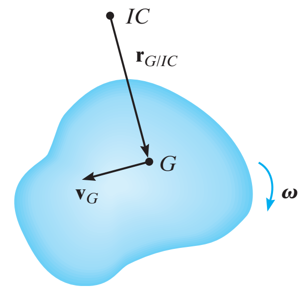
18–1.

At a given instant the body of mass m has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body determined about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



SOLUTION

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

$$= \frac{1}{2}m(\omega r_{G/IC})^2 + \frac{1}{2}I_G\,\omega^2$$

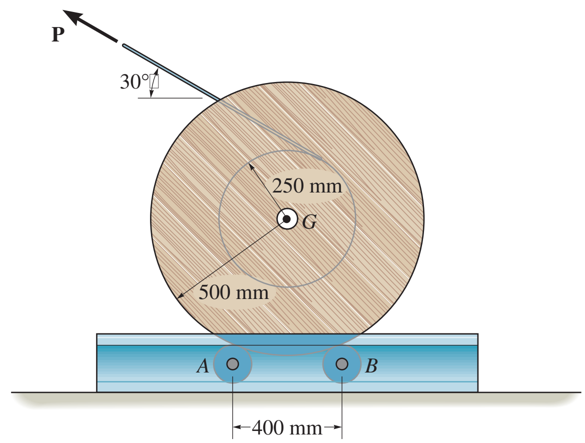
$$= \frac{1}{2}\left(mr_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } mr_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2}I_{IC}\,\omega^2$$

Q.E.D.

18–2.

A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.

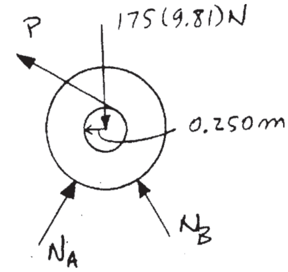


SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$

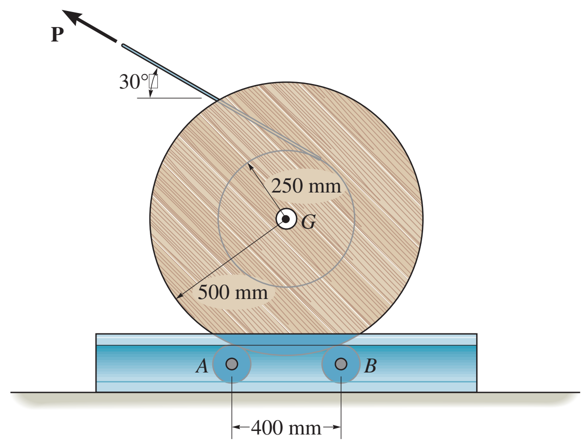
$\omega = 2.02$ rad/s **Ans.**



Ans:
$\omega = 2.02$ rad/s

18–3.

A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn without slipping on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the cable. Each roller can be considered as an 18-kg cylinder, having a radius of 0.1 m. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



SOLUTION

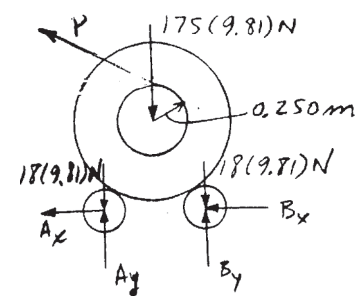
System:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$[0 + 0 + 0] + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2 + 2\left[\frac{1}{2}(18)(0.1)^2\right]\omega_r^2$$

$$v = \omega_r(0.1) = \omega(0.5)$$

$$\omega_r = 5\omega$$



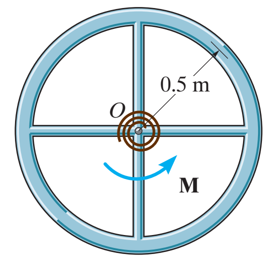
Solving:

$\omega = 1.78$ rad/s **Ans.**

Ans:
$\omega = 1.78$ rad/s

***18–4.**

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2 \text{ N} \cdot \text{m/rad}$, and the wheel is rotated until the torque $M = 25 \text{ N} \cdot \text{m}$ is developed, determine the maximum angular velocity of the wheel if it is released from rest.



SOLUTION

Kinetic Energy and Work: The mass moment of inertia of the wheel about point O is

$$I_O = m_R r^2 + 2\left(\frac{1}{12} m_r l^2\right)$$

$$= 5(0.5^2) + 2\left[\frac{1}{12}(2)(1^2)\right]$$

$$= 1.5833 \text{ kg} \cdot \text{m}^2$$

Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}(1.5833)\,\omega^2 = 0.79167\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$. The torque developed is $M = k\theta = 2\theta$. Here, the angle of rotation needed to develop a torque of $M = 25 \text{ N} \cdot \text{m}$ is

$$2\theta = 25 \qquad \theta = 12.5 \text{ rad}$$

The wheel achieves its maximum angular velocity when the spacing is unwound that is when the wheel has rotated $\theta = 12.5$ rad. Thus, the work done by $\underset{\infty}{\mathbf{M}}$ is

$$U_M = \int M\,d\theta = \int_0^{12.5 \text{ rad}} 2\theta\,d\theta$$

$$= \theta^2 \Big|_0^{12.5 \text{ rad}} = 156.25 \text{ J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

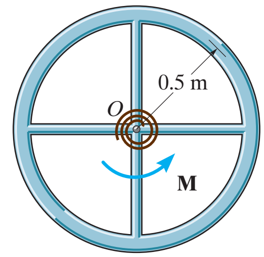
$$0 + 156.25 = 0.79167\,\omega^2$$

$$\omega = 14.0 \text{ rad/s}$$ **Ans.**

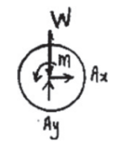
Ans:
$\omega = 14.0 \text{ rad/s}$

18–5.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



SOLUTION



$$I_o = 2\left[\frac{1}{12}(2)(1)^2\right] + 5(0.5)^2 = 1.583$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{4\pi} 2\theta\, d\theta = \frac{1}{2}(1.583)\,\omega^2$$

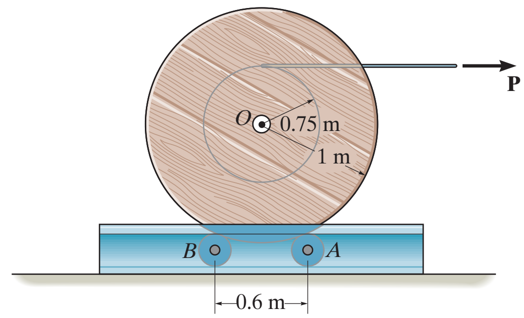
$$(4\pi)^2 = 0.7917\omega^2$$

$\omega = 14.1$ rad/s **Ans.**

Ans:
$\omega = 14.1$ rad/s

18–6.

A force of $P = 60$ N is applied to the cable, which causes the 200-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. Assume the radius of gyration of the reel about its center axis remains constant at $k_O = 0.6$ m.



SOLUTION

Kinetic Energy. Since the reel is at rest initially, $T_1 = 0$. The mass moment of inertia of the reel about its center O is $I_0 = mk_0^2 = 200(0.6^2) = 72.0 \text{ kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(72.0)\omega^2 = 36.0\,\omega^2$$

Work. Referring to the FBD of the reel, Fig. a, only force **P** does positive work. When the reel rotates 2 revolution, force **P** displaces $S = \theta r = 2(2\pi)(0.75) = 3\pi$ m. Thus

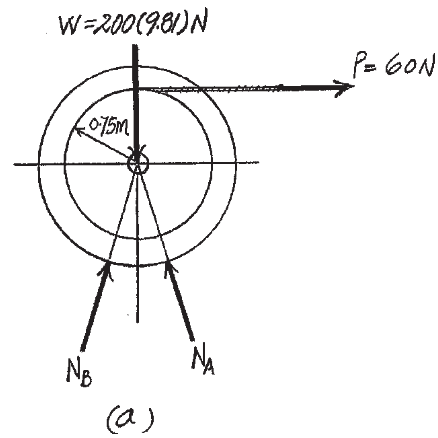
$$U_p = P_s = 60(3\pi) = 180\pi \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

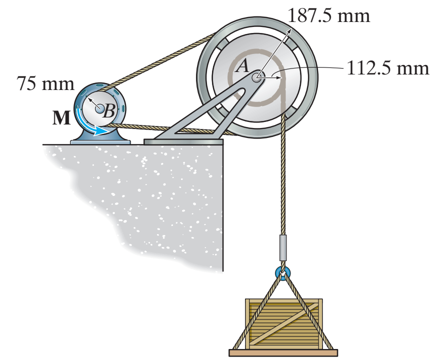
$$0 + 180\pi = 36.0\,\omega^2$$

$$\omega = 3.9633 \text{ rad/s} = 3.96 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 3.96$ rad/s

18–7. The wheel and the attached reel have a combined mass of 25 kg and a radius of gyration about their center of $k_A = 150$ mm. If pulley B attached to the motor is subjected to a torque of $M = 60(2 - e^{-0.1\theta})$ N·m, where θ is in radians, determine the velocity of the 100-kg crate after it has moved upwards a distance of 1.5 m, starting from rest. Neglect the mass of pulley B.



SOLUTION

Kinetic Energy and Work: Since the wheel rotates about a fixed axis, $v_C = \omega r_C = \omega(0.1125)$. The mass moment of inertia of A about its mass center is $I_A = mk_A^2 = (25)(0.15^2) = 0.5625\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A \omega^2 + \frac{1}{2} m_C v_C^2$$

$$= \frac{1}{2}(0.5625)\omega^2 + \frac{1}{2}(100)[\omega(0.1125)]^2$$

$$= 0.91406\omega^2$$

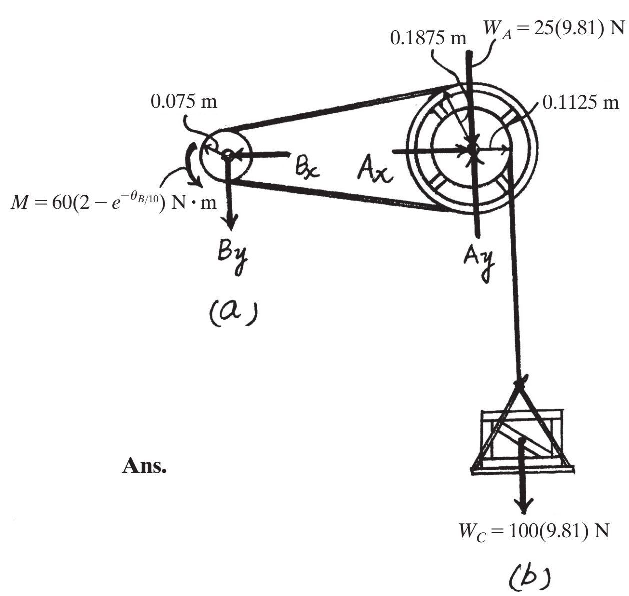
Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When crate C moves 1.5 m upward, wheel A rotates through an angle of $\theta_A = \dfrac{s_C}{r} = \dfrac{1.5}{0.1125} = 13.333$ rad. Then, pulley B rotates through an angle of $\theta_B = \dfrac{r_A}{r_B}\theta_A = \left(\dfrac{0.1875}{0.075}\right)(13.333) = 33.33$ rad. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = \int M d\theta_B = \int_0^{33.33\text{ rad}} 60\left(2 - e^{-0.1\theta}\right)d\theta$$

$$= \left[60\left(2\theta + 10e^{-0.1\theta}\right)\right]\Big|_0^{33.33\text{ rad}}$$

$$= 3421.01\text{ J}$$

$$U_{W_C} = -W_C s_C = -100(9.81)(1.5) = -1471.5\text{ J}$$



Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [3421.01 - 1471.5] = 0.91406\omega^2$$

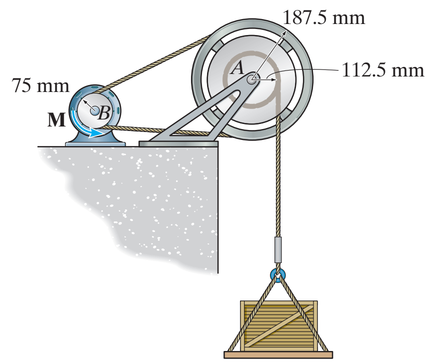
$$\omega = 46.18\text{ rad/s}$$

Thus,

$$v_C = 46.18(0.1125) = 5.20\text{ m/s} \uparrow$$ **Ans.**

Ans:
$v_C = 5.20$ m/s ↑

***18–8.** The wheel and the attached reel have a combined mass of 25 kg and a radius of gyration about their center of $k_A = 150$ mm. If pulley B attached to the motor is subjected to a torque of $M = 75$ N·m, determine the velocity of the 100-kg crate after the pulley has turned 5 revolutions. Neglect the mass of the pulley.



SOLUTION

Kinetic Energy and Work: Since the wheel at A rotates about a fixed axis, $v_C = \omega r_C = \omega(0.1125)$. The mass moment of inertia of wheel A about its mass center is $I_A = mk_A{}^2 = (25)(0.15^2) = 0.5625$ kg·m². Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A \omega^2 + \frac{1}{2} m_C v_C{}^2$$

$$= \frac{1}{2}(0.5625)\omega^2 + \frac{1}{2}(100)[\omega(0.1125)]^2$$

$$= 0.91406\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When pulley B rotates $\theta_B = (5 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 10\pi$ rad, the wheel rotates through an angle of $\theta_A = \frac{r_B}{r_A}\theta_B = \left(\frac{0.075}{0.1875}\right)(10\pi) = 4\pi$. Thus, the crate displaces upwards through a distance of $s_C = r_C\,\theta_A = 0.1125(4\pi) = 0.45\pi$ m. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = M\theta_B = 75(10\pi) = 750\pi \text{ J}$$

$$U_{W_C} = -W_C\,s_C = -100(9.81)(0.45\pi) = -441.45\pi \text{ J}$$

Principle of Work and Energy:

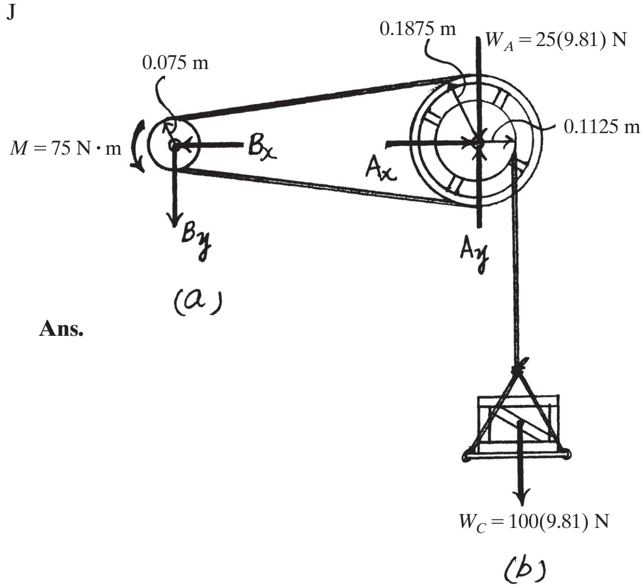
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [750\pi - 441.45\pi] = 0.91406\omega^2$$

$$\omega = 32.56 \text{ rad/s}$$

Thus,

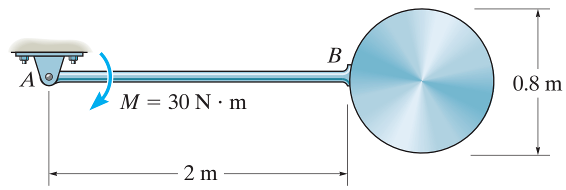
$$v_C = 32.56(0.1125) = 3.66 \text{ m/s} \uparrow$$ **Ans.**



Ans:
$v_C = 3.66$ m/s ↑

18–9.

The pendulum consists of a 10-kg uniform disk and a 3-kg uniform slender rod. If it is released from rest in the position shown, determine its angular velocity when it rotates clockwise 90°.



SOLUTION

Kinetic Energy. Since the assembly is released from rest, initially, $T_1 = 0$. The mass moment of inertia of the assembly about A is

$I_A = \left[\frac{1}{12}(3)(2^2) + 3(1^2)\right] + \left[\frac{1}{2}(10)(0.4^2) + 10(2.4^2)\right] = 62.4\ \text{kg}\cdot\text{m}^2$. Thus,

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(62.4)\,\omega^2 = 31.2\ \omega^2$$

Work. Referring to the FBD of the assembly, Fig. *a*. Both $\mathbf{W}_r$ and $\mathbf{W}_d$ do positive work, since they displace vertically downward $S_r = 1$ m and $S_d = 2.4$ m, respectively. Also, couple moment **M** does positive work

$$U_{W_r} = W_r S_r = 3(9.81)(1) = 29.43\ \text{J}$$

$$U_{W_d} = W_d S_d = 10(9.81)(2.4) = 235.44\ \text{J}$$

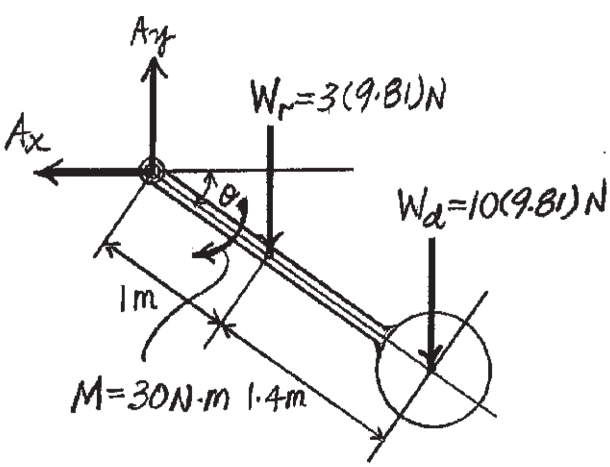
$$U_M = M\theta = 30\left(\frac{\pi}{2}\right) = 15\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 29.43 + 235.44 + 15\pi = 31.2\ \omega^2$$

$$\omega = 3.1622\ \text{rad/s} = 3.16\ \text{rad/s}$$ **Ans.**

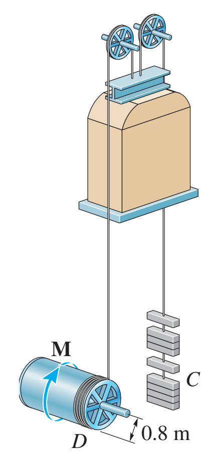


(a)

Ans:
$\omega = 3.16$ rad/s

18–10.

A motor supplies a constant torque $M = 6\ \text{kN}\cdot\text{m}$ to the winding drum that operates the elevator. If the elevator has a mass of 900 kg, the counterweight C has a mass of 200 kg, and the winding drum has a mass of 600 kg and radius of gyration about its axis of $k = 0.6$ m, determine the speed of the elevator after it rises 5 m starting from rest. Neglect the mass of the pulleys.

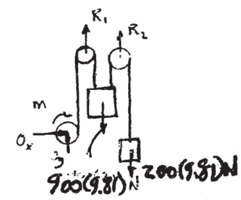


SOLUTION

$$v_E = v_C$$

$$\theta = \frac{s}{r} = \frac{5}{0.8}$$

$$T_1 + \Sigma U_{1-2} = T_2$$



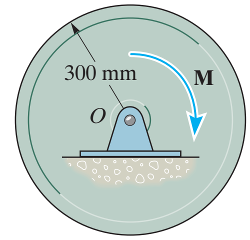
$$0 + 6000\left(\frac{5}{0.8}\right) - 900(9.81)(5) + 200(9.81)(5) = \frac{1}{2}(900)(v)^2 + \frac{1}{2}(200)(v)^2 + \frac{1}{2}[600(0.6)^2]\left(\frac{v}{0.8}\right)^2$$

$$v = 2.10\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 2.10$ m/s

18–11.

The disk, which has a mass of 20 kg, is subjected to the couple moment of $M = (2\theta + 4)$ N·m, where θ is in radians. If it starts from rest, determine its angular velocity when it has made two revolutions.



SOLUTION

Kinetic Energy. Since the disk starts from rest, $T_1 = 0$. The mass moment of inertia of the disk about its center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.3^2) = 0.9\text{ kg}\cdot\text{m}^2$. Thus

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(0.9)\omega^2 = 0.45\omega^2$$

Work. Referring to the FBD of the disk, Fig. *a*, only couple moment **M** does work, which it is positive

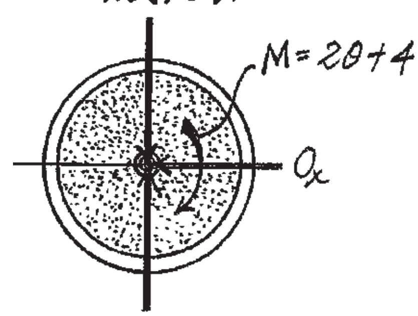
$$U_M = \int M\,d\theta = \int_0^{2(2\pi)}(2\theta + 4)d\theta = \theta^2 + 4\theta\Big|_0^{4\pi} = 208.18\text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 208.18 = 0.45\omega^2$$

$$\omega = 21.51\text{ rad/s} = 21.5\text{ rad/s}$$ **Ans.**

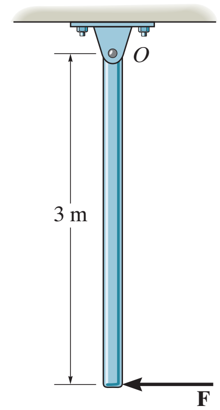


(a)

Ans:
$\omega = 21.5$ rad/s

***18–12.**

The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 90° clockwise from the position shown. The force is always perpendicular to the rod.



SOLUTION

Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O *is* $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0 \text{ kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(30.0)\,\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. *a*, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 90°$, $S_W = 1.5$ m and $S_F = \theta r = \left(\frac{\pi}{2}\right)(3) = \frac{3\pi}{2}$ m. Thus

$$U_F = 150\left(\frac{3\pi}{2}\right) = 225\pi \text{ J}$$

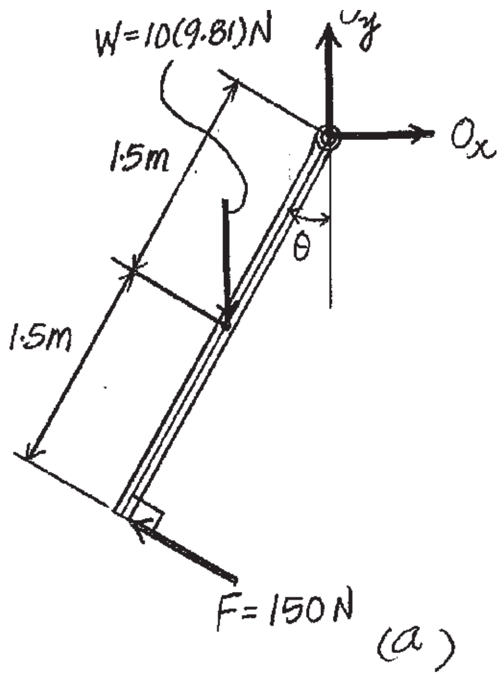
$$U_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 225\pi + (-147.15) = 15.0\,\omega^2$$

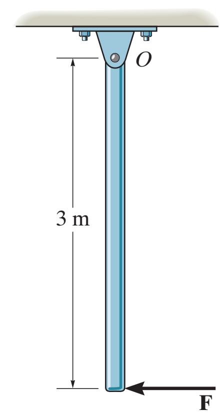
$$\omega = 6.1085 \text{ rad/s} = 6.11 \text{ rad/s}$$ **Ans.**



Ans:
$\omega = 6.11$ rad/s

18–13.

The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 180° clockwise from the position shown. The force is always perpendicular to the rod.



SOLUTION

Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O is $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0 \text{ kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(30.0)\,\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. *a*, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 180°$, $S_W = 3$ m and $S_F = \theta r = \pi(3) = 3\pi$ m. Thus

$$U_F = 150(3\pi) = 450\pi \text{ J}$$

$$U_W = -10(9.81)(3) = -294.3 \text{ J}$$

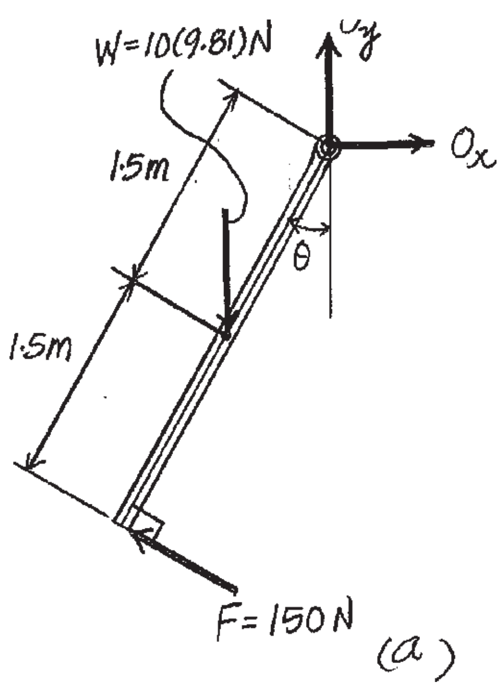
Principle of Work and Energy. Applying Eq. 18,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 450\pi + (-294.3) = 15.0\,\omega^2$$

$$\omega = 8.6387 \text{ rad/s} = 8.64 \text{ rad/s}$$

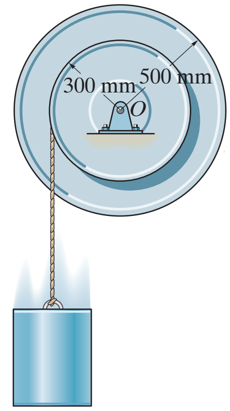
Ans.



Ans:
$\omega = 8.64$ rad/s

18–14.

The spool has a mass of 40 kg and a radius of gyration of $k_O = 0.3$ m. If the 10-kg block is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 15$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

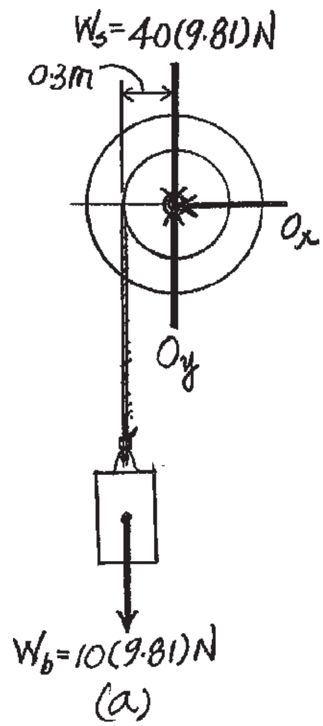
Kinetic Energy. Since the system is released from rest, $T_1 = 0$. The final velocity of the block is $v_b = \omega r = 15(0.3) = 4.50$ m/s. The mass moment of inertia of the spool about O is $I_0 = mk_0^2 = 40(0.3^2) = 3.60$ Kg · m². Thus

$$T_2 = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(3.60)(15^2) + \frac{1}{2}(10)(4.50^2)$$

$$= 506.25 \text{ J}$$

For the block, $T_1 = 0$ and $T_2 = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(10)(4.50^2) = 101.25$ J



Work. Referring to the FBD of the system Fig. *a*, only $\mathbf{W}_b$ does work when the block displaces *s* vertically downward, which it is positive.

$$U_{W_b} = W_b s = 10(9.81)s = 98.1\,s$$

Referring to the FBD of the block, Fig. *b*. $\mathbf{W}_b$ does positive work while **T** does negative work.

$$U_T = -Ts$$

$$U_{W_b} = W_b s = 10(9.81)(s) = 98.1\,s$$

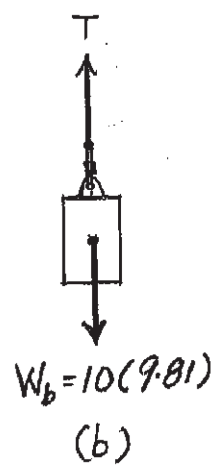
Principle of Work and Energy. For the system,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1s = 506.25$$

$$s = 5.1606 \text{ m} = 5.16 \text{ m} \qquad \textbf{Ans.}$$

For the block using the result of *s*,



$$T_1 + \Sigma U_{1-2} = T_2$$

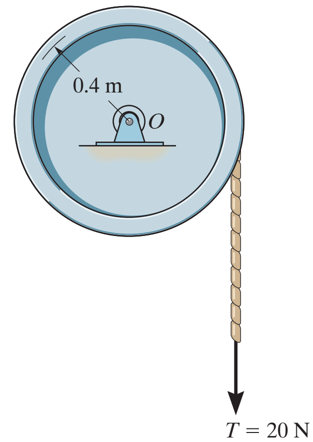
$$0 + 98.1(5.1606) - T(5.1606) = 101.25$$

$$T = 78.48 \text{ N} = 78.5 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$s = 5.16$ m
$T = 78.5$ N

18–15.

The force of $T = 20$ N is applied to the cord of negligible mass. Determine the angular velocity of the 20-kg wheel when it has rotated 4 revolutions starting from rest. The wheel has a radius of gyration of $k_O = 0.3$ m.



SOLUTION

Kinetic Energy. Since the wheel starts from rest, $T_1 = 0$. The mass moment of inertia of the wheel about point O is $I_0 = mk_0^2 = 20\left(0.3^2\right) = 1.80 \text{ kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(1.80)\,\omega^2 = 0.9\,\omega^2$$

Work. Referring to the FBD of the wheel, Fig. *a*, only force **T** does work. This work is positive since **T** is required to displace vertically downward, $s_T = \theta r = 4(2\pi)(0.4) = 3.2\pi$ m.

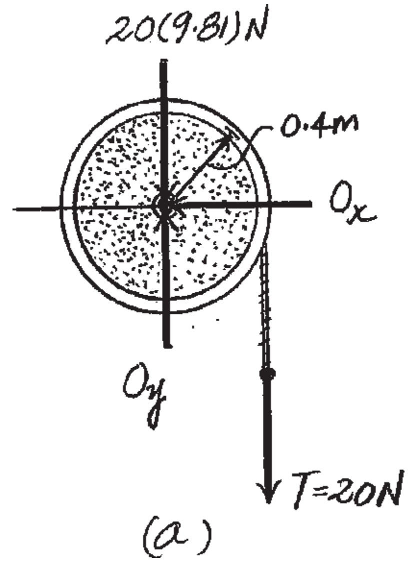
$$U_T = Ts_T = 20(3.2\pi) = 64\pi \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 64\pi = 0.9\,\omega^2$$

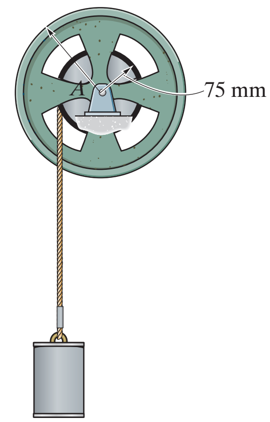
$$\omega = 14.94 \text{ rad/s} = 14.9 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 14.9$ rad/s

***18–16.**

Determine the velocity of the 50-kg cylinder after it has descended a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg and a radius of gyration about its center of mass A of $k_A = 125$ mm.

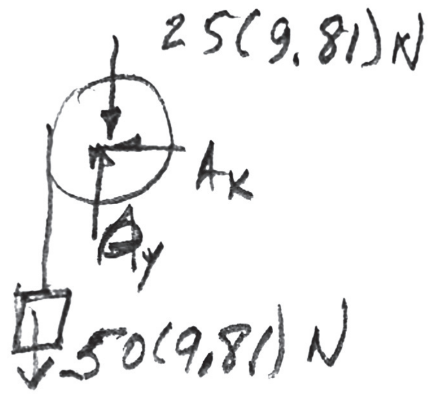


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

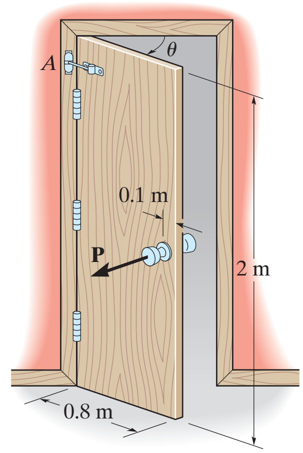
$$0 + 50(9.81)(2) = \frac{1}{2}[(25)(0.125)^2]\left(\frac{v}{0.075}\right)^2 + \frac{1}{2}(50)\,v^2$$

$$v = 4.05 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 4.05$ m/s

18–17. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80\ \text{N}\cdot\text{m/rad}$, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint*: For a torsional spring $M = k\theta$, where k is the stiffness and θ is the angle of twist.



SOLUTION

Given:

$M = 20\ \text{kg}$ $\quad a = 0.8\ \text{m}$

$k = 80\ \text{N·m/rad}$ $\quad b = 0.1\ \text{m}$

$\omega = 12\ \text{rad/s}$ $\quad c = 2\ \text{m}$

$P = 0\ \text{N}$

Guess $\quad \theta_0 = 1\ \text{rad}$

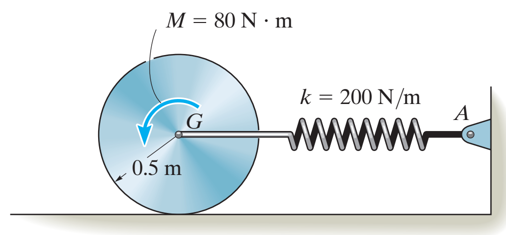
Given

$$\int_{\theta_0+90°}^{\theta_0} -k\,\theta\,d\theta = \frac{1}{2}\frac{1}{3}Ma^2\omega^2 \qquad \theta_0 = \text{Find}(\theta_0) \qquad \theta_0 = 1.66\ \text{rad}\quad \textbf{Ans.}$$

Ans:
$\theta_0 = 1.66\ \text{rad}$

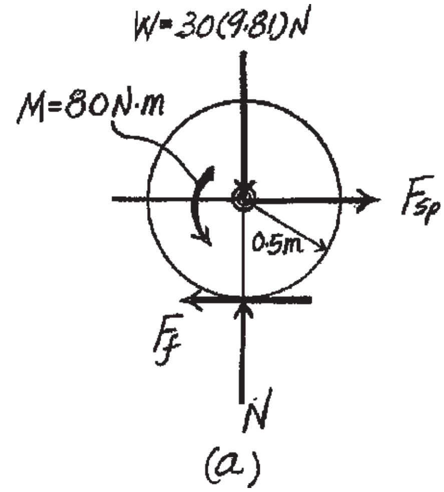
18–18.

The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment of $M = 80\ \text{N} \cdot \text{m}$ is then applied to the disk as shown. Determine its angular velocity when its mass center G has moved 0.5 m along the plane. The disk rolls without slipping.



SOLUTION

Kinetic Energy. Since the disk is at rest initially, $T_1 = 0$. The disk rolls without slipping. Thus, $v_G = \omega_r = \omega(0.5)$. The mass moment of inertia of the disk about its center of gravity G is $I_G = \frac{1}{2}mr = \frac{1}{2}(30)(0.5^2) = 3.75\ \text{kg} \cdot \text{m}^2$. Thus,



$$T_2 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}Mv_G^2$$

$$= \frac{1}{2}(3.75)\omega^2 + \frac{1}{2}(30)[\omega(0.5)]^2$$

$$= 5.625\ \omega^2$$

Work. Since the disk rolls without slipping, the friction $\mathbf{F}_f$ does no work. Also when the center of the disk moves $S_G = 0.5$ m, the disk rotates $\theta = \frac{s_G}{r} = \frac{0.5}{0.5} = 1.00$ rad. Here, couple moment $\mathbf{M}$ does positive work whereas the spring force does negative work.

$$U_M = M\theta = 80(1.00) = 80.0\ \text{J}$$

$$U_{F_{sp}} = -\frac{1}{2}kx^2 = -\frac{1}{2}(200)(0.5^2) = -25.0\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

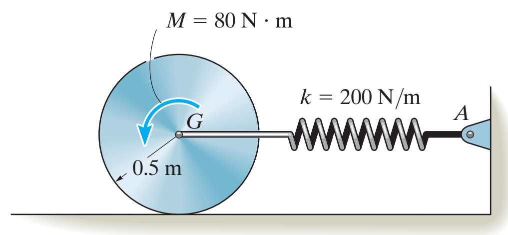
$$0 + 80 + (-25.0) = 5.625\ \omega^2$$

$$\omega = 3.127\ \text{rad/s} = 3.13\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 3.13$ rad/s

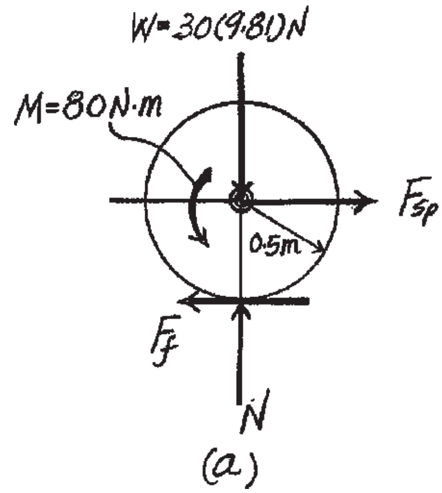
18–19.

The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment $M = 80 \text{ N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels along the plane before it momentarily stops. The disk rolls without slipping.



SOLUTION

Kinetic Energy. Since the disk is at rest initially and required to stop finally, $T_1 = T_2 = 0$.



Work. Since the disk rolls without slipping, the friction $\mathbf{F}_f$ does no work. Also, when the center of the disk moves s_G, the disk rotates $\theta = \dfrac{s_G}{r} = \dfrac{s_G}{0.5} = 2\,s_G$. Here, couple moment $\mathbf{M}$ does positive work whereas the spring force does negative work.

$$U_M = M\theta = 80(2s_G) = 160\,s_G$$

$$U_{F_{sp}} = -\frac{1}{2}kx^2 = -\frac{1}{2}(200)\,s_G^2 = -100\,s_G^2$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 160\,s_G + \left(-100\,s_G^2\right) = 0$$

$$160\,s_G - 100\,s_G^2 = 0$$

$$s_G(160 - 100\,s_G) = 0$$

Since $s_G \neq 0$, then

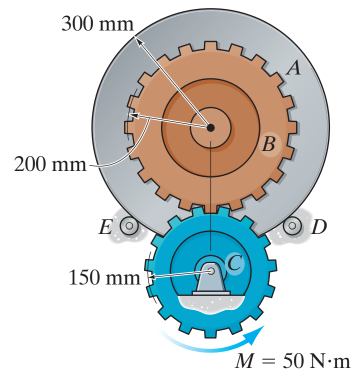
$$160 - 100\,s_G = 0$$

$$s_G = 1.60 \text{ m}$$ **Ans.**

Ans:
$s_G = 1.60 \text{ m}$

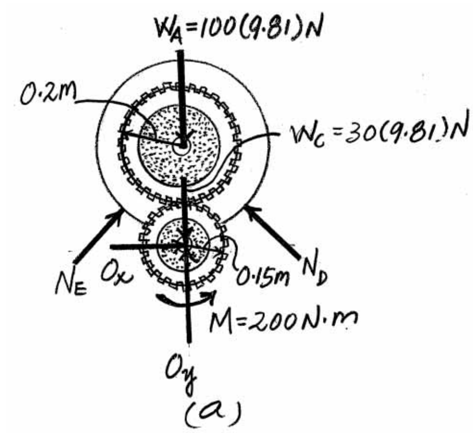
***18–20.**

Gear B is rigidly attached to drum A and is supported by two small rollers at E and D. Gear B is in mesh with gear C and is subjected to a torque of $M = 50\ \text{N}\cdot\text{m}$. Determine the angular velocity of the drum after C has rotated 10 revolutions, starting from rest. Gear B and the drum have 100 kg and a radius of gyration about their rotating axis of 250 mm. Gear C has a mass of 30 kg and a radius of gyration about its rotating axis of 125 mm.



SOLUTION

Kinetic Energy and Work: Since gear B is in mesh with gear C and both gears rotate about fixed axes, $\omega_C = \left(\dfrac{r_B}{r_C}\right)\omega_A = \left(\dfrac{0.2}{0.15}\right)\omega_A = 1.333\omega_A$. The mass moment of the drum and gear C about their rotating axes are $I_A = m_A k^2 = 100(0.25^2) = 6.25\ \text{kg}\cdot\text{m}^2$ and $I_C = m_C k^2 = 30(0.125^2) = 0.46875\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is



$$T = T_A + T_C$$

$$= \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_C\omega_C^2$$

$$= \frac{1}{2}(6.25)\omega_A^2 + \frac{1}{2}(0.46875)(1.333\omega_A)^2$$

$$= 3.5417\omega_A^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, **M** does positive work. When the gear C rotates $\theta = (10\ \text{rev})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 20\pi$, the work done by **M** is

$$U_M = 50(20\pi) = 1000\pi\ \text{J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

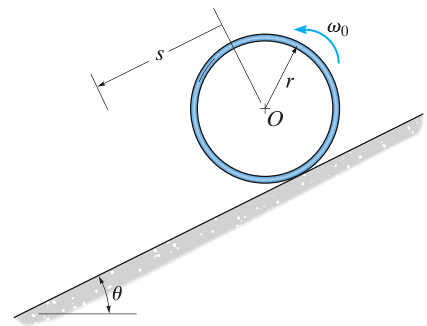
$$0 + 1000\pi = 3.5417\omega_A^2$$

$$\omega_A = 29.8\ \text{rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_A = 29.8\ \text{rad/s}$

18–21.

The center O of the thin ring of mass m is given an angular velocity of ω_0. If the ring rolls without slipping, determine its angular velocity after it has traveled a distance of s down the plane. Neglect its thickness.

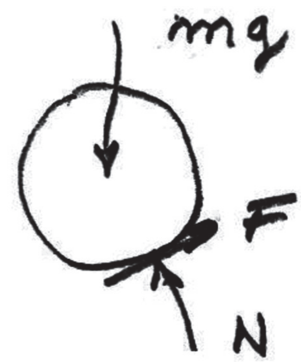


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(mr^2 + mr^2)\omega_0^2 + mg(s\sin\theta) = \frac{1}{2}(mr^2 + mr^2)\omega^2$$

$$\omega = \sqrt{\omega_0^2 + \frac{g}{r^2}s\sin\theta}$$ **Ans.**

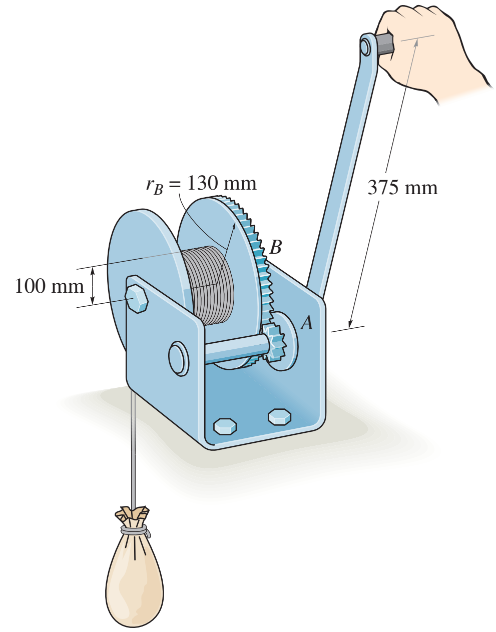


Ans:

$$\omega = \sqrt{\omega_0^2 + \frac{g}{r^2}s\sin\theta}$$

18–22.

The hand winch is used to lift the 50-kg load. Determine the work required to rotate the handle five revolutions. The gear at A has a radius of 20 mm.



SOLUTION

$$20(\theta_A) = \theta_B(130)$$

When $\theta_A = 5 \text{ rev.} = 10\,\pi$

$$\theta_B = 4.8332 \text{ rad}$$

Thus load moves up

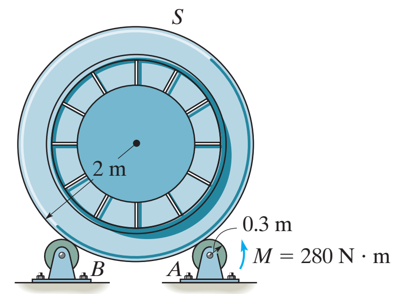
$$s = 4.8332(0.1 \text{ m}) = 0.48332 \text{ m}$$

$$U = 50(9.81)(0.48332) = 237 \text{ J}$$ **Ans.**

Ans:
$U = 237$ J

18–23.

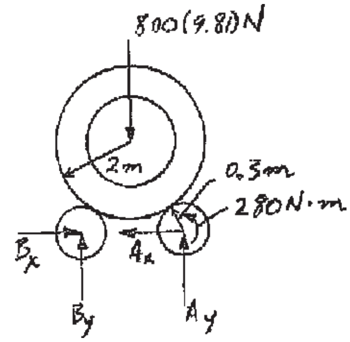
The rotary screen S is used to wash limestone. When empty it has a mass of 800 kg and a radius of gyration of $k_G = 1.75$ m. Rotation is achieved by applying a torque of $M = 280\ \text{N} \cdot \text{m}$ about the drive wheel at A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated 5 revolutions. Neglect the mass of A and B.



SOLUTION

$$T_S + \Sigma U_{1-2} = T_2$$

$$0 + 280(\theta_A) = \frac{1}{2}[800(1.75)^2]\,\omega^2$$



$$\theta_S(2) = \theta_A(0.3)$$

$$5(2\pi)(2) = \theta_A(0.3)$$

$$\theta_A = 209.4 \text{ rad}$$

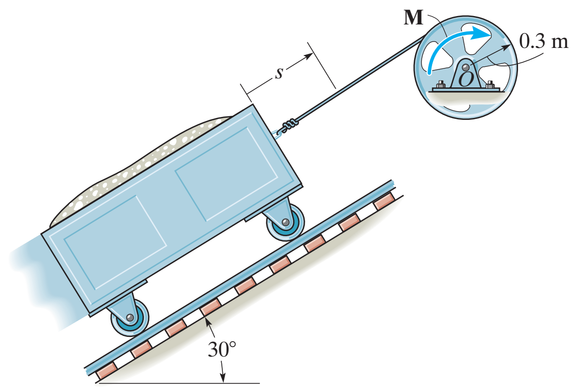
Thus

$$\omega = 6.92 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 6.92$ rad/s

***18–24.**

The wheel has a mass of 100 kg and a radius of gyration of $k_O = 0.2$ m. A motor supplies a torque $M = (40\theta + 900)\ \text{N} \cdot \text{m}$, where θ is in radians, about the drive shaft at O. Determine the speed of the loading car, which has a mass of 300 kg, after it travels $s = 4$ m. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.



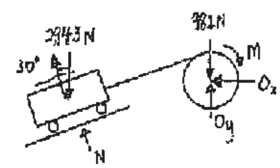
SOLUTION

$s = 0.3\theta = 4$

$\theta = 13.33$ rad

$T_1 + \Sigma U_{1-2} = T_2$

$$[0 + 0] + \int_0^{13.33} (40\theta + 900)d\theta - 300(9.81)\sin 30°\,(4) = \frac{1}{2}(300)v_C^2 + \frac{1}{2}\left[100(0.20)^2\right]\left(\frac{v_C}{0.3}\right)^2$$

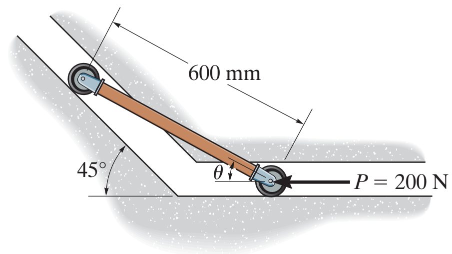


$v_C = 7.49$ m/s **Ans.**

Ans:
$v_C = 7.49$ m/s

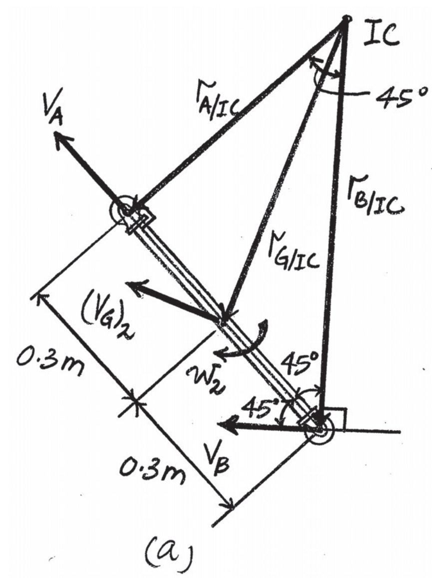
18–25.

If $P = 200$ N and the 15-kg uniform slender rod starts from rest at $\theta = 0°$, determine the rod's angular velocity at the instant just before $\theta = 45°$.



SOLUTION

Kinetic Energy and Work: Referring to Fig. *a*,



$$r_{A/IC} = 0.6 \tan 45° = 0.6 \text{ m}$$

Then

$$r_{G/IC} = \sqrt{0.3^2 + 0.6^2} = 0.6708 \text{ m}$$

Thus,

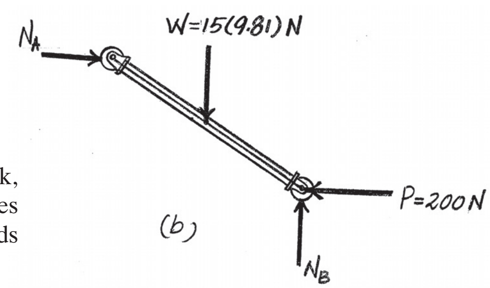
$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(0.6708)$$

The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(15)(0.6^2) = 0.45 \text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\,\omega_2{}^2$$

$$= \frac{1}{2}(15)[w_2(0.6708)]^2 + \frac{1}{2}(0.45)\,\omega_2{}^2$$

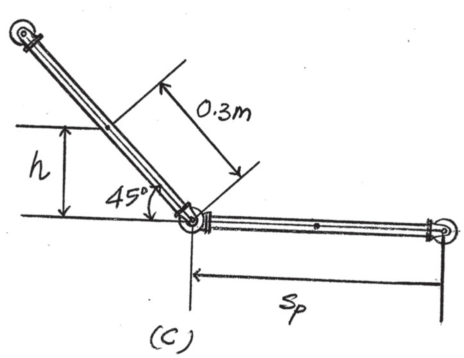
$$= 3.6\omega_2{}^2$$



Since the rod is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work and **W** does negative work. When $\theta = 45°$, **P** displaces through a horizontal distance $s_P = 0.6$ m and W displaces vertically upwards through a distance of $h = 0.3 \sin 45°$, Fig. *c*. Thus, the work done by **P** and **W** is

$$U_P = Ps_P = 200(0.6) = 120 \text{ J}$$

$$U_W = -Wh = -15(9.81)(0.3 \sin 45°) = -31.22 \text{ J}$$



Principle of Work and Energy:

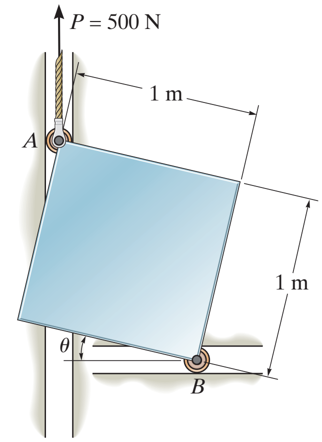
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [120 - 31.22] = 3.6\omega_2{}^2$$

$$\omega_2 = 4.97 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_2 = 4.97$ rad/s

18–26. If corner A of the 60-kg plate is subjected to a vertical force of $P = 500$ N, and the plate is released from rest when $\theta = 0°$, determine the angular velocity of the plate when $\theta = 45°$.



SOLUTION

Kinetic Energy and Work: Since the plate is initially at rest, $T_1 = 0$. Referring to Fig. a,

$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1 \cos 45°) = 0.7071\omega_2$$

The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(60)\left(1^2 + 1^2\right) = 10 \text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}m(60)(0.7071\omega_2)^2 + \frac{1}{2}(10)\omega_2{}^2$$

$$= 20\omega_2{}^2$$

Referring to Fig. b, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work, and **W** does negative work. When $\theta = 45°$, **W** and **P** displace upwards through a distance of $h = 1 \cos 45° - 0.5 = 0.2071$ m and $s_P = 2(1 \cos 45°) - 1 = 0.4142$ m. Thus, the work done by **P** and **W** is

$$U_P = Ps_P = 500(0.4142) = 207.11 \text{ J}$$

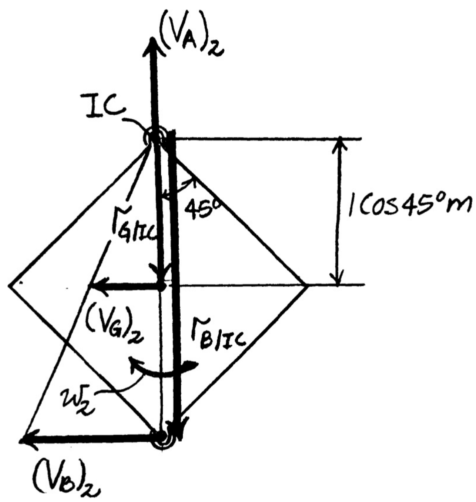
$$U_W = -Wh = -60(9.81)(0.2071) = -121.90 \text{ J}$$

Principle of Work and Energy:

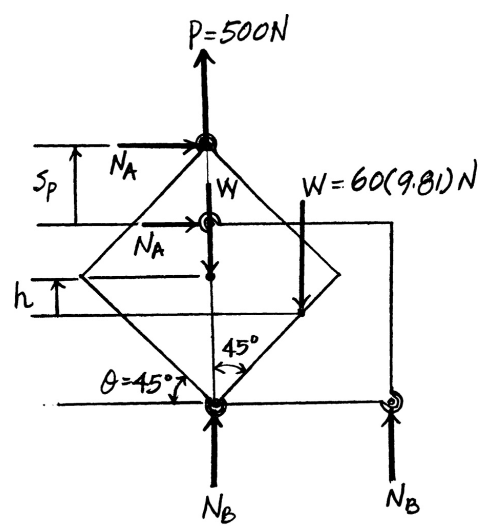
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [207.11 - 121.90] = 20\omega_2{}^2$$

$$\omega_2 = 2.06 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

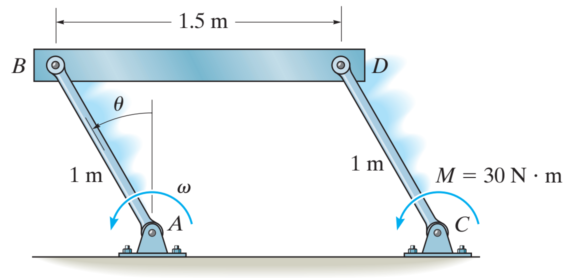


(b)

Ans:
$\omega_2 = 2.06$ rad/s

18–27.

The linkage consists of two 6-kg rods AB and CD and a 20-kg bar BD. When $\theta = 0^\circ$, rod AB is rotating with an angular velocity $\omega = 2$ rad/s. If rod CD is subjected to a couple moment of $M = 30\ \text{N}\cdot\text{m}$, determine ω_{AB} at the instant $\theta = 90^\circ$.



SOLUTION

Kinetic Energy. The mass moment of inertia of each link about the axis of rotation is $I_A = \frac{1}{12}(6)(1^2) + 6(0.5^2) = 2.00\ \text{kg}\cdot\text{m}$. The velocity of the center of mass of the bar is $v_G = \omega r = \omega(1)$. Thus,

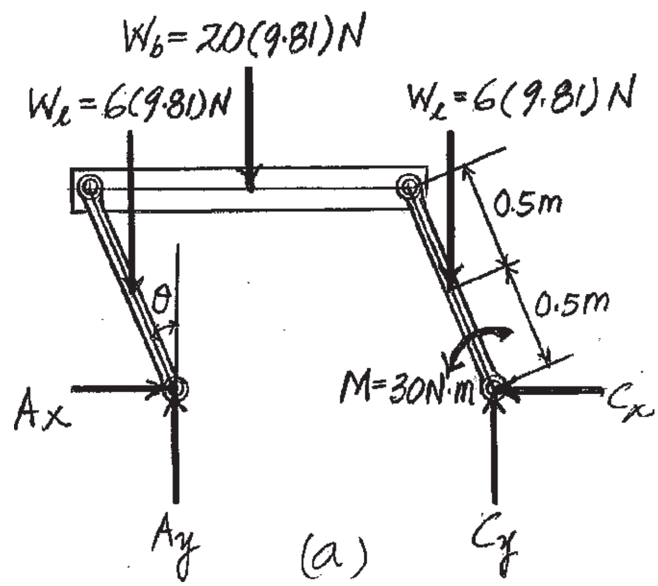
$$T = 2\left(\frac{1}{2}I_A\omega^2\right) + \frac{1}{2}M_b v_G^2$$

$$= 2\left[\frac{1}{2}(2.00)\omega^2\right] + \frac{1}{2}(20)[\omega(1)]^2$$

$$= 12.0\,\omega^2$$

Initially, $\omega = 2$ rad/s. Then

$$T_1 = 12.0(2^2) = 48.0\ \text{J}$$



Work. Referring to the FBD of the assembly, Fig. *a*, the weights W_b, W_c and couple moment **M** do positive work when the links undergo an angular displacement θ. When $\theta = 90^\circ = \frac{\pi}{2}$ rad,

$$U_{W_b} = W_b s_b = 20(9.81)(1) = 196.2\ \text{J}$$

$$U_{W_c} = W_c s_c = 6(9.81)(0.5) = 29.43\ \text{J}$$

$$U_M = M\theta = 30\left(\frac{\pi}{2}\right) = 15\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

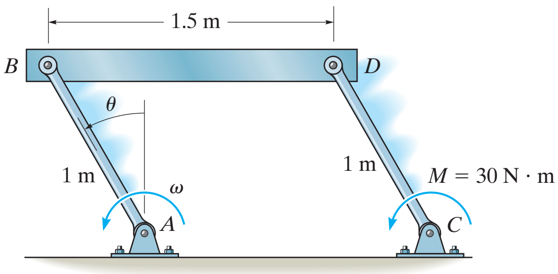
$$48.0 + [196.2 + 2(29.43) + 15\pi] = 12.0\,\omega^2$$

$$\omega = 5.4020\ \text{rad/s} = 5.40\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 5.40$ rad/s

***18–28.**

The linkage consists of two 6-kg rods AB and CD and a 20-kg bar BD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega = 2$ rad/s. If rod CD is subjected to a couple moment $M = 30\ \text{N}\cdot\text{m}$, determine ω at the instant $\theta = 45°$.



SOLUTION

Kinetic Energy. The mass moment of inertia of each link about the axis of rotation is $I_A = \frac{1}{12}(6)(1^2) + 6(0.5^2) = 2.00\ \text{kg}\cdot\text{m}^2$. The velocity of the center of mass of the bar is $v_G = \omega r = \omega(1)$. Thus,

$$T = 2\left(\frac{1}{2}I_A\omega_A^2\right)^2 + \frac{1}{2}m_b v_G^2$$

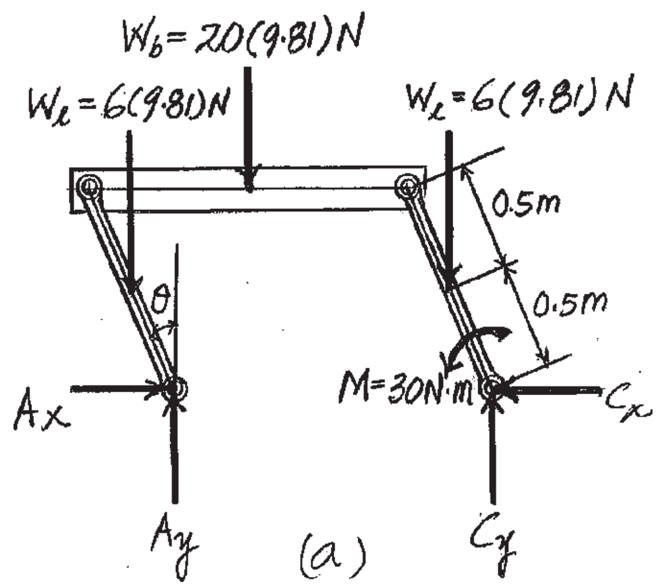
$$= 2\left[\frac{1}{2}(2.00)\omega^2\right] + \frac{1}{2}(20)[\omega(1)]^2$$

$$= 12.0\,\omega^2$$

Initially, $\omega = 2$ rad/s. Then

$$T_1 = 12.0(2^2) = 48.0\ \text{J}$$

Work. Referring to the FBD of the assembly, Fig. a, the weights W_b, W_c and couple moment $\mathbf{M}$ do positive work when the links undergo an angular displacement θ. when $\theta = 45^\circ = \frac{\pi}{4}$ rad,



$$U_{W_b} = W_b s_b = 20(9.81)(1 - \cos 45^\circ) = 57.47\ \text{J}$$

$$U_{W_c} = W_c s_c = 6(9.81)[0.5(1 - \cos 45°)] = 8.620\ \text{J}$$

$$U_M = M\theta = 30\left(\frac{\pi}{4}\right) = 7.5\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

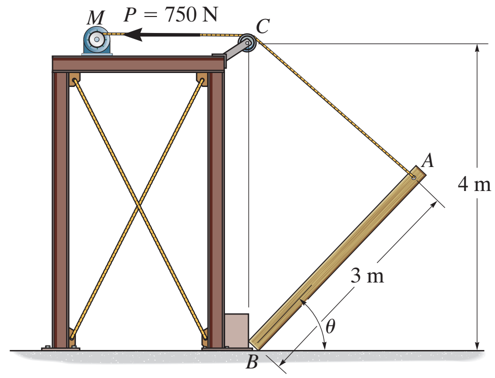
$$48.0 + [57.47 + 2(8.620) + 7.5\pi] = 12.0\,\omega^2$$

$$\omega = 3.4913\ \text{rad/s} = 3.49\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 3.49$ rad/s

18–29.

Motor M exerts a constant force of $P = 750$ N on the rope. If the 100-kg post is at rest when $\theta = 0°$, determine the angular velocity of the post at the instant $\theta = 60°$. Neglect the mass of the pulley and its size, and consider the post as a slender rod.



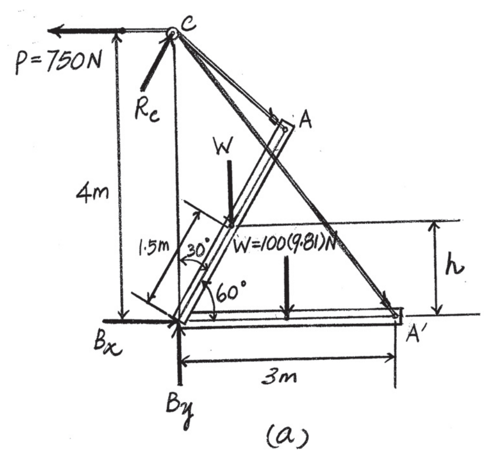
SOLUTION

Kinetic Energy and Work: Since the post rotates about a fixed axis, $v_G = \omega r_G = \omega(1.5)$. The mass moment of inertia of the post about its mass center is $I_G = \frac{1}{12}(100)(3^2) = 75\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the post is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$
$$= \frac{1}{2}(100)[\omega(1.5)]^2 + \frac{1}{2}(75)\omega^2$$
$$= 150\omega^2$$

This result can also be obtained by applying $T = \frac{1}{2}I_B\omega^2$, where $I_B = \frac{1}{12}(100)(3^2) + 100(1.5^2) = 300\ \text{kg}\cdot\text{m}^2$. Thus,

$$T = \frac{1}{2}I_B\omega^2 = \frac{1}{2}(300)\omega^2 = 150\omega^2$$



Since the post is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{B}_x$, $\mathbf{B}_y$, and $\mathbf{R}_C$ do no work, while $\mathbf{P}$ does positive work and $\mathbf{W}$ does negative work. When $\theta = 60°$, $\mathbf{P}$ displaces $s_P = A'C - AC$, where $AC = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 30°} = 2.053$ m and $A'C = \sqrt{4^2 + 3^2} = 5$ m. Thus, $s_P = 5 - 2.053 = 2.947$ m. Also, $\mathbf{W}$ displaces vertically upwards through a distance of $h = 1.5 \sin 60° = 1.299$ m. Thus, the work done by $\mathbf{P}$ and $\mathbf{W}$ is

$$U_P = Ps_P = 750(2.947) = 2210.14\ \text{J}$$
$$U_W = -Wh = -100(9.81)(1.299) = -1274.36\ \text{J}$$

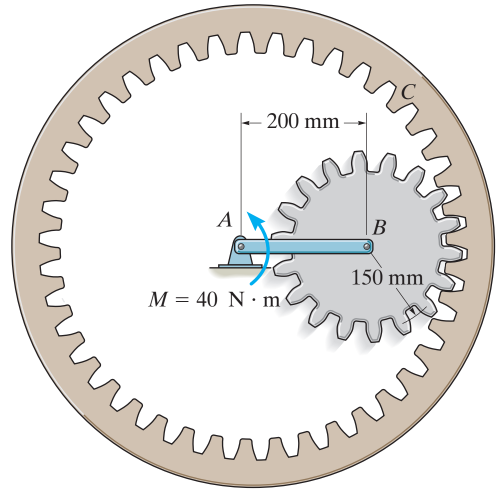
Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$
$$0 + [2210.14 - 1274.36] = 150\omega^2$$
$$\omega = 2.50\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 2.50$ rad/s

18–30.

The link AB is subjected to a couple moment of $M = 40\ \text{N}\cdot\text{m}$. If the ring gear C is fixed, determine the angular velocity of the 15-kg inner gear when the link has made two revolutions starting from rest. Neglect the mass of the link and assume the inner gear is a disk. Motion occurs in the vertical plane.



SOLUTION

Kinetic Energy. The mass moment of inertia of the inner gear about its center B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15^2) = 0.16875\ \text{kg}\cdot\text{m}^2$. Referring to the kinematics diagram of the gear, the velocity of center B of the gear can be related to the gear's angular velocity, which is

$$v_B = \omega r_{B/IC}; \qquad v_B = \omega(0.15)$$

Thus,

$$T = \frac{1}{2}I_B\omega^2 + \frac{1}{2}Mv_G^2$$

$$= \frac{1}{2}(0.16875)\,\omega^2 + \frac{1}{2}(15)[\omega(0.15)]^2$$

$$= 0.253125\,\omega^2$$

Since the gear starts from rest, $T_1 = 0$.

Work. Referring to the FBD of the gear system, we notice that **M** does positive work whereas **W** does no work, since the gear returns to its initial position after the link completes two revolutions.

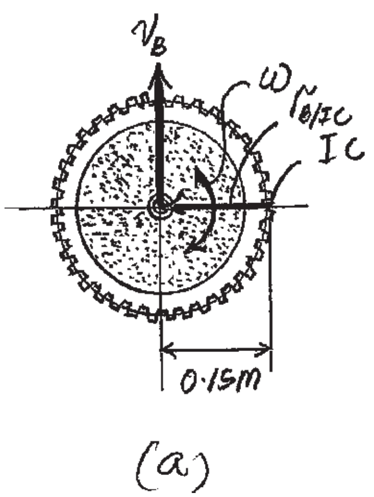
$$U_M = M_\theta = 40[2(2\pi)] = 160\pi\ \text{J}$$

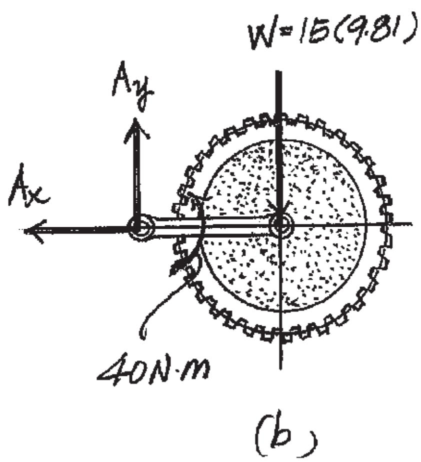
Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 160\pi = 0.253125\,\omega^2$$

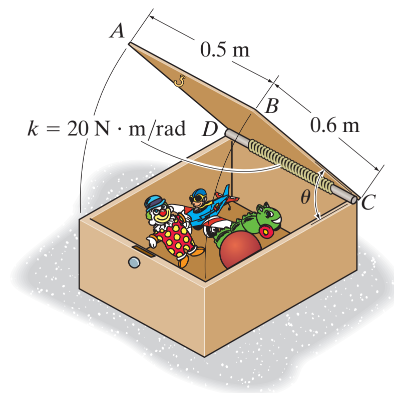
$$\omega = 44.56\ \text{rad/s} = 44.6\ \text{rad/s} \qquad \textbf{Ans.}$$





Ans:
$\omega = 44.6\ \text{rad/s}$

18–31. The 6-kg lid on the box is held in equilibrium by the torsional spring at $\theta = 60°$. If the lid is forced closed, $\theta = 0°$, and then released, determine its angular velocity at the instant it opens to $\theta = 45°$.



SOLUTION

Equilibrium: Here, $M = k\theta_0 = 20\theta_0$, where θ_0 is the initial angle of twist for the torsional spring. Referring to Fig. *a*, we have

$+\Sigma M_C = 0; \qquad 6(9.81)\cos 60°(0.3) - 20\theta_0 = 0 \qquad \theta_0 = 0.44145 \text{ rad}$

Kinetic Energy and Work: Since the cover rotates about a fixed axis passing through point C, the kinetic energy of the cover can be obtained by applying $T = \frac{1}{2} I_C\, \omega^2$, where $I_C = \frac{1}{3} mb^2 = \frac{1}{3}(6)\left(0.6^2\right) = 0.72 \text{ kg} \cdot \text{m}^2$. Thus,

$$T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}(0.72)\omega^2 = 0.36\omega^2$$

Since the cover is initially at rest ($\theta = 0°$), $T_1 = 0$. Referring to Fig. *b*, C_x and C_y do no work. **M** does positive work, and W does negative work. When $\theta = 0°$ and $45°$, the angles of twist for the torsional spring are $\theta_1 = 1.489$ rad and $\theta_2 = 1.489 - \frac{\pi}{4} = 0.703$ rad, respectively. Also, when $\theta = 45°$, **W** displaces vertically upward through a distance of $h = 0.3 \sin 45° = 0.2121$ m. Thus, the work done by **M** and **W** are

$$U_M = \int M\, d\theta = \int_{\theta_2}^{\theta_1} 20\theta\, d\theta = 10\theta^2 \Big|_{0.7032 \text{ rad}}^{1.4886 \text{ rad}} = 17.22 \text{ J}$$

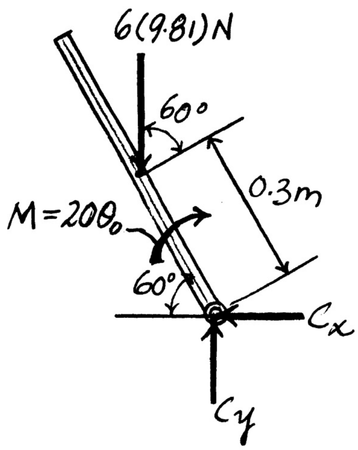
$$U_W = -Wh = -6(9.81)(0.2121) = -12.49 \text{ J}$$

Principle of Work and Energy:

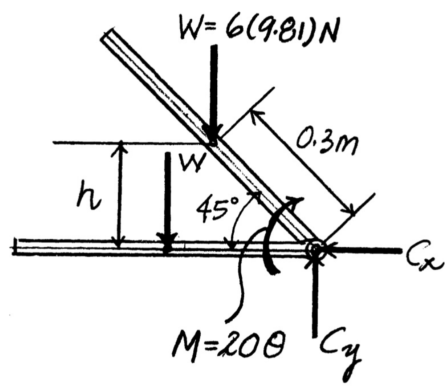
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[17.22 + (-12.49)\right] = 0.36\omega^2$$

$$\omega = 3.62 \text{ rad/s}$$ **Ans.**



(a)

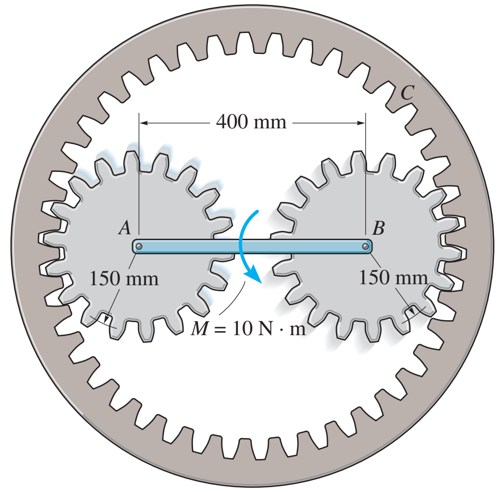


(b)

Ans:
$\omega = 3.62$ rad/s

***18–32.**

The two 2-kg gears A and B are attached to the ends of a 3-kg slender bar. The gears roll within the fixed ring gear C, which lies in the horizontal plane. If a 10-N·m torque is applied to the center of the bar as shown, determine the number of revolutions the bar must rotate starting from rest in order for it to have an angular velocity of $\omega_{AB} = 20$ rad/s. For the calculation, assume the gears can be approximated by thin disks. What is the result if the gears lie in the vertical plane?



SOLUTION

Energy equation (where G refers to the center of one of the two gears):

$$M\theta = T_2$$

$$10\theta = 2\left(\frac{1}{2} I_G \omega_{\text{gear}}^2\right) + 2\left(\frac{1}{2} m_{\text{gear}}\right)(0.200\omega_{AB})^2 + \frac{1}{2} I_{AB}\omega_{AB}^2$$

Using $m_{\text{gear}} = 2$ kg, $I_G = \frac{1}{2}(2)(0.150)^2 = 0.0225 \text{ kg}\cdot\text{m}^2$,

$I_{AB} = \frac{1}{12}(3)(0.400)^2 = 0.0400 \text{ kg}\cdot\text{m}^2$ and $\omega_{\text{gear}} = \frac{200}{150}\omega_{AB}$,

$$10\theta = 0.0225\left(\frac{200}{150}\right)^2 \omega_{AB}^2 + 2(0.200)^2\omega_{AB}^2 + 0.0200\omega_{AB}^2$$

When $\omega_{AB} = 20$ rad/s,

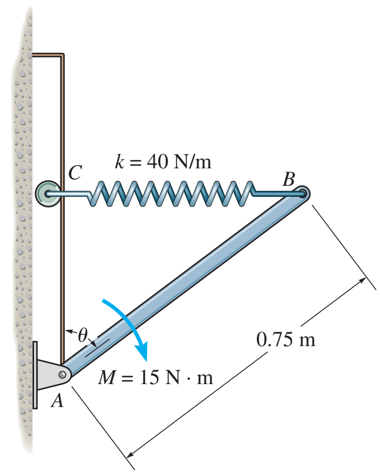
$$\theta = 5.60 \text{ rad}$$

$$= 0.891 \text{ rev, regardless of orientation}$$ **Ans.**

Ans:
$\theta = 0.891$ rev,
regardless of orientation

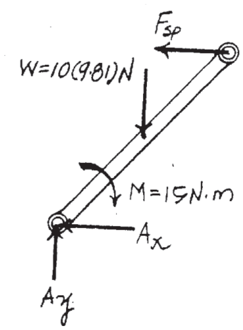
18–33.

The 10-kg rod *AB* is pin-connected at *A* and subjected to a couple moment of $M = 15$ N·m. If the rod is released from rest when the spring is unstretched at $\theta = 30°$, determine the rod's angular velocity at the instant $\theta = 60°$. As the rod rotates, the spring always remains horizontal, because of the roller support at *C*.



SOLUTION

***Free Body Diagram*:** The spring force F_{sP} does *negative* work since it acts in the opposite direction to that of its displacement s_{sp}, whereas the weight of the cylinder acts in the same direction of its displacement s_w and hence does *positive* work. Also, the couple moment **M** does positive work as it acts in the same direction of its angular displacement θ. The reactions A_x and A_y do no work since point *A* does not displace. Here, $s_{sp} = 0.75 \sin 60° - 0.75 \sin 30° = 0.2745$ m and $s_W = 0.375 \cos 30° - 0.375 \cos 60° = 0.1373$ m.



Principle of Work and Energy: The mass moment of inertia of the cylinder about point A is $I_A = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(10)\left(0.75^2\right) + 10\left(0.375^2\right) = 1.875 \text{ kg}\cdot\text{m}^2$. Applying Eq.18–13, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + Ws_W + M\theta - \frac{1}{2}ks_P^2 = \frac{1}{2}I_A\,\omega^2$$

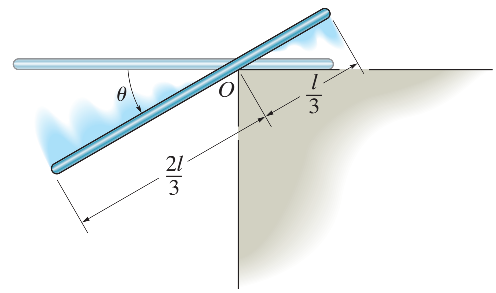
$$0 + 10(9.81)(0.1373) + 15\left(\frac{\pi}{3} - \frac{\pi}{6}\right) - \frac{1}{2}(40)\left(0.2745^2\right) = \frac{1}{2}(1.875)\,\omega^2$$

$$\omega = 4.60 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 4.60$ rad/s

18–34.

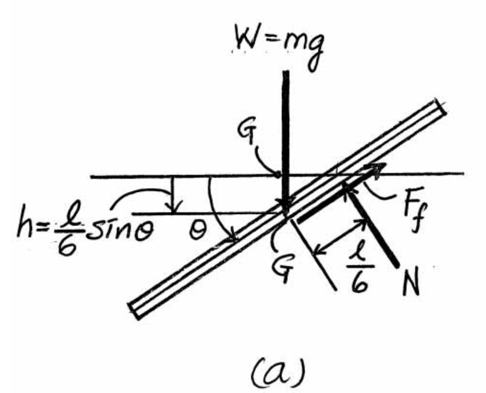
The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine its angular velocity as a function of the angle θ before it slips.



SOLUTION

Kinetic Energy and Work: Before the bar slips, the bar rotates about the fixed axis passing through point O. The mass moment of inertia of the bar about this axis is $I_O = \frac{1}{12}ml^2 + m\left(\frac{l}{6}\right)^2 = \frac{1}{9}ml^2$. Thus, the kinetic energy of the bar is

$$T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{1}{9}ml^2\right)\omega^2 = \frac{1}{18}ml^2\omega^2$$



Initially, the bar is at rest. Thus, $T_1 = 0$. Referring to the FBD of the bar, Fig. a, we notice that **N** and $\mathbf{F}_f$ do no work while **W** does positive work which is given by

$$U_W = Wh = mg\left(\frac{l}{6}\sin\theta\right) = \frac{mgl}{6}\sin\theta$$

Principle of Work and Energy:

$$T_1 + U_{1-2} = T_2$$

$$0 + \frac{mgl}{6}\sin\theta = \frac{1}{18}ml^2\omega^2$$

$$\omega^2 = \frac{3g}{l}\sin\theta$$

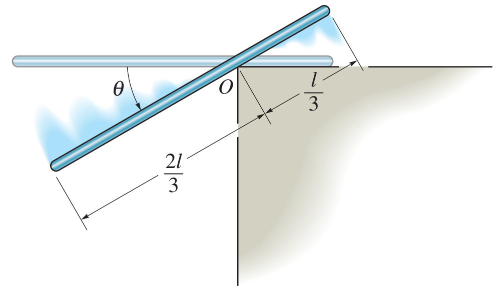
$$\omega = \sqrt{\frac{3g}{l}\sin\theta} \qquad \textbf{Ans.}$$

Ans:

$$\omega = \sqrt{\frac{3g}{l}\sin\theta}$$

18–35.

The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine the angle θ at which it first begins to slip. The coefficient of static friction at O is $\mu_s = 0.3$.

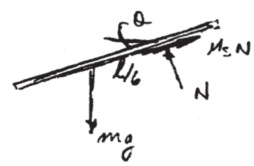


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m\,g\left(\frac{l}{6}\sin\theta\right) = \frac{1}{2}\left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\omega^2$$

$$\omega = \sqrt{\frac{3\,g\sin\theta}{l}}$$



$\zeta + \Sigma M_O = I_O\,\alpha;\qquad m\,g\cos\theta\left(\frac{1}{6}\right) = \left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\alpha$

$$\alpha = \frac{3\,g\cos\theta}{2\,l}$$

$+\ \Sigma F_n = m(a_G)_n;\quad \mu_s N - m\,g\sin\theta = m\left(\frac{3\,g\sin\theta}{l}\right)\left(\frac{l}{6}\right)$

$$\mu_s N = 1.5\,m\,g\sin\theta$$

$+\Sigma F_t = m(a_G)_t;\qquad -N + m\,g\cos\theta = m\left(\frac{3\,g\cos\theta}{2\,l}\right)\left(\frac{l}{6}\right)$

$$N = 0.75\,m\,g\cos\theta$$

Thus,

$$\mu_s = \frac{1.5}{0.75}\tan\theta$$

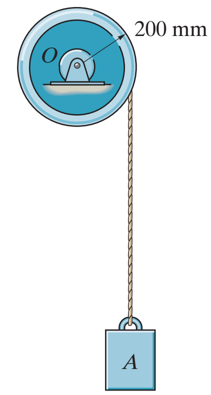
$$0.3 = 2\tan\theta$$

$$\theta = 8.53° \qquad \textbf{Ans.}$$

Ans:
$\theta = 8.53°$

***18–36.**

The spool has a mass of 20 kg and a radius of gyration of $k_O = 160$ mm. If the 15-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 8$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

Kinetic Energy. The mass moment of inertia of the spool about its center O is $I_0 = mk_0^2 = 20(0.16^2) = 0.512\ \text{kg}\cdot\text{m}^2$. The velocity of the block is $v_b = \omega_s r_s = \omega_s(0.2)$. Thus,

$$T = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(0.512)\omega_s^2 + \frac{1}{2}(15)[\omega_s(0.2)]^2$$

$$= 0.556\,\omega_s^2$$

Since the system starts from rest, $T_1 = 0$. When $\omega_s = 8$ rad/s,

$$T_2 = 0.556(8^2) = 35.584\ \text{J}$$

Potential Energy. With reference to the datum set in Fig. *a*, the initial and final gravitational potential energy of the block are

$$(V_g)_1 = m_b g y_1 = 0$$

$$(V_g)_2 = m_b g(-y_2) = 15(9.81)(-s_b) = -147.15\,s_b$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 35.584 + (-147.15\,s_b)$$

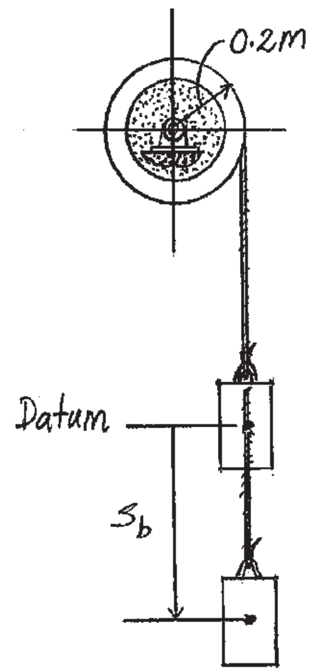
$$s_b = 0.2418\ \text{m} = 242\ \text{mm} \qquad \textbf{Ans.}$$

Principle of Work and Energy. The final velocity of the block is $(v_b)_2 = (\omega_s)_2\, r_s = 8(0.2) = 1.60$ m/s. Referring to the FBD of the block, Fig. *b* and using the result of S_b,

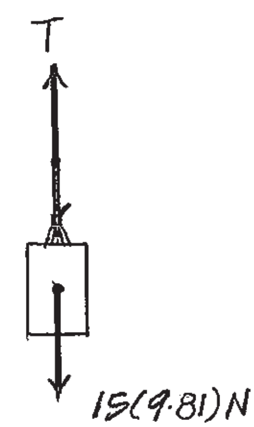
$$T_1 + \Sigma\, U_{1-2} = T_2$$

$$0 + 15(9.81)(0.2418) - T(0.2418) = \frac{1}{2}(15)(1.60^2)$$

$$T = 67.75\ \text{N} = 67.8\ \text{N} \qquad \textbf{Ans.}$$



(a)

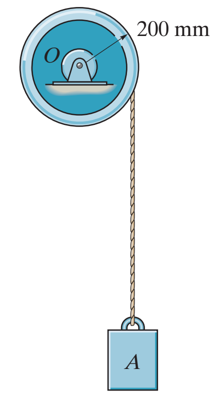


(b)

Ans:
$s_b = 242$ mm
$T = 67.8$ N

18–37.

The spool has a mass of 20 kg and a radius of gyration of $k_O = 160$ mm. If the 15-kg block A is released from rest, determine the velocity of the block when it descends 600 mm.



SOLUTION

Kinetic Energy. The mass moment of inertia of the spool about its center O is $I_0 = mk_0^2 = 20\left(0.16^2\right) = 0.512\ \text{kg}\cdot\text{m}^2$. The angular velocity of the spool is $\omega_s = \dfrac{v_b}{r_s} = \dfrac{v_b}{0.2} = 5v_b$. Thus,

$$T = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(0.512)(5v_b)^2 + \frac{1}{2}(15)v_b^2$$

$$= 13.9v_b^2$$

Since the system starts from rest, $T_1 = 0$.

Potential Energy. With reference to the datum set in Fig. *a*, the initial and final gravitational potential energies of the block are

$$(V_g)_1 = m_b g y_1 = 0$$

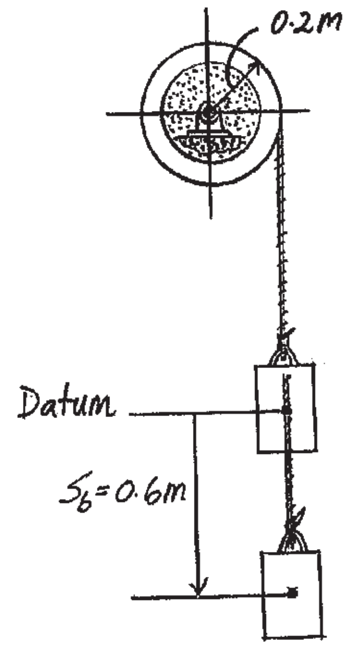
$$(V_g)_2 = m_b g y_2 = 15(9.81)(-0.6) = -88.29\ \text{J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 13.9v_b^2 + (-88.29)$$

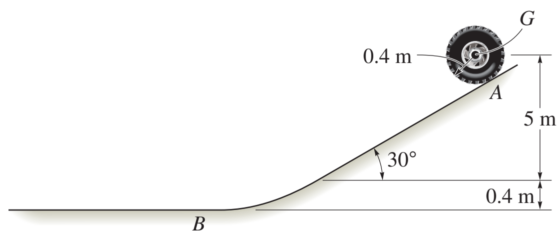
$$v_b = 2.5203\ \text{m/s} = 2.52\ \text{m/s}$$ **Ans.**



(a)

Ans:
$v_b = 2.52$ m/s

18–38. An automobile tire has a mass of 7 kg and radius of gyration of $k_G = 0.3$ m. If it is released from rest at A on the incline, determine its angular velocity when it reaches the horizontal plane. The tire rolls without slipping.



SOLUTION

$$v_G = 0.4\omega$$

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

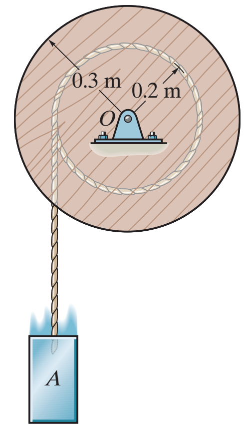
$$0 + 7(9.81)(5) = \frac{1}{2}(7)(0.4\omega)^2 + \frac{1}{2}[7\,(0.3)^2]\omega^2 + 0$$

$$\omega = 19.8 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 19.8$ rad/s

18–39.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

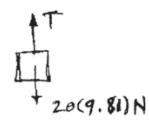
$v_A = 0.2\omega = 0.2(5) = 1$ m/s

System:

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$$

$s = 0.30071 \text{ m} = 0.301 \text{ m}$ **Ans.**



Block:

$T_1 + \Sigma U_{1-2} = T_2$

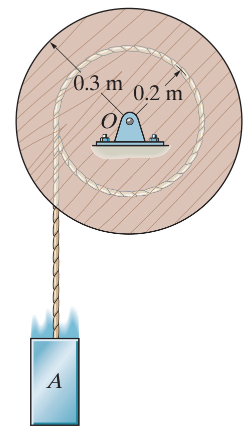
$$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$$

$T = 163$ N **Ans.**

Ans:
$s = 0.301$ m
$T = 163$ N

***18–40.**

The spool has a mass of 50 kg and a radius of gyration of $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the velocity of the block when it descends 0.5 m.



SOLUTION

Potential Energy: With reference to the datum established in Fig. *a*, the gravitational potential energy of block A at position 1 and 2 are

$$V_1 = (V_g)_1 = W_A y_1 = 20(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_A y_2 = -20(9.81)(0.5) = -98.1 \text{ J}$$

Kinetic Energy: Since the spool rotates about a fixed axis, $\omega = \dfrac{v_A}{r_A} = \dfrac{v_A}{0.2} = 5v_A$. Here, the mass moment of inertia about the fixed axis passes through point O is $I_O = mk_O^2 = 50(0.280)^2 = 3.92 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_A v_A^2$$

$$= \frac{1}{2}(3.92)(5v_A)^2 + \frac{1}{2}(20)v_A^2 = 59v_A^2$$

Since the system is at rest initially, $T_1 = 0$

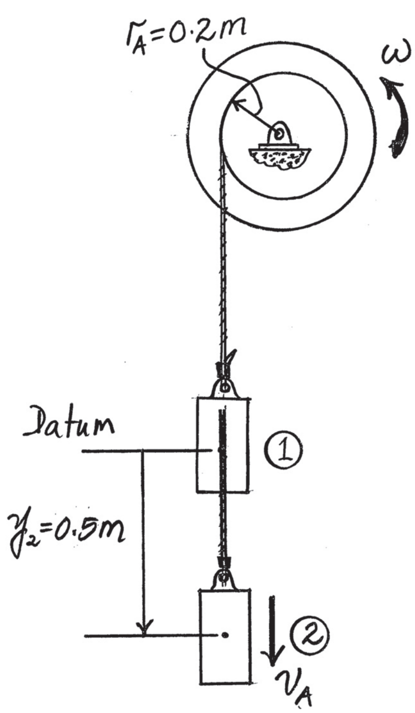
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 59v_A^2 + (-98.1)$$

$$v_A = 1.289 \text{ m/s}$$

$$= 1.29 \text{ m/s}$$ **Ans.**

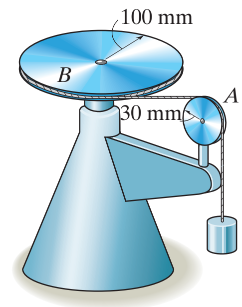


(a)

Ans:
$v_A = 1.29$ m/s

18–41.

The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right]\omega_A^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$$v_C = \omega_B(0.1) = 0.03\omega_A$$

Thus,

$$\omega_B = 10v_C$$

$$\omega_A = 33.33v_C$$

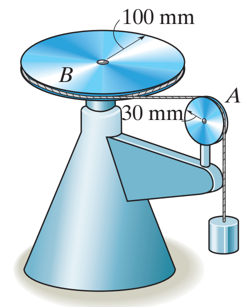
Substituting and solving yields,

$$v_C = 1.52 \text{ m/s}$$ **Ans.**

Ans:
$v_C = 1.52 \text{ m/s}$

18–42.

The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the distance the block must descend, starting from rest, in order to cause B to have an angular velocity of 6 rad/s. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



SOLUTION

$v_C = \omega_B(0.1) = 0.03\,\omega_A$

If $\omega_B = 6$ rad/s then

$\omega_A = 20$ rad/s

$v_C = 0.6$ m/s

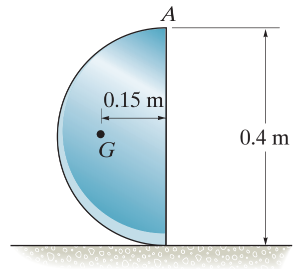
$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right](20)^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right](6)^2 + \frac{1}{2}(2)(0.6)^2 - 2(9.81)s_C$$

$s_C = 78.0$ mm **Ans.**

Ans:
$s_C = 78.0$ mm

18–43. The 15-kg semicircular segment is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is $I_G = 0.25\ \text{kg}\cdot\text{m}^2$.



SOLUTION

Given:

$M = 15\ \text{kg}$ $\quad r = 0.15\ \text{m}$

$I_G = 0.25\ \text{kg}\cdot\text{m}^2$ $\quad d = 0.4\ \text{m}$

Guesses $\quad \omega = 1\ \text{rad/s} \quad v_G = 1\ \text{m/s}$

Given $\quad Mgd = \frac{1}{2}Mv_G^{\,2} + \frac{1}{2}I_G\omega^2 + Mg(d-r) \qquad v_G = \omega\left(\frac{d}{2} - r\right)$

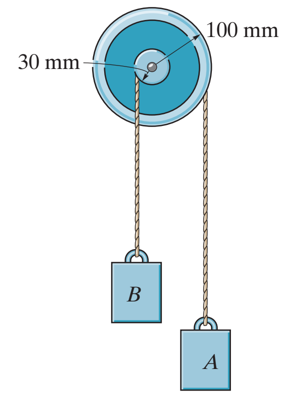
$$\begin{pmatrix}\omega \\ v_G\end{pmatrix} = \text{Find}(\omega, v_G) \qquad \omega = 12.4\ \text{rad/s} \qquad v_G = 0.619\ \text{m/s}$$

$$\mathbf{v_A} = \begin{pmatrix}0\\0\\\omega\end{pmatrix} \times \begin{pmatrix}\frac{-d}{2}\\ \frac{d}{2}\\ 0\end{pmatrix} \qquad \mathbf{v_A} = \begin{pmatrix}-2.48\\-2.48\\0.00\end{pmatrix}\text{m/s} \qquad |\mathbf{v_A}| = 3.50\ \text{m/s} \quad \textbf{Ans.}$$

Ans:
$\mathbf{v_A} = \{-2.48\mathbf{i} - 2.48\mathbf{j}\}\ \text{m/s}$
$|\mathbf{v_A}| = 3.50\ \text{m/s}$

***18–44.**

The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.

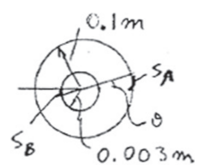


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0 + 0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$$\theta = \frac{s_B}{0.03} = \frac{s_A}{0.1}$$



$s_B = 0.3\, s_A$

Set $s_A = 0.2$ m, $s_B = 0.06$ m

Substituting and solving yields,

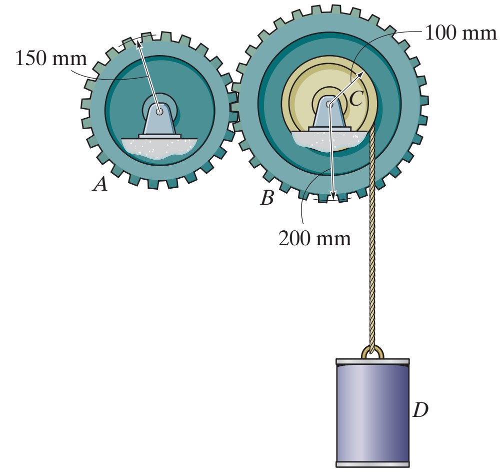
$\omega = 14.04$ rad/s

$v_A = 0.1(14.04) = 1.40$ m/s **Ans.**

Ans:
$v_A = 1.40$ m/s

18–45.

Determine the speed of the 50-kg cylinder after it has descended a distance of 2 m, starting from rest. Gear A has a mass of 10 kg and a radius of gyration of 125 mm about its center of mass. Gear B and drum C have a combined mass of 30 kg and a radius of gyration about their center of mass of 150 mm.



SOLUTION

Potential Energy: With reference to the datum shown in Fig. a, the gravitational potential energy of block D at position (1) and (2) is

$$V_1 = (V_g)_1 = W_D(y_D)_1 = 50(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_D(y_D)_2 = -50(9.81)(2) = -981 \text{ J}$$

Kinetic Energy: Since gear B rotates about a fixed axis, $\omega_B = \dfrac{v_D}{r_D} = \dfrac{v_D}{0.1} = 10v_D$.

Also, since gear A is in mesh with gear B, $\omega_A = \left(\dfrac{r_B}{r_A}\right)\omega_B = \left(\dfrac{0.2}{0.15}\right)(10v_D) = 13.33v_D$.

The mass moment of inertia of gears A and B about their mass centers are $I_A = m_A k_A^2 = 10(0.125^2) = 0.15625 \text{ kg}\cdot\text{m}^2$ and $I_B = m_B k_B^2 = 30(0.15^2) = 0.675 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 + \frac{1}{2}m_D v_D^2$$

$$= \frac{1}{2}(0.15625)(13.33v_D)^2 + \frac{1}{2}(0.675)(10v_D)^2 + \frac{1}{2}(50)v_D^2$$

$$= 72.639v_D^2$$

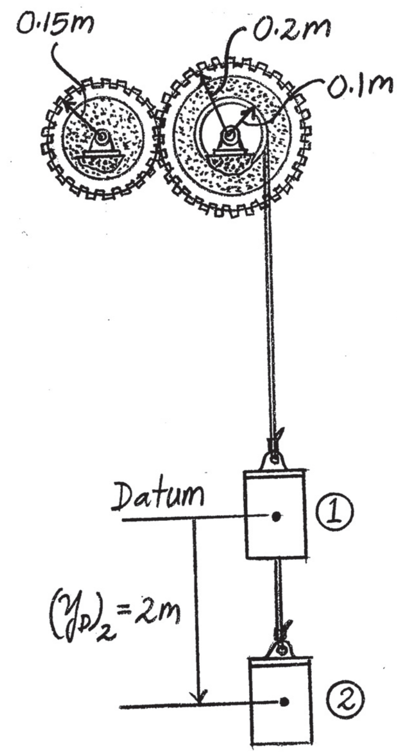
Since the system is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 72.639v_D^2 - 981$$

$$v_D = 3.67 \text{ m/s}$$ **Ans.**

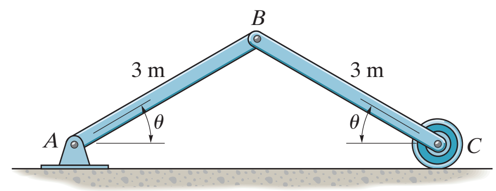


(a)

Ans:
$v_D = 3.67$ m/s

18–46.

The assembly consists of two 10-kg bars which are pin connected. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 0°$. The 5-kg disk at C has a radius of 0.5 m and rolls without slipping.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of bar BC at the final position, Fig. a, we found that IC is located at C. Thus, $(v_c)_2 = 0$. Also,

$$(v_B)_2 = (\omega_{BC})_2\, r_{B/IC}; \qquad (v_B)_2 = (\omega_{BC})_2(3)$$

$$(v_G)_2 = (\omega_{BC})_2\, r_{G/IC}; \qquad (v_G)_2 = (\omega_{BC})_2(1.5)$$

Then for bar AB,

$$(v_B)_2 = (\omega_{AB})_2\, r_{AB}; \quad (\omega_{BC})_2(3) = (\omega_{AB})_2(3)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

For the disk, since the velocity of its center $(v_c)_2 = 0$, then $(\omega_d)_2 = 0$. Thus

$$T_2 = \frac{1}{2} I_A(\omega_{AB})_2^2 + \frac{1}{2} I_G(\omega_{BC})_2^2 + \frac{1}{2} m_r(v_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}(10)[(\omega_{BC})_2(1.5)]^2$$

$$= 30.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. b, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[10(9.81)(1.5 \sin 60°)] = 254.87 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 0$$

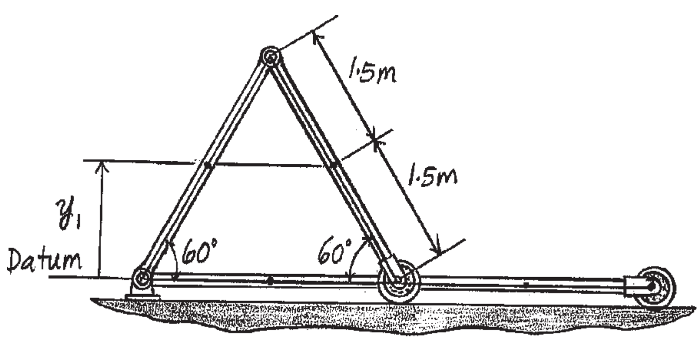
Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

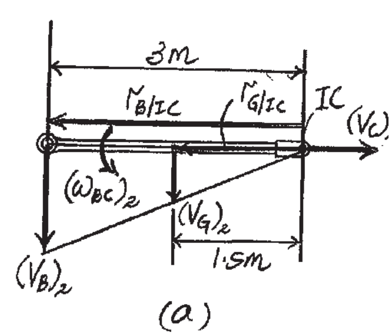
$$0 + 254.87 = 30.0(\omega_{BC})_2^2 + 0$$

$$(\omega_{BC})_2 = 2.9147 \text{ rad/s} = 2.91 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(\omega_{AB})_2 = (\omega_{BC})_2 = 2.91 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

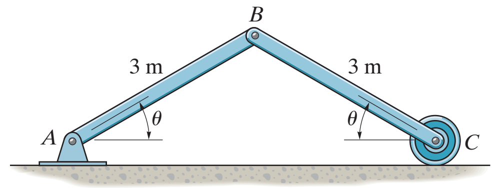


(a)

Ans:
$(\omega_{BC})_2 = 2.91$ rad/s
$(\omega_{AB})_2 = 2.91$ rad/s

18–47.

The assembly consists of two 10-kg bars which are pin connected. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. The 5-kg disk at C has a radius of 0.5 m and rolls without slipping.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of bar BC at final position with IC so located, Fig. a,

$$r_{B/IC} = r_{C/IC} = 3 \text{ m} \qquad r_{G/IC} = 3 \sin 60° = 1.5\sqrt{3} \text{ m}$$

Thus,

$$(v_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (v_B)_2 = (\omega_{BC})_2(3)$$

$$(v_C)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (v_C)_2 = (\omega_{BC})_2(3)$$

$$(v_G)_2 = (\omega_{BC})_2 r_{G/IC}; \qquad (v_G)_2 = (\omega_{BC})_2(1.5\sqrt{3})$$

Then for rod AB,

$$(v_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(3) = (\omega_{AB})_2(3)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

For the disk, since it rolls without slipping,

$$(v_C)_2 = (\omega_d)_2 r_d; \qquad (\omega_{BC})_2(3) = (\omega_d)_2(0.5)$$

$$(\omega_d)_2 = 6(\omega_{BC})_2$$

Thus, the kinetic energy of the system at final position is

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(v_G)_2^2 + \frac{1}{2}I_C(\omega_d)_2^2 + \frac{1}{2}m_d(v_C)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}(10)\left[(\omega_{BC})_2(1.5\sqrt{3})\right]^2$$

$$+ \frac{1}{2}\left[\frac{1}{2}(5)(0.5^2)\right][6(\omega_{BC})_2]^2 + \frac{1}{2}(5)[(\omega_{BC})_2(3)]^2$$

$$= 86.25(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. b, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[10(9.81)(1.5 \sin 60°)] = 254.87 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 2[10(9.81)(1.5 \sin 30°)] = 147.15 \text{ J}$$

18–47. Continued

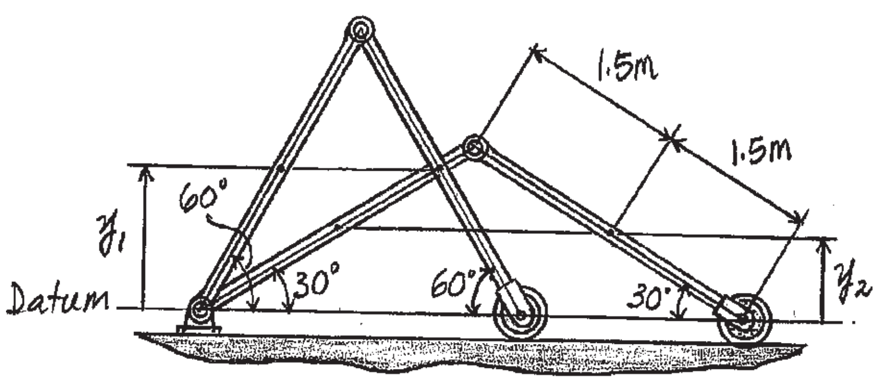
Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

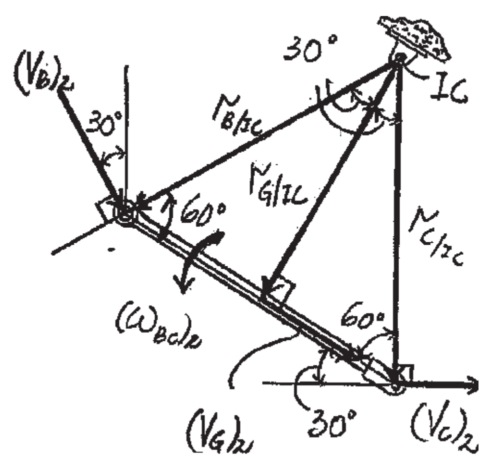
$$0 + 254.87 = 86.25(\omega_{BC})_2^2 + 147.15$$

$$(\omega_{BC})_2 = 1.1176 \text{ rad/s} = 1.12 \text{ rad/s}$$ **Ans.**

$$(\omega_{AB})_2 = (\omega_{BC})_2 = 1.12 \text{ rad/s}$$ **Ans.**



(b)

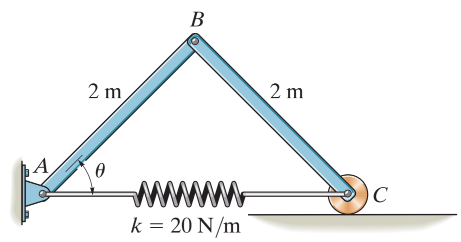


(a)

Ans:
$(\omega_{AB})_2 = (\omega_{BC})_2 = 1.12 \text{ rad/s}$

***18–48.**

The two 12-kg slender rods are pin connected and released from rest at the position $\theta = 60°$. If the spring has an unstretched length of 1.5 m, determine the angular velocity of rod BC, when the system is at the position $\theta = 0°$. Neglect the mass of the roller at C.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of rod BC at the final position, Fig. *a* we found that IC is located at C. Thus, $(v_C)_2 = 0$. Also,

$$(v_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (v_B)_2 = (\omega_{BC})_2(2)$$

$$(v_G)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (v_G)_2 = (\omega_{BC})_2(1)$$

Then for rod AB,

$$(v_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(2) = (\omega_{AB})_2(2)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

Thus,

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(v_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(12)(2^2)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right](\omega_{BC})_2^2 + \frac{1}{2}(12)[(\omega_{BC})_2(1)]^2$$

$$= 16.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. *b*, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[12(9.81)(1 \sin 60°)] = 203.90 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 0$$

The stretch of the spring when the system is at initial and final position are

$$x_1 = 2(2 \cos 60°) - 1.5 = 0.5 \text{ m}$$

$$x_2 = 4 - 1.5 = 2.50 \text{ m}$$

Thus, the initial and final elastic potential energies of the spring is

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(20)(0.5^2) = 2.50 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(20)(2.50^2) = 62.5 \text{ J}$$

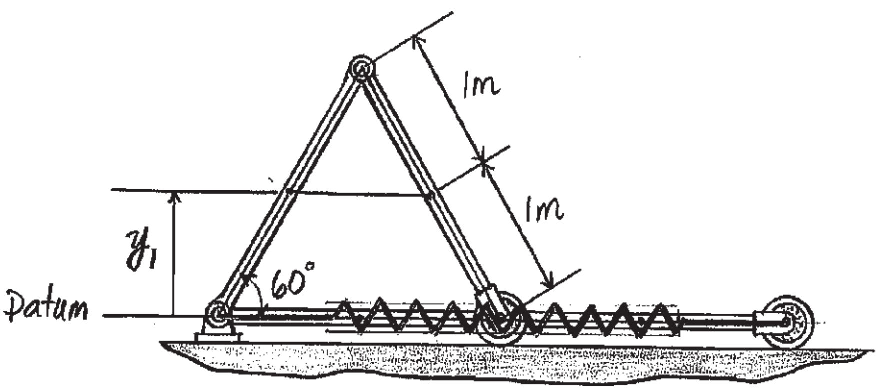
18–48. Continued

Conservation of Energy.

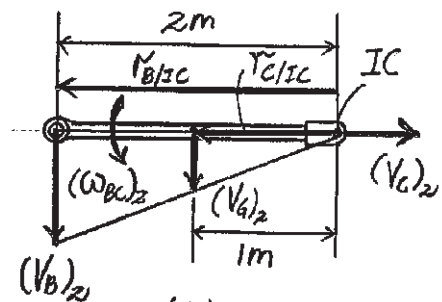
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (203.90 + 2.50) = 16.0(\omega_{BC})_2^2 + (0 + 62.5)$$

$$(\omega_{BC})_2 = 2.9989 \text{ rad/s} = 3.00 \text{ rad/s}$$ **Ans.**



(b)

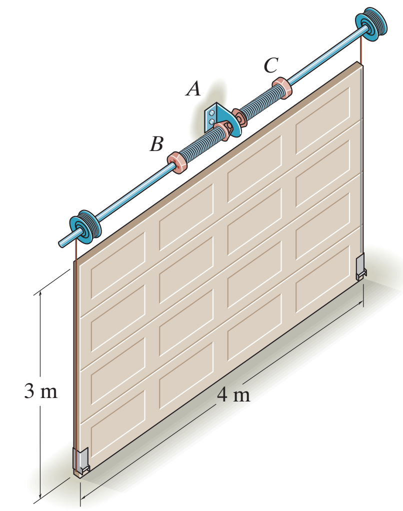


(a)

Ans:
$(\omega_{BC})_2 = 3.00$ rad/s

18–49.

The uniform garage door has a mass of 150 kg and is guided along smooth tracks at its ends. Lifting is done using the two springs, each of which is attached to the anchor bracket at *A* and to the counterbalance shaft at *B* and *C*. As the door is raised, the springs begin to unwind from the shaft, thereby assisting the lift. If each spring provides a torsional moment of $M = (0.7\theta)$ N · m, where θ is in radians, determine the angle θ_0 at which both the left-wound and right-wound spring should be attached so that the door is completely balanced by the springs, i.e., when the door is in the vertical position and is given a slight force upwards, the springs will lift the door along the side tracks to the horizontal plane with no final angular velocity. *Note:* The elastic potential energy of a torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and in this case $k = 0.7$ N · m/rad.



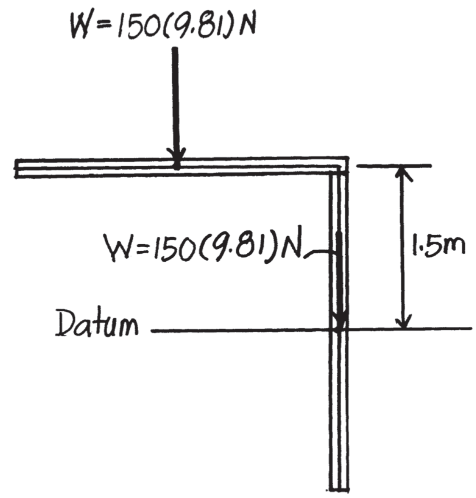
SOLUTION

Datum at initial position.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(0.7)\theta_0^2\right] + 0 = 0 + 150(9.81)(1.5)$$

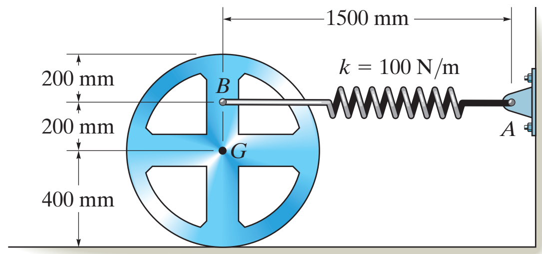
$$\theta_0 = 56.15 \text{ rad} = 8.94 \text{ rev.}$$ **Ans.**



Ans:
$\theta_0 = 56.15$ rad

18–50.

The 40-kg wheel has a radius of gyration about its center of gravity G of $k_G = 250$ mm. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 100$ N/m and an unstretched length of 500 mm. The wheel is released from rest.



SOLUTION

Kinetic Energy. The mass moment of inertia of the wheel about its center of mass G is $I_G = mk_G^2 = 40(0.25^2) = 2.50 \text{ kg}\cdot\text{m}^2$, since the wheel rolls without slipping, $v_G = \omega r_G = \omega(0.4)$. Thus

$$T = \frac{1}{2} I_G \omega^2 + \frac{1}{2} m v_G^2$$

$$= \frac{1}{2}(2.50)\omega^2 + \frac{1}{2}(40)[\omega(0.4)]^2 = 4.45\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$.

Potential Energy. When the wheel rotates 90° clockwise from position ① to ②, Fig. *a*, its mass center displaces $S_G = \theta r_G = \frac{\pi}{2}(0.4) = 0.2\pi$ m. Then $x^y = 1.5 - 0.2 - 0.2\pi = 0.6717$ m. The stretches of the spring when the wheel is at positions ① and ② are

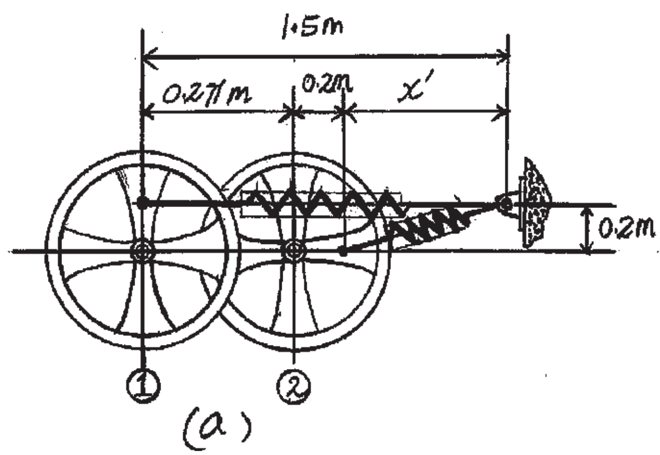
$$x_1 = 1.50 - 0.5 = 1.00 \text{ m}$$

$$x_2 = \sqrt{0.6717^2 + 0.2^2} - 0.5 = 0.2008 \text{ m}$$

Thus, the initial and final elastic potential energies are

$$(V_e)_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(100)(1^2) = 50 \text{ J}$$

$$(V_e)_2 = \frac{1}{2} k x_2^2 = \frac{1}{2}(100)(0.2008^2) = 2.0165 \text{ J}$$



Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

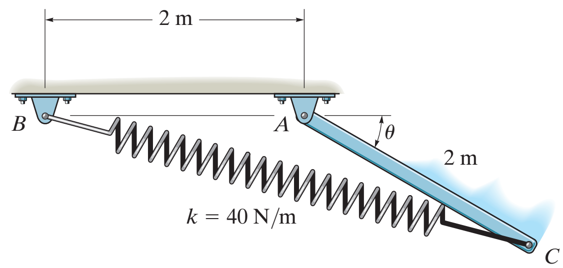
$$0 + 50 = 4.45\omega^2 + 2.0165$$

$$\omega = 3.2837 \text{ rad/s} = 3.28 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 3.28$ rad/s

18–51.

The 12-kg slender rod is attached to a spring, which has an unstretched length of 2 m. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$.



SOLUTION

Kinetic Energy. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(12)(2^2) + 12(1^2) = 16.0\ \text{kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(16.0)\omega^2 = 8.00\,\omega^2$$

Since the rod is released from rest, $T_1 = 0$.

Potential Energy. With reference to the datum set in Fig. a, the gravitational potential energies of the rod at positions ① and ② are

$$(V_g)_1 = mg(-y_1) = 12(9.81)(-1\sin 30°) = -58.86\text{ J}$$

$$(V_g)_2 = mg(-y_2) = 12(9.81)(-1) = -117.72\text{ J}$$

The stretches of the spring when the rod is at positions ① and ② are

$$x_1 = 2(2\sin 75°) - 2 = 1.8637\text{ m}$$

$$x_2 = \sqrt{2^2 + 2^2} - 2 = 0.8284\text{ m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(40)\left(1.8637^2\right) = 69.47\text{ J}$$

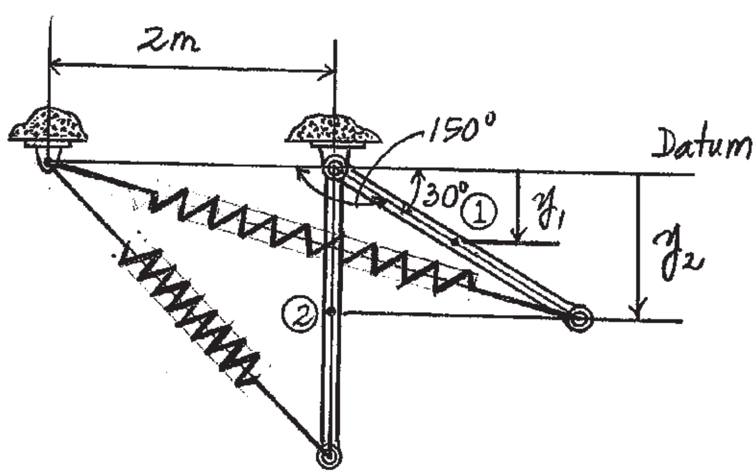
$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(40)\left(0.8284^2\right) = 13.37\text{ J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-58.86) + 69.47 = 8.00\,\omega^2 + (-117.72) + 13.73$$

$$\omega = 3.7849\text{ rad/s} = 3.78\text{ rad/s}$$ **Ans.**

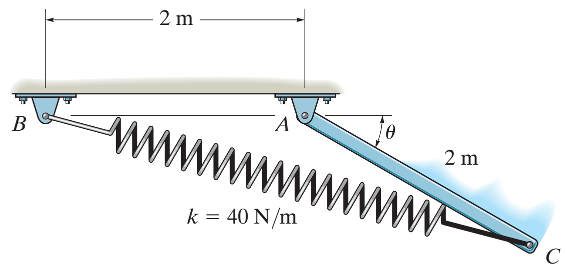


(a)

Ans:
$\omega = 3.78\text{ rad/s}$

***18–52.**

The 12-kg slender rod is attached to a spring, which has an unstretched length of 2 m. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



SOLUTION

Kinetic Energy. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(12)(2^2) + 12(1^2) = 16.0\ \text{kg} \cdot \text{m}^2$. Then

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(16.0)\omega^2 = 8.00\,\omega^2$$

Since the rod is release from rest, $T_1 = 0$.

Potential Energy. When the spring is unstretched, the rod is at position ② shown in Fig. *a*. with reference to the datum set, the gravitational potential energies of the rod at positions ① and ② are

$$(V_g)_1 = mg(-y_1) = 12(9.81)(-1 \sin 30°) = -58.86\ \text{J}$$

$$(V_g)_2 = mg(-y_2) = 12(9.81)(-1 \sin 60°) = -101.95\ \text{J}$$

The stretch of the spring when the rod is at position ① is

$$x_1 = 2(2 \sin 75°) - 2 = 1.8637\ \text{m}$$

It is required that $x_2 = 0$. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(40)\left(1.8637^2\right) = 69.47\ \text{J}$$

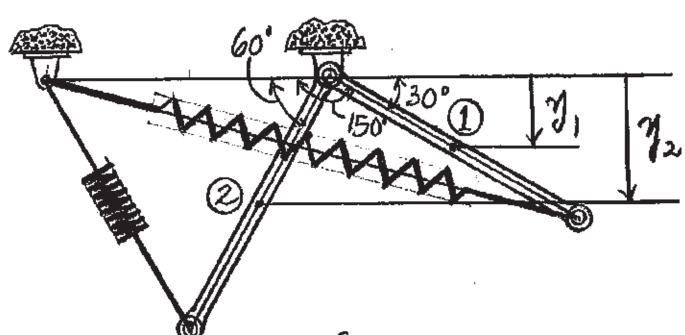
$$(V_e)_2 = \frac{1}{2} k x_2^2 = 0$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-58.86) + 69.47 = 8.00\omega^2 + (-101.95) + 0$$

$$\omega = 3.7509\ \text{rad/s} = 3.75\ \text{rad/s}$$ **Ans.**

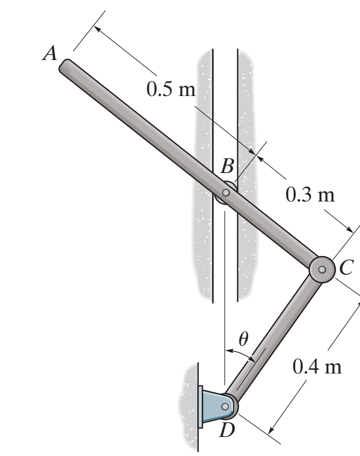


(a)

Ans:
$\omega = 3.75\ \text{rad/s}$

18–53.

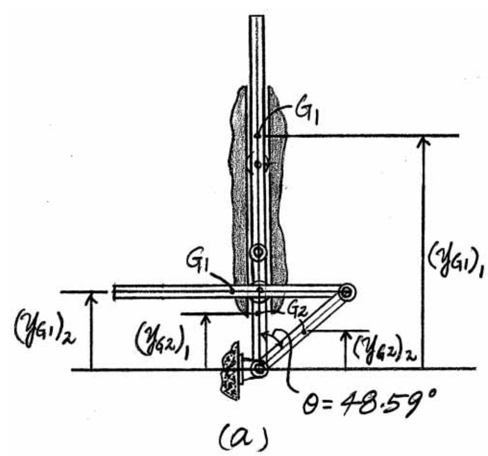
The 6-kg rod ABC is connected to the 3-kg rod CD. If the system is released from rest when $\theta = 0°$, determine the angular velocity of rod ABC at the instant it becomes horizontal.



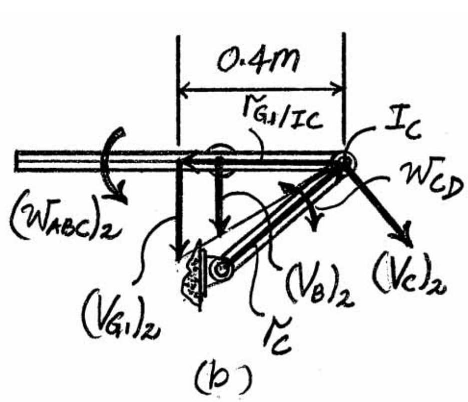
SOLUTION

Potential Energy: When rod ABC is in the horizontal position, Fig. a, $\theta = \sin^{-1}\left(\frac{0.3}{0.4}\right) = 48.59°$. With reference to the datum in Fig. a, the initial and final gravitational potential energy of the system is

$$V_1 = (V_g)_1 = W_1(y_{G1})_1 + W_2(y_{G2})_1$$
$$= 6(9.81)(0.8) + 3(9.81)(0.2) = 52.974 \text{ J}$$
$$V_2 = (V_g)_2 = W_1(y_{G1})_2 + W_2(y_{G2})_2$$
$$= 6(9.81)(0.4 \cos 48.59°) + 3(9.81)(0.2 \cos 48.59°) = 19.466 \text{ J}$$



Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $(v_{G1})_2 = (\omega_{ABC})_2 \, r_{G1/IC} = (\omega_{ABC})_2(0.4)$. Since point C is at the $IC(v_C)_2 = 0$. Then, $\omega_{CD} = \frac{(v_C)_2}{r_C} = \frac{0}{0.4} = 0$. The mass moment of inertia of rod ABC about its mass center is $I_{G1} = \frac{1}{12}(6)(0.8^2) = 0.32 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the system is



$$T_2 = \frac{1}{2} m_1 (v_{G1})_2{}^2 + \frac{1}{2} I_{G1} (\omega_{ABC})_2{}^2$$
$$= \frac{1}{2}(6)\left[(\omega_{ABC})_2(0.4)\right]^2 + \frac{1}{2}(0.32)(\omega_{ABC})_2{}^2$$
$$= 0.64\omega_{ABC}{}^2$$

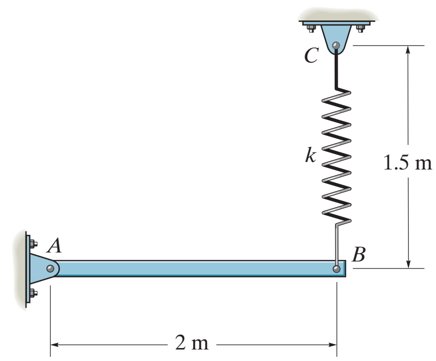
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 52.974 = 0.64\omega_{ABC}{}^2 + 19.466$$
$$(\omega_{ABC})_2 = 7.24 \text{ rad/s}$$ **Ans.**

Ans:
$(\omega_{ABC})_2 = 7.24$ rad/s

18–54.

The slender 6-kg bar AB is horizontal and at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90° after being released.



SOLUTION

Kinetic Energy. The mass moment of inertia of the bar about A is $I_A = \frac{1}{12}(6)(2^2) + 6(1^2) = 8.00\ \text{kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(8.00)\,\omega^2 = 4.00\,\omega^2$$

Since the bar is at rest initially and required to stop finally, $T_1 = T_2 = 0$.

Potential Energy. With reference to the datum set in Fig. *a*, the gravitational potential energies of the bar when it is at positions ① and ② are

$$(V_g)_1 = mgy_1 = 0$$

$$(V_g)_2 = mgy_2 = 6(9.81)(-1) = -58.86\ \text{J}$$

The stretch of the spring when the bar is at position ② is

$$x_2 = \sqrt{2^2 + 3.5^2} - 1.5 = 2.5311\ \text{m}$$

Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = 0$$

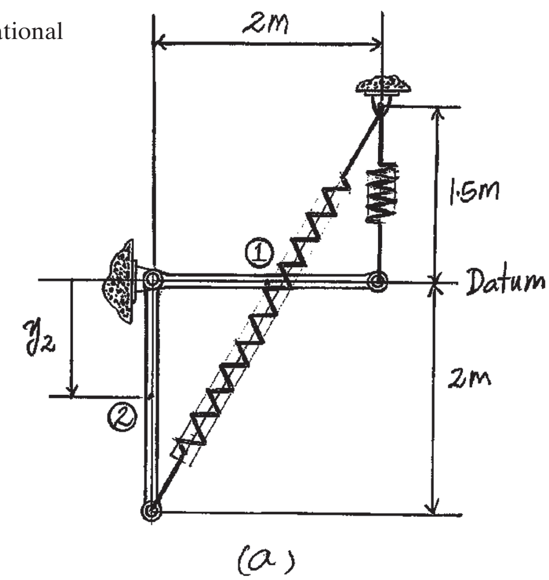
$$(V_e)_2 = \frac{1}{2}k\left(2.5311^2\right) = 3.2033k$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 0) = 0 + (-58.86) + 3.2033k$$

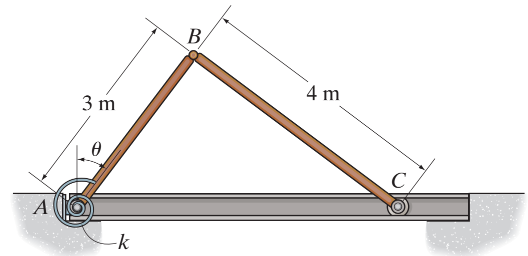
$$k = 18.3748\ \text{N/m} = 18.4\ \text{N/m}$$ **Ans.**



Ans:
$k = 18.4\ \text{N/m}$

18–55.

The torsional spring at A has a stiffness of $k = 2000\ \text{N}\cdot\text{m/rad}$ and is uncoiled when $\theta = 0°$. Determine the angular velocity of the bars, AB and BC, when $\theta = 0°$, if they are released from rest at the closed position, $\theta = 90°$. The bars have a mass per unit length of $20\ \text{kg/m}$.



SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the system at its open and closed positions is

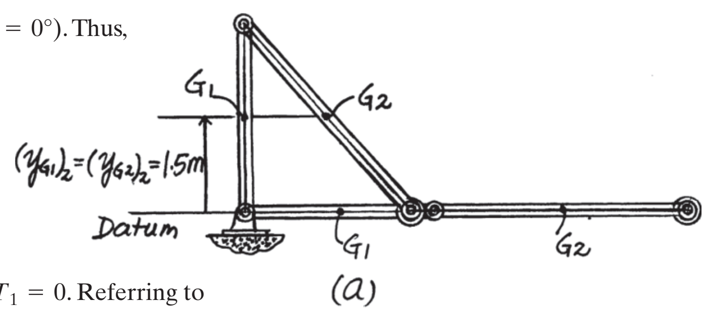
$$(V_g)_1 = W_{AB}(y_{G1})_1 + W_{BC}(y_{G2})_1$$
$$= 20(3)(9.81)(0) + 20(4)(9.81)(0) = 0$$
$$(V_g)_2 = W_{AB}(y_{G1})_2 + W_{BC}(y_{G2})_2$$
$$= 20(3)(9.81)(1.5) + 20(4)(9.81)(1.5) = 2060.1\ \text{J}$$

When the panel is in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$(V_e)_1 = \frac{1}{2}k\theta^2 = \frac{1}{2}(2000)\left(\frac{\pi}{2}\right)^2 = 250\pi^2\ \text{J}$$

The spring is uncoiled when the panel is in the open position ($\theta = 0°$). Thus,

$$(V_e)_2 = 0$$



And so,

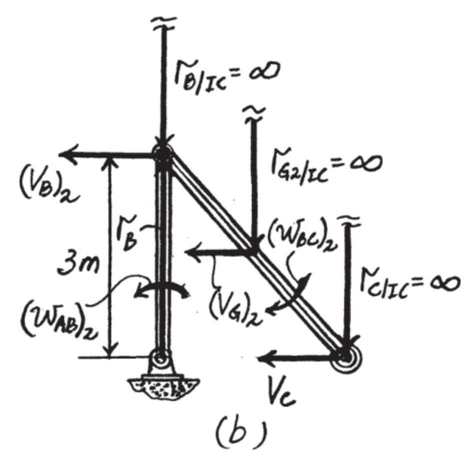
$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 250\pi^2 = 250\pi^2\ \text{J}$$
$$V_2 = (V_g)_2 + (V_e)_2 = 2060.1 + 0 = 2060.1\ \text{J}$$

Kinetic Energy: Since the panel is at rest in the closed position, $T_1 = 0$. Referring to Fig. *b*, the *IC* for *BC* is located at infinity. Thus,

$$(\omega_{BC})_2 = 0 \qquad \textbf{Ans.}$$

Then,

$$(v_G)_2 = (v_B)_2 = (\omega_{AB})_2 r_B = (\omega_{AB})_2(3)$$



The mass moments of inertia of AB about point A and BC about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[20(3)](3^2) = 180\ \text{kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[20(4)](4^2) = 106.67\ \text{kg}\cdot\text{m}^2$$

Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \frac{1}{2}m_{BC}(v_{G2})^2$$
$$= \frac{1}{2}(180)(\omega_{AB})_2{}^2 + \frac{1}{2}[20(4)]\left[(\omega_{AB})_2(3)\right]^2$$
$$= 450(\omega_{AB})_2{}^2$$

18–55. continued

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

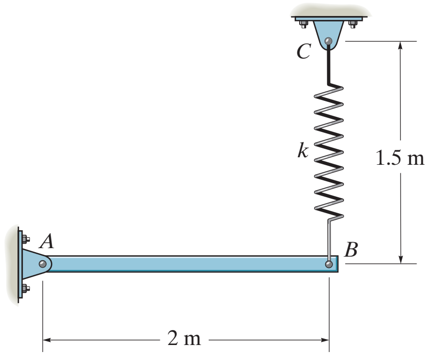
$$0 + 250\pi^2 = 450(\omega_{AB})_2{}^2 + 2060.1$$

$$(\omega_{AB})_2 = 0.951 \text{ rad/s}$$ **Ans.**

Ans:
$(\omega_{BC})_2 = 0$
$(\omega_{AB})_2 = 0.951$ rad/s

***18–56.**

The slender 6-kg bar AB is horizontal and at rest and the spring is unstretched. Determine the angular velocity of the bar when it has rotated clockwise 45° after being released. The spring has a stiffness of $k = 12$ N/m.



SOLUTION

Kinetic Energy. The mass moment of inertia of the bar about A is $I_A = \frac{1}{12}(6)(2^2) + 6(1^2) = 8.00\ \text{kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(8.00)\,\omega^2 = 4.00\,\omega^2$$

Since the bar is at rest initially, $T_1 = 0$.

Potential Energy. with reference to the datum set in Fig. a, the gravitational potential energies of the bar when it is at positions ① and ② are

$$(V_g)_1 = mgy_1 = 0$$

$$(V_g)_2 = mgy_2 = 6(9.81)(-1\sin 45°) = -41.62\ \text{J}$$

From the geometry shown in Fig. a,

$$a = \sqrt{2^2 + 1.5^2} = 2.5\ \text{m} \qquad \phi = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$$

Then, using cosine law,

$$l = \sqrt{2.5^2 + 2^2 - 2(2.5)(2)\cos(45° + 36.87°)} = 2.9725\ \text{m}$$

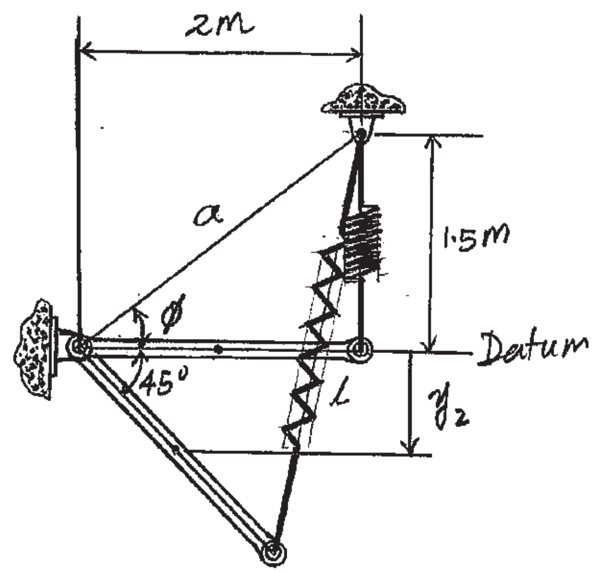
Thus, the stretch of the spring when the bar is at position ② is

$$x_2 = 2.9725 - 1.5 = 1.4725\ \text{m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = 0$$

$$(V_e)_2 = \frac{1}{2}(12)(1.4725^2) = 13.01\ \text{J}$$



(a)

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

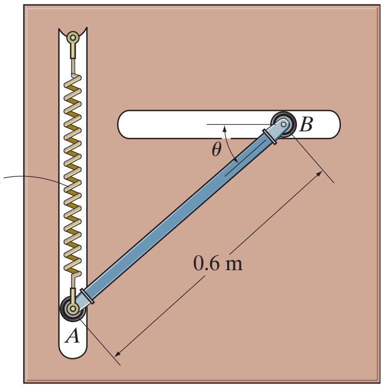
$$0 + (0 + 0) = 4.00\,\omega^2 + (-41.62) + 13.01$$

$$\omega = 2.6744\ \text{rad/s} = 2.67\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 2.67$ rad/s

18–57.

A spring having a stiffness of $k = 300$ N/m is attached to the end of the 15-kg rod, and it is unstretched when $\theta = 0°$. If the rod is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 30°$. The motion is in the vertical plane.

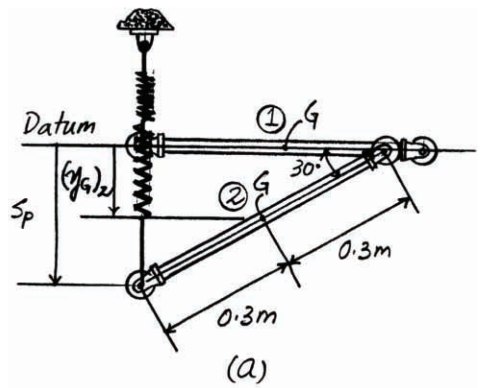


SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 15(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -15(9.81)(0.3 \sin 30°) = -22.0725 \text{ J}$$



Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 30°$, the stretch of the spring is $s_P = 0.6 \sin 30° = 0.3$ m. Thus, the final elastic potential energy of the spring is

$$\left(V_e\right)_2 = \frac{1}{2} k s_P{}^2 = \frac{1}{2}(300)\left(0.3^2\right) = 13.5 \text{ J}$$

Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 0 = 0$$

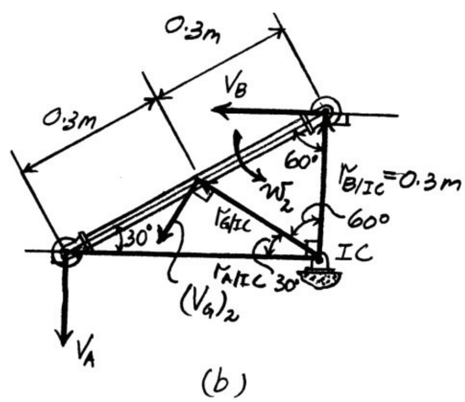
$$V_2 = (V_g)_2 + (V_e)_2 = -22.0725 + 13.5 = -8.5725 \text{ J}$$

Kinetic Energy: Since the rod is initially at rest, $T_1 = 0$. From the geometry shown in Fig. *b*, $r_{G/IC} = 0.3$ m. Thus, $(V_G)_2 = \omega_2 r_{G/IC} = \omega_2\,(0.3)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12} ml^2 = \frac{1}{12}(15)\left(0.6^2\right) = 0.45 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the rod is

$$T_2 = \frac{1}{2} m(v_G)_2{}^2 + \frac{1}{2} I_G \omega_2{}^2$$

$$= \frac{1}{2}(15)\left[\omega_2(0.3)\right]^2 + \frac{1}{2}(0.45)\omega_2{}^2$$

$$= 0.9\omega_2{}^2$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

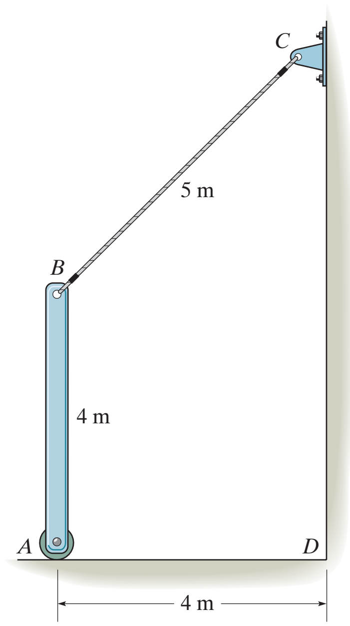
$$0 + 0 = 0.9\omega_2{}^2 - 8.5725$$

$$\omega_2 = 3.09 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_2 = 3.09$ rad/s

18–58.

The slender 15-kg bar is initially at rest and standing in the vertical position when the bottom end A is displaced slightly to the right. If the track in which it moves is smooth, determine the speed at which end A strikes the corner D. The bar is constrained to move in the vertical plane. Neglect the mass of the cord BC.



SOLUTION

$$x^2 + y^2 = 5^2$$

$$x^2 + (7 - y)^2 = 4^2$$

Thus, $y = 4.1429 \text{ m}$

$x = 2.7994 \text{ m}$

$$(5)^2 = (4)^2 + (7)^2 - 2(4)(7)\cos\phi$$

$$\phi = 44.42°$$

$$h^2 = (2)^2 + (7)^2 - 2(2)(7)\cos 44.42°$$

$$h = 5.745 \text{ m}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 147.15(2) = \frac{1}{2}\left[\frac{1}{12}(15)(4)^2\right]\omega^2 + \frac{1}{2}(15)(5.745\omega)^2 + 147.15\left(\frac{7 - 4.1429}{2}\right)$$

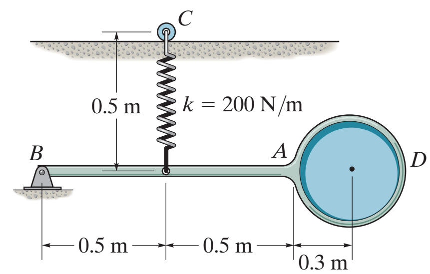
$$\omega = 0.5714 \text{ rad/s}$$

$v_A = 0.5714(7) = 4.00 \text{ m/s}$ **Ans.**

Ans:
$v_A = 4.00 \text{ m/s}$

18–59.

The pendulum consists of a 6-kg slender rod fixed to a 15-kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



SOLUTION

Kinetic Energy. The mass moment of inertia of the pendulum about B is $I_B = \left[\frac{1}{12}(6)(1^2) + 6(0.5^2)\right] + \left[\frac{1}{2}(15)(0.3^2) + 15(1.3^2)\right] = 28.025 \text{ kg} \cdot \text{m}^2$. Thus

$$T = \frac{1}{2}I_B\,\omega^2 = \frac{1}{2}(28.025)\,\omega^2 = 14.0125\,\omega^2$$

Since the pendulum is released from rest, $T_1 = 0$.

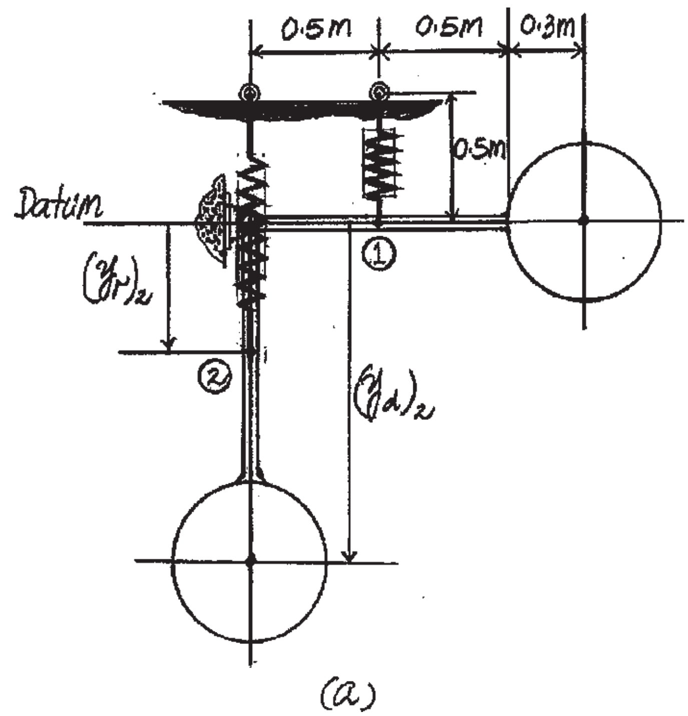
Potential Energy. with reference to the datum set in Fig. a, the gravitational potential energies of the pendulum when it is at positions ① and ② are

$$(V_g)_1 = m_r g(y_r)_1 + m_d g(y_d)_1 = 0$$

$$(V_g)_2 = m_r g(y_r)_2 + m_d g(y_d)_2$$

$$= 6(9.81)(-0.5) + 15(9.81)(-1.3)$$

$$= -220.725 \text{ J}$$



The stretch of the spring when the pendulum is at positions ① and ② are

$$x_1 = 0.5 - 0.2 = 0.3 \text{ m}$$

$$x_2 = 1 - 0.2 = 0.8 \text{ m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(200)(0.3^2) = 9.00 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(200)(0.8^2) = 64.0 \text{ J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

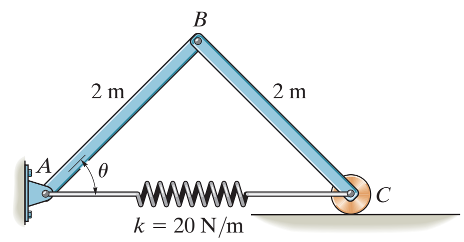
$$0 + (0 + 9.00) = 14.0125\omega^2 + (-220.725) + 64.0$$

$$\omega = 3.4390 \text{ rad/s} = 3.44 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 3.44$ rad/s

***18–60.**

The two 12-kg slender rods are pin connected and released from rest at the position $\theta = 60°$. If the spring has an unstretched length of 1.5 m, determine the angular velocity of rod BC, when the system is at the position $\theta = 30°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of rod BC at final position with IC so located, Fig. *a*

$$r_{B/IC} = r_{C/IC} = 2 \text{ m} \qquad r_{G/IC} = 2 \sin 60° = \sqrt{3} \text{ m}$$

Thus,

$$(V_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (V_B)_2 = (\omega_{BC})_2(2)$$

$$(V_C)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (V_C)_2 = (\omega_{BC})_2(2)$$

$$(V_G)_2 = (\omega_{BC})_2 r_{G/IC}; \qquad (V_G)_2 = (\omega_{BC})_2\left(\sqrt{3}\right)$$

Then for rod AB,

$$(V_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(2) = (\omega_{AB})_2(2)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

Thus,

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(V_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(12)\left(2^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(12)\left(2^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}(12)[(\omega_{BC})_2\sqrt{3}]^2$$

$$= 28.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. *b*, the initial and final gravitational potential energy of the system are

$$(V_g)_1 = 2mgy_1 = 2[12(9.81)(1 \sin 60°)] = 203.90 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 2[12(9.81)(1 \sin 30°)] = 117.72 \text{ J}$$

The stretch of the spring when the system is at initial and final position are

$$x_1 = 2(2 \cos 60°) - 1.5 = 0.5 \text{ m}$$

$$x_2 = 2(2 \cos 30°) - 1.5 = 1.9641 \text{ m}$$

Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(20)\left(0.5^2\right) = 2.50 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(20)\left(1.9641^2\right) = 38.58 \text{ J}$$

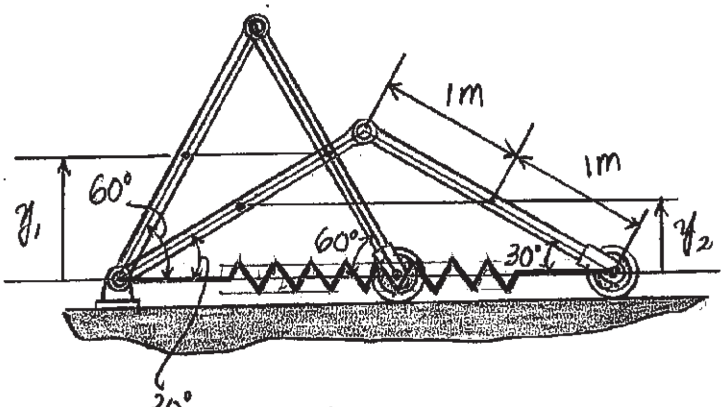
18–60. Continued

Conservation of Energy.

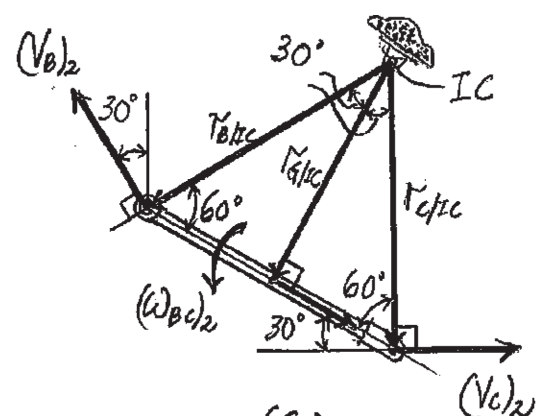
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (203.90 + 2.50) = 28.0(\omega_{BC})_2^2 + (117.72 + 38.58)$$

$$\omega_{BC} = 1.3376 \text{ rad/s} = 1.34 \text{ rad/s}$$ **Ans.**



(b)

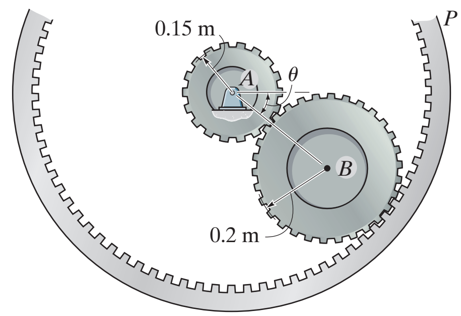


(a)

Ans:
$\omega_{BC} = 1.34$ rad/s

18–61.

If the 40-kg gear B is released from rest at $\theta = 0°$, determine the angular velocity of the 20-kg gear A at the instant $\theta = 90°$. The radii of gyration of gears A and B about their respective centers of mass are $k_A = 125$ mm and $k_B = 175$ mm. The outer gear ring P is fixed.



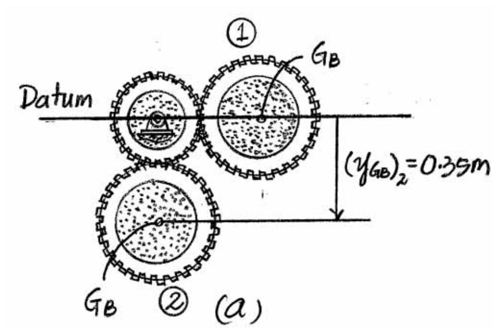
SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of gear B at positions (1) and (2) is

$$V_1 = (V_g)_1 = W_B(y_{GB})_1 = 40(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_B(y_{GB})_2 = -40(9.81)(0.35) = -137.34 \text{ J}$$

Kinetic Energy: Referring to Fig. *b*, $v_P = \omega_A r_A = \omega_A(0.15)$. Then, $\omega_B = \dfrac{v_P}{r_{P/IC}} = \dfrac{\omega_A(0.15)}{0.4} = 0.375\omega_A$. Subsequently, $v_{GB} = \omega_B r_{GB/IC} = (0.375\omega_A)(0.2) = 0.075\omega_A$. The mass moments of inertia of gears A and B about their mass centers are $I_A = m_A k_A^2 = 20(0.125^2) = 0.3125 \text{ kg}\cdot\text{m}^2$ and $I_B = m_B k_B^2 = 40(0.175^2) = 1.225 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is



$$T = T_A + T_B$$

$$= \frac{1}{2} I_A\omega_A^2 + \left[\frac{1}{2} m_B v_{GB}^2 + \frac{1}{2} I_B\omega_B^2\right]$$

$$= \frac{1}{2}(0.3125)\omega_A^2 + \left[\frac{1}{2}(40)(0.075\omega_A)^2 + \frac{1}{2}(1.225)(0.375\omega_A)^2\right]$$

$$= 0.3549\omega_A^2$$

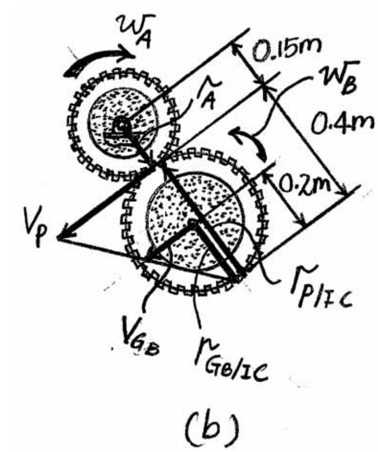
Since the system is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

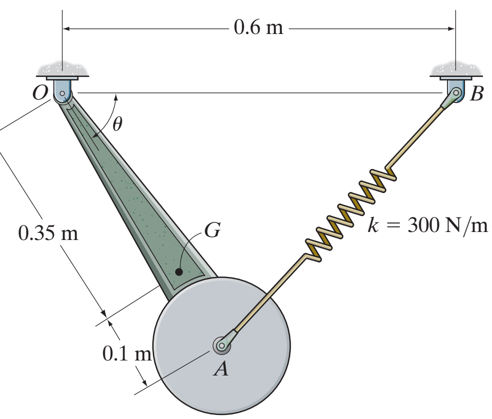
$$0 + 0 = 0.3549\omega_A^2 - 137.34$$

$$\omega_A = 19.7 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega_A = 19.7$ rad/s

18–62. The 30 kg pendulum has its mass center at G and a radius of gyration about point G of $k_G = 300$ mm. If it is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 90°$. Spring AB has a stiffness of $k = 300$ N/m and is unstretched when $\theta = 0°$.



SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the pendulum at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 30(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -30(9.81)(0.35) = -103.005 \text{ J}$$

Since the spring is unstretched initially, $(V_e)_1 = 0$. When $\theta = 90°$, the spring stretches $s = AB - A'B = \sqrt{0.45^2 + 0.6^2} - 0.15 = 0.6$ m. Thus,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(300)\left(0.6^2\right) = 54 \text{ J}$$

and

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 0$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -103.005 + 54 = -49.005 \text{ J}$$

Kinetic Energy: Since the pendulum rotates about a fixed axis, $v_G = \omega r_G = \omega(0.35)$. The mass moment of inertia of the pendulum about its mass center is $I_G = mk_G{}^2 = 30\left(0.3^2\right) = 2.7 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the pendulum is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

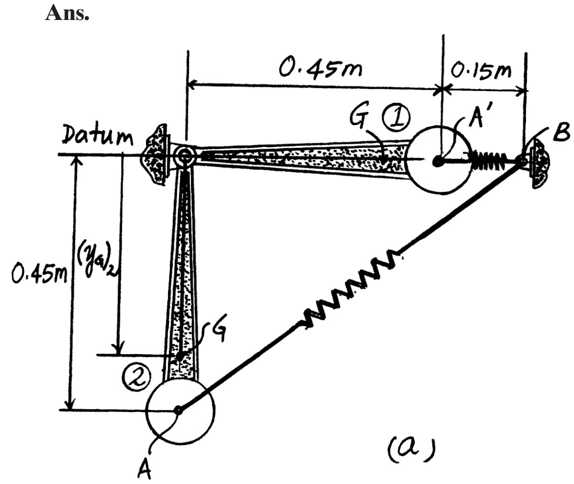
$$= \frac{1}{2}(30)\left[\omega(0.35)\right]^2 + \frac{1}{2}(2.7)\omega^2 = 3.1875\omega^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 3.1875\omega^2 - 49.005$$

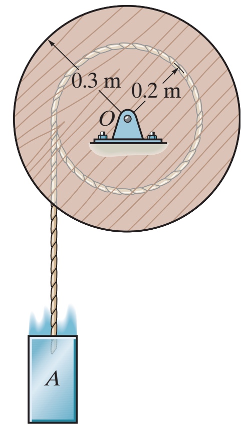
$$\omega = 3.92 \text{ rad/s}$$ **Ans.**



Ans:
$\omega = 3.92$ rad/s

18–63.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

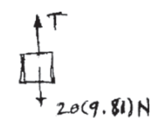
$v_A = 0.2\omega = 0.2(5) = 1 \text{ m/s}$

System:

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$$

$s = 0.30071 \text{ m} = 0.301 \text{ m}$ **Ans.**



Block:

$T_1 + \Sigma U_{1-2} = T_2$

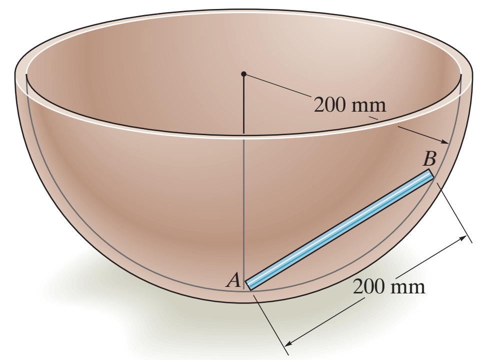
$$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$$

$T = 163 \text{ N}$ **Ans.**

Ans:
$s = 0.301$ m
$T = 163$ N

***18–64.**

The 500-g rod AB rests along the smooth inner surface of a hemispherical bowl. If the rod is released from rest from the position shown, determine its angular velocity at the instant it swings downward and becomes horizontal.



SOLUTION

Select datum at the bottom of the bowl.

$$\theta = \sin^{-1}\left(\frac{0.1}{0.2}\right) = 30^\circ$$

$$h = 0.1 \sin 30^\circ = 0.05$$

$$CE = \sqrt{(0.2)^2 - (0.1)^2} = 0.1732 \text{ m}$$

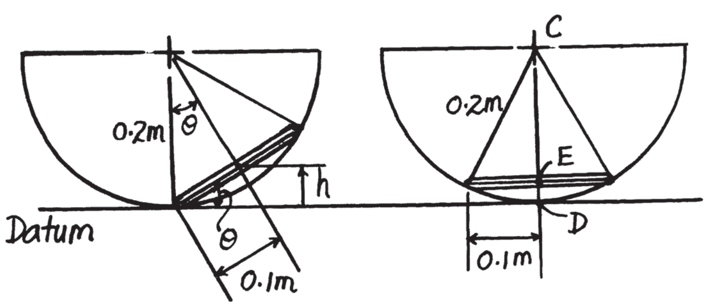
$$ED = 0.2 - 0.1732 = 0.02679$$

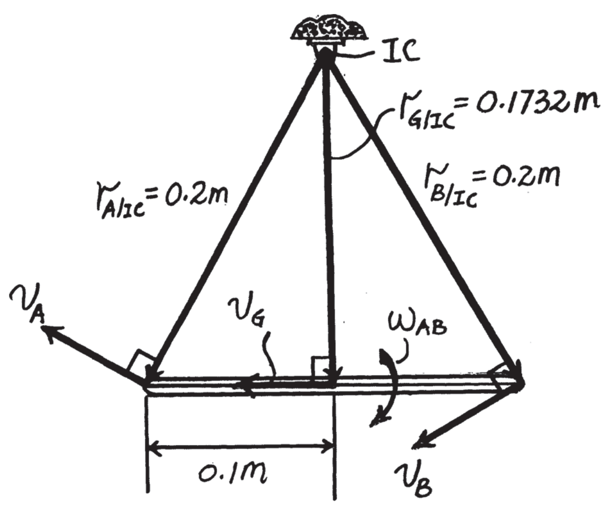
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(9.81)(0.05) = \frac{1}{2}\left[\frac{1}{12}(0.5)(0.2)^2\right]\omega_{AB}^2 + \frac{1}{2}(0.5)(v_G)^2 + (0.5)(9.81)(0.02679)$$

Since $v_G = 0.1732\omega_{AB}$

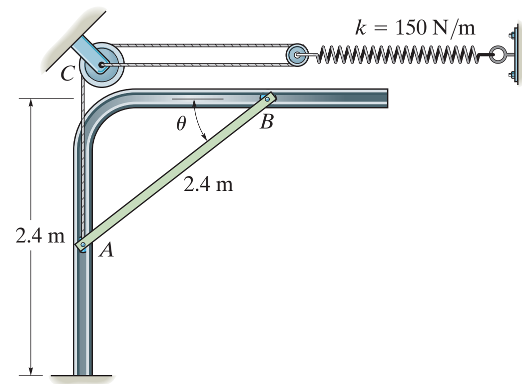
$$\omega_{AB} = 3.70 \text{ rad/s}$$ **Ans.**





Ans:
$\omega_{AB} = 3.70$ rad/s

18–65. The motion of the uniform 40-kg garage door is guided at its ends by the track. Determine the required initial stretch in the spring when the door is open, $\theta = 0°$, so that when it falls freely it comes to rest when it just reaches the fully closed position, $\theta = 90°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



SOLUTION

$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$2.4 \text{ m} = -2\Delta s_s$$

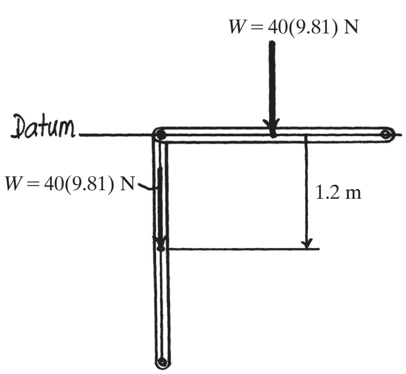
$$\Delta s_s = -1.2 \text{ m}$$

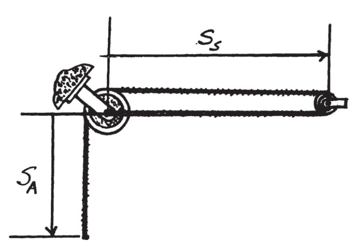
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(150)s^2\right] = 0 - 40(9.81)(1.2) + 2\left[\frac{1}{2}(150)(1.2 + s)^2\right]$$

$$150s^2 = -470.88 + 150(1.44 + 2.4s + s)^2$$

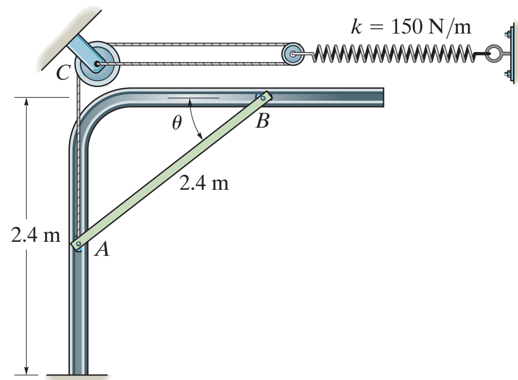
$$s = 0.708 \text{ m} \qquad \textbf{Ans.}$$





Ans:
$s = 0.708$ m

18–66. The motion of the uniform 40-kg garage door is guided at its ends by the track. If it is released from rest at $\theta = 0°$, determine the door's angular velocity at the instant $\theta = 30°$. The spring is originally stretched 0.3 m when the door is held open, $\theta = 0°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



SOLUTION

$v_G = 1.2\omega$

$s_A + 2s_s = l$

$\Delta s_A = -2\Delta s_s$

$1.2 \text{ m} = -2\Delta s_s$

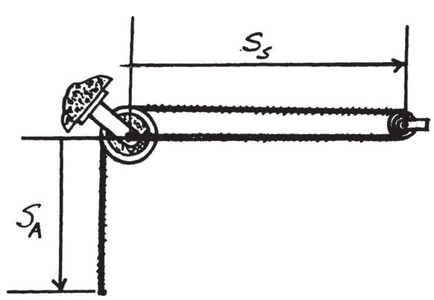
$\Delta s_s = -0.6 \text{ m}$

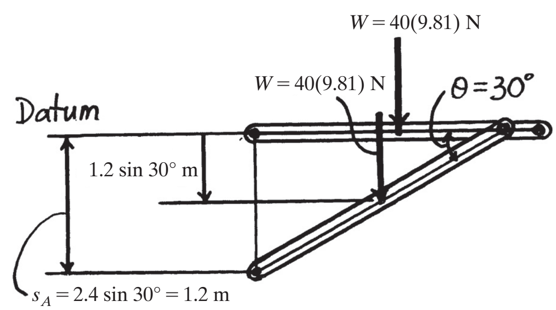
$T_1 + V_1 = T_2 + V_2$

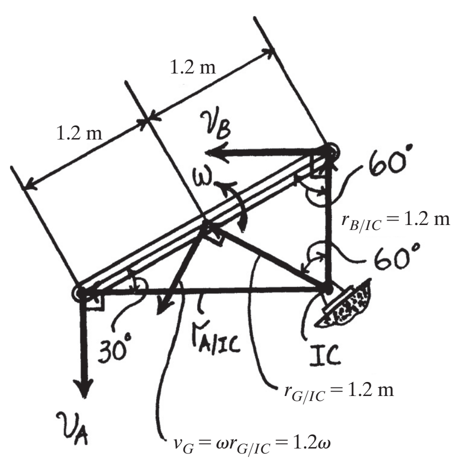
$$0 + 2\left[\frac{1}{2}150(0.3^2)\right] = \frac{1}{2}(40)(1.2\omega)^2 + \frac{1}{2}\left[\frac{1}{12}(40)(2.4^2)\right]\omega^2 - 40(9.81)(1.2\sin 30°)$$

$$+ 2\left[\frac{1}{2}(150)(0.6 + 0.3)^2\right]$$

$\omega = 1.82 \text{ rad/s}$ **Ans.**



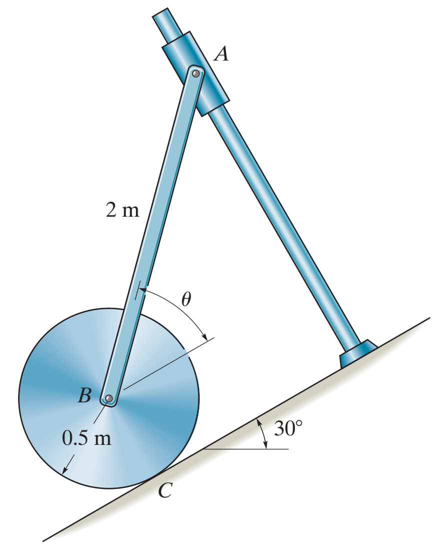




Ans:
$\omega = 1.82 \text{ rad/s}$

18–67.

The system consists of a 30-kg disk, 12-kg slender rod BA, and a 5-kg smooth collar A. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 0°$. The system is released from rest when $\theta = 45°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of the rod at its final position, Fig. *a*, we found that IC is located at B. Thus, $(v_B)_2 = 0$. Also

$$(v_A)_2 = (\omega_r)_2 r_{A/IC}; \qquad (v_A)_2 = (\omega_r)_2(2) \qquad (\omega_r)_2 = \frac{(v_A)_2}{2}$$

Then

$$(v_{Gr})_2 = (\omega_r)_2(r_{Gr/IC}); \qquad (v_{Gr})_2 = \frac{(v_A)_2}{2}(1) = \frac{(v_A)_2}{2}$$

For the disk, since the velocity of its center $(v_B)_2 = 0$, $(\omega_d)_2 = 0$. Thus,

$$T_2 = \frac{1}{2}m_r(v_{Gr})_2^2 + \frac{1}{2}I_{Gr}(\omega_r)_2^2 + \frac{1}{2}m_c(v_A)_2^2$$

$$= \frac{1}{2}(12)\left[\frac{(v_A)_2}{2}\right]^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right]\left[\frac{(v_A)_2}{2}\right]^2 + \frac{1}{2}(5)(v_A)_2^2$$

$$= 4.50(v_A)_2^2$$

Potential Energy. Datum is set as shown in Fig. *a*. Here,

$$S_B = 2 - 2\cos 45° = 0.5858 \text{ m}$$

Then

$$(y_d)_1 = 0.5858 \sin 30° = 0.2929 \text{ m}$$

$$(y_r)_1 = 0.5858 \sin 30° + 1 \sin 75° = 1.2588 \text{ m}$$

$$(y_r)_2 = 1 \sin 30° = 0.5 \text{ m}$$

$$(y_c)_1 = 0.5858 \sin 30° + 2 \sin 75° = 2.2247 \text{ m}$$

$$(y_c)_2 = 2 \sin 30° = 1.00 \text{ m}$$

Thus, the gravitational potential energies of the disk, rod and collar at the initial and final positions are

$$(V_d)_1 = m_d g(y_d)_1 = 30(9.81)(0.2929) = 86.20 \text{ J}$$

$$(V_d)_2 = m_d g(y_d)_2 = 0$$

$$(V_r)_1 = m_r g(y_r)_1 = 12(9.81)(1.2588) = 148.19 \text{ J}$$

$$(V_r)_2 = m_r g(y_r)_2 = 12(9.81)(0.5) = 58.86 \text{ J}$$

$$(V_c)_1 = m_c g(y_c)_1 = 5(9.81)(2.2247) = 109.12 \text{ J}$$

$$(V_c)_2 = m_c g(y_c)_2 = 5(9.81)(1.00) = 49.05 \text{ J}$$

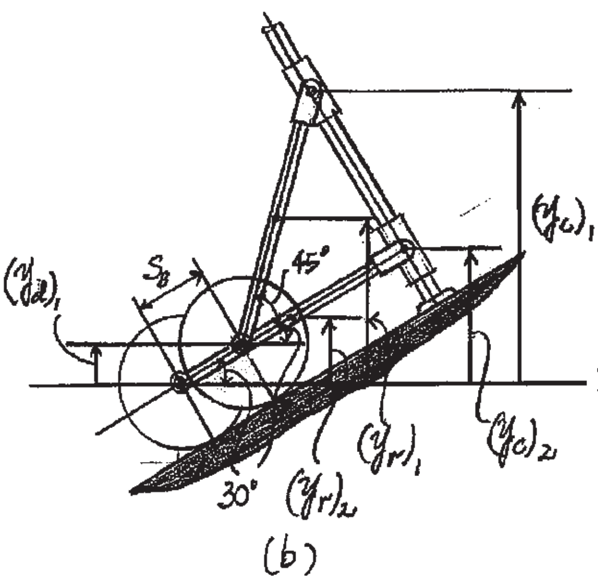
18–67. Continued

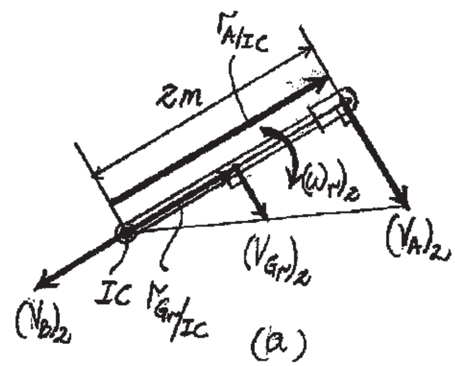
Conservation of Energy.

$T_1 + V_1 = T_2 + V_2$

$0 + (86.20 + 148.19 + 109.12) = 4.50(v_A)_2^2 + (0 + 58.86 + 49.05)$

$(v_A)_2 = 7.2357 \text{ m/s} = 7.24 \text{ m/s}$ **Ans.**

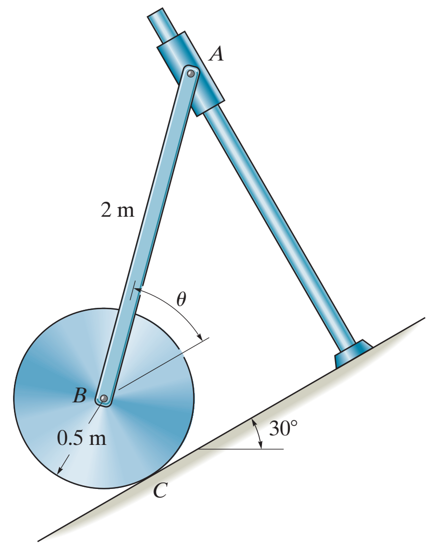




Ans:
$(v_A)_2 = 7.24 \text{ m/s}$

***18–68.**

The system consists of a 30-kg disk A, 12-kg slender rod BA, and a 5-kg smooth collar A. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of the rod at final position with IC so located, Fig. a,

$$r_{A/IC} = 2\cos 30° = 1.7321 \text{ m} \qquad r_{B/IC} = 2\cos 60° = 1.00 \text{ m}$$

$$r_{Gr/IC} = \sqrt{1^2 + 1.00^2 - 2(1)(1.00)\cos 60°} = 1.00 \text{ m}$$

Then

$$(v_A)_2 = (\omega_r)_2(r_{A/IC}); \qquad (v_A)_2 = (\omega_r)_2(1.7321) \qquad (\omega_r)_2 = 0.5774(v_A)_2$$

$$(v_B)_2 = (\omega_r)_2(r_{B/IC}); \qquad (v_B)_2 = [0.5774(v_A)_2](1.00) = 0.5774(v_A)_2$$

$$(v_{Gr})_2 = (\omega_r)_2(r_{Gr/IC}); \qquad (v_{Gr})_2 = [0.5774(v_A)_2](1.00) = 0.5774(v_A)_2$$

Since the disk rolls without slipping,

$$(v_B)_2 = \omega_d r_d; \qquad 0.5774(v_A)_2 = (\omega_d)_2(0.5)$$

$$(\omega_d)_2 = 1.1547(v_A)_2$$

Thus, the kinetic energy of the system at final position is

$$T_2 = \frac{1}{2}m_r(v_{Gr})_2^2 + \frac{1}{2}I_{Gr}(\omega_r)_2^2 + \frac{1}{2}m_d(v_B)_2^2 + \frac{1}{2}I_B(\omega_d)_2^2 + \frac{1}{2}m_c(v_A)_2^2$$

$$= \frac{1}{2}(12)[0.5774(v_A)_2]^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right][0.5774(v_A)_2]^2$$

$$+ \frac{1}{2}(3.0)[0.5774(v_A)_2]^2 + \frac{1}{2}\left[\frac{1}{2}(30)(0.5^2)\right][1.1547(v_A)_2]^2$$

$$+ \frac{1}{2}(5)(v_A)_2^2$$

$$= 12.6667(v_A)_2^2$$

Potential Energy. Datum is set as shown in Fig. a. Here,

$$S_B = 2\cos 30° - 2\cos 45° = 0.3178 \text{ m}$$

Then

$$(y_d)_1 = 0.3178 \sin 30° = 0.1589 \text{ m}$$

$$(y_r)_1 = 0.3178 \sin 30° + 1 \sin 75° = 1.1248 \text{ m}$$

$$(y_r)_2 = 1 \sin 60° = 0.8660 \text{ m}$$

$$(y_c)_1 = 0.3178 \sin 30° + 2 \sin 75° = 2.0908 \text{ m}$$

$$(y_c)_2 = 2 \sin 60° = 1.7321 \text{ m}$$

***18–68. Continued**

Thus, the gravitational potential energies of the disk, rod and collar at initial and final position are

$$(V_d)_1 = m_d g(y_d)_1 = 30(9.81)(0.1589) = 46.77 \text{ J}$$

$$(V_d)_2 = m_d g(y_d)_2 = 0$$

$$(V_r)_1 = m_r g(y_r)_1 = 12(9.81)(1.1248) = 132.42 \text{ J}$$

$$(V_r)_2 = m_r g(y_r)_2 = 12(9.81)(0.8660) = 101.95 \text{ J}$$

$$(V_c)_1 = m_c g(y_c)_1 = 5(9.81)(2.0908) = 102.55 \text{ J}$$

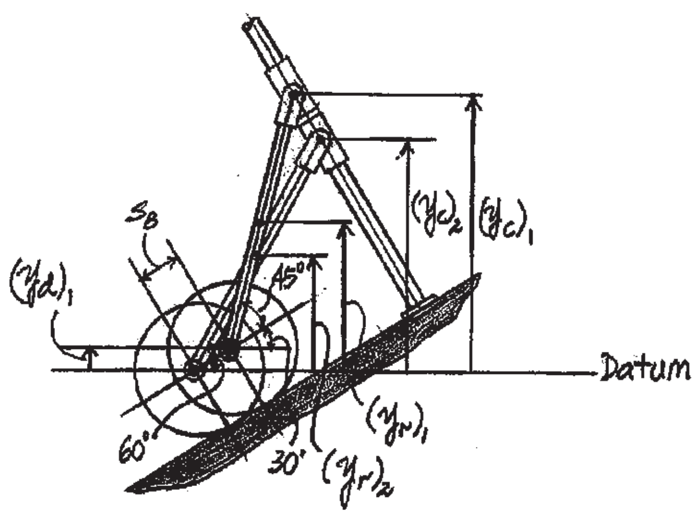
$$(V_c)_2 = m_c g(y_c)_2 = 5(9.81)(1.7321) = 84.96 \text{ J}$$

Conservation of Energy.

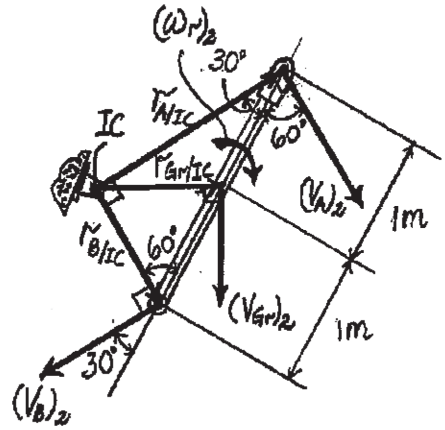
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (46.77 + 132.42 + 102.55) = 12.6667(v_A)_2^2 + (0 + 101.95 + 84.96)$$

$$(v_A)_2 = 2.7362 \text{ m/s} = 2.74 \text{ m/s}$$ **Ans.**



(b)



(a)

Ans:
$(v_A)_2 = 2.74 \text{ m/s}$