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دوسية الزيود في:

# ميكانيكا الموائع

# Fluid Mechanics


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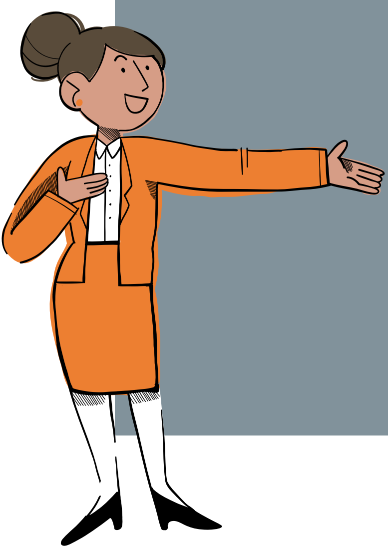
# مقدمة : ميكانيكا الموائع

ميكانيكا الموائع (بالإنجليزية: Fluid Mechanics) هو تخصص فرعي من ميكانيكا المواد المتصلة وهو معني أساسا بالموائع، التي هي أساسا السوائل والغازات، ويدرس هذا التخصص السلوك الفيزيائي الظاهر الكلي لهذه المواد، ويمكن تقسيمه من ناحية إلى إستاتيكا الموائع- أو دراستها في حالة عدم الحركة، أو ديناميكا الموائع أو دراستها في حالة الحركة، ويندرج تحتها تخصصات أخرى معينة، فهناك الديناميكيات الهوائية (أيروديناميك) والديناميكيات المائية (هيدروديناميك).[1][2][3] يسعى هذا التخصص إلى تحديد الكميات الفيزيائية الخاصة بالموائع، وذلك مثل السرعة، الضغط، الكثافة، ودرجة الحرارة، واللزوجة ومعدل التدفق، وقد ظهرت تطبيقات حسابية حديثة لإيجاد حلول للمسائل المتصلة بميكانيكا الموائع، ويسمى التخصص المعني بذلك ديناميكا الموائع الحسابية.



# مقدمة : ميكانيكا الموائع

تعتبر ميكانيكا الموائع غالبا أحد التخصصات الفرعية لميكانيكا المواد المتصلة، كما هو موضح في الجدول التالي

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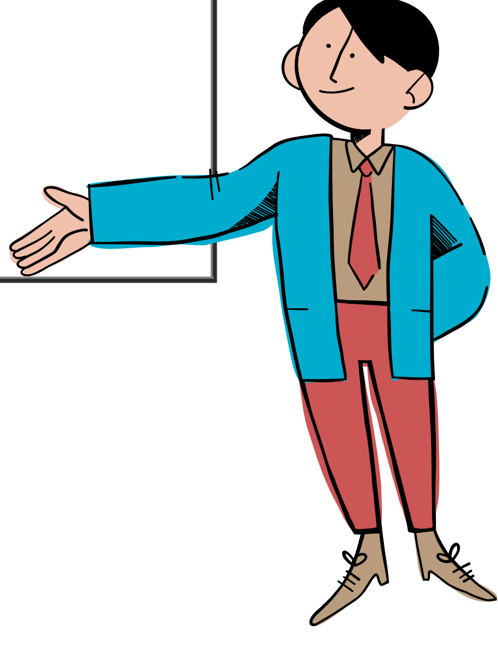
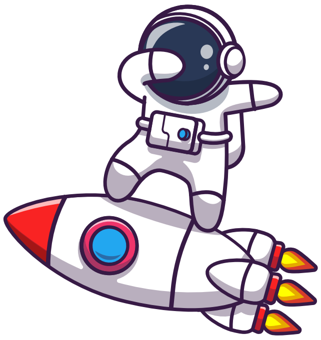




# فلنبدأ !!!

# مع المهندس محمد حسن

# ch 2: fluid properties

Hydrodynamics: deals primarily with the flow of fluids of constant density, such as the flow of liquids or the flow of liquids or the flow of gases at low speeds.

gas dynamics :deals with the flow of fluids that undergo significant density change.

Basic Units:

1inch=25mm 1ft=12inch

EX:

5inch to ft ?

Ft=$\frac{5}{12}$

Pressure

Pa=atm*1.01325*$10^5$

1mmhg=133pa

ملاحظة: كل 10متر ضغط =1ضغط جوي

Temperature: Rankine{R°}=460+ F°

T(F°)=1.8 T(C°)+32

T(K°)=T(C°)+273

K°:keliven        F°:Fahrenheit

تنقسم خصائص(propertis ) المائع الى ؟

1) Extensive properties: are properties related to the total mass of the system,example: M, W.

هي الخصائص التي تتأثر بالتغيرات التي تطرأ على المحلول

2) Intensive properties :are properties independent of the amount of fluid, example: p, T, ρ.

• توضيح:لو يوجد كوب من الماء درجة حرارته 25° وقمت بنقل جزء من هذا الماء في وعاء اخر فان الحرارة تكون متساوية في الوعائين intensive

ولكن وزن العينة اختلف extensive

مثلا كان وزن الكوب 500 عند نقل جزء من الماء اصبح كل كوب وزنه 250

## - Mass Density, ρ:unit(kg/$m^3$ )

Mass per unit volume

$$\rho=\frac{m}{v}$$

ρwater at 4C°=1000kg/$m^3$

ρair at 20C°and stander pressure=1.2

## - Specific Weight(γ) unit(N/$m^3$)

$$\gamma=\rho\ g=g*\frac{m}{v}$$

g:تسارع الجاذبية الارضية

M:mass V:volume

γwater at 20 C°=9.79

γair at 20 C° and stander pressure=11.8

## - Specific Gravity (S)

is the ratio of the specific weight of a given fluid to the specific weight of water at a standard reference temperature.

$$S=\frac{\gamma fluid}{\gamma water}=\frac{\rho fluid}{\rho water}$$

- At standard reference temp of 4 oC, γwater=9810 N/$m^3$
- To find pressure and density in ideal gas:

$$p=\rho \; R \; T$$

R:gas constant unit(J/kg.k)

P:absolute pressure unit (pa or psi or psf)

T:Temperature unit(k or R°)

اذا كانت وحدة الضغط pa فان الحرارتكون بالكيلفن

اذا كانت وحدة الضغط psfفان الحرارتكون R°

## -Specific heat (c):

قبل البدء بشرحه يجب علينا معرفة الفرق بين heat,temperature

Temperature:the way describe how hot and cold object

هي عبارة عن طريقة تصف كيف يسخن الجسم او يبرد

Heat: a form of energy unit (J)

شكل من اشكال الطاقة

• The specific heat (c): is the amount of thermal energy that must be transferred to a unit mass of a substance to raise its temperature by one degree.

هو مقدار الطاقة المنقولة للجسم لزيادة درجة حرارته

من خل التعريف نلاحظ ان العلاقة عكسية بين

Temperature, heat

• Specific heat can be given at constant pressure (cp )or at constant volume (cv ).

• The ratio cp /cv .is given by the symbol (k) and is always constant for a given gas.

## - Specific Internal Energy (u):J/kg

is the energy that a substance possesses per unit mass because of the state of the molecular activity in the substance.

## - Specific Enthalpy(h):J/kg

$$h=u+\frac{p}{\rho}$$

For an ideal gas (u) and (h) are function of temperature alone

## -viscosity (الزوجة)

A fluid is a substance that deforms continuously when subjected to a shear stress.

يتشوه المائع باستمرار عند تعرضه الى قوة القص

τ : shear stress

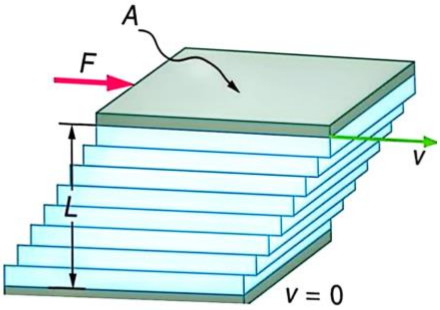
$$\tau=\mu\frac{dv}{dy}$$

μ : dynamic viscosity or absolute viscosity , dV/dy: velocity gradient, or time rate of strain, or shear strain

μ unit N*S/$m^2$ or kg/m*S

ملاحظة : اذا ذكر بالسؤال

μ=0.1فان قيمة poise

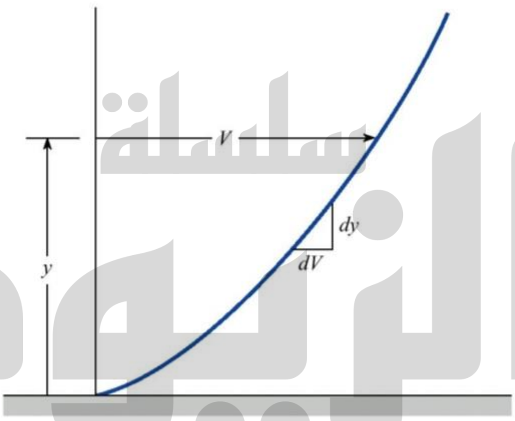



الجزء الذي يلامس السطح تكون السرعة
عنده تساوي صفر ويطلق عليه

No-slipe condition

وكلما ابتعدنا عن السطح تزداد السرعة

- The velocity distribution in a fluid near a boundary can be given as follows:




kinematic viscosity ($\nu$):unit ($m^2$/s)

$$\nu=\frac{\mu}{\rho}$$

مهم جدا:معرفة الفرق بين وحدة $\nu$و وحدة$\mu$

•The viscosity of a gas increases with temperature as given by the Sutherland's equation:

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + S}{T + S}$$

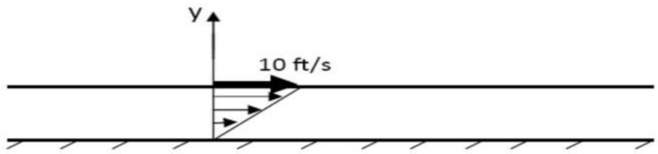
S:constant for gas , T: درجة الحرارة( keliven)

•the viscosity of a liquid decreases with temperature

$$\mu = Ce^{b/T}$$

C,b: constant

**Example**: Two plates are separated by 1/4 inch space. The lower plate is stationary, the upper plate moves at a velocity of 10 ft/s. Oil (SAE 10W-30, 150 °F) which fills the space. The variation in velocity of the oil is linear. What is the shear stress in the oil?




* نجد $\frac{dv}{dy}$ من خلال طريقتين

$$\tau = \mu \frac{dv}{dy}$$

1- من خلال الرسمة نأخذ الميل:-

$$\frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \frac{10-0}{\left(\frac{1}{(4)(12)}\right)-0} = 480 \text{ Ft/s}$$

2- نفرض معادلة (y)، معادلة الخط المستقيم ⇐ $\boxed{ax+b}$

$$v = ay + b$$

$\boxed{b=0} \Rightarrow a+y = 0 \; , \; v=0$

$a + y = \frac{1}{(4)(12)} \; , \; v = 10$

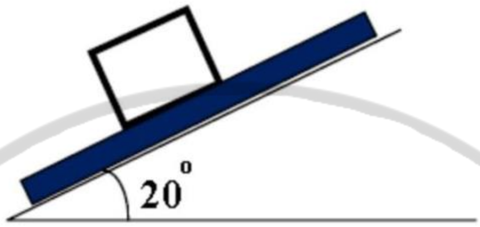
$a = 480$

$v = 480y$ $\quad \boxed{\frac{dv}{dy} = 480 \text{ Ft/s}}$

ثم نعوض بالقانون ⇐ $\tau = \mu \frac{dv}{dy}$

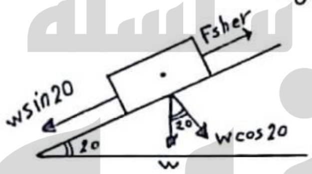
$$\tau = 5{,}2 * 10^{-4} * 480 = 0{,}25 \text{ Ib/Ft}^2$$

**Example**: A block weighing 1 kN and having dimensions 200 mm on an edge is allowable to slide down an incline on a film of oil having a thickness of 0.005 mm. If we use a linear velocity profile in the oil. What is the terminal speed of the block. The viscosity of the oil is $7 \times 10^{-3}$ N.s/m$^2$




sol.:-

نقوم بتحليل الشكل:-




* في السؤال ذكر ( terminal speed ) تعني هذه ان التسارع = صفر ( a )

$\Sigma F_x = m \cdot a$ → zero

$w\sin 20 - F_{sher} = \text{zero}$

$\Rightarrow w\sin 20 = F_{sher}$

$\Rightarrow \boxed{F_{sher} = \tau A}$

$\boxed{\tau = \mu \frac{dv}{dy}} \Rightarrow \frac{dv}{dy} = \frac{\Delta V}{\Delta y} = \frac{\Delta V}{(0.005) * 10^{-3}} = \boxed{200000\, \Delta V}$

mm → m

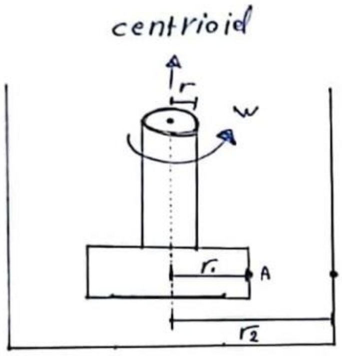
$\tau = (7 * 10^{-3})(200000\, \Delta V) = \boxed{1400 \Delta V}$

$w\sin 20 = \tau \cdot A$

$\Rightarrow (1 * 10^{-3})(\sin 20) = (1400 \Delta V)(0.2)^2$

$\boxed{\Delta V = 6.11 \text{ m/s}}$

* سؤال سنوات :-




w :- Anguler velocity

$v = w \cdot r$

$\tau = FA$

* Area at Point Ⓐ :- $A = 2\pi r_1 t$

* Area at Point Ⓑ :- $A = 2\pi r_2 t$

* لو كانت العلاقة (not-liner) :-

$v = y^2$

$\frac{dv}{dy} = 2y$

تنقسم الموائع حسب العلاقة بين shear stress وstrain الى

1) Newtonian 2) Non_ Newtonian: وتكون العلاقة طردية

•Elasticity (المرونة): compressibility of the fluid is related to the amount of deformation (expansion or contraction) for a given pressure change

## • bulk modulus of elasticity ($E_v$): N/$m^2$

$$E_v=\frac{dp}{d\rho/\rho}=\frac{dp}{dv/v}$$

P: pressure , ρ:density , V:volume

يمثل التغير في الضغط على التغير في الحجم او الكثافة

•The elasticity of an ideal gas:

1) Isothermal process: الانتقال من حالة الى اخرى دون التغير في درجة حرارة الجسم

$$E_v = \rho R T = p$$

2) adiabatic process: يكون النظام معزول

•heat transform =zero

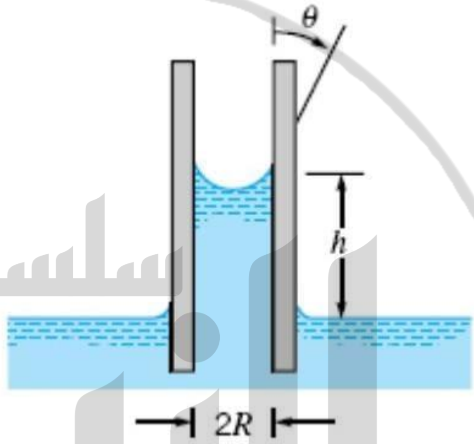
$$E_v=p\frac{Cp}{cv}$$

## - Surface Tension, σ: N/m

لو عندي وعاء بداخله ماء وقمت بالقاء بداخله عملات نقدية فان سطح الماء رح يرتفع تدريجيا لحد ما تنسكب الماء للخارج ولكن الماء لن تنسكب للخارج عند القاء اول عملة وذلك بسبب ان كل جزيء من الماء يتأثر بقوة من جميع الاتجاهات فيحدث له تزان.

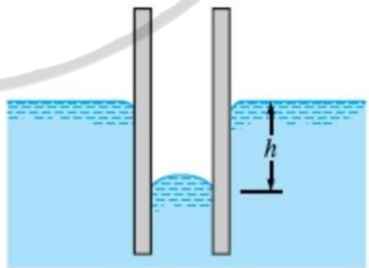
التماسك:يكون بين الجزيئات المتماثلة (cohesive)

التلاصق: يكون بين جزيئات مختلفة Adhesive




نلاحظ ان الماء يرتفع للاعلى في داخل الانبوبة وذلك بسبب قوى التلاصق بين الماء والجدار اكبر من قوى التماسك بين جزيئات الماء

Adhesive > Cohesive




Adhesive < Cohesive

# Vapour Pressure:

is the pressure at which a liquid will boil.

•The vapour pressure increases with temperature.

يحدث الغليان عندما تزداد درجة الحرارة فيعمل على زيادة vapour pressure بحيث تصبح مساوية للضغط الجوي

| |
|---|
| العلاقة بين T,p علاقة طردية |
| العلاقة بين السرعة p, علاقة عكسية |

boiling can occur at low temperatures if the pressure in the liquid is decreased to its vapour pressure.

•The effect of vapour pressure can be noticed in flowing liquids when vapour bubbles start growing in local regions of very low pressure and collapse in regions of high pressure. This phenomenon is known as cavitation

PROBLEM 2.2

Situation: Carbon dioxide is at 300 kPa and 60°C.

Find: Density and specific weight of $CO_2$.

Properties: From Table A.2, $R_{CO_2} = 189$ J/kg·K.

هنا نتعامل مع غازات لايجاد مقدار ( $\rho$ ) نطبق

$P = \rho R T$ $\qquad$ $\rho = \frac{P}{RT}$

T: Keliven / P: Pa

تحويل (T) من (C°) الى (K)

$T(k) = 60 + 273 = 333\,K$

$\rho = \frac{300 * 10^3}{(189)(333)} = 4,767\ Kg/m^3$

$\gamma = \rho . g = 4,767 * 9.81 = 46,764\ N/m^3$

Situation: Natural gas (10 °C) is stored in a spherical tank. Atmospheric pressure is 100 kPa.
Initial tank pressure is 100 kPa-gage. Final tank pressure is 200 kPa-gage.
Temperature is constant at 10 °C.

Find: Ratio of final mass to initial mass in the tank.

* لإيجاد (Mass) نستخدم العلاقة :-

$\rho = \frac{M}{V}$ $\qquad$ $M = \rho . V$

هنا نتعامل مع غازات فان ($\rho$) :-

$\rho = \frac{P}{RT}$

* وبالتحويل الى هدف المطلوب فقط النسبة

$M = V * \frac{P}{RT}$

فنلاحظ من خلال القانون ان ( M ) تتناسب بشكل طردي مع (P)

$\frac{M_2}{M_1} = \frac{P_2}{P_1} = \frac{300,3}{200,3} = 1,5$

$P_2 = 200 + 101,3$

$P_1 = 100 + 10,3$

Situation: Water and air are at $T = 100^oC$ and $p = 5$ atm.

Find: Ratio of density of water to density of air.

Properties: From Table A.2, $R_{\text{air}} = 287$ J/kg·K. From Table A.5, $\rho_{\text{water}} = 958$ kg/m$^3$.

طلب: النسبة بين ال($\rho_{water}$) و ($\rho_{air}$)

$\boxed{\rho_{water} = 458}$ ⟹ جاهزة في السؤال

$\rho_{air} = \dfrac{P}{RT}$  $P$: Pa / $T$: K°

ولكن في السؤال معطى قيمة $P$ بال (atm) نعود الى (Pa)

$P = 5(1{,}01325 \times 10^5) = \boxed{506600\ Pa}$

$T = 100 + 273 = 373\ K$

$\rho = \dfrac{506600}{(287)(373)} = 4{,}73\ Kg/m^3$

$\dfrac{\rho_{water}}{\rho_{air}} = \dfrac{458}{4{,}73} = \boxed{202}$



Situation: Air is at an absolute pressure of $p$ = 600 kPa and a temperature of $T = 50^oC$.

Find: (a) Specific weight, and (b) density

Properties: From Table A.2, $R = 287 \frac{\text{J}}{\text{kg}\cdot\text{K}}$.

APPROACH

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho_{\text{air}} &= \frac{P}{RT} \\ &= \frac{600,000}{287(50+273)} \\ &= \boxed{6.47\ \text{kg/m}^3}\end{aligned}$$

Specific weight

$$\begin{aligned}\gamma_{\text{air}} &= \rho_{\text{air}} \times g \\ &= 6.47 \times 9.81 \\ &= \boxed{63.5\ \text{N/m}^3}\end{aligned}$$

Situation: Oxygen ($p = 400$ psia, $T = 70\,°F$) fills a tank. Tank volume $= 10$ ft$^3$. Tank weight $=100$ lbf.

Find: Weight (tank plus oxygen).

Properties: From Table A.2, $R_{O_2} = 1555$ ft·lbf/(slug·$^o R$).

* المطلوب هنا (weight (oxygen + tank))

$W_{tank} = 100\,Ib$ ⇒ معطى بالسؤال

* لايجاد ($W_{oxygen}$) :-

(I) نجد $\rho$ ⇐ من خلال ⇐ $\rho = \frac{P}{RT}$

(II) $\rho = \frac{M}{V}$ ⇐ نجد $M$

(III) $W = M \cdot g$

sol:-

(I) $\rho = \frac{P}{RT}$  $P: Ib/Ft^2$  $T: R°$

$P = \frac{400\,Ib/in^2}{(12)^2} = 57600\,Ib/Ft^2$

$T = 460 + 70 = 530\,R°$  $\rho = 0{,}0689\,slugs/Ft^3$

(II) $\rho = \frac{m}{V} \Rightarrow m = 0{,}699$

(III) $W = m \cdot g = 0{,}699 \cdot 32{,}2 = 22{,}5\,Ib$

$W_{tot} = W_{oxygen} + W_{tank} = 22{,}5 + 100 = 122{,}5\,Ib$

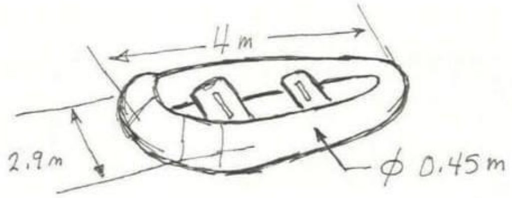
Prop(2,10):

1) volume $= \frac{\pi d^2}{4} \cdot l = \frac{\pi \cdot (0{,}45)^2}{4} \cdot 10 \Rightarrow \Delta V = 2{,}54\,m^3$

2) $\rho = \frac{P}{RT} = \frac{122 * 10^3}{(189)(290)} = 2{,}226\,kg/m^3$

mass $= \rho \cdot V = 5{,}66\,kg$

Situation: A design team needs to know how much $CO_2$ is needed to inflate a rubber raft.
Raft is shown in the sketch below.
Inflation pressure is 3 psi above local atmospheric pressure. Thus, inflation pressure is 17.7 psi = 122 kPa.




Find: (a)Estimate the volume of the raft.
(b) Calculate the mass of $CO_2$ in grams to inflate the raft.

Properties: From Table A.2, $R_{CO_2} = 189$ J/kgK.

Assumptions: 1.) Assume that the $CO_2$ in the raft is at 62 °F = 290 K.
2.) Assume that the volume of the raft can be approximated by a cylinder of diameter 0.45 m and a length of 16 m (8 meters for the length of the sides and 8 meters for the lengths of the ends plus center tubes).

APPROACH

Mass is related to volume by $m = \rho *$Volume. Density can be found using the ideal gas law.

ANALYSIS

Volume contained in the tubes.

$$\begin{aligned} \Delta V &= \frac{\pi D^2}{4} \times L \\ &= \left(\frac{\pi \times 0.45^2}{4} \times 16\right) \text{ m}^3 \\ &= 2.54 \text{ m}^3 \end{aligned}$$

$$\boxed{\Delta V = 2.54 \text{ m}^3}$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{122{,}000 \text{ N/m}^2}{(189 \text{ J/kg} \cdot \text{K})(290 \text{ K})} \\ &= 2.226 \text{ kg/m}^3 \end{aligned}$$

Mass of $CO_2$

$$\begin{aligned} m &= \rho \times \text{Volume} \\ &= \left(2.226 \text{ kg/m}^3\right) \times \left(2.54 \text{ m}^3\right) \\ &= 5.66 \text{ kg} \end{aligned}$$

$$\boxed{m = 5.66 \text{ kg}}$$

Situation: The viscosity of air is $\mu_{\text{air}}\,(15^{\circ}C) = 1.78 \times 10^{-5}$ N·s/m$^2$.

Find: Dynamic viscosity $\mu$ of air at 200 °C using Sutherland's equation.

Properties: From Table A.2, $S = 111K$.

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} * \frac{T_o + S}{T + S}$$

$$T_o = 15 + 273 = 288\,K$$

نحول (T) ⇐ من (C°) الى (K°)

$$T = 200 + 273 = 473\,K$$

$$\frac{\mu}{1.78 * 10^{-5}} = \left(\frac{473}{288}\right)^{3/2} * \frac{288 + 111}{473 + 111}$$

$$\boxed{\mu_{air} = 2.56 * 10^{-5}\ N.s/m^2}$$



Situation: Oil (SAE 10W30) fills the space between two plates. Plate spacing is $\Delta y = 1/8 = 0.125$ in.
Lower plate is at rest. Upper plate is moving with a speed $u = 25$ ft/s.

Find: Shear stress.

Properties: Oil (SAE 10W30 @ 150 °F) from Figure A.2: $\mu = 5.2 \times 10^{-4}$ lbf·s/ft$^2$.

Assumptions: 1.) Assume oil is a Newtonian fluid. 2.) Assume Couette flow (linear velocity profile).

(sher stress) المطلوب هنا

$$\boxed{\tau = \mu \frac{dv}{dy}}$$

$$\frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \frac{(25)\ Ft/s}{(0.125/12)\ Ft} = \boxed{2400\ 1/s}$$

$$\tau = (5.2 * 10^{-4})(2400) = \boxed{1.25\ Ib/Ft^2}$$

Situation: Air and water at 40 °C and absolute pressure of 170 kPa

Find: Kinematic and dynamic viscosities of air and water.

Properties: Air data from Table A.3, $\mu_{\text{air}} = 1.91 \times 10^{-5}$ N·s/m$^2$
Water data from Table A.5, $\mu_{\text{water}} = 6.53 \times 10^{-4}$ N·s/m$^2$, $\rho_{\text{water}} = 992$ kg/m$^3$.

* المطلوب (Kinemotic viscosity)
dynomic viscosity: ثابتة لكل مادة نجدها من خلال الجدول

* Air:

$\nu = \frac{\mu}{\rho}$ $\mu_{air} = 1{,}91 * 10^{-5}$ $\rho = \frac{P}{RT}$

P: Pa , T: Keliven

$\rho = \frac{170 * 10^3}{(40+273)(313,2)} = 1{,}89 \ kg/m^3$

$\nu = \frac{1{,}91 * 10^{-5}}{1{,}89} = 10{,}1 * 10^{-6} \ m^2/s$

* water: $\mu_{water} = 6{,}53 * 10^{-5}$

$\rho = 992$ ⟹ معطى بالسؤال

$\nu = \frac{\mu}{\rho} = 6{,}58 * 10^{-7} \ m^2/s$

Situation: Water flows near a wall. The velocity distribution is

$$u(y) = a\left(\frac{y}{b}\right)^{1/6}$$

where $a = 10$ m/s, $b = 2$ mm and $y$ is the distance from the wall in units of mm.

Find: Shear stress in the water at $y = 1$ mm.

Properties: Table A.5 (water at 20 °C): $\mu = 1.00 \times 10^{-3}$ N · s/m$^2$.

$\tau = \mu \frac{dv}{dy}$

$v(y) = 10\left(\frac{y}{2}\right)^{1/2}$

$\frac{dv}{dy} = \frac{10}{12}\left(\frac{2}{y}\right)^{5/6}$ ⟹ $y = 1$ نعوض

$\frac{dv}{dy} = 1485 \ s^{-1}$

$\tau = (1 * 10^{-3})(1485) = 1{,}485 \ Pa$

Situation: Water in a 1000 $\text{cm}^3$ volume is subjected to a pressure of $2 \times 10^6\ \text{N/m}^2$.

Find: Volume after pressure applied.

Properties: From Table A.5, $E = 2.2 \times 10^9$ Pa

$$E = -\Delta p \frac{V}{\Delta V}$$

$$\Delta V = -\frac{\Delta p (V)}{E} = \frac{-2 \times 10^6}{2.2 \times 10^9}(1000)$$

$$\boxed{\Delta V = -0.9091\ \text{cm}^3}$$

$$V_{Final} = V + \Delta V = 1000 - 0.9091 = \boxed{999.1\ \text{cm}^3}$$

Situation: Water is subjected to an increase in pressure.

Find: Pressure increase needed to reduce volume by 1%.

Properties: From Table A.5, $E = 2.2 \times 10^9$ Pa.

$$\boxed{\Delta V = -0.01 \cdot V}$$

$$E = -\Delta p \left(\frac{V}{\Delta V}\right)$$

$$\Delta p = -E \frac{\Delta V}{V} = -2.2 \times 10^3 \left(\frac{-0.01 V}{V}\right)$$

$$\boxed{\Delta p = 22\ \text{MPa}}$$

Situation: Very small spherical droplet of water.

Find: Pressure inside.

هنا تطبيق على (surface tension)

$$p(\pi r^2) = 2\pi r \sigma$$

$$\Rightarrow p = \frac{2\sigma}{r}$$

Situation: The application is a helium filled balloon of radius $r = 1.3\,\text{m}$.
$p = 0.89\,\text{bar} = 89\,\text{kPa}$.
$T = 22\,°\text{C} = 295.2\,\text{K}$.

Find: Weight of helium inside balloon.

Properties: From Table A.2, $R_{He} = 2077$ J/kg·K.

APPROACH

Weight is given by $W = mg$. Mass is related to volume by $m = \rho *$Volume. Density can be found using the ideal gas law.

ANALYSIS

Volume in a sphere

$$\begin{aligned}\text{Volume} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi 1.3^3\,\text{m}^3 \\ &= 9.203\,\text{m}^3\end{aligned}$$

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{89,000\,\text{N/m}^2}{(2077\,\text{J/kg}\cdot\text{K})\,(295.2\,\text{K})} \\ &= 0.145\ \text{kg/m}^3\end{aligned}$$

Weight of helium

$$\begin{aligned}W &= \rho \times \text{Volume} \times g \\ &= \left(0.145\ \text{kg/m}^3\right) \times \left(9.203\,\text{m}^3\right) \times \left(9.81\,\text{m/s}^2\right) \\ &= 13.10\,\text{N}\end{aligned}$$

$$\boxed{\text{Weight} = 13.1\ \text{N}}$$

# Ch3:fluid statics

**Pressure (p):** Normal force exerted by a fluid per unit area .

يؤثر الضغط في السوائل والغازات

اما في حالة (solid) يؤثر عليها stress

F=P/A

F:Normal force , A:Area

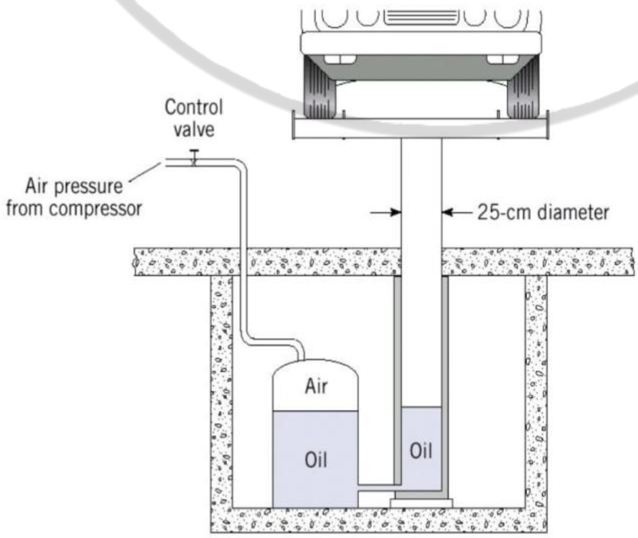
• Unit : N/$m^2$ (pa) , (psf) Ibf/$ft^2$,(psi) Ibf/$in^2$

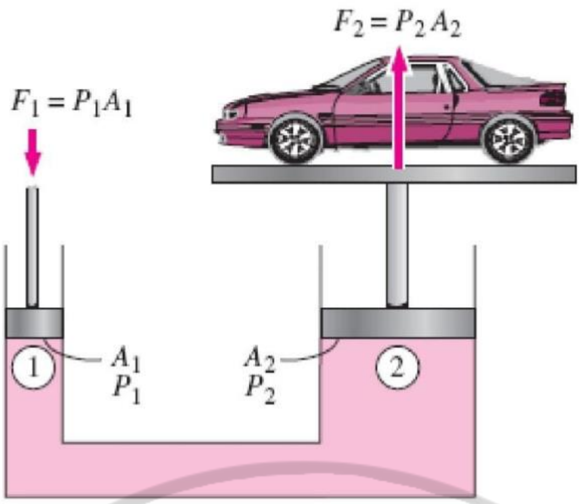
1kpa=$10^3$, 1Mpa=$10^6$ , 1Gpa=$10^9$ , 1bar=$10^5$

يمكن قياس الضغط من خلال:

1) pressure transducers
2) Bourdon – tube gages
3) Simple and differential manometers

## Pressure Transmission






سؤال سنوات على نص القانون

•Pascal law: A consequence of the pressure in a fluid remaining constant in the horizontal direction is that the pressure applied to a confined fluid increases the pressure throughout by the same amount

ينص القانون ان قيمة الضغط تكون ثابتة في المسافات الافقية (فقط في حالة statics)

$P_{abs} = p_{gage} + p_{atm}$

Absolute pressure ($p_{abs}$)

Gage pressure ($p_{gage}$):

هو الضغط الذي نقوم بمعرفته من خلال الساعة مثل: عند معرفة ضغط الهواء في الاطارات ويكون لها قيمتين

1) موجبة: عند ضغط الاطار 2) سالبة : سحب الهواء من الاطار

Atmospheric pressure ($p_{atm}$):

atmospheric pressure at sea level=101.3kpa

هذه القيمة حفظ نستخدمها في حلول الاسئلة

- If the atmospheric pressure is 101.3 kPa which is measured at sea level at T=23 oC:

1)gage pressure = 50kpa

2)gage pressure=-50kpa , find $p_{abs}$?

1) Pabs=pgage+patm=50+101.3=151.3kpa

2) Pabs=pgage+patm=-50+101.3=51.3kpa

- Pressure variation with elevation:

باسكال اكتشف انو الضغط بكون متساوي بالمسافات الافقية ولكن اكتشفوا العلماء ايضا ان الضغط يتغير قيمته بالمسافات العمودية كلما ارتفعنا للاعلى يقل الضغط

الضغط

مسافة عمودية vertical يتغير

مسافة افقية (horizontal) ثابت

تنقسم الموائع حسب density:

1) incompressible: $\rho$ is constant

2) compressible: $\rho$ not constant

لحساب الضغط في حالة (incompressible fluid):

نستخدم العلاقة:

$p + \gamma z = \text{constant}$ **OR** $\frac{p}{\gamma} + z = constant$

هنالك قانون اخر وهو افضل للحل واسرع

$P + \gamma h = p_2$

Z,h:الارتفاع

سنقوم بتوضيح طريقة استخدام القوانين بالامثلة

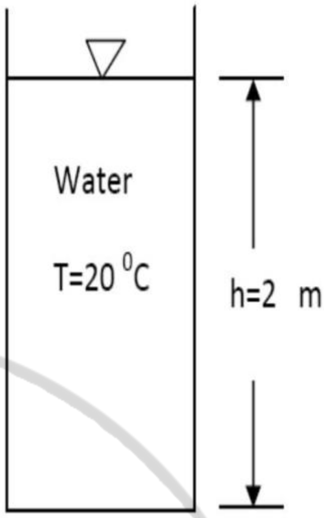
## - Pressure Variation for Compressible Fluids:

- For Ideal gas: $p=\rho RT$ or $\rho=p/RT$
- Multiply by g: $\rho g=pg/RT$, then $\gamma=pg/RT$
- $\gamma$ =fn (p, T) • $dp/dz = -\gamma$ = fn (p, T)

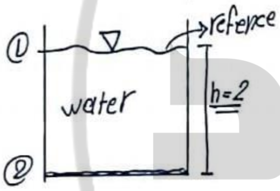
$\gamma=\frac{pg}{RT}$

**Example:** Find the pressure at the tank bottom.

$\gamma$ =9790 N.m³ (Table A.5)




* في هذا السؤال طلب قيمة الضغط في اسفل الخزان وكما تعلمنا تتغير قيمة الضغط عند قطع مسافة عمودية




* اول خطوة نقوم بها: نحدد الـ (refrence) لا يهم المكان الذي نختار فية مكان الـ (refrence) نضعه على السطح للسهولة

$$\frac{P_1}{\gamma} + Z = \frac{P_2}{\gamma} + Z$$

* في هذا القانون عندما نتحرك الى اسفل (refrence) تكون الـ (Z) سالبة واذا التحرك للاعلى تكون موجبة

refrence
-ve(Z)

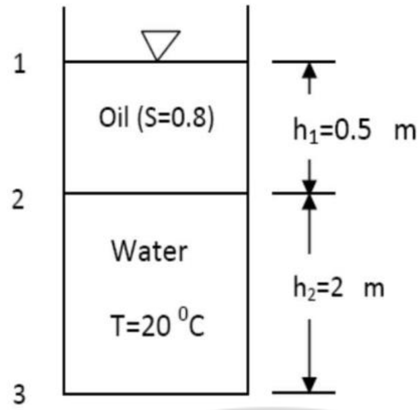
ولانه على السطح (مفتوح على الجو) ⇒ $P_1 = zero$

$$\frac{0}{\gamma} + 0 = \frac{P_2}{9790} - 2 \Rightarrow P_2 = 2(9790) = 19{,}58\ kPa\ (gauge)$$

* نلاحظ هنا احنا بنستخرج (gauge)
لو بدي (Pabs):-

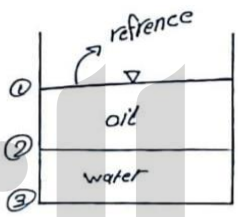
$P_{abs} = 19{,}58 + 101{,}3$

**Example:** Find the pressure at the tank bottom




* نلاحظ وجود اختلاف في السائل فيكون هنالك اختلاف في قيمة ( $\gamma$ )

Ⓘ تحديد ال (refrence)
Ⓘ Ⓘ استخدام قانونين




* الطريقة الاولى :

refrence ↓ -ve (z)

$$\frac{P_1}{\gamma_{oil}} + Z_1 = \frac{P_2}{\gamma_{oil}} + Z_2 \Rightarrow \frac{P_2}{\gamma_{water}} + Z = \frac{P_3}{\gamma_{water}} + Z$$

* الطريقة الثانية :

$$P_1 + \gamma_{water} h + \gamma_{oil} h = P_3$$

نبدأ بالطريقة الاولى :- نلاحظ ان ( $\gamma_{oil}$ ) مجهولة

$$\gamma_{water} = 9810$$

$$S = \frac{\gamma_{oil}}{\gamma_{water}} \Rightarrow \gamma_{oil} = (0.8)(9810) = 7848$$

$$\frac{P_1}{\gamma_{oil}} + Z_1 = \frac{P_2}{\gamma_{oil}} + Z_2 \Rightarrow P_2 = 3,924 \, KPa$$

($P_1$ zero, $Z_1$ zero)

$$\frac{P_2}{\gamma_{water}} + Z_2 = \frac{P_3}{\gamma_{water}} + Z$$

نطلعها من الجدول ( $\gamma_{water}$ )

$$\frac{3,924}{9790} + 0 = \frac{P_3}{9790} + (-2) \Rightarrow P_3 = 23,504 \, KPa$$

* الطريقة الثانية :-

في هذه الطريقة نهتم في البداية و النهاية

نبدأ من عند النقطة (1) و ننتهي عند النقطة (3)

refrence

$\downarrow (+ve)(\gamma)$

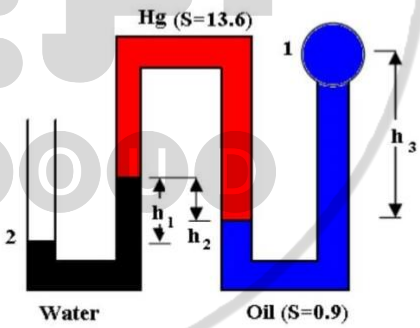
$$P_1 + \gamma_{oil} h + \gamma_{water} h = P_3$$

$$\boxed{P_1 = zero}$$

$$0 + (7848)(0,5) + (9790)(2) = P_3$$

$$P_3 = 23,504 \, kPa$$

**Example**: Find the pipe pressure if $h_1$=1.2 m, $h_2$=1 m, and $h_3$=3 m



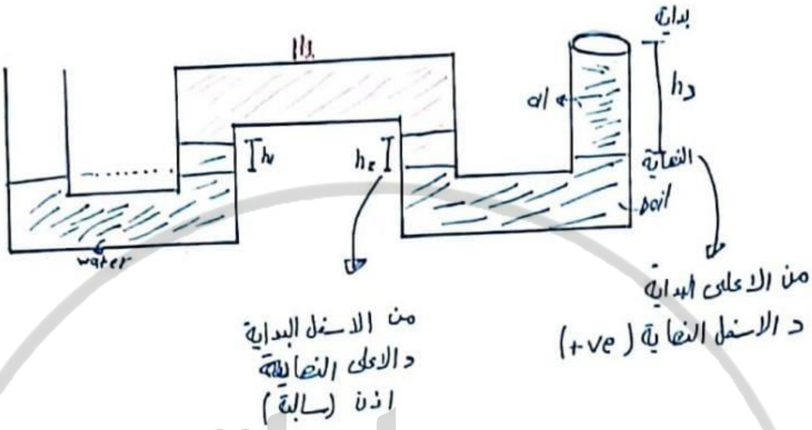
*sol :-

المطلوب (P₁)

ح نمشي في مسار بحيث البداية عند النقطة (1) والنهاية عند النقطة (2)

$$P_3 + \gamma_{oil} h_3 - \gamma_{Hg} h_2 + \gamma_{water} h_1 = P_1$$

refrence (point) ↓ +/-ve (δ)




* بدي اجد مقدار (γ) لكل (Hg) و (oil) من خلال (S) نجدها

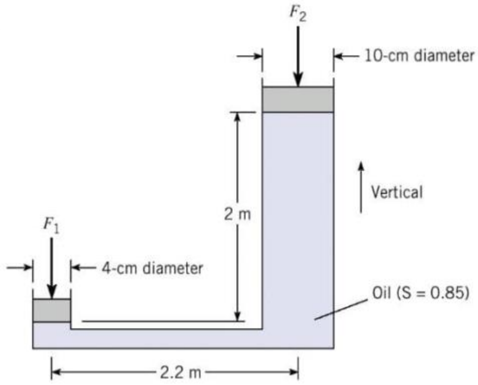
$$S = \frac{\gamma_{oil}}{\gamma_{water}} \qquad \gamma_{oil} = (0,9)(9810) \qquad S = \frac{\gamma_{Hg}}{\gamma_{water}} \Rightarrow \gamma_{Hg} = (3,6)(9810)$$

$\gamma_{water} = 9810$ ⟹ نأخذ (standar) لأنه مش محدد درجة الحرارة

$$P_1 + (0,9)(9810)(3) - (13,6)(9810)(1) + (9810)(1,2) = 0$$

$$P_1 = 95,157 \text{ Kpa}$$

**Example:** Given $F_1$= 200 N, Find the $F_2$. Neglect the weights of the pistons




sol.:

* هنا يريد مقدار $(F_2)$ $\boxed{F = P*A}$

نقوم بايجاد $(P_2)$ ثم نعوضها في القانون

$\boxed{P_1 - \gamma h = P_2}$

refrance +(ve) Z

$P_1 = \frac{F}{A} = \frac{200}{\frac{\pi}{4}(0.04)^2} = 159.23\ KPa$

$159.23*10^3 - (0.85*9810)(2) = P_2$

$\boxed{P_2 = 142.558\ KPa}$ $F_2 = (142558)(\frac{\pi}{4})(0.1)^2 = 1.119\ KN$

# •Pressure Measurements:

## Manometry:

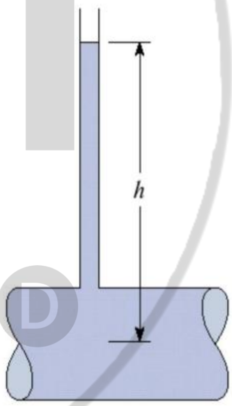
الشكل العام له: هو عبارة عن tube اقطاره صغيرة مصنوع من مادة شفافة مثل البلاستيك والزجاج يستخدم للضغوط المنخفضة

يعتمد بشكل اساسي في قياس الضغط هو تغيره بالمسبة للارتفاع

وله اشكال عدة :

a: Piezometer:

A simple manometer, or a piezometer can be attached to a pipe and the height of the liquid's column is an indicator of the pressure in the pipe

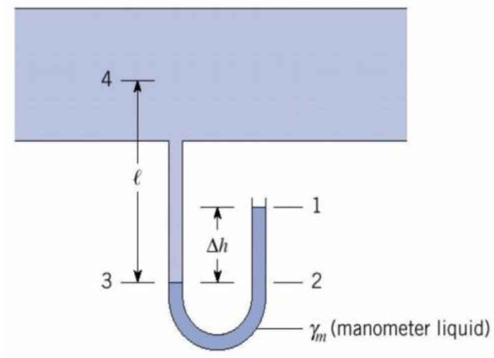



لو طلب الضغط بالاسفل

$P_1+\gamma h=p_2$ , $p_1=$ zero لانه مفتوح على الجو

**$P_2=\gamma h$**

b: U-tube Manometers :



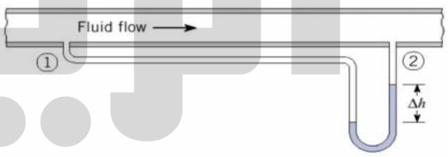

لمعرفة الضغط عند النقطة 4

$P_1+\gamma_3\ h_3-\gamma_4\ h_4=p_4$ , $h_4=L$

**$P_4=\gamma 3\ h3-\gamma 4\ L$**



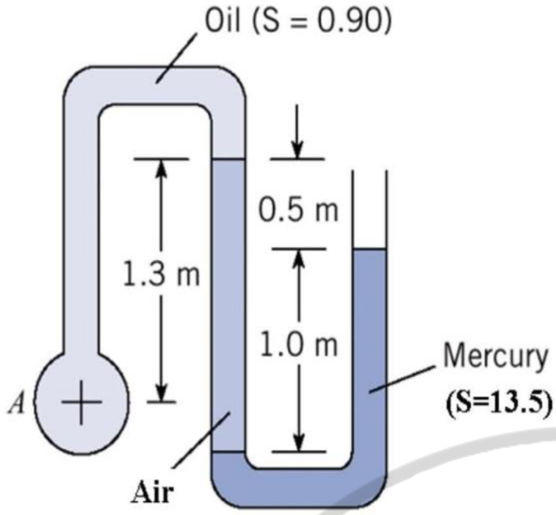
c: Differential Manometers




يستخدم لقياس الضغط بين نقتطين داخل pipe

$$\Delta p=(\gamma_m-\gamma_f)\,\Delta h$$

**Example:** Find the pipe pressure.




الفكرة في هذا السؤال هي ان $\boxed{\gamma_{air} \approx 0}$

sol.: هي لا تساوي صفر بالضبط ولكن نعوض قيمتها صفر بسبب ان قيمة $(\gamma)$ كبيرة جداً مقارنة بـ $\boxed{\gamma_{air} = 1}$ فنعوضها (zero)

$\downarrow (+ve)$

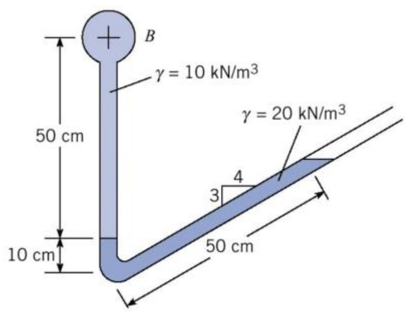
$$\boxed{P_A - \gamma_{oil} h + \gamma_{air} h - \gamma_{mercury} h = 0}$$

$$\boxed{S = \frac{\gamma_{Fluid}}{\gamma_{water}}}$$

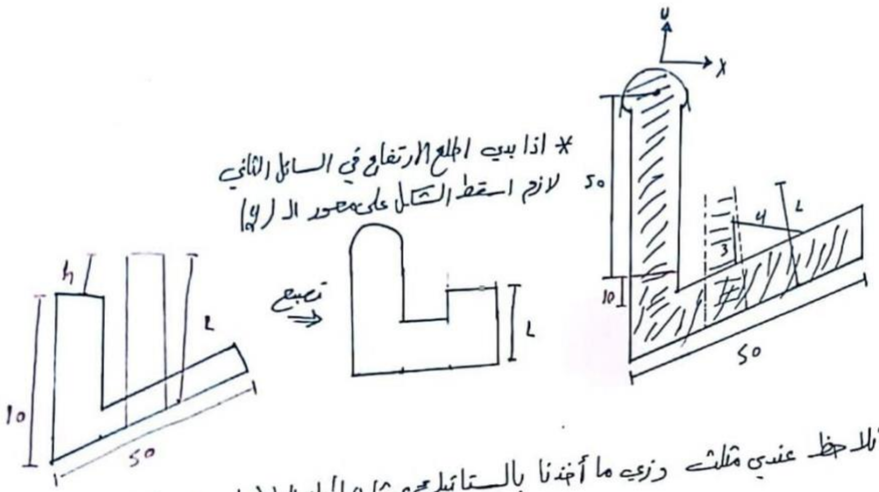
$$P_A - (0.9)(9810)(1.3) + 0 - (13.5)(9810)(1) = 0$$

$$\boxed{P_A = 143.912 \text{ KPa}}$$

- **Example:** Find the pipe pressure



* اذا بدي اطلع الارتفاع في السائل الثاني لازم اسقط الشكل على محور ال (y)




* نلاحظ عندي مثلث وزي ما أخذنا بالستاتيك عشان اطلع الطول على محور (y) :-

طول الجسم المائل * $\frac{\text{الرقم الموازي لمحور (y)}}{\text{الوتر}}$

الوتر $= \sqrt{(3)^2+(4)^2} = 5$

$L = (0.5)\left(\frac{3}{5}\right) = 0.3\,m$

$h = 0.3 - 0.1 = 0.2\,m$

$P_B + (10*10^3)(0.5) - (20*10^3)(0.2) = 0$

$P_B = -1\,KPa$

## 2)Bourdon-Tube Gage:

يعد من اكثر الطرق شيوعا في قياس الضغط

وهو عبارة عن ساعة وبداخلها مؤشر عند ضغط الابرة يتحرك المؤشر

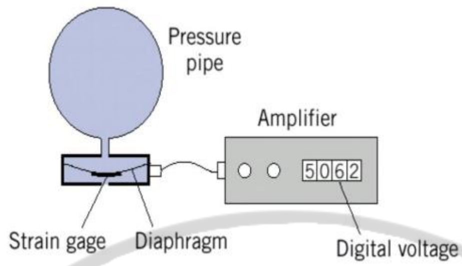
طريقة ميكانيكية

يستخدم للضغوط العالية ويتم قياس ضغط الاطارات من خلاله

## 3) Pressure Transducers:

مبدأ عمله : يقوم بتحويل الضغط الى اشارات كهربائية ويعد من اكثر الاجهزة دقة

يستخدم للضغوط العالية (طريقة كهربائية)



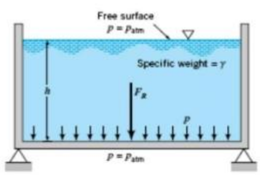

## • Hydrostatic Forces on Plane Surfaces:

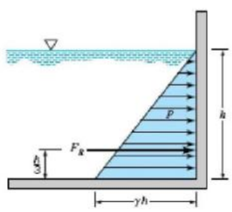
هو عبارة عن الضغط الناتج من السوائل يؤثر على البوابات او جدران السدود ولمقاومة هذه القوى بالسدود نقوم بتسليح الجدران

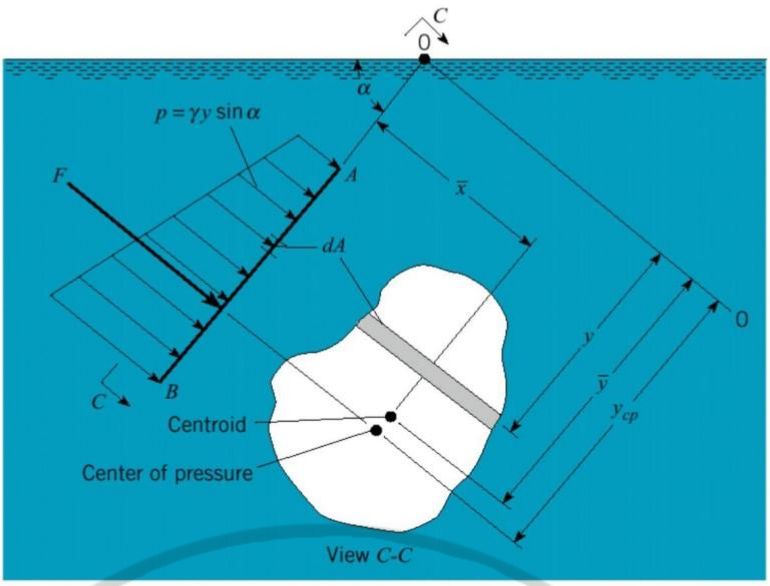
في هذا الموضوع رح نتعلم طريقة حساب ال force الناتجة من السوائل على الاسطح المستوية

في الاسطح الافقية يكون الضغط متساوي على طول السطح وتكون المحصلة في المركز

اما في الاسطح العمودية او المائلة (inclined surface) يكون الضغط غير متساوي والمحصلة القوى لاتكون بالمركز








ملاحظات على حل الاسئلة :

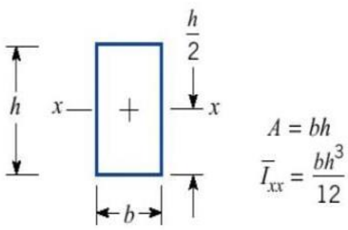
1) يكون محور (x) افقي و يتقاطع مع الجسم ومحور (y) عمودي وموازي للجسم
2) عند اعلى نقطة على السطح نضع محور (x)
3) $\alpha$: هي الزاوية المحصورة بين محور (x,y)
4) $\bar{y}$: هي المسافة من الصفر الى منتصف الشكل centroid
5) $y_{cp}$: هي المسافة من الصفر الى center of pressure ونخرجها من خلال المعادلات

$$F = \gamma\, \bar{y}\, A\, \sin\alpha = \bar{p}\, A$$

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}\, A}$$

I: second moment of area

فقط مطلوب منا حفظ (I) للمربع



* طريقة اخرى لحل الامثلة على هذا الموضوع

(I) نجد (P) :- $P=\gamma Z$

Z : المسافة من سطح الماء الى منتصف طول البوابة

(II) نجد (F) :- $F=P \cdot A$

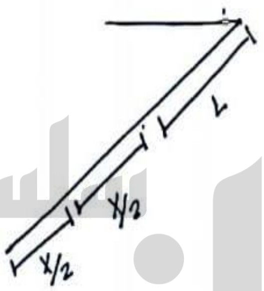
A :- مساحة البوابة

(III) نحدد موقع (F) من خلال :- نجد ($\bar{y}$) ثم نجد ($\bar{y}_{cp}$)

نأخذ مسافة من منتصف البوابة الى سطح الماء :- $\bar{y}$

$$\bar{y} = \frac{X}{2} + L$$

$$\bar{y}_{cp} - \bar{y} = \frac{I}{\bar{y}A}$$

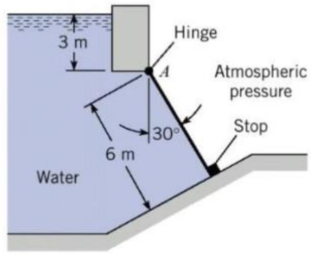


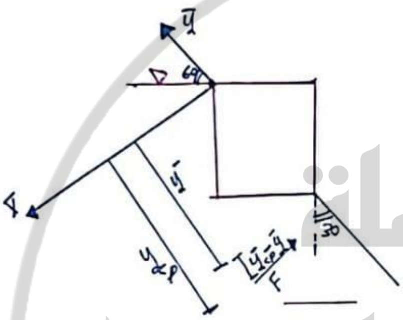

(IV) نعمل F.B.D للبداية :- نأخذ (w.b Area) :-

بحيث يكون شكل البوابة عبارة عن مسقط اذا كانت مستطيل مثلاً

نأخذ طول البوابة * العمق

- **Example:** The gate shown is rectangular and has dimensions *6 m* x *4 m*. What is the reaction at point A? Neglect the weight of the gate.




① اولخطوة :- نرسم F.B.D

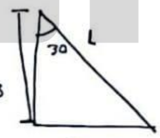
* نريد إيجاد (wgHros tails Forces)

$F = \bar{P}A$ , $\bar{P} = \bar{y}\gamma\sin\alpha$

① نجد $(\bar{P})$ :-

$\bar{y} = L+3 = 3.464+3$

$\boxed{\bar{y} = 6,464 m}$



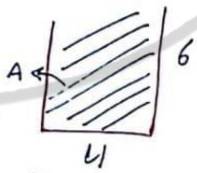

$\cos 30 = \frac{3}{L}$

$\boxed{L = 3,464 m}$

$\bar{P} = (6,464)(9810)(\sin 60)$

$\boxed{\bar{P} = 5491626 Pa}$

$F = (5491626)(6)(4) = \boxed{131799 0m}$




* الان نريد تحديد موقع (F) من خلال إيجاد $(y_{cp})$

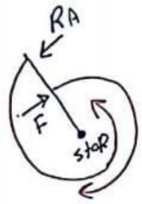
$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$ , $I = \frac{1}{12}bh^3 = (1/12)(4)(6)^3 = 72$

$y_{cp} - \bar{y} = \frac{72}{(6,464)(4)(6)} = 0,46 m$

$\Sigma M_{stop} = 0$

$F(3 - 0.464) = RA * 6$

$\boxed{RA = 5571 w}$




* كيف تحدد الاشارة في ~~كوننت~~ المومنت

* نلاحظ ان (RA) يدور عكس عقارب الساعة حول (stop) اذن يكون موجب و (F) تدور مع عقارب الساعة اذن سالب

*طريقة اخرى لحل المثال:-

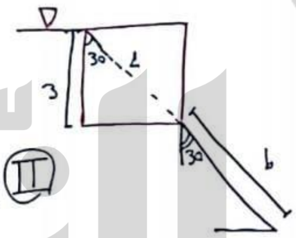
(I) نجد (P): $\boxed{P = \gamma Z}$

المسافة من سطح الماء الى منتصف البوابة: Z

*نحلل طول البوابة على محور (y) ثم نأخذ منتصف الطول

$Z = 3 + \boxed{\frac{6\cos 30}{2}} = 5.598 m$

$P = (9810)(5.598) = \boxed{54917 KN}$




(II) نجد (F):- $\boxed{F = PA}$ ⟸

$F = (549.17)(4)(6) =$

$\Rightarrow \boxed{F = 13180}$

(III) نجد موقع (F) من خلال ايجاد ($\bar{y}$) ثم ($\bar{y}_{cp}$)

نأخذ مسافة من منتصف البوابة الى سطح الماء: $\bar{y}$

$\bar{y} = \boxed{3} + L = 6.464 m$ ($\boxed{\frac{6}{2}}$)

$\bar{y}_{cp} - \bar{y} = \frac{I}{\bar{y}A} = 0.464 m$

$\cos 30 = \frac{3}{L}$

$\boxed{L = 3.464}$


* لايجاد (Force) نأخذ البوابة ونضع الـ (Force) التي تأثر عليها

$\Sigma M_{stop} = 0 \quad F(6 - 3.464) + R_A(6) = 0$

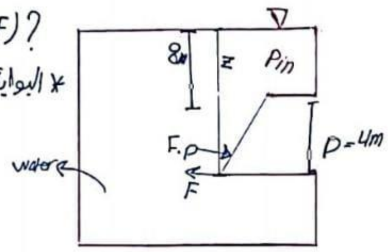
$\boxed{R_A = 5571 w}$

* مثال على البوابات المستوية (Plane surface)

Ex:

Find (F)?

* البوابة ذات الشكل البيضاوي



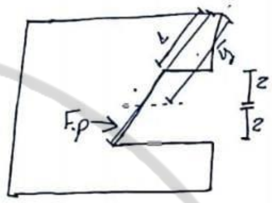

(I) $P = \gamma z = (9810)(8+2) = 98,1\ KPa$

(II) $F_P = PA = 98,1 \times 10^3 \times 15,71$

$A = \pi a . b$

$A = \pi\left(\frac{5}{2}\right)\left(\frac{4}{2}\right)$

$= 15,71\ m^2$

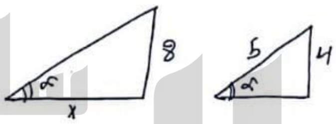



(III) موقع (FB)

نجد ($\bar{y}$) ثم ($\bar{y}_{cp}$)

$\bar{y} = 2,5 + L$

$\sin\alpha = \frac{4}{5} = \frac{8}{x} \Rightarrow x = 10$

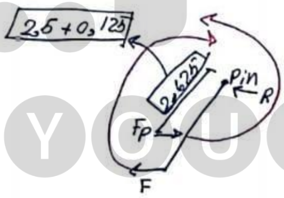



$\bar{y} = 12,5\ m$

$\bar{y}_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{\frac{\pi a^3 b}{4}}{\bar{y}\ A} = \frac{\frac{\pi(2,5)^2(2)}{4}}{(19,5)(15,71)}$

تأخذ بالجدول ← لا تحفظها

$\bar{y}_{cp} - \bar{y} = 0,125\ m$




$\circlearrowleft + \Sigma M_{pin} = 0$

$F_P(2,625) - F(5) = 0$

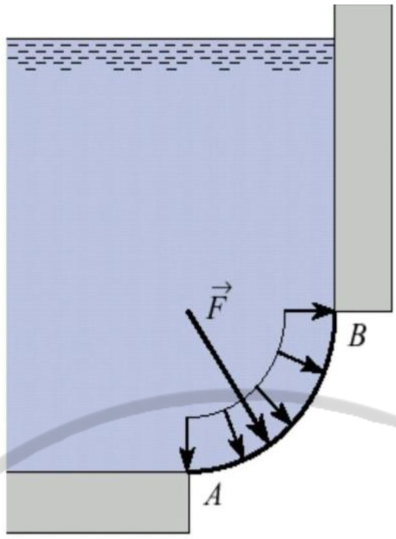
$F = 809\ KN$

# •Hydrostatic Forces on Curved Surfaces:

كنا في السابق نتعامل مع بوابات مستوية وكنا نحسب القوى المؤثرة عليها من ضغط السوائل والان رح نتعامل مع بوابات ذات اسطح منحنية

**ملاحظة:** البوابات في الاسئلة تكون على شكل مساقط ولحساب المساحة نأخذ طول البوابة والعمق

سبب صعوبة هذه المسائل هو ان توزيع الضغط ليس خطي ولذلك سنقوم بتحويل سطح البوابة من منحني الى مستوي




•خطوات حل الاسئلة :

1) نحولها من سطح مائل الى مستوي ونرسم FBD:

Horizontal force component on curved surface:

$$F_{horizontal} = F_{AC} = \bar{p}\,A$$

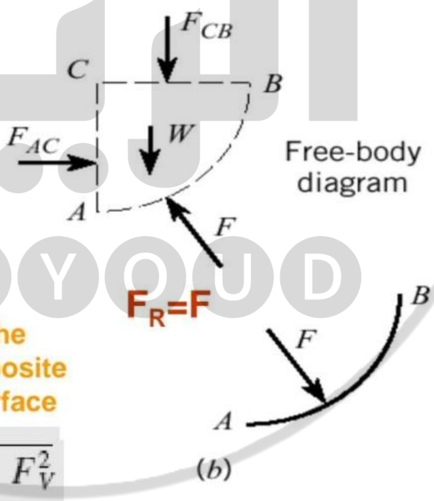
Vertical force component on curved surface:

$$F_{vertical} = W + F_{CB}$$

The resultant hydrostatic force acting on the curved solid surface is then equal and opposite to the force acting on the curved liquid surface (Newton's third law).

$$F_R = \sqrt{F_H^2 + F_V^2}$$

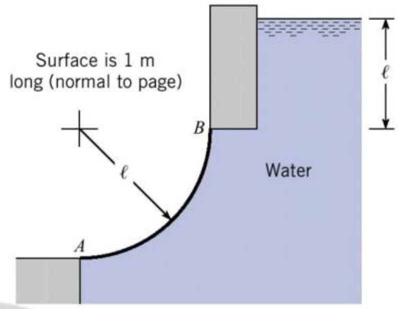
$$\tan\theta = F_V/F_H$$

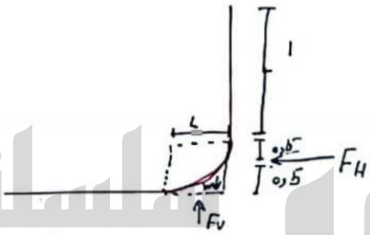



Fv: تكون دائما بال centroid , **w=γv**

V: volume , w: يكون تأثيرها بمنتصف الشكل

- **Example:** Find the vertical and horizontal forces on the given gate (L=1 m).




sol:

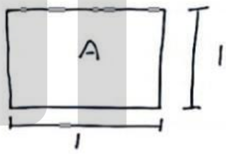
$F_H = PA \Rightarrow P = \gamma z = (9810)(1+0.5)$

$P = 14.715 \text{ kPa}$

$\bar{y} = 1 + 0.5 = 1.5 m$

$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$ , $I = \frac{1}{12} bh^3 = (\frac{1}{12})(1)(1)^3$

$I = 1/12$




$\bar{y}_{cp} = 1.5 + \frac{1/12}{(1.5)(1)^2}$

$\bar{y}_{cp} = 1.555 m$

$F_1 = PA \Rightarrow P = \gamma h = (9810)(2) = 19620 \text{ Pa}$

$F_1 = (19620)(1)^2 = 19620 N$

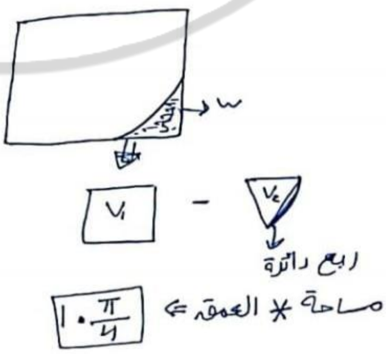
$F_V = F_1 - w$

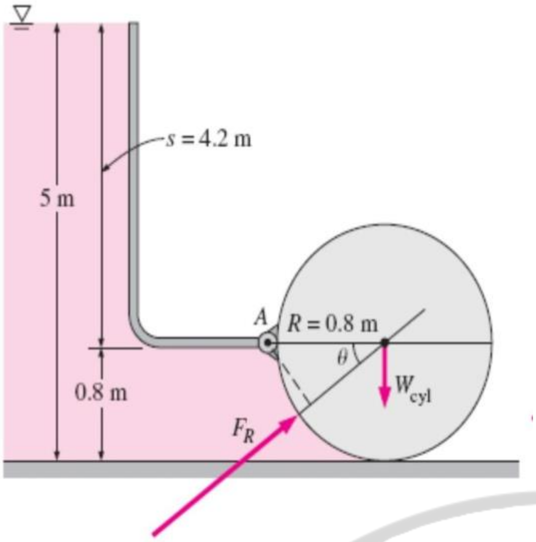
$w = \gamma V = \gamma (V_1 - V_2)$

$= \gamma (1*1*1) - \frac{\pi}{4}\frac{(2L)^2}{4}(1)$

$W = 2109.15 N$

$F_V = F_1 - w = 17.51 kN$



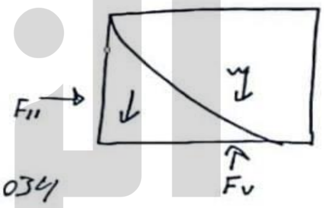



E

Find $F_H$, $F_v$, w For cylinder?
a 2m long cylinder R= 0.8

$F_H = PA = \gamma Z A = (4{,}2 + \frac{0{,}8}{2})(9810)(0{,}8)(2)$

$\Rightarrow$ $\boxed{F_H = 72{,}2\ KN}$




$\bar{y} = 4{,}6$

$\bar{y}_{cp} = \bar{y} + \frac{I}{\bar{y} A}$ , $I = (1/12)(0{,}8)(0{,}8)^3 = 0{,}034$

$\bar{y}_{cp} = 4{,}6 + \frac{0{,}034}{(4.6)(0{,}8)(2)} = \boxed{4{,}6046 m}$

$F_v = F_1 - w$

$F_1 = PA \Rightarrow P = \gamma Z A = (9810)(5)(0{,}8)(2)$

$\boxed{F_1 = 78{,}48\ KN}$

$w = \gamma V = \gamma (V_{\square} - V_{\bigcirc})$

$= 9810((0{,}8)^2(2) - \frac{\pi}{4}(0{,}8)^2(2))$

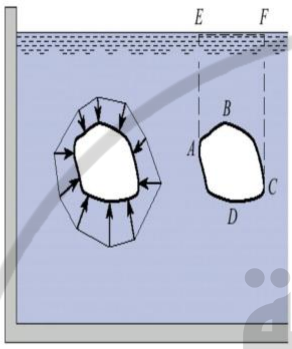
$\boxed{w = 26{,}9\ KN}$

$\boxed{F_v = 51{,}58\ KN}$

## Buoyancy (الطفو):

عند وضع كرة من الحديد واخرى من الخشب على سطح الماء نلاحظ ان الحديد يهبط للاسفل والخشب يطفو للاعلى وفي الماضي فسرنا هذه الظاهرة بسبب اختلاف الكثافات بحيث كثافة الحديد اكبر من كثافة الماء اما كثافة الخشب اقل ولكن ليس هذا هو السبب الوحيد

بحيث اكتشفوا ان سطح الماء يؤثر بقوة على الجسم وتكون عكس اتجاه الوزن تعمل على رفع الجسم وتسمى **(Buoyant force)**

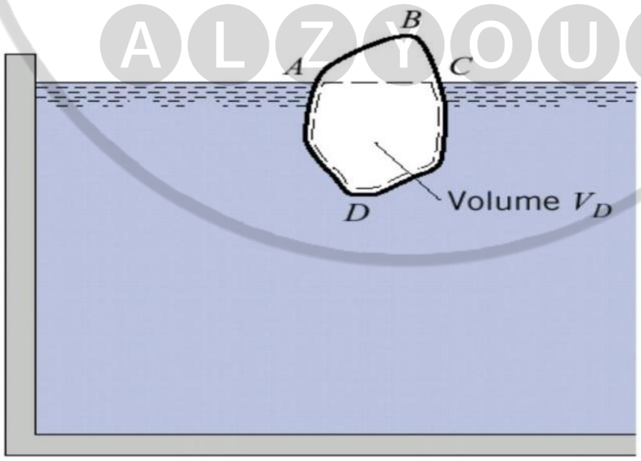



هنا الضغط عند نقطة D اكبر من نقطةB

لحساب Buoyant force:

$F_B = V_{body}\,\gamma_{fluid}$

V: volume (للجزء المغمور تحت الماء)



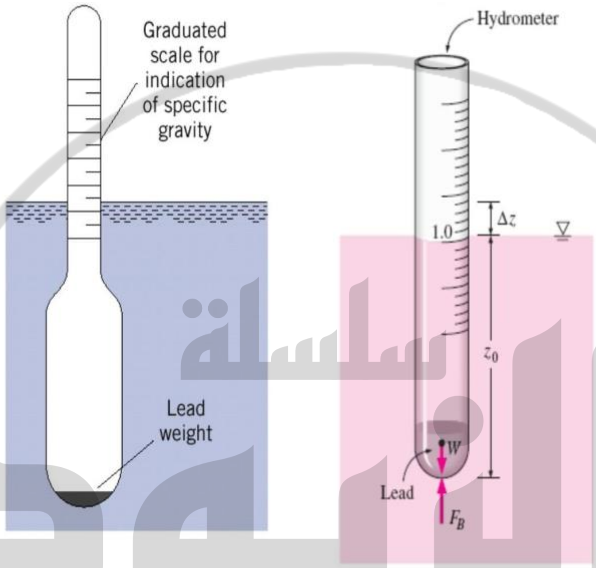

**A body partially submerged in a liquid**

$$F_B = \gamma_{fluid}\ V_D$$

Archimedes' principle: "For an object partially or completely submerged in a fluid, there is a net upward force equal to the weight of the displaced fluid."

**•Hydromety**:

Device [glass bulb] to measure the $\gamma$ or S of a liquid based on the principle of buoyancy

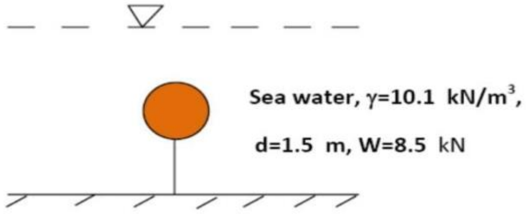



قوانين مهمة لمعرفتها:

حجم الكرة= $\frac{4}{3}\pi r^3$ , حجم اسطوانة = $r^2\pi h$

الحجم = مساحة القاعدة * الارتفاع

- **Example:** Find the tension in the given figure.




* نرسم F.B.D :-

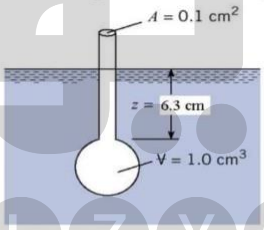

$\Sigma F_y = 0 \Rightarrow F_B - W - T = 0$

$F_B = \gamma_{Fluid} V$ $\quad$ $\boxed{V = \frac{4\pi r^3}{3}}$ $\quad$ حجم الدائرة = $\frac{\pi d^3}{6}$

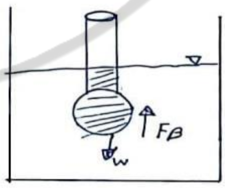
$F_B = (10.1)\frac{\pi(1.5)^3}{6} = \underline{17.85\,KN}$

$T = F_B - W = 17.85 - 8.5 = \underline{9.35\,KN}$

- **Example:** find the specific gravity of the given unknown fluid, the hydrometry weight is 0.015 N.




* نرسم F.B.D :-




$\Sigma F_y = 0 \Rightarrow F_B = W = \underline{0.015\,N}$

$S = \frac{\gamma_{oil}}{\gamma_{water}}$

$F_B = \gamma_{oil}(V) = \gamma_{oil}\left(\underbrace{(1)(10^{-2})^3}_{V_0} + \underbrace{(0.1)(10^{-2})(6.3)(10^{-2})}_{V_{cylinder}}\right)$

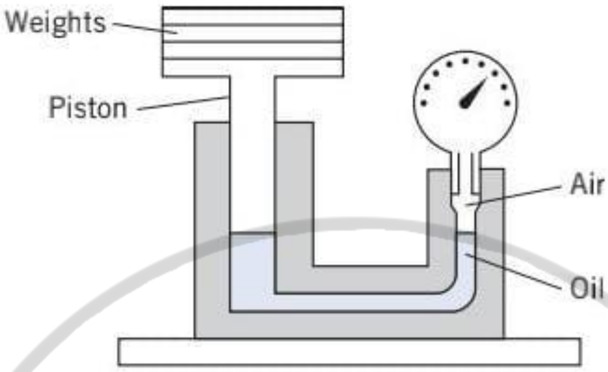
$\boxed{\gamma_{oil} = 9202\,N/m^3}$

$V_{cylinder}$ = مساحة القاعدة * الارتفاع

$\boxed{A = 0.1\,cm^2}$ ⇐ معطى بالسؤال

$\boxed{S = \frac{9202}{9810} = 0.938}$

**3.4** The Crosby gage tester shown in the figure is used to calibrate or to test pressure gages. When the weights and the piston together weigh 140 N, the gage being tested indicates 200 kPa. If the piston diameter is 30 mm, what percentage of error exists in the gage?




PROBLEM 3.4



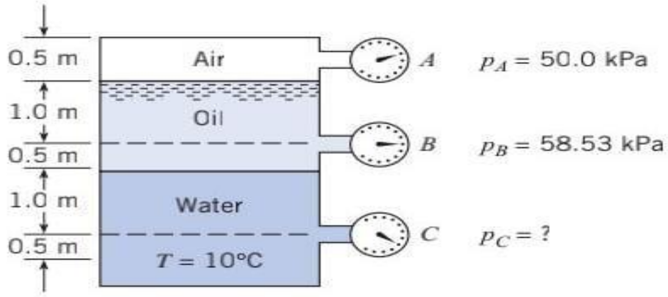
$$Error\% = \frac{(P_{recordy} - P_{tube})}{P_{tube}} * 100\%$$

$$P_{tube} = \frac{F}{A} = \frac{140}{\frac{\pi}{4}(0,03)^2} = \boxed{198{,}059\ kPa}$$

$$Error\% = \frac{200-198}{198} * 100\%$$

$$\boxed{Error\% = 1{,}01\%}$$

**3.11** For the closed tank with Bourdon-tube gages tapped into it, what is the specific gravity of the oil and the pressure reading on gage *C*?




PROBLEM 3.11

$P_1 + \gamma h = P_2$

$50 + \gamma(1) = 58{,}53$

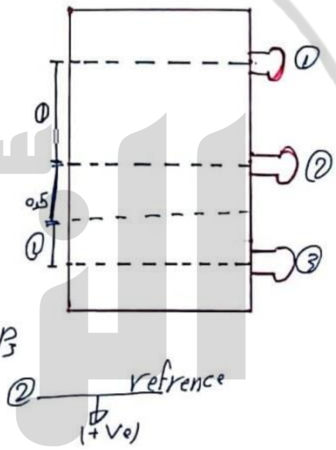
$\boxed{\gamma = 8{,}53\ kN/m^3}$

$S_{oil} = \frac{\gamma_{oil}}{\gamma_{water}} = \frac{8530}{9810} = 0{,}87$

$P_2 + \gamma_{oil} h + \gamma_{water} h = P_3$

$58{,}53 + (8{,}53)(0{,}5) + (9810)(1) = P_3$

$\boxed{P_3 = 72{,}6\ Kpa}$






**3.12** This manometer contains water at room temperature. The glass tube on the left has an inside diameter of 1 mm *(d* = 1.0 mm). The glass tube on the right is three times as large. For these conditions, the water surface level in the left tube will be (a) higher than the water surface level in the right tube, (b) equal to the water surface level in the right tube, or (c) less than the water surface level in the right tube. State your main reason or assumption for making your choice.

Ans : **a**

**3.14** Some skin divers go as deep as 50 m. What is the gage pressure at this depth in fresh water, and what is the ratio of the absolute pressure at this depth to normal atmospheric pressure? Assume $T = 20°C$.

$P_{gage} = \gamma z = (9790)(50) = 489{,}5\ KPa$

← تكون معطى بالسؤال

$$\frac{P_{abs}}{P_{atm}} = \frac{489{,}5 + 101{,}3}{101{,}3} = 5{,}83$$

**3.15** Water occupies the bottom 1.0 m of a cylindrical tank. On top of the water is 0.75 m of kerosene, which is open to the atmosphere. If the temperature is 20°C, what is the gage pressure at the bottom of the tank?



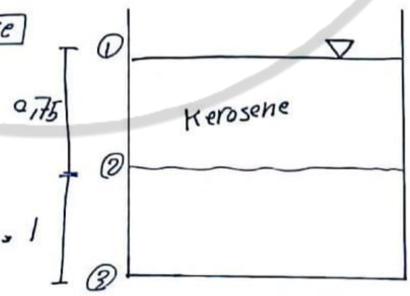
* فكرة هذا السؤال ان هنالك خزان وضع داخل مادتين:

1) water 2) kerosene

$\gamma_{water} = 9790$ , $\gamma_{kerosene} = 8010$ → معطى في السؤال

$P_{bottom} = P_3$

refrence ↓ +ve

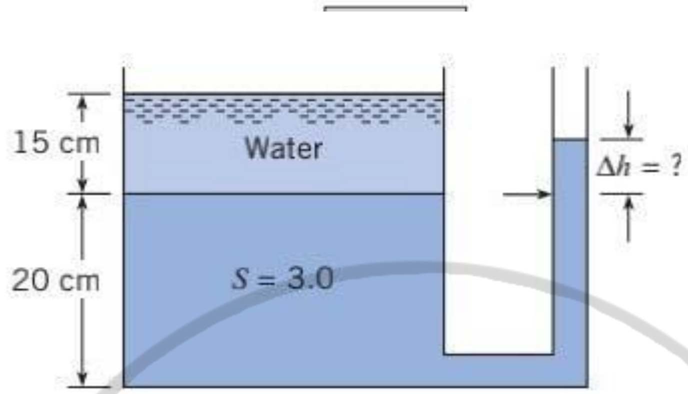



$P_1 + \gamma_1 h_1 + \gamma_2 h_2 = P_3$

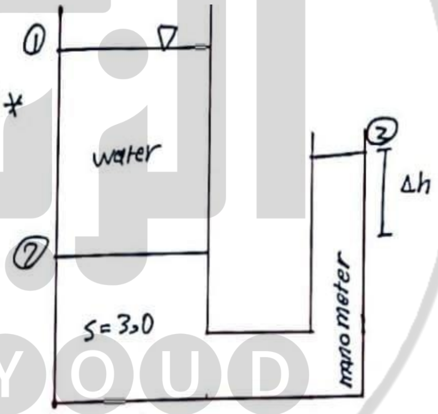
$(8010)(0{,}75) + (9790)(1) = P_3$

$P_3 = 15{,}797\ KPa$

**3.18** A tank is fitted with a manometer on the side, as shown. The liquid in the bottom of the tank and in the manometer has a specific gravity (S) of 3.0. The depth of this bottom liquid is 20 cm. A 15 cm layer of water lies on top of the bottom liquid. Find the position of the liquid surface in the manometer.




PROBLEM 3.18




* لإيجاد مقدار التغير في ارتفاع السائل

$P_1 + \gamma_1 h_1 - \gamma h_2 = P_3$

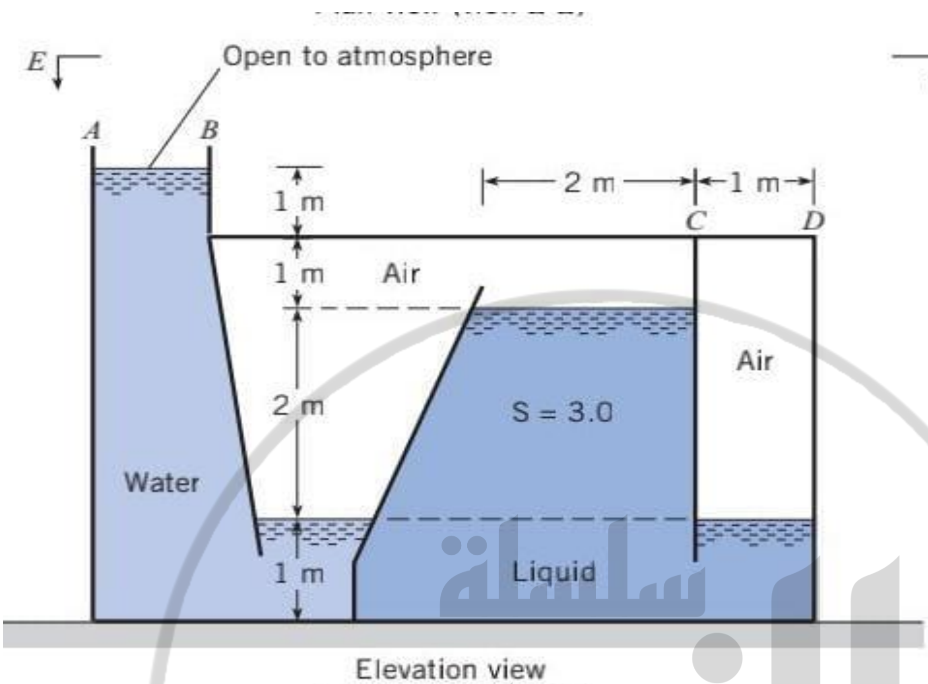
$P_1, P_3 = 0$ ⇒ لأنهم مفتوحين على السطح

$h_2 = \Delta h$ ⇒ لأنه يعني البداية والنهاية والفرق في المسافة بين النقطة ② والنقطة ③

$(9810)(0.15) - (9810)(3)(\Delta h) = 0$

$\Delta h = 0.05 m = 5 cm$

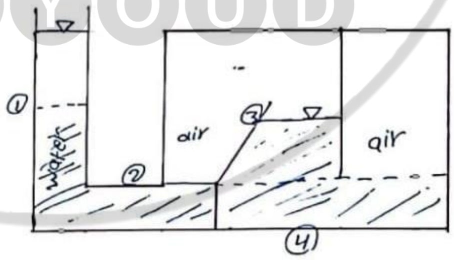
**3.21** What is the maximum gage pressure in the odd tank shown in the figure? Where will the maximum pressure occur? What is the hydrostatic force acting on the top ($CD$) of the last chamber on the right-hand side of the tank? Assume $T = 10°C$.




PROBLEM 3.21



* يزداد الضغط كلما تحركنا للاسفل، فإن ($P_m$) رح تكون في اسفل الخزان




$\overset{zero}{P_1} + \gamma_1 h_1 + \overset{zero}{\gamma_{air} h} + \gamma_3 h = P_4$

$(9810)(4) + (3)(9810)(3) = P_4$

$P_4 = 127,5\ kPa$

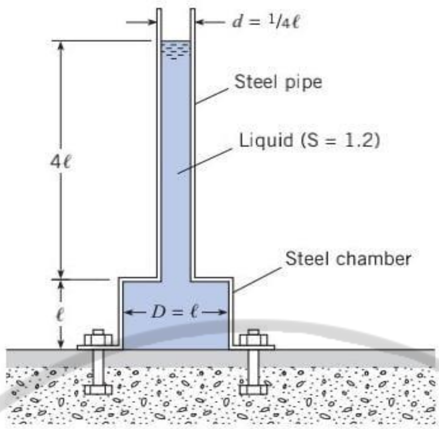
$F_{CD} = PA$

$P_{CD} + \gamma h + \gamma_{1+4} h = P_4$

$P_{CD} = 127,5 - (1)(3)(9810) = 98,07\ kPa$

$F_{CD} = (98,07)(1)^2 = \boxed{98,07\ kN}$

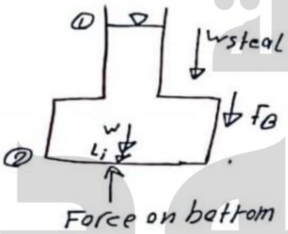
**3.22** The steel pipe and steel chamber shown in the figure together weigh 600 lbf. What force will have to be exerted on the chamber by all the bolts to hold it in place? The dimension $\ell$ is equal to 2.5 ft. *Note:* There is no bottom on the chamber—only a flange bolted to the floor.




PROBLEM 3.22

① F.B.D

* حوّل طالب الـ (Force) التي تنشأ بالـ (steaL) دتؤثر على (bolt)




$\Sigma F_y = 0 \rightarrow W_{steal} + W_{Liquid} + F_B - P_2 A_2 \; \ldots\ldots ①$

zero

$P_1 + \gamma h = P_2$

$(1,2)(9810)(5L) = P_2$

$\boxed{P_2 = 936}$ $\quad A_2 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2,5)^2 = 9,9\ Ft$

$W_{liquid} = \gamma V$

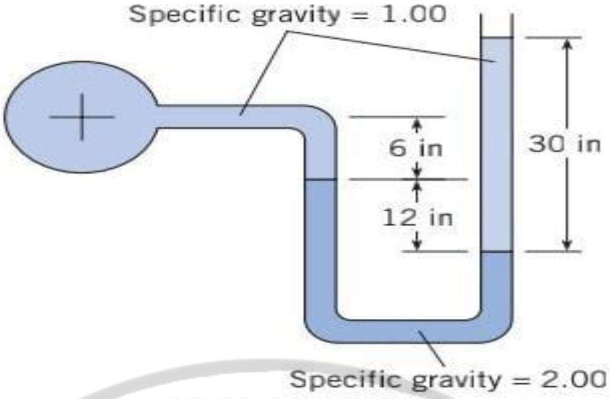
$W = \gamma_{liq}\left(A_2 L + \frac{\pi}{4} d^2 \cdot 4L\right)$ → حجم الاسطوانة = مساحة القاعدة * الارتفاع

$\boxed{W = 1148{,}7\ Ib}$

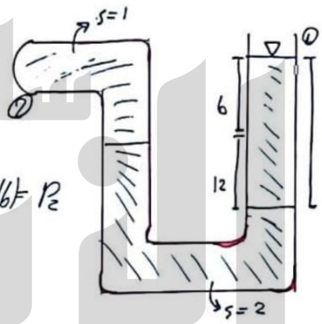
نعوض في معادلة ① لإيجاد $F_B$ :-

$\boxed{F_B = 2850\ Ib}$

**3.30** Is the gage pressure at the center of the pipe (a) negative, (b) zero, or (c) positive? Neglect surface tension effects and state your rationale.




PROBLEM 3.30

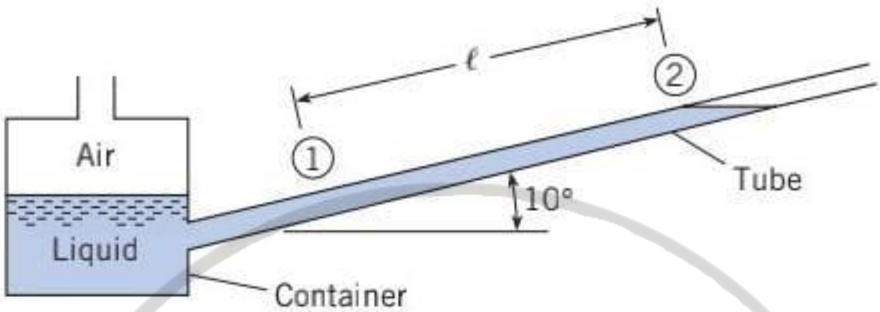



$P_1 + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_3$

$(9810)(1)(30) - (2)(9810)(12) - (9810)(1)(6) = P_2$

$P_2 = \text{zero}$

**3.34** The ratio of container diameter to tube diameter is 8. When air in the container is at atmospheric pressure, the free surface in the tube is at position 1. When the container is pressurized, the liquid in the tube moves 40 cm up the tube from position 1 to position 2. What is the container pressure that causes this deflection? The liquid density is 1200 kg/m$^3$.

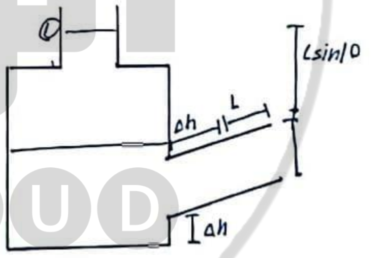



PROBLEMS 3.34, 3.35

* الفكرة في هذا السؤال هو ان مقدار التغير الحجم في السائل في (tube) = مقدار تغير حجم السائل في (container)

$V_{tube} = V_{container}$

$\frac{\pi}{4} d_{tube}^2 * L = \frac{\pi}{4} d_{container}^2 * \Delta h$




$\Delta h = (1/8)^2 \cdot 40 = 0{,}625 \text{ cm}$

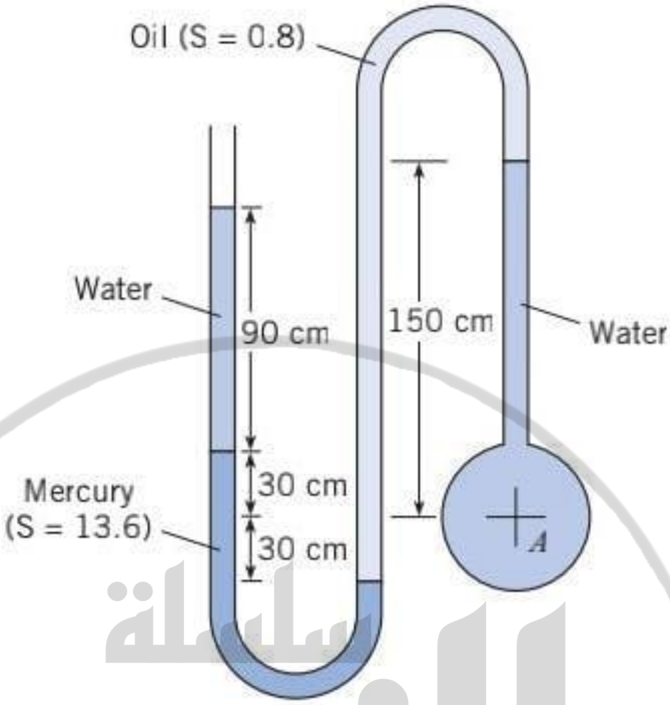
$P_{container} = \overset{zero}{P_1} + \gamma h$

$\Rightarrow (40 \sin 10 + 0{,}625)(1200)(9810)$

$\boxed{P = 891 \text{ Pa}}$

$\gamma = \rho g = (1200)(9810)$

**3.39** Find the pressure at the center of pipe $A$. $T = 10°C$.

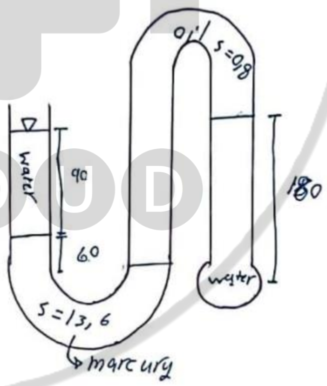



PROBLEM 3.39

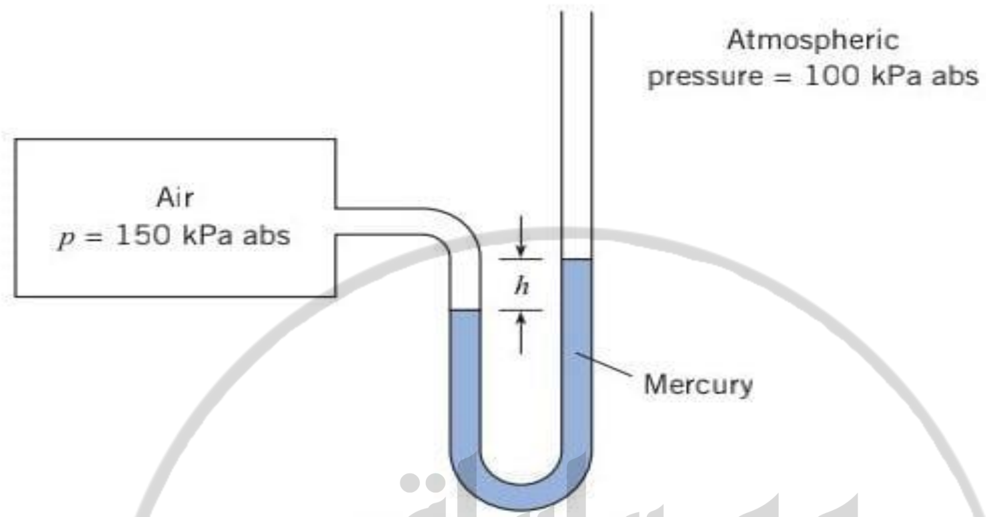
$$P_1 + \gamma h_{water} + \gamma h_{mer} - \gamma_{oil} h + \gamma_{water} h = P_A$$

$$(9810)(0,9) + (13,6)(9810)(0,6) - (0,8)(9810)(180) + (9810)(150) = P_A$$

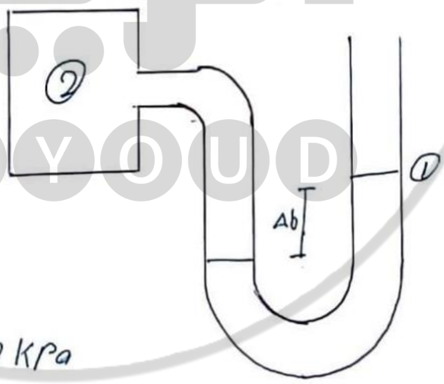
$$P_A = 147,736 \, KPa$$



**3.41** The deflection on the manometer is *h* meters when the pressure in the tank is 150 kPa absolute. If the absolute pressure in the tank is doubled, what will the deflection on the manometer be?




PROBLEM 3.41




$$P_1 + \gamma h = P_2$$

$$\gamma h = P_2 - P_1$$

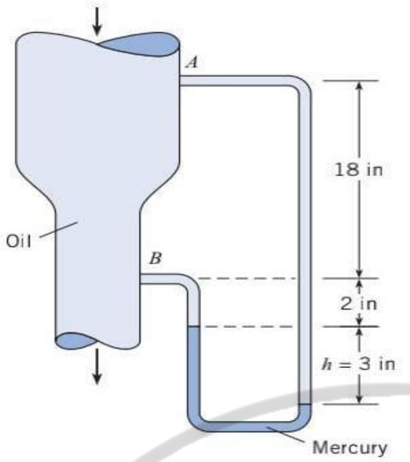
$$= 150 - 100 = 50\ kPa$$

new Pressur $= (150)(2) = 300\ kPa$

$$\gamma h_{new} = 300 - 100 = 200$$

$$\frac{\gamma h_{new}}{\gamma h} = \frac{200}{150} \Rightarrow \boxed{h_{new} = 4h}$$

**3.42** A vertical conduit is carrying oil (S = 0.95). A differential mercury manometer is tapped into the conduit at points *A* and *B*. Determine the difference in pressure between *A* and *B* when *h* = 3 in. What is the difference in piezometric head between *A* and *B*?




$\gamma_{mercurs} - 847$

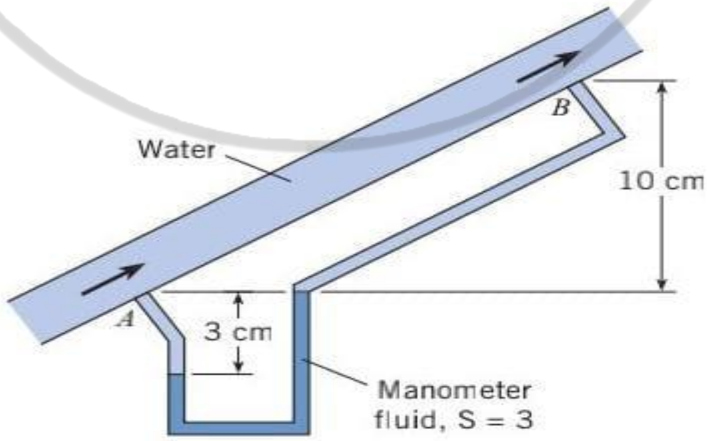
$P_A + \gamma_{oil} h - \gamma_{mercury} h + \gamma_{oil} h = P_B$

$P_A + (0.95)(9810)\left(\frac{18+2+3}{12}\right) - 847\left(\frac{3}{12}\right) + (1.095)(9810)\left(\frac{2}{12}\right) - P_B$

$\boxed{P_A - P_B - 108.01 \ Ib/Ft^2}$

**3.44** A manometer is used to measure the pressure difference between points *A* and *B* in a pipe as shown. Water flows in the pipe, and the specific gravity of the manometer fluid is 3.0. The distances and manometer deflection are indicated on the figure. Find the pressure differences $p_A - p_B$.





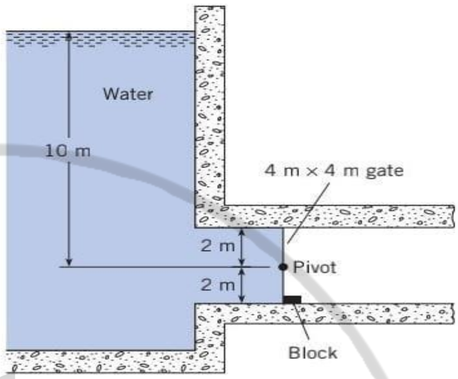

PROBLEM 3.44

$$P_A + 0{,}03\gamma_F - 0{,}03\gamma_m - 0{,}1\gamma_F = P_B$$

$$P_A - P_B = 0{,}03(\gamma_m - \gamma_F) + 0{,}1\gamma_F \Rightarrow P_A - P_B = (0{,}03)(13*9810 - 9810) + 0{,}1*9810$$

$$\boxed{P_A - P_B = 1{,}57\ kPa}$$

**3.59** Find the force of the gate on the block. See sketch.

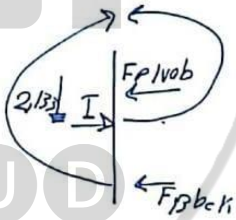



PROBLEM 3.59

$$F = PA = \gamma z A = (9810)(10)(4)^2 = 1569{,}6\ KN$$

$$\boxed{y = 10\ m}$$

$$\bar{y}_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{(1/12)(4)(4)^3}{(10)\cdot(4)^2} = \boxed{0{,}1333\ m}$$

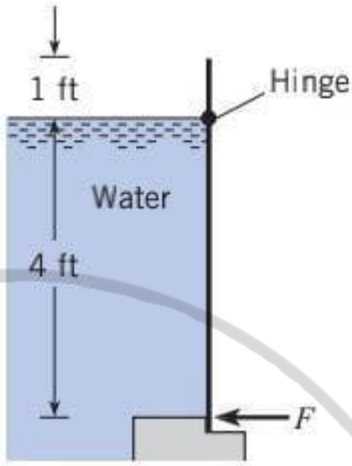



$$\circlearrowleft + \Sigma M_{pivot} = 0$$

$$(1569{,}6)(0{,}1333) - F_{Block}(2) = 0$$

$$\boxed{F_{Block} = 105\ KN}$$

**3.61** A rectangular gate is hinged at the water line, as shown. The gate is 4 ft high and 10 ft wide. The specific weight of water is 62.4 lbf/ft$^3$. Find the necessary force (in lbf) applied at the bottom of the gate to keep it closed.



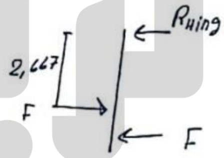

PROBLEM 3.61

$F = PA = \gamma z A = (62,4)(2)(4)(10) = 4992N$

$\bar{y} = 2m \Rightarrow \bar{y}_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{(1/12)(4)^3(10)}{(2)(4)(10)} = 0,667m$

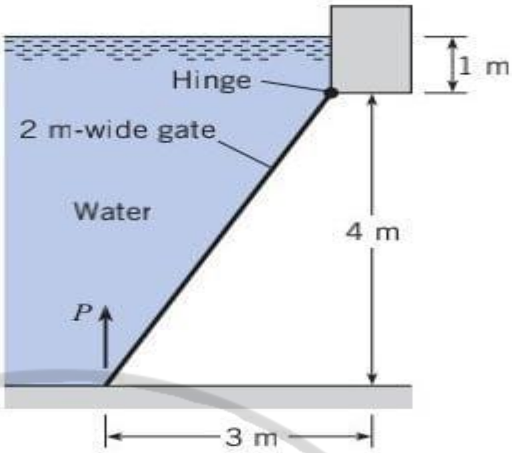
$\circlearrowleft + \sum M_{Hing} = 0$

$(4992)(2,667) - F(4) = 0$




$\boxed{F = 3328,4N}$

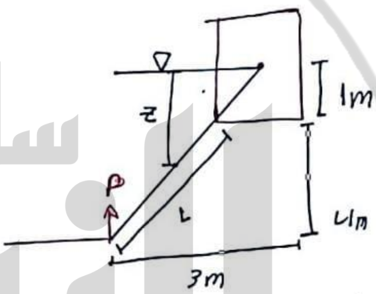
**3.63** Determine *P* necessary to just start opening the 2 m–wide gate.




PROBLEM 3.63

$F = PA = \gamma \bar{z} A = 9810\left(\frac{4}{2} + 1\right)(5)(2)$

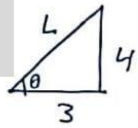
* مساحة البوابة كاتب انه (2m wide) وطولها (5) اذا المساحة (2*5)




$\bar{y} = x + \frac{L}{2} = 1.25 + \frac{5}{2} = 3.75\,m$

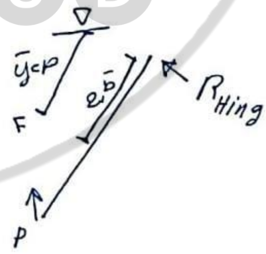
$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{(1/12)(2)(5)^3}{(3.75)(2)(5)} = 0.56\,m$

$L = 5m$




$\sin 53.1 = \frac{1}{x}$

$x = 1.25$




$\Sigma M_{Hing} = 0$

$(294.3)(2.5 + 0.56) - 3P = 0$

$P = 300\,kN$

# Ch4 :flowing fluids and pressure variation

• في هذا الشابتر رح ندرس حركة الموائع وحساب بعض المطاليب في حالة الحركة وتأثيرها على الضغط

• There are two ways of expressing the equations for fluids in motion:

–The Lagrangian approach :

في هذه الفرضية انا بتبع particleمعينة من المائع وبتبع تغيرات حركتها من نقطة الى اخرى

–The Eulerian approach:

انا هون بوخذ control volume معينة وبلاحظ التغيرات التي تحدث على particle بداخلها

• يمكن حساب السرعة من خلال الفرضيتين

The Lagrangian approach is based on recording the motion of a specific fluid particle

r(t) = x i + y j+ zk , r(t): distance

وعند اشتقاق المسافة تعطي السرعة

$$V(t) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

$$V(t) = u\,\mathbf{i} + v\,\mathbf{j} + w\,\mathbf{k}$$

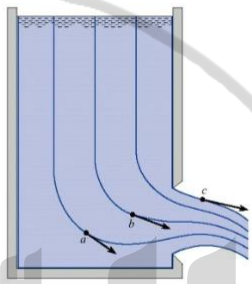
•Eulerian Approach focuses on a certain point in space and describes the motion of fluid particles passing through this point

وتعد هذه النظرية افضل

• The velocity of fluid particles will be described depending on the location of the point in passing through it in space and time:

$$V=v(s,t)$$

## Streamlines: ( the path of particle)

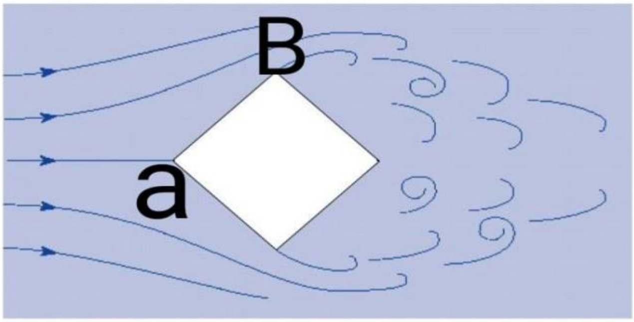


هو شكل من اشكال اظهار flow field من خلال الرسم

وتكون السرعة مماسية على stream line

A group of streamlines construct what is known as a flow pattern

اذا مر خطين stream line بجانب بعضهم نطلق عليهم اسم قناة وكل ما **زادت المسافة بينهم تكون السرعة اقل**




•At point (a) : is called stagnation point , the velocity = zero

ومن المعروف ان الضغط يتناسب عكسي مع السرعة اذ يكون الضغط اعلى قيمة له عند (a)

•At point (B) : is called separation point

•تم تصنيف الجريان بالاعتماد على :

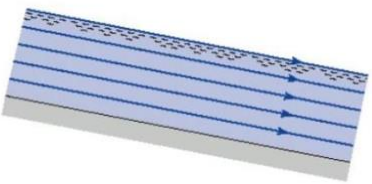
1) space :

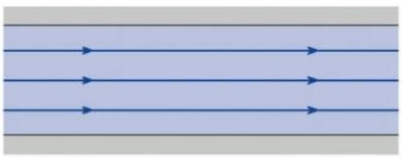
a) Uniform Flow: خصائصه

1) The velocity does not change from point to point along any of the streamlines in the flow field

2) The streamlines are straight and parallel

$$\frac{dv}{ds} = zero$$



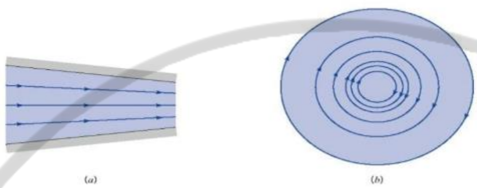


b) Non-uniform Flow:

1 )The velocity changes along the streamlines either in **direction or magnitude.**

2) The streamlines may **not be** straight and/or parallel

$$\frac{\partial V}{\partial s} \neq 0.0$$




2) time:

a) steady flow :

السرعة ثابتة zero =$\frac{dv}{dt}$

b)unsteady flow : السرعة تتغير بالنسبة للزمن

$$\frac{\partial V}{\partial t} \neq 0.0$$

تم تقسيم الموائع حسب شكل الحركة الى :

1) Laminar flow: تكون الحركة منتظمة وانسيابية
2) Turbluent flow: is mainly characterised by the mixing action throughout the flow field
تكون الحركة متداخلة واضطرابية

يمكننا التميز بينهم من خلال حساب Reynolds number (Re)

$$Re=\frac{\rho V D}{\mu}$$

$Re<2100$

OR $Re= 2100$ → **laminar**

$(Re > 2100)$ **Turbulent**

للفهم:

تؤثر العوامل الخارجية بشكل الحركة بحيث لو كان عندي pipe وكانت Re=100 من المفترض ان تكون الحركة laminar لكن لو قمت بتحريك pipe فان الحركة تصبح اضطرابية بسبب الظروف الخارجية

ولكن بالحل بعتبره laminar الا اذا ذكر السؤال انه turbulent

- Flow patterns: group of stream lines

•Methods for Developing Flow Patterns

1) Analytical methods: تعتمد على العمليات الحسابية

2) Computational Methods, CFD: مثل نيوميركال

3) Experimental Methods:

تعتمد هذه الطريقة على القيام بالتجارب العملية وتكون مكلفة

•يمكن تصور شكل التدفق (flow pattern):

a- **Pathline**: is a line drawn through the flow field in such a way that it defines the path that a given particle of fluid has taken. Ex.: PIV b- **Streakline**: is to inject dye or smoke in the flow field and to observe the dye or smoke trace as it travels downstream

هي عبارة عن صبغة توضع في حالة السوائل او دخان يتم وضعه في حالة الغازات ( التعريف سؤال سنوات)

## •Acceleration:

في التسارع سنقوم بايجادها من خلال lagrangian approach و Eulerian approch

في حالة Eulerian approach

$$a = u\frac{\partial(\;)}{\partial x} + v\frac{\partial(\;)}{\partial y} + w\frac{\partial(\;)}{\partial z} + \frac{\partial(\;)}{\partial t}$$

• الاقواس اذا طلب $a_x$ نضع u مكان الاقواس لو طلب $a_y$ نضع v لو طلب $a_z$ نضع w

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

•نلاحظ ان القانون مقسوم الى :

1) Derivatives with respect to position (**convective acceleration**).

هذا الجزء يعتمد على الموقع

$$u\frac{\partial(\,)}{\partial x}+v\frac{\partial(\,)}{\partial y}+w\frac{\partial(\,)}{\partial z}$$

2) Derivative with respect to time (**local acceleration**)

$$\frac{\partial(\,)}{\partial t}$$ يعتمد على زمن

- **Example:**

Given:

$$u = xt + 2y$$
$$v = xt^2 - yt$$
$$w = 0$$

What is the acceleration at a point $x=1$ m, $y=2$ m, and at a time $t=3$ s?

في هذا السؤال طلب

$a_x \Rightarrow \Delta t\, x = 1$

$a_y \Rightarrow \Delta t\, y = 2$

$$a = u\frac{\partial(\,)}{\partial x} + v\frac{\partial(\,)}{\partial y} + w\frac{\partial(\,)}{\partial z} + \frac{\partial(\,)}{\partial t}$$

يعتمد هذا الجزء على الزمن

و بالسؤال معطي الزمن اذن لا يساوي صفر

* نبدأ بـ ($a_x$) فلنا نضع مكان الاقواس ($u$):

w → zero

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial x} = t \quad , \quad \frac{\partial u}{\partial y} = 2 \quad , \quad \frac{\partial u}{\partial t} = x$$

$$a_x = (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x$$

نعوض ⇐ $x=1$ و $y=2$ و $t=3$ في المعادلة

$$a_x = 28\, m/s^2$$

w → zero

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial x} = t^2 \quad , \quad \frac{\partial v}{\partial y} = -t \quad , \quad \frac{\partial v}{\partial t} = 2xt - y$$

$$a_y = (xt + 2y)(t)^2 + (xt^2 - yt)(-t) + 0 + (2xt - y)$$

$$a_y = 58\, m/s^2$$

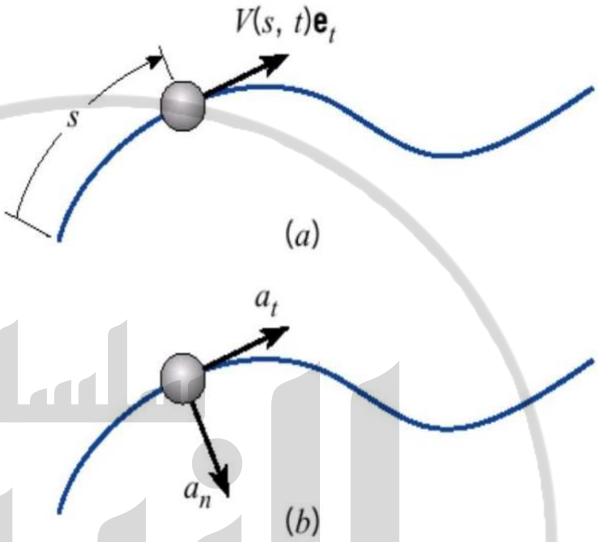
* لو طلب ($a$) على شكل (cartesine vector)

$$a = 28\hat{i} + 58\hat{j} \quad m/s^2$$

## Acceleration by Applying the Lagrangian approach:

كما نعلم ان هذه الفرضية تعتمد على تتبع particle معينة وكنا نحسب سرعة هذه particle في اكثر من نقطة لملاحظة تغير السرعة والان سنتعلم طريقة حساب التسارع من هذه الفرضية .

**التسارع هو مشتقة السرعة** $a=\frac{dv}{dx}$



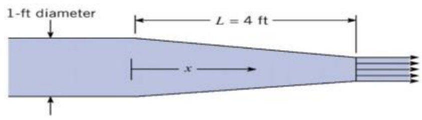

$$a=(V\frac{\partial V}{\partial s}+\frac{\partial V}{\partial t})e_t+(\frac{V^2}{r})e_n$$

$r$: is the local radius of curvature of the pathline.

V: velocity , s: space , t: time , $e_t$: tangent الجزء المماسي

$e_n$: الجزء العمودي

**Example:**
The velocity of water flow in the nozzle shown is given by the following equation:
V=2t / (1- 0.5 □ /L)², where L=4 ft. When x=0.5 L and t= 3 sec, what is the local acceleration along the centerline? What is the convective acceleration? Assuming one-dimensional flow prevails.



* المطلوب الاول هو (local acceleration) وكما نعلم انه يعتمد على الزمن

$$a_{Local} = \left( \underbrace{V\frac{dV}{ds}}_{zero} + \frac{dV}{dt} \right) e_b + \left( \underbrace{\frac{V^2}{r}}_{zero} \right) e_n$$

$$\boxed{a_{Local} = \frac{dV}{dt}} \Rightarrow V = \frac{2t}{\left(1 - \frac{0,5x}{L}\right)^2}$$

$$\frac{dV}{dt} = \frac{2}{\left(1 - \frac{0,5x}{L}\right)^2} \quad \boxed{L = 4}, \boxed{x = 0,5}$$

$$\boxed{a_{Local} = \frac{dV}{dt} = 3,56\ Ft/s}$$

* المطلوب الثاني (convective acceleration) هنا يعتمد على الموقع

اي هنا نشتق بالنسبة للمتغير (x) :-

$$a = \left( V\frac{dV}{ds} + \underbrace{\frac{dV}{dt}}_{zero} \right) e_b + \left( \underbrace{\frac{V}{r}}_{zero} \right) e_n$$

$$a_{ca} = V\frac{dV}{ds} \qquad V = 2t\left(1 - \frac{0,5x}{L}\right)^{-2}$$

$$\frac{dV}{dx} = (2t)\left(\frac{-2}{2L}\left(1 - \frac{0,5x}{L}\right)^{-3}\right) = \frac{2t/L}{\left(1 - \frac{0,5x}{L}\right)^3}$$

$$V\frac{dV}{dx} = \left(\frac{2t}{\left(1 - \frac{0,5}{L}\right)^2}\right)\left(\frac{2t/L}{\left(1 - \frac{0,5}{L}\right)^3}\right) = \frac{4t^2/L}{\left(1 - \frac{0,5}{L}\right)^5}$$

$$\boxed{a_c = 37,9\ Ft/s^2}$$

العوامل التي تؤثر بالضغط؟

1) Weight effects , 2) Acceleration , 3) Viscous resistance

لاحظنا ان التسارع من العوامل المؤثرة بقيمة الضغط بحيث يقل الضغط باتجاه التسارع , العلاقة عكسية , بالسابق كنا نحسب الضغط في حالة السكون p+γh=constant اما الان المائع في حالة حركة ولحساب التسارع : نستخدم قانون Euler

$$-\frac{\partial}{\partial l}(p+\gamma z)=\rho\, a_l$$

**Inviscid Flow**

ولكن هذه المعادلة يوجد بها خطأ واحد هو انه قام باهمال shear force

ولذلك اعتبرناه inviscid flow

ينقسم flow الى :

1)viscous flow , 2) inviscid flow

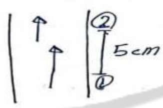
<u>•The pressure must decrease in the direction of flow</u>

ملاحظة : نطبق هذه المعادلة للمسافات القصيرة



* cases in Euler evnotion :-

<u>Case one</u> :




Find $P_2$ IF $a_z = g$ ?

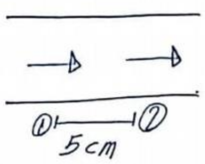
$$\frac{d}{dl}(P+\gamma z) = \rho a_l$$

$$-\frac{d}{dz}(P+\gamma z) = \rho a_z \Rightarrow \frac{dP}{dz} + \gamma \frac{dz}{dz} = -\rho g \to \gamma$$

$$\frac{dP}{dz} = \frac{P_2-P_1}{z_2-z_1} \Rightarrow \frac{P_2-P_1}{z_2-z_1} = -2\gamma$$

$$z_2-z_1 = \frac{5}{100} \Rightarrow P_2-P_1 = -0.1\gamma$$

* case (2):



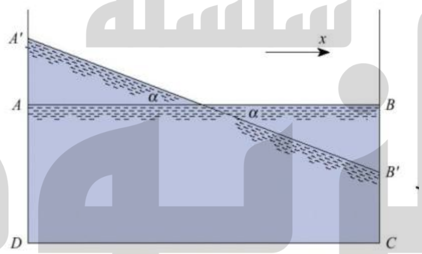

Find P, $a_x = g$

$$-\frac{d}{dx}(P+\gamma z) = \rho(g)$$

$$\frac{dP}{dx} + \gamma \frac{dz}{dx} = -\gamma$$ (zero)

$$\frac{P_2 - P_1}{5/100} = -\gamma \Rightarrow P_2 - P_1 = -0.05\,\gamma$$

Case 3:




* لو كان (statics) اي $a = 0$ فإن $P_D = P_C$
ولكن هنا يوجد حركة

$$\frac{dP}{dx} = -\rho a_x$$

$$\frac{P_D - P_C}{x_2 - x_1} = -\rho a_x$$

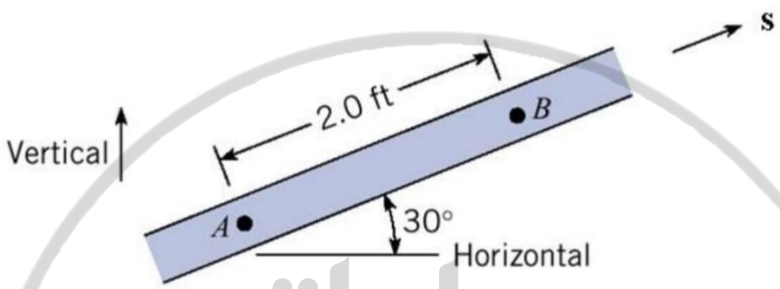
* لو طلب (P) من D الى A' ؟

$$-\frac{d}{dz}(P+\gamma z) = \rho a_z \Rightarrow a_z = \text{zero}$$

$$P + \gamma z = \text{constant}$$

## Example:

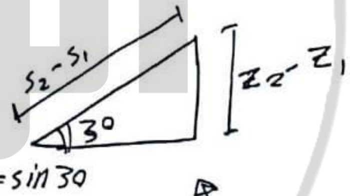
A liquid with a specific weight of 100 lbf/ft$^3$ is in the conduit. This is a special kind of liquid that has zero viscosity. The pressures at points A and B are 170 psf and 100 psf, respectively. Find the acceleration.






*

$$\frac{d}{ds}(p+\gamma z) = -\rho a_s$$




$$\frac{dp}{ds} + \gamma \frac{dz}{ds} = -\rho a_s \Rightarrow \frac{P_2 - P_1}{s_2 - s_1} + \gamma \frac{z_2 - z_1}{s_2 - s_1} = -\rho a_s$$

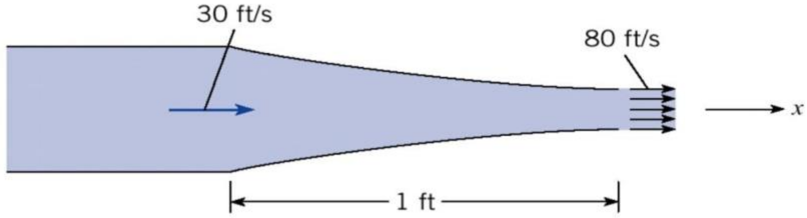
$$\frac{100-170}{2} + \left(\frac{1}{2}\right)100 = -\rho a_s$$

$$-35 + 50 = -\rho a_s$$

$$\boxed{a_s = \frac{-15}{\rho}}$$

## Example:

If the velocity varies linearly with distance through this water nozzle, what will be the pressure gradient , dp/dx, halfway through the nozzle? **Assume steady and inviscid flow**




inviscid Flow ⇒ (Euler) تعني انه رح نستخدم معادلة

$$-\frac{d}{dx}(p+\gamma z)=\rho a_x$$

$a_x$ ⇒ معلومة

zero ⇒ steady لانه

$$a_x = u\frac{du}{dx} + \underbrace{v\frac{du}{dy}}_{zero} + \underbrace{w\frac{du}{dz}}_{zero} + \underbrace{\frac{du}{dt}}_{zero}$$

* الان نريد ايجاد الاقتران (u) لكي نشتقه

* نلاحظ انه الحركة في خط مستقيم اذن معادلة الخط المستقيم :

$$u = ax + b$$

لايجاد a و b نلاحظ ان

at $x = 0$ , $u = 30$

at $x = 1$ , $u = 80$

$$b = 30 \; , \; a = 50$$

$u = 50x + 30$ → at $x = 1/2$ ⇒ (half way) لانه طلب

$u = 55$ , $\frac{du}{dx} = 50$

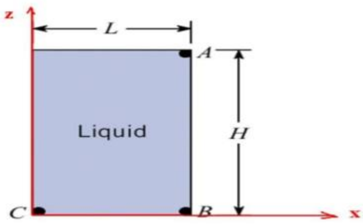
$$a_x = u\frac{du}{dx} = (55)(50) = 2750\ ft/s^2$$

$$\frac{dp}{dx} + \gamma \underbrace{\frac{dz}{dx}}_{zero} = \rho a_x$$

$$\frac{dp}{dx} = (-1.94)(275) = -5355\ psf/ft$$

- **Example:** The closed tank shown, which is full of liquid, is accelerated downward at *2/3 g* and to the right at one *g*. Here *L=2* m, *H=3* m, and the liquid has a specific gravity of *1.3*.

Determine $p_C - p_A$ and $p_B - p_A$.




* هون طالب الضغط بين النقطة (A) و (C) إحنا كنا متعودين انو تكون المسافة عمودية او أفقية ويكون المسار مستقيم
ح نوخذ مسار مستقيم رح نمشي من (C ← B) ومن (B ← A)
ومجموعهم بعطي الفرق بين (A و C)

* C−B :-

$$-\frac{d}{dx}(P+\gamma z) = \rho a_x$$

$$\frac{dP}{dx} + \gamma \underbrace{\frac{dz}{dx}}_{zero} = -\rho a_x \quad , \quad \boxed{\frac{dP}{dx} = -\rho g = -\gamma = -S\gamma_{water}}$$

$$\frac{P_C - P_B}{2} = (1,3)(9810) \quad , \quad \boxed{P_C - P_B = +25,506} \text{ --- ①}$$

حولهما موجب لان $(P_C > P_B)$ بسبب التسارع

* A−B :-

$$-\frac{d}{dz}(P+\gamma z) = \rho a_z$$

نعوض التسارع سالب لأنه بتسارع للأسفل ونحن نمشي عكس التسارع

$$\frac{dP}{dz} + \gamma \underbrace{\frac{dz}{dz}}_{1} = -\rho \times \frac{2}{3} g$$

$$\frac{dP}{dz} = \frac{2}{3}\gamma - \gamma = -\frac{1}{3}\gamma = -4251$$

ذهبح موجب لان $(P_B > P_A)$ بسبب التسارع

$$\frac{P_B - P_A}{3} = -4251 \Rightarrow \boxed{P_B - P_A = +12,75\ kPa}$$

$$\boxed{P_B = (P_A - 12,753)} \Rightarrow$$ نعوض في ①

$$P_C - (P_A - 12,753) = 25,506 \Rightarrow \boxed{P_C - P_A = 38,26\ kPa}$$

# •Bernoulli Equation:

استطاع برنولي من خلال هذه المعادلة اثبات ان الطاقة محفوظة خلال حجم معين من السوائل

وتعد هذه المعادلة من معادلات الحركة وتحتوي على 3 انواع من الطاقة:

1) kinetic energy , 2) potential energy , 3) flow energy

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C$$

نستخدم هذه المعادلة في حالة Inviscid flow

ملاحظة: لايجاد التسارع نستخدم Euler equation

ولايجاد السرعة نستخدم Bernoulli equation

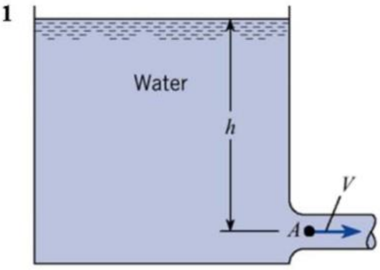
شروط تطبيق معادلة برنولي:

1) applied along streamline

2) The flow is steady , 3) The flow is incompressible

4) The flow is inviscid (viscous effects negligible)

- **Example:** The velocity in the outlet pipe from this reservoir is 6 m/s and h=15 m. Because of the rounded entrance to the pipe, the flow is assumed to be irrotational. Under these conditions, what is the pressure at A?




* لإيجاد (Pressure) نطبق معادلة (Bernoulli) لأنه معطى السرعة بالسؤال

$$\frac{\cancel{P_1}}{\gamma} + \cancel{z_1} + \frac{\cancel{V_1^2}}{2g} = \frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g}$$

(zero, zero, zero)

g : تسارع الجاذبية الارضية

$$\frac{P_A}{9810} + (-15) + \frac{(6)^2}{2g} = -15 + \frac{36}{2 \times 9.8}$$

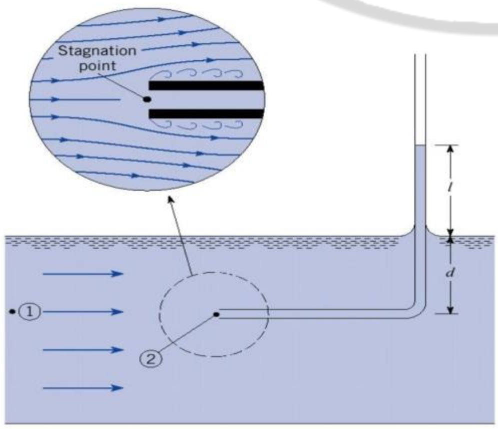
$\boxed{P_A = 129.15 \text{ KPa}}$

* وضعنا (z) بالسالب (-ve) refrence (كما تعلمنا في شابتر(3))



هنالك بعض الاجهزة تعمل على مبدأ برنولي ونجد من خلالها السرعة او الضغط:

1) Stagnation Tube: simple device that can be used for measuring the velocity



* لو طلب إيجاد السرعة عند النقطة ① ؟

* نطبق معادلة برنولي

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

* نلاحظ ان $Z_2 = Z_1$ لان النقطتين على نفس المستوى

* وان $V_2 = 0$ لان النقطة ② (stagnation point)

$P_1 = \gamma h = \gamma d \Rightarrow$ لانها تتأثر بضغط السائل في الوعاء نأخذ الارتفاع لحد سطح الماء بالوعاء

$P_2 = \gamma h = \gamma(d + l) \Rightarrow$ لانها بتتأثر بضغط السائل داخل (Tube)

نعوض في معادلة برنولي

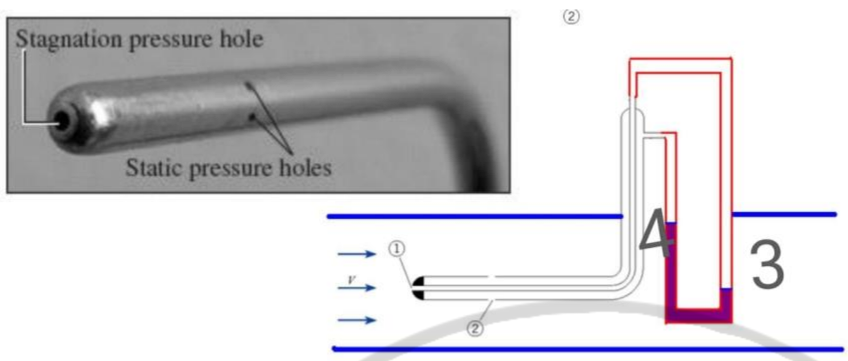
$$\frac{\gamma d}{\gamma} + \frac{V_1^2}{2g} = \frac{\gamma(d+l)}{\gamma}$$

$$\frac{V_1^2}{2g} = d + l - d \Rightarrow V_1 = \sqrt{2gl}$$

* عدم حفظ القانون

* فهم آلية التطبيق

2) Pitot Tube: measuring the velocity of the flow extremely useful in pressurised pipes and for gases




point

Point 1: stagnation point

Point 2 : static point

* لو طلب إيجاد الضغط عند النقطة ② ؟

* تطبق معادلة برنولي

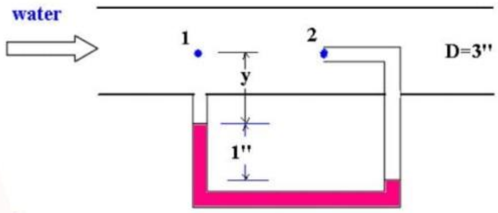
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\boxed{P_1 + \gamma h_{13} - \gamma h_{34} + \gamma_w h = P_2}$$

$$\Downarrow$$

نطبقها بمعادلة برنولي و نجد السرعة

- **Example:** Find the velocity at the pipe center.




* المطلوب هنا السرع ولايجاد السرع نطبق معادلة برنولي

$$\frac{P_1}{\gamma} + \overset{zero}{\cancel{Z_1}} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \overset{zero}{\cancel{Z_2}} + \frac{\overset{zero}{\cancel{V_2}}^2}{2g}$$

zero ⇒ stagnation Point

$$\boxed{\frac{P_2 - P_1}{\gamma} = \frac{V_1^2}{2g}} \quad ---(1)$$

* نلاحظ ان (pipe) موصول ب (manometer) نجد من خلاله $(P_2 - P_1)$:

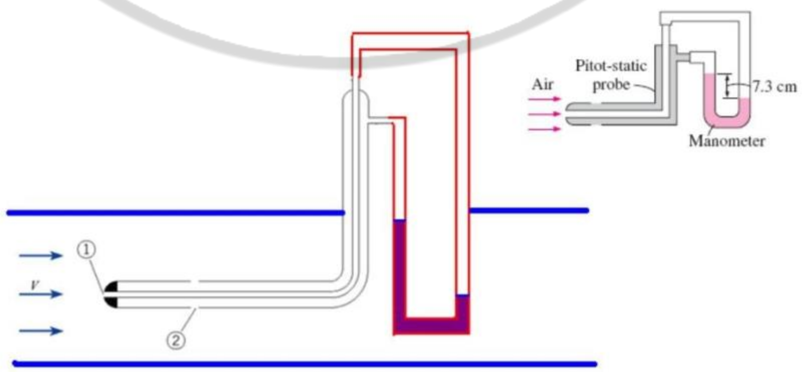
$$P_1 + \gamma_{water} h + \gamma_{mercury} h - \gamma_{water} h = P_2 \Rightarrow P_2 - P_1 = (9810)(y) + (13.6)(9810)\left(\frac{1}{12}\right) - (9810)\left(\frac{1+y}{12}\right)$$

نحولها Ft ← Inch

$$\boxed{P_2 - P_1 = 65.52\ Ib/Ft^2}$$

$$\boxed{V_1 = \sqrt{\frac{(P_2 - P_1)(2g)}{\gamma}} = 8.2\ Ft/s}$$

- **Example:** A pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm. Determine the air velocity. Take the density of air to be 1.25 kg/m$^3$ .



* لإيجاد السرعة 2 نطبق برنولي

$$\frac{P_1}{\gamma_{air}} + \overset{zero}{\cancel{Z_1}} + \overset{zero}{\cancel{\frac{V_1^2}{2g}}} = \frac{P_2}{\gamma_{air}} + \overset{zero}{\cancel{Z_2}} + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{(P_2 - P_1)(2\cancel{g})}{\rho_{air}\cancel{g}}} = \sqrt{\frac{2(P_2 - P_1)}{\rho_{air}}}$$

* لنجد الضغط من خلال (manometer)

$$P_1 - \gamma_{water} h = P_2 \Rightarrow P_1 - (9810)(0,073) = P_2$$

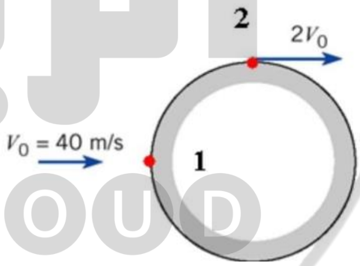
$$\boxed{P_2 - P_1 = 716,3 \ Pa}$$

$0 = \gamma_{air}$ ⇐ مملوء كامل بالهواء (manometer)

نأخذ الجزء الذي يوجد فيه ماء ونحسب الارتفاع

$$\boxed{V_2 = 33,8 \ m/s}$$

- **Example:** The maximum velocity of the flow past a circular cylinder is twice the approach velocity. What is Δp between the point of highest pressure and point of lowest pressure in a 40 m/s wind? Assume irrotational flow and the air density is 1.2 kg/m³.




* هون معطى السرعة وطلب (P) نطبق برنولي :-

$$\frac{P_1}{\gamma} + \overset{zero}{\cancel{Z_1}} + \overset{zero}{\cancel{\frac{V_1^2}{2g}}} = \frac{P_2}{\gamma} + \overset{zero}{\cancel{Z_2}} + \frac{V_2^2}{2g}$$

* بالسؤال كاتب ان $(P_1)$ highst ⇐ اي تعني ان النقطة (1) هي (stagnition point) اي $\boxed{V_1 = zero}$

$$P_1 - P_2 = \gamma \frac{V_2^2}{2g} \Rightarrow P_1 - P_2 = (\rho \cancel{g}) \left(\frac{V_2^2}{2\cancel{g}}\right) = \rho \frac{V_2^2}{2}$$

$$P_1 - P_2 = (1,2)\frac{(80)^2}{2} = \boxed{3,84 \ KPa}$$

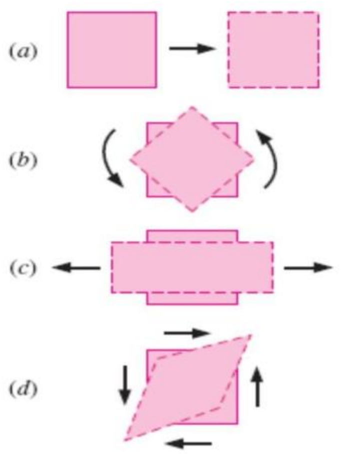
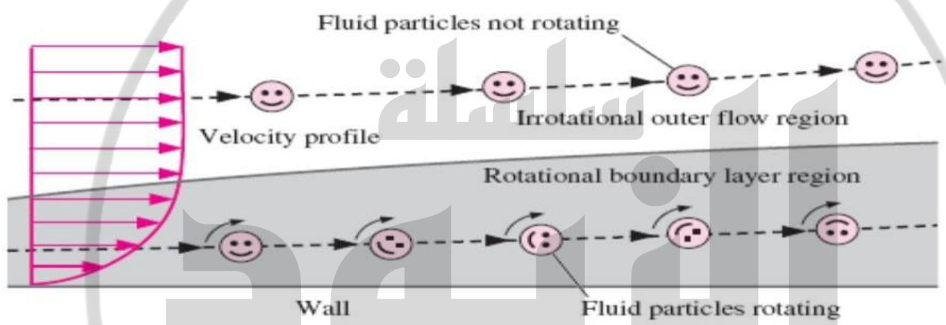



**FIGURE 4–34**
Fundamental types of fluid element motion or deformation: (*a*) translation, (*b*) rotation, (*c*) linear strain, and (*d*) shear strain.




الجزء العلوي لا يدور irrotation لانه لا يوجد shear force

الجزء السفلي يدور rotation بسبب وجود shear force

الدوران rotation: هو دوران particle حول نفسها

$$\Omega = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$$

$\Omega$: rate of rotation

$$\omega = 2\Omega = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$$

$\omega$: vorticity , is twice the rate of rotation

- **Example:** Is the following equation irrotational?

$$V = (-x^2 y)_i + (xy^2)_j$$

* نلاحظ وجود محور (x) ومحور (y) وفقدان محور (z)

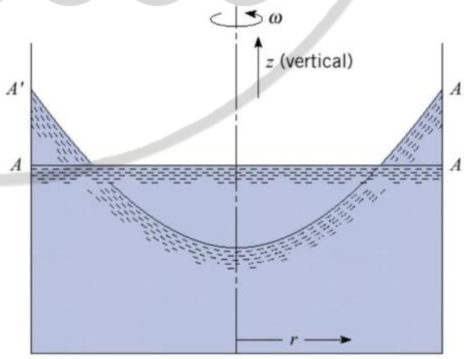
اذا كان $\boxed{\Omega_z = zero}$ ← يكون (irrotational)

$$\Omega_z = \frac{dv}{dx} - \frac{du}{dy}$$

$$\boxed{y^2 + x \neq zero}$$

* the flow is rotational

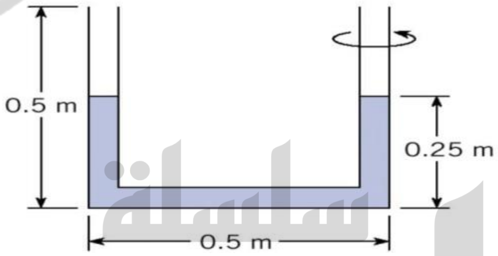
•Pressure Distribution in Rotating Flows:



$$p + \gamma z - \frac{\rho r^2 \omega^2}{2} = C$$

$V = \omega r$ → For a liquid rotating as a rigid body

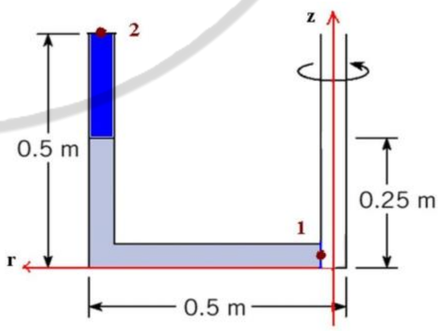
- **Example:** A U-tube is rotated about one leg, before being rotated the liquid in the tube fills 0.25 m of each leg. The length of the base of the U-tube is 0.5 m, and each leg is 0.5 m long. What would be the maximum rotation rate (in rad/sec) to ensure that no liquid is expelled from the outer leg?




$$\overset{zero}{p_1} + \overset{zero}{\gamma z_1} - \frac{\rho r_1^2 w^2}{2} = \overset{zero}{p_2} + \gamma z_2 - \frac{\rho r_2^2 w^2}{2}$$

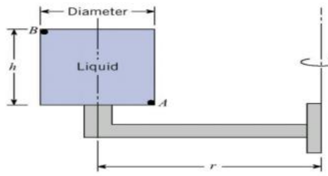
$z_1 = 0$, $\boxed{z_2 = 0.5\ m}$

$$w = \sqrt{\frac{2g z_2}{r_2^2}} = \boxed{6.264\ rad/s}$$



## Example:

**A tank of liquid (S=0.8) that is I ft in diameter and 1 ft high (h=1 ft) is rigidly fixed (as shown) to a rotating arm having a 2 ft radius. The arm rotates such that the speed at point A is 20 ft/s. If the pressure at A is 25 psf, what is the pressure at B?**




* لإيجاد (P) نطبق العلاقة:-

$$P_A + \gamma z_A - \frac{\rho w^2 r_A^2}{2} = P_B + \gamma z_B - \frac{\rho w^2 r_B^2}{2}$$

$\boxed{z_A = zero}$ , $\boxed{z_B = -1}$ ↓(+ve) * نأخذ (refrence) عند النقطة (A)

$\boxed{w = \frac{v}{r}} \Rightarrow \boxed{w_A = w_B} \Rightarrow$ معطى ($v_A$)

$$w = \frac{v_A}{r_A} = \frac{20}{r - 0{,}5d} = \frac{20}{2 - (0{,}5)(1)} = 13{,}33 \ rad/s$$

$\boxed{r_B = r + 0{,}5d = 2{,}5 \ m}$

* نطبق بالعلاقة

~~25 + 0~~

$$\rho = S \cdot \rho_{water} = (0{,}8)(1{,}94) = 1{,}552$$

$$\gamma = S \cdot \gamma_{water} = (9810)(0{,}8) = 7848$$

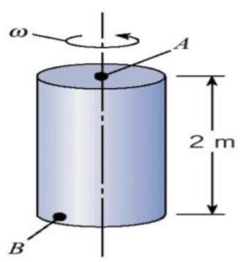
$$25 + 0 - \frac{(1{,}552)(13{,}33)^2(1{,}5)^2}{2} = P_B + 7848(-1) - \frac{(1{,}552)(13{,}33)^2(2{,}5)^2}{2}$$

$\boxed{P_B = 526{,}6 \ PSF}$



## Example:

**A closed tank of liquid (S=1.2) is rotated about a vertical axis, and at the same time the entire tank is accelerated upward at 4 m/s². If the rate of rotation is 10 rad/s, what is the difference in pressure between points A and B ($P_B$-$P_A$)? Points B is at the bottom of the tank at a radius of 0.5 m from the axis of rotation, and point A is at the top on the axis of rotation.**



(I) رح نفرض النقطة (c) على محور الدوران وعلى
نفس خط النقطة (A)

(II) رح انجد (P) من B الى C $(P_B - P_C)$ من خلال معادلة الدوران

(III) رح نطبق معادلة (Euler) لإيجاد P من C الى A

طبقنا (Euler) لانه عند المحور لايوجد دوران (irrotational)

(I) $P_B + \gamma z_B - \frac{\rho r^2 w^2}{2} = P_c + \gamma z_c - \frac{\rho r^2 w^2}{2}$

* النقطة (C) والنقطة (B) على نفس المستوى اذن $\boxed{z_c = z_B = zero}$

$\boxed{r_B = 0,5}$ $\boxed{r_c = 0}$

$\boxed{w = 10\ rad/sec}$ ، $\rho = S\rho_{water} = (1,2)(1000)$

* في السؤال هون فارض انه $\boxed{\rho_{water} = 1000}$ $\boxed{\rho = 1200}$

الان رح نطبق على القانون

$P_B + 0 - \frac{(1200)(0,5)(10)}{2} = P_c + 0 - 0$

$\boxed{P_B - P_c = 15\ KPa}$

(II) $-\frac{d}{dz}(P + \gamma z) = \rho a_z$

$\frac{dP}{dz} + \gamma \frac{dz}{dz} = -\rho a_z$

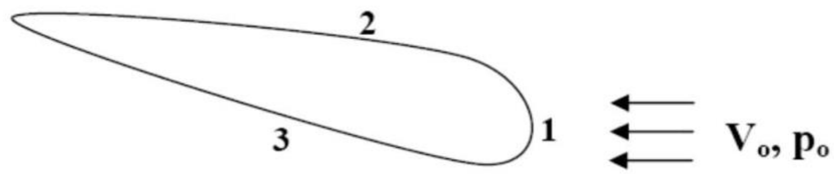
$\Rightarrow \frac{P_A - P_c}{2} = -\rho(4)$

$P_A - P_c = -2(1,2)(9810) - (2)(4)(1200) = 33,144\ KPa$

$\boxed{P_c - P_A = 33,144\ KPa}$ $\boxed{P_c = P_B - 15}$

$\boxed{P_B - P_A = 48,14\ KPa}$

Pressure Coefficient, Cp: static pressure over dynamic pressure




At point 1 : stagnation point , Cp=1

**For liquid:** $$C_p = \frac{h - h_o}{V_o^2/2g} = 1 - (\frac{V}{V_o})^2$$

**For Gas:** $$C_p = \frac{p - p_o}{\frac{1}{2}\rho V_o^2} = 1 - (\frac{V}{V_o})^2$$

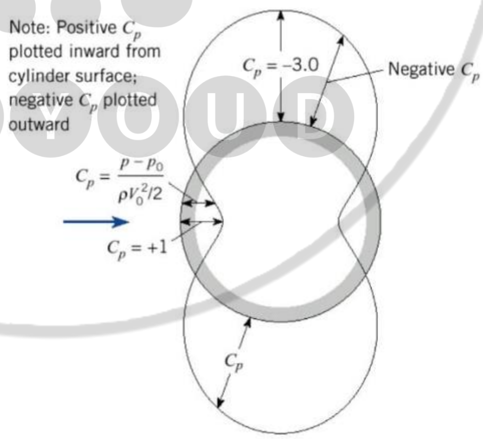
**Where: h=piezometric head=p/γ +z**

**$V_o$,$p_o$= reference velocity,pressure**

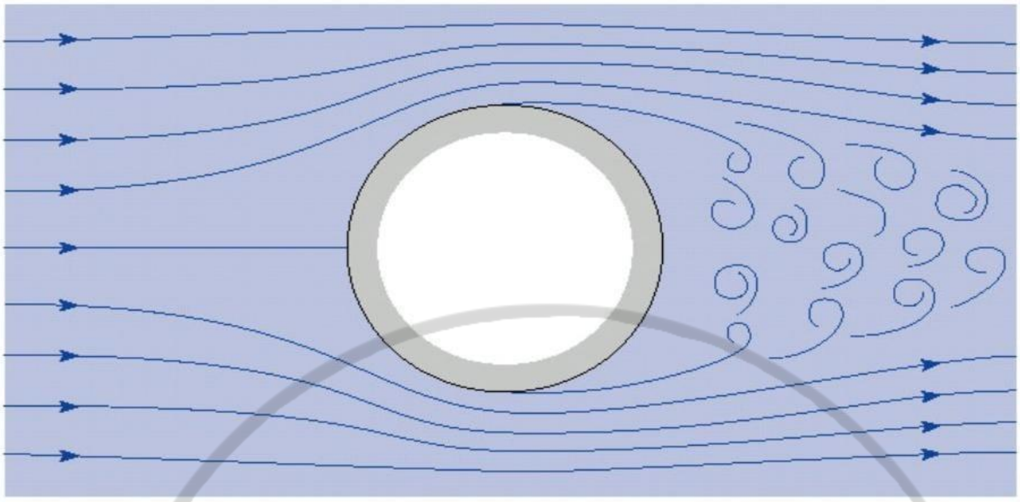
(Cp): untliess (ليس لها وحدة)

كلما زاد مقدار Cp بالسالب تكون السرعة اعلى والضغط اقل

- This is further illustrated in the figure showing the distribution of Cp:
  - Positive Cp is drawn inward.
  - Negative Cp is drawn outward.



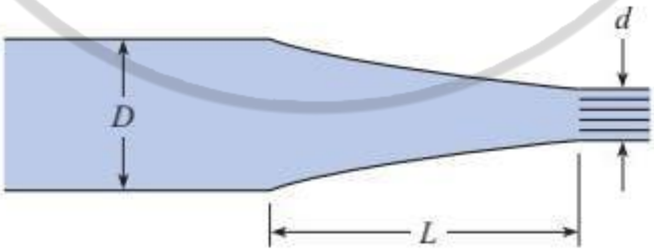
Separation: Phenomenon occurs when the flow separate from the boundary and a recirculation pattern is generated in the region



في الطائرات يمنع اقلاع طائرة صغيرة خلف طائرة كبيرة وذلك بسبب الموجات التي تحدث خلف الطائرة الكبيرة بسبب ظاهرة الانفصال separation

**4.20** The nozzle in the figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming quasi–one-dimensional flow, what is the convective acceleration midway between the base and the tip if the velocity is 1 ft/s at the base and 4 ft/s at the tip? Nozzle length is 18 inches.






PROBLEMS 4.20, 4.21

convective acceleration ⇒ هو التسارع الذي يعتمد على الموقع

$$a_x = u\frac{du}{dx} + \underbrace{v\frac{du}{dy}}_{zero} + \underbrace{w\frac{du}{dz}}_{zero} + \underbrace{\frac{du}{dt}}_{zero}$$

$$\boxed{a_x = u\frac{du}{dx}} \qquad \boxed{u = ax + b} \Rightarrow$$

* لانه يسير في خط مستقيم نستخدم معادلة الخط المستقيم

at $\boxed{x = 0} \Rightarrow u = 1\ Ft/s$

$1 = a(0) + b \Rightarrow \boxed{b = 1}$

at $\boxed{x = \frac{18}{12}} \Rightarrow u = 4\ F/s$

$4 = 1.5a + 1 \Rightarrow \boxed{a = 2}$

$$\boxed{u \text{ at } x = \frac{1.5}{2} = 0.75}$$

$a_c \Rightarrow$ at mid point $= u\frac{du}{dx}$

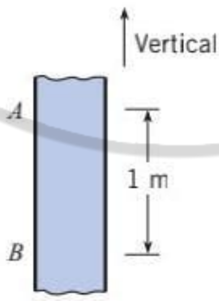
$u = (1.5)(0.75) + 1 = 2.5$ $\qquad \boxed{\frac{du}{dx} = 2}$

$$\boxed{a_c = (2.5)(2) = 5\ Ft/s^2}$$

* لو طلب (local acceleration) ⇐ $\boxed{a_c = 0}$



**4.29** The hypothetical liquid in the tube shown in the figure has zero viscosity and a specific weight of 10 kN/m³. If $p_B - p_A$ is equal to 12 kPa, one can conclude that the liquid in the tube is being accelerated (a) upward, (b) downward, or (c) neither: acceleration = 0.



* بمشي من النقطة (A) الى (B) $(A) \xrightarrow{\text{refrance}} (B)$ (+ve)

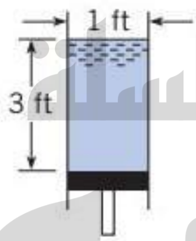
$$-\frac{d}{dz}(P+\gamma z) = \rho a_z \qquad \boxed{\rho = \frac{\gamma}{g}}$$

$$\frac{dP}{dz} + \gamma\frac{dz}{dz} = \frac{-\gamma}{g} a_z \Rightarrow P_A - P_B + \gamma = \frac{-\gamma}{g} a_z$$

$$\frac{-g}{\gamma}\left(\frac{-12*10^3}{1} + \gamma\right) = a_z \Rightarrow -g\left(\frac{-12*10^3}{\gamma} + \frac{\gamma}{\gamma}\right) = a_z$$

$$\Rightarrow g(1{,}2-1) = a_z \rightarrow \boxed{a_z = 0{,}2g}$$ accelerat upwered

**4.30** If the piston and water ($\rho = 62.4 \text{ lbm/ft}^3$) are accelerated upward at a rate of $0.5g$, what will be the pressure at a depth of 2 ft in the water column?




PROBLEM 4.30

(I) نحسب (P) من خلال (Euler)
(II) نضربها بالعمق

$$-\frac{d}{dz}(P+\gamma z) = \rho a_z$$

$$\frac{dP}{dz} + \gamma = \frac{-\gamma}{g}(0{,}5)g$$

$$P_2 - P_1 \quad \frac{dP}{dz} = -1{,}5\gamma \Rightarrow \frac{dP}{dz} = 1{,}5\gamma \Rightarrow$$ الضغط يتناقص كلما ارتفعنا للاعلى بهذا المقدار

$$\boxed{P_2 = 1{,}5 * 2\gamma = 3\gamma} \qquad \boxed{\gamma = \rho * g}$$

posite to flow direction) at a rate of 0.3g?

**4.28** What pressure gradient is required to accelerate kerosene ($S = 0.81$) vertically upward in a vertical pipe at a rate of $0.3\,g$?

$$\frac{dP}{dz} + \gamma \frac{dz}{dz} = -\rho a_z$$

$$\frac{dP}{dz} + \gamma = -\rho\, 0.3g \qquad \gamma = \gamma_{water} \cdot S = (32.2)(0.81)$$

$$\frac{dP}{dz} = \gamma(-1-0.3) = (-26)(1.3)$$

$$\frac{dP}{dz} = -33.8$$

Q) air gas constant = 287 J/kg.K and T = 55 C°, specific internal energy = 1000. Find specific enthalpy?

$T = 55 + 273 = 328 K°$

$h = u + \frac{P}{\rho} = u + \frac{\rho R T}{\rho} = 95136$

**Two plates are separated by 8 mm space. The lower plate is moving at a velocity of 5 m/s, the upper plate moves at a velocity of 10 m/s. Oil with a density $\rho = 630.2\ kg/m^3$ and viscosity of $1 \times 10^{-6}\ m^2/s$ which fills the space. The variation in velocity of the oil is linear. What is the shear stress in the oil ($N/m^2$)?**

$v = \frac{\mu}{\rho} \Rightarrow \mu = v \cdot \rho$

$\mu = (6,302 \times 10^{-4}) \left(\frac{10-5}{0,008}\right) = 0,39$



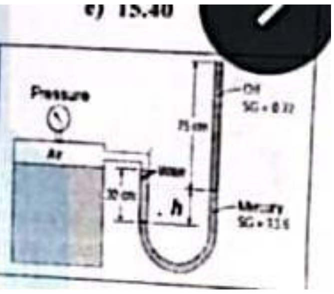
Q9) What the pressure increase *(in MPa)* that must be applied to water to reduce its volume by 0.5%? Water Bulk modulus = 2.2 GPa

$$E = \frac{\Delta P}{\Delta V} \Rightarrow 2,2 \times 10^9 = \frac{\Delta P}{0,5/100}$$

$$\boxed{\Delta P = 11 m Pa}$$

**Q10) *The gage pressure* of the air in the tank shown is measured to be 65 kPa. If the specific gravity (SG) for Mercury = 13.6 and SG for Oil = 0.72, also Water height = 30cm and oil height = 75cm (as shown on graph). What is the differential height *h (in cm)* of the mercury column?**

a) 43.21 b) 39.46 c) 32.36 d) 35.71 e) 47.00



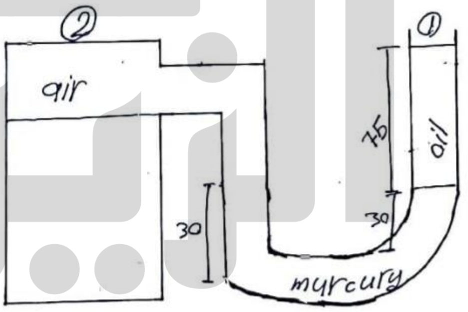

**The following paragraph applies to Q11 and Q12**

Consider a fluid between two parallel ...

$$P_1 + \gamma_{oil} h + \gamma_{mercury} h - \gamma_{water} h = P_2$$

$$(0,72)(9810)(0,75) + (13,5)(9810)(h) - (9810)(0,3) = 65 \times 10^3$$

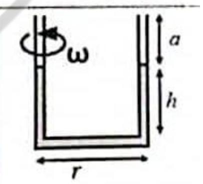
$$\boxed{h = 0,47 m = 47 cm}$$






**Water density = 1000 kg/m$^3$, g = 9.81 m/s$^2$**

**Q 1)** A manometer is rotated around one leg, as shown. The liquid in the manometer is oil [S=0.82] and the dimensions shown are as follows: [*r* = *20 cm, h*=19 *cm and h+a=29 cm*]. **What is the maximum allowable speed of rotation, ω, [in rad/s] so that liquid will not spill out of the manometer?**




$$\overset{zero}{P_1} + \overset{zero}{\gamma z_1} + \overset{zero}{\frac{\rho w^2 r_1^2}{2}} = \overset{zero}{P_2} + \gamma z_2 - \frac{\rho w^2 r_2^2}{2}$$

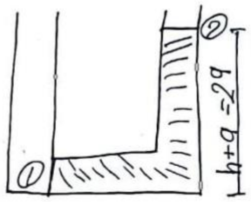
$$w = \sqrt{\frac{(\gamma z_2)(2)}{r_2^2}}$$

$\gamma = \rho g$

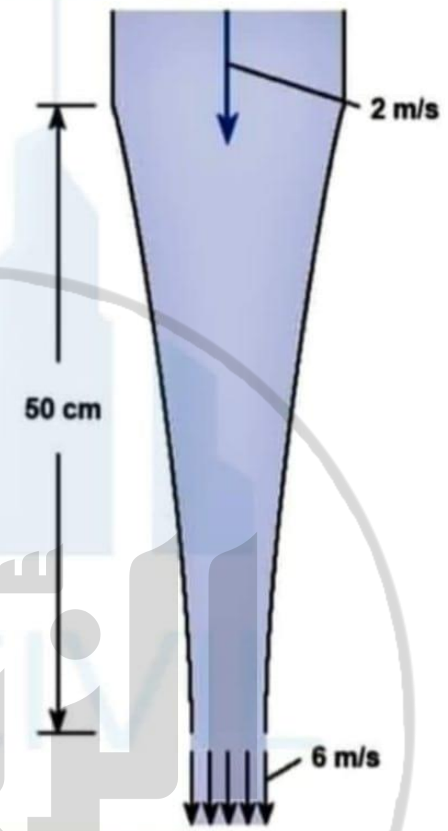
$r_2 = 0,2$

$z_2 = 0,29$

$\boxed{w = 11,54 rad/s}$



If the velocity varies linearly with distance through this fluid nozzle (vertical nozzle), where the specific gravity of the fluid is S=0.46, what will be the pressure gradient, halfway through the nozzle? Assume steady and inviscid flow






$-\frac{d}{dz}(P+\gamma z) = \rho a_z$

$a_z \Rightarrow \boxed{w = az+b}$ at $z=0$, $w=2$, $\boxed{b=2}$; at $z=0.5$, $w=6$, $\boxed{a=8}$

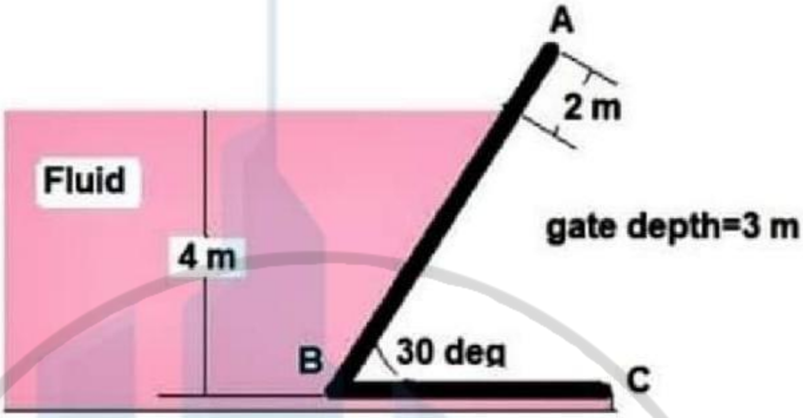
$\boxed{w=8z+2} \Rightarrow \frac{dw}{dz} = 8$, $w \Rightarrow \boxed{z=1/4} \Rightarrow \boxed{w=4}$

$a_z = w\frac{dw}{dz} = (4)(8) = \boxed{32\ m/s^2}$

$\frac{dP}{dz} + \gamma \frac{dz}{dz}^{(1)} = -\rho(32) \Rightarrow \frac{dP}{dz} + (0.46)(9810) = -S\rho_{water}(32)$, with $\rho_{water} = 1000$

$\frac{P_2 - P_1}{z_2 - z_1} + 4512.6 = -14720 \rightarrow \boxed{P_2 - P_1 = -9616.3\ Pa}$, with $z_2 - z_1 \rightarrow 0.5$

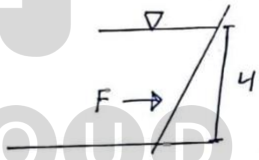
What is the hydrostatic force (F) in N that affect the the gate A-B, the fluid has a specific gravity S=0.58






$F = PA = \gamma z * A = (0.58)(9810)(2)(L \cdot 3)$

$\boxed{F = 273110.4}$ ©




$\sin 30 = \frac{4}{L}$

$\boxed{L = 8}$

ميكانيكا الموائع  هو تخصص فرعي من ميكانيكا المواد المتصلة وهو معني أساسا بالموائع، التي هي أساسا السوائل والغازات، ويدرس هذا التخصص السلوك الفيزيائي الظاهر الكلي لهذه المواد، ويمكن تقسيمه من ناحية إلى إستاتيكا الموائع- أو دراستها في حالة عدم الحركة، أو ديناميكا الموائع أو دراستها في حالة الحركة..

إعداد: محمد حسن

# Ch 5: Control Volume Approach and Continuity Principle

في هذا الشابتر رح نتحدث عن اول قانون يتحكم في حركة الموائع وهو قانون حفظ الكتلة (Conservation of mass)

•Rate of flow:

رح نتحدث عن تغير الكتلة او الحجم بالنسبة للزمن

1) Discharge or Volume flow rate, Q {m3 /s}:

For a fluid with constant velocity: **Q =V.A**

V: velocity , (السرعة تكون عمودية على المساحة)

- في حالة كانت السرعة متغيرة

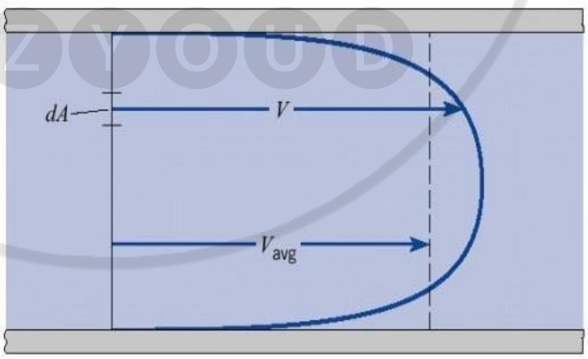
**$Q=\int V.dA$**

2)Mass Flow Rate,m° , {kg/s}:

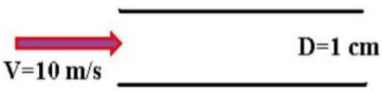
Constant velocity: $m°=\rho V.A$

Variable velocity and constant density: **$m° = \rho \int VdA=\rho Q$**

-Mean Velocity $\bar{V}$ =Vavg =Q /A



- **Example:** Find the volume and mass flow rate of water.




* السرعة هنا ثابتة

* السرعة عمودية على مساحة الدائرة

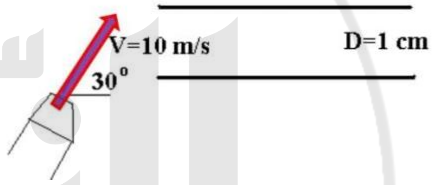
$Q = V \cdot A = V \cdot \frac{\pi}{4} d^2$

$V \Rightarrow \perp D = 1\,cm$ (لـ pipe)

$Q = (10)(\frac{\pi}{4})(0.01)^2 = 7.85 \times 10^{-4}\,m^3/s$

$\dot{m} = \rho Q = (1000)(7.85 \times 10^{-4}) = \boxed{0.785\,kg/s}$

- **Example:** Find the volume and mass flow rate of water.

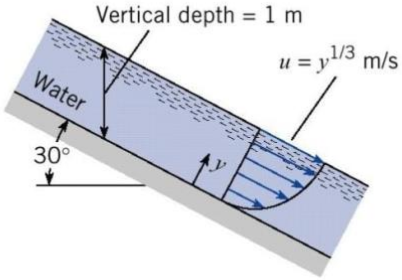



* السرعة يلي بعوضها في القانون هي يلي على محور الـ (x)
لأنه هذا الجزء يلي رح يدخل بالـ (pipe)

$Q = V \cdot A = (10)(\cos 30)(\frac{\pi}{4}(0.01)^2) = 6.798 \times 10^{-4}\,m^3/s$

$\boxed{\dot{m} = \rho Q = 0.679\,kg/s}$

- **Example:** The rectangular channel is 2 m wide. What is the discharge in the channel?

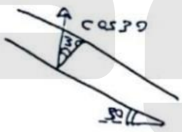



$\boxed{Q = V \cdot A}$ ولكن نلاحظ ان السرعه متغيرة ، رح نستخدم التعامل في السؤال هنا السرعة تتغير على محور (y) فقط

$Q = \int V\,dA = \int_0^y \int_0^x V\,dA$

$\boxed{\int_0^x V\,dA = 2} \Longrightarrow \boxed{x = 2} \Longrightarrow \int_0^2 V\,dA$

$Q = \int_0^{1\cos 30} y^{1/3}\,(2\,dy) = 1.24\ m^3/s$




* هنالك طريقة اخرى لإيجاد (Q) في حالة كانت متغيره :-

(I) افرض النقطة في منتصف (pipe)

(II) $, u = (y^{1/3})^{1/3} \Rightarrow (r)^{1/3}$

(III) $\int V(r)\,2\pi r\,dr$


$\Rightarrow Q = \int_0^{0.5\cos 30} V(r) * 2\pi r\,dr \Rightarrow \int_0^{0.5\cos 30} (r)^{1/3} * 2\pi r$

$\Rightarrow \int_0^{0.5\cos 30} 2\pi (r^{4/3}) \longrightarrow Q = 1.24\ m^3/s$

•**A control volume:** is a selected volumetric region in space.

•**A control surface:** is the surface enclosing the control volume.

للتميز بينهم : لو عندي بالون وبداخله هواء فان الهواء الذي بداخل البالون نطلق عليه control volume وجدار البالون نطلق عليه اسم

Control surface

• The mass within the control volume can change with time, and a control volume can deform with time, and move and rotate in space (open system)

•In contrast with the control volume, a system is defined as a continuous mass of fluid that always contains the same fluid particles ( close system)

**Continuity Equation**

$$\frac{dM_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

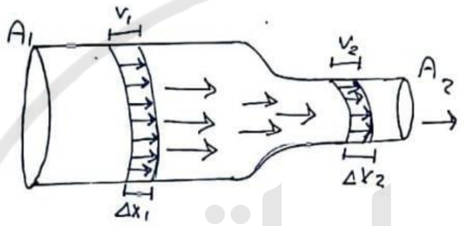
* continuity equation: ( معادلة الاستمرارية )

* تعتمد هذه المعادلة على مبدأ حفظ الكتلة (conservation of mass)

* بحيث ان الشرط الاساسي لها هو مجموع الكتل الداخلة يساوي مجموع الكتلة الخارجية

$m_{in} = m_{out}$

*steady + incompressible




كمية الماء الخارجة

*نلاحظ ان كمية الماء الداخلة مساوية لكمية الماء الخارجة خلال زمن معين (Δt)

* $m_{in} = m_{out}$

لانه نفس السائل $\rho V_1 = \rho V_2 \Rightarrow A_1 \Delta x_1 = A_2 \Delta x_2 \Rightarrow$ بقسم $(\Delta t)$

$$A_1 \left(\frac{\Delta x_1}{\Delta t}\right) = A_2 \left(\frac{\Delta x_2}{\Delta t}\right) \Rightarrow A_1 V_1 = A_2 V_2$$

V: velocity

*تستخدم في حالة (steady incompressible)

* في حالة كان (unsteady) القانون هو:-

$$\frac{d}{dt}\int \rho\, dV + \int \rho V dA$$

$\frac{d}{dt}\int \rho\, dV$ ← هذا الشق يمثل controvolume

$\int \rho V dA$ ← هذا الشق يمثل control surface

$\int \rho V dA = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}$

$$\boxed{\frac{dM_{c.v}}{dt} = \rho \cdot volume}$$

* ملاحظة:-

steady $\Rightarrow \frac{dv}{dt} = 0$ / unsteady $\Rightarrow \frac{dv}{dt} \neq 0$

$$\boxed{volume = A \cdot h}$$

A:- Area

h:- height (الارتفاع)

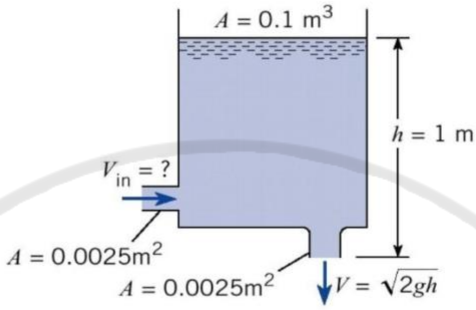
# Reynolds Transport Theorem

$$\underbrace{\frac{dB_{sys}}{dt}}_{\text{Lagrangian}} = \underbrace{\frac{d}{dt}\int_{cv} b\rho\, d\forall + \int_{cs} b\rho V dA}_{\text{Eulerian}}$$

$$B_{cv} = \int b\, dm = \int b\rho\, d\forall$$

| Extensive Property, B | Intensive property, b | Result |
|---|---|---|
| Mass: M | 1 | Continuity Equation |
| Momentum: M V | V | Momentum Equation |
| Energy: E | e | Energy Equation |

- **Example**: A tank has a hole in the bottom with a cross-sectional area of 0.0025 m$^2$. The cross-sectional area of the tank is 0.1 m$^2$. The velocity of the liquid flowing out the bottom hole is V=(2gh)$^{0.5}$, where *h* is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1 m and rising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet.




*في هذا السؤال هو (unsteady) لان (volume) في الخزان متغير، نطبق العلاقة :-

$$\boxed{\frac{dm_{c.v}}{dt} = \dot{m}_{in} - \dot{m}_{out}}$$

$$A \times \rho \frac{dh}{dt} = \rho V_{in} A - \rho V_{out} A$$

بما ان (incompressible) اذن (ρ) ⇒ constant

$$A \frac{dh}{dt} = V_{in} A - V_{out} A$$

$\boxed{h = 1m,\ g = 9.81}$ ⇒ معطى في السؤال

$$(0,1)\left(\frac{0,1}{100}\right) = V_{in}(0,0025) - (2gh)^{0,5}(0,0025)$$

التحويل من cm ← m

$$(0,1)\left(\frac{0,1}{100}\right) = V_{in}(0,0025) - \sqrt{(2)(9.81)(1)}\,(0,0025)$$

$$\boxed{V_{in} = 4,468\ m/s}$$

* لو اعطى $\left(\frac{dh}{dt} = 2\right)$ ، طلب التغير في الزمن :-

$$\frac{dh}{dt} = 2 \Rightarrow dh = 2dt$$

$$\boxed{dt = \frac{dh}{2}}$$

- **Example:** steady, incompressible flow of water through the device.

  Given:

  $A_1=0.2\ m^2$

  $A_2=0.2\ m^2$

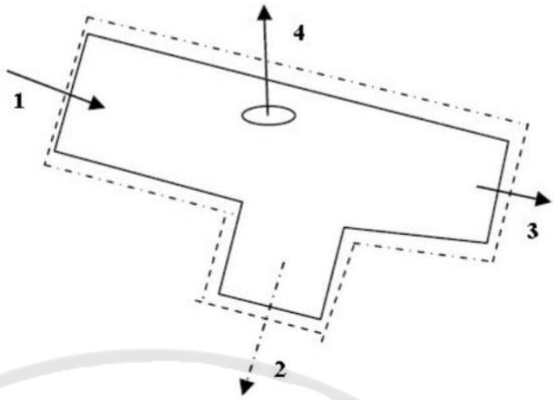
  $A_3=0.15\ m^2$

  $V_1=5\ m/s$

  $V_3=12\ m/s$

  $Q_4=0.1\ m^3/s$

  $\rho = 999\ kg/m^3$




*Find $V_2$ and it's direction.*

$$\frac{\partial m \cdot V}{\partial t} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \Rightarrow \rho \Rightarrow (constant)$$ * ثابتة

zero لانه (steady)

$$\dot{m}_{in} = \dot{m}_{out} = \sum \rho V_{in} A_{in} - \sum \rho V_{out} A_{out}$$

$$\boxed{Q = VA}$$

$$V_1 A_1 = V_2 A_2 + V_3 A_3 + V_4 A_4$$

$$\boxed{Q_4 = 0.1}$$

$$\Rightarrow (5)(0.2) = V_2 (0.2) + (12)(0.15) + Q_4$$

$$\boxed{V_2 = -4.5\ m/s}$$

* الاشارة السالبة تعني ان الاتجاه (V2) عكس الفرض في السؤال

- **Example:** Find the $V_{max}$ for the given steady, incompressible flow through pipe 3.

  Given:

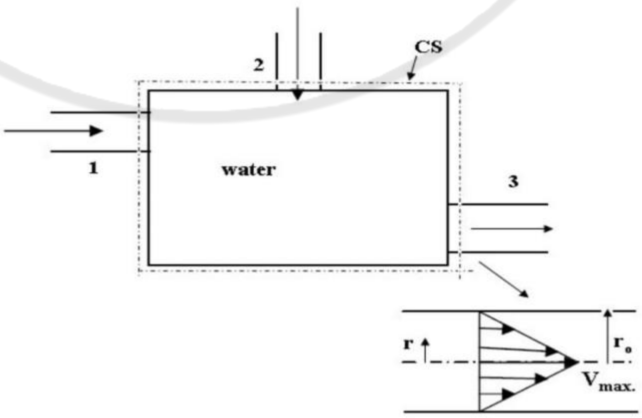
  $A_1=0.1\ m^2$

  $V_1=5\ m/s$

  $A_2=0.1\ m^2$

  $V_2=3\ m/s$

  $A_3=0.1256\ m^2$



$\sum \dot{m}_{in} = \sum \dot{m}_{out}$ $\Rightarrow$ $A_1 V_1 + A_2 V_2 = \int V_3 \, dA$ ---- ①

$V_3 = a + br$ $\Rightarrow$ لإيجاد $(V_3)$

at $r = 0$, $V_3 = V_{max}$, $a = V_{max}$

at $r = r_0$, $V = 0$, $b = -\frac{V_{max}}{r}$

$$V_3 = V_{max} - \frac{V_{max}}{r} = V_{max}(1 - 1/r)$$

$$\dot{m}_3 = \int_0^{r_0} V_{max}(1 - 1/r_0)\, 2\pi r\, dr$$

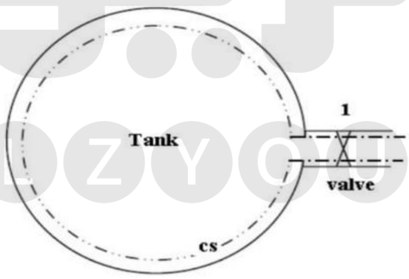
$$\dot{m}_3 = \frac{\pi}{3} r_0^2 V_{max}$$ , $r_0 \Rightarrow A = \frac{\pi}{4} d^2 = \pi r_0^2$

$r_0 = 0.2\,m$ $\Rightarrow$ نعوض في ①

$$(5)(0.1) + (3)(0.1) = \frac{\pi}{3}(0.2)^2 V_{max}$$

$$V_{max} = 19.1\,m/s$$

- **Example:** Tank of a volume of 0.05 $m^3$ contains air. At t=0.0, air escapes through a valve. Air leaves with speed V=300 m/s and density of 6 $kg/m^3$ through area of 65 $mm^2$. Find the rate of change of air density in the tank at t=0.0.




:

*هنا مطلوب $\left(\frac{d\rho}{dt}\right)$

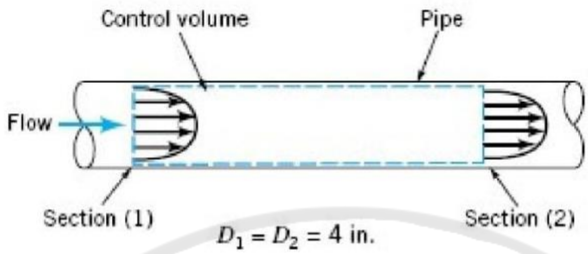
* هنا المسألة (unsteady) لان الحجم ثابت وبما ان الكثافة متغيرة لكن السرعة ثابتة بنلغي التكامل

$$\rho_1 V_1 A_1 = -\frac{d\rho}{dt} \forall \rightarrow \frac{d\rho}{dt} = \frac{-\rho_1 V_1 A_1}{\forall} = \frac{-(6)(300)(65)(10^{-6})}{0.05}$$

$$\frac{d\rho}{dt} = -2.34\,(kg/m^3)/s$$

## Example:

Air flows steadily between two sections in a long, straight portion of 4-in inside diameter pipe. The uniformly distributed temperature and pressure at each section are given. If the average air velocity (nonuniform velocity distribution) at section (2) is 1000 ft/s, calculate the average air velocity at section (1).




$D_1 = D_2 = 4$ in.

$p_1 = 100$ psia
$T_1 = 540$ °R

$p_2 = 18.4$ psia
$T_2 = 453$ °R
$V_2 = 1000$ ft/s

* هنا طلب (averag velocity) ⇐ بنعامله معاملة الثابت

(average) ⇒ نلغي التكامل

$\dot{m}_1 = \dot{m}_2$

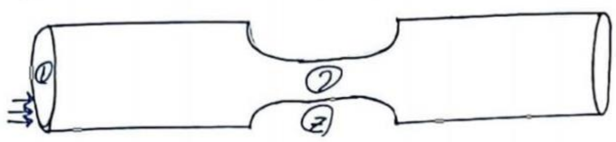
$\rho \bar{V}_1 A_1 = \rho \bar{V}_2 A_2$ , $\rho = \frac{P}{RT}$

$\bar{V}_1 = \frac{P_2 T_1 \bar{V}_1}{P_1 T_2}$

$\bar{V}_1 = \frac{(18,4)(540)(1000)}{(100)(453)} = \boxed{219 \text{ Ft/s}}$

Ex:

$P_1 = 100$ KPa  $P_2 = 1.227$ KPa (absolute)



Find Q ?

$Q = VA$

* لكن السرعة عند النقطة (2) مجهولة نستخدم برنولي لإيجاد السرعة

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1}^{zero} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}^{zero}$$

$$P_2 = P_{gage} = P_{absolute} - P_{atm} = 1.227 - 101.3 = -100 \text{ KPa}$$

* نلاحظ في القانون وجود مجهولين ($V_1$ و $V_2$)

$Q = V_1 A_1$ و $Q = V_2 A_2$

$\frac{Q}{A_1} = V_1$  $\frac{Q}{A_2} = V_2$

نعوضهم بالقانون ونجد مقدار (Q)

• **Cavitation:** is the phenomenon that occur when the fluid pressure is reduced to the local vapor pressure

تحدث هذه الظاهرة في المناطق التي تكون السرعة عندها عالية

ونستنتج انه كلما قل مساحة pipe تزداد السرعة والضغط يقل

-If the pipe area is reduced, the velocity is increased according to the continuity equation and the pressure is reduced as dictated by the Bernoulli equation

## •Differential Form of the Continuity Equation:

نتعامل معه في حالة اذا اردت ايجاد السرعة داخل(C.v)(inside C.v)

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}+\frac{\partial \rho}{\partial t}=0$$

## • Integral form:

نلجأ له في حالة اذا كنت اتعامل على حدود (C.v) (outside C.v)

$$\int_{cs} \rho V.dA=-\frac{d}{dt}\int_{cv} \rho\, d\forall$$

$$\frac{dM_{cv}}{dt}=\dot{m}_{in}-\dot{m}_{out}$$

•اتى سؤال سنوات على الفرق بينهم

• **Example:** check the following equation if it satisfies the continuity equation for incompressible flow. $V=(-x^2y)_i+(xy^2)_j$

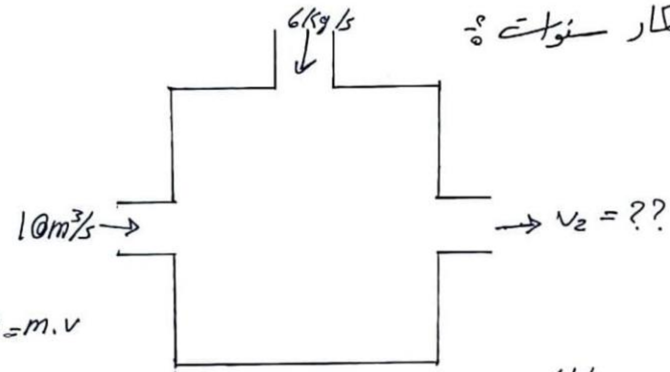
يجب ان يكون $\frac{du}{dx}+\frac{dv}{dy}=0$

$u=-x^2y \Rightarrow \frac{du}{dx}=-2xy$

$v=xy^2 \Rightarrow \frac{dv}{dy}=2xy$

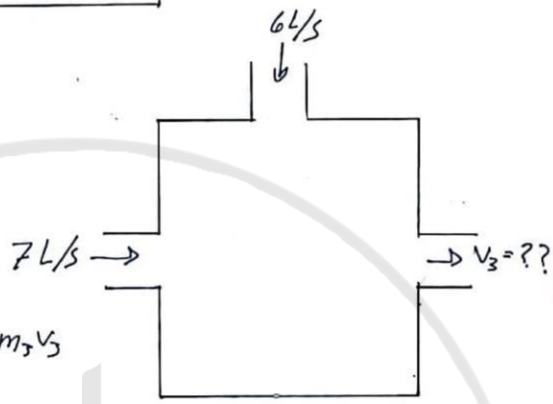
*it satisfies the continuity equation

* افكار سنوات :




$+6 + 10\rho = m.v$

---




L :- Letter

$$\boxed{1L = 10^{-3} m^3}$$

$$6L/s \times 10^{-3} = 6\times10^{-3} m^3/s$$

$$7L/s \times 10^{-3} = 7\times10^{-3} m^3/s$$

$$(\rho)(7\times10^{-3}) + (\rho)(6\times10^{-3}) = m_3 V_3$$

**5.5** The discharge of water in a 25 cm diameter pipe is 0.05 $m^3/s$. What is the mean velocity?

$$\bar{V} = \frac{Q}{A} = \frac{0.05}{(\pi/4)(0.25)^2} = 1.018\ m/s$$

**5.8** A pipe whose diameter is 8 cm transports air with a temperature of 20°C and pressure of 200 kPa absolute at 20 m/s. Determine the mass flow rate.

$$\boxed{R = 287}$$

$\dot{m} = \rho V A \Rightarrow$ كثافة (ρ) مجهولة

$$P = \rho R T \Rightarrow \rho = \frac{P}{RT} = \frac{200\times10^3}{(287)(20+273)} = 2.378\ kg/m^3$$

$$\dot{m} = (2.378)(20)(\pi/4)(0.08)^2$$

$$\boxed{\dot{m} = 0.239\ kg/s}$$

**5.9** Natural gas (methane) flows at 20 m/s through a pipe with a 1 m diameter. The temperature of the methane is 15°C, and the pressure is 150 kPa gage. Determine the mass flow rate.

$$R = 518$$

$$\dot{m} = \rho V A$$

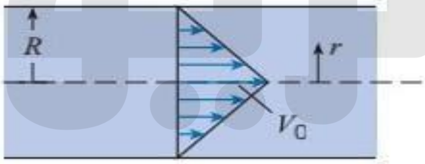
$$\rho = \frac{P}{RT} = \frac{150 \times 10^3 + 101 \times 10^3}{(518)(273+15)} = 1.682 \text{ kg/m}^3$$

$$\dot{m} = 1.682 \times 20 \times \frac{\pi}{4} \times (1)^2 = 26.4 \text{ kg/s}$$

**5.12** The hypothetical velocity distribution in a circular duct is

$$\frac{v}{V_0} = 1 - \frac{r}{R}$$

where $r$ is the radial location in the duct, $R$ is the duct radius, and $V_0$ is the velocity on the axis. Find the ratio of the mean velocity to the velocity on the axis.




PROBLEM 5.12

$$\bar{V} = \frac{Q}{A} \Rightarrow Q = \int_0^R v(r)\, 2\pi r\, dr$$

$$\frac{v(r)}{V_0} = 1 - \frac{r}{R} \Rightarrow v(r) = V_0\left(1 - \frac{r}{R}\right)$$

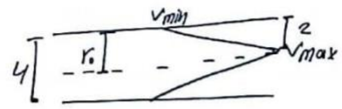
$$Q = \underbrace{(V_0 2\pi)}_{\text{constant}} \left[\left(\frac{r^2}{2}\right) - \left(\frac{r^3}{3R}\right)\right]_0^R$$

$$= \frac{1}{3}\pi V_0 R^2$$

$$\bar{V} = \frac{\frac{1}{3}\pi V_0 R^2}{\pi R^2} \Rightarrow \bar{V} = \frac{1}{3} V_0 \Rightarrow \frac{\bar{V}}{V_0} = \frac{1}{3}$$

**5.14** Water flows in a pipe that has a 4 ft diameter and the following hypothetical velocity distribution: The velocity is maximum at the centerline and decreases linearly with $r$ to a minimum at the pipe wall. If $V_{max} = 15$ ft/s and $V_{min} = 12$ ft/s, what is the discharge in cubic feet per second and in gallons per minute?

* بحكيلك في السؤال انه السرعة تتغير بشكل خطي
معادلة الخط المستقيم :  $\boxed{v(r) = ar + b}$




at $r = 0$ , $v = v_{max} = 15$

$b = 15$

at $r = r_0$ , $v = 12$

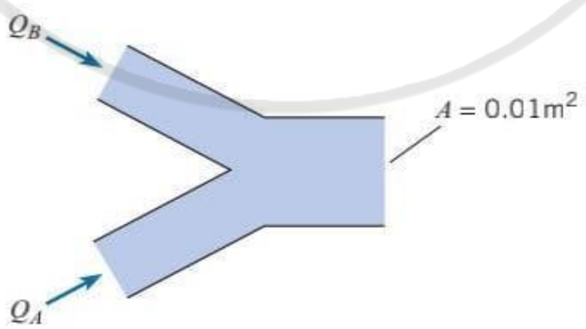
$12 = ar_0 + 15 \Rightarrow \boxed{a = \frac{-3}{r_0}}$ , $v(r) = \frac{-3r}{r_0} + 15$ , $\boxed{r_0 = 12}$

$Q = \int_0^{r_0} v(r) 2\pi r d$

$Q = \int_0^2 \left(\frac{3r}{2} + 15\right) 2\pi r = 2\pi \int_0^2 \frac{-3r^2}{2} + 15 r dr$   * تكامل على الالة حاسبة

$\boxed{Q = 163.4}$

**5.46** Two streams discharge into a pipe as shown. The flows are incompressible. The volume flow rate of stream $A$ into the pipe is given by $Q_A = 0.02t$ m$^3$/s and that of stream $B$ by $Q_B = 0.008t^2$ m$^3$/s, where $t$ is in seconds. The exit area of the pipe is 0.01 m$^2$. Find the velocity and acceleration of the flow at the exit at $t = 1$ s.




PROBLEM 5.46

a) $Q_{exit} = Q_A + Q_B = 0{,}026 + 0{,}008t^2$

$\boxed{V = \frac{Q}{A}}$

$V_{exit} = \frac{0{,}026}{0{,}01} + \frac{0{,}008t^2}{0{,}01}$

$\boxed{V = 2t + 0{,}8t^2}$

Velocity at $\boxed{t=1} \Rightarrow V = 2(1) + (0{,}8)(1)^2$

$\boxed{V = 2{,}8 \text{ m/s}}$

b) هنا بتحرك على محور (X) فقط

$a = V\frac{dv}{dx} + \frac{dv}{dt}$ $\boxed{\frac{dv}{dt} \neq 0}$

$\frac{dv}{dt}$ → unsteady لانه

ولكن بالسؤال طالب تغير السرعة بالنسبة للزمن وليس للموقع

ولانه ايضا لا يوجد متغير (X) في الاقتران (V)

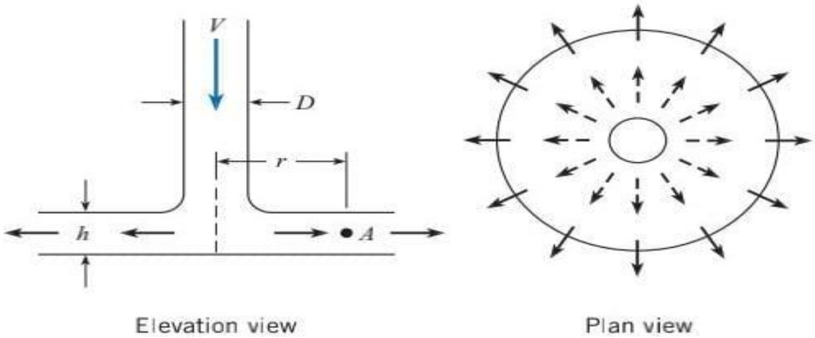
$\boxed{\frac{dv}{dx} = 0}$

$a = \frac{dv}{dt} = 2 + 1{,}6t = 2 + (1{,}6)(1)$

$\boxed{a = 3{,}6 \text{ m/s}^2}$



**5.47** Air discharges downward in the pipe and then outward between the parallel disks. Assuming negligible density change in the air, derive a formula for the acceleration of air at point *A*, which is a distance *r* from the center of the disks. Express the acceleration in terms of the constant air discharge *Q*, the radial distance *r*, and the disk spacing *h*. If $D = 10$ cm, $h = 0.6$ cm, and $Q = 0.380$ m$^3$/s, what are the velocity in the pipe and the acceleration at point *A* where $r = 20$ cm?




PROBLEMS 5.47, 5.48

$$V_{pipe} = \frac{Q}{A_{pipe}} = \frac{0.38}{(\frac{\pi}{4})(0.1)^2} = 48.4 m/s$$

$$a = V_r \frac{dv}{dr} + \frac{dv}{dt} \text{ (zero)} \quad , \quad \boxed{V_r = \frac{Q}{2\pi r h}}$$

$$V_r = \frac{Q r^{-1}}{2\pi h} \Rightarrow \frac{dv}{dr} = \frac{-Q r^{-2}}{2\pi h} = \frac{-Q}{2\pi h r^2}$$

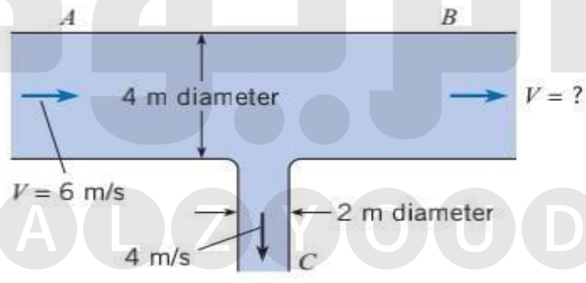
$$= \frac{-Q}{(2\pi r h)^2}$$

$$a = \left(\frac{Q}{2\pi r h}\right)\left(\frac{-Q}{(2\pi r^2 h)^2}\right) = \frac{-Q}{(r(2\pi r h)^2)}$$

$$a = \frac{(-0.38)^2}{(0.2)(2\pi * 0.2 * 0.05^2)}$$

$$\boxed{a = -18288 m/s^2}$$

**5.58** What is the velocity of the flow of water in leg $B$ of the tee shown in the figure?




PROBLEM 5.58

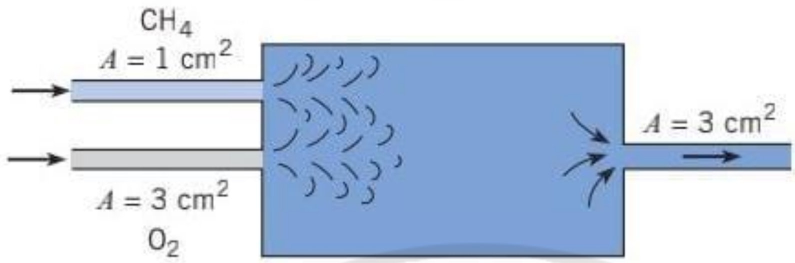
$$\dot{m}_{in} = \dot{m}_{out}$$

$$V_A A_A = (V_B * A_B) + (V_C * A_C)$$

$$V_B = \frac{V_A A_A - V_C A_C}{A_B} = \frac{((6)(\frac{\pi}{4})(4)^2) - ((4)(\frac{\pi}{4})(2)^2)}{\frac{\pi}{4}(4)^2}$$

$$\boxed{V_B = 5 m/s}$$

**5.67** Oxygen and methane are mixed at 250 kPa absolute pressure and 100°C. The velocity of the gases into the mixer is 5 m/s. The density of the gas leaving the mixer is 2.2 kg/m$^3$. Determine the exit velocity of the gas mixture.




PROBLEM 5.67

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\rho_{O_2} V A + \rho_{CH_4} V A = \rho V A$$

نريد ايجاد $(\rho) \Leftarrow (CH_4 , O_2)$ (I)

$$\rho_{O_2} = \frac{P}{RT} = \frac{250 * 10^3}{(260 * 373)} = 2{,}57\ kg/m^3$$

$$\rho_{CH_4} = \frac{250 * 10^3}{(218 * 373)} = 1{,}29\ kg/m^3$$

(II) $\boxed{V = 4{,}87\ m/s}$

# Ch6 : momentum equation

بنعرف انه حركة الموائع بتحكم بها 3 قوانين :

1) قانون حفظ الكتلة ونستنتج منها (continuity equation)
2) قانون حفظ الزخم نستنتج منه (momentum equation)
3) قانون حفظ الطاقة نستنتج منه (energy equation), (Bernoulli equation)

نستنتج معادلة الزخم من قانون نيوتن الثاني : F=ma

$$\sum \mathbf{F} = \frac{d}{dt}\int_{cv} \mathbf{v}\,\rho\, d\forall + \int_{cs} \mathbf{v}\,\rho\, \mathbf{V}.d\mathbf{A}$$

لو كانت المعادلة steady:

$$\frac{d}{dt}\int_{cv} \mathbf{v}\,\rho\, d\forall$$

هذا الحد يصبح يساوي صفر

- **V**: fluid velocity relative to the CS at the location where the flow is crossing the surface
- **v**: the velocity relative to an inertial frame; that is a frame which does not rotate and can either be fixed or moving at a constant velocity

- **The momentum equation states that**:
  The sum of external forces acting on the material in the CV = the rate of momentum change inside the CV + the net rate at which momentum flows out of the CV

# Force Terms ( $\sum F$ )

- These forces can be either:
  - **Body forces:** (gravity, electrostatic, magnetic).
  - **Surface forces:** (pressure, shear, supports…etc.).

لو كانت المعادلة steady رح نستخدم القانون الاتي :

$\sum F=\sum m^{\circ} v$

واغلب المسائل في هذا الشابتر تكون steady + incompressible

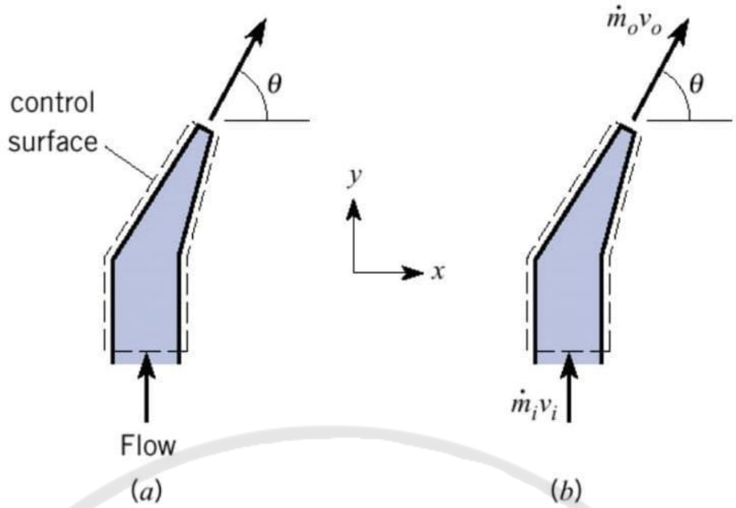
- تحديد الاشارات:

1) اشارة ($m^{\circ}$): اذا كانت داخلة بالجسم **سالبة**
   واذا كانت خارجة من الجسم تكون **موجبة**
2) السرعة ($v$): تكون موجبة باتجاه محور ($x,y$) الموجب
3) القوى($F$): تكون موجبة باتجاه محور ($x,y$) الموجب

ملاحظة: $m^{\circ}$ لاتحلل

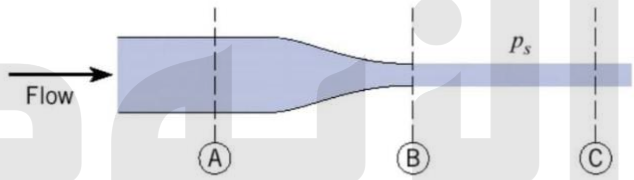
($F,V$): تحلل

## Nozzle

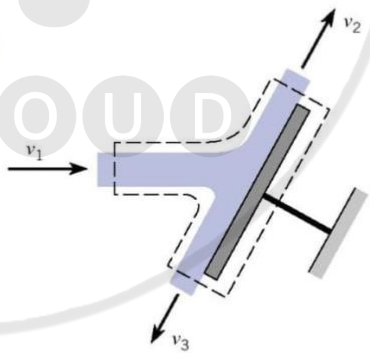



الهدف من Nozzle: هو زيادة السرعة

## Fluid jet




## Fluid jet striking a flat vane

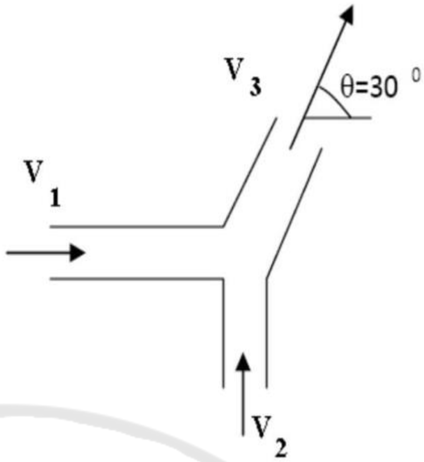



P1=P2=P3=zero

V1=V2=V3

بشرط اهمال الارتفاعات

- **Example**: find the momentum flow $\int_{cs} \mathbf{v}\,\rho\,\mathbf{V}.d\mathbf{A}$




* احنا بنتعامل مع كل محور لوحده :-

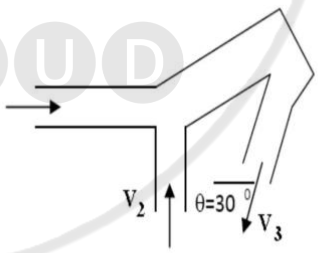
x-axis :-

$$V_1(-\dot{m}_1) + V_3(\dot{m}_3)\cos 30 = \Sigma F_x$$

y-axis :-

$$\Sigma F_y = V_2(-\dot{m}_2) + (V_3 \sin 30)(\dot{m}_3)$$

- **Example**: find the momentum flow $\int_{cs} \mathbf{v}\,\rho\,\mathbf{V}.d\mathbf{A}$




x-axis :-

$$\Sigma F_x = V_1(-\dot{m}_1) - (V_3 \cos 30)(\dot{m}_3)$$

y-axis :-

$$\Sigma F_y = V_2(-\dot{m}_2) - (V_3 \sin 30)(\dot{m}_3)$$

- **Example:** Steady, uniform flow at each section, incompressible, and neglect weight of 90 $^0$ reducing elbow and water. Determine the forced required to hold the elbow in place.

Given:

**$A_1$=0.01 $m^2$**

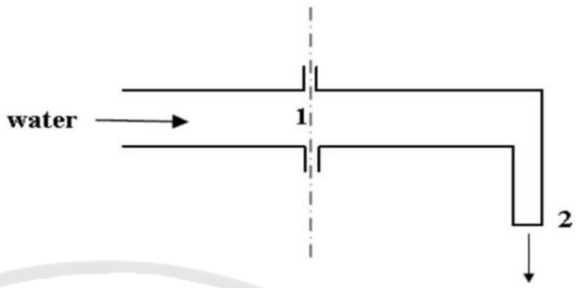
**$p_1$=119 kpa**

**$A_2$=0.0025 $m^2$**

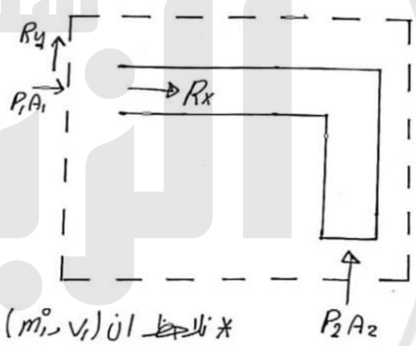
**$V_2$=16 m/s**

**$p_2$=$p_{atm}$**

**$\rho$=1000 kg/ $m^3$**




* اول خطوة :- نحدد (control surface)




$$\Sigma F_x = \frac{d}{dt}\overset{0}{\int_{CV}} V_x \rho \, d\forall + \int_{CS} V_x \rho V \, dA$$

$$R_x + P_1A_1 = V_{x1}(-\dot{m}_1) + \overset{zero}{V_{x2}}(\dot{m}_2)$$

$$R_x = -(P_1A_1 + V_1\dot{m}_1)$$

* نلاحظ ان $(\dot{m}_1, V_1)$ مجهولة، نستخدم معادلة الاستمرارية لإيجادهم

$$\boxed{\dot{m}_1 = \dot{m}_2}$$

$$\rho V_1A_1 = \rho V_2A_2$$

$$V_1 = \frac{V_2A_2}{A_1} = \frac{(16)(0.0025)}{0.01} = 4 \ m/s$$

$$\dot{m}_1 = \rho V_1A_1 = (1000)(4)(0.01) = 40 \ kg/s$$

$$R_x = -((119)(10^3)(0.01) + (4)(40)) = -1350 N$$

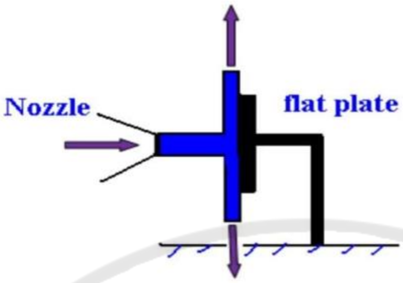
* الاشارة السالبة تعني عكس الاتجاه الذي فرضناه

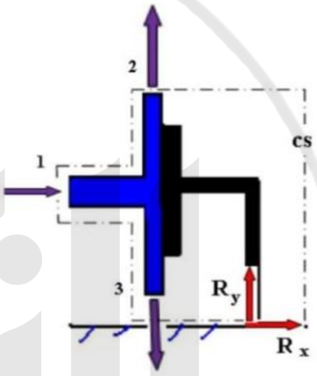
$$\Sigma F_y = \frac{d}{dt}\int_{CV} V_y \rho \, d\forall + \int_{CS} V_y \rho V \, dA = \Sigma V(\dot{m})$$

$$R_y = \overset{zero}{V_{y1}}(-\dot{m}_1) + V_{y2}(\dot{m}_2)$$

$$R_y = -V_2 \dot{m}_3 \Rightarrow R_y = (-16)(40) = \boxed{-640N}$$

- **Example:** The water leaves the nozzle at 15 m/s ($A_{nozzle}$=0.01 m$^2$). Assuming steady, incompressible, and neglect the weight of jet and the plate, change in elevation is also neglected. Determine the reaction forces on the support.




$$\sum F_x = \int_{cs} V_x \rho V dA = \sum \dot{m} V$$

$$R_x = V_{x1}(-\dot{m}_1) + \overset{zero}{V_{x2}}(\dot{m}_2) + \overset{zero}{V_{x3}}(\dot{m}_3)$$

$$\boxed{R_x = -V_1 \dot{m}_1}$$

$$\dot{m}_1 = \rho V_1 A_1 = (1000)(15)(0.01) = 150 \text{ kg/s}$$

$$R_x = (-15)(150) = \boxed{-2.25 \text{ kN}}$$

$$\sum F_y = \int_{cs} V_y \rho V dA = \sum \dot{m} V$$

$$R_y = V_{1y}(-\dot{m}_1) + (V_{y2})(\dot{m}_2) + (V_{y3})(\dot{m}_3)$$

$$R_y = V_2 \dot{m}_2 + V_3 \dot{m}_3$$

* هنا عندي ($V_2$ و $V_3$) مجهولة، لإيجادهم رح اطبق برنولي، ولكن رح اهمل الارتفاعات (elevation) لو بدي اخذ النقطة (1) و (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \qquad \boxed{Z_1 = Z_2}, \boxed{P_1 = P_2 = 0}$$

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} \Rightarrow \boxed{V_1 = V_2} \quad \boxed{V_1 = V_2 = V_3}$$

* لإيجاد ($\dot{m}_2$)، ($\dot{m}_3$) :-

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \Rightarrow \boxed{\dot{m}_1 = 2\dot{m}_2} \Rightarrow$$ لإن ($\dot{m}_3$، $\dot{m}_2$) نفس السرعة ونفس المساحة

$$\boxed{R_y = V_2(\dot{m}_2 - \dot{m}_3) = zero}$$

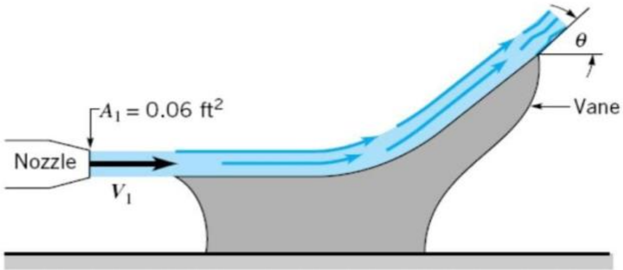
- **Example:** Determine the anchoring force needed to hold the vane stationary. The problem is steady, incompressible, neglect the gravity.

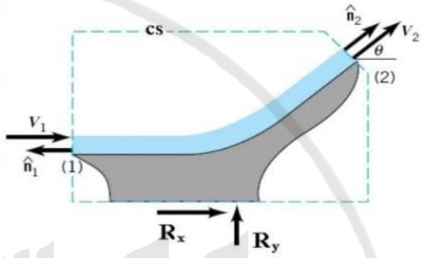
Given:

$V_1=10$ ft/s

$A_1=0.06$ ft$^2$

$\rho_{fluid}=1.94$ slug/ft$^3$




$$\Sigma F_x = \int v_x \rho V \cdot dA = \Sigma \dot{m} V$$

$$R_x = V_{x_1}(-\dot{m}_1) + V_{x2}(\dot{m}_2) \qquad V_{x2} \rightarrow V_2 \cos\theta$$

$P_1 = P_2 = 0$ $\quad$ $V_1 = V_2$ $\quad$ نطبق برنولي ب نهمل الارتفاع

$\dot{m}_1 = \dot{m}_2$

$$\dot{m}_1 = \rho V_1 A = (1{,}94)(10)(0{,}06) = 1{,}64 \text{ slugs/sec}$$

$R_x = 11{,}64(\cos\theta - 1)$ lbF $\qquad \Sigma F_y = \int_{cs} v_y \rho V \cdot dA$

$$R_y = V_{y_1}(-\dot{m}_1) + V_{y2}(\dot{m}_2) \qquad V_{y2} \rightarrow V_2 \sin\theta$$

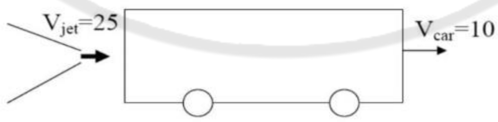
$R_y = 11{,}64 \sin\theta$



- Moving Control Volumes.

$$\vec{V}_r = \vec{V} - \vec{V}_{c.v}$$

example:




**$V_{r1}=25$ i-10 i=15 i**

**In case if the car velocity is to the left**

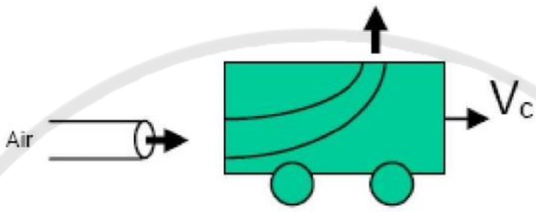
**$V_{r1}=25$ i-(-10 i)=35 i**

**The relative velocity is the fluid velocity relative to the moving control volume-the fluid velocity seen by an observer riding along on the control volume.**

**The absolute velocity is the fluid velocity as seen by a stationary observer in a fixed coordinate system.**

## Example:

**A jet of air traveling at 12 m/s is directed at a 90-degree curved passage in a cart that is moving at constant speed $V_c$=5 m/s. The curved passage has an inlet diameter of 5 cm and outlet diameter of 1.5 cm. The jet diameter is 5 cm, what is the jet velocity (m/s) of air at the outlet of the curved passage?**




$\dot{m}_1 = \dot{m}_2$ $\qquad$ $\boxed{V_r = V_j - V_c}$

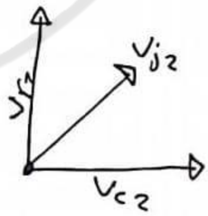
$\rho_1 V_{r_1} A_1 = \rho V_{r_2} A_2 \Rightarrow V_{r_1} A_1 = V_{r_2} A_2$

$(V_{j1} - V_{c1}) A_1 = (V_{j2} - V_{c2}) A_2$

$(12 - 5)\left(\frac{\pi}{4}(0{,}05)^2\right) = (V_{j2} - V_{c2})\left(\frac{\pi}{4}(0{,}015)^2\right)$

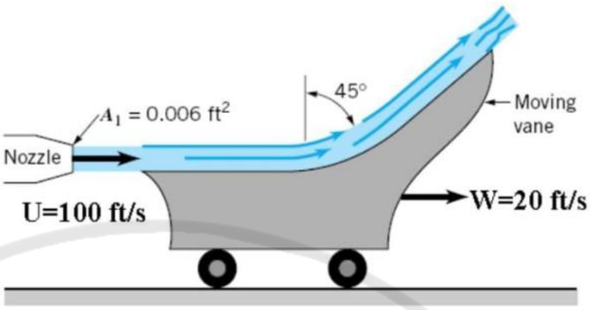
$(V_{j2} - V_{c2}) = \boxed{V_{r2} = 77{,}78 m/s}$ $\qquad$ $\boxed{V_{c1} = V_{c2}$}

$\boxed{V_{j2} = (5\hat{i} + 77{,}78\hat{j})\, m/s}$

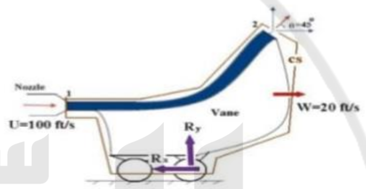


- **Example:** Determine the reaction forces for a moving vane (W=20 ft/s). The jet velocity is U=100 ft/s. The problem is steady, incompressible, neglect the gravity effect.

Given:
$A_1=0.006\ ft^2$
$\rho_{fluid}=1.94\ slug/ft^3$




$$V_1 = U_1 - W_1 = 100 - 20 = 80\ Ft/s$$




$$\boxed{V_1 = V_2} \qquad \boxed{\dot{m}_1 = \dot{m}_2}$$

$$\dot{m}_1 = \rho V_1 A_1 = (1.94)(80)(0.006) = 0.9312\ slugs/sec$$

$$\Sigma F_x = \int_{cs} V_x \rho V dA$$

$$-R_x = V_{x1}(-\dot{m}_1) + V_{x2}(\dot{m}_2)$$
$V_{x2} \rightarrow V_2 \cos 45$

$$\boxed{R_x = 21.8\ lbF}$$

$$\Sigma F_y = \int_{cs} V_y \rho V dA = \Sigma \dot{m} v$$

$$R_y = \overset{zero}{V_{y1}}(-\dot{m}_1) + V_{y2}(\dot{m}_2)$$
$V_{y2} \rightarrow V_2 \sin \theta$

$$\boxed{R_y = 52.67\ lbF}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2} = 57\ lbF \Rightarrow$$ لإيجاد المحصلة

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 67.5^\circ$$

# 6.6: Navier-Stokes Equations

غير مطلوب الحل عليه

They are a differential form equations of momentum based on a control volume of infinitesimal size.

For incompressible and constant viscosity flow:

**X-direction**

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho.g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

acceleration — flow force — body force — shear force

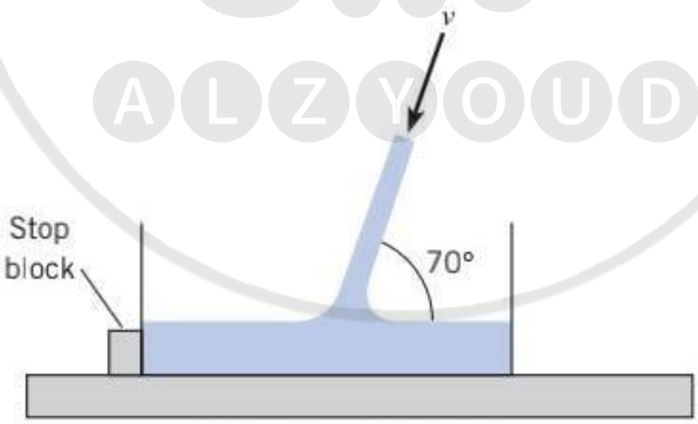
**Y-direction**

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho.g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

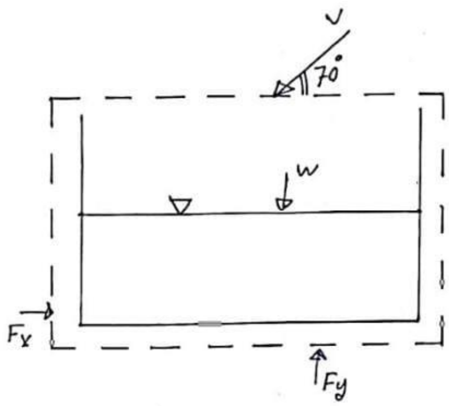
**Z-direction**

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho.g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

**6.7** A water jet of diameter 30 mm and speed $v = 20$ m/s is filling a tank. The tank has a mass of 20 kg and contains 20 liters of water at the instant shown. The water temperature is 15°C. Find the force acting on the bottom of the tank and the force acting on the stop block. Neglect friction.




PROBLEM 6.7, 6.8




$\Sigma F_x = \Sigma \dot{m} v$

$F_x = -V_1(-\dot{m}_1)\cos 70$

$= \rho V_1^2 A \cos 70$

$= (1000)\left(\frac{\pi(0.03)^2}{4}\right)(20)^2$

$= \boxed{96.6\,N}$

$\Sigma F_y = \Sigma \dot{m} v \Rightarrow F_y - W = -V\sin 70(-\dot{m})$

$F_y = W + V\sin 70(\dot{m})$

$W = W_{tank} + W_{water}$

$\boxed{W = mg}$ $= W_{tank} = (20)(9.81) = \boxed{196.2\,N}$

* في السؤال عندي حجم السائل بديه اطلع منه الوزن

$\boxed{m = \rho V}$

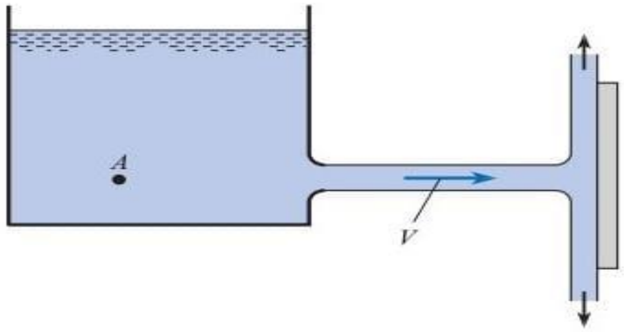
$W = mg = \rho V g = \gamma V \Rightarrow (0.02)(9810) = \boxed{196.2N}$

$\boxed{W = 392.4N}$

$F_y = 392.4 + \rho V^2 A \sin(70)$

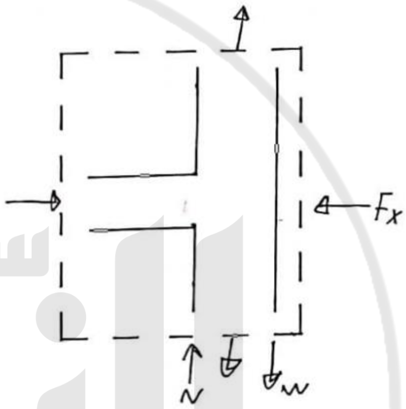
$\boxed{F_y = 658\,N}$

**6.10** A horizontal water jet at 70°F impinges on a vertical-perpendicular plate. The discharge is 2 cfs. If the external force required to hold the plate in place is 200 lbf, what is the velocity of the water?

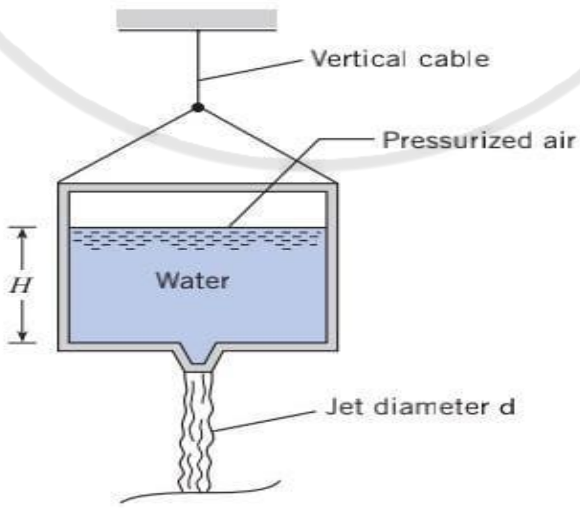



$\rho = 1.94$

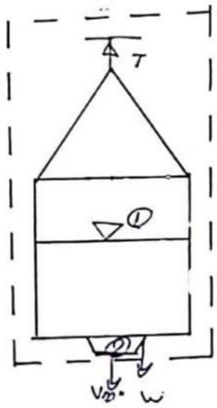
$$\Sigma F_x = \Sigma \dot{m} V$$
$$-F_x = (-\dot{m})V$$
$$F_x = (\rho Q)V$$
$$V = \frac{F_x}{\rho Q} = \frac{200}{(1.94)(2)} = 51.5 \text{ ft/s}$$




**6.15** A tank of water (15°C) with a total weight of 200 N (water plus the container) is suspended by a vertical cable. Pressurized air drives a water jet ($d$ = 12 mm) out the bottom of the tank such that the tension in the vertical cable is 10 N. If $H$ = 425 mm, find the required air pressure in units of atmospheres (gage). Assume the flow of water is irrotational.



* لإيجاد الـ (Pressure) نطبق برنولي




$$P_1 + \gamma z_1 + \rho \frac{V_1^2}{2g} = P_2 + \gamma z_2 + \rho \frac{V_2^2}{2g}$$

$V_1 = 0$ , $V_2 = ??$ , $P_2 = 0$

$z_1 = 0$ , $z_2 = 0.425$ نريد إيجاد السرعة ($V_2$)

$$\Sigma F_y = \Sigma v \dot{m}$$

$$T - W = (-v)(\dot{m}) = 10 - 200 = \rho A_2 V_2^2$$

$$-190 = (1000)\left(\frac{\pi}{4}(0.012)^2\right)(V_2)^2$$

$V_2 = 41 \, m/s$

* النقطة (2) عند (Jet)

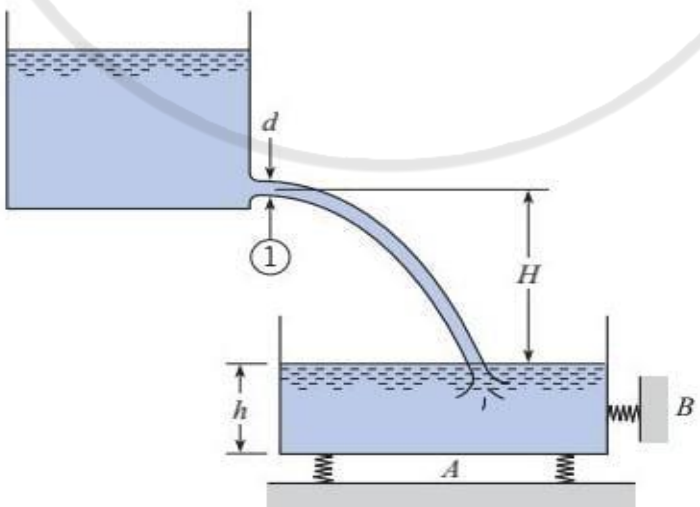
* نطبق برنولي

$$P_1 = \frac{\rho V_2^2}{2} - \gamma z_2$$

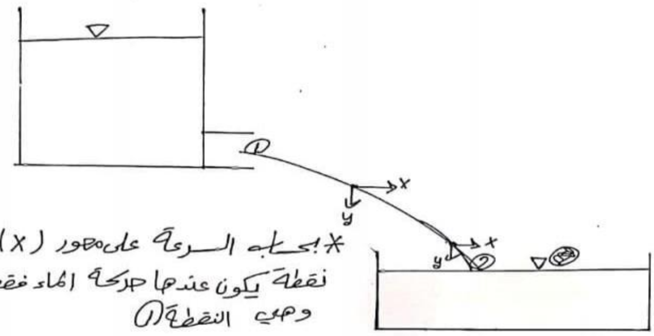
$$P_1 = \frac{835900 \, Pa}{101.3 \times 10^3} = 8.25 \, atm$$

**6.16** A jet of water (60°F) is discharging at a constant rate of 2.0 cfs from the upper tank. If the jet diameter at section 1 is 4 in., what

forces will be measured by scales *A* and *B*? Assume the empty tank weighs 300 lbf, the cross-sectional area of the tank is 4 ft$^2$, $h = 1$ ft, and $H = 9$ ft.




PROBLEM 6.16




* لحساب السرعة على محور (X) بدي اخذ نقطة يكون عندها حركة الماء فقط على محور (X) وهي النقطة (1)

* لحساب السرعة على محور (y) بدي اخذ نقطة يتحرك عندها الماء على محور (y) فقط وهي نقطة (2)

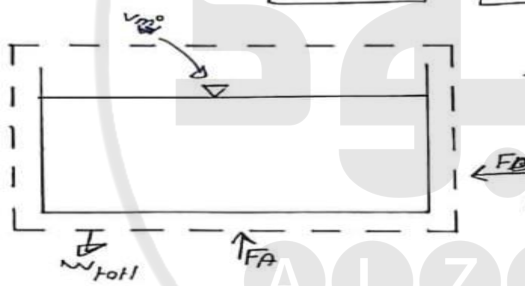
* نطبقه برنولي بين (1) و (2)

السرعة على (y) = zero

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (P_1, V_1, P_2, z_2 = \text{zero})$$

$$h = \frac{V_2^2}{2g} \Rightarrow V_2 = \sqrt{2gh}$$

$\boxed{\rho = 1.94}$ , $\boxed{\gamma = 62.37}$ , $\boxed{g = 32.2}$




* نلاحظ ان الماء يسقط سقوط حر من النقطة (1)، سرعتها على محور (X)، نجدها من خلال (Q)

$$\dot{m} = \rho Q = (1.94)(2) = \boxed{3.88}$$

$$V_1 = V_x = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} d^2} = \frac{2}{(\frac{\pi}{4})(\frac{4}{12})^2} = 22.9$$

$$V_y = \sqrt{2gH} = \sqrt{(2)(32.2)(9)} = 24.1 \text{ Ft/s}$$

$$\Sigma F_x = \Sigma \dot{m} V \Rightarrow -F_B = V(-\dot{m})$$

$$F_B = (3.88)(22.9) = \boxed{88.9 \text{ IbF}}$$

$$\Sigma F_y = \Sigma \dot{m} V \Rightarrow F_A - W_{tot} = -V(-\dot{m})$$

$$F_A = W_{tot} + V_y(\dot{m})$$

$$W_{tot} = W_{H_2O} + W_{tank} = (\gamma V) + 300$$

$$F_A = ((62.37)(4)(1)) + 300) + (3.88)(24.1)$$

$$F_A = 643 \text{ IbF}$$

# Ch7 : Energy principle

في هذا الشابتر رح نتكلم عن القانون الثالث الذي يتحكم بحركة الموائع

First law of thermodynamics:

$\Delta E = Q - W$

Q: Heat transferred to the system

وله نوعين: 1) يؤدي الى تغيير درجة حرارة الجسم

2) يؤدي الى تغير في حالة المادة مثل تحول المادة من الحالة السائلة الى الصلبة

W: Work done by the system on the surroundings

(Q,W): تكون على control surface

E: the energy of a system

$E = E_u + E_k + E_p$

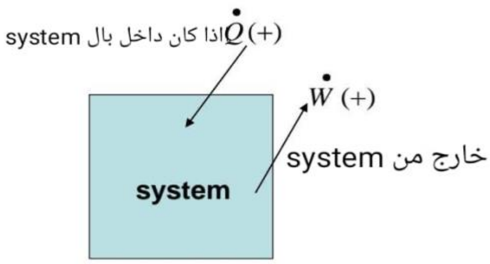
$E_u$: internal energy (atoms)

$E_k$ : kinetic energy, $E_p$ : potential energy

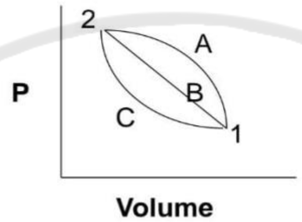
هنا بالطاقة احنا بنتعامل مع
معدل للطاقة لانه اذا بدي اشوف الطاقة لكل particle في المائع رح يكون صعب احنا بنوخذ C.v معينة وبشوف عندها معدل الطاقة

## In terms of rate of energy:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$




**Q and W are path function**




Q, W path function:

لاني بقدر احقق الهدف ب اكثر من طريقة

مثلا لو نلاحظ بالشكل بقدر انتقل من النقطة 1 الى 2 بأكثر من طريقة

اقل work عند c

اعلى work عند A

W°=power=$\frac{w}{t}$ , work rate

**Work can be divided into:**

- **Shaft work (through turbine or pump).**
- **Flow work (due to pressure).**

$$\dot{W} = \dot{W}_s + \dot{W}_f$$

**Shaft work rate** **flow work rate**

- For steady flow:

$$\dot{Q} - \dot{W}_s = \int_{cs} \left(\frac{V^2}{2} + gz + h\right)\rho \mathbf{V}.d\mathbf{A}$$

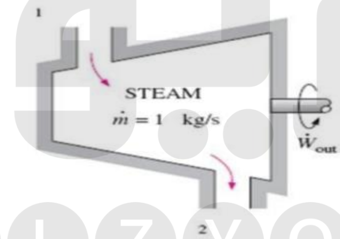
- For steady and uniform properties:

$$\dot{Q} - \dot{W}_s = \sum_{cs} \left(\frac{V^2}{2} + gz + h\right)\rho \mathbf{V}.\mathbf{A}$$

h: enthalpy , $h = u + \frac{p}{\rho}$

بقسم على 1000لكي تصبح الوحد بالقانون مساوية لوحدة h(Kj/kg)

**Example:** A turbine receives steam at 1.8 Mpa, 500 °C (h=3470 kJ/kg) at a velocity of 5 m/s. The steam exits at an enthalpy 0f 2630 kJ/kg with a velocity of 70 m/s. The steam flows through at a rate of 1 kg/s, and the turbine develops 830 kW. Calculate the heat transfer from the turbine. Neglect the potential energy due to the elevation difference.




$$Q^\circ - w_s^\circ = \sum \left(\frac{V^2}{2} + gz + h\right)\underbrace{\rho V \cdot A}_{m^\circ}$$

* المطلوب هنا حساب Q :

$$Q^\circ - 830 = \left(\frac{V_1^2}{2} + h_1\right)(-m_1^\circ) + \left(\frac{V_2^2}{2} + h_2\right)(m_2^\circ)$$

$$Q - 830 = \left(\frac{(5)^2}{2*1000} + 3470\right)(-1) + \left(\frac{(70)^2}{2*1000} + 2630\right)(1)$$

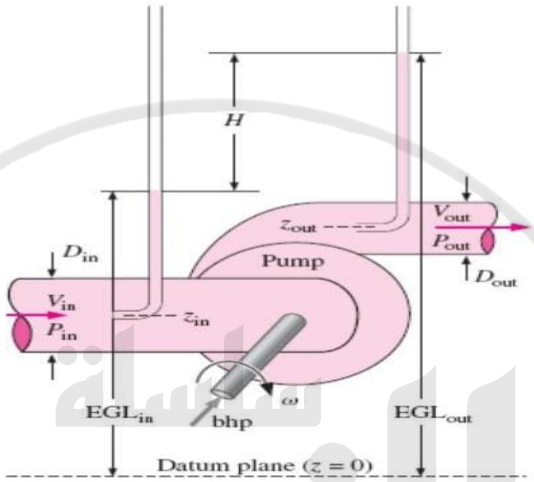
$$\boxed{Q = -7,6\ Kw}$$

* قسمنا على (1000) حتى تصبح الوحد مساوية لوحدة (h: Kj/Kg)

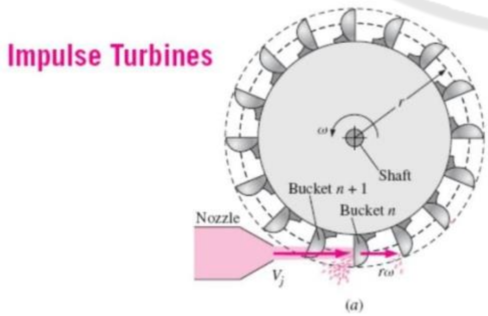
المضخة (pump): هو عبارة عن جهاز يستخدم لنقل السوائل من مكان الى اخر عن طريق زيادة ضغط السوائل

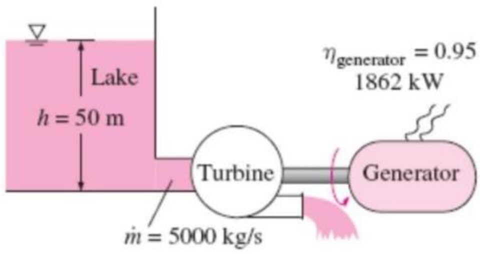
الهدف الرئيسي للمضخات: increase pressure

وتعمل على تزويد السوائل بالطاقة




Turbine: هو جهاز يعمل على امتصاص الطاقة



في المضخات يكون الخط الداخل اكبر من الخط الخارج وذلك لكي تحدث عملية التكهيف (cavitation)

تم اكتشاف معادلة لايجاد مطاليب معينة في وجود pump او turbine

وسميت هذه المعادلة ب energy equation

$$\frac{p_1}{\gamma}+z_1+\alpha_1\frac{\bar{V}_1^2}{2g}+h_p=\frac{p_2}{\gamma}+z_2+\alpha_2\frac{\bar{V}_2^2}{2g}+h_t+h_L$$

Mechanical energy

Thermal energy

α =2 Laminar flow α =1 Turbulent flow

- Where: $h_p=\frac{\dot{W}_p}{\dot{m}g}=\frac{\dot{W}_p}{\gamma Q}$ Pump Head

$h_t=\frac{\dot{W}_t}{\dot{m}g}=\frac{\dot{W}_t}{\gamma Q}$ Turbine Head

$h_L=\frac{u_2-u_1}{g}-\frac{\dot{Q}}{\dot{m}g}$ Head Loss

Note: The bar over V is usually omitted

وهذه المعادلة يوجد شرطين لتطبيقها:(for viscous flow)

1)steady flow , 2) incompressible

α: kinetic energy correction factor

وضعناه بالقانون لتسهيل علينا لانه في اشتقاق القانون يوجد تكاملات قمنا ب ازالة التكاملات ووضعنا مكانه α

•والذي يميز هذه المعادلة عن معادلة برنولي هو وجود head loss

بحيث انه بمعادلة برنولي كان مهمل الاحتكاك

•لتطبيق هذه المعادلة لازم اخذ نقتطين وامشي باتجاه الflow

• في هذه المعادلة تكون الوحدات طولية (ft,m,mm,inch,...)

**Pumps and turbines lose energy due to:**

**1- mechanical friction**

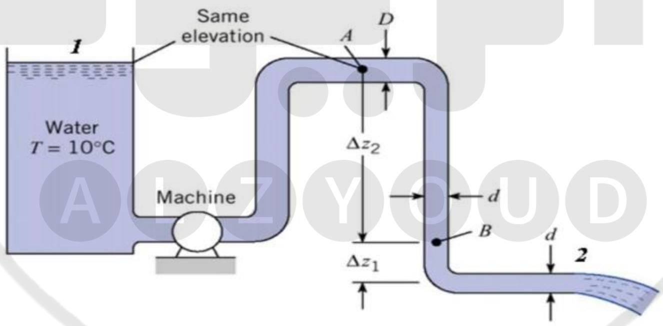
**2- viscous dissipation**

**3- leakage**

$$\eta_p = \frac{\dot{W}_{fluid}}{\dot{W}_{shaft}} = \frac{\gamma Q h_p}{w\, T_{shaft}}$$ **Pump effeciency**

$$\eta_t = \frac{\dot{W}_{shaft}}{\dot{W}_{fluid}} = \frac{\dot{W}_{shaft}}{\gamma Q h_t}$$ **Turbine effeciency**

**Example**: In this system, d=6 in., D=12 in., $\Delta z_1$=6 ft, and $\Delta z_2$=12 ft . The discharge of water in the system is 10 cfs. Is the machine a pump or a turbine? What are the pressures at point A and B? **Neglect head losses**. Assume $\alpha$=1.



* طالب بالسؤال مطلوبين :-

1) حدد نوع الالة

2) الضغط عند النقطة (A)، (B)

1) لتحديد نوع الالة رح افرض انها (pump) واحسب مقدار (hp)

($h_t = 0$) اذا كانت قيمة (hp) موجبة يكون الفرض صحيح

اذا كانت قيمة (hp) سالبة يكون (turbine)

* رح امشي من النقطة (1) الى (2)

$$\cancel{\frac{P_1}{\gamma}}^{zero} + \alpha_1 \cancel{\frac{V_1^2}{2g}}^{zero} + Z_1 + h_p = \cancel{\frac{P_2}{\gamma}}^{zer} + \alpha_2 \frac{V_2^2}{2g} + \cancel{Z_2}^{zero} + \cancel{h_t}^{zero} + \cancel{h_L}^{zero}$$

$$Z_1 + h_p = \alpha_2 \frac{V_2^2}{2g}$$

$$V_2 = \frac{Q}{A_2} = \frac{10}{\frac{\pi}{4}\left(\frac{67}{12}\right)^2} = 50,95 \text{ Ft/s}$$

$$\boxed{Z_1 = 18} \Rightarrow \Delta Z_2 + \Delta Z_1$$

$$18 + h_p = (1)\left(\frac{(50,95)^2}{32,2}\right) \Rightarrow \boxed{h_p = 22,31 \text{ Ft}}$$

* نأخذ من (B) الى (2) :- $\boxed{V_B = V_2}$

$$\frac{P_B}{\gamma} + \alpha_B \frac{V_B^2}{2g} + Z_B + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + Z_2 + h_t + h_L$$

($h_p$, $\frac{V_B^2}{2g}$, $\frac{P_2}{\gamma}$, $\frac{V_2^2}{2g}$, $Z_2$, $h_t$, $h_L$ crossed out; $Z_2$ = zero, $P_2$ = zero)

* اخذنا من (B) الى (2) لكي لا يكون مسارنا تحت تأثير المضخة ($h_p = 0$)

$$\frac{P_B}{62.4} + 6 = 0 \Rightarrow P_B = -374.4 \,(Ib/Ft^2)$$

* نأخذ مسار من (A) الى (2)

$$\frac{P_A}{\gamma} + \alpha_A \frac{V_A^2}{2g} + Z_A + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + Z_2 + h_t + h_L$$

($\frac{P_2}{\gamma}$ = zero, $Z_2$ = zero, $h_t$, $h_L$ crossed out)

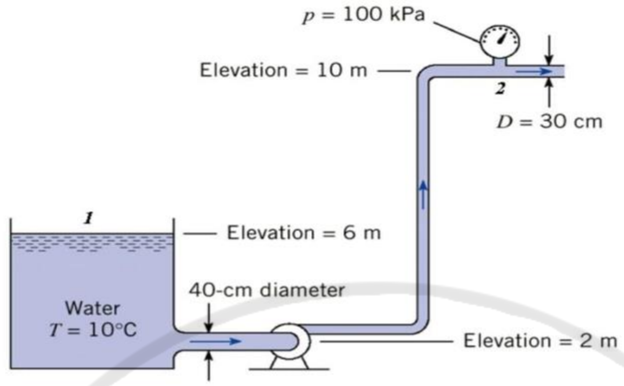
* نريد حساب سرعة (A) من خلال معادلة الاستمرارية

$$V_A A_A = V_B A_B \Rightarrow \boxed{V_A = 12.74 \, Ft/s}$$

$$\frac{P_A}{62.4} + (1)\frac{(12.74)^2}{2(32.2)} + 18 = (1)\frac{(50.95)^2}{(2)(32.2)}$$

$$P_A = 1235 \, Ib/Ft^2$$

**Example**: Water is flowing at a rate of 0.25 m³/s, and it is assumed that $h_L = 2V^2/2g$ from the reservoir to the gage, where V is the velocity in the 30-cm pipe. What power must the pump supply?




* هنا في السؤال طالب (Power)

$$\dot{W} = Q\gamma h_p$$

نطبق العلاقة :-

$$\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

($P_1/\gamma$ = zero, $V_1^2/2g$ = zero, $h_t$ = zero)

$$V_2 = \frac{Q}{A} = \frac{0,25}{\frac{\pi}{4}(0,3)^2} = 3,54\ m/s$$

$Z_1 = 6m$ ⇒ اخذنا (refrance)

لإيجاد (α) نحن بحاجة لإيجاد (Re) لمعرفة نوع الحركة

$$Re = \frac{\rho V D}{\mu} = \frac{(1000)(3,54)(0,3)}{(1,31)(10)^{-3}} = 81068772100$$

Turbelent

(μ, ρ) ⇒ تعطى بالسؤال

$\alpha = 1$

$$6 + h_P = \frac{100 * 10^3}{9810} + (1)\frac{(3,54)^2}{2(9810)} + 10 + \frac{2(3,54)^2}{2(9810)}$$

$h_p = 16,1m$

$$\dot{W}_p = (0,25)(9810)(16,1)$$

$$\dot{W}_p = 39,5\ Kw$$

الان رح نبدأ بشرح موضوع (losses):

هي تمثل الطاقة المفقودة من عملية الاحتكاك

تنقسم (losses):

**1) major loss:** الاحتكاك بهذه الحالة يحدث بشكل مستمر على طول pipe

تنشأ عند التغير في اقطار pipe

**2)minor loss:**

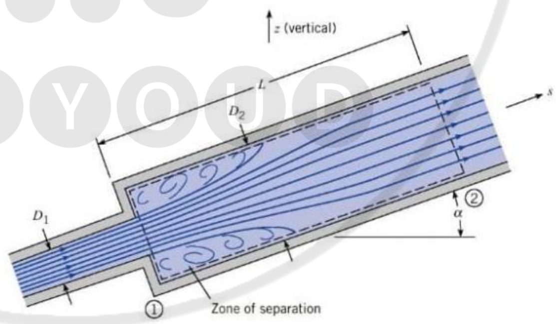
ينشأ الاحتكاك عند نقطة معينة وينشأ عند المداخل والمخارج والاكواع

ولكن في هذا الشابتر مطلوب فقط حالتين

Case1 : Abrupt Expansion

- Using this equation along with the energy equation and the continuity equation:

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

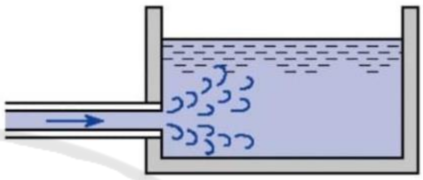


## Case 2 :

## Discharge into a Reservoir

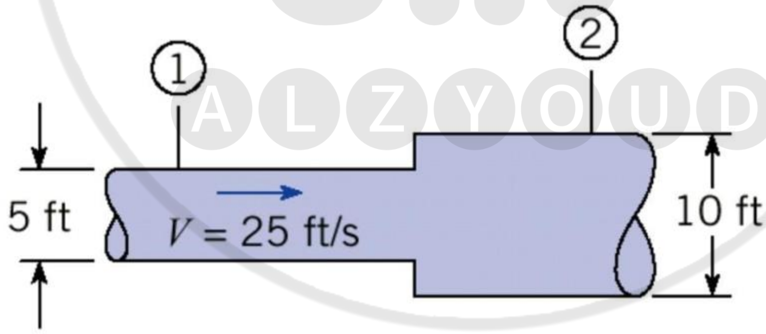
- When a pipe discharges into a reservoir, $V_2=0$:

$$h_L = \frac{V^2}{2g}$$



- The energy is dissipated by viscous action of the liquid.

**Example**: This abrupt expansion is to be used to dissipate the high-energy flow of water in the 5-ft diameter penstock.
a- What power (in horsepower) is lost through the expansion
b- If the pressure at section 1 is 5 psig, what is the pressure at section 2?
c- What force is needed to hold the expansion in place?




psig: Pounds per Square Inch

* الفكرة من هذا السؤال هو معرفة كيفية حساب (losses)
يوجد في السؤال هنا (one minor loss) هو على الحالة الأولى :-

a) $P = Q\gamma h_L$

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

ولكن ($V_2$) مجهولة نجدها من معادلة الاستمرارية

$V_1 A_1 = V_2 A_2 \rightarrow V_2 = 6,25\ Ft$

$$h_L = \frac{(25 - 6,25)^2}{(2)(32,2)} = 5,46\ Ft$$

$Q = V_1 A_1 = 490,9\ Ft^3/s$

$$P = \frac{Q\gamma h_L}{550} = \frac{(490,9)(62,4)(5,46)}{550} = 304,hp$$

قسمنا على (550) لكي نحول الى وحدة (horse)

b) لمعرفة (Pressure) نطبق معادلة الطاقة

$$\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + hp = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + ht + h_L$$

($z_1$ = zero, $z_2$ = zero)

نريد إيجاد ($\alpha_1$، $\alpha_2$) من خلال إيجاد ($R_e$)

$R_{e(1)} = \frac{\rho V D}{\mu} = 888278 4 \Rightarrow Turb \Rightarrow \alpha_1 = 1$

$R_{e(2)} = 4441342 \Rightarrow Turb \Rightarrow \alpha_2 = 1$

$P_1 = 5\ Psig = (5)(144)\ Psfg$

$P_2 = 946,6\ Psfg$

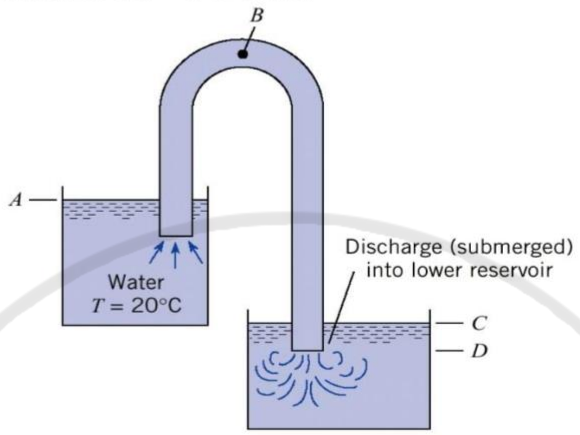
c) $\Sigma F_x = \Sigma \dot{m} v \Rightarrow$ momuntem

$P_1 A_1 - P_2 A_2 + F_x = V_2 \dot{m}_2 - V_1 \dot{m}_1 \Rightarrow \dot{m}_1 = \dot{m}_2$

$\dot{m} = \rho Q = (1,94)(490,9) = 951,8$

$F_x = 42292\ IbF$

**Example**: For this siphon the elevations at A, B, C, and D are 30 m, 32 m, 27 m, and 26 m, respectively. The head loss between the inlet and point B is ¾ of the velocity head, and the head loss in the pipe itself between point B and the end of the pipe is ¼ of the velocity head. For these conditions, what is the discharge and what is the pressure at point B? The diameter=30 cm.




$\boxed{\text{Velocity head} = \frac{V^2}{2g}}$ (major minor) يوجد بالسؤال *

Point A to C: $h_L = \left(\frac{3}{4} + \frac{1}{4}\right)\frac{V^2}{2g}$

Point A to B: $h_L = \frac{3}{4}\frac{V^2}{2g}$

Point B to C: $h_L = \frac{1}{4}\frac{V^2}{2g}$

$h_{total}(\text{major}) = \frac{V^2}{2g}$

$\text{minor} = \frac{V^2}{2g}$

$h_{total} = \frac{2V^2}{2g}$

$$\frac{P_A}{\gamma} + Z_A + \alpha\frac{V_A^2}{2g} + h_P = \frac{P_C}{\gamma} + Z_C + \alpha\frac{V_C^2}{2g} + h_t + h_L$$

(zero: $P_A$, $\frac{V_A^2}{2g}$, $h_P$, $P_C$, $\frac{V_C^2}{2g}$, $h_t$)

$$30 = 27 + \frac{2V^2}{2g} \Rightarrow \boxed{V = 5.42 \text{ m/s}}$$

$$Q = VA = (5.42)\left(\frac{\pi}{4}(0.3)^2\right) = \boxed{0.383 \text{ m}^3/\text{s}}$$

$$\frac{P_A}{\gamma} + \alpha\frac{V_A^2}{2g} + Z_A + h_P = \frac{P_B}{\gamma} + \alpha\frac{V_B^2}{2g} + Z_B + h_t + h_L$$

($P_A = 0$, $V_A = 0$, $h_P = 0$, $h_t = 0$, $\boxed{Z_A = 30}$, $\boxed{V_B = 5.42}$, $\boxed{Z_B = 32}$, $h_L = \frac{3}{4}\frac{V^2}{2g}$)

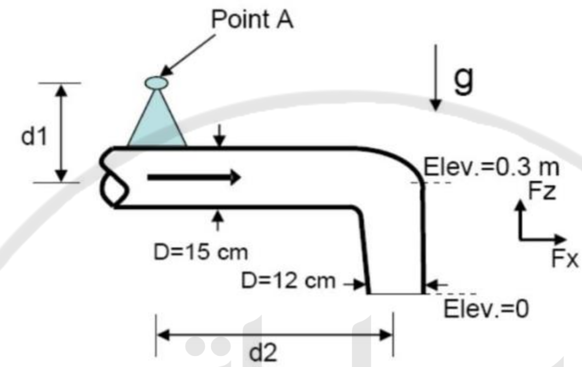
$$\boxed{P_B = -45.3 \text{ kPa}}$$

# EXAMPLE:

Water discharges to the atmosphere from a large faucet (control valve) as shown in the figure with a pressure at the faucet inlet is 250 kPa, gauge. The faucet is held stationary at point A. Let $\alpha$'s are equal to one, the total head loss in the faucet is 10 m and the inlet/outlet diameters are 15 & 12 cm, respectively. Neglect the weight of faucet and the water inside it. $d_1 = 0.35$ m and $d_2 = 0.5$ m.

What are the inlet and outlet velocities? What are the components of the force to hold the faucet stationary?

What is the torque (moment) necessary to keep the faucet from twisting?




1 velocity

$\dot{m}_1 = \dot{m}_2 = \rho V_1 A_1 = \rho V_2 A_2$

$V_1 = 0.64 V_2$

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$\frac{(250)(10)^3}{9810} + \frac{(0.64V_2)^2}{2(9.810)} + 3 = \frac{V_2^2}{(2)(9.81)} + 10$$

$V_2 = 22.9\ m/s$ , $V_1 = 14.7\ m/s$

2 Force:

$P_1 A_1 + F_x = -\dot{m}_1 (V)$

$\dot{m}_1 = \rho V_1 A_1 = 259.8\ kg/s$

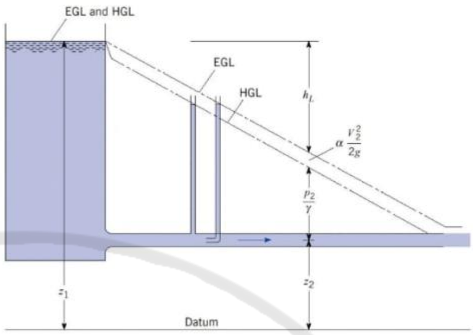
$F_x = -8.24\ kN$

$F_z = (-V_2)(\dot{m}_2) = -5.95\ kN$

# Hydraulic & Energy Grade Lines

- Recalling the energy equation between the surface of the reservoir and the downstream section:

$$z_1 = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{\overline{V}_2^2}{2g} + h_L$$




اذا كان الخطين متوازين فأن السرعة ثاتبة وتعني ان القطر ثابت

EGL اعلى منHGL

تكون الخطوط نازلة باتجاه الحركة

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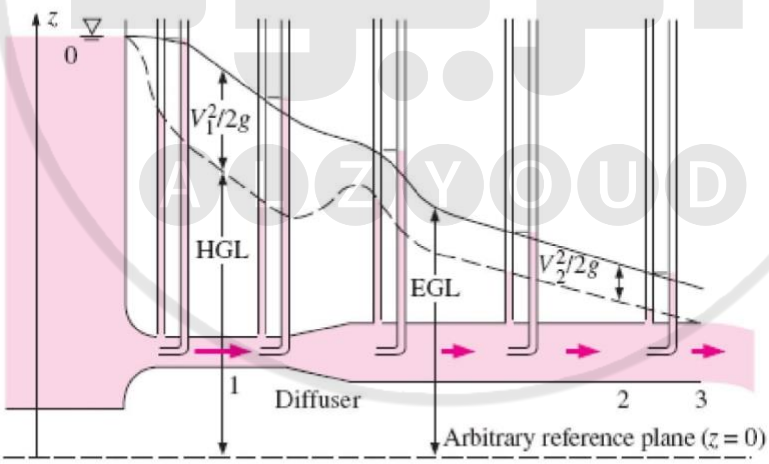


اذا قلت المسافة بين الخطين فأن السرعة تقل ،
والقطر يكون أكبر

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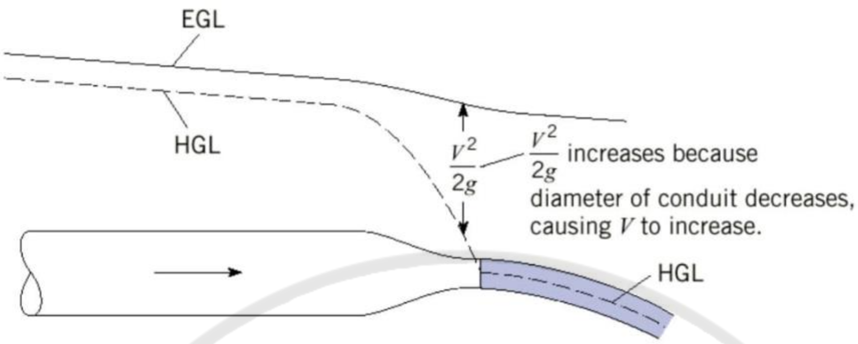
# Hydraulic & Energy Grade Lines




هنا السرعة زادت لان المسافة بين الخطين قلت

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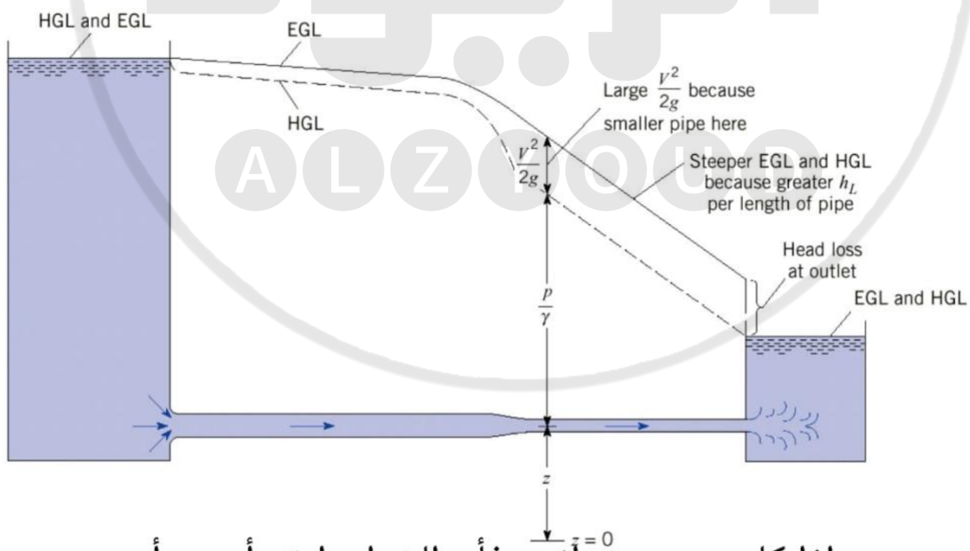
# Hydraulic & Energy Grade Lines




اذا كان يوجد خزانين فأن الخطوط تبدأ من رأس
الخزان وتنتهي ب رأس الخزان

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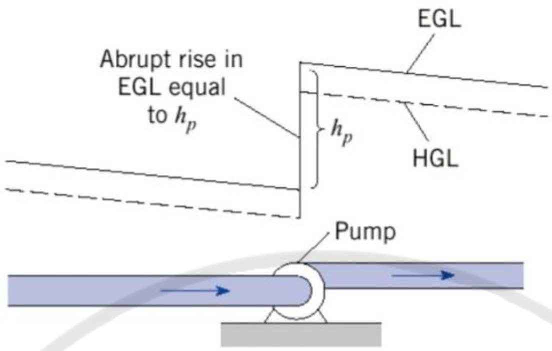
# Hydraulic & Energy Grade Lines




عند وجود المضخة ترتفع الخطوط للاعلى

يزاد hp

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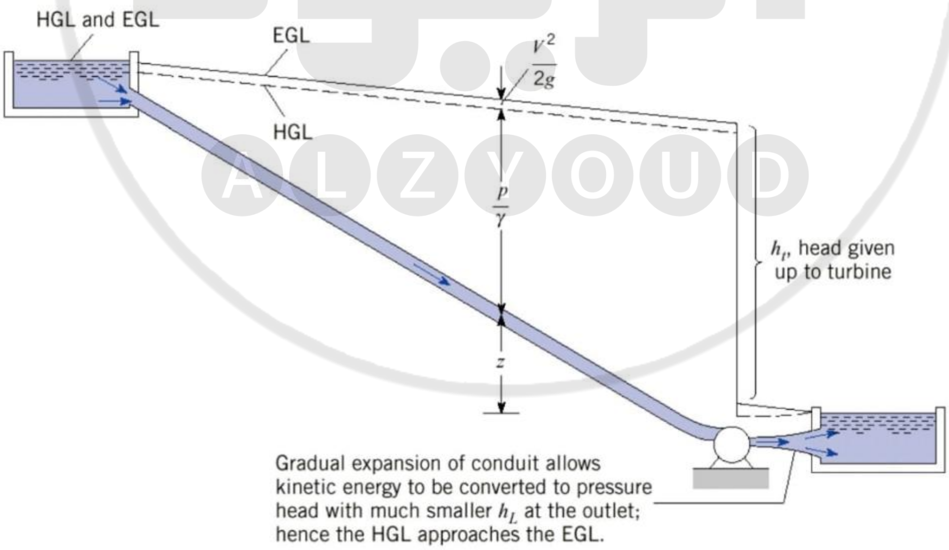
# Hydraulic & Energy Grade Lines




عند وجود turbine فأن الخطوط تهبط للاسفل

50

**HGL=Hydraulic grade line= line to describe the piezometric head**

**EGL= Energy grade line= line to describe the total head**

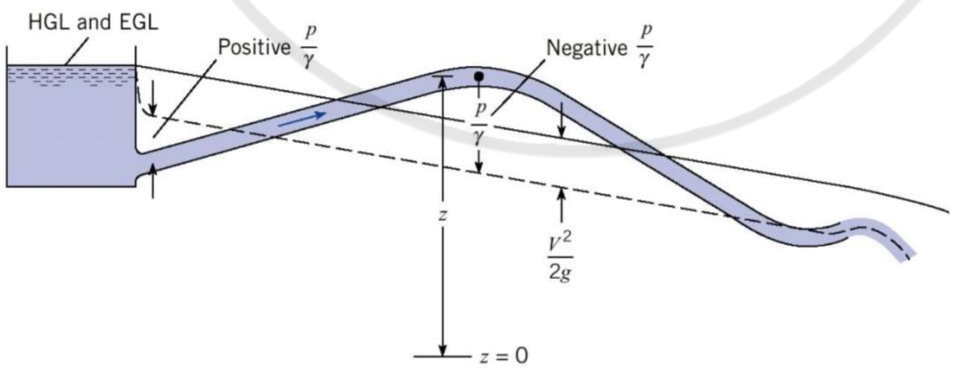
قد يأتي بالامتحان اسئلة يطلب بها رسم خطوط (EGL,HGL)

ملاحظات على رسم الخطوط:

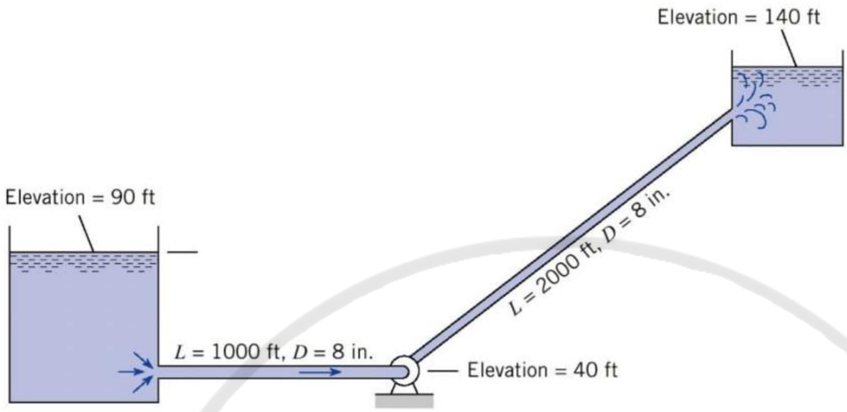
1) EGL اعلى من HGL
2) اذا كان الخطين متوازين فان السرعة ثابتة وتعني ان قطر pipeثابت
3) اتجاه نزول الخطوط يدل على اتجاه الحركة
4) اذا كان يوجد خزان فان الخطوط تبدأ او تنتهي برأس الخزان
5) اذا قلت المسافة بين EGL,HGL فان السرعة تقل والقطر pipe يكون اكبر
6) عند المضخة يزداد الارتفاع hp
7) عند turbine تهبط الخطوط للاسفل
8) $\frac{p}{\gamma}$ نقوم بقياسها من خلال قياس المسافة بين center of pipe الى HGL لو كان pipe اعلى من HGL فانها تكون سالبة والعكس صحيح

# Hydraulic & Energy Grade Lines





**Example:** What horsepower must be supplied to the water to pump 3.0 cfs at 68 °F from the lower to the upper reservoir ? Assume that the head loss in the pipes is given by $h_L=0.015(L/D)(V^2/2g)$, where L is the length of the pipe in feet and D is the pipe diameter in feet. Sketch the HGL and the EGL.




في السؤال هنا طلب :- 1) مقدار (horse Power)
2) رسم (HGL) ، (EGL)

$$\boxed{P = Q\gamma h_p}$$

$$\frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + Z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + Z_2 + h_t + h_L \quad \ldots ①$$

major $\frac{0.015 L V^2}{2gD}$ — نلاحظ ان ($h_p$) مجهولة و ($h_L$) ، لإيجاد ($h_L$) يوجد في السؤال

minor : عند النقطة ② $= \frac{V^2}{2g}$

$$h_L = \text{major} + \text{minor} = \frac{0.015 L V^2}{2gD} + \frac{V^2}{2g}$$

لكن (V) مجهولة نجد (V) من خلال (Q) :-

$$V = \frac{Q}{A} = \frac{3}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2} = \boxed{8.59 \text{ Ft/s}}$$

$$h_L = \frac{(0.015)(3000)(8.59)^2}{(2)(32.2)\left(\frac{8}{12}\right)} + \frac{(8.59)^2}{2(32.2)}$$

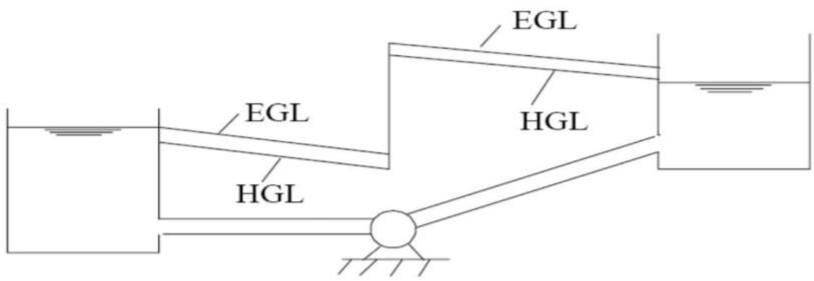
$\boxed{h_L = 78.5 \text{ Ft}}$ ⟹ نعوض في معادلة ①

$$90 + h_p = 140 + 78.5$$

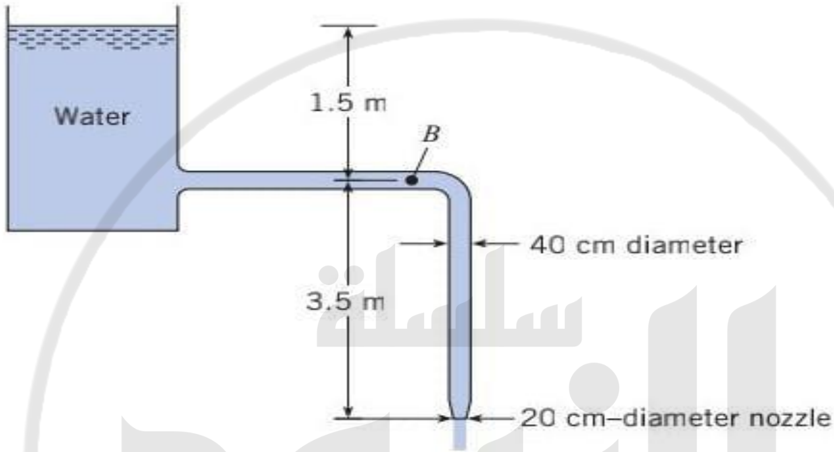
$$\boxed{h_p = 128.5 \text{ Ft}}$$

$P = Q\gamma h_p$ ⟹ بدي اقسم على (550) عشان يصير بوحدة (horse Power)

$$P = \frac{Q\gamma h_p}{550} = \boxed{43.7 \text{ hp}}$$




**7.21** Determine the discharge in the pipe and the pressure at point *B*. Neglect head losses. Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.21



$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 = \frac{V_2^2}{2g} - 5 \qquad \boxed{V_2 = 9.9\ m/s}$$

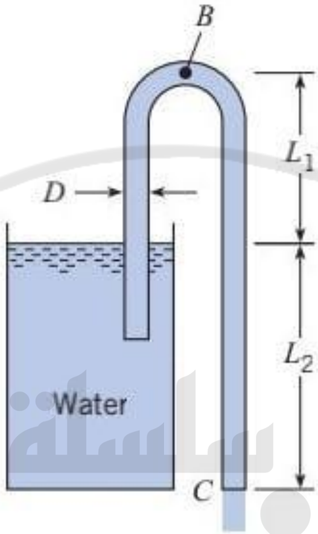
$$Q = VA = (9.9)\left(\frac{\pi}{4}\right)(0.2)^2 \Rightarrow \boxed{Q_2 = 0.311\ m^3/s}$$

$$\boxed{Q_2 = Q_B} \qquad V_B = \frac{Q_B}{A_B} = \boxed{2.48\ m/s}$$

$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_B}{\gamma} + \alpha\frac{V_B^2}{2g} + z_2 + h_t + h_L$$

$$\frac{p_B}{\gamma} + \frac{(2.48)^2}{2g} - 1.5 = 0 \qquad \boxed{p_B = 11.7\ kPa}$$

**7.32** The discharge in the siphon is 2.80 cfs, $D = 8$ in., $L_1 = 3$ ft, and $L_2 = 3$ ft. Determine the head loss between the reservoir surface and point $C$. Determine the pressure at point $B$ if three-quarters of the head loss (as found above) occurs between the reservoir surface and point $B$. Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.32

$$\frac{p_1}{\gamma} + \alpha \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_c}{\gamma} + \alpha \frac{V_c^2}{2g} + z_c + h_t + h_L$$

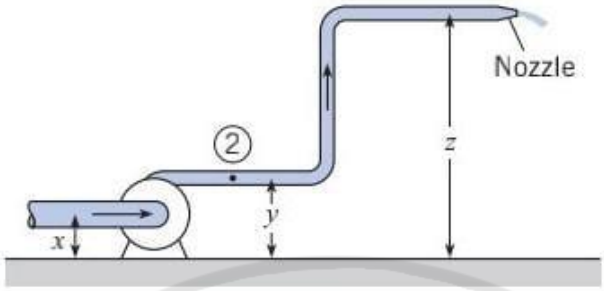
$$V_c = \frac{Q}{A} = \frac{2.8}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2} = \boxed{8.02\ Ft/s} \Rightarrow 3 = \frac{(8.02)^2}{64.4} + h_L \rightarrow \boxed{h_L = 2Ft}$$

$$\frac{p_1}{\gamma} + \alpha \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_B}{\gamma} + \alpha \frac{V_B^2}{2g} + z_B + h_t + h_L$$

$$\boxed{V_B = V_c}$$

$$h_L = \left(\frac{3}{4}\right)(2), \boxed{\gamma = 62.4} \Rightarrow P_B = -343\ PsFg$$

**7.24** For this system, the discharge of water is 0.1 $m^3/s$, $x = 1.0$ m, $y = 2.0$ m, $z = 7.0$ m, and the pipe diameter is 30 cm. Neglecting head losses, what is the pressure head at point 2 if the jet from the nozzle is 10 cm in diameter? Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.24

$$\text{Pressure head} = \frac{P}{\gamma}$$

$$\frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_p = \frac{P_3}{\gamma} + \alpha\frac{V_3^2}{2g} + z_3 + h_t + h_L$$

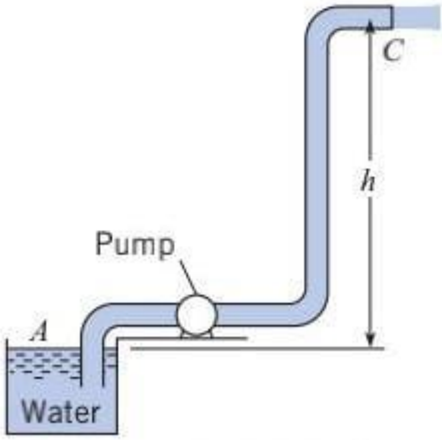
$$V_2 = \frac{Q}{A} = \frac{0,1}{\frac{\pi}{4}(0,3)^2} = 1,41 \, m/s$$

$$V_3 = \frac{0,1}{\frac{\pi}{4}(0,1)^2} = 12,73 \, m/s$$


$$\frac{P_2}{\gamma} + \frac{(1,41)^2}{2g} + 2 = \frac{(12,73)^2}{2g} + 7 \Rightarrow \frac{P_2}{\gamma} = 13,16 \, m$$



**7.44** A pump draws water through an 8 in. suction pipe and discharges it through a 4 in. pipe in which the velocity is 12 ft/s. The 4 in. pipe discharges horizontally into air at *C*. To what height *h* above the water surface at *A* can the water be raised if 25




PROBLEMS 7.44, 7.45

$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$h_p = h + \frac{3(12)^2}{2g} \Rightarrow$$ نجدها من خلال (efficiency)

$$\eta_p = \frac{\dot{W}_f}{\dot{W}_s} = \frac{\gamma Q h_p}{\dot{W}_s} = \eta \Rightarrow h_p = \frac{\dot{W}_s(\eta_p)}{\gamma Q}$$

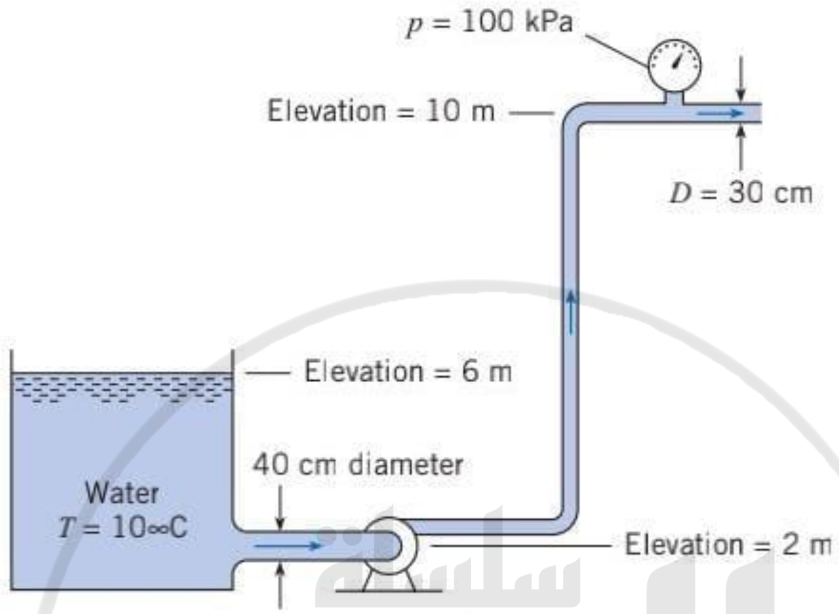
$$Q = V_c A_c = (12)\left(\frac{\pi}{4}\right)\left(\frac{4}{12}\right)^2 = \boxed{1.047\ ft^3/s}$$

$$h_p = \frac{(25)(550)(0.6)}{(1.047)(62.4)} = 126.3\ ft$$

$$h_p = h + 3\frac{(12)^2}{2g}$$

$$\boxed{h = 120\ ft}$$

**7.50** Water (10°C) is flowing at a rate of 0.35 $m^3/s$, and it is assumed that $h_L = 2V^2/2g$ from the reservoir to the gage, where $V$ is the velocity in the 30-cm pipe. What power must the pump supply? Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.50

$\boxed{Q = 0{,}25}$

$P = Q\gamma h_p$

$h_L = \frac{2V_2^2}{2g}$

$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

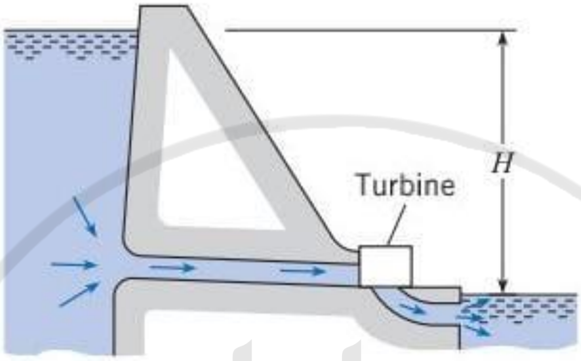
$$V_2 = \frac{Q}{A} = \frac{0{,}25}{\frac{\pi}{4}(0{,}3)^2} = \boxed{3{,}54\ m/s}$$

$\boxed{h_p = 16{,}1\ m}$

$P = \dot{W} = (0{,}25)(9{,}81)(16{,}1)$

$\boxed{P = 39{,}5\ kW}$

**7.53** A small-scale hydraulic power system is shown. The elevation difference between the reservoir water surface and the pond water surface downstream of the reservoir, *H*, is 15 m. The velocity of the water exhausting into the pond is 5 m/s, and the discharge through the system is 1 m³/s. The head loss due to friction in the penstock is negligible. Find the power produced by the turbine in kilowatts.




PROBLEM 7.53

$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

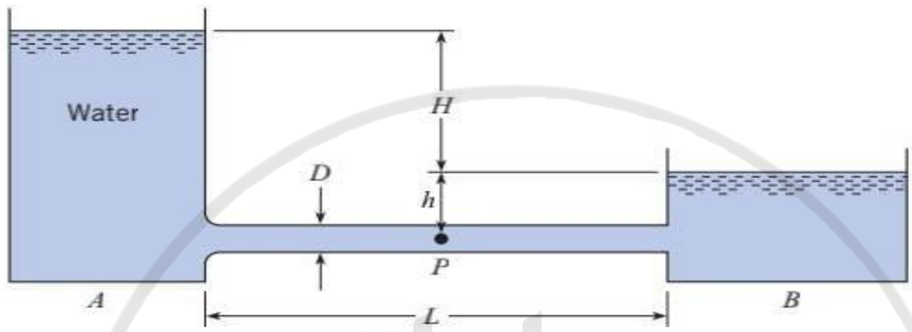
$$h_L = \text{minor loss} = \frac{V^2}{2g}$$

$$h_t = 15 - \frac{(5)^2}{2g} \qquad h_t = 13.73\,m$$

$$P = Q\gamma h_t$$

$$\Rightarrow (1)(9810)(13.73) = 134.6\ kW$$

**7.81** Water flows from reservoir $A$ to reservoir $B$. The water temperature in the system is 10°C, the pipe diameter $D$ is 1 m, and the pipe length $L$ is 300 m. If $H = 16$ m, $h = 2$ m, and the pipe head loss is given by $h_L = 0.01(L/D)(V^2/2g)$, where $V$ is the velocity in the pipe, what will be the discharge in the pipe? In your solution, include the head loss at the pipe outlet, and sketch the HGL and the EGL. What will be the pressure at point $P$ halfway between the two reservoirs? Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.81

1 minor → 1 major

$$\frac{P_A}{\gamma} + \alpha\frac{V_A^2}{2g} + z_A + h_p = \frac{P_B}{\gamma} + \alpha\frac{V_B^2}{2g} + z_B + h_t + \sum h_L$$

$$16 = (0.01)(300) * \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\boxed{V = 8.86 \text{ m/s}}$$

$$Q = VA = (8.86)\left(\frac{\pi}{4}\right)(1)^2 = \boxed{6.96 \text{ m}^3/\text{s}}$$

to Find Pressure :-

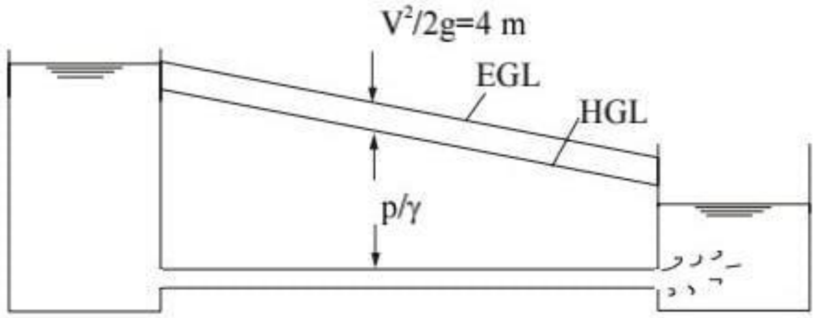
1 major

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_p}{\gamma} + \alpha\frac{V_p^2}{2g} + z_p + h_t + \sum h_L$$

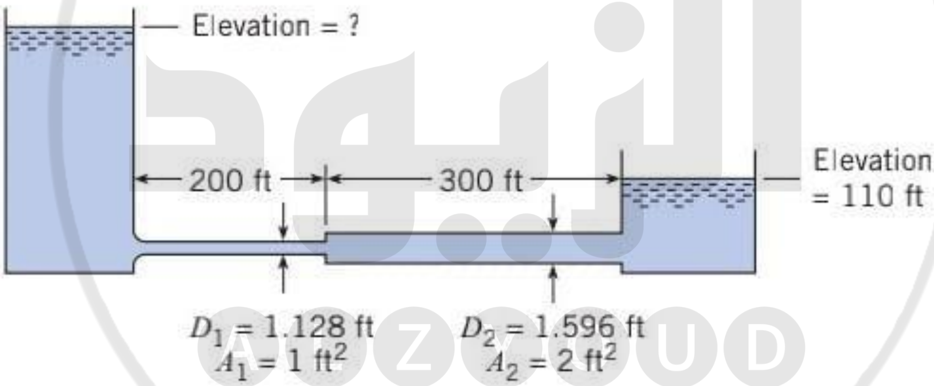
$$16 = \frac{P}{\gamma} + \frac{(8.86)^2}{2g} - 2 + (0.01)(150)\frac{(8.86)}{2g}$$

$$\boxed{P_p = 78.5 \text{ kPa}}$$

#




**7.82** Water flows from the reservoir on the left to the reservoir on the right at a rate of 16 cfs. The formula for the head losses in the pipes is $h_L = 0.02(L/D)(V^2/2g)$. What elevation in the left reservoir is required to produce this flow? Also carefully sketch the HGL and the EGL for the system. *Note:* Assume the head-loss formula can be used for the smaller pipe as well as for the larger pipe. Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.82

$V_1 = \frac{Q}{A} = \frac{16}{1} = 16\ Ft/s$

$V_2 = \frac{Q}{A} = 8\ Ft/s$

* عندي في السؤال:
2 minor + 2 major

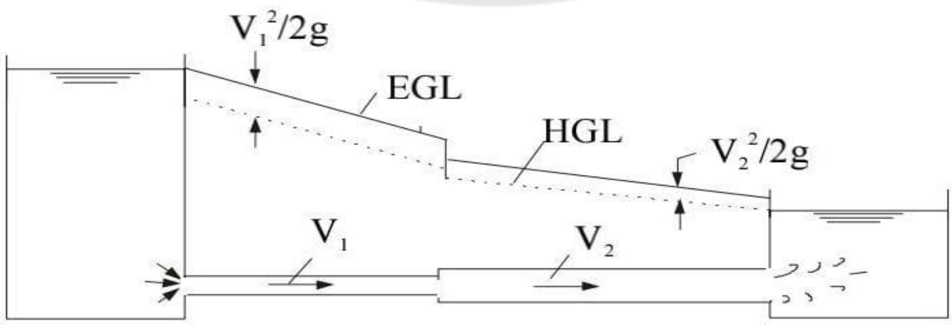
$h_L = \Sigma major + \Sigma minor$

$= \left(0.02 \frac{L_1}{D_1}\left(\frac{V_1^2}{2g}\right)\right) + 0.02 \frac{L_2}{D_2}\left(\frac{V_2^2}{2g}\right) + \underbrace{\frac{(V_1 - V_2)^2}{2g} + \frac{V_2^2}{2g}}_{minor}$

$\frac{P_L}{\gamma} + \alpha\frac{V_L^2}{2g} + Z_L + h_P = \frac{P_R}{\gamma} + \alpha\frac{V_R^2}{2g} + Z_R + h_t + h_L$

$Z_L = 110 + \frac{0.02(200)}{1.128}\left(\frac{16^2}{2g}\right) + (0.02)\left(\frac{300}{1.596}\right)\left(\frac{(8)^2}{2g}\right) + \frac{(16-8)^2}{2g} + \frac{(8)^2}{2g}$

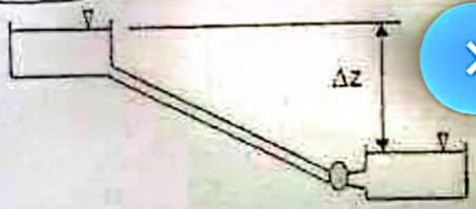
$Z_L = 128.6\ Ft$



8- What is the power generated (kW) from a hydraulic turbine when it is operated between two water reservoirs that have $\Delta z = 35$ m difference in elevation and the flow rate is 7 $m^3/s$? Let the head loss (major loss) be 4 m and neglect the abrupt losses.

a) 2737.0 b) 912.3 c) 1520.6 d) 2128.8

19




$$\dot{W} = Q\gamma h_t$$

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$35 = h_t + 4 \Rightarrow \boxed{h_t = 31\,m}$$

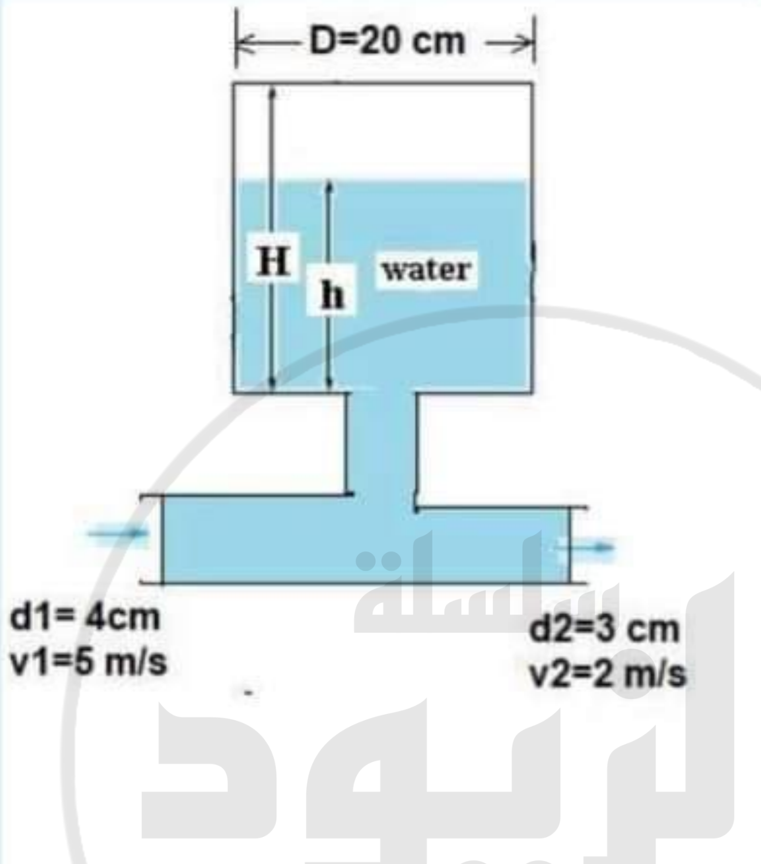
$$\dot{W} = (7)(9810)(31) = \boxed{2128.8}$$

The continuity principle is applicable for the case

a) incompressible/compressible b) viscous/inviscid
c) irrotational/rotational d) all

4

What time (in second) is required to fill the circular tank with incompressible water from h= 2 m to H=7.3 m?



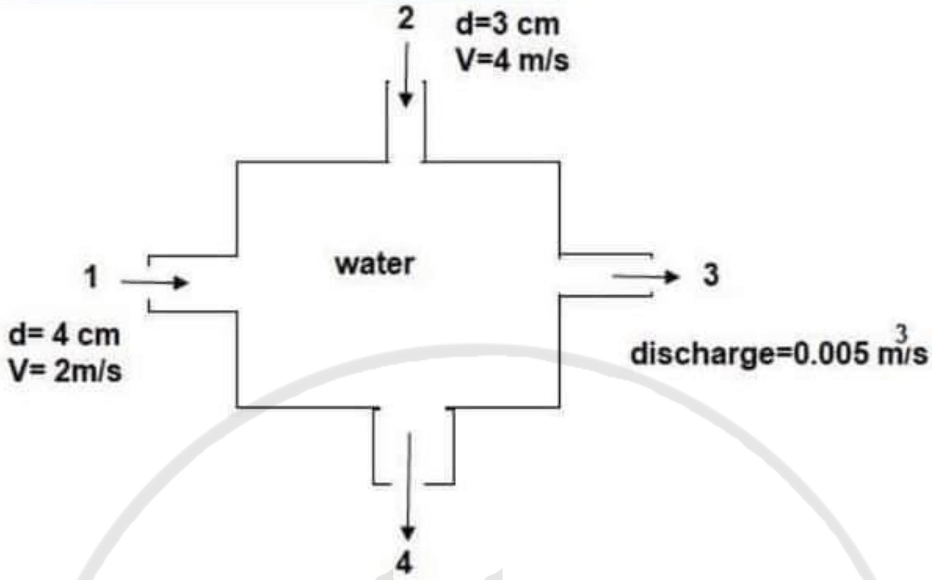

$$\frac{d m_{CV}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\rho \, volume = \dot{m}_1 - \dot{m}_2 \Rightarrow \rho A \frac{dh}{dt} = \rho v_1 A_1 - \rho v_2 A_2$$

$$\frac{\pi}{4}(0.2)^2 \frac{dh}{dt} = 5\left(\frac{\pi}{4}\right)(0.04)^2 - 2\left(\frac{\pi}{4}\right)(0.03)^2$$

$$\boxed{\frac{dh}{dt} = 0.155} \Rightarrow dt = \frac{dh}{0.155} = \frac{7.3-2}{0.155} = 34.19$$

What is the velocity (m/s) at section 4, if the d4=11.0 cm? The problem is steady and incompressible.



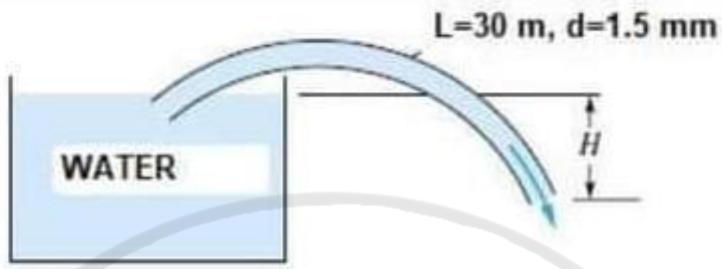

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$V_1 A_1 + V_2 A_2 = V_3 A_3 + V_4 A_4$$

$$2\left(\frac{\pi}{4}\right)(0{,}04)^2 + 4\left(\frac{\pi}{4}\right)(0{,}03)^2 = 0{,}005 + V_4\left(\frac{\pi}{4}\right)(0{,}11)^2$$

$$\boxed{V_4 = 0{,}0359}$$

Water with a viscosity of $\mu$=0.032 kg/m.s and density =1000 kg/$m^3$ is to be siphoned through a tube 30 m long and 1.5 mm in diameter, as shown. The flow is to be laminar. What is the Reynolds number if H = 12 m? Assume the total head loss is 10 m.




$Re = \frac{\rho V D}{\mu}$

نطبق معادلة (energy) لإيجاد (V)

$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$

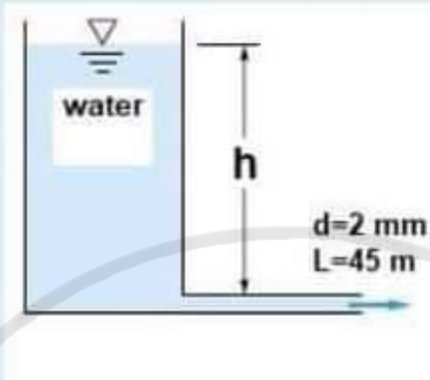
$\boxed{\alpha = 2}$

$12 = (2)\frac{V_2^2}{2g} + 10$

$\Rightarrow 2 = 2\frac{V_2^2}{2g} \Rightarrow V_2 = \sqrt{2g} \Rightarrow V_2 = 4,43\ m/s$

$Re = \frac{(1000)(4,43)(1.5\times10^{-3})}{0,032} = 207,6303$

What level h (m) must be maintained to keep the Reynold number at 1600 through the commercial-steel pipe? The total head loss hL=4.8 m. Water (viscosity of 0.01 kg/m.s and density of 1000 kg/m$^3$)




$$Re = \frac{\rho V D}{\mu}$$

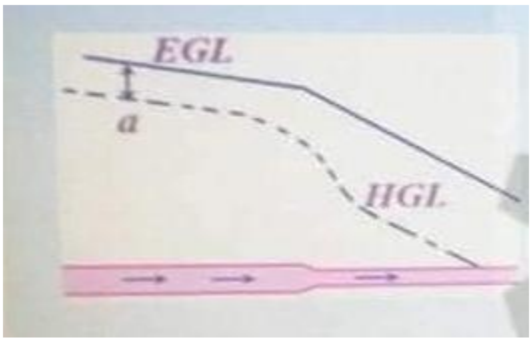
$$\boxed{V = 8\,m/s}$$

$$\frac{P_1}{\gamma} + z_1 + \alpha_1\frac{V_1^2}{2g} + h_p = \frac{P_2}{\gamma} + z_2 + \alpha_2\frac{V_2^2}{2g} + h_t + h_L$$

$$z_1 = 4.8 + 2 * \frac{8^2}{2(9.81)} = \underline{\underline{11.32}}$$



The figure shows EGL , HGL of a turbulent water flow in a pipe 6 cm diameter and small diameter is 3.5cm what the velocity in smaller section , a=0.2m ?



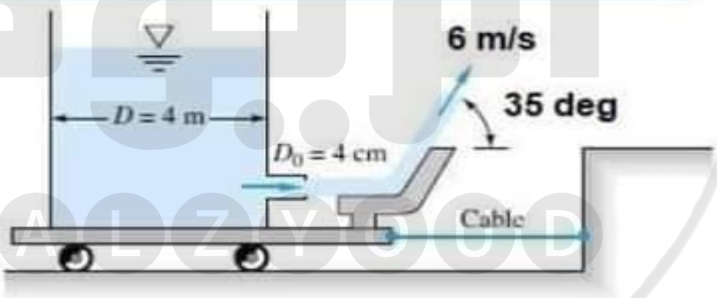
$$Q_1 = Q_2 \Rightarrow V_1A_1 = V_2A_2 \qquad \frac{V_2^2}{2g} = 0{,}2$$

$$\Rightarrow \boxed{V_2 = 1{,}98}$$

$$V_1\frac{\pi}{4}(3{,}5)^2 = \frac{\pi}{4}(6)^2(1{,}98)$$

$$\boxed{V_1 = 5{,}8\ m/s}$$

A tank (contains a fluid of density=3684.0 kg/m3) stands on a frictionless cart (The cart is fixed) and feeds a jet of diameter 4 cm and velocity 6 m/s, which is deflected θ=35 degree by a vane. Compute the tension in the supporting cable.



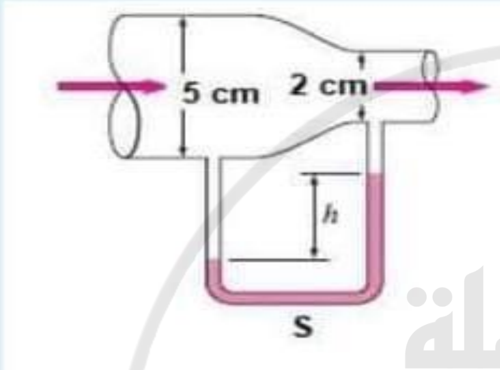

$$\Sigma F = \Sigma V\dot{m}$$

$$\dot{m} = \rho VA = (3684)(6)\left(\frac{\pi}{4}\right)(0{,}04)^2$$

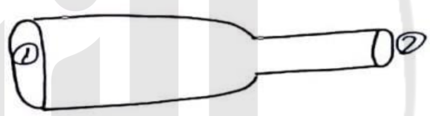
$$= 27{,}78$$

$$R_x = 6\cos(35)\,(27{,}78) \Rightarrow \boxed{R_x = 136{,}53\,N}$$

Water flows through a horizontal pipe at a rate of 0.01 $m^3/s$. The pipe consists of two sections of diameters 5 cm and 2 cm with a smooth reducing section. The pressure difference between the two pipe sections is measured by a fluid manometer that has S=12.3. Neglecting the viscous effects, determine the differential height of fluid manometer h (m) between the two pipe sections.




$$\Delta P = (\gamma_m - \gamma_F)h$$




$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1}^{0} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2}^{0}$$

$$\boxed{Q_1 = Q_2}$$

$$V_1 = \frac{Q}{A} = \frac{0.01}{(\frac{\pi}{4})(0.05)^2} = 5\ m/s$$

$$V_2 = \frac{0.01}{(\frac{\pi}{4})(0.02)^2} = 31.8\ m/s$$

$$\frac{P_2 - P_1}{\gamma} = \frac{V_2^2 - V_1^2}{2g} \Rightarrow \boxed{\Delta P = 493120\ Pa}$$

$$\Delta P = (\gamma_m - \gamma_F)h \quad , \quad \gamma_m = (12.3)(9810) = 120663$$

$$h = \frac{493120}{(120663 - 9810)} = 4.45$$

A garden hose attached with a nozzle is used to fill a 20 Liters bucket. The inner diameter of the hose is $d_{hose}$=8 cm, and it reduces to $d_{nozzle}$= 3.63 cm at the nozzle exit. If it takes 50 seconds to fill the bucket with water, determine average velocity (m/s) of water at the nozzle exit.

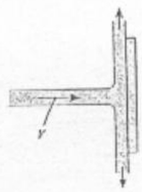

$$\bar{V} = \frac{Q}{A_{Nozzel}} = \frac{20 \times 10^{-3}}{50} = 4 \times 10^{-4}$$

$$d_{Nozzel} = \frac{3.63}{100} = 0.0363$$

$$\bar{V} = \frac{4 \times 10^{-4}}{\frac{\pi}{4}(0.0363)^2} = 0.3865 \, m/s$$

1- A jet of water 3 cm in diameter strikes normal to a plate as in the shown figure. If the force required to hold the plate in place is 23 N, then the jet velocity is:

a) 2.8 m/s
b) 5.7 m/s
c) 8.1 m/s
d) 4.0 m/s
e) 6.4 m/s



2- A two dimensional velocity field is given by the formula:

$$V = (x^2 - y^2 + x)\,\mathbf{i} - (C\,xy + y)\,\mathbf{j}$$

In order for this field to satisfy the continuity principle, the value of the constant C is equal to:

a) 0.5
b) 1
c) 2
d) 2.5
e) 3

3- A rectangular air duct 20 cm by 50 cm carries a flow of 1.44 $m^3$/s. The mean velocity of air in the duct is equal to:

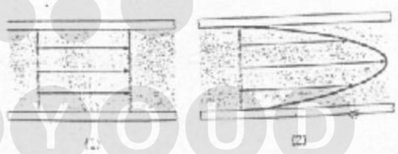
a) 0.72 m/s
b) 7.2 m/s
c) 28.8 m/s
d) 1.44 m/s
e) 14.4 m/s

4- A water tank with a square cross section of square 1x1m is being filled through a 12 cm pipe that discharges water at a velocity of 3 m/s. The rate at which the water level in the tank rises is:

a) 0.014 m/s
b) 0.034 m/s
c) 0.122 m/s
d) 0.063 m/s
e) 0.056 m/s

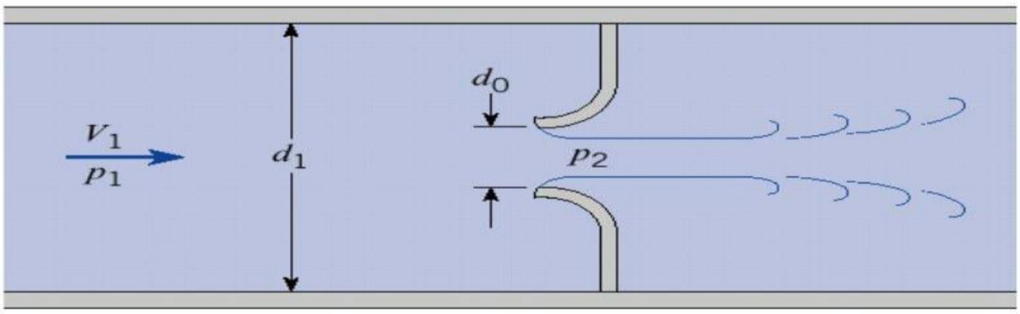
5- For the velocity profiles shown, the value of $\alpha$ (kinetic energy correction factor) is:

a) 1 for the first and 2 for the second
b) 2 for the first and 1 for the second
c) 2 for both of them
d) 0.5 for the first and 0.25 for the second
e) 1 for the first and 0.5 for the second



# Ch 8 : : Dimensional Analysis and Similitude

في هاذ الشابتر بدي اعرف كيف تؤثر المفاهيم ببعض مثلا كيف يؤثر السرعة على الضغط ونوع العلاقة بينهم




في هذا pipe نلاحظ ان الضغط يتأثر( بالكثافة و السرعة واقطار pipe)

$P_1 - p_2 = \Delta p = f(\rho, V, d_1, d_0)$

مثلا لو بدي ادرس تأثير القطر( $d_0$) على الضغط p فلازم اثبت مقدار ($\rho, d_1, v$) وتعد هذه العملية مكلفة وتضيع الكثير من الوقت

- قام العلماء باكتشاف طريقة اخرى وهي انه بنعاملهم كمجموعات ونطلق عليهم اسم( dimensional group)

$$\frac{\Delta p}{\frac{1}{2}\rho V_1^2} = \varphi\left(\frac{d_1}{d_o}\right)$$

وكل مجموعة تكون (unit less) ليس لها وحدة

في حركة اي مائع فأنه يتأثر ب ($\rho, \mu$)

$\mu$ لا توجد بحالة :1) كانت الحركة دورانية يظهر بدالها w

2) اذا كانت inviscid

## 8.2 Dimensions and Equations

| Description | Dimensions |
|---|---|
| Mass(m) | M |
| Length | L |
| Time | T |
| Temperature | θ |

BASIC DIMENSIONS (M, L, T, θ)

Area:............................... $L^2$
Diameter:..................... $L$
Pressure:..................... $M/LT^2$
Acceleration:................ $L/T^2$
Work:.......................... $ML^2/T^2$
Mass flow rate:.............. $M/T$
Volume flow rate:............ $L^3/T$
Force:.......................... $ML/T^2$
Velocity:........................ $L/T$
Gas constant(R):............ $L^2/\theta T^2$
Density(ρ):....................... $M/L^3$
Dynamic viscosity(μ):........ $M/LT$
Kinematic viscosity(ν):...... $L^2/T$
Angular speed(ω):............ $1/T$
Specific weight (γ)...........$M/L^2T^2$
Surface Tension(σ)..............$M/T^2$

حفظ

See Appendix A-1



**•The Buckingham Π Theorem:**

The number of independent dimensionless groups of variables (dimensionless parameters)= n - m

Π = pi

n: number of variables, m:number of basic dimensions

m:يكون عددهم 4 ولكن اذا كان لا يوجد بالمسألة حرارة يصبح عددهم 3

Dimensional variables:

$y_1 = f(y_2, y_3, \ldots, y_n)$

$$p_1 - p_2 = \Delta p = f(\rho, V_1, d_1, d_o)$$

Dimensionless parameters (Π-groups):

$\Pi_1 = \varphi(\Pi_2, \Pi_3, \ldots, \Pi_{n-m})$

$$\frac{\Delta p}{\frac{1}{2}\rho V_1^2} = \varphi\left(\frac{d_1}{d_o}\right)$$

طرق تحويل (variables) الى (dimensional group):

## 1) The step-by-step method:

سنقوم بتوضيح هذه الطريقة بالمثال

**Example:**
A thin rectangular plate having a width **w** and a height **h** is located so that it is normal to a moving stream of fluid. Assume the drag force $F_D$, that the fluid exerts on the plate is a function of **w** and **h**, the fluid viscosity $\mu$ and density $\rho$, and the velocity **V** of the fluid approaching the plate. Determine a suitable set of pi terms using the step-by-step method to study this problem experimentally.

**Solution:** $F_D = f(w, h, \mu, \rho, V)$

$F_D = ML/T^2$
$w = L$
$h = L$
$\mu = M/LT$
$\rho = M/L^3$
$V = L/T$

# of Pi terms(dimensionless groups)= 6-3= 3

$n - m = 6 - 3 = 3 \Rightarrow$ Three $\pi$ term

$F_D = F(w, h, \rho, V)$

* بدي اتخلص من:- L ①، M ②، T ③

بدي اشوف (L) لحالها عندي (w، h) بختار واحد منهم وبقسم على الباقي

| variable | [ ] |
|---|---|
| $F_D$ | $mL/T^2$ |
| $w$ | $L$ |
| $h$ | $L$ |
| $\mu$ | $m/LT$ |
| $\rho$ | $m/L^3$ |
| $V$ | $L/T$ |
| $F_D/w$ | $m/T^2$ |
| $h/w$ | $0$ |
| $\mu * w$ | $m/T$ |
| $\rho * w^3$ | $m$ |
| $V/w$ | $1/T$ |
| $F_D/\rho w^3$ | $1/T^2$ |
| $\mu/\rho w^3$ | $1/T$ |
| $V/w$ | $1/T$ |

$h/w$ → 0 $\Longrightarrow$ * تعني ($\pi$ term)

* بدي ($\mu$) لحالها بختار ($\rho * w^3$)

$\mu/\rho w^3$ و $V/w$ → بختار واحدة منهم

$F_D/(\rho w^4)(w^2/V) = F_D/\rho w^2 V^2$ → 0

$h/w$ → 0

$\mu/\rho w V$ → 0

$$\frac{F_D}{\rho w^2 V^2} = \Phi\left(\frac{w}{h}, \frac{\mu}{\rho h V}\right)$$

**Example:** It is known that the pressure developed by a centrifugal pump, **Δp**, is a function of the diameter **D** of the impeller, the speed of rotation **n**, the discharge **Q**, and the fluid density ρ. By dimensional analysis, determine the π groups relating these variables. Use step-by-step method.

**Solution:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Delta p$ | $\frac{M}{LT^2}$ | $\Delta p D$ | $\frac{M}{T^2}$ | $\frac{\Delta p}{\rho D^2}$ | $\frac{1}{T^2}$ | $\frac{\Delta p}{n^2 \rho D^2}$ | 0 |
| $D$ | $L$ | | | | | | |
| $n$ | $\frac{1}{T}$ | $n$ | $\frac{1}{T}$ | $n$ | $\frac{1}{T}$ | | |
| $Q$ | $\frac{L^3}{T}$ | $\frac{Q}{D^3}$ | $\frac{1}{T}$ | $\frac{Q}{D^3}$ | $\frac{1}{T}$ | $\frac{Q}{nD^3}$ | 0 |
| $\rho$ | $\frac{M}{L^3}$ | $\rho D^3$ | $M$ | | | | |

$$\frac{\Delta p}{n^2 \rho D^2} = f\left(\frac{Q}{nD^3}\right)$$

## 2) The Exponent Method

**Example:**

A thin rectangular plate having a width **w** and a height **h** is located so that it is normal to a moving stream of fluid. Assume the drag force $F_D$, that the fluid exerts on the plate is a function of **w** and **h**, the fluid viscosity μ and density ρ, and the velocity **V** of the fluid approaching the plate. Determine a suitable set of pi terms using the exponent method to study this problem experimentally.

$5-3=2 \Rightarrow$ Have $\Pi$ group

$F_D = f(w, h, \mu, \rho, V)$

(I) $F_D = w^a h^b \mu^c \rho^d V^e$

(II) (dimensional) نضع

$MLT^{-2} = (L)^a (L)^b (ML^{-1}T^{-1})^c (ML^{-3})^d (LT^{-1})^e$

$M: \; 1 = c + d$ , $T: \; -2 = -c - e$

$L: \; 1 = a + b - c - 3d + e$

* نلاحظ ان (c) الاكثر تكرار بحليها على يمين المعادلة

$\boxed{d = 1 - c}$ ... (1) $\boxed{e = 2 - c}$ ... (2) $\boxed{a = 2 - c - b}$ ... (3)

نعوض في قيمهم (e, d)

(III) $F_D = w^{2-c-b} h^b \mu^c \rho^{1-c} V^{2-c}$

$F_D = w^2 w^{-c} w^{-b} h^b \mu^c \rho^1 \rho^{-c} V^2 V^{-c}$

$F_D = (w^2 \rho V^2)(h/w)^b (\mu/\rho w V)^c$

$$\boxed{\frac{F_D}{w^2 \rho V^2} = \phi(h/w, \; \mu/\rho w V)}$$



## •Common Dimensionless Numbers

Some common established nondimensional parameters or $\Pi$'s encountered in fluid mechanics and heat transfer*

| Name | Definition | Ratio of Significance |
|---|---|---|
| Drag coefficient | $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$ | $\frac{\text{Drag force}}{\text{Dynamic force}}$ |

| | | |
|---|---|---|
| ➡ Lift coefficient | $C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$ | $\frac{\text{Lift force}}{\text{Dynamic force}}$ |
| ➡ Mach number | $\text{Ma (sometimes } M) = \frac{V}{c}$ | $\frac{\text{Flow speed}}{\text{Speed of sound}}$ |
| ➡ Pressure coefficient | $C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V^2}$ | $\frac{\text{Static pressure difference}}{\text{Dynamic pressure}}$ |
| ➡ Reynolds number | $\text{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$ | $\frac{\text{Inertial force}}{\text{Viscous force}}$ |

**•Similitude( التشابه) :**

فكرته باختصار عندما اريد تصميم جسر فبعد تحديد الاحمال التي تقع عليه وانشاء التصميم المناسب اقوم ببناء مجسم واختبار مقاومة الجسم للاحمال وهذا يسمى (model)

وعند تنفيذ الجسر على ارض الواقع اطلق عليه اسم (prototype)

وتعد هذه الطريقة من افضل الطرق لتوفير المال والوقت

**Model**: the replica of the structure on which the tests are made. Experimental testing is often performed with a small scale replica يكون حجمه اصغر (النموذج في المختبر)

**prototype**: Full-scale structure employed in the actual engineering design حجمه اكبر (النموذج الاصلي)

**Model:**

$\Pi_{1m} = \varphi(\Pi_{2m}, \Pi_{3m}, \ldots, \Pi_{nm})$

**Prototype:**

$\Pi_{1p} = \varphi(\Pi_{2p}, \Pi_{3p}, \ldots, \Pi_{np})$

$\Pi_{1m} = \Pi_{1p}$
$\Pi_{2m} = \Pi_{2p}$
$\Pi_{3m} = \Pi_{3p}$
.
.
.
$\Pi_{nm} = \Pi_{np}$

Example: $F_D = f(w, h, \mu, \rho, V)$

Solution:

$$\frac{F_D}{\rho w^2 V^2} = \varphi\left(\frac{h}{w}, \frac{\mu}{\rho w V}\right)$$

**Geometry Similitude**

$$\left(\frac{h}{w}\right)_m = \left(\frac{h}{w}\right)_p$$

$$\left(\frac{F_D}{\rho w^2 V^2}\right)_m = \left(\frac{F_D}{\rho w^2 V^2}\right)_p$$

$$\left(\frac{\mu}{\rho w V}\right)_m = \left(\frac{\mu}{\rho w V}\right)_p$$

**Dynamic Similitude**

## Model Scales

**Length scale or scale model:** $L_m/L_p = \lambda_L$

**Velocity scale:** $V_m/V_p = \lambda_V$

**Density scale:** $\rho_m/\rho_p = \lambda_\rho$

**Viscosity scale:** $\mu_m/\mu_p = \lambda_\mu$

**Temperature scale:** $T_m/T_p = \lambda_T$

Example: length scale =1/10 scale mode or 1:10 scale model

$L_m/L_p = 1/10$

Notes:

1- sometime as an example: $\rho_m = \rho_p$ , or $\mu_m = \mu_p$, or $g_m = g_p$ or $T_m = T_p$, ……….

2- $$\frac{Q_p}{Q_m} = \frac{V_p A_p}{V_m A_m} = \left(\frac{V_p}{V_m}\right)\left(\frac{d_p}{d_m}\right)^2$$

$$\frac{V_p}{V_m} = \frac{L_p}{L_m}\frac{t_m}{t_p}$$

$$\frac{p_p}{p_m} = \frac{\rho_p}{\rho_m}\frac{T_p}{T_m} \text{ (for ideal gas)}$$

**Example**: The drag on a submarine moving below the free surface is to be determined by a test on a **1/20 scale model** in a water tunnel. The velocity of prototype in sea water **($\rho$=1015 kg/m$^3$, $\nu$=1.4 x 10$^{-6}$ m$^2$/s)** is **2m/s**. The test is done in pure water at **20 $^0$C**. Determine the speed of the water in the water tunnel for dynamic similitude and the ratio of drag force on the model to the drag force on the prototype.

(I) اول شي بدي اجد (dimensional group)

$F_D = F(L, D, \mu, \rho, V)$

يشبه هذا المثال المثال السابقه

$$\frac{F_D}{\rho L^2 V^2} = \phi\left(\frac{D}{L}, \frac{\mu}{\rho L V}\right)$$

* لايجاد السرعة تكون ($R_e$) متساوية ⇐ $\boxed{Re_p = Re_m}$

$$\frac{\rho_p L_p V_p}{\mu_p} = \frac{\rho_m L_m V_m}{\mu_m} \Rightarrow \frac{L_p V_p}{\nu_p} = \frac{L_m V_m}{\nu_m}$$

$$V_m = \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right) V_p = 20\left(\frac{1*10^{-6}}{1.4*10^{-6}}\right) 2 = 28.6 \, m/s$$

تكون معطى بالسؤال

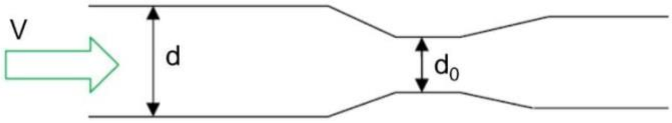
(II) لإيجاد ($F_D$):-

$$\frac{F_{Dm}}{\rho_m L_m^2 V_m^2} = \frac{F_{Dp}}{\rho_p L_p^2 V_p^2}$$

$$\frac{F_{Dm}}{F_{Dp}} = \left(\frac{\rho_m}{\rho_p}\right)\left(\frac{V_m}{V_p}\right)^2\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{998}{1015}\right)\left(\frac{28.6}{2}\right)^2\left(\frac{1}{20}\right)^2$$

$$\Rightarrow \boxed{0.503}$$

**Example**: A large venturi meter is calibrated by means of a 1/10 scale model using the prototype liquid. What is the discharge ratio $Q_m/Q_p$ for dynamic similarity? If a pressure difference of 300 kpa is measured across ports in the model for a given discharge, what pressure difference will occur between similar ports in the prototype for dynamically similar conditions?




* الضغط بتأثر بـ$(d, d_0, \mu, \rho, V)$

بدي احولهم لـ (dimensional group)

$$\frac{\Delta P}{\rho V^2} = \phi\left(\frac{d}{d_0}, \frac{\rho V d}{\mu}\right)$$

$$Re_m = Re_p \Rightarrow \frac{V_m d_m}{\nu_m} = \frac{V_p d_p}{\nu_p}$$

$$\Rightarrow \boxed{\frac{V_m}{V_p} = \left(\frac{d_p}{d_m}\right)\left(\frac{\nu_m}{\nu_p}\right)}$$

$$\frac{Q_m}{Q_p} = \frac{V_m A_m}{V_p A_p} \Rightarrow \boxed{A = \frac{\pi}{4} d^2}$$

$$\left(\frac{V_m}{V_p}\right)\left(\frac{d_m}{d_p}\right)^2 = \left(\frac{d_p}{d_m}\right)\left(\frac{\nu_m}{\nu_p}\right)\left(\frac{d_m}{d_p}\right)^2$$

$$\Rightarrow \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{d_m}{d_p}\right) = \frac{1}{10}$$

$$\frac{\Delta P_m}{\rho_m V_m^2} = \frac{\Delta P_p}{\rho_p V_p^2} \Rightarrow \Delta P_p = \Delta P_m \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{d_m}{d_p}\right)^2\left(\frac{\nu_p}{\nu_m}\right)^2$$

$$\Delta P_p = (300)(1)(1/10)^2(1)^2 = \underline{3\ kPa}\ \#$$

# Ch9: surface Resistance

في هذا الشابتر رح نتحدث external flow ولكن على اسطح مستوية فقط مثل جدران المنازل

اشكال الجريان الخارجي (external flow):

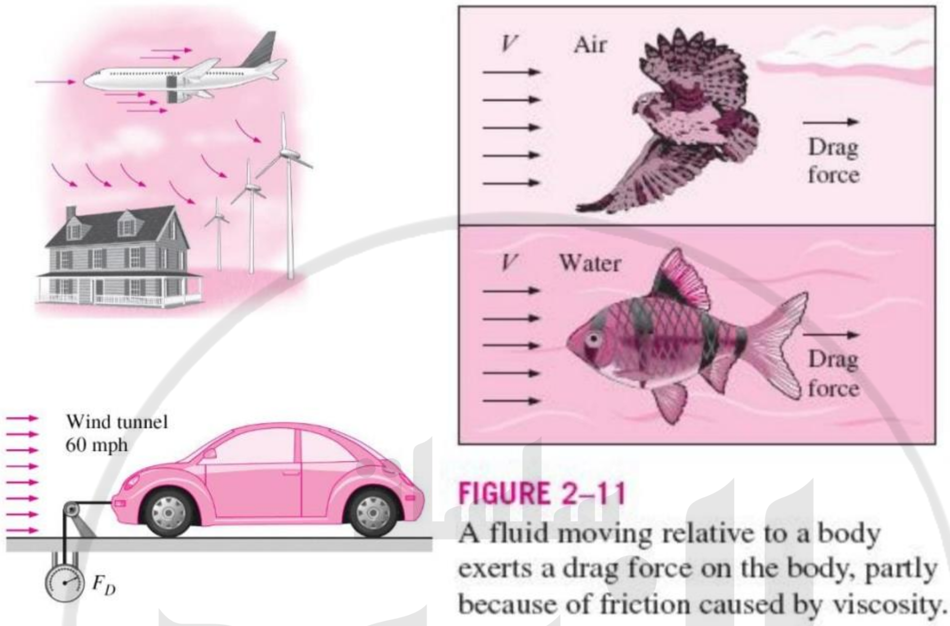
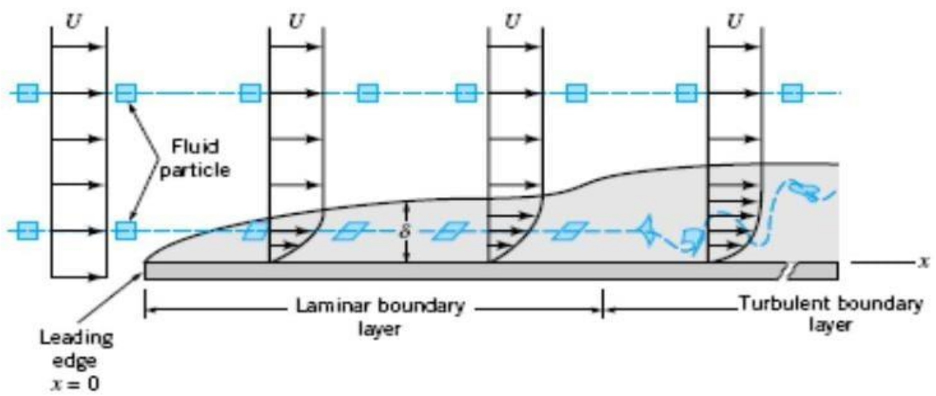



FIGURE 2–11
A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

- The boundary layer: is the layer of fluid near the surface where there is change in velocity due to the shear stress at the surface.

هي عبارة عن طبقة رقيقة تتغير فيها سرعة المائع بسبب نشوء Shear stress

بنعرف انه حركة الموائع بتنقسم الى: laminar , turbulent



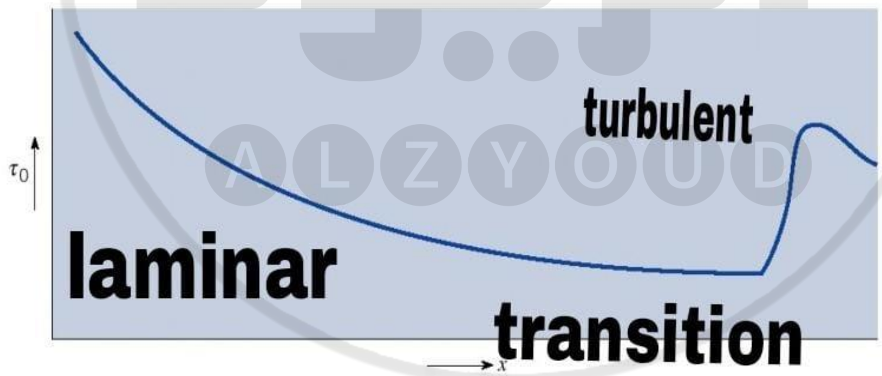

نلاحظ في البداية ان السرعة كانت منتظمة قبل ملامسة المائع للسطح

وعند ملامسة المائع للسطح بدأت السرعة تقل بحيث السرعة على السطح تكون صفر و ادى هذا الى تغير شكل الحركة ايضا بحيث كانت في البداية

Laminar ثم transition ثم turbulent

Transition: فترة التحول

ويمكن ايجاد علاقة بين المسافة المقطوعة على الطبقة و shear stress




$$\tau_0 = \mu \frac{\partial u}{\partial y}$$

This equation is valid for laminar and turbulent

ولتحديد نوع الحركة نقوم بحساب مقدار Reynolds number (Re)

$$Re_x = U_o x / v$$

$U_o$: velocity , X: المسافة الموازية لحركة المائع

V:السرعة وتكون معطى بالسؤال

قيم Re للجريان الخارجي تختلف عن الجريان الداخلي

$$Re_x, Re_L < 5\times10^5 \Rightarrow \text{Laminar}$$
$$Re_x, Re_L \geq 5\times10^5 \Rightarrow \text{Turbulent}$$

ومن العوامل الؤثرة على الجريان سماكة الطبقة بحيث كلما زادت السماكة يصبح الجريان غير مستقر وتقل السرعة ايضا ويقل shear stress

- لحساب السماكة ($y=\delta$)

$$\delta = \frac{5x}{Re_x^{1/2}}$$

اتعرفنا على boundary layer وقدرنا انميز نوع الحركة عليها واتعلمنا كيف نحسب بعض المطاليب عليها في حال كانت الحركة laminar

احنا اعرفنا انه بنشأ stressعند احتكاك المائع ب boundary layer

لحساب shear force:

$$F_s = 0.664\, B\, \mu U_0\, \mathrm{Re}_L^{\;1/2}$$

$Re_L$: Reynolds number على طول الطبقة

$$\mathrm{Re}_L = U_o L / \nu$$

B:width , L: length

$C_f$: average shear stress coefficient

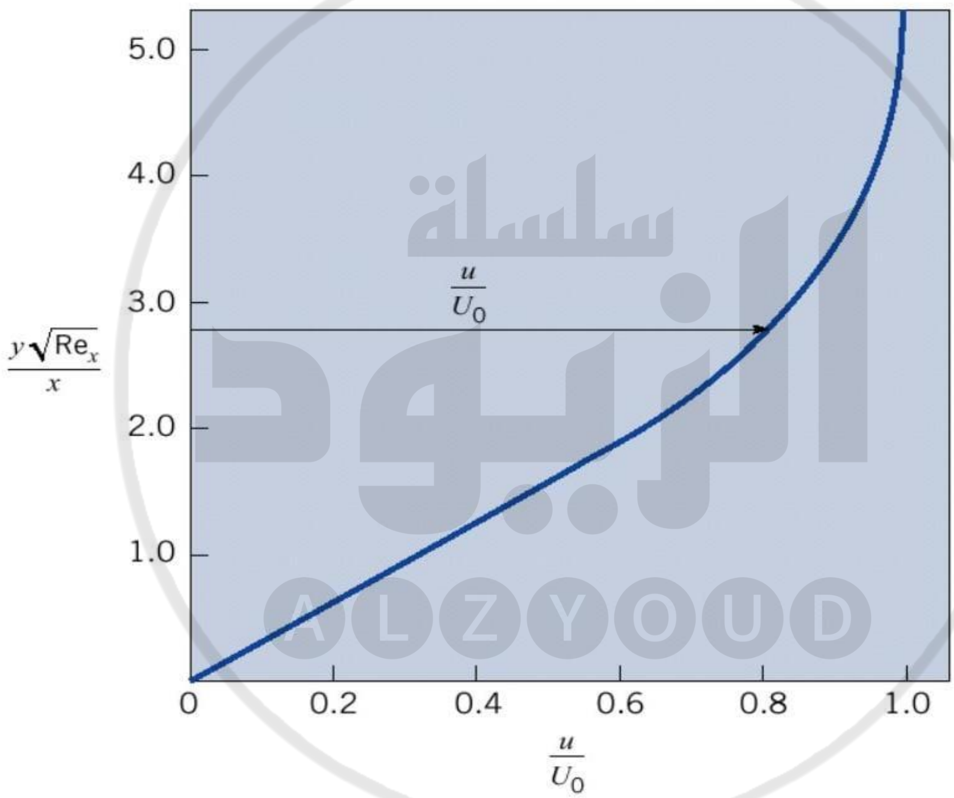
ويمكن من خلال هذا القانون حساب (fs)

$$C_f = \frac{F_s}{BL\rho U_0^2 / 2}$$

$$C_f = \frac{1.33}{\mathrm{Re}_L^{\;1/2}}$$

لايجاد(Cf) عند نقطة معينة:

$$c_f = \frac{\tau_0}{\rho U_0^2 / 2} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$




## •Turbulent boundary layer:

يعد هذا العلم غامض وغير مفهوم بالنسبة للعلماء وهو اكثر تعقيدا

القوانين المستخدمة في هذه الحالة:

- Power – law equation:

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$$

لحساب thickness:

$$\delta = \frac{0.16x}{\mathrm{Re}_x^{\ 1/7}}$$

- And the shear stress at the boundary by:

$$\tau_0 = \rho \frac{U_0^2}{2} \frac{(0.027)}{\mathrm{Re}_x^{\ 1/7}}$$

- Integrating over the area of the plate, the total shear force is equal to:

$$F_s = \frac{0.032BL}{\mathrm{Re}_L^{\ 1/7}} \rho \frac{U_0^2}{2}$$

- The average shear stress coefficient can be given as:

$$C_f = \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$$

- Where,

$$C_f = \frac{F_s}{BL\rho U_0^2/2}$$

Local shear stress coefficient →

$$c_f = \frac{\tau_0}{\rho U_0^2/2} = \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)}$$

القوانين الغير مطلوبة حفظها تكون معطاة بالامتحان

Table 9.3 SUMMARY OF EQUATIONS FOR BOUNDARY LAYER ON A FLAT PLATE

| | Laminar Flow $\mathrm{Re}_x$, $\mathrm{Re}_L < 5 \times 10^5$ | Turbulent Flow $\mathrm{Re}_x$, $\mathrm{Re}_L \geq 5 \times 10^5$ |
|---|---|---|
| Boundary-Layer Thickness, $\delta$ | $\delta = \frac{5x}{\mathrm{Re}_x^{1/2}}$ | $\delta = \frac{0.16x}{\mathrm{Re}_x^{1/7}}$ |
| Local Shear-Stress Coefficient, $c_f$ | $c_f = \frac{\tau_0}{\rho U_0^2/2} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$ | $c_f = \frac{\tau_0}{\rho U_0^2/2} = \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)}$ |
| Average Shear-Stress Coefficient, $C_f$ | $C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}}$ | $C_f = \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$ |

ينقسم friction الى نوعين:

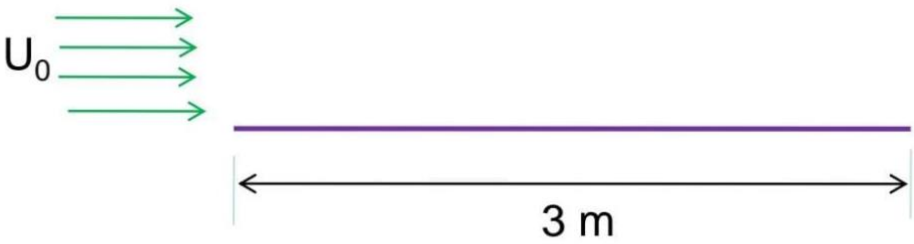
1) skin – drag:

ينشأ بسبب الاحتكاك مع boundary layer وينتج shear force

2) form – drag:(pressure drag)

ينتج بسبب التغير في قيمة الضغط ويكون تأثيره كبير

**Example**: A plate has a total length of 3 m parallel to the flow direction and it is 1 m wide. If the approach velocity is 1 m/s what is the skin –friction drag (shear force) on one side of the plate.(Given $\upsilon=2\text{x}10^{-5}$ m²/s, $\rho$=1000 kg/m³)




* اول شي بدي اعرفه نوع الحركة من خلال حساب (Re) عشان اعرف القوانين يلي رح اشتغل عليه

* المطلوب في السؤال مقدار ال (shear force)

$$\boxed{F_s = \frac{C_F \cdot B \cdot L \cdot \rho \cdot U_0^2}{2}}$$

* نلاحظ ان ( $C_F$ ) مجهولة نجدها من خلال القانون :-

$$\boxed{C_F = \frac{1{,}33}{\sqrt{Re_L}}}$$

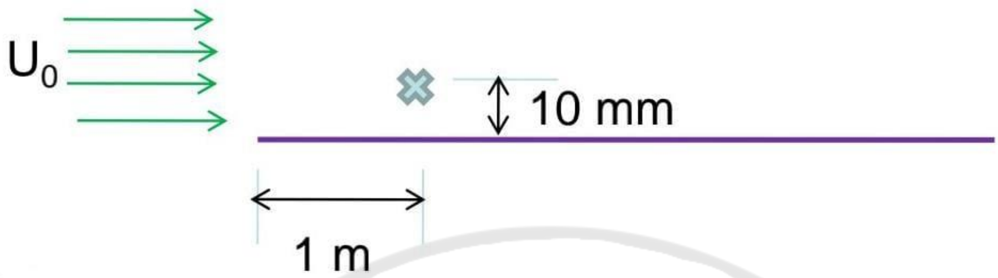
$$Re_L = \frac{U_0 \cdot L}{\nu} = \frac{1*3}{2*10^{-5}} = 1{,}5*10^{5}$$

$$C_F = \frac{1{,}33}{\sqrt{1{,}5*10^{5}}} = 0{,}00343$$

$$F_s = \frac{0{,}00343*1*3*(1)^2*1000}{2}$$

$$\boxed{F_s = 5{,}15 N}$$

**Example**: Oill ($\upsilon = 10^{-4}$ m$^2$/s) flows tangentially past a thin plate. If the free-stream velocity is 6 m/s, what is the velocity 1 m downstream from the leading edge and 10 mm away from the plate?




Ex ⇒ Page (19)

* في السؤال هنا طالب مقدار السرعة عند نقطة محددة

① تحدد نوع الحركة

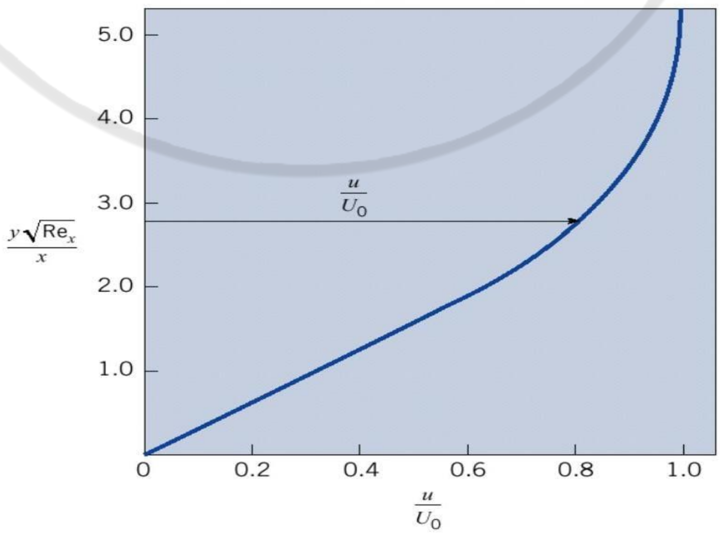
② نعوض في القانون $\boxed{\frac{u}{u_0} = 0,72}$

① $Re_x = \frac{U_0 x}{\nu} = \frac{6 * 1}{10^{-4}} = 6 * 10^4$ Laminar

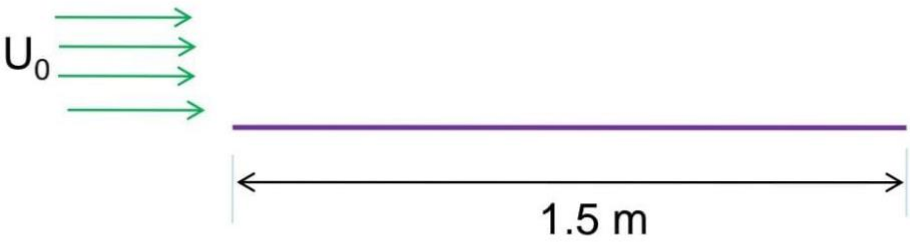
$\frac{u}{u_0} = 0,72 \Rightarrow$ (4,6) نجدها من خلال الرسمة

$u = 6 * 0,72 = \boxed{4,3 \text{ m/s}}$

$\frac{y (Re_x)^{1/2}}{x} = \frac{0,01 \sqrt{(6 \times 10^4)}}{1} \Rightarrow 2,45$



**Example**: A flat plate 1.5 m long and 1 m wide is towed in water in the direction of its length at a speed of 20 cm/s. Determine the resistance of the plate and the boundary-layer thickness at its aft end.(Given $\upsilon=10^{-6}$ $m^2/s$, $\rho=1000$ $kg/m^3$)




* المطلوب مقدار (shear Force) عند النهاية ⇐ $x=1,5$

$$F_s = \frac{C_F \cdot B \cdot L \cdot \rho \cdot U_0^2}{2}$$

* المجهول هنا مقدار الـ ($C_F$) :-

$B=1m$ , $L=1,5\ m$ , $\rho=1000$ , $U_0=0,2$

(I) اول شي بدي احدد نوع الحركة

(II) اجد مقدار الـ ($C_F$)

(I) $R_e = \frac{U_0 \cdot L}{\nu} = \frac{0,2 * 1,5}{10^{-6}} = 3*10^5$ Laminar

(II) $C_F = \frac{1,33}{\sqrt{R_{ex}}} = \frac{1,33}{\sqrt{3*10^5}} = 0,00243$

$F_s = \frac{(0,00243)(1)(1,5)(0,2)^2}{2} \Rightarrow F_s = 0,146\ N$

* المطلوب الثاني هو مقدار سماكة الطبقة ($y$) :

$\delta = y = \frac{5x}{\sqrt{R_{ex}}} = \frac{5 \cdot 1,5}{\sqrt{3\times10^5}} \Rightarrow y = 0,0137\ m$

**Example**: A liquid flows tangentially past a flat plate. The fluid properties are $\mu=10^{-5}$ N.s/m$^2$ and $\rho=1.5$ kg/m$^3$. Find the boundary layer thickness at the trailing edge, the skin-friction drag per unit width if the plate is 2 m long and the approach velocity is 20 m/s. Also, what is the velocity gradient at a point that is 1 m downstream of the leading edge and just next to the plate (y=0)?


① نحدد نوع الحركة

① $Re = \frac{\rho U_0 L}{\mu} = \frac{(1.5)(20)(2)}{10^{-5}} \Rightarrow \boxed{Re = 6\times10^{6}\ \text{Turbulent}}$

* المطلوب الاول:- مقدار سماكة الطبقة (y) :-

$\delta = y = \frac{0.16x}{(Re_x)^{1/7}} = \frac{0.16 * 2}{(6*10^{6})^{1/7}} \Rightarrow \boxed{y = 0.0344\ m}$

* المطلوب الثاني:- مقدار $\left(\frac{F_s}{B}\right)$ :-

$\boxed{F_s = \frac{C_F L B \rho U_0^2}{2}}$

① نجد ال $(C_F)$

① $C_F = \frac{0.523}{(\ln(0.06 Re_L))^2} - \frac{1520}{Re_L} \Rightarrow \boxed{C_F = 0.00294}$

$\frac{F_s}{B} = \frac{(0.00294)(2)(1.5)(20)^2}{2} = \boxed{1.764\ N/m} \rightarrow$ For one side

Layer $F_{s(1)}$, $F_{s(2)}$

For two side $\Rightarrow (2)(1.764) \Rightarrow \boxed{3.528\ N/m}$

* المطلوب الثالث :- $\boxed{\frac{du}{dy}}$ $\boxed{at \Rightarrow 1m}$ $\boxed{\tau = \mu \frac{du}{dy}}$

* نريد معرفة نوع الحركة على بعد (1m)

$Re = \frac{\rho U_0 L}{\mu} = \frac{(1.5)(20)(1)}{10^{-5}} = 3*10^{6}$ (Turbulent)

* نجد $(\tau)$ من خلال $\boxed{C_F = \frac{2\tau}{\rho U_0^2}}$ . ولكن ال $(C_F)$ مجهولة :-

$C_F = \frac{0.455}{\ln^2(0.06 Re_x)} = \frac{0.455}{\ln^2(0.06*3*10^{6})}$

$\boxed{C_F = 0.031}$

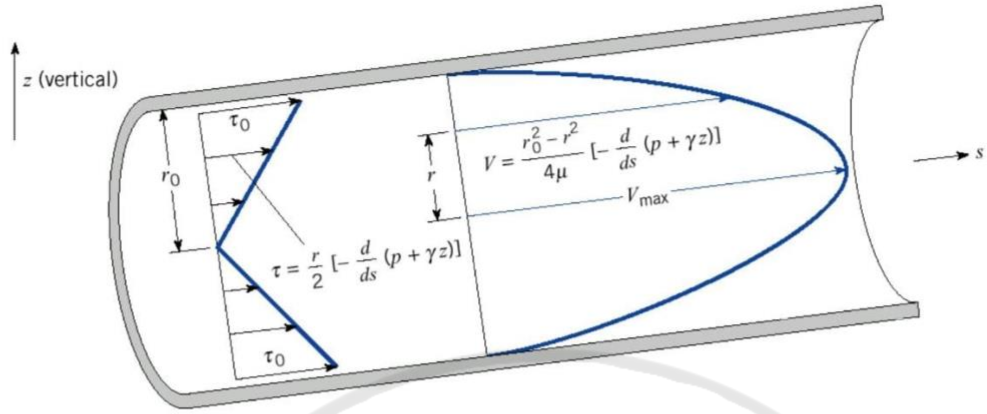
$\tau = \frac{C_F \rho U_0^2}{2} = \frac{(0.031)(1.5)(20)^2}{2}$

$\boxed{\tau = 0.932\ N/m^2\ (Pa)}$

$\frac{du}{dy} = \frac{\tau}{\mu} = \frac{0.932}{10^{-5}} = \boxed{9.322\times10^{4}\ s^{-1}}$

# Ch 10 : flow conduits

في هذا الشابتر رح ندرس حركة الموائع داخل pipe ورح نتعلم طريقة حساب losses



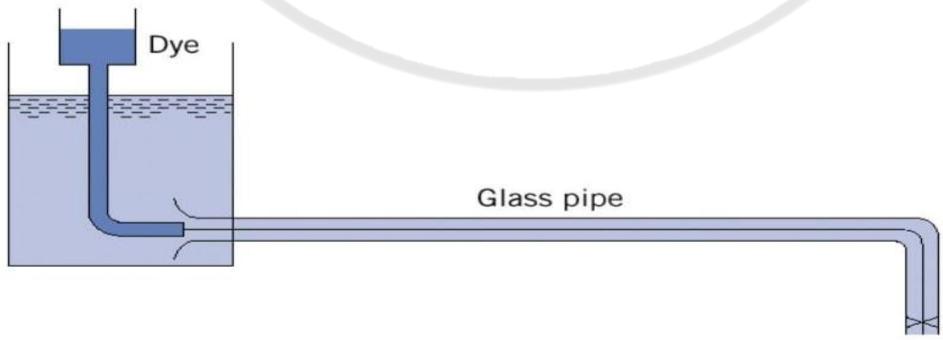

نلاحظ ان السرعة تكون اعلى قيمة بالمنتصف وذلك بسبب ان shear stress = صفر وتكون اعلى قيمة له على الاطراف

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d(p+\gamma z)}{ds}\right]$$ **Parabolic eqn**

بنعرف انه حركة الموائع بتنقسم الى :

1) laminar , 2) turbulent

وتعلمنا انه بنميز بينهم من خلال حساب قيمة Re وتم اكتشاف هذا الرقم من خلال التجربة الاتية :



قام باستخدام هذا الجهاز وقام بوضع صبغة مع السائل ولاحظ بالبداية

ان حركة جزيئات السائل منتظمة وقام بزيادة ارتفاع السائل وعند الوصل الى ارتفاع معين بدأت تتداخل جزيئات السائل

$Re=\frac{\rho V D}{\mu}$

تكلمنا في شابتر 7 عن losess ولكن كان يعطينا مقدار $h_L$ جاهزة في السؤال ولكن في هذا الشابتر رح نتعلم طريقة حسابها $h_L$

بنعرف انه تتقسم  losses الى :

1)major , 2)minor

طريقة حساب major:

نحسبه من خلال معادلة (Darcy-Weisbach equation)

**Major loss due to internal friction inside conduits**

$$h_f = f \frac{L}{D}\frac{V^2}{2g}$$ → Valid for laminar and turbulent flow

- $f$: resistance coefficient or friction coefficient
- For laminar flow it can be easily shown that:

$$f = 64/\text{Re}$$

لحساب( f) في حال كانت الحركة turbulent

رح يختلف مقدارها حسب نوع المادة  وتنقسم الى :

1)smooth pipe: مثل الزجاج والبلاستيك

$$\frac{1}{\sqrt{f}} = 2\log(\text{Re}\sqrt{f}) - 0.8$$

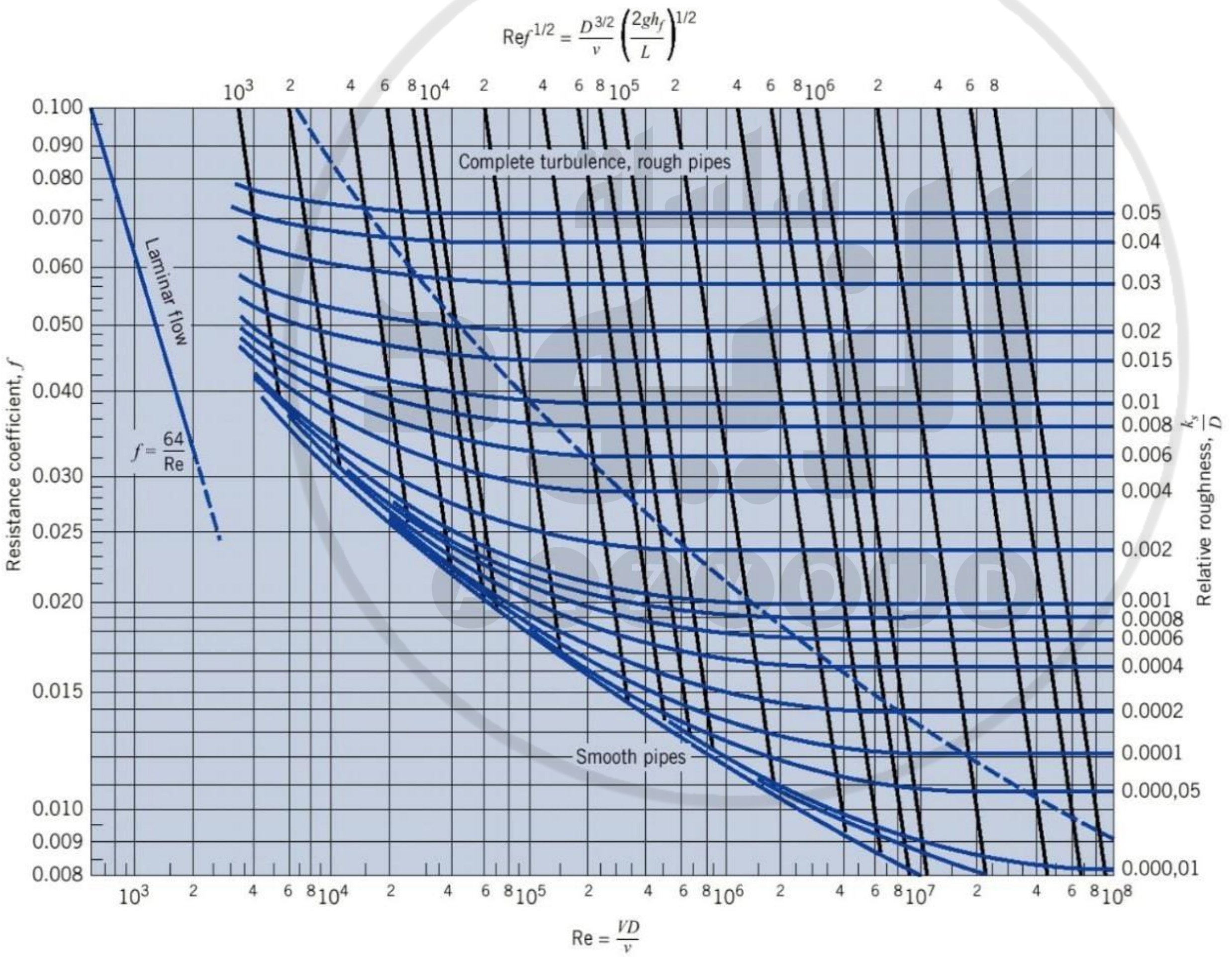
2)Rough pipes: تمثل جميع المعادن ما عدا الزجاج والبلاستيك

نجد f في هذه الحالة من خلال

1) المعادلات:Colebrook-White equation

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

2)moody diagram:



بالبداية اقوم بايجاد مقادر Re

ثم اجد مقدار (Ks/D) : Relative of Roughness

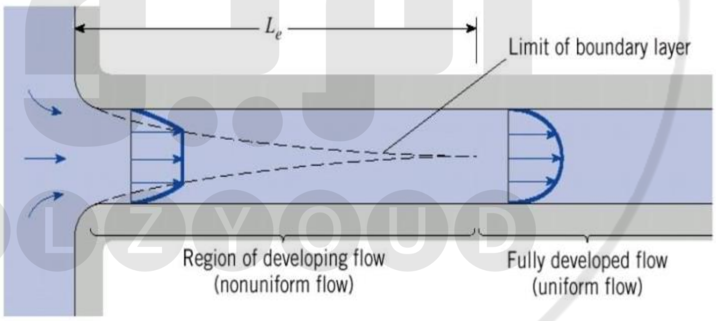
Ks: grain size (ولها جدول خاص بها)

D: diameter pipe

TABLE 10.2 EQUIVALENT SAND GRAIN ROUGHNESS, $k_s$, FOR VARIOUS PIPE MATERIALS

| Boundary Material | $k_s$, millimeters | $k_s$, inches |
|---|---|---|
| Glass, plastic | Smooth | Smooth |
| Copper or brass tubing | 0.0015 | $6 \times 10^{-5}$ |
| Wrought iron, steel | 0.046 | 0.002 |
| Asphalted cast iron | 0.12 | 0.005 |
| Galvanized iron | 0.15 | 0.006 |
| Cast iron | 0.26 | 0.010 |
| Concrete | 0.3 to 3.0 | 0.012–0.12 |
| Riveted steel | 0.9–9 | 0.035–0.35 |
| Rubber pipe (straight) | 0.025 | 0.001 |

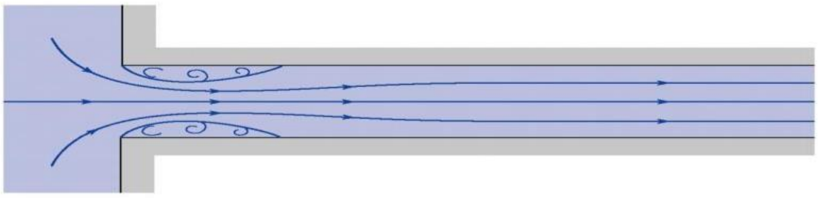
والان رح ننتقل لشرح minor losses

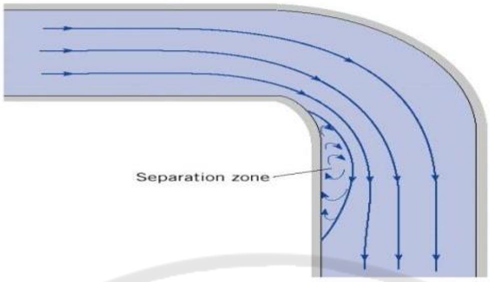



• The length of the boundary layer development region can be given approximately by:

Le= 0.05 D Re, for laminar flow

Le= 50 D, for turbulent flow , **Le : entrance Length**






بنعرف انه minor loss يقع عند الاكواع وبعد المداخل والمخارج

بنلاحظ وجود دوامات وهذه تدل على فقدان الطاقة

لحساب minor loss:

$$h_L = K\frac{V^2}{2g}$$

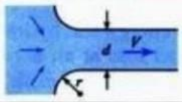
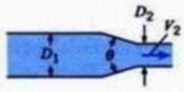
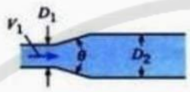
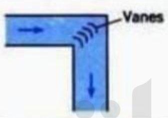
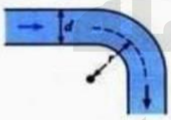
– Where $K$ is the loss coefficient.

قيمة k تختلف فنجدها من خلال الجدول الاتي


| | $D_1/D_2$ | $K_E$ $\theta = 20°$ | $K_E$ $\theta = 180°$ | |
|---|---|---|---|---|
| Expansion | 0.0 | | 1.00 | (2) |
| | 0.20 | 0.30 | 0.87 | |
| | 0.40 | 0.25 | 0.70 | |
| | 0.60 | 0.15 | 0.41 | |
| $h_L = K_E V_1^2/2g$ | 0.80 | 0.10 | 0.15 | |

Note: in The Expansion if the angle is θ =180° you have two choices for the head loss: **1- Use the above equation 2- Use the abrupt expansion equation**

TABLE 10.3 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

| Description | Sketch | Additional Data | | K | Source |
|---|---|---|---|---|---|
| Pipe entrance |  | $r/d$ | | $K_e$ | (2)* |
| | | 0.0 | | 0.50 | |
| | | 0.1 | | 0.12 | |
| $h_L = K_e V^2/2g$ | | >0.2 | | 0.03 | |
| Contraction |  | $D_2/D_1$ | $K_C$ $\theta = 60°$ | $K_C$ $\theta = 180°$ | (2) |
| | | 0.0 | 0.08 | 0.50 | |
| | | 0.20 | 0.08 | 0.49 | |
| | | 0.40 | 0.07 | 0.42 | |
| | | 0.60 | 0.06 | 0.27 | |
| | | 0.80 | 0.06 | 0.20 | |
| $h_L = K_C V_2^2/2g$ | | 0.90 | 0.06 | 0.10 | |
| Expansion |  | $D_1/D_2$ | $K_E$ $\theta = 20°$ | $K_E$ $\theta = 180°$ | (2) |
| | | 0.0 | | 1.00 | |
| | | 0.20 | 0.30 | 0.87 | |
| | | 0.40 | 0.25 | 0.70 | |
| | | 0.60 | 0.15 | 0.41 | |
| $h_L = K_E V_1^2/2g$ | | 0.80 | 0.10 | 0.15 | |
| 90° miter bend |  | Without vanes | | $K_b = 1.1$ | (39) |
| | | With vanes | | $K_b = 0.2$ | (39) |
| 90° smooth bend |  | $r/d$ | | | (5) and (15) |
| | | 1 | | $K_b = 0.35$ | |
| | | 2 | | 0.19 | |
| | | 4 | | 0.16 | |
| | | 6 | | 0.21 | |
| | | 8 | | 0.28 | |
| | | 10 | | 0.32 | |
| Threaded pipe fittings | Globe valve—wide open | | | $K_v = 10.0$ | (39) |
| | Angle valve—wide open | | | $K_v = 5.0$ | |
| | Gate valve—wide open | | | $K_v = 0.2$ | |
| | Gate valve—half open | | | $K_v = 5.6$ | |
| | Return bend | | | $K_b = 2.2$ | |
| | Tee | | | | |
| | straight-through flow | | | $K_t = 0.4$ | |
| | side-outlet flow | | | $K_t = 1.8$ | |
| | 90° elbow | | | $K_b = 0.9$ | |
| | 45° elbow | | | $K_b = 0.4$ | |

# Summary of the Energy equation

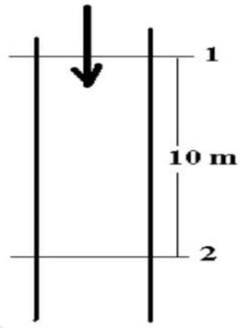
$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

**Total loss= Major loss + Minor Loss**

$$h_L = h_{L,\,major} + h_{L,\,minor}$$

$$h_L = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_j \frac{V_j^2}{2g}$$

**Example:** Liquid flows downward in a 1-cm, vertical, smooth pipe with a mean velocity of 2.0 m/s. The liquid has a density of 1000 kg/m$^3$ and a viscosity of 0.06 N.s/m$^2$. If the pressure at a given section is 600 kpa, what will be the pressure at a section 10 m below that section?




1 major loss , 0 minor loss

* لإيجاد (P) بدي اطبق معادلة (energy)

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_P = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$V_1 = V_2$

نريد إيجاد مقدار ($h_L$)

$h_L = \Sigma major + \Sigma minor$

انا عندي بس (major)

$$h_L = F\frac{L}{D}\frac{V^2}{2g}$$

* لإيجاد (F) لازم احدد نوع الحركة

$$Re = \frac{\rho V D}{\mu} = \frac{(1000)(2)(0{,}01)}{0{,}06} = 333{,}33 \text{ (Lamier)}$$

$$F = \frac{64}{Re} = 0{,}192$$

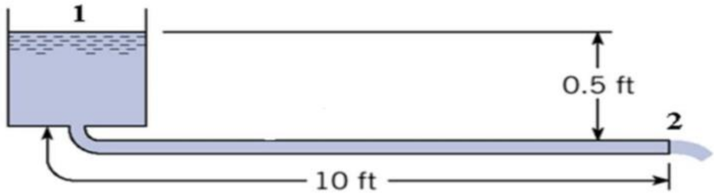
$$h_L = (0{,}192)\left(\frac{10}{0{,}01}\right)\left(\frac{(2)^2}{(2)(9{,}81)}\right) = 39{,}14\,m$$

* نعوض مقدار ($h_L$) في معادلة (energy)

$$\frac{600 \times 10^3}{9810} = \frac{P_2}{9810} + (-10) + 39{,}14$$

$$P_2 = 314{,}1 \text{ KPa}$$

**Example:** Kerosene (S=0.8 and T=68 $^0$F) flows from the tank shown and through 3/8 inch diameter (ID) tube. Determine the mean velocity in the tube and the discharge. Hint: include the major loss only.




I major, I minor    ولكن بالسؤال طالب انه نهمل (minor)

المطلوب هنا مقدار (Q)

$Q = V.A$    لحساب (V) نستخدم معادلة (energy)

$$\frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + Z_1 + h_P = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + Z_2 + h_T + h_L$$

$$h_L = \Sigma \text{major} + \Sigma \text{minor}$$

$$h_L = F\frac{LV^2}{D2g}$$ , F: * تعتمد على نوع الحركة ، نفرض الحركة (Laminar)

$$F = \frac{64}{Re} \Rightarrow Re = \frac{\rho V D}{\mu}$$

$$h_L = \frac{64}{\frac{\rho V D}{\mu}} * \frac{LV^2}{D2g} = \frac{64\mu V L}{2g D^2 \rho}$$

$$h_L = \frac{(64)(4\times10^{-5})(V)(10)}{(2)(32.2)\left(\frac{3}{8\times12}\right)^2(1.94\times0.8)} = \frac{8.45V}{g}$$ → نعوضها في معادلة (energy)

$$\left(\left(\frac{V^2}{2g}\right) + \left(\frac{8.45V}{g}\right) - (0.5) = 0\right) \times 2g$$

$$V^2 + 16.9V - 32.2 = 0$$

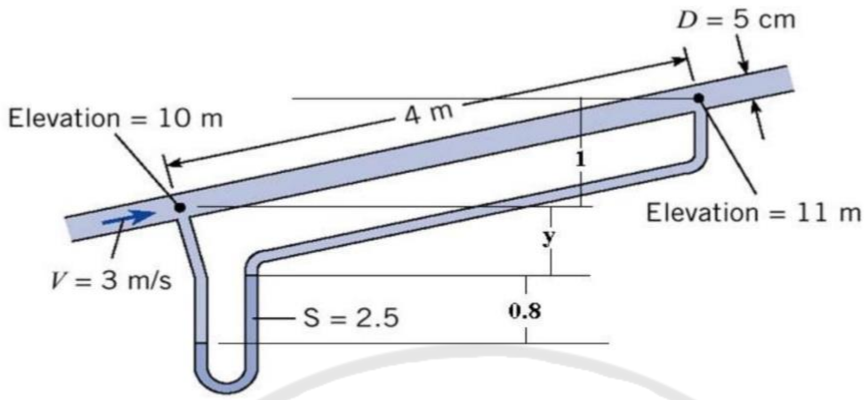
$V = 1.602$ Ft/s

$Re = \frac{\rho V D}{\mu} = 1942 \rightarrow$ Laminar

* بدنا نتأكد من الحركة اذا (Laminar) او (Turbular)

$Q = V.A = 1.228\times10^{-3}$ cF

**Example:** Water flows in the pipe shown, and the manometer deflects 80 cm. What is the resistance coefficient (friction coefficient) for the pipe if V=3 m/s?




1major , 0 minor

* انا باخذ بس (pipe) المستقيم اما (pipe) السفلي هو عباره عن وسيلة لقياس الضغط ما يتم بال (losses) عنده

* طالب مقدار (F) وبنعرفه انه قانونها بعتمد على نوع الحركة ولكن (μ) مجهولة ما بقدر احسب (Re)

* رح اطبق معادلة (energy) ومن خلال قيمة ($h_L$) بحسبها

$$\frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

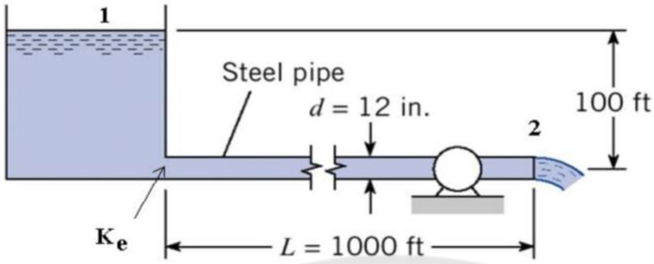
($z_1 \to 10$, $h_p \to 0$, $z_2 \to 11$, $h_t \to 0$; velocity-head terms crossed out)

$$\frac{P_2 - P_1}{\gamma} = 10 - 11 - h_L \Rightarrow \frac{P_2 - P_1}{\gamma} = -1 - h_L$$

$$\Delta P = (\gamma_m - \gamma_f)\Delta h = \underline{21582\,Pa}$$

$$\frac{21582}{\gamma} = -1 - \frac{f L V^2}{D\,2g} \quad (L \to 4,\ V \to 3,\ D \to 0.05) \Rightarrow \boxed{F = 0.033}$$

**Example:** A water turbine is connected to a reservoir as shown. The flow rate in this system is 5 cfs. What power can be delivered by the turbine if its efficiency is 80%? Assume a temperature of 70 $^0$F.




1 major , 1 minor $\quad \boxed{P = Q\gamma h_t \eta}$

$$\frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$h_L = \Sigma minor + \Sigma major$

$$K_e\frac{V^2}{2g} + \frac{fLV^2}{D2g} \qquad V = \frac{Q}{A} = \frac{5}{\frac{\pi}{4}\left(\frac{12}{12}\right)^2} = \boxed{6.369 \text{ Ft/s}}$$

$$Re = \frac{VD}{\nu} = \frac{6.369\left(\frac{12}{12}\right)}{1.0615 \times 10^{-5}} = 6 \times 10^5 \Rightarrow \text{Turbulent}$$

($\nu$) لازم تكون معطى في السؤال

$$\frac{K_s}{D} = \frac{0.002}{12} = 0.00016 \qquad K_s \Rightarrow$$ نجدها من الجدول $\boxed{(r/d) = 0}$

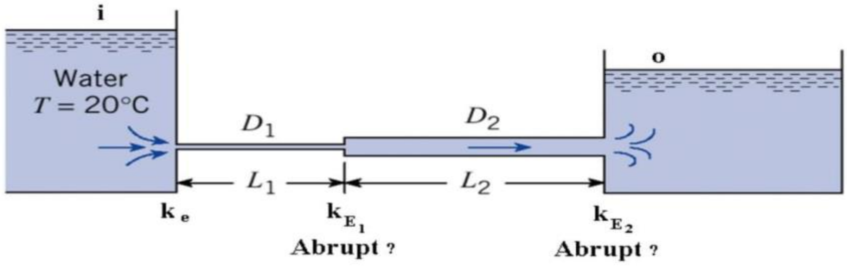
F ⇒ (moody digram) نجدها من ⇒ $\boxed{f = 0.015}$

$$h_L = 0.5\frac{(6.369)^2}{2 * 32.2} + (0.015)(1000)\frac{(6.369)^2}{2 * 32.2} \Rightarrow$$ نعوض في معادلة (energy)

$$0 = \frac{(1)(6.369)^2}{2 * 32.2} - (100) + h_t + h_L \Rightarrow \boxed{h_t = 89.6 \text{ Ft}}$$

$$P = \frac{(5)(62.4)(89.6)(0.8)}{550} = 40.66 \text{ hp}$$

**Example:** Both pipes shown have an equivalent sand roughness $k_s$ of 0.1 mm and a discharge of o.1 $m^3/s$. Also $D_1$=15 cm, $L_1$=50 m, $D_2$=30 cm, and $L_2$=160 m. determine the difference in the water-surface elevation between the two reservoirs.




2 major , 3 minar

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$\boxed{z_1 - z_2 = h_L}$ $\qquad h_L = \Sigma \text{major} + \Sigma \text{minor}$

$$\Rightarrow K_e \frac{V_{15}^2}{2g} + K_{E_1} \frac{V_{15}^2}{2g} + K_{E_2} \frac{V_{30}^2}{2g} + \left(\frac{FLV^2}{2Dg}\right)_{15} + \left(\frac{FLV^2}{2Dg}\right)_{30}$$

$Re_{15} = \frac{V_{15} D_{15}}{\nu} \Rightarrow V_{15} = \frac{Q}{A} = 5,659 \, m/s$ , $V_{30} = 1,415 \, m/s$

($\nu$) تعطى في السؤال

$Re_{15} = \frac{(5,859)(0,015)}{10^{-6}} = 8,49 \times 10^5 \Rightarrow$ (Turbulent)

$F \Rightarrow$ (moody digram) نجدها من

$\frac{k_s}{D_{15}} = \frac{0,1}{150} = 0,00067$ , $\boxed{F_{15} = 0,0185}$

$Re_{30} = \frac{V_{30} D}{\nu} = 4,25 \times 10^5$ , $\frac{k_s}{D_{30}} = \frac{0,1}{300} = 0,00033$

$\boxed{F_{30} = 0,0165}$ $\quad \boxed{K_e = 0,5}$ $\quad \boxed{r/d = 0}$

$K_{E_1} \rightarrow$ (I) $\frac{D_1}{D_2} = 0,5$ , $\theta = 180°$ (Expansion)

لا توجد هذه القيمة في الجدول

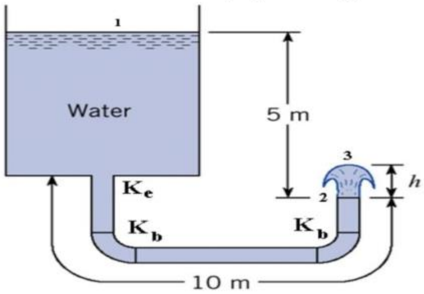
نجدها من خلال (interpolarion)

$\frac{0,41 - 0,7}{0,6 - 0,7} = \frac{X - 0,7}{0,5 - 0,4}$ $\qquad \boxed{X = 0,55 = K_{E_1}}$

$K_{E_2} = 1 \Rightarrow$ (Expansion) $\boxed{D_1/D_2 = 0}$ , $\theta = 180°$

$$\boxed{\Delta z = 12,787 \, m}$$

**Example:** A tank and piping system is shown. The pipe diameter is 2 cm and the total length of pipe is 10 m. The two 90⁰ elbows are threaded fittings. The vertical distance from the water surface to the pipe outlet is 5 m. The velocity of the water in the tank is negligible. Find a) the exit velocity of the water and, b) the height (h) the water jet would rise on exiting the pipe. Assume the pipe is galvanized iron




1 major , 3 minor

* هون بدي افرض نوع الحركة ونلاحظ وجود عدد كبير من الاكواع رح افرضها (turbulent)

* قاعدة :- عند وجود عدد كبير من الاكواع وغير معروف نوع الحركة نفرض (turbuent)

$$\frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$5 = \frac{V_2^2}{2g} + h_L$$

pipe entraence ← → elbowthered ($\theta = 90$)

$$h_L = K_e\frac{V^2}{2g} + 2K_b\frac{V^2}{2g} + \frac{FLV^2}{2Dg}$$

$r/d = 0$ , $K_e = 0.5$ , $K_b = 0.9$

F ⇒ عندي مجهولة و مجهول (Re) بدي أفرض قيمة لها من خلال ايجاد $\left(\frac{K_s}{D}\right)$ و ثم من (moody dicramy)

$$\frac{K_s}{D} = \frac{0.15 \times 10^{-3}}{0.02} = 0.0075$$ اجد الفرض

* بعدي امشي من (0.0075) بشكل مستقيم للقيمة المقابلة لها في المخطط

$F = 0.035$  $V = 2.17\ m/s$  اعوض ($h_L$) بدلالة (V) في معادلة الطاقة

$$Re = \frac{VD}{\nu} = \frac{2.17 \times 0.02}{10^{-6}} = 4.34 \times 10^4 \text{ (Turbulent)}$$

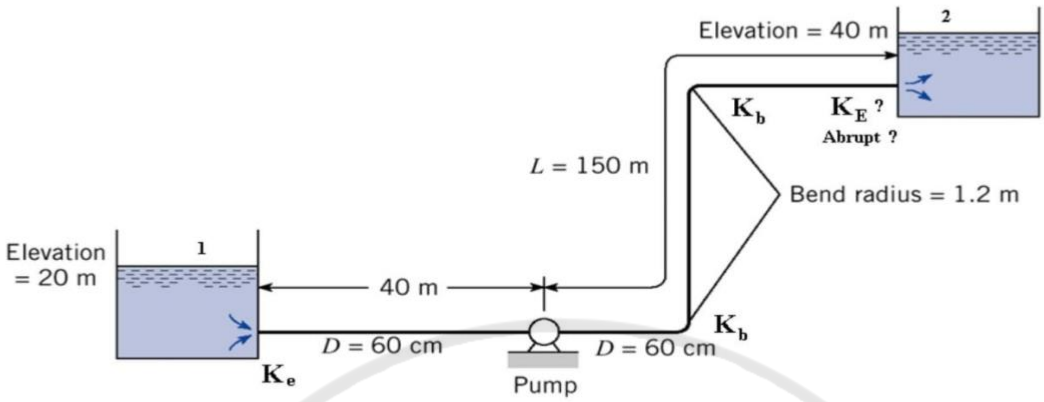
$F = 0.036$

a) $V_2 = 2.15\ m/s$

b) $$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$z_3 - z_2 = h = \frac{2.15}{2g} = 0.24\ m$$

**Example:** If the pump efficiency is 70%, what power must be supplied to the pump in order to pump fuel oil (S=0.94) at a rate of 1.2 $m^3/s$ up to the high reservoir? Assume that the conduit is a steel pipe and the viscosity is 5 x $10^{-5}$ $m^2/s$.




4 minor , 1 major

$$\boxed{P = \frac{Q \gamma h_p}{\eta}}$$

$$V = \frac{Q}{A} = 4.15$$

$$R_e = \frac{VD}{\nu} = 5.1 \times 10^4 \text{ (Turbulent)}$$

$$\frac{k_s}{D} = \frac{0.046 \times 10^{-3}}{0.6} = 0.00008 \text{ , } F = 0.021$$

$$h_L = K_e \frac{V^2}{2g} + 2K_b \frac{V^2}{2g} + \frac{V^2}{2g} + \frac{FLV^2}{2Dg}$$

$K_e = 0.5$, Pipe entrance $r/d = 0$

$K_b = 0.19$, smooth bend $r/d = \frac{1.2}{0.6} = 2.0$, $\theta = 90^\circ$

$$\boxed{V = 4.15}$$

(energy) نعوض في معادلة

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

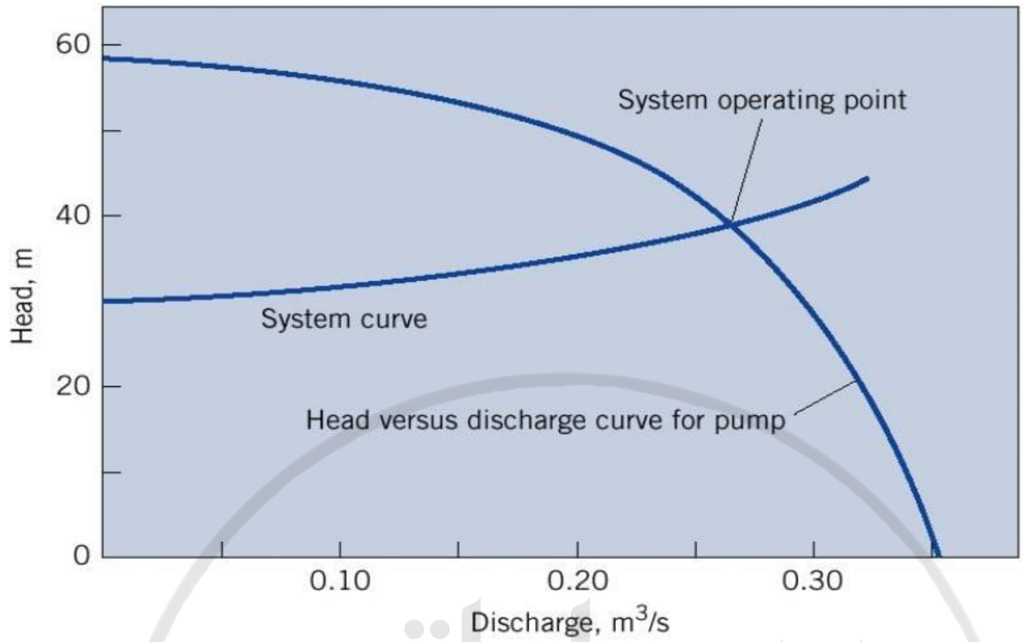
$$h_p = z_2 - z_1 + h_L \quad (z_2 \to 40,\ z_1 \to 20)$$

$$\boxed{h_p = 27.9 \text{ m}}$$

$$P = \frac{(1.2)(0.94)(9810)(27.9)}{0.7}$$

$$\boxed{P = 441 \text{ kW}}$$

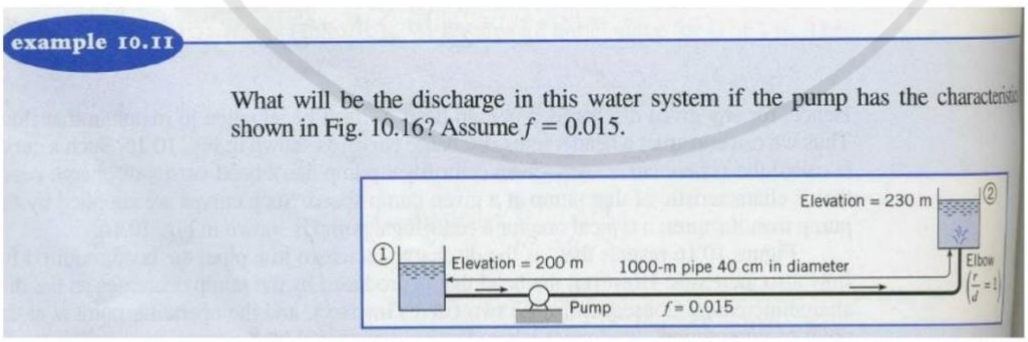
## •pipe Systems:




يمثل هذا المنحنى العلاقة بين (head Vs. Discharge)

الفكرة منه : هو انه يوجد منحنى جاهز وخاص لكل مضخة يسمى (system curve) ومنحنى نقوم نحن بايجاده ورسمه ونقطة تقاطع المنحنين هي النقطة التي تشتغل عندها المضخة وتسمى

**(System operating point)**


example 10.11

What will be the discharge in this water system if the pump has the characteristi shown in Fig. 10.16? Assume $f = 0.015$.

1 major , 3 minor

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_T + h_L$$

$$200 + h_p = 230 + \left(f\frac{L}{D} + K_e + K_b + K_E\right)\frac{V^2}{2g}$$

$$K_e = 0.5 ,\quad K_b = 0.35 ,\quad K_E = 1$$

$$V = \frac{Q}{A} \qquad h_p = 30 + 127Q^2 \text{ m}$$

* الفكرة في يدي أطلع (hp) بدلالة (Q) ثم اجد المنحنى

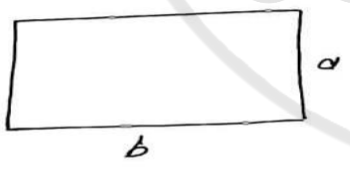
**•Turbulent Flow in Non-Circular Conduits:**

احنا في هذه المادة كنا نتعامل مع (pipe) الدائري ولكن في هذا الموضوع بدنا نتعلم كيفية التعامل مع (pipe) اذا ما كان دائري وحساب diameter

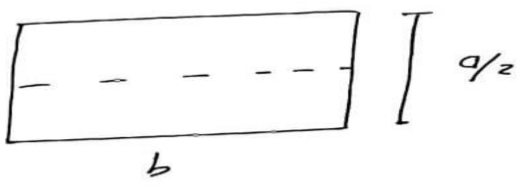
$$D_h = \frac{4A}{P}$$

**P: Wetted perimeter**
**A: cross-sectional area**

$D_h$ (hydraulic diameter)




$$D_h = \frac{4 * a \times b}{2(a+b)}$$

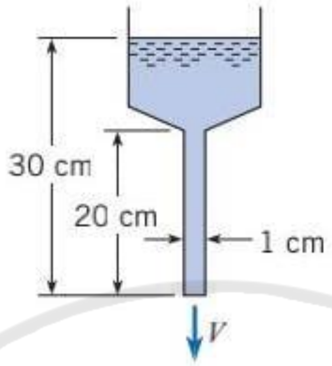



$$D_h = \frac{(4)(a/2)(b)}{2(a/2) + 2b}$$


$$D_h = \frac{(4)(\pi/4)D^2}{\pi D}$$

$$D_h = D$$

**10.24** Glycerine ($T = 20°C$) flows through a funnel as shown. Calculate the mean velocity of the glycerine exiting the tube. Assume the only head loss is due to friction in the tube.




$h_L$ :- major

$$F = \frac{64}{Re} \Rightarrow h_L = \frac{F L V^2}{2 D g}$$

$$h_L = \frac{32 \mu L V}{\gamma D^2} \Rightarrow$$ نعوضها في معادلة (energy)

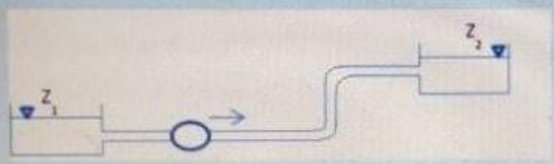
$$0.3 = 2\left(\frac{V_2^2}{2 * 9.81}\right) + \frac{(32)(1.41)(0.2)(V_2)}{(12300)(0.01)^2}$$

$$V_2 = 4.087 * 10^{-2} \ m/s$$

لازم نتأكد من الفرض

$$Re = \frac{V D \rho}{\mu} = 0.365 \Rightarrow \text{laminar}$$

What is the required shaft power (kW) that must be supplied to a pump to pump water at flow rate $0.012 m^3/s$ through pipe that has a diameter of 6.0 cm? Total length of pipes is 100m. The friction factor is 0.02. Exclude all component losses (minor). The pump efficiency is 85%. $Z_1$=8m, $Z_2$=40.




Select one:

- a. $Power_{sh}$ = 10.19 kW
- b. $Power_{sh}$ =13.27 kW
- c. $Power_{sh}$ =8.67 kW
- d. $Power_{sh}$ =6.15 kW
- e. $Power_{sh}$ =7.33 kW

$$P = \frac{Q\gamma h_p}{\eta}$$

1 major, 4 minor

$$\frac{P_1}{\gamma_1} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$10 + h_p = 45 + h_L$$

$$h_L = \text{major} + \text{minor}$$

$$\frac{fLV^2}{2Dg} + K_e\frac{V^2}{2g} + K_{E_1}\frac{V^2}{2g} + 2K_b\frac{V^2}{2g}$$

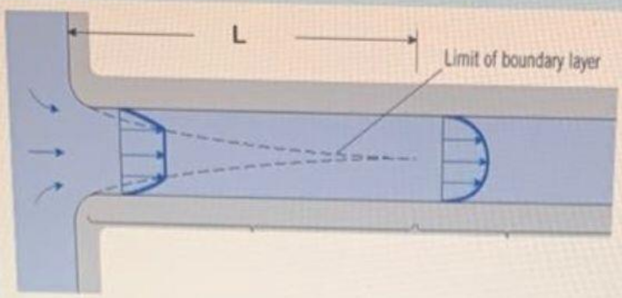
$r/d = 4$, $K_e = 0.5$

$D_1/D_2 = 0$, $K_{E_1} = 1$

$0.9$

نجد مقدار ( $h_L$ ) وثم نجد ( $h_p$ ) ونعوض في معادلة ( Power )

Water (density of 1000 kg/$m^3$ and viscosity $\nu$ of 0.067 $m^2$/s) flows inside a pipe of a diameter of 5 cm as shown with a flow rate of 1.963495 x $10^{-4}$ $m^3$/s, what is the distance L (m) that is required to achieve the fully developed flow case?




$l = \frac{\rho v D}{\mu}$

$v = \frac{\mu}{\rho} \Rightarrow \mu = 0,0094 * 1000$

$\boxed{\mu = 9,4}$

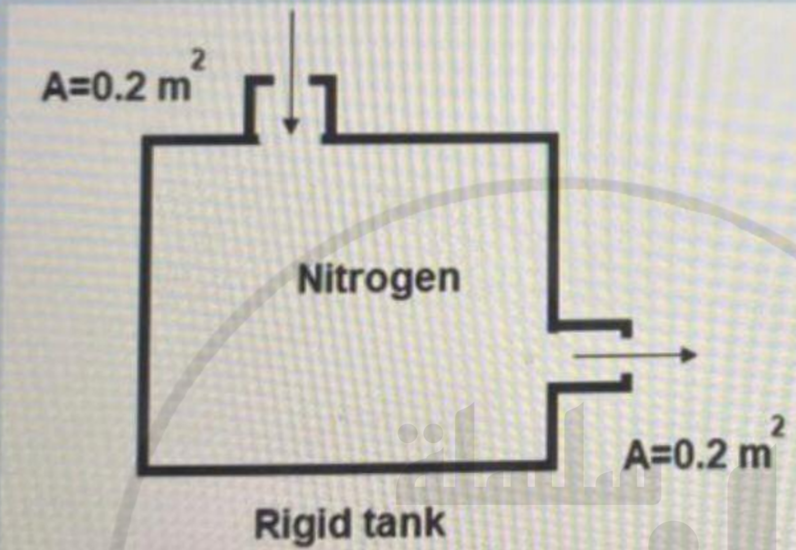
$Q = \frac{V}{A} \Rightarrow V = QA = 1,963495 * 10^{-4} * \frac{\pi}{4} (0,05)^2$

$\boxed{V = 3,86 * 10^{-7} \, m/s}$

$Re = 2,05 * 10^{-6}$ (Laminer)

$L_e = 0,05 D * Re$ $\quad \boxed{L_e = 5,125 * 10^{-9}}$

A rigid rectangular tank of a volume of $V=14.2\ m^3$ contains Nitrogen. At t=0.0, Nitrogen enters the tank with a speed of 50 m/s and a density of 7 $kg/m^3$ and escapes the tank with speed V=120 m/s and density of 6 $kg/m^3$. Find the rate of change of Nitrogen density in the tank at t=0.0.





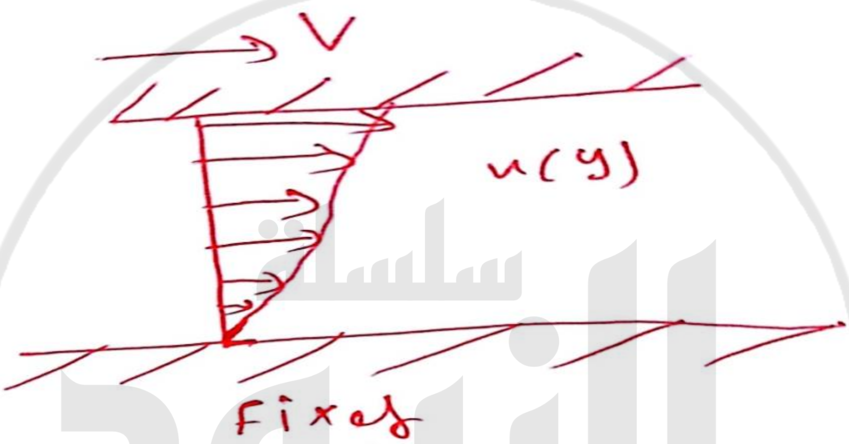

$$\int \rho V \, \partial A = \frac{\partial}{\partial t} \int \rho \, \partial \forall$$

$$\rho_1 V_1 A_1 - \rho_2 V_2 A_2 = \frac{-\partial \rho}{\partial t} \forall$$

$$7 \times 50 \times 0.2 - 120 \times 6 \times 0.2 = \frac{-\partial \rho}{\partial t} \times 5$$

$$\boxed{\frac{\partial \rho}{\partial t} = 14.8}$$

The velocity distribution of fluid (S=0.7) between a fixed and moving plate is shown. The upper plate moves at constant velocity of 10 m/s. The shear stress at the middle is equal to 5 N/m$^2$. What is the kinematic viscosity (m$^2$/s) of the fluid?



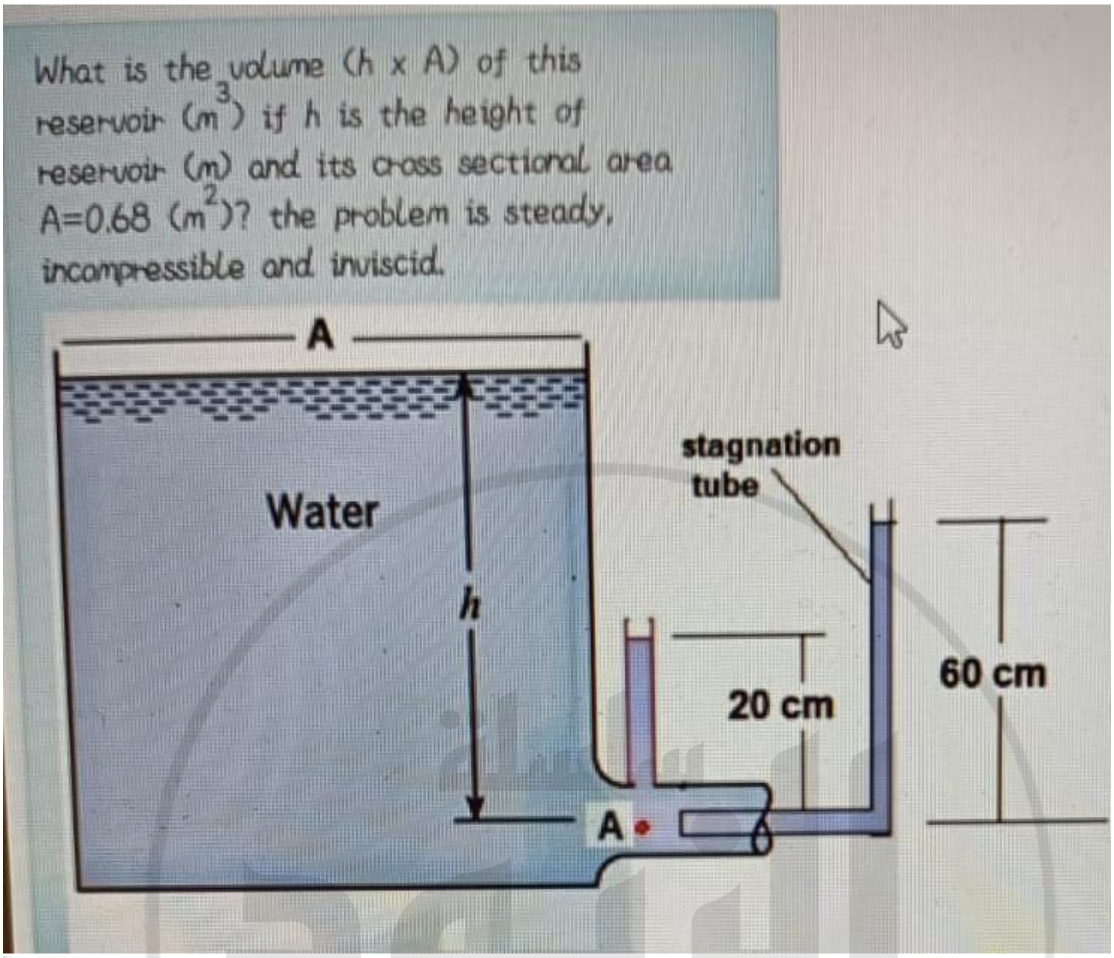

$\tau = \mu \dfrac{dv}{dy}$

$v = ax + b$ at $x = 0, v = 0 \rightarrow b = v$
at $x = y, v = 10$

$v = ay$
$\dfrac{dv}{dy} = a \Rightarrow \boxed{a = \dfrac{10}{y}}$

at $x = (1/2)y$, $v = ??$
$v = a(\frac{1}{2} y) = \dfrac{10}{y} \times \dfrac{y}{2} \Rightarrow \boxed{v = 5 m/s}$

$5 = \mu 5$ $\boxed{\mu = 1}$
$\nu = \dfrac{\mu}{\rho} = \dfrac{1}{(0.7)1000} = 1.43 \times 10^{-3} N/m^2$



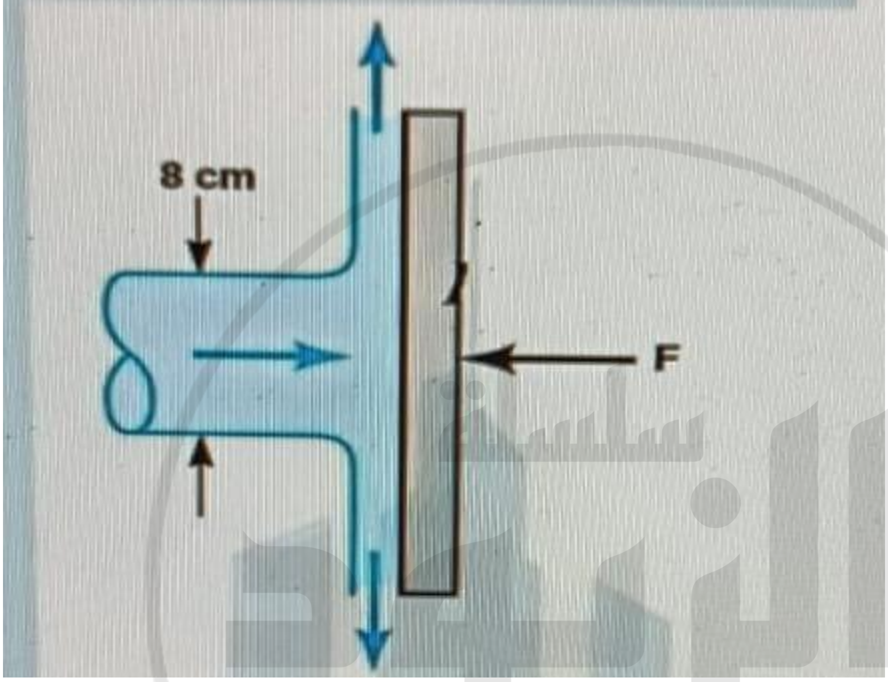

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$h = 0.6$

$\text{volume}(A \times h) = (0.6)(0.68)$

$$\boxed{V = 0.41\ m^3}$$

A 8 cm diameter horizontal jet of liquid that has a specific gravity S=0.62 strikes a flat plate as shown. What is the jet velocity (m/s) if a 12 N horizontal force is required to hold the plate stationary?




$$F = m v$$

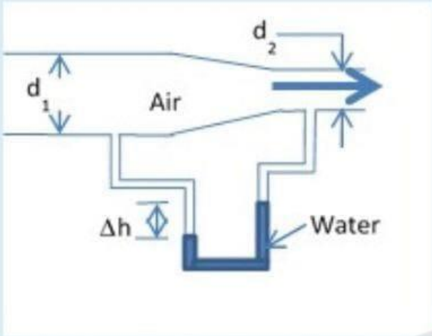
$$12 = \rho A v^2$$

$$v^2 = \frac{12}{\rho A}$$

$$v^2 = \frac{12}{(0{,}62 * 1000)(\frac{\pi}{4} * 0{,}08^2)}$$

$$\boxed{v = 1{,}96 \ m/s}$$

What is the air velocity (m/s) in the pipe at section 2 when the deflection in the water manometer is 12cm? $d_1$=6cm, $d_2$=2.5cm. Air density=1.2 kg/$m^3$ .




$$\frac{\Delta P}{\gamma} = \frac{V_2^2 - (0{,}17 V_2)^2}{2g}$$

$$\frac{1175{,}8}{11{,}77} = \frac{0{,}9711 V_2^2}{2g}$$

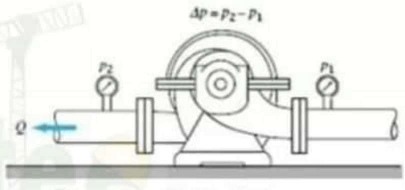
$$\boxed{V = 45 \text{ m/s}}$$

The pressure rise, Δp, across a centrifugal pump of a given shape can be expressed as shown below:

$\Delta p = f_n(D, w, \rho, Q)$. Where D is the impeller diameter, w the angular velocity of the impeller ($sec^{-1}$), ρ the fluid density, and Q the volume flow rate of the flow through the pump.

What is the value of **Qm** **($ft^3/s$)** for the model if the value of Qp for prototype is Qp=19 **($ft^3/s$)**?

| Prototype | Model |
|---|---|
| D = 12 in | D = 8 in |
| ω = 60π rad/s | ω = 40π rad/s |
| ρ= 1.94 slugs/ft³ | ρ=1.94 slugs/ft³ |
| Qp | Qm |




Centrifugal pump

$$ML^{-1}T^{-2} = (L)^a (T^{-1})^b (ML^{-3})^c (MT^{-1})^d$$

$M: 1 = c + d \qquad L: -1 = a - 3c \qquad T: -2 = -b - d$

$\boxed{d = 1 - c}$ ... ① $\boxed{a = 3c - 1}$ ... ② $\boxed{b = d - 2}$ ... ③

$$\Delta p = D^{3c-1} (w)^{d-2} (\rho)^c (Q)^{1-c}$$

$$\Delta p = D^{3c} D^{-1} w^{d} w^{-2} \rho^c Q Q^{-c}$$

$$\Delta p = (D^{-1} w^{-1} Q)(D^{3c} \rho^c Q^{-c})$$

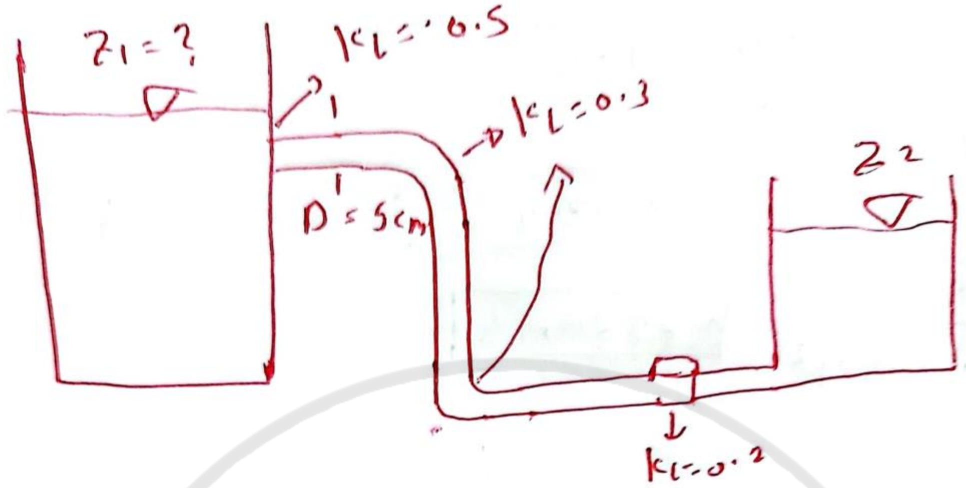
$$\frac{\Delta P \cdot D \cdot w}{Q} = \frac{D^3 \rho}{Q}$$

$$\frac{D_p^3 \rho_p}{Q_p} = \frac{D_m^3 \rho_m}{Q_m}$$

$$Q_m = \frac{D_m^3 \rho_m Q_p}{D_p^3 \rho_p} = \frac{(8)^3 \cancel{(1.94)} (19)}{(12)^3 \cancel{(1.94)}}$$

$$\boxed{Q_m = 5,65}$$

Water flows from a large reservoir to a smaller one through a 5-cm diameter cast iron piping system. The density and dynamic viscosity of water are 999.7 kg/m$^3$ and 1.307 x 10$^{-3}$ kg/m · s. Determine the elevation z1 (m) for a flow rate of 6 L/s. Here z2= 59 m




$$\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$Z_1 = 59 + h_L$ 1 major , 4 minor

$$h_L = \frac{f L V^2}{2Dg} + K_{L_1}\frac{V^2}{2g} + K_{L_2}\frac{V^2}{2g} + K_{L_3}\frac{V^2}{2g} + K_{L_4}\frac{V^2}{2g}$$

(0,5 ; 0,3 ; 0,5 ; 0,2)

$V = 1{,}178 \times 10^{-6}$

$$Re = \frac{\rho V D}{\mu} = 0{,}45 \text{ (laminar)}$$

$$V = QA \Rightarrow 6 \times 10^{-3} \times \frac{\pi}{4}(0{,}05)^2 = V$$

$V = 1{,}178 \times 10^{-5}$

$f = \frac{64}{Re} = 142$

نحسب مقدار ($h_L$) ونعوض بمعادلة الطاقة ونجد (Z)