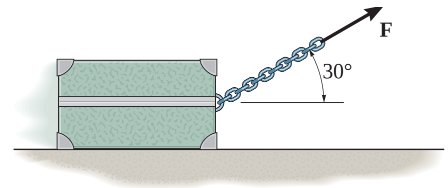
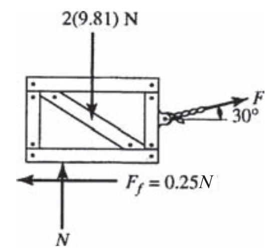
14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30° - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

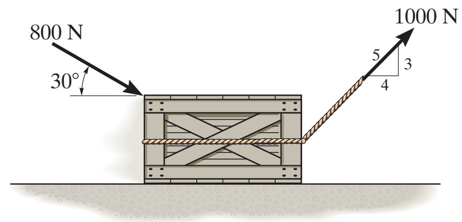
$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30°\, ds - \int_{15\text{ m}}^{25\text{ m}} 36.55\, ds = \frac{1}{2}(20)v^2$$

$v = 10.7$ m/s **Ans.**

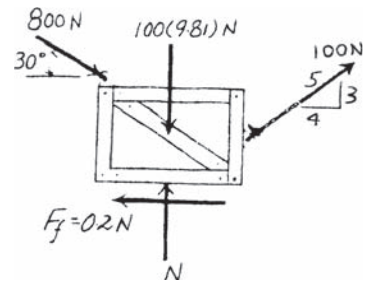
Ans:
$v = 10.7$ m/s

14–2.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30^\circ - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$

Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

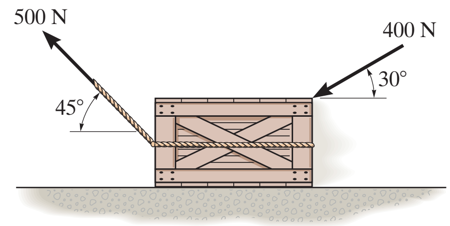
$$0 + 800 \cos 30^\circ(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m}$$ **Ans.**

Ans:
$s = 1.35$ m

14–3.

The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8\ \text{m/s}$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

Work. Consider the force equilibrium along the y axis by referring to the FBD of the crate, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N + 500 \sin 45° - 100(9.81) - 400 \sin 30° = 0$$

$$N = 827.45\ \text{N}$$

Thus, the friction is $F_f = \mu_k N = 0.2(827.45) = 165.49$ N. Here, F_1 and F_2 do positive work whereas F_f does negative work. W and N do no work

$$U_{F_1} = 400 \cos 30°\ s = 346.41\ s$$

$$U_{F_2} = 500 \cos 45°\ s = 353.55\ s$$

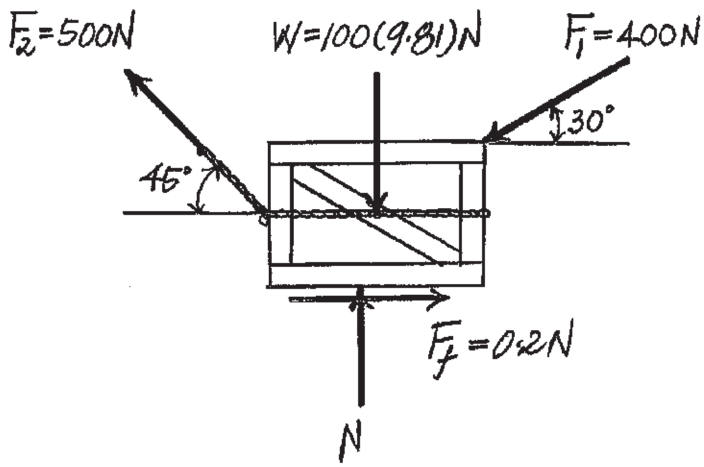
$$U_{F_f} = -165.49\ s$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 346.41\ s + 353.55\ s + (-165.49\ s) = \frac{1}{2}(100)(8^2)$$

$$s = 5.987\ \text{m} = 5.99\ \text{m} \qquad \textbf{Ans.}$$

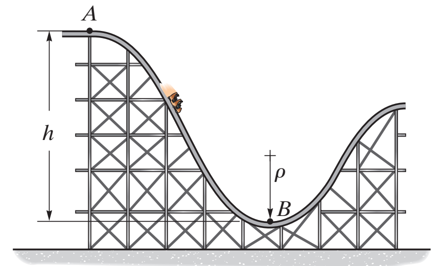


(a)

Ans:
$s = 5.99$ m

***14–4.**

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill A it will reach a speed of 100 km/h when it comes to the bottom B. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4mg = (39.24m)$ N? Neglect the size of the car and passenger.



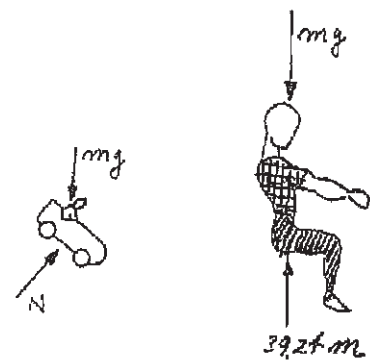
SOLUTION

$$100 \text{ km/h} = \frac{100(10^3)}{3600} = 27.778 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m(9.81)h = \frac{1}{2}m(27.778)^2$$

$$h = 39.3 \text{ m} \qquad \textbf{Ans.}$$

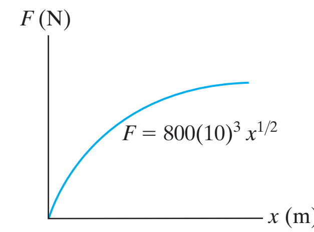


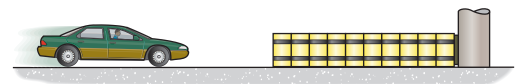
$$+\uparrow \Sigma F_n = ma_n; \qquad 39.24\,m - mg = m\left(\frac{(27.778)^2}{\rho}\right)$$

$$\rho = 26.2 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 39.3$ m
$\rho = 26.2$ m

14–5. For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = [800(10^3)x^{1/2}]$ N, where x is in m, determine the car's maximum penetration in the barrier. The car has a mass of 2 Mg and it is traveling with a speed of 20 m/s just before it hits the barrier.





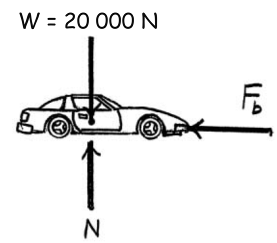
SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 20$ m/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. *a*, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20\,000}{9.81}\right)(20^2) + \left[-\int_0^{x_{max}} 800(10^3)x^{1/2}dx\right] = 0$$

$$x^{max} = 0.825\,\text{m}$$ **Ans.**

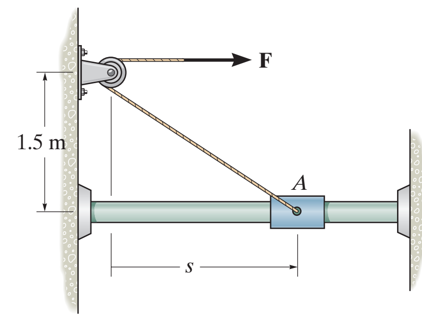


(a)

Ans:
$x^{max} = 0.825$ m

14–6.

The force of $F = 50$ N is applied to the cord when $s = 2$ m. If the 6-kg collar is orginally at rest, determine its velocity at $s = 0$. Neglect friction.



SOLUTION

Work. Referring to the FBD of the collar, Fig. *a*, we notice that force F does positive work but W and N do no work. Here, the displacement of F is $s = \sqrt{2^2 + 1.5^2} - 1.5 = 1.00$ m

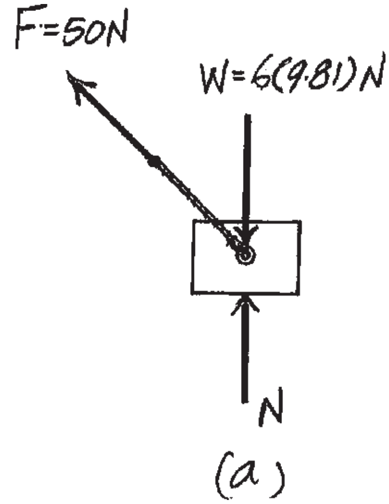
$$U_F = 50(1.00) = 50.0 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 50 = \frac{1}{2}(6)v^2$$

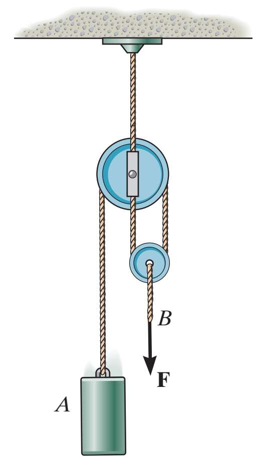
$$v = 4.082 \text{ m/s} = 4.08 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 4.08$ m/s

14–7.

A force of $F = 250$ N is applied to the end at B. Determine the speed of the 10-kg block when it has moved 1.5 m, starting from rest.



SOLUTION

Work. with reference to the datum set in Fig. a,

$$S_W + 2s_F = l$$

$$\delta S_W + 2\delta s_F = 0 \qquad \textbf{(1)}$$

Assuming that the block moves upward 1.5 m, then $\delta S_W = -1.5$ m since it is directed in the negative sense of S_W. Substituted this value into Eq. (1),

$$-1.5 + 2\delta s_F = 0 \qquad \delta s_F = 0.75 \text{ m}$$

Thus,

$$U_F = F\delta S_F = 250(0.75) = 187.5 \text{ J}$$

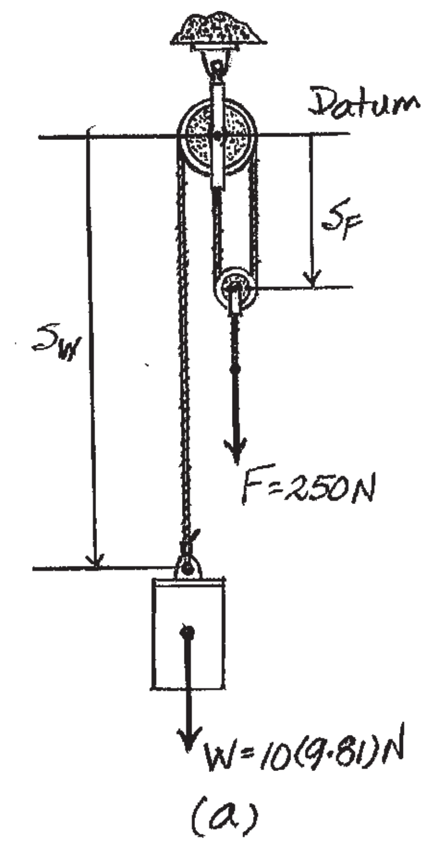
$$U_W = -W\delta S_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + U_{1-2} = T_2$$

$$0 + 187.5 + (-147.15) = \frac{1}{2}(10)v^2$$

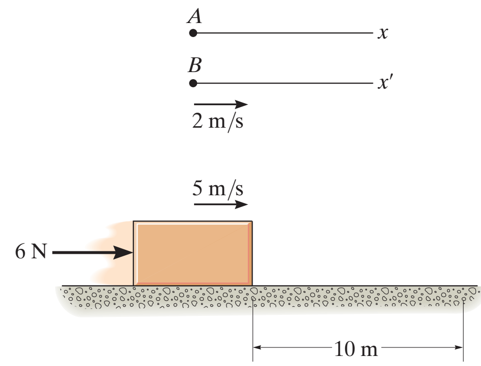
$$v = 2.841 \text{ m/s} = 2.84 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 2.84$ m/s

***14–8.**

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

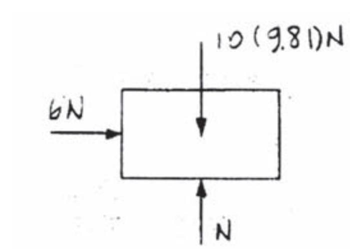
$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$v_2 = 6.08 \text{ m/s}$ **Ans.**

Observer B:

$$F = ma$$

$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$



$$(\stackrel{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

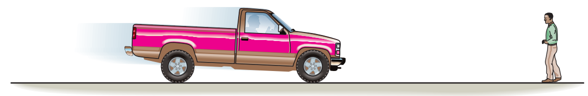
$v_2 = 4.08 \text{ m/s}$ **Ans.**

Note that this result is 2 m/s less than that observed by A.

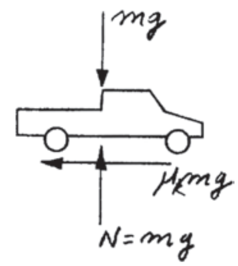
Ans:
Observer A: $v_2 = 6.08$ m/s
Observer B: $v_2 = 4.08$ m/s

14–9.

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



SOLUTION



$40 \text{ km/h} = \frac{40(10^3)}{3600} = 11.11 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$

$\mu_k g = 20.576$

$T_1 + \Sigma U_{1-2} = T_2$

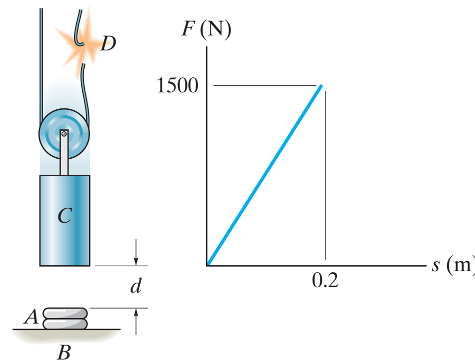
$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$

$d = 12 \text{ m}$ **Ans.**

Ans:
$d = 12$ m

14–10.

The “air spring” A is used to protect the support B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the air spring as a function of its deflection is shown by the graph. If the block has a mass of 20 kg and is suspended a height $d = 0.4$ m above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



SOLUTION

Work. Referring to the FBD of the tensioning weight, Fig. *a*, W does positive work whereas force F does negative work. Here the weight displaces downward $S_W = 0.4 + x_{max}$ where x_{max} is the maximum compression of the air spring. Thus

$$U_W = 20(9.81)(0.4 + x_{max}) = 196.2(0.4 + x_{max})$$

The work of F is equal to the area under the F-S graph shown shaded in Fig. b, Here $\frac{F}{x_{max}} = \frac{1500}{0.2}$; $F = 7500x_{max}$. Thus

$$U_F = -\frac{1}{2}(7500\,x_{max})(x_{max}) = -3750x_{max}^2$$

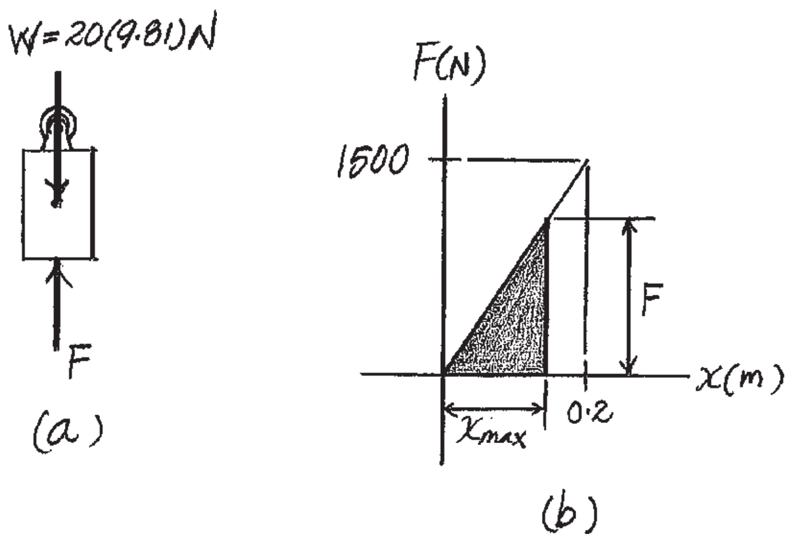
Principle of Work And Energy. Since the block is at rest initially and is required to stop momentarily when the spring is compressed to the maximum, $T_1 = T_2 = 0$. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 196.2(0.4 + x_{max}) + \left(-3750x_{max}^2\right) = 0$$

$$3750x_{max}^2 - 196.2x_{max} - 78.48 = 0$$

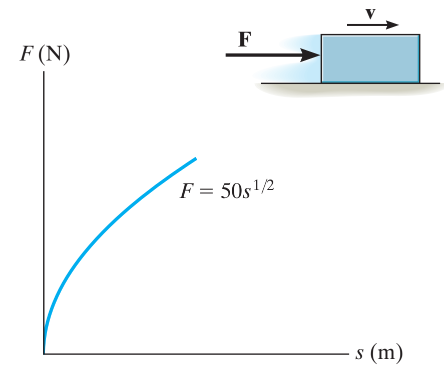
$$x_{max} = 0.1732 \text{ m} = 0.173 \text{ m} < 0.2 \text{ m} \quad \textbf{(O.K!)}$$ **Ans.**



Ans:
$x_{max} = 0.173$ m

14–11.

The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When $s = 0$ the block is moving to the right at $v = 6$ m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



SOLUTION

Work. Consider the force equilibrium along y axis, by referring to the FBD of the block, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) = 0 \qquad N = 196.2\text{ N}$

Thus, the friction is $F_f = \mu_k N = 0.3(196.2) = 58.86$ N. Here, force F does positive work whereas friction F_f does negative work. The weight W and normal reaction N do no work.

$$U_F = \int F ds = \int_0^s 50 s^{\frac{1}{2}}\, ds = \frac{100}{3} s^{\frac{3}{2}}$$

$$U_{F_f} = -58.86\, s$$

Principle of Work And Energy. Applying Eq. 14–7,

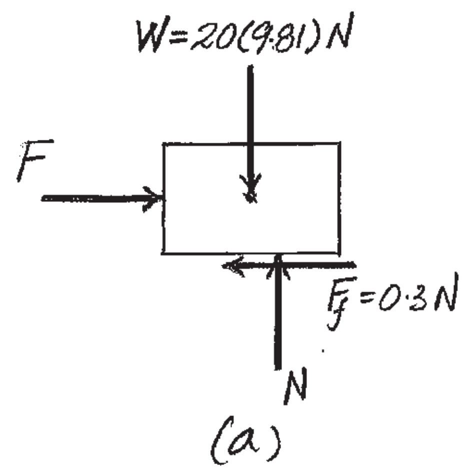
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(6^2) + \frac{100}{3}s^{\frac{3}{2}} + (-58.86s) = \frac{1}{2}(20)(15^2)$$

$$\frac{100}{3}s^{\frac{3}{2}} - 58.86s - 1890 = 0$$

Solving numerically,

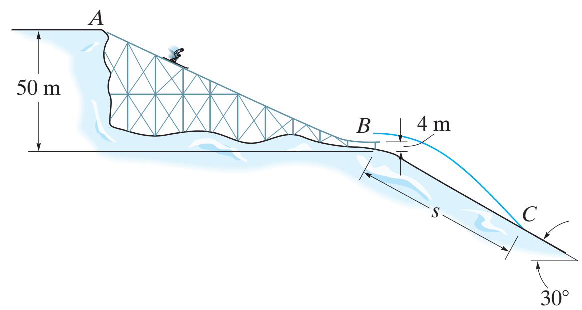
$$s = 20.52\text{ m} = 20.5\text{ m} \qquad \textbf{Ans.}$$



Ans:
$s = 20.5$ m

***14–12.**

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.

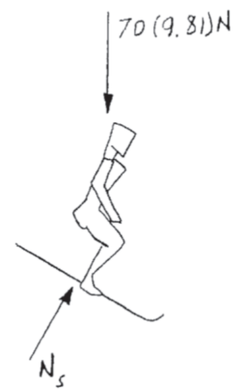


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$s \cos 30° = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

Eliminating t,

$$s^2 - 122.67s - 981.33 = 0$$

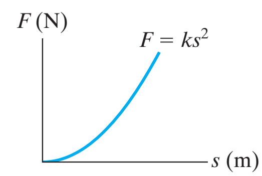
Solving for the positive root

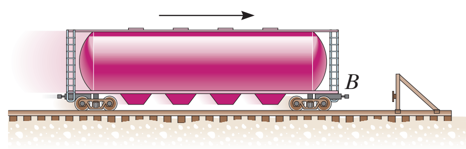
$$s = 130 \text{ m}$$ **Ans.**

Ans:
$v_B = 30.0$ m/s
$s = 130$ m

14–13.

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



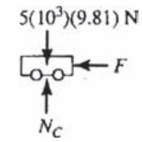


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$$

$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

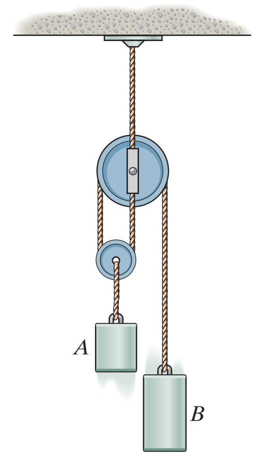
$$k = 15.0\ \text{MN/m}^2$$ **Ans.**



Ans:
$k = 15.0\ \text{MN/m}^2$

14–14.

The 8-kg cylinder A and 3-kg cylinder B are released from rest. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

Kinematics: Express the length of cord in terms of position coordinates s_A and s_B by referring to Fig. *a*

$$2s_A + s_B = l \qquad \textbf{(1)}$$

Thus

$$2\Delta s_A + \Delta s_B = 0 \qquad \textbf{(2)}$$

If we assume that cylinder A is moving downward through a distance of $\Delta s_A = 2$ m, Eq. (2) gives

$$(+\downarrow) \qquad 2(2) + \Delta s_B = 0 \qquad \Delta s_B = -4\text{ m} = 4\text{ m}\uparrow$$

Taking the time derivative of Eq. (1),

$$(+\downarrow) \qquad 2v_A + v_B = 0 \qquad \textbf{(3)}$$

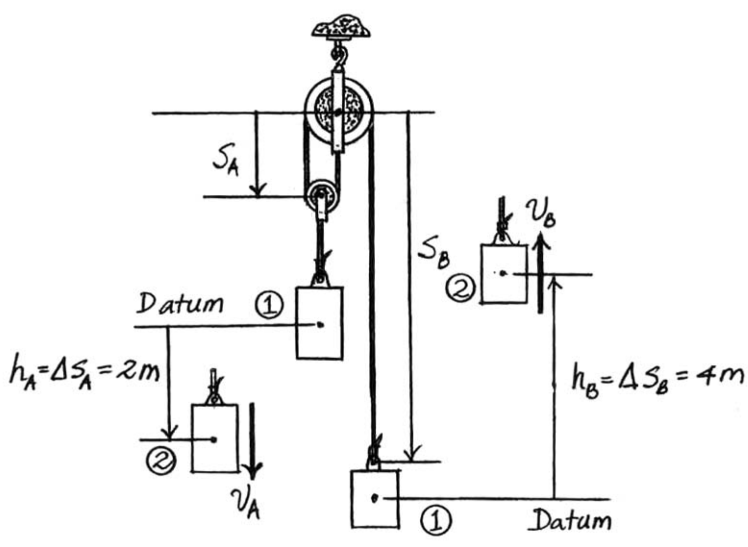
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$0 + 8(2)9.81 - 3(4)9.81 = \frac{1}{2}(8)v_A^2 + \frac{1}{2}(3)v_B^2$$

Positive net work on left means assumption of A moving down is correct. Since $v_B = -2v_A$,

$$v_A = 1.98\text{ m/s}\downarrow \qquad \textbf{Ans.}$$

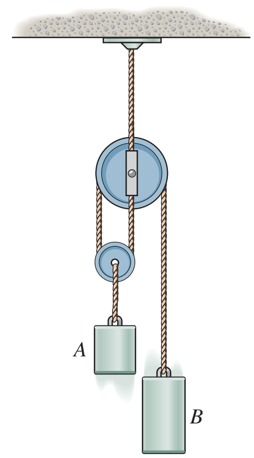
$$v_B = -3.96\text{ m/s} = 3.96\text{ m/s}\uparrow$$



(a)

Ans:
$v_A = 1.98\text{ m/s}\downarrow$
$v_B = 3.96\text{ m/s}\uparrow$

14–15. Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

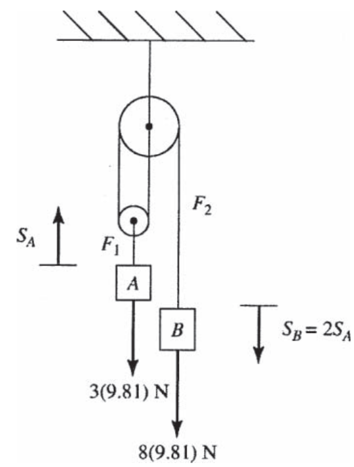
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

$$255.06 = 17.5v_A^2$$

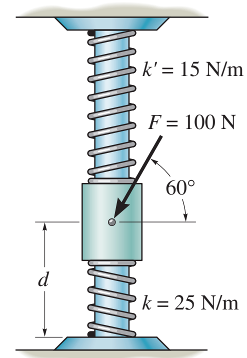
$$v_A = 3.82 \text{ m/s}$$ **Ans.**



Ans:
$v_A = 3.82 \text{ m/s}$

***14–16.**

The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

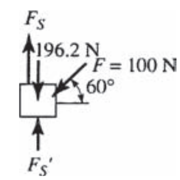


SOLUTION

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

$$- \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

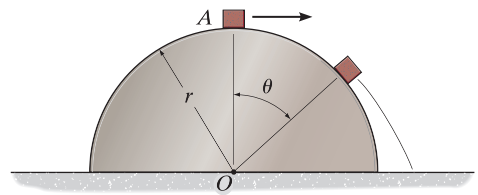


$v_C = 2.36$ m/s **Ans.**

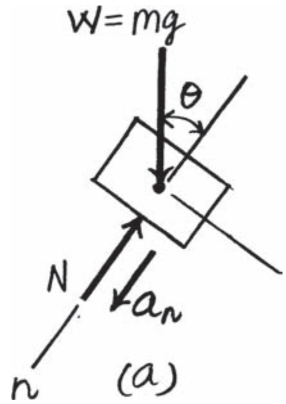
Ans:
$v_C = 2.36$ m/s

14–17.

A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



SOLUTION



Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \qquad (1)$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\frac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

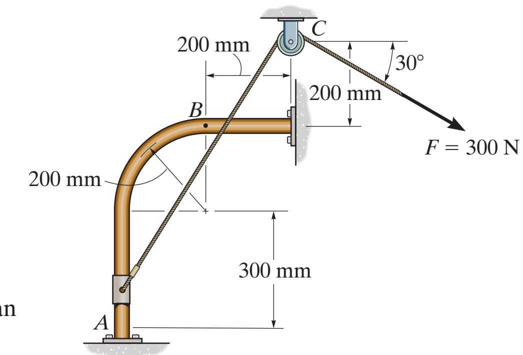
$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 41.4°$

14–18.

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

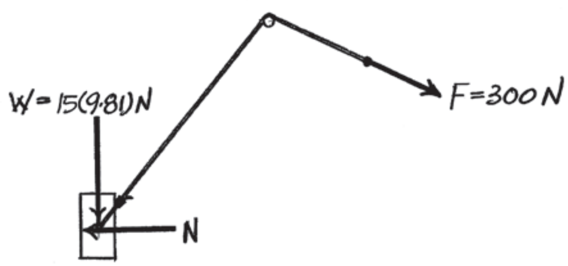
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)\text{ m} = 0.5\text{ m}$, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234\text{ m}$. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335\text{ m/s} = 3.34\text{ m/s}$$ **Ans.**

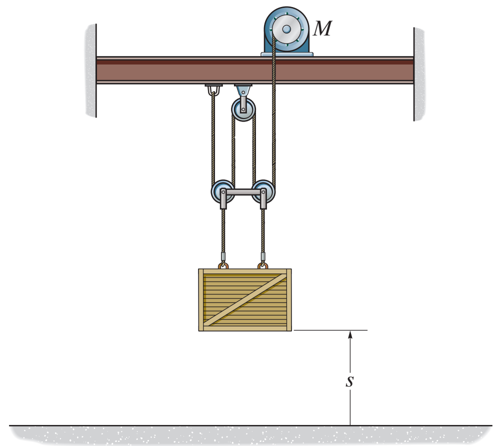


(a)

Ans:
$v_B = 3.34$ m/s

14–19.

If the force exerted by the motor M on the cable is 250 N, determine the speed of the 100-kg crate when it is hoisted to $s = 3$ m. The crate is at rest when $s = 0$.



SOLUTION

Kinematics: Expressing the length of the cable in terms of position coordinates s_C and s_P referring to Fig. a,

$$3s_C + (s_C - s_P) = l$$

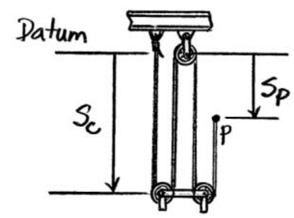
$$4s_C - s_P = l \qquad \textbf{(1)}$$

Using Eq. (1), the change in position of the crate and point P on the cable can be written as

$$(+\downarrow) \qquad 4\Delta s_C - \Delta s_P = 0$$

Here, $\Delta s_C = -3$ m. Thus,

$$(+\downarrow) \qquad 4(-3) - \Delta s_P = 0 \qquad \Delta s_p = -12 \text{ m} = 12 \text{ m} \uparrow$$



(a)

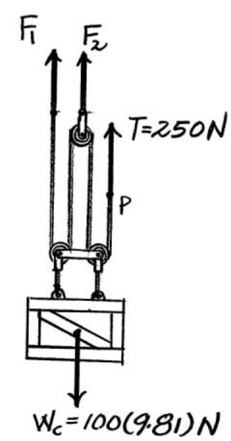
Principle of Work and Energy: Referring to the free-body diagram of the pulley system, Fig. b, $\mathbf{F}_1$ and $\mathbf{F}_2$ do no work since it acts at the support; however, $\mathbf{T}$ does positive work and $\mathbf{W}_C$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + T\Delta s_P + [-W_C \Delta s_C] = \frac{1}{2} m_C v^2$$

$$0 + 250(12) + [-100(9.81)(3)] = \frac{1}{2}(100)v^2$$

$$v = 1.07 \text{ m/s} \qquad \textbf{Ans.}$$

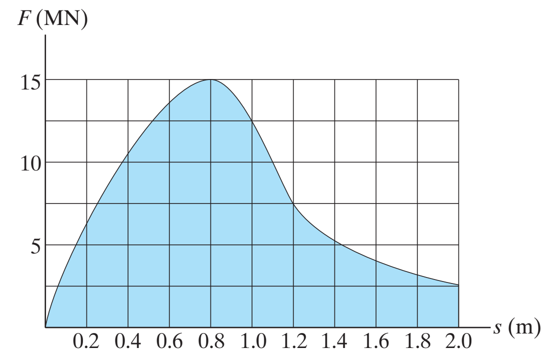


(b)

Ans:
$v = 1.07$ m/s

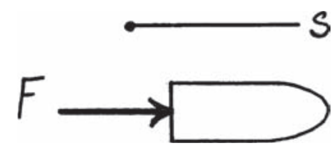
***14–20.**

When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



SOLUTION

The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5\left(10^6\right)(0.2)$,



$$T_1 + \Sigma U_{1-2} = T_2$$

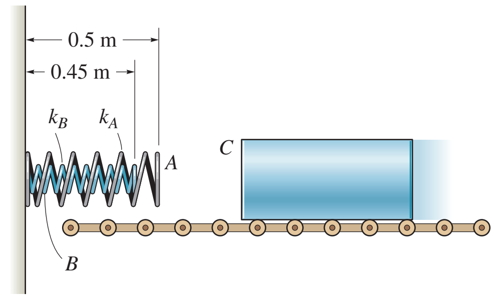
$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121 \text{ m/s} = 2.12 \text{ km/s}$ (approx.) **Ans.**

Ans:
$v_2 = 2.12 \text{ km/s}$

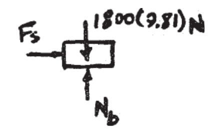
14–21.

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

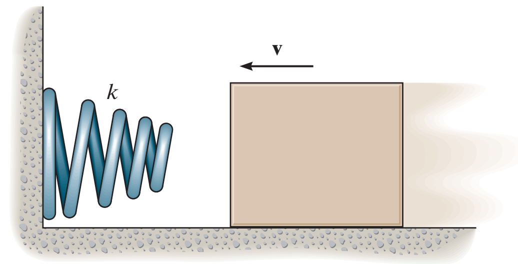


$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45 - 0.3)^2 = 0$$

$k_B = 11.1$ kN/m **Ans.**

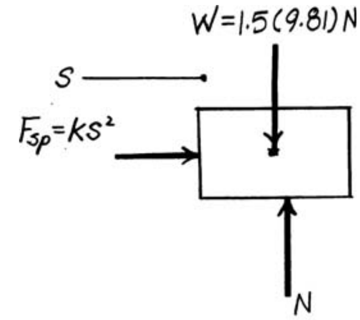
Ans:
$k_B = 11.1$ kN/m

14–22. The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of $v = 4$ m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900$ N/m^2. Determine the speed of the block after it has compressed the spring $s = 0.2$ m.



SOLUTION

Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have



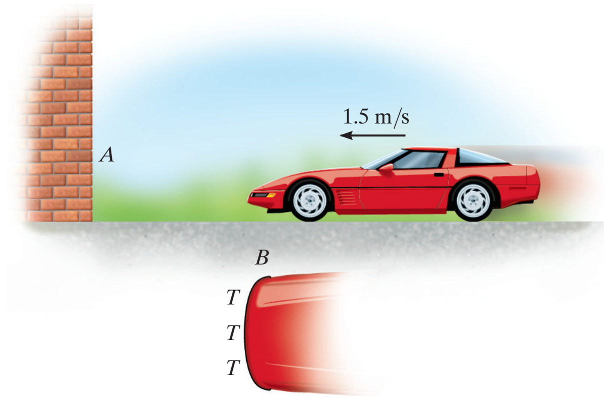
$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900s^2\,ds\right] = \frac{1}{2}(1.5)\,v^2$$

$$v = 3.58\text{ m/s}$$ **Ans.**

Ans:
$v = 3.58$ m/s

14–23. A car is equipped with a bumper B designed to absorb collisions. The bumper is mounted to the car using pieces of flexible tubing T. Upon collision with a rigid barrier at A, a constant horizontal force **F** is developed which causes a car deceleration of $3g = 29.43\ \text{m/s}^2$ (the highest safe deceleration for a passenger without a seatbelt). If the car and passenger have a total mass of 1.5 Mg and the car is initially coasting with a speed of 1.5 m/s, determine the magnitude of **F** needed to stop the car and the deformation x of the bumper tubing.



Units Used:

$$\text{Mm} = 10^3\ \text{kg}$$

$$\text{kN} = 10^3\ \text{N}$$

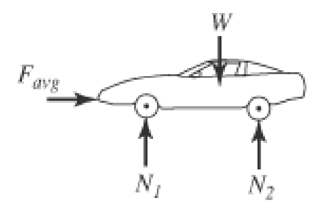
SOLUTION

Given:

$$M = 1.5\,10^3\ \text{kg}$$

$$v = 1.5\ \frac{\text{m}}{\text{s}}$$

$$k = 3$$



The average force needed to decelerate the car is

$$F_{avg} = M k g \qquad F_{avg} = 44.1\ \text{kN} \qquad \textbf{Ans.}$$

The deformation is

$$T_1 + U_{12} = T_2$$

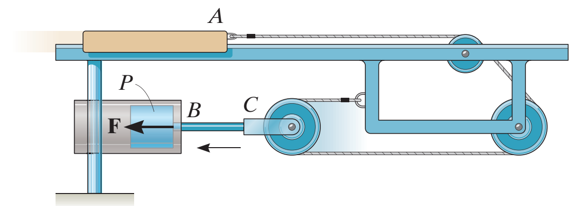
$$\frac{1}{2} M v^2 - F_{avg} x = 0$$

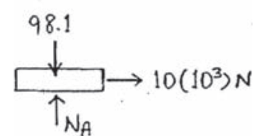
$$x = \frac{1}{2} M \left(\frac{v^2}{F_{avg}}\right) \qquad x = 38.2\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F_{avg} = 44.1\ \text{kN}$
$x = 38.2\ \text{mm}$

***14–24.**

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.





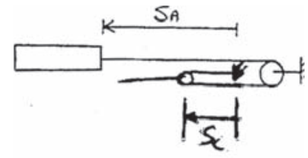
SOLUTION

$2\,s_C + s_A = l$

$2\,\Delta\,s_C + \Delta\,s_A = 0$

$2(0.2) = -\,\Delta\,s_A$

$-0.4 = \Delta\,s_A$



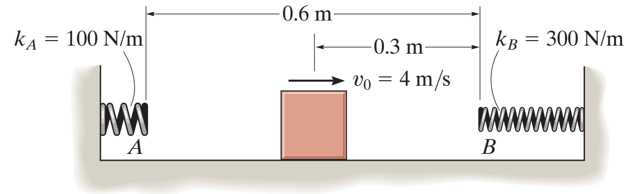
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$v_A = 28.3$ m/s **Ans.**

Ans:
$v_A = 28.3$ m/s

14–25. The 12-kg block has an initial speed of $v_0 = 4$ m/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction is $f_f = M_k N = 0.4[12(9.81)] = 47.088$ N. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(12)(4^2) - 47.088(0.3 + s_1) - \frac{1}{2}(300)\, s_1^2 = 0$$

$$s_1 = 0.5983 \text{ m}$$

Assume the block bounces back and stops without striking spring A. The spring force does positive work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \Sigma U_{2-3} = T_3$$

$$0 + \frac{1}{2}(300)(0.5983^2) - 47.088(0.5983 + s_2) = 0$$

$$s_2 = 0.5421 \text{ m}$$

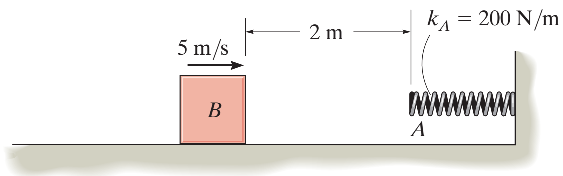
Since $s_2 = 0.5421 \text{ m} < 0.6$ m, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 0.3 = 2(0.5983) + 0.5421 + 0.3 = 2.039 \text{ m} = 2.04 \text{ m} \quad \textbf{Ans.}$$

Ans:
$s_{\text{Tot}} = 2.04$ m

14–26.

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



SOLUTION

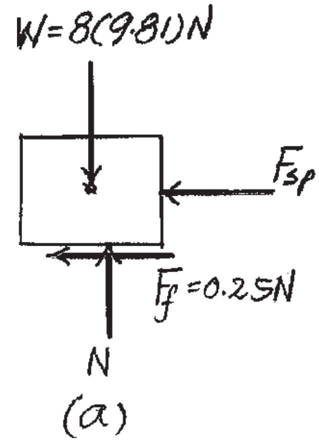
Work. Consider the force equilibrium along y axis by referring to the FBD of the block, Fig. a

$$+\uparrow \Sigma F_y = 0; \qquad N - 8(9.81) = 0 \qquad N = 78.48\text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62\text{ N}$ and $F_{sp} = kx = 200\,x$. Here, the spring force F_{sp} and F_f both do negative work. The weight W and normal reaction N do no work.

$$U_{F_{sp}} = -\int_0^x 200\,x\,dx = -100\,x^2$$

$$U_{F_f} = -19.62(x + 2)$$



(a)

Principle of Work And Energy. It is required that the block stopped momentarily, $T_2 = 0$. Applying Eq. 14–7

$$T_1 + \Sigma\, U_{1-2} = T_2$$

$$\frac{1}{2}(8)(5^2) + \left(-100x^2\right) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0$$

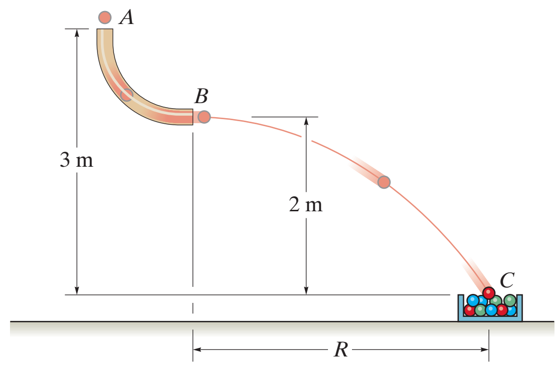
Solved for positive root,

$$x = 0.6875\text{ m} = 0.688\text{ m}$$ **Ans.**

Ans:
$x = 0.688$ m

14–27.

Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

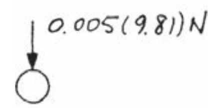


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

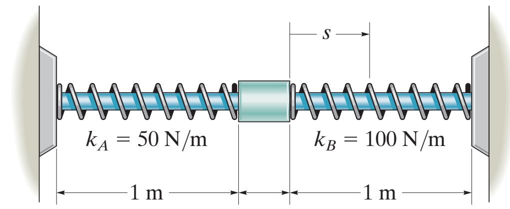
$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

Ans:
$R = 2.83$ m
$v_C = 7.67$ m/s

***14–28.**

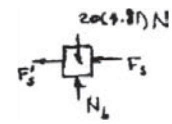
The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

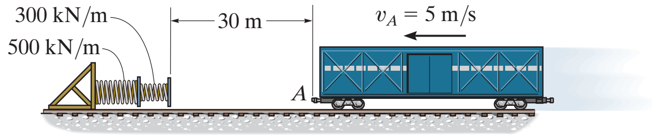


$s = 0.730$ m **Ans.**

Ans:
$s = 0.730$ m

14–29.

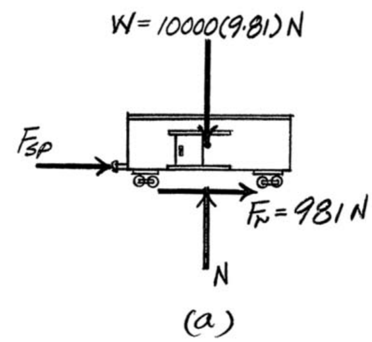
The train car has a mass of 10 Mg and is traveling at 5 m/s when it reaches A. If the rolling resistance is 1/100 of the weight of the car, determine the compression of each spring when the car is momentarily brought to rest.



SOLUTION

Free-Body Diagram: The free-body diagram of the train in contact with the spring is shown in Fig. *a*. Here, the rolling resistance is $F_r = \frac{1}{100}[10\,000(9.81)] = 981$ N. The compression of springs 1 and 2 at the instant the train is momentarily at rest will be denoted as s_1 and s_2. Thus, the force developed in springs 1 and 2 are $\left(F_{sp}\right)_1 = k_1 s_1 = 300(10^3)s_1$ and $\left(F_{sp}\right)_2 = 500(10^3)s_2$. Since action is equal to reaction,

$$\left(F_{sp}\right)_1 = \left(F_{sp}\right)_2$$
$$300(10^3)s_1 = 500(10^3)s_2$$
$$s_1 = 1.6667 s_2$$



(a)

Principle of Work and Energy: Referring to Fig. *a*, **W** and **N** do no work, and $\mathbf{F}_{sp}$ and $\mathbf{F}_r$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10\,000)(5^2) + [-981(30 + s_1 + s_2)] + \left\{-\frac{1}{2}\left[300(10^3)\right]s_1^2\right\} + \left\{-\frac{1}{2}\left[500(10^3)\right]s_2^2\right\} = 0$$

$$150(10^3)s_1^2 + 250(10^3)s_2^2 + 981(s_1 + s_2) - 95570 = 0$$

Substituting Eq. (1) into Eq. (2),

$$666.67(10^3)s_2^2 + 2616 s_2 - 95570 = 0$$

Solving for the positive root of the above equation,

$$s_2 = 0.3767 \text{ m} = 0.377 \text{ m}$$

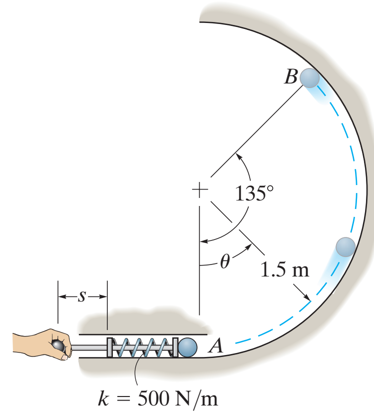
Substituting the result of s_2 into Eq. (1),

$$s_1 = 0.6278 \text{ m} = 0.628 \text{ m}$$ **Ans.**

Ans:
$s_1 = 0.628$ m

14–30.

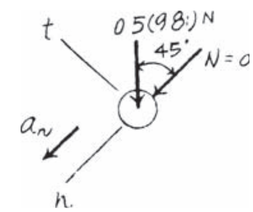
The 0.5-kg ball is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135^\circ$.



SOLUTION

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45^\circ = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41\ \text{m}^2/\text{s}^2$$



Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5 \sin 45^\circ = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-B} = T_B$$

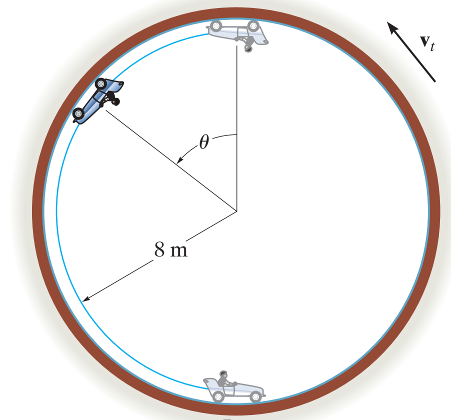
$$0 + \int_0^s 500(s + 0.08)\,ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789\ \text{m} = 179\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$s = 179$ mm

14–31.

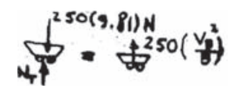
The “flying car” is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car’s brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3$ m/s. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(250)(3)^2 + 250(9.81)(16) = \frac{1}{2}(250)(v_B)^2$$



$v_B = 17.97 = 18.0$ m/s **Ans.**

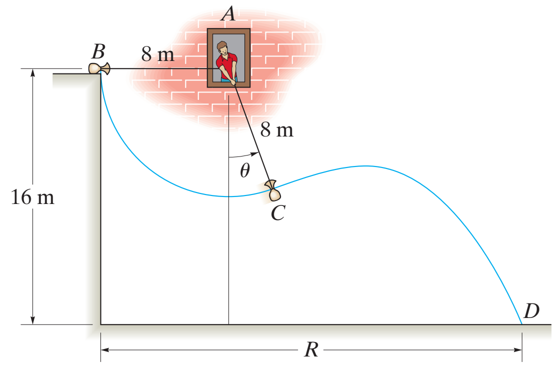
$+\uparrow \Sigma F_n = ma_n \qquad N_B - 250(9.81) = 250\left(\frac{(17.97)^2}{8}\right)$

$N_B = 12.5$ kN **Ans.**

Ans:
$v_B = 18.0$ m/s
$N_B = 12.5$ kN

***14–32.**

The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



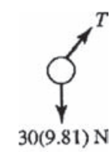
SOLUTION

$T_B + \Sigma U_{B-C} = T_C$

$$0 + 30(9.81)8 \cos 30° = \frac{1}{2}(30)v_C^2$$

$v_C = 11.659 \text{ m/s}$

$T_B + \Sigma U_{B-D} = T_D$



$$0 + 30(9.81)(16) = \frac{1}{2}(30)\,v_D^2$$

$v_D = 17.7 \text{ m/s}$ **Ans.**

During free flight:

$$(+\downarrow)\; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8 \cos 30° - 11.659 \sin 30° t + \frac{1}{2}(9.81)t^2$$

$t^2 - 1.18848\,t - 1.8495 = 0$

Solving for the positive root:

$t = 2.0784 \text{ s}$

$(\overset{+}{\rightarrow})\; s = s_0 + v_0\, t$

$s = 8 \sin 30° + 11.659 \cos 30°(2.0784)$

$s = 24.985 \text{ m}$

Thus,

$R = 8 + 24.985 = 33.0 \text{ m}$ **Ans.**

Also,

$(v_D)_x = 11.659 \cos 30° = 10.097 \text{ m/s}$

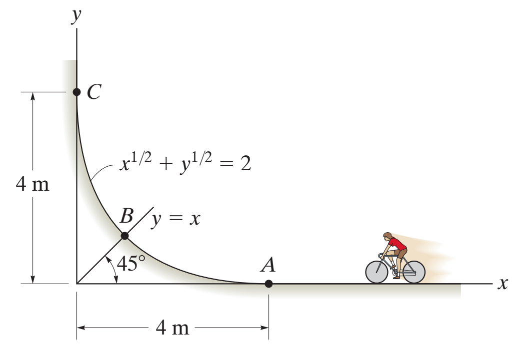
$(+\downarrow)\;(v_D)_x = -11.659 \sin 30° + 9.81(2.0784) = 14.559 \text{ m/s}$

$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 17.7 \text{ m/s}$ **Ans.**

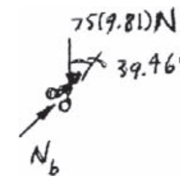
Ans:
$v_D = 17.7 \text{ m/s}$
$R = 33.0 \text{ m}$
$v_D = 17.7 \text{ m/s}$

14–33.

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4\text{ m/s}$. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



SOLUTION



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$$y = 0.81549\text{ m} = 0.815\text{ m}$$ **Ans.**

$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033\text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46^\circ$$

$\nearrow + \Sigma F_n = m\,a_n;\qquad N_b - 9.81(75)\cos 39.46^\circ = 0$

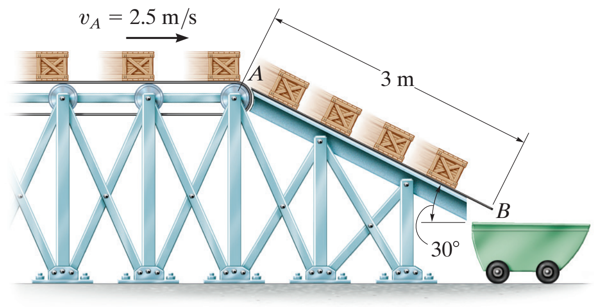
$$N_b = 568\text{ N}$$ **Ans.**

$+\searrow \Sigma F_t = m\,a_t;\qquad 75(9.81)\sin 39.46^\circ = 75\,a_t$

$$a = a_t = 6.23\text{ m/s}^2$$ **Ans.**

Ans:
$y = 0.815\text{ m}$
$N_b = 568\text{ N}$
$a = a_t = 6.23\text{ m/s}^2$

14–34. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's velocity is $v_A = 2.5\text{ m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.



SOLUTION

Given:

$M = 12\text{ kg}$

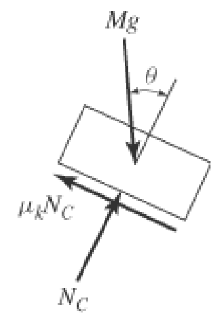
$v_A = 2.5\text{ m/s}$

$\mu_k = 0.3$

$g = 9.81\text{ m/s}^2$

$\theta = 30^\circ$

$a = 3\text{ m}$



$$N_c = Mg\cos(\theta)$$

$$\frac{1}{2}Mv_A^{\,2} + (Mga)\sin(\theta) - \mu_k N_c a = \frac{1}{2}Mv_B^{\,2}$$

$$v_B = \sqrt{v_A^{\,2} + (2ga)\sin(\theta) - (2\mu_k g)\cos(\theta)a}$$

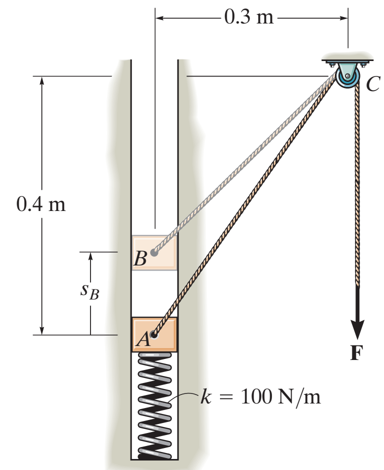
$v_B = 4.52\text{ m/s}$ **Ans.**

Ans:

$v_B = 4.52\text{ m/s}$

14–35.

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

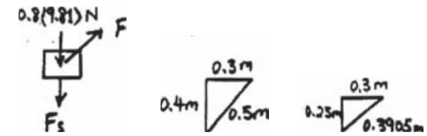
$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

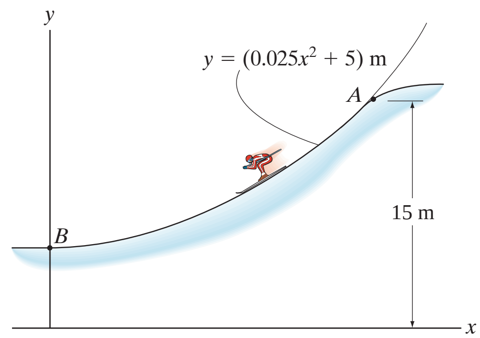
$$F = 43.9 \text{ N}$$ **Ans.**



Ans:
$F = 43.9$ N

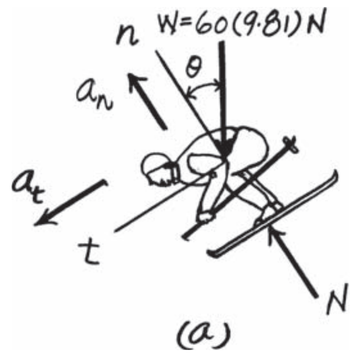
***14–36.**

If the 60-kg skier passes point A with a speed of 5 m/s, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10$ m and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87 \text{ m/s} = 14.9 \text{ m/s} \qquad \textbf{Ans.}$$

$$dy/dx = 0.05x$$

$$d^2y/dx^2 = 0.05$$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20 \text{ m}$$

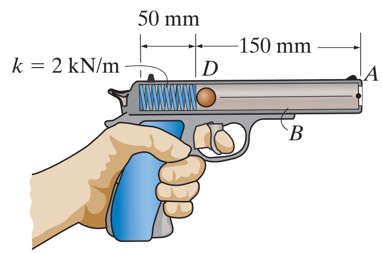
$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$v_B = 14.9$ m/s
$N = 1.25$ kN

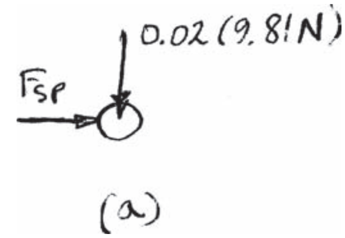
14–37.

The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



SOLUTION

Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}ks_1^{\,2} - \frac{1}{2}ks_2^{\,2}\right] = \frac{1}{2}mv_A^{\,2}$$

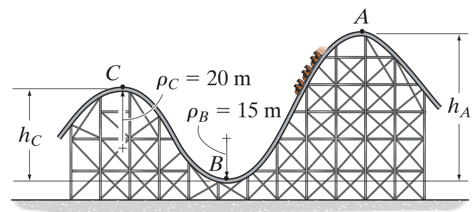
$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A^{\,2}$$

$$v_A = 10.5 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 10.5$ m/s

14–38.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



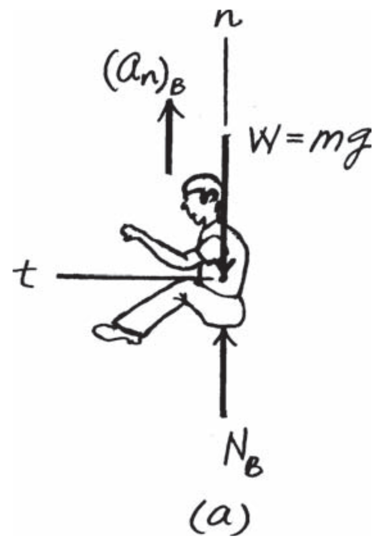
SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

$$v_B{}^2 = 45g$$



At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

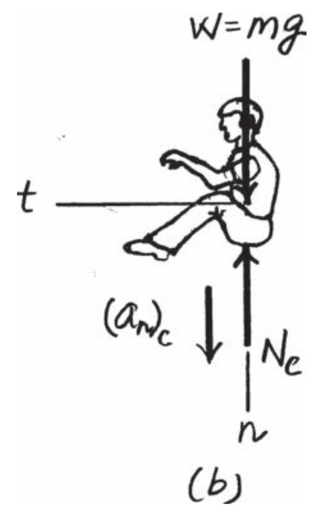
Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

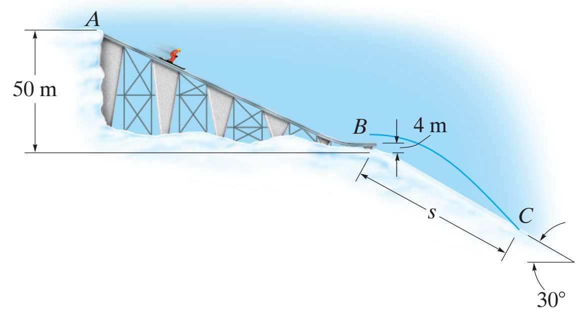
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h_A = 22.5$ m
$h_C = 12.5$ m

14–39. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass 75 kg.



SOLUTION

Guesses $\quad v_B = 1\ \dfrac{m}{s} \qquad t = 1\ s \qquad d = 1\ m$

Given $\quad M g(h_1 - h_2) = \dfrac{1}{2} M v_B^2 \qquad v_B t = d\cos(\theta) \qquad -h_2 - d\sin(\theta) = \dfrac{-1}{2} g t^2$

Given:

$M = 75\ kg$

$h_1 = 50\ m$

$h_2 = 4\ m$

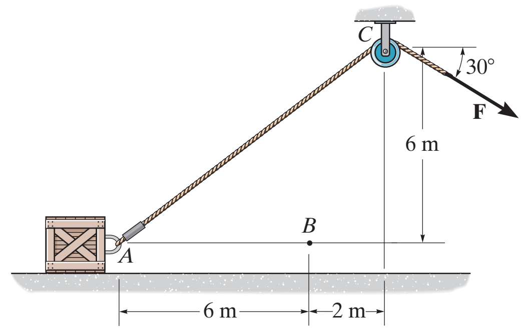
$\theta = 30°$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \qquad t = 3.754\ s \qquad v_B = 30.0\ m/s \qquad d = 130.2\ m \qquad \textbf{Ans.}$$

Ans:
$v_B = 30.0\ m/s$
$d = 130.2\ m$

***14–40.**

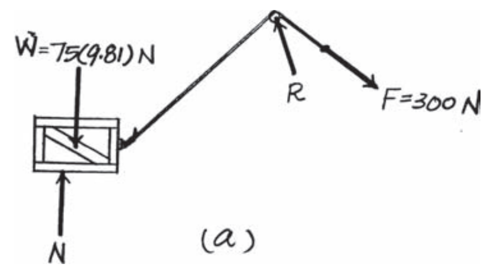
If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.



$$T_1 + \Sigma U_{1-2} = T_2$$

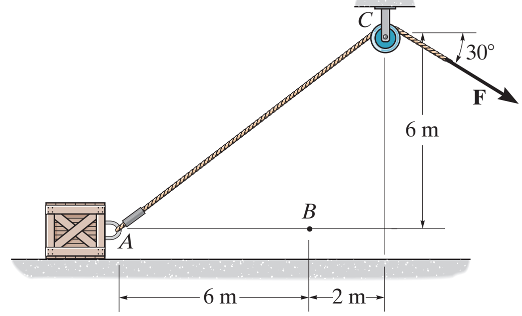
$$0 + 300(3.675) = \frac{1}{2}(75)v_B^2$$

$$v_B = 5.42 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 5.42$ m/s

14–41.

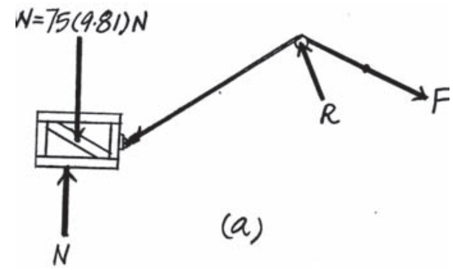
If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367 \text{ N}$$ **Ans.**

Ans:
$F = 367$ N

14–42. A spring having a stiffness of 5 kN/m is compressed 400 mm. The stored energy in the spring is used to drive a machine which requires 90 W of power. Determine how long the spring can supply energy at the required rate.

SOLUTION

Given: $k = 5\ \text{kN/m}$ $\quad \delta = 400\ \text{mm}$ $\quad P = 90\ \text{W}$

$U_{12} = \frac{1}{2}k\delta^2 = P\,t$ $\quad t = \frac{1}{2}k\left(\frac{\delta^2}{P}\right)$ $\quad t = 4.44\ \text{s}$ **Ans.**

Ans:
$t = 4.44\ \text{s}$

14–43.

To dramatize the loss of energy in an automobile, consider a car having a weight of 25 000 N that is traveling at 56 km/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy.

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{56\text{ km}}{\text{h}}\right) \times \left(\frac{1000\text{ m}}{1\text{ km}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 15.56 m/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{25\,000}{9.81}\right)\left(15.56^2\right) = 308\,504\text{ J}$$

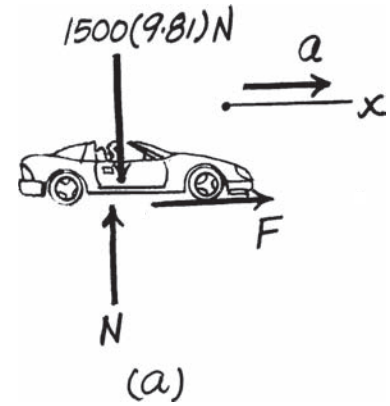
The power of the bulb is $P_{bulb} = 100$ W.

Thus,

$$t = \frac{U}{P_{bulb}} = \frac{308\,504}{100} = 3085.04\text{ s} = 51.4\text{ min} \qquad \textbf{Ans.}$$

Ans:
$t = 51.4$ min

***14–44.** If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.



SOLUTION

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{out} = \mathbf{F} \cdot \mathbf{v}$$

$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

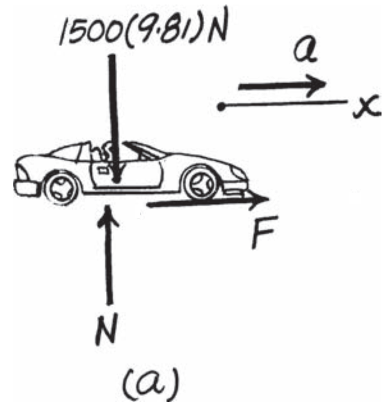
$$\int_0^{200\text{ m}} 10ds = \int_0^v v^2\,dv$$

$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^v$$

$$v = 18.7\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 18.7\text{ m/s}$

14–45. If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.



SOLUTION

Equations of Motion: Here, $a = v\frac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{out} = \mathbf{F} \cdot \mathbf{v}$$

$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

$$\int_0^{200\text{ m}} 10ds = \int_0^v v^2\, dv$$

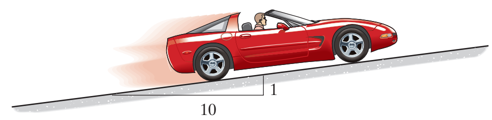
$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^v$$

$$v = 18.7 \text{ m/s}$$ **Ans.**

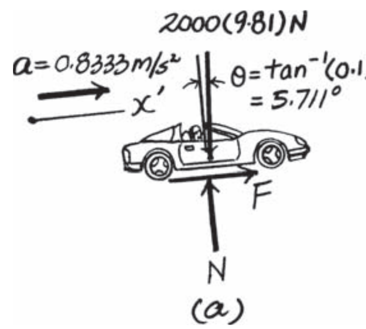
Ans:
$v = 18.7$ m/s

14–46.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\varepsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$$25 = 0 + a_c (30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$\Sigma F_{x'} = ma_{x'};\quad F - 2000(9.81) \sin 5.711° = 2000(0.8333)$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

Ans:
$P_{\max} = 113 \text{ kW}$
$P_{\text{avg}} = 56.5 \text{ kW}$

14–47. A car has a mass m and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

SOLUTION

Power: Since the power output is constant, then the traction force F varies with v. Applying Eq. 14-10, we have

$$P = Fv \qquad F = \frac{P}{v}$$

Equation of Motion: $\frac{P}{v} = ma \qquad a = \frac{P}{Mv}$

Kinematics: Applying equation $ds = \frac{v dv}{a}$, we have

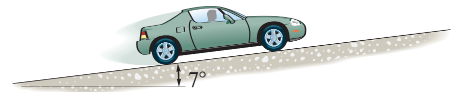
$$\int_0^s 1\, ds = \int_0^v \frac{mv^2}{P}\, dv \qquad s = \frac{mv^3}{3P} \qquad \textbf{Ans.}$$

Ans:

$$s = \frac{mv^3}{3P}$$

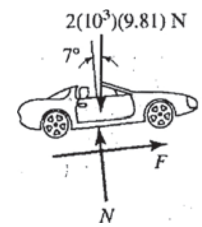
***14–48.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100\text{ km/h}$. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\varepsilon = 0.65$.



SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08\text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78\text{ m/s}$. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

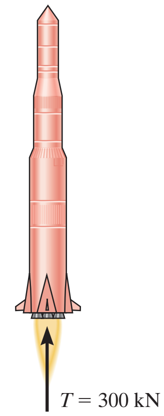
Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

Ans:
power input $= 102\text{ kW}$

14–49. A rocket having a total mass of 8 Mg is fired vertically from rest. If the engines provide a constant thrust of $T = 300$ kN, determine the power output of the engines as a function of time. Neglect the effect of drag resistance and the loss of fuel mass and weight.



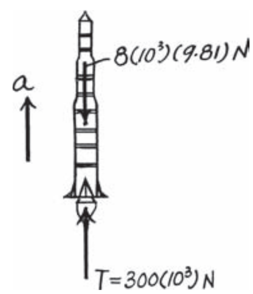
SOLUTION

$+\uparrow \Sigma F_y = ma_y;\qquad 300(10^3) - 8(10^3)(9.81) = 8(10^3)a \qquad a = 27.69\text{ m/s}^2$

$(+\uparrow)\qquad v = v_0 + a_c t$

$= 0 + 27.69t = 27.69t$

$P = \mathbf{T}\cdot\mathbf{v} = 300\,(10^3)\,(27.69t) = 8.31t\ \text{MW}$ **Ans.**

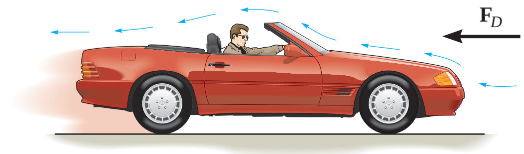


Ans:

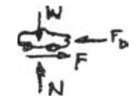
$P = 8.31t$ MW

14–50.

The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\varepsilon = 0.68$.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x;\qquad F - 0.3v^2 = 2.3(10^3)(5)$

$$F = 0.3v^2 + 11.5(10^3)$$

At $v = 28$ m/s

$$F = 11\,735.2 \text{ N}$$

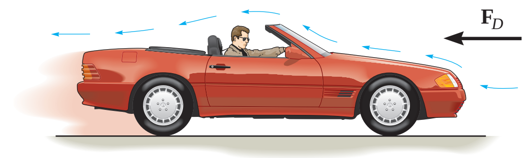
$$P_O = (11\,735.2)(28) = 328.59 \text{ kW}$$

$$P_i = \frac{P_O}{\varepsilon} = \frac{328.59}{0.68} = 438 \text{ kW}$$ **Ans.**

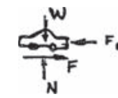
Ans:
$P_i = 483$ kW

14–51.

The sports car has a mass of 2.3 Mg and accelerates at 6 m/s^2, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\varepsilon = 0.68$.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad F - 10v = 2.3(10^3)(6)$

$F = 13.8(10^3) + 10\,v$

$(\overset{+}{\rightarrow})\ v = v_0 + a_c\,t$

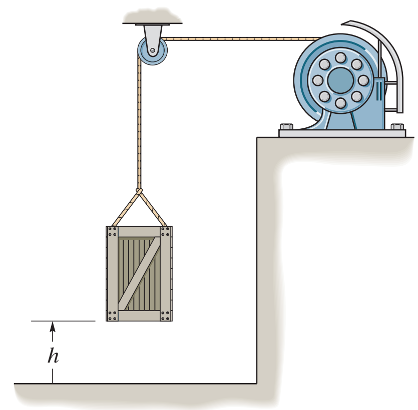
$v = 0 + 6(5) = 30 \text{ m/s}$

$P_O = \mathbf{F}\cdot\mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0 \text{ kW}$

$P_i = \dfrac{P_O}{\varepsilon} = \dfrac{423.0}{0.68} = 622 \text{ kW}$ **Ans.**

Ans:
$P_i = 622 \text{ kW}$

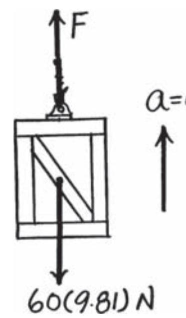
***14–52.** A motor hoists a 60-kg crate at a constant velocity to a height of $h = 5$ m in 2 s. If the indicated power of the motor is 3.2 kW, determine the motor's efficiency.



SOLUTION

Equations of Motion:

$$+\uparrow \Sigma F_y = ma_y; \qquad F - 60(9.81) = 60(0) \qquad F = 588.6 \text{ N}$$



Power: The crate travels at a constant speed of $v = \dfrac{5}{2} = 2.50$ m/s. The power output can be obtained using Eq. 14–10.

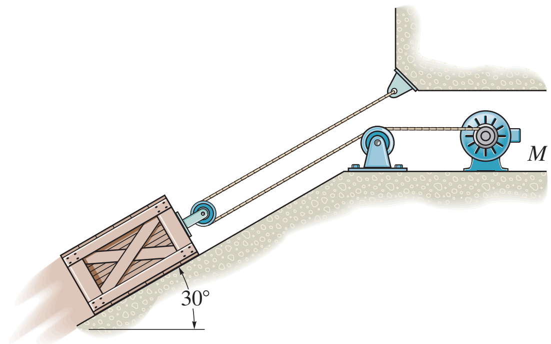
$$P = \mathbf{F} \cdot \mathbf{v} = 588.6\,(2.50) = 1471.5 \text{ W}$$

Thus, from Eq. 14–11, the efficiency of the motor is given by

$$\varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{1471.5}{3200} = 0.460$$ **Ans.**

Ans:
$\varepsilon = 0.460$

14–53. The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant. Neglect friction along the plane. The motor has an efficiency of $\varepsilon = 0.74$.

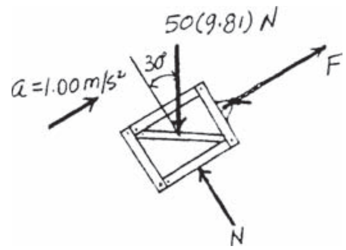


SOLUTION

Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00 \text{ m/s}^2$$

Equations of Motion:



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30° = 50(1.00) \qquad F = 295.25 \text{ N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 295.25\,(4) = 1181 \text{ W} = 1.181 \text{ kW}$$

Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

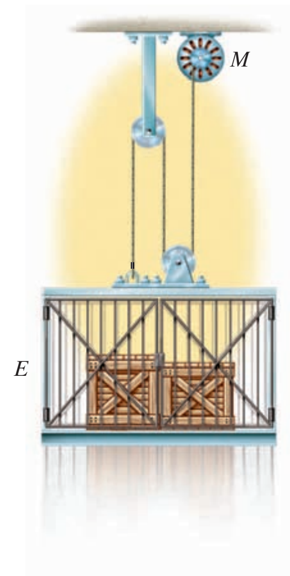
$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60 \text{ kW}$$ **Ans.**

Ans:
$P_{in} = 1.60$ kW

14–54.

The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\ \text{m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



SOLUTION

$+\uparrow \Sigma F_y = m\, a_y; \qquad 3T - 500(9.81) = 500(2)$

$T = 1968.33\ \text{N}$

$3s_E - s_P = l$

$3\, v_E = v_P$

When $t = 3$ s,

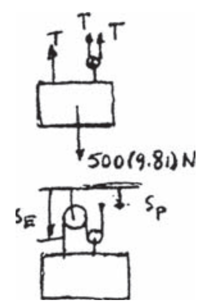
$(+\uparrow)\ v_0 + a_c\, t$

$v_E = 0 + 2(3) = 6\ \text{m/s}$

$v_P = 3(6) = 18\ \text{m/s}$

$P_O = 1968.33(18)$

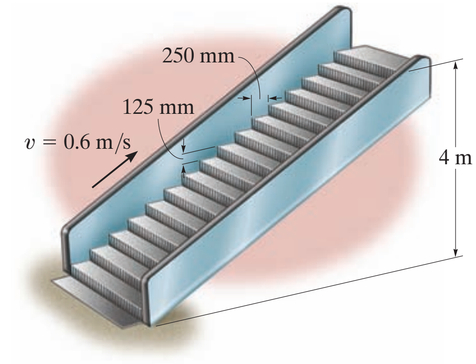
$P_O = 35.4\ \text{kW}$ **Ans.**



Ans:
$P_O = 35.4\ \text{kW}$

14–55.

The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



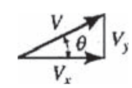
SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then



$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

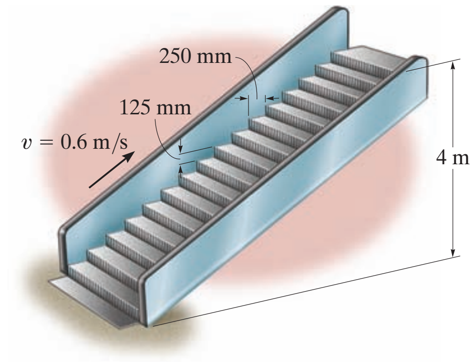
Also,

$$P = \mathbf{F} \cdot \mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

Ans:
$P = 12.6$ kW

***14–56.**

If the escalator in Prob. 14–55 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



SOLUTION

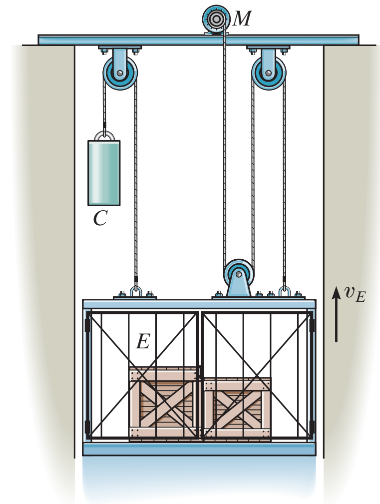
$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4 \text{ s}$$

$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285 \text{ m/s}$$ **Ans.**

Ans:
$v = 0.285$ m/s

14–57.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\varepsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$

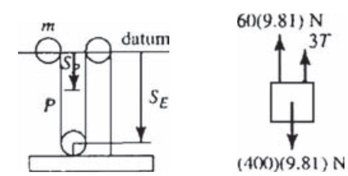
$T = 1111.8$ N

$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

Since $v_E = -4$ m/s, $\qquad v_P = -12$ m/s

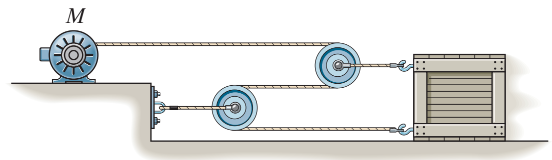
$$P_i = \frac{\mathbf{F}\cdot\mathbf{v}_P}{\varepsilon} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans.**



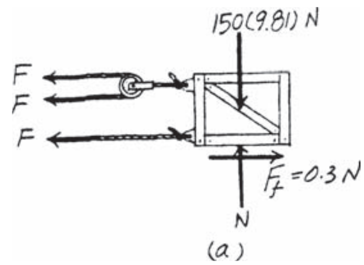
Ans:
$P_i = 22.2$ kW

14–58.

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



SOLUTION

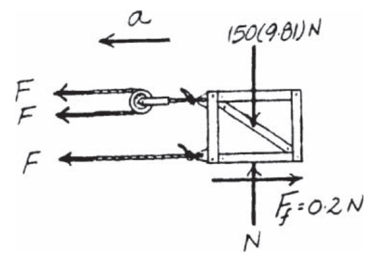


Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867 \text{ s}$$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150(0) \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2(1471.5) - 3\left(8t^2 + 20\right) = 150(-a)$$

$$a = \left(0.160t^2 - 1.562\right) \text{ m/s}^2$$

Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867 \text{ s}}^5 \left(0.160t^2 - 1.562\right) dt$$

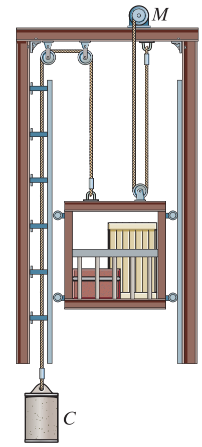
$$v = 1.7045 \text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 3(220)(1.7045) = 1124.97 \text{ W} = 1.12 \text{ kW}$$ **Ans.**

Ans:
$P = 1.12$ kW

14–59. The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. At a given instant, the hoist has an upward velocity of 2 m/s and an acceleration of 1.5 m/s^2. Determine the power generated by the motor M at this instant if it operates with an efficiency of $\varepsilon = 0.8$.



SOLUTION

Equations of Motion: Here, $a = 1.5$ m/s^2. By referring to the free-body diagram of the hoist and counterweight shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(1.5) \qquad \textbf{(1)}$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(1.5)$$

Solving,

$$T' = 1246.5 \text{ N}$$

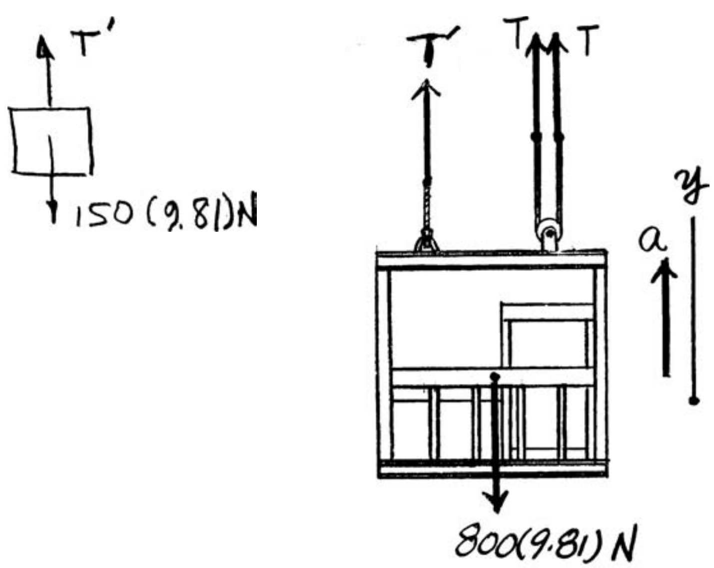
$$T = 3900.75 \text{ N}$$

Power:

$$P_{\text{out}} = 2\mathbf{T} \cdot \mathbf{v} = 2(3900.75)(2) = 15\,603 \text{ W}$$

Thus,

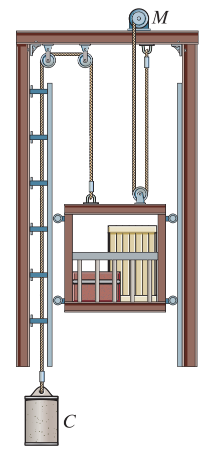
$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{15603}{0.8} = 19.5(10^3) \text{ W} = 19.5 \text{ kW} \qquad \textbf{Ans.}$$



(a)

Ans:
$P_{\text{in}} = 19.5$ kW

***14–60.** The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s, determine the average power generated by the motor M during this time. The motor operates with an efficiency of $\varepsilon = 0.8$.



SOLUTION

Kinematics: The acceleration of the hoist can be determined from

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667\ \text{m/s}^2$$

Equations of Motion: Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(0.6667)$$

Solving,

$$T' = 1371.5\ \text{N}$$

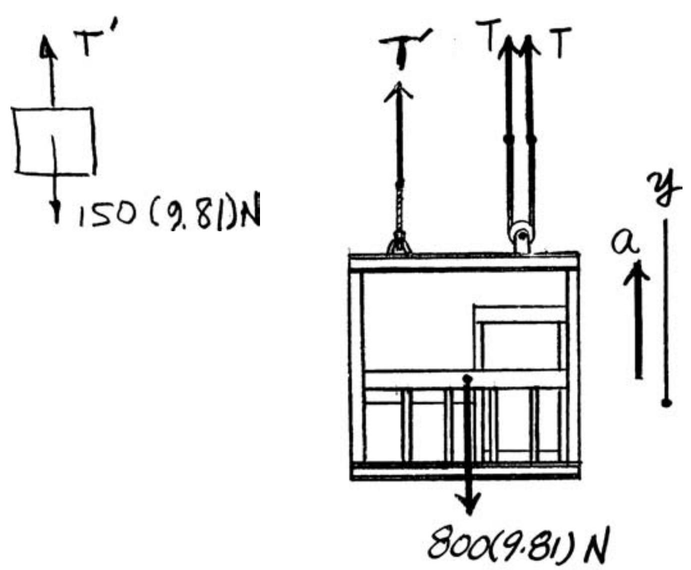
$$T = 3504.92\ \text{N}$$

Power:

$$(P_{\text{out}})_{\text{avg}} = 2\mathbf{T}\cdot\mathbf{v}_{\text{avg}} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8\ \text{W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3\ \text{W} = 8.76\ \text{kW} \qquad \textbf{Ans.}$$

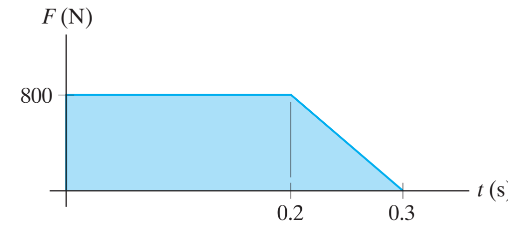


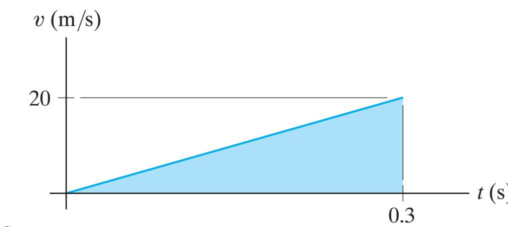
(a)

Ans:
$P_{\text{in}} = 8.76$ kW

14–61.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





SOLUTION

For $0 \leq t \leq 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67t$$

$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans.**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans.**

$$U = \int_0^{0.3} P\, dt$$

$$U = \int_0^{0.2} 53.3t\, dt + \int_{0.2}^{0.3} (160t - 533t^2)\, dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans.**

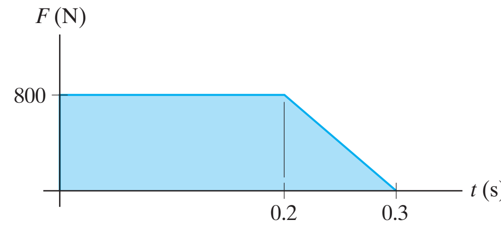
Ans:

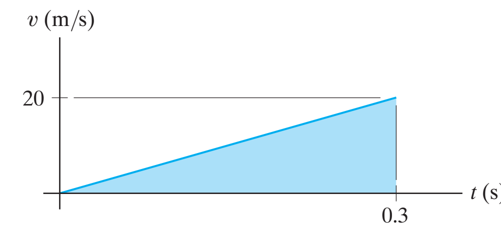
$P = \left\{160\, t - 533t^2\right\}$ kW

$U = 1.69$ kJ

14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





SOLUTION

See solution to Prob. 14–62.

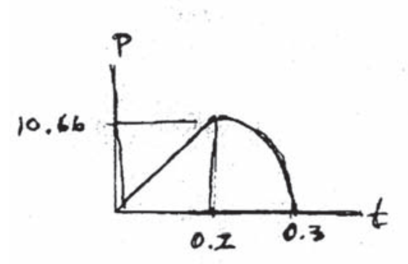
$$P = 160\,t - 533\,t^2$$

$$\frac{dP}{dt} = 160 - 1066.6\,t = 0$$

$$t = 0.15 \text{ s} < 0.2 \text{ s}$$

Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7$ kW **Ans.**

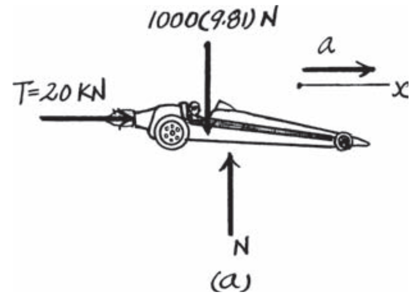


Ans:
$P_{max} = 10.7$ kW

14–63.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.

SOLUTION



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$

Kinematics: The velocity of the dragster can be determined from

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

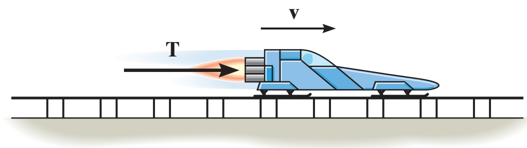
$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

$$= \left[400(10^3)t\right] \text{ W} \qquad \textbf{Ans.}$$

Ans:
$P = \left\{400(10^3)t\right\}$ W

***14–64.**

The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Neglect the loss of fuel mass and air resistance.



SOLUTION

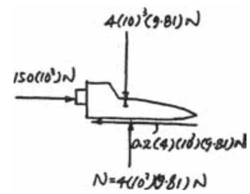
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3\, a$

$a = 35.54 \text{ m/s}^2$

$(\xrightarrow{+})\ v = v_0 + a_c t$

$= 0 + 35.54t = 35.54t$

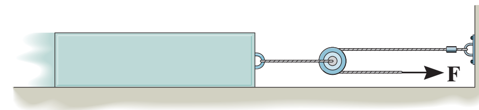
$P = \mathbf{T} \cdot \mathbf{v} = 150(10)^3\,(35.54t) = 5.33t \text{ MW}$ **Ans.**



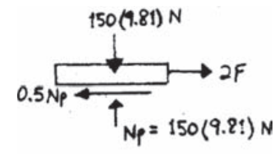
Ans:
$P = 5.33t$ MW

14–65.

The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F - 0.5(150)(9.81) = 0$

$F = 367.875 = 60t^2$

$t = 2.476 \text{ s}$

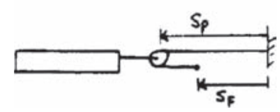
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$a_p = 0.8t^2 - 3.924$

$dv = a\,dt$

$$\int_0^v dv = \int_{2.476}^{5} \left(0.8t^2 - 3.924\right) dt$$

$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t \Big|_{2.476}^{5} = 19.38 \text{ m/s}$$



$s_P + (s_P - s_F) = l$

$2v_P = v_F$

$v_F = 2(19.38) = 38.76 \text{ m/s}$

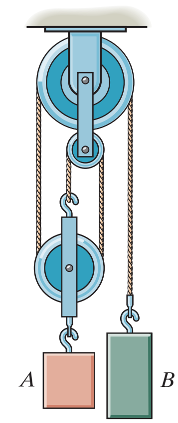
$F = 60(5)^2 = 1500 \text{ N}$

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1 \text{ kW}$ **Ans.**

Ans:
$P = 58.1 \text{ kW}$

14–66.

The assembly consists of two blocks A and B, which have a mass of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



SOLUTION

$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

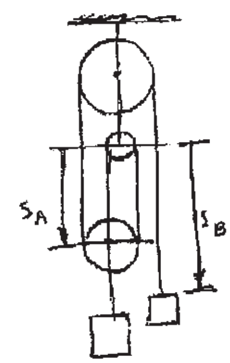
$3v_A = -v_B$

$v_B = -9 \text{ m/s}$

$T_1 + V_1 = T_2 + V_2$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$

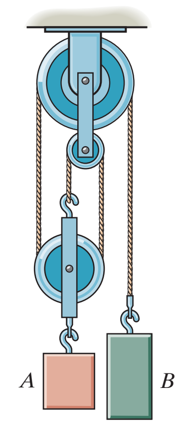
$s_B = 5.70 \text{ m}$ **Ans.**



Ans:
$s_B = 5.70 \text{ m}$

14–67.

The assembly consists of two blocks A and B which have a mass of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



SOLUTION

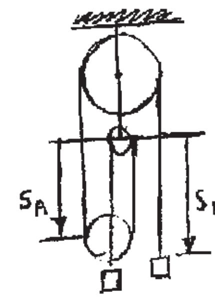
$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

$3v_A = -v_B$

$T_1 + V_1 = T_2 + V_2$

$$(0+0)+(0+0)=\frac{1}{2}(20)(v_A)^2+\frac{1}{2}(30)(-3v_A)^2+20(9.81)\left(\frac{1.5}{3}\right)-30(9.81)(1.5)$$



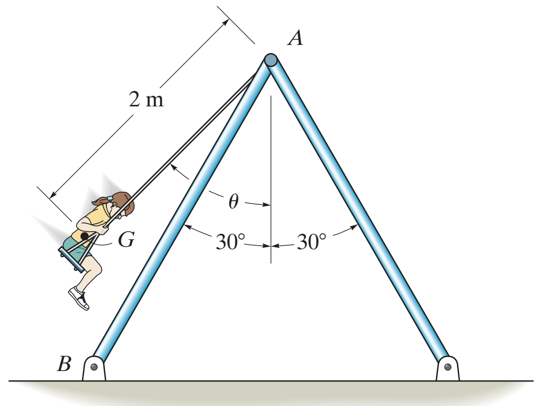
$v_A = 1.54\text{ m/s}$ **Ans.**

$v_B = 4.62\text{ m/s}$ **Ans.**

Ans:
$v_A = 1.54\text{ m/s}$
$v_B = 4.62\text{ m/s}$

***14–68.**

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



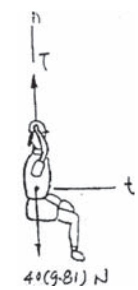
SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

$$T_1 + V_1 = T_2 + V_2$$

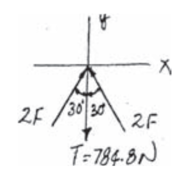
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$



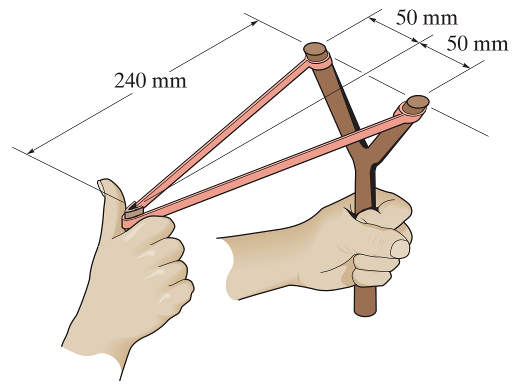
$$+\uparrow \Sigma F_n = ma_n; \qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \qquad T = 784.8 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(2F)\cos 30° - 784.8 = 0 \qquad F = 227 \text{ N}$$ **Ans.**



Ans:
$F = 227$ N

14–69. Each of the two elastic rubber bands of the slingshot has an unstretched length of 180 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 30-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 80$ N/m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

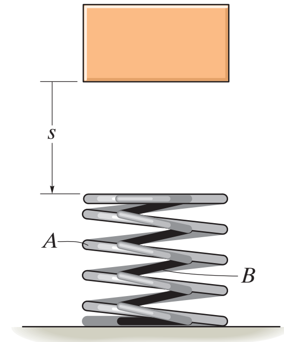
$$0 + 2\left(\frac{1}{2}\right)(80)[\sqrt{(0.05)^2 + (0.240)^2} - 0.18]^2 = 0 + 0.030(9.81)h$$

$$h = 1.154 \text{ m} = 1154 \text{ mm}$$ **Ans.**

Ans:
$h = 1154$ mm

14–70.

Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

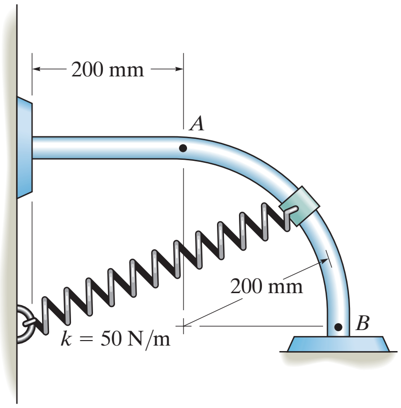
$$0 + 0 = 0 - 2(9.81)(0.5 + 0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$$

$$k_B = 287 \text{ N/m}$$ **Ans.**

Ans:
$k_B = 287$ N/m

14–71.

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels down along the smooth guide. Determine the speed of the collar when it reaches point B, which is located just before the end of the curved portion of the rod. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

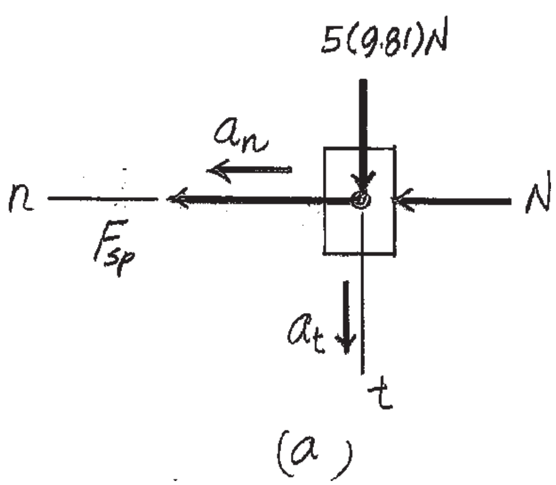
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s}$$ **Ans.**

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95 \text{ N} = 694 \text{ N}$$ **Ans.**

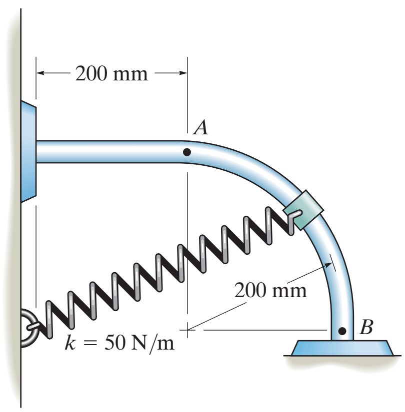


(a)

Ans:
$v_B = 5.33$ m/s
$N = 694$ N

***14–72.**

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels along the smooth guide. Determine its speed when its center reaches point B and the normal force it exerts on the rod at this point. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

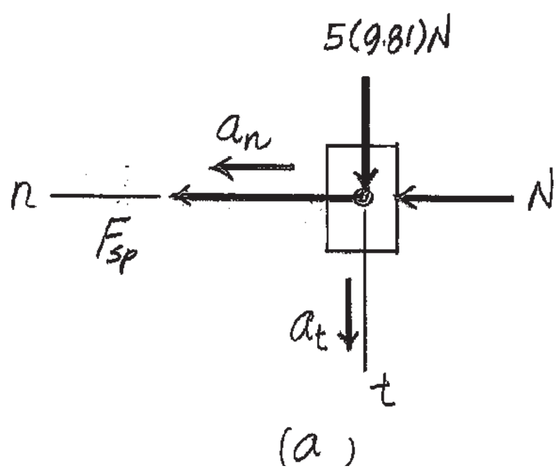
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. *a*,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

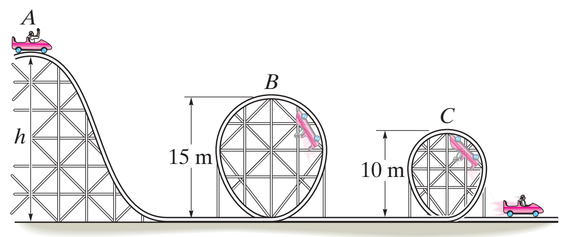
$$N = 693.95 \text{ N} = 694 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$v_B = 5.33 \text{ m/s}$
$N = 694 \text{ N}$

14–73.

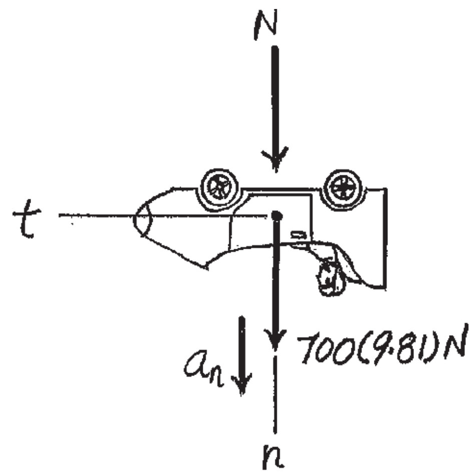
The roller coaster car has a mass of 700 kg, including its passenger. If it starts from the top of the hill A with a speed $v_A = 3\text{ m/s}$, determine the minimum height h of the hill crest so that the car travels around the inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5\text{ m}$ and $\rho_C = 5\text{ m}$.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$



(a)

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5\ m$ and $\rho_C = 5\ m$. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575\text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05\text{ m}^2/\text{s}^2$$

Judging from the above results, the coster car will not leave the loop at C if it safely passes through B. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. With $v_B^2 = 73.575\text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(700)(3^2) + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.29\text{ m} = 18.3\text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_c^2 + 700(9.81)(10)$$

$$v_c^2 = 171.675\text{ m}^2/\text{s}^2 > 49.05\text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5\text{ m}$,

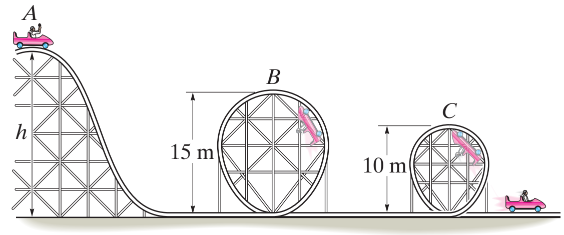
$$N_c + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)\text{ N} = 17.2\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 0$
$h = 18.3\text{ m}$
$N_c = 17.2\text{ kN}$

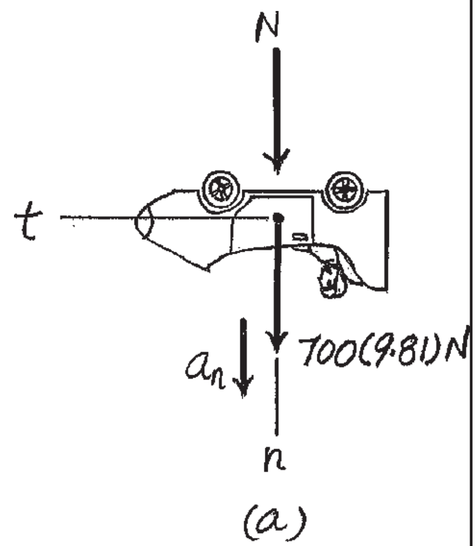
14–74.

The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5$ m and $\rho_C = 5$ m. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05 \text{ m}^2/\text{s}^2$$

Judging from the above result the coaster car will not leave the loop at C provided it passes through B safely. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. Applying Eq. 14– from A to B with $v_B^2 = 73.575 \text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$0 + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.75 \text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_C^2 + 700(9.81)(10)$$

$$v_C^2 = 171.675 \text{ m}^2/\text{s}^2 > 49.05 \text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5$ m,

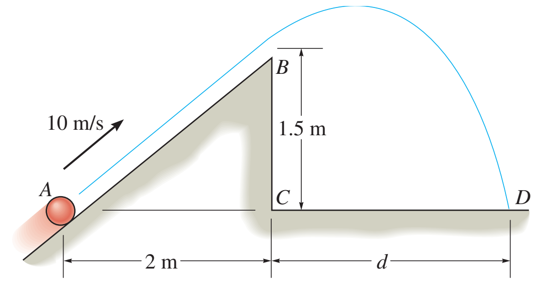
$$N_C + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)N = 17.2 \text{ kN} \qquad \textbf{Ans.}$$

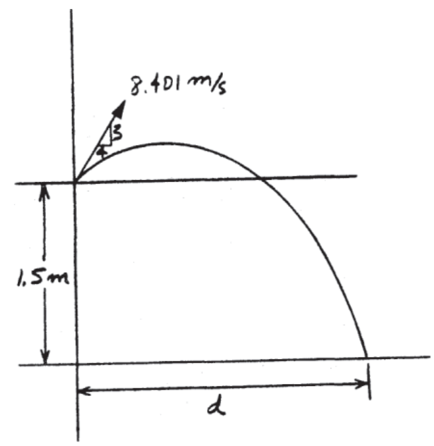
Ans:
$N_B = 0$
$h = 18.75$ m
$N_C = 17.2$ kN

14–75.

The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?



SOLUTION



Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$$

$$v_B = 8.401 \text{ m/s}$$

$$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$$

$$d = 0 + 8.401\left(\frac{4}{5}\right)t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$$

$$-4.905t^2 + 5.040t + 1.5 = 0$$

Solving for the positive root,

$$t = 1.269 \text{ s}$$

$$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53 \text{ m}$$ **Ans.**

Datum at A:

$$T_A + V_A = T_D + V_D$$

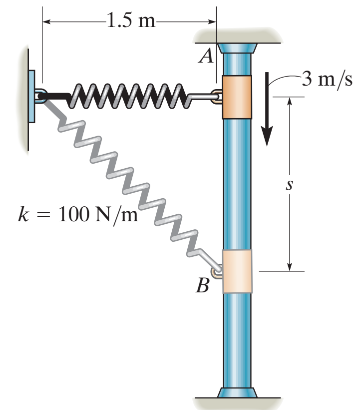
$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$$

$$v_D = 10 \text{ m/s}$$ **Ans.**

Ans:
$d = 8.53$ m
$v_D = 10$ m/s

***14–76.**

The 4-kg smooth collar has a speed of 3 m/s when it is at $s = 0$. Determine the maximum distance s it travels before it stops momentarily. The spring has an unstretched length of 1 m.



SOLUTION

Potential Energy. With reference to the datum set through A the gravitational potential energies of the collar at A and B are

$$(V_g)_A = 0 \qquad (V_g)_B = -mgh_B = -4(9.81)\,S_{max} = -39.24\,S_{max}$$

At A and B, the spring stretches $x_A = 1.5 - 1 = 0.5$ m and $x_B = \sqrt{S_{max}^2 + 1.5^2} - 1$.
Thus, the elastic potential Energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(100)\left(0.5^2\right) = 12.5\text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(100)\left(\sqrt{S_{max}^2 + 1.5^2} - 1\right)^2 = 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

Conservation of Energy. Since the collar is required to stop momentarily at B, $T_B = 0$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(4)\left(3^2\right) + 0 + 12.5 = 0 + (-39.24\,S_{max}) + 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

$$50\,S_{max}^2 - 100\sqrt{S_{max}^2 + 1.5^2} - 39.24\,S_{max} + 132 = 0$$

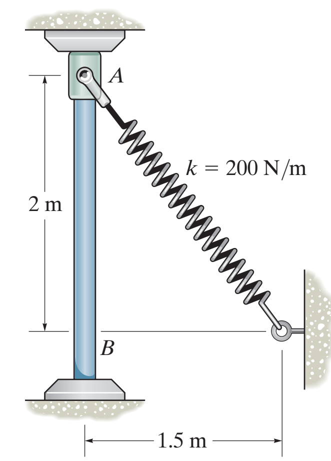
Solving numerically,

$$S_{max} = 1.9554\text{ m} = 1.96\text{ m}$$ **Ans.**

Ans:
$S_{max} = 1.96$ m

14–77.

The spring has a stiffness $k = 200$ N/m and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



SOLUTION

Potential Energy. With reference to the datum set through B, the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 3(9.81)(2) = 58.86 \text{ J}$$
$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{1.5^2 + 2^2} - 0.5 = 2.00$ m and $x_B = 1.5 - 0.5 = 1.00$ m. Thus, the elastic potential energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(200)\left(2.00^2\right) = 400 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(200)\left(1.00^2\right) = 100 \text{ J}$$

Conservation of Energy. Since the collar is released from rest at A, $T_A = 0$.

$$T_A + V_A = T_B + V_B$$

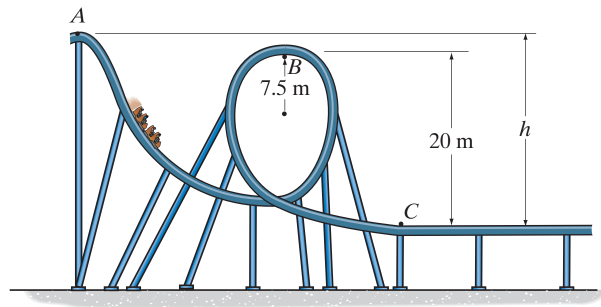
$$0 + 58.86 + 400 = \frac{1}{2}(3)v_B^2 + 0 + 100$$

$$v_B = 15.47 \text{ m/s} = 15.5 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 15.5$ m/s

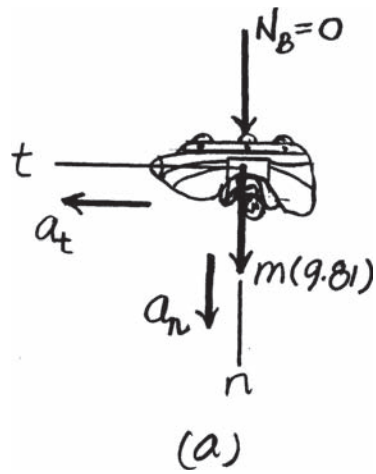
14–78.

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \frac{v_B^2}{\rho_B} = \frac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $(V_g)_A = mgh_A = m(9.81)h = 9.81mh$, $(V_g)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $(V_g)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

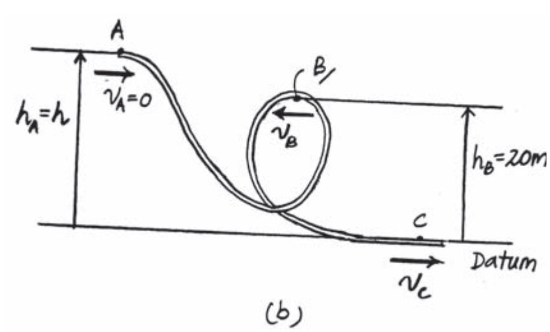
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

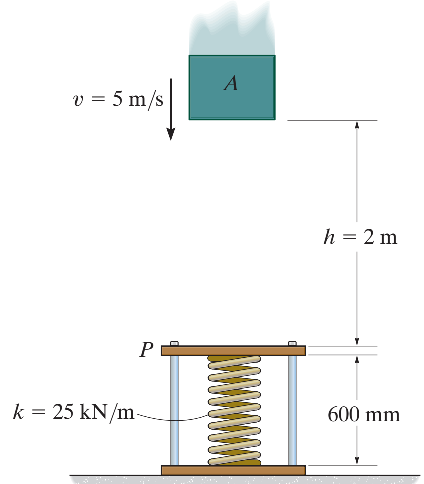
$$v_C = 21.6 \text{ m/s}$$ **Ans.**



Ans:
$h = 23.75$ m
$v_C = 21.6$ m/s

14–79.

A 750-mm-long spring is compressed and confined by the plate P, which can slide freely along the vertical 600-mm-long rods. The 40-kg block is given a speed of $v = 5$ m/s when it is $h = 2$ m above the plate. Determine how far the plate moves downwards when the block momentarily stops after striking it. Neglect the mass of the plate.

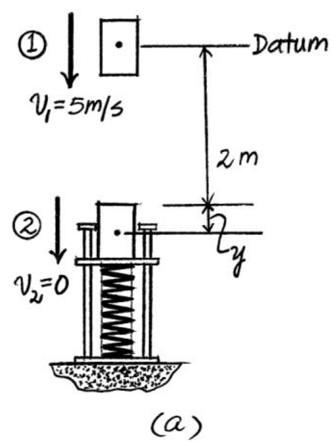


SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 40(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 40(9.81)[-(2 + y)] = [-392.4(2 + y)]$, respectively. The compression of the spring when the block is at positions (1) and (2) are $s_1 = (0.75 - 0.6) = 0.15$ m and $s_2 = s_1 + y = (0.15 + y)$ m. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25)(10^3)(0.15^2) = 281.25 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25)(10^3)(0.15 + y)^2$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \left[(V_g)_1 + (V_e)_1\right] = \frac{1}{2}mv_2^2 + \left[(V_g)_2 + (V_e)_2\right]$$

$$\frac{1}{2}(40)(5^2) + (0 + 281.25) = 0 + [-392.4(2 + y)] + \frac{1}{2}(25)(10^3)(0.15 + y)^2$$

$$12500y^2 + 3357.6y - 1284.8 = 0$$

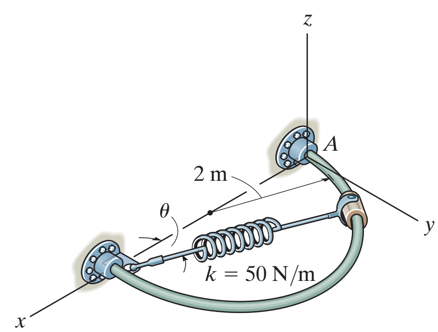
Solving for the positive root of the above equation,

$$y = 0.2133 \text{ m} = 213 \text{ mm}$$ **Ans.**

Ans:
$y = 213$ mm

***14–80.**

The spring has a stiffness $k = 50\ \text{N/m}$ and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0°$), determine the speed of the collar when $\theta = 60°$. The motion occurs in the horizontal plane. Neglect the size of the collar.

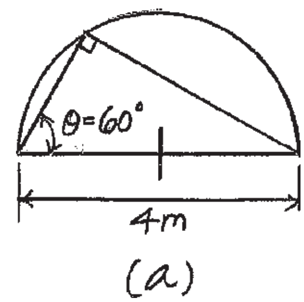


SOLUTION

Potential Energy. Since the motion occurs in the horizontal plane, there will be no change in gravitational potential energy when $\theta = 0°$, the spring stretches $x_1 = 4 - 0.3 = 3.7\ \text{m}$. Referring to the geometry shown in Fig. *a*, the spring stretches $x_2 = 4\cos 60° - 0.3 = 1.7\ \text{m}$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(50)(3.7^2) = 342.25\ \text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(50)(1.7^2) = 72.25\ \text{J}$$



Conservation of Energy. Since the collar is released from rest when $\theta = 0°$, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

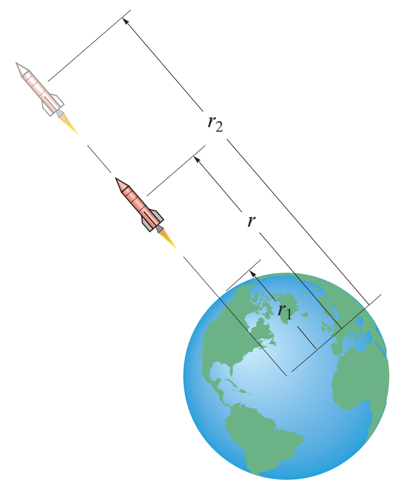
$$0 + 342.25 = \frac{1}{2}(2)v^2 + 72.25$$

$$v = 16.43\ \text{m/s} = 16.4\ \text{m/s}$$ **Ans.**

Ans:
$v = 16.4\ \text{m/s}$

14–81.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \rightarrow \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m \int_{r1}^{r2} \frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \rightarrow \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

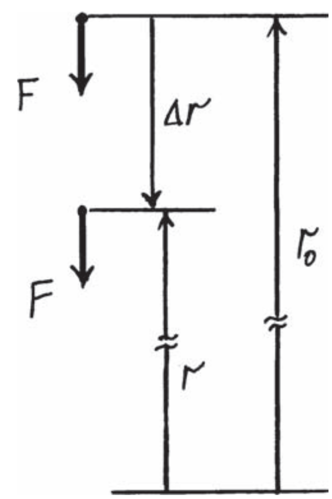
$$V_g \rightarrow \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2}$$

Q.E.D.

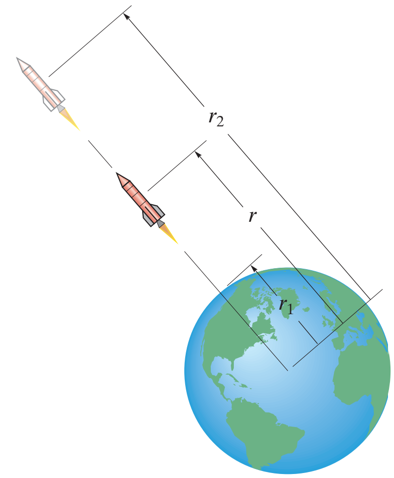


Ans:

$$F = \frac{-G\,M_e\,m}{r^2}$$

14–82.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

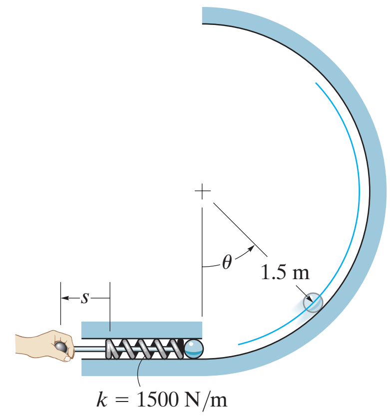
$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

Ans:

$$F = GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

14–83.

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Potential Energy. With reference to the datum set through the center of the circular track, the gravitational potential energies of the ball when $\theta = 0°$ and $\theta = 60°$ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = -mgh_2 = -0.3(9.81)(1.5 \cos 60°) = -2.20725 \text{ J}$$

When $\theta = 0°$, the spring compress $x_1 = 0.1$ m and is unstretched when $\theta = 60°$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)\left(0.1^2\right) = 7.50 \text{ J}$$

$$(V_e)^2 = 0$$

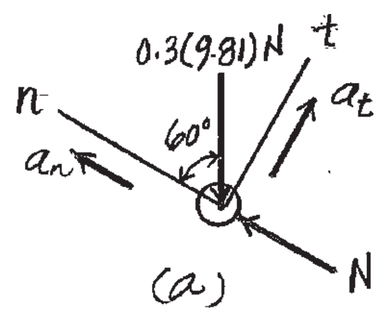
Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + (-2.20725) + 0$$

$$v^2 = 35.285 \text{ m}^2/\text{s}^2$$

$$v = 5.94 \text{ m/s}$$ **Ans.**



Equation of Motion. Referring to the FBD of the ball, Fig. *a*,

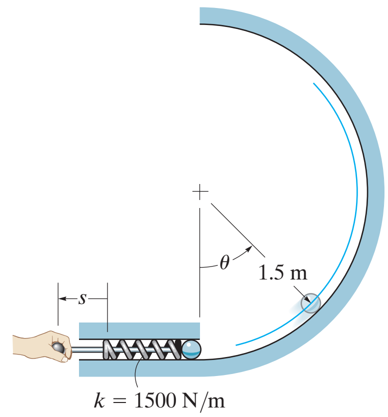
$$\Sigma F_n = ma_n; \qquad N - 0.3(9.81)\cos 60° = 0.3\left(\frac{35.285}{1.5}\right)$$

$$N = 8.5285 \text{ N} = 8.53 \text{ N}$$ **Ans.**

Ans:
$v = 5.94$ m/s
$N = 8.53$ N

***14–84.**

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Equation of Motion. It is required that the ball leaves the track, and this will occur provided $\theta > 90°$. When this happens, $N = 0$. Referring to the FBD of the ball, Fig. *a*

$$\Sigma F_n = ma_n; \qquad 0.3(9.81)\sin(\theta - 90°) = 0.3\left(\frac{v^2}{1.5}\right)$$

$$v^2 = 14.715 \sin(\theta - 90°) \qquad \textbf{(1)}$$

Potential Energy. With reference to the datum set through the center of the circular track Fig. *b*, the gravitational potential Energies of the ball when $\theta = 0°$ and θ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = mgh_2 = 0.3(9.81)[1.5 \sin(\theta - 90°)]$$

$$= 4.4145 \sin(\theta - 90°)$$

When $\theta = 0°$, the spring compresses $x_1 = 0.1$ m and is unstretched when the ball is at θ for max height. Thus, the elastic potential energies in the spring when $\theta = 0°$ and θ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)(0.1^2) = 7.50 \text{ J}$$

$$(V_e)_2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + 4.4145 \sin(\theta - 90°) + 0$$

$$v^2 = 20.57 - 29.43 \sin(\theta - 90°) \qquad \textbf{(2)}$$

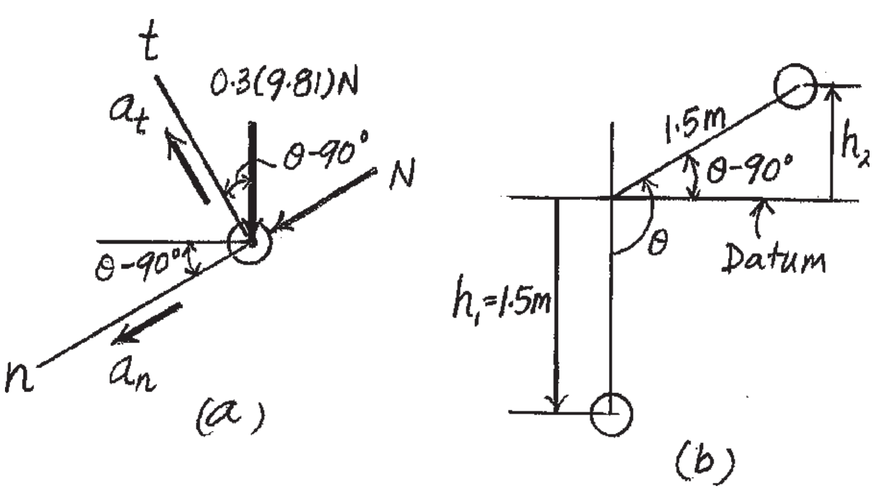
Equating Eqs. (1) and (2),

$$14.715 \sin(\theta - 90°) = 20.57 - 29.43 \sin(\theta - 90°)$$

$$\sin(\theta - 90°) = 0.4660$$

$$\theta - 90° = 27.77°$$

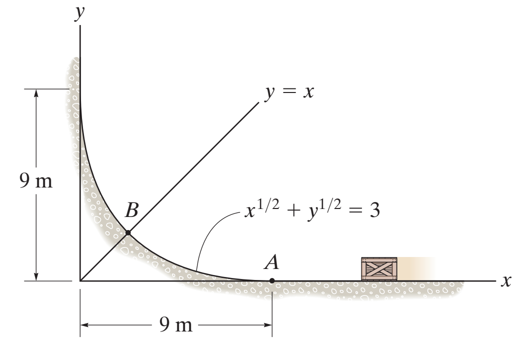
$$\theta = 117.77° = 118° \qquad \textbf{Ans.}$$



Ans:
$\theta = 118°$

14–85.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



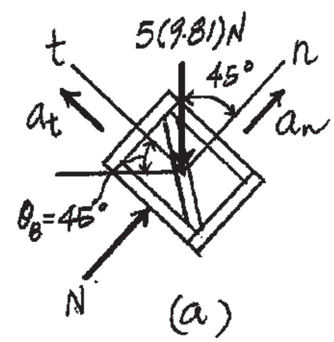
SOLUTION

Conservation of Energy. At point B, $y = x$

$$x^{\frac{1}{2}} + x^{\frac{1}{2}} = 3$$

$$x = \frac{9}{4}\text{ m}$$

Then $y = \frac{9}{4}$ m. With reference to the datum set to coincide with the x axis, the gravitational potential energies of the box at points A and B are



$$(V_g)_A = 0 \qquad (V_g)_B = mgh_B = 5(9.81)\left(\frac{9}{4}\right) = 110.3625\text{ J}$$

Applying the energy equation,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(5)(10^2) + 0 = \frac{1}{2}(5)v_B^2 + 110.3625$$

$$v_B^2 = 55.855\text{ m}^2/\text{s}^2$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\frac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \frac{3}{x^{\frac{1}{2}}}$ and $\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{3}{2}} = \frac{3}{2x^{\frac{3}{2}}}$. At point B, $x = \frac{9}{4}$ m. Thus,

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=\frac{9}{4}\text{ m}} = 1 - \frac{3}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = -1 \qquad \theta_B = -45° = 45°$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{9}{4}\text{ m}} = \frac{3}{2\left(\frac{9}{4}\right)^{\frac{3}{2}}} = 0.4444$$

The radius of curvature at B is

$$P_B = \frac{[1 + (dy/dx)^2]^{\frac{3}{2}}}{|d^2y/dx^2|} = \frac{[1 + (-1)^2]^{\frac{3}{2}}}{0.4444} = 6.3640\text{ m}$$

Referring to the FBD of the box, Fig. *a*

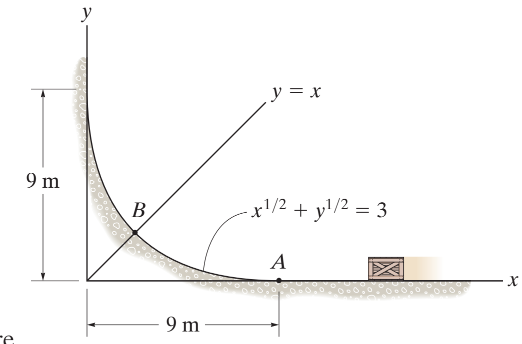
$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 45° = 5\left(\frac{55.855}{6.3640}\right)$$

$$N = 78.57\text{ N} = 78.6\text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 78.6\text{ N}$

14–86.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine how high the box reaches up the surface before it comes to a stop. Also, what is the resultant normal force on the surface at this point and the acceleration? Neglect friction and the size of the box.



SOLUTION

Conservation of Energy. With reference to the datum set coincide with x axis, the gravitational potential energy of the box at A and C (at maximum height) are

$$(V_g)_A = 0 \qquad (V_g)_C = mgh_c = 5(9.81)(y) = 49.05y$$

It is required that the box stop at C. Thus, $T_c = 0$

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(5)(10^2) + 0 = 0 + 49.05y$$

$$y = 5.0968\text{ m} = 5.10\text{ m} \qquad \textbf{Ans.}$$

Then,

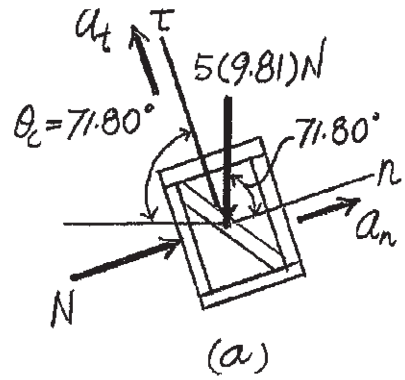
$$x^{\frac{1}{2}} + 5.0968^{\frac{1}{2}} = 3 \qquad x = 0.5511\text{ m}$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{\frac{1}{2}}\right)$ $= \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{\frac{-3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$. At point C, $x = 0.5511\text{ m}$.

Thus

$$\tan\theta_c = \left.\frac{dy}{dx}\right|_{x=0.5511\text{ m}} = 1 - \frac{3}{0.5511^{\frac{1}{2}}} = -3.0410 \qquad \theta_C = -71.80° = 71.80°$$

Referring to the FBD of the box, Fig. a,



$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 71.80° = 5\left(\frac{0^2}{\rho_C}\right)$$

$$N = 15.32\text{ N} = 15.3\text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t; \qquad -5(9.81)\sin 71.80° = 5a_t$$

$$a_t = -9.3191\text{ m/s}^2 = 9.32\text{ m/s}^2 \searrow$$

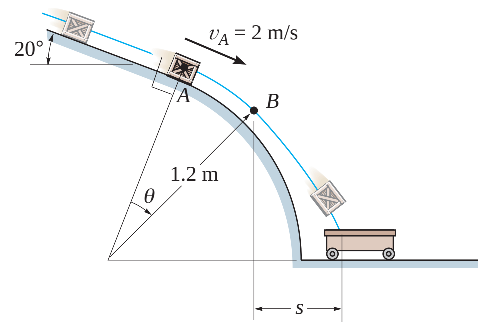
Since $a_n = 0$, Then

$$a = a_t = 9.32\text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$

Ans:
$y = 5.10\text{ m}$
$N = 15.3\text{ N}$
$a = 9.32\text{ m/s}^2 \searrow$

14–87.

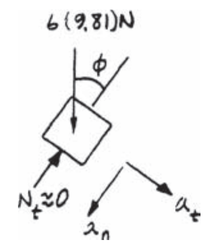
When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:

$$+\swarrow \Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$



Datum at bottom of curve:

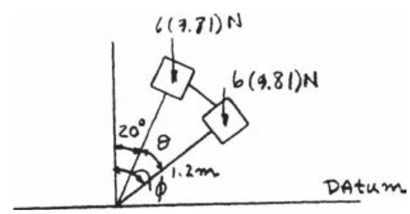
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$$v_B = 2.951 \text{ m/s}$$



Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

$$\theta = \phi - 20° = 22.3° \qquad \textbf{Ans.}$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$$

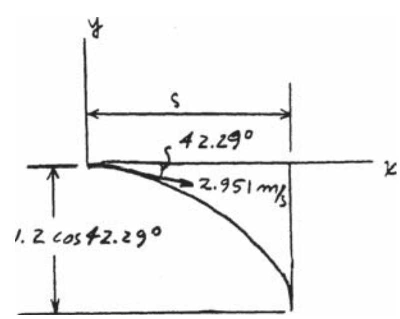
$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s

$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

$$s = 0 + (2.951\cos 42.29°)(0.2687)$$

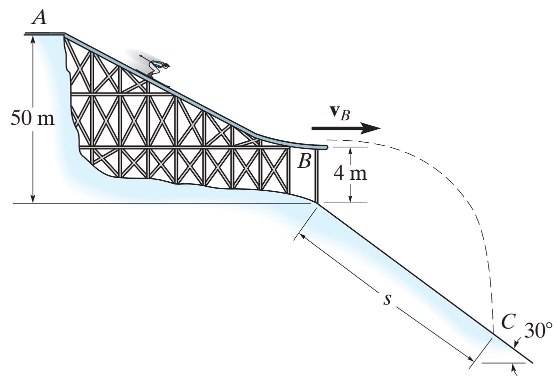
$$s = 0.587 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$\theta = 22.3°$
$s = 0.587$ m

***14–88.**

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)v^2 + 0$$

$$v = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**

$$(+\downarrow)\ s_y = (s_y)_0 + (v_0)_y t + \frac{1}{2}a_c t^2$$

$$4 + s \sin 30° = 0 + 0 + \frac{1}{2}(9.81)t^2 \quad \textbf{(1)}$$

$$(\overset{+}{\leftarrow})\ s_x = v_x t$$

$$s \cos 30° = 30.04t \quad \textbf{(2)}$$

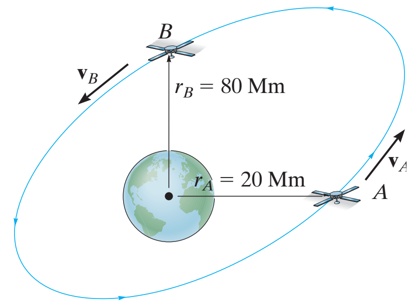
$$s = 130 \text{ m}$$ **Ans.**

$$t = 3.75 \text{ s}$$

Ans:
$v = 30.0$ m/s
$s = 130$ m

14–89.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–81, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})$ m^3/(kg · s^2).



SOLUTION

$$v_A = 40 \text{ Mm/h} = 11\,111.1 \text{ m/s}$$

Since $V = -\dfrac{GM_e m}{r}$

$$T_1 + V_1 = T_2 + V_2$$

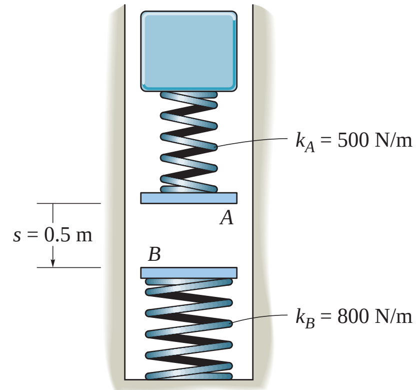
$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672 \text{ m/s} = 34.8 \text{ Mm/h}$ **Ans.**

Ans:
$v_B = 34.8$ Mm/h

14–90.

The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



SOLUTION

Datum at initial position:

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = 0 + \frac{1}{2}(500)s_A^2 + \frac{1}{2}(800)s_B^2 + 20(9.81)\left[-(s_A + s_B) - 0.5\right] \quad \textbf{(1)}$$

Also, $F_s = 500s_A = 800s_B \qquad s_A = 1.6s_B$ **(2)**

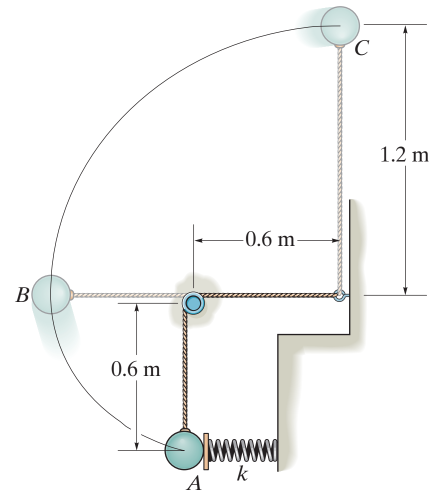
Solving Eqs. (1) and (2) yields:

$s_B = 0.638$ m **Ans.**

$s_A = 1.02$ m **Ans.**

Ans:
$s_B = 0.638$ m
$s_A = 1.02$ m

14–91. The 0.75-kg bob of a pendulum is fired from rest at position A by a spring which has a stiffness $k = 6$ kN/m and is compressed 125 mm. Determine the speed of the bob and the tension in the cord when the bob is at positions B and C. Point B is located on the path where the radius of curvature is still 0.6 m, i.e., just before the cord becomes horizontal.



SOLUTION

Given:

$M = 0.75$ kg

$k = 6$ kN/m

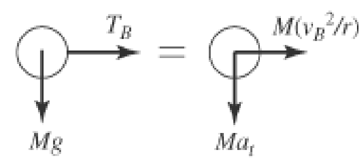
$\delta = 125$ mm

$r = 0.6$ m

At B:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}M v_B^2 + M g r$$

$$v_B = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 2g r} \qquad v_B = 10.6 \text{ m/s} \quad \textbf{Ans.}$$

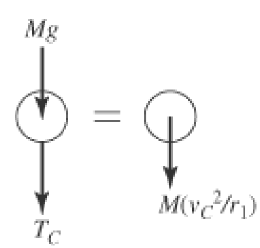


$$T_B = M\left(\frac{v_B^2}{r}\right) \qquad T_B = 142 \text{ N} \quad \textbf{Ans.}$$

At C:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}M v_C^2 + M g 3r$$

$$v_C = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 6g r} \qquad v_C = 9.47 \text{ m/s} \quad \textbf{Ans.}$$



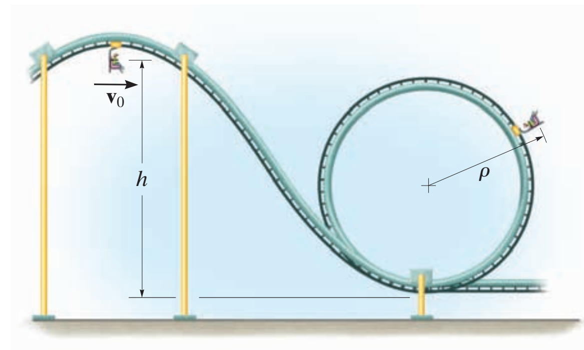
$$T_C + M g = M\left(\frac{v_C^2}{2r}\right)$$

$$T_C = M\left(\frac{v_C^2}{2r} - g\right) \qquad T_C = 48.7 \text{ N} \quad \textbf{Ans.}$$

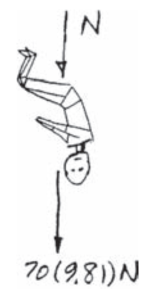
Ans:
$v_B = 10.6$ m/s, $T_B = 142$ N
$v_C = 9.47$ m/s, $T_C = 48.7$ N

***14–92.**

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at $v_0 = 4$ m/s when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the 70-kg passenger on his seat at this instant. The car has a mass of 50 kg. Take $h = 12$ m, $\rho = 5$ m. Neglect friction and the size of the car and passenger.



SOLUTION



Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(120)(4)^2 + 120(9.81)(12) = \frac{1}{2}(120)(v_1)^2 + 120(9.81)(10)$$

$v_1 = 7.432$ m/s **Ans.**

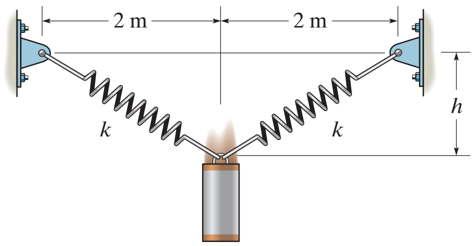
$+\downarrow \Sigma F_n = ma_n; \qquad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$

$N = 86.7$ N **Ans.**

Ans:
$v_1 = 7.432$ m/s
$N = 86.7$ N

14–93.

If the 20-kg cylinder is released from rest at $h = 0$, determine the required stiffness k of each spring so that its motion is arrested or stops when $h = 0.5$ m. Each spring has an unstretched length of 1 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k(2 - 1)^2\right] = 0 - 20(9.81)(0.5) + 2\left[\frac{1}{2}k\left(\sqrt{(2)^2 + (0.5)^2} - 1\right)^2\right]$$

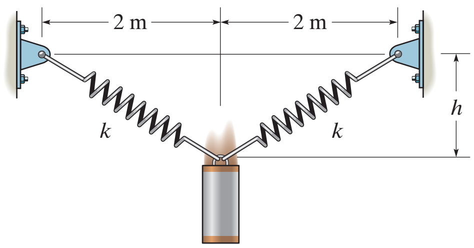
$$k = -98.1 + 1.12689\,k$$

$$k = 773 \text{ N/m}$$ **Ans.**

Ans:
$k = 773$ N/m

14–94.

The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. Each spring has a stiffness $k = 40$ N/m and an unstretched length of 2 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

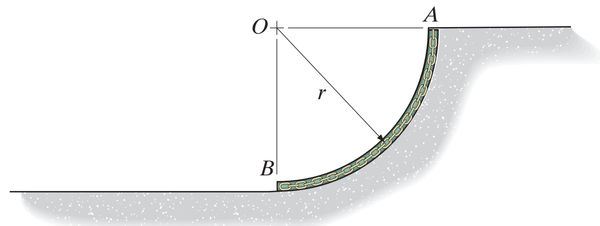
$$0 + 0 = 0 + 2\left[\frac{1}{2}(40)\left(\sqrt{3^2 + 2^2} - 2^2\right)\right] - 20(9.81)(3) + \frac{1}{2}(20)v^2$$

$$v = 6.97 \text{ m/s}$$ **Ans.**

Ans:
$v = 6.97$ m/s

14–95.

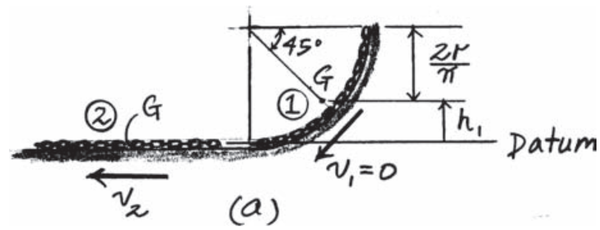
A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



SOLUTION

Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. a. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. a the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0r^2g$$



and

$$\left(V_g\right)_2 = mgh_2 = 0$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2^2 + \left(V_g\right)_2$$

$$0 + \left(\frac{\pi - 2}{2}\right)m_0r^2g = \frac{1}{2}\left(\frac{\pi}{2}m_0r\right)v_2^2 + 0$$

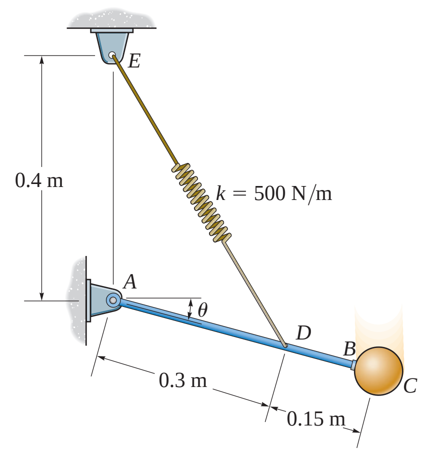
$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr}$$ **Ans.**

Ans:

$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr}$

***14–96.**

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the sphere at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0.45) = 44.145$ J and $\left(V_g\right)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \frac{100}{500} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3$ m, and the elastic potential energy of the spring is $\left(V_e\right)_1 = \frac{1}{2}ks_1{}^2 = \frac{1}{2}(500)(0.2^2) = 10$ J. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4$ m, and the elastic potential energy of the spring is $\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(500)(0.4^2) = 40$ J.

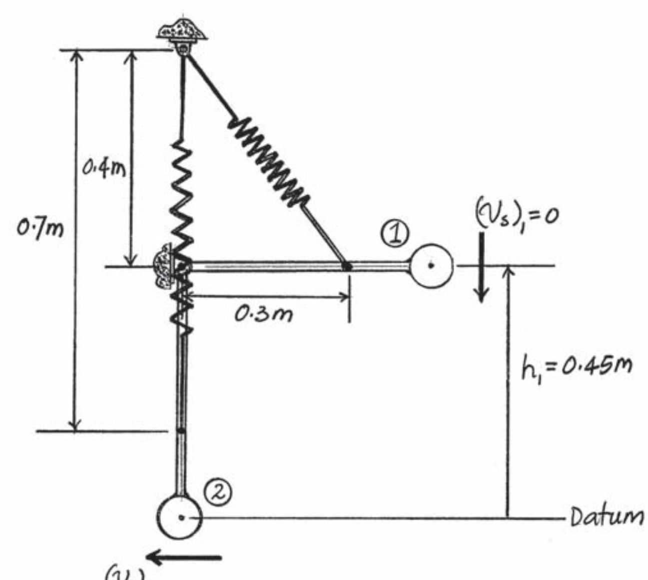
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s\left(v_s\right)_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_s\left(v_s\right)_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2^2 + (0 + 40)$$

$$(v_s)_2 = 1.68 \text{ m/s}$$ **Ans.**

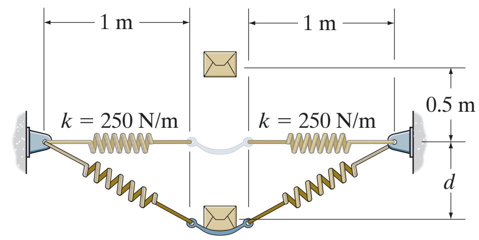


(a)

Ans:
$v = 1.68$ m/s

14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 10(9.81)\left[-\left(0.5 + d\right)\right] = -98.1\left(0.5 + d\right)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $\left(V_e\right)_1 = (2)\frac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$\left(V_e\right)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2+1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

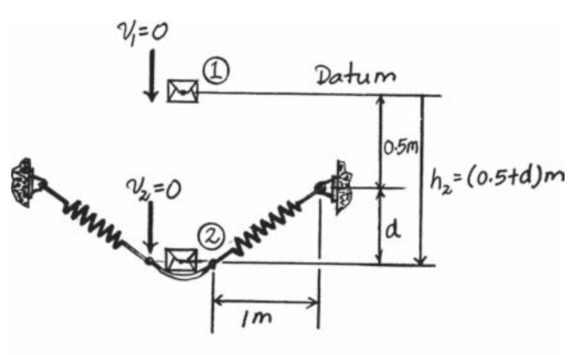
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 10\right) = 0 + \left[-98.1\left(0.5 + d\right) + 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2+1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$d = 1.34$ m