Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant* $a = a_c$
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a\, ds = v\, dv$	$v^2 = v_0^2 + 2a_c 1s - s_0 2$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$ | $a_t = \dot{v} = v \dfrac{dv}{ds}$

$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant* $\alpha = \alpha_c$
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\, d\omega = \alpha\, d\theta$	$\omega = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \qquad v = \omega r \qquad a_t = \alpha r \qquad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} \times (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $I = \int r^2\, dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F \cos\theta\, ds$
Constant force	$U_F = (F_c \cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F}\, dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F}\, dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G\, dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

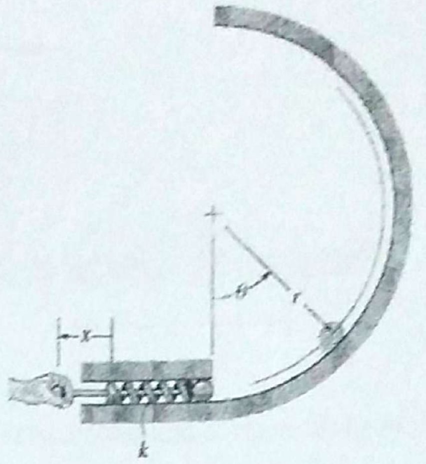
Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

Rod; $\bar{I} = \frac{1}{12} mL^2$, Plate $\bar{I} = \frac{1}{12} m(a^2 + b^2)$

Question (1):

A small ball of mass, M is fired up the vertical circular smooth track using the shown spring plunger. The plunger keeps the spring compressed a distance δ when $x = 0$. Let $M = 0.5$ kg, $\delta = 50$ mm, $r = 1.5$ m and $k = 500$ N/m.



Q1) How far x must the plunger be pulled back and released so that the ball will begin to leave the track at $\theta = \mathbf{150}^{\circ}$?

(a) 0.215 m (b) 0.192 m (c) 0.182 m (d) 0.179 m (e) 0.161 m

Questions (2-3):

A 90-kg man and a 60-kg woman stand at opposite ends of a 330-kg boat, ready to dive, each with a 10 m/s velocity **relative** to the boat from corresponding end.

Q2) If the man dives first, then the velocity of the boat, in *m/s* after they have both dived is:

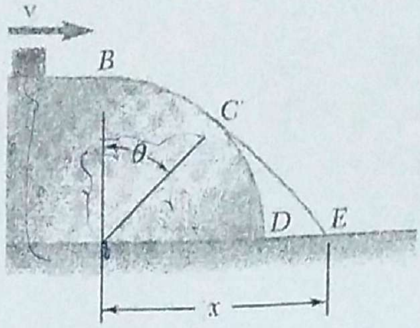
(a) -1.147 (b) -0.893 (c) -0.315 (d) -0.625 (e) -0.337

Q3) If they both dive at same time, then the velocity of the boat, in *m/s* after they have dived is:

(a) -1.147 (b) -0.893 (c) -0.315 (d) -0.625 (e) -0.337

Questions (4-6):

A small block of 3 kg mass slides on a horizontal fixed smooth surface at a speed of $v = 5$ m/s at a height $h = 3$ m above ground, as shown in the figure.



Q4) The **angular momentum** of the block about the center of the cylindrical surface, in $kg.m^2/s$ at point, ***B*** is: (where ***k*** : ccw ; **-*k*** : cw)

(a) -45 ***k*** (b) - 30 ***k*** (c) – 15 ***k*** (d) 30 ***k*** (e) 45 ***k***

Q5) The angle θ at which it will leave the smooth cylindrical surface *BCD* is:

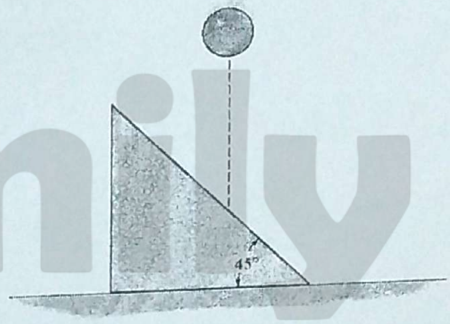
(a) 48.2° (b) 74.1° (c) 68.2° (d) 18.2° (e) 45.0°

Q6) The velocity of the block, in ***m/s*** at point ***C*** will be:

(a) 5.0 (b) 5.3 (c) 4.8 (d) 5.9 (e) 3.5

Questions (7-9):

A sphere of **2-kg** mass falls from rest at height, ***h*** above impact point and strikes a **6-kg** triangular block with a vertical velocity, $v = 5$ ***m/s***. If the triangular block rests on a smooth surface and the coefficient of restitution, $e = 1$, then find the following:



Q7) The height of the ball, ***h*** above impact point, **before** the impact is:

(a) 2.17 m (b) 2.71 m (c) 1.72 m (d) 1.27 m (e) 5.00 m

Q8) The velocity of **the block**, in ***m/s*** just after the impact is

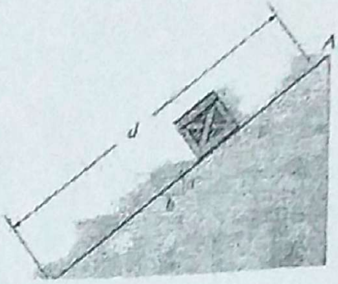
(a) 4.34 (b) 1.42 (c) 3.43 (d) 5.00 (e) 2.41

Q9) Assume that the velocity of the sphere after impact is equal to **5 m/s** in the **horizontal** direction, and if the time of impact between the sphere and the block is **0.003 s**, then the average impulsive **force (in kN)** exerted on the sphere by the block is:

(a) 7.07 (b) 14.14 (c) 4.71 (d) 3.53 (e) zero

Questions (10):

A block of 3 kg mass rests initially at the **bottom** of an inclined smooth surface of length, ***d*** = 10 m making an angle of 30° with the horizontal. This block is subjected to a force, ***F*** **parallel** to the inclined surface, for only **one second,** then the block continues its motion such that it reached the top of the inclined surface at point, ***A,*** where its velocity is zero.



Q10) What is the **minimum** value of force, ***F*** ?

(a) 14.715 N (b) 25.312 N (c) 37.968 N (d) 50.625 N (e) 12.656 N