

CH12- Introduction to fluid ↓

- Fluid mechanics is the science that deals with the action of forces on the fluid substance

- Fluid is liquid or gas.

- Fluid is a substance that cannot resist shear forces
i.e. it will continue to deform as long as the shear force is small.

- Hydraulic

- Aerodynamics

- Newtonian fluid ↔ linear relationship between shear stress and shear rate

- Incompressible → flow (density) $\frac{d\rho}{dt} = \frac{d\rho}{dx} = \frac{d\rho}{dy} = \frac{d\rho}{dz} = 0$
→ fluid

- Modulus of compressibility very high for water.

CH28- Fluid properties ↓

- extensive energy, density, & no depends on mass
- Intensive temperature, specific, ρ No doesn't depend on mass

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{kg}}{\text{m}^3}$$

for water Density = $\rho_{H_2O} = 1000 \text{ kg/m}^3$
 $1 \text{ m}^3 = 1000 \text{ liter}$

$$\text{Density} = \rho_{H_2O} = 1 \text{ kg/liter} = 1 \text{ g/cm}^3$$

$$\rho_{H_2O} = 1000 \frac{\text{kg}}{\text{m}^3} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = 1 \text{ g/cm}^3$$

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ m}^3 = 10^6 \text{ cm}^3$$

specific weight = γ

$$\gamma = \rho g = \frac{\text{Weight}}{\text{Volume}}$$

$$|\gamma| = \frac{N}{\text{m}^3}$$

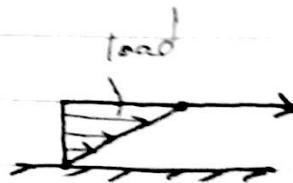
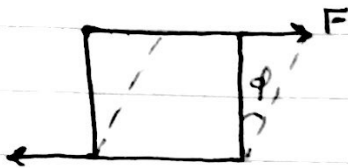
- specific gravity = S

$$S = \left(\frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}} \right)_{T_{\text{standard}}}$$

أو $S = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}}$ عند درجة حرارة قياسية

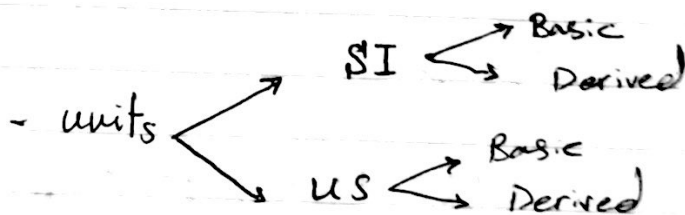
$$S = \frac{(\rho_g)_{\text{fluid}}}{(\rho_g)_{\text{fluid standard}}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}}$$

- Viscosity, μ أو η



Solid Mechanics :- shear force of strain

Fluid Mechanics :- shear force of rate of strain.



Basic Units

SI

length [m]

mass [kg]

Time [s]

U.S

length [ft]

Weight [lb_f] force

Time [s]

Derived units

SI

Force = ma

$$[N] = \text{kg} \cdot \frac{\text{m}}{\text{s}^2}$$

Energy = force * distance

$$J = N \cdot m$$

$$= \text{kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

power = Energy / time

$$= \text{N} \cdot \text{m} / \text{s}$$

$$W = \text{kg} \cdot \frac{\text{m}^2}{\text{s}^3}$$

Energy = Power * time

$$J = W \cdot s$$

$$kJ = kW \cdot s$$

$$kW \cdot hr = kW \cdot 3600s$$

$$= 3600 kW \cdot s$$

$$= 3600 kJ$$

$$1 kW \cdot hr = 3.6 MJ$$

US units

$$1 ft = 0.3048 m$$

$$1 ft = 12 inches$$

$$1 lb_m = 0.45 kg$$

Temperature

- Absolute Temp:-

$$K = 273.15 + ^\circ C$$

$$^{\circ}R = 459.67 + ^{\circ}F$$

$$^{\circ}F = 32 + \frac{9}{5} ^\circ C$$

$$^{\circ}R = \frac{9}{5} K$$

* Extensive properties

* Intensive properties

* Density $[\rho]$

* specific weight $[\gamma = \rho g]$

* specific gravity $[S = \frac{\gamma_{fluid}}{\gamma_{standard fluid}}]_T$

$$S' = \frac{(\rho g)_{liquid}}{(\rho g)_{H_2O}}$$

$$S = \frac{(\rho g)_{gas}}{(\rho g)_{air}}$$

• Viscosity: μ

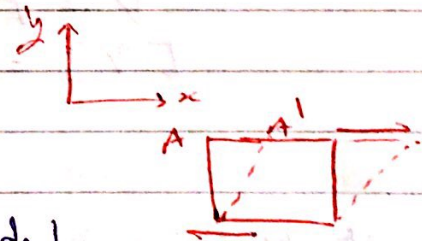
• Absolute viscosity: μ

- shear stress $\tau \propto$ strain (solids)

• $\tau \propto$ rate of shear strain (fluid)

• shear strain = $\phi = \frac{x}{y}$

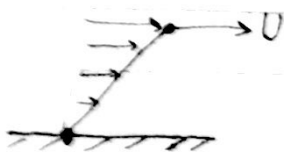
$$\tau \propto \frac{du}{dy}$$



$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

$$V = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

$$\vec{V} = V \hat{i} + V \hat{j} + V \hat{k}$$



$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

constant

where:-

μ = viscosity of the fluid

$\frac{du}{dy}$ = Velocity gradient
time rate of shear strain

y = coord. axis $\perp$ to the wall

$$\mu = \frac{\tau}{du/dy}$$

μ is a property of the fluid which determine its resistance to shear force

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau}{du/dy}$$

$$[\mu] = \frac{\text{Pa}}{\frac{\text{m}}{\text{s} \cdot \text{m}}} = \text{Pa} \cdot \text{s} = \frac{\text{N}}{\text{m}^2} \cdot \text{s} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

$$1 \text{ poise} = \frac{\text{dyne} \cdot \text{s}}{\text{cm}^2} = \frac{10^{-5} \text{ N} \cdot \text{s}}{10^{-4} \text{ m}^2} = 0.1 \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

$$\text{centi poise} = 0.01 \text{ poise}$$

kinematic viscosity ν

$$\nu = \mu / \rho$$

$$[\nu] = \frac{\text{kg}}{\text{m} \cdot \text{s}} \cdot \frac{\text{m}^3}{\text{kg}} = \frac{\text{m}^2}{\text{s}}$$

$$1 \text{ stoke} = \frac{\text{cm}^2}{\text{s}}$$

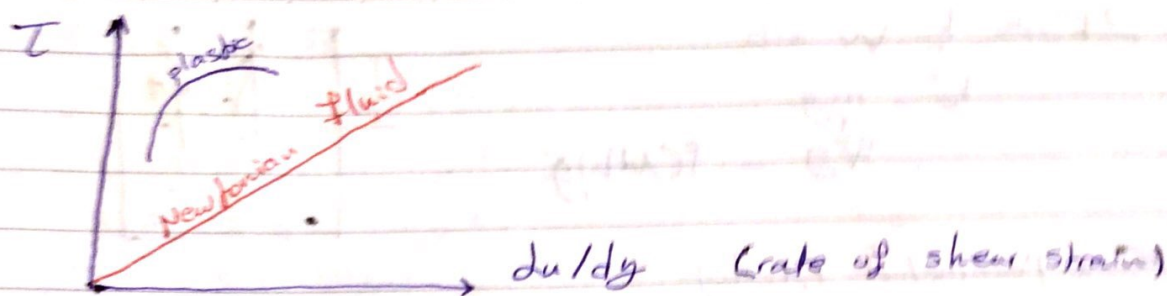
$[\mu]$ in US units

$$[\mu] = \frac{\text{lbm}}{\text{ft} \cdot \text{s}} = \frac{\text{slug}}{\text{ft} \cdot \text{s}}$$

$$1 \text{ slug} = 32.2 \text{ lbm}$$

$$g = 9.81 \text{ m/s}^2$$

Newtonian & Non-newtonian fluid



$\tau = \mu \, du/dy$ this is for all fluids that is considered Newtonian.

If $\tau \neq \mu \, du/dy \Rightarrow$ non-newtonian fluid.

$\Rightarrow$ Elasticity $\rightarrow$

$$dp \propto \frac{dV}{V}, \quad V = \text{volume}$$

$$dp = -E_v \left(\frac{dV}{V} \right)$$

$\rightarrow$ Bulk Modulus of Elasticity

for example 3. $E_v = 2.2 \, \text{GN/m}^2$

- To change volume of water by 0.05% we need to apply pressure of $1 \, \frac{\text{MN}}{\text{m}^2}$

$\Rightarrow$ For Ideal Gas

$$PV = nRT$$

$$PV = n \bar{R} T$$

$$PV = RT$$

$$\Rightarrow P dV + V dp = 0 \quad (\text{Isothermal})$$

$$dV = -V \frac{dp}{P}$$

$$dp = -E_v \left(\frac{V dp}{P V} \right)$$

$$\Rightarrow E_v = P$$

$\Rightarrow$ surface Tension σ

$$[\sigma] = \text{N/m}$$

It is the result of molecular attraction near a free surface causing the surface to act like a stretched membrane.

$$\sum F \Rightarrow F_c - W = 0$$

$$F_c = mg$$

$$\downarrow \rightarrow p(A \Delta h)g$$

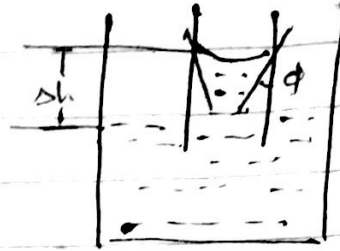
cos θ

$$\text{Parameter} = p A \Delta h g$$

- If the solid surface is Tube

$$A = \frac{\pi}{4} d^2 \quad p = \pi d$$

$$\Delta h = \frac{6 \times \pi d}{\frac{\pi}{4} d^2 \times g} = \frac{46}{(p g) d}$$



* Vapor pressure

the pressure of which liquid boils

2.6

$$S = 1.7$$

$$\rho = ? \text{ kg/m}^3$$

$$\gamma = ?? \text{ N/m}^3$$

Solution

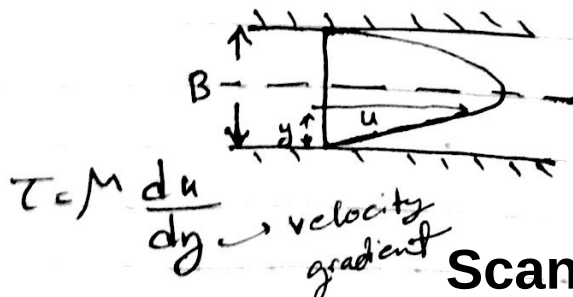
$$S = \frac{\gamma_{\text{Liquid}}}{\gamma_{\text{stand. fluid}}}$$

$$1.7 = \frac{\gamma_{\text{liquid}}}{9810} \Rightarrow \gamma_{\text{liq}} = (1.7 \times 9810) \text{ N/m}^3$$

$$\gamma = \rho g \Rightarrow \rho = \frac{\gamma}{g} = \frac{9810 \times 1.7}{9.81} = 1700 \text{ kg/m}^3$$

$$\gamma_{\text{water}} = \rho g = 1000 \times 9.81 = 9810 \text{ N/m}^3$$

2.31, 2.32, 2.33



2.32 Crut oil at 37.8°C

$$\mu = 3.83 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$u = 100y(0.1 - y) \text{ m/s}$$

$$u = 10y - 100y^2$$

$$\frac{du}{dy} = 10 - 200y$$

$$\tau = \mu \frac{du}{dy} = \mu [10 - 200y]$$

$$\tau|_{y=0} = 3.83 \times 10^{-3} [10 - 0]$$

y (mm)	V (m/s)
0	0
1	1.0
2	1.99
3	2.98

$$\mu = 10^{-3} \text{ N}\cdot\text{s/m}^2$$

$$\frac{du}{dy} = \lim_{\Delta y \rightarrow 0} \frac{\Delta u}{\Delta y}$$

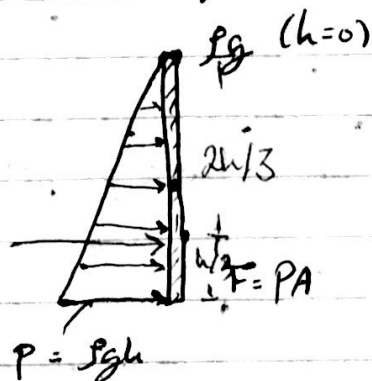
2.33 $\frac{dp}{dx} = -1.6 \text{ kN/m}^3$ pressure gradient $B = 5 \text{ cm}$

$$u = \frac{-1}{2\mu} \left(\frac{dp}{dx} \right) (By - y^2)$$

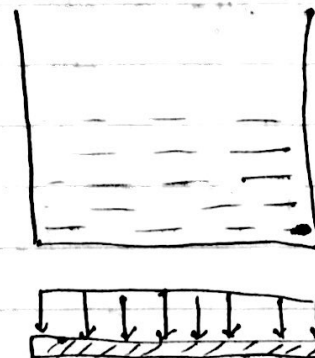
$$u, \tau|_{y=12 \text{ mm}}$$

$$\tau = \mu \frac{du}{dy} = -\frac{1}{2} \left(\frac{dp}{dx} \right) (B - 2y)$$

CH3 3- Fluid statics



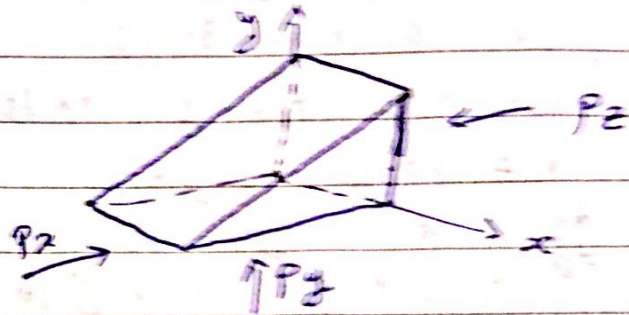
$$\bar{P} = \rho g \frac{h}{2}$$



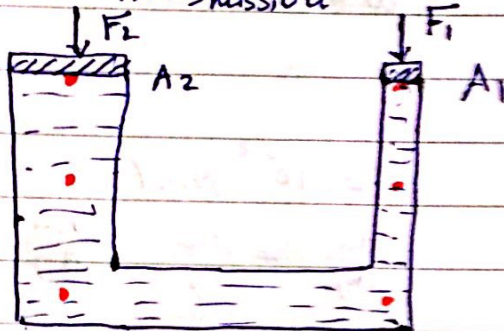
$$\begin{aligned} F &= PA \\ &= \rho g h A \\ &= \rho g V \\ &= mg \\ &= W \end{aligned}$$

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

$$P_x = P_y = P_z$$



Pressure Transmission



$$[P] = \left[\frac{F}{A} \right] = N/m^2$$

$$= \text{Pascal} = Pa$$

$$= \frac{kg \cdot m}{s^2 \cdot m^2}$$

$$= \frac{kg}{m \cdot s^2} = Pa$$

$$[P] = PSI \text{ (pound / in}^2\text{)}$$

$$= Ib/in^2$$

$$[P] = 14.69 PSI$$

Absolute, gauge and Atmospheric Pressure

$$P_{atm} = 101.325 kPa$$

$$= 1.01 \text{ bar}$$

$$= 760 \text{ mmHg}$$

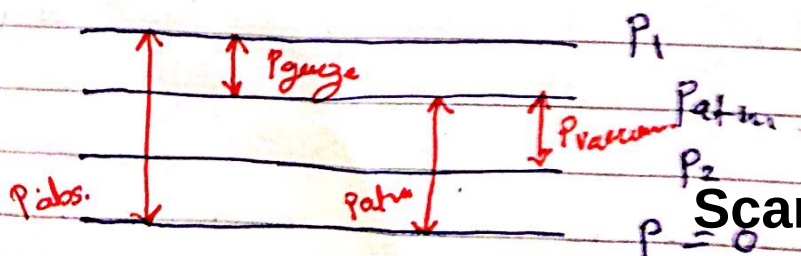
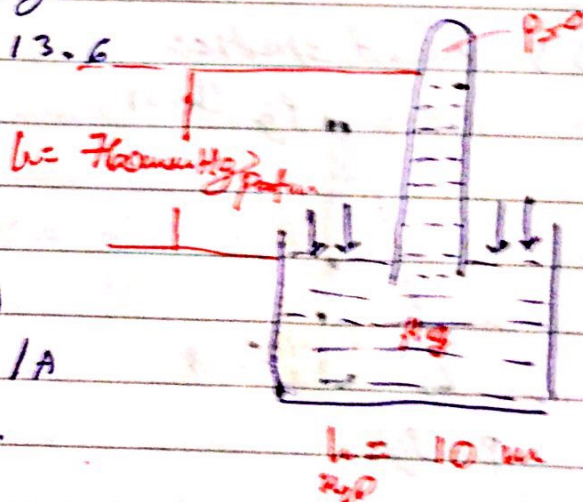
$$1 \text{ bar} = 10^5 Pa$$

$$= 100 kPa$$

$$S_{Hg} = \frac{\rho_{Hg}}{\rho_{H2O}} = 13.6$$

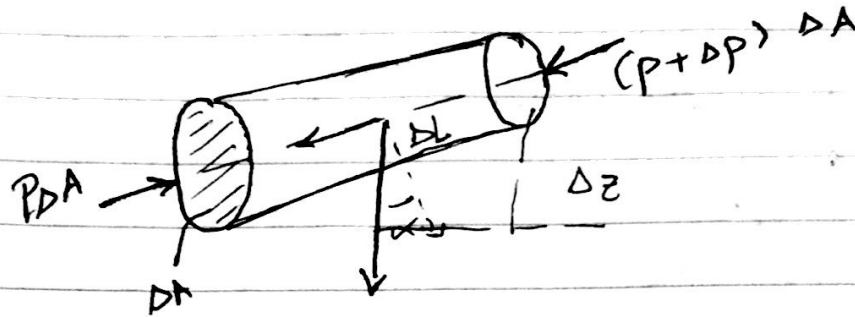
$$P = \frac{F}{A} = \frac{W}{A}$$

$$= \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h$$



$$P_{abs} = P_{atm} + P_{gauge}$$

- Pressure Variation with Elevation :-



$$\begin{aligned} W &= mg \\ &= \rho V g \\ &= \rho DA DL g \end{aligned}$$

$$\sum F_L = 0$$

$$P DA - (P + dP) DA - \rho g DA DL \sin \alpha = 0$$

$$-dP = \rho g DL$$

$$-dp = \rho g dL \frac{dz}{dL}$$

$$dp = -\rho g dz$$

$$\frac{dp}{dz} = -\rho g = -\gamma$$

$$p = -\rho g \int dz$$

$$dp = -\gamma dz$$

$$p = -\gamma z + \text{constant}$$

$$p + \gamma z = \text{constant}$$

$$p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{constant}$$

$$\rho = \text{constant} \quad \text{"The same fluid"}$$

$$\sin \alpha = \frac{dz}{dL}$$

→ Case of Ideal gas

$$pV = mRT \rightarrow \rho = \frac{P}{RT}$$

$$pV = RT \quad R = \frac{R}{M.w.t}$$

$$pV = nRT$$

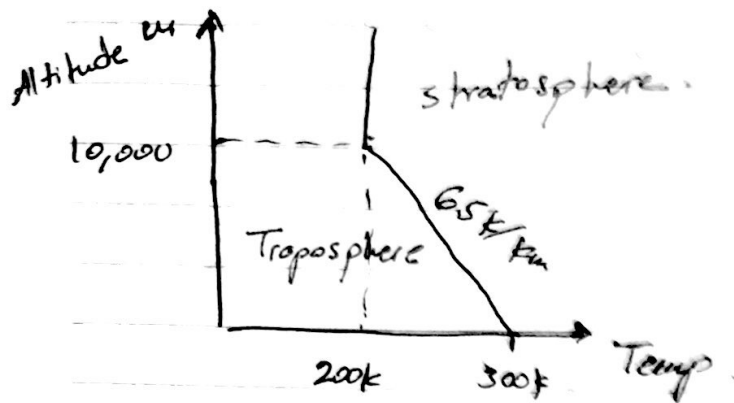
$$\frac{dp}{dz} = -\gamma = -\rho g$$

$$\frac{dp}{dz} = -\frac{P}{RT} g$$

$$\frac{dp}{p} = -\frac{g}{RT} dz$$

Lecture 5 27-6-2019

1) Variation of Temperature & pressure in the atmosphere.



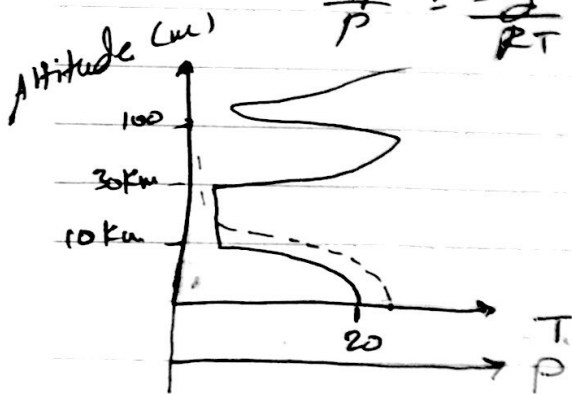
2) pressure Variation with elevation

$$\frac{dp}{dz} = -\rho = -\rho g$$

- For Ideal gas -

$$\frac{dp}{dz} = -\frac{\rho}{RT} g$$

$$\frac{dp}{p} = -\frac{g}{RT} dz$$



3) The atmosphere is divided in two layers

① Troposphere

0 → 10,769 m

- For this layer the temp. decreases at a rate (e.g. 6.5 K/km)

0 → 3600 Poles

0 → 18,000 equator

② stratosphere

z: 10,769 → 32.8 Km

- For this layer, T = constant

$$T = T_0 - \alpha (z - z_0)$$

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

$$\frac{dp}{p} = -\frac{g}{RT} dz$$

$$\frac{dp}{p} = \frac{-g}{R[T_0 - \alpha(z - z_0)]} dz$$

$$\frac{p}{p_0} = \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{g/\alpha R}$$

→ pressure variation in stratosphere

$$\frac{dp}{p} = -\int \frac{g}{RT} dz \quad T = \text{constant}$$

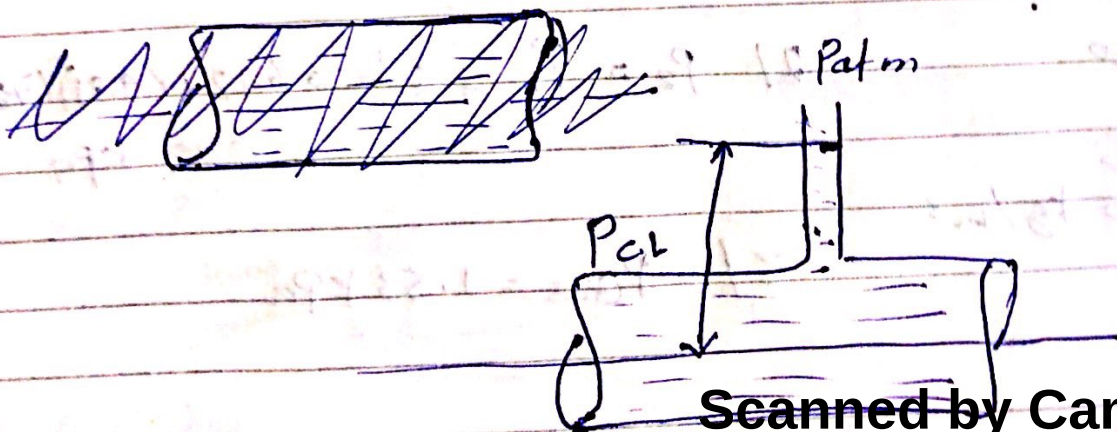
$$\ln p \Big|_{p_0}^p = -\frac{g}{RT} z + C$$

$$\frac{p}{p_0} = \exp\left(-\frac{(z - z_0)g}{RT}\right)$$

10 Pressure Measurements

- Barometry
- pressure gauge (Bourdon Tube gauge)
- pressure Transducers

- principle of Manometry

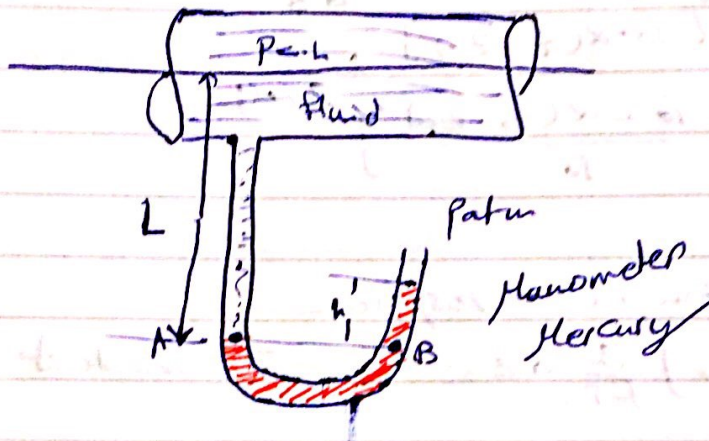


1) simple Manometer

$$P_{CL} = P_{atm} + \rho h$$

$$P_{CL \text{ gage}} = \rho h$$

2) Differential or U-tube manometer

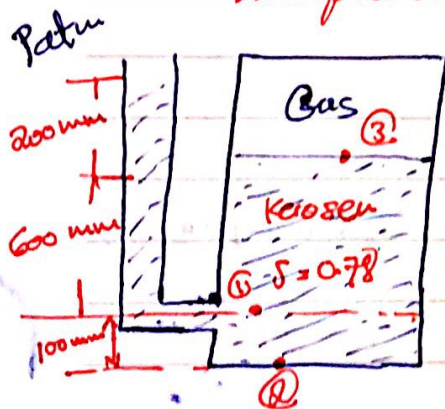


$$P_{CL} + \rho_f L = P_{atm} + \rho_m h$$

$$P_{CL \text{ gage}} = \rho_m h - \rho_f L$$

Example 8-

- Find
- Pressure at Tank entry
 - Pressure at Tank base
 - Pressure of the gas.



solution

$$1/ P_1 = P_{atm} + \rho h$$

$$P_{1 \text{ gage}} = (9.81)(1000)(0.78)(0.2)$$

$$= 6.12 \text{ kPa}$$

$$S = \frac{(\rho_g)_k}{(\rho_g)_{H_2O}}$$

$$0.78 = \frac{\rho_k}{1000}$$

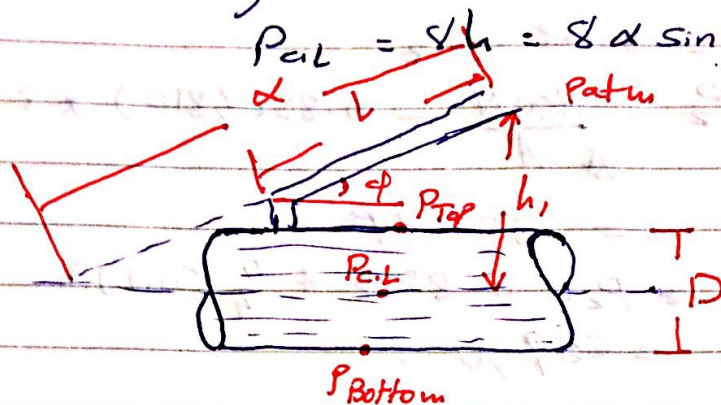
$$\rho_k = 780 \text{ kg/m}^3$$

$$2/ P_2 = P_1 + \rho h = 6.12 + (9.81)(780)(0.1)$$

$$= 6.89 \text{ kPa}$$

$$3/ P_{\text{Gas}} = 1.53 \text{ kPa}$$

- Inclined Manometers



Example 8-

$$D = 100 \text{ mm}$$

$$\alpha = 350 \text{ mm}$$

$$\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$$

$$\phi = 30^\circ$$

Find 1/ $P_{c.L}$ 2/ P_{Top} 3/ P_{Bottom}

Solution ↓

$$\begin{aligned} P_{c.L} &= \rho h_{c.L} = \rho \alpha \sin \phi \\ &= 9.81 \times 1000 \times 0.35 \sin 30 \\ &= 1.72 \text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\text{Top}} &= \rho h_{\text{Top}} \\ &= 9.81 \times 1000 \times (0.35 \sin 30 - 0.05) \\ &= 1.23 \text{ kPa} \end{aligned}$$

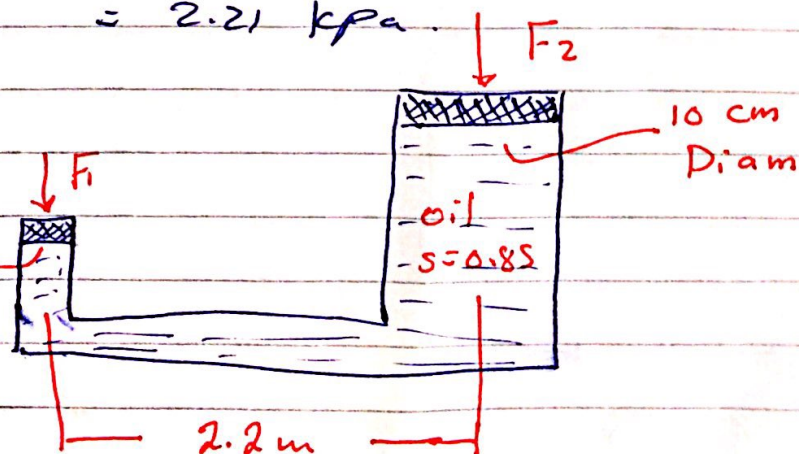
$$\begin{aligned} P_{\text{Bottom}} &= \rho h_{\text{Bottom}} \\ &= 9.81 \times 1000 \times 0.35 \sin 30 + 0.05 \\ &= 2.21 \text{ kPa} \end{aligned}$$

Example 8-

$$F_1 = 200 \text{ N}$$

$$F_2 = ??$$

10 cm
Diam



* Neglect Piston weight

No:

Date:

solution!

$$P_1 - \rho \Delta z = P_2$$

$$P_2 = \frac{200 \text{ N}}{A} - 0.85(9.810) \times 2$$

$$= 85.2 \text{ kPa}$$

$$F_2 = P_2 A = 85.2 + \frac{\pi}{4} (0.1)^2$$
$$= 669 \text{ N}$$

solution!

$$P_1 - \rho \Delta z = P_2$$

$$P_2 = \frac{200 \text{ N}}{A} - 0.85(9.810) \times 2$$

$$= 85.2 \text{ kPa}$$

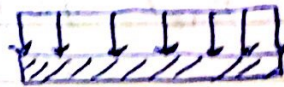
$$F_2 = P_2 A = 85.2 \times \frac{\pi}{4} (0.1)^2$$

$$= 669 \text{ N}$$

Hydrostatic forces on submerged surface

i) plane surface

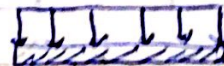
ii) curved surface



- plane surface :-

a) horizontal

b) vertical



$$p = \rho g h$$



c) Inclined

$$p = \rho g y \sin \alpha$$

$$\bar{y} = \int y dA$$

$$F = \int p dA$$

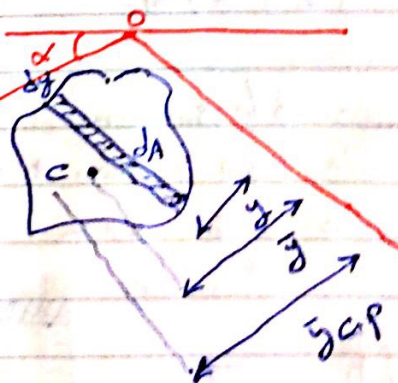
$$= \int \rho g y \sin \alpha dA$$

$$= \rho \sin \alpha \int y dA$$

$$= \rho \sin \alpha \bar{y} A$$

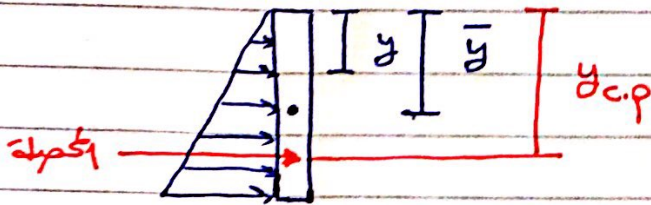
$$F = \rho (\bar{y} \sin \alpha) A$$

$$F = \bar{p} \cdot A$$



→ vertical location of line of action of resultant hydrostatic force

→ This line lies below the centroid because pressure increase with depth.



$$\begin{aligned} F \cdot y_{cp} &= \int dF \cdot y \\ &= \int p y \, dA \\ &= \int (\rho g y \sin \alpha) y \, dA \\ &= \rho g \sin \alpha \int y^2 \, dA \end{aligned}$$

→ second moment of Area (area moment of inertia)

$$F \cdot y_{cp} = \rho g \sin \alpha I_o$$

$$I_o = \bar{I} + \bar{y}^2 A \quad \text{"parallel axis theorem"}$$

$$F \cdot y_{cp} = \rho g \sin \alpha (\bar{I} + \bar{y}^2 A)$$

$$F = \bar{p} A = \rho g \sin \alpha (\bar{y} A)$$

$$\rho g \sin \alpha A \bar{y} \cdot y_{cp} = \rho g \sin \alpha (\bar{I} + \bar{y}^2 A)$$

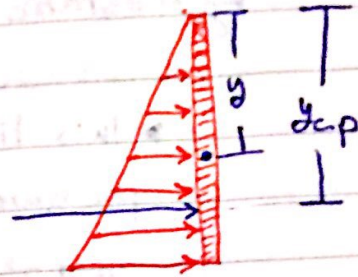
$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y} A} \quad \Rightarrow \quad y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A}$$

- Hydrostatic force on submerged surfaces

$$F = \bar{P} \cdot A$$

$$= (\gamma \bar{y}) \cdot A$$

$$\hookrightarrow \gamma \sin \alpha$$



$$y_{c.p} = \bar{y} = \frac{\bar{I}}{\bar{y}A}$$

example —

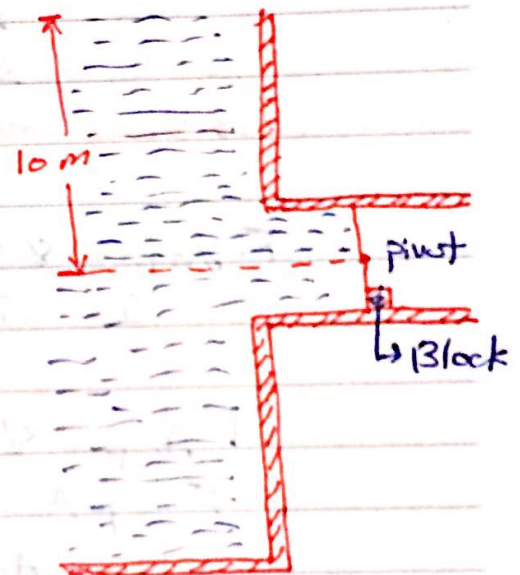
Find the force of the gate on the Block

$$F = \bar{P} \cdot A$$

$$= \gamma (\bar{y} \sin \alpha) A$$

$$= 9800 (10) (4 \times 4)$$

$$= 1.5696 \text{ MN}$$

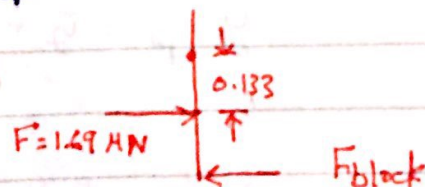
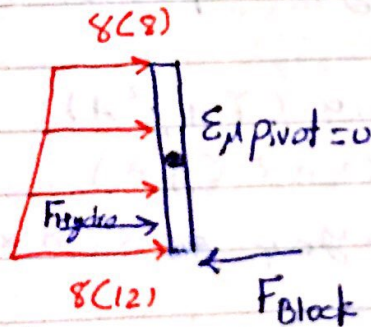


$$\gamma = 9800 \text{ N/m}^3$$

$$y_{c.p} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$

$$y_{c.p} - \bar{y} = \frac{\frac{1}{12} b h^3}{10 (4) (4)}$$

$$y_{c.p} - \bar{y} = \frac{\frac{1}{12} (4) (4)^3}{10 (4) (4)} = 0.133 \text{ m}$$



$$\Sigma M_{\text{pivot}} = 0$$

$$F_{\text{block}} \times 2 = F_{\text{Hydr}} \times 0.133$$

$$F_{\text{block}} = \frac{1.5696 \times 0.133}{2}$$

$$= 104.378 \text{ N}$$

* Example 2-

Find h when the gate is automatically open

$$F_{AB} = \bar{P}_{AB} \cdot A_{AB}$$

$$= \rho \bar{y}_{A1}$$

$$= \rho \frac{h}{2} (h(1))$$

↳ depth assume that depth F_{AB} = 1

$$= \rho \frac{h^2}{2}$$

$$F_{BC} = \bar{P}_{BC} \cdot A$$

$$= \rho \bar{y}_{A2}$$

$$= \rho (h) (4 \times 1)$$

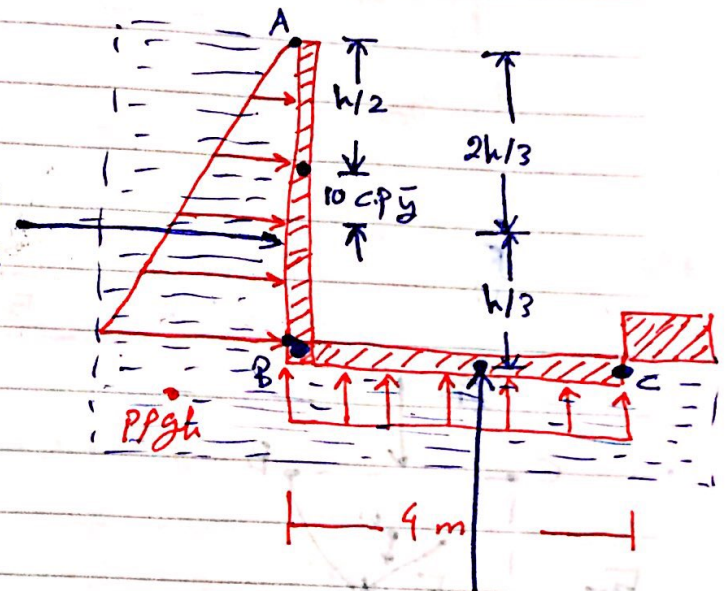
$$= 4 \rho h$$

$$\Sigma M_B = 0$$

$$\rho \frac{h^2}{2} (h/3) = 4 \rho h (2)$$

$$h^2 = 48$$

$$h = \sqrt{48} = 6.93 \text{ m}$$



Note

$$y_{c.p.} - \bar{y} = \frac{\bar{I}}{\bar{y} A}$$

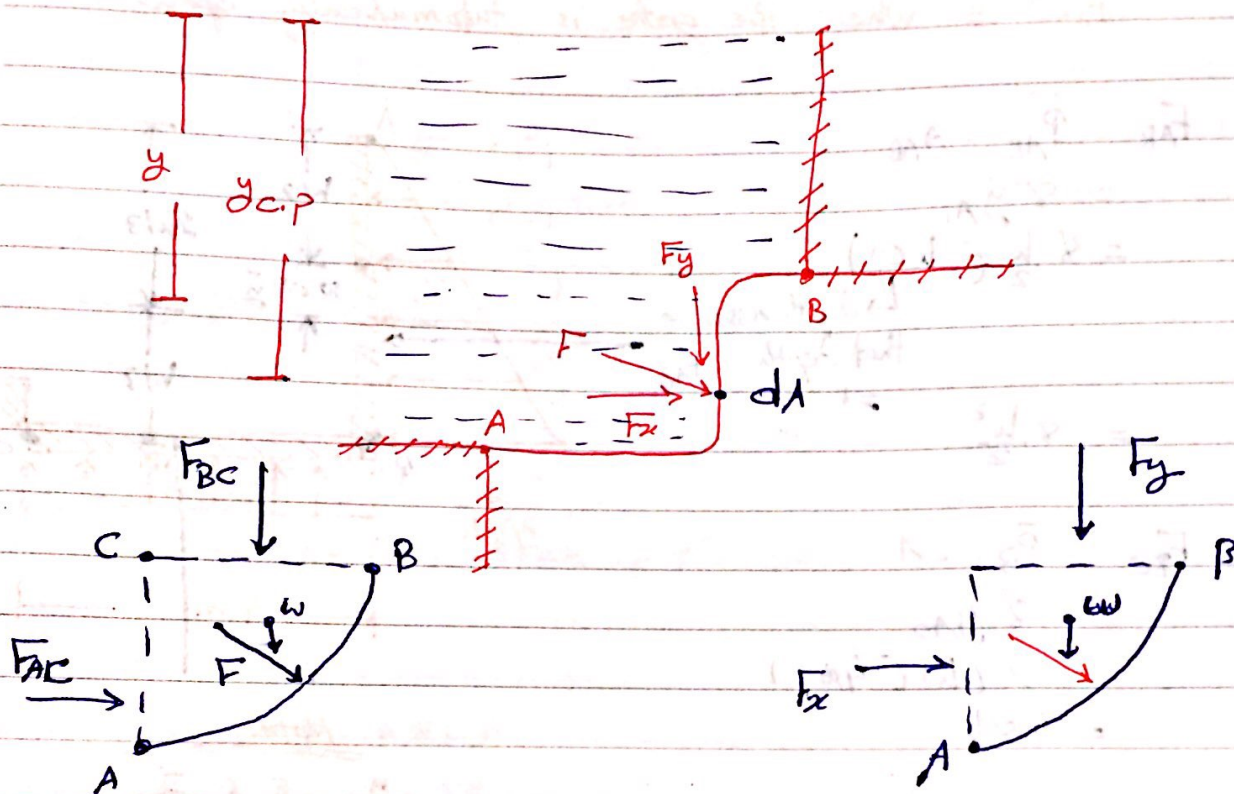
$$= \frac{\frac{1}{12} b h^3}{\frac{h}{2} (h \times b)}$$

$$= \frac{h}{6}$$

$$= \frac{h}{6}$$

$$y_{c.p.} - \frac{h}{2} = \frac{h}{6} \Rightarrow \frac{2h}{3}$$

→ Hydrostatics forces on curved surfaces



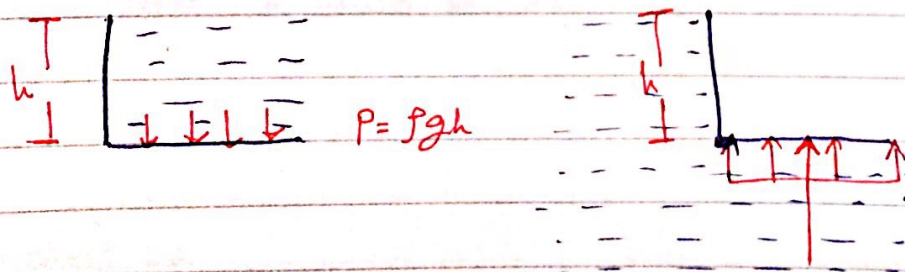
$$F_{AC} = F_{BC} = \bar{P}_v \cdot A_v$$

$$y_{cp} = \bar{y}_v = \frac{\bar{I}_v}{\bar{y}_v \cdot A_v}$$

$F_y = \gamma \cdot V$ "weight of liquid above a surface AB"

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\phi = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$



Example 3. Find →

F_v ? , line of action ?!

F_H ? , line of action ?!

$$\begin{aligned}
 F &= \bar{p} \cdot A \\
 &= 8\bar{y} \sin \alpha A \\
 &= 8\bar{y} A \\
 &= 8(1.5L) A
 \end{aligned}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A}$$

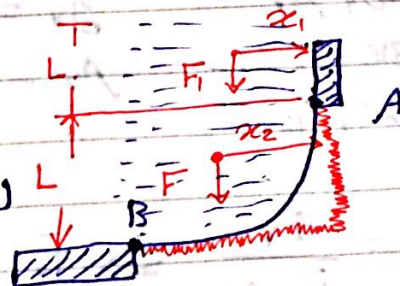
$$\begin{aligned}
 F &= PA \\
 &= 8y A \\
 &= 8(2L) A
 \end{aligned}$$

$$\omega = 8V$$

→ Another solution!

$$L = 1 \text{ m}$$

$$\begin{aligned}
 F_H = \bar{p} A &= 8\bar{y} A = 9,810(1.5)(1)(1) \\
 &= 14,715 \text{ N}
 \end{aligned}$$



$$y_{cp} \cdot \bar{y} = \frac{\bar{I}}{\bar{y} A} = \frac{\frac{1}{12} * (1 * 1^3)}{1.5 * 1 * 1} = 0.055$$

$$y_{cp} = 1.555 \text{ m}$$

$$F_v = 8V$$

$$= 9810 \left(1 * 1 * 1 + \frac{\pi}{4} (1^2 * 1) \right) = 17515 \text{ N}$$

Take Moments-

$$F_y \cdot x = F_1 x_1 + F_2 x_2$$

$$x = \frac{F_1 x_1 + F_2 x_2}{F_y}$$

$$\int_0^1 F_2 dx = 9810 \int_0^1 \sqrt{1-x^2} \propto d\alpha \frac{dx}{d\alpha}$$

ہیلے کتبھا لکھو

$$x = \frac{1 * 9810 \int_0^1 \sqrt{1-x^2} x dx}{17515} = 0.467 \text{ m}$$

$$F_{12} = \sqrt{F_x^2 + F_y^2} = \sqrt{14715^2 + 17515^2} = 22876 \text{ N}$$

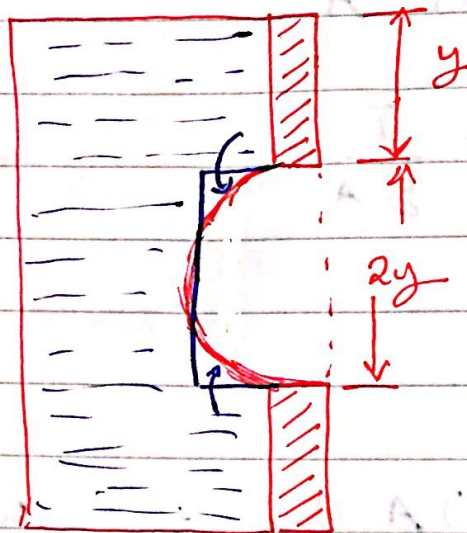
$$\phi = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 40^\circ$$

Example 3-

$$y = 1 \text{ m}$$

$$y_2 = 2 \text{ m}$$

$$T = 10^\circ \text{C}$$



$$F_H = 8 \bar{y}_v A_v$$

$$= 9810 (2 \times \pi)$$

$$= 61064 \text{ kN}$$

$$\bar{y}_v = 1 + 1 = 2 \text{ m}$$

$$A_v = \pi r^2 = \pi$$

$$y_{cp} - \bar{y} = \frac{\bar{I}_v}{\bar{y}_v A_v} \Rightarrow \bar{I}_v = \frac{2 \pi r^4}{8}$$

$$= 0.125 \text{ m}$$

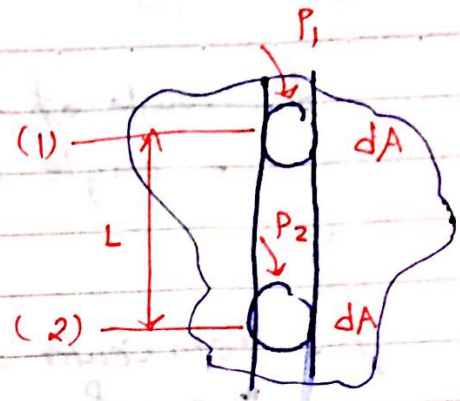
$$F_v = 8v = 9810 \left[\frac{4}{3} \pi r^3 \times \frac{1}{2} \right] = 20555 \text{ kN}$$

Buoyancy

Difference in force between
(1) & (2) = dF

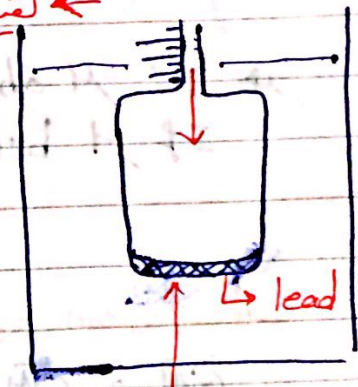
$$\begin{aligned} dF &= (P_2 - P_1) dA \\ &= \rho g L dA \\ &= \rho g dV \end{aligned}$$

$$F = \rho g V$$

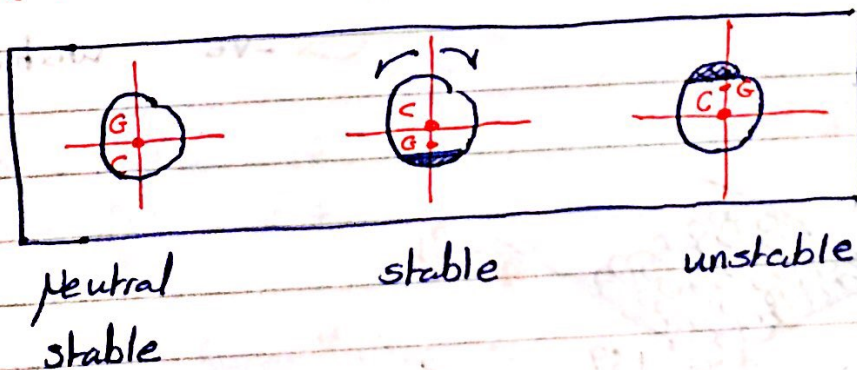


Hydrometry ← لقياس كثافة السائل

★ The hydrometer will float deeper or shallower depending on the specific weight of the liquid

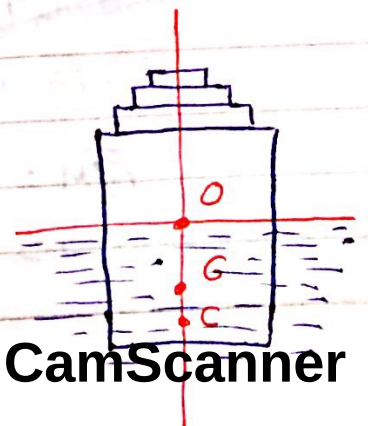
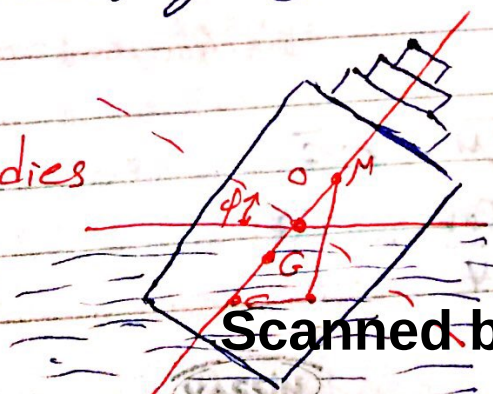


(1) Stability of Immersed or floating body
(*) Immersed bodies



★ If the center of gravity below Centroid "stable"
★ If the center of gravity above centroid "unstable"

(*) Floating bodies



ϕ = angle of tilt of the ship
= angle of heel

M = Meta center = the point of intersection of the lines of action of the buoyant force before and after the tilt (heel).

$\overline{GM}$:- Metacentric height

If M above G $\rightarrow$ stable, i.e. $\overline{GM}$ is positive

If M below G $\rightarrow$ un-stable, i.e. $\overline{GM}$ is negative

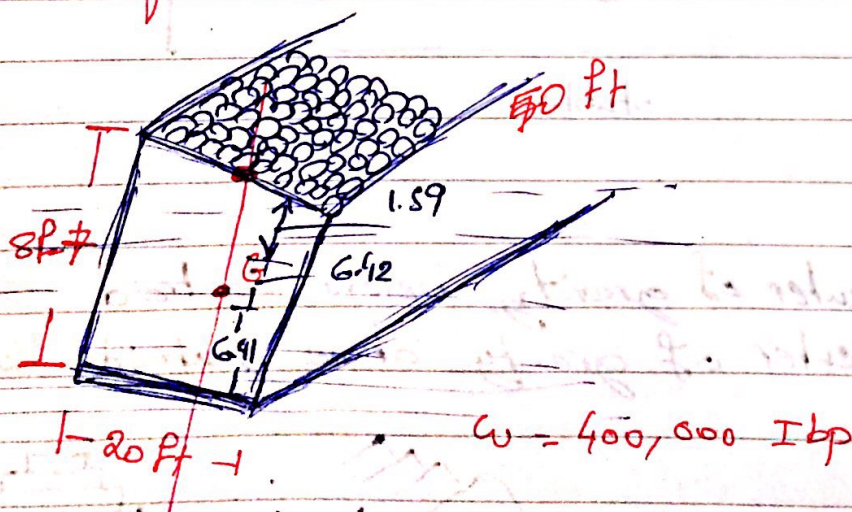
Calculation of $\overline{GM}$

$$\overline{GM} = \overline{CM} - \overline{CG}$$

$$\overline{GM} = \frac{I_{ov}}{V} - \overline{CG}$$

$\rightarrow$ +ve stable
 $\rightarrow$ 0 Neutral stable
 $\rightarrow$ -ve unstable

Example 3:-



$$\begin{aligned}\overline{GM} &= \overline{CM} - \overline{CG} \\ &= \frac{I_{ov}}{V} - \overline{CG}\end{aligned}$$

$$400,000 = 8 \rho V = 62.4 (50 \times 20 \times h)$$

$$h = 6.41 \text{ ft} / 8 \text{ ft}$$

$$V = 20 \times 60 \times 6.41$$

$$\overline{CG} = \frac{6.41}{2} + 1.59$$

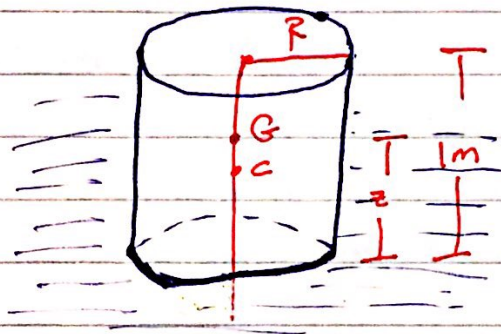
$$I = \frac{bh^3}{12} = \frac{50 \times 20^3}{12} \quad \text{or} \quad \frac{20 \times 50^3}{12}$$

$$GM = \left(\frac{50(20)^3}{12} \times \frac{1}{6.41 \times 50 \times 20} - \left(8 - \frac{6.41}{2} \right) \right)$$

$$= +0.4 \text{ ft}$$

will float stable

↳ Example



$$D = 1 \text{ m}$$

$$\rho = 8000 \text{ N/m}^3 \text{ (wood)}$$

$$\overline{GM} = \overline{CM} - \overline{CG}$$

$$= \frac{I_{ov}}{V} - \overline{CG}$$

$$\rho_{\text{wood}} V = \rho_{\text{water}} V$$

$$8000 (\pi 12^2 L) = 9810 (\pi 12^2 h)$$

$$h = 0.8155 \text{ m}$$

$$C \text{ from bottom} = 0.4077 \text{ m}$$

$$G \text{ from bottom} = 0.5 \text{ m}$$

$$\overline{CG} = 0.5 - 0.4077 = 0.0922 \text{ m}$$

$$\overline{GM} = \frac{(\pi 12^4 / 4)}{(\pi 12^2 h)} - 0.0922 = 0.077 - 0.0922 = -0.0152$$

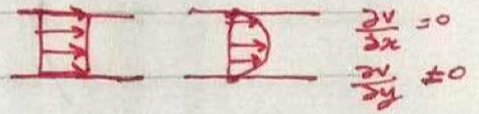
↳ The Block is unstable with axis vertical ↓

CH 4 8- Fluids in Motion.

Bernoulli Eq. and Pressure variation.

classification of flow

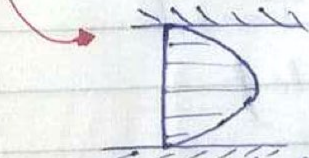
- * uniform flow $\partial v / \partial s = 0$
- * Non-Uniform flow $\partial v / \partial s \neq 0$
- * steady flow $\partial v / \partial t = 0$
- * unsteady flow $\partial v / \partial t \neq 0$



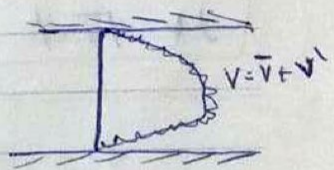
- ① uniform steady flow طابق مع السرعة
- ② uniform unsteady flow تغير مع الزمن
- ③ non-uniform steady flow
- ④ non-uniform unsteady flow

laminar & Turbulent flow

- laminar flow 3- ① Flow has a very smooth appearance
- ② Flowing in layer above each other
 - ③ No mixing between layers



- Turbulent flow 3- ① Violent motion
- ② Eddy motion
 - ③ High mixing



- Transition flow 3- flow between laminar & Turbulent.

$$Tu = \frac{v'}{\bar{v}}$$

Re = Reynolds Number

$$Re = \frac{\rho V L}{\mu}$$

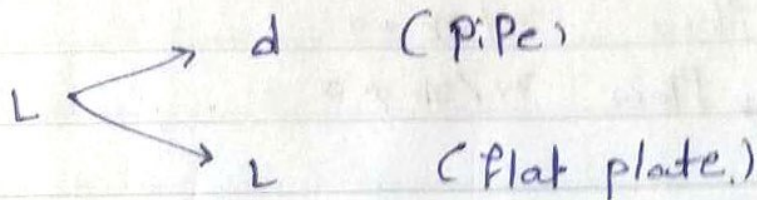
where 3.

ρ = density

v = flow velocity

L = characteristic length

μ = viscosity



$$Re = \frac{\text{Inertia Force}}{\text{Viscous force}}$$
$$= \frac{\rho v^2 L}{\mu v / L^2} = \rho v L / \mu$$

→ Pipe Flow:-

If $Re \leq 2300 \Rightarrow$ laminar flow

$2300 < Re < 4000 \Rightarrow$ Transition

$Re \geq 4000 \Rightarrow$ Turbulent

→ Flat plate:-

If $Re \leq 500,000 \Rightarrow$ laminar flow

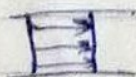
$Re > 500,000 \Rightarrow$ Turbulent

→ Hydraulic Diam

$$DA = \frac{4 \times \text{Flow Area}}{\text{parameter}}$$

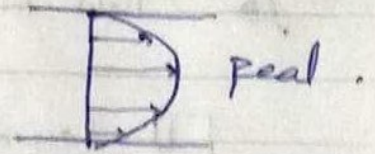
→ Real & Ideal fluid

- Ideal fluid: No viscosity, No shear stress



No friction.

→ Real fluid is μ very small



(*) Compressible & Incompressible flow (*)

→ $\rho \neq \text{constant}$

→ $\rho = \text{constant}$

→ One —, Two — and Three dimensional flow

1-D → If the parameters such as velocity and pressure vary along the direction of flow.

2-D

3-D

→ Rotational & Irrotational flow —

Rotational flow → Fluid particles rotate about its center as it flows.

Irrotational flow

→ Velocity is —

$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

$$= u \hat{i} + v \hat{j} + w \hat{k}$$

$$\vec{V} = V_r \hat{r} + V_\phi \hat{\phi} + V_z \hat{k}$$

In General

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

Two methods exist to determine $\mathbf{v}$

[1] Lagrangian Approach

Select fluid Particle and follow the motion of this particle at any instant.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt}(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

[2] Eulerian Approach

Select any point (x, y, z) in the flow field and observe the fluid flow passing through that point.

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

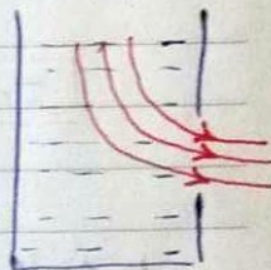
$$V = \sqrt{u^2 + v^2 + w^2}$$

→ streamlines and flow patterns.

construction of these lines to indicate the speed & direction of flow is called flow patterns.

→ stream lines, streaklines, Path lines

streamlines: is a line that is everywhere tangent to the velocity field.



c.g. 2-D flow: $\frac{dy}{dx} = \frac{v}{u}$

It is used in ~~analytical~~ analytical work.

→ A streak lines: it consists of all particles in a flow that have previously passed through a common point

It is more of laborating tool then an analytical tool.

A path line is a line traced out by a given particle as it flows from one point to another

- The path line is a lagrangian concept.

⊗ For steady flow $\Rightarrow$ Path lines, streak lines & streamlines are the same

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

where:-

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

$$\vec{a} =$$

$$\begin{aligned} a_x &= \frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ a_y &= \frac{dv}{dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ a_z &= \frac{dw}{dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \end{aligned}$$

$$V = V(s, t) \hat{e}_t$$

$$a = \frac{dV}{dt} = \left(\frac{dV}{dt} \right) \hat{e}_t + V \frac{d\hat{e}_t}{dt}$$

$$a_t = \frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s} \frac{ds}{dt} = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial s}$$

$$a = a_t \hat{e}_t + a_n \hat{e}_n$$

$$= \left(\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial s} \right) \hat{e}_t + \frac{V^2}{r} \hat{e}_r$$

Example 3. $u = -u_0 \left(1 - \frac{r_0^3}{x^3}\right)$

$a_x = ??$

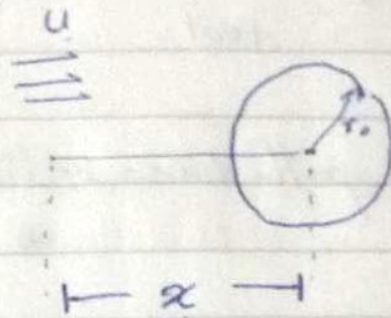
Solution!

$$a_x = \cancel{\frac{\partial u}{\partial t}} + u \frac{\partial u}{\partial x} + \cancel{v \frac{\partial u}{\partial y}} + \cancel{\omega \frac{\partial u}{\partial z}}$$

$$a_x = u \frac{\partial u}{\partial x}$$

$$= -u_0 \left(1 - \frac{r_0^3}{x^3}\right) \cdot \left(-3u_0 \frac{r_0^3}{x^4}\right)$$

$$3u_0 \left(1 - \frac{r_0^3}{x^3}\right) \left(u_0 \frac{r_0^3}{x^4}\right)$$



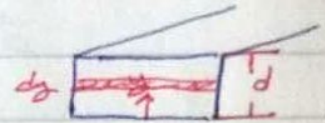
Example 2:- $u = u_{max} \left(\frac{y}{d}\right)^n$

where $n = 1/6$

$u_{max} = 3 \text{ m/s}$

$d = 1.2 \text{ m}$

$a_x = ??$ $\bar{v}_{avg} = ??$



Solution!

$$a_x = \cancel{\frac{\partial u}{\partial t}} + u \frac{\partial u}{\partial x} + \cancel{v \frac{\partial u}{\partial y}} + \cancel{\omega \frac{\partial u}{\partial z}}$$

Flow rate or Discharge Q :-

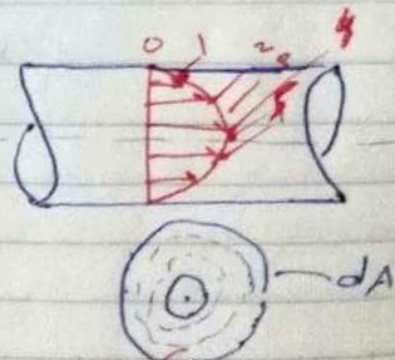
$$Q = \bar{A} \cdot \bar{v} \rightarrow \text{Average velocity.}$$

$$Q = \int_A v dA$$

$$\dot{m} = \rho Q = \rho \int_A v dA$$

$\rightarrow$ mass flow rate.

r	v (m/s)
0.5	5
1.5	4
2.5	3
3.5	2
4.5	1



$$Q = \int_A V dA$$

$$= \int_0^R V (2\pi r) dr = \sum_{i=1}^N (2\pi r_i) V_i \Delta r_i$$

$$V_{avg} = \frac{Q}{A} = \frac{\int V dA}{A}$$

$$Q = \int_0^d V_{max} \left(\frac{y}{d} \right)^{1/6} (1 dy)$$

$$= \int_0^{1.2} 3 \left(\frac{y}{1.2} \right)^{1/6} dy = 3.09 \text{ m}^3/\text{s}$$

$$\bar{V} = \frac{Q}{A} = \frac{3.09}{1 \times 1.2} = 2.57 \text{ m/s}$$

4.5 Euler Equation

(*) Pressure variation in flowing fluid :-

1. Pressure varies with elevation due to gravity
($\Delta P = \rho g \Delta z$)

2. Acceleration on fluids $\sum F = ma$

$$P_1 A_1 \rightarrow P_1 > P_2 \leftarrow P_2 A_2$$

3. Viscous resistance and shear stress

$$F_{sh.} = \tau A = \mu \left(\frac{dv}{dy} \right) A$$

(*) Pressure Variation due to weight & Acceleration :-

$$\sum F_L = ma_L$$

$$P \Delta A - \left(P + \frac{\partial P}{\partial L} \Delta L \right) \Delta A - \Delta W \sin \alpha = ma_L$$

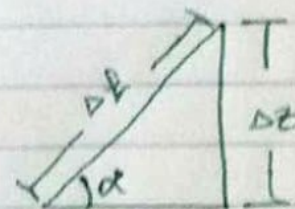
$$P \Delta A - \left(P + \frac{\partial P}{\partial L} \Delta L \right) \Delta A - \rho g \Delta A \Delta L \frac{\partial z}{\partial L} = \underbrace{\rho \Delta A \Delta L}_{\text{mass} = m} a_L$$

$$-\frac{\partial P}{\partial L} \Delta L \Delta A - \rho \Delta A \Delta L \frac{\partial z}{\partial L} = \rho \Delta A \Delta L a_L$$

$$-\frac{\partial P}{\partial L} - \rho \frac{\partial z}{\partial L} = \rho a_L$$

$$\left(-\frac{\partial}{\partial L} (P + \rho z) \right) = \rho a_L$$

Euler equation



$$\sin \alpha = \frac{\Delta z}{\Delta L}$$

(*) special cases :-

1. Velocity = constant

$$a_L = 0$$

$$-\frac{\partial}{\partial L}(P + \gamma z) = 0$$

$$P + \gamma z = \text{constant}$$

$$P_1 + \gamma z_1 = P_2 + \gamma z_2$$

$$\Delta P = -\gamma \Delta z$$

2. Uniform acceleration:-

*(Patm) high level (A'B') is the reference *

(a) select L-direction as shown.

$$-\frac{\partial}{\partial L}(P + \gamma z) = \rho a_L$$

$$\Rightarrow \frac{\partial P}{\partial L} = 0$$

$$-\frac{\partial}{\partial L}(\gamma z) = \rho a_L$$

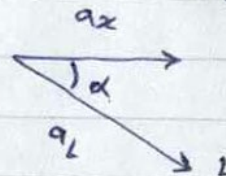
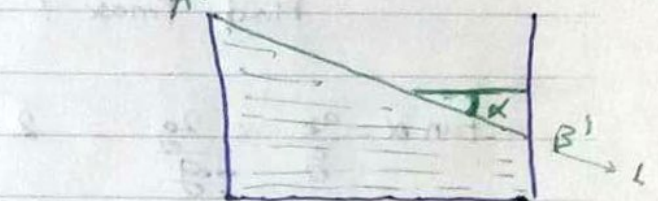
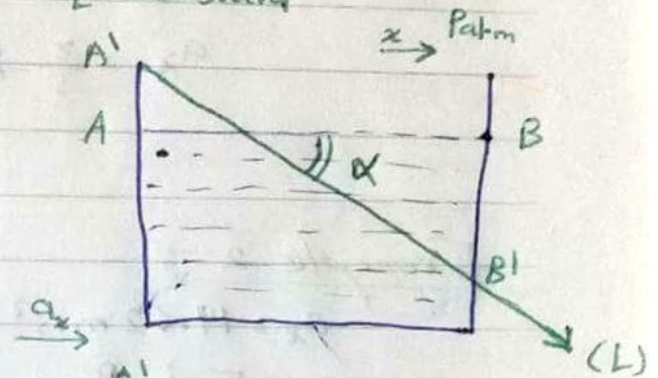
$$-\gamma \frac{\partial z}{\partial L} = \rho a_L$$

$$-\gamma \frac{\partial z}{\partial L} = \rho a_x \cos \alpha$$

$$\rho g \sin \alpha = \rho a_x \cos \alpha$$

$$\Rightarrow \tan \alpha = \frac{a_x}{g}$$

$a_L = \text{constant}$



$$a_L = a_x \cos \alpha$$

$$-\frac{\partial z}{\partial L} = \sin \alpha$$

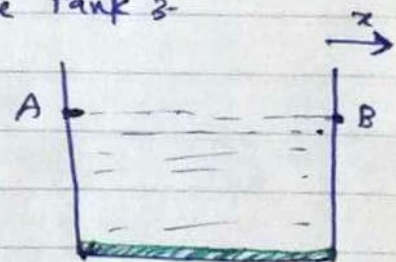
(b) select L-direction as the bottom of the tank:-

$$\Delta P = \rho g \Delta z$$

$$\Delta P = \rho a_L \Delta L$$

$$= \rho a_x \Delta z$$

$$= \rho a_y \Delta y$$



$$-\frac{\partial}{\partial L}(P + \gamma z) = \rho a_L$$

$$-\frac{\partial}{\partial x}(P + \gamma z) = \rho a_x \Rightarrow -\frac{\partial P}{\partial x} = \rho a_x$$

→ The Pressure must decrease in the direction of acc.

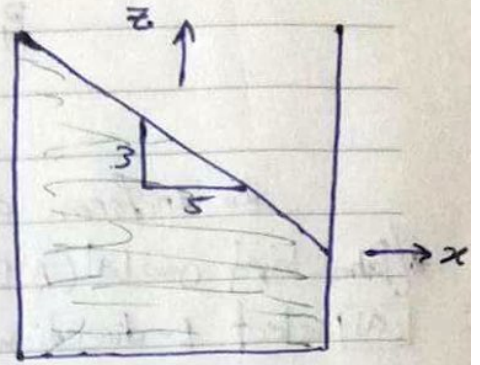
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Example 1 :-

Find the acc. of the tank -

$$\tan \alpha = \frac{a_x}{g} = \frac{3}{5} = \frac{a_x}{g}$$

$$a_x = \frac{3}{5} g$$

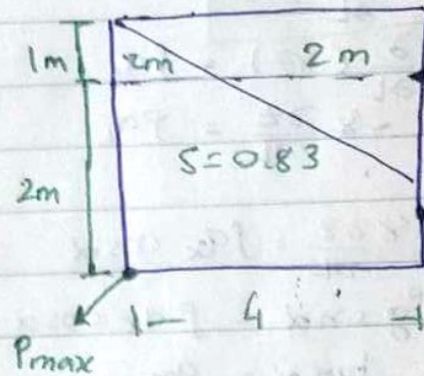


Example 2 :-

$$a_x = 19.62 \text{ m/s}^2 = 2g$$

Find $P_{\max}$?

$$\tan \alpha = \frac{a_x}{g} = \frac{2g}{g} = 2$$



$$= \frac{1}{g} \left[\frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) + \frac{\partial}{\partial x} \left(\frac{v^2}{2} \right) \right]$$

$$\frac{\partial}{\partial x} (h) = \frac{1}{g} \frac{\partial}{\partial x} \left[\frac{u^2}{2} + \frac{v^2}{2} \right]$$

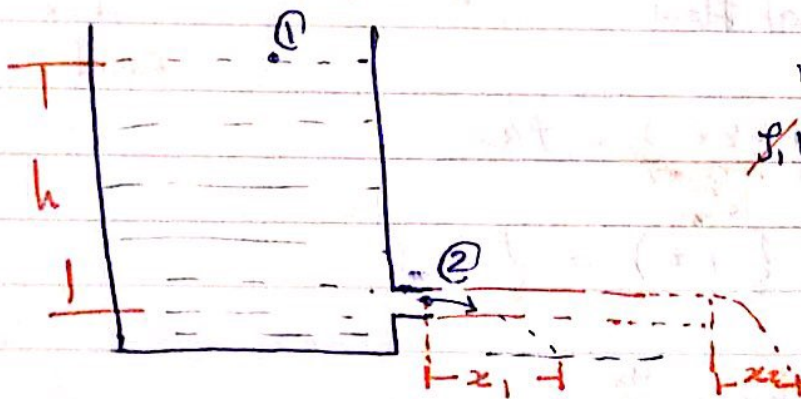
$$-\frac{\partial}{\partial x} \left(\frac{p}{\rho} + z \right) = \frac{1}{g} \frac{\partial}{\partial x} \left[\frac{v^2}{2} \right]$$

$$\frac{\partial}{\partial x} \left[\frac{p}{\rho} + \frac{v^2}{2g} + z \right] = 0$$

$$\frac{p}{\rho} + \frac{v^2}{2g} + z = \text{constant}$$

$$\frac{p_1}{\rho} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2g} + z_2$$

2-D
Steady
Inviscid
Irrotational



$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$V_1 = \frac{V_2 A_2}{A_1}$$

$$0 + 0 + h = 0 + \frac{v^2}{2g} + 0$$

$$v_2 = \sqrt{2gh}$$

$$0 + \frac{\left(v_2 \frac{A_2}{A_1} \right)^2}{2g} + h = \frac{v_2^2}{2g}$$

$$h = \frac{v_2^2}{2g} \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right]$$

$$v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1} \right)^2}}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 + h_f \quad \text{h losses}$$

h_f $\begin{cases} \text{major losses} \\ \text{minor losses} \end{cases}$

~~$h_{f \text{ major}} = f \frac{L}{D} \frac{V^2}{2g}$~~

$h_{f \text{ major}} = f \frac{L}{D} \frac{V^2}{2g}$

Friction Factor

$$h_{f \text{ minor}} = \sum K \frac{V^2}{2g}$$

Application of the Bernoulli equation

① stagnation tube

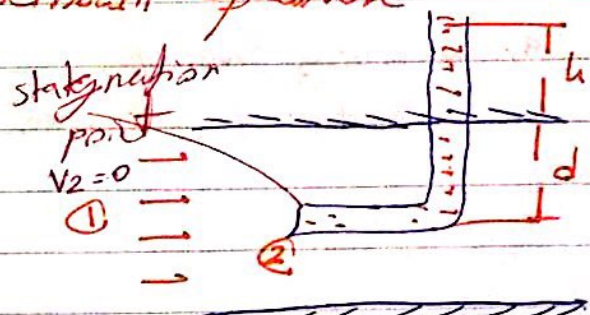
$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho} \Rightarrow V_1 = \sqrt{2gh}$$

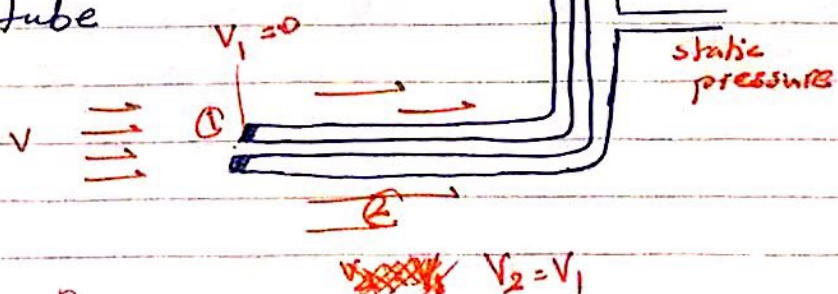
$$P_1 = \rho g d$$

$$P_2 = \rho g (d + h)$$

$$\Delta P = \rho g h$$



② pitot tube



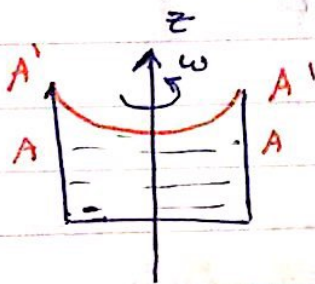
$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

$$V_2 = \sqrt{\frac{2g}{\gamma} (p_1 - p_2)}$$

$$V_2 = \sqrt{2g (h_1 - h_2)}$$



⇒ Pure Rotation of tank of liquid



* constant Rotation

$$-\frac{\partial}{\partial r} (P + \gamma z) = \rho a_r$$

where $v_r = \omega r$

$$a_r = v^2/r = \omega^2 r = -a_r$$

$$-\frac{\partial}{\partial r} (P + \gamma z) = \rho (-\omega^2 r)$$

$a_r \rightarrow$ away from the center of Curvature,

$$\frac{\partial}{\partial r} (P + \gamma z) = \rho \omega^2 r$$

$$P + \gamma z - \rho \omega^2 r = \text{constant}$$

$$P + \gamma z - \rho \frac{v^2}{r} = \text{constant}$$

2D Bernoulli Equation

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

1D Euler's Equation

$$-\frac{\partial}{\partial x} \left(\frac{P}{\gamma} + z \right) = \int a_x$$

1D Pure Rotation

$$\frac{P_1}{\gamma} - \frac{V_1^2}{r} + z_1 = \frac{P_2}{\gamma} - \frac{V_2^2}{r} + z_2$$

Example 2-

$$S = 0.8$$

$$D = 1 \text{ ft}$$

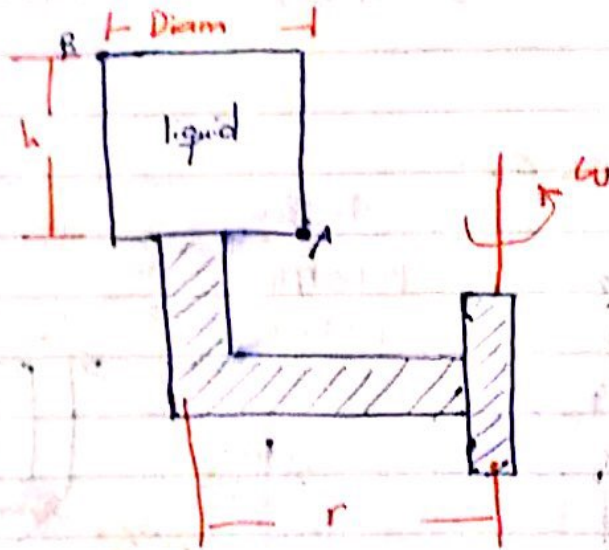
$$h = 1 \text{ ft}$$

$$r = 2 \text{ ft}$$

$$V_A = 20 \text{ ft/s}$$

$$P_A = 25 \text{ psf}$$

$$P_B = ??$$



solution

$$V_B = \omega r$$

$$V_A = \omega r \Rightarrow \omega = V_A / r \quad \text{or} \quad \omega = \frac{20 \text{ ft/s}}{2 \text{ ft}} = 10 \text{ rad/s}$$

$$V_B = \omega r = 10 \times 2 = 20 \text{ ft/s}$$

$$P_B = 526 \text{ psf}$$

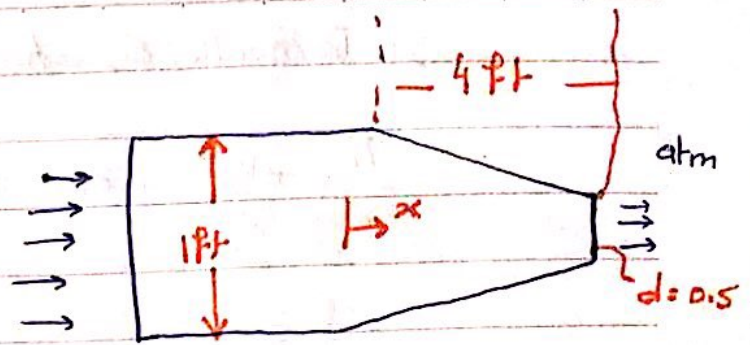
Date: _____

Example 2 3-

$$V = \frac{2t}{(1 - 0.5 \frac{x}{L})^2}$$

$$Q = 20 \text{ ft}^3/\text{s}$$

$$P_{\text{pipe}} = ??$$



Solution ↓

$$Q = VA \Rightarrow V = Q/A$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

$$Q_1 = V_1 A_1 = V_2 A_2$$

$$V_1 = Q/A_1 = 25.5 \text{ ft/s}$$

$$V_2 = Q/A_2 = 101.9 \text{ ft/s}$$

Example 3 3-

P 4.70

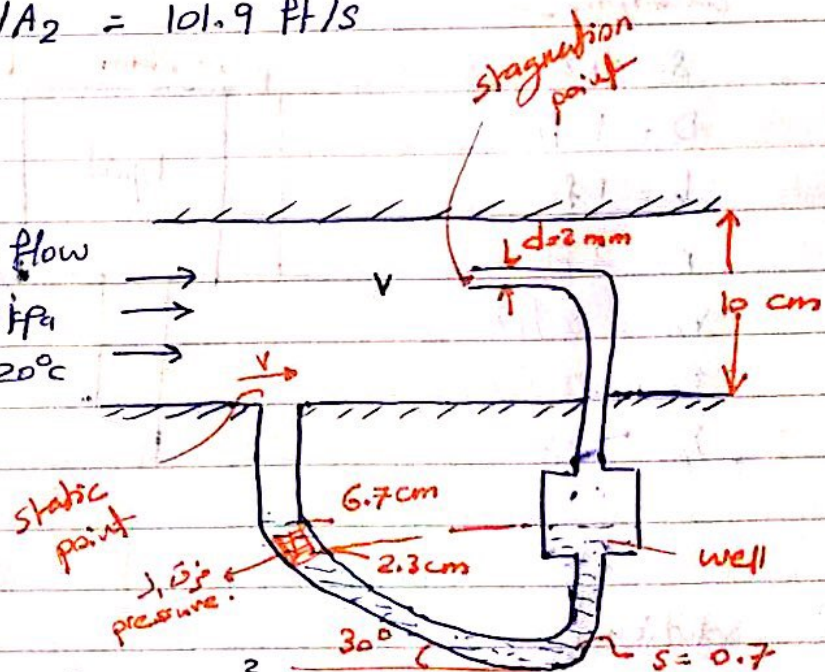
$$\dot{m} = ??$$

$$V = ??$$

Air flow

$$P = 150 \text{ kPa}$$

$$T = 20^\circ\text{C}$$



Solution ↓

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

$$\frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g}$$

$$V_2 = \sqrt{2 \frac{\Delta P}{\rho}}$$

$$\rho = \frac{P}{RT} = \frac{150}{0.287 \times 293.15} = 1.78 \frac{\text{kg}}{\text{m}^3}$$

$$\Delta h = (6.7 - 2.3) \times 10^{-2} \times \sin 80^\circ$$

$$= 4.4 \times 10^{-2} \times \frac{1}{2}$$

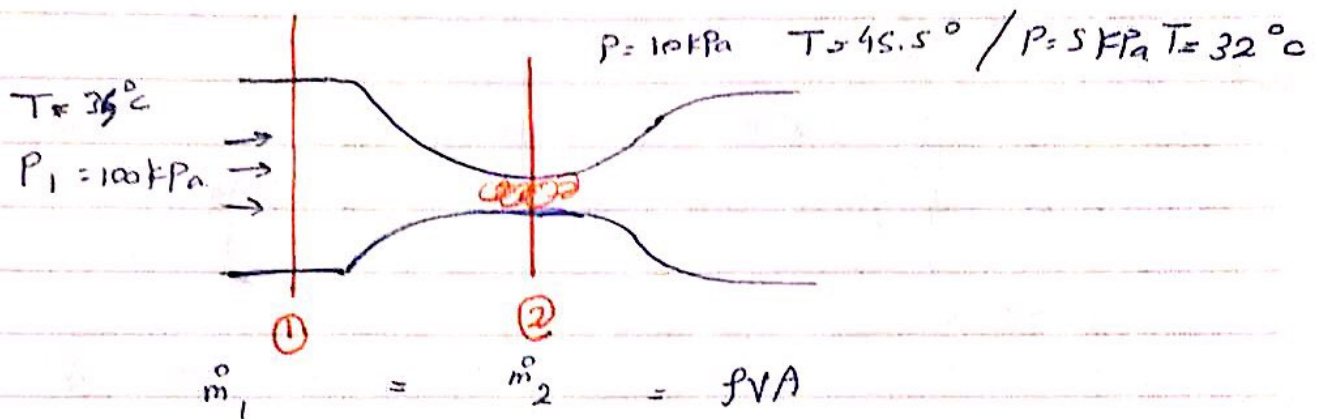
$$= 2.2 \text{ cm}$$

$$\rho_{\text{liquid}} = 6.7 \times 1000 = 700 \text{ kg/m}^3$$

$$V = 13 \text{ m/s}$$

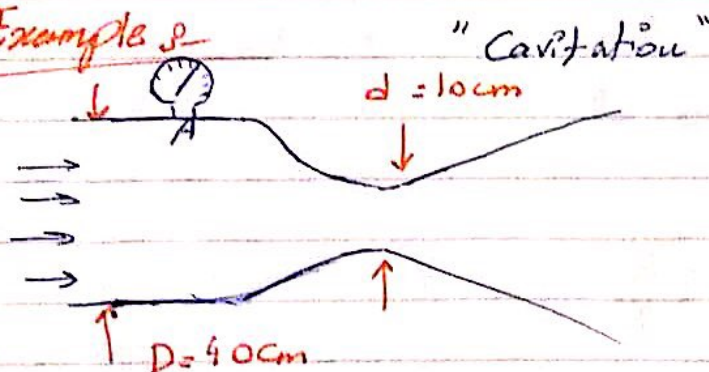
$$\dot{m} = \rho A V = 1.78 \times \pi (0.1)^2 \times 13$$

$$= 0.18 \text{ kg/s}$$



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Example 8



$$P_A = 100 \text{ kPa (gauge)}$$

$$P_{\text{atm}} = 100 \text{ kPa}$$

$$T_{\text{water}} = 10^\circ \text{C}$$

$$P_{\text{sat}} = 1.23 \text{ kPa}$$

$$Q = ??$$

Solution

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

$$P_B = 1.23 \text{ kPa}$$

$$P_{B \text{ gauge}} = -98.77$$

$$P_A = 100 + 100 = 200 \text{ kPa}$$

$$P_{A \text{ gauge}} = 100 \text{ kPa}$$

$$V_A A_A = Q = V_B A_B$$

$$V_A = \frac{Q}{A_A}$$

$$V_B = \frac{Q}{A_B}$$

$$Q = 0.157 \text{ m}^3/\text{s}$$

$$A_A = \frac{\pi}{4} D_A^2$$

$$A_B = \frac{\pi}{4} D_B^2$$

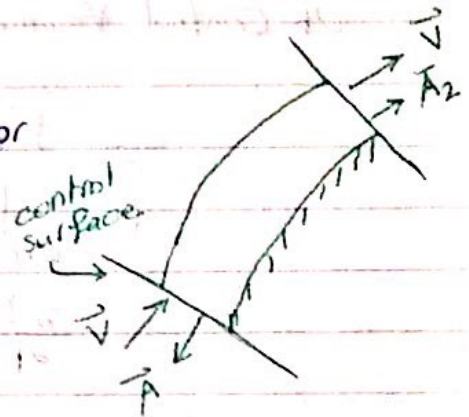
Control Volume approach & continuity Equation.

Control Volume

3 conservation principles

- CH5 1) Mass conservation principle $\rightarrow$ continuity Eq. $\rightarrow$ 1 eq.
- CH6 2) Momentum conservation principle $\rightarrow$ M-S Eq. $\rightarrow$ 3 eq.
- CH7 3) Energy conservation principle $\rightarrow$ Energy Eq. $\rightarrow$ 1 eq.

Area (A) is considered as a vector $\vec{A}$ where $\vec{A}$ is always pointing out of the CV surface.



$$Q_v = \dot{V} = \text{volume flow rate}$$

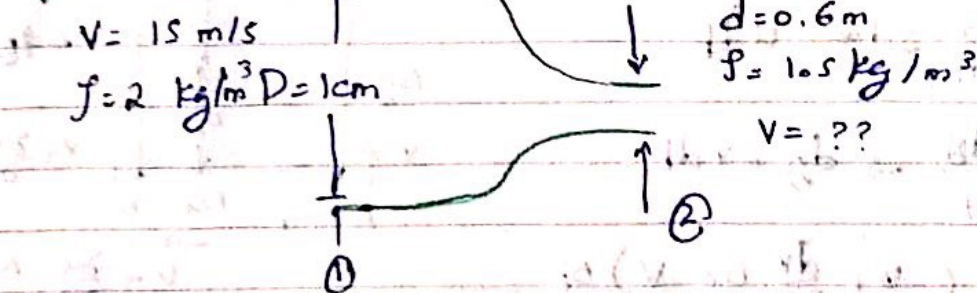
$$Q_1 = \vec{V}_1 \cdot \vec{A}_1 = V_1 A_1$$

$$Q_2 = \vec{V}_2 \cdot \vec{A}_2 = V_2 A_2$$

Flow rate out - Flow rate in

$$\begin{aligned} &= \vec{V}_2 \cdot \vec{A}_2 + \vec{V}_1 \cdot \vec{A}_1 \\ &= V_2 A_2 (1) + V_1 A_1 (-1) = V_2 A_2 - V_1 A_1 \\ &= \sum_{CS} \vec{V} \cdot \vec{A} \end{aligned}$$

- Example -



$$\begin{aligned} \rho_1 Q_1 &= \rho_2 Q_2 \\ \dot{m}_1 &= \dot{m}_2 \end{aligned}$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$V_2 = \frac{\rho_1}{\rho_2} \cdot \frac{A_1}{A_2} \cdot V_1$$

$$V_2 = \frac{2}{1.05} \cdot \frac{1.02^2}{0.6^2} \cdot 15 =$$

→ If the flow is unsteady, i.e. if the flow rate in $\neq$ flow rate out, then

$$\sum_{CS} \rho \vec{V} \cdot \vec{A} = \sum \text{Flowrate} - \sum \text{Flowrate in}$$

$$= -\frac{d}{dt} \int_{CV} \rho dV$$

$$\text{or} \quad \int_{CS} \rho \vec{V} \cdot d\vec{A} = -\frac{d}{dt} \int_{CV} \rho dV$$

→ Control Volume Approach

$$\sum_{CS} \rho \vec{V} \cdot \vec{A} = 0 \quad (\text{steady state})$$

$$\left(\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \right) \div \text{Volume}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

→ If the flow is steady:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

→ Rotation of fluid

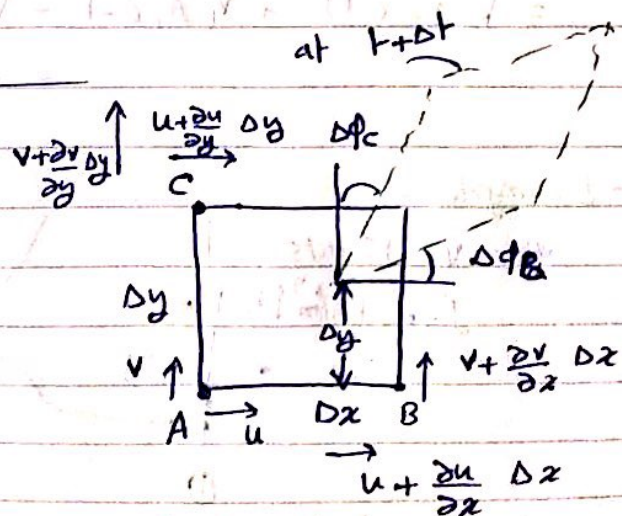
$$\Delta \phi_B = (\Delta y_B - \Delta y_A) / \Delta x$$

$$* u = \frac{dx}{dt} \Rightarrow dx = u dt$$

$$* v = \frac{dy}{dt} \Rightarrow dy = v dt$$

$$\Delta \phi_B = \left(v + \frac{\partial v}{\partial x} \Delta x - v \right) dt$$

$$= \frac{\partial v}{\partial x} \Delta x \cdot dt$$



$$\omega_{AB} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \phi_R}{\Delta t}$$

$$\omega_{AB} = \frac{\partial v}{\partial x}$$

→ similarity

$$\omega_{AC} = \frac{\partial v}{\partial y}$$

$$\omega_{AZ} = \frac{\omega_{AB} + \omega_{AC}}{2}$$

$$\omega_{AZ} = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial u}{\partial z} \right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right)$$

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

∴ ~~Vorticity~~ Vorticity = $\zeta = 2\omega$

$$\zeta = \left(\frac{\partial w}{\partial y} - \frac{\partial u}{\partial z} \right) \hat{i} + \left(\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} \right) \hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$$

∴ Vortex :-

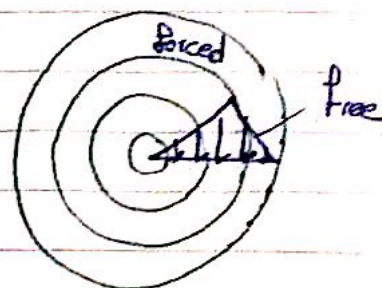
Two Types

1) Forced Vortex

$$v = k_1 r$$

2) Free Vortex

$$v = k_2 / r$$



Tornadoes and whirlpools are a combination of forced and free vortex

→ Principle —

Consider a C.V. for a given direction

$$\sum_{\text{out}} \text{Moment out of the C.V.} - \sum_{\text{in}} \text{Moment in the C.V.} + \text{Rate at which Momentum is stored within the C.V.} = \sum F_{\text{external}}$$

→ using cartesian coordinates

$$x\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{ox} - \sum_{\text{in}} \dot{m}_i V_{ix} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_x dV \right) = \sum F_x$$

$$y\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{oy} - \sum_{\text{in}} \dot{m}_i V_{iy} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_y dV \right) = \sum F_y$$

$$z\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{oz} - \sum_{\text{in}} \dot{m}_i V_{iz} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_z dV \right) = \sum F_z$$

$$\left(\sum_{\text{out}} \dot{m}_o V_o - \sum_{\text{in}} \dot{m}_i V_i \right) + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V dV \right) = \sum F$$

$$\int_{\text{C.V.}} \frac{\partial \rho}{\partial t} dV + \int_{\text{C.V.}} \frac{\partial (\rho u_i)}{\partial x_i} dV = 0$$

$$\frac{\partial m}{\partial t} + \underbrace{\sum_{\text{C.S.}} \rho u_i A_i}_{\dot{m}_o - \dot{m}_i} = 0$$

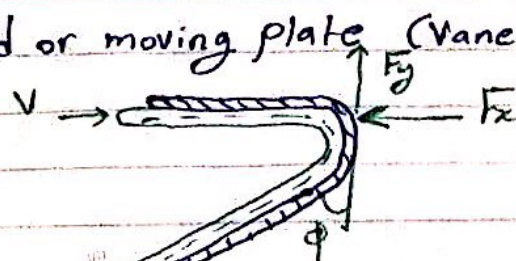
$$\dot{m}_o - \dot{m}_i = - \frac{dm}{dt}$$

$$\int \left[\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x_i} \right] = - \frac{\partial p}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_j$$

→ Applications :-

[1] Jet deflected by fixed or moving plate (Vane)

V = Velocity of the jet.



x-direction:

steady mass $\rightarrow$ $\frac{d}{dt} \int_{c.v} \rho dV$

$$\sum \dot{m}_0 V_{0x} - \sum \dot{m}_i V_{ix} + \frac{d}{dt} \left(\int_{c.v} V_x \rho dV \right) = \sum F_x$$

$$-\dot{m} V \sin \phi - \dot{m} V = -F_x$$

$$F_x = \dot{m} V (1 + \sin \phi)$$

$$\dot{m}_0 = \dot{m}_i = \dot{m}$$

$$\rho_0 V_0 A_0 = \rho_i V_i A_i$$

$$V_0 = V_i$$

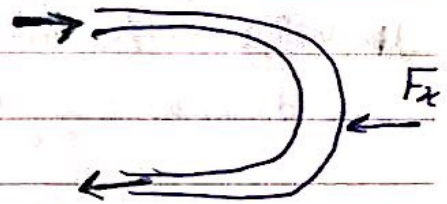
y-direction:

$$\sum \dot{m}_0 V_{0y} - \sum \dot{m}_i V_{iy} + \frac{d}{dt} \left(\int_{c.v} V_y \rho dV \right) = \sum F_y$$

$$-\dot{m} V \cos \phi - 0 = F_y$$

$$F_y = -\dot{m} V \cos \phi$$

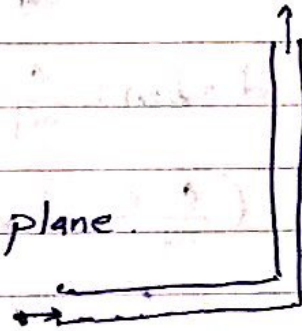
"if the vane is in horizontal plane"



$$F_x = 2\dot{m} V$$

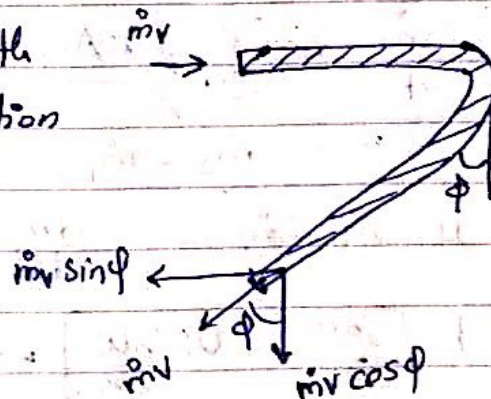
→ if the vane is in vertical plane

$$-\dot{m} V \cos \phi = F_y - \text{Weight of liquid + plane}$$



$$F_x = \dot{m} V$$

→ if the plate is moving with velocity v in the x -direction
Relative Velocity $(V-v)$.



Q6-32

$d = 6 \text{ cm}$, $T = 10^\circ \text{C}$, $v = 20 \text{ m/s}$

$V_p = 7 \text{ m/s}$

* neglect friction

Solution ↓

$$\dot{m} = \rho V A$$

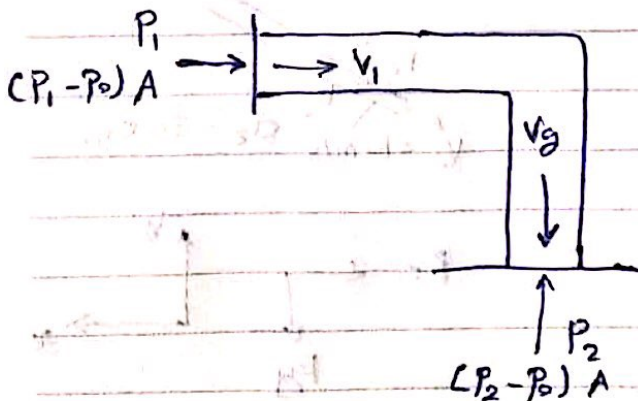
$$= (1000) \left(\frac{\pi}{4} \right) (0.06)^2 (20 - 7)$$

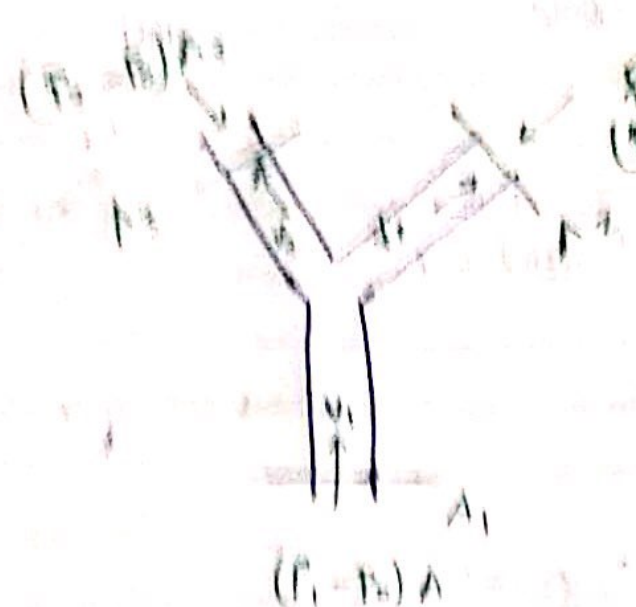
$$\begin{aligned} \dot{m} V_{rel.} &= \dot{m} V_{rel.} - V_p \\ &= 20 - 7 \end{aligned}$$

$$\begin{aligned} \sum \dot{m}_0 V_{0x} - \sum \dot{m}_i V_{ix} + \frac{\partial}{\partial t} \int V_x dV &= \sum F_x \\ - \dot{m} (V_j - V_p) - \dot{m} (V_j - V_p) &= -F_x \\ F_x &= 815.3 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum \dot{m}_0 V_{0y} - \sum \dot{m}_i V_{iy} + \frac{\partial}{\partial t} \int V_y dV &= \sum F_y \\ (36.76)(13) \sin 45^\circ - 0 &= F_y \\ F_y &= (36.76)(13) \sin 45^\circ \end{aligned}$$

Flow in Bends ($P_0 = P_{atm}$)





gauge pressure
(since the Patm distributed everywhere).



Assume

$$\rho = 1000 \text{ kg/m}^3$$

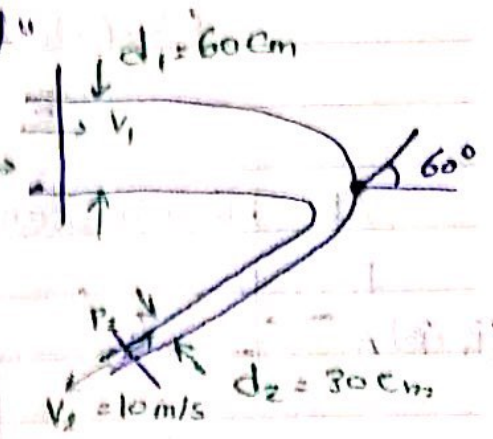
Horizontal Plane "weight doesn't affect"

dx = direction x

$$\int \rho V_1 dx = \int \rho V_2 dx + \int \rho V_2 dx = \rho F_{2x}$$

$$= \rho V_2 \cos 60^\circ = \rho V_1$$

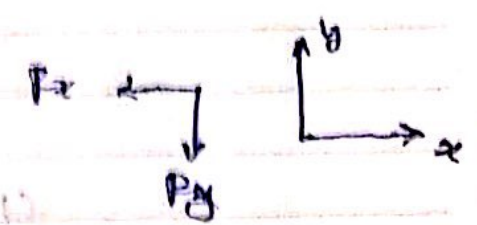
$$P_2 + \rho A_2 + P_2 A_2 \cos 60^\circ = P_1 A_1$$



$$\rho_1 = \rho_2, V_1 = V_2$$

$$(\rho V A)_1 = (\rho V A)_2$$

$$V_1 = \frac{V_2 A_2}{A_1}$$



"as you like"

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

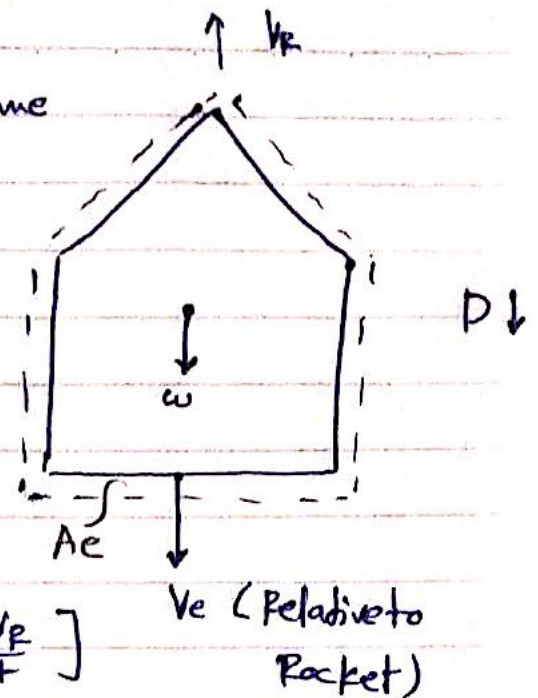
(P1, A1 and P2) use this equation =

M = mass of the Rocket at anytime

$$\sum_{out} \dot{m} V_{Dy} - \sum_{in} \dot{m} V_{iy} + \frac{\partial}{\partial t} \int \rho V_y dV = \sum F_y$$

$$\hookrightarrow \dot{m} = \rho V_e A_e$$

$$-(\rho V_e A_e)(V_e - V_R) - 0 + \frac{\partial}{\partial t} [V_R M] \\ - D - W + (P_e - P_o) A_e$$



$$\rho V_e A_e V_R - \rho V_e^2 A_e + \left[V_R \frac{\partial M}{\partial t} + M \frac{\partial V_R}{\partial t} \right]$$

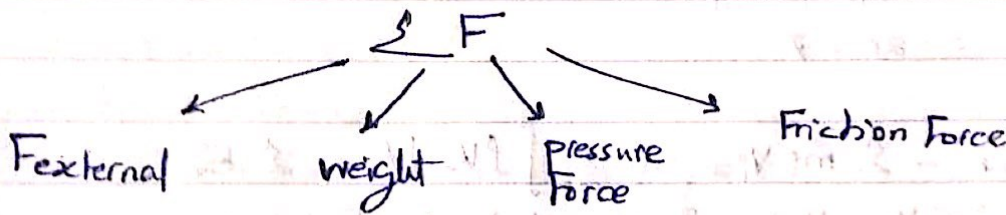
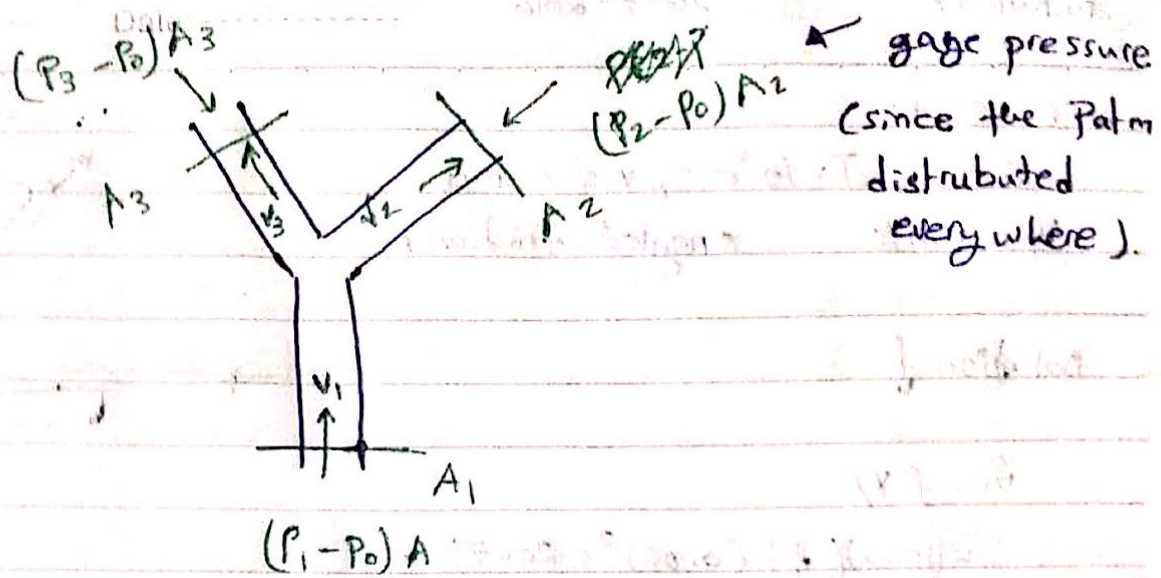
$$= -D - W + (P_e - P_o) A_e$$

$$\hookrightarrow M \frac{\partial V_R}{\partial t} = -D - W + \underbrace{(P_e - P_o) A_e + \rho V_e^2 A_e}_{\text{Thrust of Rocket engine}}$$

Thrust of Rocket engine

$$\frac{dM}{dt} = \dot{m}_o - \dot{m}_i$$

$$-D - W + T = M \frac{dV_R}{dt}$$



* 6-45 *

$$\rho = 1000 \text{ kg/m}^3$$

Horizontal Plane "weight doesn't affect"

↳ x-direction :-

$$\sum \dot{m}_o V_{ox} - \sum \dot{m}_i V_{ix} + \int \rho V_x dV = \sum F_x$$

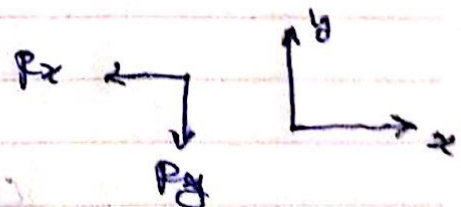
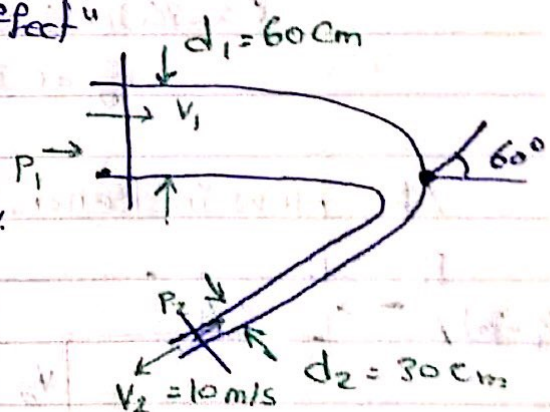
$$- \dot{m} V_2 \cos 60 - \dot{m} V_1 =$$

$$R_x + P_1 A_1 + P_2 A_2 \cos 60 + \text{zero}$$

$$\dot{m}_1 = \dot{m}_2, \quad V_1 = ?$$

$$(\rho V A)_1 = (\rho V A)_2$$

$$V_1 = \frac{V_2 A_2}{A_1}$$



"as you like !"

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

→ (To find P_1) use this equation -

monday
25/7/2016

Chapter 6 Momentum Principle

Momentum = mass x velocity = $m \vec{V} = m(u\hat{i} + v\hat{j} + w\hat{k})$

Principle: consider a C.V., for a given direction

$$\sum_{\text{out}} \text{Moment out of the C.V.} - \sum \text{Moment in the C.V.} + \text{Rate at which momentum is stored with the C.V.}$$

$$= \sum F_{\text{external}}$$

$$\frac{d}{dt}(mV) = m \frac{dv}{dt} + v \frac{dm}{dt}$$

$$= \dot{m}V + m \frac{dv}{dt}$$

Using Cartesian coordinates

$$x\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{ox} - \sum_{\text{in}} \dot{m}_i V_{ix} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_x dV \right) = \sum F_x$$

$$y\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{oy} - \sum_{\text{in}} \dot{m}_i V_{iy} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_y dV \right) = \sum F_y$$

$$z\text{-direction: } \sum_{\text{out}} \dot{m}_o V_{oz} - \sum_{\text{in}} \dot{m}_i V_{iz} + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V_z dV \right) = \sum F_z$$

$$\left(\sum_{\text{out}} \dot{m}_o V_o - \sum_{\text{in}} \dot{m}_i V_i \right) + \frac{d}{dt} \left(\int_{\text{C.V.}} \rho V dV \right) = \sum F$$

$$\int_{\text{C.V.}} \frac{d\rho}{dt} dV + \int_{\text{C.V.}} \frac{d(\rho V_i)}{dt} dV = 0$$

$$\frac{d}{dt} \int_{\text{C.V.}} dV + \int_{\text{C.S.}} \rho V_i dA_i = 0$$

$$\frac{dM}{dt} + \sum_{\text{C.S.}} \rho V_i A_i = 0 \quad \boxed{\dot{m}_o - \dot{m}_i = -\frac{dM}{dt}}$$

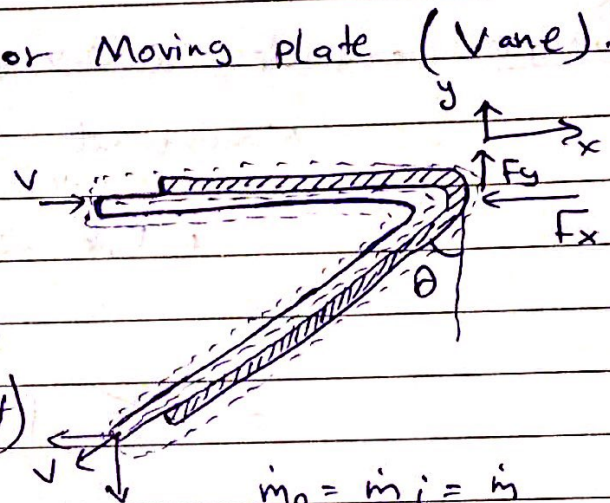
Five Apple

$$\rho \left[\frac{d u_i}{dt} + u_i \frac{d u_i}{d x_i} \right] = - \frac{d P}{d x_i} - \frac{d \tau_{ij}}{d x_j} + \rho g_i$$

Applications:

① Jet deflected by fixed or Moving plate (Vane).

V = velocity of the jet.



x-direction:

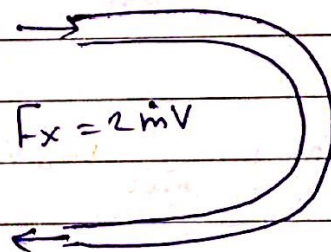
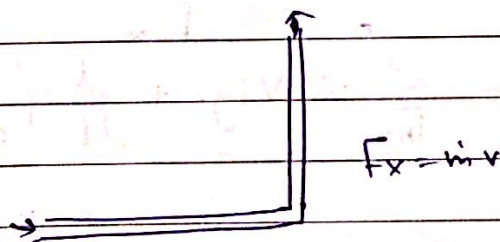
$$\sum \dot{m}_0 V_{0x} - \sum \dot{m}_i V_{ix} + \frac{d}{dt} \left(\int_{cv} V_x \rho dV \right) = \sum F_x$$

$$\dot{m}_0 = \dot{m}_i = \dot{m}$$

$$\rho_0 V_0 A_0 = \rho_i V_i A_i$$

$$-\dot{m} V \sin \theta - \dot{m} V = -F_x$$

$$F_x = \dot{m} V (1 + \sin \theta)$$



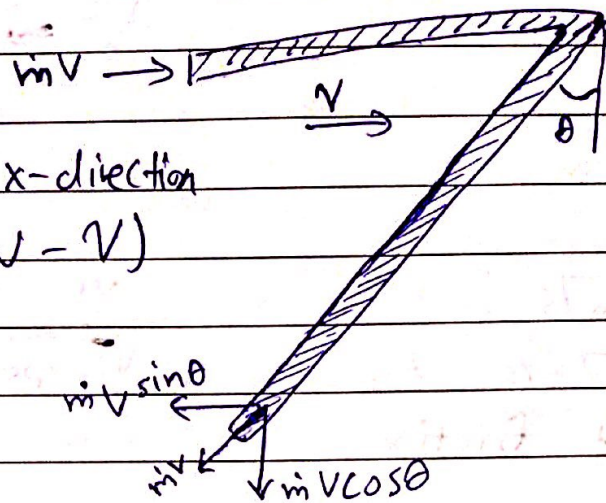
y-direction: $\sum \dot{m}_0 V_{0y} - \sum \dot{m}_i V_{iy} + \frac{d}{dt} \left(\int_{cv} V_y \rho dV \right) = \sum F_y$

$$-\dot{m} V \cos \theta - 0 = F_y, \quad F_y = -\dot{m} V \cos \theta \quad (\text{if the vane is in horizontal plane})$$

* if the ^{vane} ~~plane~~ is in vertical plane:

$$-\dot{m} V \cos \theta = F_y - \text{weight of liquid \& plate}$$

If the plate is moving
with velocity V in the x-direction
Relative velocity ($V - V$)



Tuesday
26/7/2016

Prob: 6.29:

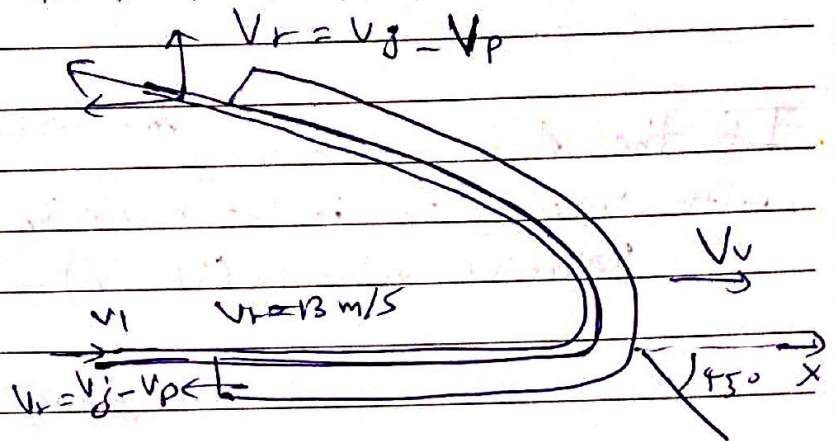
$$d = 6 \text{ cm}$$

$$T = 10^\circ \text{C}$$

$$V = 20 \text{ m/s}$$

$$V_p = 7 \text{ m/s}$$

* neglect friction



$$\sum \dot{m}_o V_{ox} - \sum \dot{m}_i V_{ix} + \frac{d}{dt} \left(\int_{c.v} V_x \rho dV \right) = \sum F_x$$

~~$$36.76 (20) - 36.76 (20 \cos 45) = -F_x$$~~

$$F_x = 41.98 \text{ N}$$

$$\dot{m} = \rho V A = 1000 \times (20 - 7) \times \frac{\pi}{4} (0.06)^2 = 36.76 \text{ kg/s}$$

$$- \dot{m} (V_1 - V_p) \cos 45 = \dot{m} (V - V_p)$$

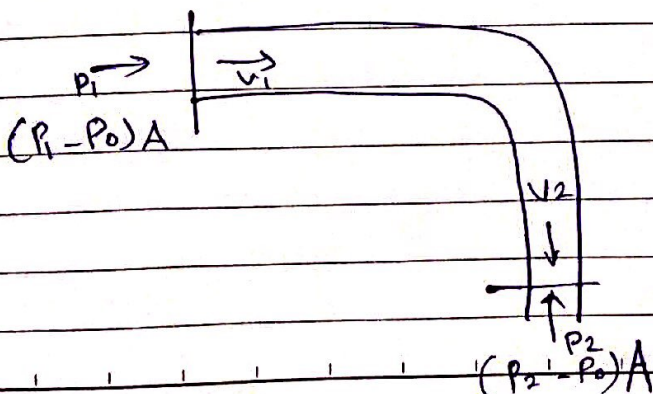
$$F_x = 815.3 \text{ N}$$

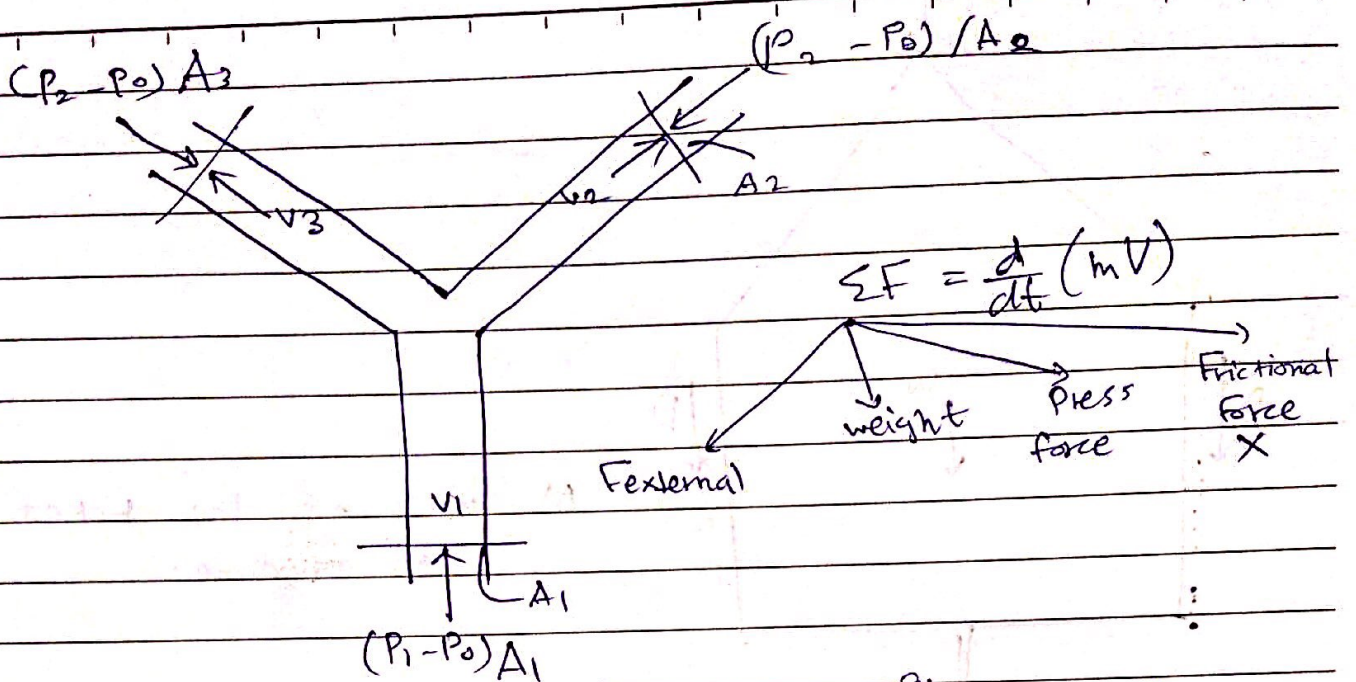
$$\sum \dot{m}_o V_{oy} - \sum \dot{m}_i V_{iy} + \frac{d}{dt} \left(\int_{c.v} V_y \rho dV \right)$$

$$\dot{m}_o V_{oy} - 0 = F_y \quad \dot{m}_i = \dot{m}_o$$

$$36.76 \times 13 \sin 45 = F_y, \quad F_y =$$

Flow in bends $P_0 = P_{atm}$





Prob: 6.45

$$\rho = 1000 \text{ kg/m}^3$$

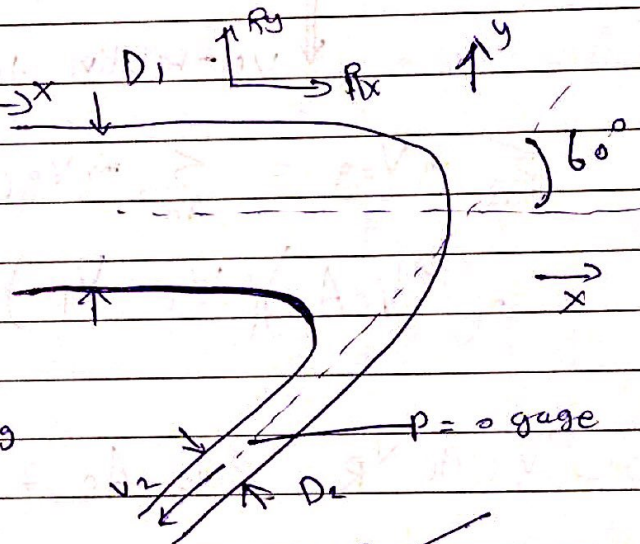
$$V = 0.25 \text{ m}^3$$

$$D_1 = 60 \text{ cm}$$

$$D_2 = 30 \text{ cm}$$

$$V_2 = 10 \text{ m/s} \quad m = 250 \text{ kg}$$

$$m_i = m_o = m$$



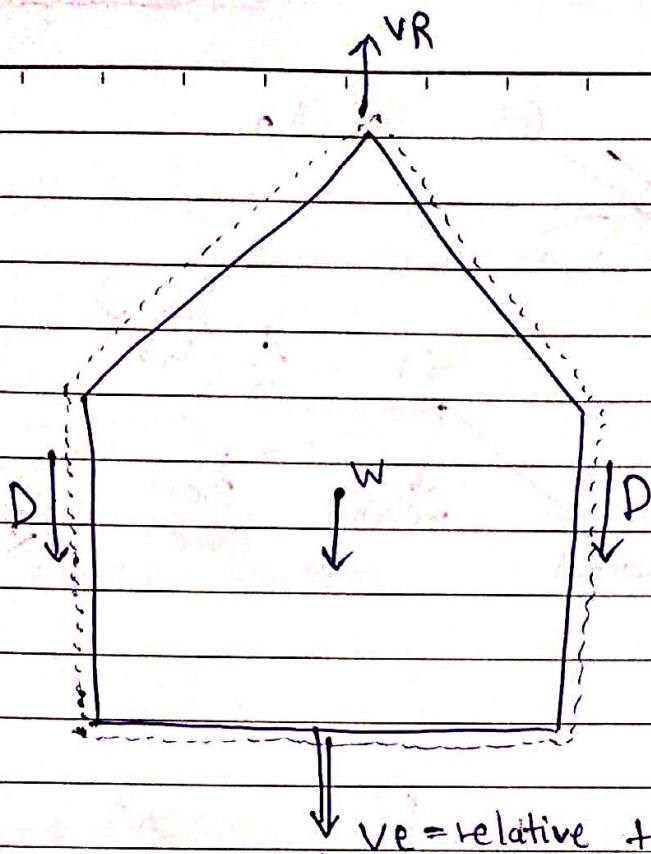
x-direction: $\sum m_o V_{ox} - \sum m_i V_{ix} + \frac{d}{dt} \int V_x \rho dV = \sum F_x$

$$-m V_2 \cos 60 - m V_1 = R_x + P_1 A_1 + P_2 A_2 \cos \theta$$

$$\cancel{m_1 = m_2} \quad \cancel{P_1 A_1} = \cancel{P_2 A_2}$$

$$\cancel{\frac{P_1 \rho}{\rho g} + \frac{V_1^2}{2g} + z_1} = \cancel{\frac{P_2 \rho}{\rho g} + \frac{V_2^2}{2g} + z_2}$$

$$V = \frac{V_2}{V_1} A_2$$



M = mass of the rocket at any time.

$$\sum_{out} \dot{m} V_{og} = \sum_{in} \dot{m} V_{gi} + \frac{d}{dt} \int V_g \rho dV = \sum F_y$$

$$-(\rho V_e A_e)(V_e - V_R) - 0 + \frac{d}{dt} [V_R M] = -D - W + (P_e - P_o) A_e$$

$$\rho V_e A_e V_R = \rho V_e^2 A_e + \left[V_R \frac{dM}{dt} + M \frac{dV_R}{dt} \right]$$

$$= -D - W + (P_e - P_o) A_e$$

$$M \frac{dV_R}{dt} = -D - W + (P_e - P_o) A_e + \rho V_e^2 A_e$$

Thrust T of rocket engine

$$\boxed{-D - W + T = M \frac{dV_R}{dt}}$$

$$\frac{dM}{dt} = \dot{m}_o - \dot{m}_i$$

wednesday

27/7/2016

* Chapter ⑦: Energy equation:

$$\dot{Q} - \dot{W} = \Delta E \quad E: \text{total energy}$$

$$E = I.E + k.E + P.E$$

$$\frac{\dot{Q}}{t} - \frac{\dot{W}}{t} = \frac{\Delta E}{t}$$

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{d}{dt} \int_{c.v} e(P dv) + \sum_{c.s} e(P \cdot v \cdot A)$$

$$E = U + k.E + P.E, \quad e = u + e_k + e_p$$

$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{c.v} e(P dv) + \sum_{c.s} e(P \cdot v \cdot A)$$

$$= \frac{d}{dt} \int_{c.v} (u + e_k + e_p) P dv + \sum_{c.s} (u + e_k + e_p) (P \cdot v \cdot A)$$

$\dot{W} = \text{shaft power } (\dot{W}_s) + \text{flow work } (\dot{W}_o)$

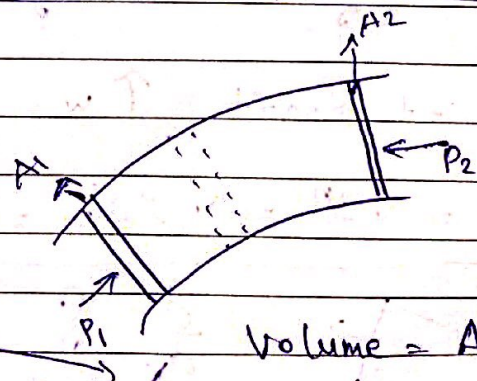
$$\dot{W}_R = P \cdot \dot{V} \cdot A$$

$$\text{work} = P \cdot \dot{V}$$

$$\frac{\text{work}}{\text{time}} = \frac{P \cdot A \cdot \Delta L}{\text{time}} = P \cdot A \cdot \dot{V}$$

$$\dot{W}_P = \sum_{\text{out}} P \vec{A} \cdot \vec{V} - \sum_{\text{in}} P \vec{A} \cdot \vec{V}$$

$$= \sum_{c.s} P \cdot \vec{V} \cdot \vec{A}$$



Volume = A dt

$$\dot{Q} - \dot{W}_s = \sum_{c.s} P \vec{A} \cdot \vec{V} = \frac{d}{dt} \int_{c.v} (u + \frac{1}{2} v^2 + g z) P dv +$$

$$\sum_{c.s} (u + \frac{v^2}{2} + g z) P v A$$

Five Apple

$$\Rightarrow \dot{Q} - \dot{w}_s = \frac{d}{dt} \int_{c.v} \left(u + \frac{1}{2} v^2 + gz \right) \rho dV$$

$$+ \sum_{c.s} \left(u + \frac{v^2}{2} + gz + \frac{p}{\rho} \right) \rho V A$$

* special cases:

(1) Steady flow: $\dot{Q} - \dot{w}_s = \sum_{c.s} \left(\frac{p}{\rho} + \frac{v^2}{2} + gz + u \right) \rho V A$

$$= \sum_{out} \dot{m} \left(\frac{p}{\rho} + \frac{v^2}{2} + gz + u \right) - \sum_{in} \dot{m} \left(\frac{p}{\rho} + \frac{v^2}{2} + gz + u \right)$$

(2) Steady 1-D: incompressible flow

$$\frac{1}{\rho} (\dot{Q} - \dot{w}) - \frac{u_1}{g} + \frac{u_2}{g} = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 - \frac{p_2}{\gamma} - \frac{v_2^2}{2g} - z_2$$

→ * when there is losses.

* If it was ideal $\Rightarrow$ no losses.

→ we use Bernoulli equation:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 - \left(\frac{\dot{Q} - \dot{w}}{\dot{m}} \right) + \left(\frac{u_2 - u_1}{g} \right)$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

Power of shaft = $\dot{W} = \dot{w}_t - \dot{w}_p$

$$\dot{w}_p = \dot{V} \Delta P = \dot{V} \rho \Delta h = \dot{m} g \Delta h$$

$$\frac{d}{dx} \int_{c.v} \rho dV = \frac{d}{dx} \int \int \int \rho dx dy dz$$

$$\int \int \rho dy dz = \rho$$

equation: ①: $\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 - \frac{w_p}{mg} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$

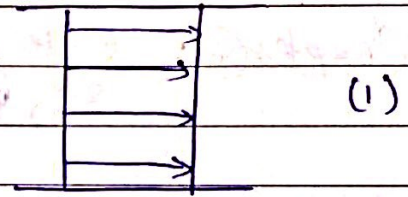
$+ h_L + \frac{w_T}{mg}$

equation: ②: $\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L + h_T$

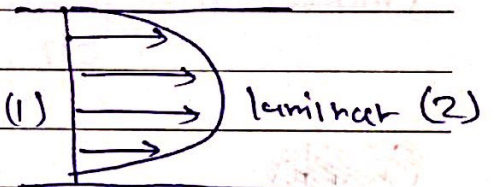
$w_p = Q \Delta P = \dot{m} g (h_p) \rightarrow \text{head}$

α = factor related to velocity

$\gamma = \frac{\rho V dA}{A \bar{V}}$

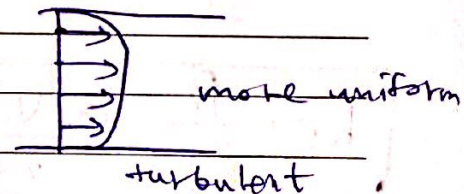


$\bar{V} = \frac{\rho \int V dA}{A} = \frac{\sum V \Delta A}{\sum \Delta A} = 1$ in state (1)

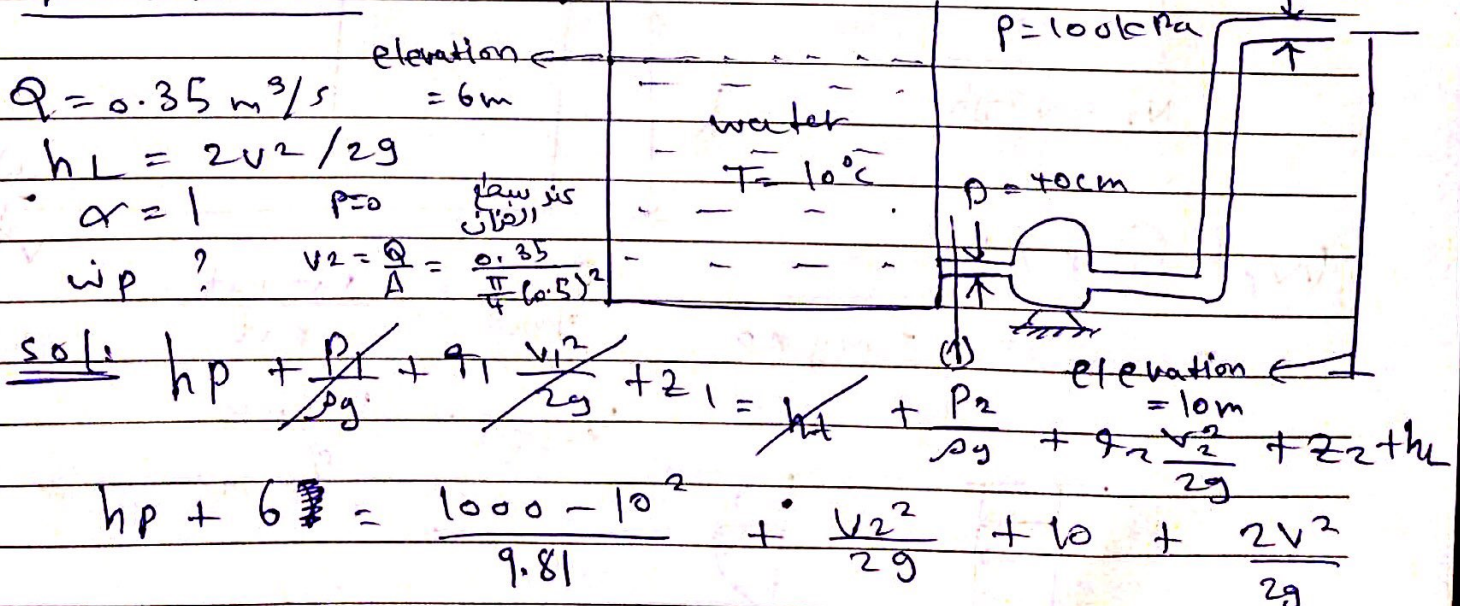


$\bar{V} = \frac{\rho \int V^2 dA}{A \bar{V}^2}$ $\alpha > 1$ in state (2)

$\alpha = \begin{cases} 2: \text{laminar (parabolic)} \\ 0.05: \text{Turbulent flow} \end{cases}$



Problem: 2.52:



$Q = 0.35 \text{ m}^3/\text{s}$

elevation = 6m

$h_L = 2v^2/2g$

$\alpha = 1$ $P=0$ $\text{law is } (1/2)^{1/2}$

$w_p ?$ $v_2 = \frac{Q}{A} = \frac{0.35}{\frac{\pi}{4}(0.5)^2}$

sol: $h_p + \frac{P_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g} + z_2 + h_L$

$h_p + 6 = \frac{1000 - 10^2}{9.81} + \frac{v_2^2}{2g} + 10 + \frac{2v^2}{2g}$

$h_p = 10.3 + 10 - 6 - 3 \frac{v^2}{2g}$

$w_p = \dot{m} g \Delta h = Q \gamma h_p$

* Problem: 7-38:

$$P_A = 0$$

$$V_A = 0$$

$$z_A = 30$$

$$h_{A \rightarrow B} = \frac{3}{4} \frac{V^2}{2g}$$

$$h_{B \rightarrow D} = \frac{1}{4} \frac{V^2}{2g}$$

$$\text{diameter} = 25 \text{ cm}$$

$$\alpha = 1$$

$$Q = ??$$

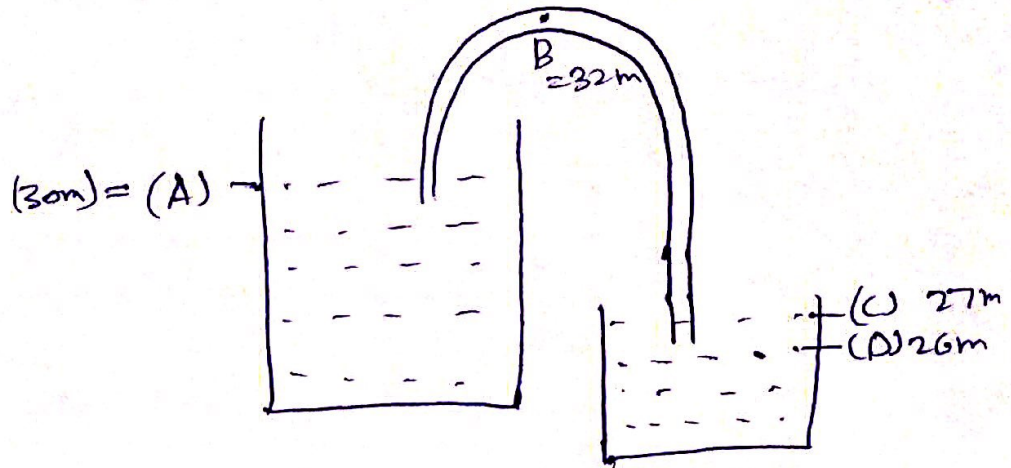
$$P_B = ??$$

Sol:

$$\cancel{P} + \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \cancel{P} + \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$P_B = -305 \text{ kPa}$$



* إذا ما ذكر α بالفرص انو $\alpha=1$

Sunday
31/7/2016

Suggested chapter ④:

4 - 14, 28, 43, 46, 51, 55, 70, 87, 81, 86, 92, 99, 105, 108.

Chapter ⑤: 5 - 15, 25, 52, 55, 64, 72, 80, 81, 82, 83, 84, 85, 99, 107

Chapter ⑥: 6 - 10, 6 - 13, 16, 19, 24, 25, 29, 32, 38, 46, 48, 60, 65, 75.

Chapter ⑦: 24, 28, 33, 37, 39, 40, 43, 45, 52, 57, 66, 69

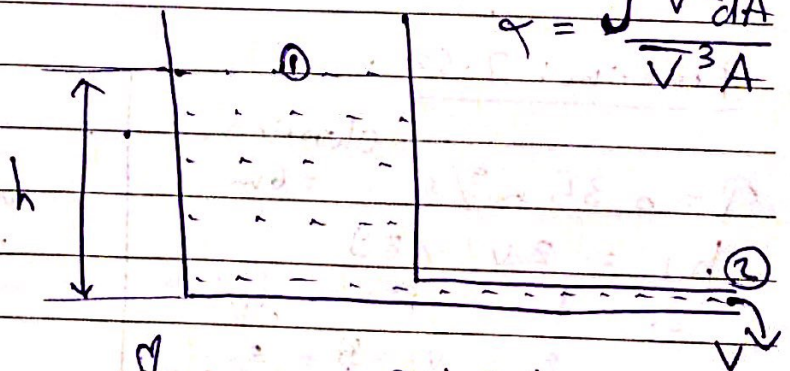
Chapter ⑦

Energy equation

$$h_p + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$h = \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2gh}$$

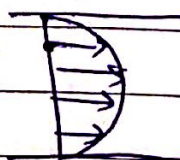
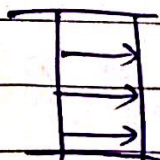


$$\dot{W}_p = Q \Delta P$$

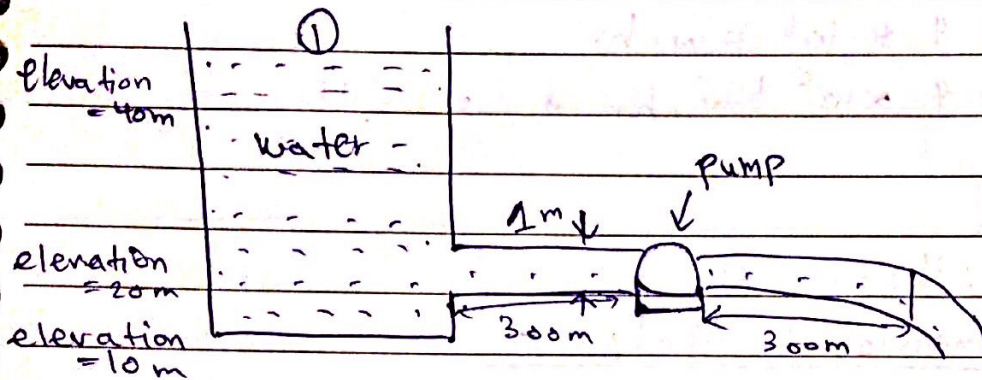
$$= \rho Q g h_p = m g h_p$$

$$\eta_{\text{pump}} = \frac{\text{out put}}{\text{in put}}$$

$$\eta_{\text{elect}} = \frac{m g h_p}{\dot{W}_{\text{pump}}}$$



Prob: 7 - 52



$$Q = 8 \text{ m}^3/\text{s}, h_L = 7 \frac{V^2}{2g}$$

$$h_p + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = h_p + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$40 = \frac{V_2^2}{2g} + 20 + 7 \frac{V^2}{2g}$$

$$20 = \frac{8V^2}{2g}, V_2 = \sqrt{50} = 7.1 \text{ m/s}$$

$$Q = \frac{\pi}{4} D^2 \times V = \frac{\pi}{4} \sqrt{50} = 5.5 \text{ m}^3/\text{s}$$

$$h_p + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$h_p + 40 = \frac{V_2^2}{2g} + 20 + 7 \frac{V_2^2}{2g}$$

$$h_p = \frac{8V_2^2}{2g} - 20 = \frac{8(Q/A)^2}{2g} - 20$$

$$= \frac{8(32/\pi)^2}{2g} - 20 = 22.3 \text{ m}$$

$$\text{Power} = Q * \rho g h_p = 8 * 9810 * 22.3 = 1750104 \text{ watt}$$

$$= 1750.104 \text{ kW}, 1.750104 \text{ MW}$$

$$\text{Power}_{\text{elec}} = \frac{1.75 \times 10^6}{\eta_{\text{mech}}} = 2 \text{ MW}$$

$$\text{Energy} = \text{Power} \times \text{time}$$

$$= 2000 \text{ kW} \times 8760 \text{ hour/year}$$

$$= 17.4 \times 10^6 \text{ kW.h}$$

$$\text{cost} = 17.4 \times 10^6 \text{ kW.h} \times \frac{0.25 \text{ ¢}}{\text{kW.h}}$$

Chapter 8

Dimensional Analysis :

$$Re = \frac{\rho V D}{\mu}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

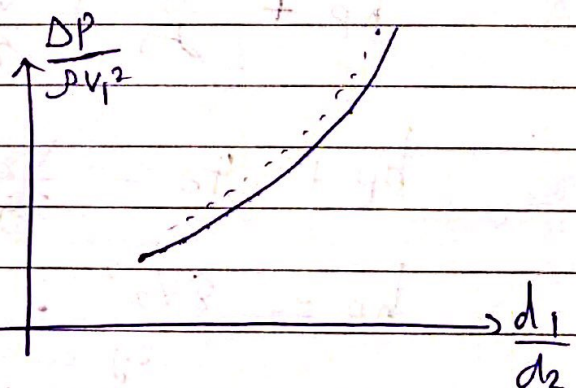
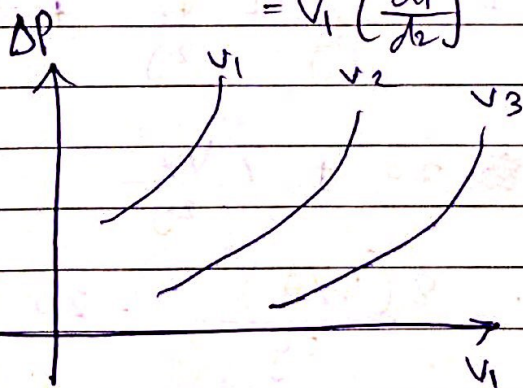
$$\frac{\rho}{2} (V_1^2 - V_2^2) = (P_2 + \rho g z_2) - (P_1 + \rho g z_1)$$

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{\frac{\pi}{4} (d_1)^2}{\frac{\pi}{4} (d_2)^2} = V_1 \left(\frac{d_1}{d_2} \right)^2$$

$$\frac{\Delta P}{\rho V_1^2} = \frac{d_2^2}{d_1^2} - 1$$

↳ Pressure coefficient = C_p



Dimensional Analysis of fluid system

Force F	} F L T system	$F = ma = M \frac{L}{T^2} = N$
Length L		
Time T		
mass M	} M L T system	$P = \frac{N}{m^2} = \frac{M}{LT^2}$
length L		
Time T		

$$\rho = \frac{\text{kg}}{\text{m}^3} = \frac{M}{L^3}$$

Π -Theorem (Buckingham theo.)

No. of Non-Dimensional groups
= No. of variables - No. of Basic dimensions

$$\Pi = h - m$$

↓ max = 3

Example: $P_1 + \frac{\rho V_1^2}{2} + \gamma z_1 = P_2 + \frac{\rho V_2^2}{2} + \gamma z_2$

$$C_p = \frac{\Delta P}{\rho V_1^2 / 2}$$

$\Delta P, V_1, \rho, z, V_2$

↓
 V_2

$$h = 5$$

$$m = 3$$

$$\Pi = 5 - 3 = 2$$

Rules for selecting the predicting variables

- ① The predicting variables must include among them all basic dimensions (M L T)
- ② The repeating variables are selected such that one is Geometrical (Length), one is flow variable such as velocity, pressure ...
one is fluid variables: $\rho, \mu, \dots$
- ③ The dependent variable must not be selected as a repeating variables.

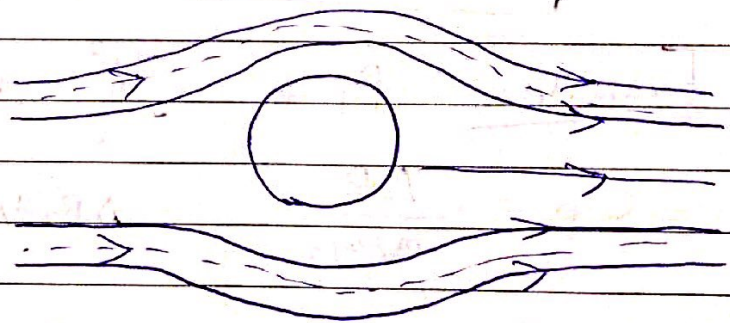
Ex: consider a fluid flow ^{through a} cylinder with diameter d , the pressure drop per unit length is $\left(\frac{\Delta P}{L} \right)$ (and length L)
a function of $\frac{\Delta P}{L} = f(D, K, V, \rho, \mu)$

roughness

$$n = 6$$

$$m = 3$$

$$\Pi = n - m = 6 - 3 = 3$$



$$\Pi_1 = D, V, \rho, \Delta P/L$$

$$\Pi_2 = D, V, \rho, K$$

$$\Pi_3 = D, V, \rho, \mu$$

$$D = L$$

$$K_s = L$$

$$V = L/T$$

$$\rho = M/L^3$$

$$\mu = M/(L \cdot T)$$

$$\frac{\Delta P}{L} = N$$

$$\frac{\Delta P}{L} = \frac{N}{m^2 \cdot m}$$

$$= \frac{N}{m^3} = \frac{kg \cdot m}{s^2 \cdot m^3}$$

$$= \frac{F}{L^3} = \frac{kg}{s \cdot m^3}$$

$$\Pi_1 = D^x \cdot \rho^y \cdot V^z \cdot \frac{\Delta P}{L} = L^x \frac{M^y}{L^3} \frac{L^2}{T^2} \frac{M}{L^2 T^2}$$

$$= (M^0, L^0, T^0)$$

$$= L^{x-3y+z-2} M^{y+1} T^{-z-2}$$

$$-z-2=0 \Rightarrow \boxed{z=-2}$$

$$y+1=0 \Rightarrow \boxed{y=-1}$$

$$x-3y+z-2=0, x=3(-1)+2+2=1$$

$$\Pi_1 = D^1 \rho^{-1} V^{-2} \frac{\Delta P}{L} = \frac{\Delta P}{L} \cdot \frac{D}{\rho V^2} \left[= \frac{\Delta P}{\rho V^2} \cdot \frac{D}{L} \right]$$

$$\Pi_3 = D^x \cdot \rho^y \cdot V^z \cdot M$$

$$[M] = \frac{kg}{m \cdot s}$$

$$(M^0, L^0, T^0) = L^x \frac{M^y}{L^3} \frac{L^2}{T^2} \frac{M}{LT}$$

$$= L^{x-3y+z-1} M^{y+1} T^{-z-1}$$

$$-z-1=0 \Rightarrow \boxed{z=-1}, y+1=0 \Rightarrow \boxed{y=-1}$$

$$x-3y+z-1=0, x=3(-1)+(-1)+1=-1$$

$$\Pi_3 = D^{-1} \rho^{-1} V^{-1} M = \frac{M}{\rho D V} = \frac{1}{Re}$$

$$\Pi_2 = D^x \cdot \rho^y \cdot V^z \cdot k_s$$

$$(M^0, L^0, T^0) = L^x \frac{M^y}{L^3} \frac{L^2}{T^2} L = L^{x-3y+z+1} M^y T^{-2y}$$

$$y=0, z=0, x-0+0+1=0 \Rightarrow \boxed{x=-1}$$

$$\Pi_2 = D^{-1} \rho^0 V^0 k_s = k_s / D$$

$$\frac{\Delta P/L}{\rho V^2/D}$$

$$= f(Re, \frac{k_s}{D})$$

$$\frac{\Delta P}{\rho V^2} \cdot \frac{D}{L}$$

↓

$$\frac{\Delta P/L}{\rho V^2/D}$$

$$(\text{cp})_{\text{per unit length}} = f(Re, k_s/D)$$



$$\textcircled{1} \frac{\text{Inertia force}}{\text{Viscous force}} = Re = \rho V D / \mu$$

$$\textcircled{2} \frac{\text{Inertia force}}{\text{gravity force}} = \frac{V^2}{Lg} = \text{Froude number}$$

$$= \frac{V}{\sqrt{Lg}}$$

$$\textcircled{3} \frac{\text{Inertia force}}{\text{surface tension}} = \frac{\rho L V^2}{\sigma} = \text{Weber number}$$

$$Ma = \frac{V}{c} = \text{Mach number}$$

(17)

{Chapter 5:}

Example: The velocity components for 2-D flow

are given by: $u = -\frac{x}{x^2+y^2}$ and $v = -\frac{y}{x^2+y^2}$

a) Is the continuity equation satisfied?

b) Is this flow rotational?

c) Find a_x at $(1, 1)$?

sol: a) If $\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0$

$$\frac{du}{dx} = \frac{(x^2+y^2)(-1) + x(2x)}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\frac{dv}{dy} = \frac{(x^2+y^2)(-1) + y(2y)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{(x^2-y^2)}{(x^2+y^2)^2} + \frac{(y^2-x^2)}{(x^2+y^2)^2} + 0 = 0 \Rightarrow \text{continuity is satisfied}$$

b) $\omega_z = \frac{1}{2} \left(\frac{dv}{dx} - \frac{du}{dy} \right) = 0$

$$\frac{dv}{dx} = \frac{y(2x)}{(x^2+y^2)^2}, \quad \frac{du}{dy} = \frac{x(2y)}{(x^2+y^2)^2}$$

$$= \frac{1}{2} \left(\frac{2yx}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2} \right) = 0 \Rightarrow \text{flow is irrotational}$$

c) $a_x = \frac{du}{dt} = \frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz}$

$$= \left(-\frac{x}{x^2+y^2} \right) \left(\frac{x^2-y^2}{(x^2+y^2)^2} \right) + \frac{-y}{x^2+y^2} \left(\frac{y^2-x^2}{(x^2+y^2)^2} \right) + 0$$

a) $a_x = 0 + \frac{-1}{2} \left(\frac{2x(x)}{(x^2+y^2)^2} \right) = -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4} \text{ m/s}^2$

Sunday
7/8/2016

Chapter 10

Flow in conduits, pipe, & Internal flow

$$\frac{h_p}{\rho g} + \frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = h_f + \frac{P_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_L$$

$$h_L = h_{L \text{ major}} + h_{L \text{ minor}}$$

↓
straight
pipe

↓
fittings, elbows, valves...

$$h_L = \underbrace{f}_{\text{friction factor}} \frac{L}{D} \frac{V^2}{2g} + \sum_{i=1}^n \frac{K_i V^2}{2g}$$

$$h_{L \text{ major}} = f \frac{L}{D} \frac{V^2}{2g}, \quad h_{L \text{ minor}} = \sum (K_i) \frac{V_i^2}{2g}$$

→ table 10.5

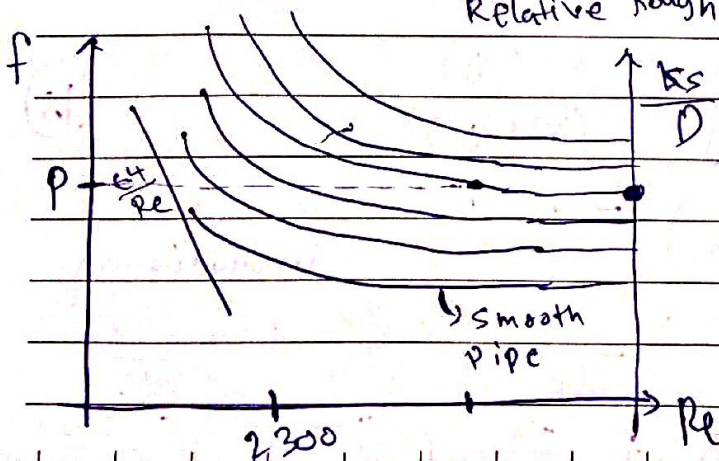
$$h_L = h_{L \text{ major}} + h_{L \text{ minor}}$$

$$f = \frac{64}{Re} \quad \text{if the flow is laminar } [Re < 2300]$$

$$f = \frac{0.25}{\left[\log_{10} \left(\frac{K_s}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^2} \quad (\text{Turbulent flow})$$

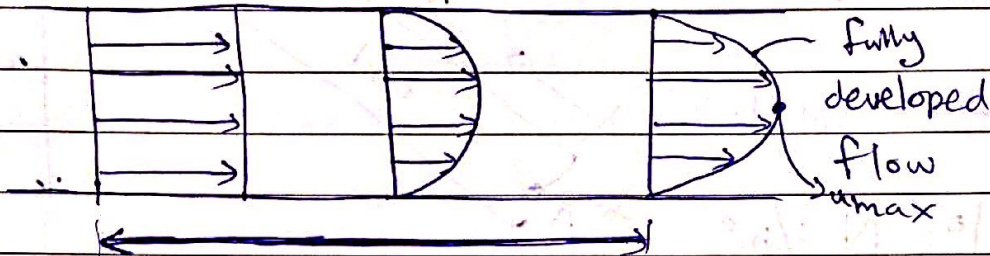
K_s = Pipe wall
Roughness (mm)

Relative Roughness

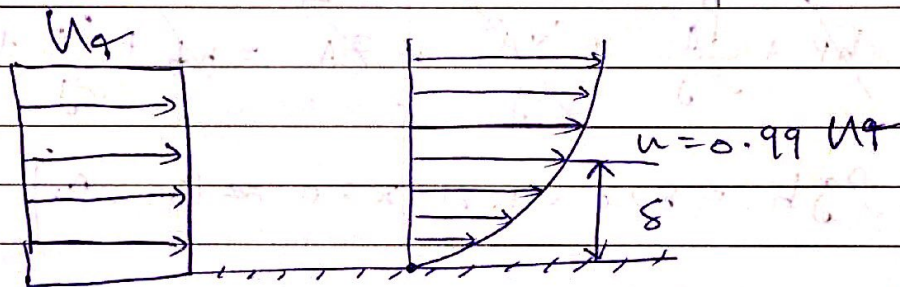


(19)

$$D_H = \frac{4 \text{ Flow Area}}{\text{wetted Parameter}} = \frac{4 A_c}{P}$$



Characteristic flow
Development of Boundary layer.



Shear stress distribution

$$\frac{U}{U_{max}} = \frac{r_0^2 - r^2}{r_0^2} = 1 - \left(\frac{r}{r_0}\right)^2 \quad \text{Parabolic distribution}$$

$$U = U_{max} \left[1 - \left(\frac{r}{r_0}\right)^2 \right], \quad h_L = \frac{32 \mu L \bar{V}}{\gamma D^2}$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} = \frac{32 \mu L \bar{V}}{\rho g D^2} \quad \text{laminar flow}$$

$$f = \frac{32 \mu (2)}{\rho D V} = \frac{64}{\rho V D / \mu} = \frac{64}{Re}$$

monday

8/8/2016

Example:

$$Q = 0.03 \text{ m}^3/\text{s}$$

oil: $s = 0.82$ at 10°C

$$\mu = 10^{-2} \text{ N}\cdot\text{s}/\text{m}^2$$

$$P_B = 250 \text{ kPa}$$

$D = 15 \text{ cm}$
(steel pipe)

$$P_A = ??$$

Sol:
$$\cancel{h_P} + \cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \cancel{h_P} + \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}} + z_B + h_L$$

$$\times [P = \rho g h = 820 \times 9.81 \times 20 \approx 160 \text{ kPa}]$$

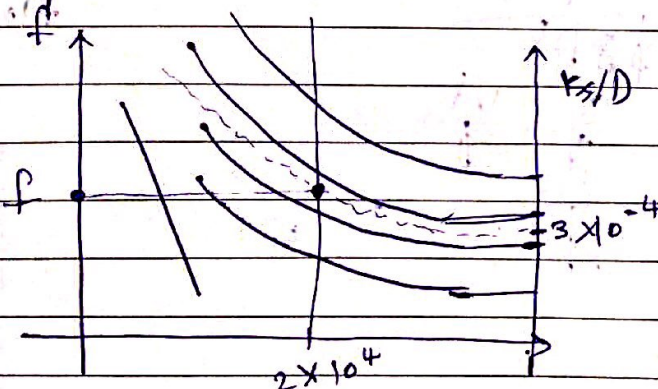
$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$V = \frac{Q}{A} = \frac{0.03}{\frac{\pi}{4}(0.15)^2}$$

$$f = f(Re, \frac{k_s}{D}), \quad Re = \frac{\rho V D}{\mu} = \frac{820 \times 1.7 \times 0.15}{10^{-2}}$$

$$= 2.09 \times 10^4 > 2300 \Rightarrow \text{Turbulent}$$

$$\frac{k_s}{D} = \frac{0.046 \text{ mm}}{150 \text{ mm}} = 3.067 \times 10^{-4}$$



$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$= 0.027 \times \frac{1000}{0.15} \times \frac{1.7^2}{2(9.81)} = 26.4 \text{ m}$$

$$P_A = P_B + \rho g (z_B + h_L)$$

$$= 250 \text{ kPa} + \frac{820(9.81)(20 + 26.4)}{1000}$$

$$= 623 \text{ kPa}$$

$$f = 0.027$$

(2)

Five Apple

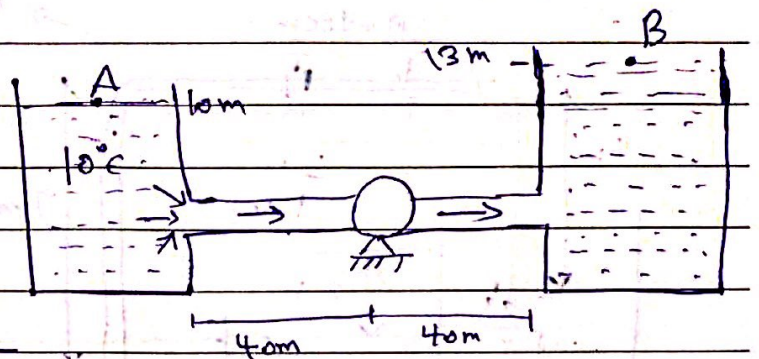
Scanned by CamScanner

prop: $10^{-6} (10^{th})$

$Q = 0.1 \text{ m}^3/\text{s}$

$h_p + \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A =$

$\frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_L$



$h_p = (z_B - z_A) + h_L$

$h_p = (13 - 10) + h_L = 3 + h_L$

$h_L = h_{L \text{ major}} + h_{L \text{ minor}}$

$= f \frac{L}{D} \frac{V^2}{2g} + \sum k_i \frac{V^2}{2g}$

$V = \frac{Q}{A} = \frac{0.1}{\frac{\pi}{4} (0.15)^2}$

$Re = \frac{\rho V D}{\mu}$
 $\mu \rightarrow 1.31 \times 10^{-3} \text{ N.s/m}^2 \text{ (Table A-5)}$
 $\rho \rightarrow 1000 \text{ kg/m}^3 \text{ (Table A-5)}$

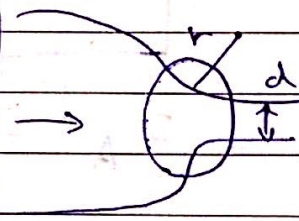
$= \frac{1000 (5.66) \times 0.15}{1.31 \times 10^{-3}}$

$= 6.4 \times 10^5 > 2300 \Rightarrow \text{Turbulent}$

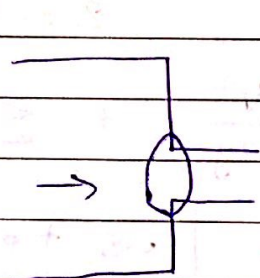
$\frac{f_s}{D} = \frac{0.046}{150} = 3.06 \times 10^{-4}$

$f = 0.016, h_L = 0.016 \times \frac{80}{0.15} \times \frac{5.66^2}{2(9.81)} + [0.5 + 1] \times \frac{5.66^2}{2(9.81)}$

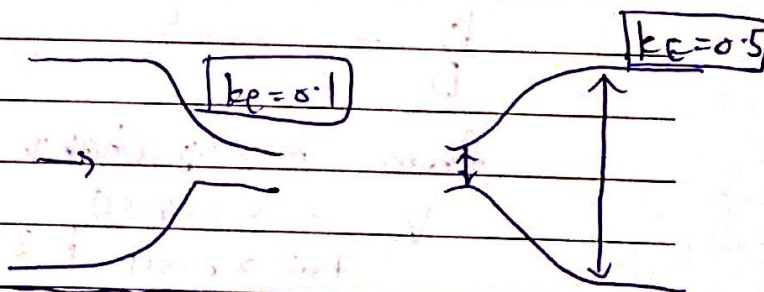
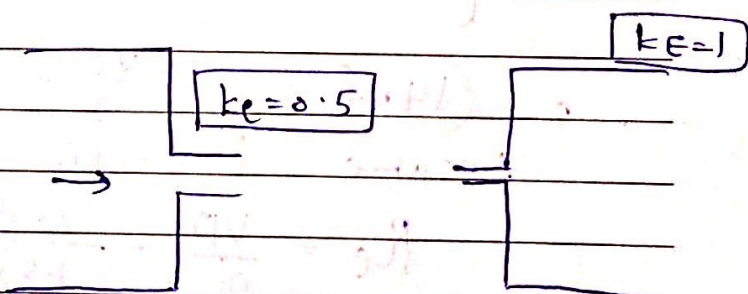
$P = Q \times \gamma h_p$



$h_L = k \frac{V^2}{2g}$

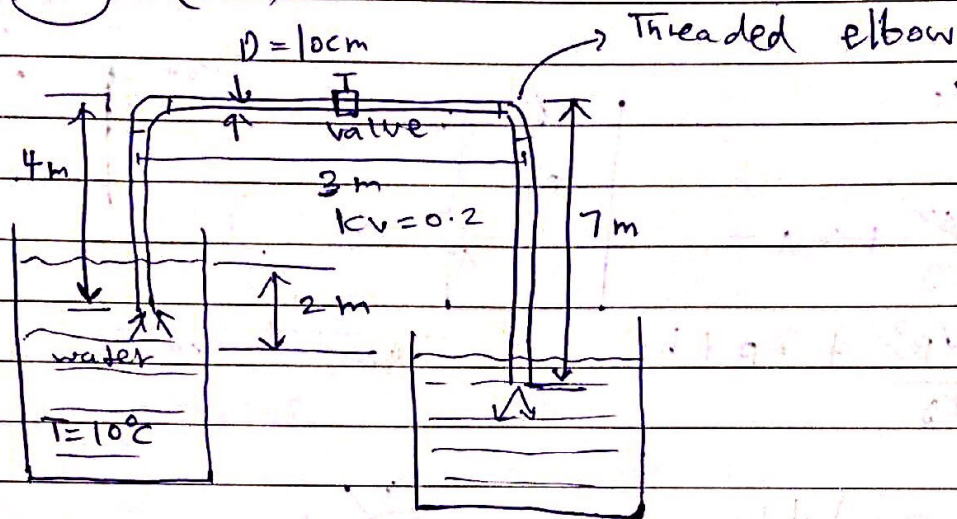


$\frac{h}{d} = k$
 $0 = 0.5$



Tuesday
9/8/2016

Prob (78) : (10th)



$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

$$Z = h_L$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} + \sum k_i \frac{V_i^2}{2g}$$

$$= \left\{ f \frac{14}{0.1} + [0.5 + 0.9 + 0.9 + 1] \right\} \frac{V^2}{2g}$$

$$Z = (140f + 3.5) \frac{V^2}{2g}$$

Assume $f = 0.015 \Rightarrow V = 2.65 \text{ m/s}$

$$Re = \frac{VD}{\nu} = \frac{2.65 \times 0.1}{1.3 \times 10^{-6}} = 2 \times 10^5$$

$$\frac{k_s}{D} = 0.0005$$

from moody chart $\Rightarrow f = 0.019$

$$V = \frac{2 \times 2(9.8)}{140 \times 0.019 + 3.5} = 2.52$$

$$Re = 1.9 \times 10^5 \quad \checkmark \text{ (OK)}$$

$$V = 1.26 \text{ m/s}$$

$$Re = 9.5 \times 10^4$$

$$f = 0.021$$

$$Z = [140 \times 0.021 + 3.3 + k_v] \frac{1.26^2}{2(9.8)}$$

$$k_v = 18.5$$

suggested problems chapter (10):

10-18, 22, 51, 54, 58, 63, 64, 65, 66, 68, 69,
81, 82.

Chapter (8): 8-9, 10, 11, 13, 16, 17, 20, 22, 23, 21