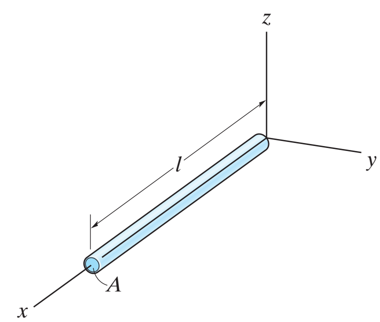
17–1.

Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



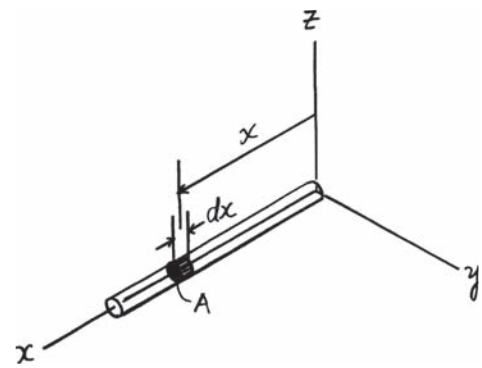
SOLUTION

$$I_y = \int_M x^2 \, dm$$

$$= \int_0^l x^2 (\rho A \, dx)$$

$$= \frac{1}{3} \rho A l^3$$

$$m = \rho A l$$



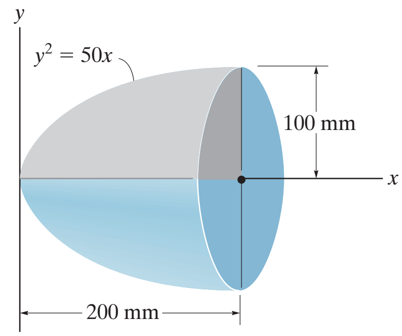
Thus,

$$I_y = \frac{1}{3} m l^2$$ **Ans.**

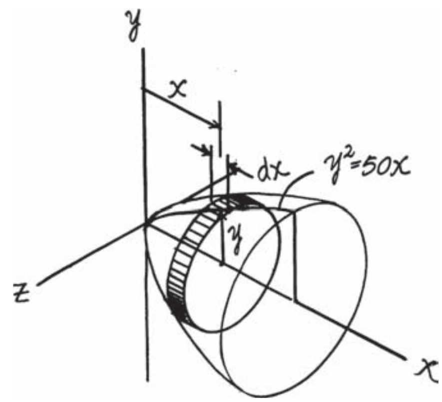
Ans:
$I_y = \frac{1}{3} m l^2$

17–2.

The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \text{Mg/m}^3$.



SOLUTION



$$dm = \rho\,\pi\,y^2\,dx = \rho\,\pi\,(50x)\,dx$$

$$I_x = \int \frac{1}{2} y^2\,dm = \frac{1}{2}\int_0^{200} 50\,x\,\{\pi\,\rho\,(50x)\}\,dx$$

$$= \rho\,\pi\left(\frac{50^2}{2}\right)\left[\frac{1}{3}x^3\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50^2}{6}\right)(200)^3$$

$$m = \int dm = \int_0^{200} \pi\,\rho\,(50x)\,dx$$

$$= \rho\,\pi\,(50)\left[\frac{1}{2}x^2\right]_0^{200}$$

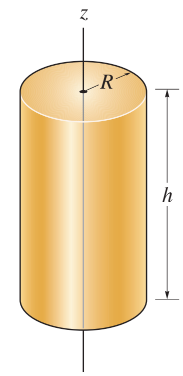
$$= \rho\,\pi\left(\frac{50}{2}\right)(200)^2$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{50}{3}(200)} = 57.7\ \text{mm}$$ **Ans.**

Ans:
$k_x = 57.7\ \text{mm}$

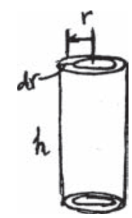
17–3.

The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



SOLUTION

Consider a shell element of radius r and mass



$$dm = \rho\, dV = \rho(2\pi\, r\, dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi\, r\, dr)h$$

$$m = 2\pi\, h(\frac{kR^2}{2} + \frac{aR^4}{4})$$

$$m = \pi\, h\, R^2(k + \frac{aR^2}{2}) \qquad \textbf{Ans.}$$

$$dI = r^2\, dm = r^2(\rho)(2\pi\, r\, dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi\, r\, dr)\, h$$

$$I_z = 2\pi h \int_0^R (k\, r^3 + a\, r^5)\, dr$$

$$I_z = 2\pi h[\frac{k\, R^4}{4} + \frac{aR^6}{6}]$$

$$I_z = \frac{\pi\, h\, R^4}{2}[k + \frac{2\, aR^2}{3}] \qquad \textbf{Ans.}$$

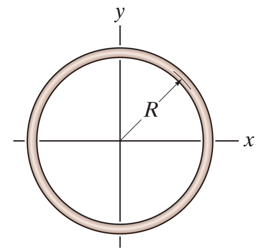
Ans:

$$m = \pi\, h\, R^2\left(k + \frac{aR^2}{2}\right)$$

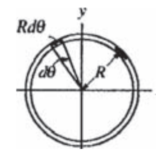
$$I_z = \frac{\pi\, h\, R^4}{2}\left[k + \frac{2\, aR^2}{3}\right]$$

***17–4.**

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



SOLUTION



$$I_z = \int_0^{2\pi} \rho A(R\,d\theta)R^2 = 2\pi\,\rho\,A\,R^3$$

$$m = \int_0^{2\pi} \rho\,A\,R\,d\theta = 2\pi\,\rho\,A\,R$$

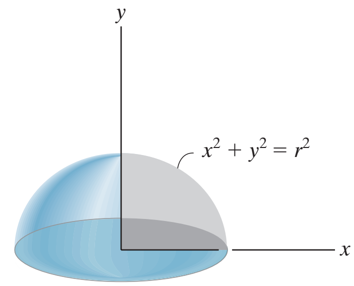
Thus,

$$I_z = m R^2$$ **Ans.**

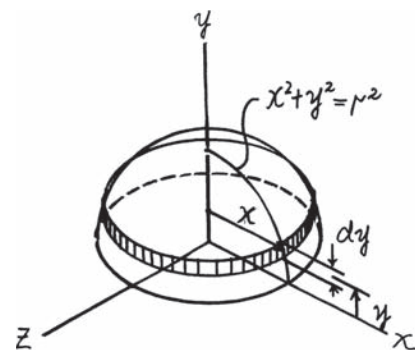
Ans:
$I_z = mR^2$

17–5.

The hemisphere is formed by rotating the shaded area around the y axis. Determine the moment of inertia I_y and express the result in terms of the total mass m of the hemisphere. The material has a constant density ρ.



SOLUTION



$$m = \int_V \rho\, dV = \rho \int_0^r \pi\, x^2\, dy = \rho\pi \int_0^r (r^2 - y^2) dy$$

$$= \rho\pi \left[r^2\, y - \frac{1}{3}\, y^3 \right]_0^r = \frac{2}{3} \rho\pi\, r^3$$

$$I_y = \int_m \frac{1}{2}\, (dm)\, x^2 = \frac{\rho}{2} \int_0^r \pi x^4\, dy = \frac{\rho\pi}{2} \int_0^r (r^2 - y^2)^2\, dy$$

$$= \frac{\rho\pi}{2} \left[r^4 y - \frac{2}{3}\, r^2\, y^3 + \frac{y^5}{5} \right]_0^r = \frac{4\rho\pi}{15}\, r^5$$

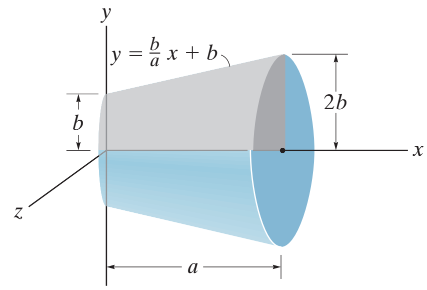
Thus,

$$I_y = \frac{2}{5}\, m\, r^2$$ **Ans.**

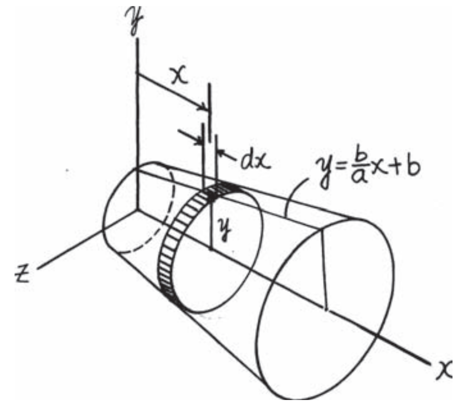
Ans:
$I_y = \dfrac{2}{5} m r^2$

17–6.

The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density ρ.



SOLUTION



$$dm = \rho\, dV = \rho\pi y^2\, dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\rho\pi y^4\, dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4\,b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4\,b^4}{a}x + b^4\right)dx$$

$$= \frac{31}{10}\rho\pi ab^4$$

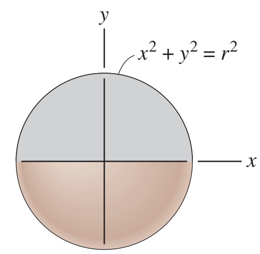
$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi ab^2$$

$$I_x = \frac{93}{70}mb^2$$ **Ans.**

Ans:
$I_x = \frac{93}{70}mb^2$

17–7.

The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



SOLUTION

$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\,\pi(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\;dx$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho\,r^5$$

$$m = \int_{-r}^{r} \rho\,\pi(r^2 - x^2)\,dx$$

$$= \frac{4}{3}\rho\,\pi\,r^3$$

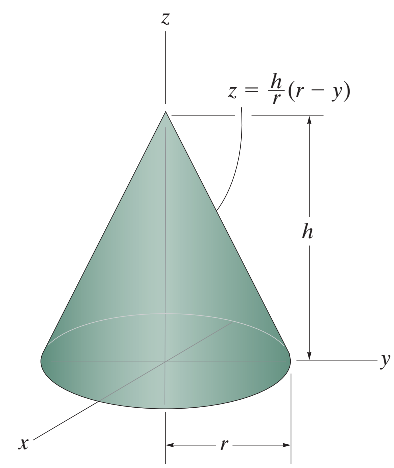
Thus,

$$I_x = \frac{2}{5}m\,r^2$$ **Ans.**

Ans:
$I_x = \frac{2}{5}m r^2$

***17–8.**

Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



SOLUTION

Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho\, dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

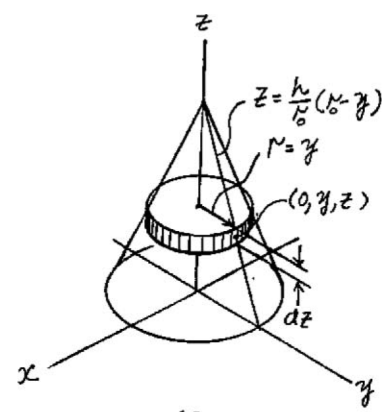
Mass Moment of Inertia: Integrating dI_z, we obtain

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$

From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2$$ **Ans.**



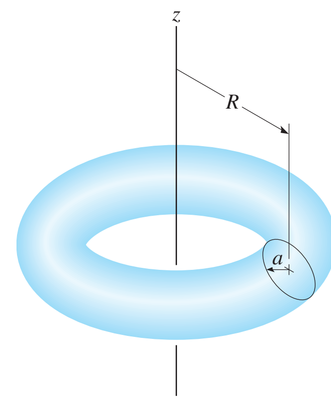
(a)

Ans:

$I_z = \frac{3}{10}mr_o^2$

17–9.

Determine the moment of inertia I_z of the torus. The mass of the torus is m and the density ρ is constant. *Suggestion:* Use a shell element.



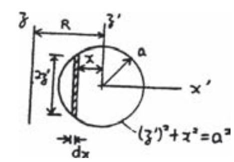
SOLUTION

$$dm = 2\pi(R - x)(2z'\rho\, dx)$$

$$dI_z = (R - x)^2 dm$$

$$= 4\pi\rho[(R^3 - 3R^2 x + 3Rx^2 - x^3)\sqrt{a^2 - x^2}\, dx]$$

$$I_z = 4\pi\rho[R^3\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - 3R^2\int_{-a}^{a}x^3\sqrt{a^2 - x^2}\, dx + 3R\int_{-a}^{a}\sqrt{a^2 - x^2}\, dx - \int_{-a}^{a}x^3\sqrt{a^2 - x^2}$$



$$= 2\pi^2\rho Ra^2(R^2 + \frac{3}{4}a^2)$$

Since $m = \rho V = 2\pi R\rho\pi a^2$

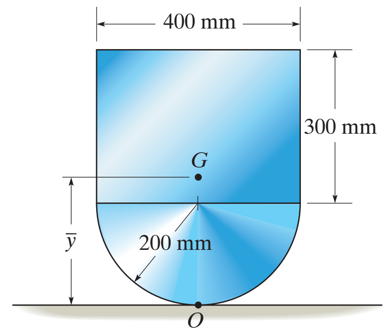
$$I_z = m(R^2 + \frac{3}{4}a^2)$$ **Ans.**

Ans:

$I_z = m(R^2 + \frac{3}{4}a^2)$

17–10.

Determine the location $\bar{y}$ of the center of mass G of the assembly and then calculate the moment of inertia about an axis perpendicular to the page and passing through G. The block has a mass of 3 kg and the semicylinder has a mass of 5 kg.



SOLUTION

Moment inertia of the semicylinder about its center of mass:

$$(I_G)_{cyc} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199mR^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{\left[0.2 - \dfrac{4(0.2)}{3\pi}\right](5) + 0.35(3)}{5 + 3} = 0.2032 \text{ m} = 0.203 \text{ m} \qquad \textbf{Ans.}$$

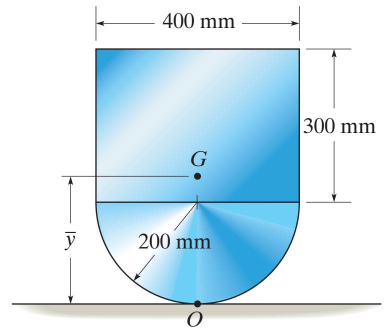
$$I_G = 0.3199(5)(0.2)^2 + 5\left[0.2032 - \left(0.2 - \frac{4(0.2)}{3\pi}\right)\right]^2 + \frac{1}{12}(3)\left(0.3^2 + 0.4^2\right) + 3(0.35 - 0.2032)^2$$

$$= 0.230 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$I_G = 0.230 \text{ kg}\cdot\text{m}^2$

17–11.

Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O. The block has a mass of 3 kg, and the semicylinder has a mass of 5 kg.



SOLUTION

$$(I_G)_{cyl} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199\, mR^2$$

$$I_O = 0.3199(5)(0.2)^2 + 5\left(0.2 - \frac{4(0.2)}{3\pi}\right)^2 + \frac{1}{12}(3)\left((0.3)^2 + (0.4)^2\right) + 3(0.350)^2$$

$$= 0.560 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Also from the solution to Prob. 17–22,

$$I_O = I_G + md^2$$

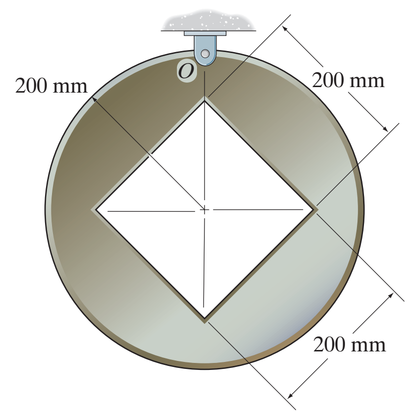
$$= 0.230 + 8(0.2032)^2$$

$$= 0.560 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_O = 0.560 \text{ kg}\cdot\text{m}^2$

***17–12.**

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



SOLUTION

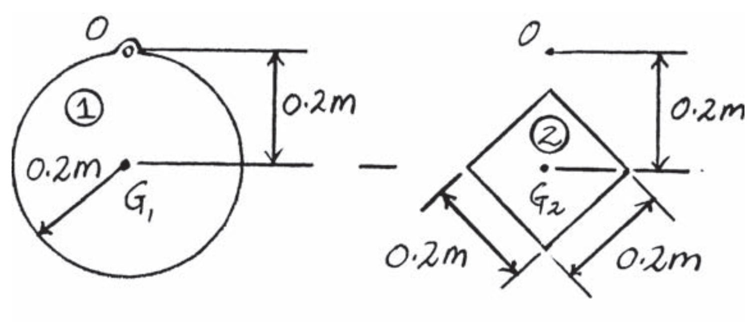
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**

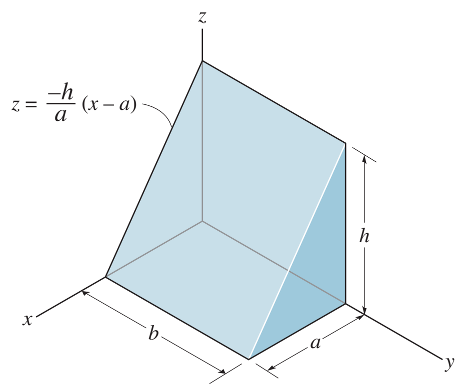


(a)

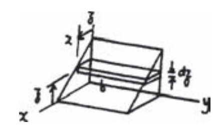
Ans:
$I_O = 0.113\ \text{kg}\cdot\text{m}^2$

17–13.

Determine the moment of inertia of the homogeneous triangular prism with respect to the *y* axis. Express the result in terms of the mass *m* of the prism. *Hint:* For integration, use thin plate elements parallel to the *x–y* plane and having a thickness *dz*.



SOLUTION



$$dV = bx\,dz = b(a)(1 - \frac{z}{h})\,dz$$

$$dI_y = dI_y + (dm)[(\frac{x}{2})^2 + z^2]$$

$$= \frac{1}{12}dm(x^2) + dm(\frac{x^2}{4}) + dmz^2$$

$$= dm(\frac{x^2}{3} + z^2)$$

$$= [b(a)(1 - \frac{z}{h})dz](\rho)[\frac{a^2}{3}(1 - \frac{z}{h})^2 + z^2]$$

$$I_y = ab\rho\int_0^k [\frac{a^3}{3}(\frac{h - z}{h})^3 + z^2(1 - \frac{z}{h})]dz$$

$$= ab\rho[\frac{a^2}{3h^3}(h^4 - \frac{3}{2}h^4 + h^4 - \frac{1}{4}h^4) + \frac{1}{h}(\frac{1}{3}h^4 - \frac{1}{4}h^4)]$$

$$= \frac{1}{12}abh\rho(a^2 + h^2)$$

$$m = \rho V = \frac{1}{2}abh\rho$$

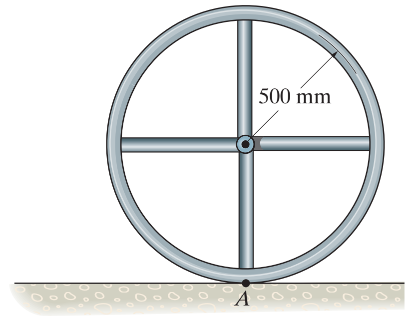
Thus,

$$I_y = \frac{m}{6}(a^2 + h^2)$$ **Ans.**

Ans:
$I_y = \frac{m}{6}(a^2 + h^2)$

17–14.

The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods, each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



SOLUTION

$$I_A = I_o + md^3$$

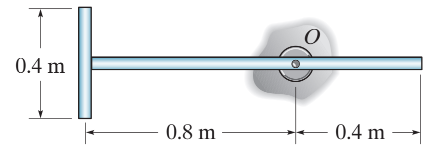
$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$I_A = 7.67 \text{ kg} \cdot \text{m}^2$

17–15.

The assembly is made of the slender rods that have a mass per unit length of 3 kg/m. Determine the mass moment of inertia of the assembly about an axis perpendicular to the page and passing through point O.



SOLUTION

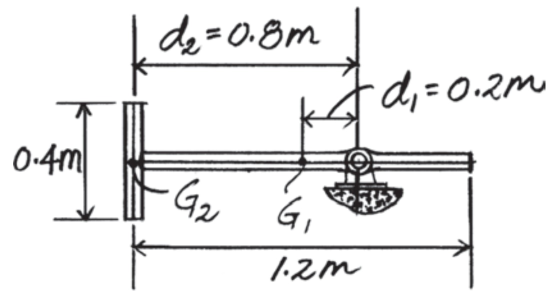
Using the parallel axis theorem by referring to Fig. *a*,

$$I_O = \Sigma\left(I_G + md^2\right)$$

$$= \left\{\frac{1}{12}[3(1.2)]\left(1.2^2\right) + [3(1.2)]\left(0.2^2\right)\right\}$$

$$+ \left\{\frac{1}{12}[3(0.4)]\left(0.4^2\right) + [3(0.4)]\left(0.8^2\right)\right\}$$

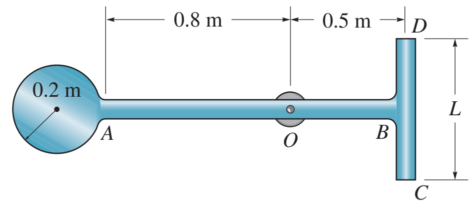
$$= 1.36 \text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

Ans:
$I_O = 1.36 \text{ kg}\cdot\text{m}^2$

***17–16.** The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass per unit length of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through O?



SOLUTION

Measured from the right side,

$$y = \frac{6(1.5) + 2(1.3)(0.65)}{6 + 1.3(2) + L(2)} = 0.5$$

$L = 6.39$ m **Ans.**

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(6.39)(6.39)^2 + 2(6.39)(0.5)^2$$

$I_O = 53.2\ \text{kg} \cdot \text{m}^2$ **Ans.**

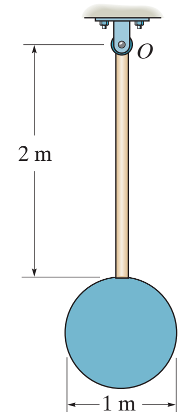
Ans:

$L = 6.39$ m

$I_O = 53.2\ \text{kg} \cdot \text{m}^2$

17–17.

The pendulum consists of a 4-kg circular plate and a 2-kg slender rod. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



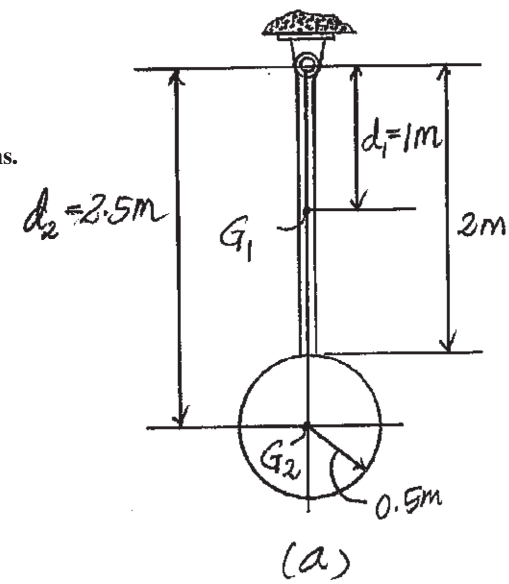
SOLUTION

Using the parallel axis theorem by referring to Fig. *a*,

$$I_O = \Sigma\left(I_G + md^2\right)$$
$$= \left[\frac{1}{12}(2)\left(2^2\right) + 2\left(1^2\right)\right] + \left[\frac{1}{2}(4)\left(0.5^2\right) + 4\left(2.5^2\right)\right]$$
$$= 28.17 \text{ kg} \cdot \text{m}^2$$

Thus, the radius of gyration is

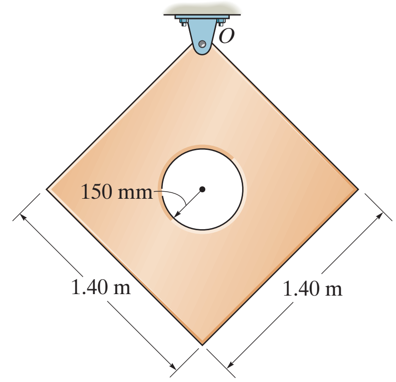
$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{28.17}{4+2}} = 2.167 \text{ m} = 2.17 \text{ m}$$ **Ans.**



Ans:
$k_O = 2.17$ m

17–18.

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50\ \text{kg/m}^3$.



SOLUTION

$$I_G = \frac{1}{12}\left[50(1.4)(1.4)(0.05)\right]\left[(1.4)^2 + (1.4)^2\right] - \frac{1}{2}\left[50(\pi)(0.15)^2(0.05)\right](0.15)^2$$

$$= 1.5987\ \text{kg}\cdot\text{m}^2$$

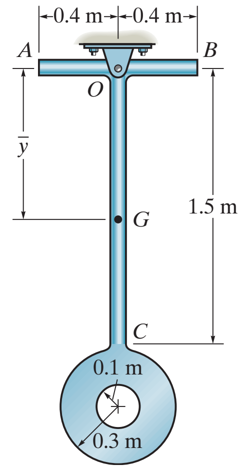
$$I_O = I_G + md^2$$

$$m = 50(1.4)(1.4)(0.05) - 50(\pi)(0.15)^2(0.05) = 4.7233\ \text{kg}$$

$$I_O = 1.5987 + 4.7233(1.4 \sin 45°)^2 = 6.23\ \text{kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_O = 6.23\ \text{kg}\cdot\text{m}^2$

17–19. The pendulum consists of two slender rods AB and OC which have a mass per unit length of 3 kg/m. The thin circular plate has a mass per unit area of 12 kg/m². Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$= 0.8878 \text{ m} = 0.888 \text{ m} \qquad \textbf{Ans.}$$

$$I_G = \frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.8878)^2$$

$$+\frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.8878)^2$$

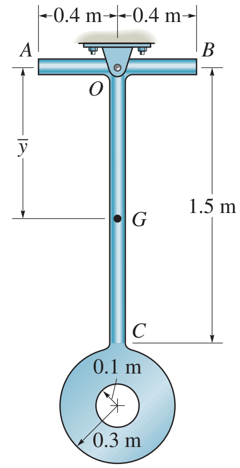
$$+\frac{1}{2}[\pi(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8 - 0.8878)^2$$

$$-\frac{1}{2}[\pi(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8 - 0.8878)^2$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = 0.888$ m
$I_G = 5.61$ kg·m²

***17–20.** The pendulum consists of two slender rods AB and OC which have a mass per unit length of 3 kg/m. The thin circular plate has a mass per unit area of 12 kg/m^2. Determine the moment of inertia of the pendulum about an axis perpendicular to the page and passing through the pin at O.



SOLUTION

$$I_o = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 = 13.4 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Also, from the solution to Prob. 17–16,

$$m = 3(0.8 + 1.5) + 12[\pi(0.3)^2 - \pi(0.1)^2] = 9.916 \text{ kg}$$

$$I_o = I_G + m\,d^2$$

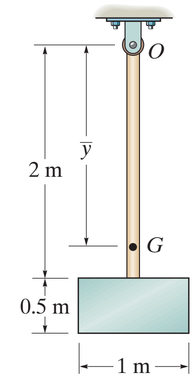
$$= 5.61 + 9.916(0.8878)^2$$

$$= 13.4 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_o = 13.4 \text{ kg} \cdot \text{m}^2$

17–21.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{\Sigma\, \bar{y}m}{\Sigma\, m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m}$$ **Ans.**

$$I_G = \Sigma \bar{I}_G + md^2$$

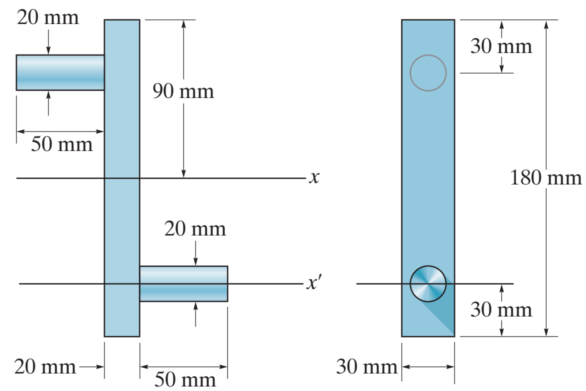
$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$\bar{y} = 1.78$ m
$I_G = 4.45$ kg $\cdot$ m^2

17–22.

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85(10^3)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_\rho = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

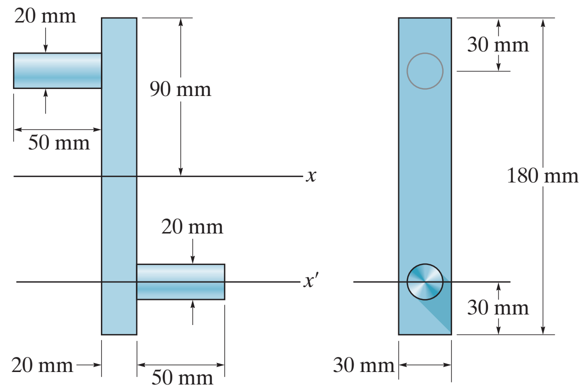
$$+\left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_x = 3.25\ \text{g}\cdot\text{m}^2$

17–23.

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85\left(10^3\right)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

$$m_p = 7.85\left(10^3\right)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

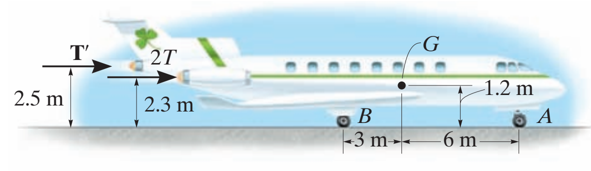
$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**

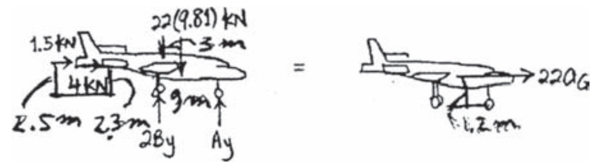
Ans:
$I_{x'} = 7.19\ \text{g}\cdot\text{m}^2$

***17–24.**

The jet aircraft has a total mass of 22 Mg and a center of mass at G. Initially at take-off the engines provide a thrust $2T = 4$ kN and $T' = 1.5$ kN. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m a_x;\qquad 1.5 + 4 = 22 a_G$

$+\uparrow \Sigma F_y = 0;\qquad 2B_y + A_y - 22(9.81) = 0$

$\circlearrowleft + \Sigma M_B = \Sigma (M_K)_B;\qquad 4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22 a_G (1.2)$

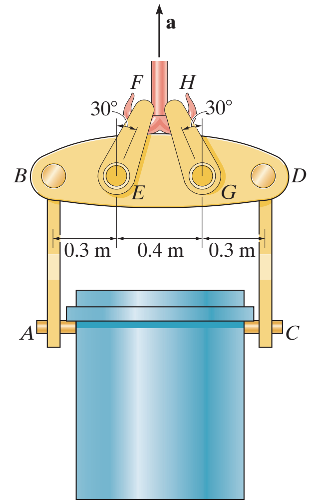
$A_y = 72.6$ kN **Ans.**

$B_y = 71.6$ kN **Ans.**

$a_G = 0.250\ \text{m/s}^2$ **Ans.**

Ans:
$A_y = 72.6$ kN
$B_y = 71.6$ kN
$a_G = 0.250\ \text{m/s}^2$

17–25. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.



SOLUTION

Canister:

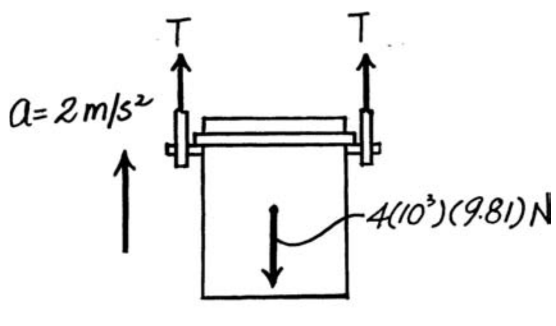
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T - 4(10^3)(9.81) = 4(10^3)(2)$

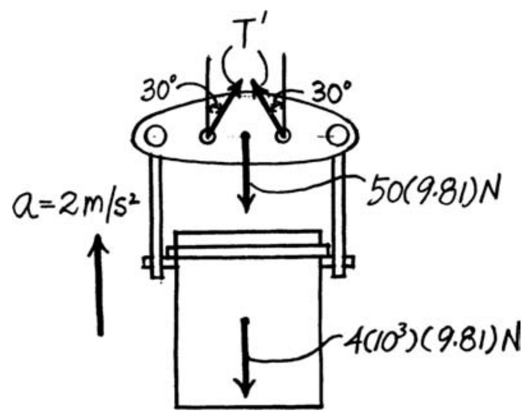
$T_{AB} = T_{CD} = T = 23.6\text{ kN}$ **Ans.**

System:

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T' \cos 30° - 4050(9.81) = 4050(2)$

$T_{EF} = T_{GH} = T' = 27.6\text{ kN}$ **Ans.**

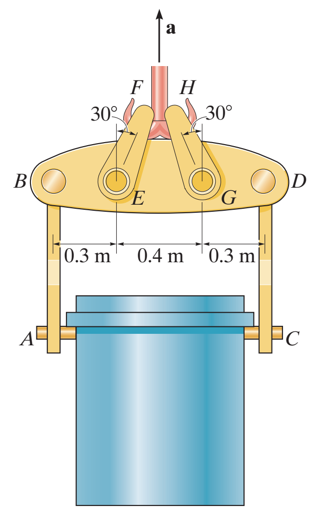




Ans:

$T_{AB} = T_{CD} = T = 23.6\text{ kN}$
$T_{EF} = T_{GH} = T' = 27.6\text{ kN}$

17–26. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration **a** of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.



SOLUTION

Canister:

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2(30)\left(10^3\right) - 4\left(10^3\right)(9.81) = 4\left(10^3\right)a$

$a = 5.19 \text{ m/s}^2$

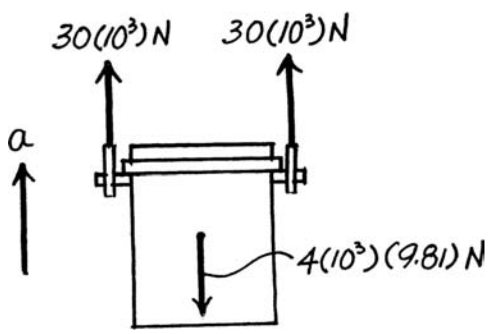
System:

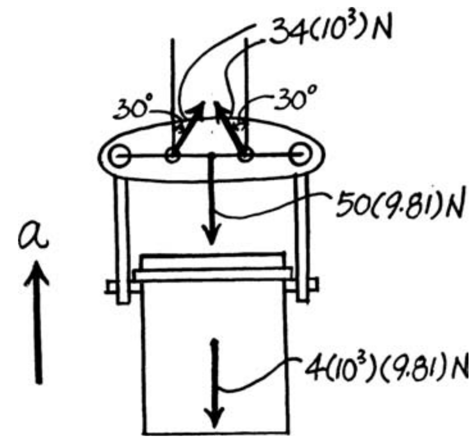
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2\left[34\left(10^3\right)\cos 30°\right] - 4050(9.81) = 4050a$

$a = 4.73 \text{ m/s}^2$

Thus,

$a_{max} = 4.73 \text{ m/s}^2$ **Ans.**

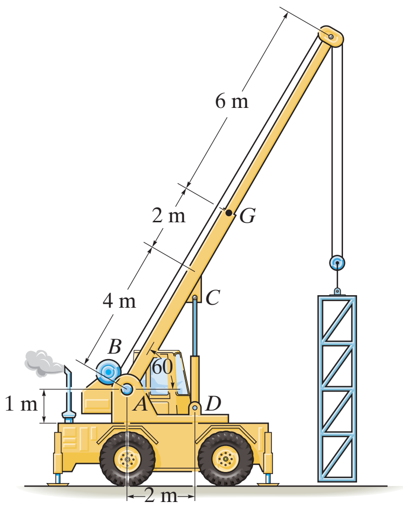




Ans:
$a_{max} = 4.73 \text{ m/s}^2$

17–27.

The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at B draws in the cable with an acceleration of 2 m/s^2, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at G.



SOLUTION

$s_B + 2s_L = l$

$a_B = -2a_L$

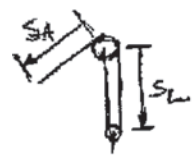
$2 = -2a_L$

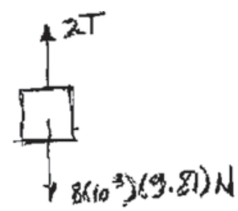
$a_L = -1 \text{ m/s}^2$

Assembly:

$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24 \text{ kN}$

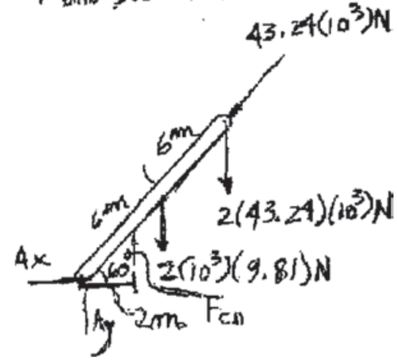




Boom:

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(2) - 2(10^3)(9.81)(6 \cos 60°) - 2(43.24)(10^3)(12 \cos 60°) = 0$

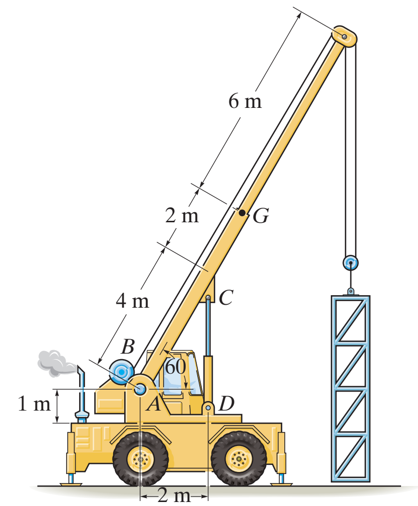
$F_{CD} = 289 \text{ kN}$ **Ans.**



Ans:
$F_{CD} = 289 \text{ kN}$

***17–28.**

The assembly has a mass of 4 Mg and is hoisted using the winch at B. Determine the greatest acceleration of the assembly so that the compressive force in the hydraulic cylinder supporting the boom does not exceed 180 kN. What is the tension in the supporting cable? The boom has a mass of 2 Mg and mass center at G.



SOLUTION

Boom:

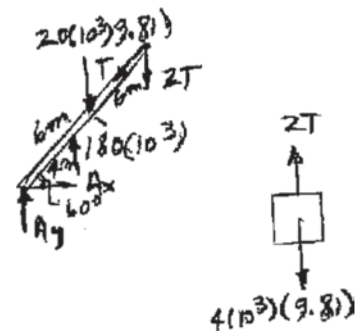
$\zeta + \Sigma M_A = 0;$ $\quad 180(10^3)(2) - 2(10^3)(9.81)(6 \cos 60°) - 2T(12 \cos 60°) = 0$

$T = 25\,095 \text{ N} = 25.1 \text{ kN}$ **Ans.**

Assembly:

$+\uparrow \Sigma F_y = ma_y;$ $\quad 2(25\,095) - 4(10^3)(9.81) = 4(10^3)\,a$

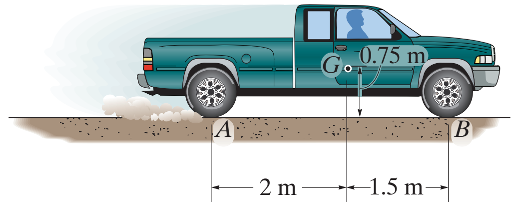
$a = 2.74 \text{ m/s}^2$ **Ans.**



Ans:
$a = 2.74 \text{ m/s}^2$
$T = 25.1 \text{ kN}$

17–29.

Determine the shortest time possible for the rear-wheel drive, 2-Mg truck to achieve a speed of 16 m/s with a constant acceleration starting from rest. The coefficient of static friction between the wheels and the road surface is $\mu_s = 0.8$. The front wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Equations of Motion: The maximum acceleration of the truck occurs when its rear wheels are on the verge of slipping. Thus, $F_A = \mu_s N_A = 0.8N_A$. Referring to the free-body diagram of the truck shown in Fig. *a*, we can write

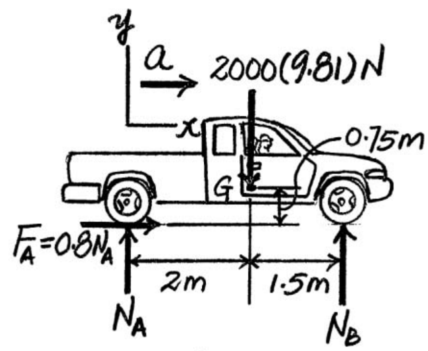
$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 0.8N_A = 2000a$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 2000(9.81) = 0$ **(2)**

$+\Sigma M_G = 0; \quad N_B(1.5) + 0.8N_A(0.75) - N_A(2) = 0$ **(3)**

Solving Eqs. (1), (2), and (3) yields

$N_A = 10\,148.28 \text{ N} \qquad N_B = 9471.72 \text{ N} \qquad a = 4.059 \text{ m/s}^2$



(a)

Kinematics: Since the acceleration of the truck is constant, we can apply

$(\overset{+}{\rightarrow}) \qquad v = v_0 + at$

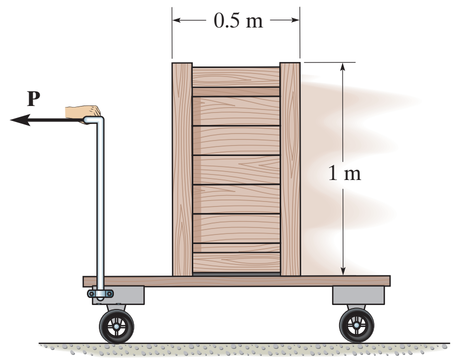
$16 = 0 + 4.059t$

$t = 3.94 \text{ s}$ **Ans.**

Ans:
$t = 3.94 \text{ s}$

17–30.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to slip or tip on the cart. The coefficient of static friction between the crate and cart is $\mu_s = 0.2$.



SOLUTION

Equation of Motion. Assuming that the crate slips before it tips, then $F_f = \mu_s N = 0.2\,N$. Referring to the FBD and kinetic diagram of the crate, Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150\,(9.81) = 150\,(0) \qquad N = 1471.5 \text{ N}$$

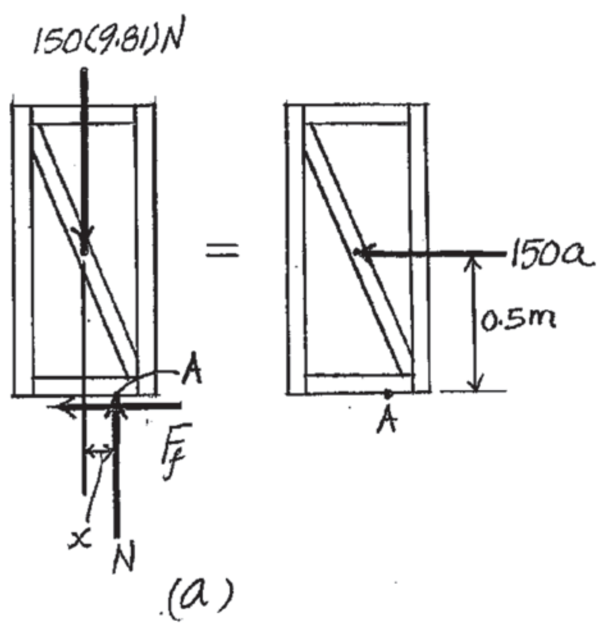
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2(1471.5) = 150\,a \qquad a = 1.962 \text{ m/s}^2$$

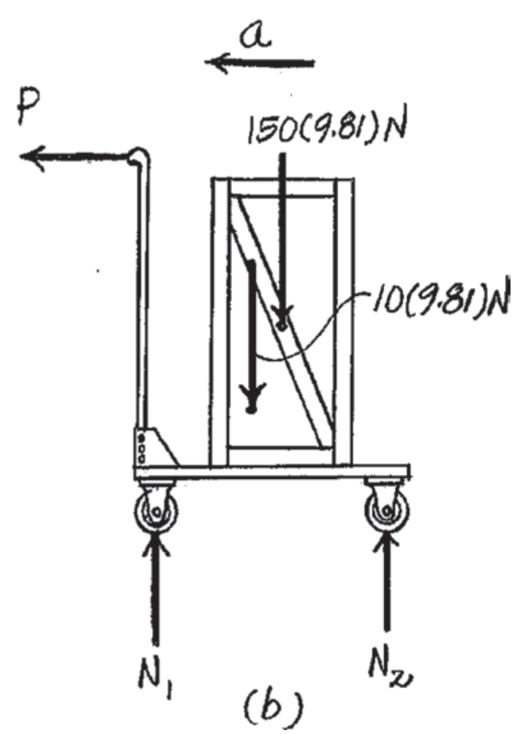
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(x) = 150(1.962)(0.5)$$

$$x = 0.1 \text{ m}$$

Since $x = 0.1 \text{ m} < 0.25 \text{ m}$, the crate indeed slips before it tips. Using the result of a and refer to the FBD of the crate and cart, Fig. b,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = (150 + 10)(1.962) = 313.92 \text{ N} = 314 \text{ N} \qquad \textbf{Ans.}$$

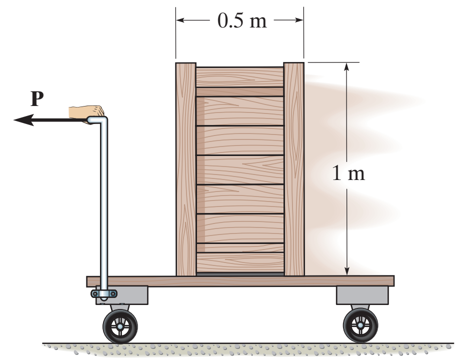




Ans:
$P = 314$ N

17–31.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to tip on the cart. Slipping does not occur.



SOLUTION

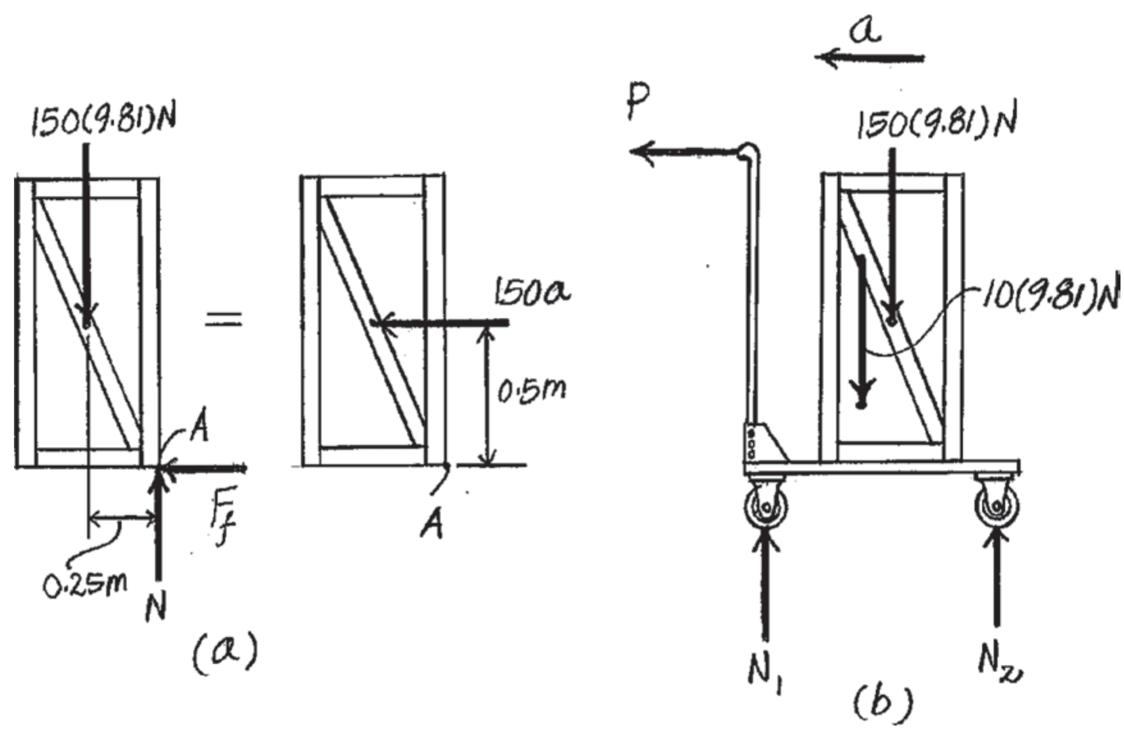
Equation of Motion. Tipping will occur about edge A. Referring to the FBD and kinetic diagram of the crate, Fig. a,

$$\zeta+\Sigma M_A = \Sigma(M_K)_A; \qquad 150(9.81)(0.25) = (150a)(0.5)$$

$$a = 4.905 \text{ m/s}^2$$

Using the result of a and refer to the FBD of the crate and cart, Fig. b,

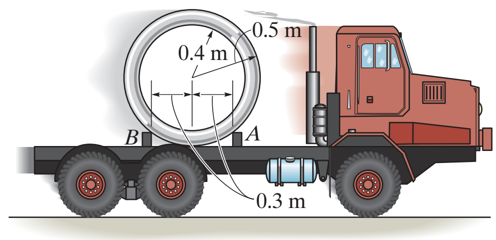
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x \qquad P = (150 + 10)(4.905) = 784.8 \text{ N} = 785 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$P = 785$ N

***17–32.**

The pipe has a mass M and is held in place on the truck bed using the two boards A and B. Determine the acceleration of the truck so that the pipe begins to lose contact at A and the bed of the truck and starts to pivot about B. Assume board B will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board B exert on the pipe during the acceleration?



SOLUTION

Units Used: $\quad kN = 10^3\ N$

Given:

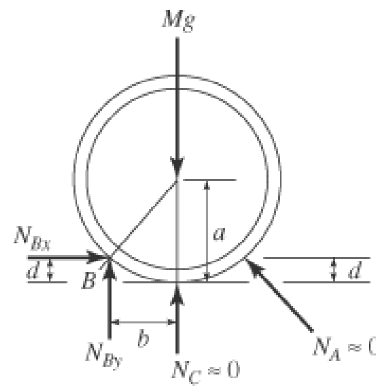
$M = 460\ kg$

$a = 0.5\ m$

$b = 0.3\ m$

$c = 0.4\ m$

$g = 9.81\ m/s^2$



$$\theta = \operatorname{asin}\left(\frac{b}{a}\right)$$

Guesses

$N_{Bx} = 1\ N \qquad N_{By} = 1\ N \qquad a_t = 1\ m/s^2$

Given

$N_{Bx} = M a_t \qquad N_{By} - M g = 0 \qquad N_{Bx}(a)\cos(\theta) - N_{By} b = 0$

$$\begin{pmatrix} N_{Bx} \\ N_{By} \\ a_t \end{pmatrix} = \text{Find}(N_{Bx}, N_{By}, a_t) \qquad \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} = \begin{pmatrix} 3.384 \\ 4.513 \end{pmatrix} kN \qquad a_t = 7.36\ m/s^2 \quad \textbf{Ans.}$$

$$\left|\begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix}\right| = 5.64\ kN \quad \textbf{Ans.}$$

Ans:

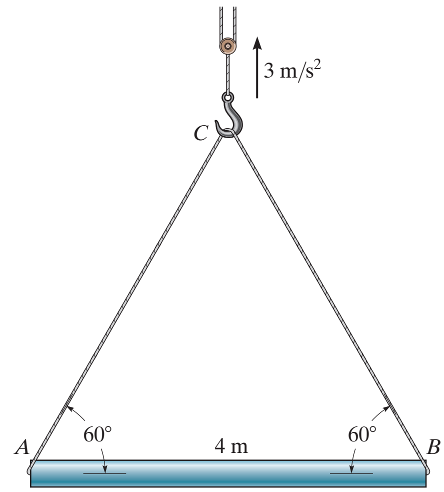
$a_t = 7.36\ m/s^2$

$N_{Bx} = 5.64\ kN$

$N_{By} = 5.64\ kN$

17–33.

The uniform girder AB has a mass of 8 Mg. Determine the internal axial, shear, and bending-moment loadings at the center of the girder if a crane gives it an upward acceleration of 3 m/s^2.



SOLUTION

Girder:

$+\uparrow \Sigma F_y = ma_y; \qquad 2T \sin 60° - 8000(9.81) = 8000(3)$

$T = 59\,166.86 \text{ N}$

Segment:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 59\,166.86 \cos 60° - N = 0$

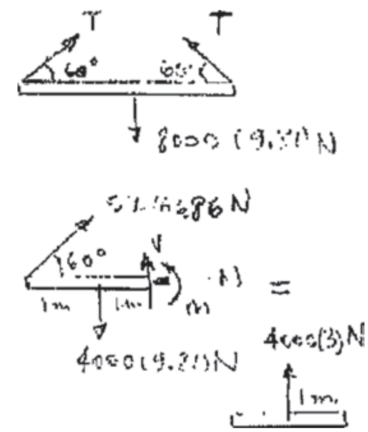
$N = 29.6 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = ma_y; \qquad 59\,166.86 \sin 60° - 4000(9.81) + V = 4000(3)$

$V = 0$ **Ans.**

$\circlearrowleft + \Sigma M_C = \Sigma (M_k)_C; \qquad M + 4000(9.81)(1) - 59\,166.86 \sin 60°(2) = -4000(3)(1)$

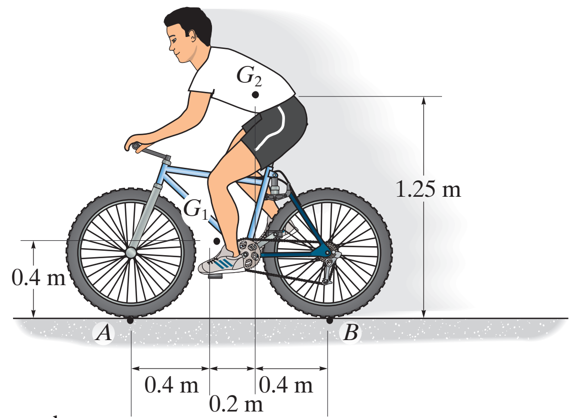
$M = 51.2 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$N = 29.6 \text{ kN}$
$V = 0$
$M = 51.2 \text{ kN} \cdot \text{m}$

17–34.

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. Determine the maximum deceleration when the brake is applied to the front wheel, without causing the rear wheel B to leave the road. Assume that the front wheel does not slip. Neglect the mass of all the wheels.

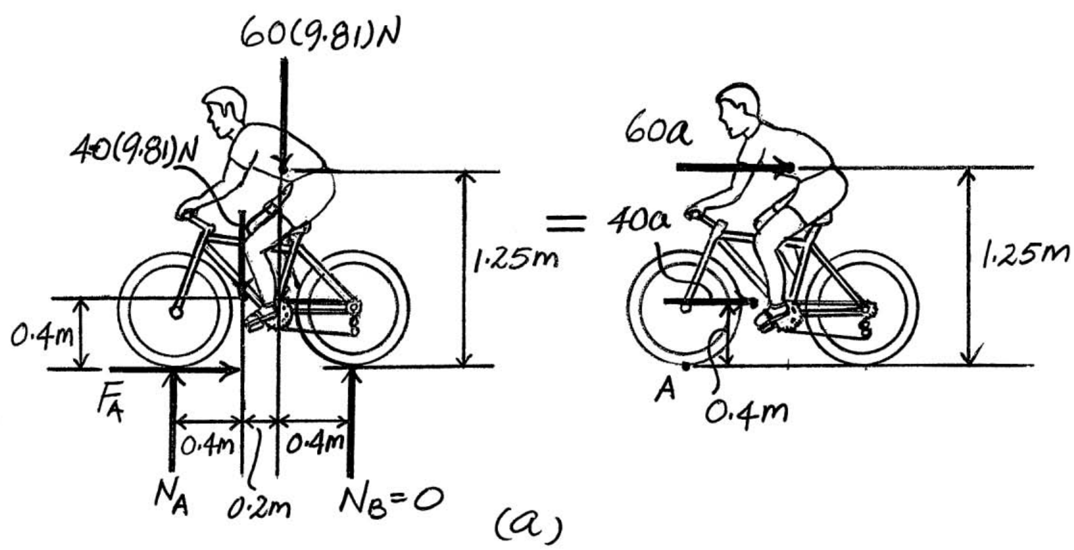


SOLUTION

Equations of Motion: Since the rear wheel B is required to just leave the road, $N_B = 0$. Thus, the acceleration **a** of the bike can be obtained directly by writing the moment equation of motion about point A.

$$+\Sigma M_A = (M_k)_A; \quad -40(9.81)(0.4) - 60(9.81)(0.6) = -40a(0.4) - 60a(1.25)$$

$$a = 5.606 \text{ m/s}^2 = 5.61 \text{ m/s}^2 \qquad \textbf{Ans.}$$

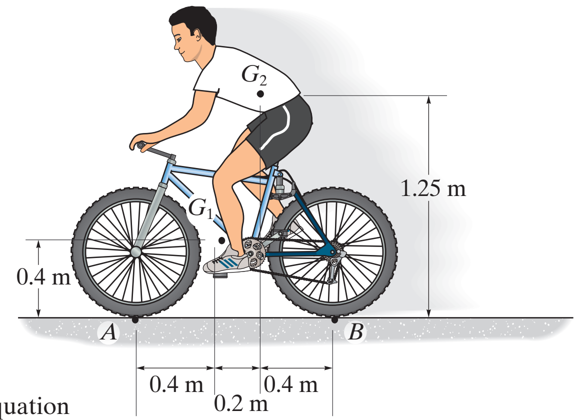


(a)

Ans:
$a = 5.61 \text{ m/s}^2$

17–35.

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. When the brake is applied to the front wheel, it causes the bike to decelerate at a constant rate of 3 m/s^2. Determine the normal reaction the road exerts on the front and rear wheels. Assume that the rear wheel is free to roll. Neglect the mass of all the wheels.



SOLUTION

Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

$+\Sigma M_A = (M_k)_A;$

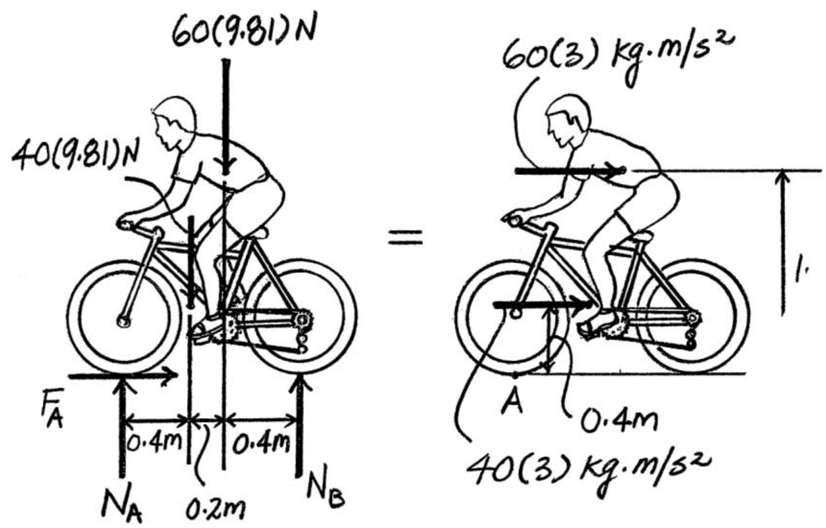
$N_B(1) - 40(9.81)(0.4) - 60(9.81)(0.6) = -60(3)(1.25) - 40(3)(0.4)$

$N_B = 237.12 \text{ N} = 237 \text{ N}$ **Ans.**

Using this result and writing the force equations of motion along the y axis,

$+\uparrow \Sigma F_y = m(a_G)_y;$ $N_A + 237.12 - 40(9.81) - 60(9.81) = 0$

$N_A = 743.88 \text{ N} = 744 \text{ N}$ **Ans.**

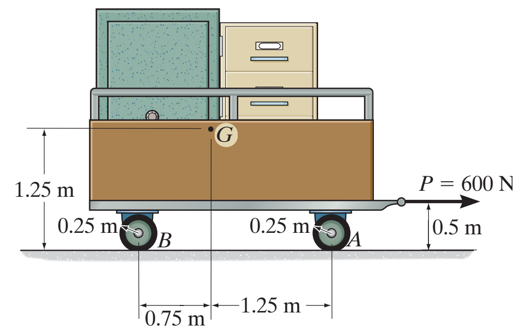


Ans:

$N_B = 237 \text{ N}$
$N_A = 744 \text{ N}$

***17–36.**

The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



SOLUTION

Equations of Motion: Writing the force equation of motion along the x axis,

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 600 = 150a \qquad a = 4 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Using this result to write the moment equation about point A,

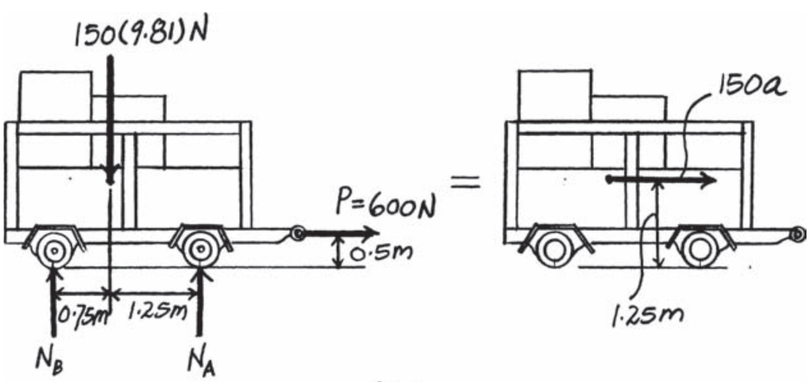
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$$

$$N_B = 1144.69 \text{ N} = 1.14 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to write the force equation of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)$$

$$N_A = 326.81 \text{ N} = 327 \text{ N} \qquad \textbf{Ans.}$$

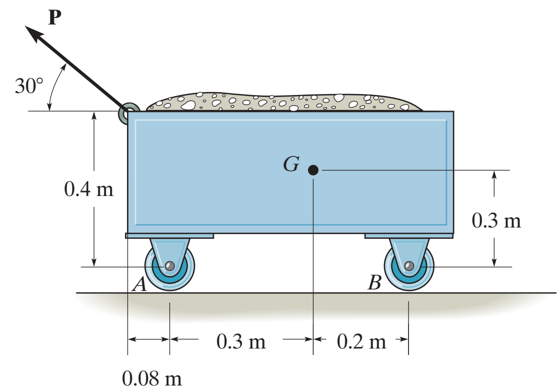


(a)

Ans:
$a = 4 \text{ m/s}^2 \rightarrow$
$N_B = 1.14$ kN
$N_A = 327$ N

17–37.

A force of $P = 300$ N is applied to the 60-kg cart. Determine the reactions at both the wheels at A and both the wheels at B. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Referring to the FBD of the cart, Fig. *a*,

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 300 \cos 30° = 60a$

$a = 4.3301 \text{ m/s}^2 = 4.33 \text{ m/s}^2 \leftarrow$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B + 300 \sin 30° - 60(9.81) = 60(0)$ **(1)**

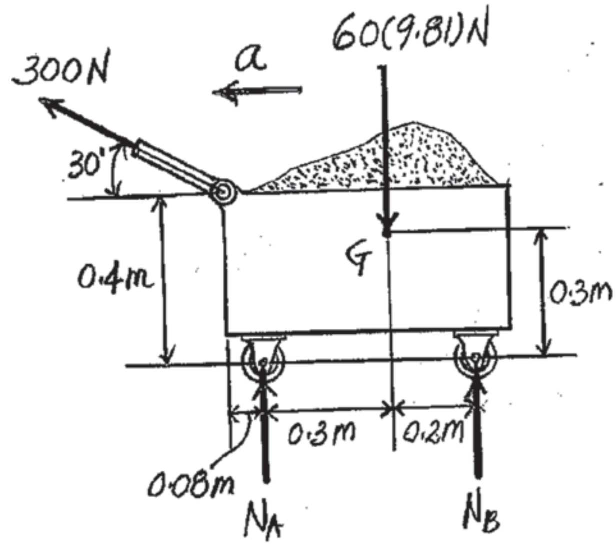
$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(0.2) - N_A(0.3) + 300 \cos 30°(0.1)$

$- 300 \sin 30°(0.38) = 0$ **(2)**

Solving Eqs. (1) and (2),

$N_A = 113.40 \text{ N} = 113 \text{ N}$ **Ans.**

$N_B = 325.20 \text{ N} = 325 \text{ N}$ **Ans.**

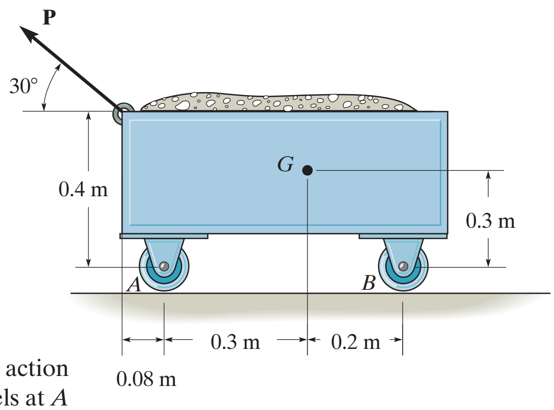


(a)

Ans:
$a = 4.33 \text{ m/s}^2 \leftarrow$
$N_A = 113 \text{ N}$
$N_B = 325 \text{ N}$

17–38.

Determine the largest force **P** that can be applied to the 60-kg cart, without causing one of the wheel reactions, either at A or at B, to be zero. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Since $(0.38\text{ m})\tan 30° = 0.22\text{ m} > 0.1\text{ m}$, the line of action of **P** passes *below* G. Therefore, **P** tends to rotate the cart clockwise. The wheels at A will leave the ground before those at B. Then, it is required that $N_A = 0$. Referring, to the FBD of the cart, Fig. *a*

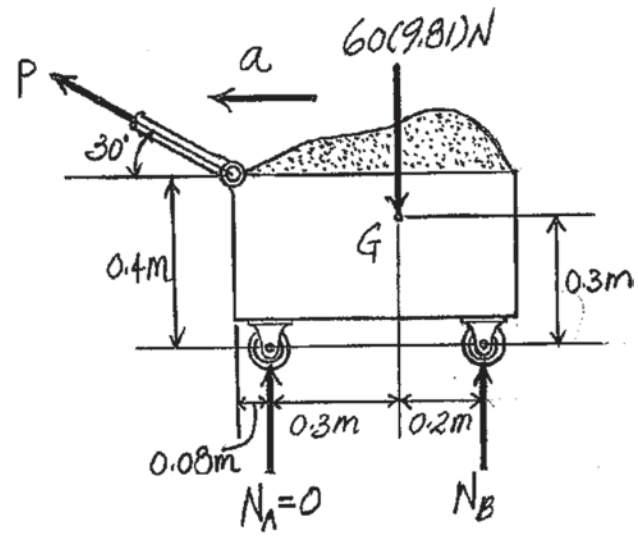
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + P\sin 30° - 60(9.81) = 60(0) \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad P\cos 30°(0.1) - P\sin 30°(0.38) + N_B(0.2) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$P = 578.77\text{ N} = 579\text{ N} \qquad \textbf{Ans.}$$

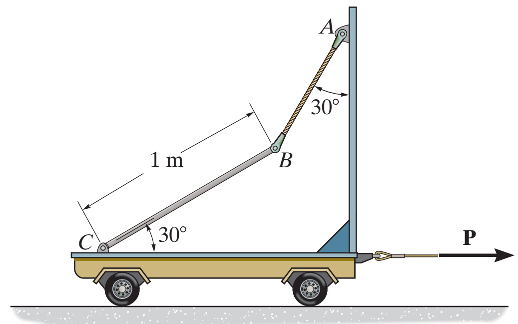
$$N_B = 299.22\text{ N}$$



Ans:
$P = 579$ N

17–39.

If the cart's mass is 30 kg and it is subjected to a horizontal force of $P = 90$ N, determine the tension in cord AB and the horizontal and vertical components of reaction on end C of the uniform 15-kg rod BC.



SOLUTION

Equations of Motion: The acceleration **a** of the cart and the rod can be determined by considering the free-body diagram of the cart and rod system shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 90 = (15 + 30)a \qquad a = 2 \text{ m/s}^2$

The force in the cord can be obtained directly by writing the moment equation of motion about point C by referring to Fig. *b*.

$+\Sigma M_C = (M_k)_C; \qquad F_{AB} \sin 30°(1) - 15(9.81) \cos 30°(0.5) = -15(2) \sin 30°(0.5)$

$F_{AB} = 112.44 \text{ N} = 112 \text{ N}$ **Ans.**

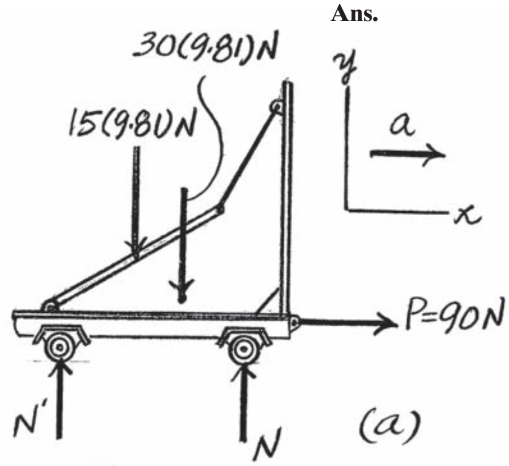
Using this result and applying the force equations of motion along the x and y axes,

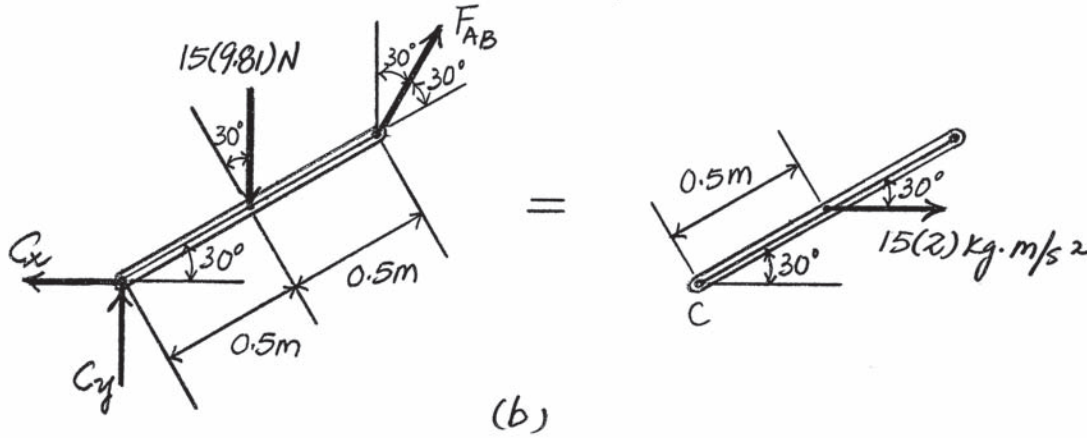
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad -C_x + 112.44 \sin 30° = 15(2)$

$C_x = 26.22 \text{ N} = 26.2 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y + 112.44 \cos 30° - 15(9.81) = 0$

$C_y = 49.78 \text{ N} = 49.8 \text{ N}$ **Ans.**

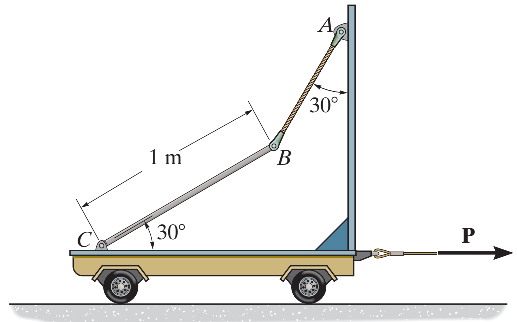




Ans:
$F_{AB} = 112$ N
$C_x = 26.2$ N
$C_y = 49.8$ N

***17–40.**

If the cart's mass is 30 kg, determine the horizontal force P that should be applied to the cart so that the cord AB just becomes slack. The uniform rod BC has a mass of 15 kg.



SOLUTION

Equations of Motion: Since cord AB is required to be on the verge of becoming slack, $F_{AB} = 0$. The corresponding acceleration **a** of the rod can be obtained directly by writing the moment equation of motion about point C. By referring to Fig. a.

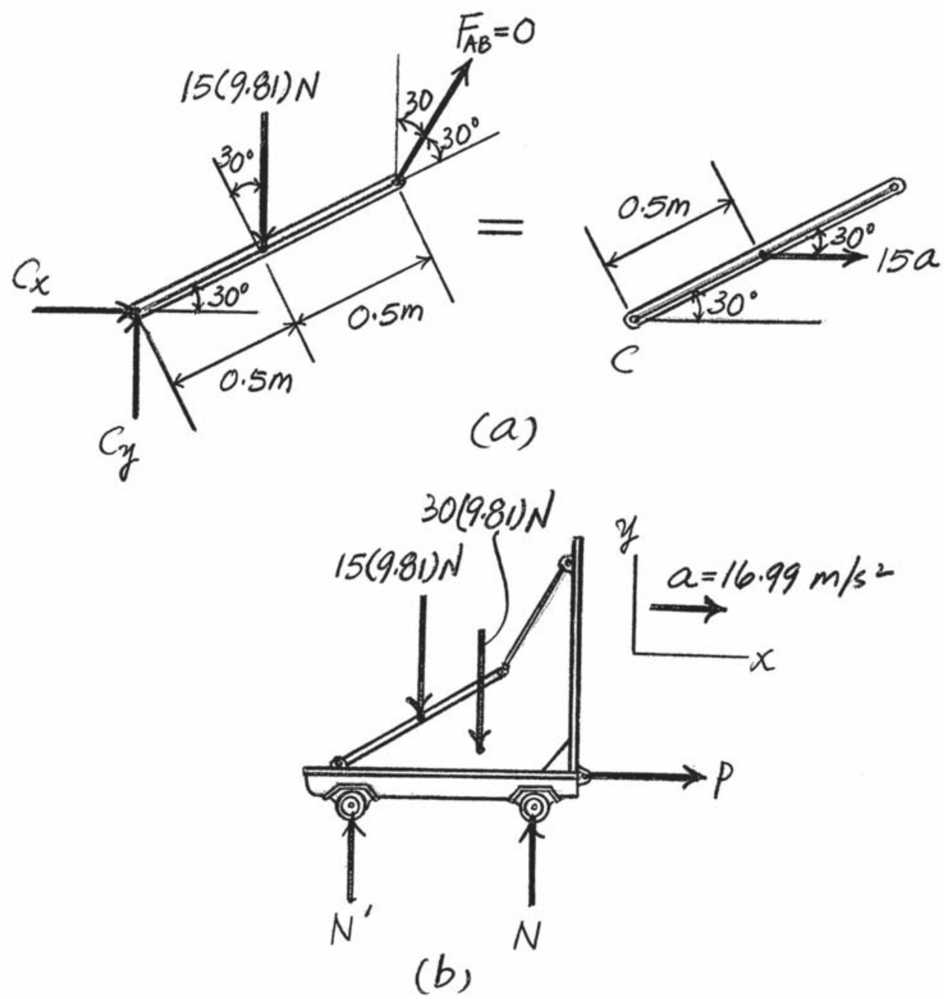
$$+\Sigma M_C = \Sigma(M_C)_A; \qquad -15(9.81)\cos 30°(0.5) = -15a\sin 30°(0.5)$$

$$a = 16.99 \text{ m/s}^2$$

Using this result and writing the force equation of motion along the x axis and referring to the free-body diagram of the cart and rod system shown in Fig. b,

$$(\overset{+}{\rightarrow})\Sigma F_x = m(a_G)_x; \qquad P = (30 + 15)(16.99)$$

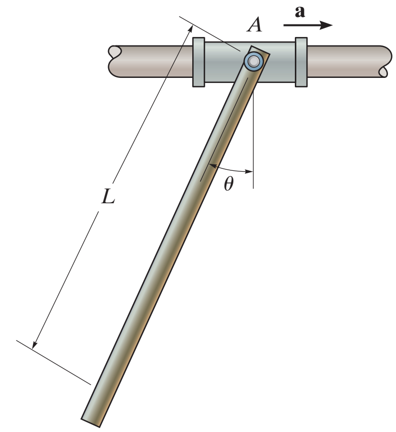
$$= 764.61 \text{ N} = 765 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$P = 765$ N

17–41.

The uniform bar of mass m is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of **a**, determine the bar's inclination angle θ. Neglect the collar's mass.

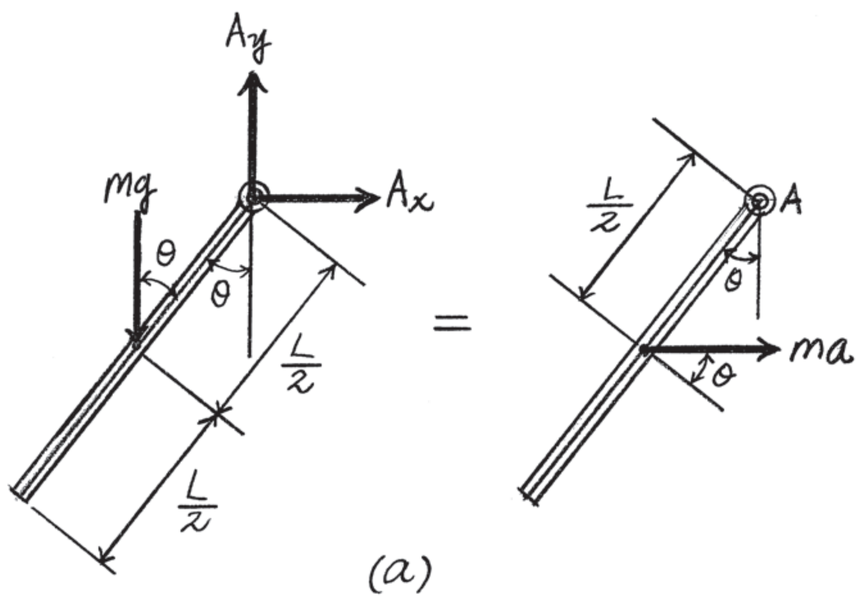


SOLUTION

Equations of Motion: Writing the moment equation of motion about point A,

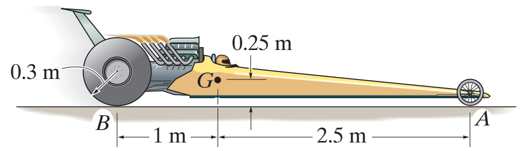
$$+\Sigma M_A = (M_k)_A; \quad mg\sin\theta\left(\frac{L}{2}\right) = ma\cos\theta\left(\frac{L}{2}\right)$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \qquad \textbf{Ans.}$$



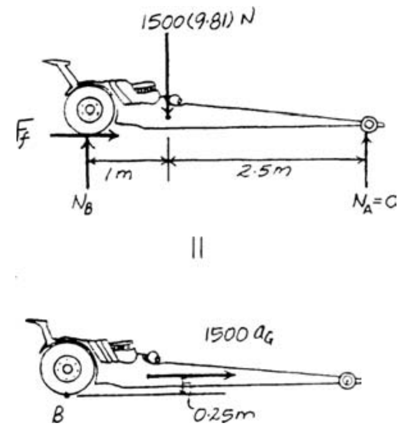
Ans:
$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

17–42. The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear drive wheels are slipping. Neglect the mass of the wheels and assume that the front wheels are free to roll.



SOLUTION

If the front wheels A lift off the ground, then $N_A = 0$.



$\zeta + \Sigma M_B = \Sigma(M_k)_B; \qquad -1500(9.81)(1) = -1500a_G(0.25)$

$$a_G = 39.24 \text{ m/s}^2$$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 1500(39.24) = 58860 \text{ N}$

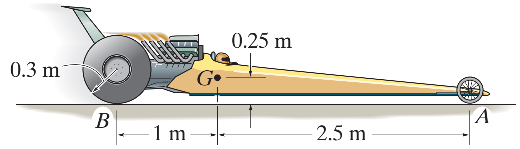
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B - 1500(9.81) = 0 \qquad N_B = 14715 \text{ N}$

Since the required friction $F_f > (F_f)_{max} = \mu_k N_B = 0.6(14715) = 8829$ N, **it is not possible to lift the front wheels off the ground**. **Ans.**

Ans:

Since the required friction $F_f > (F_f)_{max} = \mu_k N_B = 0.6(14715) = 8829$ N, **it is not possible to lift the front wheels off the ground**.

17–43. The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the frictional force $\mathbf{F}_B$ which must be developed at each of the rear drive wheels B in order to create an acceleration of $a = 6\ \text{m/s}^2$. What are the normal reactions of each wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.



SOLUTION

$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B;\qquad 2N_A(3.5) - 1500(9.81)(1) = -1500(6)(0.25)$

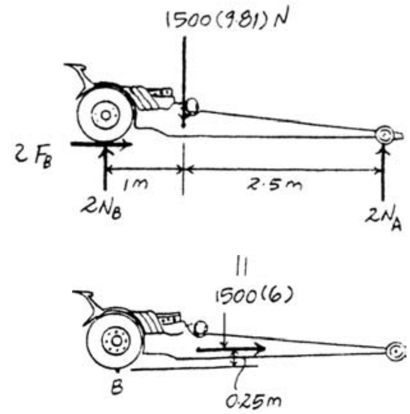
$N_A = 1780.71\ \text{N} = 1.78\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2N_B + 2(1780.71) - 1500(9.81) = 0$

$N_B = 5576.79\ \text{N} = 5.58\ \text{kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad 2F_B = 1500(6)$

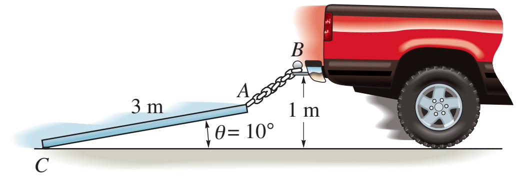
$F_B = 4500\ \text{N} = 4.50\ \text{kN}$ **Ans.**



Ans:
$N_A = 1.78\ \text{kN}$
$N_B = 5.58\ \text{kN}$
$F_B = 4.50\ \text{kN}$

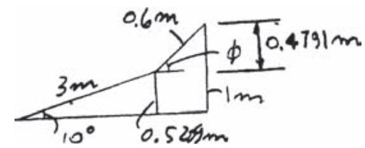
***17–44.**

The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is $\mu_k = 0.4$, determine the acceleration of the truck if the angle $\theta = 10°$ with the road as shown.



SOLUTION

$$\phi = \sin^{-1}\left(\frac{0.4791}{0.6}\right) = 52.98°$$



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad T\cos 52.98° - 0.4N_C = 500a_G$

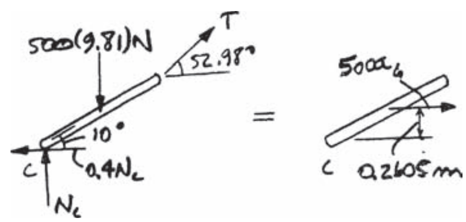
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_C - 500(9.81) + T\sin 52.98° = 0$

$\circlearrowleft + \Sigma M_C = \Sigma(M_k)_C;\qquad -500(9.81)(1.5\cos 10°) + T\sin(52.98° - 10°)(3) = -500a_G(0.2605)$

$T = 3.39\text{ kN}$

$N_C = 2.19\text{ kN}$

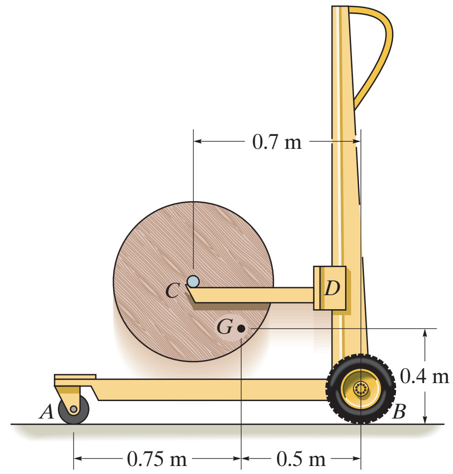
$a_G = 2.33\text{ m/s}^2$ **Ans.**



Ans:
$a_G = 2.33\text{ m/s}^2$

17–45.

The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of $3\ \text{m/s}^2$, determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the movable arm CD.



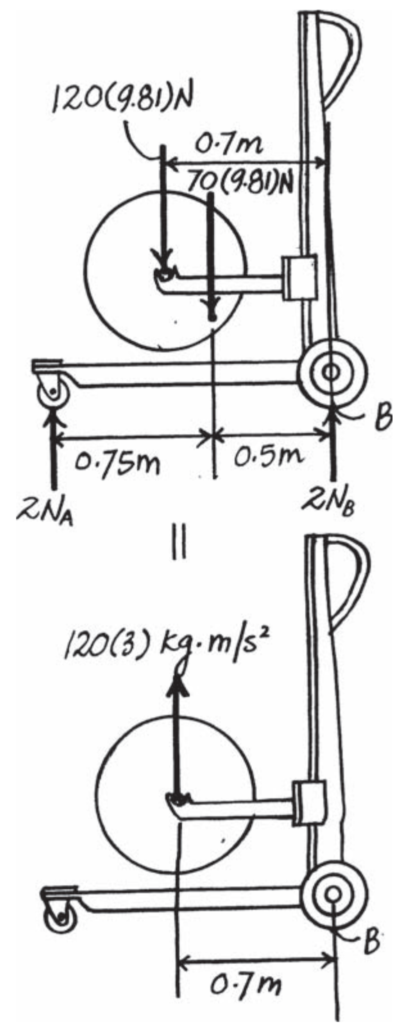
SOLUTION

$$\zeta+\Sigma M_B = \Sigma(M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25) = -120(3)(0.7)$$

$$N_A = 567.76\ \text{N} = 568\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$$

$$N_B = 544\ \text{N} \qquad \textbf{Ans.}$$

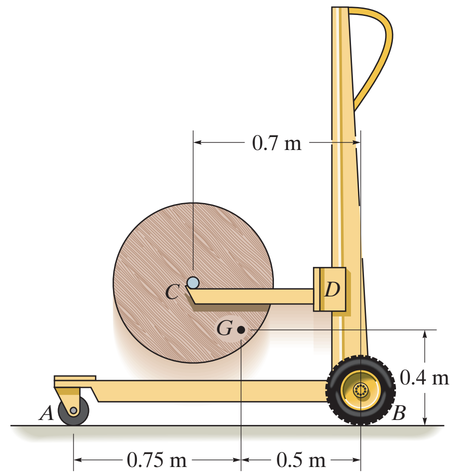


Ans:

$N_A = 568\ \text{N}$
$N_B = 544\ \text{N}$

17–46.

The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction on the wheels exceeds 600 N.



SOLUTION

Assume $N_A = 600$ N.

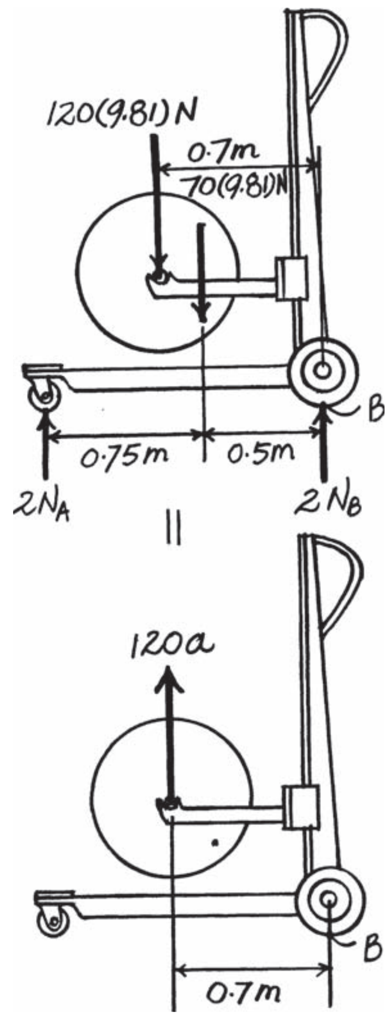
$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B$; $70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$a = 3.960 \text{ m/s}^2$

$+\uparrow \Sigma F_y = m(a_G)_y$; $2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600 \text{ N}$ **OK**

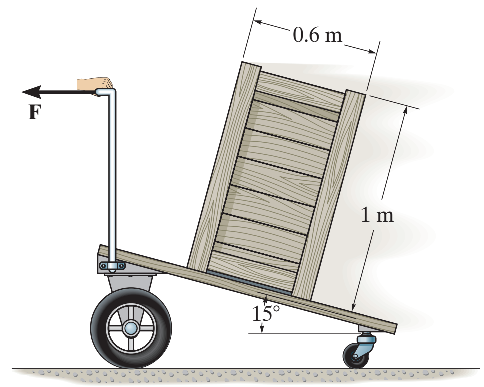
Thus $a = 3.96 \text{ m/s}^2$ **Ans.**



Ans:
$a = 3.96 \text{ m/s}^2$

17–47.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



SOLUTION

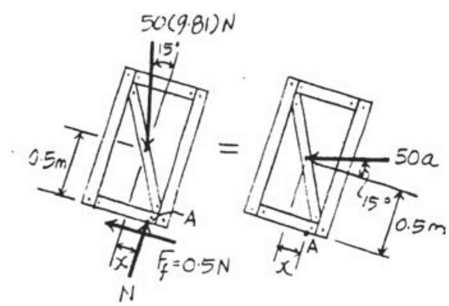
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;$ $\quad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$

$= 50a\cos 15°(0.5) + 50a\sin 15°(x)$ **(1)**

$+\nearrow \Sigma F_{y'} = m(a_G)_{y'};$ $\quad N - 50(9.81)\cos 15° = -50a\sin 15°$ **(2)**

$\searrow + \Sigma F_{x'} = m(a_G)_{x'};$ $\quad 50(9.81)\sin 15° - 0.5N = -50a\cos 15°$ **(3)**



Solving Eqs. (1), (2), and (3) yields

$N = 447.81 \text{ N} \qquad x = 0.250 \text{ m}$

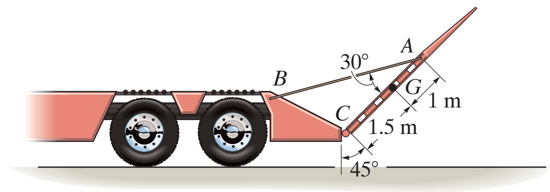
$a = 2.01 \text{ m/s}^2$ **Ans.**

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

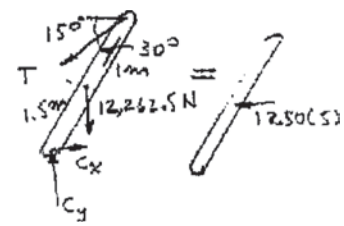
Ans:
$a = 2.01 \text{ m/s}^2$
The crate slips.

***17–48.**

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at 5 m/s^2. Also, what are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\zeta + \Sigma M_C = \Sigma (M_k)_C;\quad T \sin 30°(2.5) - 12\,262.5(1.5 \cos 45°) = 1250(5)(1.5 \sin 45°)$

$T = 15\,708.4 \text{ N} = 15.7 \text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad -C_x + 15\,708.4 \cos 15° = 1250(5)$

$C_x = 8.92 \text{ kN}$ **Ans.**

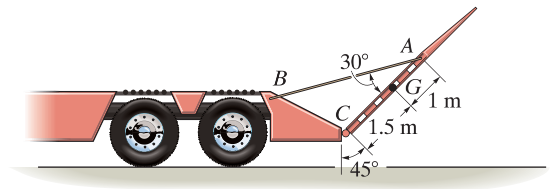
$+\uparrow \Sigma F_y = m(a_G)_y;\quad C_y - 12\,262.5 - 15\,708.4 \sin 15° = 0$

$C_y = 16.3 \text{ kN}$ **Ans.**

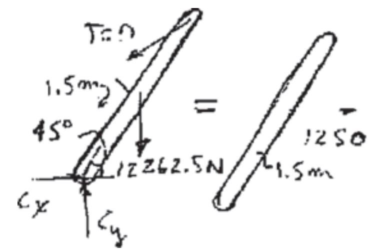
Ans:
$T = 15.7$ kN
$C_x = 8.92$ kN
$C_y = 16.3$ kN

17–49.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the maximum deceleration of the truck so that the gate does not begin to rotate forward. What are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\circlearrowleft + \Sigma M_C = \Sigma (M_k)_C;$ $\quad -12\,262.5(1.5 \cos 45°) = -1250(a)(1.5 \sin 45°)$

$a = 9.81 \text{ m/s}^2$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\quad C_x = 1250(9.81)$

$C_x = 12.3 \text{ kN}$ **Ans.**

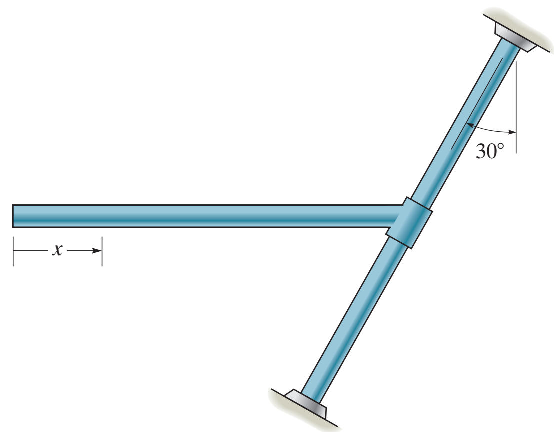
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad C_y - 12\,262.5 = 0$

$C_y = 12.3 \text{ kN}$ **Ans.**

Ans:
$a = 9.81 \text{ m/s}^2$
$C_x = 12.3 \text{ kN}$
$C_y = 12.3 \text{ kN}$

17–50.

The bar has a weight per length w and is supported by the smooth collar. If it is released from rest, determine the internal normal force, shear force, and bending moment in the bar as a function of x.



SOLUTION

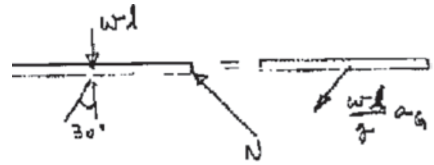
Entire bar:

$$\Sigma F_{x'} = m(a_G)_{x'}; \qquad wl\cos 30° = \frac{wl}{g}(a_G)$$

$$a_G = g\cos 30°$$

Segment:

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \qquad N = (wx\cos 30°)\sin 30° = 0.433wx \qquad \textbf{Ans.}$$

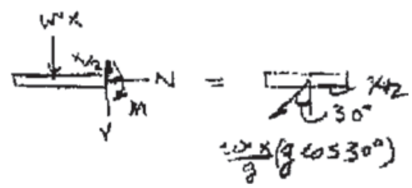


$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad wx - V = wx\cos 30°(\cos 30°)$$

$$V = 0.25wx \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_S = \Sigma(M_k)_S; \qquad wx\left(\frac{x}{2}\right) - M = wx\cos 30°(\cos 30°)\left(\frac{x}{2}\right)$$

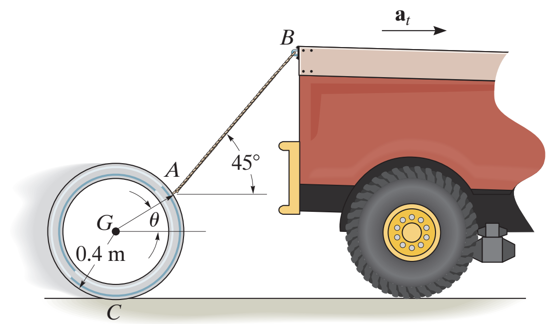
$$M = 0.125wx^2 \qquad \textbf{Ans.}$$



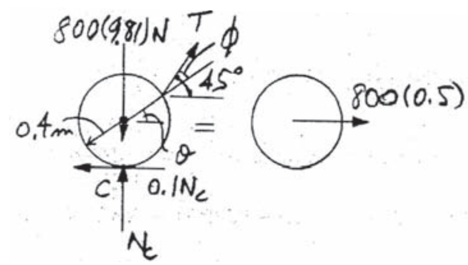
Ans:
$N = 0.433wx$
$V = 0.25wx$
$M = 0.125wx^2$

17–51.

The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T\sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$

$N_C = 6770.9\ \text{N}$

$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans.**

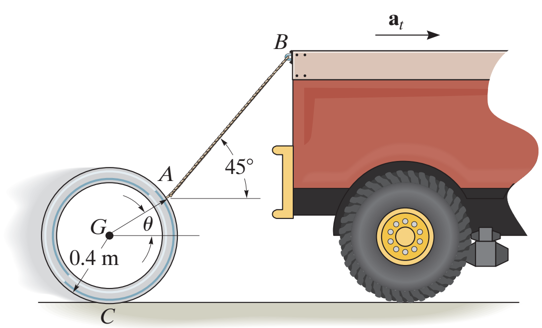
$\sin\phi = \dfrac{0.1(6770.9)}{1523.24} \qquad \phi = 26.39°$

$\theta = 45° - \phi = 18.6°$ **Ans.**

Ans:
$T = 1.52\ \text{kN}$
$\theta = 18.6°$

***17–52.**

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad T \cos 45° - 0.1N_C = 800a$

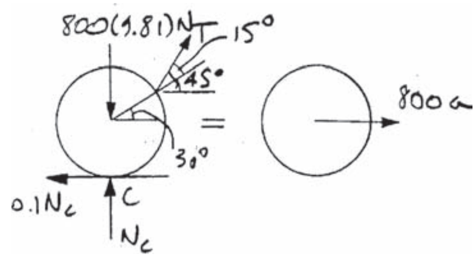
$+\uparrow \Sigma F_y = ma_y; \quad N_C - 800(9.81) + T \sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \quad T \sin 15°(0.4) - 0.1N_C(0.4) = 0$

$N_C = 6161 \text{ N}$

$T = 2382 \text{ N} = 2.38 \text{ kN}$ **Ans.**

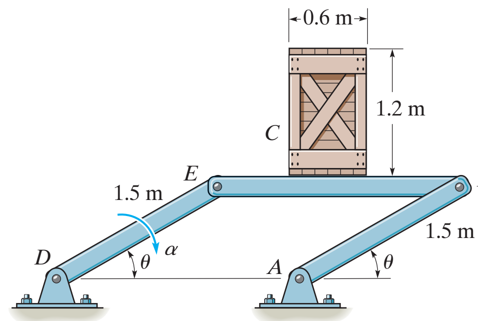
$a = 1.33 \text{ m/s}^2$ **Ans.**



Ans:
$T = 2.38 \text{ kN}$
$a = 1.33 \text{ m/s}^2$

17–53.

The 100-kg uniform crate C rests on the elevator floor where the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest at $\theta = 90°$, without causing the crate to slip. No tipping occurs.



SOLUTION

Equations of Motion. The crate undergoes curvilinear translation. At $\theta = 90°$, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. However; $(a_G)_t = \alpha r = \alpha(1.5)$. Assuming that the crate slides before it tips, then, $F_f = \mu_s N = 0.4\,N$.

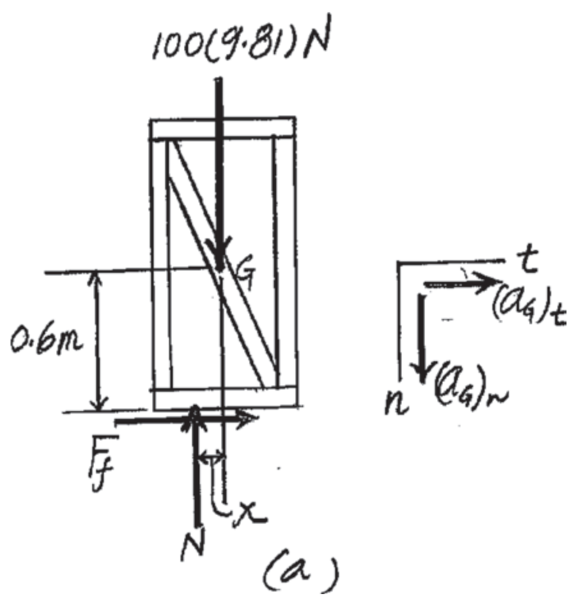
$\Sigma F_n = m(a_G)_n;\quad 100(9.81) - N = 100(0) \quad N = 981\text{ N}$

$\Sigma F_t = m(a_G)_t;\quad 0.4(981) = 100[\alpha(1.5)] \quad \alpha = 2.616\text{ rad/s}^2 = 2.62\text{ rad/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0;\quad 0.4(981)(0.6) - 981(x) = 0$

$x = 0.24\text{ m}$

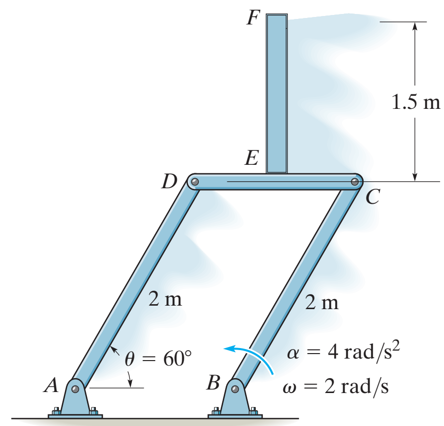
Since $x < 0.3$ m, the crate indeed slides before it tips, as assumed.



Ans:
$\alpha = 2.62\text{ rad/s}^2$

17–54.

The two uniform 4-kg bars DC and EF are fixed (welded) together at E. Determine the normal force N_E, shear force V_E, and moment M_E, which DC exerts on EF at E if at the instant $\theta = 60°$ BC has an angular velocity $\omega = 2$ rad/s and an angular acceleration $\alpha = 4 \text{ rad/s}^2$ as shown.



SOLUTION

Equations of Motion. The rod assembly undergoes curvilinear motion. Thus, $(a_G)_t = \alpha r = 4(2) = 8 \text{ m/s}^2$ and $(a_G)_n = \omega^2 r = (2^2)(2) = 8 \text{ m/s}^2$. Referring to the FBD and kinetic diagram of rod EF, Fig. a

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad V_E = 4(8)\cos 30° + 4(8)\cos 60°$

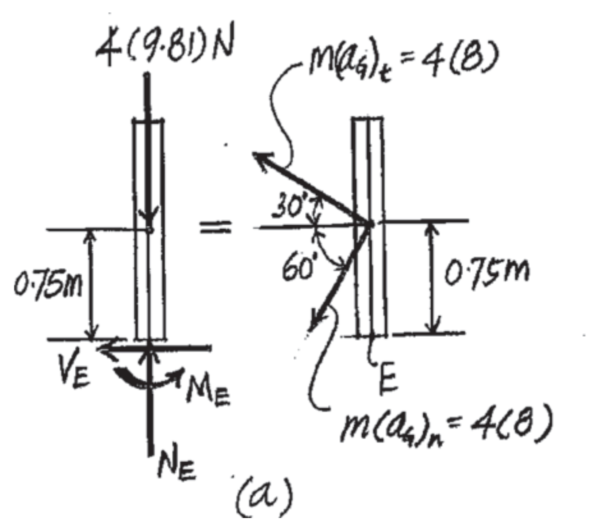
$= 43.71 \text{ N} = 43.7 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_E - 4(9.81) = 4(8)\sin 30° - 4(8)\sin 60°$

$N_E = 27.53 \text{ N} = 27.5 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = \Sigma(M_k)_E;\quad M_E = 4(8)\cos 30°(0.75) + 4(8)\cos 60°(0.75)$

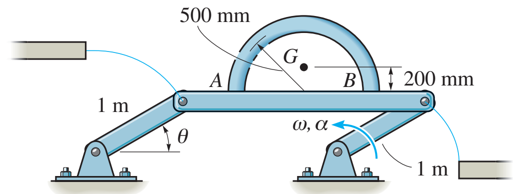
$= 32.78 \text{ N}\cdot\text{m} = 32.8 \text{ N}\cdot\text{m}$ **Ans.**



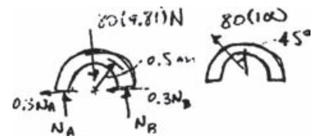
Ans:
$V_E = 43.7$ N
$N_E = 27.5$ N
$M_E = 32.8 \text{ N}\cdot\text{m}$

17–55.

The arched pipe has a mass of 80 kg and rests on the surface of the platform for which the coefficient of static friction is $\mu_s = 0.3$. Determine the greatest angular acceleration α of the platform, starting from rest when $\theta = 45°$, without causing the pipe to slip on the platform.



SOLUTION



$a_G = (a_G)_t = (1)(\alpha)$

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad N_B(1) - 80(9.81)(0.5) = 80(1\alpha)(\sin 45°)(0.2) + 80(1\alpha)(\cos 45°)(0.5)$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.3N_A + 0.3N_B = 80(1\alpha)\sin 45°$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B - 80(9.81) = 80(1\alpha)\cos 45°$

Solving,

$\alpha = 5.95 \text{ rad/s}^2$ **Ans.**

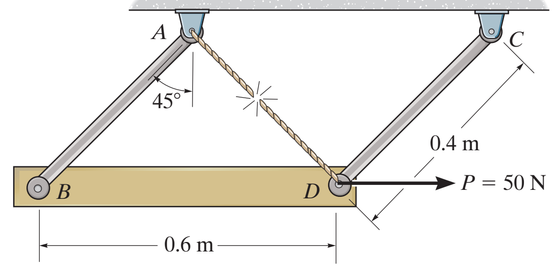
$N_A = 494 \text{ N}$

$N_B = 628 \text{ N}$

Ans:
$\alpha = 5.95 \text{ rad/s}^2$

***17–56.**

Determine the force developed in the links and the acceleration of the bar's mass center immediately after the cord fails. Neglect the mass of links AB and CD. The uniform bar has a mass of 20 kg.



SOLUTION

Equations of Motion: Since the bar is still at rest at the instant the cord fails, $v_G = 0$. Thus, $(a_G)_n = \dfrac{v_G^2}{r} = 0$. Referring to the free-body diagram of the bar, Fig. *a*,

$\Sigma F_n = m(a_G)_n;\qquad T_{AB} + T_{CD} - 20(9.81)\cos 45° + 50\cos 45° = 0$

$\Sigma F_t = m(a_G)_t;\qquad 20(9.81)\sin 45° + 50\sin 45° = 20(a_G)_t$

$+\Sigma M_G = 0;\qquad T_{CD}\cos 45°(0.3) - T_{AB}\cos 45°(0.3) = 0$

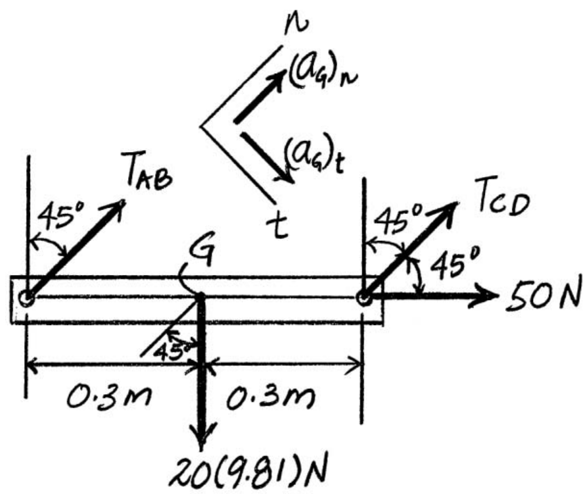
Solving,

$T_{AB} = T_{CD} = 51.68\text{ N} = 51.7\text{ N}$ **Ans.**

$(a_G)_t = 8.704\text{ m/s}^2$

Since $(a_G)_n = 0$, then

$a_G = (a_G)_t = 8.70\text{ m/s}^2 \searrow$ **Ans.**

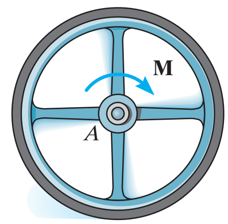


(a)

Ans:
$T_{AB} = T_{CD} = 51.7\text{ N}$
$a_G = (a_G)_t = 8.70\text{ m/s}^2 \searrow$

17–57.

The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

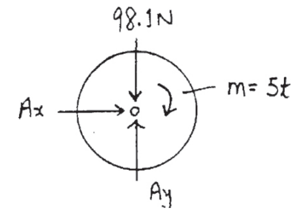


SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 10(9.81) = 0$

$\circlearrowright + \Sigma M_A = I_a \alpha; \qquad 5t = 10(0.2)^2\alpha$



$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\, dt = \frac{12.5}{2}(3)^2$$

$\omega = 56.2$ rad/s **Ans.**

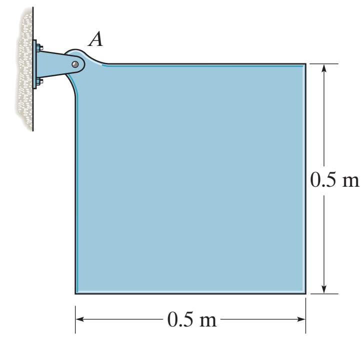
$A_x = 0$ **Ans.**

$A_y = 98.1$ N **Ans.**

Ans:
$\omega = 56.2$ rad/s
$A_x = 0$
$A_y = 98.1$ N

17–58.

The uniform 24-kg plate is released from rest at the position shown. Determine its initial angular acceleration and the horizontal and vertical reactions at the pin A.



SOLUTION

Equations of Motion. The mass moment of inertia of the plate about its center of gravity G is $I_G = \frac{1}{12}(24)(0.5^2 + 0.5^2) = 1.00 \text{ kg} \cdot \text{m}^2$. Since the plate is at rest initially $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Here $r_G = \sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2}$ m. Thus, $(a_G)_t = \alpha r_G = \alpha(0.25\sqrt{2})$. Referring to the FBD and kinetic diagram of the plate,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \quad -24(9.81)(0.25) = -24[\alpha(0.25\sqrt{2})](0.25\sqrt{2}) - 1.00\,\alpha$$

$$\alpha = 14.715 \text{ rad/s}^2 = 14.7 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where $I_A = \frac{1}{12}(24)(0.5^2 + 0.5^2) + 24(0.25\sqrt{2})^2 = 4.00 \text{ kg} \cdot \text{m}^2$:

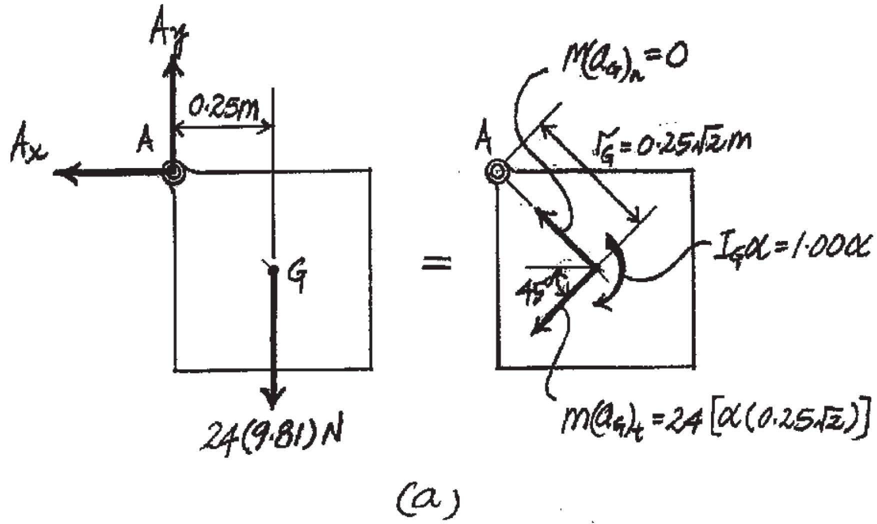
$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \quad -24(9.81)(0.25) = -4.00\,\alpha$$

$$\alpha = 14.715 \text{ rad/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad A_x = 24[14.715(0.25\sqrt{2})] \cos 45° = 88.29 \text{ N} = 88.3 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad A_y - 24(9.81) = -24[14.715(0.25\sqrt{2})] \sin 45°$$

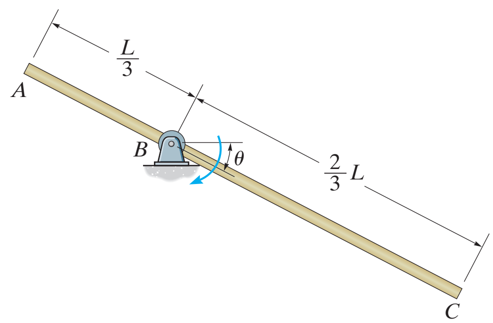
$$A_y = 147.15 \text{ N} = 147 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$\alpha = 14.7 \text{ rad/s}^2$
$A_x = 88.3$ N
$A_y = 147$ N

17–59.

The uniform slender rod has a mass m. If it is released from rest when $\theta = 0°$, determine the magnitude of the reactive force exerted on it by pin B when $\theta = 90°$.



SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point B, $(a_G)_t = \alpha r_G = \alpha\left(\frac{L}{6}\right)$ and $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{6}\right)$. The mass moment of inertia of the rod about its G is $I_G = \frac{1}{12}mL^2$. Writing the moment equation of motion about point B,

$$+\Sigma M_B = \Sigma(M_k)_B; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -m\left[\alpha\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

This equation can also be obtained by applying $\Sigma M_B = I_B\alpha$, where $I_B = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$. Thus,

$$+\Sigma M_B = I_B\alpha; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

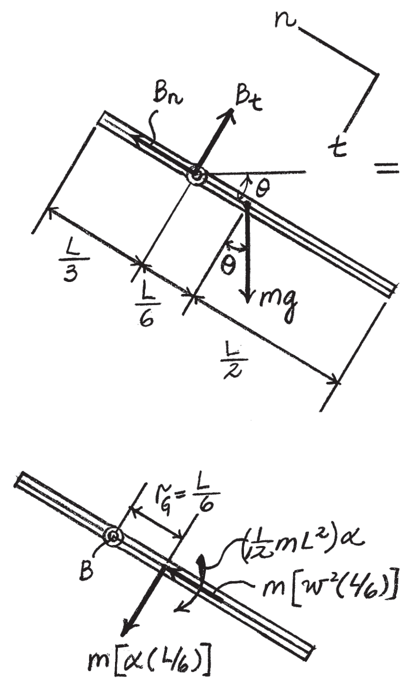
Using this result and writing the force equation of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \quad mg\cos\theta - B_t = m\left[\left(\frac{3g}{2L}\cos\theta\right)\left(\frac{L}{6}\right)\right]$$

$$B_t = \frac{3}{4}mg\cos\theta \qquad \textbf{(1)}$$

$$\Sigma F_n = m(a_G)_n; \quad B_n - mg\sin\theta = m\left[\omega^2\left(\frac{L}{6}\right)\right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg\sin\theta \qquad \textbf{(2)}$$



(a)

Kinematics: The angular velocity of the rod can be determined by integrating

$$\int \omega\, d\omega = \int \alpha\, d\theta$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{3g}{2L}\cos\theta\, d\theta$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

When $\theta = 90°$, $\omega = \sqrt{\frac{3g}{L}}$. Substituting this result and $\theta = 90°$ into Eqs. (1) and (2),

$$B_t = \frac{3}{4}mg\cos 90° = 0$$

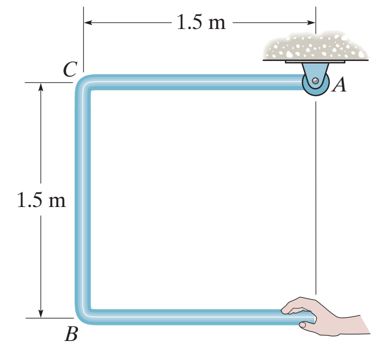
$$B_n = \frac{1}{6}m\left(\frac{3g}{L}\right)(L) + mg\sin 90° = \frac{3}{2}mg$$

$$F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg \qquad \textbf{Ans.}$$

Ans:
$F_A = \frac{3}{2}mg$

***17–60.**

The bent rod has a mass of 2 kg/m. If it is released from rest in the position shown, determine its initial angular acceleration and the horizontal and vertical components of reaction at A.



SOLUTION

Equations of Motion. Referring to Fig. a, the location of center of gravity G of the bent rod is at

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{2[0.75(1.5)(2)] + 1.5(2)(1.5)}{3(1.5)(2)} = 1.00 \text{ m}$$

$$\bar{y} = \frac{1.5}{2} = 0.75 \text{ m}$$

The mass moment of inertia of the bent rod about its center of gravity is $I_G = 2\left[\frac{1}{12}(3)(1.5^2) + 3(0.25^2 + 0.75^2)\right] + \left[\frac{1}{12}(3)(1.5^2) + 3(0.5^2)\right] = 6.1875 \text{ kg}\cdot\text{m}^2$. Here, $r_G = \sqrt{1.00^2 + 0.75^2} = 1.25$ m. Since the bent rod is at rest initially, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.25)$. Referring to the FBD and kinetic diagram of the plate,

$\circlearrowleft + \Sigma M_A = (M_k)_A;\qquad 9(9.81)(1) = 9[\alpha(1.25)](1.25) + 6.1875\,\alpha$

$$\alpha = 4.36 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where

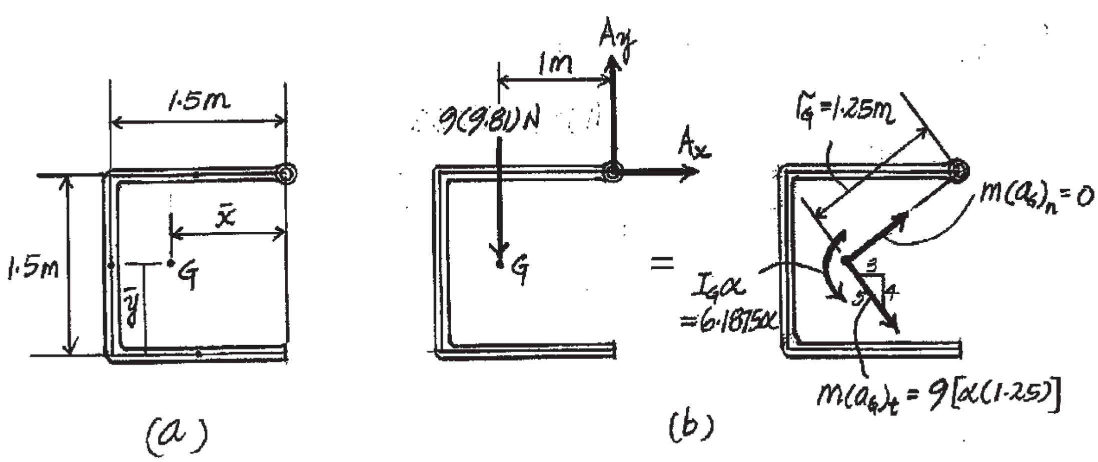
$$I_A = \frac{1}{12}(3)(1.5^2) + 3(0.75^2) + \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2) + \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2) = 20.25 \text{ kg}\cdot\text{m}^2:$$

$\circlearrowleft + \Sigma M_A = I_A\alpha,\qquad 9(9.81)(1) = 20.25\,\alpha \qquad \alpha = 4.36 \text{ rad/s}^2$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad A_x = 9[4.36(1.25)]\left(\frac{3}{5}\right) = 29.43 \text{ N} = 29.4 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad A_y - 9(9.81) = -9[4.36(1.25)]\left(\frac{4}{5}\right)$

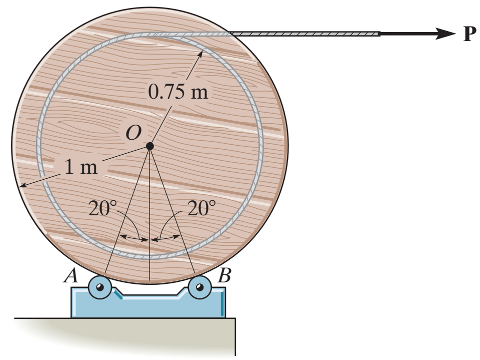
$A_y = 49.05 \text{ N} = 49.1 \text{ N}$ **Ans.**



Ans:
$\alpha = 4.36 \text{ rad/s}^2 \circlearrowleft$
$A_x = 29.4$ N
$A_y = 49.1$ N

17–61.

If a horizontal force of $P = 100$ N is applied to the 300-kg reel of cable, determine its initial angular acceleration. The reel rests on rollers at A and B and has a radius of gyration of $k_O = 0.6$ m.



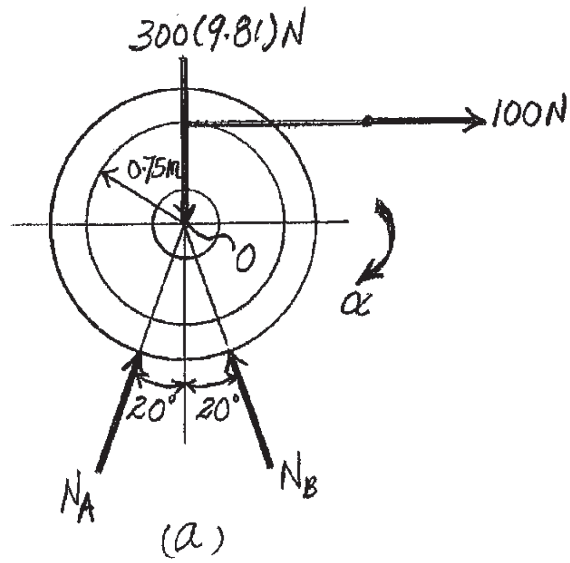
SOLUTION

Equations of Motions. The mass moment of inertia of the reel about O is $I_O = Mk_O^2 = 300(0.6^2) = 108\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. a,

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -100(0.75) = 108(-\alpha)$$

$$\alpha = 0.6944\ \text{rad/s}^2$$

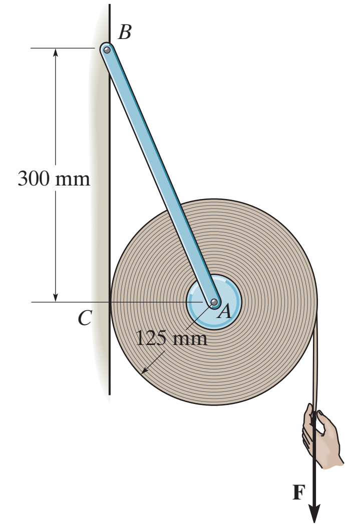
$$= 0.694\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\alpha = 0.694\ \text{rad/s}^2$

17–62.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad N_C - T_{AB}\cos 67.38^\circ = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - 30 = 0$

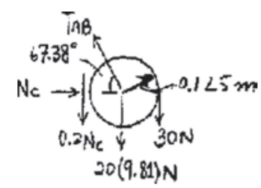
$\zeta + \Sigma M_A = I_A\alpha; \qquad -0.2N_C(0.125) + 30(0.125) = 20(0.09)^2\alpha$

Solving:

$N_C = 103$ N

$T_{AB} = 267$ N

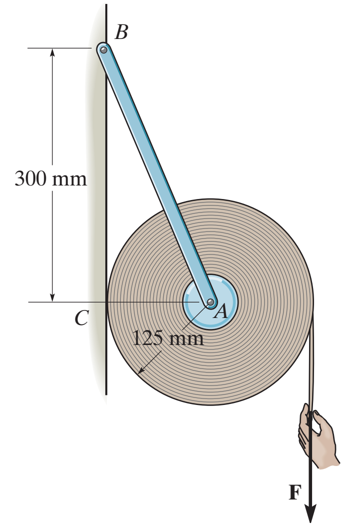
$\alpha = 7.28 \text{ rad/s}^2$ **Ans.**



Ans:
$\alpha = 7.28 \text{ rad/s}^2$

17–63.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$, determine the constant vertical force F that must be applied to the roll to pull off 1 m of paper in $t = 3$ s starting from rest. Neglect the mass of paper that is removed.



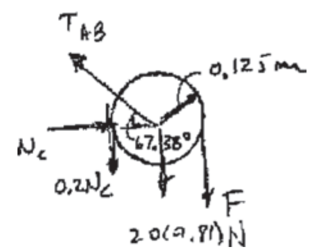
SOLUTION

$$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_C t^2$$

$$1 = 0 + 0 + \frac{1}{2}a_C(3)^2$$

$$a_C = 0.222\ \text{m/s}^2$$

$$\alpha = \frac{a_C}{0.125} = 1.778\ \text{rad/s}^2$$



$$\xrightarrow{+}\ \Sigma F_x = m(a_G)_x; \qquad N_C - T_{AB}\cos 67.38^\circ = 0$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - F = 0$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad -0.2N_C(0.125) + F(0.125) = 20(0.09)^2(1.778)$$

Solving:

$$N_C = 99.3\text{ N}$$

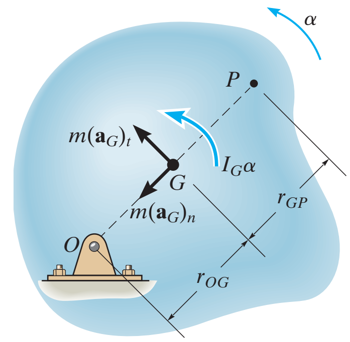
$$T_{AB} = 258\text{ N}$$

$$F = 22.1\text{ N} \qquad \textbf{Ans.}$$

Ans:
$F = 22.1$ N

***17–64.**

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



SOLUTION

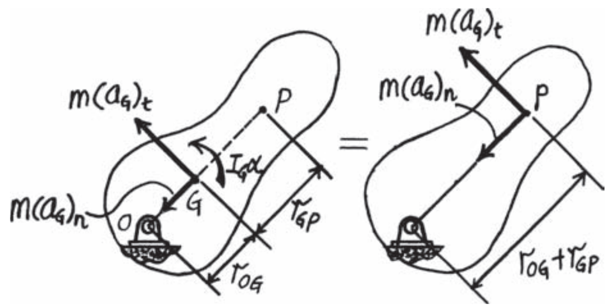
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$

However,

$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

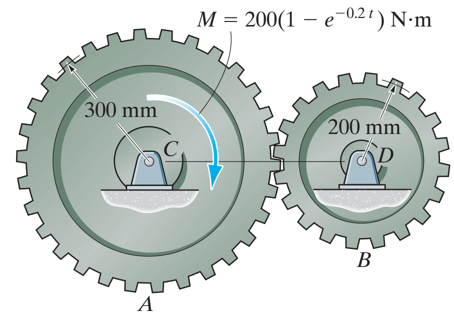
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



Ans:
$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t(r_{OG} + r_{GP})$

17–65.



Gears A and B have a mass of 50 kg and 15 kg, respectively. Their radii of gyration about their respective centers of mass are $k_C = 250$ mm and $k_D = 150$ mm. If a torque of $M = 200(1 - e^{-0.2t})$ N · m, where t is in seconds, is applied to gear A, determine the angular velocity of both gears when $t = 3$ s, starting from rest.

SOLUTION

Equations of Motion: Since gear B is in mesh with gear A, $\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{0.3}{0.2}\right)\alpha_A = 1.5\alpha_A$. The mass moment of inertia of gears A and B about their respective centers are $I_C = m_A k_C^2 = 50(0.25^2) = 3.125\ \text{kg}\cdot\text{m}^2$ and $I_D = m_B k_D^2 = 15(0.15^2) = 0.3375\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about the gears' center using the free-body diagrams of gears A and B, Figs. a and b,

$$\circlearrowleft + \Sigma M_C = I_C\alpha_A; \quad F(0.3) - 200(1 - e^{-0.2t}) = -3.125\alpha_A \qquad \textbf{(1)}$$

and

$$\circlearrowleft + \Sigma M_D = I_D\alpha_B; \quad F(0.2) = 0.3375(1.5\alpha_A) \qquad \textbf{(2)}$$

Eliminating F from Eqs. (1) and (2) yields

$$\alpha_A = 51.49(1 - e^{-0.2t})\ \text{rad/s}^2$$

Kinematics: The angular velocity of gear A can be determined by integration.

$$\int d\omega_A = \int \alpha_A\, dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 51.49(1 - e^{-0.2t})dt$$

$$\omega_A = 51.49(t + 5e^{-0.2t} - 5)\ \text{rad/s}$$

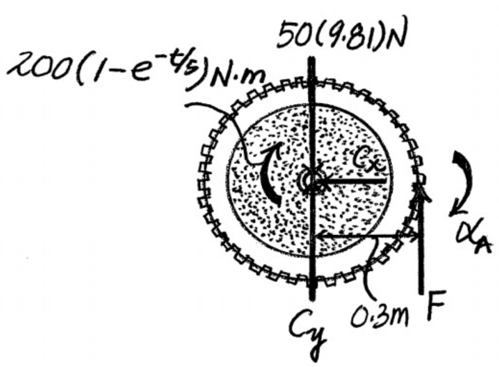
When $t = 3$ s,

$$\omega_A = 51.49(3 + 5e^{-0.2(3)} - 5) = 38.31\ \text{rad/s} = 38.3\ \text{rad/s} \qquad \textbf{Ans.}$$

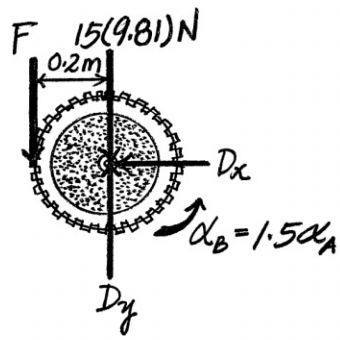
Then

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{0.3}{0.2}\right)(38.31)$$

$$= 57.47\ \text{rad/s} = 57.5\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

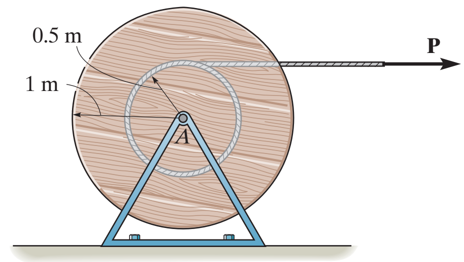


(b)

Ans:
$\omega_A = 38.3$ rad/s
$\omega_B = 57.5$ rad/s

17–66.

The reel of cable has a mass of 400 kg and a radius of gyration of $k_A = 0.75$ m. Determine its angular velocity when $t = 2$ s, starting from rest, if the force $\mathbf{P} = (20t^2 + 80)$ N, when t is in seconds. Neglect the mass of the unwound cable, and assume it is always at a radius of 0.5 m.



SOLUTION

Equations of Motion. The mass moment of inertia of the reel about A is $I_A = Mk_A^2 = 400(0.75^2) = 225\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. *a*

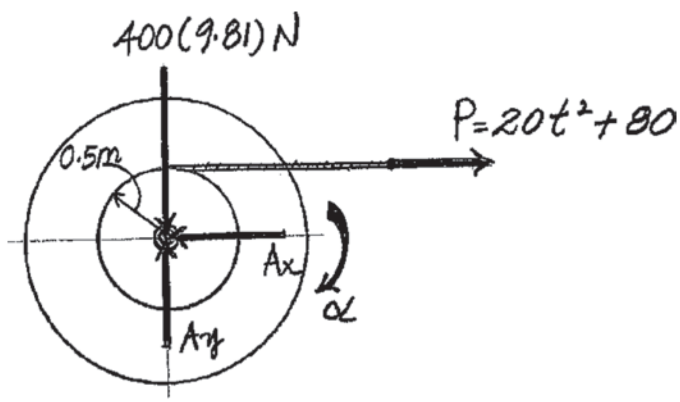
$$\circlearrowleft +\Sigma M_A = I_A\alpha; \quad -(20t^2 + 80)(0.5) = 225(-\alpha)$$

$$\alpha = \frac{2}{45}(t^2 + 4)\ \text{rad/s}^2$$

Kinematics. Using the result of α, integrate $d\omega = \alpha dt$, with the initial condition $\omega = 0$ at $t = 0$,

$$\int_0^\omega d\omega = \int_0^{2s} \frac{2}{45}(t^2 + 4)\, dt$$

$$\omega = 0.4741\ \text{rad/s} = 0.474\ \text{rad/s} \qquad \textbf{Ans.}$$

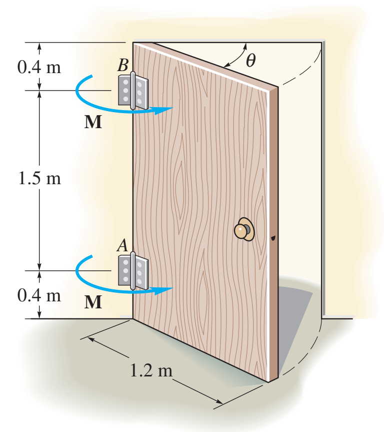


(a)

Ans:
$\omega = 0.474$ rad/s

17–67.

The door will close automatically using torsional springs mounted on the hinges. Each spring has a stiffness $k = 50\ \text{N}\cdot\text{m/rad}$ so that the torque on each hinge is $M = (50\theta)\ \text{N}\cdot\text{m}$, where θ is measured in radians. If the door is released from rest when it is open at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

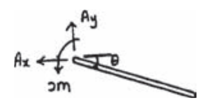
$$I_{AB} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(70)(1.2)^2 + 70(0.6)^2 = 33.6\ \text{kg}\cdot\text{m}^2$$

$$\Sigma M_{AB} = I_{AB}\,\alpha; \qquad 2(50\theta) = -33.6(\alpha) \qquad \alpha = -2.9762\theta$$

$$\omega d\omega = \alpha d\theta$$

$$\int_0^\omega \omega d\omega = -\int_{\frac{\pi}{2}}^{0} 2.9762\theta d\theta$$

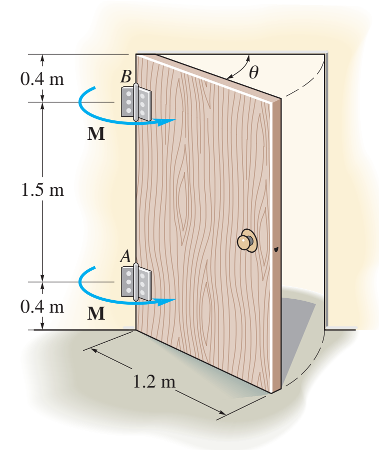
$$\omega = 2.71\ \text{rad/s}$$ **Ans.**



Ans:
$\omega = 2.71\ \text{rad/s}$

***17–68.**

The door will close automatically using torsional springs mounted on the hinges. If the torque on each hinge is $M = k\theta$, where θ is measured in radians, determine the required torsional stiffness k so that the door will close ($\theta = 0°$) with an angular velocity $\omega = 2$ rad/s when it is released from rest at $\theta = 90°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

$$\Sigma M_A = I_A\alpha; \qquad 2M = -\left[\frac{1}{12}(70)(1.2)^2 + 70(0.6)^2\right](\alpha)$$

$$M = -16.8\alpha$$

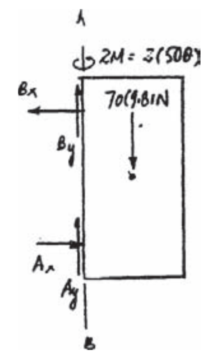
$$k\theta = -16.8\alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-k\int_{\frac{\pi}{2}}^{0} \theta d\theta = 16.8\int_{0}^{2} \omega\, d\omega$$

$$\frac{k}{2}\left(\frac{\pi}{2}\right)^2 = \frac{16.8}{2}(2)^2$$

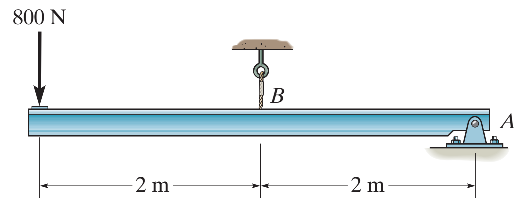
$$k = 27.2\ \text{N}\cdot\text{m/rad} \qquad \textbf{Ans.}$$



Ans:
$k = 27.2\ \text{N}\cdot\text{m/rad}$

17–69.

If the cord at B suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin A, and the angular acceleration of the 120-kg beam. Treat the beam as a uniform slender rod.



SOLUTION

Equations of Motion. The mass moment of inertia of the beam about A is $I_A = \frac{1}{12}(120)(4^2) + 120(2^2) = 640\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(2) = 2\alpha$. Referring to the FBD of the beam, Fig. *a*

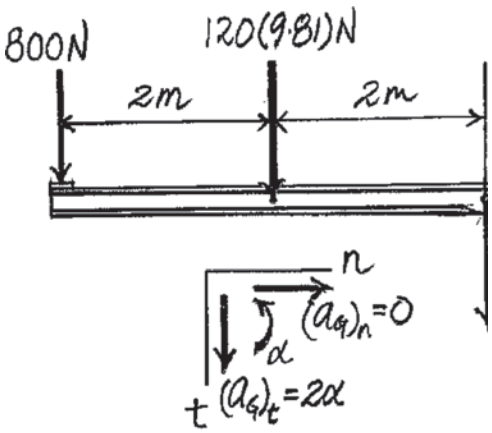
$\zeta + \Sigma M_A = I_A\alpha;\qquad 800(4) + 120(9.81)(2) = 640\,\alpha$

$\alpha = 8.67875\ \text{rad/s}^2 = 8.68\ \text{rad/s}^2$ **Ans.**

$\Sigma F_n = m(a_G)_n;\qquad A_n = 0$ **Ans.**

$\Sigma F_t = m(a_G)_t;\qquad 800 + 120(9.81) + A_t = 120[2(8.67875)]$

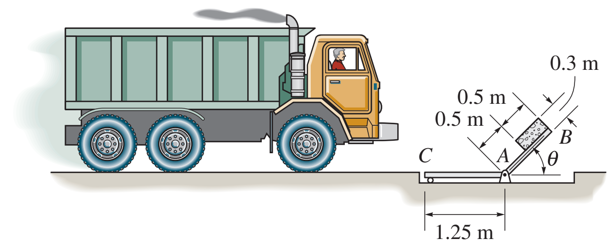
$A_t = 105.7\ \text{N} = 106\ \text{N}$ **Ans.**



Ans:
$\alpha = 8.68\ \text{rad/s}^2$
$A_n = 0$
$A_t = 106\ \text{N}$

17–70.

The device acts as a pop-up barrier to prevent the passage of a vehicle. It consists of a 100-kg steel plate AC and a 200-kg counterweight solid concrete block located as shown. Determine the moment of inertia of the plate and block about the hinged axis through A. Neglect the mass of the supporting arms AB. Also, determine the initial angular acceleration of the assembly when it is released from rest at $\theta = 45°$.



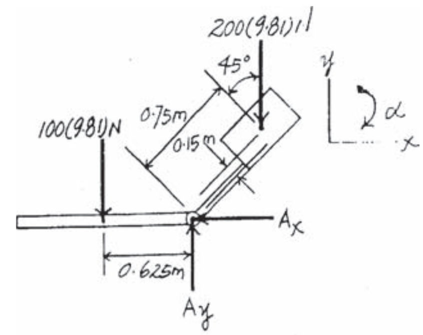
SOLUTION

Mass Moment of Inertia:

$$I_A = \frac{1}{12}(100)\left(1.25^2\right) + 100\left(0.625^2\right)$$

$$+ \frac{1}{12}(200)\left(0.5^2 + 0.3^2\right) + 200\left(\sqrt{0.75^2 + 0.15^2}\right)^2$$

$$= 174.75 \text{ kg}\cdot\text{m}^2 = 175 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



Equation of Motion: Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_A = I_A \alpha; \qquad 100(9.81)(0.625) + 200(9.81)\sin 45°(0.15)$$

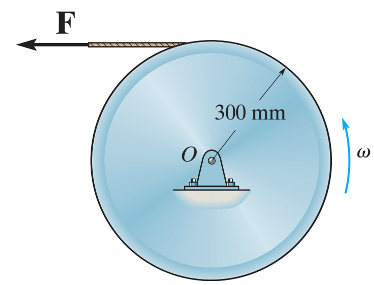
$$-200(9.81)\cos 45°(0.75) = -174.75\alpha$$

$$\alpha = 1.25 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$I_A = 175 \text{ kg}\cdot\text{m}^2$
$\alpha = 1.25 \text{ rad/s}^2$

17–71.

A cord is wrapped around the outer surface of the 8-kg disk. If a force of $F = (\frac{1}{4}\theta^2)$ N, where θ is in radians, is applied to the cord, determine the disk's angular acceleration when it has turned 5 revolutions. The disk has an initial angular velocity of $\omega_0 = 1$ rad/s.



SOLUTION

Equations of Motion. The mass moment inertia of the disk about O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(8)(0.3^2) = 0.36\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the disk, Fig. a,

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad \left(\frac{1}{4}\theta^2\right)(0.3) = 0.36\,\alpha$$

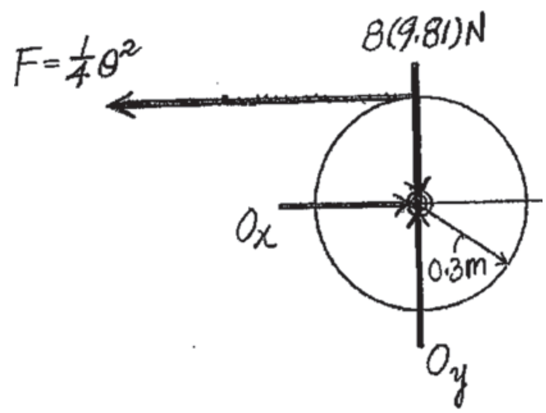
$$\alpha = (0.2083\,\theta^2)\ \text{rad/s}^2$$

Kinematics. Using the result of α, integrate $\omega d\omega = \alpha d\theta$ with the initial condition $\omega = 0$ when $\theta = 0$,

$$\int_1^{\omega} \omega d\omega = \int_0^{5(2\pi)} 0.2083\,\theta^2\,d\theta$$

$$\left(\frac{1}{2}\right)(\omega_2 - 1) = 0.06944\,\theta^3\Big|_0^{5(2\pi)}$$

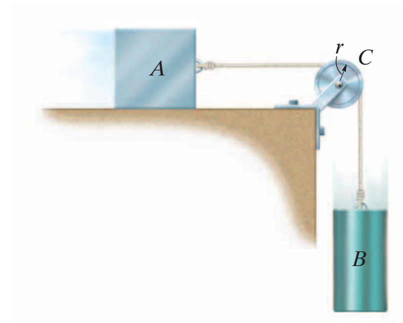
$$\omega = 65.63\ \text{rad/s} = 65.6\ \text{rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 65.6$ rad/s

***17–72.**

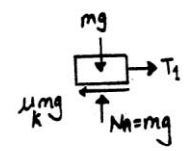
Block A has a mass m and rests on a surface having a coefficient of kinetic friction μ_k. The cord attached to A passes over a pulley at C and is attached to a block B having a mass $2m$. If B is released, determine the acceleration of A. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius r and mass $\frac{1}{4}m$. Neglect the mass of the cord.



SOLUTION

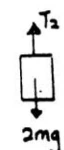
Block A:

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad T_1 - \mu_k mg = ma \qquad \textbf{(1)}$$



Block B:

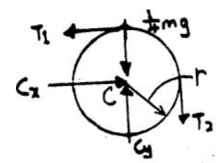
$$+\downarrow \Sigma F_y = ma_y; \qquad 2mg - T_2 = 2ma \qquad \textbf{(2)}$$



Pulley C:

$$\circlearrowleft + \Sigma M_C = I_G\alpha; \qquad T_2 r - T_1 r = \left[\frac{1}{2}\left(\frac{1}{4}m\right)r^2\right]\left(\frac{a}{r}\right)$$

$$T_2 - T_1 = \frac{1}{8}ma \qquad \textbf{(3)}$$



Substituting Eqs. (1) and (2) into (3),

$$2mg - 2ma - (ma + \mu_k mg) = \frac{1}{8}ma \qquad (2 - \mu_k)g = \frac{25}{8}a$$

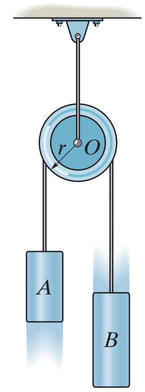
$$2mg - \mu_k mg = \frac{1}{8}ma + 3ma \qquad a = \frac{8}{25}(2 - \mu_k)g \qquad \textbf{Ans.}$$

Ans:

$$a = \frac{8}{25}(2 - \mu_k)g$$

17–73.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point O is

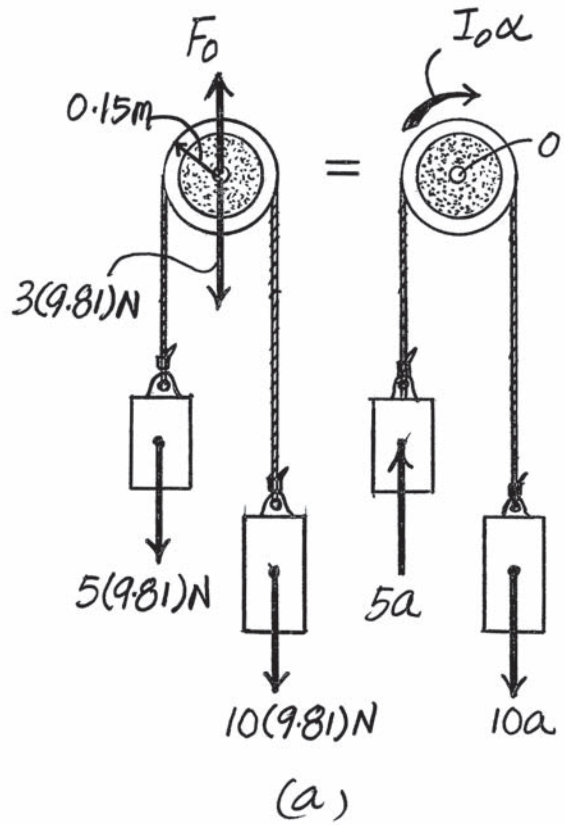
$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375 \text{ kg} \cdot \text{m}^2$$

Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a,

$$\circlearrowleft + \Sigma M_o = \Sigma(M_k)_o; \qquad 5(9.81)(0.15) - 10(9.81)(0.15)$$

$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

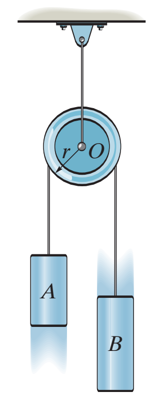
$$a = 2.973 \text{ m/s}^2 = 2.97 \text{ m/s}^2 \qquad \textbf{Ans.}$$



Ans:
$a = 2.97 \text{ m/s}^2$

17–74.

The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



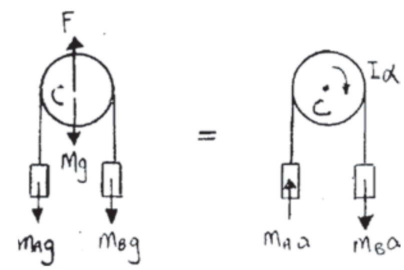
SOLUTION

$$a = \alpha r$$

$$\circlearrowleft +\Sigma M_C = \Sigma(M_k)_C; \qquad m_B g(r) - m_A g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + m_B r^2\alpha + m_A r^2\alpha$$

$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)} \qquad \textbf{Ans.}$$

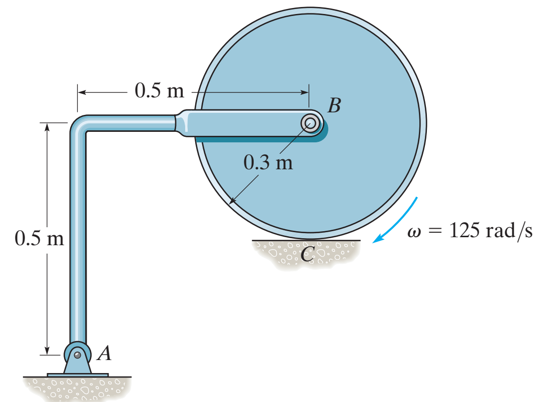


Ans:

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)}$$

17–75.

The 30-kg disk is originally spinning at $\omega = 125$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of force which the member AB exerts on the pin at A during this time? Neglect the mass of AB.



SOLUTION

Equations of Motion. The mass moment of inertia of the disk about B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, $F_f = \mu_C N_C = 0.5\ N_C$. Referring to the FBD of the disk, Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad 0.5N_C - F_{AB}\cos 45^\circ = 30(0)$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_C - F_{AB}\sin 45^\circ - 30(9.81) = 30(0)$ **(2)**

Solving Eqs. (1) and (2),

$N_C = 588.6\ \text{N} \qquad F_{AB} = 416.20\ \text{N}$

Subsequently,

$\circlearrowleft + \Sigma M_B = I_B\alpha;\qquad 0.5(588.6)(0.3) = 1.35\alpha$

$\alpha = 65.4\ \text{rad/s}^2 \circlearrowright$

Referring to the FBD of pin A, Fig. b,

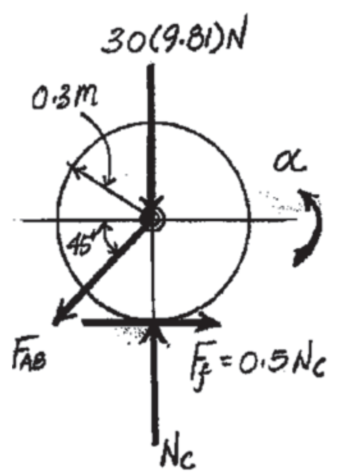
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 416.20\cos 45^\circ - A_x = 0 \qquad A_x = 294.3\ \text{N} = 294\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 416.20\sin 45^\circ - A_y = 0 \qquad A_y = 294.3\ \text{N} = 294\ \text{N}$ **Ans.**

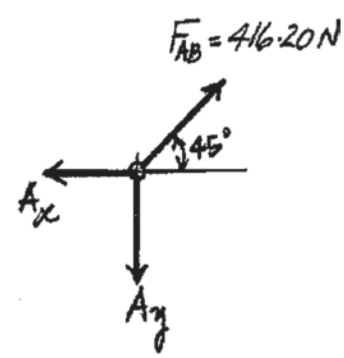
Kinematic. Using the result of α,

$+\circlearrowright\ \omega = \omega_0 + \alpha t;\qquad 0 = 125 + (-65.4)t$

$t = 1.911\ \text{s} = 1.91\ \text{s}$ **Ans.**



(a)

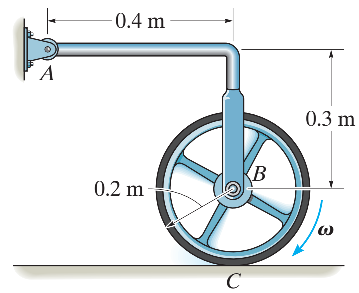


(b)

Ans:
$A_x = 294$ N
$A_y = 294$ N
$t = 1.91$ s

***17–76.**

The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.



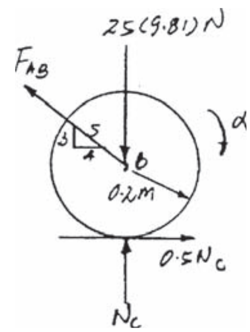
SOLUTION

$I_B = mk_B^2 = 25(0.15)^2 = 0.5625\ \text{kg}\cdot\text{m}^2$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad \left(\frac{3}{5}\right) F_{AB} + N_C - 25(9.81) = 0$ **(1)**

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.5N_C - \left(\frac{4}{5}\right) F_{AB} = 0$ **(2)**

$\circlearrowleft + \Sigma M_B = I_B\alpha; \quad 0.5N_C(0.2) = 0.5625(-\alpha)$ **(3)**



Solvings Eqs. (1), (2) and (3) yields:

$F_{AB} = 111.48\ \text{N} \qquad N_C = 178.4\ \text{N}$

$\alpha = -31.71\ \text{rad/s}^2$

$A_x = \frac{4}{5} F_{AB} = 0.8(111.48) = 89.2\ \text{N}$ **Ans.**

$A_y = \frac{3}{5} F_{AB} = 0.6(111.48) = 66.9\ \text{N}$ **Ans.**

$\omega = \omega_0 + \alpha_c t$

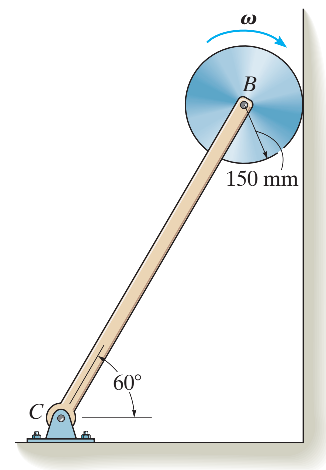
$0 = 40 + (-31.71)\,t$

$t = 1.26\ \text{s}$ **Ans.**

Ans:
$A_x = 89.2$ N
$A_y = 66.9$ N
$t = 1.25$ s

17–77.

The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad F_{CB}\sin 30° - N_A = 0$

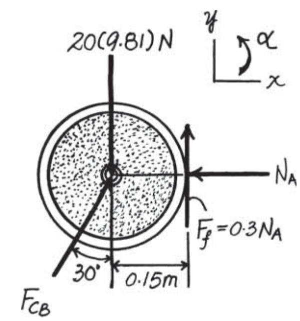
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad F_{CB}\cos 30° - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha;\qquad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$N_A = 96.6\text{ N}$

$F_{CB} = 193\text{ N}$ **Ans.**

$\alpha = 19.3\text{ rad/s}^2$



$\circlearrowright + \qquad \omega = \omega_0 + \alpha_c\,t$

$0 = 60 + (-19.3)\,t$

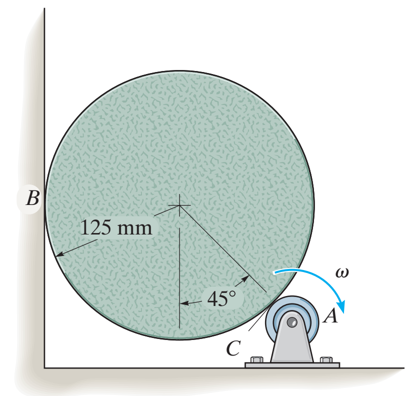
$t = 3.11\text{ s}$ **Ans.**

Ans:

$F_{CB} = 193\text{ N}$
$t = 3.11\text{ s}$

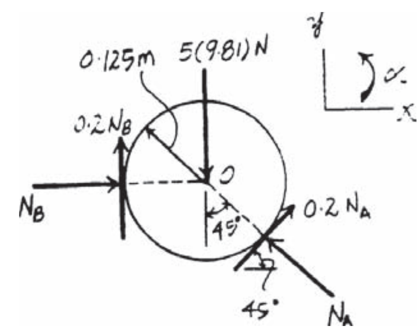
17–78.

The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



SOLUTION

Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad N_B + 0.2N_A \cos 45° - N_A \sin 45° = 0$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 0.2N_B + 0.2N_A \sin 45° + N_A \cos 45° - 5(9.81) = 0$ **(2)**

$\circlearrowleft + \Sigma M_O = I_O\,\alpha;\qquad 0.2N_A\,(0.125) - 0.2N_B\,(0.125) = 0.0390625\alpha$ **(3)**

Solving Eqs. (1), (2), and (3) yields;

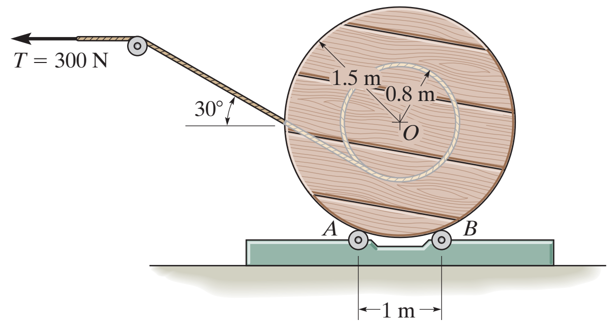
$$N_A = 51.01\ \text{N} \qquad N_B = 28.85\ \text{N}$$

$$\alpha = 14.2\ \text{rad/s}^2$$ **Ans.**

Ans:
$\alpha = 14.2\ \text{rad/s}^2$

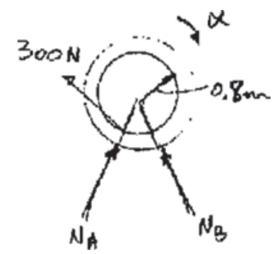
17–79.

Cable is unwound from a spool supported on small rollers at A and B by exerting a force $T = 300$ N on the cable. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



SOLUTION

$I_O = mk_O^2 = 600(1.2)^2 = 864 \text{ kg} \cdot \text{m}^2$



$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 300(0.8) = 864(\alpha) \qquad \alpha = 0.2778 \text{ rad/s}^2$

The angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad.

$$\theta = \theta_0 + \omega_0 r + \frac{1}{2}\alpha_c t^2$$

$$6.25 = 0 + 0 + \frac{1}{2}(0.27778)t^2$$

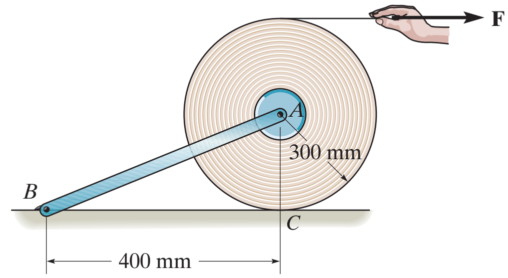
$$t = 6.71 \text{ s}$$

Ans.

Ans:
$t = 6.71$ s

***17–80.**

The 20-kg roll of paper has a radius of gyration $k_A = 120$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. The roll rests on the floor, for which the coefficient of kinetic friction is $\mu_k = 0.2$. If a horizontal force $F = 60$ N is applied to the end of the paper, determine the initial angular acceleration of the roll as the paper unrolls.



SOLUTION

Equations of Motion. The mass moment of inertia of the paper roll about A is $I_A = mk_A^2 = 20(0.12^2) = 0.288\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, the friction is $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the paper roll, Fig. a

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.2\text{ N} - F_{AB}\left(\frac{4}{5}\right) + 60 = 20(0) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \text{N} - F_{AB}\left(\frac{3}{5}\right) - 20(9.81) = 20(0) \qquad \textbf{(2)}$$

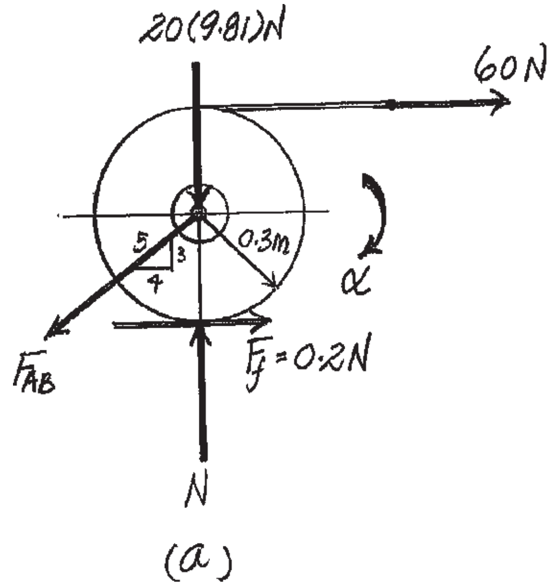
Solving Eqs. (1) and (2)

$$F_{AB} = 145.94\text{ N} \qquad \text{N} = 283.76\text{ N}$$

Subsequently

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad 0.2(283.76)(0.3) - 60(0.3) = 0.288(-\alpha)$$

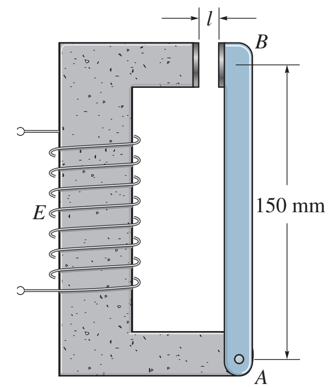
$$\alpha = 3.3824\text{ rad/s}^2 = 3.38\text{ rad/s}^2$$ **Ans.**



Ans:
$\alpha = 3.38\text{ rad/s}^2$

17–81.

The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.



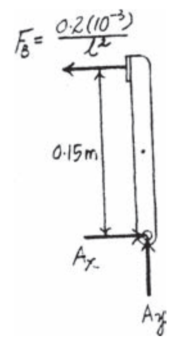
SOLUTION

Equation of Motion: The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)(0.15^2) + 0.2(0.075^2) = 1.50(10^{-3})\text{kg}\cdot\text{m}^2$

Applying Eq. 17–16, we have

$$\circlearrowleft +\Sigma M_A = I_A\alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50(10^{-3})\,\alpha$$

$$\alpha = \frac{0.02}{l^2}$$



Kinematic: From the geometry, $l = 0.02 - 0.15\theta$. Then $dl = -0.15\,d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

$$v\,dv = -0.15\,\alpha\,dl$$

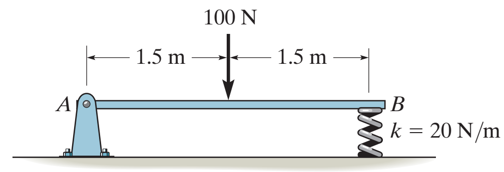
$$\int_0^v v\,dv = \int_{0.02\text{ m}}^{0.01\text{ m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548\text{ m/s}$$ **Ans.**

Ans:
$v = 0.548$ m/s

17–82.

The 4-kg slender rod is initially supported horizontally by a spring at B and pin at A. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the 100-N force is applied.



SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(4)(3^2) + 4(1.5^2) = 12.0\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.5)$. The force developed in the spring before the application of the 100 N force is $F_{sp} = \frac{4(9.81)\ \text{N}}{2} = 19.62$ N. Referring to the FBD of the rod, Fig. a,

$$\circlearrowleft + M_A = I_A\alpha; \qquad 19.62(3) - 100(1.5) - 4(9.81)(1.5) = 12.0(-\alpha)$$

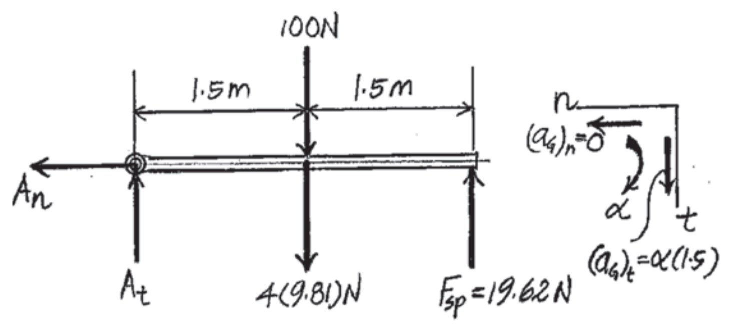
$$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then

$$(a_G)_t = 12.5(1.5) = 18.75\ \text{m/s}^2 \downarrow$$

Since $(a_G)_n = 0$. Then

$$a_G = (a_G)_t = 18.75\ \text{m/s}^2 \downarrow \qquad \textbf{Ans.}$$

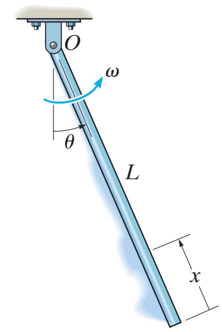


(a)

Ans:
$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright$
$a_G = 18.75\ \text{m/s}^2 \downarrow$

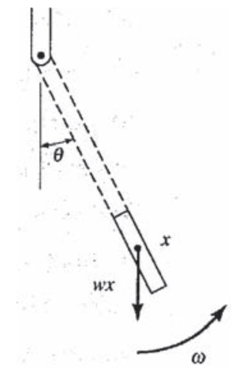
17–83.

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.



SOLUTION

$$a = \omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow}$$



Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow} = N\overset{\theta}{\nwarrow} + S\angle\theta + wx\downarrow \qquad \textbf{(1)}$$

Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

$$O = M - \frac{1}{2}Sx \qquad \textbf{(2)}$$

Solving (1) and (2),

$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right] \qquad \textbf{Ans.}$$

$$V = wx\sin\theta \qquad \textbf{Ans.}$$

$$M = \frac{1}{2}wx^2\sin\theta \qquad \textbf{Ans.}$$

Ans:

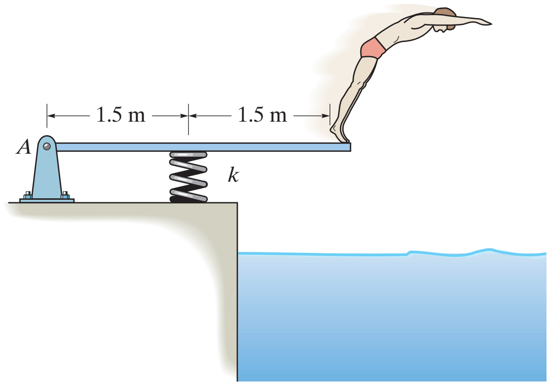
$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right]$$

$$V = wx\sin\theta$$

$$M = \frac{1}{2}wx^2\sin\theta$$

***17–84.**

Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



SOLUTION

$$\zeta + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

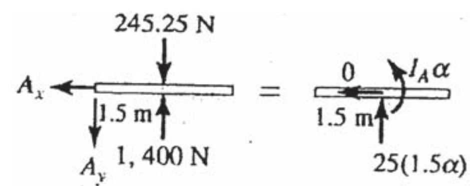
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \qquad A_x = 0$$

Solving,

$A_x = 0$ **Ans.**

$A_y = 289$ N **Ans.**

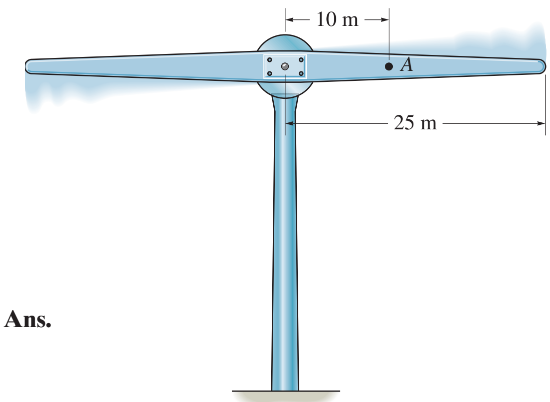
$\alpha = 23.1$ rad/s^2 **Ans.**



Ans:
$A_x = 0$
$A_y = 289$ N
$\alpha = 23.1$ rad/s^2

17–85.

The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



SOLUTION

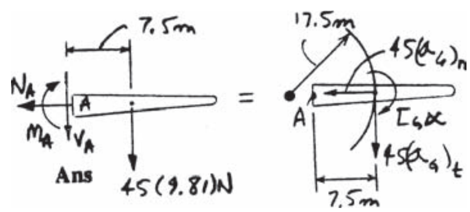
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad N_A = 45(15)^2(17.5) = 177\text{kN}$ **Ans.**

$+\downarrow \Sigma F_t = m(a_G)_t; \qquad V_A + 45(9.81) = 45(8)(17.5)$

$V_A = 5.86 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

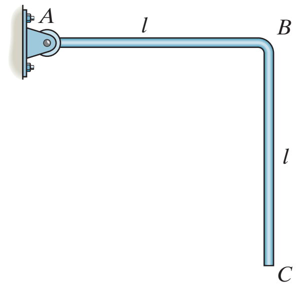
$M_A = 50.7 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$N_A = 177 \text{ kN}$
$V_A = 5.86 \text{ kN}$
$M_A = 50.7 \text{ kN} \cdot \text{m}$

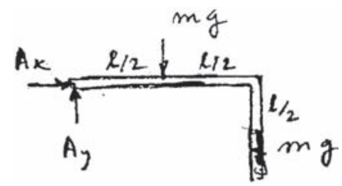
17–86.

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.



SOLUTION

Assembly:

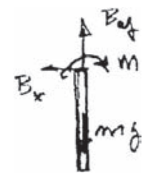


$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$$

$$= 1.667\ ml^2$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$$

$$\alpha = \frac{0.9\ g}{l}$$

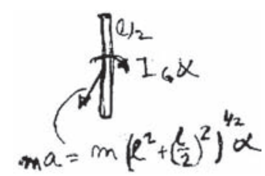


Segment BC:

$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \qquad M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\alpha(\frac{l/2}{l^2 + (\frac{l}{2})^2})(\frac{l}{2})$$

$$M = \frac{1}{3}ml^2\,\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$$

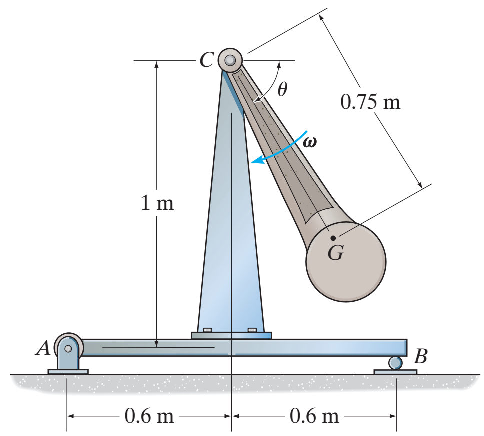
$$M = 0.3gml$$ **Ans.**



Ans:
$M = 0.3gml$

17–87.

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 90°$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

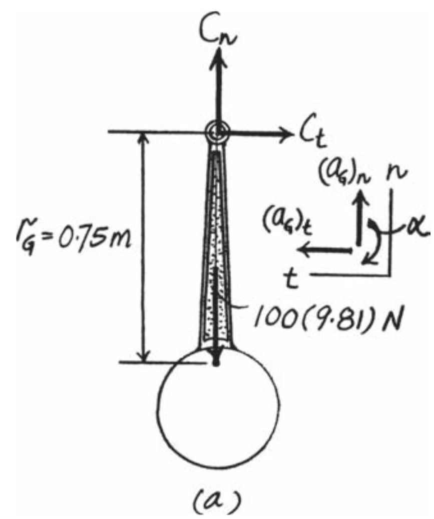
Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 8^2(0.75) = 48\text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram of the pendulum, Fig. a, we have

$\circlearrowleft +\Sigma M_C = I_C\alpha;\qquad 0 = 62.5\alpha \qquad \alpha = 0$

Using this result to write the force equations of motion along the n and t axes,

$\overset{+}{\leftarrow} \Sigma F_t = m(a_G)_t;\qquad -C_t = 100[0(0.75)] \qquad C_t = 0$

$+\uparrow \Sigma F_n = m(a_G)_n;\qquad C_n - 100(9.81) = 100(48) \qquad C_n = 5781\text{ N}$



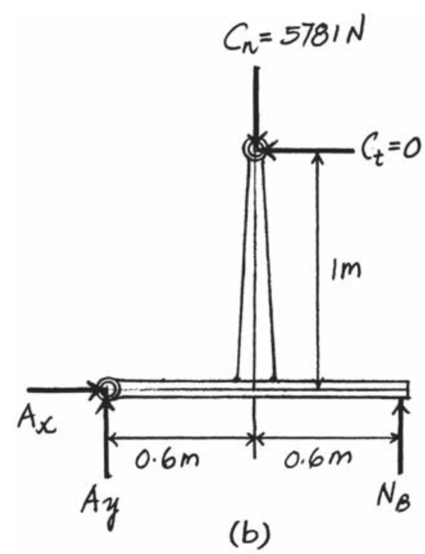
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b, we have

$+\Sigma M_A = 0;\qquad N_B(1.2) - 5781(0.6) = 0 \qquad N_B = 2890.5\text{ N} = 2.89\text{ kN}$ **Ans.**

Using this result to write the force equations of equilibrium along the x and y axes, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

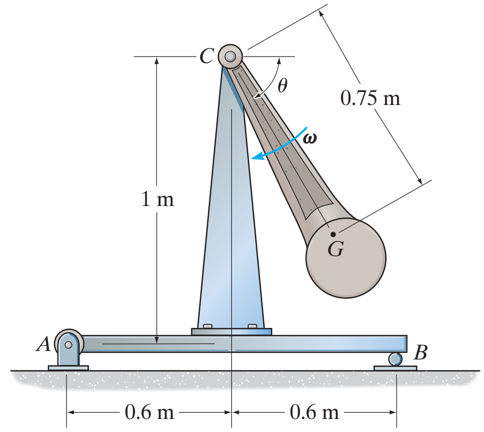
$+\uparrow \Sigma F_y = 0;\qquad A_y + 2890.5 - 5781 = 0 \qquad A_y = 2890.5\text{ N} = 2.89\text{ kN}$ **Ans.**



Ans:
$N_B = 2.89$ kN
$A_x = 0$
$A_y = 2.89$ kN

***17–88.**

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 0°$ when the pendulum is rotating at $\omega = 4$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12\text{ m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram shown in Fig. *a*,

$$\circlearrowleft +\Sigma M_C = I_C\alpha; \qquad -100(9.81)(0.75) = -62.5\alpha \qquad \alpha = 11.772\text{ rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes, we have

$$+\uparrow \Sigma F_t = m(a_G)_t; \qquad C_t - 100(9.81) = -100[11.772(0.75)] \qquad C_t = 98.1\text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad C_n = 100(12) \qquad C_n = 1200\text{ N}$$

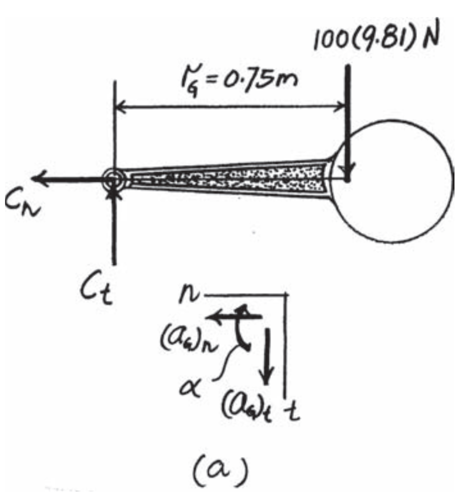
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. *b*,

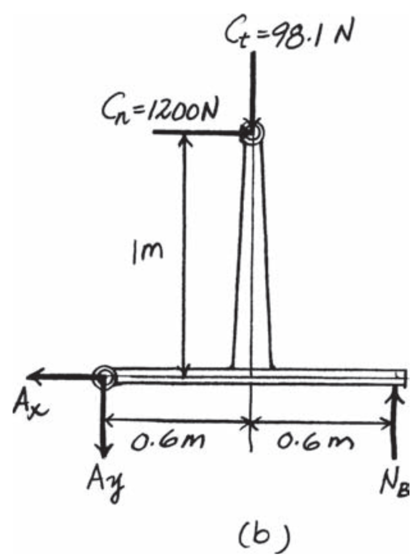
$$+\Sigma M_A = 0; \qquad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \qquad N_B = 1049.05\text{ N} = 1.05\text{ kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - A_x = 0 \qquad A_x = 1200\text{ N} = 1.20\text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1049.05 - 98.1 - A_y = 0 \qquad A_y = 950.95\text{ N} = 951\text{ N} \qquad \textbf{Ans.}$$

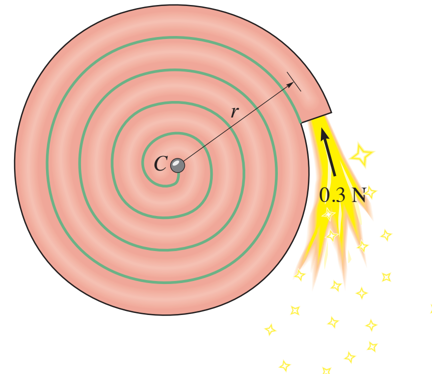




Ans:
$N_B = 1.05$ kN
$A_x = 1.20$ kN
$A_y = 951$ N

17–89.

The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such as that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of r = 75 mm. For the calculation, consider the wheel to always be a thin disk.

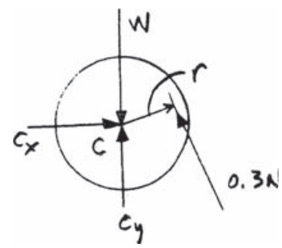


SOLUTION

Mass of wheel when 75% of the powder is burned = 0.025 kg

Time to burn off 75 % $= \dfrac{0.075\text{ kg}}{0.02\text{ kg/s}} = 3.75\text{ s}$

$$m(t) = 0.1 - 0.02t$$



Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\text{ kg}}{\pi(0.075\text{ m})^2} = 5.6588\text{ kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$

$$+\Sigma M_C = I_C\alpha; \qquad 0.3r = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$d\omega = \alpha\, dt$$

$$\int_0^\omega d\omega = 2.530\int_0^t [0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$$\omega = 800\text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 800$ rad/s

17–90.

If the disk in Fig. 17–19 *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, IC, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

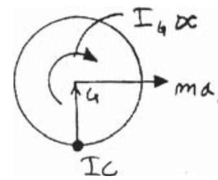
SOLUTION

$\zeta + \Sigma M_{IC} = \Sigma(M_K)_{IC}; \qquad \Sigma M_{IC} = I_G\alpha + (ma_G)r$

Since there is no slipping, $a_G = \alpha r$

Thus, $\Sigma M_{IC} = \left(I_G + mr^2\right)\alpha$

By the parallel–axis thoerem, the term in parenthesis represents I_{IC}. Thus,

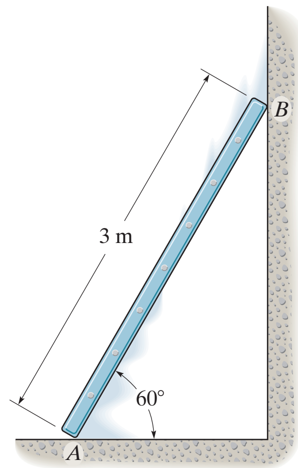


$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D**.

Ans:
$\Sigma M_{IC} = I_{IC}\alpha$

17–91.

The slender 12-kg bar has a clockwise angular velocity of $\omega = 2$ rad/s when it is in the position shown. Determine its angular acceleration and the normal reactions of the smooth surface A and B at this instant.



SOLUTION

Equations of Motion. The mass moment of inertia of the rod about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(3^2) = 9.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the rod, Fig. a

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \qquad N_B = 12(a_G)_x \tag{1}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 12(9.81) = -12(a_G)_y \tag{2}$$

$$\circlearrowleft + \Sigma M_O = (M_k)_O; \quad -12(9.81)(1.5\cos 60°) = -12(a_G)_x(1.5\sin 60°) - 12(a_G)_y(1.5\cos 60°) - 9.00\alpha$$

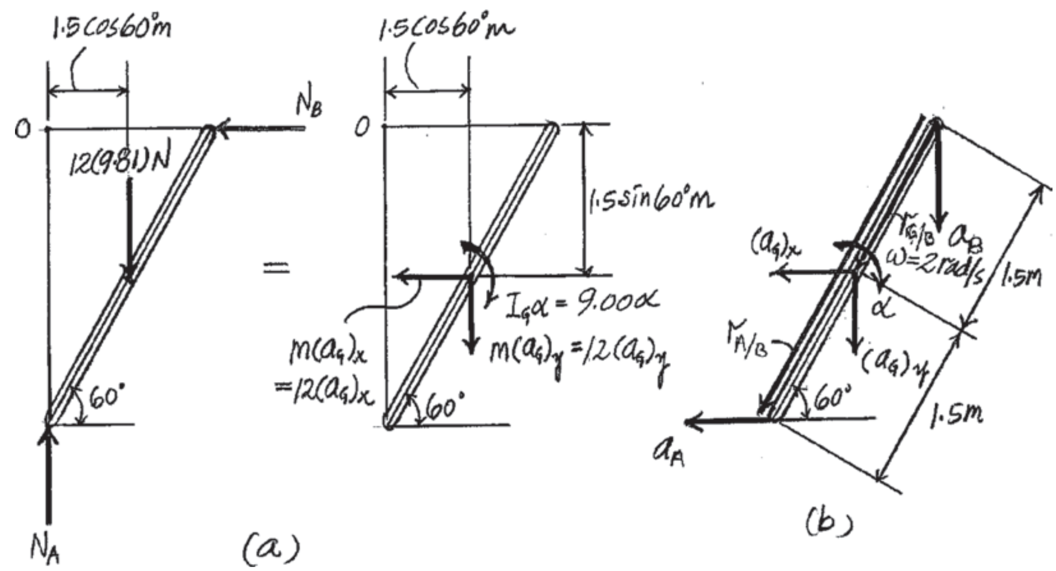
$$\sqrt{3}(a_G)_x + (a_G)_y + \alpha = 9.81 \tag{3}$$

Kinematics. Applying the relative acceleration equation relating $\mathbf{a}_G$ and $\mathbf{a}_B$ by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B} - \omega^2\mathbf{r}_{G/B}$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-1.5\cos 60°\mathbf{i} - 1.5\sin 60°\mathbf{j}) - 2^2(-1.5\cos 60°\mathbf{i} - 1.5\sin 60°\mathbf{j})$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (3 - 0.75\sqrt{3}\alpha)\mathbf{i} + (0.75\alpha - a_B + 3\sqrt{3})\mathbf{j}$$



17–91. Continued

Equating **i** and **j** components,

$$-(a_G)_x = 3 - 0.75\sqrt{3}\alpha \qquad \textbf{(4)}$$

$$-(a_G)_y = 0.75\alpha - a_B + 3\sqrt{3} \qquad \textbf{(5)}$$

Also, relate $\mathbf{a}_B$ and $\mathbf{a}_A$,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-2^2(-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-a_A\mathbf{i} = (6 - 1.5\sqrt{3}\alpha)\mathbf{i} + (1.5\alpha - a_B + 6\sqrt{3})\mathbf{j}$$

Equating **j** components,

$$0 = 1.5\alpha - a_B + 6\sqrt{3}; \qquad a_B = 1.5\alpha + 6\sqrt{3} \qquad \textbf{(6)}$$

Substituting Eq. (6) into (5)

$$(a_G)_y = 0.75\alpha + 3\sqrt{3} \qquad \textbf{(7)}$$

Substituting Eq. (4) and (7) into (3)

$$\sqrt{3}\left(0.75\sqrt{3}\alpha - 3\right) + 0.75\alpha + 3\sqrt{3} + \alpha = 9.81$$

$$\alpha = 2.4525 \text{ rad/s}^2 = 2.45 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (4) and (7)

$$-(a_G)_x = 3 - (0.75\sqrt{3})(2.4525); \quad (a_G)_x = 0.1859 \text{ m/s}^2$$

$$(a_G)_y = 0.75(2.4525) + 3\sqrt{3}; \quad (a_G)_y = 7.0355 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2)

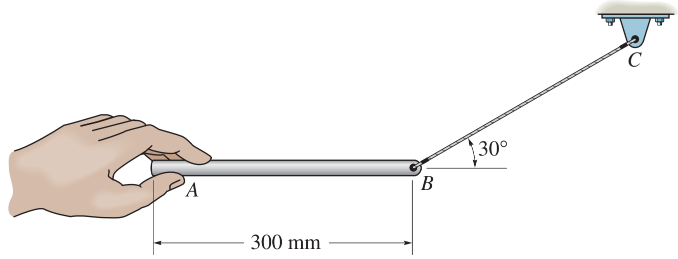
$$N_B = 12(0.1859); \quad N_B = 2.2307 \text{ N} = 2.23 \text{ N} \qquad \textbf{Ans.}$$

$$N_A - 12(9.81) = -12(7.0355); \quad N_A = 33.2937 \text{ N} = 33.3 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 2.45 \text{ rad/s}^2$
$N_B = 2.23$ N
$N_A = 33.3$ N

***17–92.**

The 2-kg slender bar is supported by cord BC and then released from rest at A. Determine the initial angular acceleration of the bar and the tension in the cord.

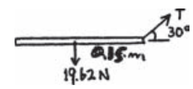


SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad T \cos 30° = 2(a_G)_x$

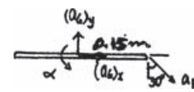
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T \sin 30° - 19.62 = 2(a_G)_y$

$\zeta + \Sigma M_G = I_G\alpha; \qquad T \sin 30°(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$



$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B \sin 30°\mathbf{i} - a_B \cos 30°\mathbf{j} = (a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} + \alpha(0.15)\mathbf{j}$



$(\xrightarrow{+}) \qquad (a_B) \sin 30° = (a_G)_x$

$(+\uparrow) \qquad (a_B) \cos 30° = -(a_G)_y - \alpha(0.15)$

Thus,

$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$

$T = 5.61 \text{ N}$ **Ans.**

$(a_G)_x = 2.43 \text{ m/s}^2$

$(a_G)_y = -8.41 \text{ m/s}^2$

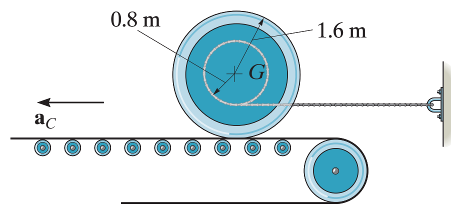
$\alpha = 28.0 \text{ rad/s}^2$ **Ans.**

Ans:

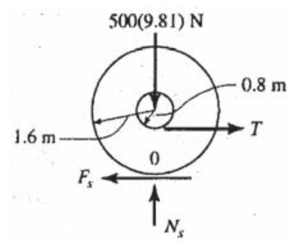
$T = 5.61 \text{ N}$
$\alpha = 28.0 \text{ rad/s}^2$

17–93.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.4$. If the conveyor accelerates at $a_C = 1\ \text{m/s}^2$, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.



SOLUTION



$\xrightarrow{+} \sum F_x = m(a_G)_x; \qquad -F_s + T = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

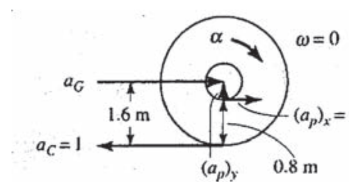
$\circlearrowright + \sum M_G = I_G\alpha; \qquad F_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$$\mathbf{a}_p = \mathbf{a}_G + \mathbf{a}_{p/G}$$

$$(a_p)_y\mathbf{j} = a_G\,\mathbf{i} - 0.8\alpha\mathbf{i}$$

$$\alpha_G = 0.8\alpha$$

$$N_s = 4905\text{ N}$$



Assume no slipping

$$\alpha = \frac{ac}{0.8} = \frac{1}{0.8} = 1.25\text{ rad/s}$$ **Ans.**

$$a_G = 0.8(1.25) = 1\text{ m/s}^2$$

$$T = 2.32\text{ kN}$$ **Ans.**

$$F_s = 1.82\text{ kN}$$

Since

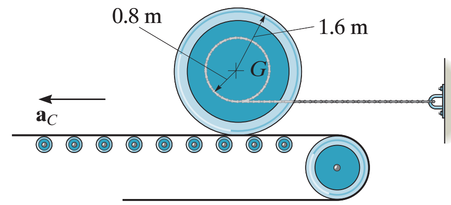
$$(F_s)_{max} = 0.5(4.905) = 2.45 > 1.82$$

(No slipping occurs)

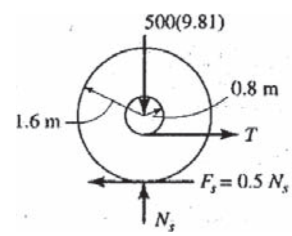
Ans:
$\alpha = 1.25\text{ rad/s}$
$T = 2.32\text{ N}$

17–94.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



SOLUTION



$\overset{+}{\rightarrow} \sum F_x = m(a_G)_x; \qquad T - 0.5N_S = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

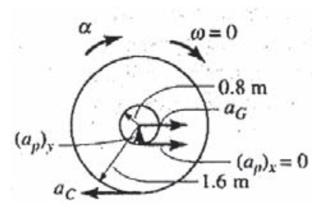
$\circlearrowleft + \sum M_G = I_G\alpha, \qquad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_p = a_C + a_{p/G}$

$(a_p)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$

Solving;



$N_s = 4905$ N

$T = 3.13$ kN **Ans.**

$\alpha = 1.684$ rad/s **Ans.**

$a_G = 1.347$ m/s^2

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$ac = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$

$a_C = 1.35$ m/s^2 **Ans.**

Also,

$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha; \qquad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

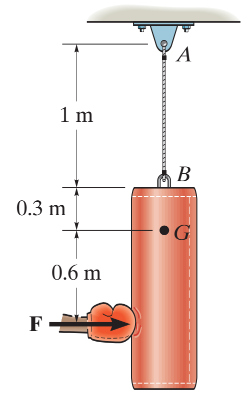
Since $N_S = 4905$ N

$\alpha = 1.684$ rad/s

Ans:
$T = 3.13$ kN,
$\alpha = 1.684$ rad/s
$a_C = 1.35$ m/s^2

17–95.

The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



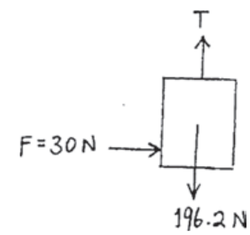
SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 30(0.6) = 20(0.4)^2\alpha$

$\alpha = 5.62 \text{ rad/s}^2$ **Ans.**



$(a_G)_x = 1.5 \text{ m/s}^2$

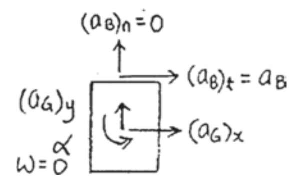
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B\,\mathbf{i} = (a_G)_y\,\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$

$(+\uparrow) \qquad (a_G)_y = 0$

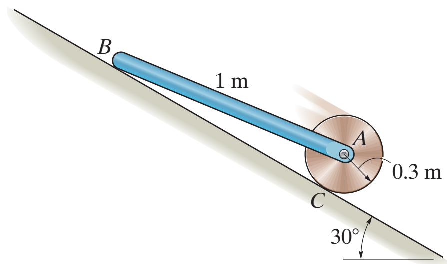
Thus,

$T = 196 \text{ N}$ **Ans.**



Ans:
$\alpha = 5.62 \text{ rad/s}^2$
$T = 196 \text{ N}$

***17–96.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



SOLUTION

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

$m_b = 10\ \text{kg}$ $\quad r = 0.3\ \text{m}$

$\mu_s = 0.6$ $\quad \theta = 30°$

$\mu_k = 0.4$ $\quad g = 9.81\ \text{m/s}^2$

$$\phi = \text{asin}\left(\frac{r}{L}\right)$$

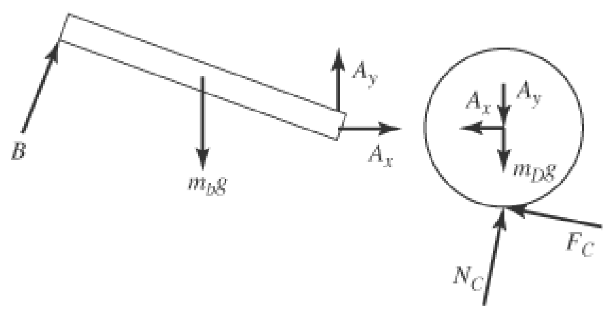
Assume no slip

Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$

$\alpha = 1\ \text{rad/s}^2$ $\quad a_A = 1\ \text{m/s}^2$

$F_{max} = 1\ \text{N}$



Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

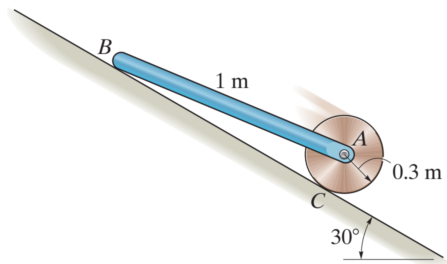
$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 109.04 \\ 16.05 \\ 65.43 \end{pmatrix}\text{N} \qquad \alpha = 13.38\ \text{rad/s}^2 \quad \textbf{Ans.}$$

Since $F_C = 16.05\ \text{N} < F_{max} = 65.43\ \text{N}$ then our no-slip assumption is correct.

Ans:
$\alpha = 13.38\ \text{rad/s}^2$
Since $F_C = 16.05\ \text{N} < F_{max} = 65.43\ \text{N}$
then our no-slip assumption is correct.

17–97. Solve Prob. 17–96 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



SOLUTION

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

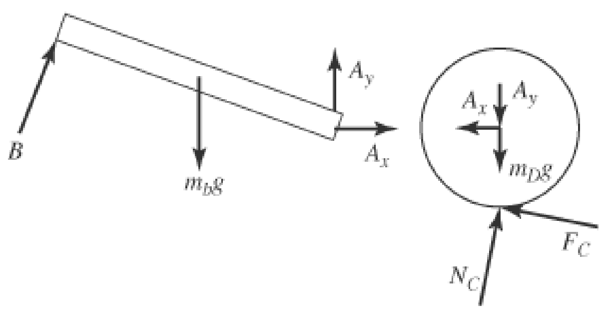
$m_b = 0\ \text{kg}$ $\quad r = 0.3\ \text{m}$

$\mu_s = 0.15$ $\quad \theta = 30°$

$\mu_k = 0.1$ $\quad g = 9.81\ \text{m/s}^2$

$$\phi = \operatorname{asin}\left(\frac{r}{L}\right)$$

Assume no slip



Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$

$\alpha = 1\ \text{rad/s}^2$ $\quad a_A = 1\ \text{m/s}^2$

$F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.97 \\ 13.08 \\ 10.19 \end{pmatrix}\text{N} \qquad \alpha = 10.90\ \text{rad/s}^2$$

Since $F_C = 13.08\ \text{N} > F_{max} = 10.19\ \text{N}$ then our no-slip assumption is wrong and we know that slipping does occur.

17–97. Continued

Guesses

$N_C = 1\ \text{N} \quad F_C = 1\ \text{N} \quad \alpha = 1\ \text{rad/s}^2 \quad a_A = 1\ \text{m/s}^2 \quad F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

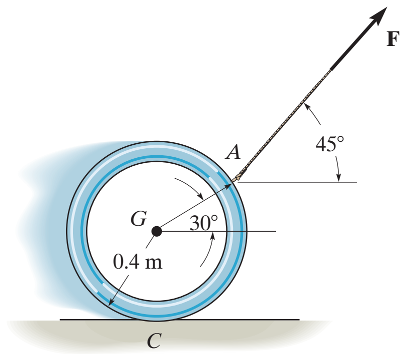
$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad F_{max} = \mu_s N_C \qquad F_C = \mu_k N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.97 \\ 6.80 \\ 10.19 \end{pmatrix}\text{N} \qquad \alpha = 5.66\ \text{rad/s}^2$$ **Ans.**

Ans:
$\alpha = 5.66\ \text{rad/s}^2$
Since $F_C = 13.08\ \text{N} > F_{max} = 10.19\ \text{N}$
then our no-slip assumption is wrong and we know that slipping does occur.

17–98.

A force of $F = 10$ N is applied to the 10-kg ring as shown. If slipping does not occur, determine the ring's initial angular acceleration, and the acceleration of its mass center, G. Neglect the thickness of the ring.



SOLUTION

Equations of Motion. The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60 \text{ kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the ring, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = (\mu_k)_C; \quad (10 \sin 45°)(0.4 \cos 30°) - (10 \cos 45°)[0.4(1 + \sin 30°)]$$

$$= -(10a_G)(0.4) - 1.60\alpha$$

$$4a_G + 1.60\alpha = 1.7932 \quad \textbf{(1)}$$

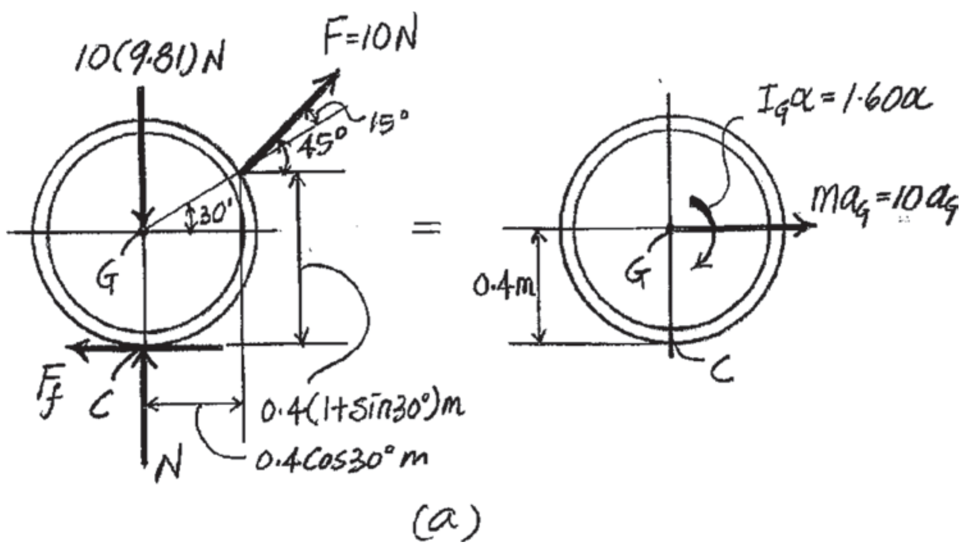
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \quad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\alpha = 0.5604 \text{ rad/s}^2 = 0.560 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

$$a_G = 0.2241 \text{ m/s}^2 = 0.224 \text{ m/s}^2 \rightarrow \quad \textbf{Ans.}$$

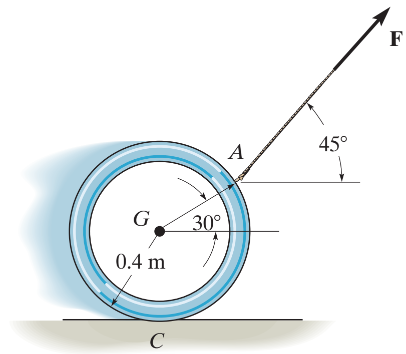


(a)

Ans:
$\alpha = 0.560 \text{ rad/s}^2 \circlearrowright$
$a_G = 0.224 \text{ m/s}^2 \rightarrow$

17–99.

If the coefficient of static friction at C is $\mu_s = 0.3$, determine the largest force **F** that can be applied to the 5-kg ring, without causing it to slip. Neglect the thickness of the ring.



SOLUTION

Equations of Motion: The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60\ \text{kg}\cdot\text{m}^2$. Here, it is required that the ring is on the verge of slipping at C, $F_f = \mu_s N = 0.3\ \text{N}$. Referring to the FBD and kinetic diagram of the ring, Fig. *a*

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F \sin 45° + N - 10(9.81) = 10(0) \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F \cos 45° - 0.3\ \text{N} = 10a_G \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad F \sin 15°(0.4) - 0.3\ \text{N}(0.4) = -1.60\alpha \qquad \textbf{(3)}$$

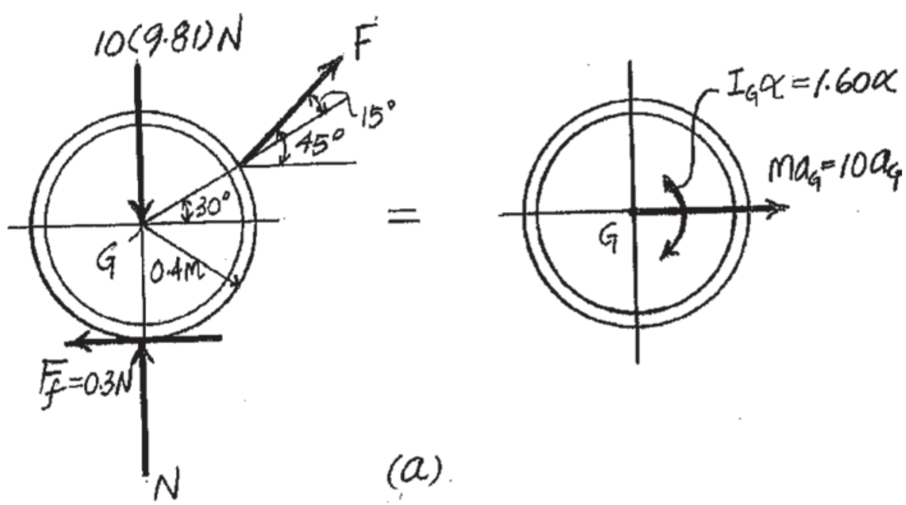
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$F = 42.34\ \text{N} = 42.3\ \text{N} \qquad \textbf{Ans.}$$

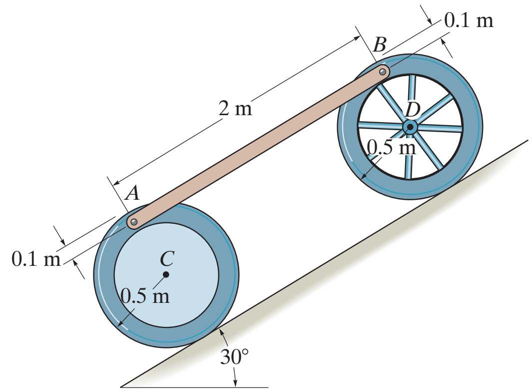
$N = 68.16\ \text{N} \qquad \alpha = 2.373\ \text{rad/s}^2 \circlearrowright \qquad a_G = 0.9490\ \text{m/s}^2 \rightarrow$



Ans:
$F = 42.3\ \text{N}$

***17–100.**

Wheel C has a mass M_1 and a radius of gyration k_C, whereas wheel D has a mass M_2 and a radius of gyration k_D. Determine the angular acceleration of each wheel at the instant shown. Neglect the mass of the link and assume that the assembly does not slip on the plane.



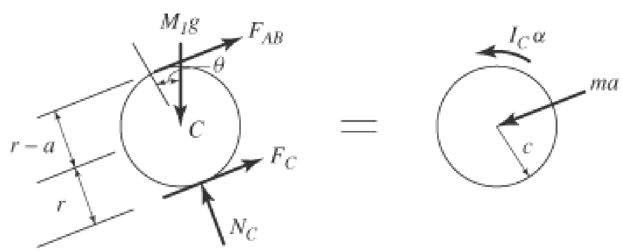
Given:

$M_1 = 60 \text{ kg}$ $\quad r = 0.5 \text{ m}$

$M_2 = 40 \text{ kg}$ $\quad a = 0.1 \text{ m}$

$k_C = 0.4 \text{ m}$ $\quad b = 2 \text{ m}$

$k_D = 0.35 \text{ m}$ $\quad \theta = 30°$



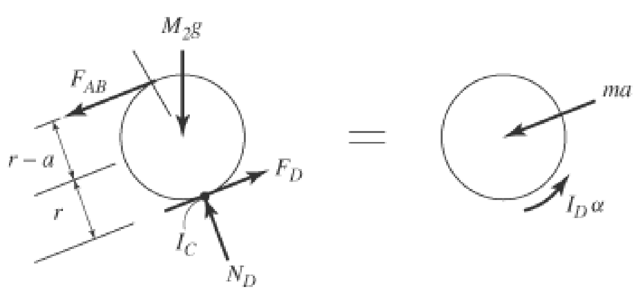
Solution:

Both wheels have the same angular acceleration.

Guesses

$F_{AB} = 1 \text{ N}$

$\alpha = 1 \text{ rad/s}^2$



Given

$$-F_{AB}(2r - a) + M_1 g \sin(\theta) r = M_1 k_C^2 \alpha + M_1 (r\alpha) r$$

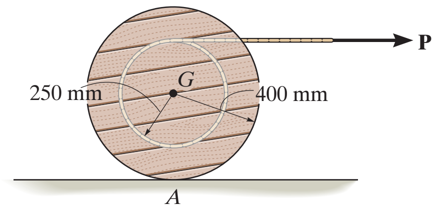
$$F_{AB}(2r - a) + M_2 g \sin(\theta) r = M_2 k_D^2 \alpha + M_2 (r\alpha) r$$

$$\begin{pmatrix} F_{AB} \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, \alpha) \qquad F_{AB} = -6.21 \text{ N} \qquad \alpha = 6.21 \text{ rad/s}^2 \quad \textbf{Ans.}$$

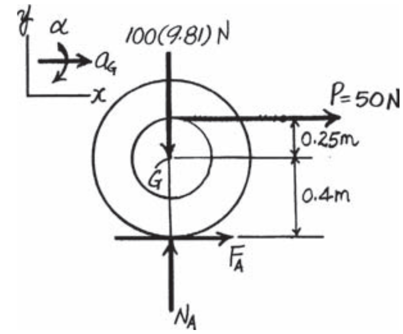
Ans:
$\alpha = 6.21 \text{ rad/s}^2$

17–101.

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

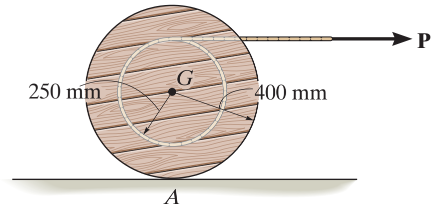
$a_G = 0.520 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

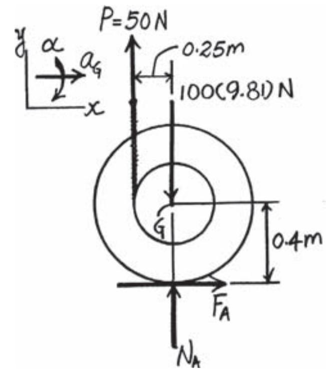
Ans:
$\alpha = 1.30 \text{ rad/s}^2$

17–102.

Solve Prob. 17–101 if the cord and force $P = 50\text{ N}$ are directed vertically upwards.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)x; \quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

$\alpha = 0.500\text{ rad/s}^2$ **Ans.**

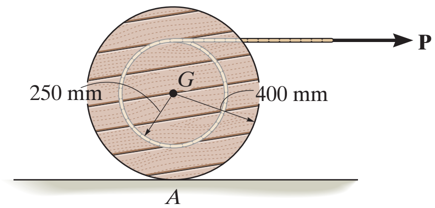
$a_G = 0.2\text{ m/s}^2 \qquad N_A = 931\text{ N} \qquad F_A = 20\text{ N}$

Since $(F_A)_{max} = 0.2(931) = 186.2\text{ N} > 20\text{ N}$ **OK**

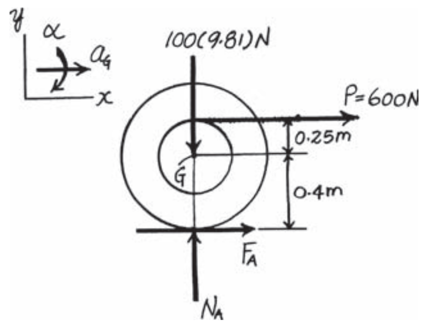
Ans:
$\alpha = 0.500\text{ rad/s}^2$

17–103.

The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G\,\alpha;\qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 15.6\text{ rad/s}^2$ **Ans.**

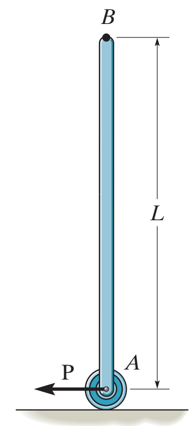
$a_G = 6.24\text{ m/s}^2 \qquad N_A = 981\text{ N} \qquad F_A = 24.0\text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2\text{ N} > 24.0\text{ N}$ **OK**

Ans:
$\alpha = 15.6\text{ rad/s}^2$

***17–104.**

The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



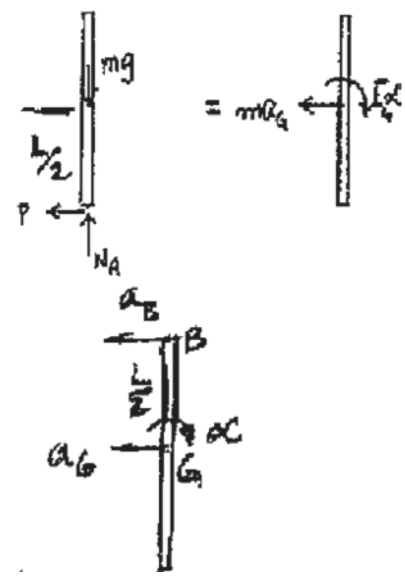
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = ma_G$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad P\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right)\alpha$

$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$ **Ans.**



$a_G = \frac{P}{m}$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\,\mathbf{i}$

$(\overset{+}{\leftarrow}) \qquad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

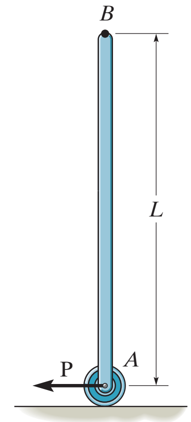
$= \frac{P}{m} - \frac{L}{2}\left(\frac{6P}{mL}\right)$

$a_B = -\frac{2P}{m} = \frac{2P}{m}$ **Ans.**

Ans:
$\alpha = \frac{6P}{mL}$
$a_B = \frac{2P}{m}$

17–105.

Solve Prob. 17–104 if the roller is removed and the coefficient of kinetic friction at the ground is μ_k.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P - \mu_k N_A = m a_G$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad (P - \mu_k N_A)\dfrac{L}{2} = \left(\dfrac{1}{12} m L^2\right)\alpha$

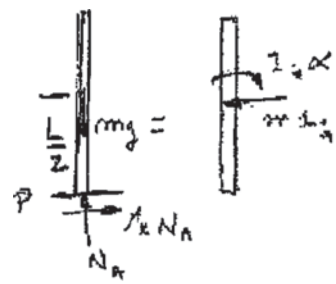
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - mg = 0$

Solving,

$$N_A = mg$$

$$a_G = \frac{L}{6}\alpha$$

$$\alpha = \frac{6(P - \mu_k mg)}{mL} \qquad \textbf{Ans.}$$



$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$(\overset{+}{\rightarrow}) \quad a_B = -\dfrac{L}{6}\alpha + \dfrac{L}{2}\alpha$

$a_B = \dfrac{L\alpha}{3}$

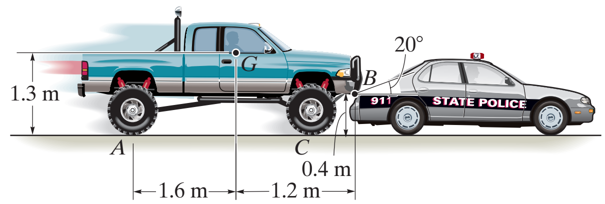
$$a_B = \frac{2(P - \mu_k mg)}{m} \qquad \textbf{Ans.}$$

Ans:

$\alpha = \dfrac{6(P - \mu_k mg)}{mL}$

$a_B = \dfrac{2(P - \mu_k mg)}{m}$

17–106. A "lifted" truck can become a road hazard since the bumper is high enough to ride up a standard car in the event the car is rear-ended. As a model of this case consider the truck to have a mass of 2.70 Mg, a mass center G, and a radius of gyration about G of $k_G = 1.45$ m. Determine the horizontal and vertical components of acceleration of the mass center G, and the angular acceleration of the truck, at the moment its front wheels at C have just left the ground and its smooth front bumper begins to ride up the back of the stopped car so that point B has a velocity of $v_B = 8$ m/s at 20º from the horizontal. Assume the wheels are free to roll, and neglect the size of the wheels and the deformation of the material.



SOLUTION

$\text{Mg} = 10^3 \text{ kg}$

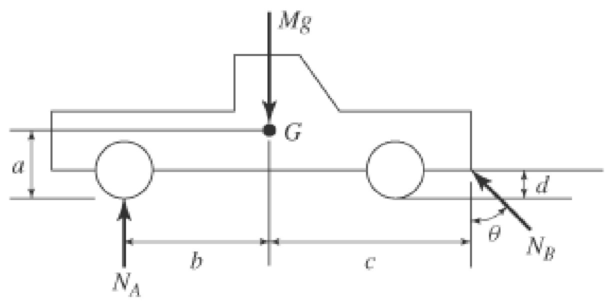
$\text{kN} = 10^3 \text{ N}$

$M = 2.70 \text{ Mg}$ $\quad a = 1.3 \text{ m}$

$\theta = 20 \text{ deg}$ $\quad b = 1.6 \text{ m}$

$k_G = 1.45 \text{ m}$ $\quad c = 1.2 \text{ m}$

$v_B = 8 \text{ m/s}$ $\quad d = 0.4 \text{ m}$



Guesses

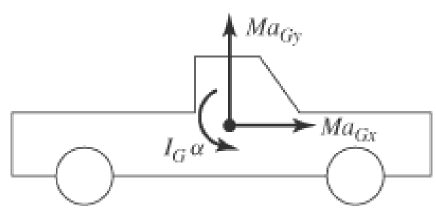
$v_A = 1 \text{ m/s}$ $\quad \omega = 1 \text{ rad/s}$

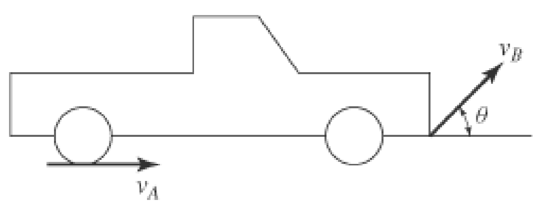
$a_B = 1 \text{ m/s}^2$ $\quad \alpha = 1 \text{ rad/s}^2$

$a_{Gx} = 1 \text{ m/s}^2$ $\quad a_{Gy} = 1 \text{ m/s}^2$

$N_A = 1 \text{ N}$ $\quad N_B = 1 \text{ N}$

$a_A = 1 \text{ m/s}^2$





Given

$$N_A + N_B\cos(\theta) - Mg = Ma_{Gy} \qquad -N_B\sin(\theta) = Ma_{Gx}$$

17–106. Continued

$$N_B\cos(\theta)c - N_B\sin(\theta)(a-d) - N_A b = M k_G^{\,2}\alpha$$

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} v_B\cos(\theta) \\ v_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ \omega \\ a_A \\ a_B \\ a_{Gx} \\ a_{Gy} \\ \alpha \\ N_A \\ N_B \end{pmatrix} = \text{Find}(v_A, \omega, a_A, a_B, a_{Gx}, a_{Gy}, \alpha, N_A, N_B)$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \end{pmatrix} = \begin{pmatrix} -1.82 \\ -1.69 \end{pmatrix} \text{m/s}^2$$ **Ans.**

$$\alpha = -0.283 \text{ rad/s}^2$$ **Ans.**

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 8.38 \\ 14.39 \end{pmatrix} \text{kN}$$

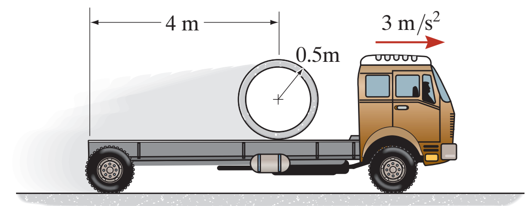
$$\omega = 0.977 \text{ rad/s}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -0.663 \\ -3.431 \end{pmatrix} \text{m/s}^2$$

Ans:
$a_{Gx} = -1.82 \text{ m/s}^2$
$a_{Gy} = -1.69 \text{ m/s}^2$
$\alpha = -0.283 \text{ rad/s}^2$

17–107.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of 3 m/s^2, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125 \text{ kg} \cdot \text{m}^2$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 0 = 125\alpha - 500 a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3 \text{ m/s}^2$. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} = 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

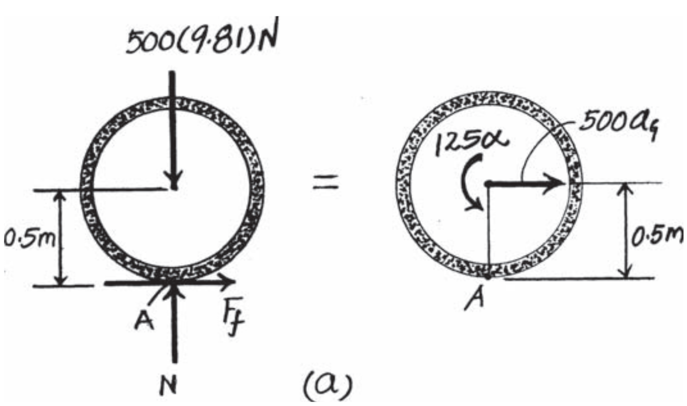
Equating the **i** components,

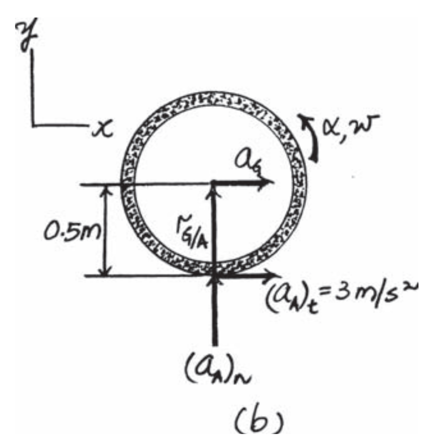
$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 1.5 \text{ m/s}^2 \rightarrow$$

$$\alpha = 3 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

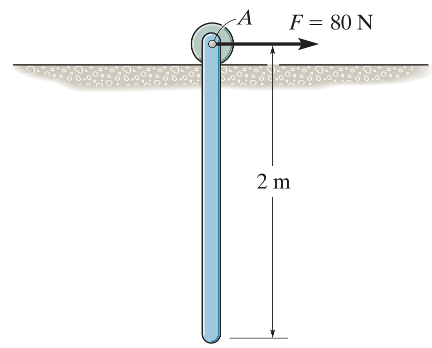




Ans:
$\alpha = 3 \text{ rad/s}^2$

***17–108.**

The 12-kg uniform bar is supported by a roller at A. If a horizontal force of $F = 80$ N is applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. Neglect the weight and the size of the roller.



SOLUTION

Equations of Motion. The mass moment of inertia of the bar about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(2^2) = 4.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the bar, Fig. a

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 80 = 12(a_G)_x \quad (a_G)_x = 6.6667\ \text{m/s}^2 \rightarrow$

$\circlearrowleft + \Sigma M_A = (\mu_k)_A;\quad 0 = 12(6.6667)(1) - 4.00\,\alpha \quad \alpha = 20.0\ \text{rad/s}^2 \circlearrowright$

Kinematic. Since the bar is initially at rest, $\omega = 0$. Applying the relative acceleration equation by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

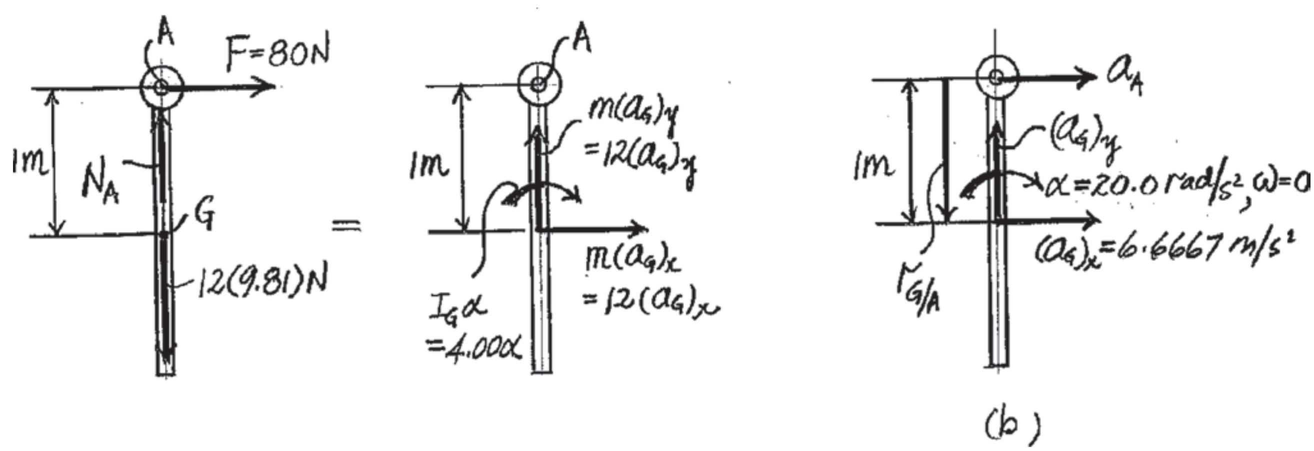
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = a_A\mathbf{i} + (-20.0\mathbf{k}) \times (-\mathbf{j}) - \mathbf{0}$$

$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = (a_A - 20)\mathbf{i}$$

Equating **i** and **j** components,

$6.6667 = a_A - 20;\quad a_A = 26.67\ \text{m/s}^2 = 26.7\ \text{m/s}^2 \rightarrow$ **Ans.**

$(a_G)_y = 0$

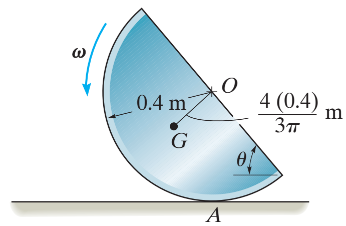


(b)

Ans:
$a_A = 26.7\ \text{m/s}^2 \rightarrow$

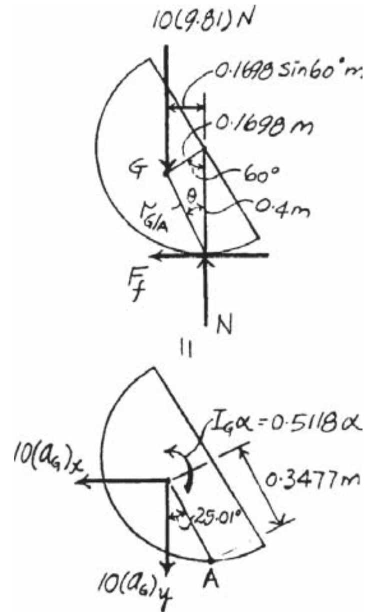
17–109.

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\left(0.1698^2\right) = 0.5118 \text{ kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m Also, using law of sines, $\frac{\sin\theta}{0.1698} = \frac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698 \sin 60°) = 0.5118\alpha$$

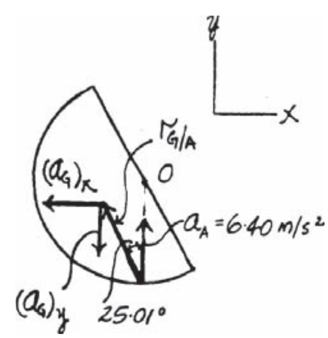
$$+ 10(a_G)_x \cos 25.01°(0.3477)$$

$$+ 10(a_G)_y \sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477 \sin 25.01°\mathbf{i} + 0.3477 \cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have



$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

$$\alpha = 13.85 \text{ rad/s}^2 \qquad (a_G)_x = 2.012 \text{ m/s}^2 \qquad (a_G)_y = 0.6779 \text{ m/s}^2$$

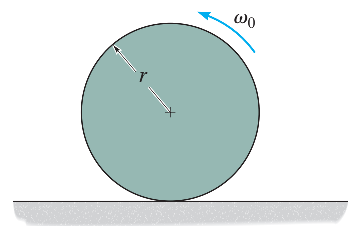
$$F_f = 20.12 \text{ N} \qquad N = 91.32 \text{ N}$$

Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

Ans:
Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**.

17–110.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

Equations of Motion. Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$

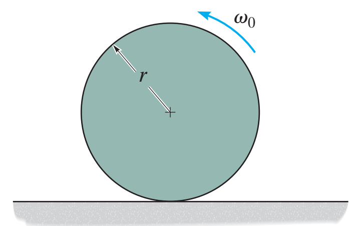
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g \leftarrow$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad -\mu_k(mg)r = \left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r} \circlearrowright$ **Ans.**

Ans:
$a_G = \mu_k g \leftarrow$
$\alpha = \frac{2\mu_k g}{r} \circlearrowright$

17–111.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is μ_k.



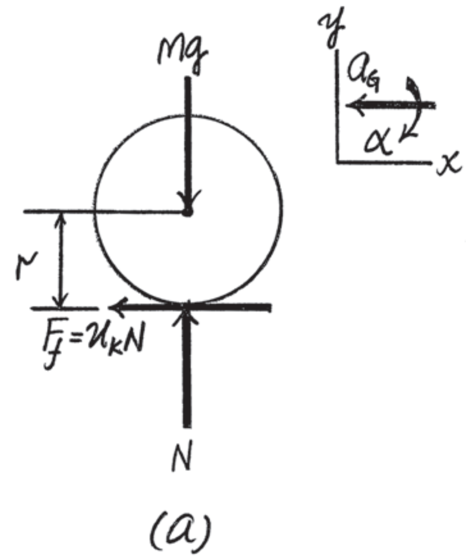
SOLUTION

Equations of Motion: Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$.

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N - mg = 0 \qquad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad \mu_k(mg) = ma_G \qquad a_G = \mu_k g$

$+\Sigma M_G = I_G\alpha;\quad -\mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r}$



Kinematics: At the instant when the disk rolls without slipping, $v_G = \omega r$. Thus,

$(\overset{+}{\leftarrow})$

$$v_G = (v_G)_0 + a_G t$$

$$\omega r = 0 + \mu_k g t$$

$$t = \frac{\omega r}{\mu_k g} \qquad \textbf{(1)}$$

and

$$\omega = \omega_0 + \alpha t$$

$(\circlearrowleft +)$

$$\omega = \omega_0 + \left(-\frac{2\mu_k g}{r}\right)t \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

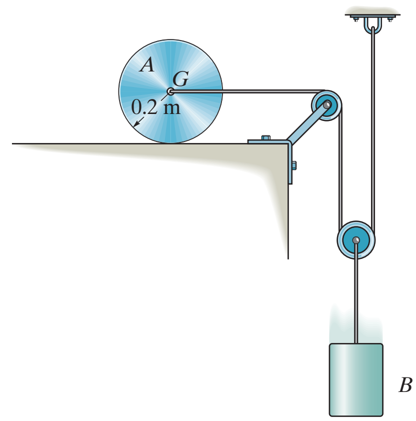
$$\omega = \frac{1}{3}\omega_0 \qquad t = \frac{\omega_0 r}{3\mu_k g} \qquad \textbf{Ans.}$$

Ans:

$\omega = \frac{1}{3}\omega_0$

$t = \frac{\omega_0 r}{3\mu_k g}$

***17–112.** The 20-kg disk A is attached to the 10-kg block B using the cable and pulley system shown. If the disk rolls without slipping, determine its angular acceleration and the acceleration of the block when they are released. Also, what is the tension in the cable? Neglect the mass of the pulleys.



SOLUTION

Equation of Motions:

Disk:

$$\circlearrowleft +\Sigma M_{IC} = \Sigma(M_k)_{IC}; \qquad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha \qquad \textbf{(1)}$$

Block:

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 10(9.81) - 2T = 10a_B \qquad \textbf{(2)}$$

Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \qquad \textbf{(3)}$$

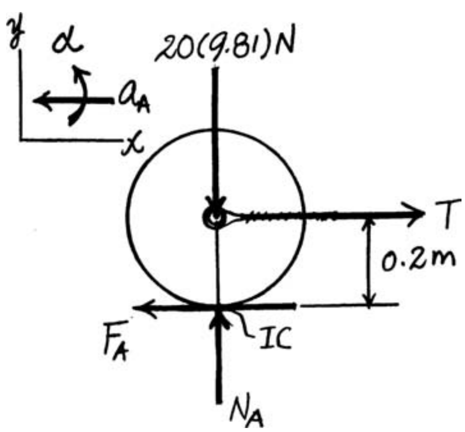
Note the direction for α and a_B are the same for all equations.

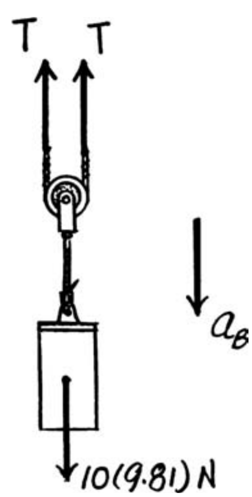
Solving Eqs. (1) through (3):

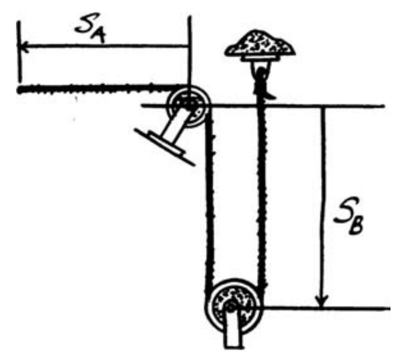
$$a_B = 0.755 \text{ m/s}^2 = 0.755 \text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$\alpha = -7.55 \text{ rad/s}^2 = 7.55 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$T = 45.3 \text{ N} \qquad \textbf{Ans.}$$



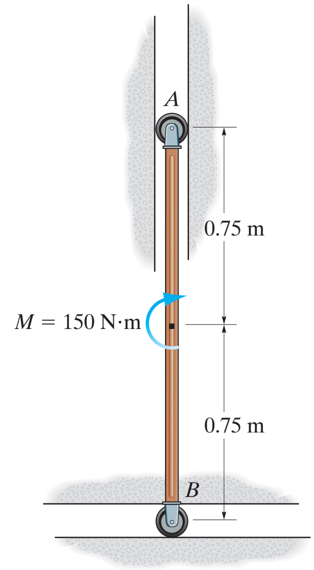




Ans:
$a_B = 0.755 \text{ m/s}^2 \downarrow$
$\alpha = 7.55 \text{ rad/s}^2 \circlearrowright$
$T = 45.3 \text{ N}$

17–113.

The 30-kg uniform slender rod AB rests in the position shown when the couple moment of $M = 150\ \text{N}\cdot\text{m}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625\ \text{kg}\cdot\text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -150 = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 150 \qquad \textbf{(1)}$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

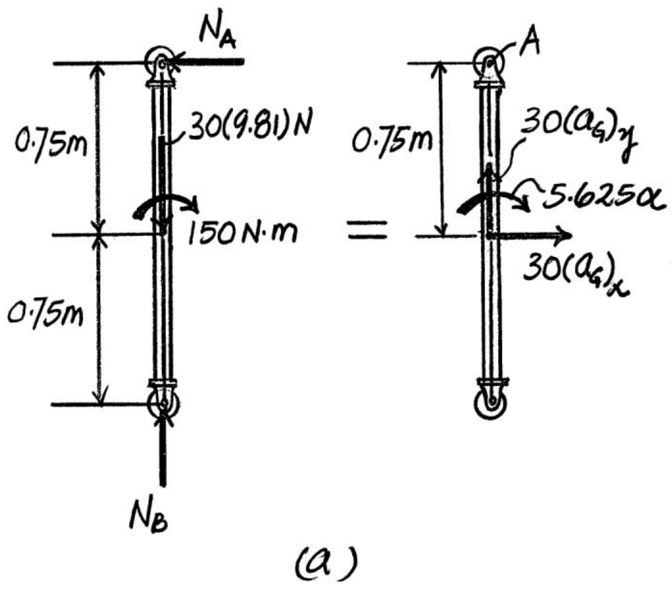
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

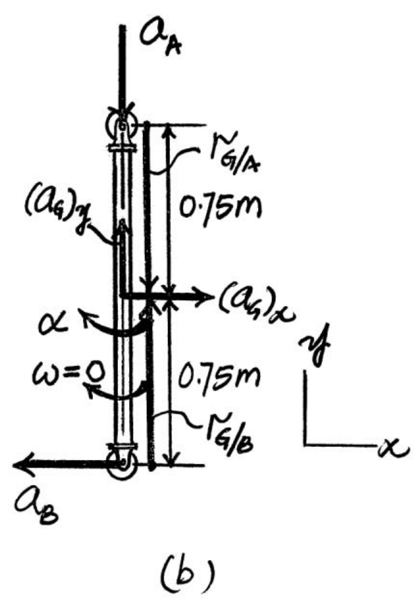
Equating the **i** components,

$$(a_G)_x = -0.75\alpha \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1),

$$\alpha = 6.667\ \text{rad/s}^2 = 6.67\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

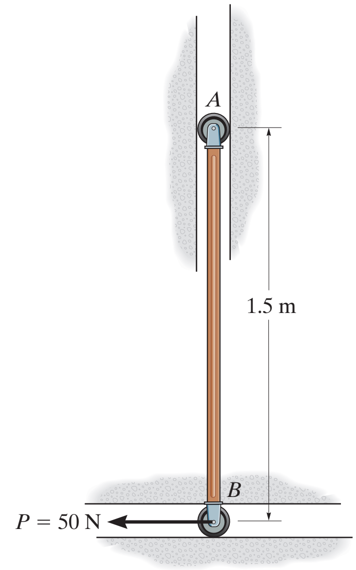




Ans:
$\alpha = 6.67\ \text{rad/s}^2$

17–114.

The 30-kg slender rod AB rests in the position shown when the horizontal force $P = 50$ N is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625 \text{ kg} \cdot \text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -50(0.15) = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 75 \qquad (1)$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

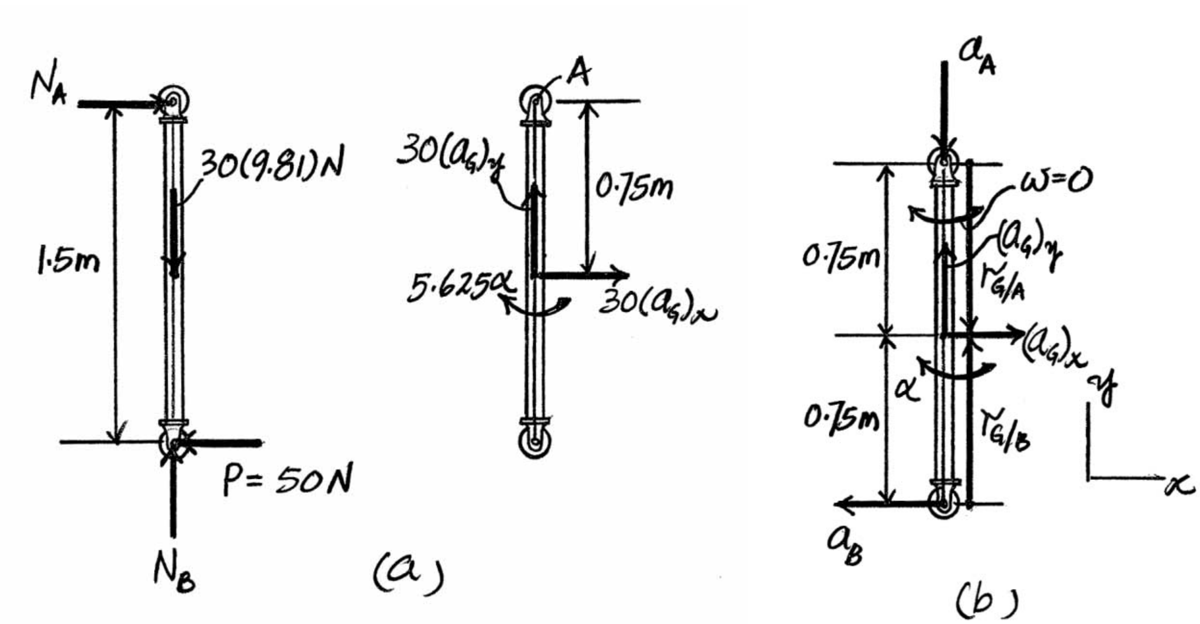
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

Equating the **i** components,

$$(a_G)_x = -0.75\alpha \qquad (2)$$

Substituting Eq. (2) into Eq. (1),

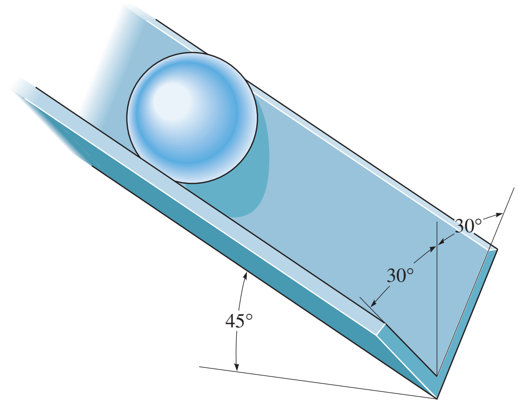
$$\alpha = 3.333 \text{ rad/s}^2 = 3.33 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\alpha = 3.33 \text{ rad/s}^2$

17–115.

The solid ball of radius r and mass m rolls without slipping down the 60° trough. Determine its angular acceleration.

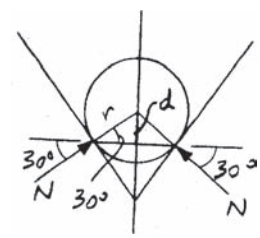


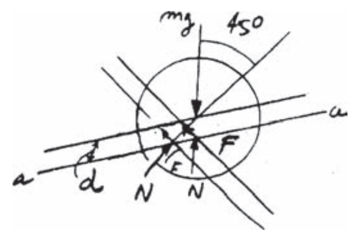
SOLUTION

$$d = r \sin 30^\circ = \frac{r}{2}$$

$$\Sigma M_{a-a} = \Sigma (M_k)_{a-a}; \qquad mg \sin 45^\circ \left(\frac{r}{2}\right) = \left[\frac{2}{5}mr^2 + m\left(\frac{r}{2}\right)^2\right]\alpha$$

$$\alpha = \frac{10g}{13\sqrt{2}\,r} \qquad \textbf{Ans.}$$



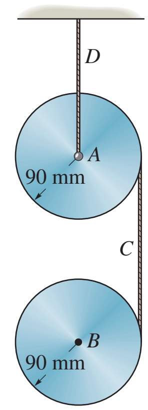


Ans:

$$\alpha = \frac{10g}{13\sqrt{2}\,r}$$

***17–116.**

A cord is wrapped around each of the two 10-kg disks. If they are released from rest, determine the angular acceleration of each disk and the tension in the cord C. Neglect the mass of the cord.



SOLUTION

For A:

$$\circlearrowleft +\Sigma M_A = I_A\alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad \textbf{(1)}$$

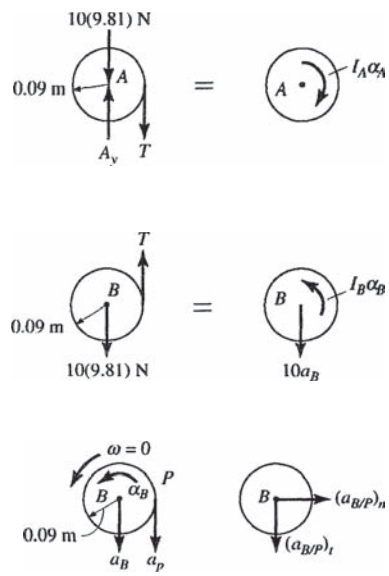
For B:

$$\circlearrowright +\Sigma M_B = I_B\alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad \textbf{(2)}$$

$$+\downarrow \Sigma F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad \textbf{(3)}$$

$$\mathbf{a}_B = \mathbf{a}_P + (\mathbf{a}_{B/P})_t + (\mathbf{a}_{B/P})_n$$

$$(+\downarrow)\ a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \qquad \textbf{(4)}$$



Solving,

$a_B = 7.85\ \text{m/s}^2$

$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

$T = 19.6\ \text{N}$ **Ans.**

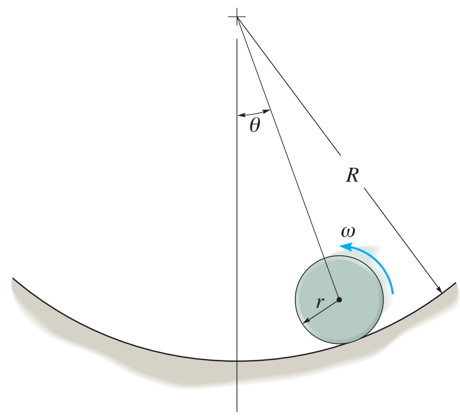
$A_y = 10(9.81) + 19.62$

$= 118\ \text{N}$

Ans:
$\alpha_A = 43.6\ \text{rad/s}^2 \circlearrowright$
$\alpha_B = 43.6\ \text{rad/s}^2 \circlearrowleft$
$T = 19.6\ \text{N}$

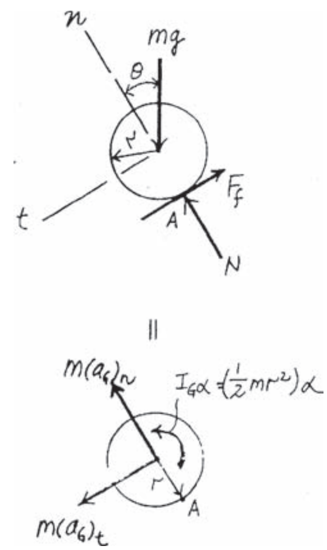
17–117.

The disk of mass m and radius r rolls without slipping on the circular path. Determine the normal force which the path exerts on the disk and the disk's angular acceleration if at the instant shown the disk has an angular velocity of $\boldsymbol{\omega}$.



SOLUTION

Equation of Motion: The mass moment of inertia of the disk about its center of mass is given by $I_G = \frac{1}{2}mr^2$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad mg \sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(a_G)_t\,(r) \qquad \textbf{[1]}$$

$$\Sigma F_n = m(a_G)_n; \qquad N - mg\cos\theta = m(a_G)_n \qquad \textbf{[2]}$$

Kinematics: Since the semicircular disk does not slip at A, then $v_G = \omega r$ and $(a_G)_t = \alpha r$. Substitute $(a_G)_t = \alpha r$ into Eq. [1] yields

$$mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(\alpha r)(r)$$

$$\alpha = \frac{2g}{3r}\sin\theta \qquad \textbf{Ans.}$$

Also, the center of the mass for the disk moves around a circular path having a radius of $\rho = R - r$. Thus, $(a_G)_n = \frac{v_G^2}{\rho} = \frac{\omega^2 r^2}{R - r}$. Substitute into Eq. [2] yields

$$N - mg\cos\theta = m\left(\frac{\omega^2 r^2}{R - r}\right)$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right) \qquad \textbf{Ans.}$$

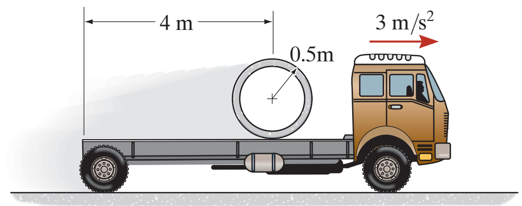
Ans:

$$\alpha = \frac{2g}{3r}\sin\theta$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right)$$

17–118.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of 3 m/s^2, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125 \text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3 \text{ m/s}^2$. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} - 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

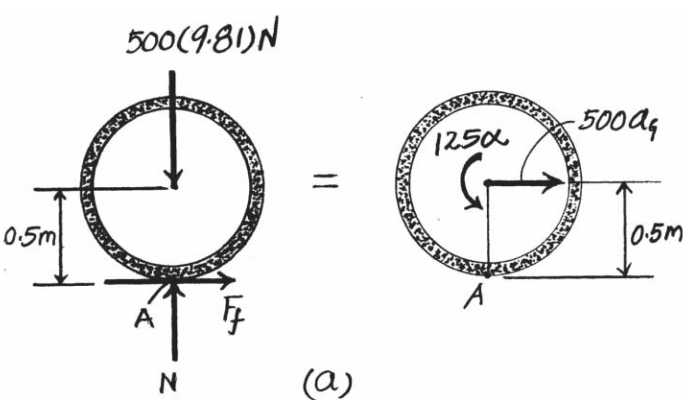
Equating the **i** components,

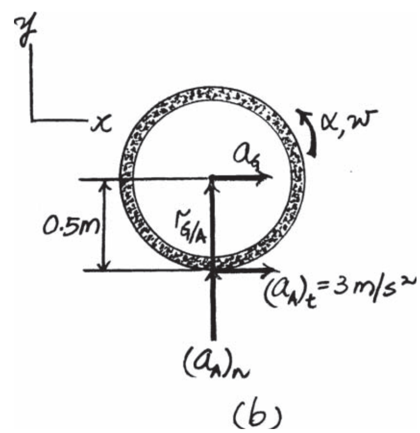
$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 1.5 \text{ m/s}^2 \rightarrow$$

$$\alpha = 3 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

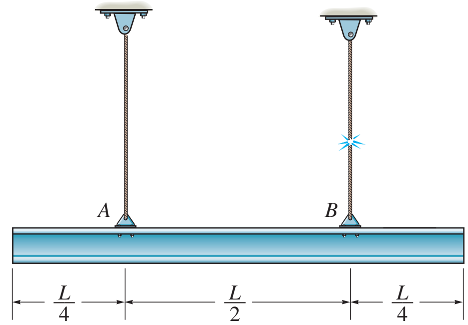




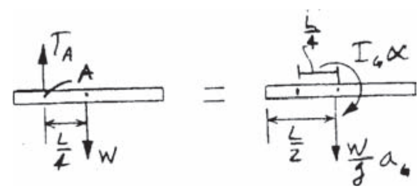
Ans:
$\alpha = 3 \text{ rad/s}^2$

17–119.

The uniform beam has a weight W. If it is originally at rest while being supported at A and B by cables, determine the tension in cable A if cable B suddenly fails. Assume the beam is a slender rod.



SOLUTION



$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad W\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha + \frac{W}{g}\left(\frac{L}{4}\right)\alpha\left(\frac{L}{4}\right)$$

$$1 = \frac{1}{g}\left(\frac{L}{4} + \frac{L}{3}\right)\alpha$$

Since $a_G = \alpha\left(\frac{L}{4}\right)$.

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = W - \frac{W}{g}(\alpha)\left(\frac{L}{4}\right) = W - \frac{W}{g}\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)\left(\frac{L}{4}\right)$$

$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Also,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_G = I_G\,\alpha; \qquad T_A\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha$$

Since $a_G = \frac{L}{4}\alpha$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\alpha$$

$$\frac{1}{3}\left(\frac{W}{g}\right)L\alpha - W = -\frac{W}{g}\left(\frac{L}{4}\right)\alpha$$

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)$$

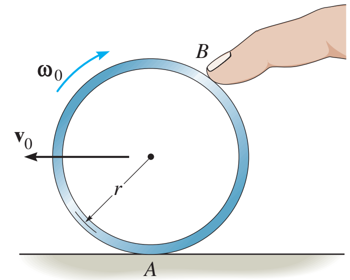
$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Ans:

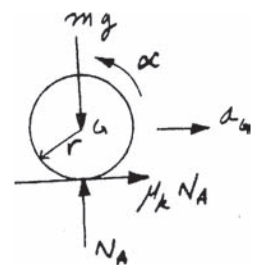
$T_A = \frac{4}{7}W$

***17–120.**

By pressing down with the finger at B, a thin ring having a mass m is given an initial velocity $\mathbf{v_0}$ and a backspin $\boldsymbol{\omega}_0$ when the finger is released. If the coefficient of kinetic friction between the table and the ring is μ_κ, determine the distance the ring travels forward before backspinning stops.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad N_A - mg = 0$

$$N_A = mg$$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k\,(mg) = m(a_G)$

$$a_G = \mu_k g$$

$\zeta + \Sigma M_G = I_G\alpha; \qquad \mu_k\,(mg)r = mr^2\alpha$

$$\alpha = \frac{\mu_k\, g}{r}$$

$(\zeta +) \qquad \omega = \omega_0 + \alpha_c\, t$

$$0 = \omega_0 - \left(\frac{\mu_k\, g}{r}\right)t$$

$$t = \frac{\omega_0\, r}{\mu_k\, g}$$

$\left(\overset{+}{\leftarrow}\right) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c\, t^2$

$$s = 0 + v_0\left(\frac{\omega_0\, r}{\mu_k\, g}\right) - \left(\frac{1}{2}\right)(\mu_k\, g)\left(\frac{\omega_0^2\, r^2}{\mu_k^2\, g^2}\right)$$

$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right) \qquad \textbf{Ans.}$$

Ans:

$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right)$$