Dynamics

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MechFamily
كُن أنت التغيير..
Notebooks

Dynamics

first
7/3

chapter 12 - Kinematics of particles -

3 kinds of motion based on speed / acceleration relationship :

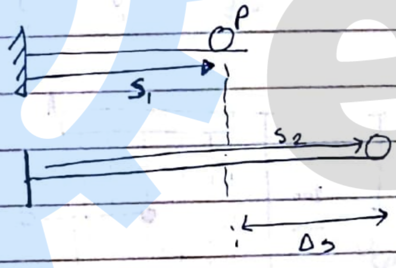
① constant speed
② constant Acceleration
③ variable Acceleration

* before solving any problem we have to → reference difine

Motion based on particles :

① Rectilinear motion
② curvilinear motion

position m



Displacement (ΔS)

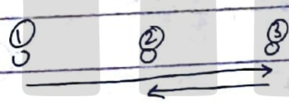
$\Delta S = S_2 - S_1$

Velocity (v) (m/s)

$\boxed{V_{avg} = \frac{\Delta s}{\Delta t}}$ $V_{inst.} = \frac{ds}{dt}$

Average speed

$\boxed{(V_{sp})_{avg} = \frac{S_T}{\Delta t}}$ → total distance



avg velocity
$1 \rightarrow 2$

avg speed
$1 \rightarrow 3 + 2 \rightarrow 3$

Acc.

$\boxed{a_{avg} = \frac{\Delta V}{\Delta t}}$ → velocity   it can be + or -

$a = \frac{dv}{dt}$

imp.

$\boxed{a \, ds = v \, dv}$

$a = \frac{dv}{dt} \rightarrow dt = \frac{ds}{v} \rightarrow v = \frac{ds}{dt}$

$a = \frac{dv}{\frac{ds}{v}}$

$\frac{a \, ds}{v} = dv \rightarrow a \, ds = v \, dv$

For constant acceleration **ONLY**:

we should be careful about the units

1. $v = v_0 + a_c t$
2. $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
3. $v^2 = v_0^2 + 2a_c(s - s_0)$

displacement
مسافة

Ex 12.1

$v = 0.6t^2 + t \quad m/s$

$s = ?! \quad a = ?! \quad t = 3\,s$

$t = 0 \quad s = 0$

$a = \frac{dv}{dt}$

$a\big|_{t=3} = 1.2t + 1$

$v = \frac{ds}{dt}$

$\int_0^s ds = \int_0^t v\,dt$

$\longrightarrow a \longrightarrow$ اشتقاق

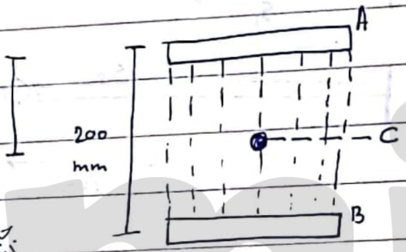
$\longrightarrow s \longrightarrow$ تكامل

?! a.a أولى

Ex 12.4

$a = 4s \quad m/s^2 \quad / \quad v_0 = 0$

$v_b = 2$



Variable acc. so we can't use newtons laws

@ $v_0 = 0$

$v_B = 2$

(b) $t = ?!$

$C \rightarrow B$

so we use this relation

$\int a\,ds = \int v\,dv$

$\int_{0.1}^{s} 4s\,ds = \int_0^{v_b} v\,dv$

حبيت الحدود من وين بتيجي المسافة

@ $s = 0.2$

$v = 346\,mm/s$

$2s^2\Big]_{0.1}^{s} = \frac{1}{2} v_b^2$

$2s^2 - 2(0.1)^2 = \frac{1}{2} v_b^2$

$\boxed{v^2 = 4s^2 - 0.04}$

we can use it when in need

(b) $v = \frac{ds}{dt}$

$dt = \frac{ds}{v}$

$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{4s^2 - 0.04}}$

$t(s) = \ldots$

6/2 ②

ex 12.5

$V = 3t^2 - 6t \quad m/s$

distance traveled = ?! $\qquad t = 3.5\ s$

$V_{avg} = ?!$ $\qquad (V_{sp})_{avg} = ?!$

imp.

$S = t^3 - 3t^2$

$S\big|_{t=3.5} = 6.125\ m$

$V_{avg} = \frac{\Delta S}{\Delta t} = \frac{6.125 - 0}{3.5 - 0} = 1.75\ m/s$

$V_{sp} = \frac{S_T}{\Delta t}$

عشان أعرف اذا غير اتجاهه
بشوف اذا صارت السرعة صفر
عشان يعتبر يغير

$V = 0 = 3t^2 - 6t$

$t_1 = 0$

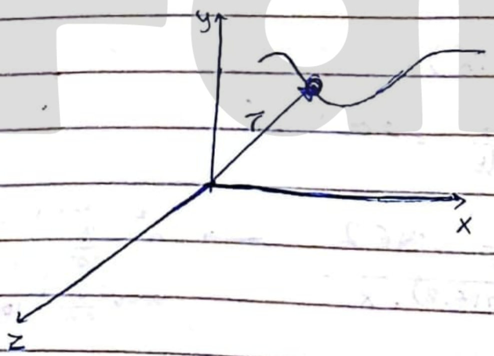
$t_2 = 2\,s$

$S\big|_{t=2} = -4\,m$

$-4\,m \qquad x$

$V_{sp} = \frac{4 + 4 + 6.125}{3.5}$

* Curvilinear motion



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{v} = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

$\dot{x} = v_x \qquad \dot{y} = v_y \qquad \dot{z} = v_z$

$\vec{a} = \frac{dr^2}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$

$\ddot{x} \qquad \ddot{y} \qquad \ddot{z}$

$\dot{v}_x \qquad \dot{v}_y \qquad \dot{v}_z$

$a_x \qquad a_y \qquad a_z$

ex 12.9

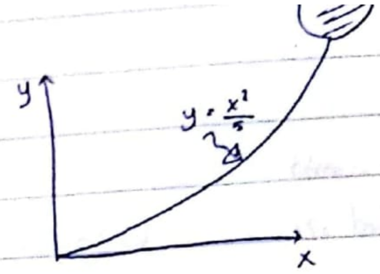
$x = 2t$

$v = ?!$

$a = ?!$

$t = 2\,s$

مفترض نحط معادلة بدلالة
الثانية ثم نشتق ونعوض
بس هون منستخدم ال
chain rule



$v_x = \frac{dx}{dt} = 2$

use the chain rule $\quad v_y = \frac{dy}{dt}\left(\frac{x^2}{5}\right) = \frac{dy}{dx}\cdot\frac{dx}{dt} = \left(\frac{2x}{5}\right)(\dot{x}) = \frac{2(4)(2)}{5} = 3.2\ m/s$

$a_x = 0$

$a_y =$ with respect to time

$\frac{d}{dt}\left(\frac{2x\dot{x}}{5}\right) = \frac{2\dot{x}\dot{x}}{5} + \cancel{\frac{2x\ddot{x}}{5}}^{\,0} = \frac{2(2)(2)}{5} = 1.6\ m/s^2$

$v = \sqrt{v_x^2 + v_y^2} = \ldots$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$a = \sqrt{a_y^2}$

$\theta =$ in the y axis since we dont have $\underline{\underline{a}}$ in the x direction

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ex 12.10

$y = 0.01x^2$

$v_y = 10\ m/s$ [constant]

$v = ?!$

$a = ?!$

$y = 100\ m$

$v_y = \frac{d(0.01x^2)}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = 0.02x\ .\ \dot{x}$

$x = \sqrt{\frac{100}{0.01}} = 316.2 \longrightarrow x^2 = \frac{y}{0.01}$

$x = \sqrt{\frac{y}{0.01}} \rightarrow 100$

$10 = 0.02\,(316.2)\,.\,\dot{x}$

$\dot{x} = 15.8\ m/s$

$v_x = 15.8\ m/s$

$v_y = 10\ m/s$

imp ↓ $\quad v = \sqrt{15.8^2 + 10^2} = \ldots$

$\theta = \ldots$

$v_y = $ constant ← $a_y = 0$

$a_y = 0.002\,\dot{x}\dot{x} + 0.002\,x\ddot{x}$

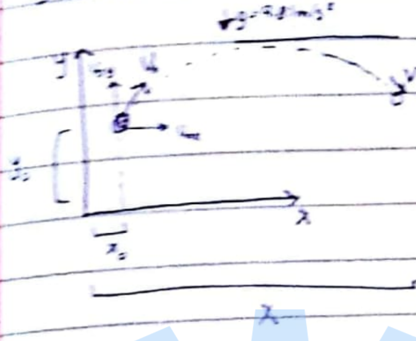
$0 = 0.002\,(15.8)^2 + 0.002\,(316.2)\,\underset{a_x}{\ddot{x}}$

$a_x = 0.791\ m/s^2$

projectile motion:

- the only acceleration is in the $-y$ direction
- there are other things

② Motion of a projectile



x:

$$\boxed{\begin{aligned} V_x &= V_{0x} \\ V_x^2 &= V_{0x}^2 \longrightarrow V_x = V_{0x} \\ x &= x_0 + V_x t \end{aligned}}$$

its constant in the x direction so $a = 0$

y:

$$\boxed{\begin{aligned} V_y &= V_{y_0} - gt \\ V_y^2 &= V_{0y}^2 - 2g(y - y_0) \\ y &= y_0 + V_y t - \tfrac{1}{2} g t^2 \end{aligned}}$$

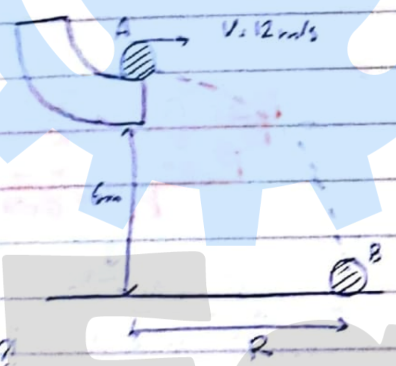
a is constant and equals 9.81

$V = V_0 + a_c t$

$V^2 = V_0^2 + 2a_c(S - S_0)$

$S = S_0 + V_0 t + \frac{1}{2} a_c t^2$

ex: 12.11



$t = ?$

$R = ??$

point A is my reference

we use 3rd eqn

$y = y_0 + V_y t - \frac{1}{2} g t^2$

$-6 = 0 + 0 - \frac{1}{2}(9.81) t^2$

$t = 1.11\,s$

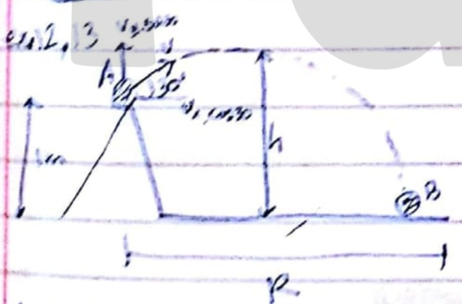
here we'll work with the x direction

$x = x_0 + V_{0x} t$

$R = 0 + 12 * 1.11$

$R = 13.3\,m$

ex: 12.13



$t = 1.5\,s$

$V_A = ??$

$R = ?$

$h = ?$

$y = y_0 + V_y t + -\frac{1}{2} a t^2$

$-1 = 0 + V_{Ay} \sin 30° * 1.5 - \frac{1}{2} * 9.81 * (1.5)^2$ . . . then find V_A

$V_A = 13.4\,m/s$

$x = x_0 + V_{0x} t$

$R = 0 + 13.4 * \cos 30° * 1.5$

$R = 17.4\,m$

we use eq ② → منقدر نطلع t من الاولى
بدين نعوض ب③ بس طارق
طويلة

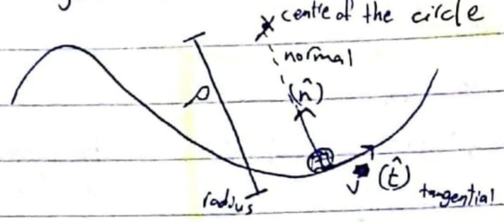
$V_{y_c}^2 = V_{Ay}^2 - 2g(y_c - y_A)$

$= (13.4 \sin 30°)^2 - 2(9.81)([h - 1] - 0)$

$h = 3.28\,m$

Curvilinear Motion:

Tangential & Normal components



v is tangent to the path

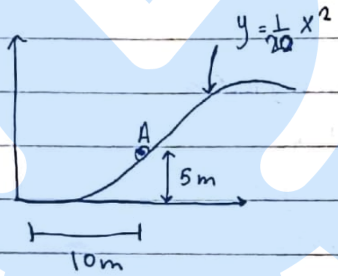
$\boxed{v = v(\hat{t})}$ in the direction of t

$\boxed{a_t = \dot{v}(\hat{t})}$

$\boxed{a_n = \frac{v^2}{\rho}(\hat{n})}$ direction of $\hat{n}$

$$\boxed{a = \sqrt{a_t^2 + a_n^2}}$$

11/2 (4) 12.14



$\theta_v = ??$

$|a|$ & $\theta_a =$

$v = 6\ m/s$

$a_t = 2\ m/s$

$a_n = \frac{v^2}{\rho} \rightarrow \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

the slope $\frac{dy}{dx} = \frac{1}{10}x$

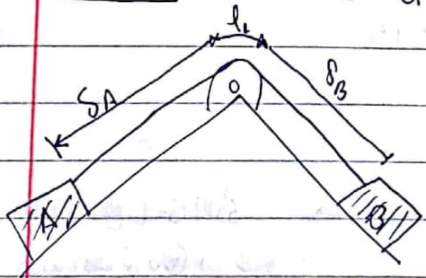
$\left.\frac{dy}{dx}\right|_{x=10} = 1 \rightarrow \theta_v = \tan^{-1}(1)$
$= \frac{\pi}{4} = 45°$

$\frac{d^2y}{dx^2} = \frac{1}{10}$

$\left.\rho\right|_{x=10} = \left[\frac{1+(1)^2}{1/10}\right]^{3/2} = 28.28\ m$

$a_n = \frac{6^2}{28.28} = \checkmark \rightarrow a$

$\boxed{12.9}$ Absolute dependent Motion



الطول الصغير فوق البكرة (ثابت)

length $= S_A + l_1 + S_B$
0

$\frac{d}{dt}(\text{length}) = \frac{d}{dt}(S_A + l_1 + S_B)$ مش مهم

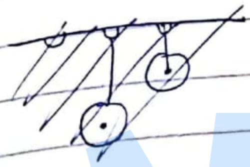
$0 = v_A + v_B$

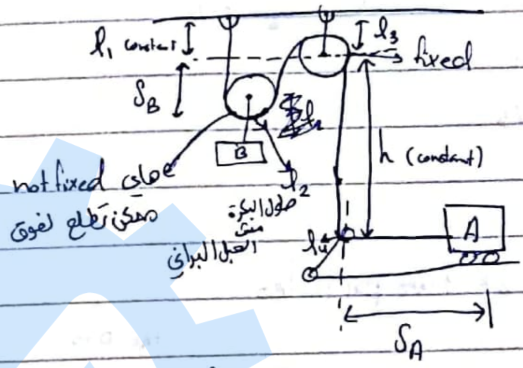
$\boxed{v_A = -v_B}$

Scanned by CamScanner

$$\frac{d}{dt}(v_A + v_B = v)$$

$$\boxed{a_A = -a_B}$$





$$\text{length} = l_1 + \delta_B + l_2 + \delta_B + l_3 + \cancel{h} + l_4 + \delta_A$$

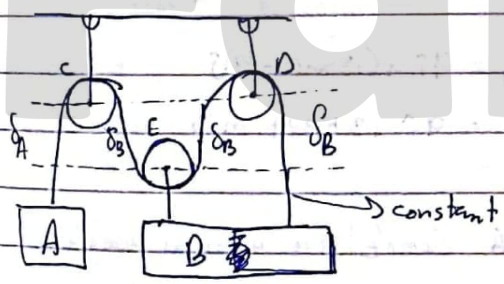
$$\frac{d}{dt}(\text{length}) = 0 \Rightarrow$$

$$v_B + v_B + v_A = 0$$

$$\boxed{2v_B = -v_A}$$

$$\frac{d}{dt}(2v_B + v_A = 0)$$

$$\boxed{2a_B = -a_A}$$



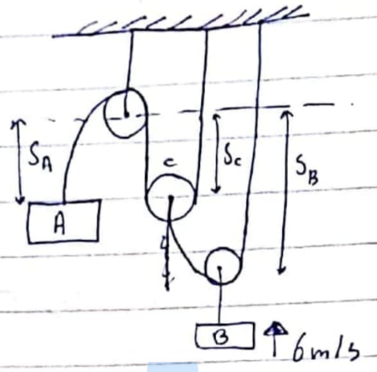
$$\text{length} = \delta_A + \delta_B + \delta_B + \delta_B$$

$$\frac{d}{dt}\text{length} = \frac{d\delta_A}{dt} + 3\frac{d\delta_B}{dt}$$

$$0 = v_A + 3v_B$$

$$v_A = \checkmark$$

13/2 (5) ex. 12.22



$L_1 = S_A + 2S_c$

$L_2 = S_B + (S_B - S_c)$

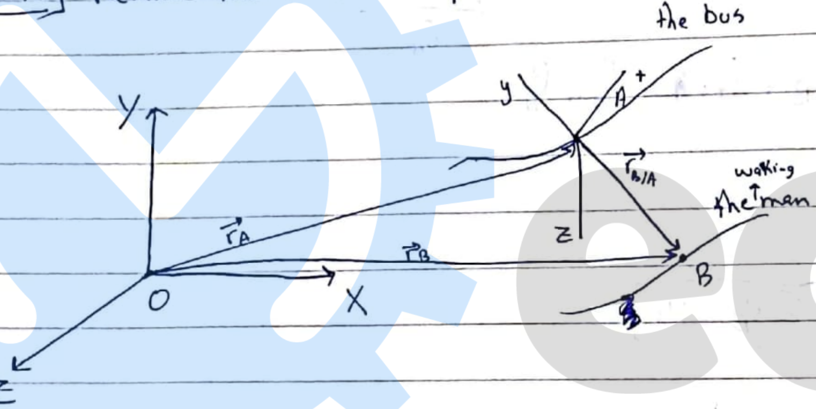
$= 2S_B - S_c$

$\rightarrow V_A = -2V_c$

$\rightarrow V_c = 2V_B$

الباقي const. مامنطلهم عشان
بس نشتق رح يروحوا

12-10 Relative motion of two particles

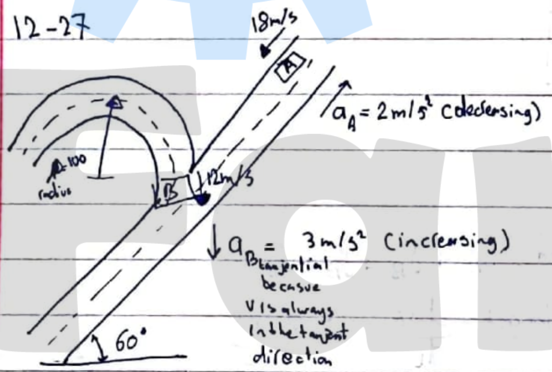


$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

ex 12-27



$a_A = 2\,m/s^2$ (decreasing)

$a_B = 3\,m/s^2$ (increasing) tangential because V is always in the tangent direction

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$-12\hat{j} = (-18\cos 60°\hat{i} - 18\sin 60\hat{j}) + V_{B/A}$

$\vec{V}_{B/A} = 9\hat{i} + (18\sin 60 - 12)\hat{j}$

$= 9\hat{i} + 3.588\hat{j}\ m/s$

Vehicle A doesn't have a normal acceleration

$(\rho = \text{infinity})$ لانه خط مستقيم لا يوجد مماس زي الدايرة

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_B = a_{Bn}\hat{i} + a_{Bt}\hat{j}$

$= -\frac{V^2}{\rho}\hat{i} - 3\hat{j}$

$= -1.44\hat{i} - 3\hat{j}\ m/s$

لسه بدون جذر

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$-1.44\hat{i} - 3\hat{j} = (2\cos 60°\hat{i} + 2\sin 60\hat{j}) + \vec{a}_{B/A}$

$a_{B/A} = (-1.44 - 1)\hat{i} + (-3 - 2\sin 60°)\hat{j}$

$= -2.44\hat{i} - 4.732\hat{j}\ m/s^2$

$|a_{B/A}| = \sqrt{2.44^2 + 4.732^2}$

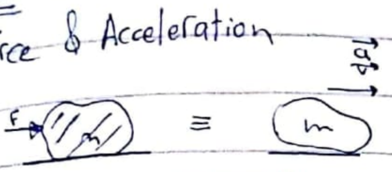
$\theta = \tan^{-1}\left(\frac{4.732}{2.44}\right)$

Sugg. problems:

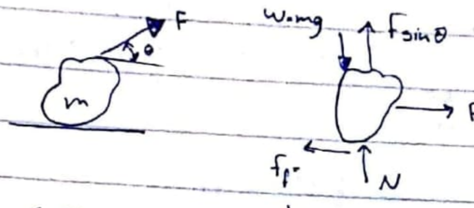
3 / 8 / 12 / 19 / 21 / 70 / 75 / 81
87 / 90 / 96 / 98 / 107 / 116 / 121 / 126
133 / 145 / 179 / 201 / 204 / 212 / 215 / 217 / 220 / 224 / 225 / 229 / 232

5/2 ⑥ ch13 Kinetics of particles

Force & Acceleration



2nd law $\sum \vec{F} = m\vec{a}$



$\sum F_x = m\vec{a}_x$

$\sum F_y = m\vec{a}_y$

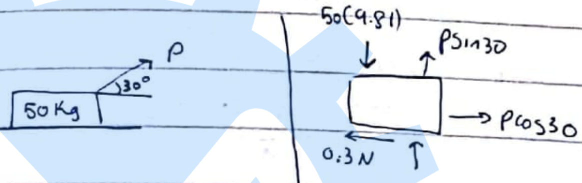
$\sum F_z = m\vec{a}_z$

$F_f = \mu N$
- μ_s : static
- μ_k : Kinetic

حسب اذا واقف او بيتحرك

problem 13.7

- $v_0 = 0$
- $v = 4\ m/s$ @ $S = 5m$
- $P = ?!$
- $\mu_k = 0.3$



$\overset{+}{\rightarrow} \sum F_x = m a_x$ | $P\cos 30° - 0.3N = 50 a_x$ ...... ①

$+\uparrow \sum F_y = 0$ | $N + P\sin 30° - 50(9.81) = 0$ ...... ②

كرفائدة a ثابت constant بلحظة معينة

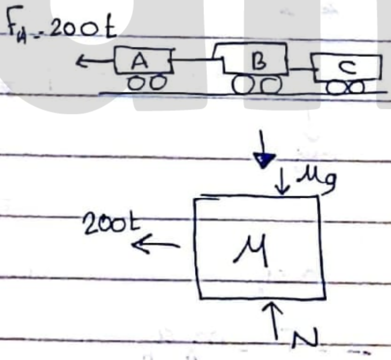
$v^2 = v_0^2 + 2a(S - S_0)$

$v = 4 \quad S_0 = 0$

$v_0 = 0 \quad S = 5 \qquad a_x = 1.6\ m/s^2$ .... ③

ex 13.3

- $m_A = 450\ kg$
- $m_B = 275\ kg$
- $m_C = 160\ kg$
- $v = ?!$ @ $t = 2s$ / $v_0 = 0$



the same velocity and acceleration for all

$F_{A-B} = ?!$ coupling

$M = M_A + M_B + M_C$

$\overset{+}{\leftarrow} \sum F_x = M \vec{a}_x$

$200t = (450 + 275 + 160) \times a_x$

$a_x = 0.226\, t$

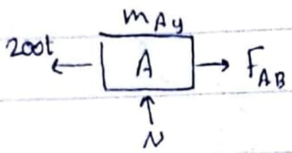
$a = \frac{dv}{dt} \rightarrow \int_0^v dv = \int_0^t a\,dt$

$v = \int 0.226\, t \cdot dt = \frac{0.226\, t^2}{2}$

$v|_{2s} = ✓$

part 2

F_{A-B} coupling



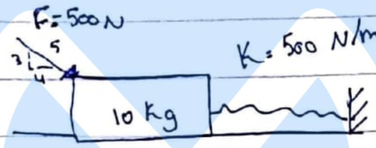
we take the B cart and replace it with the coupling force

$\overset{+}{\leftarrow} \sum F = m_A a_x$

$200k - \underset{coupling}{F_{AB}} = 450(0.226t)$

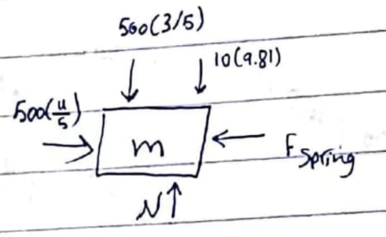
$F_{AB} \Big|_{t=2} = \checkmark$

13.3



$v = ??$ @ $s = 0.5$

$s = 0$ { block @ rest, spring is uncompressed



acceleration isn't constant , $F = Ks$
لا يزيد بتزيد ال F المطلوبة

we cant use newtons laws

$\xrightarrow{+} \sum F_x = m a_x$

$400 - \underset{\text{spring force}}{500 s} = 10 a_x$

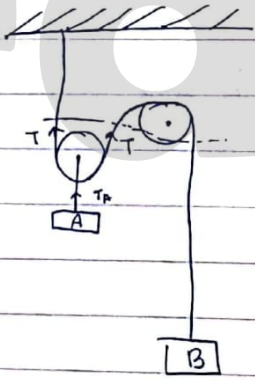
$a_x = 40 - 50s$

$a\,ds = v\,dv$

$\int_0^s (40 - 50s)\,ds = \int_0^v v\,dv$

$v(s) = \ldots$

18/2 (7) Ex 13.5



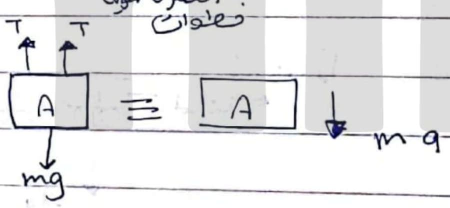
$m_A = 100$ kg

$m_B = 20$ kg

$v_B = ?$ $t = 2s$

$T_A = T + T$

اتغير بنفس الاتجاه



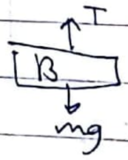
موجب الـ acceleration السالبة تعني تغير في الاتجاه متوقع decleration

$+\downarrow \sum F_y = m\vec{a}_A$

$m_A g - 2T = m_A a_A$

$100(9.81) - 2T = 100\, \vec{a}_A$

T is the same because its the same robe



$+\downarrow \sum F_y = m_B \vec{a}_B$

$M_B g - T = m_B \vec{a}_B$

$20(9.81) - T = 20 a_B$

$2S_A + S_B = 0$

$2v_A = -v_B$

$2a_A = -a_B$

$a_B = \checkmark$ 6.54

$T = \checkmark$

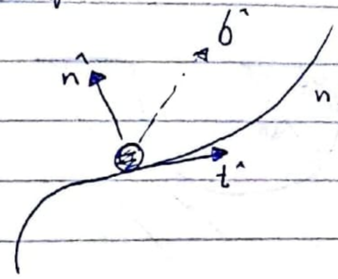
$v_B = \cancel{v_B} + a_B t$

Equations of motion : Normal & Tangential comp.

$\Sigma F_n = m\vec{a}_n$

$\Sigma F_t = m\vec{a}_t$

$\Sigma F_b = 0$

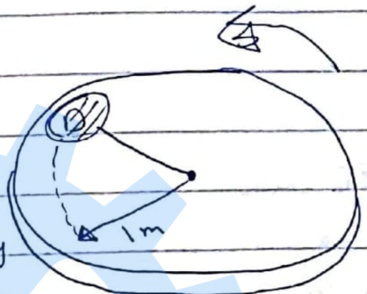


binomial axis

n/t عمودي على السطح الي فيه

a_t & rate of increase

ex: 13.7

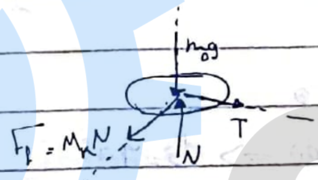


$M_D = 3\,Kg$

$t = ?! \rightarrow V_D$ = great energy to torent the cord

$T_{max} = 100\,N$

$\mu_k = 0.1$



$T = ma_n = \frac{3V^2}{(1)}$ --- ① $\rightarrow 100 = 3V^2$

$V_{cn} = 5.77\ m/s$

$\mu_t N = ma_t = 3a_t$ --- ②

$N - m_D g = 0$ --- ③

$N = 3(9.81)$

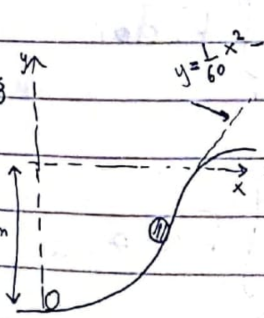
$a_t = \frac{\mu_t N}{3} = \frac{0.1(3(9.81))}{3} = 0.981$

Constant

$V = V_0 + a_t t$

5.77 ....

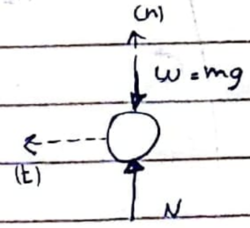
2012 ⑧ Ex 13.8



$m = 70\,Kg$

$N = ?!$ @ A

$a = ?!$



$\overset{+}{\rightarrow} \Sigma F_x = ma_t$

$0 = a_t$

$+\uparrow \Sigma F_n = ma_n$

$N - mg = ma_n$

$N - 70(9.81) = 70\left(\frac{20}{\rho}\right)$

$\frac{dy}{dx} = \frac{1}{30}x$

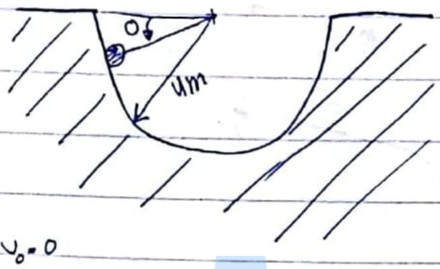
$\frac{d^2y}{dx^2} = \frac{1}{30}$

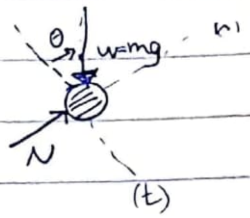
$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$, $x = 0$

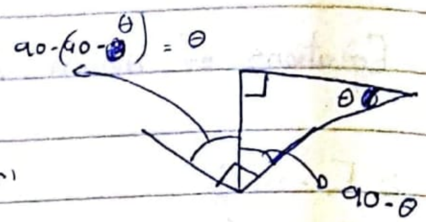
$\rho = 30$

$\frac{(1+0)^{3/2}}{\frac{1}{30}} = 30$

Ex 13.9







$\theta_0 = 0 \quad v_0 = 0$

$N = ?! \quad \theta = 60^\circ$

$\overset{+}{\searrow} \sum F_t = m a_t$ $\qquad$ $\overset{+}{\nearrow} \sum F_n = m a_n$

$\cancel{m} g \cos\theta = \cancel{m} a_t$ $\qquad$ $N - mg \sin\theta = m \frac{v^2}{\rho}$

$\boxed{a_t = g\cos\theta}$ $\qquad$ $N - 60(9.81)\sin 60 = 60 \frac{v^2}{\rho}$

$a\,ds = v\,dv$ $\qquad$ $s = r\theta$

$g\cos\theta\, ds = v\,dv$ $\qquad$ $ds = r\,d\theta \Longrightarrow ds = 4\,d\theta$

$\int g\cos\theta\,(4\,d\theta) = \int v\,dv$

$\int_0^\theta 4 g\cos\theta \,.\, d\theta = \int_0^v v\,dv$

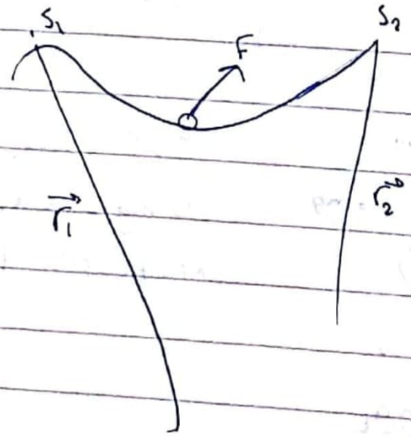
$v^2(\theta) = \checkmark \qquad \theta = 60^\circ$

$v^2 = 67.96\ m^2/s^2$

ch.14 Kinetics of particles
Work and energy

Force (F) will do work on the particle only when the ~~part of~~ particle undergoes in a displacement in the direction of the force

(1) Varience force



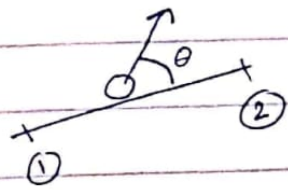
$$U_{1.2} = \int_{r_1}^{r_2} F \cdot dr$$

$$= \int_{s_1}^{s_2} F \cos\theta \,.\, ds$$

Scanned by CamScanner

(2) Constant Force

$$U_{1-2} = F\cos\theta\,(\overset{\Delta S}{S_2 - S_1})$$



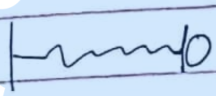
$$U_{1-2} = F(\Delta s \cos\theta)$$

(3) weight

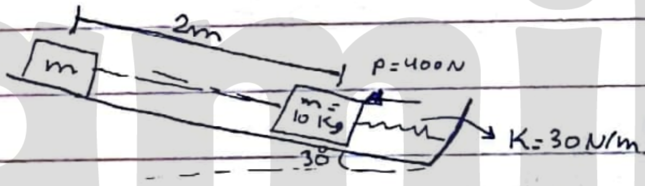
$$U_{w_{1-2}} = -W\Delta y$$

(4) work of a spring

$s = 0$ (unstreched / uncompressed)



$$U_{s_{1-2}} = -\frac{1}{2}K(s_2^2 - s_1^2)$$



Ex 14.1

$\delta_1 = 0.5\,m$

work ← $U_{total} = ?!$

$\delta_2 = 0.5 + 2 = 2.5$

$$U_{total} = U_P + U_W + U_s$$

$U_P = (P\cos 30)(2) = (P)(2\cos 30)$

$U_w = -W \Delta y = -mg(2\sin 30°)$

$= -10(9.81)(2\sin 30°) = -98.1\,J$

$U_s = -\frac{1}{2}K(s^2 - s_1^2) = -\frac{1}{2} \times 30 \times ((2+0.5)^2 - (0.5)^2)$

principle of work & energy.

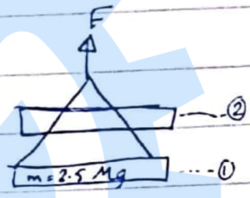
$$\underbrace{T_1}_{\text{initial K.E}} + \underbrace{\Sigma U_{1-2}}_{\text{Total work done... from ① to ②}} = \underbrace{T_2}_{\text{Final K.E}}$$

K.E

Kinetic energy $\rightarrow T = \frac{1}{2} m v^2$

---

ex 14.3



$F = (28 + 3s^2)\ kN$

$v = ?$

$t = ?$

$s = 3\ m$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\int_0^s (28 + 3s^2) \times 10^3\, ds - 2500(9.81)(s) = \frac{1}{2}(2500)\, v^2$$

$s = 3 \rightarrow v = \checkmark$

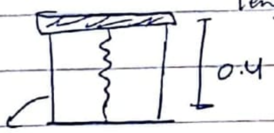
$v = (\cdot \text{---} s)^{\frac{1}{2}}$

$v = \frac{ds}{dt} \Rightarrow \int dt = \int \frac{ds}{v}$ ← fraction of s

---

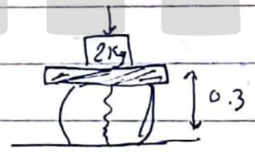
ex 14.4

length of the spring = 1.0 m

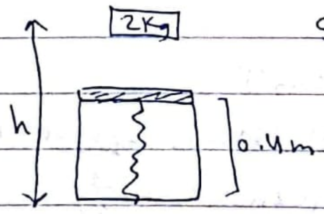


ropes مثبتة للزنبرك لحتى يضل مضغوط

state 0



state 1



بس نفلته والوزن بطير

$T_1 + \Sigma U_{1-2} = T_2$

released from rest so sth zero ($V_1 = 0$)

$$U_s + U_w = \frac{1}{2} m v^2$$

$$-\frac{1}{2}k(S_2^2 - S_1^2) + (-W\Delta y) = 0$$

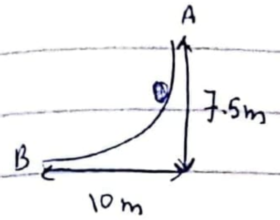
compressed and it was 1

$$-\frac{1}{2}(200)\left((0.6)^2 - (0.7)^2\right)$$

$$-2(9.81)(h - 0.3) = 0$$

Ex 14.5

$m = 40\,kg$



$V_B = ??$. $V_? = ??$

$N_B = ??$

$T_{A_1} + \Sigma V_{A_1 B} = T_B$ $\quad y_B - y_A$

$\frac{1}{2}m\cancel{v^2}_0 + -40(9.81)(0-7.5) = \frac{1}{2}(40)v_B^2$

$-mg\,\Delta y$

$V_B = \checkmark$ m/s $\qquad y = 0.075x^2$

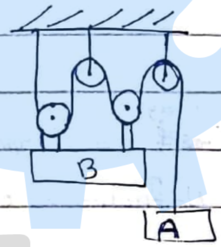
$\boxed{14.6}$

$M_A = 10\,kg$

$M_B = 100\,kg$

$S_B = ??$

$V_B = 2\,m/s$



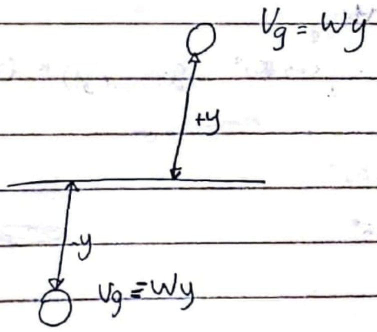
$T_1 + \Sigma V_{1-2} = T_2$

$\frac{1}{2}m\cancel{v_{A_1}}^{0\,2} + \frac{1}{2}m\cancel{v_{B_1}}^{0\,2} + \Sigma V_{1-2} = \frac{1}{2}mv_{A_2}^2 + \frac{1}{2}mv_{B_2}^2$

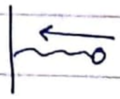
$+W_B\Delta S_B - W_A\Delta S_A = \frac{1}{2}m_A V_A^2 + \frac{1}{2} \ldots$

14.5 Conservation of energy

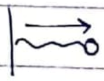
potential energy (P.E)



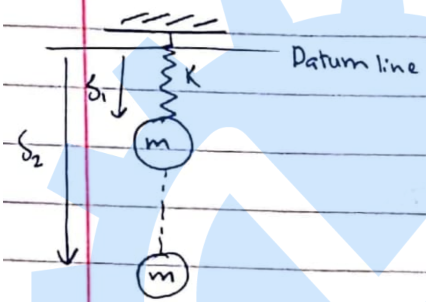
elastic (P.E) (V_e)



$V_e = \frac{1}{2} K s^2$



$V_e = \frac{1}{2} K s^2$



because below the datum line

$V_1 = V_{g_1} + V_{e1} = (-) W s_1 + \frac{1}{2} K s_1^2$

$V_2 = V_{g_2} + V_{e_2} = -W s_2 + \frac{1}{2} K s_2^2$

$V_1 - V_2 = -W(S_1 - S_2) + \frac{1}{2} K (S_1^2 - S_2^2)$

$\boxed{W(S_2 - S_1) - \frac{1}{2} K (S_2^2 - S_1^2)}$

↓ down
Positive work

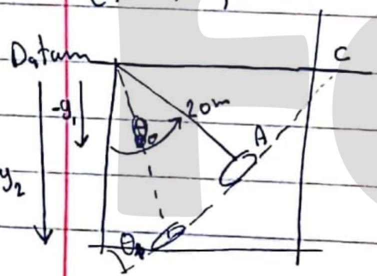
$\Sigma V_{1-2} = V_w + V_s$

$\boxed{T_1 + V_1 = T_2 + V_2}$ → Only when conservative forces (weight and spring force)

K.E P.E K.E 2 P.E 2

No Friction, no external force

ex 14.9



$m = 8\ Mg$

$\theta_0 = 60°$

$V = ?!$

$\theta = 15°$

$y_1 = 20\cos 60$

$y_2 = 20\cos 15$

$\cancel{T_1}^{0} + V_1 = T_2 + V_2$

$-W y_1 = \frac{1}{2} m v^2 - W y_2$

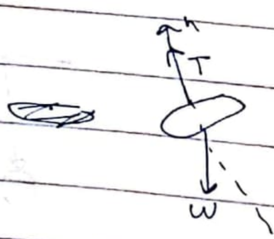
$-8000 (9.81)(20\cos 60°) = \frac{1}{2} \cos 8 v^2 - 8000 (9.81) \times (20\cos 15°)$

part 2 $\Sigma F_n = m a_n$

$= T - W\cos 15°$

$= m \frac{v^2}{(20)}$

$T =$ ___



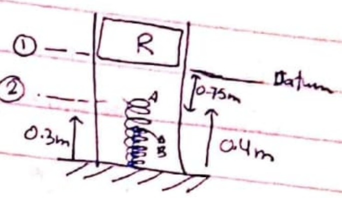
7/2 11

Ex: 14.10

$m = 100\ Kg$

$K_A = 12\ KN/m$

$K_B = 15\ KN/m$



$T_1 + V_1 = T_2 + V_2 = 0$

$T_1 = 0$: release from rest; $V_1 = 0$: its going stop; $T_2 = 0$: Datum line at first position

$-W^*(0.75 + S_A) + \frac{1}{2} K_A S_A^2 + \frac{1}{2} K_B (S_A - 0.1)^2$ (S_B)

$13500\ S_A^2 - 2481\ S_A - 660.5 = 0$

$S_A = 0.331\ m$

$S_A = -ve$ X

$S_B = 0.231\ m$

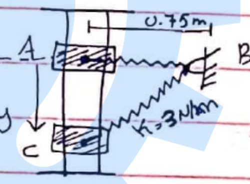
14.11

$m = 2\ Kg$

$V_c = ?$ $y = 1m$

a) $V_A = 0$

b) $V_A = 2\ m/s$



$S = (\sqrt{1^2 + 0.75^2}) - (0.75) = 0.5\ m$

$T_A + V_A = T_c + B_c$

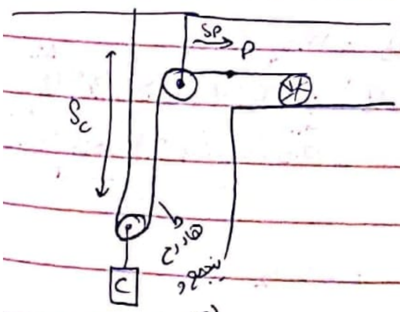
$= \frac{1}{2} m v_c^2 - Wy + \frac{1}{2} K s^2$

$= \frac{1}{2}(2) v_c^2 - 2 * 9.81 * 1 + \frac{1}{2} * 3 * (0.5)^2 = 0$

14.4 Power & efficiency

$$P = \frac{dU}{dt} = \frac{d(F.r)}{dt} = F.\frac{dr}{dt} = F.V$$

$$\varepsilon = \frac{P_{output}}{P_{input}} = \frac{\text{output energy}}{\text{input energy}}$$



$m = 35\ Kg$

$a_p = 1.2\ m/s^2$

$P_{input} = ??$ $V_p = 0.6\ m/s$

$\varepsilon = 0.85$

$+\uparrow \sum F = m a_c$

$2T - mg = m a_c$

$2T - 35(9.81) = 35 a_c$

$T = 182.2\ N$

$P_{output} = F.V$

$\text{out power} = T.V$

$= 182.2 * (0.6)$

$= 109.3\ W$

$2S_c + S_p = l$

$2\Delta S_c + \Delta S_p = 0$

$2V_c + V_p = 0$

$2a_c + a_p = 0$

$a_c = -0.6\ m/s^2$

خارج

بنحوط

تعديت يرجع يتوزع من البداية موزعاً

free body diagram

29/2 ⑫

Ch 15

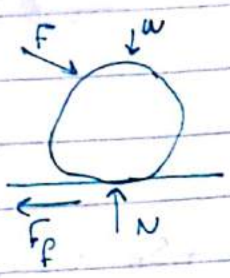
Kinetics of particles
(Impulse & momentum) دفع

$\Sigma F = m\vec{a}$

$\Sigma F = m\frac{dv}{dt}$

$\int_0^t \Sigma F\, dt = \int_{v_1}^{v_2} m\, dv$

$\int_0^t \Sigma F . dt = mv_2 - mv_1$



$mv_1 + \int_0^t \Sigma F . dt = mv_2$

Initial momentum — Impulse — Final momentum

x:

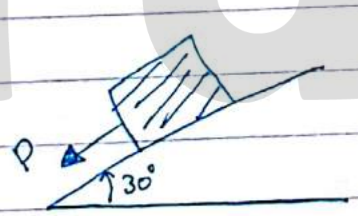
$mv_{1x} + \int_0^t \Sigma F_x . dt = mv_{2x}$

y:

$mv_{1y} + \int_0^t \Sigma F_y . dt = mv_{2y}$

z:

$mv_{1z} + \int_0^t \Sigma F_z . dt = mv_{2z}$



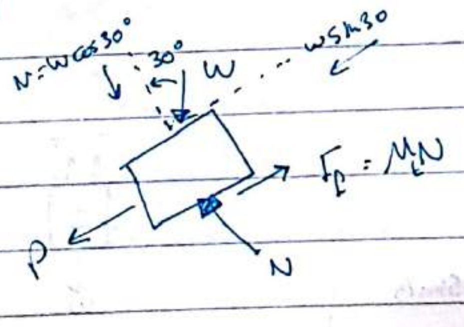
$P = 100t$ N

$V = ?!$ $t = 2$ s

$V_1 = 1$ m/s

$\mu_K = 0.3$

$m = 25$ Kg



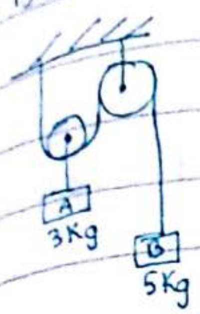
$mv_1 + \int \Sigma F . dt = mv_2$

$25(1) + \left(\int_0^2 100t . dt\right) + 25(9.81)\sin(30°)\underset{\Delta t}{(2)}$

$- \underset{\mu_k}{0.3}\underbrace{(25)(9.81)\cos30°}_{N}\underset{\Delta t}{(2)}$

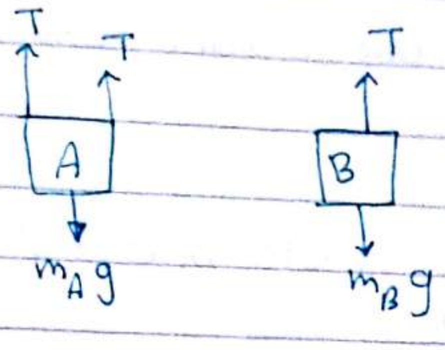
$= 25v_2$

ex: 15.3



$v_B = ?$

$t = 6\,s$



$\underline{A}$: $\cancel{m_A v_{A_1}}^{0} + \int \Sigma F . dt = m_A v_{A_2}$

$\underset{3(9.81)}{m_A g}(6) - 2T(6) = \overset{3}{\cancel{m_A}}\, v_{A_2} \quad \text{......} \; ①$

$B: \cancel{m_B v_{B_1}}^{0} + \int_0^t \Sigma F . dt = m_B v_{B_2}$

$5(9.81)(6) - T(6) = 5\overset{??}{v_{B_2}} \quad \text{......} \; ②$

$2 S_A + S_B = L$

$2 \Delta S_A + \Delta S_B = 0$

$2 v_A + v_B = 0$

$2 a_A + a_B = 0$

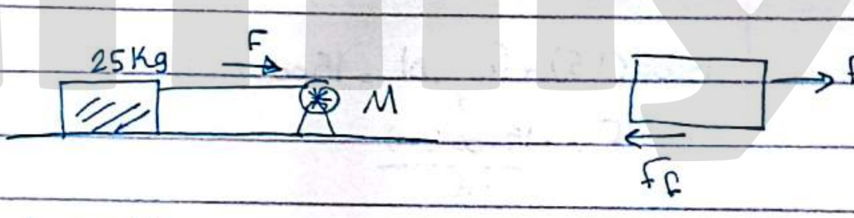
$v_A = \ominus \frac{1}{2} v_B \quad \text{......} \; ③$

بس وحدة منهم لازم تكون سالبة عشان

ما يلغوا بعض، بالتالي بفرض التنين لتحت و السالب

(in dependent motion) حاله يطلع

F.15.3



$F = 20t^2$

$t = 4\,s \Rightarrow v = ?!$

$\mu_s = 0.3 \quad \mu_k = 0.25$

$\cancel{m v_1}^{0} + \int \Sigma F . dt = m v_2$

$\int_{1.9}^{4} 20t^2 \, dt - [0.25][25(9.81)]\underbrace{[4 - 1.9]}_{\Delta t}$

$20t^2 = \mu_s N$

$20t^2 = 0.3(25)(9.81)$

$t = 1.9\,s$ this is the time when it starts moving

لازم يتغلب عليها و بليش بالتحرك، بالتالي ما بطلش

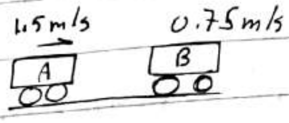
الحل عأساس انه بتحرك من $t = 0$

4/3 (13) Conservation of linear - momentum

* when the summation of external impulses acting on the system of particle is zero

$$\sum m_i v_{i_{①}} = \sum m_i v_{i②}$$

Ex: 15.4



$m_A = 15\ Mg = 15000\ Kg$

ml gram

$m_B = 12\ Mg = 12000\ Kg$

A + B

① velocity after they couple

② F_{avg} coupling $\Delta t = 0.8\ s$

① $\sum m_i v_{i①} = \sum m_i v_{i②}$

$m_A v_{A_1} + m_B (v_{B①}) = (m_A + m_B) v_2$

negative

$15000(1.5) - 12000(0.75) = 27000\, v_2$

$v_2 = ✓$

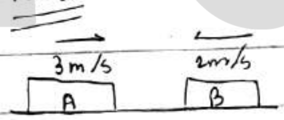
② $F_{c_{avg}} = m_A v_{A_1} + \int \sum F.dt = m_A v_{A②}$

$15000(1.5) + F_{c_{avg}} . \Delta t = 15000(0.5)$

$$F = \frac{-15000(+1)}{0.8}$$

- area under the curve of find Favg and multiply by time

Ex 15.5



$m_A = m_B = 150\ Kg$

No energy is lost.

$m_A v_{A①} + m_B v_{B②} = m_A v_{A②} + m_B v_{B②}$

$(150)^* 3 - 150(2) = 150\, v_{A_2} + 150\, v_{B_2}$

$$\boxed{v_{A_2} + v_{B_2} = 1}$$

bc no energy loss:

$T_1 + V_1 = T_2 + V_2$

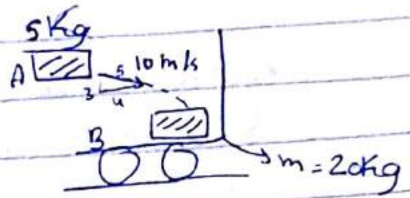
$T_1 = T_2 \rightarrow \frac{1}{2} m_A v_{A_1}^2 + \frac{1}{2} m_B v_{B_2}^2 = \frac{1}{2} m_A v_{A_2}^2 + \frac{1}{2} m_B v_{B_2}^2$

$3^2 + 2^2 = v_{A_2}^2 + v_{B_2}^2$

$$\boxed{v_{A_2}^2 + v_{B_2}^2 = 13}$$

if they coupled and no energy lost then v_2 for B = 3 and v_2 for A = 2 in the negative direction

F. 15.8



$$m_A v_{A_1} + \cancel{m_B v_B}^{\,0} = (m_A + m_B) V_2$$

x: $5 \left(10 \cdot \frac{4}{5}\right) = (5+20) V_2$

$V_2 = \checkmark$

after the throw of the package the car moved right in a new velocity

F. 15.11

$m_A = 15\,Kg$

$m_B = 10\,Kg$

$\delta = ?!$

$$T_1 + \cancel{V_1}^{\,0} = \cancel{T_2}^{\,0} + V_2$$

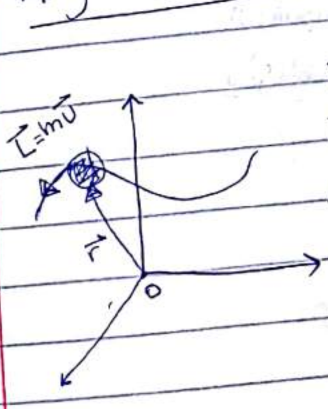
$$\frac{1}{2} m v^2 = \frac{1}{2} K \delta^2 \longrightarrow \delta = \checkmark$$

$$m_A U_{A_1} + m_B U_{B_1} = (m_A + m_B) V$$

$$0 + 10(15) = (15+10) V$$

$$V = \frac{150}{25} = 6\,m/s$$

6/3/19 Angular momentum



$\vec{r}$: position vector

$\vec{L}$: Linear momentum

Angular momentum = "The moment" of the linear momentum around the point O

$$\vec{H}_o = \vec{r} \times \vec{L} = \vec{r} \times m\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix} \begin{matrix} \\ \to \vec{r} \\ \to \vec{L} \end{matrix}$$

Relation Between Angular momentum & moment of a force.

$\Sigma \vec{F} = m\vec{a} = m\dot{\vec{v}}$

$M_o = \vec{r} \times \Sigma\vec{F} = \vec{r} \times m\dot{\vec{v}} \quad \cdots ①$

$H_o = \vec{r} \times m\vec{v}$

$\dfrac{d(H_o)}{dt} = \dfrac{d}{dt}(\vec{r} \times m\vec{v})$

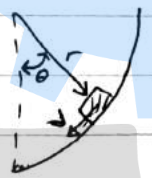
$\dot{\vec{H}}_o = \dot{\vec{r}} \times \cancel{m\vec{v}}^{\,0} + \vec{r} \times m\dot{\vec{v}}$

v

اذا ضربناهم دفعنا
الافقي بعطينا zero
عشان الزاوية بينهم = zero

then: $\boxed{\dot{\vec{H}}_o = M_o}$

Ex: 15.12

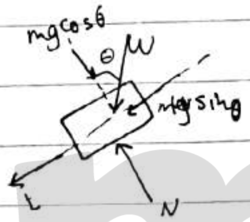


$H_o = ?$

$a_t = ?$

$H_o = r m v$

⋮



※ when a subject sliding on a smooth surface, its acceleration is always $g \boxed{\sin\theta}$

N / mg cos θ

ما رح يأثروا ع الحركة

الـ tangential بس رح

يأثر عليها (mg sinθ)

$\dot{\vec{H}}_o = M_o$

$r m\dot{v} = (mg\sin\theta)(r)$

$\dot{v} = a_t = g\sin\theta$

⋮ ✓

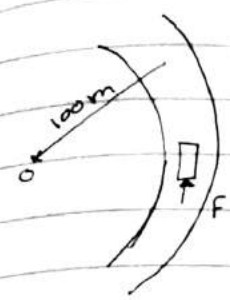
※ principle of Angular impulse and momentum

Angular Impulse $= \int_{t_1}^{t_2} M_o \, dt = \int_{t_1}^{t_2} (\vec{r} \times \vec{F}) \, dt$

$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o \, dt = (H_o)_2$

$\boxed{(H_o)_1 = (H_o)_2}$ when $\Sigma \int_{t_1}^{t_2} M_o \, dt$ is zero

ex: 15.13



$m = 1.5\ Mg = 1500\ kg$

$F = 150t^2\ N$

$t = 5\ s \rightarrow v = ??$

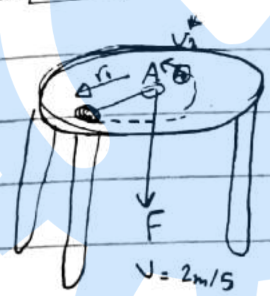
$v_0 = 5\ m/s$

$$(H_o)_1 + \Sigma \int M_o\,dt = (H_o)_2$$

$$r m v_0 + \int_0^5 (150t^2)(100)\,dt = r m v_2$$

$$100(1500)(5) + 5000t^3\Big]_0^5 = 100(150)v_2$$

ex: 15.14



$m = 0.8\ kg$

$r_1 = 0.875$

$v_1 = 2\ m/s$

① $v_2 = ??$ $r_2 = 0.3m$

② U_f

* there are no angular impulses

$\Sigma \square = zero$ impulses

① بس يسحب الحبل بيقرب الدائرة للمركز

$$(H_o)_1 = (H_o)_2$$

$$r_1 m v_1 = r_2 m v_2^*$$

the tangential

من الداخلة لجوا للمركز

$$(0.875)(0.8)(2) = (0.3)(0.8)v_2^*$$

$$v_2 = \sqrt{(5.833)^2 + (2)^2} = \checkmark$$

②

$$U_f = T_2 - T_1$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= \frac{1}{2} \times 0.8 \times 6.56^2 - \frac{1}{2} \times 0.8 \times 2^2$$

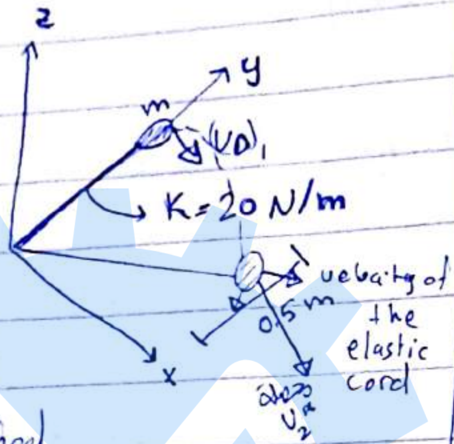
ex: $\boxed{15.15}$

$m = 2\,kg$

$(v_0)_1 = 1.5\ m/s$

① Determine the rate at which the cord is being stretched

② $v = ?!$ $\quad s = 0.2\,m$



$(H_o)_1 = (H_o)_2$

$r_1 m v_1 = r_2 m v_2$

$(0.5)(2)(1.5) = (0.5 + 0.2)(2)\,v_2$

$v_2 = 1.07\ m/s$

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2} m v_1^2 + 0 = \frac{1}{2} m v_2^{*2} + \frac{1}{2} K s^2$

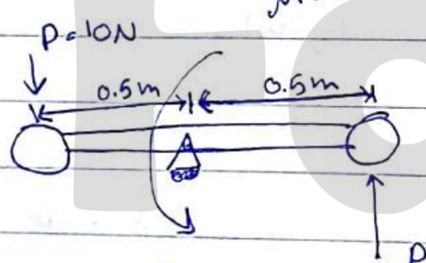
$\frac{1}{2} \cdot 2 \cdot 1.5^2 + 0 = \frac{1}{2} \cdot 2 \cdot v_2^{*2} + \frac{1}{2} \cdot 20 \cdot 0.2^2$

$v_2^* = 1.36\ m/s$

$\text{Velocity}_{cord}^2 + v_2^2 = v_2^{*2} \longrightarrow \text{Velocity}_{cord} = \sqrt{v_2^{*2} - v_2^2}$

F. 15 – 24

$M = 8t\ N.m$



$m = 10\ Kg$

$v = ?!\quad t = 4\,s$

Bc velocity is zero

$\cancel{(H_o)_1} + \Sigma \int M.dt = (H_o)_2$

$\int_0^{4} 8t.dt + \left[2 * \overset{P}{10} \cdot \overset{r}{0.5}\right] \overset{4s}{\Delta t} = (0.5)(10)\,v_2$

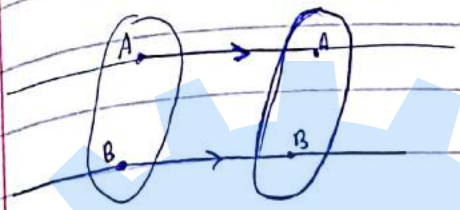
Ch 16

[Kinematics of Rigid bodies]

Types of motions

① Translation

Ⓐ Rectilinear motion



Rectilinear motion (straight path + the points don't move, orientation of A/B is const.)

Ⓑ Curvilinear motion

(same orientation + curved path)

② Rotation about a fixed axis

Angular displacement [rad]	Angular velocity (ω) $\quad \omega = \frac{d\theta}{dt}$ (rad/sec)

Angular Acc (α)

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad (rad/s^2)$$

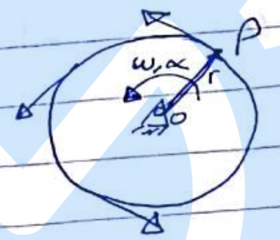
$$\boxed{\alpha \, d\theta = \omega \, d\omega}$$

In case constant Angular $\underline{\underline{Acc}}$ only

① - $\omega = \omega_o + \alpha_c t$

② $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha_c t^2$

③ $\omega^2 = \omega_o^2 + 2\alpha_c (\theta - \theta_o)$



$\vec{V}_p = \vec{\omega} \times \vec{r}$

$a_p = \frac{dv_p}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times [\vec{\omega} \times \vec{r}]$

$[\vec{\alpha} \times \vec{r} - \vec{\omega}^2 \vec{r}]$

$\sigma = x\hat{i} + y\hat{j}$

$\omega = \omega\hat{k}$

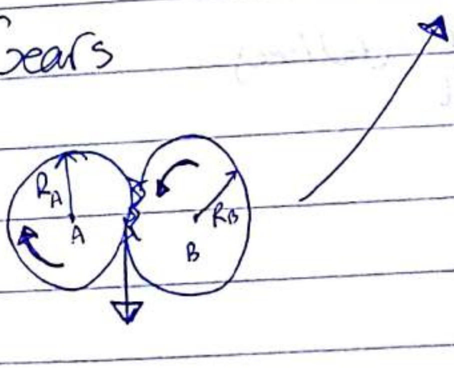
$V_p = \omega_A r_A = \omega_B r_B$ / $a_{p_t} = \alpha_A r_A = \alpha_B r_B$

a_{p_t} V_p

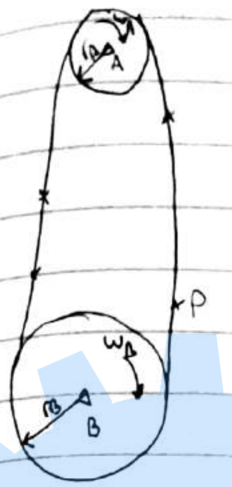
p = contact point (one point for both, so it will have one velocity / acc. / magnitude)

15/3 ⑰

Gears

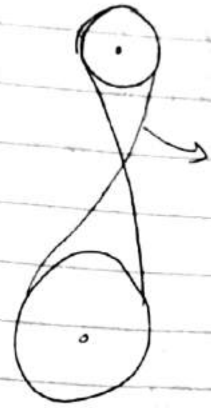


pulley & Belts



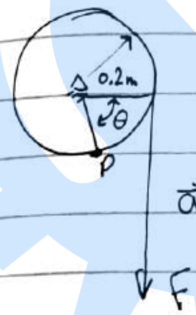
$V_P = \omega_A r_A = \omega_B r_B$

$a_{P_t} = \alpha_A r_A = \alpha_B r_B$



هيك بصير
اخلي كل pully
تتحرك باتجاه
عكس الثانية

ex 16.1



$\omega_0 = 0$, $\theta = 0$

(a) ω

(b) θ

$\vec{a} = 4t$ m/s²

$\alpha = ?!$

$a = \alpha r$

$4t = 0.2\alpha$

$\alpha = 20t$ rad/s²

$\alpha = \frac{d\omega}{dt} \rightarrow \int d\omega = \int \alpha\, dt$

$\boxed{\omega = 10t^2}$

$\theta = ?!$

$\int d\theta = \int \omega\, dt$

$\theta = \int 10t^2\, dt = 3.33t^3$

$\omega = \frac{d\theta}{dt}$

$\boxed{16\text{-}2}$

$\omega_{A_0} = 0$, $\alpha_A = 2$ rad/s²

$|v_P| = ?!$ after A turnes 2 rev

$|a_P| = ?!$

$\theta = 2 * 2\pi$

$\boxed{\theta = 12.57 \text{ rad}}$

$\omega_A^2 = \omega_{A_0}^2 + 2\alpha_A(\Delta\theta)$

$\omega_A^2 = 0 + 2*2*12.57$

$\omega_A = 7.09$ rad/s

$\omega_A r_A = \omega_B r_B$ ← we need it to find the acc

$7.09 * 0.15 = \omega_B * 0.4$

$\omega_B = $ — rad/s

لانه بيتغير عشان اطلع ال a_n ومعها ال resultant

$v_P = \omega_A r_A = (7.09)(0.15) = $ —

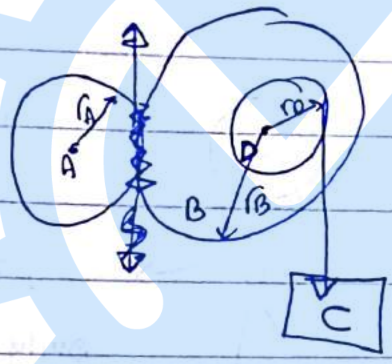
or $\omega_B r_B$

$a_{P_t} = \alpha_A r_A = 2 * 0.15 = 0.3$ m/s²

$a_{P_n} = \omega_B^2 r_B = $ —

$a_P = \sqrt{a_{P_t}^2 + a_{P_n}^2}$

F16-6



$r_A = 75\,mm$

$r_B = 225\,mm$

$r_D = 125\,mm$

$\alpha_A = 4.5\ rad/s^2$

$\omega_{oA} = 0$

$V_c = ?!$ $t = 3\ s$

$y_c = ?!$

$a_{At} = a_{Bt}$

$\alpha_A R_A = \alpha_B R_B$

$\frac{75 \times 4.5}{225} = \alpha_B$

$\alpha_B = 1.5\,rad/s^2$

$\omega_B = \omega_D$

$\omega_B = \omega_{oB} + \alpha t$

$= 0 + 1.5 \times 3$

$\omega_B = 4.5\ m/s$

$V_c = \omega_B \times r_D = 4.5 \times 125 \times 10^{-3}$

$a_c = \alpha_B \times r_D$

$= 4.5 \times 125 \times 10^{-3}$

$y = \cancel{y_0} + \cancel{v t} + \frac{1}{2} a t^2$

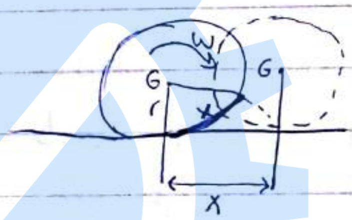
18/3 (18)

General plane Motion

- Relative Motion Analysis $(v+a)$
- I'C

- Absolute Motion Analysis:

$\overset{x}{d} = f(\theta)$



unfolding the arc length

$\boxed{x = r\theta}$

$v = \frac{dx}{dt} = r\omega$

$a = \frac{dv}{dt} = r\alpha$

* at contact point $v = \text{zero}$

Ex 16.5

$\omega = ?$

$\alpha = ?$

$\theta = 30^\circ$

$\frac{ds}{dt} = 0.5\ m/s$

we use the cos law:

$\boxed{S^2 = a^2 + b^2 - 2ab\cos\theta}$

$S^2 = 5 - 4\cos\theta$

Keep it so we can differentiate and find the other things.

$S|_{\theta=30°} = \checkmark = 1.239m$

$(2S)\frac{ds}{dt} = (4\sin\theta)\frac{d\theta}{dt}$

$2 * 1.239(0.5) = 4\sin\theta|_{\theta=30°}(\omega)$

$\omega = 0.62$ rad/s

$2S\dot{S} = 4\sin\theta(\omega)$

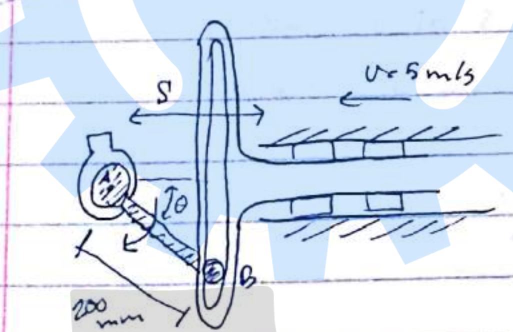
$2\dot{S}^2 + 2S\overset{\to 0}{\ddot{S}} = 4\alpha\sin\theta + 4\cos\theta\,\omega^2$

bc $\ddot{S} = 0$ — need to find it

$\to 2(0.5)^2 + 0 = 4*(0.62^2)(\cos(30)) + 4\sin(30)\alpha$

$\alpha = \checkmark$ rad/s²

P 16.40



$\theta = 60°$

$a = 2 m/s^2$

$\alpha = ?!$ link AB

$\omega = ?!$

this method is about to find a function for the displacement.

$S = (200)\times10^{-3}(\cos\theta)$

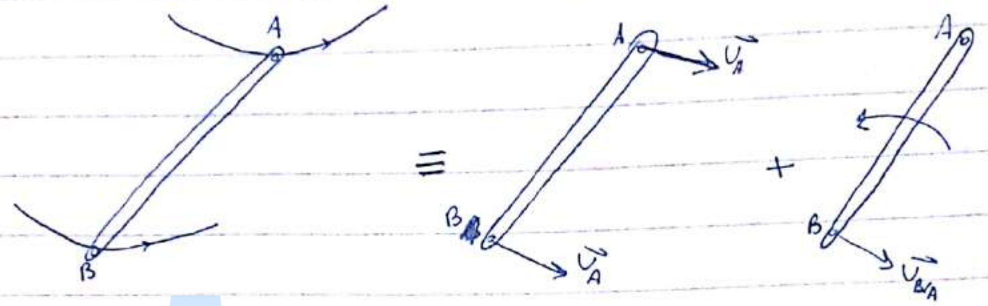
$\frac{ds}{dt} = -(0.2\sin\theta)\omega$

$-5 = -(0.2\sin 60)\omega \to \omega = \checkmark$

$\frac{d^2s}{dt^2} = a = -0.2\cos\theta\,\omega^2 - (0.2\sin\theta)\alpha$

$-2 = -0.2\cos(60)\omega^2 - 0.2\sin(60)\alpha \Rightarrow \alpha = \checkmark$

20/3 (19) <u>Relative Motion Analysis</u>

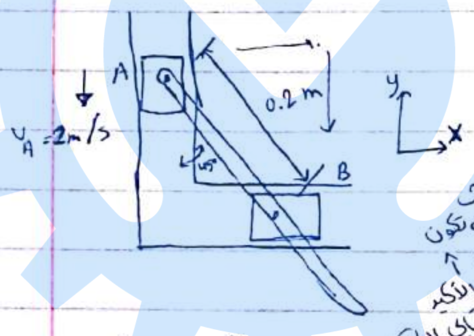


$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$V_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$
$= \vec{V}_B + \vec{\omega} \times \vec{r}_{AB}$

Ex: 16.6



$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$= -2j + \omega\hat{k} \times [0.2\sin 45°\hat{i} - 0.2\cos 45°\hat{j}]$

$V_B\hat{i} = \underbrace{-2\hat{j} + 0.2\omega\sin 45°\hat{j}}_{zero} + 0.2\,\omega\cos 45°\hat{i}$

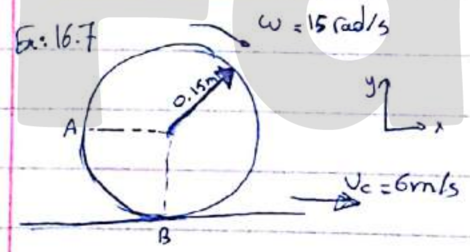
مش رح تكون غير ، لأكيد لأني الـ(j) على الصفحة تساوي صفر

$\omega_{AB} = \omega\hat{k}$ positive

$\omega = \dfrac{2}{0.2\sin 45°} =$

$V_B = 2\,m/s$

Ex: 16.7



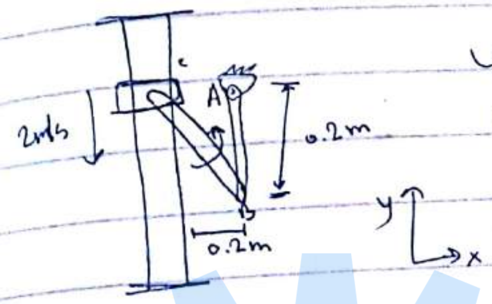
no slipping

So, ~~...~~ $V_C = V_B$
اني بساوي اقلت

$\vec{V}_A = \vec{V}_B + \vec{\omega} \times \vec{r}_{AB}$

$= 0.6\hat{i} + [-15\hat{k}] \times [-0.15\hat{i} + 0.15\hat{j}]$

$= 0.6\hat{i} + (0.15)(15)\hat{j} + (0.15)(15)\hat{i}$



$v_B = \vec{V}_C + \vec{\omega}_{BC} \times \vec{r}_{BC}$

$v_B = -2\hat{j} + \vec{\omega}_{BC}\hat{k} \times [0.2\hat{i} - 0.2\hat{j}]$

$v_B\hat{i} = -2\hat{j} + [0.2\omega\hat{j} + 0.2\omega\hat{i}]$

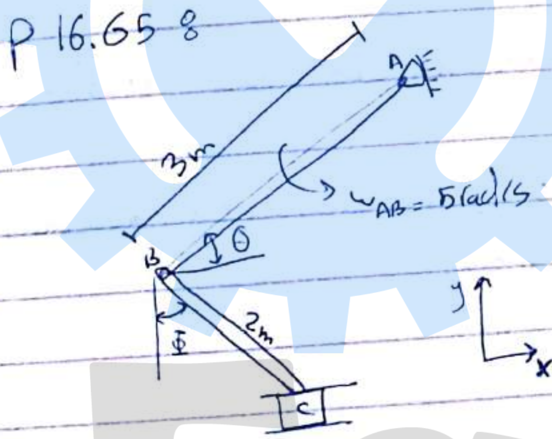
$0.2\omega - 2 = 0$

$\omega = \frac{2}{0.2} = 10\ rad/s$

v_B will move in $\hat{i}$ only because AB is vertical at that instant

$v_{B\hat{i}} = 2\ m/s$

P 16.65 :



$V_C = ?!$

$\omega_{BC} = ?!$

$\theta = 45°$

$\Phi = 30°$

$v_B = v_A + \omega_{AB} \times r_{B/A}$

$= 0 + 5\hat{k} \times [-3\cos 45° \hat{i} - 3\sin 45° \hat{j}]$

$v_{B\hat{i}} + v_{B\hat{j}} =$

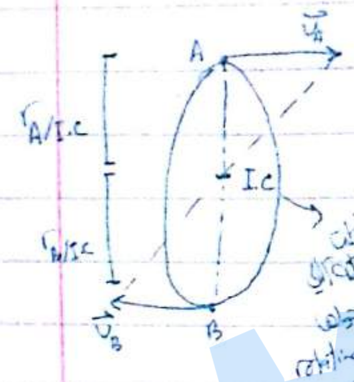
$V_C = V_B + \omega_{BC} \times r_{C/B}$

$V_C\hat{i} = (V_{B\hat{i}} + V_{B\hat{j}}) + \omega_{BC}\hat{k} \times [2\sin 30\hat{i} - 2\cos 30\hat{j}]$

$v = 2\omega$ $\qquad V_B\hat{j} + 2\omega_{BC}\sin 30° = 0$

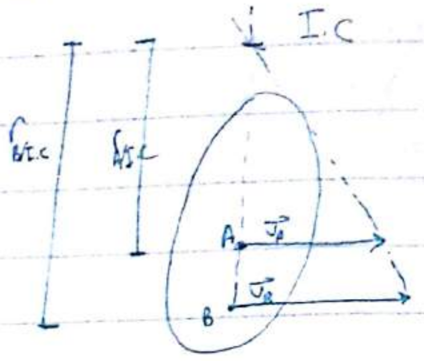
17?

(20) Instintaneous Center of zero Velocity I.C



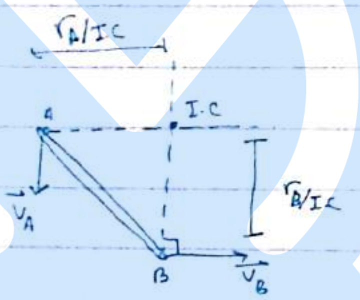
النقطة اللي بصير حولها ال rotating

$$\omega = \frac{v_A}{r_{A/I.C}} = \frac{v_B}{r_{B/I.C}}$$

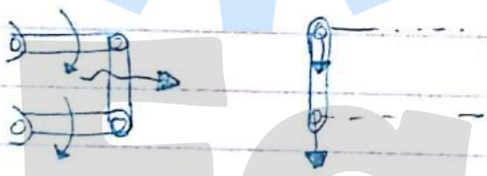


نوصل النهايات ثم رؤوس ال vectors ونقطة التقاطع تكون I.C

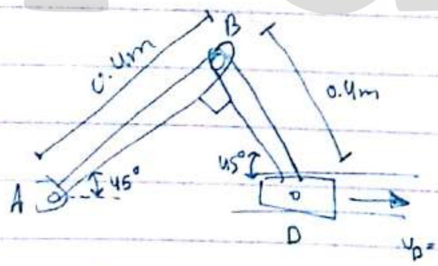
* The point chosen to be the I.C point can only be used at the instant

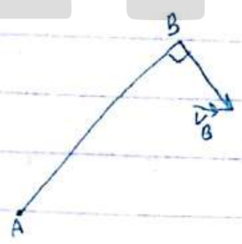


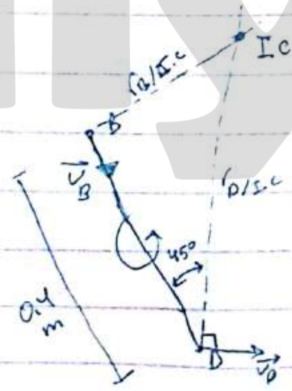
on $\underline{v}$ we construct a perpendecular point



Ex 16.10







$\omega_{AB} = ??$

$\omega_{BD} = ??$

$$\omega_{BD} = \frac{v_D}{r_{D/IC}} = \frac{3}{0.4/\cos 45^\circ} = \dots$$

$$v_B = \omega_{BD} \times r_{B/IC} = (\dots)(0.4 \tan 45^\circ) = \dots$$

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = (\dots)$$

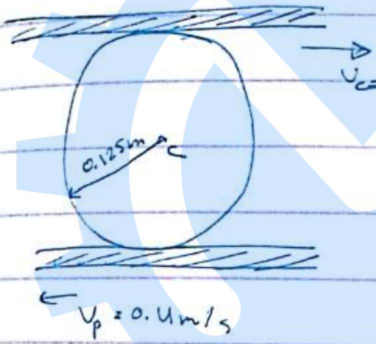
$r_{B/A} = 0.4$

bc its rotating about A not about I.C point

$$\tan 45^\circ = \frac{r_{B/I.C}}{0.4}$$

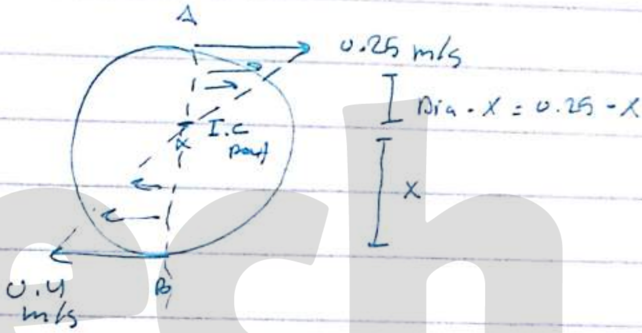
$$\rightarrow r_{B/I.C} = 0.4 \tan 45^\circ$$

$$r_{D/I.C} \cos 45^\circ = 0.4 \rightarrow r_{D/I.C} = \frac{0.4}{\cos 45^\circ}$$



$\omega = ?!$

$V_c = ?!$



$V_A = \omega(0.25 - x)$

$V_B = \omega(x) = 0.4$

$\omega = 0.4/x$

$0.25x = 0.4(0.25 - x)$ ....

$x = 0.1538$ m

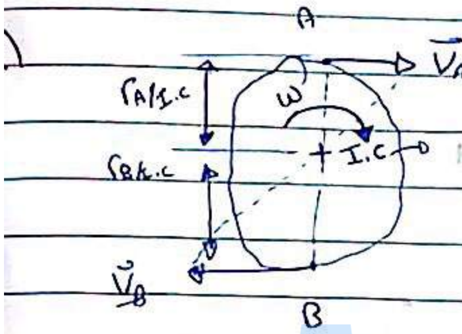
$V_c = \omega\ E/I.C$

$\omega = \dfrac{0.4}{0.1538} = 2.6$ rad/s

$V_c = (2.6)(0.1538 - 0.125)$

Instantaneous centre of zero velocity : I.C

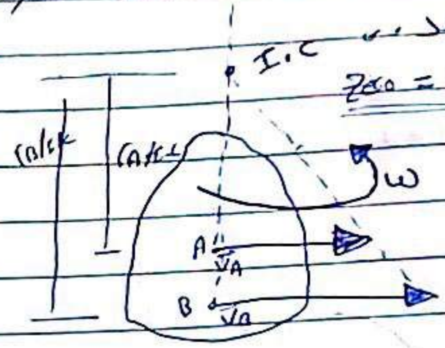
1)



بيكون أقرب للقيمة الأصغر.

$$\omega = \frac{V_A}{r_{A/I.C}} = \frac{V_B}{r_{B/I.C}}$$

2)

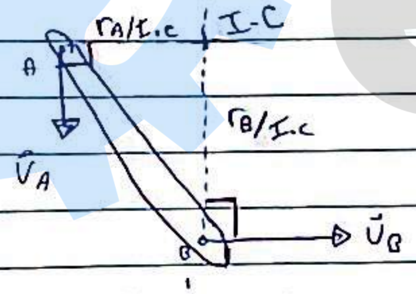


برا الشكل بس عندها V = Zero

$$\omega = \frac{V_A}{r_{A/I.C}} = \frac{V_B}{r_{B/I.C}}$$

* The point chosen to be I.C point can only be used At the instant considerd.

3)



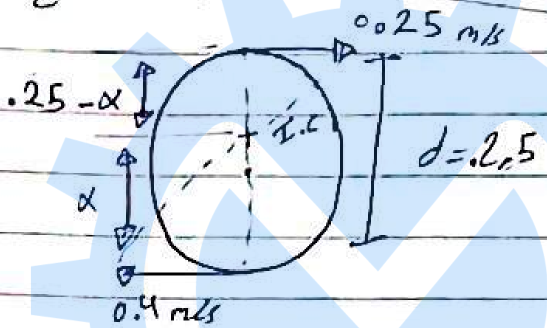
(Link.)

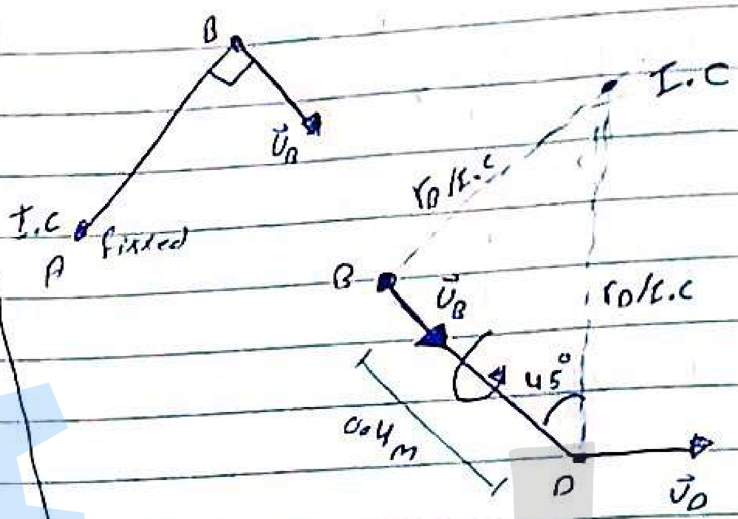
Ex. #. page. (16/10)

$\omega_{AB} = ??$

$\omega_{BD} = ??$

Ex. 16.11:





$v_A = \omega\,(0.25 - x) = .25$

$v_B = \omega.(x) = .4$

$\omega = 0.4/x$

$0.25x = 0.4\,(0.25 - x)$

$x = \text{--} - 0\boxed{.1538\,m}$

$v_C = \omega \cdot C_C/I.C$

$\omega = \frac{0.4}{.1538} = 2.6\ \text{rad/s}$

$v_C = 2.6\,(0.1538 - 0.125)$

$\tan 45° = \frac{r_B/I.C}{0.4}$

$r_B/I.C = .4 \tan 45°$

$r_D/I.C \cos 45 = 0.4$

$r_D/I.C = \frac{0.4}{\cos 45°}$

$\omega_{BD} = \frac{v_D}{r_D/I.C} = \frac{3}{\frac{0.4}{\cos 45°}} = \omega_{BD}$ rad/s

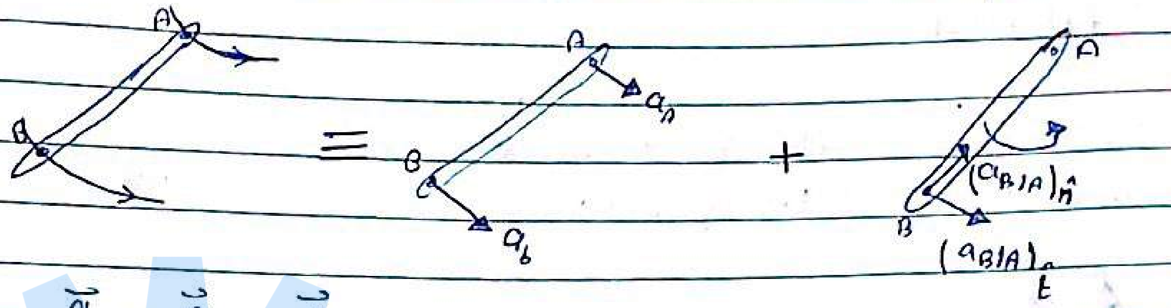
$v_B = \omega_{BD} \cdot r_B/I.C = \text{--}$ rad/s

$\omega_{AB} = \frac{v_B}{r_B/A\ (0.4)} = \text{--}$ rad/s

Relative Motion Analysis:-

Acceleration:

translation. rotation



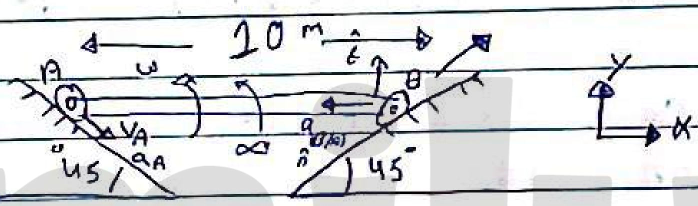
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$= \vec{a}_A + (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n$$

$$\vec{a}_B = \vec{a}_A + \underbrace{\vec{\alpha}_{AB} \times \vec{r}_{B/A}}_{\text{tangential Angular Acc}} - \underbrace{\omega^2_{A/B}\, r_{B/A}}_{\text{normal Acc}}$$

∴ دوال ~~vector~~ $\vec{\alpha}$ $\vec{\omega}$ من قوانين Abs relative وبعد ما نجيبها هنعوض ونجيب a_{acc} .

Ex. 16.13



$V_A = 2\ m/s$, $a_A = 3\ m/s^2$

$\alpha_{AB} = ??$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2_{AB}\, \vec{r}_{B/A}$$

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$V_B \cos 45^\circ\, \hat{i} + V_B \sin 45^\circ\, \hat{j} = \overset{2}{V_A} \cos 45^\circ\, \hat{i} - \overset{2}{V_A} \sin 45^\circ\, \hat{j} + \omega_{AB}\, \hat{k} \times 10\hat{i}$$

$$V_B \cos 45^\circ\, \hat{i} = 2 \cos 45^\circ \quad \therefore V_B = 2\ m/s$$

$$\overset{2}{\cancel{V_B}} \sin 45^\circ\, \hat{j} = -2 \sin 45^\circ + 10\, \omega_{AB}$$

$$\omega = 0.283\ rad/s$$

assume

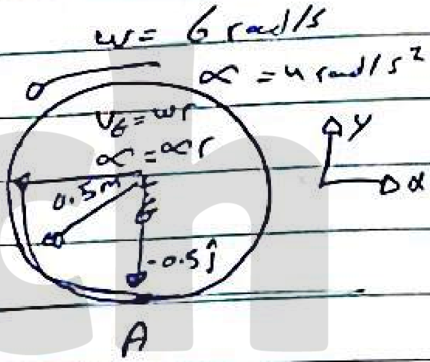
$a_B \cos 45\,\hat{i} + a_B \sin 45\,\hat{j} = 3\cos 45\,\hat{j} - 3\sin 45\,\hat{j} + \alpha_{AB}\,\hat{k} \times 10\,\hat{i}$

$-(2.83)^2 (10\,\hat{j})$

$a_B \cos 45 = 3\cos 45 - (2.28)^2(10) \quad a_B = \cancel{3}\ 1.87\ m/s^2$

$a_B \sin 45 = -3\sin 45\,\hat{j} + 10\alpha \quad \therefore \alpha = 0.344\ rad/s$

Ex 16.14



$a_A = ??$

$\vec{a}_A = \vec{a}_G + (\bar{a}_{A/G})_t + (\bar{a}_{A/G})_n$

$\vec{a}_A = -2\hat{i} + \vec{\alpha} \times \vec{r}_{A/G} - \omega^2 \cdot \vec{r}_{A/G}$

$V_G = 6 \cdot (0.5) = -3\hat{i}$

$a_G = 4(0.5) = -2\hat{i}$

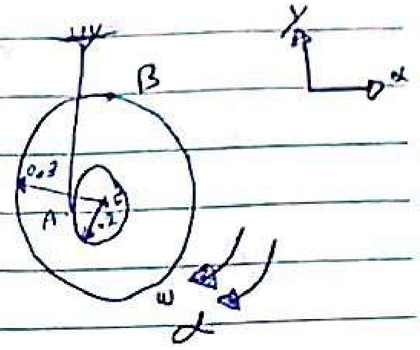
$\vec{a}_A = -2\hat{i} + (4\hat{k} \times -0.5\hat{j}) - (6^2 \cdot -0.5\hat{j})$

$\vec{a}_A = -\cancel{2\hat{i}} + \cancel{2\hat{i}} + 18\hat{j}$

$\vec{a}_A = 18\hat{j}$

Ex 16.15.

$\omega = -3\hat{k}$, $\alpha = -4\hat{k}$



$\omega = 3$ rad/s

$\alpha = 4$ rad/s

$\alpha_B = ??$

$a_G = \alpha (0.2) = 4 \cdot (0.2)$

$= 0.8 \, m/s^2 \downarrow$

- on the centre

$a_G = \alpha \cdot r$

$\vec{a}_B = \vec{a}_G + (\vec{a}_{B/G})_t + (a_{B/G})_N$

$\vec{a}_G + \alpha \times \vec{r}_{B/G} - \omega^2 \cdot \vec{r}_{B/G}$

$a_B = -0.8 j + (-4\hat{k} \times 0.3 \hat{j}) + [-(3)^2 (0.3\hat{j})]$

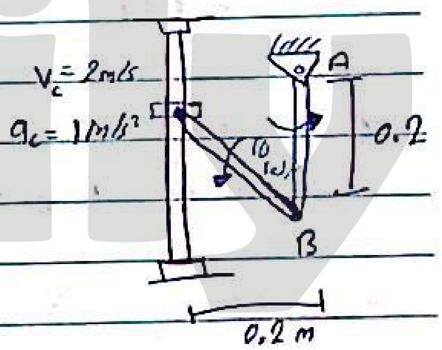
$= 1.2\hat{i} - 3.5\hat{j}$

$|a_B| = \sqrt{\quad\quad} = \_\_$

$\theta = \_$

Ex 16.16

α ليها ω علشان بيتحرك



$\alpha_{AB} = ??$

$\alpha_{CB} = ??$

$a_B = a_A + a_{(B/A)t} + a_{(B/A)N}$

$a_B = \cancel{a_A}^{zero} + \alpha_{AB} \times \vec{r}_{B/A} - \omega^2 \cdot \vec{r}_{B/A}$

$= \alpha \hat{k} \times -0.2\hat{j} - (10)^2 \cdot (-0.2\hat{j})$

$\vec{a}_B = .2\alpha \hat{i} + 20\hat{j}$

$\vec{a}_c = \vec{a}_B + \alpha_{CB} \times \vec{r}_{C/B} - \omega_{CB}^2 \cdot \vec{r}_{C/B}$

$-1\hat{j} = 0.2\alpha_{AB}\hat{i} + 20\hat{j} + \alpha_{CB}\hat{k} \times [-0.2\hat{i} + 0.2\hat{j}] - (10)^2 [-0.2\hat{i} + 0.2\hat{j}]$

$-1\hat{j} = 0.2\,\alpha_{AB}\,\hat{i} + 20\hat{j} - 0.2\,\alpha_{CB}\,\hat{j} - 0.2\,\alpha_{CB}\,\hat{i} + 20\,\hat{i} - 20\hat{j}$

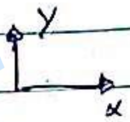
$\hat{j}$:- $-1 = \cancel{20} - 0.2\,\alpha_{CB} - \cancel{20}$

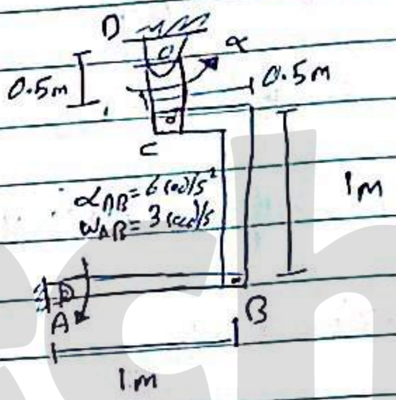
$\alpha_{CB} = +5\ \text{rad/s}^2$

$\hat{i}$:- $0 = 0.2\,\alpha_{AB} + 20 - 2\alpha_{CB}$

$\alpha_{AB} = $ —

Problem 16.115





$\omega_{AB} = -3\hat{k}$, $\alpha_{AB} = -6\hat{k}$

$r_{B/A} = 1\hat{i}$

$v_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$= -3\hat{k} \times 1\hat{i}$

$= -3\hat{j}$

$\vec{a}_B = (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n$

$= \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$

$= -6\hat{k} \times 1\hat{i} - (3)^2(1\hat{i})$, $\vec{a}_B = -9\hat{i} - 6\hat{j}$

$v_C = \omega_{CD}\; r_{C/D}$

$= \omega_{CD}\,\hat{k} \times \vec{r}_{C/D} = \omega_{C/D}\,\hat{k} \times -0.5\hat{j}$

$= 0.5\,\omega_{CD}\,\hat{i}$

$a_C = (\vec{a}_{C/D})_t + (\vec{a}_{C/D})_n$

$= \alpha_{CD}\,\hat{k} \times -0.5\hat{j} - \omega_{CD}^2\,(-0.5\hat{j})$

$\vec{a}_C = 0.5\,\alpha_{CD}\,\hat{i} + 0.5\,\omega_{CD}^2\,\hat{j}$

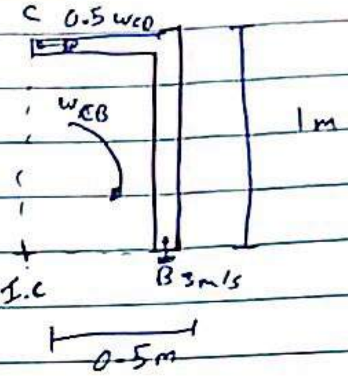
$\vec{a}_C = \vec{a}_B + (a_{C/B})_t + (\vec{a}_{C/B})_n$

$= \vec{a}_B + \vec{\alpha}_{CB} \times \vec{r}_{C/B} - \underbrace{\omega_{CB}^2}_{\text{missing } \omega}\,\vec{r}_{C/B}$

$v_B = \omega_{CB}\, r_{B/I.C}$

$3 = \omega_{CB} \cdot (0.5)$

$\omega_{CB} = 6 \text{ rad/s}$



$v_C = \omega_{CB} \cdot r_{I.C}$

$6 \cdot (1) = 6 \text{ m/s}$

$v_C = 0.5\, \omega_{CD}$

$6 = 0.5\, \omega_{CD}$

$\omega_{CD} = 12 \text{ rad/s}$

$\bar{a}_C = \bar{a}_B + (\bar{a}_{C/B})_t + (\bar{a}_{C/B})_n$

$= \bar{a}_B + \vec{\alpha}_{CB} \times \vec{r}_{C/B} - \omega_{CB}^2\, \vec{r}_{C/B}$

tangential

$0.5\, \alpha_{CD}\, \hat{i} + 0.5\, \underset{(12)}{\omega_{CD}^2}\, \hat{j} = -9\hat{i} - 6\hat{j} + \alpha_{CB}\, \hat{k} \times (-0.5\hat{i} + \hat{j})$

$\omega^2 r$

$- [(6)^2 (-0.5\hat{i} + \hat{j})]$

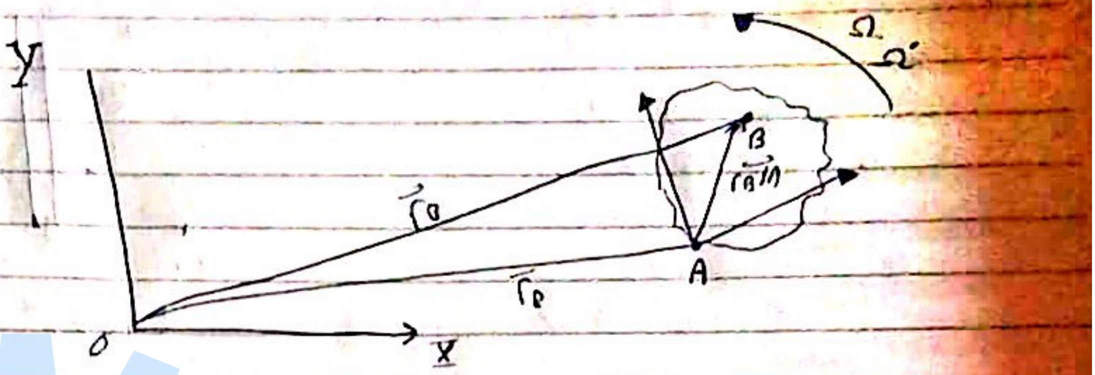
normal

$0.5\, \alpha_{CD}\, \hat{i} + 0.5\,(12)^2\, \hat{j} = -9\hat{i} - 6\hat{j} + [-\alpha_{CB}\, \hat{i} - 0.5\, \alpha_{CB}\, \hat{j}] + [18\hat{i} - 36\hat{j}]$

$\hat{j}$:- $72 = -6 - 0.5\, \alpha_{CB} - 36 \Rightarrow \alpha_{CB} = \text{—}$

$\hat{i}$:- $0.5\, \alpha_{CD} = -9 - \alpha_{CB} + 18 \Rightarrow \alpha_{CD} = \text{—}$

Sec 1.8 Relative motion Analysis using Rotating Axis.



$r_{B/A} = x i + y j$

$\Omega \equiv$ Angular velocity

$\dot{\Omega} \equiv$ Angular acc

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\frac{d}{dt}(\text{—}) =$

$\vec{V}_B = \vec{V}_A + \frac{d}{dt}(\vec{r}_{B/A})$

$\frac{d}{dt}(\vec{r}_{B/A}) = \frac{dx}{dt} i + x \frac{di}{dt} + \frac{dy}{dt} j + y \frac{dj}{dt}$

$\frac{d}{dt}(\vec{r}_{B/A}) = (V_{B/A})_{xyz} + \Omega \times \vec{r}_{B/A}$

$\vec{V}_B = \vec{V}_A + \Omega \times \vec{r}_{B/A} + (V_{B/A})_{xyz}$

ABs. velocity of B in XYZ — $\vec{V}_A$: of A — Angular motion effect caused By rotating xyz — Velocity of B w.r.t A in xyz

$\frac{d}{dt}(\text{—}) = \theta$

$$\frac{d\,\vec{a}_B}{} = \vec{a}_A + \dot{\Omega} \times r_{B/A} + \Omega \times \frac{d}{dt}(\vec{r}_{B/A}) + \frac{d}{dt}(V_{B/A})_{xyz}$$

عوض من تحت

$$\Omega \times \frac{d\,\vec{r}_{B/A}}{dt} = \vec{\Omega} \times \vec{V}_{(B/A)_{xyz}} + \vec{\Omega} \times (\vec{\Omega} \times r_{B/A})$$

$$\frac{d(\vec{V}_{B/A})_{xyz}}{dt} = \frac{d^2x}{dt^2}\, i + \frac{dx}{dt}\frac{di}{dt} + \frac{d^2y}{dt^2}\, j + \frac{dy}{dt}\frac{dj}{dt}$$

$$V_{B/A} = \overset{\frac{dx}{dt}}{V_x}\, i + V_y$$

اشتق ضرب

$$= (a_{B/A})_{xyz} + (\vec{\Omega} \times \vec{V}_{B/A})$$

حل الأقواس أولاً

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{v}_{B/A}) + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A})$$

$$+ \vec{\Omega} \times (\vec{V}_{B/A})$$

same

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \underbrace{2\vec{\Omega} \times \vec{V}_{B/A}}_{\text{corioulus Acc.}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + (a_{B/A})_{xy}$$

Ex 16.18

$\Omega = -3\hat{k}$, $\dot{\Omega} = -2\hat{k}$

Determine coriolous Acc :- $\rightarrow 2\vec{\Omega} \times (V_{C/O})$

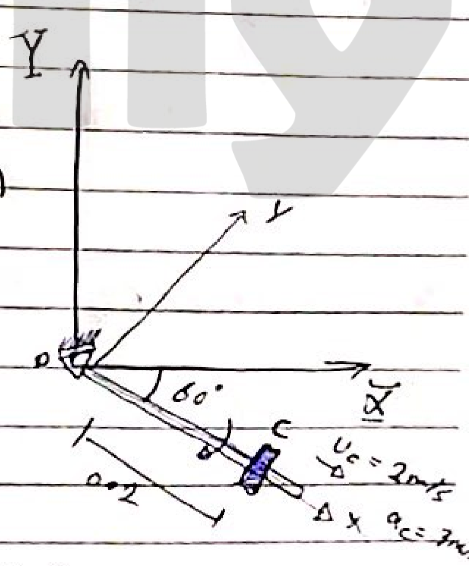
$$\vec{V}_C = \overset{\text{zero}}{\vec{V}_O} + \vec{\Omega} \times r_{C/O} + (\vec{V}_{C/O})_{xyz}$$

$$= -3\hat{k} \times [0.2\hat{i}] + 2i$$

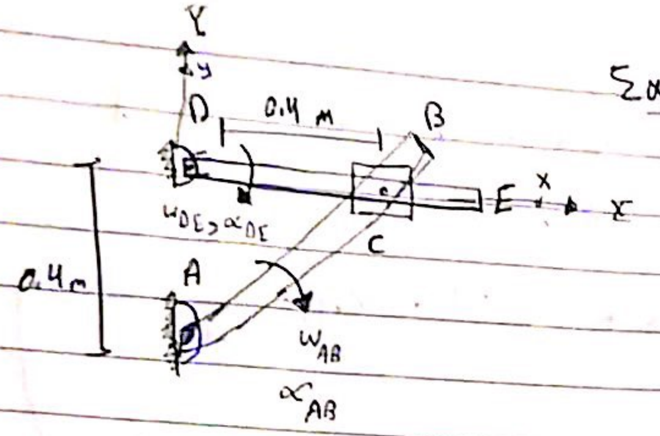
$$= 2i - 0.2\hat{j}$$

$$a_C = \overset{\text{zero}}{a_O} + \dot{\Omega} \times \vec{r}_{C/O} + 2\vec{\Omega} \times V_{C/O} + \vec{\Omega} \times (\vec{\Omega} \times r_{C/O})$$

$$= -2 \times 0.2\hat{i} + 2(-3\hat{k}) \times (2\hat{i}) + (-3\hat{k}) \times (-3\hat{k} \times 0.2\hat{i}) + 3\hat{i}$$



Ex: 16.191



$\omega_{AB} = 3$ rad/s

$\alpha_{AB} = 4$ rad/s^2

find : ω_{DE}, α_{DE}

$\vec{V}_c = \overset{\text{zero}}{\vec{V}_D} + \boxed{\vec{\Omega}} \times \boxed{\vec{r}_{c/D}} + \boxed{(V_{c/D})_{xyz}}$

$\vec{a}_c = \overset{\text{zero}}{\vec{a}_D} + \boxed{\dot{\vec{\Omega}}} \times \vec{r}_{c/D} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{c/D}) + 2\vec{\Omega} \times (V_{c/D})_{xyz} + \boxed{(a_{c/D})_{xyz}}$

$\vec{\Omega} = -\omega_{DE}\hat{k}$ $\qquad \dot{\vec{\Omega}} = -\alpha_{DE}\hat{k}$

$\vec{r}_{c/D} = 0.4\hat{i}$ $\qquad (a_{c/D})_{xyz} = (a_{c/D})_{xyz}\hat{i}$

$(V_{c/D})_{xyz} = (V_{c/D})_{xyz}\hat{i}$

$\vec{V}_c = \overset{\text{zero}}{\vec{V}_A} + \vec{\omega}_{AB} \times r_{c/A}$

$\vec{V}_c = -3\hat{k} \times (0.4\hat{i} + 0.4\hat{j}) = 1.2\hat{i} - 1.2\hat{j}$

$1.2\hat{i} - 1.2\hat{j} = (-\omega_{DE}\hat{k} \times 0.4\hat{j}) + (V_{c/D})_{xyz}\hat{i}$

$\hat{i}: (V_{c/D})_{xyz} = 1.2$

$\hat{j}: -1.2\hat{j} = -0.4\,\omega_{DE} \Rightarrow \omega_{DE} = 3$ rad/s

$\vec{a}_c = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{c/A} - \omega^2 \cdot \vec{r}_{c/A}$

$= -4\hat{k} \times [0.4\hat{i} + 0.4\hat{j}] - (3)^2 (0.4\hat{i} + 0.4\hat{j})$

$= 1.6\hat{i} - 1.6\hat{j} - 3.6\hat{i} - 3.6\hat{j}$

$\vec{a}_c = -2\hat{i} - 5.2\hat{j}$

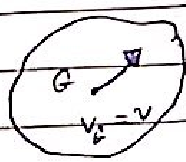
CH. 18 Kinatics of Rigid bodies

Work and energy.

- Kinatic energy.

* Translation :-

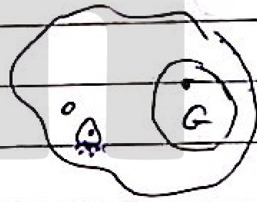
$$T = \frac{1}{2} . m . v_o^2$$



[For both Rectilinear and curvilinear]

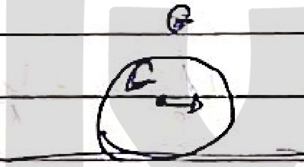
* Rotation about fixed axis

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G . w^2$$



$$T = \frac{1}{2} I_o w^2$$

* General plane Motion:-



$$T = \frac{1}{2} m V_o^2 + \frac{1}{2} I_G . w^2$$

$$T = \frac{1}{2} . I_{I_c} . w^2$$

work. , work by force (const), weight , spring, variable

work of couple moment.

$$U_M (M \text{ variable}) = \int M . d\theta$$

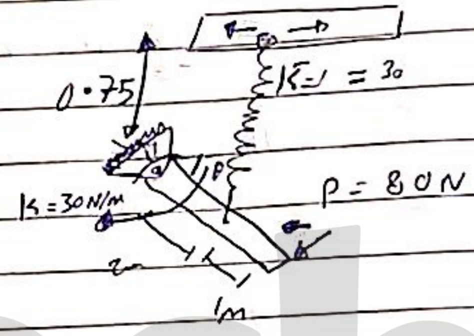
$$U_M = M (\Delta\theta)_{\theta_2 - \theta_1}$$

Forces doesn't exert work:

1) weight & normal (perpendicular)

2) F_f if there is slippin

3) external Forces.



Ex 18.1

$m = 10\ Kg$

$\theta = 0^\circ \rightarrow \theta = 90^\circ$

$U_{tot} = ??$

$U_w = W \cdot \Delta y$

$98.1\ (1.5) = 147.2\ J$

$U_M = M \cdot \Delta\theta = 50 \cdot \frac{\pi}{2} = 78.5\ J$

$U_s = -\frac{1}{2} K (\Delta S^2)$ $\quad \overset{\Delta S}{(2.75 - .5)^2}$

$= -\frac{1}{2} \times 30 \times (2.25)^2$

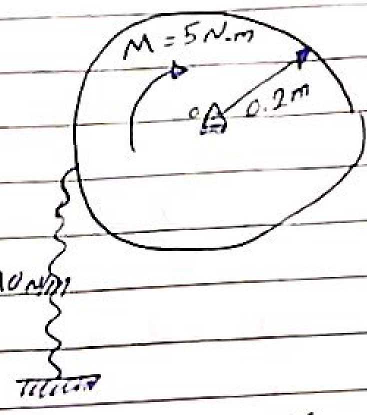
$\cancel{U_P = 80 \cos\theta}$

$P \cdot \Delta S$

$U_P = 80 \cdot (3\ \frac{\pi}{2}) = 377\ J$

Principle of Work and Energy

$T_1 + \Sigma U_{1-2} = T_2$



Ex 18/2

$m = 30\ Kg$

$\theta = ??$ to attain $w = 2\ rad/s$

$T_1 = 0 \qquad T_2 = \frac{1}{2} I_o w^2$

$I_{o\,disk} = \frac{1}{2} m r^2$

$= \frac{1}{2} \cdot 30 \cdot (.2$

$= 0.6\ Kg.m$

$T_2 = \frac{1}{2} \cdot (.6) \cdot (2)^2 = 1.2\ J$

$\Sigma U_{1-2} = U_M + U_S$

$S_2 = 0.2\,\theta$ ($r\theta$)

$U_m = M \cdot \Delta\theta = 5\theta$

$U_s = \frac{1}{2} \cdot K \cdot S^2 = \frac{1}{2} \cdot 10 \cdot (0.2 \cdot \theta)^2 = -0.2\,\theta^2$

$T_1 + \Sigma U_{1-2} = T_2$

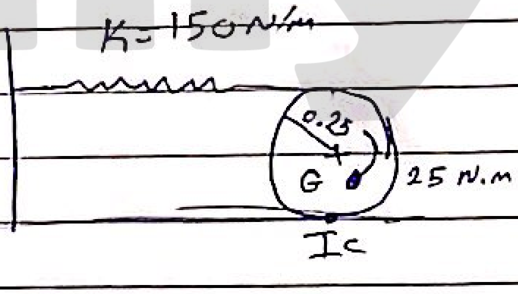
$0 + \quad 0.2\,\theta^2 - 5\theta + 1.2 = 0$

$\boxed{\theta = .24\ rad}$

Ex 18.3

General Plane motion

مش Fix خارج يتحرك

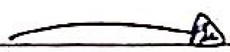


$m = 20\ Kg$

$k_G = 0.2\ m$

From rest , No slipping

$W = ??$ G moves $0.18\ m \rightarrow$



$T_1 = 0$

$T_2 = \frac{1}{2} \cdot m \cdot V_G^2 + \frac{1}{2} I_G \cdot w^2 = \frac{1}{2} I_{Ic} \cdot w^2$

$\boxed{T_2 = 1.025\, w^2}$

$I_{Ic} = I_G + m \cdot r^2$

$= m k^2 + m \cdot r^2$

$20 \cdot (0.2)^2 + 20 \cdot (0.25)^2$

$= \cancel{1.025}\ Kg \cdot m^2$

2.05

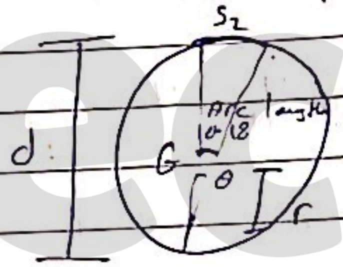
$T_1 + \Sigma U_{1-2} = T_2$

$0 + U_m + U_s = T_2$

$U_m = M \cdot \Delta\theta = 25 \cdot (0.72) = -$

$U_s = \frac{1}{2} \cdot K \cdot S_2^2$

لازم نجيب θ و S_2 من النقطة الي بلف حولها $I_{.c}$



$\frac{1}{2} \cdot (150)(.36)^2 = -$

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 25(.72) - \frac{1}{2} 150 (.36)^2 = 1.025 w^2$

$w = -$

$0.18 = r \cdot \theta$

$0.18 = 0.25 \cdot \theta$

$\boxed{\theta = .72 \text{ rad}}$

$S_2 = d\,\theta$

$S_2 = (.5) \cdot (.72) = \boxed{.36}$

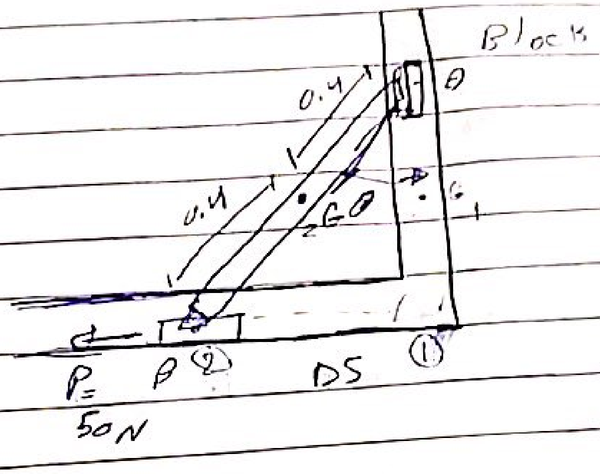
Ex 18.5

A and B are massless

$m_r = 10$ kg

$\theta_1 = 0$ @ rest

$\omega = ??$ $\theta_2 = 45°$



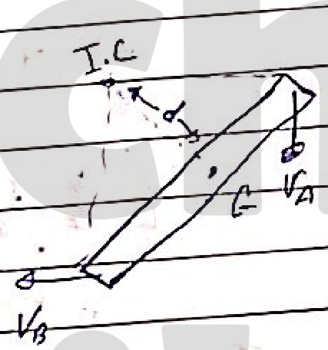
① $\theta = 0$ ② $\theta = 45$

$\underbrace{T_1}_{zero} + \sum U_{1-2} = T_2$

$T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \cdot \omega^2$

$T_2 = \frac{1}{2} I_{I.C} \cdot \omega^2$

$= \frac{1}{2} \cdot \left(I_G + m d^2 \right) \cdot \omega^2 =$ ($d = 0.4$)



$T_2 = 1.0667\ \omega^2$

$\tan 45 = \frac{d}{0.4}$

$\boxed{d = 0.4}$

$\sum U_{1-2} = U_P + U_{weight}$

$U_P = P \cdot \Delta S$

$50 \cdot (0.8 \sin 45) = 28.28$ J

$V_G = \omega \cdot d \boxed{= 0.4 \cdot}$

$U_w = \underbrace{10 (9.81)}_{w} \cdot \underbrace{(0.4 - 0.4 \cos 45°)}_{\Delta y} = 11.55$ J

$T_1 + \sum U_{1-2} = T_2$

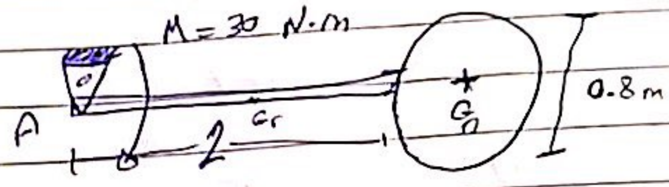
$\omega_2 =$

Scanned by CamScanner

problem 18.9

$M_D = 10$ Kg

$m_r = 3$ Kg



$w = ??$

rotates $\theta = 90°$ from rest

$T_1 + \Sigma U_{1-2} = T_2$

$T_2 = \frac{1}{2} I_{r_A} \cdot w^2 + \frac{1}{2} I D_A \cdot w^2$

$= \frac{1}{2} (I_{r_A} + I D_A) \cdot w^2$

$\frac{1}{2} ([\cancel{\frac{1}{2}} I_{G} + m(1)^2] + [\cancel{\frac{1}{2}} I_{G_D} + m_D \cdot (2.4)^2]) \; w^2$

$\frac{1}{12} m L^2$

$T_2 =$

$\Sigma U_{1-2} = U_{w_r} + U_M + U_{w_D}$

$U_M = M \cdot \Delta\theta = 30 \cdot (\frac{\pi}{2}) = 15\pi$

$U_{w_r} = 3(9.81)[1] = —$

$U_{w_D} = 10(9.81)[2.4] = —$

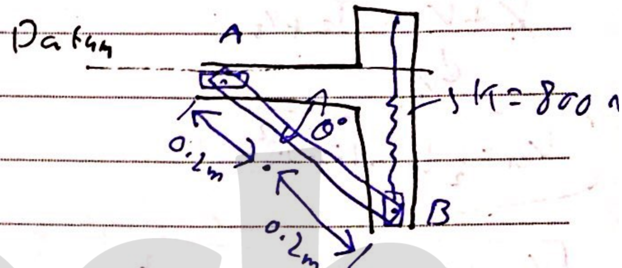
Scanned by CamScanner

Conservation of energy :-

$$T_1 + V_1 = T_2 + V_2$$

$V_1 \rightarrow V_{e_1}, V_{g_1}$; $V_2 \rightarrow V_{e_2}, V_{g_2}$

example 18.6 :-



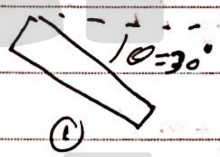
A & B massless

$m_r = 10$ Kg

when $\theta = 0°$, the spring is unstretched

Relleased from rest

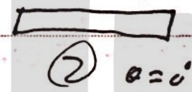
$\theta = 30°$, Find $\omega_{AB} = ?$



$$\text{Sol}^n: \underbrace{T_1}_{zero} + V_1 = T_2 + V_2 \quad (V_{g_2} = zero,\ V_{e_2} = zero)$$

$$zero + V_g + V_e = T_2$$

$$V_{g_1} = -W\Delta y = -10(9.81) * (0.2 \sin 30)$$



$$V_{e_1} = \frac{1}{2} K s^2 = \frac{1}{2} * 800 * (0.4 \sin 30)^2$$

$$\boxed{V_1 = 6.19 \text{ J}}$$

$$T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_{I.c} \omega^2$$

$$T_2 = \frac{1}{2} * 10 * (0.2\omega)^2 + \frac{1}{2}\left(\frac{1}{12} * 10 * (0.4)^2\right)\omega^2 \Rightarrow \text{---}\ \omega^2$$

راح تكون حول ال Ic

لانه لما تكون $V = 0$, $\theta = 0$

و يبدا يغير اتجاه حركته.

$$V_1 = T_2$$

$$\boxed{6.19 = \text{---}\ \omega^2}$$

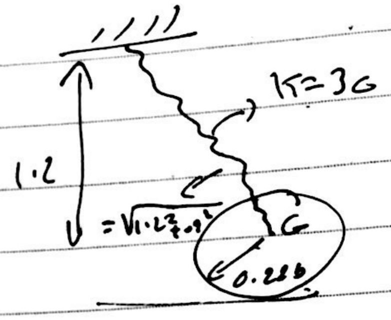
example 18.7 $\circ$-

$k_G = 0.2m$

$m = 15\ kg$

$S_{unstretched} = 0.3$

$\omega = ?$



sol^n:

$T_1 + V_1 = T_2 + V_2$

$$\boxed{V_{e_1} = \frac{1}{2}(30)(1.5-0.3)^2}$$

$$\boxed{V_2 = V_{e_2} = \frac{1}{2}(30)(1.2-0.3)^2}$$

$T_2 = \frac{1}{2} I_{I_c} \omega^2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$

$I_{I_c} = I_G + m d^2$

$= m \cdot k_G^2 + m r^2$

$= 15(0.2)^2 + (0.225)^2$

$$\boxed{T_2 = \_\_\ \omega^2}$$

$V_{e_1} = V_{e_2} + T_2$

example 18.8 :-

$m_D = 10\ kg$

$m_r = 5\ kg$

~~rest~~ at rest when $\theta = 60^\circ$

$\theta = 0$, $\omega_{AD} = ?$

Sol :- $\underbrace{\cancel{T_1}}_{zero} + V_1 = T_2 + V_2$

$V_1 = W_r y = 5(9.81)(0.3 \sin 60) = 12.74\ J$

$V_2 = 0$

$$T_2 = \Big(\underbrace{\tfrac{1}{2} m_D V_{G_D}^2}_{zero} + \underbrace{\tfrac{1}{2} I_{G_D} \omega_D^2}_{zero}\Big) + \Big(\tfrac{1}{2} m_r V_{G_r}^2 + \tfrac{1}{2} I_{G_r} \omega_r^2\Big)$$

$$T_2 = \tfrac{1}{2}\, 5(\omega_r\, 0.3)^2 + \tfrac{1}{2}\Big(\tfrac{1}{12}(5)(0.6)^2 \omega^2 = \text{--}\ \omega_r$$