$

$$\boxed{\frac{dB_{sys}}{dt} = \frac{d}{dt}\left(\int_{c.v} b\,\rho\, d\forall + \sum_{c.s} b\,\rho\,\vec{U}\cdot\vec{A}\right.} \quad \text{----} \ (3.21)$$

non steady, 2D, flow

for steady flow $\frac{d}{dt} = 0$

$$\boxed{\frac{dB_{sys}}{dt} = \int_{c.s} b\,\rho\,\vec{U}\cdot d\vec{A}}$$ steady

special case

Special case: incompressible flow $\rho = c$

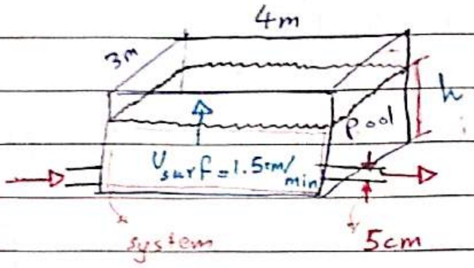
$$\cancel{\rho_1} U_1 A_1 = \cancel{\rho_2} U_2 A_2 \qquad \boxed{\dot{\forall}_1 = \dot{\forall}_2}$$

No.

example :-

find Volumetric flow rate $\boxed{\dot{V}_{in}}$???

$\dot{V}_{in} = ??$



$U_{out} = 5\ m/s$

solution :-

conservation of mass

$$\dot{m}_{in} - \dot{m}_{out} = \frac{d\, m_{sys}}{dt}$$

$$\dot{m}_{in} = \rho \dot{V}_{in} \quad , \dot{m}_{out} = \rho\, U_{out} A_{out} = \rho\ 5 * \frac{\pi * (0.05)^2}{4}$$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}(\rho V_{sys}) = \rho \frac{d}{dt} V_{sys} = \rho \frac{d}{dt}(h * 3 * 4)$$

$$= 12\rho \boxed{\frac{dh}{dt}} \equiv U_{surf} = 12\rho\ \frac{0.015}{60}$$

$$\cancel{\rho}\dot{V}_{in} - \cancel{\rho}\frac{5\pi}{4}(0.05)^2 = \cancel{\rho}\left(\frac{12}{60} * 0.015\right)$$

$$\dot{V}_{in} = 0.0128\ m^3/s$$

No.

6/4/2014

** CHAPTER 6 **
The momentum principle

Recall $$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{C.V} b.\rho \, d\forall + \int_{cs} b.\rho \vec{V}.d\vec{A}$$

put $B \equiv$ momentum , MV

$$b \equiv \frac{B}{M} = V$$

$$\frac{d(MV)_{sys}}{dt} = \frac{d}{dt}\int_{C.V} V\rho \, d\forall + \int_{C.S} \vec{V}\rho \vec{V}.d\vec{A} \quad \text{--- } 5.21$$

also $$\frac{d(MV)_{sys}}{dt} = \sum \vec{F}$$ sum of external forces On the system

Note that

eq $\boxed{5.21}$

* $\frac{d}{dt}\int_{C.V} V\rho \, d\forall$ is the rate of change of momentum inside C.V

* $\int_{cs} \vec{V}\rho\vec{V}.dA$ is the net momentum flow out of C.V

$$\rightarrow \frac{dMV_{sys}}{dt} = \sum \vec{F} = \sum F_B + \sum F_s$$

Body forces — surface forces

No.

* if the velocity is uniform :-

$$\sum \vec{F_s} + \sum F_B = \sum \vec{v} \rho \vec{V} \cdot d\vec{A} + \frac{d}{dt} \int \vec{v} \rho \, d\forall .$$

* limitations and assumptions

$\boxed{A}$ → forces on the fluid

$\boxed{1}$ - F_s could Be pressure force

$F_{pressure} = PA$ normal to area

or friction force ((parallel to surface))

$\boxed{2}$ F_B : usually a gravity with electric field

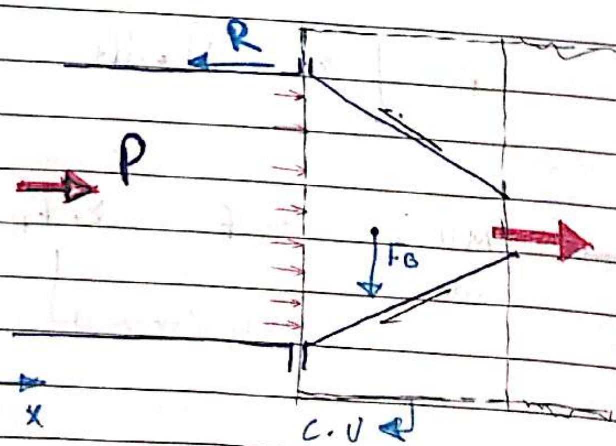
$\boxed{B}$ Velocity Reference

↓

(8/4/2014)

forces on the fluid

$$\underbrace{\sum F_s + \sum F_B}_{} = \sum_{c.s} \vec{v} \rho \vec{V} \cdot d\vec{A} + \frac{d}{dt} \left(\int_{c.v} \vec{v} \rho \, d\forall \right)$$



No.

* Velocity reference :-

(1) $\vec{V}$ always normal to C.V , absolute velocity
→ $V \cdot dA$ is referenced to $\boxed{C.V}$

(2) $\vec{v}$ some times inclined , relative to C.V
$\vec{v}$ → referenced to internal ref.

$\boxed{C}$ * unsteadiness :- occures when conditions changes with time.

$\boxed{D}$ Non uniformity of velocity :- if the flow is uniform :-

→ V_1 is constant every across the section

→ V_2 " " " " " "

* if we have components of forces in the directions

$$\sum F_x = \sum_{c.s} v_x (\rho \vec{V} \cdot \vec{A}) + \frac{d}{dt} \int_{c.v} v_x \, \rho \, d\forall$$

$$\sum F_y = \sum_{c.s} v_y (\rho \vec{V} \cdot \vec{A}) + \frac{d}{dt} \int_{c.v} v_y \, \rho \, d\forall$$

Example :- at same graph above :- $P_2 = 1$ atm $V_2 = 25$ m/s
$d_2 = \frac{\cancel{8}}{2}$ cm $d_1 = 10$ cm $\rho = 999$

water flows at 15°c througth the nozzle,
Calculate P_1 , V_1 and force By the nozzle on the fluid???

No.

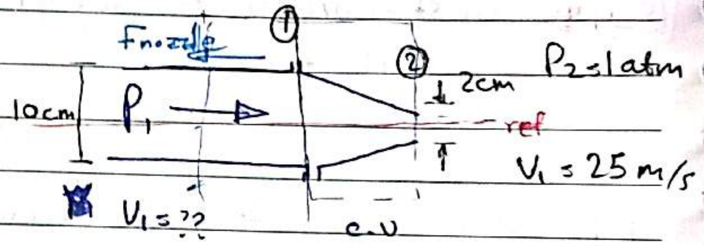
solution :- apply Bernoulli Between 1 and 2

ⓐ $P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2}$

also $Q_1 = Q_2$

$V_1 A_1 = V_2 A_2$

$V_1 = V_2 \frac{d_2^2}{d_1^2}$



$= 25 \left(\frac{2}{10}\right)^2$

$\boxed{V_1 = 1\ m/s}$

$P_1 = P_2 + \frac{\rho}{2}(V_2^2 - V_1^2) \Rightarrow \boxed{P_1 = 311.2\ \text{kPa gage}}$

ⓑ force ??

steady $\rightarrow \frac{d}{dt} \int_{c.v} \vec{v}\, \rho\, d\forall = 0$

also neglect ΣF_B

$\Sigma F_s = \sum_{c.s} \vec{v}\, \rho\, \vec{V} \cdot \vec{A} = F_{nozzle} + P_1 A_1$

$= \underset{\cos(180)}{\ominus} V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2$

No.

$$F_{nozzle} = -P_1 A_1 + \dot{m}(V_2 - V_1)$$

$$= -311188.5 * \frac{\pi}{4}(0.1)^2 + 7.846 * (25-1)$$

$$= \boxed{-22.56\ N}$$

$$\rho V_2 A_2 = \dot{m} = const$$

$$= 999 * 25 * \frac{\pi}{4}(0.02)^2$$

$$= 7.846\ kg/s$$

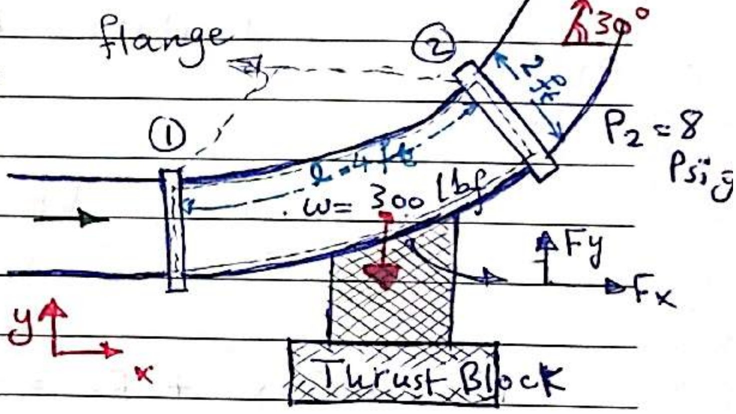
10/4/2014 Example :- $\boxed{P\ 6-44}$ British units

what's the force Fy that holds the nozzle in it's position ???

$Q = 31.4\ m^3/s$

w:- for empty pipe

$P_1 = 10\ Psig$



solution :-

① establish the Coord. system
② Draw a control Volume such that crosses the system Boundary where Reaction are given and Unknowns are required, Draw all force on C.V
③ Apply mom equation

$$y\text{-mom :- } \cancel{F_{1y}}^{\,0} - F_{2y} + F_y - W = \overset{\cos 180}{\cancel{v_{1y}}}\rho V_1 A_1 + v_{2y}\rho V_2 A_2$$

$$0 - P_2 A_2 (\overset{\sin}{\cancel{\cos}}\ 30) + F_y - \overset{W}{\cancel{300}} = 0 + V_2 \sin 30\, \rho \boxed{V_2 A_2} \quad ①$$

solving for Fy $\quad F_y = V_2 \sin 30\, \rho Q + P_2 A_2 \sin 30 + W$

$$F_y = \underbrace{1.94}_{slug/ft^3}(31.4) * 9.995 * \sin 30 + 8\left(\frac{\pi}{4}2^2\right)\underbrace{144}_{\text{التحويل}} * \sin 30 + \cancel{300}\ W$$

$F_y =$

$$Q = V_1 A_1 = V_2 A_2$$

$$V_2 = \frac{Q}{A_2} = \frac{31.4}{\frac{\pi}{4}(2)^2}$$

$$= 9.995\ ft/s$$

No.

now W :-

$$W_{tot} = \rho \cdot g \cdot \frac{\pi}{4}(2)^2 * l + W$$

$$= 62.4 * \frac{\pi}{4} 4 * 4 + 300$$

$$= 1084 \text{ lbf}$$

$F_y = 3198$ lbf $\boxed{\text{upward}}$

extension :- calculate F_x

$$\Sigma F_x = \sum_{c.v} u_x \rho V A + \cancel{\frac{d}{dt}\int_{c.s} u_x \rho \, d\forall}$$ steady flow

$$F_1 x - F_2 x + F_x = -V_1 \rho V_1 A_1 + (V_2 x) * \rho A_2 V_2$$

$$P_1 A_1 - P_2 A_2 \cos 30 + F_x = -U_1 \rho Q + U_2(\cos 30) \rho A_2 U_2$$

$$F_x = A(P_2 \cos 30 - P_1) + V\rho Q(\cos 30 - 1)$$

$Q = V_2 A_2 = U_1 A_1$

$\rightarrow A_1 = A_2$

$U_2 = U_1 =$

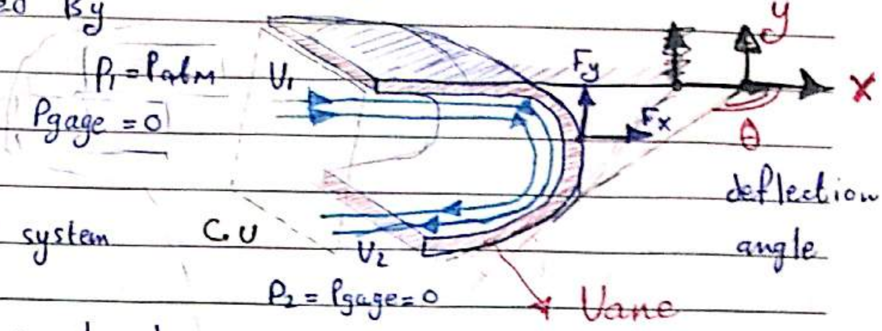
$V = 9.999$ ft/s

$F_x = -1.471$ lbf

No.

13/4/2014 **Deflection of Jet By a fixed Vane :**

find the Force exerted By Vane on Jet.



solution :

[1] Establish a C.V

[2] " " coord system

[3] $\left.\begin{aligned} P_1 &= P_2 = 0 \\ U_1 &= U_2 \end{aligned}\right\}$ also; steady

Apply X-mom eq

steady $\frac{d}{dt} = 0$

$$\sum F_x = \sum_{c.s} v_x \, \rho \, \vec{U} \cdot \vec{A} + 0$$

$$\cancel{F_1} - \cancel{F_{2x}} + F_x + \cancel{W} = -U_{1x} \, \rho \, U_1 A_1 + U_{2x} \, \rho \, U_2 A_2$$

since $U_1 = U_2 = U_{1x} = U$

and $U_{2x} = U_2 \cos\theta$

and $U_1 A_1 = U_2 A_2 = Q$

also $\rho Q = \dot{m}$

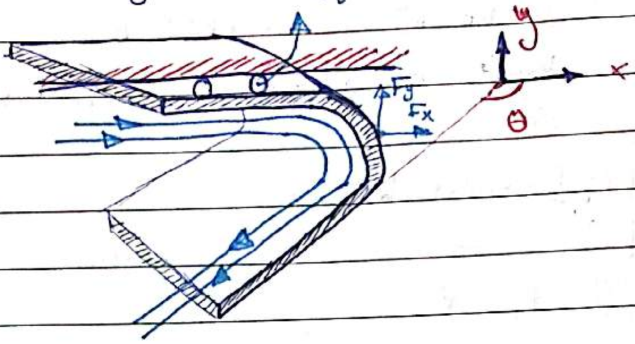
$$\boxed{F_x = \dot{m} \, U (\cos\theta - 1)}$$

let $\theta = 90^\circ \rightarrow \boxed{F_x = -\dot{m} U}$

let $\theta = 180^\circ \rightarrow \boxed{F_x = -2 \dot{m} U}$

No.

* Moving Vane with Const Velocity :-



Relative motion
$U - U_v$

→ x-mom equation

$$0 + F_x = V_{1x}\,\rho\,(V_{1x} A_1) + U_{2x}(\rho\, U_{2x} A_2)$$

$$F_x = (U_1 - U_v)(\rho (U_1 - U_v) A_1) + (U - U_v)\cos\theta\,\rho\,((U_2 - U_v) A_2)$$

since $A = A_1 = A_2$

$$\boxed{F_x = \rho\,(V - U_v)^2 A\,(\cos\theta - 1)}$$

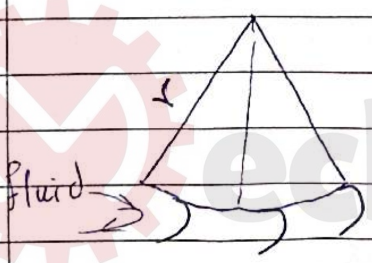
↑ force on the fluid

now moment $= F_x * r$

Power $= F_x \times \vec{r} \times \vec{\omega}$

↳ angular velocity

↓ radius of the wheel



No.

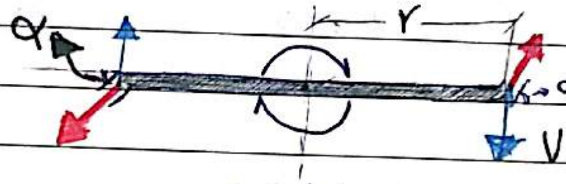
15/4/2014

Moment of momentum

remark
$$\frac{dB_{sys}}{dt} = \sum_{c.s} \beta \rho \vec{V}\cdot\vec{A} + \frac{d}{dt}\int_{c.v} \beta \rho \, d\forall \quad \boxed{5.22}$$

put $\beta = \vec{r} \times \vec{V}$; angular momentum per unit mass

$$\vec{B} = \int_{c.v} \beta \, dm$$



sprinkler
((رشاش مياه))

$$\vec{B} = \int_{c.v} \beta \rho \, d\forall$$

$$= \int_{c.v} (\vec{r}\times\vec{V}) \rho \, d\forall \quad \text{.... put in 5.22}$$

$$\underbrace{\frac{d\left(\int(\vec{r}\times\vec{V})\rho\, d\forall\right)_{sys}}{dt}}_{\text{rate of change of ang. mom within the system}} = \underbrace{\sum_{c.s}(\vec{r}\times\vec{V})\rho\,\vec{V}\cdot\vec{A}}_{\text{Net flow of angular mom OUT of C.S}} + \underbrace{\frac{d}{dt}\int_{c.v}(\vec{r}\times\vec{V})\rho\, d\forall}_{\text{rate of change of ang. mom. within the C.V}}$$

$$\sum \vec{M} = \sum_{c.s} (\vec{r}\times\vec{V})\rho\,\vec{V}\cdot\vec{A} + \frac{d}{dt}\int_{c.v}(\vec{r}\times\vec{V})\rho\, d\forall \quad .... \boxed{5.27}$$

Example :- sprinkler, find the expression for the moment on the fluid (M) interms of (α, ω, R, Q) ???

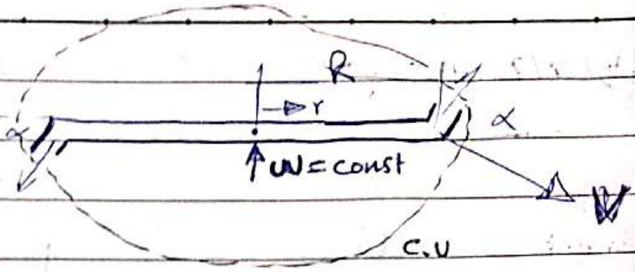
what is M at $\boxed{\omega = 0}$??

" " ω at $\boxed{M = 0}$???

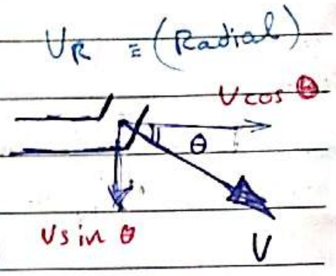
No.

Solution :-

① select the C.V
② apply eq. of mom of momentum $\boxed{6.27}$
③ assuming the flow is steady



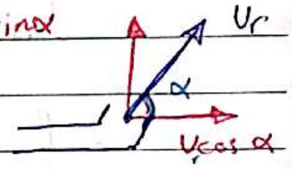
steady $\rightarrow \dfrac{d}{dt}\displaystyle\int_{c.v} (\vec{r} \times \vec{u})\, \rho\, d\forall = 0$



But at inlet $\boxed{r = 0}$

$$\sum \vec{M} = 0 + R\, \underbrace{v\sin\theta}_{V_t}\, \rho\, \underbrace{UA}_{Q}$$

$$V_t = V_r \sin\alpha - \omega R$$



note that
$V_r \sin\alpha = V\cos\theta$

$$\rightarrow M = R\rho Q\,(V_r \sin\alpha - \omega R)$$

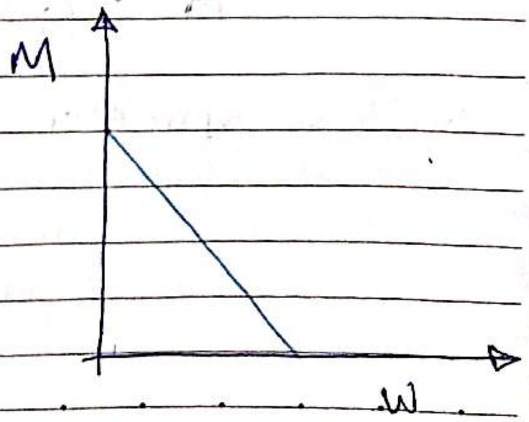
But $V_r = \dfrac{Q}{A} = \dfrac{Q}{\frac{\pi}{4}d^2 * 2} = \dfrac{Q}{\pi d^2/2}$

Hence :- $\boxed{M = R\rho Q\left(\dfrac{Q}{\pi d^2/2} \sin\alpha - \omega R\right)}$

put $\omega = 0$:-

$$M_{max} = \frac{R\rho Q^2 \sin\alpha}{\pi d^2/2}$$

Put $M = 0 \rightarrow \omega = \dfrac{Q}{(\pi d^2/2) . R}$



2^{nd} exam is on Thursday 8/5/2014 Up to CH 7

Hw # 4 on CH 6

Quiz 24/4/2014

20, 26, 28, 45, 55, 87

No.

17/4/2014

* CHAPTER 7 *

The Energy Principle.

7.1 Energy eqn

recall eqn 5.22

$$\frac{dB_{sys}}{dt} = \sum_{c.s} b\rho\vec{V}\cdot\vec{A} + \frac{d}{dt}\int_{c.v} b\rho\, d\forall$$

put $\boxed{B = E}$ and $\boxed{b = \frac{E}{M} = e}$

$$\frac{dE_{sys}}{dt} = \sum_{c.s} e\rho\vec{V}\cdot\vec{A} + \frac{d}{dt}\int_{c.v} e\rho\, d\forall$$

~~For 1-D case $\sum$ Becomes $\int$~~

But from 1^{st} law of thermodynamics

$$\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}$$

heat in — work out

$e = ke + pe + u$

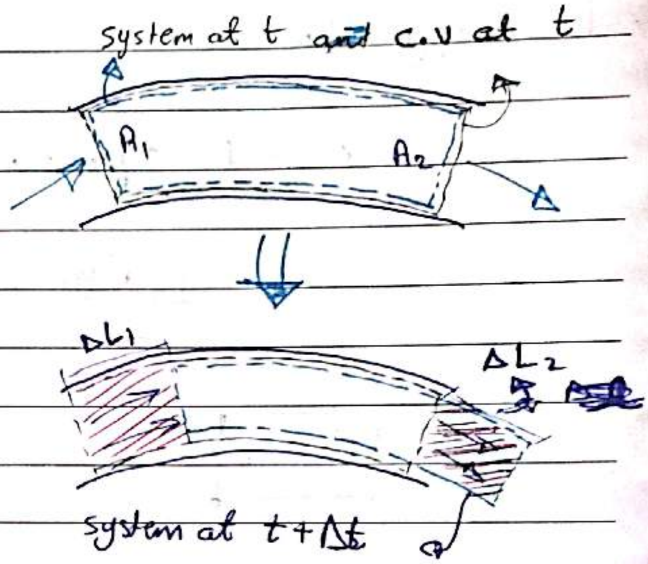
$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{c.v} e\rho\, d\forall + \sum_{c.s} e\rho\vec{V}\cdot\vec{A}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{c.v} (e_k + e_p + e_u)\rho\, d\forall + \sum_{c.s} (e_k + e_p + e_u)\rho\vec{V}\cdot\vec{A}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{c.v} \left(\frac{V^2}{2} + gz + u\right)\rho\, d\forall + \sum_{c.s} \left(\frac{V^2}{2} + gz + u\right)\rho\vec{V}\cdot\vec{A}$$

No.

* The rate of work is divided into two parts
① flow work
②



force acting on fluid to the right :-

- Force acting on the surface at right of c.v $= P_2 A_2$

- distance travelled $= \Delta L_2 = V_2 \Delta t$

work done on the surronding fluid :-

$$\Delta W_{f_2} = P_2 A_2 V_2 \Delta t$$

the rate of work $= \frac{P_2 A_2 V_2 \Delta t}{\Delta t} = \dot{W}_{f_2} = P_2 \vec{V_2} \cdot \vec{A_2}$

→ similarly for the fluid on the left

$$\dot{W}_{f_1} = P_1 \vec{V_1} \cdot A_1$$

hence $\dot{W}_f = \dot{W}_{f_1} + \dot{W}_{f_2} = \sum_{c.s} P \vec{V} \cdot \vec{A}$

② Shaft work, W_s

$$\dot{W} = \dot{W}_s + \dot{W}_f$$

$$\dot{Q} - \left(\dot{W}_s + \frac{\rho \sum_{c.s} P \vec{V} \cdot \vec{A}}{\rho}\right) = \frac{d}{dt} \int_{c.v} \left(\frac{V^2}{2} + gz + u\right) \rho \, d\forall + \sum_{c.s} \left(\frac{V^2}{2} + gz + u\right) \rho \vec{V} \cdot \vec{A}$$

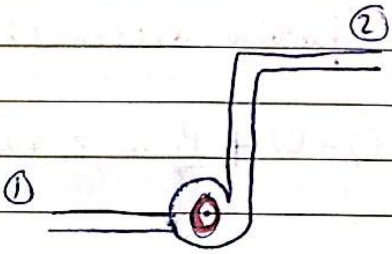
No.

For 2-D case :-

$$\boxed{\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{c.v}\left(\frac{V^2}{2} + gz + u\right)\rho\, d\forall + \int_{c.s}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz + u\right)\rho\, \vec{V}\cdot d\vec{A}}$$

Basic form of energy eqn

* Simplifications:-



① steady $\longrightarrow \frac{d}{dt} = 0$

apply on the inlet and outlet of a pump

$$\dot{Q} - \dot{W}_s + \int_{A_1}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + u_1\right)\rho\, \vec{V}_1\cdot d\vec{A}_1 = \int_{A_2}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + u_2\right)\rho_2\, V_2\cdot dA$$

* dividing the term $\frac{V^2}{2}$ from the integral

also; $\frac{P_1}{\rho} + gz_1$ is a constant value, also u.

The equation Becomes:-

$$\dot{Q} - \dot{W}_s + \left(\frac{P_1}{\rho} + gz_1 + u_1\right)\int_{A_1}\rho\, \vec{V}_1\cdot d\vec{A}_1 + \int_{A_1}\rho\frac{V_1^3}{2}\, dA_1$$

$$= \left(\frac{P_2}{\rho} + gz_2 + u_2\right)\int_{A_2}\rho\, \vec{V}_2\cdot d\vec{A}_2 + \int_{A_2}\rho\frac{V_2^3}{2}\, dA_2$$

$\Rightarrow \int \rho\, V\cdot dA = \dot{m}$

* suppose $\int_{A_1}\rho\frac{V_1^3}{2}\, dA = \alpha_1\,\rho\,\frac{V_1^3}{2}A_1$ / $\int_{A_2}\rho\frac{V_2^3}{2}\, dA_2 = \alpha_2\,\rho\,\frac{V_2^3}{2}A_2$

No.

where, α_1, α_2 are the Kinetic energy correcting factors

take $\rho U A$ as $\dot{m}$

equation Becomes:-

$$\dot{Q} - \dot{W} + \left(\frac{P_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{V_1^2}{2}\right)\dot{m} = \left(\frac{P_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{V_2^2}{2}\right)\dot{m}$$

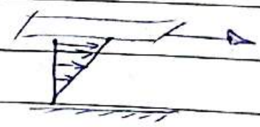
Divide By $\dot{m}$

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}) + \frac{P_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{V_1^2}{2} = \frac{P_2}{\rho} + gz_2 + u_2 + \alpha \frac{U_2^2}{2}$$

if $\boxed{\alpha = 1}$ → The ~~flow~~ velocity is uniform over the Cross section

if $\boxed{\alpha < 1}$ → " " " non " " " " "

if $\boxed{\alpha = 2}$ → Flow is laminar →



if $\boxed{\alpha = 1.05}$ → Flow is turbulant →

Take commonly $\boxed{\alpha = 1}$:-

also Take $\dot{W}_s = \dot{W}_t - \dot{W}_p$ and divide By $\boxed{g}$

$$\boxed{\frac{\dot{W}_p}{\dot{m}g}} + \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \boxed{\frac{\dot{W}_t}{\dot{m}g}} + \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left[\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g}\right]$$

h_p:- Pump head (m) . h_t:- turbine head (m)

No. ______

$\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g} \Rightarrow h_L$:- head loss

* if there is no heat addition during the process

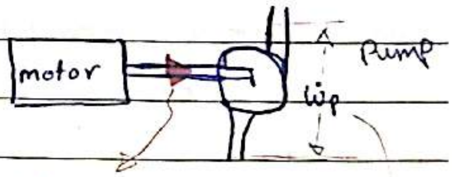
then $\boxed{\frac{\dot{Q}}{\dot{m}g} = 0}$

Finally :- Energy equation :-

$$\boxed{\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_t + h_L} \quad --- \quad \boxed{7.24}$$

22/4/2014 Note on Power and efficiency :-

* if the machine is a pump



$h_p = \frac{\dot{W}_p}{\dot{m}g}$

Power delivered to pump

pump Power :- $\dot{W}_p = \dot{m}g(h_p)$ power delivered to liquid

$$= \boxed{Q \rho g h_p = Q \gamma h_p}$$

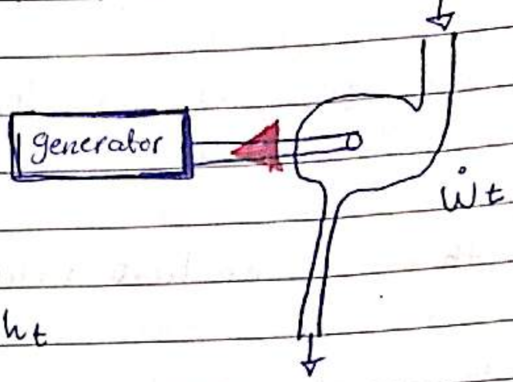
which is power delivered to the liquid ((hydraulic power))

pump efficiency : $\eta_p = \frac{\text{Power delivered to liquid}}{\text{Power delivered to pump By Motor}}$

No.

* if the machine is turbine :-

$$h_t = \frac{\dot{W}_t}{\dot{m} g}$$



turbine power :- $\dot{W}_t = \dot{m} g h_t$

$$= \boxed{Q \rho g h_t = Q \gamma h_t}$$

Q :- volumetric flow rate

which is power delivered By the liquid

Turbine efficiency :- $\eta_t = \frac{\text{Power delivered to the generator}}{\text{power delivered By the liquid}}$

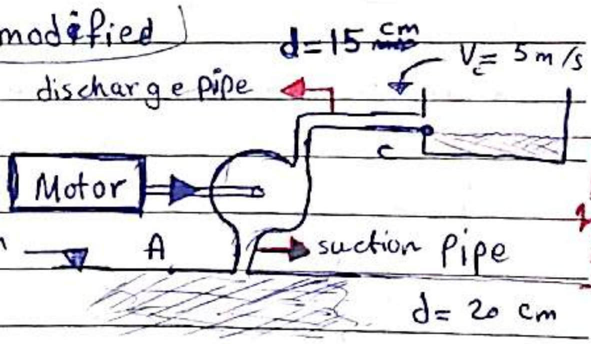
$$\boxed{\eta_t < 1}$$

Example, Problem 58 ((10 th ed)) modified

Given :- power delivered to pump = 35 Kw

$\eta_p = 70\ \%$

$h_L = 2V_c^2/2g$



find $\boxed{h}$???

Solution :- Apply energy equation Between A, C

$$\cancel{\frac{P_1}{\gamma}} + \overset{0}{\cancel{z_1}} + \cancel{\frac{V_1^2}{2g}} + h_p = \cancel{\frac{P_2}{\gamma}} + \boxed{z_2} + \frac{V_2^2}{2g} + \cancel{h_t} + h_L$$

atmospheric $P_1 = P_2$ reservoir surface no turbine

$$h_p = \frac{V_2^2}{2g} + h_L + 2V_2^2/2g \implies h = h_p - \frac{3V_2^2}{2g}$$

No.

$$h_p = \frac{\dot{W}_P}{\dot{m} g} * \eta_p \quad \rightarrow \quad \dot{m} = \rho U A = 1000 * 5 * \frac{\pi}{4}(0.15)^2 = 88.63 \text{ kg/s}$$

$$h_p = \frac{35 * 1000}{88.63 * 9.81} * 0.70 = 28.26 \text{ m}$$

$$h = 28.26 - 3(5^2)/2 * 9.81 = 24.4 \text{ m}$$

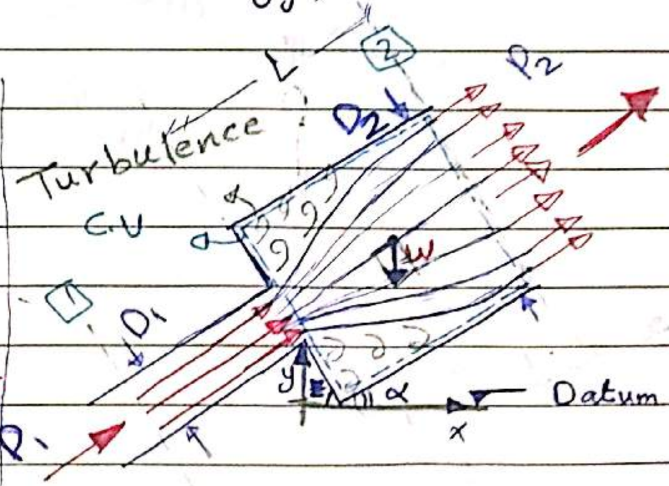
Notes :- ON CHAPTER $\boxed{6}$

$\boxed{1}$ if the calculations involving $\boxed{h}$ we must use Bernoulli P.(28)

$\boxed{2}$ if there is [a Flange] we will add the term (PA) to calculations P-45 , P-55

4/2014 $\boxed{7.6}$ other applications on energy, momentum and continuity equations :-

Sudden expansion
find an expression for head loss (h_L)



$$\frac{P_1}{\gamma} + \frac{U_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{U_2^2}{2g} + Z_2 + \boxed{h_L} \quad \text{--- (1)}$$

$\Rightarrow$ modified Bernoulli equation

now, momentum eq$\underline{n}$

$$\Sigma F = \sum_{cv} U \rho \vec{V} \cdot \vec{A} \quad \text{--- (2)}$$

$$P_1 A_1 - P_2 A_2 - W \sin\alpha = -\rho U_1^2 A_1 + \rho U_2^2 A_2$$

Put $W = \gamma \forall = \gamma A_2 L$

substituting in (2) and dividing By γA_2 $\longrightarrow$

No.

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - L\sin\alpha = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}\left(\frac{A_1}{A_2}\right)$$

Cont $\frac{V_2}{V_1} = \frac{A_1}{A_2}$

combining with $\boxed{\text{I}}$ and solving for h_L

$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$

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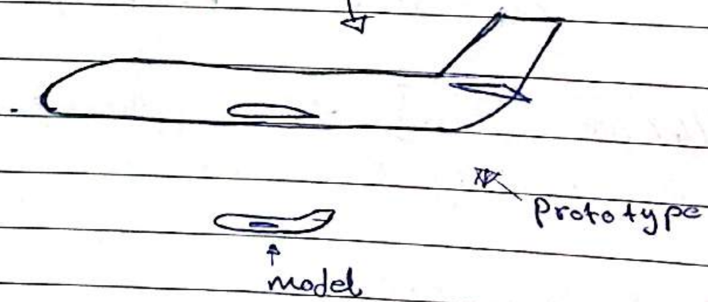
** CHAPTER 8 **

Dimensional analysis and Similitude



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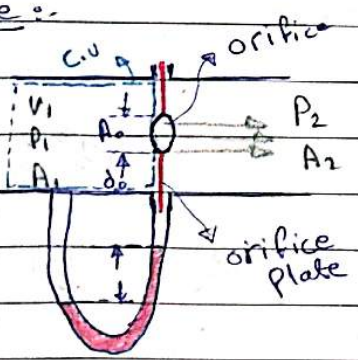
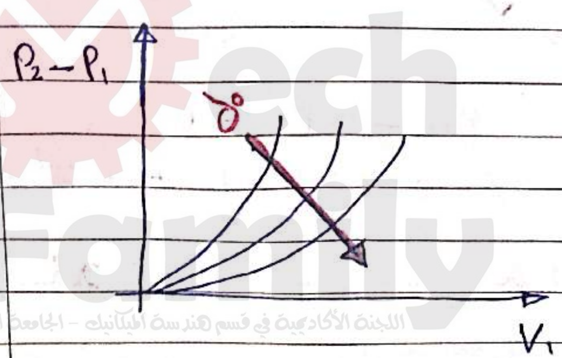




No.

29/4/2014 Fundamental eqn

$$\frac{P_1}{\cancel{g}} + \rho\frac{V_1^2}{2} = P_2 + \rho\frac{V_2^2}{2}$$ Bernoulli eqn.

$$\boxed{\frac{P_1 - P_2}{\rho V_1^2/2}} = \frac{V_2^2}{V_1^2} - 1$$

$$\left[\frac{P_1 - P_2}{\rho V_1^2/2}\right] = \frac{N/m^2}{kg/m^3 \cdot m^3/s^2} = \frac{\frac{\cancel{kg \cdot m}}{\cancel{s^2}} \cdot \frac{1}{\cancel{m^2}}}{\frac{\cancel{kg}}{\cancel{m^2}} \cdot \frac{\cancel{m^2}}{\cancel{s^2}}}$$

→ Dimensionless

$\equiv C_p$ [pressure coffecient]

$$\frac{V_2}{V_1} = \frac{A_1}{A_2} = f\left(\frac{d_1}{d_2}\right)^2$$

$$\frac{P_1 - P_2}{\rho V_1^2/2} = \left(f\left(\frac{d_1}{d_2}\right)^2 - 1\right)$$

Graph :-

we converted the relation from 5 variables $(\Delta P, d_1, d_2, V, \rho)$

To 2 variables $(C_P, \frac{d_1}{d_2})$

No.

8.3 Buckingham's π theorem

→ if the number of variables involved in a process is $\boxed{n}$, and the number of Basic dimensions that form these variables are $\boxed{m}$, then the number of Dimensionless groups needed to ~~calculate~~ correlate these variables is $\boxed{n-m}$

2 methods :-

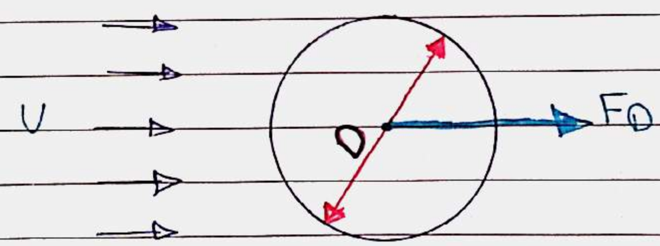
① Step - By - step method ← Example 8.2

② Exponent metod

find the ~~number of~~ dimensionless groups that express The drag force in terms of other physical parameters which ar: (U, ρ, μ, D)

Solution :-

ie. given $F_D = f_1(U, \rho, \mu, D)$



- find the dimensions of each variable in terms of system of Basic units, such MLT system

$[F_D] = ML/T^2$　　　$[\rho] = M/L^3$

$[U] = L/T$　　　$[\mu] = M/(L.T)$

$[D] = L$

- ~~eleminate~~ eleminate Basic dimensions of the dependent variables, one By one, starting with M for example, By dividing By avariable contains M such as ρ

No.

$$\frac{F_D}{\rho} = f_2\left(V, \frac{\mu}{\rho}, D\right)$$

$$\left[\frac{F_D}{\rho}\right] = \frac{ML/T^3}{M/L^3} = \frac{L^4}{T^2}$$

$$\left[\frac{\mu}{\rho}\right] = \frac{M/LT}{M/L^3} = L^2/T$$

- eleminate time dimension By dividing By V^2 for example

$$\frac{F_D}{\rho V^2} = f_3\left(\frac{\mu}{\rho V^{\cancel{2}}}, D\right) \qquad \left[\frac{F_D}{\rho V^2}\right] = \frac{L^4/T^2}{L^2/T^2} = L^2$$

$$\left[\frac{\mu}{\rho V^{\cancel{2}}}\right] = \frac{L^2/T}{L^{\cancel{2}}/T^{\cancel{2}}} = \cancel{T}\, L$$

-eleminate L By dividing By D^2

$$\underbrace{\frac{F_D}{\rho V^2 D^2}}_{\text{Dimensionless}} = f_4\Bigg(\underbrace{\frac{\mu}{\rho V D}}_{\text{Dimensionless}}\Bigg) \equiv f\left(\frac{1}{Re}\right)$$

Reynold's number $= \dfrac{\rho V D}{\mu}$

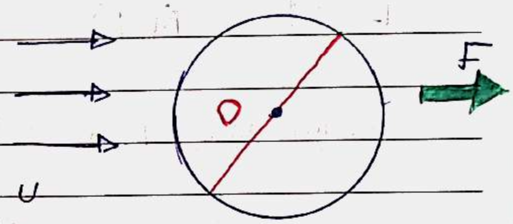
No.

4/5/2014 **The exponent method**

- Do the example $\boxed{8.2}$ using exp. method using FLT dimensions

- Start with the functional relationship

$$F = f[U, \rho, \mu, D]$$



- Put the functional Relationship inthe form

$$F = f[U^a * \rho^b * \mu^c * D^d]$$

- substitute and collect similar dimensions

$$[U] = \frac{L}{T} \qquad [\rho] = \frac{[F]/a}{L^3} = \frac{FT^2}{L^4} \qquad [\mu] = \frac{[N]\cdot s}{L^2} = \frac{FT}{L^2}$$

$$[D] = L$$

$$F = \left(\frac{L}{T}\right)^a \left(\frac{FT^2}{L^4}\right)^b \left(\frac{FT}{L^2}\right)^c (L)^d$$

$$F = \frac{L^{a-4b-2c+d} * F^{b+c} *}{T^{a-2b-c}}$$

- collect similar exponents

No.

for $\boxed{F}$:- $b + c = 1$ --- $\boxed{1}$

for $\boxed{L}$:- $a - 4b - 2c + d = 0$ --- $\boxed{2}$

for $\boxed{T}$:- $a - 2b - c = 0$ --- $\boxed{3}$

solve the $\boxed{\text{3 equations}}$ in terms of 1 unknown, which appears more frequently, ie $\boxed{C}$

- put the eqn's in matrix form :-

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 1 & -2 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ d \end{pmatrix} = \begin{pmatrix} 1-c \\ 2c \\ c \end{pmatrix}$$

check that the determinate of the matrix $\neq 0$

$$0(0+2) - 1(0-1) + 0(-2+4) \neq 0 \quad ✓ \text{ ok}$$

→ a unique solution is obtainable.

from $\boxed{1}$ $b = 1 - c$

from $\boxed{2}$ $a = 2(1-c) + c = 2 - c$

from $\boxed{3}$ $d = -2 + c + 4(1-c) + 2c = 2 - c$

$\therefore$ $\boxed{a = d}$

- substitute in the functional relationship trying to form a dimensionless groups

$$F = f\left(U^{2-c} \cdot \rho^{1-c} \cdot \mu^{c} \cdot D^{2-c}\right)$$

No.

$$F = V^2 \rho D^2 \left(\frac{\mu}{\rho V D} \right)$$

$$\hookrightarrow \frac{1}{Re}$$

$$\boxed{\frac{F}{\rho V^2 D^2}} = f_1\left(\frac{1}{Re}\right)$$

C_p

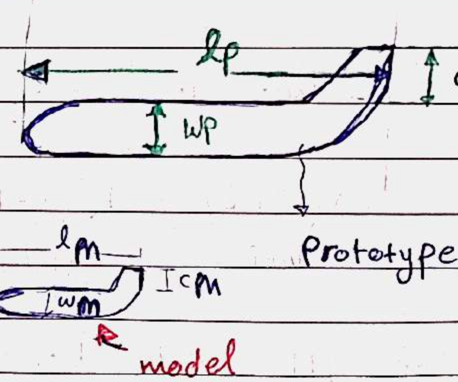
$\boxed{6/5/2014}$

$\boxed{8.6}$ Similitude

→ Two types:- 1- geometrical 2- Dynamic

[1] Geometrical

* $\frac{L_m}{L_p} = \frac{W_m}{W_p} = \frac{C_m}{C_p} \equiv$ scale ratio $((L_r))$



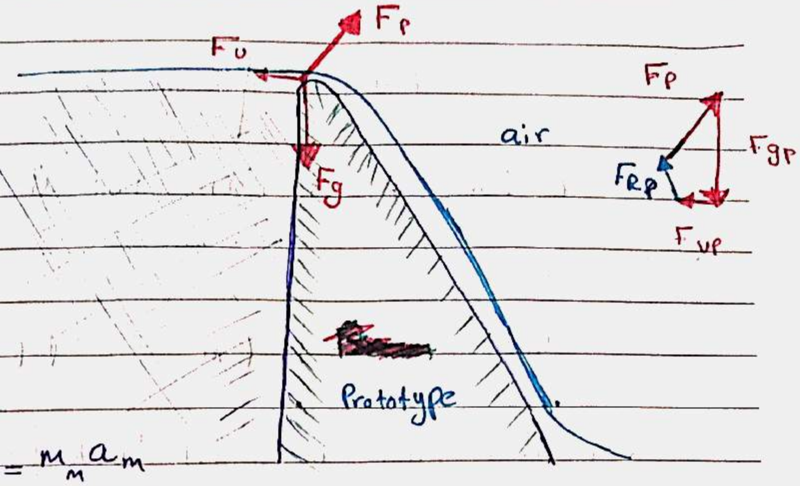
* $A_r = \frac{A_m}{A_p} = (L_r)^2$

* $V_r = \frac{V_m}{V_p} = (L_r)^3$

The Three conditions must ~~Be~~ exists to Be Geomitric similitude

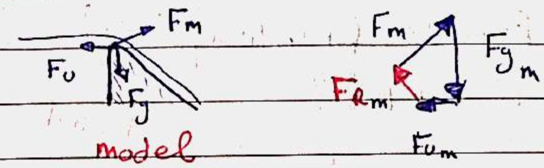
2 Dynamic :-

for example, a spillway



$$\frac{F_m}{F_p} = \text{const}$$

$$* \quad \frac{F_{Rm}}{F_{Rp}} = \frac{F_{gm}}{F_{gp}} = \cancel{\frac{F_{vm}}{F_{vg}}} = \frac{m_m a_m}{m_p a_p}$$



$$* \quad \text{inertitic forces} = \frac{\gamma_m L_m^3 a_m}{\gamma_p L_p^3 a_p} = \frac{\rho_m L_m^3 (V_m/t_m)}{\rho_p L_p^3 (V_p/t_p)}$$

$$\frac{\rho_m V_m}{\rho_p V_p} = \frac{\gamma_m t_m}{\gamma_p t_p} \Longrightarrow \frac{V_m}{g_m t_m} = \frac{V_p}{g_p t_p}$$

$$\text{But} \quad \frac{t_m}{t_p} = \frac{L_m/V_m}{L_p/V_p} \qquad \sqrt{\frac{V_m^2}{g_m L_m}} = \sqrt{\frac{V_p^2}{g_p L_p}}$$

$$\boxed{\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}} \quad \text{: Dimensionless}$$

$\rightarrow$ Froud No.

$$\boxed{Fr_m = Fr_p}$$

* applies for partially submerged.

such as ((ships)).

Suggested problems on $\boxed{CH\ 7}$

7.27 / 7.33 / 7.50 / 7.60 / 7.65

No.

also $\boxed{Re_m = Re_p}$ Reynald's No

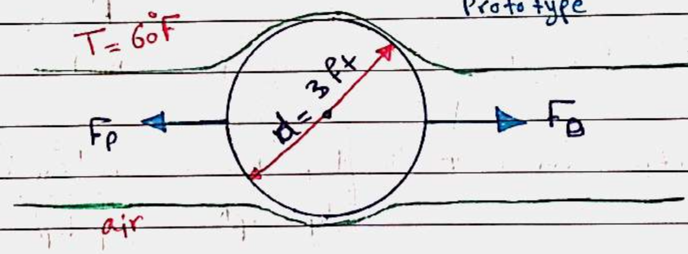
→ for objects are totally submerged

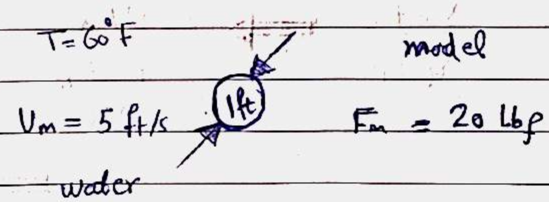
and $\boxed{C_{p_m} = C_{p_p}}$ Pressure Coef`

* in this case we must have $\boxed{Fr_m = Fr_p}$ and $\boxed{Re_m = Re_p}$

Example Problem $\boxed{8.35}$ Ballon

Find Fp ??





$\boxed{10/5/2014}$ solution :-

Dynamic similarity :- $Re_m = Re_p$ ---(1)

$C_{p_m} = C_{p_p}$ ---(2)

$Re = \frac{\rho U D}{\mu} \equiv \frac{UD}{\nu}$; $\mu = \rho \nu$ (Dinamic, Kinematic vescocity)

$\frac{V_m D_m}{\nu_m} = \frac{V_p D_p}{\nu_p} \rightarrow \frac{V_p}{V_m} = \frac{D_m}{D_p} * \frac{\nu_m}{\nu_p}$

No.

$$\frac{V_p}{V_m} = \frac{1}{3} * \left(\frac{1.58 * 10^{-4}}{1.22 * 10^{-5}} \right) \quad ①$$

<u>Now</u>: from second cond $C_{p_m} = C_{p_p}$

$$\frac{\Delta P_m}{\rho_m V_m^2/2} = \frac{\Delta P_p}{\rho_p V_p^2/2} \rightarrow \frac{\Delta P_p}{\Delta P_m} = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p^2}{V_m^2}\right)$$

$$* \frac{F_p}{F_m} = \frac{\Delta P_p * A_p}{\Delta P_m * A_m} = \left(\frac{A_p}{A_m}\right) \frac{\rho_p}{\rho_m} \frac{V_p}{V_m}$$

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m} \frac{\nu_p}{\nu_m} = \frac{0.00237}{\underbrace{1.97}_{slug/ft^3}} \left(\frac{1.58 * 10^{-4}}{1.22 * 10^{-5}}\right) = 0.2049$$

$$\boxed{F_p = F_m * 0.2049 = 20 * 0.2408 = 18.23 \text{ N}}$$

* Physical meaning of Dimensionless Numbers

	symbol	Physical mean	applications
* Pressure coeff.	$C_P = \frac{\Delta P}{\rho V^2/2}$	$\frac{\text{Pressure}}{K.E}$	flow around Bodies
* Drag coeff.	$C_D = \frac{F_D}{\rho V^2 A/2}$	$\frac{\text{Resistance}}{K.E}$	flow around Bodies
* Reynold's number	$Re = \frac{\rho V D}{\mu}$	$\frac{\text{inertia force}}{\text{Viscous force}}$	flow around Bodies in flat planes
* Froud Number	$\frac{V}{\sqrt{gL}}$	$\frac{K.E}{\text{gravity}}$	flow on partially submerged Bodies
* Relative Raughness	$\frac{\varepsilon}{D}$	—	flow inside pipes
* Mach number	$M = \frac{V}{C}$		flow

C: sound velocity

No.

13/V/2014

** CHAPTER 10 **

Flow in conduits

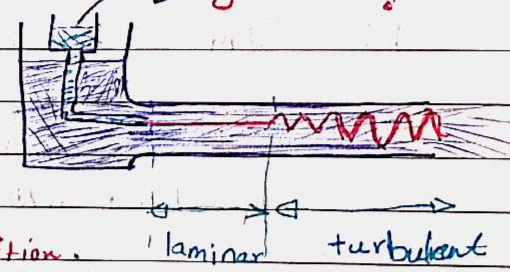
- Modefied Bernoulli eqn

$$\frac{P_1}{\gamma} + \cancel{\frac{U_1^2}{2g}} + z_1 = \frac{P_2}{\gamma} + \cancel{\frac{U_2^2}{2g}} + z_2 + h_L$$

if no change in diameter $\rightarrow \boxed{V = C}$

$$\frac{P_1}{\gamma} + z_1 = \frac{P_2}{\gamma} + z_2 + h_L$$

* Osborn Reynolds experiment :- → Dye (صبغة)



if $\frac{\rho U D}{\mu} < 2000 \rightarrow$ laminar.

if $\frac{\rho U D}{\mu} > 2000$ and $< 3000 \rightarrow$ Transition.

if $\frac{\rho U D}{\mu} > 3000 \rightarrow$ Turbulant.

* Darcy - Weisbach equation :-

$$h_f = f \frac{L}{D} \cdot \frac{U^2}{2g}$$

friction coeff

$h_f \equiv h_L \equiv$ Major loss

* $f = 64/Re$ for laminar flow

* $\frac{1}{\sqrt{f}} = 2 \log (Re \sqrt{f}) - 0.8$ for turbulant

* minor loss :- from fittings

No.

also: $f = (0.79 \ln Re_D - 1.64)^{-2}$ But only for

$5 \times 10^{6} \geq Re \geq 3000$

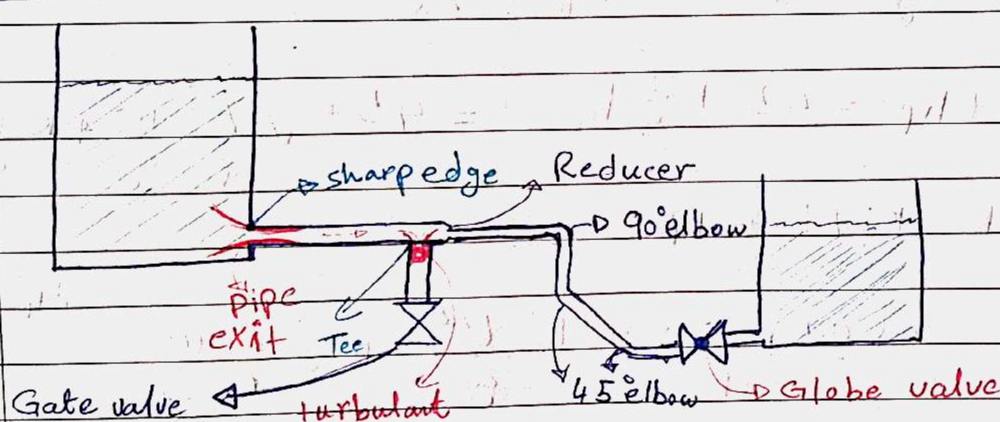
* ColeBruck-White equation :- expressed By Swamee-Jain eq.

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{K_s}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

k_s :- Raughness

or use Moody chart, Fig $\boxed{10.13}$

15/5/2014 $\boxed{10.5}$ Minor losses



$$h_L = k\frac{U^2}{2g}$$

loss coeff

for tank exit or pipe inlet

$$h_L = k_e \frac{U^2}{2g}$$

$k_e = 0.5$ for sharpedge

$k_e = 0.1$ = smooth edge

for 90° elbow

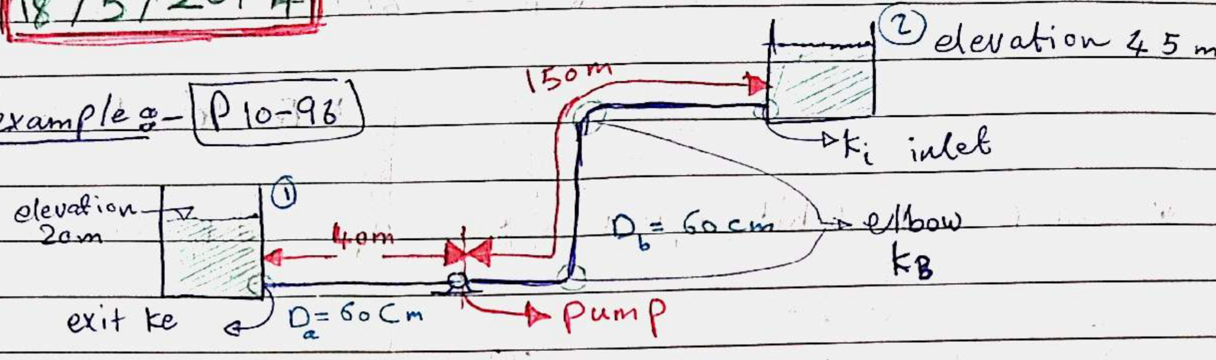
r/d	kb
1	0.35
2	0.19
4	0.16

→ Table 10.5

No.

18/5/2017

example :- P 10-96



$Q = 1\ m^3/s$, fuel oil , $S.G = 0.94$ $r/d = 2$ for elbow

steel pipe , $\eta_p = 70\%$ find Pump power ??

Solution :- energy eqn

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{0} + Z_1 + h_p = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}}^{0} + Z_2 + h_L + \cancel{h_t} \quad \sum h_L$$

$$20 + h_p = 45 + \sum h_L = 45 + f\frac{L}{D}\frac{V^2}{2g} + k\frac{V^2}{2g}$$

→ $k = K_i + K_e + 2K_B$ ((Minor losses))

$$h_p = 25\ \underbrace{(0.5 + 2 * 0.19 + 1)}_{k}\ \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} \quad 3.54^2$$

$Re = \frac{\rho V D}{\mu}$

$V = \frac{Q}{A} = 3.54\ m/s$

$\frac{V^2}{2g} = 0.639$

$Re = 4.25 * 10^4$ turbulant

$\frac{k_s}{D} = \frac{0.0514}{0.6}$ → steel $= 0.00009$

$f = 0.021$

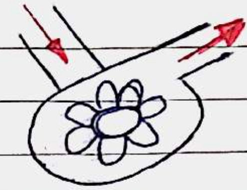
$h_p = 30.4\ m$

$$P = \frac{Q\gamma h_p}{\eta_p} = \frac{1 * 9810 * 0.94 * 30.4}{0.7}$$

$= 400$ KW

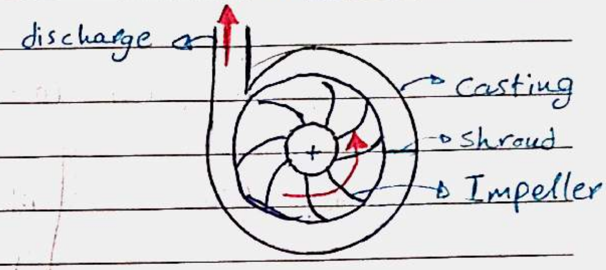
No.

20/5/2014 Modeling a cenrifugal pump



Blades:-
- forward curved
- Backward curved
- radial ((straight))

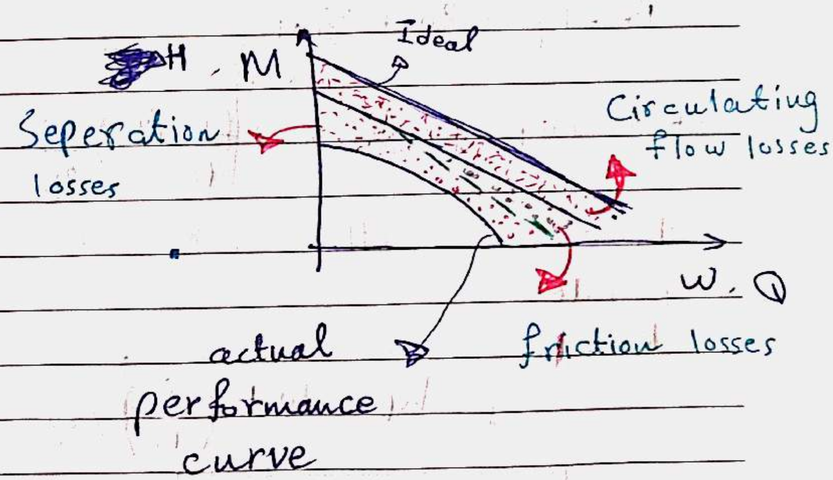
front View :-



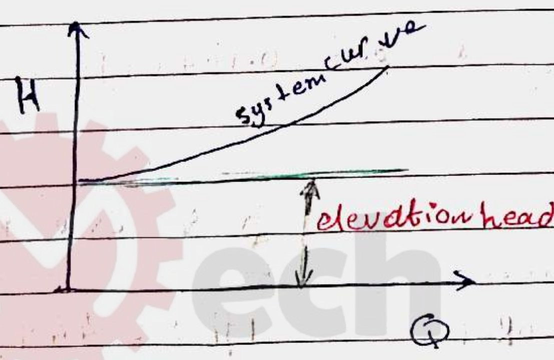
* fig 10-19 بالكتاب الرسمة أوضح

Theory :- Recall moment of momentum

$$M = \sum \vec{r} \times \vec{U} \rho \vec{U} . \vec{A} \quad -- \boxed{6.32}$$

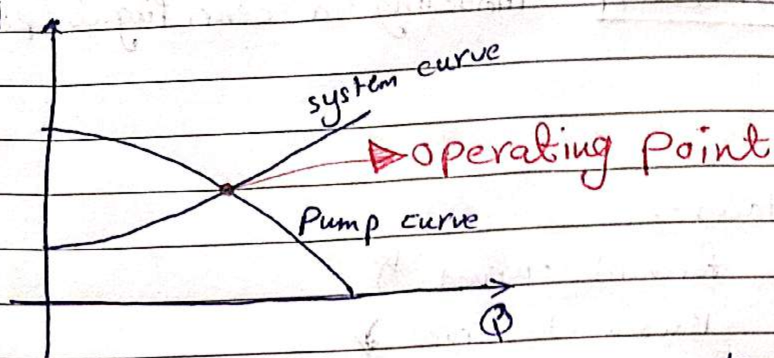


* system curve:-

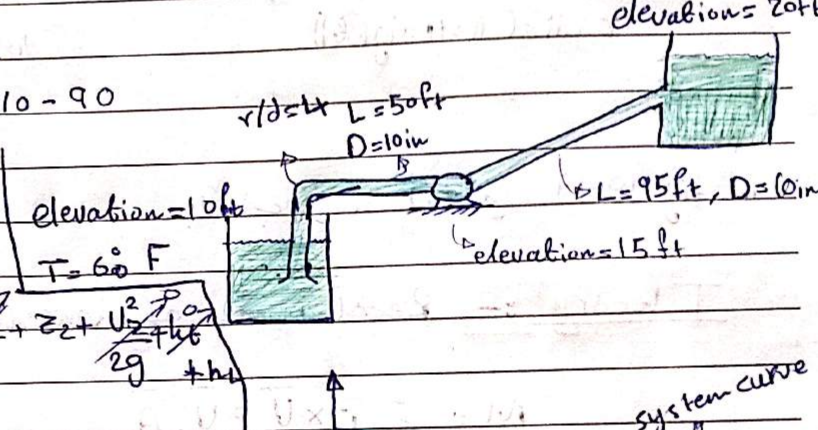


*كيف بدنا نعرف أذا الضخة ملائمة للنظام (system)؟!
أرسم كلا المنحنيات على نفس الرسمة

No.



22/5/2014 Example prob 10-90



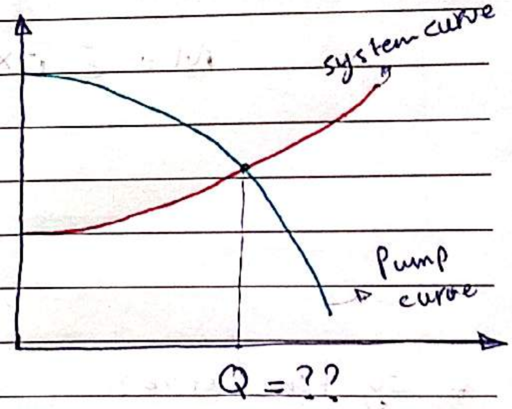
Energy equation

$$\cancel{\frac{P_1}{\gamma}}^{0} + Z_1 + \cancel{\frac{U_1^2}{2g}}^{0} + h_p = \cancel{\frac{P_2}{\gamma}}^{0} + Z_2 + \cancel{\frac{U_2^2}{2g}}^{0} + h_L$$

$$10 + h_p = 20 + \sum h_L$$

$$10 + h_p = 20 + \sum\left(\underbrace{k_i + k_e}_{\text{inlet, elbow}} + \frac{fL}{D} + \underbrace{k_o}_{\text{outlet}}\right)\frac{U^2}{2g}$$

$k_i = 0.03$
$k_e = .16$
$k_o = 1$



$$h_p = 10 + \left(\frac{Q}{A}\right)^2 / 2g \ast \left(0.03 + 0.16 + 1 + \frac{fL}{D}\right)$$

$$\boxed{h_p = 10 + 1.32\,Q^2}$$

$$\boxed{Q = 2900 \text{ gpm}}$$

Q =	1000	2000	3000
hp =	16.55	36.2	68.95

$1\ ft^3/s = 449$ gpm

wish you all luck

Ali Abu Yehea