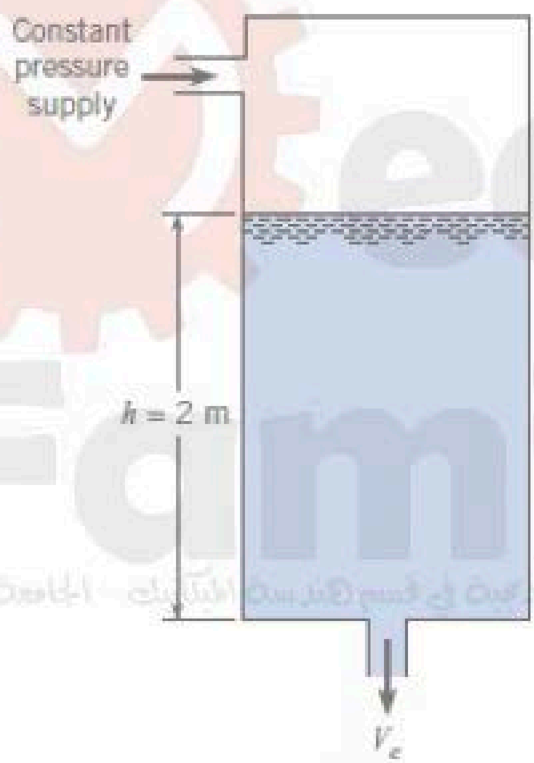
5.76 Water is draining from a pressurized tank as shown in the figure. The exit velocity is given by

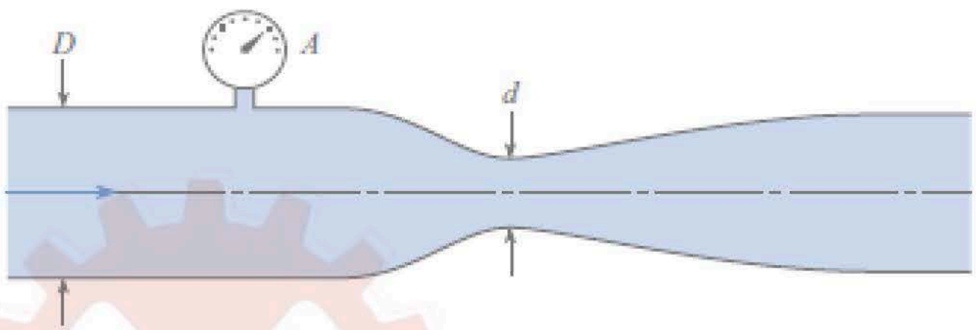
$$V_e = \sqrt{\frac{2p}{\rho} + 2gh}$$

where p is the pressure in the tank, ρ is the water density, and h is the elevation of the water surface above the outlet. The depth of the water in the tank is 2 m. The tank has a cross-sectional area of 1 m^2, and the exit area of the pipe is 10 cm^2. The pressure in the tank is maintained at 10 kPa. Find the time required to empty the tank. Compare this value with the time required if the tank is not pressurized.



PROBLEM 5.76

5.96 When gage A indicates a pressure of 120 kPa gage, thei cavitation just starts to occur in the venturi meter. If $D = 4($ cm and $d = 10$ cm, what is the water discharge in the system fo this condition of incipient cavitation? The atmospheric pressur is 100 kPa gage, and the water temperature is 10°C. Neglec gravitational effects.



PROBLEM 5.96

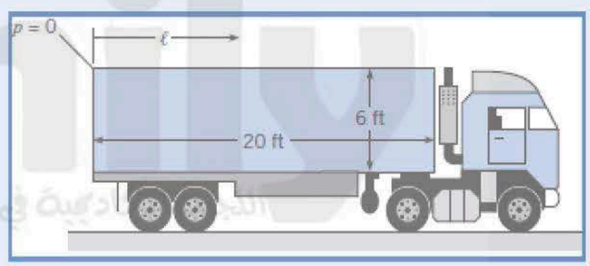
EXAMPLE 4.3 PRESSURE IN A DECELERATING TANK OF LIQUID

The tank on a trailer truck is filled completely with gasoline, which has a specific weight of 42 lbf/ft^3 (6.60 kN/m^3). The truck is decelerating at a rate of 10 ft/s^2 (3.05 m/s^2).

a. If the tank on the trailer is 20 ft (6.1 m) long and if the pressure at the top rear end of the tank is atmospheric, what is the pressure at the top front?

b. If the tank is 6 ft (1.83 m) high, what is the maximum pressure in the tank?

Sketch:



Problem Definition

Situation: Decelerating tank of gasoline with pressure equal to zero gage at top rear end.

Find:

1. Pressure (psfg and kPa, gage) at top front of tank.
2. Maximum pressure (psfg and kPa, gage) in tank.

Assumptions:

1. Deceleration is constant.
2. Gasoline is incompressible.

Properties: $\gamma = 42\ \text{lbf/ft}^3\ (6.60\ \text{kN/m}^3)$

Plan

1. Apply Euler's equation, Eq. (4.8), along top of tank. Elevation, z, is constant.
2. Evaluate pressure at top front.
3. Maximum pressure will be at front bottom. Apply Euler's equation from top to bottom at front of tank.
4. Using result from step 2, evaluate pressure at front bottom.

Solution

1. Euler's equation along the top of the tank

$$\frac{dp}{d\ell} = -\rho a_\ell$$

Integration from back (1) to front (2)

$$p_2 - p_1 = -\rho a_\ell \Delta\ell = -\frac{\gamma}{g} a_\ell \Delta\ell$$

2. Evaluation of p_2 with $p_1 = 0$

$$p_2 = -\left(\frac{42\ \text{lbf/ft}^3}{32.2\ \text{ft/s}^2}\right) \times (-10\ \text{ft/s}^2) \times 20\ \text{ft}$$

$$= \boxed{261\ \text{psfg}}$$

In SI units

$$p_2 = -\left(\frac{6.60\ \text{kN/m}^3}{9.81\ \text{m/s}^2}\right) \times (-3.05\ \text{m/s}^2) \times 6.1\ \text{m}$$

$$= \boxed{12.5\ \text{kPa, gage}}$$

3. Euler's equation in vertical direction

$$\frac{d}{dz}(p + \gamma z) = -\rho a_z$$

4. For vertical direction, $a_z = 0$. Integration from top of tank (2) to bottom (3):

$$p_2 + \gamma z_2 = p_3 + \gamma z_3$$

$$p_3 = p_2 + \gamma(z_2 - z_3)$$

$$p_3 = 261\ \text{lbf/ft}^2 + 42\ \text{lbf/ft}^3 \times 6\ \text{ft} = \boxed{513\ \text{psfg}}$$

In SI units

$$p_3 = 12.5\ \text{kN/m}^2 + 6.6\ \text{kN/m}^3 \times 1.83\ \text{m}$$

$$p_3 = \boxed{24.6\ \text{kPa, gage}}$$

6

Problem No. 3:

(6 Points)

A flow has the following velocity field:

$$V = (10t + x)\vec{i} - yz\vec{j} + 5t^2\vec{k}$$

a) Does the above equation satisfy the continuity equation? Why?

b) What is the acceleration of a particle at position (3, 1,0) m and at time t = 1 s?

a) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$, must be zero to satisfy continuity equation

$\Rightarrow \frac{\partial u}{\partial x} = 1$, $\frac{\partial v}{\partial y} = -z$, $\frac{\partial w}{\partial z} = 0$

$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 - z + 1 = 1 - z$

$\Rightarrow$ not satisfy continuity equation because $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \neq 0$

b) $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$= (10t + x)(1) + (yz)(0) + 0 + 10$

$= 10t + x + 10 = 10 + 3 + 10 = \boxed{23}$ m/s²

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$= (10t + x)(0) + (yz)(-z) + (5t^2)(-y) + 0$

$= 0 + 0 + -5 + 0 = \boxed{-5}$ m/s²

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

$= (10t + x)(0) + (yz)(0) + (5t^2)(0) + 10t$

$= 0 + 0 + 0 + 10 = \boxed{10}$ m/s²

$\vec{a} = 23\hat{i} - 5\hat{j} + 10\hat{k}$ $\quad |a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$|a| = \sqrt{23^2 + (-5)^2 + (10)^2} = 25.573$ m/s²

5.76: PROBLEM DEFINITION

Situation:

Water drains from a pressurized tank.

$V_e = \sqrt{\frac{2p}{\rho} + 2gh}$, $h_o = 2$ m.

$A = 1\,\mathrm{m}^2$, $A_e = 10\,\mathrm{cm}^2$.

Find:

Time for the tank to empty with given supply pressure.

Time for the tank to empty if supply pressure is zero.

Properties:

$p = 10$ kPa.

Water, Table A.5: $\rho = 1000$ kg/m^3.

PLAN

Apply the continuity equation. Define a control surface coincident with the tank walls and the top of the fluid in the tank.

SOLUTION

Continuity equation

$$\rho \frac{dV}{dt} = -\rho A_e V_e$$

Density is constant. The differential volume is Adh so the above equation becomes

$$-\frac{Adh}{A_e V_e} = -dt$$

or

$$-\frac{Adh}{A_e \sqrt{\frac{2p}{\rho} + 2gh}} = dt$$

Integrating this equation gives

$$-\frac{A}{A_e}\frac{1}{g}\left(\frac{2p}{\rho} + 2gh\right)^{1/2}\Big|_{h_o}^{0} = \Delta t$$

or

$$\Delta t = \frac{A}{A_e}\frac{1}{g}\left[\left(\frac{2p}{\rho} + 2gh_o\right)^{1/2} - \left(\frac{2p}{\rho}\right)^{1/2}\right]$$

and for $A = 1$ m^2, $A_e = 10^{-3}$ m^2, $h_o = 2$ m, $p = 10$ kPa and $\rho = 1000$ kg/m^3 results in

$$\boxed{\Delta t = 329\,\mathrm{s} \text{ or } 5.48 \text{ min}} \quad \text{(supply pressure of 10 kPa)}$$

For zero pressure in the tank, the time to empty is

$$\Delta t = \frac{A}{A_e}\sqrt{\frac{2h_o}{g}} = 639 \text{ s or}$$

$$\boxed{\Delta t = 10.6 \text{ min}} \quad \text{(supply pressure of zero)}$$

5.96: PROBLEM DEFINITION

Situation:

Cavitation in a venturi section.

$D = 40\,\text{cm}$, $d = 10\,\text{cm}$.

Find:

Discharge for incipient cavitation.

Properties:

Water (10 °C), Table A.5: $\rho = 1000\,\text{kg/m}^3$.

$p_A = 120\,\text{kPa}$, $p_{atm} = 100\,\text{kPa}$.

PLAN

Apply the continuity equation and the Bernoulli equation.

SOLUTION

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs).

Bernoulli equation

$$p_A + \frac{\rho V_A^2}{2} = p_{\text{throat}} + \frac{\rho V_{\text{throat}}^2}{2}$$

where $V_A = Q/A_A = Q/((\pi/4) \times 0.40^2)$

Continuity equation

$$V_{\text{throat}} = \frac{Q}{A_{\text{throat}}} = \frac{Q}{\pi/4 \times (0.10\,\text{m})^2}$$

$$\frac{\rho}{2}(V_{\text{throat}}^2 - V_A^2) = p_A - p_{\text{throat}}$$

$$\frac{\rho Q^2}{2}\left[\frac{1}{((\pi/4) \times (0.10\,\text{m})^2)^2} - \frac{1}{((\pi/4) \times (0.40\,\text{m})^2)^2}\right]$$

$$= 220,000\,\text{Pa} - 1,230\,\text{Pa}$$

$$500Q^2(16,211 - 63) = 218,770\,\text{Pa}$$

$$\boxed{Q = 0.165\ \text{m}^3/\text{s}}$$