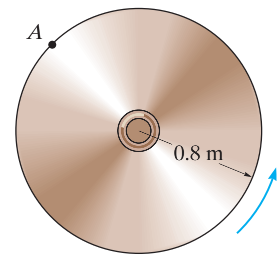
16–1.

The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.



SOLUTION

$$\omega = (5\,t^2 + 2)\text{ rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10\,t$$

$$t = 0.5\text{ s}$$

$$\omega = 3.25\text{ rad/s}$$

$$\alpha = 5\text{ rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60\text{ m/s}$$ **Ans.**

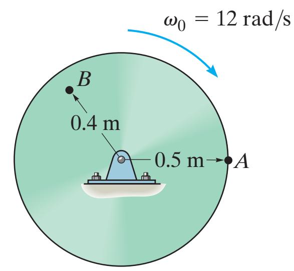
$$a_z = \alpha r = 5(0.8) = 4\text{ m/s}^2$$

$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45\text{ m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35\text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 2.60\text{ m/s}$
$a_A = 9.35\text{ m/s}^2$

16–2. The angular acceleration of the disk is defined by $\alpha = (3t^2 + 12)$ rad/s^2, where t is in seconds. If the disk is originally rotating at $\omega_0 = 12$ rad/s, determine the magnitude of the velocity and the n and t components of acceleration of point A on the disk when $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 12$ rad/s at $t = 0$.

$$\int_{12\text{ rad/s}}^{\omega} d\omega = \int_0^{2\text{ s}} (3t^2 + 12)dt$$

$$\omega - 12 = (t^3 + 12t)\Big|_0^{2\text{ s}}$$

$$\omega = 44.0 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 44.0(0.5) = 22.0 \text{ m/s}$$ **Ans.**

At $t = 2$ s, $\alpha = 3(2^2) + 12 = 24$ rad/s^2. Thus, the tangential and normal components of the acceleration are

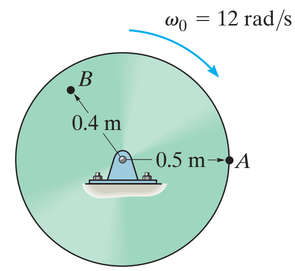
$$(a_A)_t = \alpha r_A = 24(0.5) = 12.0 \text{ m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r_A = (44.0^2)(0.5) = 968 \text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 22.0$ m/s
$(a_A)_t = 12.0$ m/s^2
$(a_A)_n = 968$ m/s^2

16–3.

The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega = \omega_0 + \alpha_c t; \qquad \omega = 12 + 20(2) = 52 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 52(0.5) = 26.0 \text{ m/s}$$ **Ans.**

The tangential and normal component of acceleration are

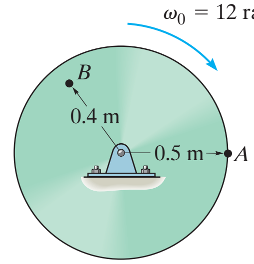
$$(a_A)_t = \alpha r = 20(0.5) = 10.0 \text{ m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r = (52^2)(0.5) = 1352 \text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 26.0$ m/s
$(a_A)_t = 10.0$ m/s^2
$(a_A)_n = 1352$ m/s^2

***16–4.**

The disk is originally rotating at $\omega_0 = 12\text{ rad/s}$. If it is subjected to a constant angular acceleration of $\alpha = 20\text{ rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0); \qquad \omega^2 = 12^2 + 2(20)[2(2\pi) - 0]$$

$$\omega = 25.43\text{ rad/s}$$

Motion of Point B. The magnitude of the velocity is

$$v_B = \omega r_B = 25.43(0.4) = 10.17\text{ m/s} = 10.2\text{ m/s} \qquad \textbf{Ans.}$$

The tangential and normal components of acceleration are

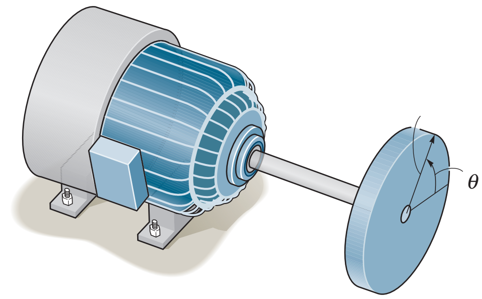
$$(a_B)_t = \alpha r_B = 20(0.4) = 8.00\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_n = \omega^2 r_B = \left(25.43^2\right)(0.4) = 258.66\text{ m/s}^2 = 259\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 10.2\text{ m/s}$
$(a_B)_t = 8.00\text{ m/s}^2$
$(a_B)_n = 259\text{ m/s}^2$

16–5.

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s,

$$\theta = 20(90) + 4\left(90^2\right) = (34200 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev} \qquad \textbf{Ans.}$$

Angular Velocity: Applying Eq. 16–1, we have

$$\omega = \frac{d\theta}{dt} = 20 + 8t \bigg|_{t=90\text{ s}} = 740 \text{ rad/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2, we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\theta = 5443$ rev
$\omega = 740$ rad/s
$\alpha = 8$ rad/s^2

16–6.

A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(15)^2 = (10)^2 + 2(3)(\theta - 0)$$

$$\theta = 20.83 \text{ rad} = 20.83\left(\frac{1}{2\pi}\right) = 3.32 \text{ rev} \qquad \textbf{Ans.}$$

$$\omega = \omega_0 + \alpha_c t$$

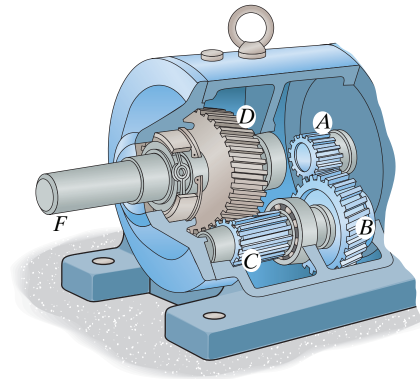
$$15 = 10 + 3t$$

$$t = 1.67 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\theta = 3.32$ rev
$t = 1.67$ s

16–7.

If gear A rotates with a constant angular acceleration of $\alpha_A = 90\ \text{rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27\ \text{rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27\ \text{rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9\ \text{rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\frac{600\ \text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 20\pi\ \text{rad/s}$. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98\ \text{s} \qquad \textbf{Ans.}$$

and

$$\omega_D{}^2 = (\omega_D)_0{}^2 + 2\alpha_D[\theta_D - (\theta_D)_0]$$

$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

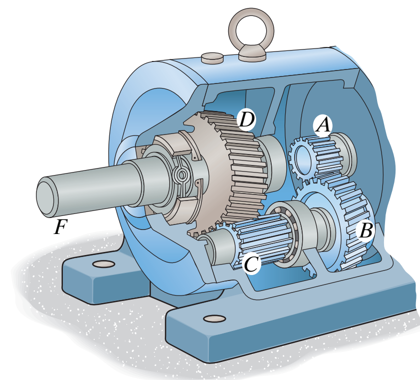
$$\theta_D = (219.32\ \text{rad})\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$= 34.9\ \text{rev} \qquad \textbf{Ans.}$$

Ans:
$t = 6.98\ \text{s}$
$\theta_D = 34.9\ \text{rev}$

***16–8.**

If gear A rotates with an angular velocity of $\omega_A = (\theta_A + 1)$ rad/s, where θ_A is the angular displacement of gear A, measured in radians, determine the angular acceleration of gear D when $\theta_A = 3$ rad, starting from rest. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Motion of Gear A:

$$\alpha_A d\theta_A = \omega_A d\omega_A$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d(\theta_A + 1)$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d\theta_A$$

$$\alpha_A = (\theta_A + 1)$$

At $\theta_A = 3$ rad,

$$\alpha_A = 3 + 1 = 4 \text{ rad/s}^2$$

Motion of Gear D: Gear A is in mesh with gear B. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(4) = 1.20 \text{ rad/s}^2$$

Since gears C and B share the same shaft $\alpha_C = \alpha_B = 1.20 \text{ rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

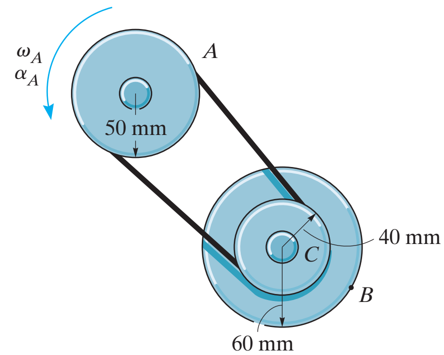
$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(1.20) = 0.4 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_D = 0.4 \text{ rad/s}^2$

16–9.

At the instant $\omega_A = 5$ rad/s, pulley A is given an angular acceleration $\alpha = (0.8\theta)$ rad/s^2, where θ is in radians. Determine the magnitude of acceleration of point B on pulley C when A rotates 3 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. The angular velocity of pulley A can be determined by integrating $\omega\, d\omega = \alpha\, d\theta$ with the initial condition $\omega_A = 5$ rad/s at $\theta_A = 0$.

$$\int_{5\text{ rad/s}}^{\omega_A} \omega\, d\omega = \int_0^{\theta_A} 0.8\theta d\theta$$

$$\frac{\omega^2}{2}\Bigg|_{5\text{ rad/s}}^{\omega_A} = \left(0.4\theta^2\right)\Bigg|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{5^2}{2} = 0.4\theta_A^2$$

$$\omega_A = \left\{\sqrt{0.8\theta_A^2 + 25}\right\} \text{ rad/s}$$

At $\theta_A = 3(2\pi) = 6\pi$ rad,

$$\omega_A = \sqrt{0.8(6\pi)^2 + 25} = 17.585 \text{ rad/s}$$

$$\alpha_A = 0.8(6\pi) = 4.8\pi \text{ rad/s}^2$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 17.585(50)$$

$$\omega_C = 21.982 \text{ rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = (4.8\pi)(50)$$

$$\alpha_C = 6\pi \text{ rad/s}^2$$

Motion of Point B. The tangential and normal components of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 6\pi(0.06) = 1.1310 \text{ m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = \left(21.982^2\right)(0.06) = 28.9917 \text{ m/s}^2$$

Thus, the magnitude of a_B is

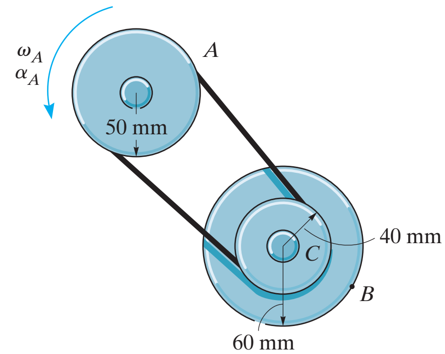
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(1.1310)^2 + (28.9917)^2}$$

$$= 29.01 \text{ m/s}^2 = 29.0 \text{ m/s}^2$$ **Ans.**

Ans:
$a_B = 29.0 \text{ m/s}^2$

16–10.

At the instant $\omega_A = 5$ rad/s, pulley A is given a constant angular acceleration $\alpha_A = 6$ rad/s^2. Determine the magnitude of acceleration of point B on pulley C when A rotates 2 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. Since the angular acceleration of pulley A is constant, we can apply

$$\omega_A^2 = (\omega_A)_0^2 + 2\alpha_A[\theta_A - (\theta_A)_0]$$

$$\omega_A^2 = 5^2 + 2(6)[2(2\pi) - 0]$$

$$\omega_A = 13.2588 \text{ rad/s}$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 13.2588(50)$$

$$\omega_C = 16.5735 \text{ rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = 6(50)$$

$$\alpha_C = 7.50 \text{ rad/s}^2$$

Motion of Point B. The tangential and normal component of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 7.50(0.06) = 0.450 \text{ m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = (16.5735^2)(0.06) = 16.4809 \text{ m/s}^2$$

Thus, the magnitude of a_B is

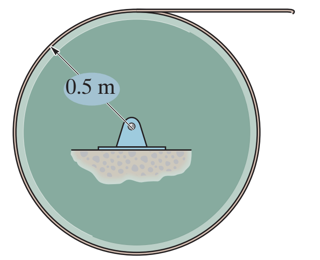
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(0.450)^2 + (16.4809)^2}$$

$$= 16.4871 \text{ m/s}^2 = 16.5 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 16.5 \text{ m/s}^2$

16–11.

The cord, which is wrapped around the disk, is given an acceleration of $a = (10t)\ \text{m/s}^2$, where t is in seconds. Starting from rest, determine the angular displacement, angular velocity, and angular acceleration of the disk when $t = 3$ s.



SOLUTION

Motion of Point P. The tangential component of acceleration of a point on the rim is equal to the acceleration of the cord. Thus

$$(a_t) = \alpha r; \qquad 10t = \alpha(0.5)$$

$$\alpha = \{20t\}\ \text{rad/s}^2$$

When $t = 3$ s,

$$\alpha = 20(3) = 60\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 0$ at $t = 0$.

$$\int_0^\omega d\omega = \int_0^t 20t\, dt$$

$$\omega = \{10t^2\}\ \text{rad/s}$$

When $t = 3$ s,

$$\omega = 10(3^2) = 90.0\ \text{rad/s} \qquad \textbf{Ans.}$$

The angular displacement of the disk can be determined by integrating $d\theta = \omega\, dt$ with the initial condition $\theta = 0$ at $t = 0$.

$$\int_0^\theta d\theta = \int_0^t 10t^2\, dt$$

$$\theta = \left\{\frac{10}{3}t^3\right\}\ \text{rad}$$

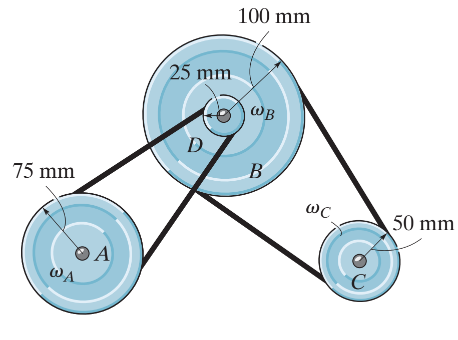
When $t = 3$ s,

$$\theta = \frac{10}{3}(3^3) = 90.0\ \text{rad} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 60\ \text{rad/s}^2$
$\omega = 90.0\ \text{rad/s}$
$\theta = 90.0\ \text{rad}$

***16–12.**

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = (20t + 40)$ rad/s, where t is in seconds, determine the angular velocities of the generator pulley B and the air-conditioning pulley C when $t = 3$ s.



SOLUTION

When $t = 3$ s

$$\omega_A = 20(3) + 40 = 100 \text{ rad/s}$$

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 100(0.075) = 7.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{7.5}{0.025} = 300 \text{ rad/s} \qquad \textbf{Ans.}$$

The speed of a point P' on the belt wrapped around the outer periphery of B is

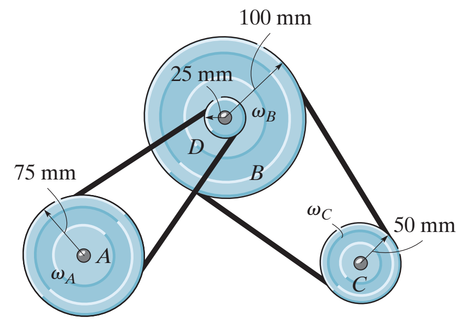
$$v'_p = \omega_B r_B = 300(0.1) = 30 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{30}{0.05} = 600 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 300$ rad/s
$\omega_C = 600$ rad/s

16–13.

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = 60$ rad/s, determine the angular velocities of the generator pulley B and the air-conditioning pulley C. The hub at D is rigidly *connected* to B and turns with it.



SOLUTION

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 60(0.075) = 4.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{4.5}{0.025} = 180 \text{ rad/s}$$ **Ans.**

The speed of a point P' on the belt wrapped around the outer periphery of B is

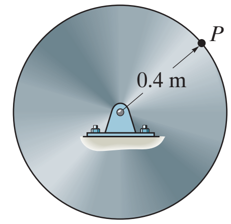
$$v'_P = \omega_B r_B = 180(0.1) = 18 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{18}{0.05} = 360 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 180$ rad/s
$\omega_C = 360$ rad/s

16–14.

The disk starts from rest and is given an angular acceleration $\alpha = (2t^2)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.



SOLUTION

$$\alpha = \frac{d\omega}{dt} = 2t^2$$

$$\int_0^\omega d\omega = \int_0^t 2t^2 dt$$

$$\omega = \frac{2}{3}t^3\Big|_0^t$$

$$\omega = \frac{2}{3}t^3$$

When $t = 4$ s,

$$\omega = \frac{2}{3}(4)^3 = 42.7\ \text{rad/s}$$ **Ans.**

$$\int_0^\theta d\theta = \int_0^t \frac{2}{3}t^3 dt$$

$$\theta = \frac{1}{6}t^4$$

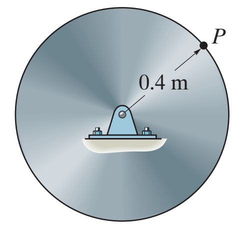
When $t = 4$ s,

$$\theta = \frac{1}{6}(4)^4 = 42.7\ \text{rad}$$ **Ans.**

Ans:
$\omega = 42.7$ rad/s
$\theta = 42.7$ rad

16–15.

The disk starts from rest and is given an angular acceleration $\alpha = (5t^{1/2})\ \text{rad/s}^2$, where t is in seconds. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 2$ s.



SOLUTION

Motion of the Disk: Here, when $t = 0, \omega = 0$.

$$d\omega = a dt$$

$$\int_0^\omega d\omega = \int_0^t 5t^{\frac{1}{2}} dt$$

$$\omega\Big|_0^\omega = \frac{10}{3}t^{\frac{3}{2}}\Big|_0^t$$

$$\omega = \left\{\frac{10}{3}t^{\frac{3}{2}}\right\}\ \text{rad/s}$$

When $t = 2$ s,

$$\omega = \frac{10}{3}\left(2^{\frac{3}{2}}\right) = 9.428\ \text{rad/s}$$

When $t = 2$ s,

$$\alpha = 5\left(2^{\frac{1}{2}}\right) = 7.071\ \text{rad/s}^2$$

Motion of point P: The tangential and normal components of the acceleration of point P when $t = 2$ s are

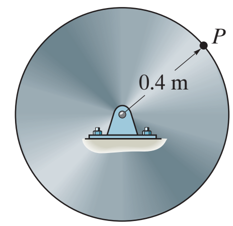
$$a_t = \alpha r = 7.071(0.4) = 2.83\ \text{m/s}^2$$ **Ans.**

$$a_n = \omega^2 r = 9.428^2(0.4) = 35.6\ \text{m/s}^2$$ **Ans.**

Ans:
$a_t = 2.83\ \text{m/s}^2$
$a_n = 35.6\ \text{m/s}^2$

***16–16.**

The disk starts at $\omega_0 = 1$ rad/s when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 1$ rev.



SOLUTION

$\alpha = 0.3\theta$

$$\int_1^{\omega} \omega d\omega = \int_0^{\theta} 0.3\theta d\theta$$

$$\frac{1}{2}\omega^2\Big|_1^{\omega} = 0.15\theta^2\Big|_0^{\theta}$$

$$\frac{\omega^2}{2} - 0.5 = 0.15\theta^2$$

$$\omega = \sqrt{0.3\theta^2 + 1}$$

At $\theta = 1 \text{ rev} = 2\pi$ rad

$$\omega = \sqrt{0.3(2\pi)^2 + 1}$$

$$\omega = 3.584 \text{ rad/s}$$

$$a_t = \alpha r = 0.3(2\pi) \text{ rad/s}^2(0.4 \text{ m}) = 0.7540 \text{ m/s}^2$$ **Ans.**

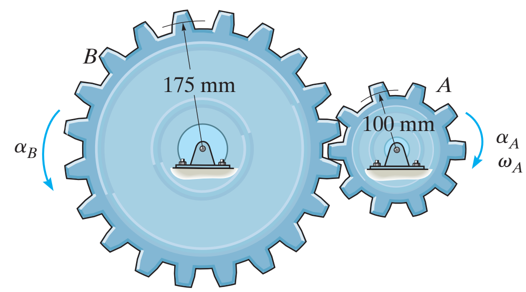
$$a_n = \omega^2 r = (3.584 \text{ rad/s})^2(0.4 \text{ m}) = 5.137 \text{ m/s}^2$$ **Ans.**

$$a_p = \sqrt{(0.7540)^2 + (5.137)^2} = 5.19 \text{ m/s}^2$$

Ans:
$a_t = 0.7540 \text{ m/s}^2$
$a_n = 5.137 \text{ m/s}^2$

16–17.

A motor gives gear A an angular acceleration of $\alpha_A = (2 + 0.006\,\theta^2)\ \text{rad/s}^2$, where θ is in radians. If this gear is initially turning at $\omega_A = 15\ \text{rad/s}$, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



SOLUTION

Angular Motion. The angular velocity of the gear A can be determined by integrating $\omega\, d\omega = \alpha\, d\theta$ with initial condition $\omega_A = 15\ \text{rad/s}$ at $\theta_A = 0$.

$$\int_{15\ \text{rad/s}}^{\omega_A} \omega\, d\omega = \int_0^{\theta_A} \left(2 + 0.006\,\theta^2\right) d\theta$$

$$\left.\frac{\omega^2}{2}\right|_{15\ \text{rad/s}}^{\omega_A} = \left.\left(2\theta + 0.002\,\theta^3\right)\right|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{15^2}{2} = 2\theta_A + 0.002\,\theta_A^3$$

$$\omega_A = \sqrt{0.004\,\theta_A^3 + 4\,\theta + 225}\ \text{rad/s}$$

At $\theta_A = 10(2\pi) = 20\pi$ rad,

$$\omega_A = \sqrt{0.004(20\pi)^3 + 4(20\pi) + 225}$$

$$= 38.3214\ \text{rad/s}$$

Since gear B is meshed with gear A,

$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 38.3214(100)$$

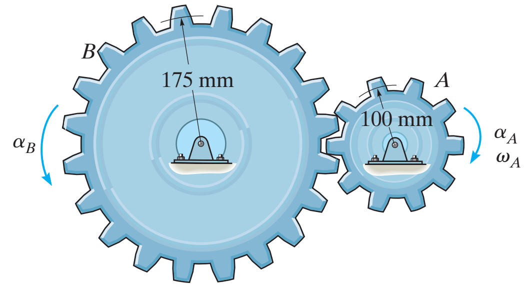
$$\omega_B = 21.8979\ \text{rad/s}$$

$$= 21.9\ \text{rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 21.9\ \text{rad/s} \circlearrowleft$

16–18.

A motor gives gear A an angular acceleration of $\alpha_A = (2t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $\omega_A = 15$ rad/s, determine the angular velocity of gear B when $t = 3$ s.



SOLUTION

Angular Motion. The angular velocity of gear A can be determined by integrating $d\omega = \alpha\, dt$ with initial condition $\omega_A = 15$ rad/s at $t = 0$ s.

$$\int_{15\ \text{rad/s}}^{\omega_A} d\omega = \int_0^t 2t^3\, dt$$

$$\omega_A - 15 = \frac{1}{2}t^4\Big|_0^t$$

$$\omega_A = \left\{\frac{1}{2}t^4 + 15\right\}\ \text{rad/s}$$

At $t = 3$ s,

$$\omega_A = \frac{1}{2}(3^4) + 15 = 55.5\ \text{rad/s}$$

Since gear B meshed with gear A,

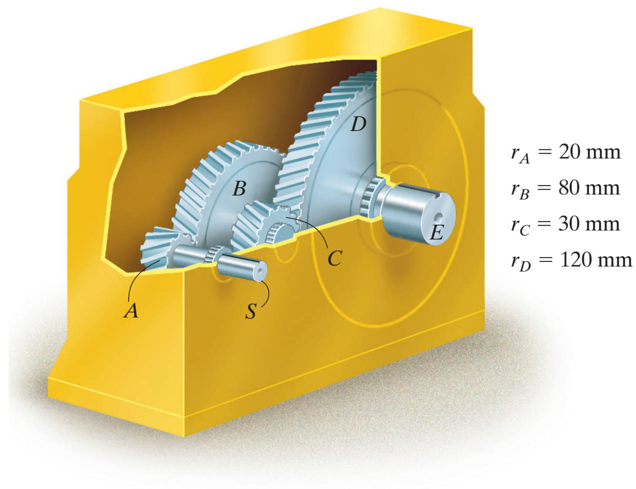
$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 55.5(100)$$

$$\omega_B = 31.7143\ \text{rad/s}$$

$$= 31.7\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 31.7\ \text{rad/s} \circlearrowright$

16–19. Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = (4\omega^{-3})$ rad/s^2, where ω is in rad/s, determine the angular velocity of shaft E at time $t = 2$ s after starting from an angular velocity 1 rad/s when $t = 0$. The radius of each gear is listed in the figure. Note that gears B and C are fixed connected to the same shaft.



SOLUTION

Given:

$r_A = 20$ mm

$r_B = 80$ mm

$r_C = 30$ mm

$r_D = 120$ mm

$\omega_0 = 1$ rad/s

$k = 4$ rad/s^5

$t_1 = 2$ s

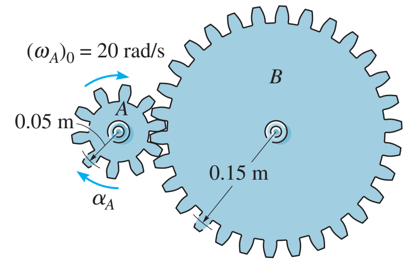
Guess $\omega_1 = 1$ rad/s Given $\int_0^{t_1} k\, dt = \int_{\omega_0}^{\omega_1} \omega^3\, d\omega$ $\omega_1 = \text{Find}(\omega_1)$

$\omega_1 = 2.397$ rad/s $\omega_E = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega_1$ $\omega_E = 0.150$ rad/s **Ans.**

Ans:
$\omega_E = 0.150$ rad/s

***16–20.**

A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20\ \text{rad/s}$, determine the angular velocity of gear B when $t = 2$ s.



SOLUTION

$$\alpha_A = 4\,t^3$$

$$d\omega = \alpha\,dt$$

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A\,dt = \int_0^t 4\,t^3\,dt$$

$$\omega_A = t^4 + 20$$

When $t = 2$ s,

$$\omega_A = 36\ \text{rad/s}$$

$$\omega_A r_A = \omega_B r_B$$

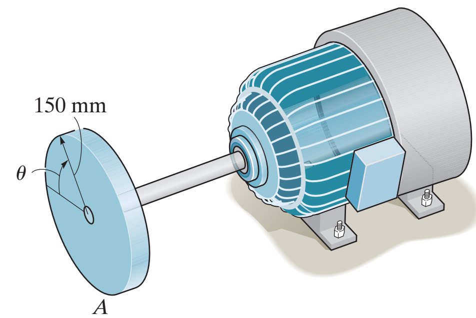
$$36(0.05) = \omega_B(0.15)$$

$$\omega_B = 12\ \text{rad/s}$$ **Ans.**

Ans:
$\omega_B = 12\ \text{rad/s}$

16–21.

The motor turns the disk with an angular velocity of $\omega = (5t^2 + 3t)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and the n and t components of acceleration of the point A on the disk when $t = 3$ s.



SOLUTION

Angular Motion. At $t = 3$ s,

$$\omega = 5(3^2) + 3(3) = 54 \text{ rad/s}$$

The angular acceleration of the disk can be determined using

$$\alpha = \frac{d\omega}{dt}; \qquad \alpha = \{10t + 3\} \text{ rad/s}^2$$

At $t = 3$ s,

$$\alpha = 10(3) + 3 = 33 \text{ rad/s}^2$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 54(0.15) = 8.10 \text{ m/s} \qquad \textbf{Ans.}$$

The tangential and normal component of acceleration are

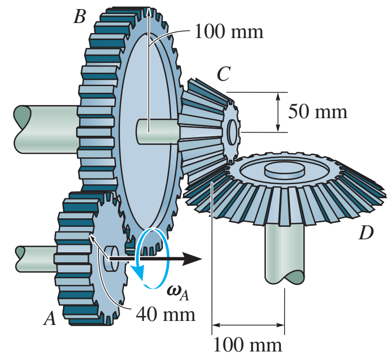
$$(a_A)_t = \alpha r_A = 33(0.15) = 4.95 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_A)_n = \omega^2 r_A = (54^2)(0.15) = 437.4 \text{ m/s}^2 = 437 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_A = 8.10$ m/s
$(a_A)_t = 4.95$ m/s^2
$(a_A)_n = 437$ m/s^2

16–22.

If the motor turns gear A with an angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 20\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(20) = 8.00\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(2) = 0.800\ \text{rad/s}^2$$

Since gear C is attached to gear B, then $\omega_C = \omega_B = 8\ \text{rad/s}$ and $\alpha_C = \alpha_B = 0.8\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

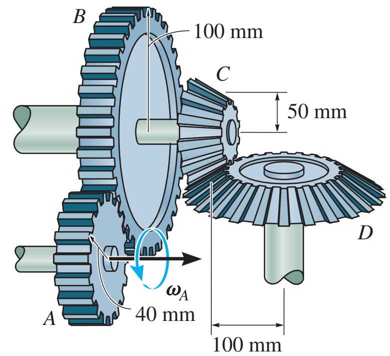
$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(8.00) = 4.00\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(0.800) = 0.400\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_D = 4.00\ \text{rad/s}$
$\alpha_D = 0.400\ \text{rad/s}^2$

16–23.

If the motor turns gear A with an angular acceleration of $\alpha_A = 3\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 60\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(60) = 24.0\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(3) = 1.20\ \text{rad/s}^2$$

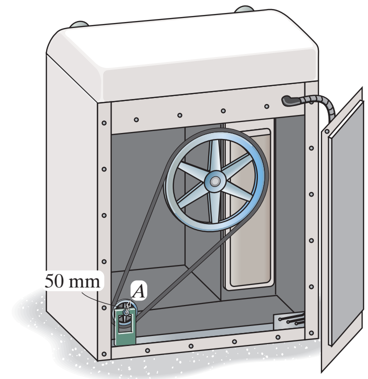
Since gear C is attached to gear B, then $\omega_C = \omega_B = 24.0\ \text{rad/s}$ and $\alpha_C = \alpha_B = 1.20\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(24.0) = 12.0\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(1.20) = 0.600\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_D = 12.0\ \text{rad/s}$
$\alpha_D = 0.600\ \text{rad/s}^2$

***16–24.** The 50-mm-radius pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (27\theta_A^{1/2})$ rad/s^2, where θ_A is in radians. Determine its angular acceleration when $t = 1$ s, starting from rest.



SOLUTION

Motion of Pulley *A*: The angular velocity of pulley A can be determined from

$$\int \omega_A\, d\omega_A = \int \alpha_A\, d\theta_A$$

$$\int_0^{\omega_A} \omega_A d\omega_A = \int_0^{\theta_A} 27\theta_A{}^{1/2} d\theta_A$$

$$\left.\frac{\omega_A{}^2}{2}\right|_0^{\omega_A} = 18\theta_A{}^{3/2}\Big|_0^{\theta_A}$$

$$\omega_A = \left(6\theta_A{}^{3/4}\right) \text{ rad/s}$$

Using this result, the angular displacement of A as a function of t can be determined from

$$\int dt = \int \frac{d\theta_A}{\omega_A}$$

$$\int_0^t dt = \int_0^{\theta_A} \frac{d\theta_A}{6\theta_A{}^{3/4}}$$

$$t\Big|_0^t = \left.\frac{2}{3}\theta_A{}^{1/4}\right|_0^{\theta_A}$$

$$t = \left(\frac{2}{3}\theta_A{}^{1/4}\right) \text{s}$$

$$\theta_A = \left(\frac{3}{2}t\right)^4 \text{ rad}$$

When $t = 1$ s

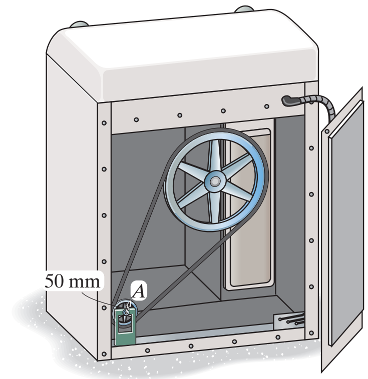
$$\theta_A = \left[\frac{3}{2}(1)\right]^4 = 5.0625 \text{ rad}$$

Thus, when $t = 1$ s, α_A is

$$\alpha_A = 27\left(5.0625^{1/2}\right) = 60.8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_A = 60.8$ rad/s^2

16–25. If the 50-mm-radius motor pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (10 + 50t)\ \text{rad/s}^2$, where t is in seconds, determine its angular velocity when $t = 3$ s, starting from rest.



SOLUTION

Motion of Pulley *A*: The angular velocity of pulley A can be determined from

$$\int d\omega_A = \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (10 + 50t)dt$$

$$\omega_A\Big|_0^{\omega_A} = \left(10t + 25t^2\right)\Big|_0^t$$

$$\omega_A = \left(10t + 25t^2\right)\text{rad/s}$$

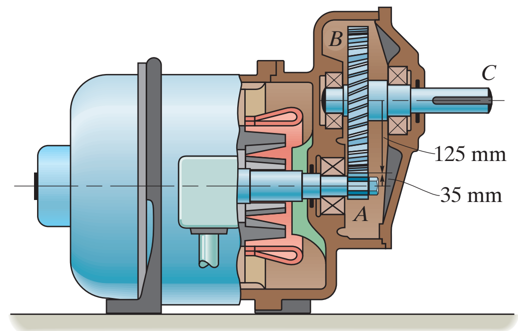
When $t = 3$ s

$$\omega_A = 10(3) + 25\left(3^2\right) = 225\ \text{rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_A = 225\ \text{rad/s}$

16–26.

The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3\ \text{rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2\ \text{s}$ starting from rest. The shaft is fixed to B and turns with it.



SOLUTION

$\omega = \omega_0 + \alpha_c t$

$\omega_A = 0 + 3(2) = 6\ \text{rad/s}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

$\theta_A = 0 + 0 + \frac{1}{2}(3)(2)^2$

$\theta_A = 6\ \text{rad}$

$\omega_A r_A = \omega_B r_B$

$6(35) = \omega_B(125)$

$\omega_C = \omega_B = 1.68\ \text{rad/s}$ **Ans.**

$\theta_A r_A = \theta_B r_B$

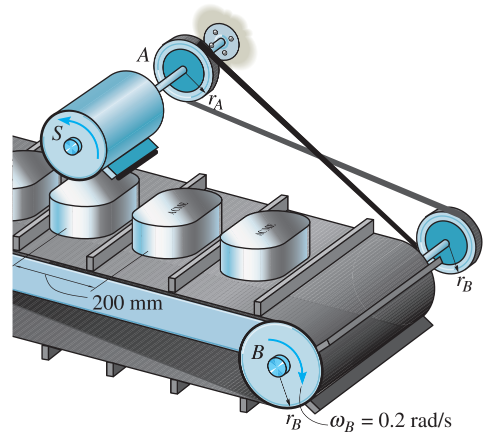
$6(35) = \theta_B(125)$

$\theta_C = \theta_B = 1.68\ \text{rad}$ **Ans.**

Ans:
$\omega_C = 1.68\ \text{rad/s}$
$\theta_C = 1.68\ \text{rad}$

16–27.

A stamp S, located on the revolving drum, is used to label canisters. If the canisters are centered 200 mm apart on the conveyor, determine the radius r_A of the driving wheel A and the radius r_B of the conveyor belt drum so that for each revolution of the stamp it marks the top of a canister. How many canisters are marked per minute if the drum at B is rotating at $\omega_B = 0.2$ rad/s? Note that the driving belt is twisted as it passes between the wheels.



SOLUTION

$l = 2\pi(r_A)$

$r_A = \dfrac{200}{2\pi} = 31.8$ mm **Ans.**

For the drum at B:

$l = 2\pi(r_B)$

$r_B = \dfrac{200}{2\pi} = 31.8$ mm **Ans.**

In $t = 60$ s

$\theta = \theta_0 + \omega_0 t$

$\theta = 0 + 0.2(60) = 12$ rad

$l = \theta r_B = 12(31.8) = 382.0$ mm

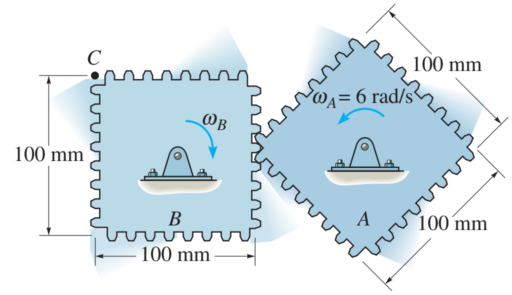
Hence,

$n = \dfrac{382.0}{200} = 1.91$ canisters marked per minute **Ans.**

Ans:
$r_A = 31.8$ mm
$r_B = 31.8$ mm
1.91 canisters per minute

***16–28.**

At the instant shown, gear A is rotating with a constant angular velocity of $\omega_A = 6\text{ rad/s}$. Determine the largest angular velocity of gear B and the maximum speed of point C.



SOLUTION

$$(r_B)_{max} = (r_A)_{max} = 50\sqrt{2}\text{ mm}$$

$$(r_B)_{min} = (r_A)_{min} = 50\text{ mm}$$

When r_A is max., r_B is min.

$$\omega_B(r_B) = \omega_A r_A$$

$$(\omega_B)_{max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$$(\omega_B)_{max} = 8.49\text{ rad/s}$$ **Ans.**

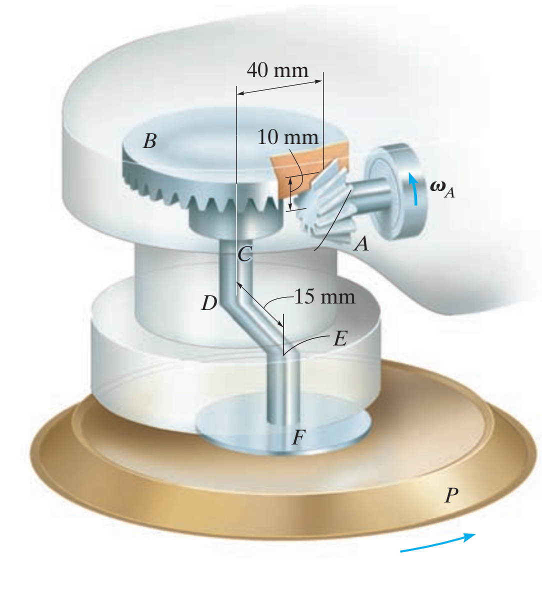
$$v_C = (\omega_B)_{max}r_C = 8.49(0.05\sqrt{2})$$

$$v_C = 0.6\text{ m/s}$$ **Ans.**

Ans:
$(\omega_B)_{max} = 8.49\text{ rad/s}$
$(v_C)_{max} = 0.6\text{ m/s}$

16–29.

For a short time a motor of the random-orbit sander drives the gear A with an angular velocity of $\omega_A = 40(t^3 + 6t)$ rad/s, where t is in seconds. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius of 15 mm. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF when $t = 2$ s after starting from rest.



SOLUTION

$$\omega_A r_A = \omega_B r_B$$

$$\omega_A(10) = \omega_B(40)$$

$$\omega_B = \frac{1}{4}\omega_A$$

$$v_E = \omega_B r_E = \frac{1}{4}\omega_A(0.015) = \frac{1}{4}(40)\left(t^3 + 6t\right)(0.015)\Big|_{t=2}$$

$$v_E = 3 \text{ m/s}$$ **Ans.**

$$\alpha_A = \frac{d\omega_A}{dt} = \frac{d}{dt}\left[40\left(t^3 + 6t\right)\right] = 120t^2 + 240$$

$$\alpha_A r_A = \alpha_B r_B$$

$$\alpha_A(10) = \alpha_B(40)$$

$$\alpha_B = \frac{1}{4}\alpha_A$$

$$(a_E)_t = \alpha_B r_E = \frac{1}{4}\left(120t^2 + 240\right)(0.015)\Big|_{t=2}$$

$$(a_E)_t = 2.70 \text{ m/s}^2$$ **Ans.**

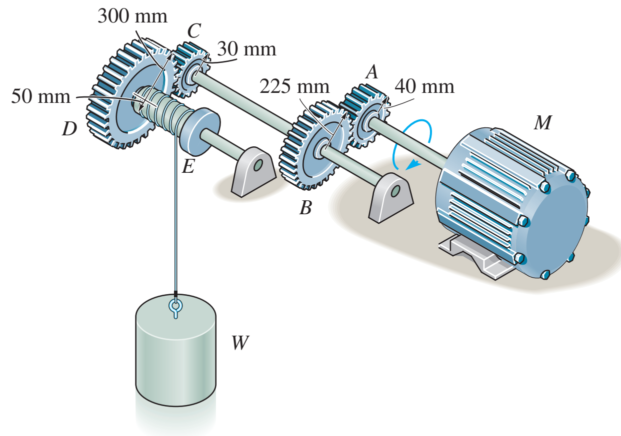
$$(a_E)_n = \omega_B^2 r_E = \left[\frac{1}{4}(40)\left(t^3 + 6t\right)\right]^2(0.015)\Big|_{t=2}$$

$$(a_E)_n = 600 \text{ m/s}^2$$ **Ans.**

Ans:
$v_E = 3$ m/s
$(a_E)_t = 2.70 \text{ m/s}^2$
$(a_E)_n = 600 \text{ m/s}^2$

16–30.

Determine the distance the load W is lifted in $t = 5$ s using the hoist. The shaft of the motor M turns with an angular velocity $\omega = 100(4 + t)$ rad/s, where t is in seconds.



SOLUTION

Angular Motion: The angular displacement of gear A at $t = 5$ s must be determined first. Applying Eq. 16–1, we have

$$d\theta = \omega dt$$

$$\int_0^{\theta_A} d\theta = \int_0^{5\text{ s}} 100(4 + t)\, dt$$

$$\theta_A = 3250 \text{ rad}$$

Here, $r_A \theta_A = r_B \theta_B$. Then, the angular displacement of gear B is given by

$$\theta_B = \frac{r_A}{r_B}\theta_A = \left(\frac{40}{225}\right)(3250) = 577.78 \text{ rad}$$

Since gear C is attached to the same shaft as gear B, then $\theta_C = \theta_B = 577.78$ rad. Also, $r_D \theta_D = r_C \theta_C$, then, the angular displacement of gear D is given by

$$\theta_D = \frac{r_C}{r_D}\theta_C = \left(\frac{30}{300}\right)(577.78) = 57.78 \text{ rad}$$

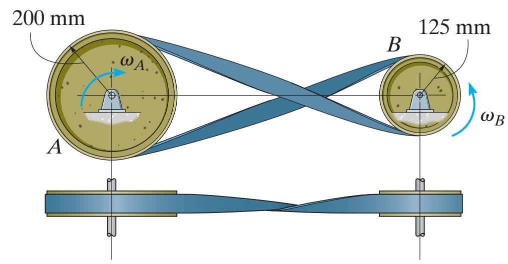
Since shaft E is attached to gear D, $\theta_E = \theta_D = 57.78$ rad. The distance at which the load W is lifted is

$$s_W = r_E\theta_E = (0.05)(57.78) = 2.89 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_W = 2.89$ m

16–31.

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)$ rad, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3$ s.



SOLUTION

Motion of Wheel A: The angular velocity and angular acceleration of wheel A can be determined from

$$\omega_A = \frac{d\theta_A}{dt} = \left(15t^2 + 20t\right) \text{ rad/s}$$

and

$$\alpha_A = \frac{d\omega_A}{dt} = \left(30t + 20\right) \text{ rad/s}^2$$

When $t = 3$ s,

$$\omega_A = 15\left(3^2\right) + 20(3) = 195 \text{ rad/s}$$

$$\alpha_A = 30(3) + 20 = 110 \text{ rad/s}^2$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(195) = 312 \text{ rad/s} \qquad \textbf{Ans.}$$

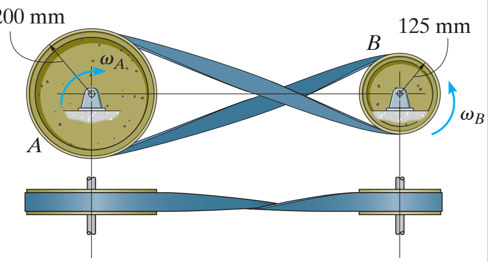
$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(110) = 176 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 312$ rad/s
$\alpha_B = 176$ rad/s^2

***16–32.**

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If A has a constant angular acceleration of $\alpha_A = 30\ \text{rad/s}^2$, determine the tangential and normal components of acceleration of a point located at the rim of B when $t = 3$ s, starting from rest.



SOLUTION

Motion of Wheel A: Since the angular acceleration of wheel A is constant, its angular velocity can be determined from

$$\omega_A = (\omega_A)_0 + \alpha_C t$$

$$= 0 + 30(3) = 90\ \text{rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(90) = 144\ \text{rad/s}$$

and

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(30) = 48\ \text{rad/s}^2$$

Thus, the tangential and normal components of the acceleration of point P located at the rim of wheel B are

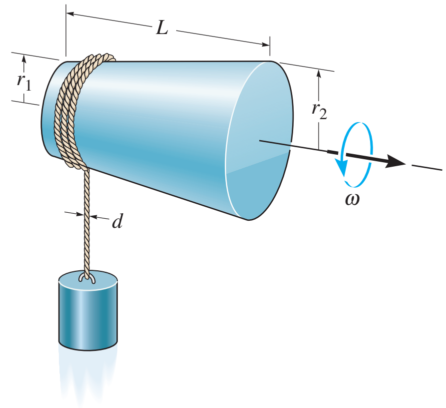
$$(a_p)_t = \alpha_B r_B = 48(0.125) = 6\ \text{m/s}^2$$ **Ans.**

$$(a_p)_n = \omega_B{}^2 r_B = (144^2)(0.125) = 2592\ \text{m/s}^2$$ **Ans.**

Ans:
$(a_p)_t = 6\ \text{m/s}^2$
$(a_p)_n = 2592\ \text{m/s}^2$

16–33.

The rope of diameter d is wrapped around the tapered drum which has the dimensions shown. If the drum is rotating at a constant rate of ω, determine the upward acceleration of the the block. Neglect the small horizontal displacement of the block.



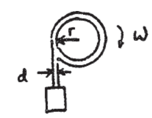
SOLUTION

$$v = \omega r$$

$$a = \frac{d(\omega r)}{dt}$$

$$= \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

$$= \omega\left(\frac{dr}{dt}\right)$$



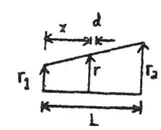
$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x$$

$$dr = \left(\frac{r_2 - r_1}{L}\right)dx$$

$$\text{But } dx = \frac{d\theta}{2\pi} \cdot d$$

$$\text{Thus } \frac{dr}{dt} = \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d\left(\frac{d\theta}{dt}\right)$$

$$= \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d\omega$$



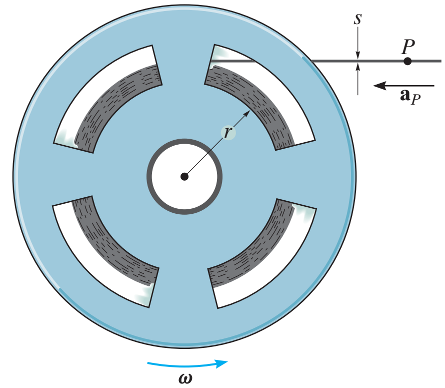
$$\text{Thus, } a = \frac{\omega^2}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d$$ **Ans.**

Ans:

$$a = \frac{\omega^2}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d$$

16–34.

A tape having a thickness s wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P of the unwrapped tape when the radius of the wrapped tape is r. *Hint*: Since $v_P = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



SOLUTION

$$v_P = \omega r$$

$$a = \frac{dv_P}{dt} = \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

Since $\frac{d\omega}{dt} = 0$,

$$a = \omega\left(\frac{dr}{dt}\right)$$

In one revolution r is increased by s, so that

$$\frac{2\pi}{\theta} = \frac{s}{\Delta r}$$

Hence,

$$\Delta r = \frac{s}{2\pi}\theta$$

$$\frac{dr}{dt} = \frac{s}{2\pi}\omega$$

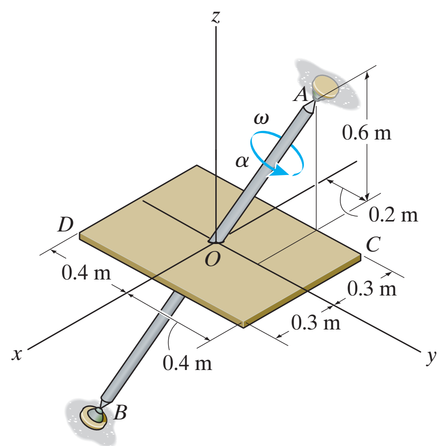
$$a = \frac{s}{2\pi}\omega^2$$ **Ans.**

Ans:

$$a = \frac{s}{2\pi}\omega^2$$

16–35.

If the shaft and plate rotates with a constant angular velocity of $\omega = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

Since ω is constant

$$\alpha = 0$$

For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}] \text{ m/s}$$ **Ans.**

and

$$\mathbf{a}_C = \alpha \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_c)$$

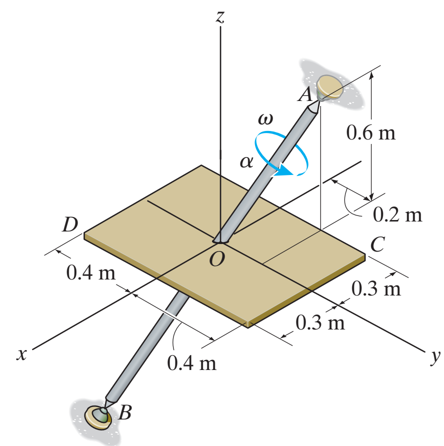
$$= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}] \text{m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}\}$ m/s^2

***16–36.**

At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}] \text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}] \text{ m/s}$$ **Ans.**

and

$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2 \mathbf{r}_D$$

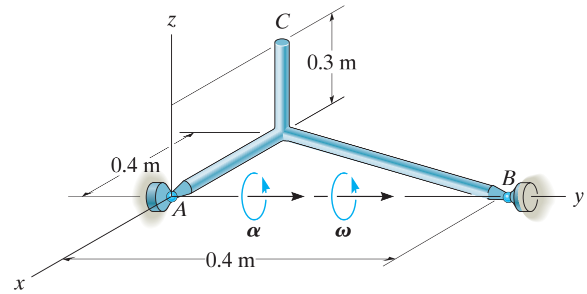
$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}] \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_D = [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]$ m/s
$\mathbf{a}_D = [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}]$ m/s^2

16–37.

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5$ rad/s and has an angular acceleration $\alpha = 8$ rad/s^2. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



SOLUTION

$$v_C = \boldsymbol{\omega} \times \mathbf{r}$$

$$v_C = 5\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) = \{1.5\mathbf{i} + 2\mathbf{k}\}\text{ m/s}$$

$$v_C = \sqrt{1.5^2 + 2^2} = 2.50\text{ m/s}$$ **Ans.**

$$a_C = \mathbf{a} \times \mathbf{r} - \omega^2\mathbf{r}$$

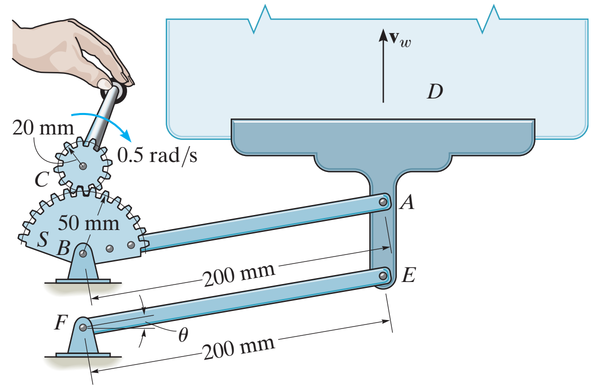
$$= 8\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) - 5^2(-0.4\mathbf{i} + 0.3\mathbf{k})$$

$$= \{12.4\mathbf{i} - 4.3\mathbf{k}\}\text{ m/s}^2$$

$$a_C = \sqrt{(12.4)^2 + (-4.3)^2} = 13.1\text{ m/s}^2$$ **Ans.**

Ans:
$v_C = 2.50$ m/s
$a_C = 13.1$ m/s^2

16–38. The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog C, which rotates the spur gear S, thereby rotating the fixed-connected lever AB which raises track D in which the window rests. The window is free to slide on the track. If the handle is wound at 0.5 rad/s, determine the speed of points A and E and the speed v_w of the window at the instant $\theta = 30°$.



SOLUTION

Given:

$\omega_c = 0.5 \text{ rad/s} \quad r_C = 20 \text{ mm}$

$\theta = 30° \quad r_s = 50 \text{ mm}$

$r_A = 200 \text{ mm}$

$v_C = \omega_c \, r_C$

$v_C = 0.01 \text{ m/s}$

$$\omega_s = \frac{v_C}{r_s}$$

$\omega_s = 0.2 \text{ rad/s}$

$v_A = v_E = \omega_s \, r_A$

$v_A = \omega_s \, r_A \qquad v_A = 40 \text{ mm/s}$ **Ans.**

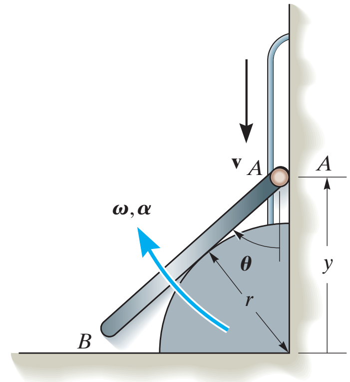
Points A and E move along circular paths. The vertical component closes the window.

$v_w = v_A \cos(\theta) \qquad v_w = 34.6 \text{ mm/s}$ **Ans.**

Ans:
$v_A = 40$ mm/s
$v_w = 34.6$ mm/s

16–39.

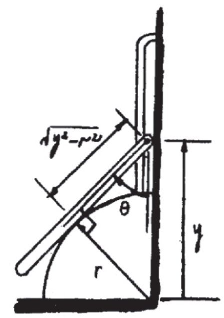
The end A of the bar is moving downward along the slotted guide with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position y.



SOLUTION

Position coordinate equation:

$$\sin\theta = \frac{r}{y}$$



Time derivatives:

$$\cos\theta\dot{\theta} = -\frac{r}{y^2}\dot{y} \text{ however, } \cos\theta = \frac{\sqrt{y^2 - r^2}}{y} \text{ and } \dot{y} = -v_A,\ \dot{\theta} = \omega$$

$$\left(\frac{\sqrt{y^2 - r^2}}{y}\right)\omega = \frac{r}{y^2}v_A \qquad \omega = \frac{rv_A}{y\sqrt{y^2 - r^2}}$$ **Ans.**

$$\alpha = \dot{\omega} = rv_A\left[-y^{-2}\dot{y}\left(y^2 - r^2\right)^{-\frac{1}{2}} + \left(y^{-1}\right)\left(-\tfrac{1}{2}\right)\left(y^2 - r^2\right)^{-\frac{3}{2}}(2y\dot{y})\right]$$

$$\alpha = \frac{rv_A^2\,(2y^2 - r^2)}{y^2(y^2 - r^2)^{\frac{3}{2}}}$$ **Ans.**

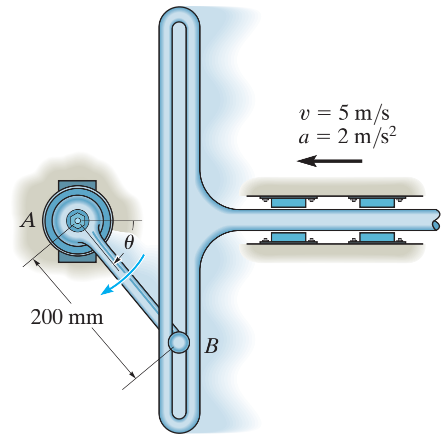
Ans:

$$\omega = \frac{rv_A}{y\sqrt{y^2 - r^2}}$$

$$\alpha = \frac{rv_A^2\left(2y^2 - r^2\right)}{y^2\left(y^2 - r^2\right)^{3/2}}$$

***16–40.**

At the instant $\theta = 60°$, the slotted guide rod is moving to the left with an acceleration of 2 m/s^2 and a velocity of 5 m/s. Determine the angular acceleration and angular velocity of link AB at this instant.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the guide rod can be related to the angular motion of the crank by relating x and θ using the geometry shown in Fig. a, which is

$$x = 0.2 \cos \theta \text{ m}$$

Time Derivatives. Using the chain rule,

$$\dot{x} = -0.2(\sin \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -0.2[(\cos \theta)\dot{\theta}^2 + (\sin \theta)\ddot{\theta}] \qquad \textbf{(2)}$$

Here $\dot{x} = v$, $\ddot{x} = a$, $\dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$ when $\theta = 60°$. Realizing that the velocity and acceleration of the guide rod are directed toward the negative sense of x, $v = -5$ m/s and $a = -2$ m/s^2. Then Eq (1) gives

$$-s = (-0.2(\sin 60°)\omega$$

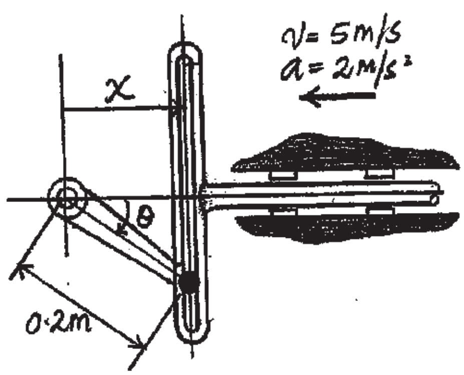
$$\omega = 28.87 \text{ rad/s} = 28.9 \text{ rad/s} ↻ \qquad \textbf{Ans.}$$

Subsequently, Eq. (2) gives

$$-2 = -0.2[\cos 60°(28.87^2) + (\sin 60°)\alpha]$$

$$\alpha = -469.57 \text{ rad/s}^2 = 470 \text{ rad/s}^2 ↺ \qquad \textbf{Ans.}$$

The negative sign indicates that $\boldsymbol{\alpha}$ is directed in the negative sense of θ.

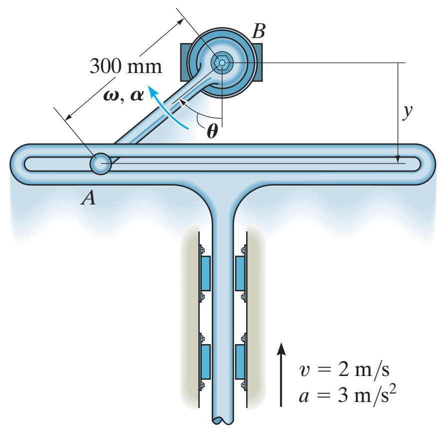


(a)

Ans:
$\omega = 28.9$ rad/s ↻
$\alpha = 470$ rad/s^2 ↺

16–41.

At the instant $\theta = 50°$, the slotted guide is moving upward with an acceleration of $3\ \text{m/s}^2$ and a velocity of $2\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant. *Note:* The upward motion of the guide is in the negative y direction.



SOLUTION

$$y = 0.3\cos\theta$$

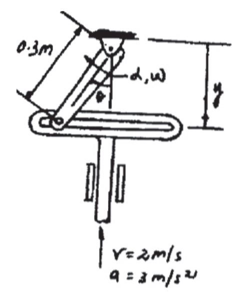
$$\dot{y} = v_y = -0.3\sin\theta\dot{\theta}$$

$$\ddot{y} = a_y = -0.3\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)$$

Here $v_y = -2\ \text{m/s}$, $a_y = -3\ \text{m/s}^2$, and $\dot{\theta} = \omega, \dot{\theta} = \omega, \ddot{\theta} = \alpha, \theta = 50°$.

$-2 = -0.3\sin 50°(\omega)$ $\qquad \omega = 8.70\ \text{rad/s}$ **Ans.**

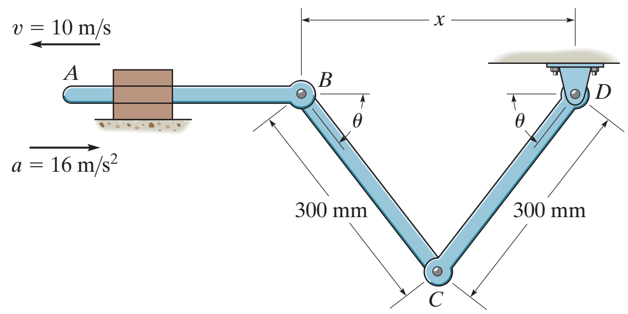
$-3 = -0.3[\sin 50°(\alpha) + \cos 50°(8.70)^2]$ $\qquad \alpha = -50.5\ \text{rad/s}^2$ **Ans.**



Ans:
$\omega = 8.70\ \text{rad/s}$
$\alpha = -50.5\ \text{rad/s}^2$

16–42.

At the instant shown, $\theta = 60^\circ$, and rod AB is subjected to a deceleration of $16\ \text{m/s}^2$ when the velocity is $10\ \text{m/s}$. Determine the angular velocity and angular acceleration of link CD at this instant.

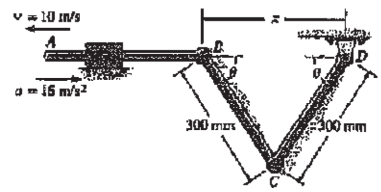


SOLUTION

$x = 2(0.3)\cos\theta$

$x = -0.6\sin\theta(\dot{\theta})$ **(1)**

$\ddot{x} = -0.6\cos\theta(\dot{\theta})^2 - 0.6\sin\theta(\ddot{\theta})$ **(2)**



Using Eqs. (1) and (2) at $\theta = 60^\circ$, $\dot{x} = 10\ \text{m/s}$, $\ddot{x} = -16\ \text{m/s}^2$.

$10 = -0.6\sin 60^\circ(\omega)$

$\omega = -19.245 = -19.2\ \text{rad/s}$ **Ans.**

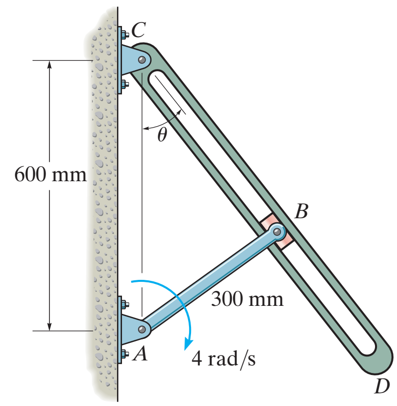
$-16 = -0.6\cos 60^\circ(-19.245)^2 - 0.6\sin 60^\circ(\alpha)$

$\alpha = -183\ \text{rad/s}^2$ **Ans.**

Ans:
$\omega = -19.2\ \text{rad/s}$
$\alpha = -183\ \text{rad/s}^2$

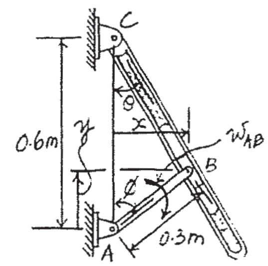
16–43.

The crank AB is rotating with a constant angular velocity of 4 rad/s. Determine the angular velocity of the connecting rod CD at the instant $\theta = 30°$.



SOLUTION

Position Coordinate Equation: From the geometry,



$$0.3 \sin \phi = (0.6 - 0.3 \cos \phi) \tan \theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$0.3 \cos \phi \frac{d\phi}{dt} = 0.6\sec^2\theta\frac{d\theta}{dt} - 0.3\left(\cos\theta \sec^2\theta \frac{d\theta}{dt} - \tan\theta \sin\theta \frac{d\phi}{dt}\right)$$

$$\frac{d\theta}{dt} = \left[\frac{0.3(\cos\phi - \tan\theta \sin\phi)}{0.3\sec^2\theta(2 - \cos\phi)}\right]\frac{d\phi}{dt} \qquad \textbf{[2]}$$

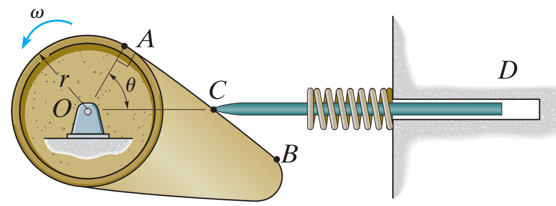
However, $\frac{d\theta}{dt} = \omega_{BC}$, $\frac{d\phi}{dt} = \omega_{AB} = 4$ rad/s. At the instant $\theta = 30°$, from Eq. [3], $\phi = 60.0°$. Substitute these values into Eq. [2] yields

$$\omega_{BC} = \left[\frac{0.3(\cos 60.0° - \tan 30° \sin 60.0°)}{0.3 \sec^2 30° (2 - \cos 60.0°)}\right](4) = 0 \qquad \textbf{Ans.}$$

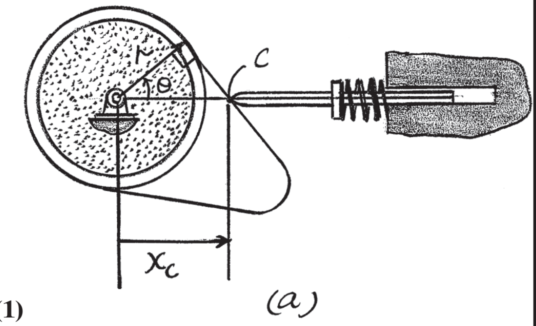
Ans:
$\omega_{AB} = 0$

***16–44.**

Determine the velocity and acceleration of the follower rod CD as a function of θ when the contact between the cam and follower is along the straight region AB on the face of the cam. The cam rotates with a constant counterclockwise angular velocity $\boldsymbol{\omega}$.



SOLUTION



Position Coordinate: From the geometry shown in Fig. a,

$$x_C = \frac{r}{\cos\theta} = r\sec\theta$$

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_C = r\sec\theta\tan\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = +\omega$ since ω acts in the positive rotational sense of θ. Thus, Eq. (1) gives

$$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_{CD} = \ddot{x}_C = r\{\sec\theta\tan\theta\ddot{\theta} + \dot{\theta}[\sec\theta(\sec^2\theta\dot{\theta}) + \tan\theta(\sec\theta\tan\theta\dot{\theta})]\}$$

$$a_{CD} = r[\sec\theta\tan\theta\ddot{\theta} + (\sec^3\theta + \sec\theta\tan^2\theta)\dot{\theta}^2]$$

Since $\dot{\theta} = \omega$ is constant, $\ddot{\theta} = \alpha = 0$. Then,

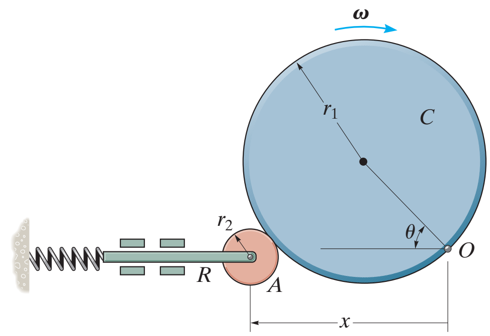
$$a_{CD} = r[\sec\theta\tan\theta(0) + (\sec^3\theta + \sec\theta\tan^2\theta)\omega^2]$$

$$= r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow \qquad \textbf{Ans.}$$

Ans:
$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow$
$a_{CD} = r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow$

16–45.

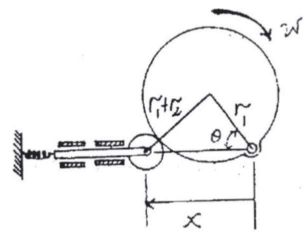
Determine the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of A on C.



SOLUTION

Position Coordinate Equation: Using law of cosine.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x \cos\theta \qquad \textbf{(1)}$$



Time Derivatives: Taking the time derivative of Eq. (1). we have

$$0 = 2x\frac{dx}{dt} - 2r_1\left(-x\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dx}{dt}\right) \qquad \textbf{(2)}$$

However $v = \dfrac{dx}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.(2),

$$0 = xv - r_1(v\cos\theta - x\omega\sin\theta)$$

$$v = \frac{r_1 x\omega\sin\theta}{r_1\cos\theta - x} \qquad \textbf{(3)}$$

However, the positive root of Eq.(1) is

$$x = r_1\cos\theta + \sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}$$

Substitute into Eq.(3), we have

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2\dot{r}_1 r_2}} + r_1\omega\sin\theta\right) \qquad \textbf{Ans.}$$

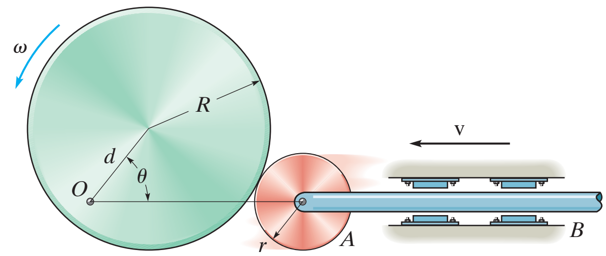
Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

Ans:

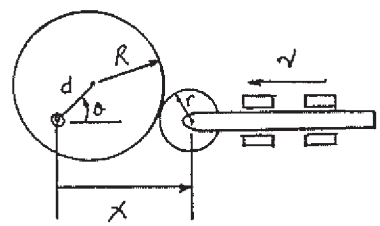
$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}} + r_1\omega\sin\theta\right)$$

16–46.

The circular cam rotates about the fixed point O with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity v of the follower rod AB as a function of θ.



SOLUTION

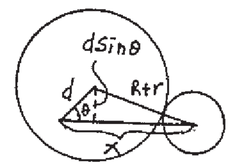


$$x = d\cos\theta + \sqrt{(R+r)^2 - (d\sin\theta)^2}$$

$$\dot{x} = v_{AB} = -d\sin\theta\dot{\theta} - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\dot{\theta} \qquad \text{Where } \dot{\theta} = \omega \text{ and } v_{AB} = -v$$

$$-v = -d\sin\theta(\omega) - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\omega$$

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right) \qquad \textbf{Ans.}$$

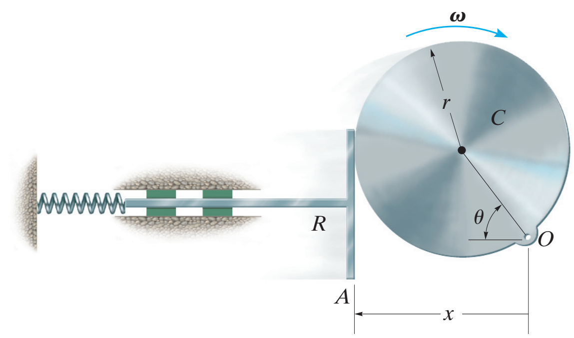


Ans:

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right)$$

16–47.

Determine the velocity of the rod R for any angle θ of cam C as the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of plate A on C.



SOLUTION

$$x = r + r\cos\theta$$

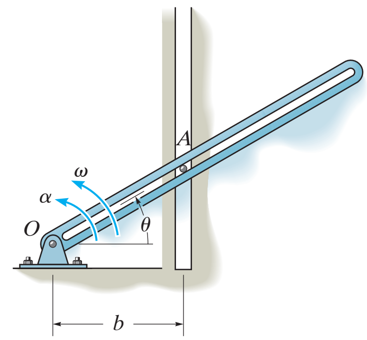
$$x = -r\sin\theta\theta$$

$$v = -r\omega\sin\theta \qquad \textbf{Ans.}$$

Ans:
$v = -r\omega\sin\theta$

***16–48.**

Determine the velocity and acceleration of the peg A which is confined between the vertical guide and the rotating slotted rod.



SOLUTION

Position Coordinate Equation. The rectilinear motion of peg A can be related to the angular motion of the slotted rod by relating y and θ using the geometry shown in Fig. a, which is

$$y = b \tan \theta$$

Time Derivatives. Using the chain rule,

$$\dot{y} = b(\sec^2 \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{y} = b[2 \sec \theta(\sec \theta \tan \theta\dot{\theta})\dot{\theta} + \sec^2 \theta\ddot{\theta}]$$

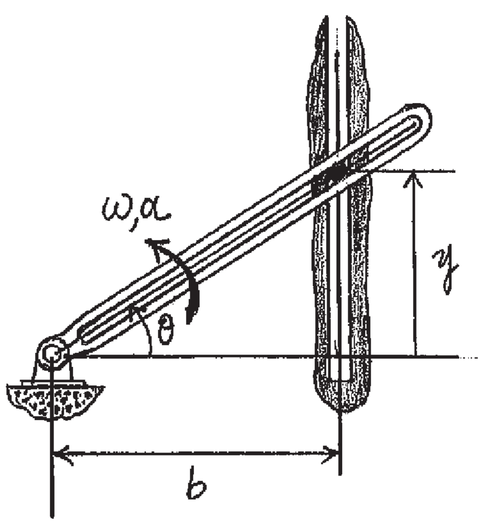
$$\ddot{y} = b(2 \sec^2 \theta \tan \theta\dot{\theta}^2 + \sec^2 \theta\ddot{\theta})$$

$$\ddot{y} = b \sec^2 \theta(2 \tan \theta\dot{\theta}^2 + \ddot{\theta}) \qquad \textbf{(2)}$$

Here, $\dot{y} = v, \ddot{y} = a, \dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$. Then Eqs. (1) and (2) become

$$v = \omega b \sec^2 \theta \qquad \textbf{Ans.}$$

$$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha) \qquad \textbf{Ans.}$$

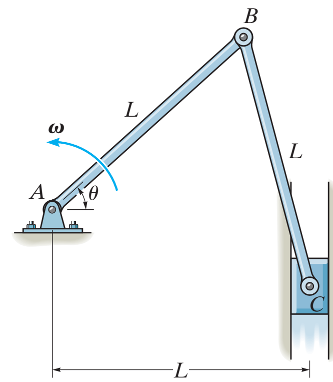


(a)

Ans:
$v = \omega b \sec^2 \theta$
$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha)$

16–49.

Bar AB rotates uniformly about the fixed pin A with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of block C, at the instant $\theta = 60°$.



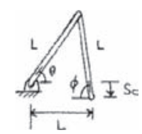
SOLUTION

$$L\cos\theta + L\cos\phi = L$$

$$\cos\theta + \cos\phi = 1$$

$$\sin\theta\,\dot{\theta} + \sin\phi\,\dot{\phi} = 0 \qquad \textbf{(1)}$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\ddot{\theta} + \sin\phi\,\ddot{\phi} + \cos\phi\,(\dot{\phi})^2 = 0 \qquad \textbf{(2)}$$



When $\theta = 60°, \phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from Eq. (1))

$\ddot{\theta} = 0$

$\ddot{\phi} = -1.155\omega^2$ (from Eq.(2))

Also, $s_C = L\sin\phi - L\sin\theta$

$$v_C = L\cos\phi\,\dot{\phi} - L\cos\theta\,\dot{\theta}$$

$$a_C = -L\sin\phi\,(\dot{\phi})^2 + L\cos\phi\,(\ddot{\phi}) - L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°, \phi = 60°$

$$s_C = 0$$

$$v_C = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega = L\omega\uparrow \qquad \textbf{Ans.}$$

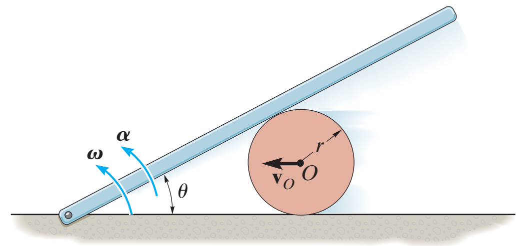
$$a_C = -L\sin 60°(-\omega)^2 + L\cos 60°(-1.155\omega^2) + 0 + L\sin 60°(\omega)^2$$

$$a_C = -0.577\,L\omega^2 = 0.577\,L\omega^2\uparrow \qquad \textbf{Ans.}$$

Ans:
$v_C = L\omega\uparrow$
$a_C = 0.577\,L\omega^2\uparrow$

16–50.

The center of the cylinder is moving to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar. Neglect the thickness of the bar.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the cylinder can be related to the angular motion of the rod by relating x and θ using the geometry shown in Fig. a, which is

$$x = \frac{r}{\tan \theta/2} = r \cot \theta/2$$

Time Derivatives. Using the chain rule,

$$\dot{x} = r\left[\left(-\csc^2 \theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\right]$$

$$\dot{x} = -\frac{r}{2}\left(\csc^2 \theta/2\right)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -\frac{r}{2}\left[2 \csc \theta/2\left(-\csc \theta/2 \cot \theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\dot{\theta} + \left(\csc^2 \theta/2\right)\ddot{\theta}\right]$$

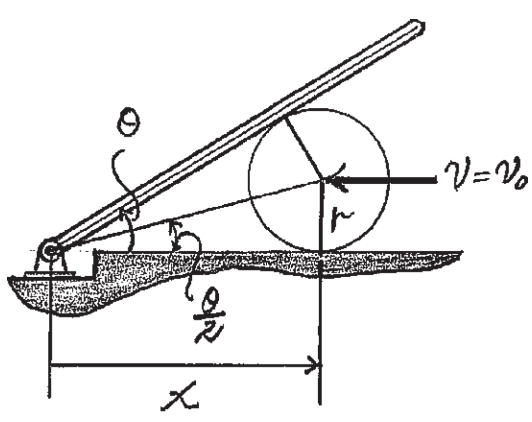
$$\ddot{x} = \frac{r}{2}\left[\left(\csc^2 \theta/2 \cot \theta/2\right)\dot{\theta}^2 - \left(\csc^2 \theta/2\right)\ddot{\theta}\right]$$

$$\ddot{x} = \frac{r \csc^2 \theta/2}{2}\left[\left(\cot \theta/2\right)\dot{\theta}^2 - \ddot{\theta}\right] \qquad \textbf{(2)}$$

Here $\dot{x} = -v_0$ since $\mathbf{v}_0$ is directed toward the negative sense of x and $\dot{\theta} = \omega$. Then Eq. (1) gives,

$$-v_0 = -\frac{r}{2}\left(\csc^2 \theta/2\right)\omega$$

$$\omega = \frac{2v_0}{r}\sin^2 \theta/2 \qquad \textbf{Ans.}$$



(a)

15–50. Continued

Also, $\ddot{x} = 0$ since v is constant and $\ddot{\theta} = \alpha$. Substitute the results of ω into Eq. (2):

$$0 = \frac{r \csc^2 \theta/2}{2}\left[(\cot \theta/2)\left(\frac{2v_0}{r}\sin^2 \theta/2\right)^2 - \alpha\right]$$

$$\alpha = (\cot \theta/2)\left(\frac{2r_0}{r}\sin^2 \theta/2\right)^2$$

$$\alpha = \left(\frac{\cos \theta/2}{\sin \theta/2}\right)\left(\frac{4v_0^2}{r^2}\sin^4 \theta/2\right)$$

$$\alpha = \frac{4v_0^2}{r^2}(\sin^3 \theta/2)(\cos \theta/2)$$

$$\alpha = \frac{2v_0^2}{r^2}(2 \sin \theta/2 \cos \theta/2)(\sin^2 \theta/2)$$

Since $\sin \theta = 2 \sin \theta/2 \cos \theta/2$, then

$$\alpha = \frac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2)$$ **Ans.**

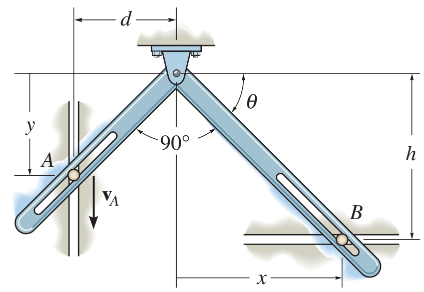
Ans:

$$\omega = \frac{2v_0}{r}\sin^2 \theta/2$$

$$\alpha = \frac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2)$$

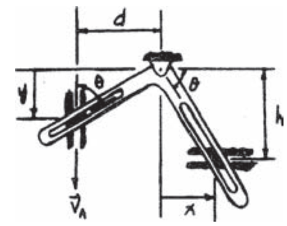
16–51.

The pins at A and B are confined to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at $\mathbf{v}_A$, determine the velocity of B at the instant shown.



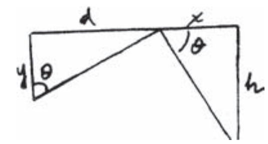
SOLUTION

Position coordinate equation:



$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$



Time derivatives:

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

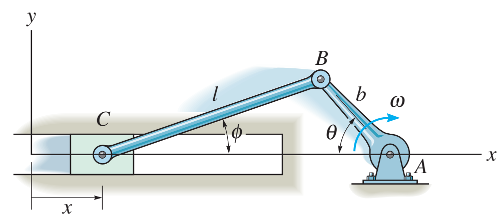
$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans.}$$

Ans:

$v_B = \left(\frac{h}{d}\right)v_A$

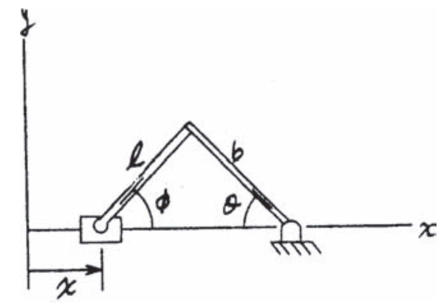
***16–52.**

The crank AB has a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of the slider at C as a function of θ. *Suggestion:* Use the x coordinate to express the motion of C and the ϕ coordinate for CB. $x = 0$ when $\phi = 0°$.



SOLUTION

$x = l + b - (L\cos\phi + b\cos\theta)$



$l\sin\phi = b\sin\theta$ or $\sin\phi = \frac{b}{l}\sin\theta$

$$v_C = \dot{x} = l\sin\phi\dot{\phi} + b\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

$$\cos\phi\dot{\phi} = \frac{b}{l}\cos\theta\dot{\theta} \qquad \textbf{(2)}$$

Since $\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}$

then,

$$\dot{\phi} = \frac{\left(\frac{b}{l}\right)\cos\theta\omega}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}} \qquad \textbf{(3)}$$

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}}\right] + b\omega\sin\theta \qquad \textbf{Ans.}$$

From Eq. (1) and (2):

$$a_C = \dot{v}_C = l\ddot{\phi}\sin\phi + l\dot{\phi}\cos\phi\dot{\phi} + b\cos\theta\left(\dot{\theta}\right)^2 \qquad \textbf{(4)}$$

$$-\sin\phi\dot{\phi}^2 + \cos\phi\ddot{\phi} = -\left(\frac{b}{l}\right)\sin\theta\dot{\theta}^2$$

$$\ddot{\phi} = \frac{\dot{\phi}^2\sin\phi - \frac{b}{l}\omega^2\sin\theta}{\cos\phi} \qquad \textbf{(5)}$$

Substituting Eqs. (1), (2), (3) and (5) into Eq. (4) and simplifying yields

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2\sin^4\theta\right)}{\left(1-\left(\frac{b}{l}\right)^2\sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right] \qquad \textbf{Ans.}$$

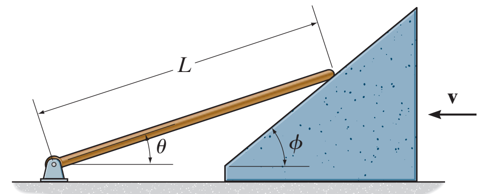
Ans:

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}}\right] + b\omega\sin\theta$$

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2\sin^4\theta\right)}{\left(1-\left(\frac{b}{l}\right)^2\sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right]$$

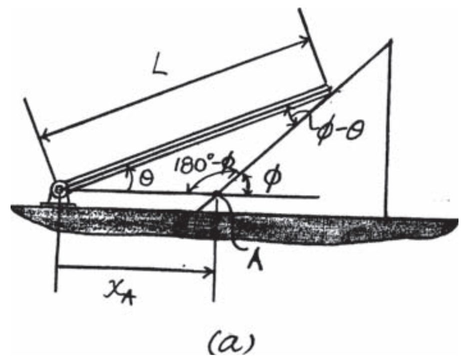
16–53.

If the wedge moves to the left with a constant velocity **v**, determine the angular velocity of the rod as a function of θ.



SOLUTION

Position Coordinates: Applying the law of sines to the geometry shown in Fig. *a*,



$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin(180^\circ - \phi)}$$

$$x_A = \frac{L\sin(\phi - \theta)}{\sin(180^\circ - \phi)}$$

However, $\sin(180^\circ - \phi) = \sin\phi$. Therefore,

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\phi}$$

Time Derivative: Taking the time derivative,

$$\dot{x}_A = \frac{L\cos(\phi - \theta)(-\dot{\theta})}{\sin\phi}$$

$$v_A = \dot{x}_A = -\frac{L\cos(\phi - \theta)\dot{\theta}}{\sin\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

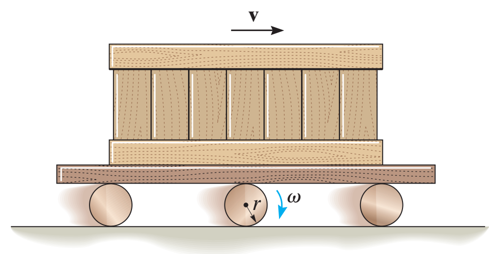
$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)} \qquad \textbf{Ans.}$$

Ans:

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)}$$

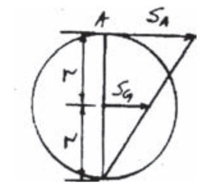
16–54.

The crate is transported on a platform which rests on rollers, each having a radius r. If the rollers do not slip, determine their angular velocity if the platform moves forward with a velocity **v**.



SOLUTION

Position coordinate equation: From Example 16.4, $s_G = r\theta$. Using similar triangles



$$s_A = 2s_G = 2r\theta$$

Time derivatives:

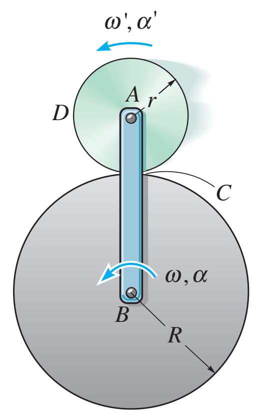
$$s_A = v = 2r\,\dot{\theta} \quad \text{Where } \dot{\theta} = \omega$$

$$\omega = \frac{v}{2r}$$ **Ans.**

Ans:
$\omega = \dfrac{v}{2r}$

16–55.

Arm AB has an angular velocity of $\boldsymbol{\omega}$ and an angular acceleration of $\boldsymbol{\alpha}$. If no slipping occurs between the disk D and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.



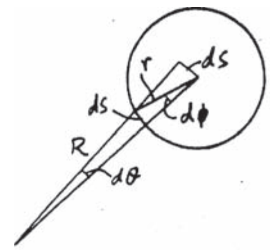
SOLUTION

$$ds = (R + r)\,d\theta = r\,d\phi$$

$$(R + r)\left(\frac{d\theta}{dt}\right) = r\left(\frac{d\phi}{dt}\right)$$

$$\omega' = \frac{(R + r)\omega}{r}$$ **Ans.**

$$\alpha' = \frac{(R + r)\alpha}{r}$$ **Ans.**



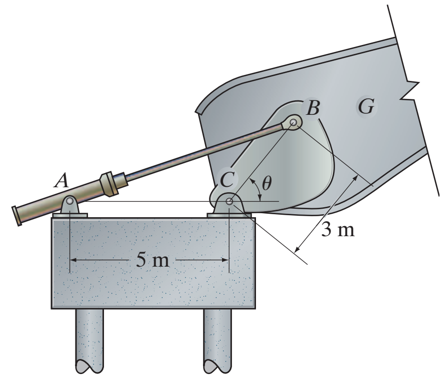
Ans:

$$\omega' = \frac{(R + r)\omega}{r}$$

$$\alpha' = \frac{(R + r)\alpha}{r}$$

***16–56.**

The bridge girder G of a bascule bridge is raised and lowered using the drive mechanism shown. If the hydraulic cylinder AB shortens at a constant rate of $0.15\,\text{m/s}$, determine the angular velocity of the bridge girder at the instant $\theta = 60°$.



SOLUTION

Position Coordinates: Applying the law of cosines to the geometry shown in Fig. *a*,

$$s^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(180°-\theta\right)$$

$$s^2 = 34 - 30\cos\left(180°-\theta\right)$$

However, $\cos\left(180°-\theta\right) = -\cos\theta$. Thus,

$$s^2 = 34 + 30\cos\theta$$

Time Derivatives: Taking the time derivative,

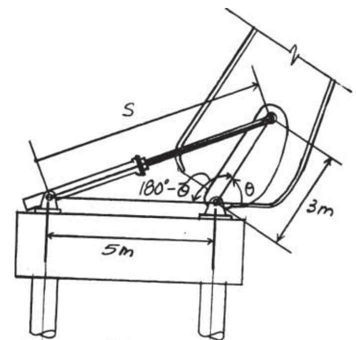
$$2s\dot{s} = 0 + 30\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = -15\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

When $\theta = 60°$, $s = \sqrt{34+30\cos 60°} = 7$ m. Also, $\dot{s} = -0.15$ m/s since $\dot{s}$ is directed towards the negative sense of s. Thus, Eq. (1) gives

$$7\left(-0.15\right) = -15\sin 60°\dot{\theta}$$

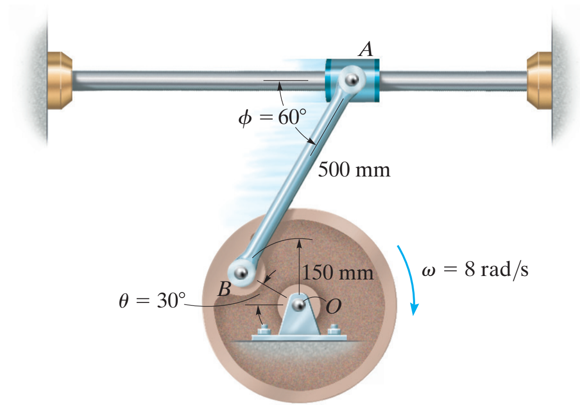
$$\omega = \dot{\theta} = 0.0808 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$\omega = 0.0808$ rad/s

16–57. The wheel is rotating with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the collar A at the instant $\theta = 30°$ and $\phi = 60°$. Also, sketch the location of bar AB when $\theta = 0°$, $30°$, and $60°$ to show its general plane motion.



SOLUTION

Given:

$\theta = 30°$

$\phi = 60°$

$\omega = 8 \text{ rad/s}$

$r_A = 500 \text{ mm}$

$r_B = 150 \text{ mm}$

Guesses $\omega_{AB} = 1 \text{ rad/s}$ $v_A = 1 \text{ m/s}$

Given

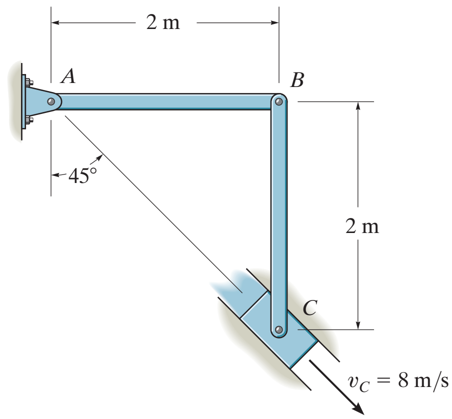
$$\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_B\cos(\theta) \\ r_B\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_A\cos(\phi) \\ r_A\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{AB} \\ v_A \end{pmatrix} = \text{Find}(\omega_{AB}, v_A) \qquad \omega_{AB} = -4.16 \text{ rad/s} \qquad v_A = 2.4 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 2.4$ m/s

16–58.

The slider block C moves at 8 m/s down the inclined groove. Determine the angular velocities of links AB and BC, at the instant shown.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$\mathbf{v}_B = (-\omega_{AB}\mathbf{k}) \times (2\mathbf{i}) = -2\omega_{AB}\mathbf{j}$$

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

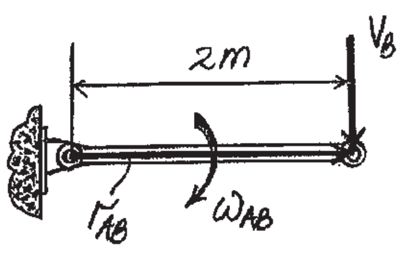
$$-2\omega_{AB}\mathbf{j} = (8 \sin 45°\mathbf{i} - 8 \cos 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (2\mathbf{j})$$

$$-2\omega_{AB}\mathbf{j} = (8 \sin 45° - 2\omega_{BC})\mathbf{i} - 8 \cos 45°\mathbf{j}$$

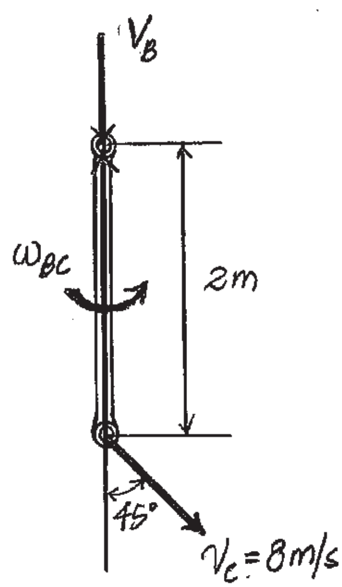
Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$$0 = 8 \sin 45° - 2\omega_{BC} \qquad \omega_{BC} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$$-2\omega_{AB} = -8 \cos 45° \qquad \omega_{AB} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



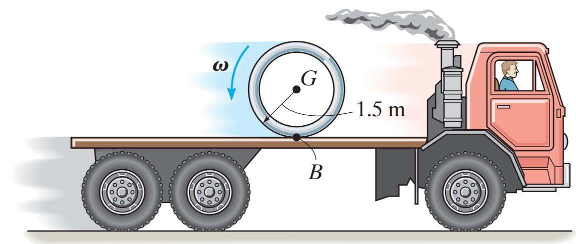
(a)



(b)

Ans:
$\omega_{BC} = 2.83 \text{ rad/s} \circlearrowleft$
$\omega_{AB} = 2.83 \text{ rad/s} \circlearrowright$

16–59. At the instant shown, the truck is traveling to the right at 3 m/s, while the pipe is rolling counterclockwise at angular $\omega = 8$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



SOLUTION

Given:

$v = 3$ m/s

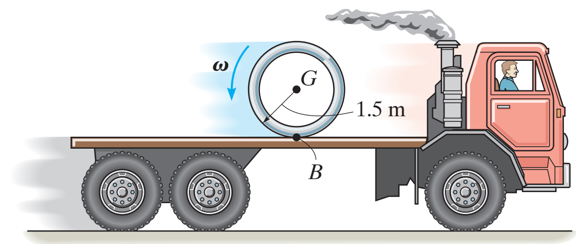
$\omega = 8$ rad/s

$r = 1.5$ m

$v_G = v + v_{GB}$ $\quad$ $v_G = v - \omega r$ $\quad$ $v_G = -9.00$ m/s $\quad$ **Ans.**

Ans:
$v_G = -9.00$ m/s

***16–60.** At the instant shown, the truck is traveling to the right at 8 m/s. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary.



SOLUTION

Given:

$v = 8\ \text{m/s}$

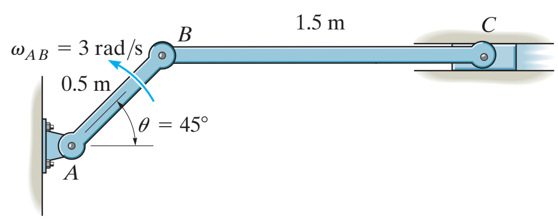
$r = 1.5\ \text{m}$

$v_G = v + v_{GB}$ $\qquad 0 = v - \omega r$ $\qquad \omega = \dfrac{v}{r}$ $\qquad \omega = 5.33\ \text{rad/s}$ **Ans.**

Ans:
$\omega = 5.33\ \text{rad/s}$

16–61.

The link AB has an angular velocity of 3 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$. Also, sketch the position of link BC when $\theta = 60°$, 45°, and 30° to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. *a*.

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{AB}$$
$$= (3\mathbf{k}) \times (0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j})$$
$$= \{-1.0607\mathbf{i} + 1.0607\mathbf{j}\} \text{ m/s}$$

General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

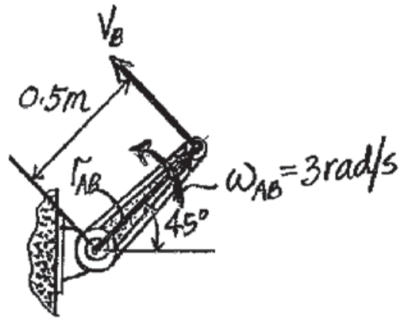
$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$
$$-v_C\mathbf{i} = (-1.0607\mathbf{i} + 1.0607\mathbf{j}) + (-\omega_{BC}\mathbf{k}) \times (1.5\mathbf{i})$$
$$-v_C\mathbf{i} = -1.0607\mathbf{i} + (1.0607 - 1.5\omega_{BC})\mathbf{j}$$

Equating **i** and **j** components;

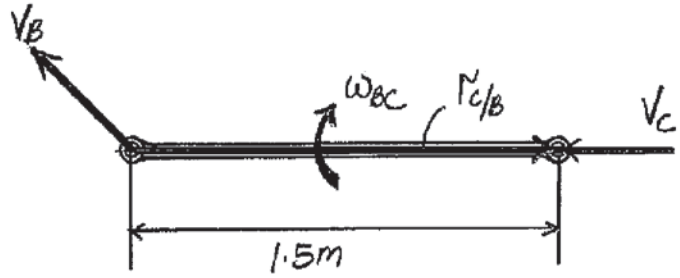
$-v_C = -1.0607$ $\qquad v_C = 1.0607 \text{ m/s} = 1.06 \text{ m/s}$ $\qquad$ **Ans.**

$0 = 1.0607 - 1.5\omega_{BC}$ $\qquad \omega_{BC} = 0.7071 \text{ rad/s} = 0.707 \text{ rad/s}$ $\qquad$ **Ans.**

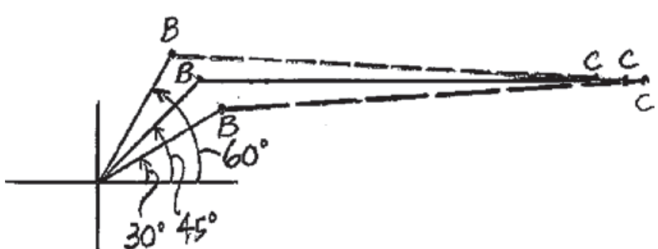
The general plane motion of link BC is described by its orientation when $\theta = 30°$, 45° and 60° shown in Fig. *c*.



(a)



(b)

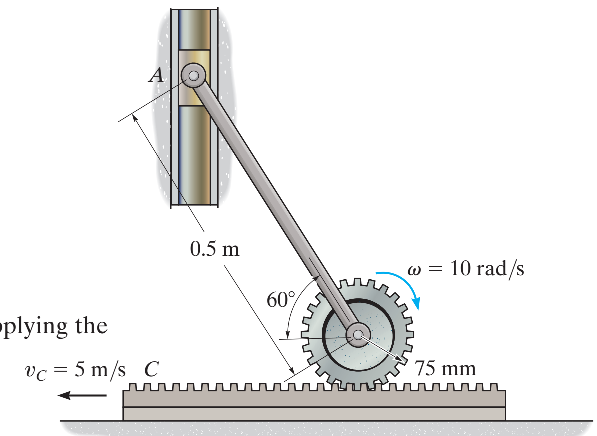


(c)

Ans:
$v_C = 1.06 \text{ m/s} \leftarrow$
$\omega_{BC} = 0.707 \text{ rad/s} \circlearrowleft$

16–62.

If the gear rotates with an angular velocity of $\omega = 10$ rad/s and the gear rack moves at $v_C = 5$ m/s, determine the velocity of the slider block A at the instant shown.



SOLUTION

General Plane Motion: Referring to the diagram shown in Fig. *a* and applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$= -5\mathbf{i} + (-10\mathbf{k}) \times (0.075\mathbf{j})$$

$$= [-4.25\mathbf{i}] \text{ m/s}$$

Then, applying the relative velocity equation to link AB shown in Fig. *b*,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{j} = -4.25\mathbf{i} + (-\omega_{AB}\mathbf{k}) \times (-0.5\cos 60°\,\mathbf{i} + 0.5\sin 60°\,\mathbf{j})$$

$$v_A\mathbf{j} = (0.4330\omega_{AB} - 4.25)\mathbf{i} + 0.25\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components, yields

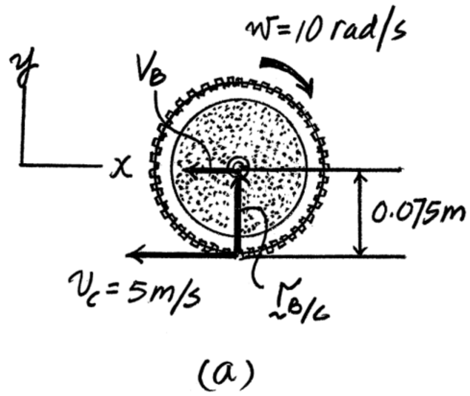
$$0 = 0.4330\omega_{AB} - 4.25 \qquad \textbf{(1)}$$

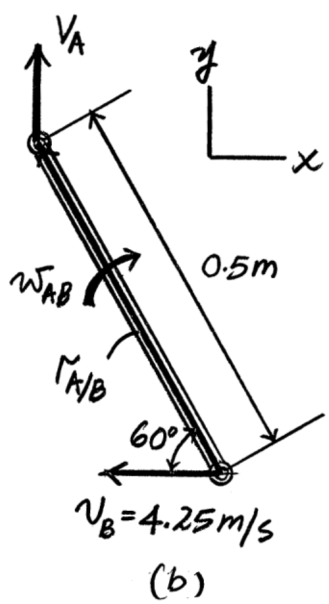
$$v_A = 0.25\omega_{AB} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\omega_{AB} = 9.815 \text{ rad/s}$$

$$v_A = 2.45 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

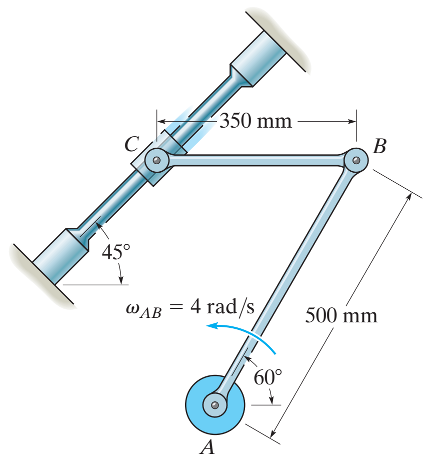




Ans:
$v_A = 2.45$ m/s $\uparrow$

16–63.

Knowing that angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.



SOLUTION

$v_B = \omega_{AB} r_{AB}$

$\quad = 4(0.5) = 2$ m/s

$\mathbf{v}_B = \{-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ m/s $\quad \mathbf{v}_C = -v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j}$

$\omega = \omega_{BC}\mathbf{k} \qquad \mathbf{r}_{C/B} = \{-0.35\mathbf{i}\}$ m

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$-v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j} = (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.35\mathbf{i})$

$-v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j} = -2\cos 30°\mathbf{i} + (2\sin 30° - 0.35\omega_{BC})\mathbf{j}$

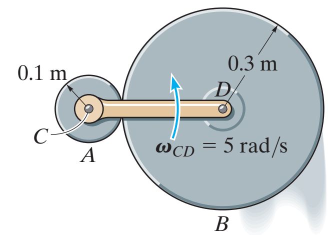
Equating the **i** and **j** components yields:

$-v_C \cos 45° = -2\cos 30° \qquad v_C = 2.45$ m/s **Ans.**

$-2.45\sin 45° = 2\sin 30° - 0.35\omega_{BC} \quad \omega_{BC} = 7.81$ rad/s **Ans.**

Ans:
$v_C = 2.45$ m/s
$\omega_{BC} = 7.81$ rad/s

***16–64.** The cylinder B rolls on the *fixed cylinder* A without slipping. If the connected bar CD is rotating with an angular velocity of $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B.



SOLUTION

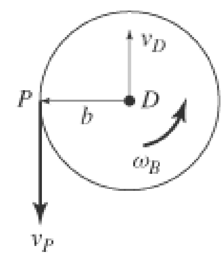
Given:

$$\omega_{CD} = 5 \text{ rad/s}$$

$$a = 0.1 \text{ m}$$

$$b = 0.3 \text{ m}$$

$$v_D = \omega_{CD}(a + b)$$

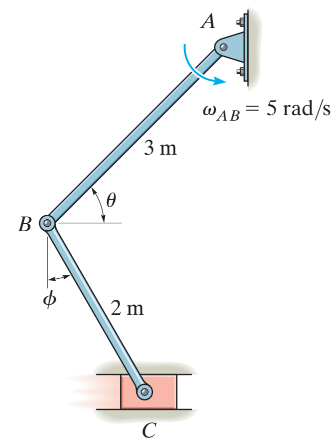


$$\omega_B = \frac{v_D}{b} \qquad \omega_B = 6.67 \text{ rad/s} \quad \textbf{Ans.}$$

Ans:
$\omega_B = 6.67$ rad/s

16–65.

The angular velocity of link AB is $\omega_{AB} = 5$ rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$ and $\phi = 30°$. Also, sketch the position of link CB when $\theta = 45°, 60°$, and $75°$ to show its general plane motion.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. *a*.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$= (5\mathbf{k}) \times (-3\cos 45°\mathbf{i} - 3\sin 45°\mathbf{j})$$

$$= \left\{\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right\}\text{ m/s}$$

General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$v_C\,\mathbf{i} = \left(\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right) + (\omega_{BC}\mathbf{k}) \times (2\sin 30°\,\mathbf{i} - 2\cos 30°\mathbf{j})$$

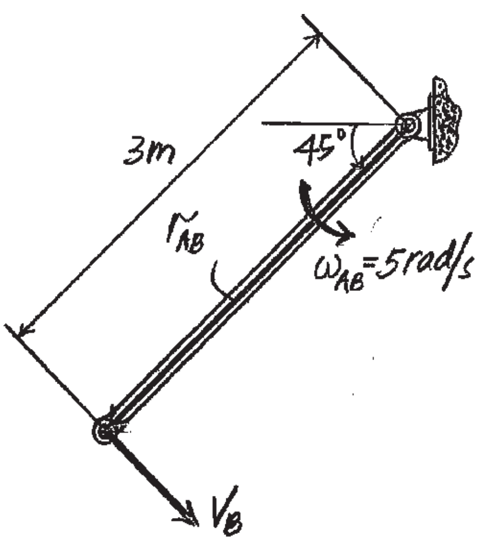
$$v_C\,\mathbf{i} = \left(\frac{15\sqrt{2}}{2} + \sqrt{3}\,\omega_{BC}\right)\mathbf{i} + \left(\omega_{BC} - \frac{15\sqrt{2}}{2}\right)\mathbf{j}$$

Equating **j** components,

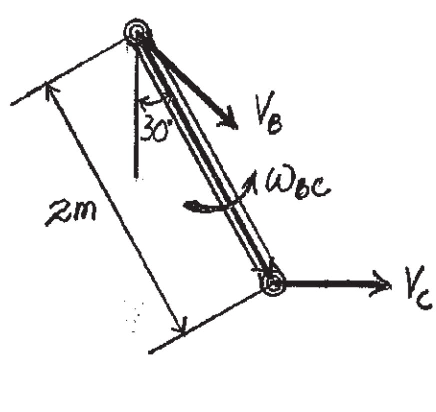
$$0 = \omega_{BC} - \frac{15\sqrt{2}}{2};\ \omega_{BC} = \frac{15\sqrt{2}}{2}\text{ rad/s} = 10.6\text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Then, equating **i** components,

$$v_C = \frac{15\sqrt{2}}{2} + \sqrt{3}\left(\frac{15\sqrt{2}}{2}\right) = 28.98\text{ m/s} = 29.0\text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



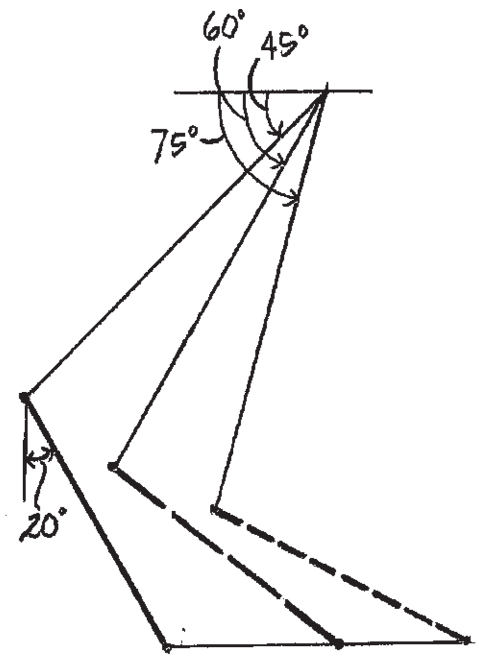
(a)



(b)

16–65. Continued

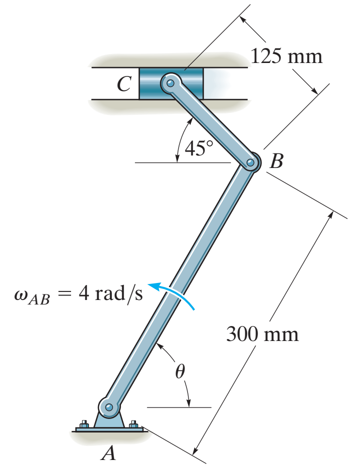
The general plane motion of link BC is described by its orientation when $\theta = 45°$, $60°$ and $75°$ shown in Fig. *c*



(c)

Ans:
$\omega_{BC} = 10.6$ rad/s ↺
$v_C = 29.0$ m/s →

16–66. The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant $\theta = 45°$, if link AB is rotating at 4 rad/s.



SOLUTION

Given:

$\theta = 60°$

$\phi = 45°$

$\omega_{AB} = 4 \text{ rad/s}$

$a = 300 \text{ mm}$

$b = 125 \text{ mm}$

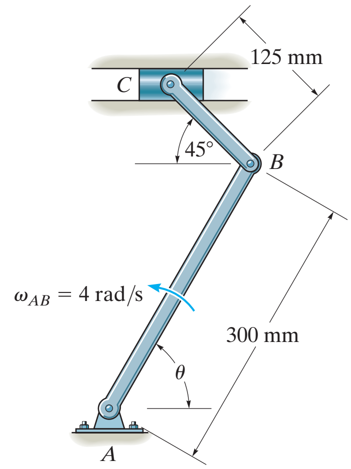
Guesses $\omega_{BC} = 1 \text{ rad/s}$ $v_C = 1 \text{ m/s}$

Given
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 6.79 \text{ rad/s} \qquad v_C = -1.64 \text{ m/s}$$ **Ans.**

Ans:
$v_C = -1.64$ m/s

16–67. Determine the velocity of the slider block at C at the instant $\theta = 45°$, if link AB is rotating at 4 rad/s.



SOLUTION

Given:

$\theta = 45°$

$\phi = 45°$

$\omega_{AB} = 4\ \text{rad/s}$

$a = 300\ \text{mm}$

$b = 125\ \text{mm}$

Guesses $\quad \omega_{BC} = 1\ \text{rad/s} \quad v_C = 1\ \text{m/s}$

Given

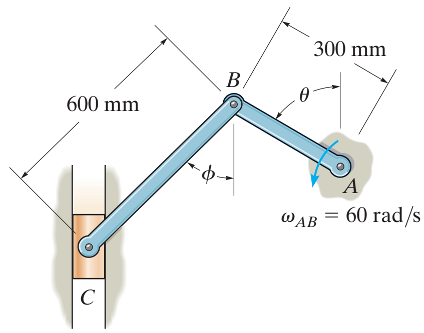
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 1.00\ \text{rad/s} \qquad v_C = -1.70\ \text{m/s}$$ **Ans.**

Ans:
$v_C = -1.70$ m/s

***16–68.**

Rod AB is rotating with an angular velocity of $\omega_{AB} = 60$ rad/s. Determine the velocity of the slider C at the instant $\theta = 60°$ and $\phi = 45°$. Also, sketch the position of bar BC when $\theta = 30°$, $60°$ and $90°$ to show its general plane motion.



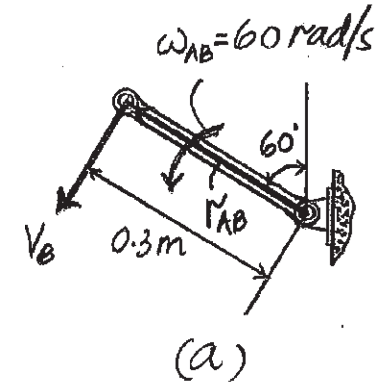
SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{V}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$= (60\mathbf{k}) \times (-0.3\sin 60°\mathbf{i} + 0.3\cos 60°\mathbf{j})$$

$$= \{-9\mathbf{i} - 9\sqrt{3}\mathbf{j}\}\ \text{m/s}$$



General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{V}_C = \mathbf{V}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$-v_C\mathbf{j} = (-9\mathbf{i} - 9\sqrt{3}\,\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.6\sin 45°\mathbf{i} - 0.6\cos 45°\mathbf{j})$$

$$-v_C\mathbf{j} = \left(0.3\sqrt{2}\omega_{BC} - 9\right)\mathbf{i} + \left(-0.3\sqrt{2}\omega_{BC} - 9\sqrt{3}\right)\mathbf{j}$$

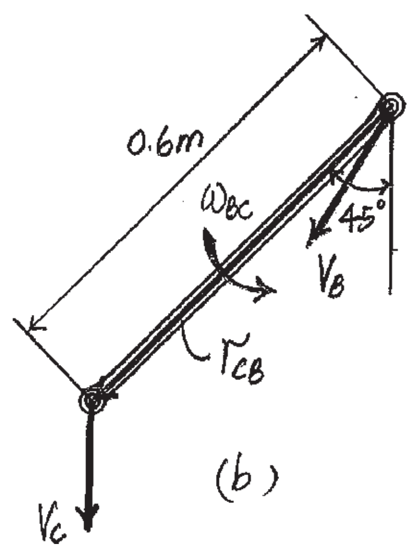
Equating $\mathbf{i}$ components,

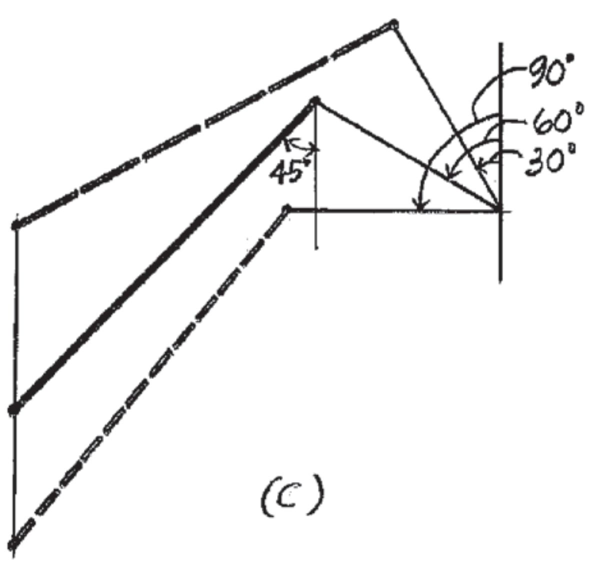
$$0 = 0.3\sqrt{2}\omega_{BC} - 9; \qquad \omega_{BC} = 15\sqrt{2}\ \text{rad/s} = 21.2\ \text{rad/s}\ ↺$$

Then, equating $\mathbf{j}$ components,

$$-v_C = \left(-0.3\sqrt{2}\right)\left(15\sqrt{2}\right) - 9\sqrt{3}; \quad v_C = 24.59\ \text{m/s} = 24.6\ \text{m/s}\downarrow \quad \textbf{Ans.}$$

The general plane motion of link BC is described by its orientation when $\theta = 30°$, $60°$ and $90°$ shown in Fig. c.

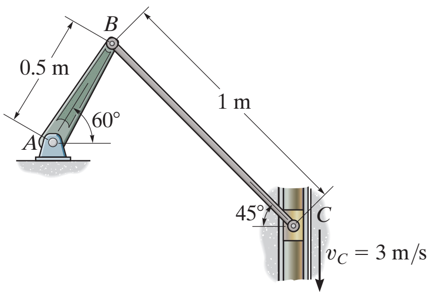




Ans:
$v_C = 24.6\ \text{m/s}\downarrow$

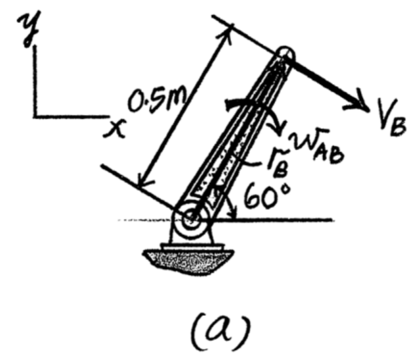
16–69.

If the slider block C is moving at $v_C = 3$ m/s, determine the angular velocity of BC and the crank AB at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. *a*,



$$v_B = \omega_{AB} \times \mathbf{r}_B$$

$$= (-\omega_{AB}\,\mathbf{k}) \times (0.5 \cos 60°\,\mathbf{i} + 0.5 \sin 60°\,\mathbf{j})$$

$$= 0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j}$$

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of link BC shown in Fig. *b*,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = -3\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (-1 \cos 45°\,\mathbf{i} + 1 \sin 45°\,\mathbf{j})$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = 0.7071\omega_{BC}\mathbf{i} + (0.7071\omega_{BC} - 3)\mathbf{j}$$

Equating the **i** and **j** components yields,

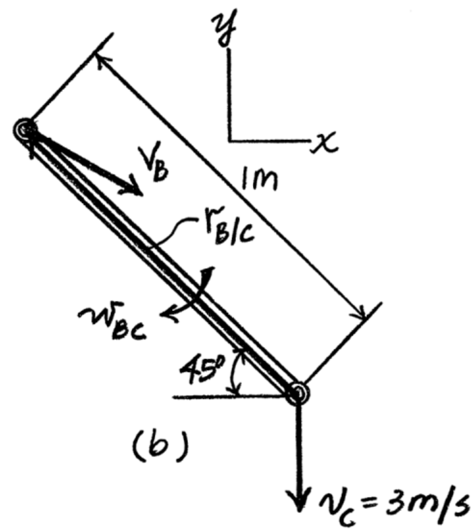
$$0.4330\omega_{AB} = 0.7071\omega_{BC}$$

$$-0.25\omega_{AB} = 0.7071\omega_{BC} - 3$$

Solving,

$$\omega_{BC} = 2.69 \text{ rad/s} \qquad \textbf{Ans.}$$

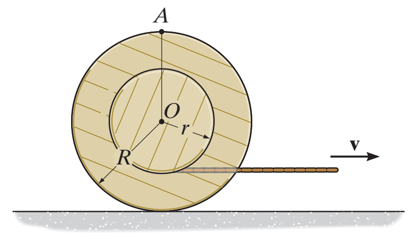
$$\omega_{AB} = 4.39 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega_{BC} = 2.69$ rad/s
$\omega_{AB} = 4.39$ rad/s

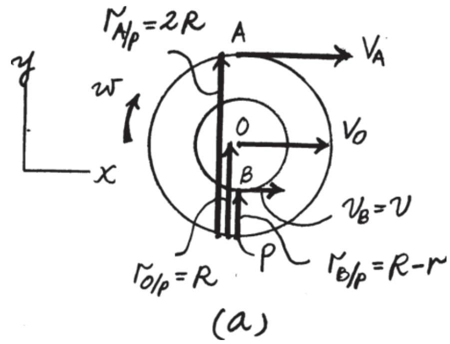
16–70.

Determine the velocity of the center O of the spool when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION

Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.



General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_O = \mathbf{v}_P + \omega \times \mathbf{r}_{O/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times R\mathbf{j}$$

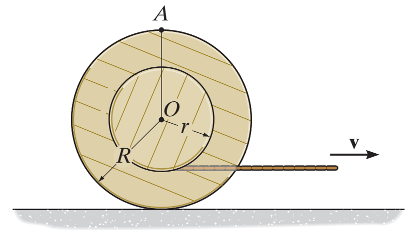
$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow \qquad \textbf{Ans.}$$

Ans:

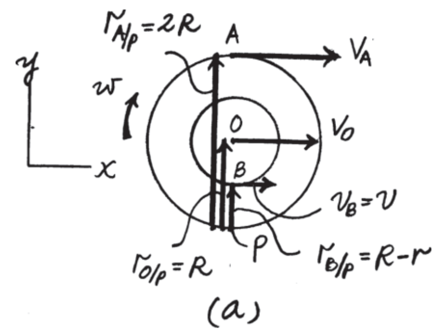
$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow$$

16–71.

Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.

General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

$$\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{A/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

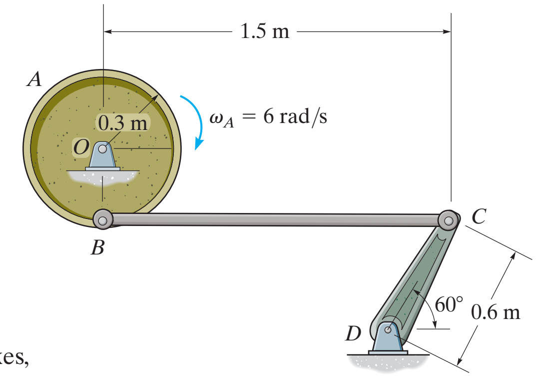
$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$ **Ans.**

Ans:

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$

***16–72.**

If the flywheel is rotating with an angular velocity of $\omega_A = 6$ rad/s, determine the angular velocity of rod BC at the instant shown.



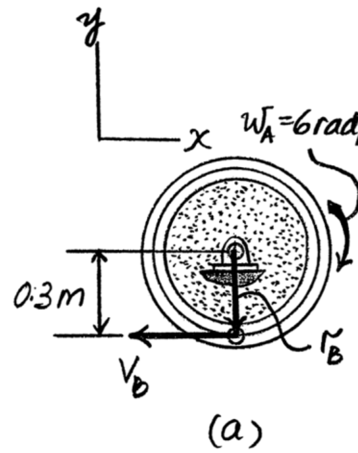
SOLUTION

Rotation About a Fixed Axis: Flywheel A and rod CD rotate about fixed axes, Figs. a and b. Thus, the velocity of points B and C can be determined from

$$v_B = \omega_A \times \mathbf{r}_B = (-6\mathbf{k}) \times (-0.3\mathbf{j}) = [-1.8\mathbf{i}] \text{ m/s}$$

$$v_C = \omega_{CD} \times \mathbf{r}_C = (\omega_{CD}\mathbf{k}) \times (0.6\cos 60°\,\mathbf{i} + 0.6\sin 60°\,\mathbf{j})$$

$$= -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j}$$



General Plane Motion: By referring to the kinematic diagram of link BC shown in Fig. c and applying the relative velocity equation, we have

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-1.5\mathbf{i})$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + (0.3\omega_{CD} - 1.5\omega_{BC})\mathbf{j}$$

Equating the **i** and **j** components

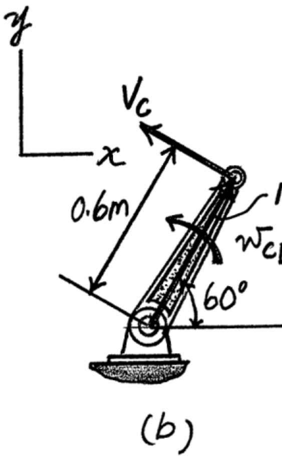
$$-1.8 = -0.5196\omega_{CD}$$

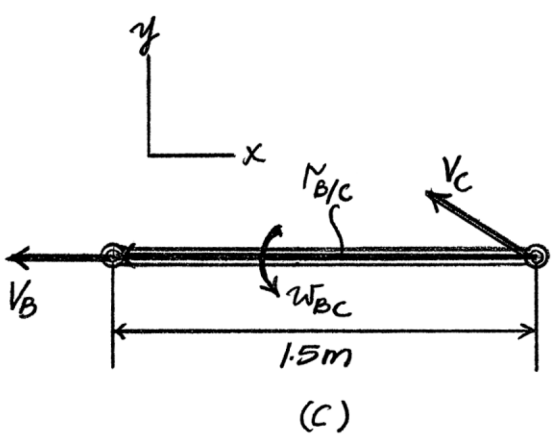
$$0 = 0.3\omega_{CD} - 1.5\omega_{BC}$$

Solving,

$$\omega_{CD} = 3.46 \text{ rad/s}$$

$$\omega_{BC} = 0.693 \text{ rad/s}$$ **Ans.**

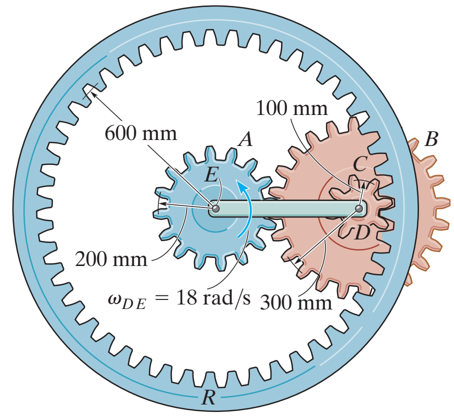




Ans:
$\omega_{BC} = 0.693$ rad/s

16–73.

The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



SOLUTION

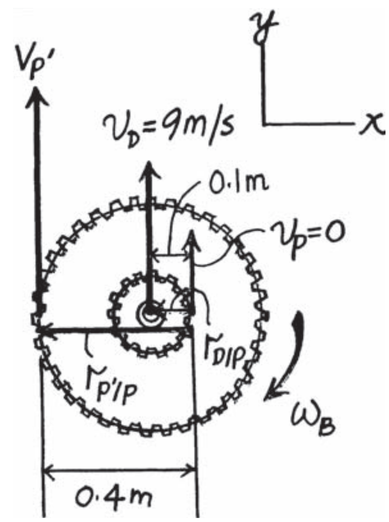
$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9 \text{ m/s} \uparrow$

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \omega \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90 \text{ rad/s} \;\circlearrowright$ **Ans.**



Let P' be the contact point between A and B.

$\mathbf{v}_{P'} = \mathbf{v}_P + \omega \times \mathbf{r}_{P'/P}$

$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

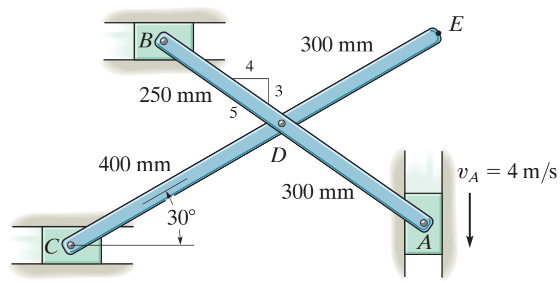
$v_{P'} = 36 \text{ m/s} \uparrow$

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180 \text{ rad/s} \;\circlearrowleft$ **Ans.**

Ans:
$\omega_B = 90 \text{ rad/s} \;\circlearrowright$
$\omega_A = 180 \text{ rad/s} \;\circlearrowleft$

16–74.

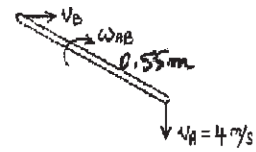
If the slider block A is moving downward at $v_A = 4\text{ m/s}$, determine the velocities of blocks B and C at the instant shown.



SOLUTION

$$v_B = v_A + v_{B/A}$$

$$\underset{\rightarrow}{v_B} = 4\downarrow + \omega_{AB}(0.55)$$



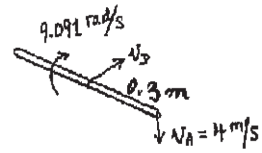
$$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{AB}(0.55)\left(\frac{3}{5}\right)$$

$$(+\uparrow) \qquad 0 = -4 + \omega_{AB}(0.55)\left(\frac{4}{5}\right)$$

Solving,

$$\omega_{AB} = 9.091\text{ rad/s}$$

$$v_B = 3.00\text{ m/s} \qquad \textbf{Ans.}$$

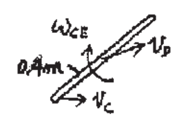


$$v_D = v_A + v_{D/A}$$

$$\mathbf{v}_D = \underset{\downarrow}{4} + \underset{{}_4\nearrow^3_5}{[(0.3)(9.091) = 2.727]}$$

$$v_C = v_D + v_{C/D}$$

$$\underset{\rightarrow}{v_C} = \underset{\downarrow}{4} + \underset{{}^5\nearrow^3_4}{2.727} + \underset{\nwarrow 30^\circ}{\omega_{CE}(0.4)}$$



$$(\overset{+}{\rightarrow}) \qquad v_C = 0 + 2.727\left(\frac{3}{5}\right) - \omega_{CE}(0.4)(\sin 30^\circ)$$

$$(+\uparrow) \qquad 0 = -4 + 2.727\left(\frac{4}{5}\right) + \omega_{CE}(0.4)(\cos 30^\circ)$$

$$\omega_{CE} = 5.249\text{ rad/s}$$

$$v_C = 0.587\text{ m/s} \qquad \textbf{Ans.}$$

Also:

$$v_B = v_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \left\{\frac{-4}{5}(0.55)\mathbf{i} + \frac{3}{5}(0.55)\mathbf{j}\right\}$$

16–74. Continued

$v_B = \omega_{AB}(0.33)$

$0 = -4 + 0.44\omega_{AB}$

$\omega_{AB} = 9.091 \text{ rad/s}$

$v_B = 3.00 \text{ m/s}$ **Ans.**

$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$

$v_D = -4\mathbf{j} + (-9.091\mathbf{k}) \times \left\{\frac{-4}{5}(0.3)\mathbf{i} + \frac{3}{5}(0.3)\mathbf{j}\right\}$

$v_D = \{1.636\mathbf{i} - 1.818\mathbf{j}\} \text{ m/s}$

$v_C = v_D + \omega_{CE} \times \mathbf{r}_{C/D}$

$v_C\mathbf{i} = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-\omega_{CE}\mathbf{k}) \times (-0.4 \cos 30°\mathbf{i} - 0.4 \sin 30°\mathbf{j})$

$v_C = 1.636 - 0.2\omega_{CE}$

$0 = -1.818 - 0.346\omega_{CE}$

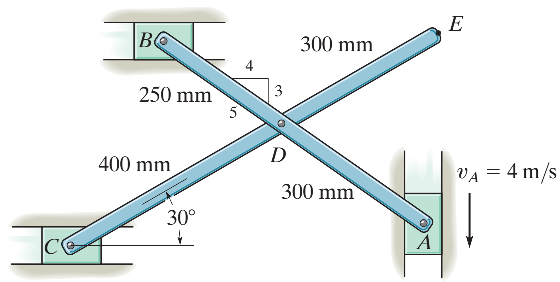
$\omega_{CE} = 5.25 \text{ rad/s}$

$v_C = 0.587 \text{ m/s}$ **Ans.**

Ans:
$v_B = 3.00 \text{ m/s}$
$v_C = 0.587 \text{ m/s}$
$v_B = 3.00 \text{ m/s}$
$v_C = 0.587 \text{ m/s}$

16–75.

If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of point E at the instant shown.

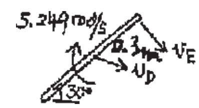


SOLUTION

See solution to Prob. 16–89.

$$v_E = v_D + v_{E/D}$$

$$\overrightarrow{v_E} = 4\downarrow + \underset{4\,\overset{3}{\nearrow}\,5}{2.727} + \underset{\nwarrow 30^\circ}{(5.249)(0.3)}$$



$$(\overset{+}{\rightarrow}) \qquad (v_E)_x = 0 + 2.727\left(\frac{3}{5}\right) + 5.249(0.3)(\sin 30^\circ)$$

$$(+\downarrow) \qquad (v_E)_y = 4 - 2.727\left(\frac{4}{5}\right) + 5.249(0.3)(\cos 30^\circ)$$

$$(v_E)_x = 2.424 \text{ m/s} \rightarrow$$

$$(v_E)_y = 3.182 \text{ m/s} \downarrow$$

$$\mathbf{v}_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7^\circ \qquad \textbf{Ans.}$$

Also:

See solution to Prob. 16–89.

$$\mathbf{v}_E = \mathbf{v}_D + \omega_{CE} \times \mathbf{r}_{E/D}$$

$$\mathbf{v}_E = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-5.25\mathbf{k}) \times \{\cos 30^\circ(0.3)\mathbf{i} - 0.4 \sin 30^\circ(0.3)\mathbf{j}\}$$

$$\mathbf{v}_E = \{2.424\mathbf{i} - 3.182\mathbf{j}\} \text{ m/s}$$

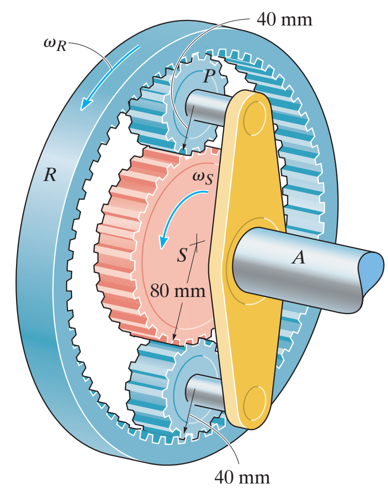
$$v_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7^\circ \qquad \textbf{Ans.}$$

Ans:
$v_E = 4.00$ m/s
$\theta = 52.7^\circ$ ◹

***16–76.**

The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



SOLUTION

$v_A = 5(80) = 400 \text{ mm/s} \leftarrow$

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p \mathbf{k}) \times (80\mathbf{j})$

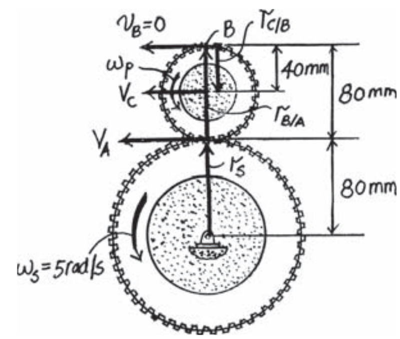
$0 = -400\mathbf{i} - 80\omega_p \mathbf{i}$

$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$ **Ans.**

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

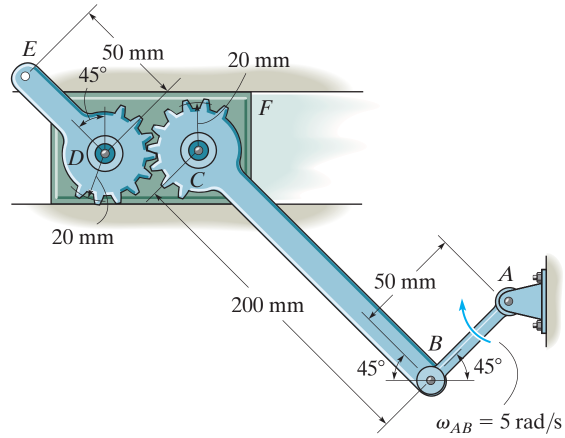
$\omega_A = \dfrac{200}{120} = 1.67 \text{ rad/s}$ **Ans.**



Ans:
$\omega_P = 5 \text{ rad/s}$
$\omega_A = 1.67 \text{ rad/s}$

16–77.

The mechanism is used on a machine for the manufacturing of a wire product. Because of the rotational motion of link AB and the sliding of block F, the segmental gear lever DE undergoes general plane motion. If AB is rotating at $\omega_{AB} = 5$ rad/s, determine the velocity of point E at the instant shown.



SOLUTION

$v_B = \omega_{AB} r_{AB} = 5(50) = 250$ mm/s 45° ⦣

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$v_C = 250 + \omega_{BC}(200)$

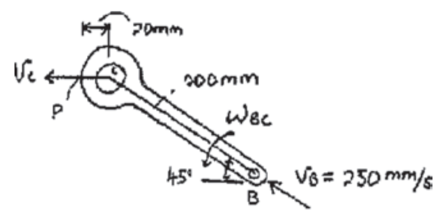
$\leftarrow$ 45° ⦣ 45° ⦢

$(+\uparrow)\quad 0 = 250 \sin 45° - \omega_{BC}(200) \sin 45°$

$(\overset{+}{\leftarrow})\quad v_C = 250 \cos 45° + \omega_{BC}(200) \cos 45°$

Solving,

$v_C = 353.6$ mm/s; $\qquad \omega_{BC} = 1.25$ rad/s



$\mathbf{v}_p = \mathbf{v}_C + \mathbf{v}_{p/C}$

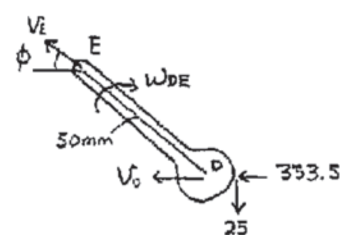
$v_p = 353.6 + [(1.25)(20) = 25]$

$\rightarrow \quad \leftarrow \qquad\qquad \downarrow$

$\mathbf{v}_D = \mathbf{v}_p + \mathbf{v}_{D/p}$

$v_D = (353.6 + 25) + 20\omega_{DE}$

$\leftarrow \quad \leftarrow \qquad \downarrow \qquad \uparrow$



$(\overset{+}{\leftarrow})\quad v_D = 353.6 + 0 + 0$

$(+\downarrow)\quad 0 = 0 + (1.25)(20) - \omega_{DE}(20)$

Solving,

$v_D = 353.6$ mm/s; $\qquad \omega_{DE} = 1.25$ rad/s

$\mathbf{v}_E = \mathbf{v}_D + \mathbf{v}_{E/D}$

$v_E = 353.6 + 1.25(50)$

ϕ ⦣ $\quad \leftarrow \qquad$ ⦢45°

$(\overset{+}{\leftarrow})\quad v_E \cos \phi = 353.6 - 1.25(50) \cos 45°$

$(+\uparrow)\quad v_E \sin \phi = 0 + 1.25(50) \sin 45°$

Solving,

$v_E = 312$ mm/s **Ans.**

$\phi = 8.13°$ **Ans.**

16–77. Continued

Also;

$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (-5\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} - 0.05 \sin 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.2 \cos 45°\mathbf{i} + 0.2 \sin 45°\mathbf{j})$

$-v_C = -0.1768 - 0.1414\omega_{BC}$

$0 = 0.1768 - 0.1414\omega_{BC}$

$\omega_{BC} = 1.25 \text{ rad/s}, \qquad v_C = 0.254 \text{ m/s}$

$\mathbf{v}_p = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C}$

$\mathbf{v}_D = \mathbf{v}_p + \omega_{DE} \times \mathbf{r}_{D/p}$

$\mathbf{v}_D = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C} + \omega_{DE} \times \mathbf{r}_{D/p}$

$v_D\mathbf{i} = -0.354\mathbf{i} + (1.25\mathbf{k}) \times (-0.02\mathbf{i}) + (\omega_{DE}\mathbf{k}) \times (-0.02\mathbf{i})$

$v_D = -0.354$

$0 = -0.025 - \omega_{DE}(0.02)$

$v_D = 0.354 \text{ m/s}, \qquad \omega_{DE} = 1.25 \text{ rad/s}$

$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$

$(v_E)_x\mathbf{i} + (v_E)_y\mathbf{j} = -0.354\mathbf{i} + (-1.25\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} + 0.05 \sin 45°\mathbf{j})$

$(v_E)_x = -0.354 + 0.0442 = -0.3098$

$(v_E)_y = 0.0442$

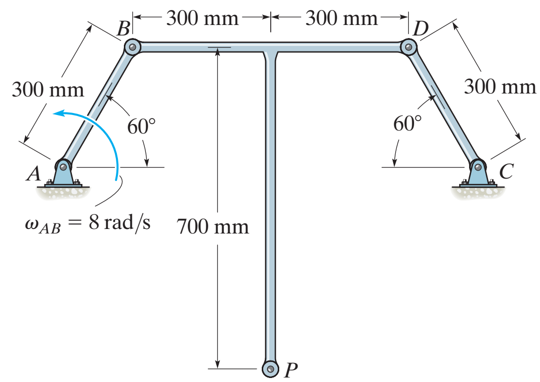
$v_E = \sqrt{(-0.3098)^2 + (0.0442)^2} = 312 \text{ mm/s}$ **Ans.**

$\phi = \tan^{-1}\left(\frac{0.0442}{0.3098}\right) = 8.13°$ **Ans.**

Ans:
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$

16–78.

The similar links AB and CD rotate about the fixed pins at A and C. If AB has an angular velocity $\omega_{AB} = 8$ rad/s, determine the angular velocity of BDP and the velocity of point P.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_B + \omega \times \mathbf{r}_{D/B}$

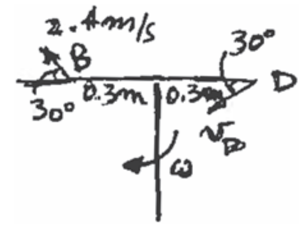
$-v_D \cos 30°\mathbf{i} - v_D \sin 30°\mathbf{j} = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (\omega\mathbf{k}) \times (0.6\mathbf{i})$

$-v_D \cos 30° = -2.4 \cos 30°$

$-v_D \sin 30° = 2.4 \sin 30° + 0.6\omega$

$v_D = 2.4$ m/s

$\omega = -4$ rad/s **Ans.**



$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$\mathbf{v}_P = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (-4\mathbf{k}) \times (0.3\mathbf{i} - 0.7\mathbf{j})$

$(v_P)_x = -4.88$ m/s

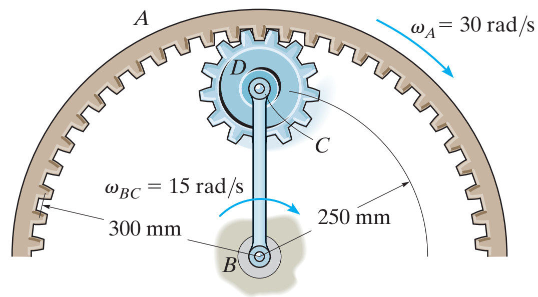
$(v_P)_y = 0$

$v_P = 4.88$ m/s $\leftarrow$ **Ans.**

Ans:
$v_P = 4.88$ m/s $\leftarrow$

16–79.

If the ring gear A rotates clockwise with an angular velocity of $\omega_A = 30$ rad/s, while link BC rotates clockwise with an angular velocity of $\omega_{BC} = 15$ rad/s, determine the angular velocity of gear D.



SOLUTION

Rotation About A Fixed Axis. The magnitudes of the velocity of Point E on the rim and center C of gear D are

$$v_E = \omega_A r_A = 30(0.3) = 9 \text{ m/s}$$

$$v_C = \omega_{BC} r_{BC} = 15(0.25) = 3.75 \text{ m/s}$$

General Plane Motion. Applying the relative velocity equation by referring to Fig. *a*,

$$\mathbf{v}_E = \mathbf{v}_C + \boldsymbol{\omega}_D \times \mathbf{r}_{E/C}$$

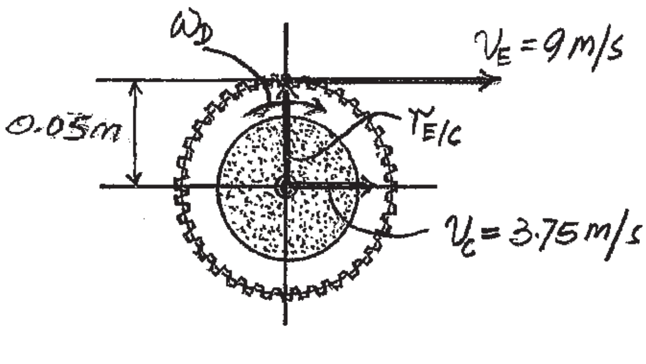
$$9\mathbf{i} = 3.75\mathbf{i} + (-\omega_D\mathbf{k}) \times (0.05\mathbf{j})$$

$$9\mathbf{i} = (3.75 + 0.05\omega_D)\mathbf{i}$$

Equating **i** component,

$$9 = 3.75 + 0.05\omega_D$$

$$\omega_D = 105 \text{ rad/s} \curvearrowright$$ **Ans.**

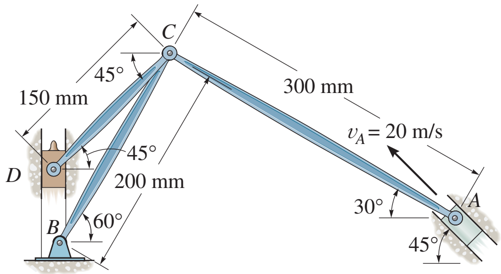


(a)

Ans:
$\omega_D = 105$ rad/s $\curvearrowright$

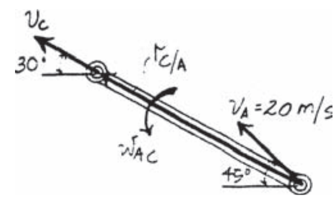
***16–80.**

The mechanism shown is used in a riveting machine. It consists of a driving piston A, three links, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at $v_A = 20$ m/s.



SOLUTION

Kinematic Diagram: Since link BC is rotating about fixed point B, then v_C is always directed perpendicular to link BC. At the instant shown. $\mathbf{v}_C = -v_C \cos 30°\mathbf{i} + v_C \sin 30°\mathbf{j} = -0.8660\, v_C\mathbf{i} + 0.500\, v_C\,\mathbf{j}$. Also, block D is moving towards the *negative* y axis due to the constraint of the guide. Then. $\mathbf{v}_D = -v_D\mathbf{j}$.

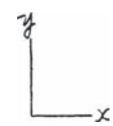


Velocity Equation: Here, $\mathbf{v}_A = \{-20\cos 45°\mathbf{i} + 20\sin 45°\mathbf{j}\}$m/s $= \{-14.14\mathbf{i} + 14.14\mathbf{j}\}$ m/s and $\mathbf{r}_{C/A} = \{-0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j}\}$m $= \{-0.2598\mathbf{i} + 0.150\mathbf{j}\}$ m. Applying Eq. 16–16 to link AC, we have

$$\mathbf{v}_C = \mathbf{v}_A + \omega_{AC} \times \mathbf{r}_{C/A}$$

$$-0.8660\, v_C\mathbf{i} + 0.500\, v_C\mathbf{j} = -14.14\mathbf{i} + 14.14\mathbf{j} + (\omega_{AC}\mathbf{k}) \times (-0.2598\mathbf{i} + 0.150\mathbf{j})$$

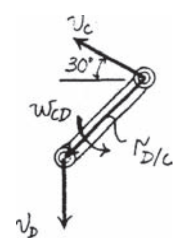
$$-0.8660\, v_C\mathbf{i} + 0.500\, v_C\mathbf{j} = -(14.14 + 0.150\,\omega_{AC})\mathbf{i} + (14.14 - 0.2598\omega_{AC})\mathbf{j}$$



Equating **i** and **j** components gives

$$-0.8660\, v_C = -(14.14 + 0.150\,\omega_{AC}) \qquad \textbf{[1]}$$

$$0.500\, v_C = 14.14 - 0.2598\,\omega_{AC} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields

$$\omega_{AC} = 17.25 \text{ rad/s} \qquad v_C = 19.32 \text{ m/s}$$

Thus, $\mathbf{v}_C = \{-19.32\cos 30°\mathbf{i} + 19.32\sin 30°\mathbf{j}\}$ m/s $= \{-16.73\mathbf{i} + 9.659\mathbf{j}\}$ m/s and $\mathbf{r}_{D/C} = \{-0.15\cos 45°\mathbf{i} - 0.15\sin 45°\mathbf{j}\}$m $= \{-0.1061\mathbf{i} - 0.1061\mathbf{j}\}$ m. Applying Eq. 16–16 to link CD, we have

$$\mathbf{v}_D = \mathbf{v}_C + \omega_{CD} \times \mathbf{r}_{D/C}$$

$$-v_D\mathbf{j} = -16.73\mathbf{i} + 9.659\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-0.1061\mathbf{i} - 0.1061\mathbf{j})$$

$$-v_D\mathbf{j} = (0.1061\omega_{CD} - 16.73)\mathbf{i} + (9.659 - 0.1061\omega_{CD})\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 0.1061\omega_{CD} - 16.73 \qquad \textbf{[3]}$$

$$-v_D = 9.659 - 0.1061\,\omega_{CD} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

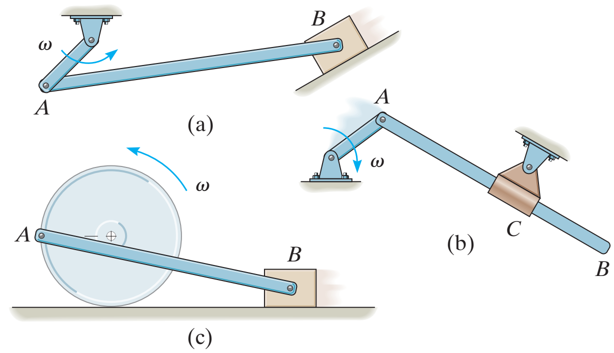
$$\omega_{CD} = 157.74 \text{ rad/s}$$

$$v_D = 7.07 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_D = 7.07$ m/s

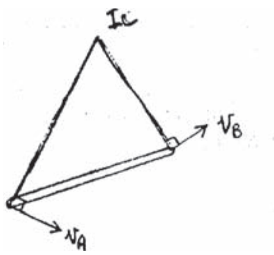
16–81.

In each case show graphically how to locate the instantaneous center of zero velocity of link AB. Assume the geometry is known.

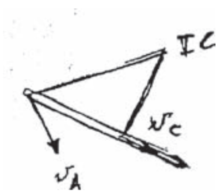


SOLUTION

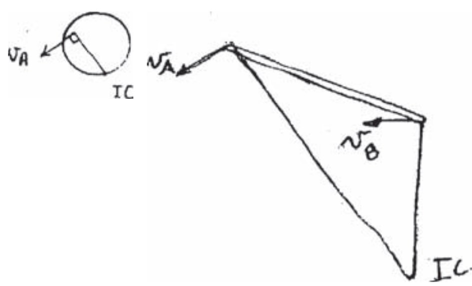
a)



b)

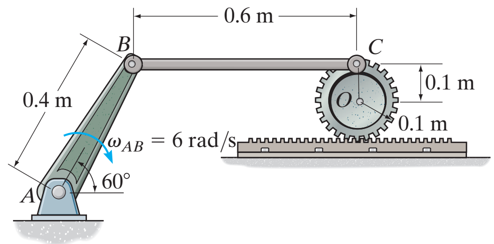


c)



16–82.

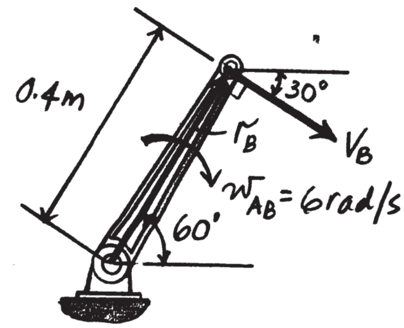
If crank AB is rotating with an angular velocity of $\omega_{AB} = 6$ rad/s, determine the velocity of the center O of the gear at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_{AB}\, r_B = 6(0.4) = 2.4 \text{ m/s}$$



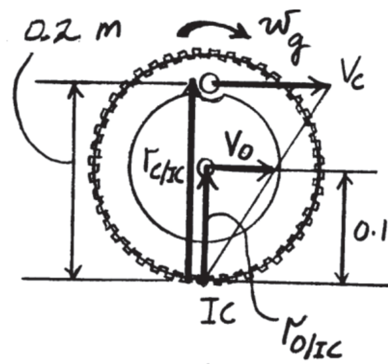
(a)

General Plane Motion: Since the gear rack is stationary, the IC of the gear is located at the contact point between the gear and the rack, Fig. *b*. Thus, $\mathbf{v}_O$ and $\mathbf{v}_C$ can be related using the similar triangles shown in Fig. *b*,

$$\omega_g = \frac{v_C}{r_{C/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_C}{0.2} = \frac{v_O}{0.1}$$

$$v_C = 2v_O$$



(b)

The location of the IC for rod BC is indicated in Fig. *c*. From the geometry shown,

$$r_{B/IC} = \frac{0.6}{\cos 60^\circ} = 1.2 \text{ m}$$

$$r_{C/IC} = 0.6 \tan 60^\circ = 1.039 \text{ m}$$

Thus, the angular velocity of rod BC can be determined from

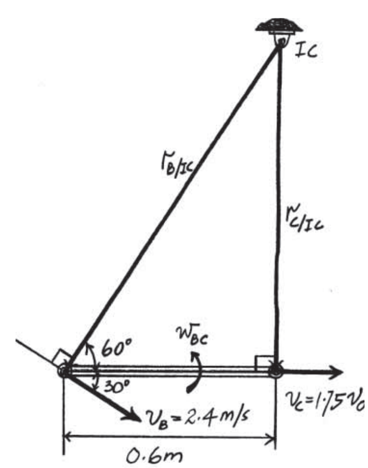
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.4}{1.2} = 2 \text{ rad/s}$$

Then,

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$2v_O = 2(1.039)$$

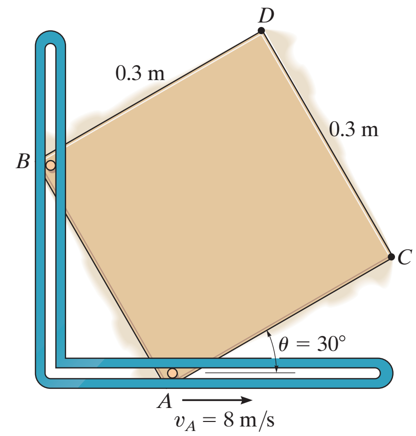
$$v_O = 1.04 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



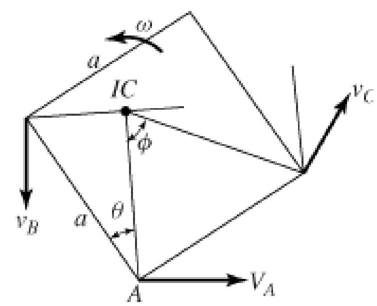
(c)

Ans:
$v_O = 1.04$ m/s $\rightarrow$

16–83. The square plate is confined within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point C at this instant.



SOLUTION



Given:

$$\theta = 30°$$

$$v_A = 8 \text{ m/s}$$

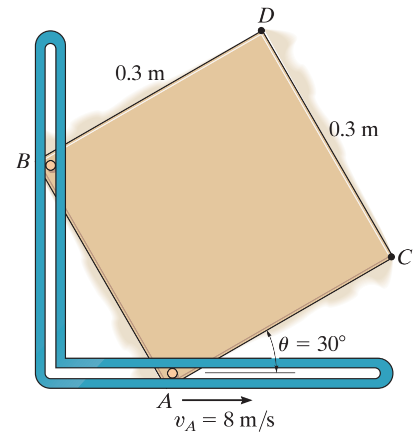
$$a = 0.3 \text{ m}$$

$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79 \text{ rad/s}$$

$$v_C = \omega\sqrt{(a\cos(\theta))^2 + (a\cos(\theta) - a\sin(\theta))^2} \qquad v_C = 8.69 \text{ m/s} \quad \textbf{Ans.}$$

Ans:
$v_C = 8.69$ m/s

***16–84.** The square plate is confined within the slots at A and B. When $\theta = 30^\circ$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point D at this instant.



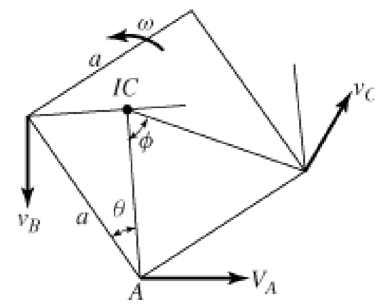
SOLUTION

Given:

$\theta = 30^\circ$

$v_A = 8$ m/s

$a = 0.3$ m



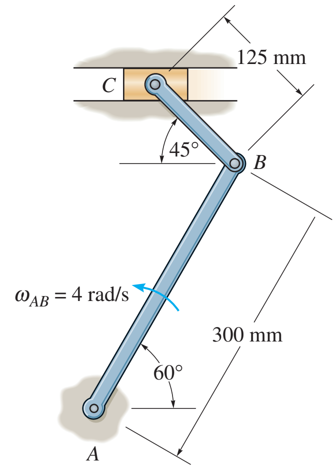
$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79 \text{ rad/s}$$

$$v_D = \omega\sqrt{(-a\sin(\theta) + a\cos(\theta))^2 + (a\sin(\theta))^2} \qquad v_D = 5.72 \text{ m/s}$$ **Ans.**

Ans:
$v_D = 5.72$ m/s

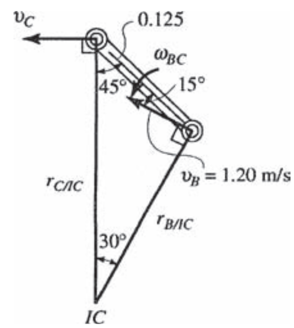
16–85.

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the angular velocity of the link CB at the instant shown, if the link AB is rotating at 4 rad/s.



SOLUTION

Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 4(0.3) = 1.20$ m/s. At the instant shown, $\mathbf{v}_B$ is directed at an angle 30° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 45°} = \frac{0.125}{\sin 30°} \qquad r_{B/IC} = 0.1768 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105°} = \frac{0.125}{\sin 30°} \qquad r_{C/IC} = 0.2415 \text{ m}$$

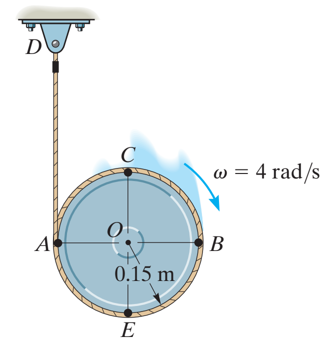
The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.1768} = 6.79 \text{ rad/s}$$ **Ans.**

Ans:
$\omega_{BC} = 6.79$ rad/s

16–86.

At the instant shown, the disk is rotating at $\omega = 4$ rad/s. Determine the velocities of points A, B, and C.



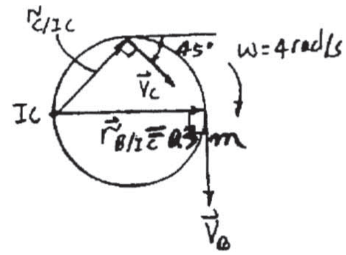
SOLUTION

The instantaneous center is located at point A. Hence, $v_A = 0$ **Ans.**

$$r_{C/IC} = \sqrt{0.15^2 + 0.15^2} = 0.2121 \text{ m} \qquad r_{B/IC} = 0.3 \text{ m}$$

$$v_B = \omega\, r_{B/IC} = 4(0.3) = 1.2 \text{ m/s}$$ **Ans.**

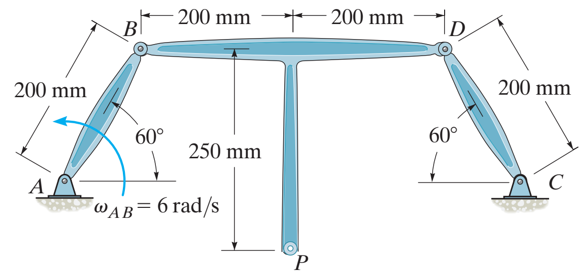
$$v_C = \omega\, r_{C/IC} = 4(0.2121) = 0.849 \text{ m/s} \quad \searrow 45^\circ$$ **Ans.**



Ans:
$v_A = 0$,
$v_B = 1.2$ m/s
$v_C = 0.849$ m/s $\searrow 45^\circ$

16–87

Member AB is rotating at $\omega_{AB} = 6 \text{ rad/s}$. Determine the velocity of point D and the angular velocity of members BPD and CD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20 \text{ m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their directions are indicated in Figs. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as show in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = r_{D/IC} = 0.4 \text{ m}$$

Then, the kinematics gives

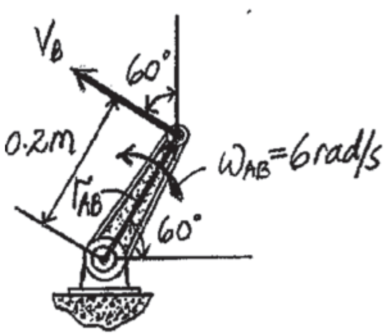
$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00 \text{ rad/s} \curvearrowright \qquad \textbf{Ans.}$$

$$v_D = \omega_{BPD} r_{D/IC} = (3.00)(0.4) = 1.20 \text{ m/s} \swarrow \qquad \textbf{Ans.}$$

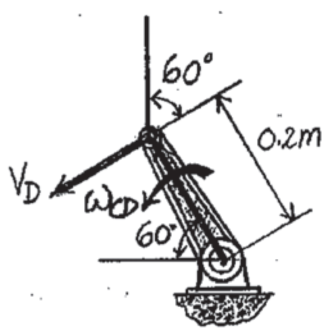
Thus,

$$v_D = \omega_{CD}(0.2); \qquad 1.2 = \omega_{CD}(0.2)$$

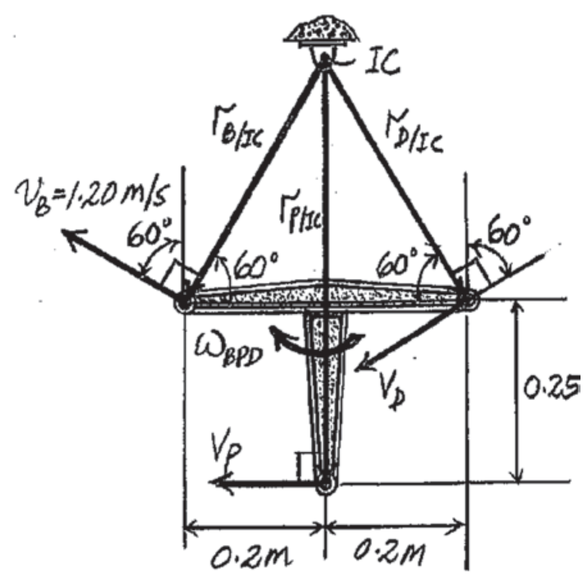
$$\omega_{CD} = 6.00 \text{ rad/s} \curvearrowleft \qquad \textbf{Ans.}$$



(a)



(b)

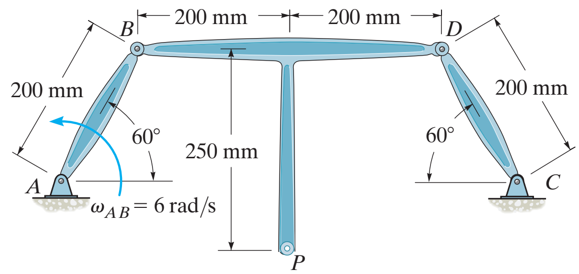


(c)

Ans:
$\omega_{BPD} = 3.00 \text{ rad/s} \curvearrowright$
$v_D = 1.20 \text{ m/s} \swarrow$
$\omega_{CD} = 6.00 \text{ rad/s} \curvearrowleft$

***16–88.**

Member AB is rotating at $\omega_{AB} = 6$ rad/s. Determine the velocity of point P, and the angular velocity of member BPD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20 \text{ m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their direction are indicated in Fig. *a* and *b*

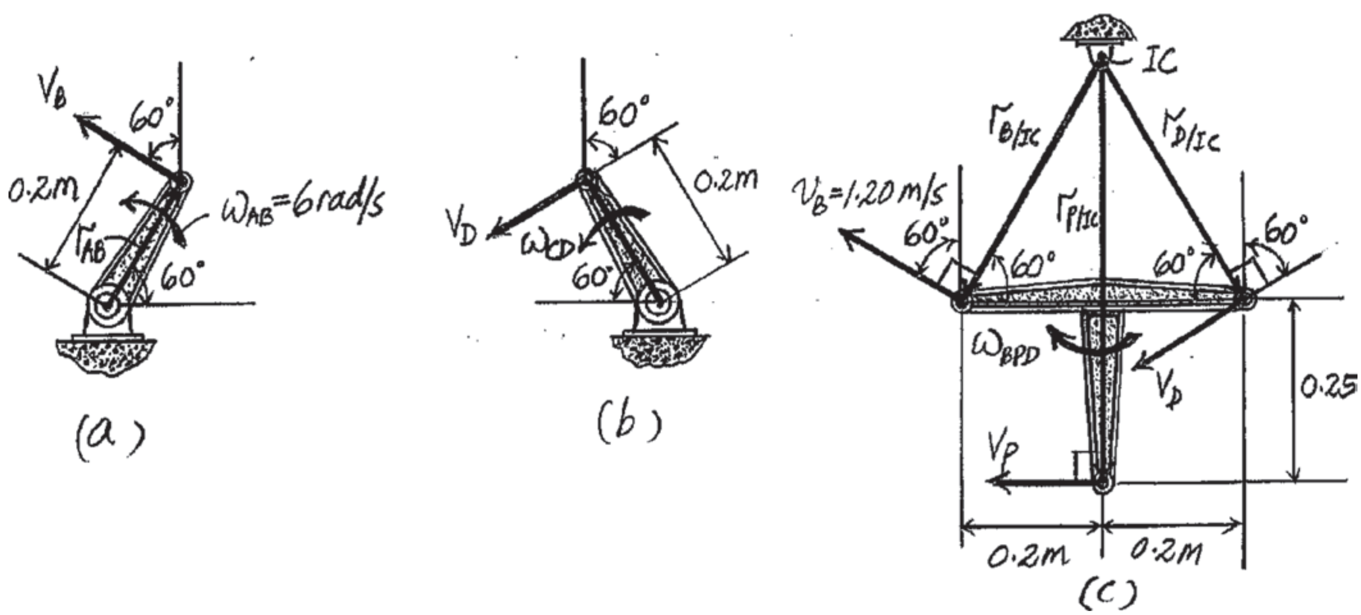
General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as shown in Fig. *c*. From the geometry of this figure

$$r_{B/IC} = 0.4 \text{ m} \qquad r_{P/IC} = 0.25 + 0.2 \tan 60^\circ = 0.5964 \text{ m}$$

Then the kinematics give

$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00 \text{ rad/s} \circlearrowright$$ **Ans.**

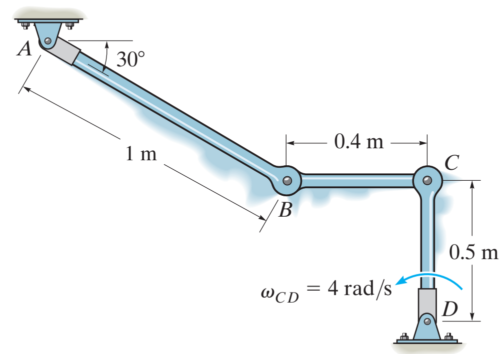
$$v_P = \omega_{BPD} r_{P/IC} = (3.00)(0.5964) = 1.7892 \text{ m/s} = 1.79 \text{ m/s} \leftarrow$$ **Ans.**



Ans:
$\omega_{BPD} = 3.00$ rad/s $\circlearrowright$
$v_P = 1.79$ m/s $\leftarrow$

16–89.

If rod CD is rotating with an angular velocity $\omega_{CD} = 4$ rad/s, determine the angular velocities of rods AB and CB at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of C and D are

$$v_C = \omega_{CD} r_{CD} = 4(0.5) = 2.00 \text{ m/s}$$

$$v_B = \omega_{AB} r_{AB} = \omega_{AB}(1)$$

And their direction are indicated in Fig. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_C$ and $\mathbf{v}_B$, the IC for link BC can be located as shown in Fig. *c*. From the geometry of this figure,

$$r_{C/IC} = 0.4 \tan 30° = 0.2309 \text{ m} \qquad r_{B/IC} = \frac{0.4}{\cos 30°} = 0.4619 \text{ m}$$

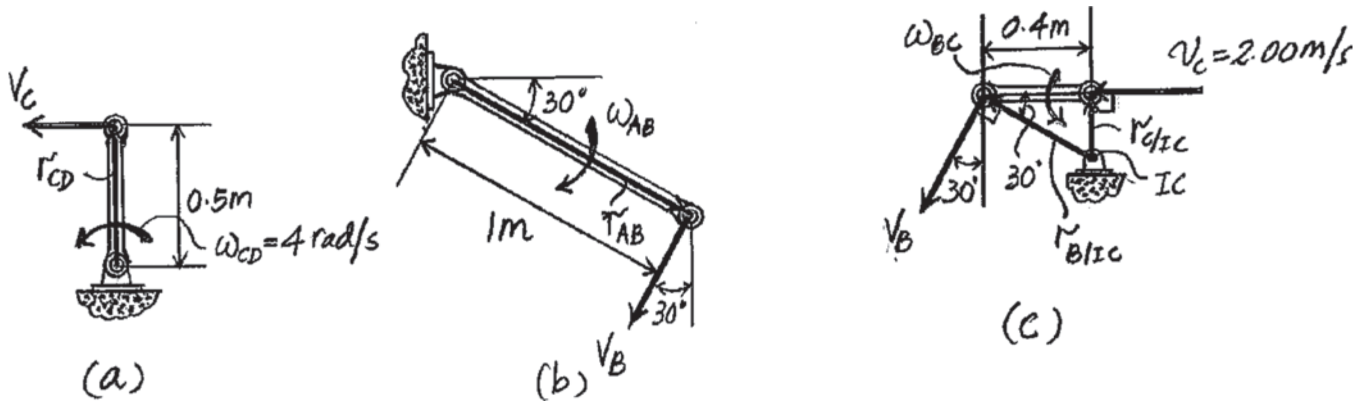
Then, the kinematics gives

$v_C = \omega_{BC} r_{C/IC};\quad 2.00 = \omega_{BC}(0.2309)$

$$\omega_{BC} = 8.6603 \text{ rad/s} = 8.66 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$v_B = \omega_{BC} r_{B/IC};\quad \omega_{AB}(1) = 8.6603(0.4619)$

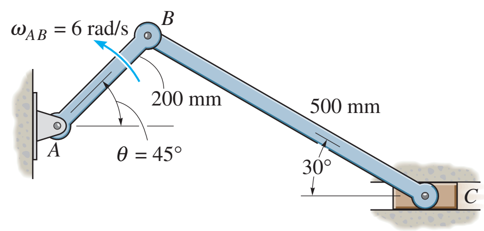
$$\omega_{AB} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



Ans:
$\omega_{BC} = 8.66$ rad/s $\circlearrowleft$
$\omega_{AB} = 4.00$ rad/s $\circlearrowright$

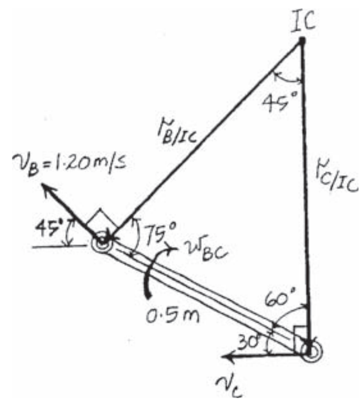
16–90.

If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



SOLUTION

Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20$ m/s. At the instant shown. $\mathbf{v}_B$ is directed with an angle 45° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.



Instantaneous Center: The instantaneous center of zero velocity of bar BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sine, we have

$$\frac{r_{B/IC}}{\sin 60^\circ} = \frac{0.5}{\sin 45^\circ} \qquad r_{B/IC} = 0.6124 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 75^\circ} = \frac{0.5}{\sin 45^\circ} \qquad r_{C/IC} = 0.6830 \text{ m}$$

The angular velocity of bar BC is given by

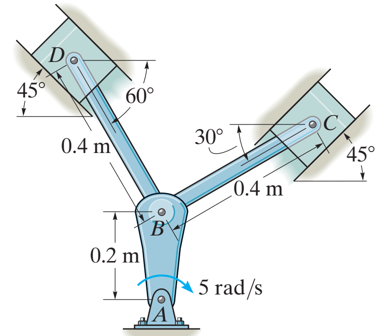
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.6124} = 1.960 \text{ rad/s}$$

Thus, the velocity of block C is

$$v_C = \omega_{BC} r_{C/IC} = 1.960(0.6830) = 1.34 \text{ m/s} \leftarrow$$ **Ans.**

Ans:
$v_C = 1.34$ m/s $\leftarrow$

16–91. The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity $\omega_{AB} = 5$ rad/s.



SOLUTION

Given:

$$\omega_{AB} = 5 \text{ rad/s}$$

$$a = 0.2 \text{ m}$$

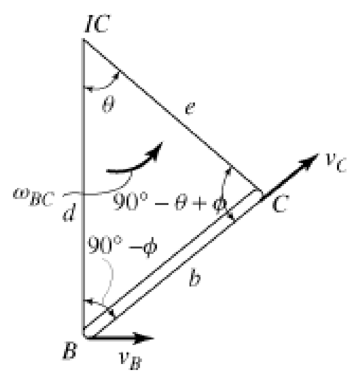
$$b = 0.4 \text{ m}$$

$$c = 0.4 \text{ m}$$

$$\theta = 45°$$

$$\phi = 30°$$

$$\beta = 45°$$



$$d = b\left(\frac{\sin(90° - \phi)}{\sin(\theta)}\right) \qquad d = 0.49 \text{ m}$$

$$e = b\left(\frac{\sin(90° + \phi - \theta)}{\sin(\theta)}\right) \qquad e = 0.55 \text{ m}$$

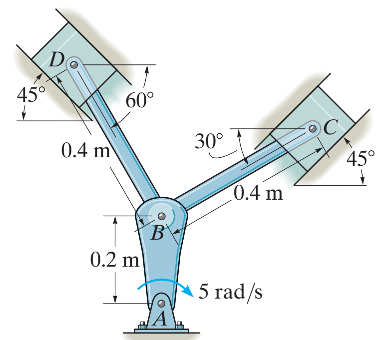
$$v_B = \omega_{AB}\, a \qquad v_B = 1.00 \text{ m/s}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83 \text{ rad/s}$$

$$v_C = \omega_{BC} d \qquad v_C = 0.897 \text{ m/s}$$ **Ans.**

Ans:
$v_C = 0.897$ m/s

***16–92.** The mechanism used in a marine engine consists of a single crank *AB* and two connecting rods *BC* and *BD*. Determine the velocity of the piston at *D* the instant the crank is in the position shown and has an angular velocity $\omega_{AB} = 5$ rad/s.



SOLUTION

Given:

$\omega_{AB} = 5$ rad/s

$a = 0.2$ m

$b = 0.4$ m

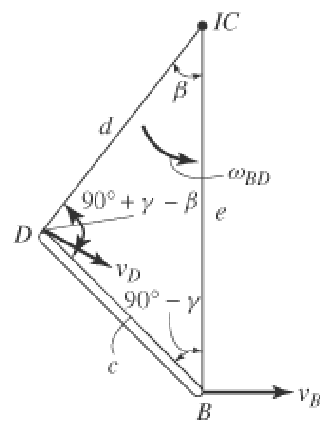
$c = 0.4$ m

$\theta = 45°$

$\phi = 30°$

$\gamma = 60°$

$\beta = 45°$



$$d = c\left(\frac{\sin(90° - \gamma)}{\sin(\beta)}\right) \qquad d = 0.28 \text{ m}$$

$$e = c\left(\frac{\sin(90° + \gamma - \beta)}{\sin(\beta)}\right) \qquad e = 0.55 \text{ m}$$

$$v_B = \omega_{AB}\, a \qquad v_B = 1.00 \text{ m/s}$$

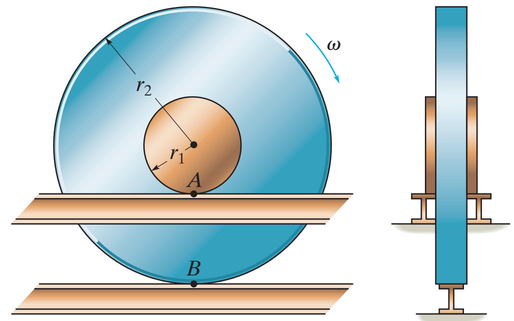
$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83 \text{ rad/s}$$

$$v_D = \omega_{BC}\, d \qquad v_D = 0.518 \text{ m/s}$$ **Ans.**

Ans:
$v_D = 0.518$ m/s

16–93.

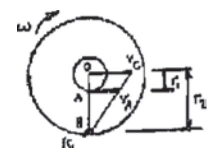
Show that if the rim of the wheel and its hub maintain contact with the three tracks as the wheel rolls, it is necessary that slipping occurs at the hub A if no slipping occurs at B. Under these conditions, what is the speed at A if the wheel has angular velocity $\boldsymbol{\omega}$?



SOLUTION

IC is at B.

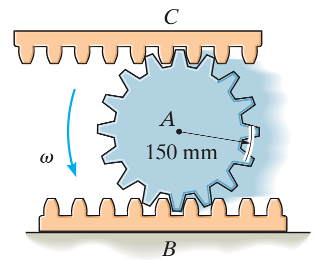
$v_A = \omega(r_2 - r_1) \rightarrow$ **Ans.**



Ans:
$v_A = \omega(r_2 - r_1)$

16–94.

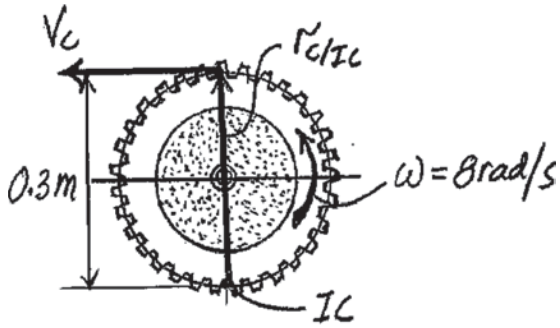
The pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the gear rack C.



SOLUTION

General Plane Motion. The location of IC for the gear is at the bottom of the gear where it meshes with gear rack B as shown in Fig. a. Thus,

$$v_C = \omega r_{C/IC} = 8(0.3) = 2.40 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

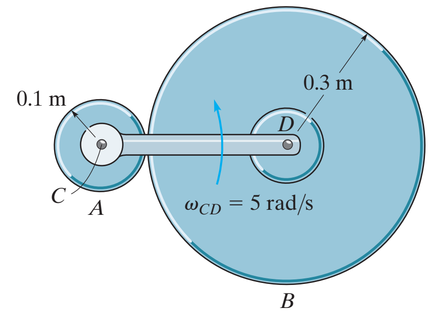


(a)

Ans:
$v_C = 2.40$ m/s ←

16–95.

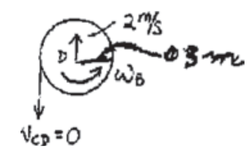
The cylinder B rolls on the fixed cylinder A without slipping. If connected bar CD is rotating with an angular velocity $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B. Point C is a fixed point.



SOLUTION

$v_D = 5(0.4) = 2$ m/s

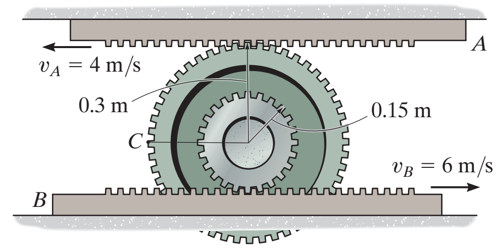
$\omega_B = \dfrac{2}{0.3} = 6.67$ rad/s **Ans.**



Ans:
$\omega_B = 6.67$ rad/s

***16–96.**

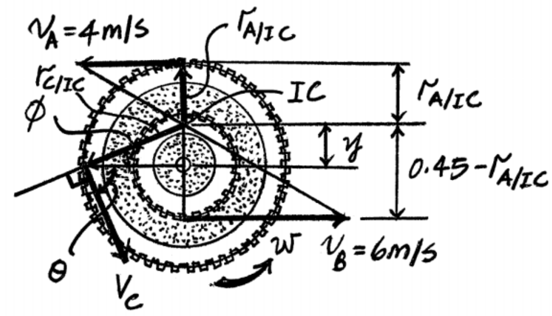
Determine the angular velocity of the double-tooth gear and the velocity of point C on the gear.



SOLUTION

General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a.

$$\frac{r_{A/IC}}{4} = \frac{0.45 - r_{A/IC}}{6} \qquad r_{A/IC} = 0.18 \text{ m}$$



(a)

Then,

$$y = 0.3 - r_{A/IC} = 0.3 - 0.18 = 0.12 \text{ m}$$

and

$$r_{C/IC} = \sqrt{0.3^2 + 0.12^2} = 0.3231 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.12}{0.3}\right) = 21.80°$$

Thus, the angular velocity of the gear can be determined from

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.18} = 22.22 \text{ rad/s} = 22.2 \text{ rad/s} \qquad \textbf{Ans.}$$

Then

$$v_C = \omega r_{C/IC} = 22.2(0.3231) = 7.18 \text{ m/s} \qquad \textbf{Ans.}$$

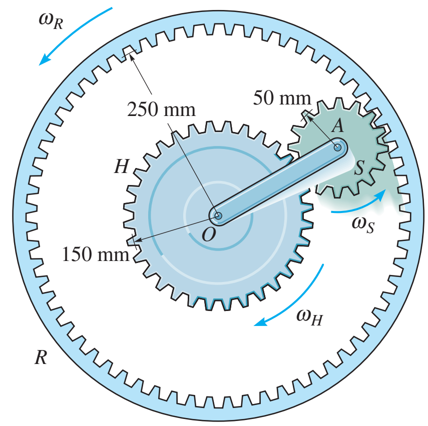
And its direction is

$$\phi = 90° - \phi = 90° - 21.80° = 68.2° \qquad \textbf{Ans.}$$

Ans:
$\omega = 22.2$ rad/s
$v_C = 7.18$ m/s
$\phi = 68.2°$

16–97.

If the hub gear H and ring gear R have angular velocities $\omega_H = 5\ \text{rad/s}$ and $\omega_R = 20\ \text{rad/s}$, respectively, determine the angular velocity ω_S of the spur gear S and the angular velocity of its attached arm OA.

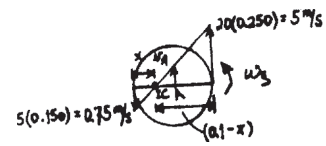


SOLUTION

$$\frac{5}{0.1 - x} = \frac{0.75}{x}$$

$$x = 0.01304\ \text{m}$$

$$\omega_S = \frac{0.75}{0.01304} = 57.5\ \text{rad/s} \ \circlearrowleft$$ **Ans.**



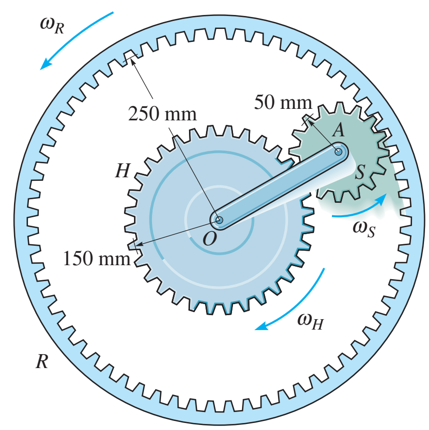
$$v_A = 57.5(0.05 - 0.01304) = 2.125\ \text{m/s}$$

$$\omega_{OA} = \frac{2.125}{0.2} = 10.6\ \text{rad/s} \ \circlearrowleft$$ **Ans.**

Ans:
$\omega_S = 57.5\ \text{rad/s}\circlearrowleft$
$\omega_{OA} = 10.6\ \text{rad/s}\circlearrowleft$

16–98.

If the hub gear H has an angular velocity $\omega_H = 5$ rad/s, determine the angular velocity of the ring gear R so that the arm OA attached to the spur gear S remains stationary ($\omega_{OA} = 0$). What is the angular velocity of the spur gear?

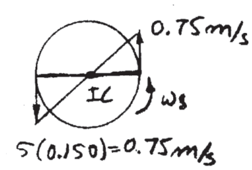


SOLUTION

The IC is at A.

$$\omega_S = \frac{0.75}{0.05} = 15.0 \text{ rad/s}$$ **Ans.**

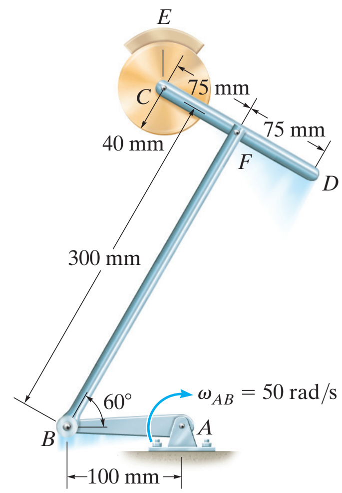
$$\omega_R = \frac{0.75}{0.250} = 3.00 \text{ rad/s}$$ **Ans.**



Ans:
$\omega_S = 15.0$ rad/s
$\omega_R = 3.00$ rad/s

16–99.

The crankshaft AB rotates at $\omega_{AB} = 50$ rad/s about the fixed axis through point A, and the disk at C is held fixed in its support at E. Determine the angular velocity of rod CD at the instant shown.



SOLUTION

$$r_{B/IC} = \frac{0.3}{\sin 30^\circ} = 0.6 \text{ m}$$

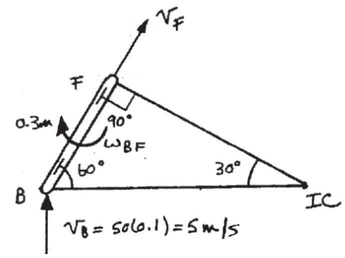
$$r_{F/IC} = \frac{0.3}{\tan 30^\circ} = 0.5196 \text{ m}$$

$$\omega_{BF} = \frac{5}{0.6} = 8.333 \text{ rad/s}$$

$$v_F = 8.333(0.5196) = 4.330 \text{ m/s}$$

Thus,

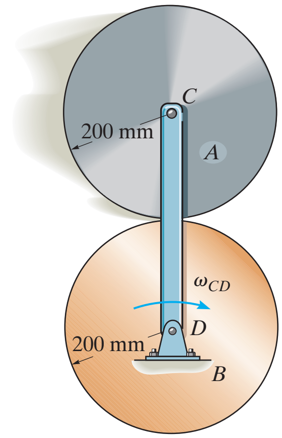
$$\omega_{CD} = \frac{4.330}{0.075} = 57.7 \text{ rad/s} \circlearrowleft$$ **Ans.**



Ans:
$\omega_{CD} = 57.7 \text{ rad/s} \circlearrowleft$

***16–100.**

Cylinder A rolls on the *fixed cylinder B* without slipping. If bar CD is rotating with an angular velocity of $\omega_{CD} = 3$ rad/s, determine the angular velocity of A.



SOLUTION

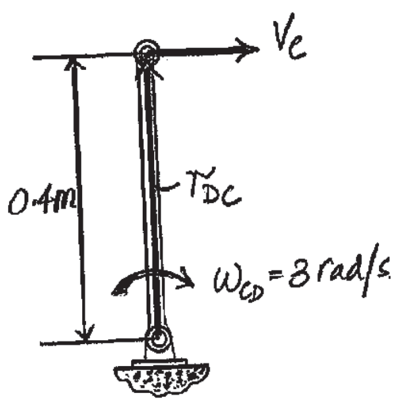
Rotation About A Fixed Axis. The magnitude of the velocity of C is

$$v_C = \omega_{CD} r_{DC} = 3(0.4) = 1.20 \text{ m/s} \rightarrow$$

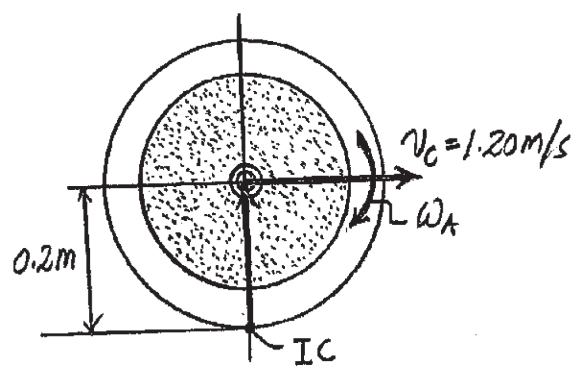
General Plane Motion. The IC for cylinder A is located at the bottom of the cylinder where it contacts with cylinder B, since no slipping occurs here, Fig. b.

$$v_C = \omega_A \, r_{C/IC}; \qquad 1.20 = \omega_A(0.2)$$

$$\omega_A = 6.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



(a)

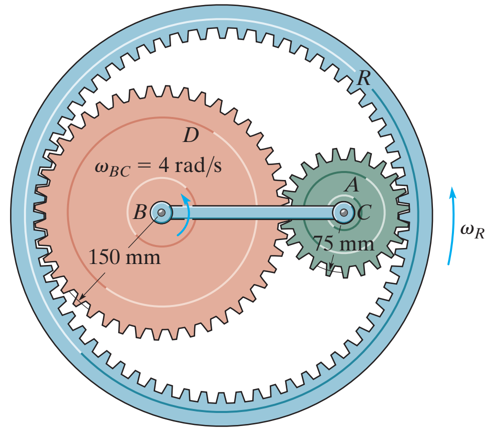


(b)

Ans:
$\omega_A = 6.00 \text{ rad/s} \circlearrowright$

16–101.

The planet gear A is pin connected to the end of the link BC. If the link rotates about the fixed point B at 4 rad/s, determine the angular velocity of the ring gear R. The sun gear D is fixed from rotating.



SOLUTION

Gear A:

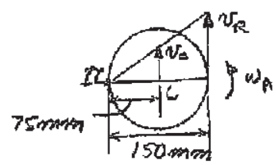
$$v_C = 4(225) = 900 \text{ mm/s}$$

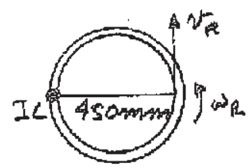
$$\omega_A = \frac{900}{75} = \frac{v_R}{150}$$

$$v_R = 1800 \text{ mm/s}$$

Ring gear:

$$\omega_R = \frac{1800}{450} = 4 \text{ rad/s}$$ **Ans.**

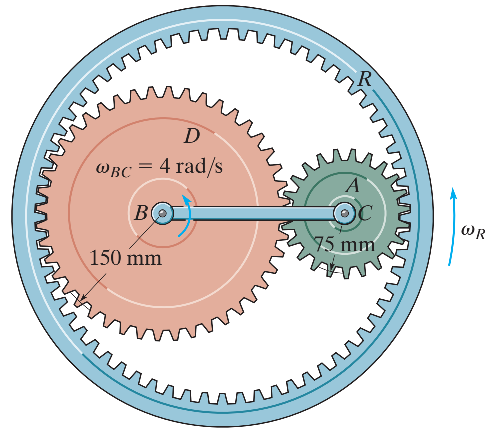




Ans:
$\omega_R = 4$ rad/s

16–102.

Solve Prob. 16–101 if the sun gear D is rotating clockwise at $\omega_D = 5$ rad/s while link BC rotates counterclockwise at $\omega_{BC} = 4$ rad/s.



SOLUTION

Gear A:

$v_P = 5(150) = 750$ mm/s

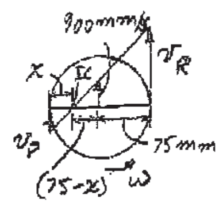
$v_C = 4(225) = 900$ mm/s

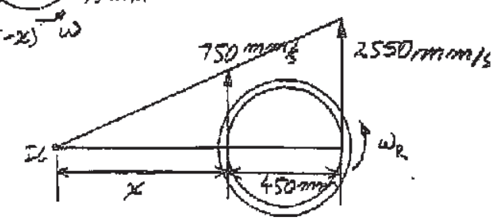
$$\frac{x}{750} = \frac{75 - x}{900}$$

$x = 34.09$ mm

$$\omega = \frac{750}{34.09} = 22.0 \text{ rad/s}$$

$v_R = [75 + (75 - 34.09)](22) = 2550$ mm/s





Ring gear:

$$\frac{750}{x} = \frac{2550}{x + 450}$$

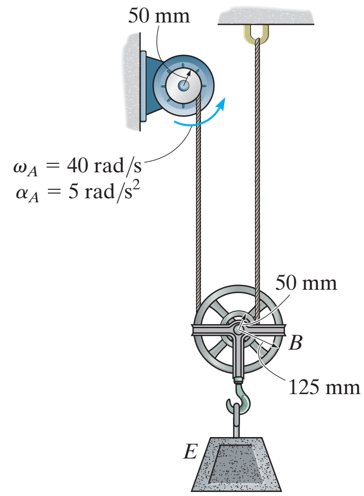
$x = 187.5$ mm

$$\omega_R = \frac{750}{187.5} = 4 \text{ rad/s} \circlearrowleft$$ **Ans.**

Ans:
$\omega_R = 4$ rad/s

16–103.

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the angular acceleration of pulley B at the instant shown.



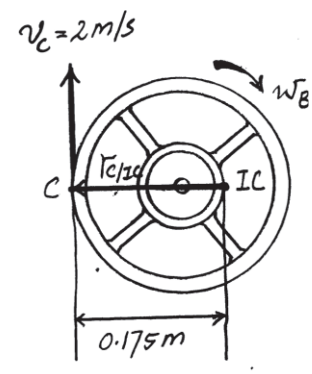
SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

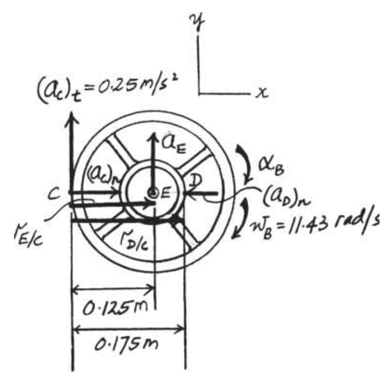
$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\,\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.43 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

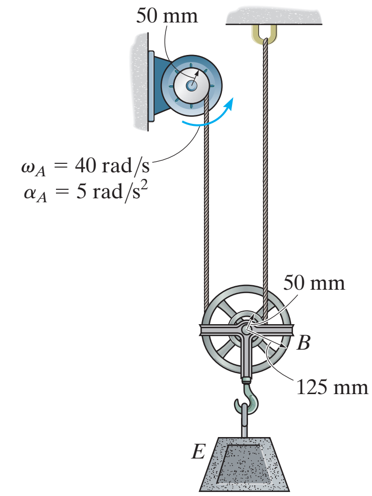


(b)

Ans:
$\alpha_B = 1.43$ rad/s^2

***16–104.**

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the acceleration of block E at the instant shown.



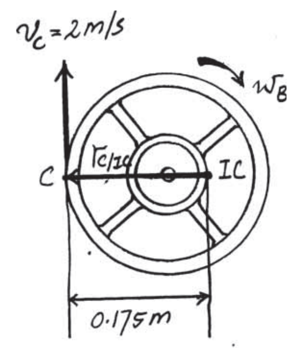
SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2\text{ m/s}\uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43\text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25\text{ m/s}^2\uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

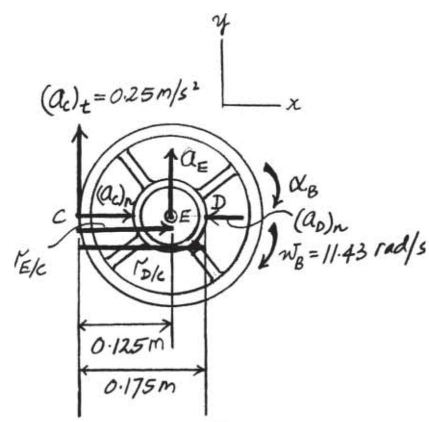
$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.429\text{ rad/s} = 1.43\text{ rad/s}^2$$



(b)

Using this result, the relative acceleration equation applied to points C and E, Fig. b, gives

$$\mathbf{a}_E = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{E/C} - \omega_B{}^2 r_{E/C}$$

$$a_E\,\mathbf{j} = [(a_C)_n\,\mathbf{i} + 0.25\mathbf{j}] + (-1.429\mathbf{k}) \times (0.125\mathbf{i}) - 11.43^2(0.125\mathbf{i})$$

$$a_E\,\mathbf{j} = [(a_C)_n - 16.33]\mathbf{i} + 0.0714\mathbf{j}$$

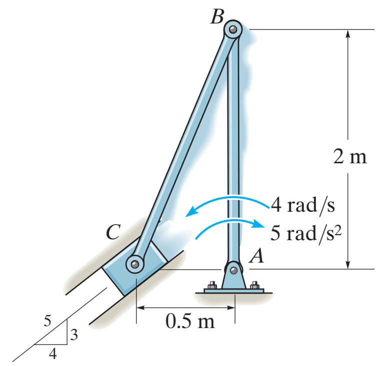
Equating the $\mathbf{j}$ components,

$$a_E = 0.0714\text{ m/s}^2\uparrow$$ **Ans.**

Ans:
$a_E = 0.0714\text{ m/s}^2\uparrow$

16–105.

Member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



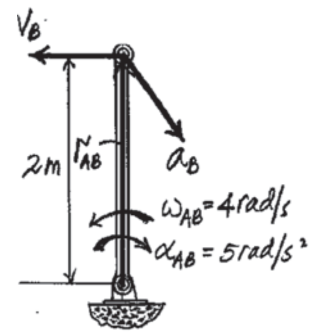
SOLUTION

Rotation About A Fixed Axis. For member AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 4(2) = 8 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-5\mathbf{k}) \times (2\mathbf{j}) - 4^2(2\mathbf{j}) = \{10\mathbf{i} - 32\mathbf{j}\} \text{ m/s}^2$$



(a)

General Plane Motion. The IC for member BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *b*. From the geometry of this figure

$$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ \qquad \theta = 90^\circ - \phi = 53.13^\circ$$

Then

$$\frac{r_{B/IC} - 2}{0.5} = \tan 53.13; \qquad r_{B/IC} = 2.6667 \text{ m}$$

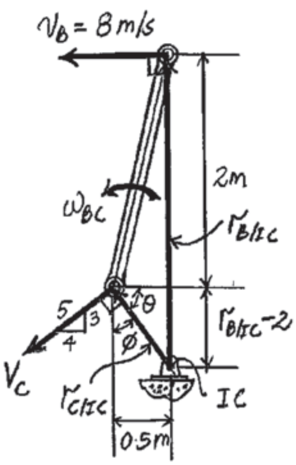
$$\frac{0.5}{r_{C/IC}} = \cos 53.13; \qquad r_{C/IC} = 0.8333 \text{ m}$$

The kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 8 = \omega_{BC}(2.6667)$$

$$\omega_{BC} = 3.00 \text{ rad/s} \circlearrowleft$$

$$v_C = \omega_{BC} r_{C/IC} = 3.00(0.8333) = 2.50 \text{ m/s} \swarrow \qquad \textbf{Ans.}$$



(b)

Applying the relative acceleration equation by referring to Fig. *c*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = (10\mathbf{i} - 32\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (-0.5\mathbf{i} - 2\mathbf{j}) - (3.00^2)(-0.5\mathbf{i} - 2\mathbf{j})$$

$$-\frac{4}{5}a_C\mathbf{i} - \frac{3}{5}a_C\mathbf{j} = (2\alpha_{BC} + 14.5)\mathbf{i} + (-0.5\alpha_{BC} - 14)\mathbf{j}$$

Equating **i** and **j** components

$$-\frac{4}{5}a_C = 2\alpha_{BC} + 14.5 \qquad \textbf{(1)}$$

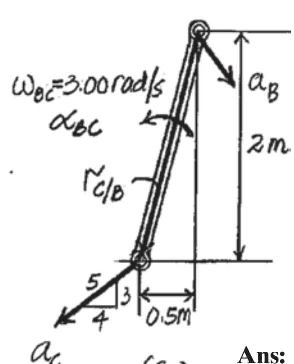
$$-\frac{3}{5}a_C = -0.5\alpha_{BC} - 14 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$a_C = 12.969 \text{ m/s}^2 = 13.0 \text{ m/s}^2 \swarrow \qquad \textbf{Ans.}$$

$$\alpha_{BC} = -12.4375 \text{ rad/s}^2 = 12.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

The negative sign indicates that α_{BC} is directed in the opposite sense from what is shown in Fig. (*c*).

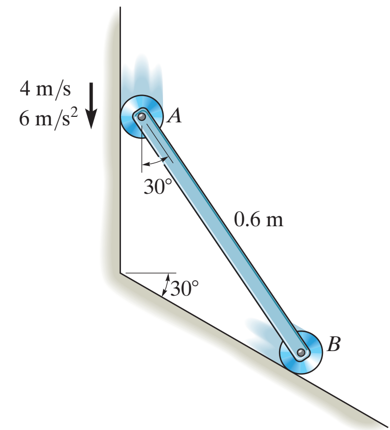


(c)

Ans:
$v_C = 2.50 \text{ m/s} \swarrow$
$a_C = 13.0 \text{ m/s}^2 \swarrow$
$\alpha_{BC} = 12.4 \text{ rad/s}^2 \circlearrowright$

16–106.

At a given instant the roller A on the bar has the velocity and acceleration shown. Determine the velocity and acceleration of the roller B, and the bar's angular velocity and angular acceleration at this instant.



SOLUTION

General Plane Motion. The IC of the bar can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. *a*. From the geometry of this figure,

$$r_{A/IC} = r_{B/IC} = 0.6 \text{ m}$$

Thus, the kinematics give

$$v_A = \omega r_{A/IC}; \qquad 4 = \omega(0.6)$$

$$\omega = 6.667 \text{ rad/s} = 6.67 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$v_B = \omega r_{B/IC} = 6.667(0.6) = 4.00 \text{ m/s} \searrow \qquad \textbf{Ans.}$$

Applying the relative acceleration equation, by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B \cos 30°\mathbf{i} - a_B \sin 30°\mathbf{j} = -6\mathbf{j} + (\alpha\mathbf{k}) \times (0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j}) - \left(6.667^2\right)(0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j})$$

$$\frac{\sqrt{3}}{2} a_B\mathbf{i} - \frac{1}{2} a_B\mathbf{j} = \left(0.3\sqrt{3}\alpha - 13.33\right)\mathbf{i} + (0.3\alpha + 17.09)\mathbf{j}$$

Equating **i** and **j** components,

$$\frac{\sqrt{3}}{2} a_B = 0.3\sqrt{3}\alpha - 13.33 \qquad \textbf{(1)}$$

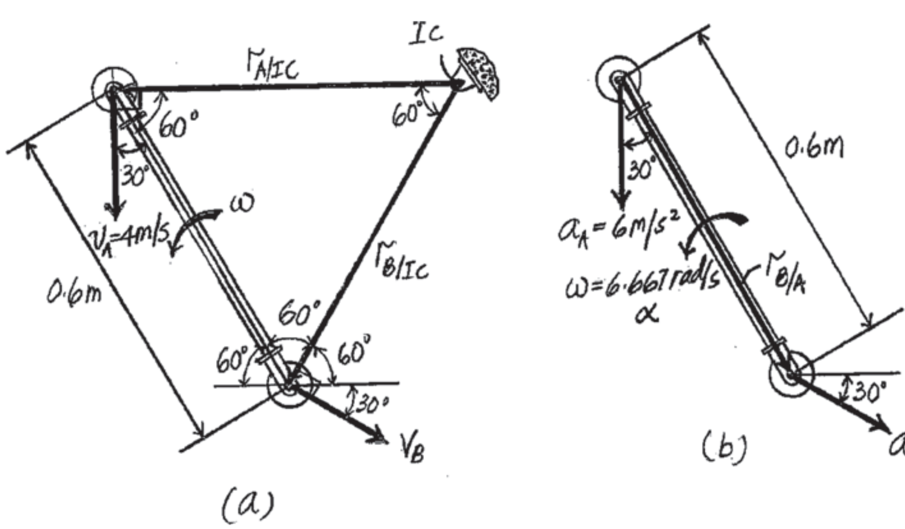
$$-\frac{1}{2} a_B = 0.3\alpha + 17.09 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\alpha = -15.66 \text{ rad/s}^2 = 15.7 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_B = -24.79 \text{ m/s}^2 = 24.8 \text{ m/s}^2 \nwarrow \qquad \textbf{Ans.}$$

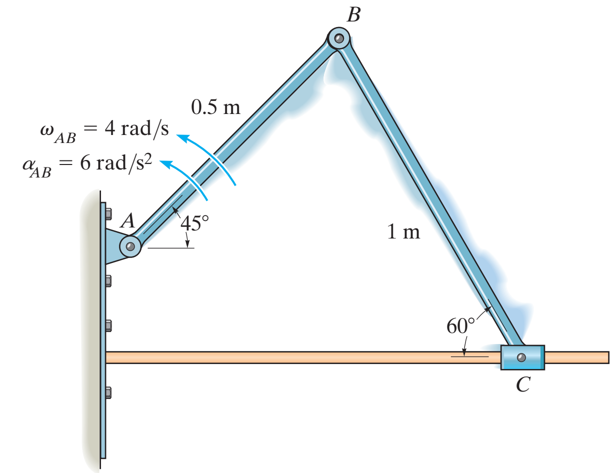
The negative signs indicate that $\boldsymbol{\alpha}$ and $\mathbf{a}_B$ are directed in the senses that opposite to those shown in Fig. *b*



Ans:
$\omega = 6.67 \text{ rad/s} \circlearrowright$
$v_B = 4.00 \text{ m/s} \searrow$
$\alpha = 15.7 \text{ rad/s}^2 \circlearrowright$
$a_B = 24.8 \text{ m/s}^2 \nwarrow$

16–107.

Bar AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

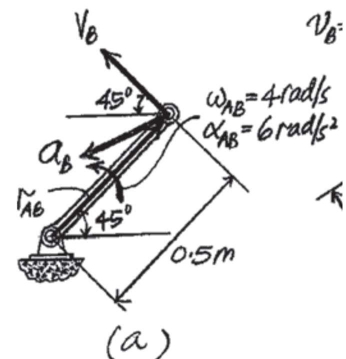
$$v_B = \omega_{AB} r_{AB} = 4(0.5) = 2.00 \text{ m/s } \angle 45^\circ$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= 6\mathbf{k} \times (0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j}) - 4^2(0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j})$$

$$= \{-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\} \text{ m/s}^2$$

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. b. From the geometry of this figure,



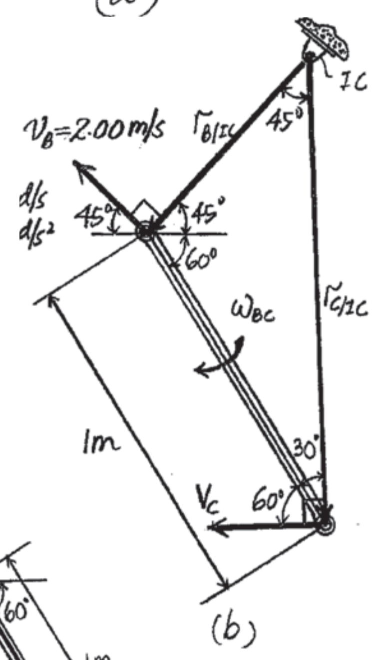
$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{1}{\sin 45^\circ}; \qquad r_{B/IC} = \frac{\sqrt{2}}{2} \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105^\circ} = \frac{1}{\sin 45^\circ}; \qquad r_{C/IC} = 1.3660 \text{ m}$$

Then the kinematics gives,

$$v_B = \omega_{BC}\, r_{B/IC}; \qquad 2 = \omega_{BC}\left(\frac{\sqrt{2}}{2}\right) \qquad \omega_{BC} = 2\sqrt{2} \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{B/IC}; \qquad v_C = \left(2\sqrt{2}\right)(1.3660) = 3.864 \text{ m/s} = 3.86 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\, r_{C/B}$$

$$-a_C\mathbf{i} = \left(-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\right) + (-\alpha_{BC}\mathbf{k}) \times (1 \cos 60^\circ \mathbf{i} - 1 \sin 60^\circ \mathbf{j})$$

$$- \left(2\sqrt{2}\right)^2(1 \cos 60^\circ \mathbf{i} - 1 \sin 60^\circ \mathbf{j})$$

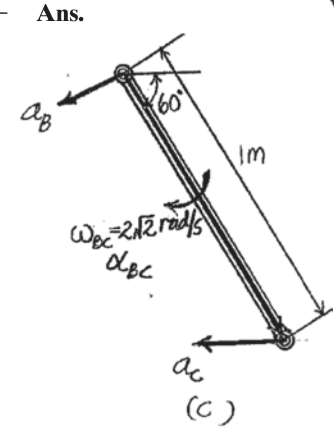
$$-a_C\mathbf{i} = \left(-\frac{\sqrt{3}}{2}\alpha_{BC} - 11.7782\right)\mathbf{i} + (3.3927 - 0.5\alpha_{BC})\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$0 = 3.3927 - 0.5\alpha_{BC}; \qquad \alpha_{BC} = 6.7853 \text{ rad/s}^2 \circlearrowright$$

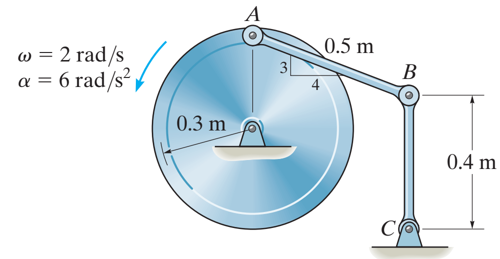
Then, $\mathbf{i}$ component gives

$$-a_C = -\frac{\sqrt{3}}{2}(6.7853) - 11.7782; \qquad a_C = 17.65 \text{ m/s}^2 = 17.7 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$



Ans:
$v_C = 3.86 \text{ m/s} \leftarrow$
$a_C = 17.7 \text{ m/s}^2 \leftarrow$

***16–108.** The flywheel rotates with angular velocity $\omega = 2\ \text{rad/s}$ and angular acceleration $\alpha = 6\ \text{rad/s}^2$. Determine the angular acceleration of links AB and BC at this instant.



SOLUTION

Given:

$\omega = 2\ \text{rad/s} \qquad a = 0.4\ \text{m}$

$\alpha = 6\ \text{rad/s}^2 \qquad b = 0.5\ \text{m}$

$r = 0.3\ \text{m} \qquad e = 3$

$d = 4$

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \frac{b}{\sqrt{e^2 + d^2}}\begin{pmatrix} d \\ -e \\ 0 \end{pmatrix} \qquad \mathbf{r_3} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1\ \text{rad/s} \qquad \omega_{BC} = 1\ \text{rad/s} \qquad \alpha_{AB} = 1\ \text{rad/s}^2 \qquad \alpha_{BC} = 1\ \text{rad/s}^2$

Given

$$\omega\mathbf{k} \times \mathbf{r_1} + \omega_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{BC}\mathbf{k} \times \mathbf{r_3} = 0$$

$$\alpha\mathbf{k} \times \mathbf{r_1} + \omega\mathbf{k} \times (\omega\mathbf{k} \times \mathbf{r_1}) + \alpha_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{AB}\mathbf{k} \times (\omega_{AB}\mathbf{k} \times \mathbf{r_2}) \ldots = 0$$
$$+ \alpha_{BC}\mathbf{k} \times \mathbf{r_3} + \omega_{BC}\mathbf{k} \times (\omega_{BC}\mathbf{k} \times \mathbf{r_3})$$

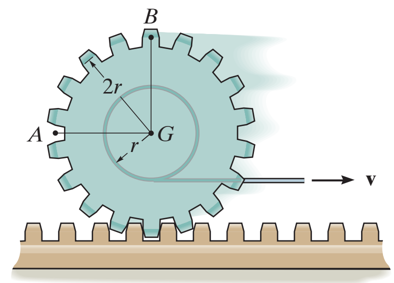
$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.50 \end{pmatrix}\ \text{rad/s}$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 3.94 \end{pmatrix}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_{AB} = 0.75\ \text{rad/s}^2$
$\alpha_{BC} = 3.94\ \text{rad/s}^2$

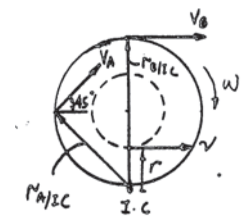
16–109.

A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity **v**, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.



SOLUTION

Velocity analysis:

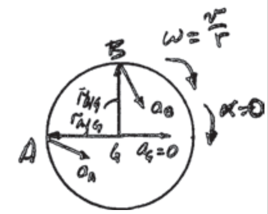


$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$ **Ans.**

$$v_A = \omega\, r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \qquad \measuredangle 45^\circ$$ **Ans.**

Acceleration equation: From Example 16–3, Since $a_G = 0, \alpha = 0$



$$\mathbf{r}_{B/G} = 2r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2\mathbf{r}_{B/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2(2r\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$

$$a_B = \frac{2v^2}{r}\downarrow$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2\mathbf{r}_{A/G}$$

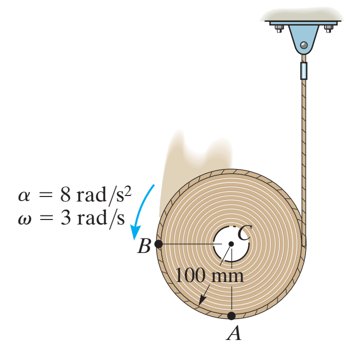
$$= 0 + 0 - \left(\frac{v}{r}\right)^2(-2r\mathbf{i}) = \frac{2v^2}{r}\mathbf{i}$$

$$a_A = \frac{2v^2}{r}\rightarrow$$ **Ans.**

Ans:
$v_B = 4v \rightarrow$
$v_A = 2\sqrt{2}v$
$\theta = 45^\circ \measuredangle$
$a_B = \frac{2v^2}{r}\downarrow$
$a_A = \frac{2v^2}{r}\rightarrow$

16–110.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point A at the instant shown.



SOLUTION

General Plane Motion. The *IC* of the reel is located as shown in Fig. *a*. Here,

$$r_{A/IC} = \sqrt{0.1^2 + 0.1^2} = 0.1414 \text{ m}$$

Then, the Kinematics give

$$v_A = \omega r_{A/IC} = 3(0.1414) = 0.4243 \text{ m/s} = 0.424 \text{ m/s} \ \measuredangle 45^\circ \qquad \textbf{Ans.}$$

Here $a_C = \alpha r = 8(0.1) = 0.8 \text{ m/s}^2 \downarrow$. Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{a}_A = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{j}) - 3^2(-0.1\mathbf{j})$$

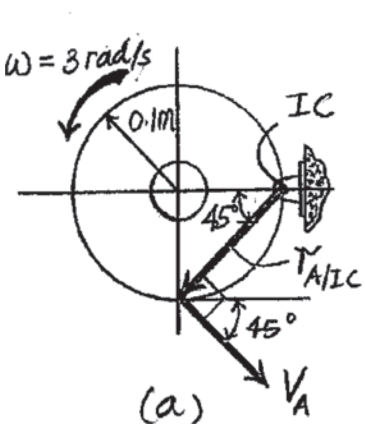
$$= \{0.8\mathbf{i} + 0.1\mathbf{j}\} \text{ m/s}^2$$

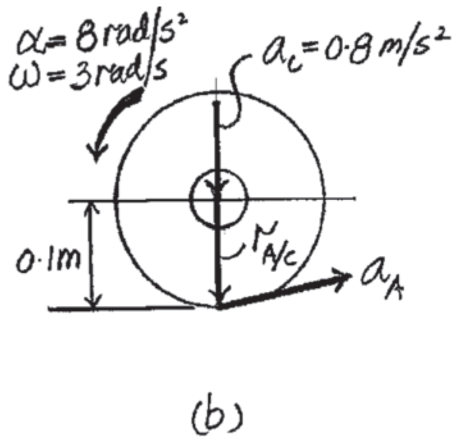
The magnitude of $\mathbf{a}_A$ is

$$a_A = \sqrt{0.8^2 + 0.1^2} = 0.8062 \text{ m/s}^2 = 0.806 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{0.1}{0.8}\right) = 7.125^\circ = 7.13^\circ \ \measuredangle \qquad \textbf{Ans.}$$

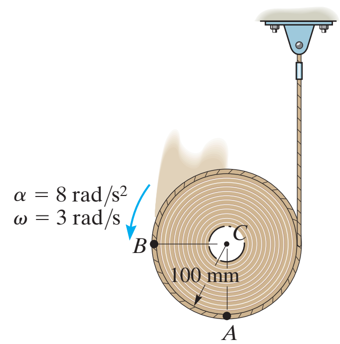




Ans:
$v_A = 0.424 \text{ m/s}$
$\theta_v = 45^\circ \measuredangle$
$a_A = 0.806 \text{ m/s}^2$
$\theta_a = 7.13^\circ \measuredangle$

16–111.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point B at the instant shown.



SOLUTION

General Plane Motion. The *IC* of the reel is located as shown in Fig. *a*. Here, $r_{B/FC} = 0.2$ m. Then the kinematics gives

$$v_B = \omega r_{B/IC} = (3)(0.2) = 0.6 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Here, $a_C = \alpha r = 8(0.1) = 0.8 \text{ m/s}^2 \downarrow$. Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$\mathbf{a}_B = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{i}) - 3^2(-0.1\mathbf{i})$$

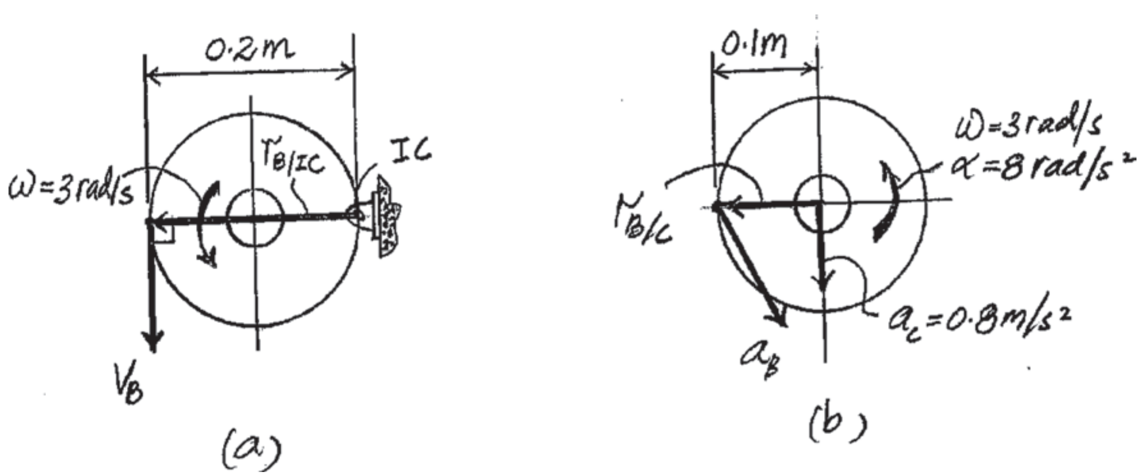
$$= \{0.9\mathbf{i} - 1.6\mathbf{j}\} \text{ m/s}^2$$

The magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{0.9^2 + (-1.6)^2} = 1.8358 \text{ m/s}^2 = 1.84 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

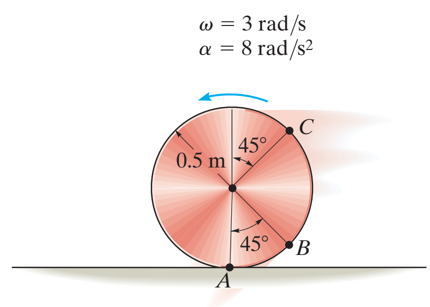
$$\theta = \tan^{-1}\left(\frac{1.6}{0.9}\right) = 60.64° = 60.6° \qquad \textbf{Ans.}$$



Ans:
$v_B = 0.6 \text{ m/s} \downarrow$
$a_B = 1.84 \text{ m/s}^2$
$\theta = 60.6°$

***16–112.**

The disk has an angular acceleration $\alpha = 8 \text{ rad/s}^2$ and angular velocity $\omega = 3 \text{ rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4 \text{ m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. *a*,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{B/O} - \omega^2 r_{B/O}$$

$$\mathbf{a}_B = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j})$$

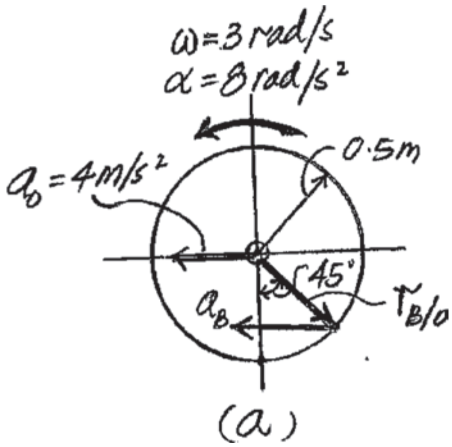
$$a_B = \{-4.354\mathbf{i} + 6.010\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{(-4.354)^2 + 6.010^2} = 7.4215 \text{ m/s}^2 = 7.42 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is given by

$$\theta = \tan^{-1}\left(\frac{6.010}{4.354}\right) = 54.08° = 54.1° \qquad \textbf{Ans.}$$

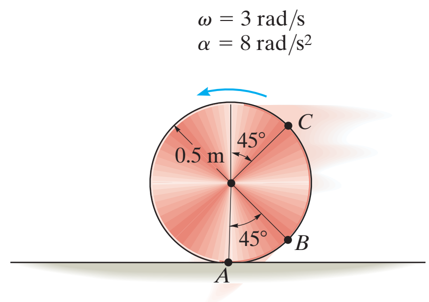


(a)

Ans:
$a_B = 7.42 \text{ m/s}^2$
$\theta = 54.1°$

16–113.

The disk has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point C.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. a,

$$\mathbf{a}_C = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{C/O} - \omega^2 \mathbf{r}_{C/O}$$

$$\mathbf{a}_C = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j})$$

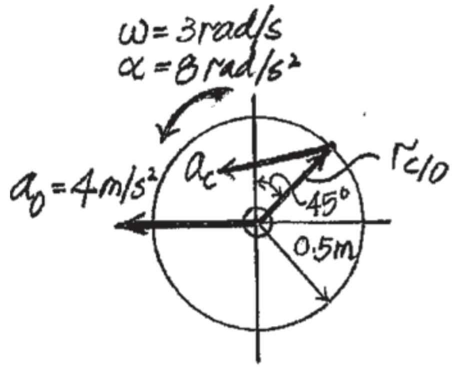
$$a_C = \{-10.0104\mathbf{i} - 0.3536\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{(-10.0104)^2 + (-0.3536)^2} = 10.017\ \text{m/s}^2 = 10.0\ \text{m/s}^2 \quad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left\{\frac{0.3536}{10.0104}\right\} = 2.023° = 2.02° \ \measuredangle \quad \textbf{Ans.}$$

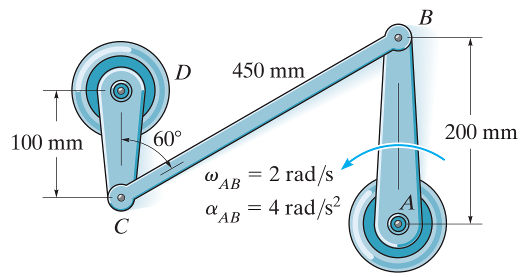


(a)

Ans:
$a_C = 10.0\ \text{m/s}^2$
$\theta = 2.02° \ \measuredangle$

16–114.

Member AB has the angular motions shown. Determine the angular velocity and angular acceleration of members CB and DC.



SOLUTION

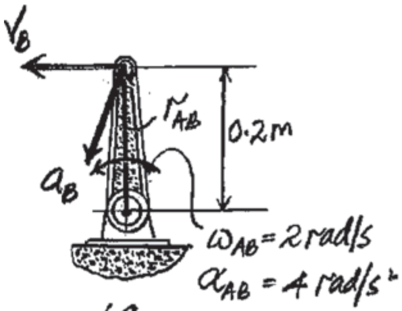
Rotation About A Fixed Axis. For crank AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 2(0.2) = 0.4 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (4\mathbf{k}) \times (0.2\mathbf{j}) - 2^2(0.2\mathbf{j})$$

$$= \{-0.8\mathbf{i} - 0.8\mathbf{j}\} \text{ m/s}^2$$



(a)

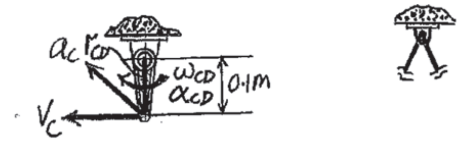
For link CD, refer to Fig. *b*.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.1)$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$= (-\alpha_{CD}\mathbf{k}) \times (-0.1\mathbf{j}) - \omega_{CD}^2(-0.1\mathbf{j})$$

$$= -0.1\alpha_{CD}\mathbf{i} + 0.1\omega_{CD}^2\mathbf{j}$$



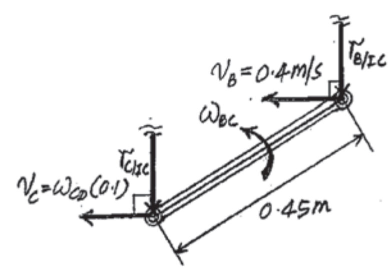
(b)

General Plane Motion. The IC of link CD can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ of which in this case is at infinity as indicated in Fig. *c*. Thus, $r_{B/IC} = r_{C/IC} = \infty$. Thus, kinematics gives

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{0.4}{\infty} = 0 \qquad \textbf{Ans.}$$

Then

$$v_C = v_B; \qquad \omega_{CD}(0.1) = 0.4 \qquad \omega_{CD} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



(c)

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.1\alpha_{CD}\mathbf{i} + 0.1(4.00^2)\mathbf{j} = (-0.8\mathbf{i} - 0.8\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (-0.45 \sin 60°\mathbf{i} - 0.45 \cos 60°\mathbf{j}) - \mathbf{0}$$

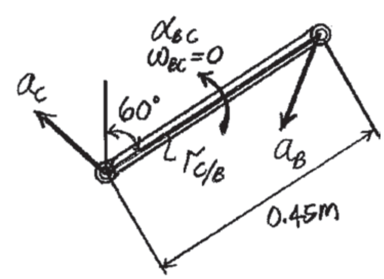
$$-0.1\alpha_{CD}\mathbf{i} + 1.6\mathbf{j} = (0.225\alpha_{BC} - 0.8)\mathbf{i} + (-0.8 - 0.3897\alpha_{BC})\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$1.6 = -0.8 - 0.3897\alpha_{BC}; \qquad \alpha_{BC} = -6.1584 \text{ rad/s}^2 = 6.16 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then $\mathbf{i}$ components give

$$-0.1\alpha_{CD} = 0.225(-6.1584) - 0.8; \qquad \alpha_{CD} = 21.86 \text{ rad/s}^2 = 21.9 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

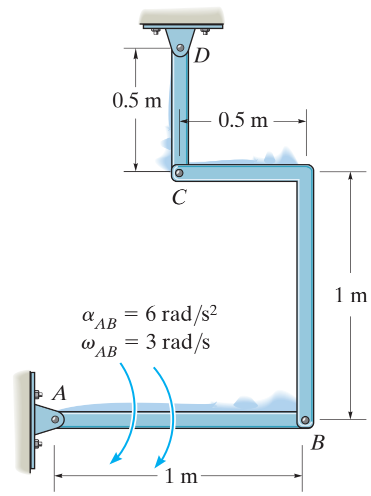


(d)

Ans:
$\omega_{BC} = 0$
$\omega_{CD} = 4.00 \text{ rad/s} \circlearrowright$
$\alpha_{BC} = 6.16 \text{ rad/s}^2 \circlearrowright$
$\alpha_{CD} = 21.9 \text{ rad/s}^2 \circlearrowright$

16–115.

Determine the angular acceleration of link CD if link AB has the angular velocity and angular acceleration shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 3(1) = 3.00 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$
$$= (-6\mathbf{k}) \times (1\mathbf{i}) - 3^2(1\mathbf{i})$$
$$= \{-9\mathbf{i} - 6\mathbf{j}\} \text{ m/s}$$

For link CD, refer to Fig. *b*

$$v_C = \omega_{CD} r_{DC} = \omega_{CD}(0.5) \rightarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{DC} - \omega_{CD}^2 \mathbf{r}_{DC}$$
$$= (\alpha_{CD}\mathbf{k}) \times (-0.5\mathbf{j}) - \omega_{CD}^2(-0.5\mathbf{j})$$
$$= 0.5\alpha_{CD}\mathbf{i} + 0.5\omega_{CD}^2\mathbf{j}$$

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. *c*. Thus

$$r_{B/IC} = 0.5 \text{ m} \quad r_{C/IC} = 1 \text{ m}$$

Then, the kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \quad 3 = \omega_{BC}(0.5) \quad \omega_{BC} = 6.00 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \quad \omega_{CD}(0.5) = 6.00(1) \quad \omega_{CD} = 12.0 \text{ rad/s} \circlearrowleft$$

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times r_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$0.5\alpha_{CD}\mathbf{i} + 0.5(12.0^2)\mathbf{j} = (-9\mathbf{i} - 6\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5\mathbf{i} + \mathbf{j}) - 6.00^2(-0.5\mathbf{i} + \mathbf{j})$$

$$0.5\alpha_{CD}\mathbf{i} + 72\mathbf{j} = (\alpha_{BC} + 9)\mathbf{i} + (0.5\alpha_{BC} - 42)\mathbf{j}$$

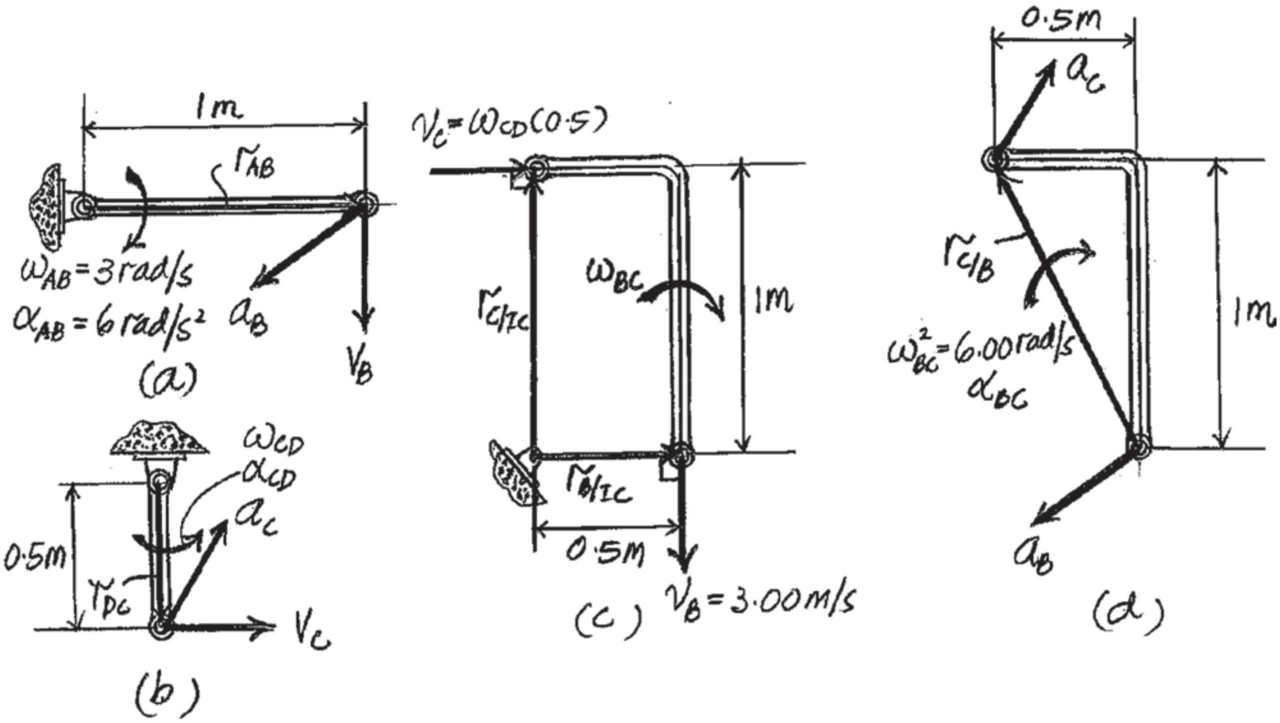
16–115. Continued

Equating **j** components,

$72 = (0.5\alpha_{BC} - 42); \quad \alpha_{BC} = 228 \text{ rad/s}^2 \circlearrowright$

Then **i** component gives

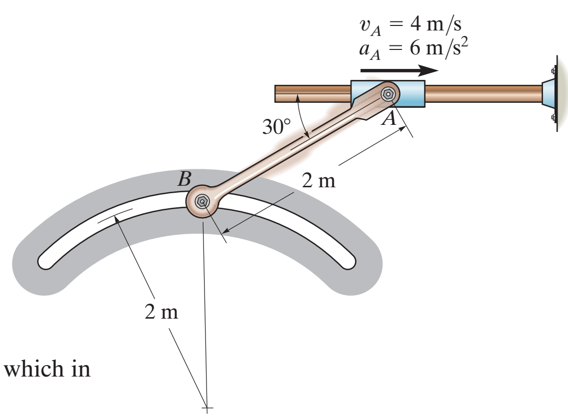
$0.5\alpha_{CD} = 228 + 9; \quad \alpha_{CD} = 474 \text{ rad/s}^2 \circlearrowleft$ **Ans.**



Ans:
$\alpha_{CD} = 474 \text{ rad/s}^2 \circlearrowleft$

***16–116.**

At a given instant the slider block A is moving to the right with the motion shown. Determine the angular acceleration of link AB and the acceleration of point B at this instant.



SOLUTION

General Plane Motion. The IC of the link can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity as shown in Fig. *a*. Thus

$$r_{A/IC} = r_{B/IC} = \infty$$

Then the kinematics gives

$$v_A = \omega\, r_{A/IC}; \quad 4 = \omega(\infty) \quad \omega = 0$$

$$v_B = v_A = 4 \text{ m/s}$$

Since B moves along a circular path, its acceleration will have tangential and normal components. Hence $(a_B)_n = \dfrac{v_B^2}{r_B} = \dfrac{4^2}{2} = 8 \text{ m/s}^2$

Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = 6\mathbf{i} + (\alpha\mathbf{k}) \times (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{j}) - 0$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = (\alpha + 6)\mathbf{i} - \sqrt{3}\alpha\mathbf{j}$$

Equating **i** and **j** componenets,

$$-8 = -\sqrt{3}\alpha; \quad \alpha = \frac{8\sqrt{3}}{3} \text{ rad/s}^2 = 4.62 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

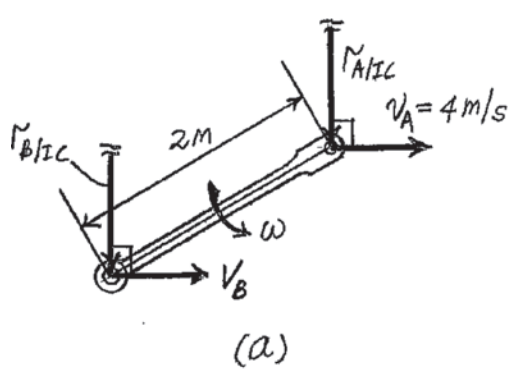
$$(a_B)_t = \alpha + 6; \quad (a_B)_t = \frac{8\sqrt{3}}{3} + 6 = 10.62 \text{ m/s}^2$$

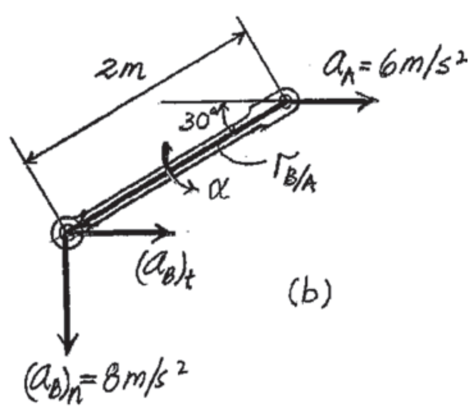
Thus, the magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{10.62^2 + 8^2} = 13.30 \text{ m/s}^2 = 13.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{8}{10.62}\right) = 36.99° = 37.0° \qquad \textbf{Ans.}$$

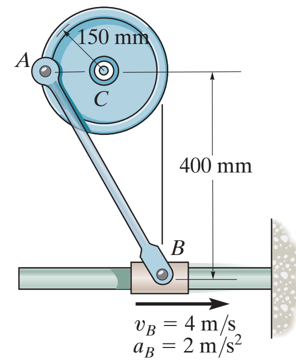




Ans:
$\alpha_{AB} = 4.62 \text{ rad/s}^2 \circlearrowleft$
$a_B = 13.3 \text{ m/s}^2$
$\theta = 37.0°$

16–117.

The slider block has the motion shown. Determine the angular velocity and angular acceleration of the wheel at this instant.



SOLUTION

Rotation About A Fixed Axis. For wheel C, refer to Fig. a.

$$v_A = \omega_C r_C = \omega_C(0.15) \downarrow$$

$$\mathbf{a}_A = \boldsymbol{\alpha}_C \times \mathbf{r}_C - \omega_C^2 \mathbf{r}_C$$

$$\mathbf{a}_A = (\alpha_C \mathbf{k}) \times (-0.15\mathbf{i}) - \omega_C^2(-0.15\mathbf{i})$$

$$= 0.15\,\omega_C^2\,\mathbf{i} - 0.15\alpha_C\mathbf{j}$$

General Plane Motion. The IC for crank AB can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. b. Here

$$r_{A/IC} = 0.3 \text{ m} \qquad r_{B/IC} = 0.4 \text{ m}$$

Then the kinematics gives

$$v_B = \omega_{AB}\, r_{B/IC}; \quad 4 = \omega_{AB}(0.4) \quad \omega_{AB} = 10.0 \text{ rad/s} ↺$$

$$v_A = \omega_{AB}\, r_{A/IC}; \quad \omega_C(0.15) = 10.0(0.3) \quad \omega_C = 20.0 \text{ rad/s} ↺ \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$2\mathbf{i} = 0.15(20.0^2)\mathbf{i} - 0.15\alpha_C\mathbf{j} + (\alpha_{AB}\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$
$$-10.0^2(0.3\mathbf{i} - 0.4\mathbf{j})$$

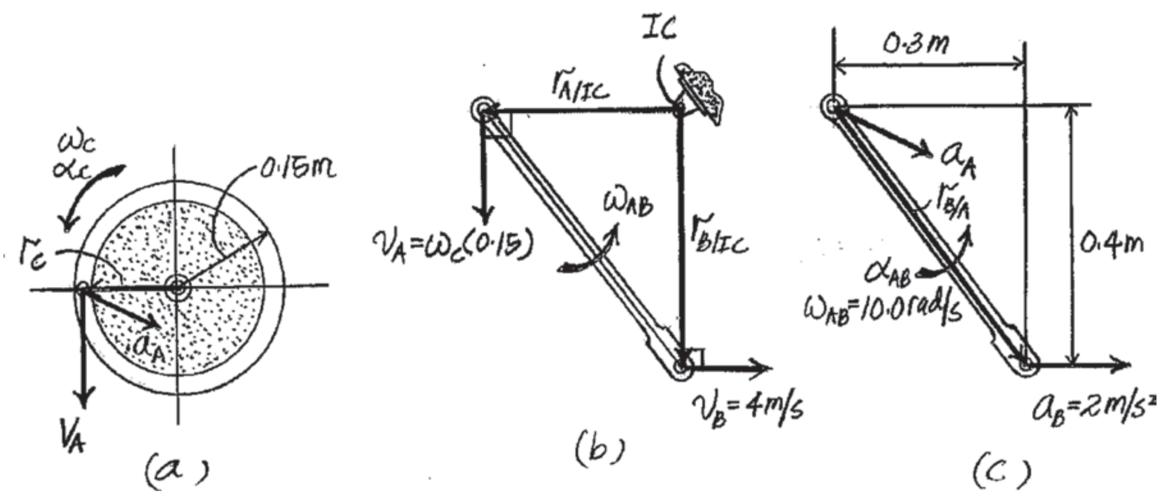
$$2\mathbf{i} = (0.4\alpha_{AB} + 30)\mathbf{i} + (0.3\alpha_{AB} - 0.15\alpha_C + 40)\mathbf{j}$$

Equating **i** and **j** components,

$$2 = 0.4\alpha_{AB} + 30; \quad \alpha_{AB} = -70.0 \text{ rad/s}^2 = 70.0 \text{ rad/s}^2 ↻$$

$$0 = 0.3(-70.0) + 0.15\alpha_C + 40; \quad \alpha_C = -126.67 \text{ rad/s}^2 = 127 \text{ rad/s}^2 ↻ \qquad \textbf{Ans.}$$

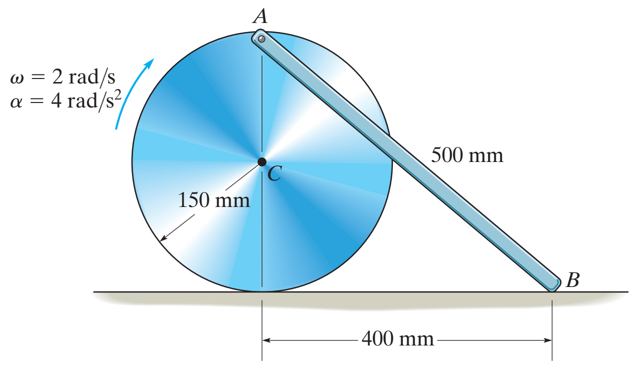
The negative signs indicate that $\boldsymbol{\alpha}_C$ and $\boldsymbol{\alpha}_{AB}$ are directed in the sense that those shown in Fig. a and c.



Ans:
$\omega_C = 20.0$ rad/s ↺
$\alpha_C = 127$ rad/s^2 ↻

16–118.

The disk rolls without slipping such that it has an angular acceleration of $\alpha = 4\ \text{rad/s}^2$ and angular velocity of $\omega = 2\ \text{rad/s}$ at the instant shown. Determine the acceleration of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, 150 mm from C.



SOLUTION

The IC is at ∞, so $\omega = 0$.

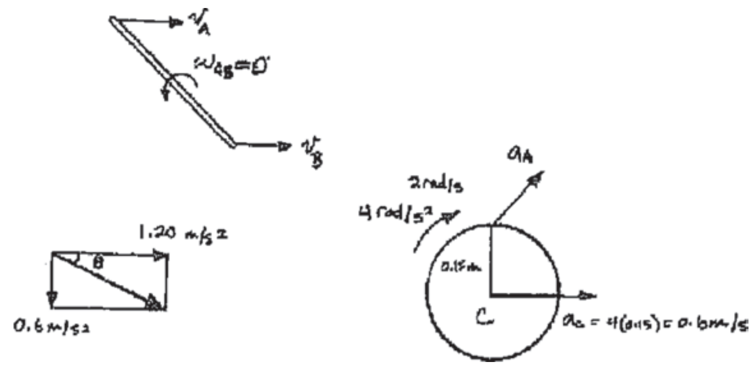
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{a}_A = 0.6\mathbf{i} + (-4\mathbf{k}) \times (0.15\mathbf{j}) - (2)^2(0.15\mathbf{j})$$

$$\mathbf{a}_A = (1.20\mathbf{i} - 0.6\mathbf{j})\ \text{m/s}^2$$

$$a_A = \sqrt{(1.20)^2 + (-0.6)^2} = 1.34\ \text{m/s}^2 \qquad \textbf{Ans.}$$

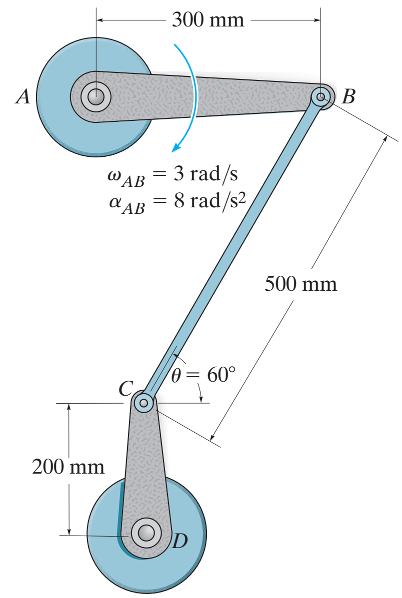
$$\theta = \tan^{-1}\left(\frac{0.6}{1.20}\right) = 26.6^\circ \searrow \qquad \textbf{Ans.}$$



Ans:
$a_A = 1.34\ \text{m/s}^2$
$\theta = 26.6^\circ \searrow$

16–119.

If member AB has the angular motion shown, determine the angular velocity and angular acceleration of member CD at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

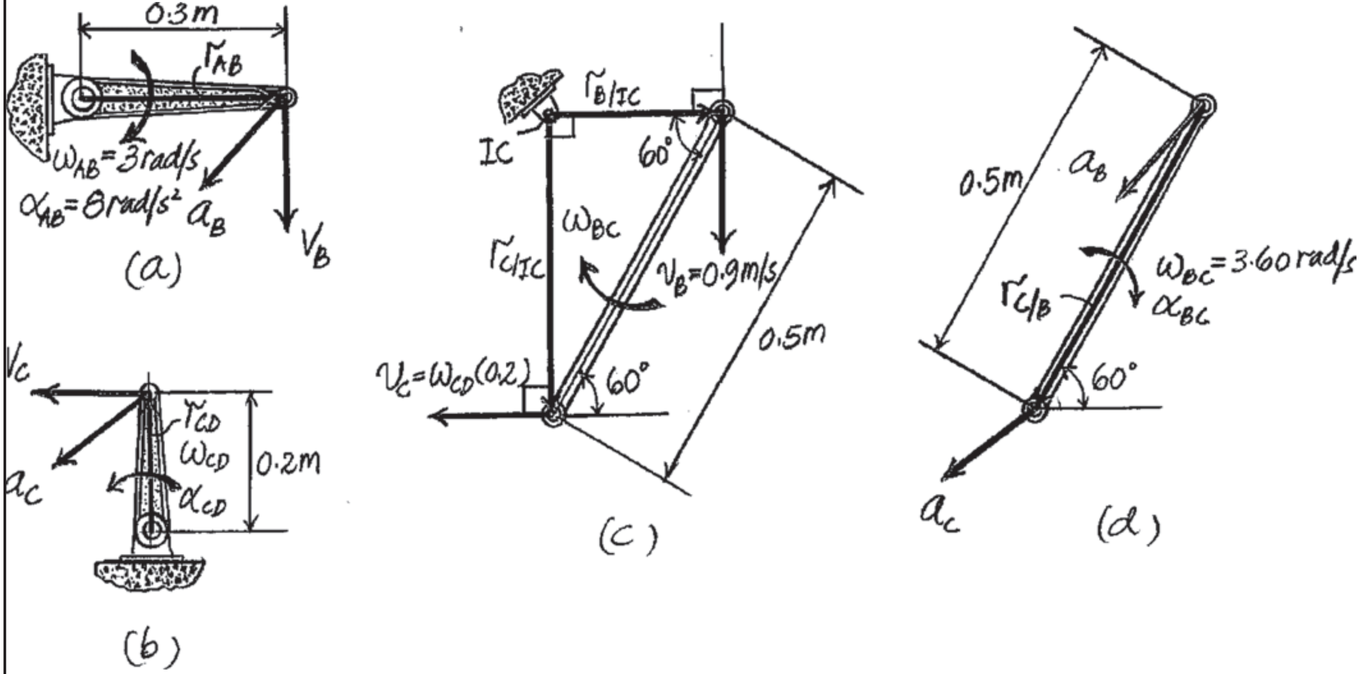
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



16–119. Continued

General Plane Motion. The *IC* of link *BC* can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$

$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the **j** components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

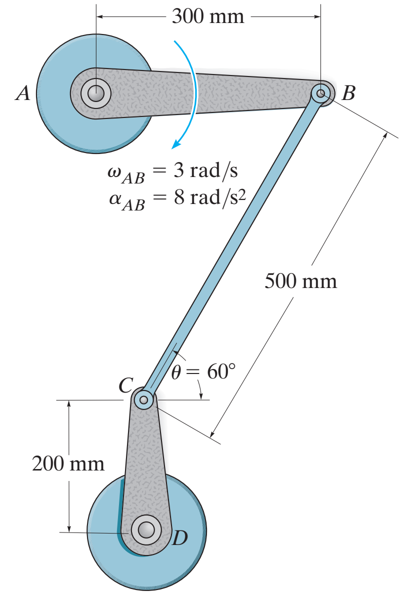
Then the **i** component gives

$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \circlearrowright$ **Ans.**

Ans:
$\omega_{CD} = 7.79 \text{ rad/s} \circlearrowleft$
$\alpha_{CD} = 136 \text{ rad/s}^2 \circlearrowright$

***16–120.**

If member AB has the angular motion shown, determine the velocity and acceleration of point C at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

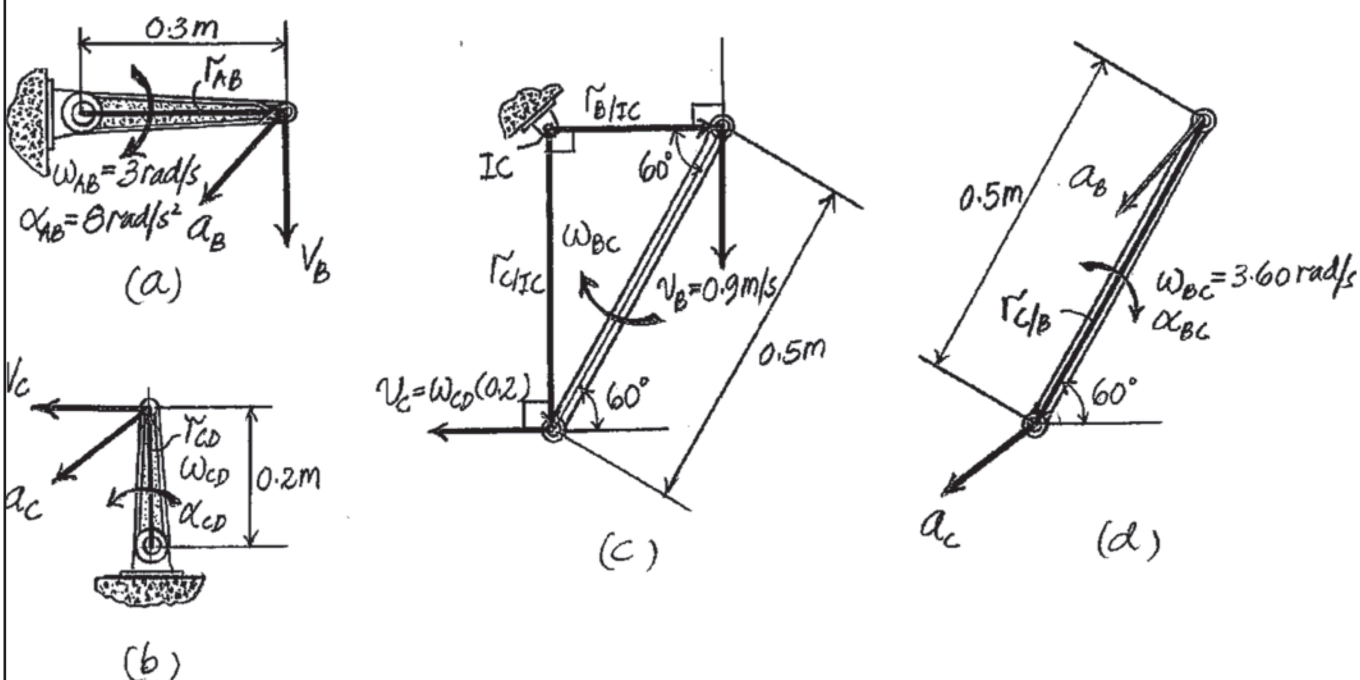
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



***16–120. Continued**

General Plane Motion. The *IC* of link *BC* can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the **j** components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

Then the **i** component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \quad \textbf{Ans.}$$

From the angular motion of *CD*,

$$v_C = w_{CD}(0.2) = (7.7942)(0.2) = 1.559 \text{ m/s} = 1.56 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = -0.2(-135.74)\mathbf{i} - 12.15\mathbf{j}$$
$$= \{27.15\mathbf{i} - 12.15\mathbf{j}\} \text{ m/s}$$

The magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{27.15^2 + (-12.15)^2} = 29.74 \text{ m/s}^2 = 29.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

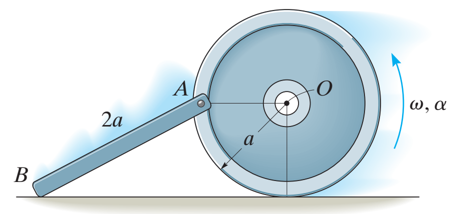
And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{12.15}{27.15}\right) = 24.11° = 24.1° \measuredangle \qquad \textbf{Ans.}$$

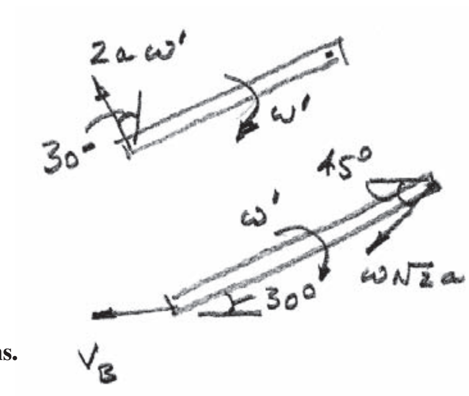
Ans:
$v_C = 1.56 \text{ m/s} \leftarrow$
$a_C = 29.7 \text{ m/s}^2$
$\theta = 24.1° \measuredangle$

16–121.

The wheel rolls without slipping such that at the instant shown it has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of point B on the rod at this instant.



SOLUTION



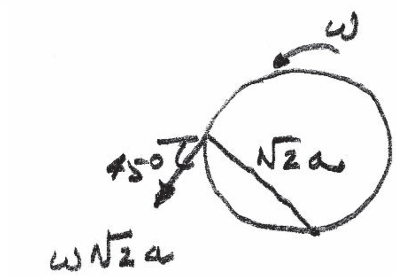
$$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A\,(Pin)}$$

$$\overset{+}{\leftarrow}\; v_B = \frac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\frac{1}{2}\right)$$

$$+\uparrow\; O = -\frac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\frac{\sqrt{3}}{2}\right)$$

$$\omega' = \frac{\omega}{\sqrt{3}}$$

$$v_B = 1.58\,\omega a \qquad \textbf{Ans.}$$

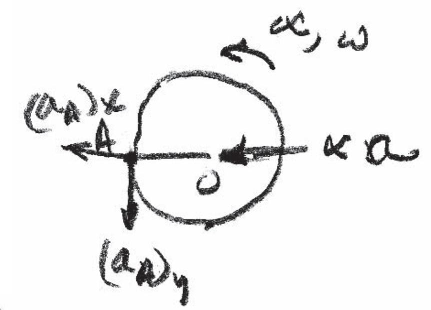


$$\bar{a}_A = \bar{a}_O + \bar{a}_{A/O\,(Pin)}$$

$$\underset{\leftarrow}{(a_A)_x} + \underset{\downarrow}{(a_A)_y} = \underset{\leftarrow}{\alpha a} + \underset{\downarrow}{\alpha(a)} + \underset{\rightarrow}{\omega^2 a}$$

$$(a_A)_x = \alpha a - \omega^2 a$$

$$(a_A)_y = \alpha a$$



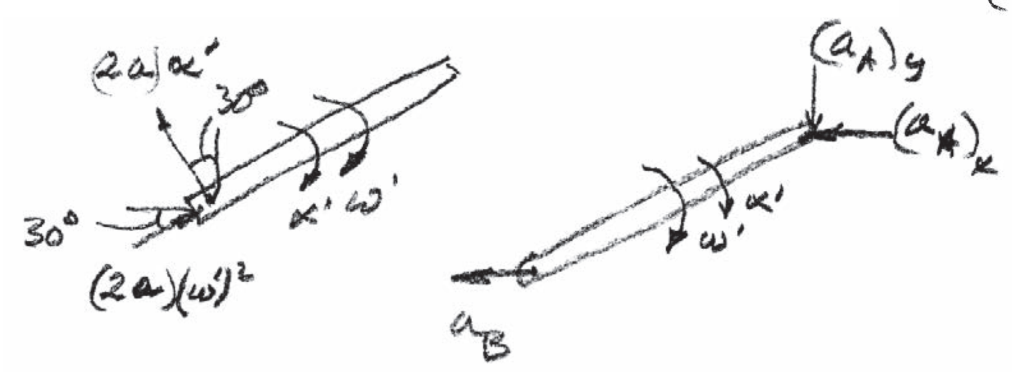
$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A\,(Pin)}$$

$$a_B = \alpha a - \omega^2 a + 2a(\alpha')\left(\frac{1}{2}\right) - 2a\left(\frac{\omega}{\sqrt{3}}\right)^2\frac{\sqrt{3}}{2}$$

$$O = -\alpha a + 2a\alpha'\left(\frac{2}{\sqrt{3}}\right) + 2a\left(\frac{\omega}{\sqrt{3}}\right)^2\left(\frac{1}{2}\right)$$

$$\alpha' = 0.577\alpha - 0.1925\omega^2$$

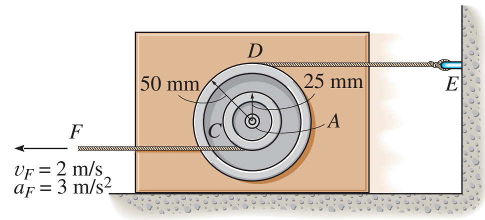
$$a_B = 1.58\alpha a - 1.77\omega^2 a \qquad \textbf{Ans.}$$



Ans:
$v_B = 1.58\omega a$
$a_B = 1.58\,\alpha a - 1.77\omega^2 a$

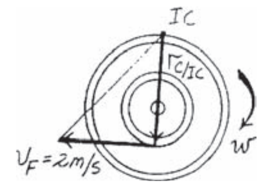
16–122.

A single pulley having both an inner and outer rim is pin-connected to the block at A. As cord CF unwinds from the inner rim of the pulley with the motion shown, cord DE unwinds from the outer rim. Determine the angular acceleration of the pulley and the acceleration of the block at the instant shown.



SOLUTION

Velocity Analysis: The angular velocity of the pulley can be obtained by using the method of instantaneous center of zero velocity. Since the pulley rotates without slipping about point D, i.e: $v_D = 0$, then point D is the location of the instantaneous center.

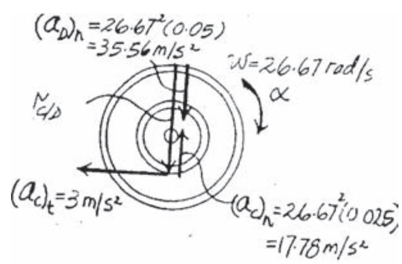


$$v_F = \omega r_{C/IC}$$

$$2 = \omega(0.075)$$

$$\omega = 26.67 \text{ rad/s}$$

Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion points C and D. Applying Eq. 16–18 with $\mathbf{r}_{C/D} = \{-0.075\mathbf{j}\}$ m, we have



$$\mathbf{a}_C = \mathbf{a}_D + \alpha \times \mathbf{r}_{C/D} - \omega^2 \mathbf{r}_{C/D}$$

$$-3\mathbf{i} + 17.78\mathbf{j} = -35.56\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.075\mathbf{j}) - 26.67^2(-0.075\mathbf{j})$$

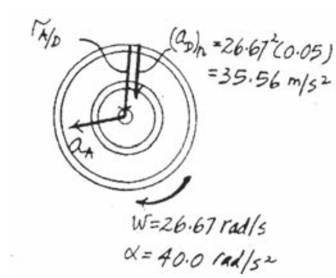
$$-3\mathbf{i} + 17.78\mathbf{j} = -0.075\,\alpha\mathbf{i} + 17.78\mathbf{j}$$

Equating **i** and **j** components, we have

$$-3 = -0.075\alpha \qquad \alpha = 40.0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$17.78 = 17.78 \; (\textbf{\textit{Check!}})$$

The acceleration of point A can be obtained by analyzing the angular motion points A and D. Applying Eq. 16–18 with $\mathbf{r}_{A/D} = \{-0.05\mathbf{j}\}$ m. we have



$$\mathbf{a}_A = \mathbf{a}_D + \alpha \times \mathbf{r}_{A/D} - \omega^2 \mathbf{r}_{A/D}$$

$$= -35.56\mathbf{j} + (-40.0\mathbf{k}) \times (-0.05\mathbf{j}) - 26.67^2(-0.05\mathbf{j})$$

$$= \{-2.00\mathbf{i}\} \text{ m/s}^2$$

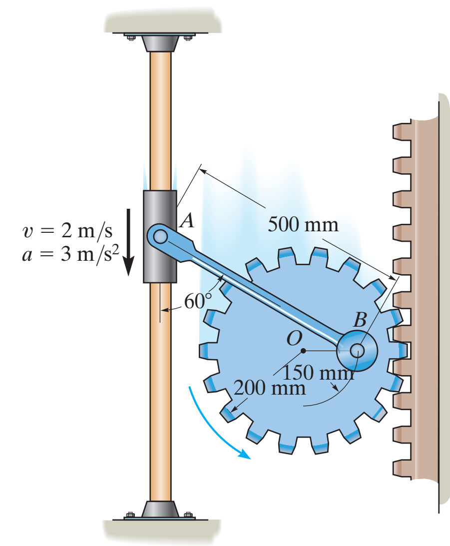
Thus,

$$a_A = 2.00 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

Ans:
$\alpha = 40.0$ rad/s²
$a_A = 2.00$ m/s² ←

16–123.

The collar is moving downward with the motion shown. Determine the angular velocity and angular acceleration of the gear at the instant shown as it rolls along the fixed gear rack.



SOLUTION

General Plane Motion. For gear C, the location of its IC is indicate in Fig. a. Thus

$$v_B = \omega_C\, r_{B/(IC)_1} = \omega_C(0.05) \downarrow \qquad \textbf{(1)}$$

The IC of link AB can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity. Thus

$$\omega_{AB} = \frac{v_A}{r_{A/(IC)_2}} = \frac{2}{\infty} = 0$$

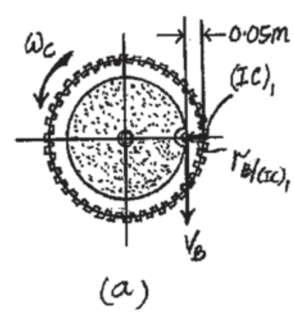
Then

$$v_B = v_A = 2\ \text{m/s} \downarrow$$

Substitute the result of v_B into Eq. (1)

$$2 = \omega_C(0.05)$$

$$\omega_C = 40.0\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

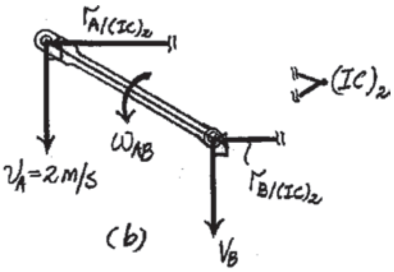


Applying the relative acceleration equation to gear C, Fig. c, with $a_O = \alpha_C r_C = \alpha_C(0.2)\downarrow$,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha}_C \times \mathbf{r}_{B/O} - \omega_C^2\,\mathbf{r}_{B/O}$$

$$\mathbf{a}_B = -\alpha_C(0.2)\mathbf{j} + (\alpha_C\mathbf{k}) \times (0.15\mathbf{i}) - 40.0^2(0.15\mathbf{i})$$

$$= -240\mathbf{i} - 0.05\alpha_C\mathbf{j}$$



For link AB, Fig. d,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

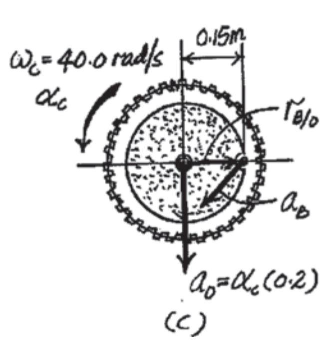
$$-240\mathbf{i} - 0.05\alpha_C\mathbf{j} = (-3\mathbf{j}) + (\alpha_{AB}\mathbf{k}) \times (0.5\sin 60°\mathbf{i} - 0.5\cos 60°\mathbf{j}) - \mathbf{0}$$

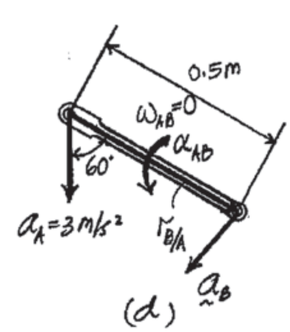
$$-240\mathbf{i} - 0.05\alpha_C = 0.25\alpha_{AB}\mathbf{i} + (0.25\sqrt{3}\alpha_{AB} - 3)\mathbf{j}$$

Equating **i** and **j** components

$$-240 = 0.25\alpha_{AB}; \quad \alpha_{AB} = -960\ \text{rad/s}^2 = 960\ \text{rad/s}^2 \circlearrowleft$$

$$-0.05\alpha_C = \left(0.25\sqrt{3}\right)(-960) - 3; \quad \alpha_C = 8373.84\ \text{rad/s}^2 = 8374\ \text{rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

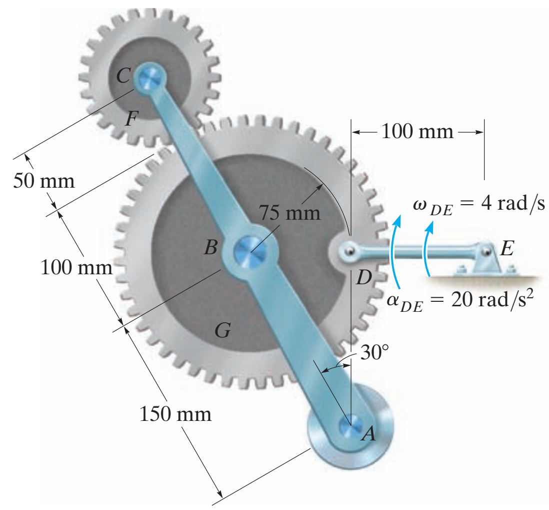




Ans:
$\omega_C = 40.0\ \text{rad/s} \circlearrowright$
$\alpha_C = 8374\ \text{rad/s}^2 \circlearrowright$

***16–124.**

The tied crank and gear mechanism gives rocking motion to crank AC, necessary for the operation of a printing press. If link DE has the angular motion shown, determine the respective angular velocities of gear F and crank AC at this instant, and the angular acceleration of crank AC.



SOLUTION

Velocity analysis:

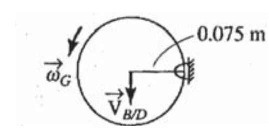
$v_D = \omega_{DE} r_{D/E} = 4(0.1) = 0.4\text{ m/s}\uparrow$

$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$

$\underset{\measuredangle 30^\circ}{v_B} = \underset{\uparrow}{0.4} + \underset{\downarrow}{(\omega_G)(0.075)}$

$(\overset{+}{\rightarrow})\qquad v_B\cos 30^\circ = 0, \qquad v_B = 0$

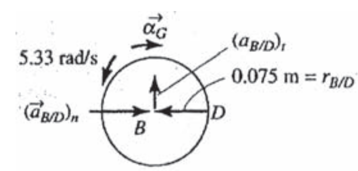
$(+\uparrow)\quad \omega_G = 5.33\text{ rad/s}$



Since $v_B = 0, \qquad v_C = 0, \qquad \omega_{AC} = 0$ **Ans.**

$\omega_F r_F = \omega_G r_G$

$\omega_F = 5.33\left(\dfrac{100}{50}\right) = 10.7\text{ rad/s}$ **Ans.**



Acceleration analysis:

$(a_D)_n = (4)^2(0.1) = 1.6\text{ m/s}^2 \rightarrow$

$(a_D)_t = (20)(0.1) = 2\text{ m/s}^2\uparrow$

$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (\mathbf{a}_D)_n + (\mathbf{a}_D)_t + (\mathbf{a}_{B/D})_n + (\mathbf{a}_{B/D})_t$

$0 + \underset{\measuredangle 30^\circ}{(a_B)_t} = \underset{\rightarrow}{1.6} + \underset{\uparrow}{2} + \underset{\rightarrow}{(5.33)^2(0.075)} + \underset{\uparrow}{\alpha_G(0.075)}$

$(+\uparrow)\quad (a_B)_t\sin 30^\circ = 0 + 2 + 0 + \alpha_G(0.075)$

$(\overset{+}{\rightarrow})\quad (a_B)_t\cos 30^\circ = 1.6 + 0 + (5.33)^2(0.075) + 0$

Solving,

$(a_B)_t = 4.31\text{ m/s}^2, \qquad \alpha_G = 2.052\text{ rad/s}^2 \circlearrowright$

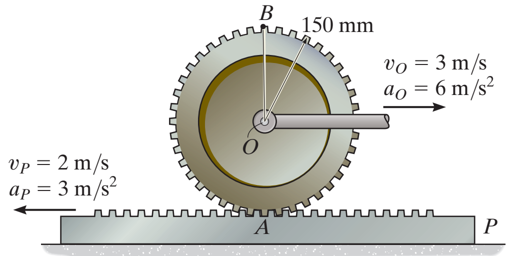
Hence,

$\alpha_{AC} = \dfrac{(a_B)_t}{r_{B/A}} = \dfrac{4.31}{0.15} = 28.7\text{ rad/s}^2 \circlearrowright$ **Ans.**

Ans:
$\omega_{AC} = 0$
$\omega_F = 10.7\text{ rad/s} \circlearrowright$
$\alpha_{AC} = 28.7\text{ rad/s}^2 \circlearrowright$

16–125.

The center O of the gear and the gear rack P move with the velocities and accelerations shown. Determine the angular acceleration of the gear and the acceleration of point B located at the rim of the gear at the instant shown.



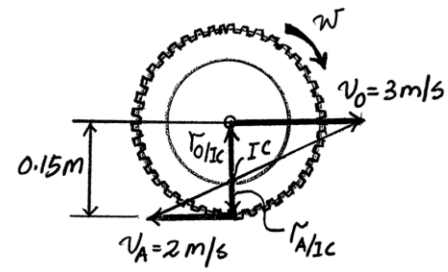
SOLUTION

Angular Velocity: The location of the IC is indicated in Fig. a. Using similar triangles,

$$\frac{3}{r_{O/IC}} = \frac{2}{0.15 - r_{O/IC}} \qquad r_{O/IC} = 0.09 \text{ m}$$

Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.09} = 33.33 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: Applying the relative acceleration equation to points O and A and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 r_{A/O}$$

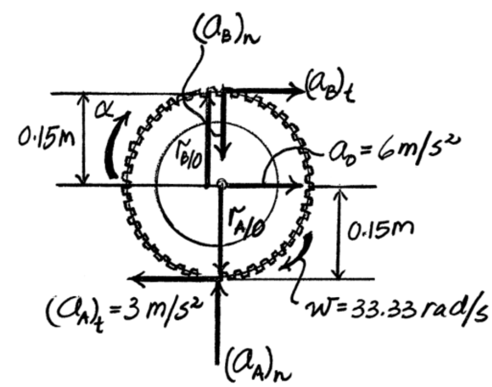
$$-3\mathbf{i} + (a_A)_n\mathbf{j} = 6\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.15\mathbf{j}) - 33.33^2(-0.15\mathbf{j})$$

$$-3\mathbf{i} + (a_A)_n\mathbf{j} = (6 - 0.15\alpha)\mathbf{i} + 166.67\mathbf{j}$$

Equating the **i** components,

$$-3 = 6 - 0.15\alpha$$

$$\alpha = 60 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(b)

Using this result, the relative acceleration equation is applied to points O and B, Fig. b, which gives

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 r_{B/O}$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 6\mathbf{i} + (-60\mathbf{k}) \times (0.15\mathbf{j}) - 33.33^2(0.15\mathbf{j})$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 15\mathbf{i} - 166.67\mathbf{j}$$

Equating the **i** and **j** components,

$$(a_B)_t = 15 \text{ m/s}^2 \qquad (a_B)_n = 166.67 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

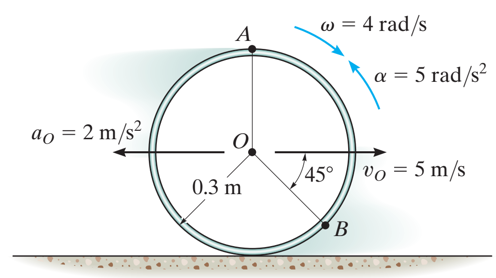
$$a_B = \sqrt{(a_B)_t^{\,2} + (a_B)_n^{\,2}} = \sqrt{15^2 + 166.67^2} = 167 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{166.67}{15}\right) = 84.9° \measuredangle$$

Ans:
$\alpha = 60 \text{ rad/s}^2$
$a_B = 167 \text{ m/s}^2$

16–126. The hoop is cast on the rough surface such that it has angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s². Also, its center has a velocity $v_0 = 5$ m/s and a deceleration $a_0 = 2$ m/s². Determine the acceleration of point A at this instant.



SOLUTION

Given:

$\omega = 4 \text{ rad/s} \quad a_0 = 2 \text{ m/s}^2$

$\alpha = 5 \text{ rad/s}^2 \quad r = 0.3 \text{ m}$

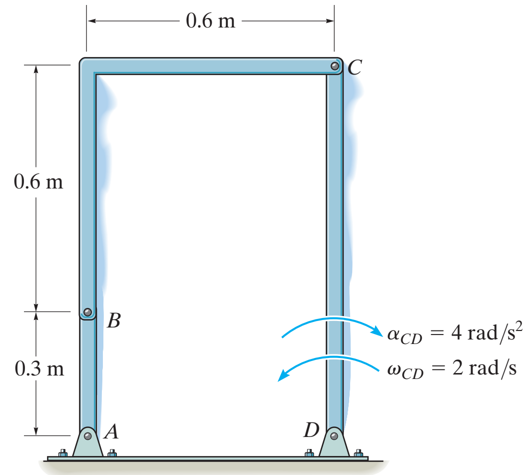
$v_0 = 5 \text{ m/s} \quad \phi = 45°$

$$\mathbf{a_A} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_A} = \begin{pmatrix} -3.50 \\ -4.80 \\ 0.00 \end{pmatrix} \text{m/s}^2 \qquad |\mathbf{a_A}| = 5.94 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{a_A} = \{-3.50\mathbf{i} - 4.80\mathbf{j}\} \text{ m/s}^2$
$|\mathbf{a_A}| = 5.94 \text{ m/s}^2$

16–127. Determine the angular acceleration of link AB if link CD has the angular velocity and angular deceleration shown.



SOLUTION

Given:

$$\alpha_{CD} = 4\ \text{rad/s}^2 \qquad a = 0.3\ \text{m}$$

$$b = 0.6\ \text{m}$$

$$\omega_{CD} = 2\ \text{rad/s} \qquad c = 0.6\ \text{m}$$

$$\omega_{BC} = 0 \qquad \omega_{AB} = \omega_{CD}\frac{a+b}{a}$$

Guesses

$$\alpha_{AB} = 1\ \text{rad/s}^2 \qquad \alpha_{BC} = 1\ \text{rad/s}^2$$

Given

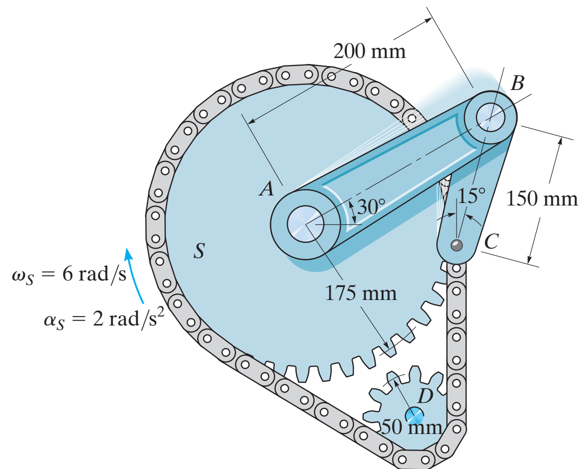
$$\begin{pmatrix}0\\0\\-\alpha_{CD}\end{pmatrix}\times\begin{pmatrix}0\\a+b\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\begin{pmatrix}0\\a+b\\0\end{pmatrix}\right]+\begin{pmatrix}0\\0\\\alpha_{BC}\end{pmatrix}\times\begin{pmatrix}-c\\-b\\0\end{pmatrix}\ \ldots = \begin{pmatrix}0\\0\\0\end{pmatrix}\text{m/s}^2$$
$$+\begin{pmatrix}0\\0\\\alpha_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}\right]$$

$$\begin{pmatrix}\alpha_{AB}\\\alpha_{BC}\end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{BC}) \qquad \alpha_{BC} = 12\ \text{rad/s}^2 \qquad \alpha_{AB} = -36\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_{AB} = -36\ \text{rad/s}^2$

***16–128.**

The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular acceleration $\alpha_S = 2\ \text{rad/s}^2$ and has an angular velocity $\omega_S = 6\ \text{rad/s}$ at the instant shown, determine the angular velocity and angular acceleration of link AB at this instant. The sprocket S is mounted on a shaft which is *separate* from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links such that it moves vertically downward.

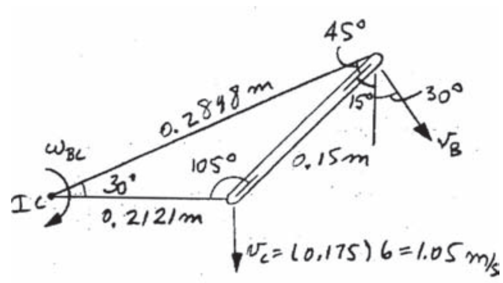


SOLUTION

$$\omega_{BC} = \frac{1.05}{0.2121} = 4.950\ \text{rad/s}$$

$$v_B = (4.95)(0.2898) = 1.434\ \text{m/s}$$

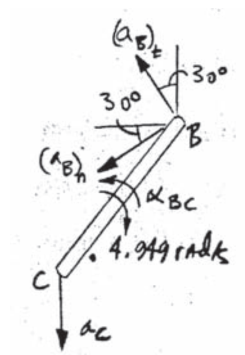
$$\omega_{AB} = \frac{1.435}{0.2} = 7.1722\ \text{rad/s} = 7.17\ \text{rad/s} \circlearrowright$$ **Ans.**



$$a_C = \alpha_S r_S = 2(0.175) = 0.350\ \text{m/s}^2$$

$$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = \mathbf{a}_C + (\mathbf{a}_{B/C})_n + (\mathbf{a}_{B/C})_t$$

$$\begin{bmatrix} (7.172)^2(0.2) \\ 30^\circ \swarrow \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ \nwarrow 30^\circ \end{bmatrix} = \begin{bmatrix} 0.350 \\ \downarrow \end{bmatrix} + \begin{bmatrix} (4.949)^2\ (0.15) \\ 15^\circ \swarrow \end{bmatrix} + \begin{bmatrix} \alpha_{BC}\ (0.15) \\ 15^\circ \nwarrow \end{bmatrix}$$



$$(\overset{+}{\rightarrow}) \quad -(10.29)\cos 30^\circ - (a_B)_t \sin 30^\circ = 0 - (4.949)^2(0.15)\sin 15^\circ - \alpha_{BC}(0.15)\cos 15^\circ$$

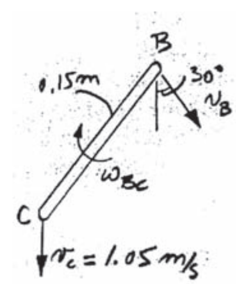
$$(+\uparrow) \quad -(10.29)\sin 30^\circ + (a_B)_t \cos 30^\circ = -0.350 - (4.949)^2(0.15)\cos 15^\circ + \alpha_{BC}(0.15)\sin 15^\circ$$

$$\alpha_{BC} = 70.8\ \text{rad/s}^2, \qquad (a_B)_t = 4.61\ \text{m/s}^2$$

Hence,

$$\alpha_{AB} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.61}{0.2} = 23.1\ \text{rad/s}^2 \circlearrowleft$$ **Ans.**

Also,



$$v_C = \omega_S r_S = 6(0.175) = 1.05\ \text{m/s} \downarrow$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$v_B \sin 30^\circ \mathbf{i} - v_B \cos 30^\circ \mathbf{j} = -1.05\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (0.15 \sin 15^\circ \mathbf{i} + 0.15 \cos 15^\circ \mathbf{j})$$

$$(\overset{+}{\rightarrow}) \quad v_B \sin 30^\circ = 0 + \omega_{BC}(0.15)\cos 15^\circ$$

$$(+\uparrow) \quad -v_B \cos 30^\circ = -1.05 - \omega_{BC}(0.15)\sin 15^\circ$$

$$v_B = 1.434\ \text{m/s}, \qquad \omega_{BC} = 4.950\ \text{rad/s}$$

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{1.434}{0.2} = 7.172 = 7.17\ \text{rad/s} \circlearrowright$$ **Ans.**

***16–128. Continued**

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$(\alpha_{AB}\mathbf{k}) \times (0.2 \cos 30°\mathbf{i} + 0.2 \sin 30°\mathbf{j}) - (7.172)^2 (0.2 \cos 30°\mathbf{i} + 0.2 \sin 30°\mathbf{j})$$

$$= -(2)(0.175)\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (0.15 \sin 15°\mathbf{i} + 0.15 \cos 15°\mathbf{j}) - (4.950)^2 (0.15 \sin 15°\mathbf{i} + 0.15 \cos 15°\mathbf{j})$$

$$(\overset{+}{\rightarrow}) \qquad -\alpha_{AB}(0.1) - 8.9108 = -0.1449\alpha_{BC} - 0.9512$$

$$(+\uparrow) \qquad \alpha_{AB}(0.1732) - 5.143 = -0.350 + 0.0388\alpha_{BC} - 3.550$$

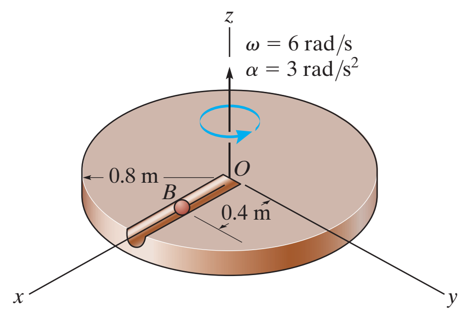
$\alpha_{AB} = 23.1 \text{ rad/s}^2 \circlearrowleft$ **Ans.**

$\alpha_{BC} = 70.8 \text{ rad/s}^2$

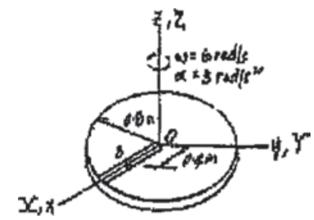
Ans:
$\omega_{AB} = 7.17 \text{ rad/s} \circlearrowright$
$\alpha_{AB} = 23.1 \text{ rad/s}^2 \circlearrowleft$

16–129.

At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



SOLUTION



Kinematic Equations:

$$v_B = v_O + \Omega \times \mathbf{r}_{B/O} + (v_{B/O})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (v_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$

$$v_O = 0$$

$$\mathbf{a}_O = 0$$

$$\Omega = \{6\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\Omega} = \{3\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{r}_{B/O} = \{0.4\mathbf{i}\}\ \text{m}$$

$$(\boldsymbol{v}_{B/O})_{xyz} = \{0.6\mathbf{i}\}\ \text{m/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\}\ \text{m/s}^2$$

Substitute the date into Eqs. (1) and (2) yields:

$$\boldsymbol{v}_B = 0 + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\}\ \text{m/s} \qquad \textbf{Ans.}$$

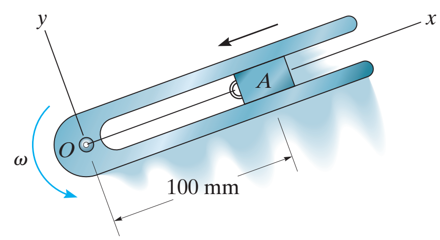
$$\mathbf{a}_B = 0 + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})] + 2(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

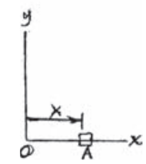
Ans:
$\mathbf{v}_B = \{0.6\mathbf{i} + 2.4\mathbf{j}\}$ m/s
$\mathbf{a}_B = \{-14.2\mathbf{i} + 8.40\mathbf{j}\}$ m/s^2

16–130.

Block A, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at O with an acceleration of $4\ \text{m/s}^2$ and its velocity is 2 m/s. Determine the acceleration of the block at this instant. The rod rotates about O with a constant angular velocity $\omega = 4$ rad/s.



SOLUTION



Motion of moving reference.

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = 4\mathbf{k}$

$\dot{\Omega} = \mathbf{0}$

Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.1\mathbf{i}$

$\mathbf{v}_{A/O} = -2\mathbf{i}$

$\mathbf{a}_{A/O} = -4\mathbf{i}$

Thus,

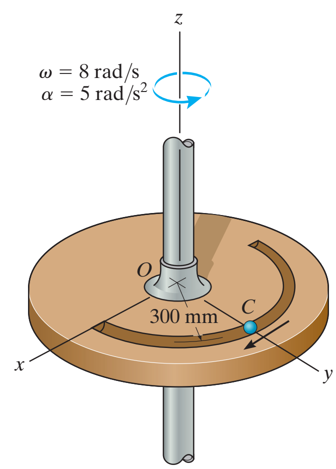
$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (4\mathbf{k}) \times (4\mathbf{k} \times 0.1\mathbf{i}) + 2(4\mathbf{k} \times (-2\mathbf{i})) - 4\mathbf{i}$$

$$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ \text{m/s}^2$$ **Ans.**

Ans:
$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ \text{m/s}^2$

16–131. Ball C moves with a speed of 3 m/s, which is increasing at a constant rate of 1.5 m/s^2, both measured relative to the circular plate and directed as shown. At the same instant the plate rotates with the angular velocity and angular acceleration shown. Determine the velocity and acceleration of the ball at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [8\mathbf{k}]\text{ rad/s} \qquad \dot{\omega} = \alpha = [5\mathbf{k}]\text{ rad/s}^2$$

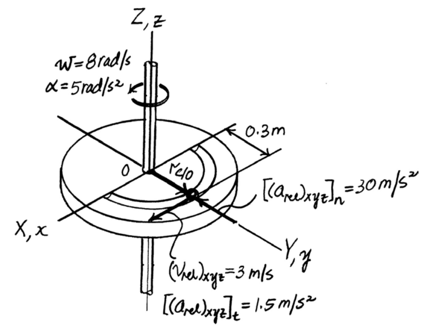
For the motion of ball C with respect to the xyz frame, we have

$$\mathbf{r}_{C/O} = [0.3\mathbf{j}]\text{ m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [3\mathbf{i}]\text{ m/s}$$

The normal component of $\left(\mathbf{a}_{\text{rel}}\right)_{xyz}$ is $\left[\left(a_{\text{rel}}\right)_{xyz}\right]_n = \dfrac{\left(v_{\text{rel}}\right)_{xyz}{}^2}{\rho} = \dfrac{3^2}{0.3} = 30\text{ m/s}^2$. Thus,

$$\left(\mathbf{a}_{\text{rel}}\right)_{xyz} = [1.5\mathbf{i} - 30\mathbf{j}]\text{ m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (8\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{i})$$
$$= [0.6\mathbf{i}]\text{ m/s} \qquad \textbf{Ans.}$$

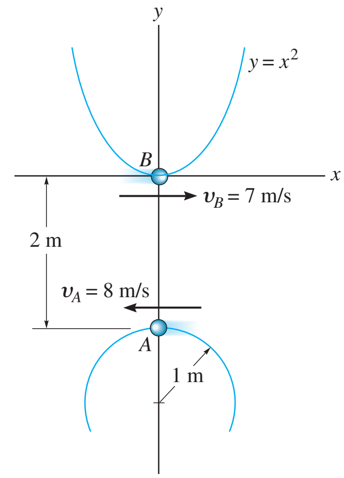
Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (5\mathbf{k}) \times (0.3\mathbf{j}) + (8\mathbf{k}) \times [(8\mathbf{k}) \times (0.3\mathbf{j})] + \mathbf{2}(8\mathbf{k}) \times (3\mathbf{i}) + (1.5\mathbf{i} - 30\mathbf{j})$$
$$= [-1.2\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = [0.6\mathbf{i}]\text{ m/s}$
$\mathbf{a}_C = [-1.2\mathbf{j}]\text{ m/s}^2$

***16–132.**

Particles B and A move along the parabolic and circular paths, respectively. If B has a velocity of 7 m/s in the direction shown and its speed is increasing at 4 m/s^2, while A has a velocity of 8 m/s in the direction shown and its speed is decreasing at 6 m/s^2, determine the relative velocity and relative acceleration of B with respect to A.



SOLUTION

$$\Omega = \frac{8}{1} = 8 \text{ rad/s}^2, \qquad \Omega = \{-8\mathbf{k}\} \text{ rad/s}$$

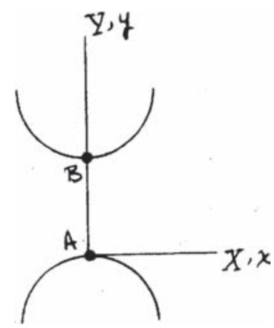
$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} + (8\mathbf{k}) \times (2\mathbf{j}) + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} - 16\mathbf{i} + (\mathbf{v}_{B/A})_{xyz}$$

$$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\} \text{ m/s}$$ **Ans.**

$$\dot{\Omega} = \frac{6}{1} = 6 \text{ rad/s}^2, \qquad \dot{\Omega} = \{-6\mathbf{k}\} \text{ rad/s}^2$$



$$(a_A)_n = \frac{(v_A)^2}{1} = \frac{(8)^2}{1} = 64 \text{ m/s}^2 \downarrow$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x\Big|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = 2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{2} = \frac{1}{2}$$

$$(a_B)_n = \frac{(7)^2}{\frac{1}{2}} = 98 \text{ m/s}^2 \uparrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + (-6\mathbf{k}) \times (2\mathbf{j}) + (8\mathbf{k}) \times (8\mathbf{k} \times 2\mathbf{j}) + 2(8\mathbf{k}) \times (31\mathbf{i}) + (\mathbf{a}_{B/A})_{xyz}$$

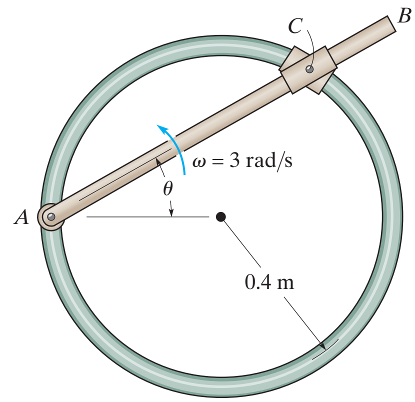
$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + 12\mathbf{i} - 128\mathbf{j} + 496\mathbf{j} + (\mathbf{a}_{B/A})_{xyz}$$

$$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\} \text{ m/s}^2$$ **Ans.**

Ans:
$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\}$ m/s
$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\}$ m/s^2

16–133.

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity of point C located on the double collar when $\theta = 30°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



SOLUTION

$r = 2(0.4 \cos 30°) = 0.6928$ m

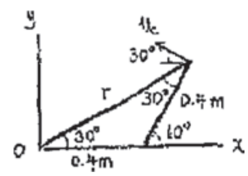
$\mathbf{r}_{C/A} = 0.6928 \cos 30°\mathbf{i} + 0.6928 \sin 30°\mathbf{j}$

$= \{0.600\mathbf{i} + 0.3464\mathbf{j}\}$ m

$\mathbf{v}_C = -0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j}$

$v_C = v_A + \Omega \times \mathbf{r}_{C/A} + (v_{C/A})_{xyz}$

$-0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j} = 0 + (3\mathbf{k}) \times (0.600\mathbf{i} + 0.3464\mathbf{j}) + (v_{C/A} \cos 30°\mathbf{i} + v_{C/A} \sin 30°\mathbf{j})$



$-0.866 v_C \mathbf{i} + 0.5 v_C \mathbf{j} = 0 - 1.039\mathbf{i} + 1.80\mathbf{j} + 0.866 v_{C/A}\mathbf{i} + 0.5 v_{C/A}\mathbf{j}$

$-0.866 v_C = -1.039 + 0.866 v_{C/A}$

$0.5 v_C = 1.80 + 0.5 v_{C/A}$

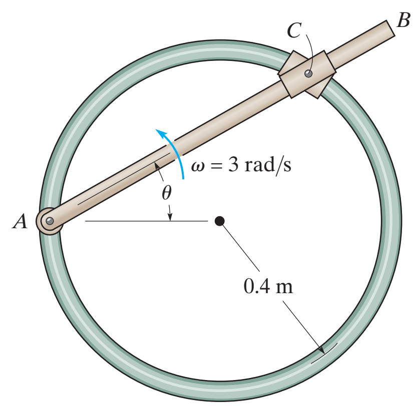
$v_C = 2.40$ m/s **Ans.**

$v_{C/A} = -1.20$ m/s

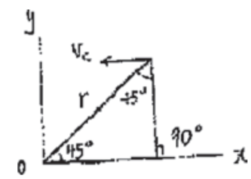
Ans:
$v_C = 2.40$ m/s
$\theta = 60°$ ◺

16–134.

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3\text{ rad/s}$. Determine the velocity and acceleration of point C located on the double collar when $\theta = 45°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



SOLUTION



$\mathbf{r}_{C/A} = \{0.400\mathbf{i} + 0.400\mathbf{j}\}$

$\mathbf{v}_C = -v_C\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-v_C\mathbf{i} = 0 + (3\mathbf{k}) \times (0.400\mathbf{i} + 0.400\mathbf{j}) + (v_{C/A}\cos 45°\mathbf{i} + v_{C/A}\sin 45°\mathbf{j})$

$-v_C\mathbf{i} = 0 - 1.20\mathbf{i} + 1.20\mathbf{j} + 0.707v_{C/A}\mathbf{i} + 0.707v_{C/A}\mathbf{j}$

$-v_C = -1.20 + 0.707v_{C/A}$

$0 = 1.20 + 0.707v_{C/A}$

$v_C = 2.40\text{ m/s}$ **Ans.**

$v_{C/A} = -1.697\text{ m/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$$-(a_C)_t\mathbf{i} - \frac{(2.40)^2}{0.4}\mathbf{j} = 0 + 0 + 3\mathbf{k} \times [3\mathbf{k} \times (0.4\mathbf{i} + 0.4\mathbf{j})] + 2(3\mathbf{k}) \times [0.707(-1.697)\mathbf{i} + 0.707(-1.697)\mathbf{j}] + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$$

$-(a_C)_t\mathbf{i} - 14.40\mathbf{j} = 0 + 0 - 3.60\mathbf{i} - 3.60\mathbf{j} + 7.20\mathbf{i} - 7.20\mathbf{j} + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$

$-(a_C)_t = -3.60 + 7.20 + 0.707a_{C/A}$

$-14.40 = -3.60 - 7.20 + 0.707a_{C/A}$

$a_{C/A} = -5.09\text{ m/s}^2$

$(a_C)_t = 0$

Thus,

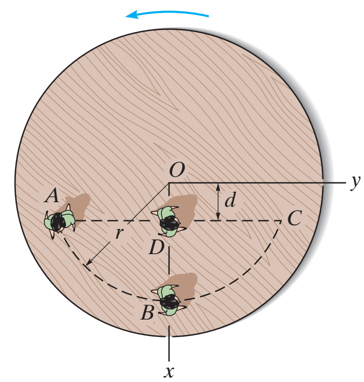
$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4\text{ m/s}^2$$

$a_C = \{-14.4\mathbf{j}\}\text{ m/s}^2$ **Ans.**

Ans:
$v_C = 2.40\text{ m/s}$
$a_C = \{-14.4\mathbf{j}\}\text{ m/s}^2$

16–135.

A girl stands at A on a platform which is rotating with an angular acceleration $\alpha = 0.2\ \text{rad/s}^2$ and at the instant shown has an angular velocity $\omega = 0.5\ \text{rad/s}$. If she walks at a constant speed $v = 0.75\ \text{m/s}$ measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.



SOLUTION

(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{D/O} + \Omega \times (\Omega \times \mathbf{r}_{D/O}) + 2\Omega \times (\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \quad \textbf{(1)}$$

Motion of moving reference

$\mathbf{a}_O = 0$

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2

Motion of D with respect to moving reference

$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m

$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s

$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.(1):

$$\mathbf{a}_B = \mathbf{0} + (0.2\mathbf{k}) \times (1\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (1\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2 \quad \textbf{Ans.}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \quad \textbf{(2)}$$

Motion of moving reference

$\mathbf{a}_O = \mathbf{0}$

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2

Motion of B with respect to moving reference

$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m

$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s

$$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\,\mathbf{i} + (a_{B/O})_t\,\mathbf{j} = -\left(\tfrac{0.75^2}{3}\right)\mathbf{i} = \{-0.1875\mathbf{i}\}\ \text{m/s}$$

Substitute the data into Eq.(2):

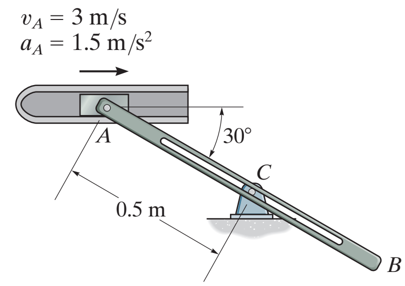
$$\mathbf{a}_B = 0 + (0.2\mathbf{k}) \times (3\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (3\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2 \quad \textbf{Ans.}$$

Ans:
$\mathbf{a}_B = \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2$
$\mathbf{a}_B = \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2$

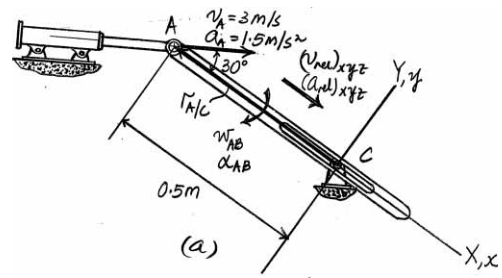
***16–136.**

If the piston is moving with a velocity of $v_A = 3$ m/s and acceleration of $a_A = 1.5$ m/s^2, determine the angular velocity and angular acceleration of the slotted link at the instant shown. Link AB slides freely along its slot on the fixed peg C.



SOLUTION

Reference Frame: The xyz reference frame centered at C rotates with link AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is



$$\mathbf{v}_C = \mathbf{a}_C = \mathbf{0} \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \alpha_{AB} = -\alpha_{AB}\mathbf{k}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/C} = [-0.5\mathbf{i}]\text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\mathbf{i}$$

The motion of point A with respect to the XYZ frame is

$$\mathbf{v}_A = 3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j} = [2.598\mathbf{i} + 1.5\mathbf{j}]\text{ m/s}$$

$$a_A = 1.5\cos 30°\mathbf{i} + 1.5\sin 30°\mathbf{j} = [1.299\mathbf{i} + 0.75\mathbf{j}]\text{ m/s}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \omega_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (v_{\text{rel}})_{xyz}\mathbf{i}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = (v_{\text{rel}})_{xyz}\mathbf{i} + 0.5\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components,

$$(v_{\text{rel}})_{xyz} = 2.598\text{ m/s}$$

$$0.5\omega_{AB} = 1.5 \qquad \omega_{AB} = 3\text{ rad/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega}_{AB} \times \mathbf{r}_{A/C} + \omega_{AB} \times (\omega_{AB} \times r_{A/C}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (-0.5\mathbf{i})] + 2(-3\mathbf{k}) \times (2.598\mathbf{i}) + (a_{\text{rel}})_{xyz}\mathbf{i}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \left[4.5 + (a_{\text{rel}})_{xyz}\right]\mathbf{i} + (0.5\alpha_{AB} - 15.59)\mathbf{j}$$

Equating the **j** components,

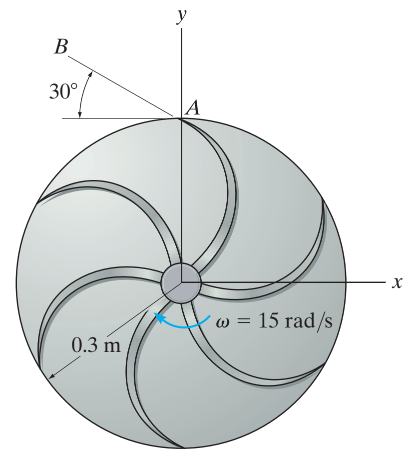
$$0.75 = 0.5\alpha_{AB} - 15.59$$

$$\alpha_{AB} = 32.68\text{ rad/s}^2 = 32.7\text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 3$ rad/s
$\alpha_{AB} = 32.7$ rad/s^2

16–137.

Water leaves the impeller of the centrifugal pump with a velocity of 25 m/s and acceleration of 30 m/s^2, both measured relative to the impeller along the blade line AB. Determine the velocity and acceleration of a water particle at A as it leaves the impeller at the instant shown. The impeller rotates with a constant angular velocity of $\omega = 15$ rad/s.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to the impeller and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

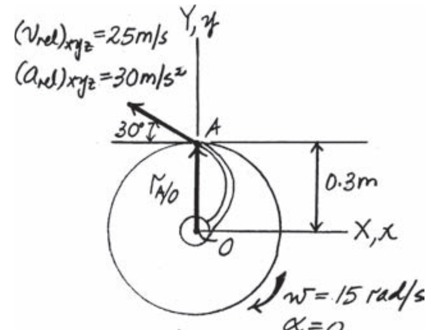
$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [-15\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \mathbf{0}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/O} = [0.3\mathbf{j}]\ \text{m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-25\cos 30°\,\mathbf{i} + 25\sin 30°\,\mathbf{j}) = [-21.65\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-30\cos 30°\,\mathbf{i} + 30\sin 30°\,\mathbf{j}) = [-25.98\mathbf{i} + 15\mathbf{j}]\ \text{m/s}^2$$



(a)

Velocity: Applying the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (-15\mathbf{k}) \times (0.3\mathbf{j}) + (-21.65\mathbf{i} + 12.5\mathbf{j})$$
$$= [-17.2\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

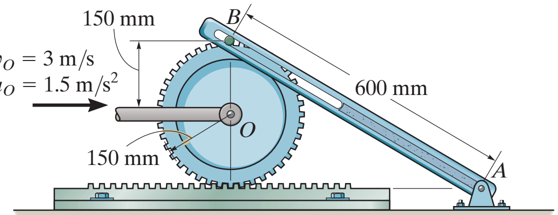
Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{A/O} + \omega \times (\omega \times \mathbf{r}_{A/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$
$$= \mathbf{0} + (-15\mathbf{k}) \times [(-15\mathbf{k}) \times (0.3\mathbf{j})] + 2(-15\mathbf{k}) \times (-21.65\mathbf{i} + 12.5\mathbf{j}) + (-25.98\mathbf{i} + 15\mathbf{j})$$
$$= [349\mathbf{i} + 597\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-17.2\mathbf{i} + 12.5\mathbf{j}\}$ m/s
$\mathbf{a}_A = \{349\mathbf{i} + 597\mathbf{j}\}$ m/s^2

16–138.

Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.



SOLUTION

Gear Motion: The *IC* of the gear is located at the point where the gear and the gear rack mesh, Fig. *a*. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$

Then,

$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. *b*,

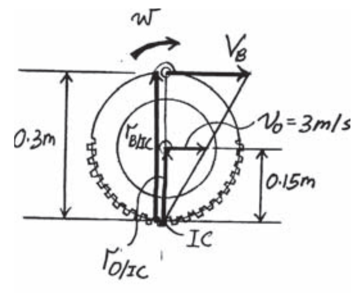
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

$$(a_B)_t\,\mathbf{i} - (a_B)_n\,\mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad (a_B)_n = 60 \text{ m/s}^2$$



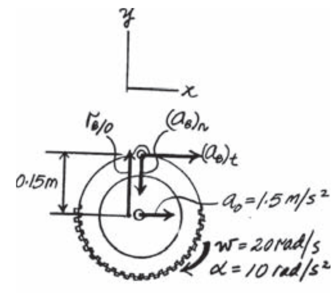
(a)

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. *c* and *d*. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30°\,\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$

$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$



(b)

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

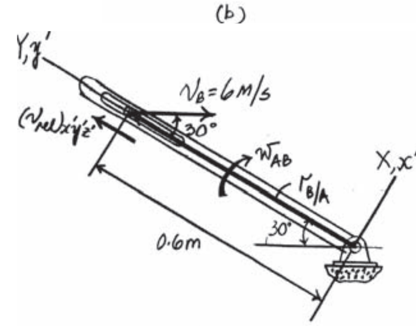
$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\,\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$

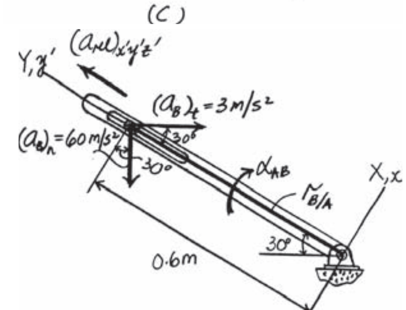


(c)

Equating the **i** and **j** components yields

$$3 = 0.6\omega_{AB} \qquad \omega_{AB} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$



(d)

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{ rel}})_{x'y'z'}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

Equating the **i** components,

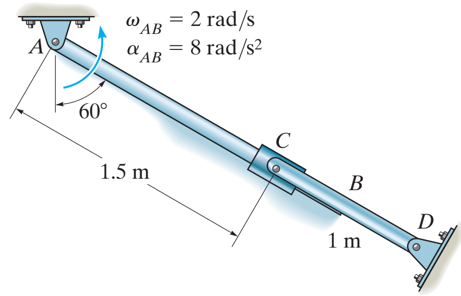
$$-50.46 = 0.6\alpha_{AB} - 51.96$$

$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 5$ rad/s ↻
$\alpha_{AB} = 2.5$ rad/s² ↻

16–139.

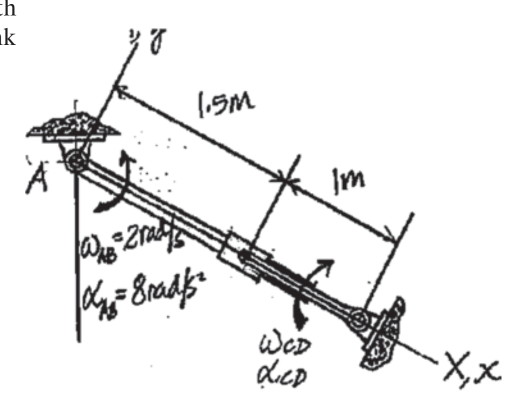
The collar C is pinned to rod CD while it slides on rod AB. If rod AB has an angular velocity of 2 rad/s and an angular acceleration of 8 rad/s^2, both acting counterclockwise, determine the angular velocity and the angular acceleration of rod CD at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate systems are set to coincide with origin at A as shown in Fig. a. Here, the $x - y$ coordinate system is attached to link AC. Thus,

Motion of moving Reference	**Motion of collar C with respect to moving Reference**
$\mathbf{v}_A = \mathbf{0}$	$\mathbf{r}_{C/A} = \{1.5\mathbf{i}\}$ m
$\mathbf{a}_A = \mathbf{0}$	
$\boldsymbol{\Omega} = \boldsymbol{\omega}_{AB} = \{2\mathbf{k}\}$ rad/s	$(\mathbf{v}_{C/A})_{xyz} = (\mathbf{v}_{C/A})_{xyz}\mathbf{i}$
$\dot{\boldsymbol{\Omega}} = \boldsymbol{\alpha}_{AB} = \{8\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{C/A})_{xyz} = (\mathbf{a}_{C/A})_{xyz}\mathbf{i}$



(a)

The motions of collar C in the fixed system are

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = (-\omega_{CD}\mathbf{k}) \times (-\mathbf{i}) = \omega_{CD}\mathbf{j}$$

$$a_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}{}^2\,\mathbf{r}_{C/D} = (-\alpha_{C/D}\mathbf{k}) \times (-\mathbf{i}) - \omega_{CD}^2(-\mathbf{i}) = \omega_{CD}^2\mathbf{i} + \alpha_{CD}\mathbf{j}$$

Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$\omega_{CD}\mathbf{j} = \mathbf{0} + (2\mathbf{k}) \times (1.5\mathbf{i}) = (v_{C/A})_{xyz}\mathbf{i}$$

$$\omega_{CD}\mathbf{j} = (v_{C/A})_{xyz}\mathbf{i} + 3\mathbf{j}$$

Equating **i** and **j** components

$$(v_{C/A})_{xyz} = 0$$

$$\omega_{CD} = 3.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Applying the relative acceleration equation,

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$3.00^2\mathbf{i} + \alpha_{CD}\mathbf{j} = \mathbf{0} + (8\mathbf{k}) \times (1.5\mathbf{i}) + (2\mathbf{k}) \times (2\mathbf{k} \times 1.5\mathbf{i}) + 2(2\mathbf{k}) \times \mathbf{0} + (a_{C/A})_{xyz}\mathbf{i}$$

$$9\mathbf{i} + \alpha_{CD}\mathbf{j} = \left[(a_{C/A})_{xyz} - 6\right]\mathbf{i} + 12\mathbf{j}$$

Equating **i** and **j** components,

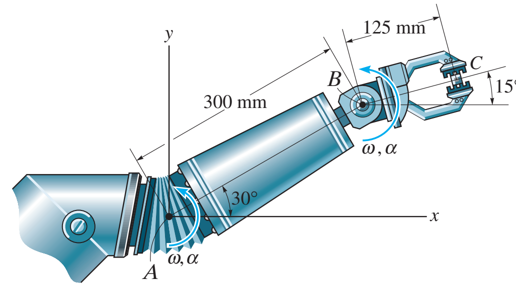
$$9 = (a_{C/A})_{xyz} - 6; \quad (a_{C/A})_{xyz} = 15 \text{ m/s}^2$$

$$\alpha_{CD} = 12.0 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 3.00$ rad/s $\circlearrowright$
$\alpha_{CD} = 12.0$ rad/s^2 $\circlearrowright$

***16–140.**

At the instant shown, the robotic arm AB is rotating counterclockwise at $\omega = 5$ rad/s and has an angular acceleration $\alpha = 2$ rad/s^2. Simultaneously, the grip BC is rotating counterclockwise at $\omega' = 6$ rad/s and $\alpha' = 2$ rad/s^2, both measured relative to a *fixed* reference. Determine the velocity and acceleration of the object held at the grip C.

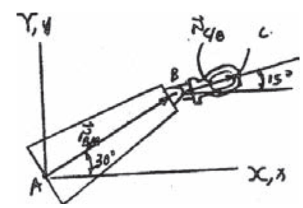


SOLUTION

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz} \quad \textbf{(2)}$$

Motion of moving reference	***Motion of C with respect to moving reference***
	$\mathbf{r}_{C/B} = \{0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}\}$ m
$\Omega = \{6\mathbf{k}\}$ rad/s	$(\mathbf{v}_{C/B})_{xyz} = 0$
$\dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{C/B})_{xyz} = 0$



Motion of *B*:

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$
$$= (5\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-0.75\mathbf{i} + 1.2990\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$
$$= (2\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j}) - (5)^2(0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-6.7952\mathbf{i} - 3.2304\mathbf{j}\} \text{ m/s}^2$$

Substitute the data into Eqs. (1) and (2) yields:

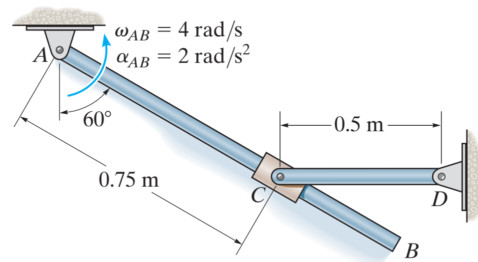
$$\mathbf{v}_C = (-0.75\mathbf{i} + 1.2990\mathbf{j}) + (6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}) + 0$$
$$= \{-0.944\mathbf{i} + 2.02\mathbf{j}\} \text{ m/s} \quad \textbf{Ans.}$$

$$\mathbf{a}_C = (-6.79527\mathbf{i} - 3.2304\mathbf{j}) + (2\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})$$
$$+ (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})] + 0 + 0$$
$$= \{-11.2\mathbf{i} - 4.15\mathbf{j}\} \text{ m/s}^2 \quad \textbf{Ans.}$$

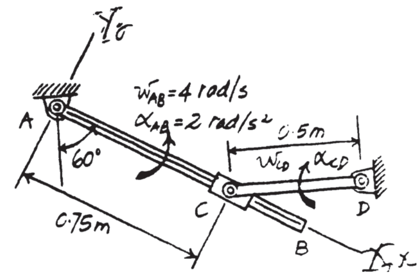
Ans:
$\mathbf{v}_C = \{-0.944\mathbf{i} + 2.02\mathbf{j}\}$ m/s
$\mathbf{a}_C = \{-11.2\mathbf{i} - 4.15\mathbf{j}\}$ m/s^2

16–141.

At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



SOLUTION



Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.

Kinematic Equation: Applying Eqs. 16–26 and 16–29, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \quad \textbf{(2)}$$

Motion of moving reference

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s^2

Motion of C with respect to moving reference

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}\text{m} = \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.(1) yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\,\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

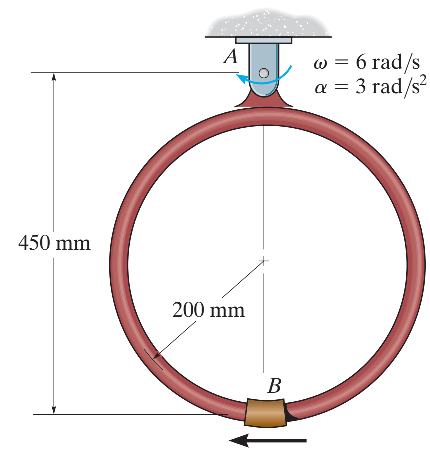
$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 6.93$ rad/s

16–142.

Collar B moves to the left with a speed of 5 m/s, which is increasing at a constant rate of 1.5 m/s^2, relative to the hoop, while the hoop rotates with the angular velocity and angular acceleration shown. Determine the magnitudes of the velocity and acceleration of the collar at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the hoop and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$v_A = a_A = \mathbf{0} \qquad \omega = [-6\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \alpha = [-3\mathbf{k}] \text{ rad/s}^2$$

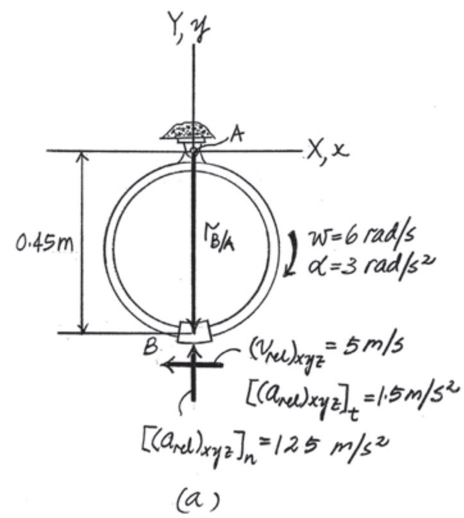
For the motion of collar B with respect to the xyz frame,

$$\mathbf{r}_{B/A} = [-0.45\mathbf{j}] \text{ m}$$

$$(v_{rel})_{xyz} = [-5\mathbf{i}] \text{ m/s}$$

The normal components of $(\mathbf{a}_{rel})_{xyz}$ is $[(a_{rel})_{xyz}]_n = \dfrac{(v_{rel})_{xyz}{}^2}{\rho} = \dfrac{5^2}{0.2} = 125 \text{ m/s}^2$. Thus,

$$(\mathbf{a}_{rel})_{xyz} = [-1.5\mathbf{i} + 125\mathbf{j}] \text{ m/s}$$



Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{rel})_{xyz}$$

$$= \mathbf{0} + (-6\mathbf{k}) \times (-0.45\mathbf{j}) + (-5\mathbf{i})$$

$$= [-7.7\mathbf{i}] \text{ m/s}$$

Thus,

$$v_B = 7.7 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{rel})_{xyz} + (\mathbf{a}_{rel})_{xyz}$$

$$= \mathbf{0} + (-3\mathbf{k}) \times (-0.45\mathbf{j}) + (-6\mathbf{k}) \times [(-6\mathbf{k}) \times (-0.45\mathbf{j})] + 2(-6\mathbf{k}) \times (-5\mathbf{i}) + (-1.5\mathbf{i} + 125\mathbf{j})$$

$$= [-2.85\mathbf{i} + 201.2\mathbf{j}] \text{ m/s}^2$$

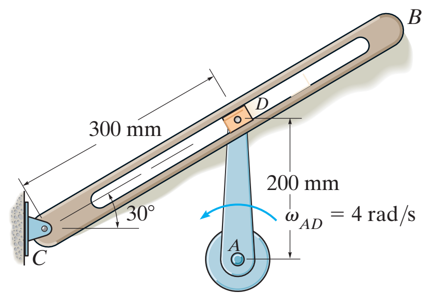
Thus, the magnitude of $\mathbf{a}_B$ is therefore

$$a_B = \sqrt{2.85^2 + 201.2^2} = 201 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.7$ m/s
$a_B = 201$ m/s^2

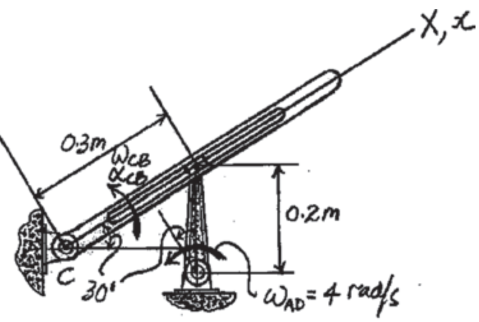
16–143.

Block D of the mechanism is confined to move within the slot of member CB. If link AD is rotating at a constant rate of $\omega_{AD} = 4$ rad/s, determine the angular velocity and angular acceleration of member CB at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate system are set to coincide with origin at C as shown in Fig. a. Here the $x - y$ coordinate system is attached to member CB. Thus



(a)

Motion of moving Reference	**Motion of Block D with respect to moving Reference**
$\mathbf{v}_C = \mathbf{0}$	$\mathbf{r}_{D/C} = \{0.3\mathbf{i}\}$ m
$\mathbf{a}_C = \mathbf{0}$	
$\boldsymbol{\Omega} = \boldsymbol{\omega}_{CB} = \omega_{CB}\mathbf{k}$	$(\mathbf{v}_{D/C})_{xyz} = (\mathbf{v}_{D/C})_{xyz}\mathbf{i}$
$\dot{\boldsymbol{\Omega}} = \boldsymbol{\alpha}_{CB} = \alpha_{CB}\mathbf{k}$	$(\mathbf{a}_{D/C})_{xyz} = (\mathbf{a}_{D/C})_{xyz}\mathbf{i}$

The Motions of Block D in the fixed frame are,

$$\mathbf{v}_D = \boldsymbol{\omega}_{A/D} \times \mathbf{r}_{D/A} = (4\mathbf{k}) \times (0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j}) = \{-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{AD} \times \mathbf{r}_{D/A} - \omega_{AD}^2(\mathbf{r}_{D/A}) = 0 - 4^2(0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j})$$
$$= \{-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j}\} \text{ m/s}^2$$

Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\Omega} \times \mathbf{r}_{D/C} + (\mathbf{v}_{D/C})_{xyz}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = \mathbf{0} + (\omega_{CB}\mathbf{k}) \times (0.3\mathbf{i}) + (v_{D/C})_{xyz}\mathbf{i}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = (v_{D/C})_{xyz}\mathbf{i} + 0.3\,\omega_{CB}\mathbf{j}$$

Equating **i** and **j** components,

$$(v_{D/C})_{xyz} = -0.4\sqrt{3} \text{ m/s}$$

$0.4 = 0.3\,\omega_{CB}; \quad \omega_{CB} = 1.3333 \text{ rad/s} = 1.33 \text{ rad/s} \circlearrowleft$ **Ans.**

Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_C + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{D/C} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{D/C}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{D/C})_{xyz} + (\mathbf{a}_{D/C})_{xyz}$$

$$-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = 0 + (\alpha_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + (1.3333\mathbf{k}) \times (1.3333\mathbf{k} \times 0.3\mathbf{i})$$
$$+ 2(1.3333\mathbf{k}) \times (-0.4\sqrt{3}\mathbf{i}) + (a_{D/C})_{xyz}\mathbf{i}$$

$$1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = [(a_{D/C})_{xyz} - 0.5333]\mathbf{i} + (0.3\alpha_{CD} - 1.8475)\mathbf{j}$$

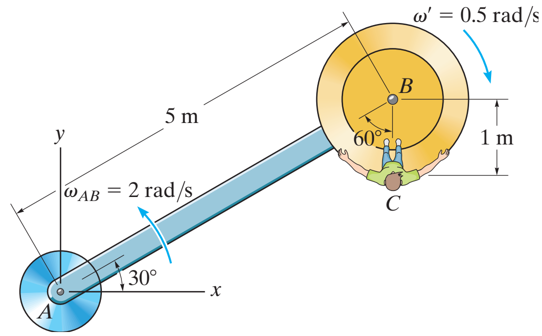
Equating **i** and **j** components

$$1.6 = [(a_{D/C})_{xyz} - 0.5333]; \quad (a_{D/C})_{xyz} = 2.1333 \text{ m/s}^2$$

$-1.6\sqrt{3} = 0.3\,\alpha_{CD} - 1.8475; \quad \alpha_{CD} = -3.0792 \text{ rad/s}^2 = 3.08 \text{ rad/s}^2 \circlearrowright$ **Ans.**

Ans:
$\omega_{CB} = 1.33 \text{ rad/s} \circlearrowleft$
$\alpha_{CD} = 3.08 \text{ rad/s}^2 \circlearrowright$

***16–144.** A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



SOLUTION

Given:

$$\omega_{AB} = 2 \text{ rad/s}$$

$$a = 5 \text{ m}$$

$$\omega' = 0.5 \text{ rad/s}$$

$$r = 1 \text{ m}$$

$$\theta = 30^\circ$$

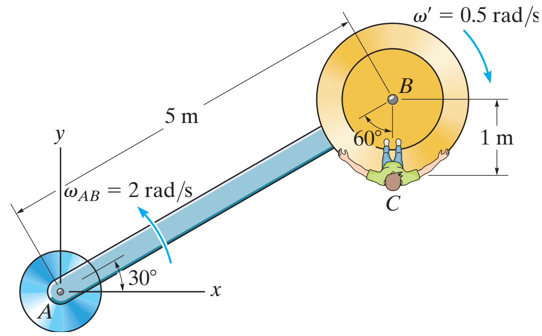
$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -3.5 \\ 8.66 \\ 0 \end{pmatrix} \text{m/s} \quad \textbf{Ans.}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_C} = \begin{pmatrix} -17.32 \\ -7.75 \\ 0.00 \end{pmatrix} \text{m/s}^2 \quad \textbf{Ans.}$$

Ans:
$\mathbf{v_C} = \{-3.5\mathbf{i} + 8.66\mathbf{j}\}$ m/s^2
$\mathbf{a_C} = \{-17.32\mathbf{i} - 7.75\mathbf{j}\}$ m/s^2

16–145. A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1\ \text{rad/s}^2$ when $\omega_{AB} = 2\ \text{rad/s}$ at the instant shown. Also at this instant the car mounted at the end of the arm has a relative angular acceleration of $\boldsymbol{\alpha}' = \{-0.6\mathbf{k}\}\ \text{rad/s}^2$ when $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}\ \text{rad/s}$. Determine the velocity and acceleration of the passenger C at this instant.



SOLUTION

Given:

$\omega_{AB} = 2\ \text{rad/s} \qquad \alpha_{AB} = 1\ \text{rad/s}^2$

$\omega' = 0.5\ \text{rad/s} \qquad \alpha' = 0.6\ \text{rad/s}^2$

$a = 5\ \text{m} \qquad r = 1\ \text{m}$

$\theta = 30°$

$$\mathbf{v}_C = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v}_C = \begin{pmatrix} -3.5 \\ 8.66 \\ 0 \end{pmatrix} \text{m/s} \qquad \textbf{Ans.}$$

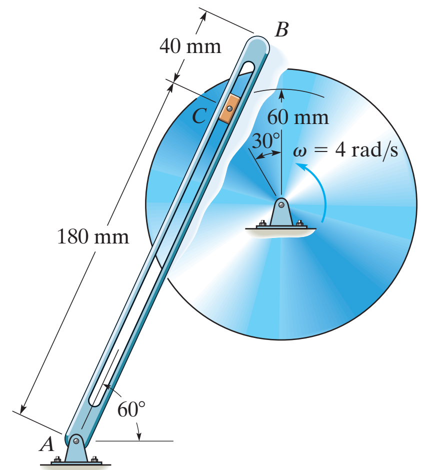
$$\mathbf{a}_C = \left[\begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right]\right] \ldots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} - \alpha' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a}_C = \begin{pmatrix} -19.42 \\ -3.42 \\ 0.00 \end{pmatrix} \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-3.5\mathbf{i} + 8.66\mathbf{j}\}\ \text{m/s}^2$
$\mathbf{a}_C = \{-19.42\mathbf{i} - 3.42\mathbf{j}\}\ \text{m/s}^2$

16–146.

If the slider block C is fixed to the disk that has a constant counterclockwise angular velocity of 4 rad/s, determine the angular velocity and angular acceleration of the slotted arm AB at the instant shown.

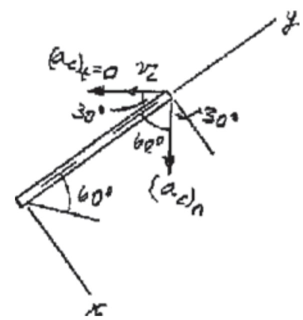


SOLUTION

$\mathbf{v}_C = -(4)(60)\sin 30°\mathbf{i} - 4(60)\cos 30°\mathbf{j} = -120\mathbf{i} - 207.85\mathbf{j}$

$\mathbf{a}_C = (4)^2(60)\sin 60°\mathbf{i} - (4)^2(60)\cos 60°\mathbf{j} = 831.38\mathbf{i} - 480\mathbf{j}$

Thus,



$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-120\mathbf{i} - 207.85\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (180\mathbf{j}) - v_{C/A}\mathbf{j}$

$-120 = -180\omega_{AB}$

$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowleft$ **Ans.**

$-207.85 = -v_{C/A}$

$v_{C/A} = 207.85 \text{ mm/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{C/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{C/A}) + 2\boldsymbol{\Omega} \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$831.38\mathbf{i} - 480\mathbf{j} = 0 + (\alpha_{AB}\mathbf{k}) \times (180\mathbf{j}) + (0.667\mathbf{k}) \times [(0.667\mathbf{k}) \times (180\mathbf{j})]$

$\quad + 2(0.667\mathbf{k}) \times (-207.85\mathbf{j}) - a_{C/A}\mathbf{j}$

$831.38\mathbf{i} - 480\mathbf{j} = -180\,\alpha_{AB}\mathbf{i} - 80\mathbf{j} + 277.13\mathbf{i} - a_{C/A}\mathbf{j}$

$831.38 = -180\alpha_{AB} + 277.13$

$\alpha_{AB} = -3.08$

Thus,

$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$ **Ans.**

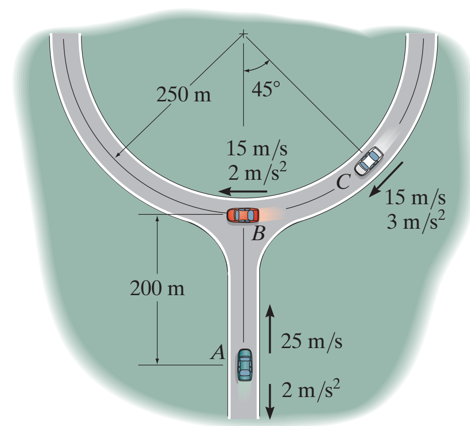
$-480 = -80 - a_{C/A}$

$a_{C/A} = 400 \text{ mm/s}^2$

Ans:
$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowleft$
$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$

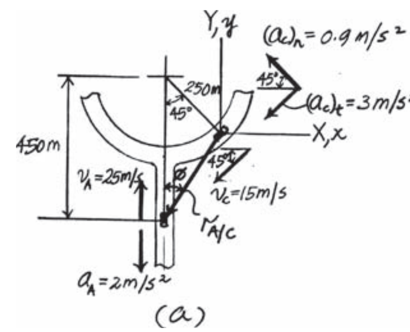
16–147.

At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of 2 m/s^2, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of 3 m/s^2. Determine the velocity and acceleration of car A with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to car C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car C moves along the circular road, its normal component of acceleration is $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$. Thus, the motion of car C with respect to the XYZ frame is



$$\mathbf{v}_C = -15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j} = [-10.607\mathbf{i} - 10.607\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\mathbf{j}]\text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame is

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\text{ rad/s} \qquad \omega = [-0.06\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\text{ rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\text{ rad/s}^2$$

The velocity and accdeleration of car A with respect to the XYZ frame is

$$\mathbf{v}_A = [25\mathbf{j}]\text{ m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}]\text{ m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/C} = -250\sin 45°\mathbf{i} - (450 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 273.22\mathbf{j}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega} \times r_{A/C} + \omega \times (\omega \times \mathbf{r}_{A/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})$$
$$+ (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

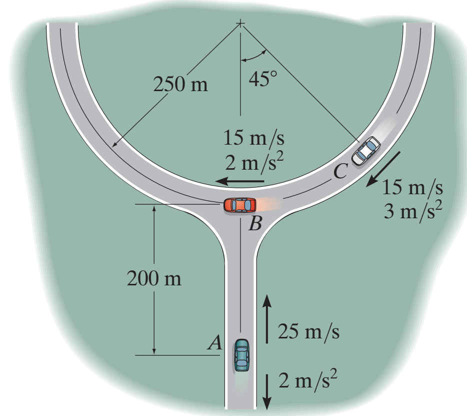
$$-2\mathbf{j} = -2.4\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\text{ m/s}$
$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\text{ m/s}^2$

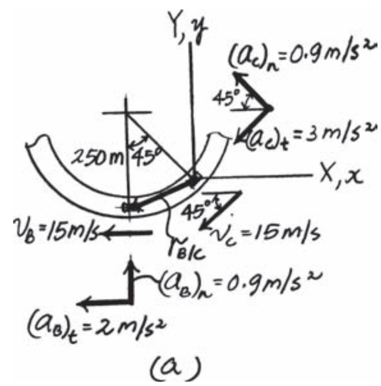
***16–148.**

At the instant shown, car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\text{m/s}^2$, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of $3\ \text{m/s}^2$. Determine the velocity and acceleration of car B with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since B and C move along the circular road, their normal components of acceleration are $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$ and $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$. Thus, the motion of cars B and C with respect to the XYZ frame are



$$\mathbf{v}_B = [-15\mathbf{i}]\ \text{m/s}$$

$$\mathbf{v}_C = [-15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j}] = [-10.607\mathbf{i} - 10.607\mathbf{j}]\ \text{m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\ \text{m/s}^2$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\,\mathbf{j}]\ \text{m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\ \text{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\ \text{rad/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{B/C} = -250\sin 45°\mathbf{i} - (250 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 73.22\,\mathbf{j}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times r_{B/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = -15\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\omega} \times \mathbf{r}_{B/C} + \omega \times (\omega \times \mathbf{r}_{B/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})$$

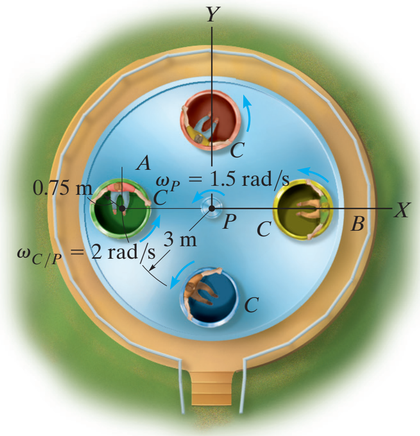
$$+(-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times \mathbf{0} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = -3\mathbf{i} + 0.9\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(a_{\text{rel}})_{xyz} = [1\mathbf{i}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0}$
$(\mathbf{a}_{\text{rel}})_{xyz} = \{1\mathbf{i}\}\ \text{m/s}^2$

16–149. A ride in an amusement park consists of a rotating platform P, having constant angular velocity $\omega_P = 1.5$ rad/s, and four cars, C, mounted on the platform, which have constant angular velocities $\omega_{C/P} = 2$ rad/s measured relative to the platform. Determine the velocity and acceleration of the passenger at B at the instant shown.



SOLUTION

Given: $\omega_P = 1.5$ rad/s $\qquad r = 0.75$ m

$\omega_{CP} = 2$ rad/s $\qquad R = 3$ m

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v_B} = \begin{pmatrix} 0.00 \\ 7.13 \\ 0.00 \end{pmatrix} \text{m/s} \qquad |\mathbf{v_B}| = 7.13 \text{ m/s}$$ **Ans.**

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} -15.94 \\ 0.00 \\ 0.00 \end{pmatrix} \text{m/s}^2 \qquad |\mathbf{a_B}| = 15.94 \text{ m/s}^2$$ **Ans.**

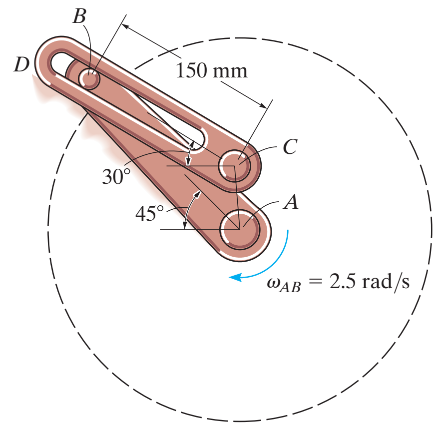
Ans:

$|\mathbf{v_B}| = 7.13$ m/s^2

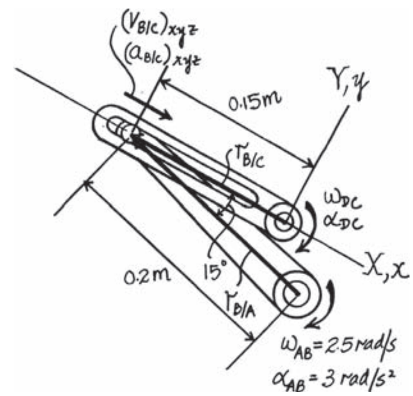
$|\mathbf{a_B}| = 15.94$ m/s^2

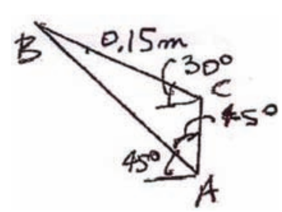
16–150.

The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s, determine the angular velocity of CD (output) at this instant.



SOLUTION





$$\frac{r_{BA}}{\sin 120^\circ} = \frac{0.15\text{ m}}{\sin 45^\circ}$$

$$\mathbf{r}_{BA} = 0.1837\text{ m}$$

$$\mathbf{v}_C = \mathbf{0}$$

$$\mathbf{a}_C = \mathbf{0}$$

$$\Omega = -\omega_{DC}\mathbf{k}$$

$$\dot{\Omega} = -\alpha_{DC}\mathbf{k}$$

$$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}\text{ m}$$

$$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\mathbf{i}$$

$$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\mathbf{i}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.1837\cos 15^\circ\mathbf{i} + 0.1837\sin 15^\circ\mathbf{j})$$

$$= \{0.1189\mathbf{i} + 0.4436\mathbf{j}\}\text{ m/s}$$

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\mathbf{i}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} + 0.15\omega_{DC}\mathbf{j}$$

Solving:

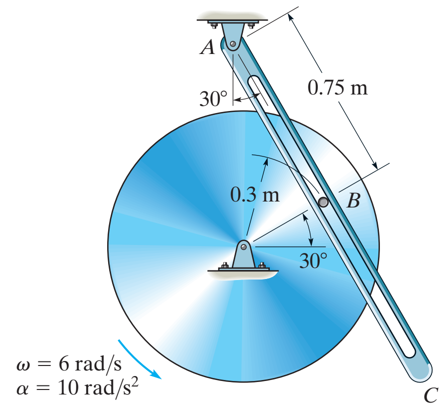
$$(v_{B/C})_{xyz} = 0.1189\text{ m/s}$$

$$\omega_{DC} = 2.96\text{ rad/s} \circlearrowright$$ **Ans.**

Ans:
$\omega_{DC} = 2.96$ rad/s $\circlearrowright$

16–151.

The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link AC at this instant. The peg at B is fixed to the disk.



SOLUTION

$$\mathbf{v}_B = -6(0.3)\mathbf{i} = -1.8\mathbf{i}$$

$$\mathbf{a}_B = -10(0.3)\mathbf{i} - (6)^2(0.3)\mathbf{j} = -3\mathbf{i} - 10.8\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{B/A} + (v_{B/A})_{xyz}$$

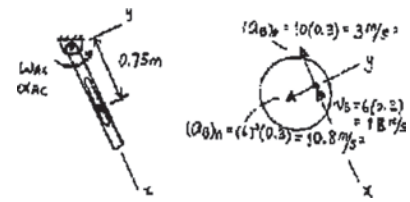
$$-1.8\mathbf{i} = 0 + (\omega_{AC}\mathbf{k}) \times (0.75\mathbf{i}) - (v_{B/A})_{xyz}\mathbf{i}$$

$$-1.8\mathbf{i} = -(v_{B/A})_{xyz}$$

$$(v_{B/A})_{xyz} = 1.8 \text{ m/s}$$

$$0 = \omega_{AC}(0.75)$$

$$\omega_{AC} = 0 \qquad \textbf{Ans.}$$



$$\mathbf{a}_B = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{B/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{B/A}) + 2\mathbf{\Omega} \times (v_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$-3\mathbf{i} - 10.8\mathbf{j} = \mathbf{0} + \alpha_{AC}\mathbf{k} \times (0.75\mathbf{i}) + \mathbf{0} + \mathbf{0} - a_{A/B}\mathbf{i}$$

$$-3 = -a_{A/B}$$

$$a_{A/B} = 3 \text{ m/s}^2$$

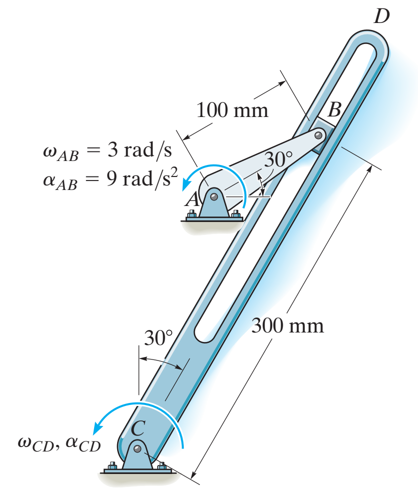
$$-10.8 = \alpha_{A/C}(0.75)$$

$$\alpha_{A/C} = 14.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{AC} = 0$
$\alpha_{AC} = 14.4 \text{ rad/s}^2 \circlearrowright$

***16–152.**

The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



SOLUTION

$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9\text{ m/s}^2$

$(a_B)_n = (3)^2(0.1) = 0.9\text{ m/s}^2$

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3\cos 60°\mathbf{i} + 0.3\sin 60°\mathbf{j} = \mathbf{0} + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\,\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s ↺ **Ans.**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

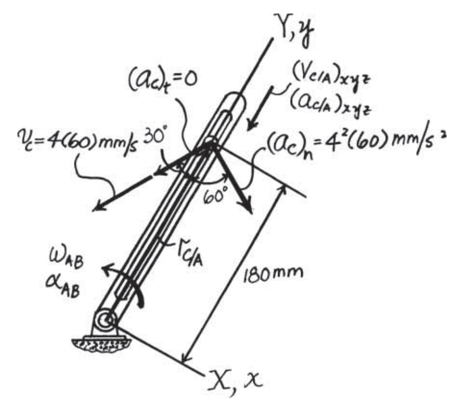
$0.9\cos 60°\mathbf{i} - 0.9\cos 30°\mathbf{i} + 0.9\sin 60°\mathbf{j} + 0.9\sin 30°\mathbf{j} = \mathbf{0} + (\alpha_{CD}\,\mathbf{k}) \times (0.3\mathbf{i})$

$+(0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\,\mathbf{i}$

$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\,\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\,\mathbf{i}$

$a_{B/C} = -0.104\text{ m/s}^2$

$\alpha_{CD} = 3.23\text{ rad/s}^2$ ↺ **Ans.**



Ans:
$\omega_{CD} = 0.866$ rad/s ↺
$\alpha_{CD} = 3.23\text{ rad/s}^2$ ↺