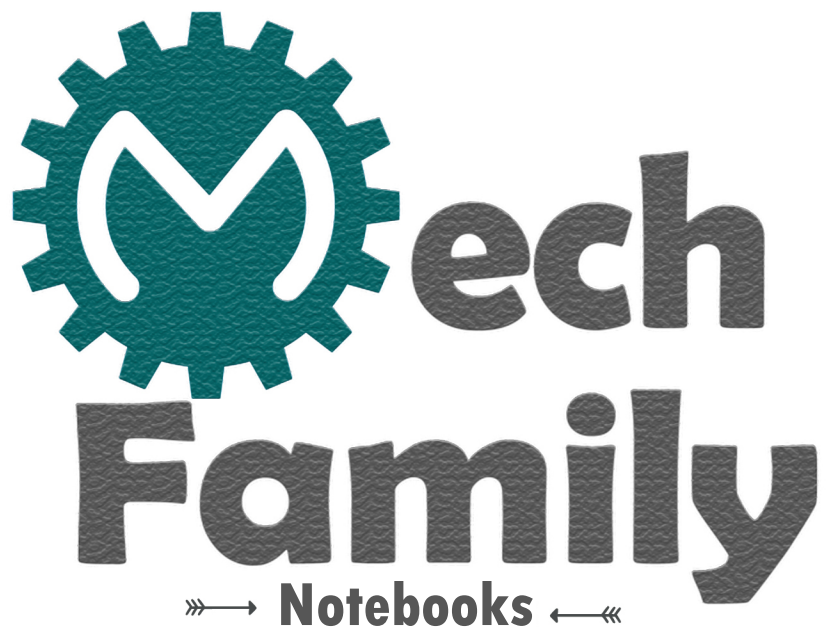


Dynamics

Dr. Ahmad Qaiseyye

1st Semester 2017



Dynamics

Kinematics

[motion parameters]

دراسة
الحركة

d, v, a

Kinetics

[cause of motion with (d, v, a)]

سبب
الحركة

particles

Rigid Body



center
of
mass



Shape
prob

size negligible

Kinematics of particles [d, v, a]

uniform motion

$v = \text{const}$

$\therefore a = 0$

constant
acceleration

$a = \text{constant}$

variable
acceleration

$a = f(t)$

$= g(x)$

particle



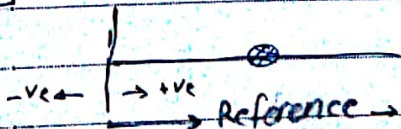
Rectilinear

وصف
الحركة



curvilinear

□



Reference

fixed

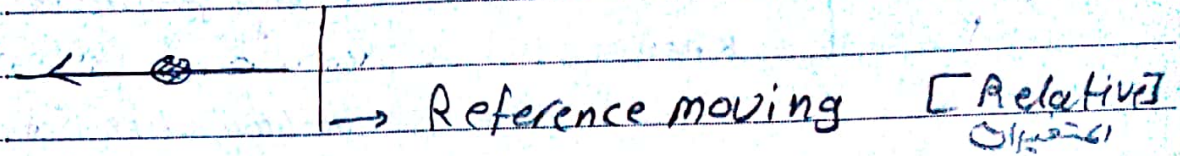
[absolute]

① Reference

②

ميكانيكا
الديناميكا

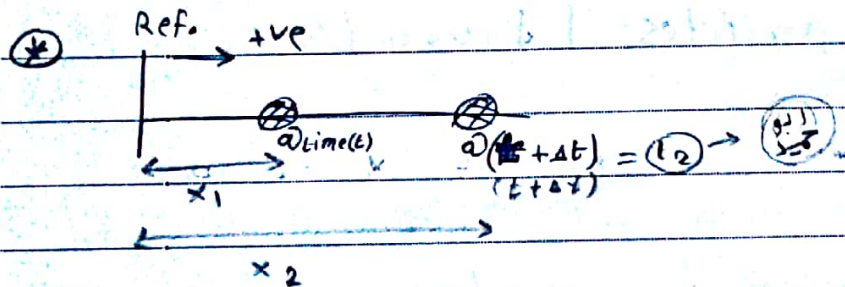
2]



عین بتحول مثال سیارگان

تتحركان و تكون احدهما Ref. الاخرى

Ref. - 60 m/s
60 m/s
80 m/s



$\Delta x = \text{change in position}$

$$\Delta x = x_2 - x_1 \text{ (m)}$$

$$\Delta v = \text{average velocity} = \frac{\Delta x}{\Delta t}$$

$$\vec{v} = \text{velocity} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

$$\therefore \vec{v} = \frac{dx}{dt} \text{ (m/s)}$$

$$\therefore \vec{v} = \frac{dx}{dt} \text{ مشتقة المسافة بالنسبة للزمن}$$

السرعة

$$\vec{a} = \text{acceleration} = \frac{d\vec{v}}{dt} \text{ (m/s}^2\text{)}$$

$$\therefore \vec{a} = \text{مشتقة السرعة بالنسبة للزمن}$$

قوانين نيوتن حركية ثابت

$$a = \frac{dv}{dt} = \text{constant}$$

$$\int_{v_0}^v dv = \int_{t_0}^t a dt \Rightarrow v - v_0 = at \quad \boxed{v = v_0 + at}$$

$$v = v_0 + at = \frac{dx}{dt} \Rightarrow \int_{x_0}^x dx = \int_0^t (v_0 + at) dt$$

$$\Rightarrow \boxed{x - x_0 = v_0 t + \frac{1}{2} at^2}$$

$$* a = \frac{dv}{dt} \Rightarrow - \frac{dv}{dx/v} = v \frac{dv}{dx} \Rightarrow \int_{x_0}^x a dx = \int_{v_0}^v v dv$$

$$* \textcircled{*} v = \frac{dx}{dt} \Rightarrow \boxed{dt = dx/v} \Rightarrow a(x - x_0) = \frac{1}{2} [v^2 - v_0^2]$$

$$\boxed{v^2 = v_0^2 + 2ax}$$

* Variable acceleration :

متغير التسارع

$$a = f(t) = \frac{dv}{dt} \Rightarrow \int_{v_0}^v dv = \int_0^t f(t) dt$$

$$\Rightarrow \boxed{v - v_0 = \int_0^t f(t) dt}$$

$$a = g(x) = v \frac{dv}{dx} \Rightarrow \int_{v_0}^v v dv = \int_{x_0}^x g(x) dx$$

$$\Rightarrow \boxed{\frac{1}{2} [v^2 - v_0^2] = \int_{x_0}^x g(x) dx}$$

Variable acceleration



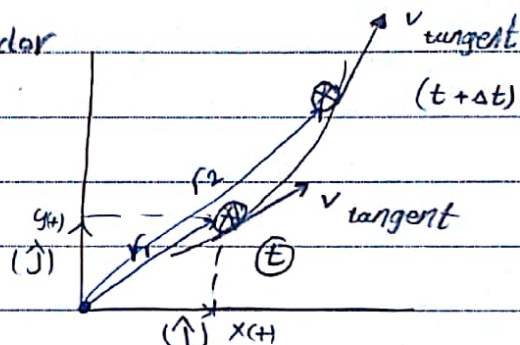
Curvilinear motion

1 Rectangular component $\rightarrow$ $\frac{dx}{dt}$ و $\frac{dy}{dt}$ $\rightarrow$ $\frac{dx}{dt}$ و $\frac{dy}{dt}$

2 curvilinear component $\rightarrow$ unit vector

$\vec{r}$ = position vector

$$\vec{r} = x\hat{i} + y\hat{j} \quad |\vec{r}| = \sqrt{x^2 + y^2}$$



$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

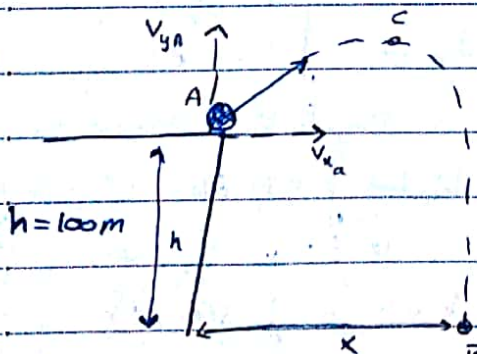
velocity in the x-direction

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

$(\vec{r}, \vec{v}, \vec{a}) \rightarrow$ kinematics

تدریس های kinematics ما به این بدو الجسم

Expo:



$\downarrow g$

ملاحظات سريعة

$$|a_y| = 9.8$$

$$|a_x| = 0$$

في تسارع بسرعة محوري x لا يوجد

$$v_x = v_{x_A} = v_{x_B}$$

given $v = 10 \text{ m/s}$, $\theta = 30^\circ$

Find $x = ??$, $v_B = ?$, $h_{\max} = ??$

Soln: motion with const. acc

$$a_y = 9.8, a_x = 0$$

$$1) y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$v_y^2 = v_{y0}^2 + 2a_y(s - s_0)$$

$$x = v_{x0}t$$

$$v = v_0 + at$$

$$v_x = v_{x0}$$

$$V = 10 \text{ m/s} \rightarrow V_x = 10 \cos 30$$

$$V_y = 10 \sin 30$$

$$x = x_0 + v_x t + \frac{1}{2} a_x t^2$$

$$x = 0 + 10 \cos 30 (t)$$

$$* -100 = 0 + 10 \sin 30 t + \frac{1}{2} (-9.8) t^2$$

ما أخذنا Ref.

$$t_1 = -ve \quad x$$

$$t_2 = +ve \quad \checkmark$$

زمن الرحلة كاملة

$$t = t_2 = +ve \rightarrow x = (10 \cos 30) t = \checkmark \text{ m}$$

$$h_{\max} \rightarrow V_y = 0$$

$$V_{cy}^2 = V_{Ay}^2 + 2 a_y (y_c - y_0)$$

$$0 = (10 \sin 30)^2 + 2 * (-9.8) (y_c - 0)$$

$$y_c = \checkmark \text{ m}$$

$$V_y = V_{Ay} + a_y t$$

1. إذا طلب الزمن والمكان
 $\Delta y = v_{iy} + \frac{1}{2} a_y t^2$

$$h_{\max} = y_c + 100 = \checkmark$$

$$V_{Bx} = V_{Ax} = 10 \cos 30 = \checkmark$$

$$V_{By} = 10 \sin 30 - 9.8(t) = \checkmark$$

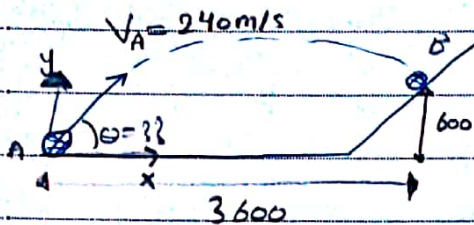
(t₂)

$$V_B = \sqrt{V_{Bx}^2 + V_{By}^2}$$

$$\begin{aligned}
 V_y &= V_{iy} + a_y t \\
 V_y^2 &= V_{iy}^2 + 2 a_y (y - y_i) \\
 y &= y_i + V_{iy} t + \frac{1}{2} a_y t^2
 \end{aligned}$$

$x = V_{ix} t$

Exp 28



find θ ??

$$x = v_x t \Rightarrow x = 240 \cos \theta t \quad \boxed{\frac{3600}{240 \cos \theta} = t} \quad [1]$$

$$y = y_0 + v_y t + \frac{1}{2} g t^2 \quad \boxed{600 = 0 + 240 \sin \theta t + \frac{1}{2} (-9.8) t^2} \quad [2]$$

[1] in [2]

$$600 = 240 \sin \theta \frac{3600}{240 \cos \theta} + \frac{1}{2} (-9.8) \times \left(\frac{3600}{240 \cos \theta} \right)^2$$

$$600 = 3600 \tan \theta - \frac{9.8}{2} \times \left(\frac{3600}{240} \right)^2 \frac{\sec^2 \theta}{(1 + \tan^2 \theta)}$$

$$a \tan^2 \theta + b \tan \theta + c = 0$$

$$\tan \theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\theta_1 = 29.9$$

$$\theta_2 = 69.6$$

[Dr.] $x = 3600 = v_A \cos \theta t \dots [1] \rightarrow t = \frac{x}{v_A \cos \theta}$

$y = 0 + v_A \sin \theta t - \frac{g}{2} t^2 \dots [2]$

$$y = v_A \sin \theta \left[\frac{x}{v_A \cos \theta} \right] - \frac{g}{2} \frac{x^2}{v_A^2 \cos^2 \theta} \quad \frac{1}{\cos^2 \theta} = \sec^2 \theta = 1 + \tan^2 \theta$$

$$y = x \tan \theta - \frac{g}{2} \frac{x^2}{v_A^2} (1 + \tan^2 \theta)$$

$$\frac{g x^2}{2 v_A^2} \tan^2 \theta - x \tan \theta - \frac{g x^2}{2 v_A^2} + y = 0 \quad a \tan^2 \theta + b \tan \theta + c = 0$$

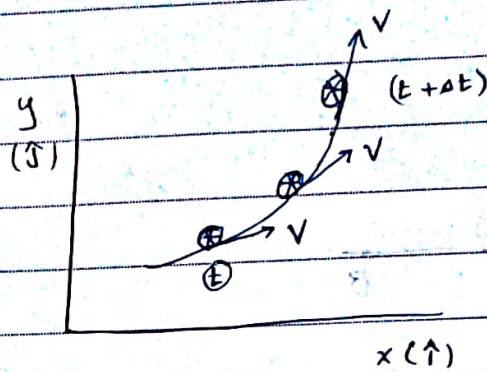
* Curvilinear coordinates

magnitude

$$\vec{V} = (\text{speed}) \vec{e}_t$$

↓

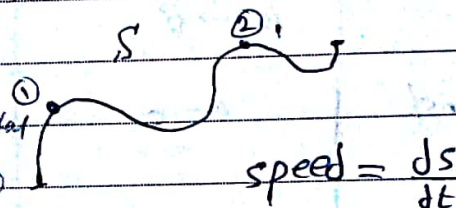
$$\vec{V} = \left(\frac{ds}{dt} \right) \vec{e}_t$$



* $\vec{e}_t$ = tangential unit vector

direction

تغير اتجاه السرعة = a centripetal

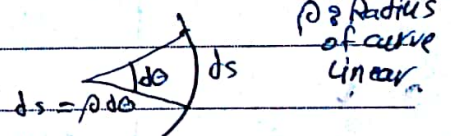


$$\vec{a} = \frac{d\vec{V}}{dt} = \left(\frac{d^2s}{dt^2} \right) \vec{e}_t + \frac{ds}{dt} \frac{d}{dt} \vec{e}_t$$

لاحقاً هناك تسارعين
واحد للسرعة وآخر للاتجاه

* $\frac{d}{dt}(\vec{e}_t) = \frac{d}{d\theta}(\vec{e}_t) \frac{d\theta}{ds} \left(\frac{ds}{dt} \right)$ speed

chain Rule



$$\frac{d}{dt}(\vec{e}_t) = \frac{ds}{dt} \cdot \frac{1}{\rho} \vec{e}_n$$

$$\vec{a} = \left[\frac{d^2s}{dt^2} \right] \vec{e}_t + \left[\left(\frac{ds}{dt} \right)^2 \frac{1}{\rho} \right] \vec{e}_n$$

a_t
(tangential)

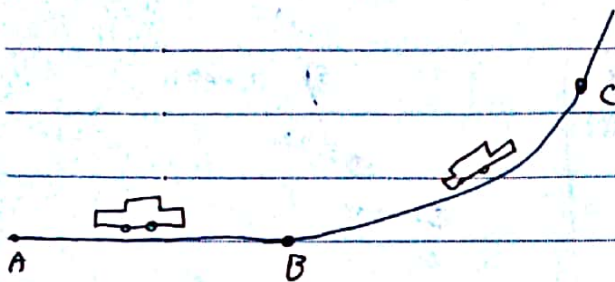
a_n
(normal)

تغير اتجاه السرعة

calculates

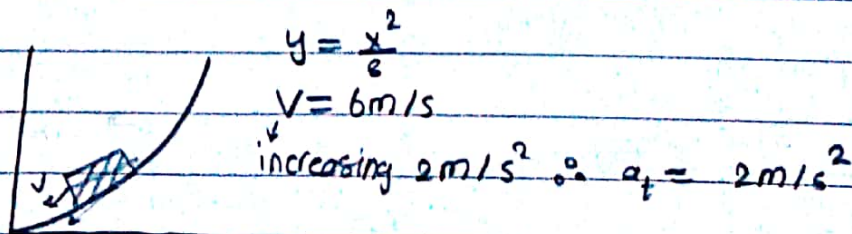
$\vec{e}_{\text{tangent}}$

اتجاه السرعة
تangent
 $\vec{e}_n$



	Linear A → B	Curve B → C
1) $v = \text{const}$ $a = 0$	$a = 0$	$a_t = 0$ $a_n = \frac{v^2}{\rho}$
2) $v = \dots \checkmark$ $a = \dots \checkmark$	$a = \checkmark$	$a_t = a$ $a_n = \frac{v^2}{\rho}$
3) v $a = f(t)$ $a = g(x)$	Variable	$a_t = f(t), g(s) \rightarrow \frac{dv}{dt} \rightarrow v \frac{dv}{ds}$ $a_n = \frac{v^2}{\rho}$ بتغير من نقطة إلى أخرى

Ex: f-12-30



Find ① direction of v

$$L \rightarrow \frac{dy}{dx} = \frac{2x}{8} \Big|_{x=3} = \frac{6}{8} = \text{slope} = \tan \theta$$

$\theta = \text{direction}$

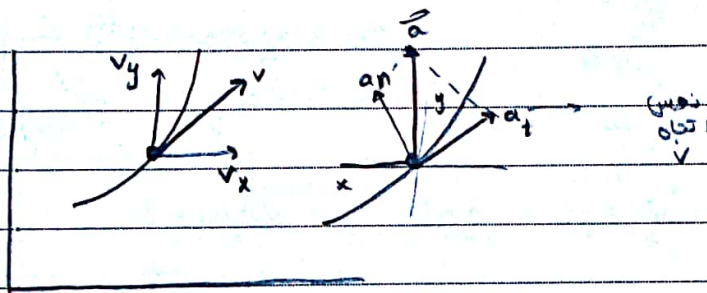
المسافة من L إلى $U_{\infty} =$

$$\boxed{2} \quad |\vec{a}| = ??$$

$$= \sqrt{a_t^2 + a_n^2}$$

$$a_n = \frac{v^2}{\rho} \rightarrow \rho = \frac{(1+y'^2)^{\frac{3}{2}}}{|y''|}$$

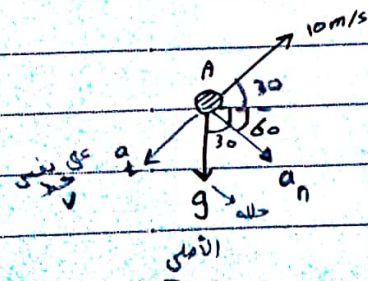
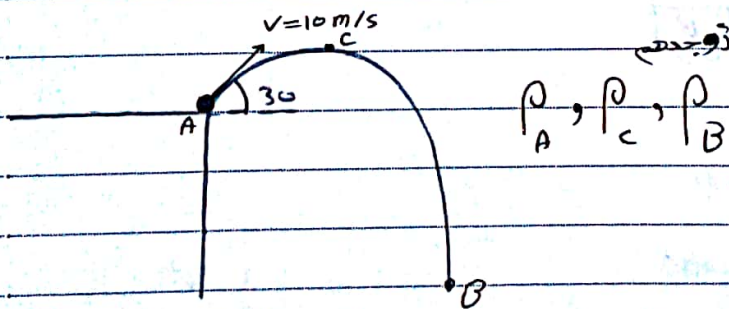
curvature
Curvature Radius منحنى



$$|V| = \text{speed} = \sqrt{v_x^2 + v_y^2}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{a_t^2 + a_n^2}$$

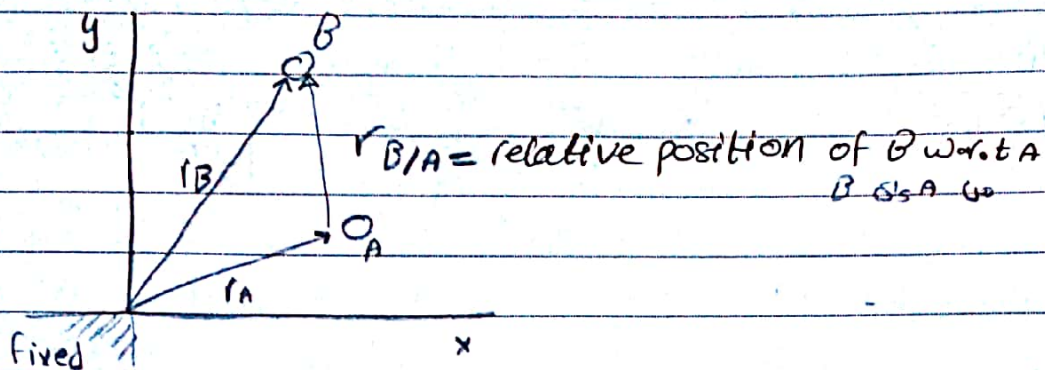
comp



$$a_n = g \cos 30 = \frac{10^2}{\rho_A} \rightarrow \rho_A = \frac{10^2}{g \cos 30}$$

component

$$g = a_n = \frac{(10 \cos 30)^2}{\rho_C} \rightarrow \rho_C = \frac{(10 \cos 30)^2}{9.8}$$



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

* Independent Motion

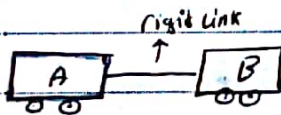
$$|\vec{r}_{B/A}| = |\vec{r}_{A/B}|$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

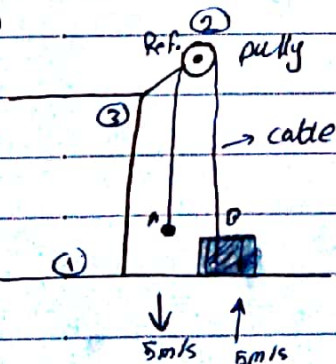
$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

$$|\vec{v}_{B/A}| = |\vec{v}_{A/B}|$$

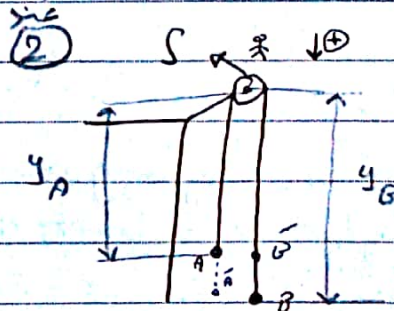
* Dependent motion



Exp



Ref.



L = cable length = const

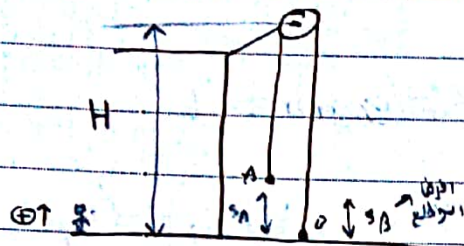
$$L = y_A + y_B + S$$

اشتقاق $0 = v_A + v_B \rightarrow v_A + v_B = 0$

اشتقاق $0 = a_A + a_B \rightarrow a_A + a_B = 0$

$v_A = 5 \text{ m/s}$
 $5 + v_B = 0$
 $v_B = -5 \text{ m/s}$

① ع



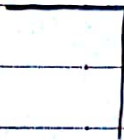
$L = \text{cable length}$

$$L = (H - s_A) + (H - s_B) + S$$

اشتقاق $0 = -v_A - v_B$

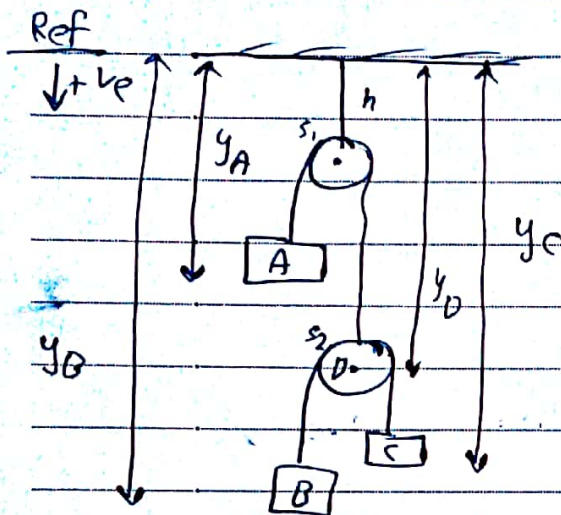
$$-5 + v_B = 0 \quad \boxed{v_B = 5 \text{ m/s}} \quad \uparrow$$

~~الارتفاع~~



* Dependent Motion

position y is $\downarrow$ particle $\downarrow$ is



cable length = L

$$L_1 = (y_A - h) + s_1 + (y_D - h)$$

$$0 = v_A + v_D \quad (1)$$

$$v_D = -v_A$$

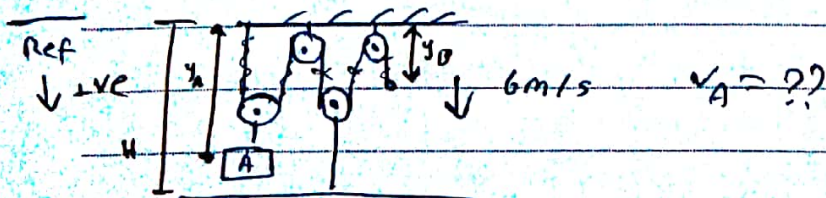
$$L_2 = (y_B - y_D) + s_2 + (y_C - y_D)$$

$$0 = v_B + v_C - 2v_D \quad (2)$$

$$0 = v_B + v_C - 2(-v_A) \quad (3)$$

$$0 = 2v_A + v_B + v_C \quad (3^*)$$

Ex-40

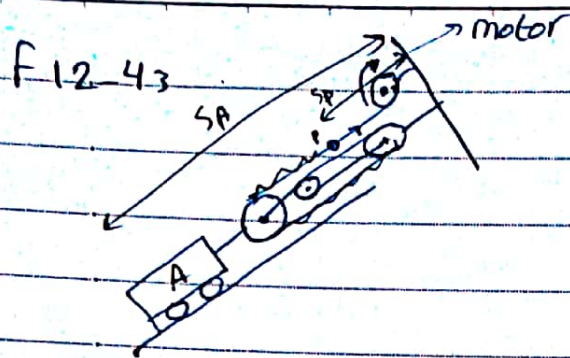


$$2y_A + 2H + y_B = L$$

$$2v_A + v_B = 0, \quad 2v_A + 6 = 0 \quad v_A = -3 \text{ m/s} = 3 \text{ m/s} \uparrow$$

position

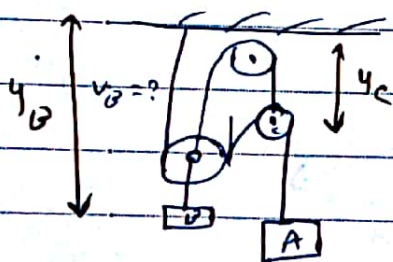
$$(y_A - d_1) + (y_A - d_1 - d_2) + (H - d_3 - d_2) + (H - d_3 - d_2) + (y_B - d_3) = L$$



$$(s_A - s) + s_A + s_A + s_A = L$$

$$4v_B - v_P = 0$$

$$v_A = \frac{-v_P}{4} = \frac{-4}{4} = -1 \text{ m/s} = 1 \text{ m/s} \uparrow$$



↓ 4 m/s

Ref
↓ +ve

$$y_B + (y_B - y_C) + (y_A - y_C) = L_1$$

$$2v_B - 2v_C + v_A = 0 \quad \text{--- (1)}$$

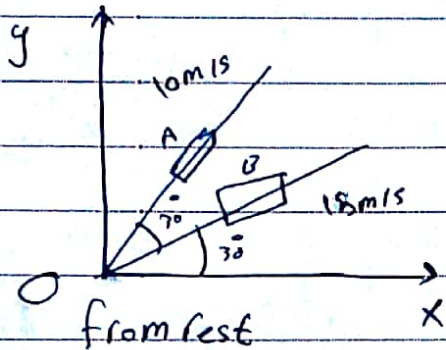
$$y_B + y_C = L_2$$

$$v_B = -v_C$$

$$2v_B - 2(-v_B) + v_A = 0$$

$$4v_B + v_A = 0 \quad 4v_B = -4$$

R12-42)

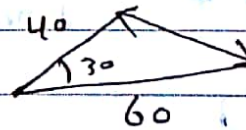


distance between A and B after 4 s

$$\vec{r}_A = 60 \cos 30^\circ \hat{i} + 60 \sin 30^\circ \hat{j} \text{ from rest}$$

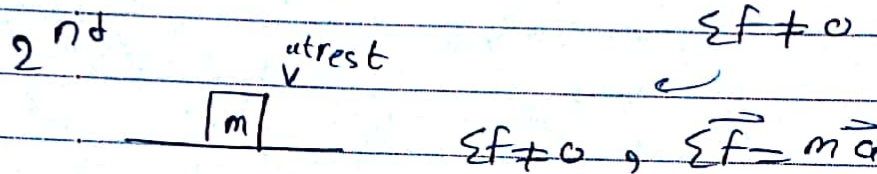
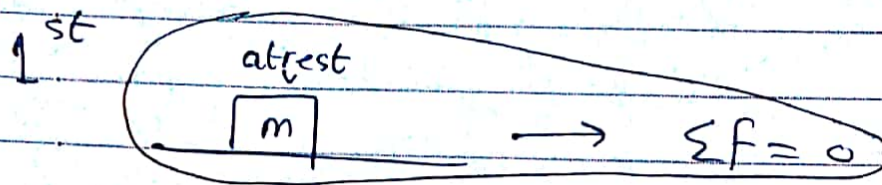
$$\vec{r}_B = 40 \cos 60^\circ \hat{i} + 40 \sin 60^\circ \hat{j}$$

$$|\vec{r}_{B/A}| = d$$

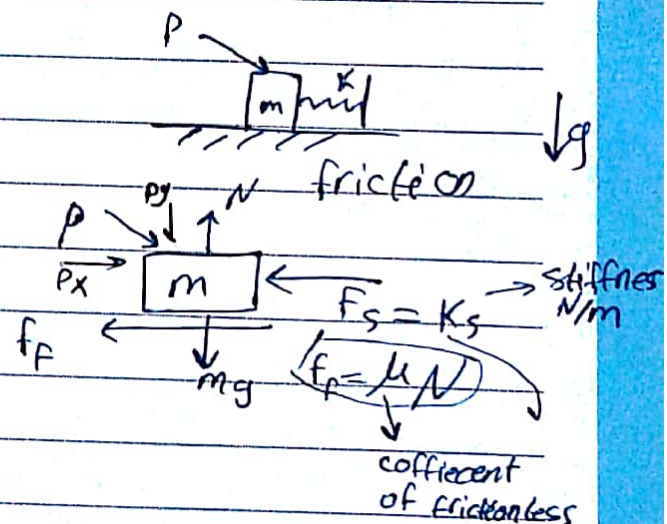


$$d = \sqrt{40^2 + 60^2 - 2(40)(60)\cos 30^\circ}$$

ch.13 Kinetics of particles Newton's 2nd law



3rd Action and reaction



$$\Sigma f_y = m a_y = 0, N - m g - P_y = 0$$

$$N = m g + P_y$$

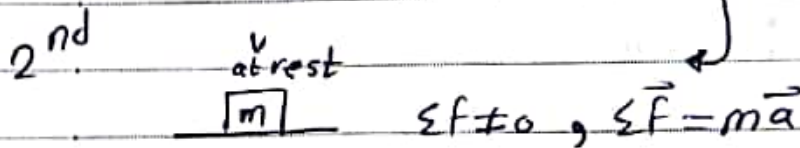
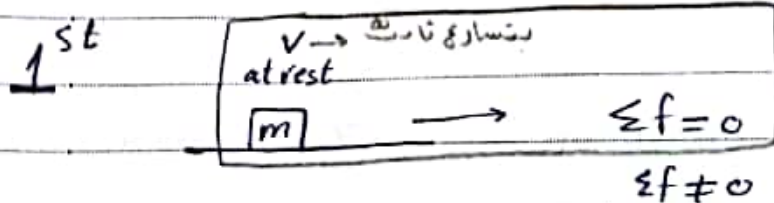
check for motion ($\rightarrow$ +ve)

$$P_x - \mu_s (m g + P_y) - K_s = 0$$

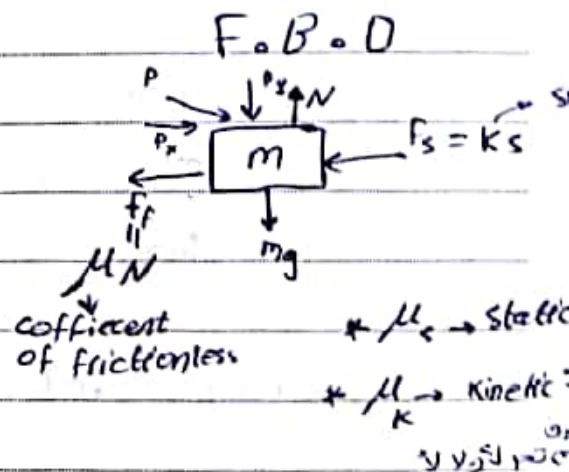
μ_s = static
 μ_k = kinetic

			$\mu_s = 0.4$ $\mu_k = 0.3$
$P = 30 N$	$30 < \mu_s N \rightarrow$	No motion $a = 0$	100
$P = 40 N$	$40 = \mu_s N \rightarrow$	threshold motion impending	$\mu_s 100$
$P = 50 N$	$50 > \mu_s N \rightarrow$	there is motion	
	$50 - \mu_k (100) = m a$		

Ch.13 Kinetics of particles Newton's 2nd law



3rd Action and reaction

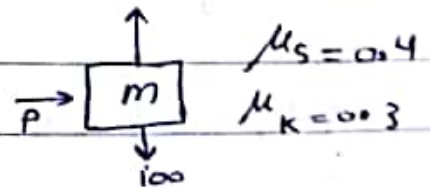


$$\Sigma f_y = m a_y = 0, N - mg - P_y = 0$$

$$N = mg + P_y$$

check for motion ($\rightarrow +ve$)

$$P_x - \mu_s (mg + P_y) - K_s = 0$$



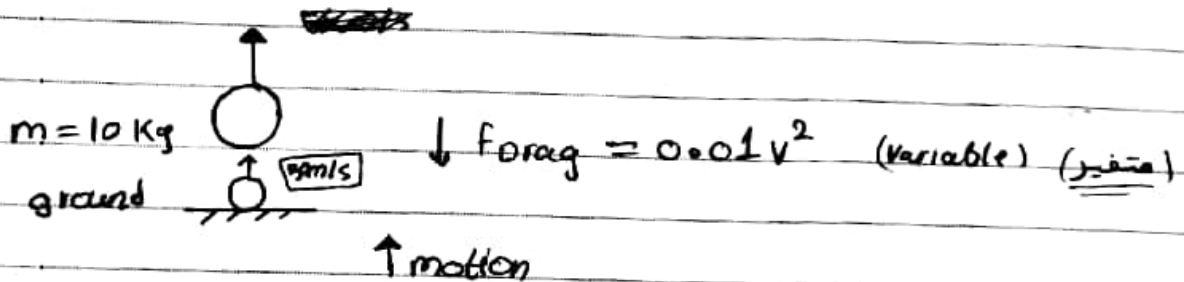
$P = 30 N$ $30 < \mu_s N = 40 \rightarrow$ No-motion $a = 0$

$P = 40 N$ $40 = \mu_s N \rightarrow$ Motion in pending

$P = 50 N$ $50 > \mu_s N \rightarrow$ there is motion $\rightarrow$ $\Sigma \vec{F} = m \vec{a}$

$$50 - \mu_k (100) = m a$$

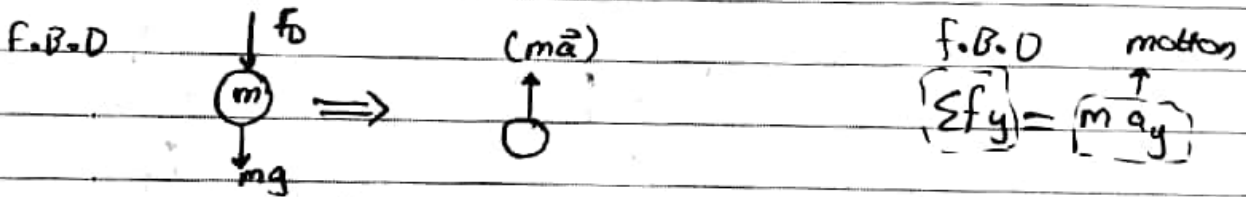
Exp 13-2



Find $h_{\text{max}} = ??$ when

1) $f_{\text{drag}} = \checkmark$

2) $f_{\text{drag}} = 0$



$$-mg - 0.01 v^2 = m \underline{a}$$

$$= m v \frac{dv}{dy} \quad \therefore$$

$$-mg - 0.01 v^2 = m v \frac{dv}{dy}$$

$$\int_0^{h_{\text{max}}} dy = \int_{50}^0 \frac{-m v dv}{(mg + 0.01 v^2)}$$

① $h_{\text{max}} = \checkmark$

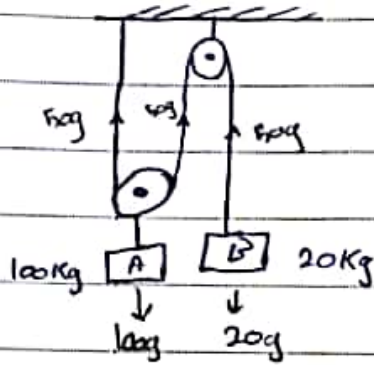
② $v^2 = v_0^2 + 2 g h_{\text{max}}$

② $h_{\text{max}2} = \checkmark$

$h_{\text{max}2} > h_{\text{max}1}$

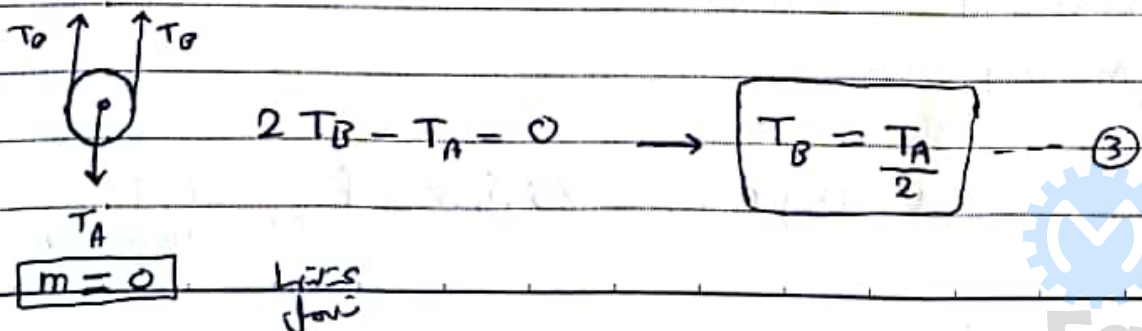
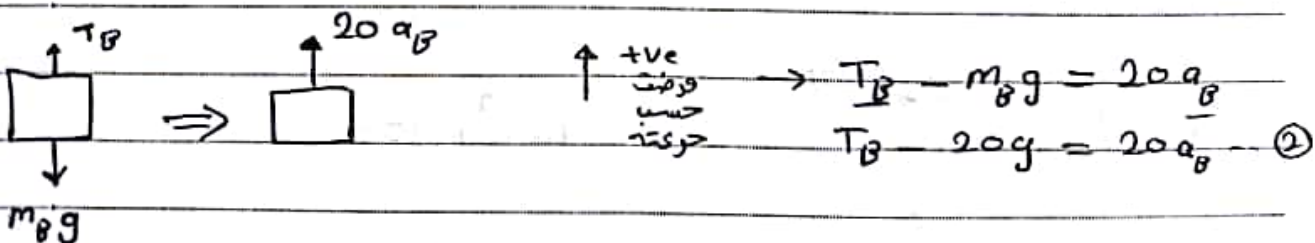
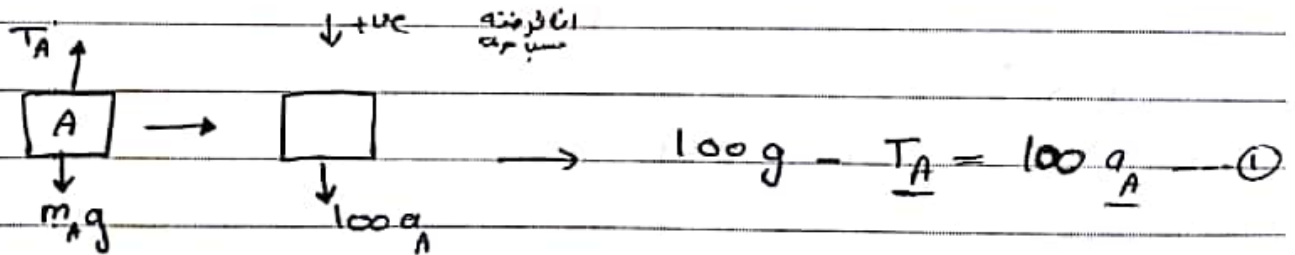
Exp (13-5)

sys. release from rest $v_B = ?$ at $t = 2s$



T في نقطتي الكبل واحد

check formation



$$2y_A + y_B = L$$

Ref
+ve
↓

$$2a_A + a_B = 0 \quad \text{--- [4]}$$

↑ v_B

$$2a_A - a_B = 0$$

$$2 \left[g - \frac{2T_0}{100} \right] - \left[\frac{T_0}{20} - g \right] = 0$$

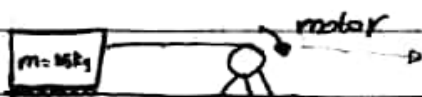
$$g - \frac{2T_0}{100} = a_A \quad \text{--- (1)}$$

$$\frac{T_0}{20} - g = a_B$$

a_B → ✓

$$v_B = v_{B0} + a_B (2)$$

F13-2



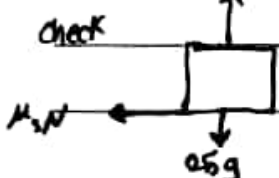
$$F = 10t^2 + 100$$

$$\mu_s = 0.3$$

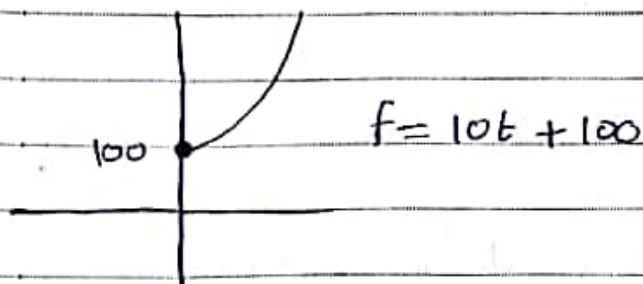
$$\mu_k = 0.25$$

$$v = ?? \text{ at } t = 4 \text{ s}$$

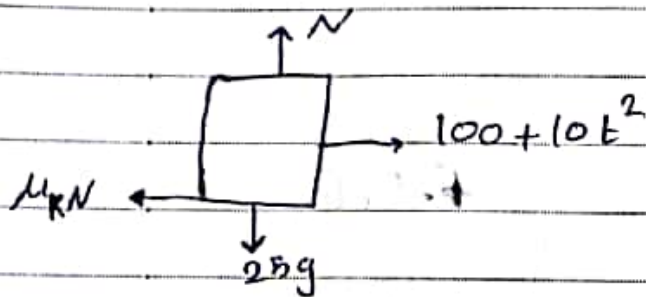
$$N = 25g$$



$$\mu_s N = 25g * 0.3 = 73.5 \text{ N} = f_{s \text{ max}} \text{ at } t=0 \text{ there is motion}$$



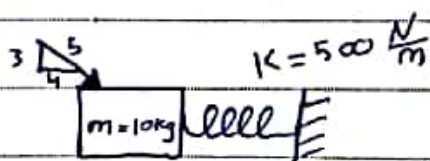
$$\boxed{25} \rightarrow a = \frac{dv}{dt}$$



$$100 + 10t^2 - 0.25(25g) = 25 \frac{dv}{dt}$$

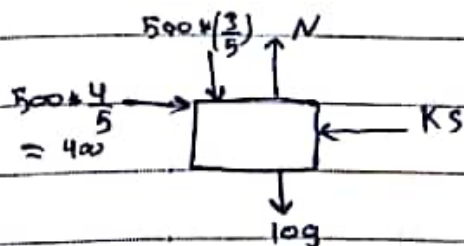
$$\int_0^4 (100 + 10t^2 - 0.25 * 25g) dt = \int_0^v 25 dv \quad (4)$$

f 1.3-2



initially uncompressed

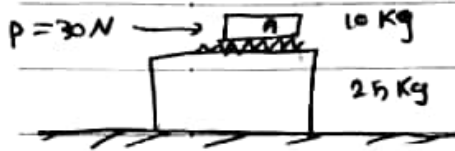
$v = ??$ at $s = 0.5 \text{ m}$



$$400 - Ks = 10v \frac{dv}{ds}$$

$$\int_0^{0.5} (400 - Ks) ds = \int_0^v 10v dv$$

f 13.6



$\mu_s = 0.4$, $\mu_k = 0.3$
smooth

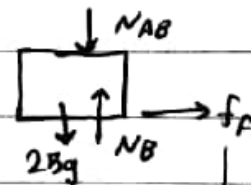
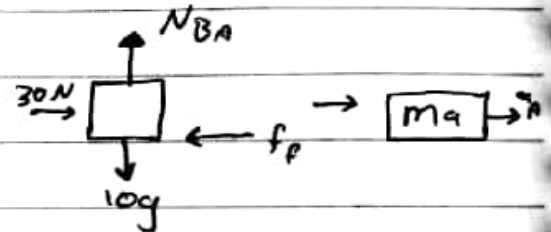
$a_A =$
 $a_B =$

check for motion

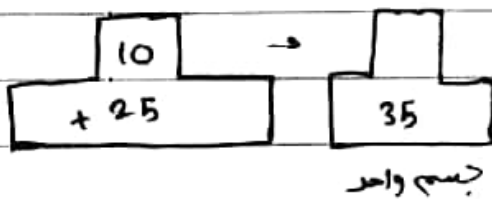
$$F_{f_{A \max}} = (0.4) 10g = 4g$$

$30 < 4g \rightarrow$ No motion
for A

F.B.D



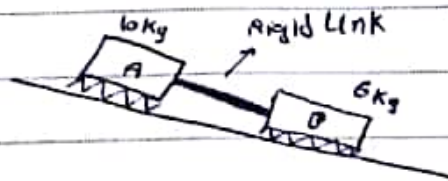
Why
does it interfere
force



$$a = \frac{30}{35}$$

If $p = 50$

13.3



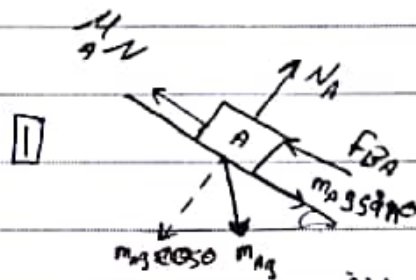
$$\mu_A = 0.1$$

$$\mu_B = 0.3$$

Find $F_{AB} = ??$

* Rigid Link *

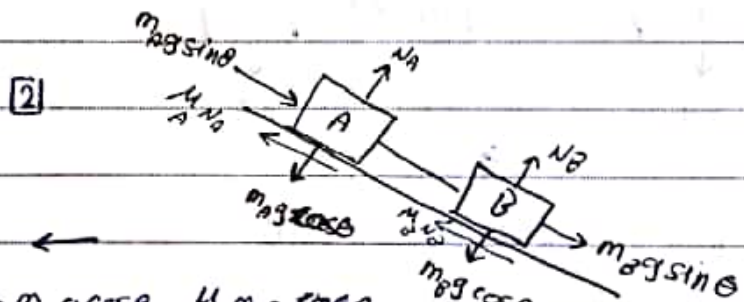
$$a_A = a_B = a$$



تدلي 1 حرك
لكن
mu



2 unknown
مجهولين
 $F_{AB} = a$



$F_{AB} = F_{BA}$
Int
f

$$(m_A + m_B) g \sin \theta - \mu_A m_A g \cos \theta - \mu_B m_B g \cos \theta = (m_A + m_B) a$$

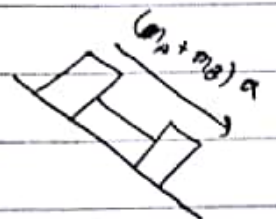
$$a = \checkmark$$

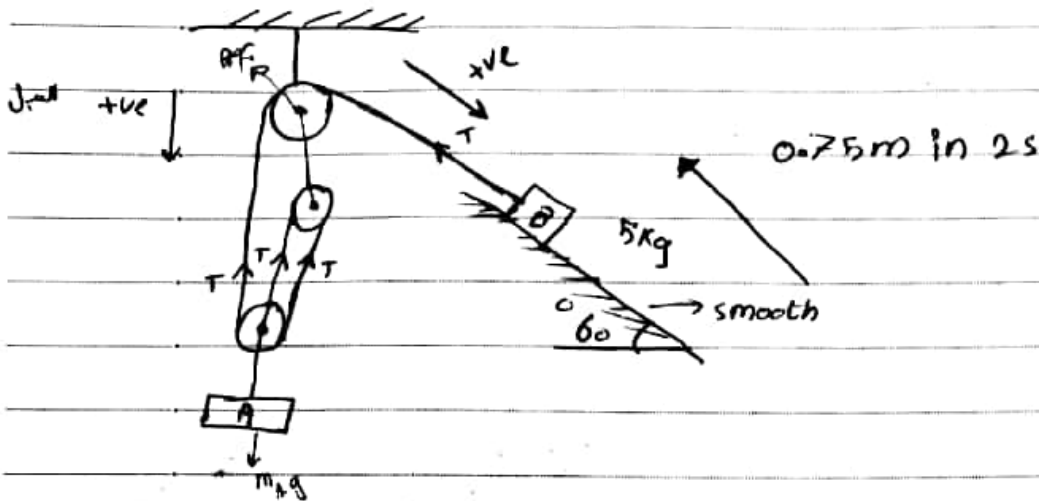
$$\Sigma f = m a, \quad m_B g \sin \theta - \mu_A m_B g \cos \theta - F_{BA} = m_B a$$

II

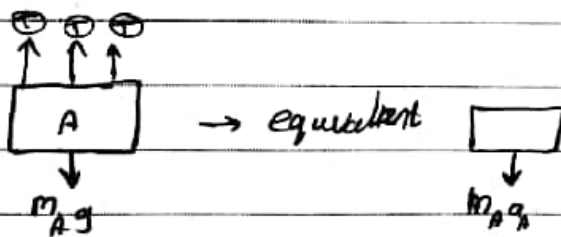
$$F_{BA} = \checkmark$$

N + $\rightarrow$ قوة
 $-$ $\rightarrow$ مركبة الوزن
 المتمايلة





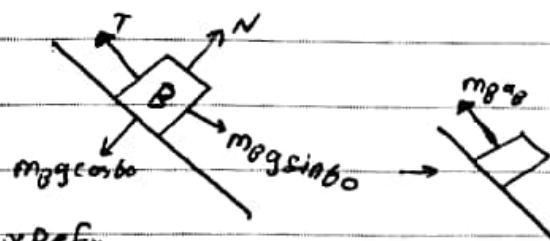
$$m_A = ??$$



cable $a_A \neq a_B$

$$m_A g - 3T = m_A a_A \quad \text{--- (1)}$$

$$T - m_B g \sin 60 = m_B a_B$$



$$s = s_0 + v_0 t + \frac{a_B}{2} t^2$$

$$0.75 = 0 + \frac{a_B}{2} (2)^2$$

$$a_B = 0.75 \text{ m/s}^2$$

solve for (1) ✓

* Ref *

$$3y_A + s_B = L$$

$$3a_A + a_B = 0$$

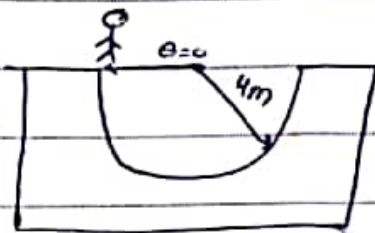
$$a_A = -\frac{1}{3} [a_B]$$

$$a_A = \frac{1}{3} [0.75] = 0.25 \text{ m/s}^2$$

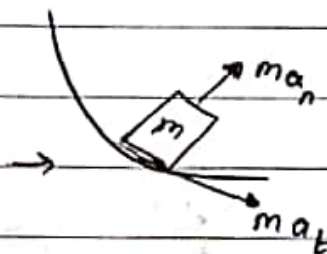
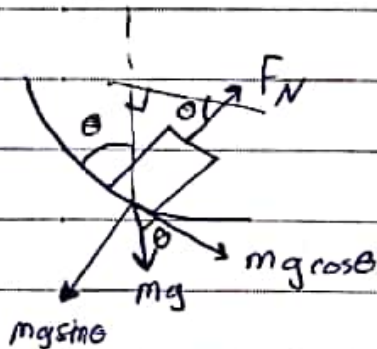
Exp 13.9

$$m = 60 \text{ kg}$$

$$\text{at } \theta = 0 \rightarrow v = 0$$



$$F_N = ?? \text{ at } \theta = 60^\circ$$



$$s = v_0 t + \frac{1}{2} a t^2 \rightarrow v^2 = v_0^2 + 2as$$

curve is in θ axis
 $a \rightarrow$ variable

$$\sum f_t = m a_t \quad (1)$$

$$\sum f_n = m a_n \quad (2)$$

$$ds = \rho d\theta$$

$$mg \cos \theta = m a_t = m v \frac{dv}{ds} \quad (1)$$

من هنا نحل المسألة

$$F_N - mg \sin \theta = m \frac{v^2}{\rho} \quad (2)$$

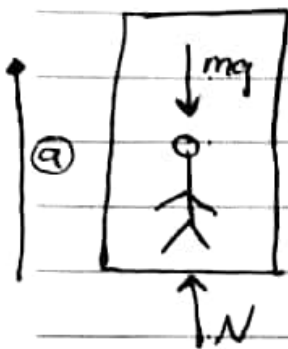
$$\int_0^{60} mg \cos \theta \rho d\theta = \int_0^v m v dv \Rightarrow \frac{1}{2} v^2$$

$$2 \rho g \sin \theta = v^2$$

$$F_N = m \left[\frac{2 \rho g \sin \theta}{\rho} \right] + mg \sin \theta$$

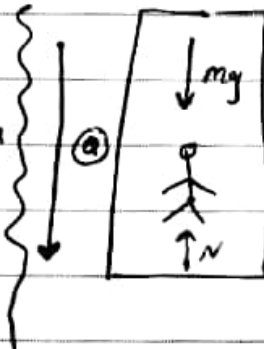
* Dynamic Eqn /

Apparent weight



$$N - mg = ma$$

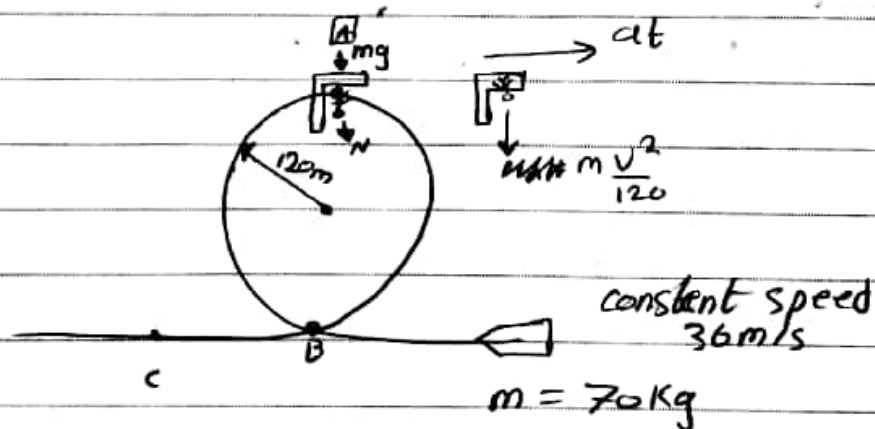
$$N = mg + ma$$



$$mg - N = ma$$

$$N = mg - ma$$

F13-9



$N = ??$
at C

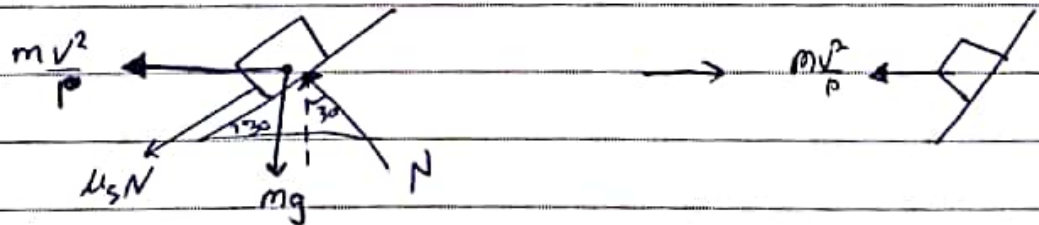
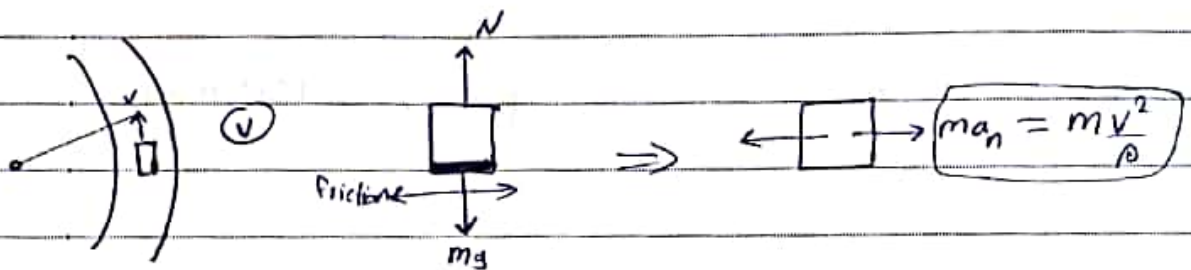
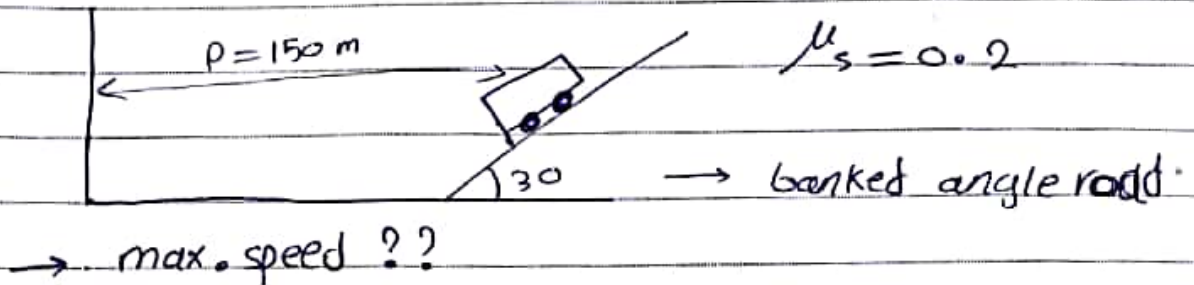
$$N_A + mg = m \frac{v^2}{120}$$

$$N_A = m \left[\frac{v^2}{120} - g \right] = 69.3 \text{ N}$$

$$N_B - mg = m \frac{v^2}{\rho} \rightarrow N_B = m \left[\frac{v^2}{\rho} + g \right] = 1442 \text{ N}$$

$$N_C = mg = 70 \times 9.81 = 686.7 \text{ N}$$

F 13-10



$$\sum F_x = \frac{m v^2}{\rho} = \mu_s N \cos 30^\circ + N \sin 30^\circ$$

$$\sum F_y = 0 = N \cos 30^\circ - \mu_s N \sin 30^\circ - mg$$

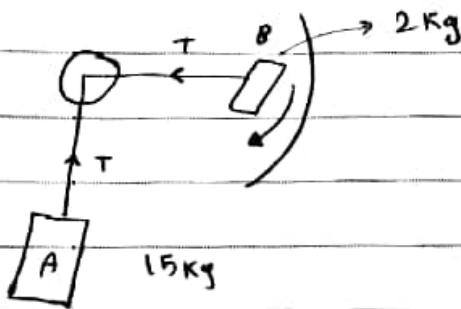
$$m = \frac{N \cos 30^\circ - \mu_s N \sin 30^\circ}{g}$$

$$v_{\max} = \sqrt{\frac{\rho g (\mu_s \cos 30^\circ + \sin 30^\circ)}{\cos 30^\circ - \mu_s \sin 30^\circ}}$$

$$v_{\max} = \sqrt{\frac{\rho g (\mu_s \cos 30^\circ + \sin 30^\circ)}{\cos 30^\circ - \mu_s \sin 30^\circ}}$$



13-58



$$T = 15g$$

$$a_n = \frac{v^2}{r} = \frac{v^2}{1.5}$$



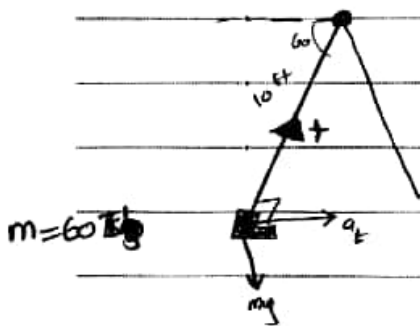
$$T = 2 \frac{v^2}{1.5}$$

$$15g = 2 \frac{v^2}{1.5}$$

$$\boxed{v = 10.5 \text{ m/s}}$$

$$15g - T = ma_y$$

13-60



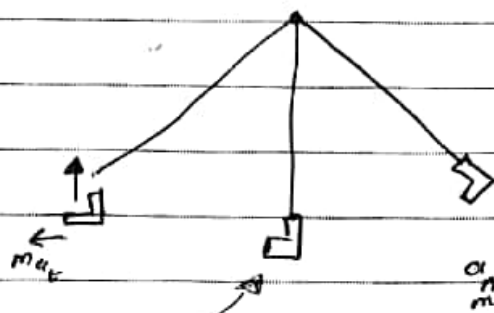
$$v = 15$$

$$a_t = ??$$

$$mg \cos 60 = ma_t$$

$$\boxed{a_t = \checkmark}$$

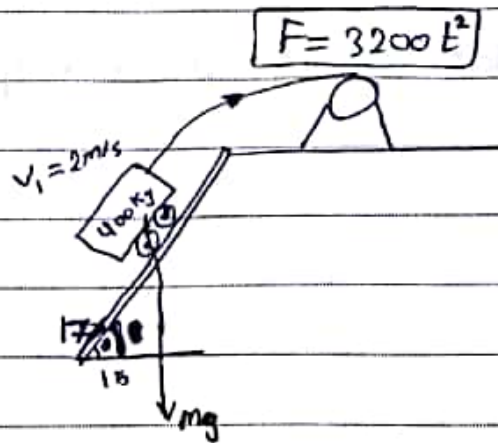
$$T - mg \sin 60 = m \frac{v^2}{r}$$



$$N - mg = ma_n = m 8g$$

$$N = 9mg$$

$$\frac{v_{\max}^2}{800} = 8g$$



$$S = ??$$

in [x]

$$3200t^2 - mg \sin \theta = m \frac{dv}{dt}$$

من اطلاق
ساعت 0
در 0

$$\int_0^t 3200t^2 - mg \sin \theta dt = \int_2^v m dv$$

$$\left[\frac{3200t^3}{3} - mg \sin \theta t \right]_0^t = m(v-2)$$

$$\frac{ds}{dt} = v = 2 + \frac{\frac{3200t^3}{3} - mg \sin \theta t}{m}$$

$$\int_0^2 (\quad) dt = \int_0^s ds$$

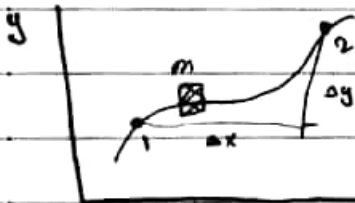
Ch. 14 "Kinetics of particles Work and Energy principle"

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\sum \vec{F} = m \vec{a}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\sum \vec{F} = m \vec{a}$$



$$u_{1 \rightarrow 2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$= \int_{s_1}^{s_2} F \cos \theta ds$$

$$u = \text{work done by force} = \int \sum \vec{F} \cdot d\vec{r}$$

العمل المكتسب

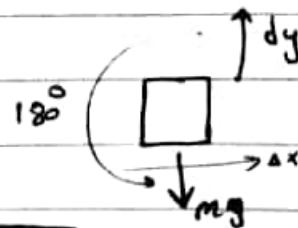
$$u_{1 \rightarrow 2} = \int_{r_1}^{r_2} \sum \vec{F} \cdot d\vec{r}$$

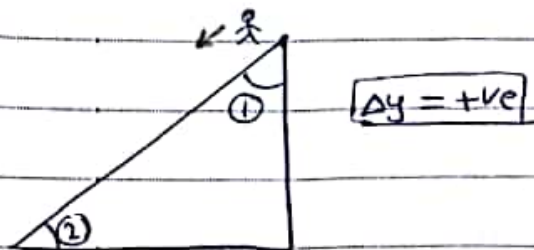
dot product

1) Gravity force

$$u_{1 \rightarrow 2} g = \int_{y_1}^{y_2} m \vec{g} \cdot d\vec{y}$$

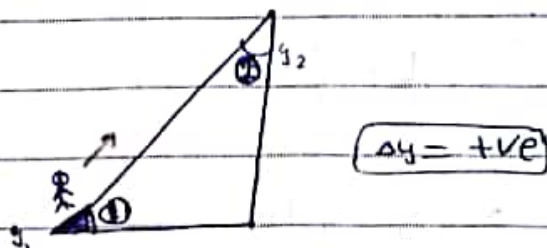
$$u_{1 \rightarrow 2} g = \int_{y_1}^{y_2} mg dy (-1) = -mg(y_2 - y_1) = \boxed{-mg \Delta y}$$





$$u_{1 \rightarrow 2} g = +ve$$

$$-mg(-\Delta y) = +ve$$

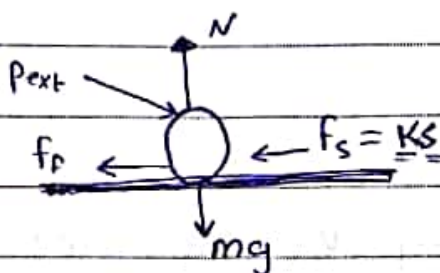


$$u_{1 \rightarrow 2} g = -ve$$

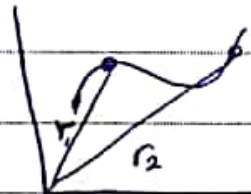
$$-mg(\Delta y) = -ve$$

* work and energy principle

$$u_{1 \rightarrow 2} = \int_{r_1}^{r_2} (\sum \vec{F}) \cdot d\vec{r}$$



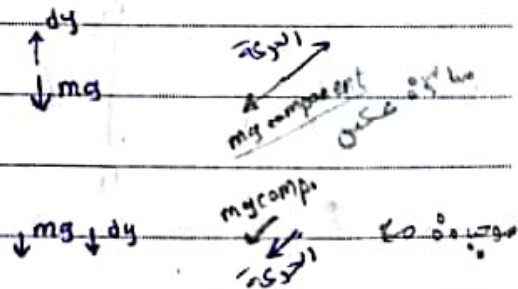
1) work done by gravity Independent to the path



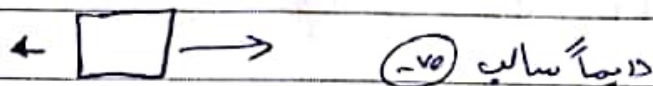
$$u_{1 \rightarrow 2} g = -mg \Delta y \quad \Delta y = y_2 - y_1$$

$$u_{1 \rightarrow 2} g = -ve \quad \Delta y = +ve$$

$$u_{1 \rightarrow 2} g = +ve \quad \Delta y = -ve$$



2) work done by friction

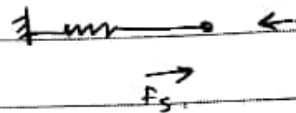
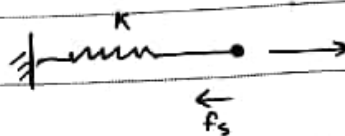


$$u_{1 \rightarrow 2} f_f = \int -(\mu_k N) \cdot \Delta x \quad u_{1 \rightarrow 2} f_f = -\mu_k N \Delta s$$

③ work done by spring = $\int F_s ds = \int -ks ds$

$U_{1 \rightarrow 2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$

$= -\frac{k}{2}(s_2^2 - s_1^2)$



④ work done by $\vec{p}$ = $\vec{p} \cdot \vec{v}$

$W = \int \vec{p} \cdot d\vec{r}$

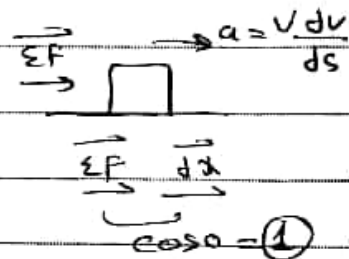
⑤ work done by $\vec{N}$ normal force

$W = 0$ $\cos 90^\circ = 0$, deformation = 0

⑥ $U_{1 \rightarrow 2} = \int_{r_1}^{r_2} (\sum \vec{F}) \cdot d\vec{r}$

if $\sum F = 0$ → ~~work done~~

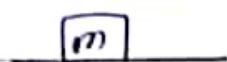
$\sum F = ma$
 $U_{1 \rightarrow 2} = \int_{r_1}^{r_2} m\vec{a} \cdot d\vec{r} = \int_{v_1}^{v_2} mvdv$
 $= \frac{m}{2}(v_2^2 - v_1^2)$



$U_{1 \rightarrow 2} = \frac{1}{2}m[v_2^2 - v_1^2]$, $T = \text{Kinetic Energy} = \frac{1}{2}mv^2$

$U_{1 \rightarrow 2} = T_2 - T_1$ $T_1 + U_{1 \rightarrow 2}^{\text{total}} = T_2$ Work and Energy principle

at rest



$$T_1 = 0$$



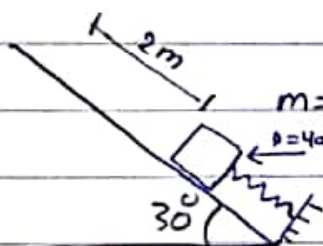
$$\Sigma f = 0 \rightarrow \boxed{\text{work} = 0}$$

$$\boxed{T_1 = T_2}$$

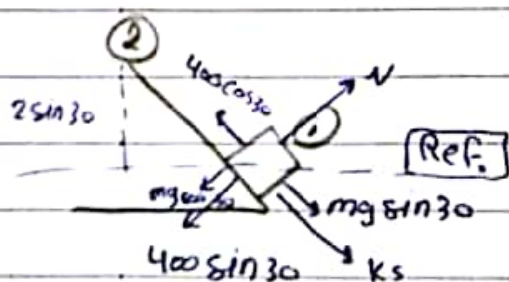
Kinetic Energy $\begin{cases} \text{is} \\ +ve \end{cases}$

$$\Sigma f \neq 0 \rightarrow \boxed{\text{work} = \checkmark} \quad T_2 = T_1 + \text{work}$$

Ex 8. 14.1



$m = 10 \text{ kg}$, $k = 30 \frac{\text{N}}{\text{m}}$, spring initial stretched by 0.5 m
Find $u_{1 \rightarrow 2} \text{ total}$



$$1) u_{1 \rightarrow 2} = -mg(y_2 - y_1) = -mg(2 \sin 30 - 0)$$

g

$$= -mg \sin 30 (2)$$

comp. to incl. initial

$$2) u_{1 \rightarrow 2} = 400 \cos 30 (2) = \checkmark$$

$$3) u_{1 \rightarrow 2) s} = -\frac{K}{2} [s_2^2 - s_1^2]$$

$$* s_2 = 0.5 + 2 = 2.5 \text{ , } s_1 = 0.5$$

$$u_{1 \rightarrow 2} = -\frac{30}{2} [2.5^2 - 0.5^2]$$

$$u_{1 \rightarrow 2) \text{ total}} = u_{1 \rightarrow 2) g} + u_{1 \rightarrow 2) s} + u_{1 \rightarrow 2) \text{ pex}}$$

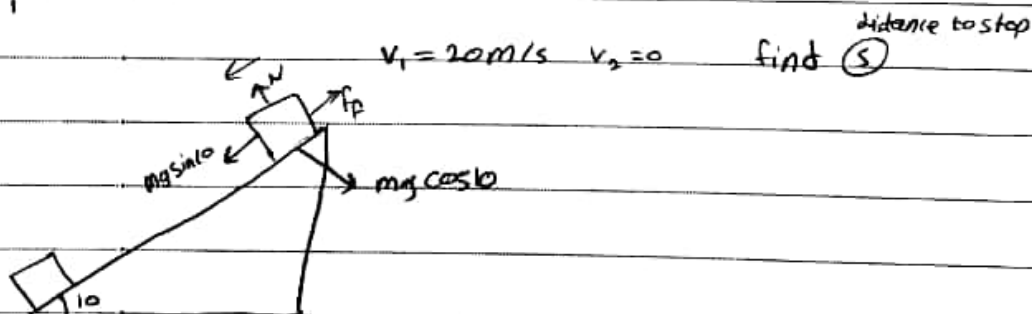
Friction's force $\otimes u_{1 \rightarrow 2) f_p} = -\mu_K N(2)$

$$N = mg \cos 30 + 400 \sin 30$$

$\boxed{\frac{V}{F}}$ cell $T_1 = 0$, $T_2 = \frac{m}{2} v_2^2$

$T_{1,2} \text{ (up)} \text{ } \&\& \text{ } T_1 + u_{1 \rightarrow 2) \text{ total}} = T_2$

Exp: 14-2



$$N = mg \cos 10$$

$$1) u_{1 \rightarrow 2) g} = -mg \sin 10 (s)$$

$$T_1 = \frac{m}{2} v_1^2$$

$$u_{1 \rightarrow 2) f_p} = -\mu_K mg \cos 10 (s)$$

$$T_2 = 0$$

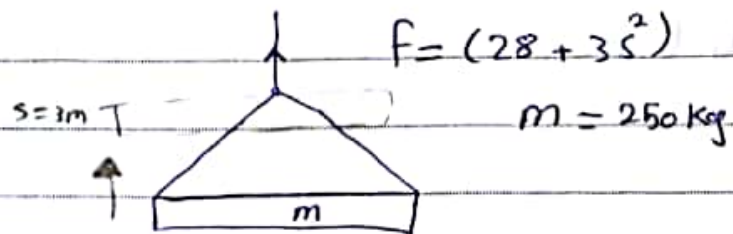
$$T_1 + u_{1 \rightarrow 2) \text{ total}} = 0 \quad \text{solve for } \textcircled{S}$$

$\&\& \&$

$$\Sigma F_s = m a_s \quad a = v$$

$$v^2 = v_0^2 + 2 a_c (s - s_0)$$

Exp: 14.3



① $V = ??$ $s = 3m$

② $t = ??$

$$T_1 = 0 \quad 0 + \int_0^s (28 + 3s^2) ds - mgs = \frac{1}{2} m v^2$$

$$v^2 = f(s), v = \sqrt{f(s)} \rightarrow v(3) = \checkmark$$

$$\Sigma F = ma \quad \frac{v dv}{ds} \quad 91$$

$$v = \checkmark$$

$$\frac{ds}{dt} = v$$

$$\int ds = \int dt$$

$$\textcircled{b} \checkmark$$

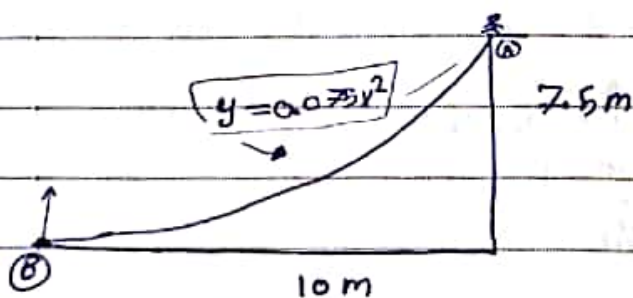
$$\int v dv = \int a ds \quad \text{find } a$$

③ $v = \frac{ds}{dt} = \sqrt{f(s)}$

$$\int_0^3 \frac{ds}{\sqrt{f(s)}} = \int_0^t dt$$

$$t = \checkmark$$

Exp: 14.5



$V_B = ??$ $N_B = ??$

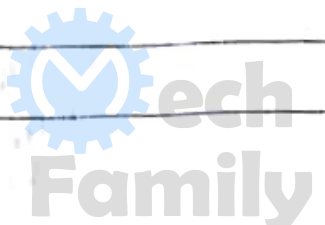
$$T_1 = 0 \quad T_2 = \frac{1}{2} m v_B^2$$

$$0 - mg(7.5) = \frac{1}{2} m v_B^2 \quad v_B = \checkmark$$

$$\Sigma F = ma_n \rightarrow \frac{v^2}{r}$$

$$N - \checkmark a_n = \checkmark m v^2 \quad \checkmark$$

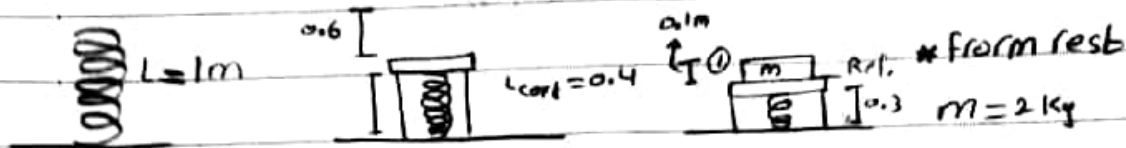
$$P_B = \frac{|1 + y'^2|^{\frac{3}{2}}}{141}$$



(*)

Exp 14.4

② → h_{max}



Find h_{max} from measured from the ground

$K=200 \text{ N/m}$

$$h = (h_{max} + 0.3) \quad \text{ground Ref.}$$

$$T_1 = 0 \quad T_2 = 0$$

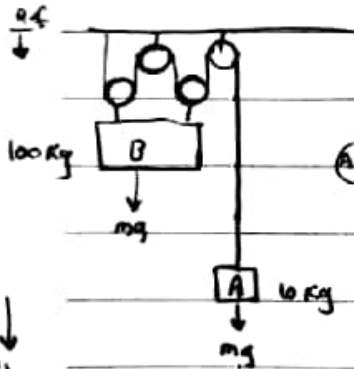
rest h_{max}

$$0 + u_{1 \rightarrow 2} g + u_{1 \rightarrow 2} s = 0$$

$$u_{1 \rightarrow 2} g = -mg [h_{max} - 0] \quad , \quad u_{1 \rightarrow 2} s = -\frac{K}{2} [0.6^2 - (0.6 + 0.1)^2]$$

solve for h_{max}

Exp 14.6



distance travel B start from rest when it $v_B = 2 \text{ m/s}$

② * single system $\uparrow T \quad \uparrow N_1 \quad \uparrow A_2$
~~reactive~~ reactive force at the support

$$\sum T_1 + \sum u_{1 \rightarrow 2} = \sum T_2$$

$$(0+0) + (-m_B g (\Delta y_B) - m_A g (\Delta y_A)) = \left(\frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2 \right)$$

$$* \quad y_A + 4y_B = L$$

$$y_{A_i} + 4y_{B_i} = L$$

$$(y_{A_i} - y_{A_f}) + 4(y_{B_i} - y_{B_f}) = 0$$

$$\Delta y_A + 4\Delta y_B = 0$$

$$\Delta y_A = -4\Delta y_B$$

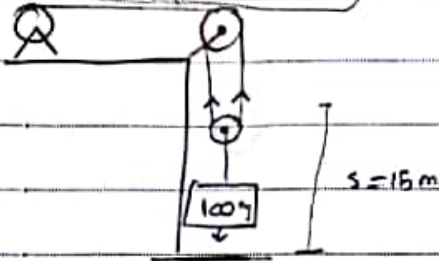
$$v_A = -4v_B$$

f 14-3

motor $F = (600 + 2s^2) \text{ N}$

$V = ??$ when $s = 15 \text{ m}$

start from rest



$T_1 = 0$

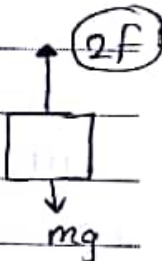
$T_2 = \frac{1}{2} m v^2$

$T = 2f$

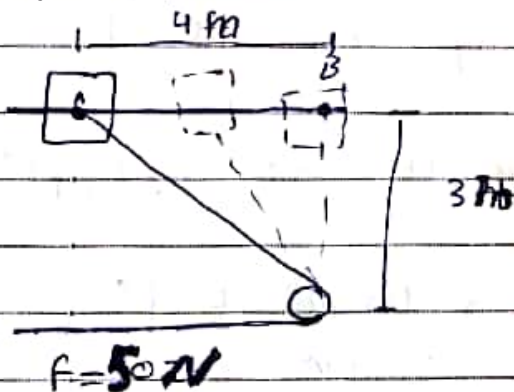
$u_{1 \rightarrow 2} \int_g = -mg(15)$

$u_{1 \rightarrow 2} \int_F = \int_0^{15} 2(600 + 2s^2) ds$

$T_1 + \Sigma K_{1 \rightarrow 2} = T_2 \quad V = \checkmark$



f. (14.6)



$u_{A \rightarrow B} \int_g = 0$

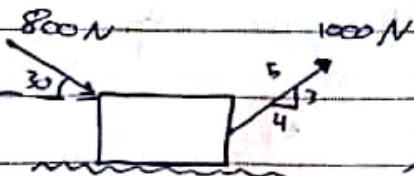
$u_{A \rightarrow B} \int_F = 50 \Delta L$

$L_A = \sqrt{3^2 + 4^2} = 5 \quad L_B = 3$

$\Delta L = 2$

$50 \Delta L = \frac{1}{2} m v^2 \quad \checkmark$

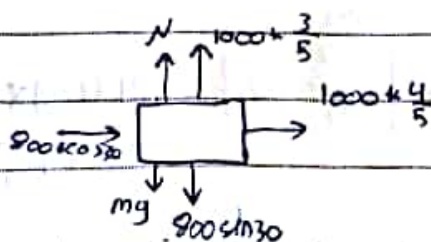
p. 14.3



originally at rest

distance = ?? at $v = 6 \text{ m/s}$

$\mu_k = 0.2$



$N + 1000 \times \frac{3}{5} - 800 \sin 30 - mg = 0$

$N = \checkmark$

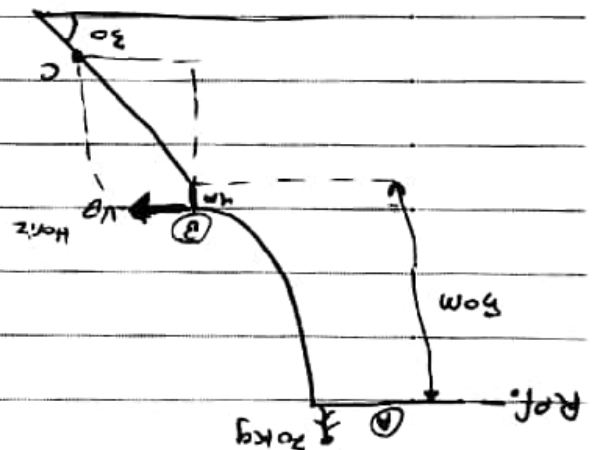
$$T_1 = 0 \quad T_2 = \frac{1}{2} m (6)^2$$

$$U_{1 \rightarrow 2} = \left(-\frac{1}{2} k N^2 \right) + 1000 \times \frac{9}{4} + 800 (\cos 30^\circ) \Delta x$$

$$T_1 + U_{1 \rightarrow 2} = T_2$$

0 +

14-12



103

$$T_A + U_{A \rightarrow B} = T_B = \frac{1}{2} m v_B^2$$

$$U_{A \rightarrow B} = -mg \Delta y = -4600$$

$$v_B = 17.3 \text{ m/s}$$

$$\Delta y = 4 + 5 \sin 30^\circ = 6.5 \text{ m}$$

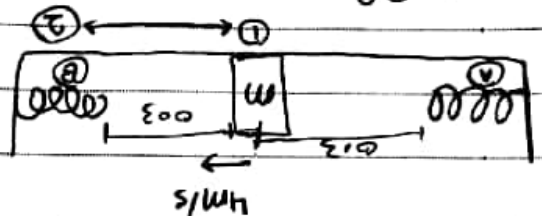
$$\Delta x = 5 \cos 30^\circ = 4.33 \text{ m}$$

$$T = \frac{5 \cos 30^\circ}{v_B} \rightarrow 4 + 5 \sin 30^\circ = \frac{1}{2} g \left[\frac{5 \cos 30^\circ}{v_B} \right]^2$$

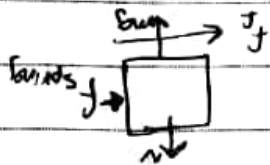
$$s = +ve \quad v =$$

$$s = -ve \quad x$$

14-25



$$x_{\text{bot}} = 0.72$$

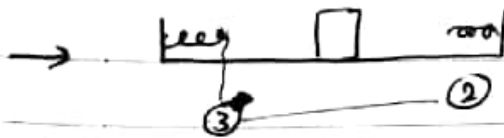


$$k_A = 100 \text{ N/m} \quad k_B = 300 \text{ N/m} \quad \mu_k = 0.4 \quad m = 12 \text{ kg}$$

$$T_{\text{max}} = \frac{1}{2} (12) (4)^2 = 96 \text{ J}$$

$$= 0 \quad (0.3 + s_2)$$

$$96 - \frac{1}{2} (300) s_2^2 - 0.4 (12g) x = 0$$



2 scenario - { يتوقف قبل
9 ن يصل
يصل الزنبر
بمنطقة

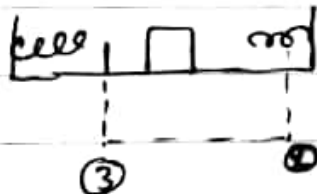
$$T_2 + u_{2 \rightarrow 3} = T_3^*$$

0 +

$$u_{2 \rightarrow 3})_{f_f} = -0.4(12g)(0.6 + 0.598) = -56.4 \text{ J}$$

$$u_{2 \rightarrow 3})_s = -\frac{300}{2} [0 - 0.598^2] = 53.64 \text{ J}$$

$$T_3^* = -ve \text{ } \circ \circ \text{ بقعة قبل أن } \circ \circ \text{ يصل } (3)$$



$$T_2 = 0 \quad T_3 = 0$$

$$u_{2 \rightarrow 3})_{f_f} = -0.4(12g)(0.598 + x)$$

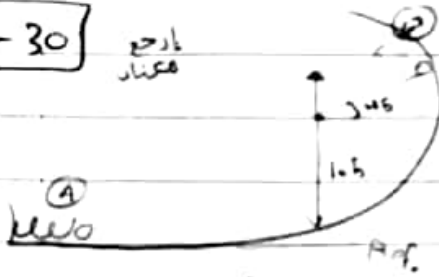
$$u_{2 \rightarrow 3})_s = 53.64 \text{ J}$$

$$53.64 - 0.4(12g)(0.598 + x) = 0 \quad x = 0.54$$

$$x_{total} = 0.3 + 0.598 + 0.598 + 0.541 = \checkmark$$

14-30

راجع
مکان



$$m = 0.5 \text{ kg}$$

$$K = 500 \text{ N/m}$$

$$\text{initial} = 0.8$$

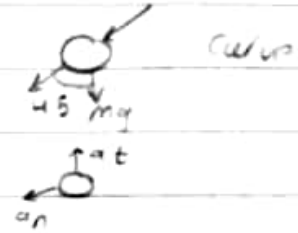
$$v_B = ??$$

$$T_1 = 0 \quad T_2 = \frac{mv^2}{r}$$

$$U_{1 \rightarrow 2} = -mg[1.5 + 1.5 \sin 45 - 0]$$

$$U_{1 \rightarrow 2} = -\frac{K}{2} [0.08^2 - (0.08 + s)^2]$$

$$mg \cos 45 = \frac{mv_B^2}{1.5} \quad v_B = \checkmark$$

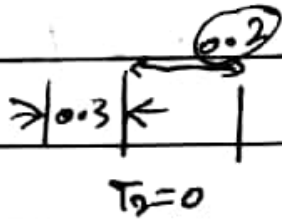


14-21



$$K_A = 5 \text{ kN/m}$$

$$K_B = ??$$



$$u_{1 \rightarrow 2} = -\frac{K}{\frac{A}{2}} [0.2^2 - 0]$$

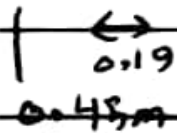
$$= -\frac{K_B}{2} [0.15^2]$$

$$1.4 \text{ kN}$$

$$T_1 = \frac{1800 (0.5)}{2}$$

(cc)

B



power and Efficiency :-

$$\text{power} = \frac{d(u)}{dt} = \frac{du}{dt} = \left\{ u = \int \vec{F} \cdot d\vec{r} \right\}$$

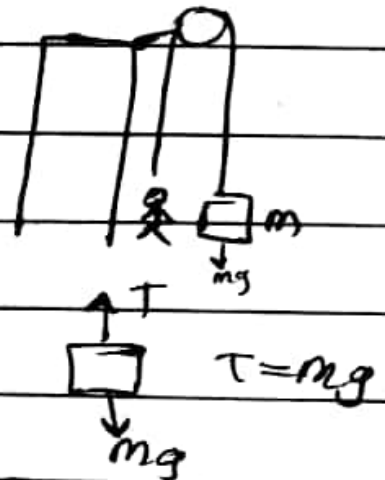
$$P = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v} = Fv \quad \text{watt}$$

ideal case

$$\text{power} = mg(v)$$

actual

$$\text{power} = F(v^*)$$



$$\text{Efficiency } (\epsilon) = \frac{\text{power output}}{\text{power input}} * 100\%$$

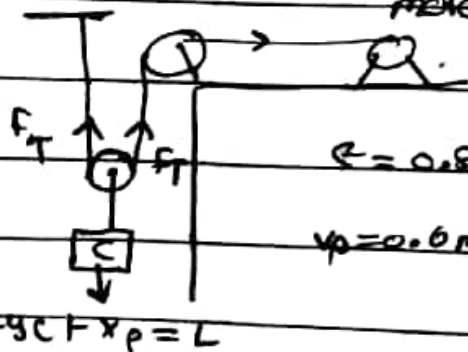
$$\epsilon < 100\%$$

Exp (14-8) مراجع الكتاب

$$P_{\text{input}} = ??$$

$$a_p = 1.2 \text{ m/s}^2 \quad \text{meter}$$

$$\text{power out} = F_T (0.6)$$



$$\epsilon = 0.85 = \frac{\text{power out}}{\text{power in}}$$

$$v_p = 0.6 \text{ m/s}$$

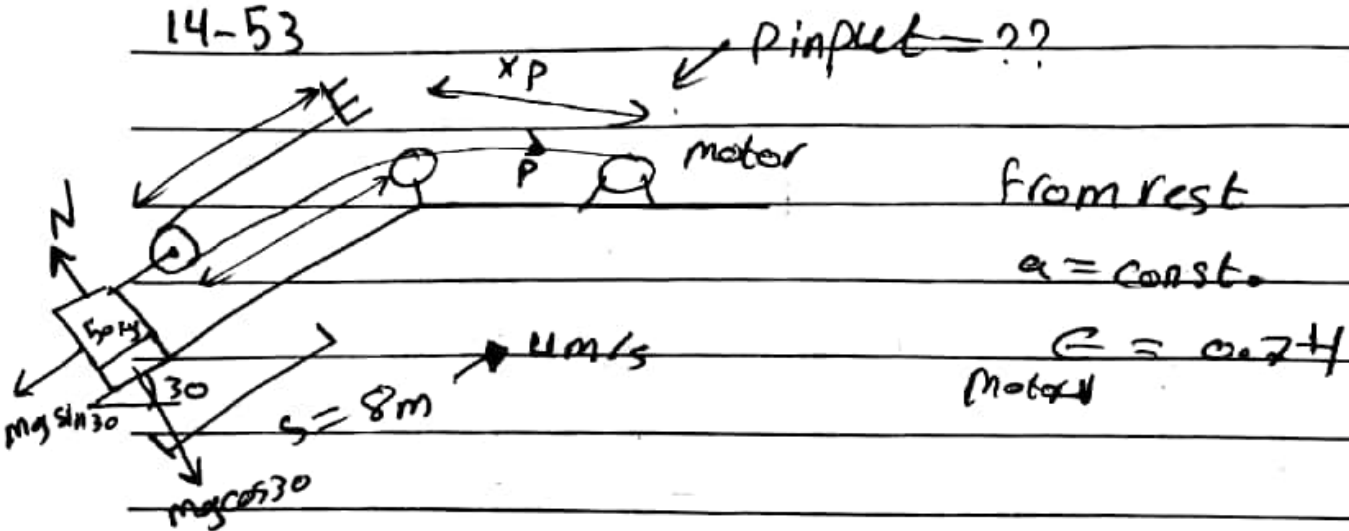
$$2F_T - 35g = 35a_c$$

$$a_c = \frac{1.2}{2} = 0.6 \text{ m/s}^2$$

$$2a_c + a_p = 0 \quad |2a_c| = |a_p|$$

$$F_T \quad \checkmark$$

14-53



$$\epsilon = \frac{\text{Power output}}{\text{Power input}} \rightarrow = (F_T) v_p$$

$$2s_c + x_p = L$$

$$2v_c + v_p = 0 \quad \therefore v_p = |2v_c|$$

$$v_p = 2 \times 4 = 8$$

$$2F_T - mg \sin 30 = ma$$

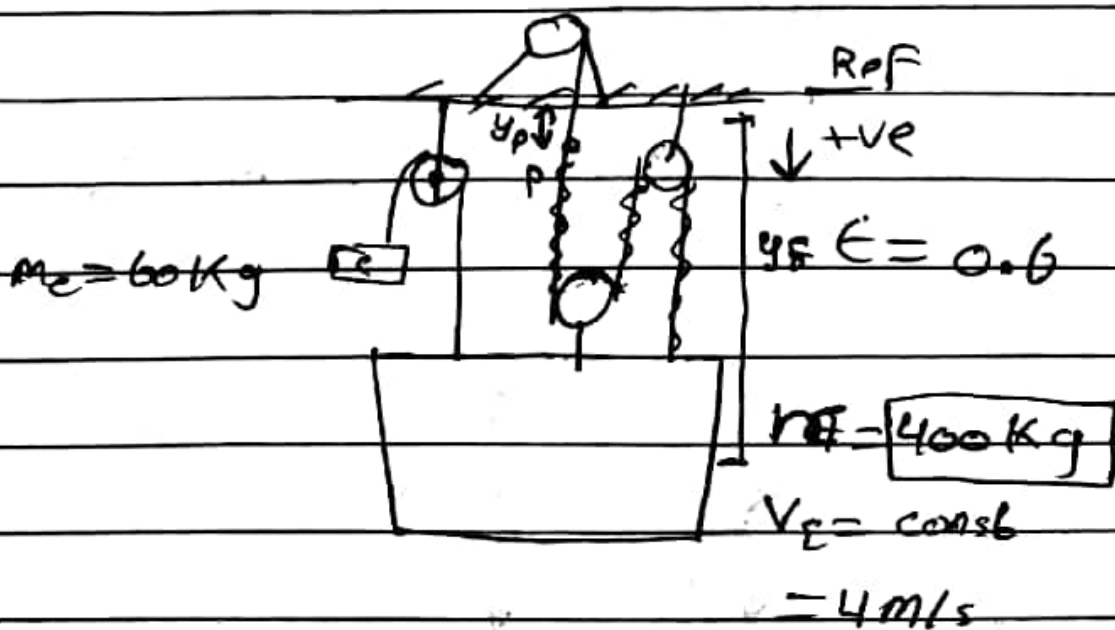
$$F_T = \frac{1}{2} [mg \sin 30 + ma]$$

$$v^2 = v_0^2 + 2as$$

$$4^2 = 2a(8)$$

$$a = 1\text{m/s}^2$$

14_57

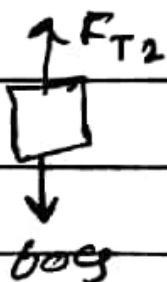


$$\sum F = 0$$

$$3F_{T1} + F_{T2} - m_1g = 0$$

$$F_{T1} = \frac{1}{3} [400g - 60g] = \frac{1}{3} 340g = 113.33g$$

Comb. speed
 constant speed



$$F_{T2} = 60g$$

$$a = \frac{\text{Product}}{\text{Pin product}} \rightarrow F_{T1}(v_p)$$

$$2y_1 + (y_1 - y_2) = L$$

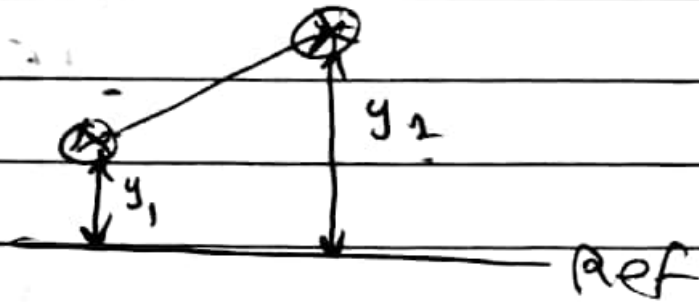
$$2v_1 + v_1 - v_2 = 0$$

$$v_p = 3v_1 = 3 \times 4 = 12 \text{ m/s}$$

⊛ conservative force

⊛ non-conservative force

for con $u_{1 \rightarrow 2} g = -mg(\Delta y) = mg(y_2 - y_1)$
as Δy is path



$$u_{1 \rightarrow 2} s = -\frac{k}{2} [s_2^2 - s_1^2]$$

non

$$u_{1 \rightarrow 2} p_{ext} = \int p_{ext} dr$$

$$u_{1 \rightarrow 2} f_f = \int \mu_c N \cdot dr$$

negative work
positive potential

$$T_1 + \cancel{u_{1 \rightarrow 2} f_f} + \cancel{u_{1 \rightarrow 2} p_{ext}} + u_{1 \rightarrow 2} g = T_2$$

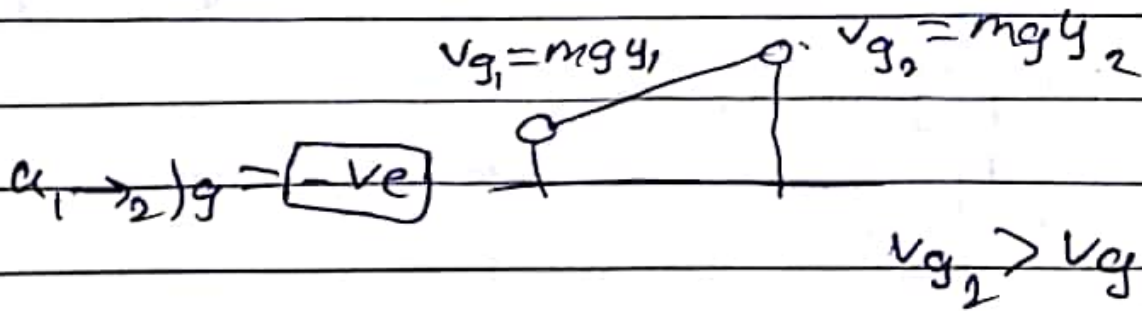
~~for~~ $u_{1 \rightarrow 2} s$

$$V_g = mgy \quad V_s = \frac{k}{2} s^2$$

conservation
of Energy
principle

$$T_1 + V_{g1} + V_{s1} = T_2 + V_{g2} + V_{s2}$$

الفرق معاً نوز الأول



f 18 12

Exp 14-10

راجع الكتاب
نوصف

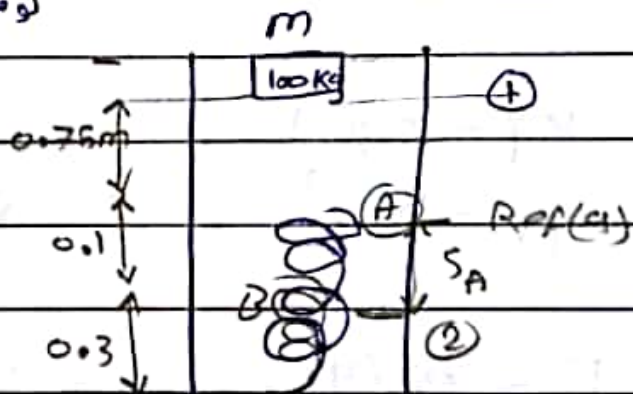
14-11

$$K_A = 12 \text{ kN/m}$$

$$K_B = 15 \text{ kN/m}$$

$$T_1 = 0, T_2 = 0$$

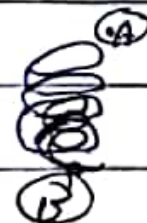
$$\bar{V}_{S,A} = 0, \bar{V}_{S,B} = 0$$



$$V_{S2A} = \frac{K_A}{2} S_A^2, V_{S2B} = \frac{K_B}{2} (S_A - 0.1)^2$$

$$V_{g1} = mg[0.75 + S_A]$$

$$V_{g2} = 0$$

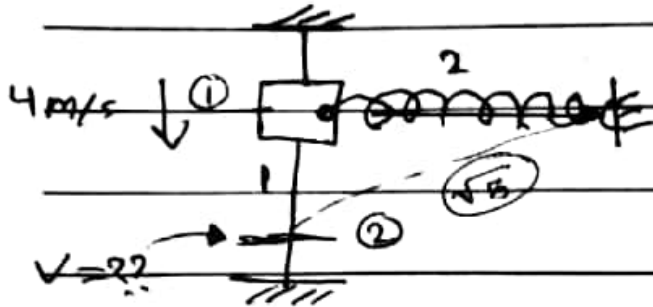


$$a S_A^2 + (b) S_A + c = 0$$

$$S_A = \sqrt{m}$$

F 14-15, 2m

$$K = 30 \frac{N}{m}$$



free length = 1m

at $s=1$

$$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$$

$$V_{1g} = mg(1)$$

$$V_{1s} = \frac{K}{2} s_1^2, \quad V_{2s} = \frac{K}{2} s_2^2$$

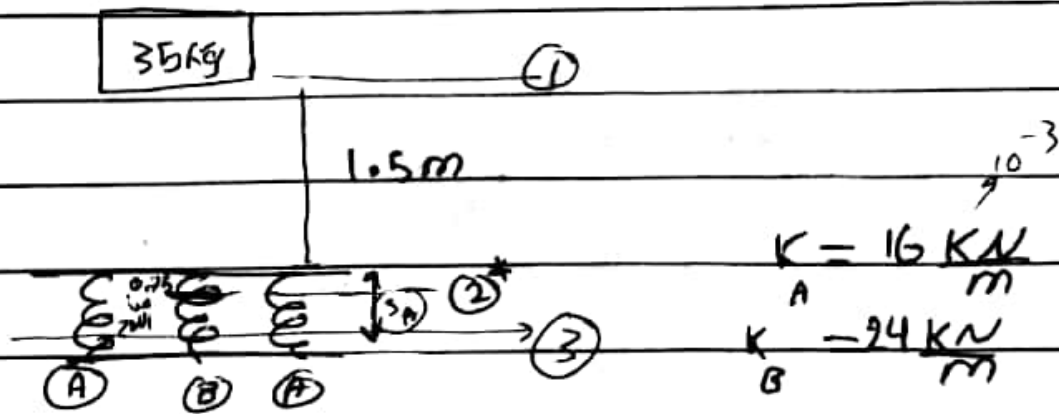
$$s_1 = 2 - l_0 = 1m$$

$$s_2 = 1m - 1$$

$$T_1 = \frac{m}{2} (4)^2, \quad T_2 = \frac{m}{2} (v_2)^2$$

$$v_2 = \sqrt{m/s}$$

F 14-17



$$T_1 = 0$$

$$V_{1g} = 35g(1.5 + 0.075) = 540.77$$

$$V_{1s} = 0$$

$$V_{2g}^* = 0$$

$$V_{2s}^* = (K_A) * 2 * (0.075)^2 = 90 \text{ J}$$

$$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$$

$$T_2 = 450 \text{ J} \quad (+ve) \quad \delta^* C_2$$

state 2 is the principle of virtual work is applied to the system in state 2 and state 3

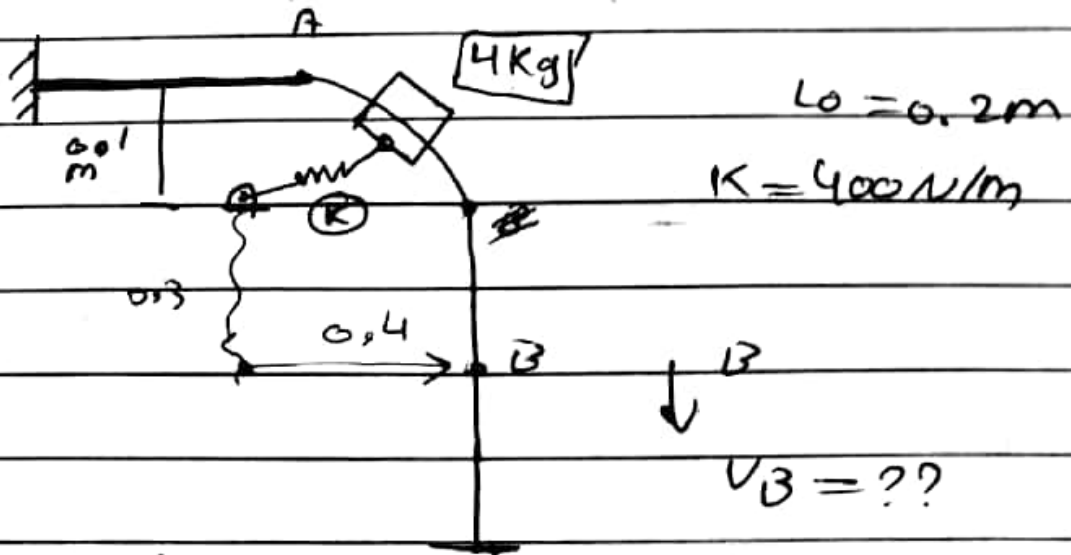
$$T_2^* + V_{2g}^* + V_{2s}^* = T_3 + V_{3g} + V_{3s}$$

$$450 + 35g s_A + 90 = 0 + 0 + 2 \left(\frac{K_A}{2} s_A^2 \right) + \frac{K_B}{2} (s_A - 0.075)^2$$

solve for $s_A = 1.1 \text{ m}$

$$s_B = s_A - 0.075 \text{ m}$$

F14-18



$$T_A = \frac{4}{2} (2)^2$$

$$T_B = \frac{4}{2} (V_B)^2$$

$$V_{As} = \frac{400}{2} [0.2 - 0.1]^2$$

$$V_{Bs} = \frac{400}{2} [0.5 - 0.2]^2$$

$$V_{Ag} = 4g(0.4)$$

$$V_{Bg} = 0$$

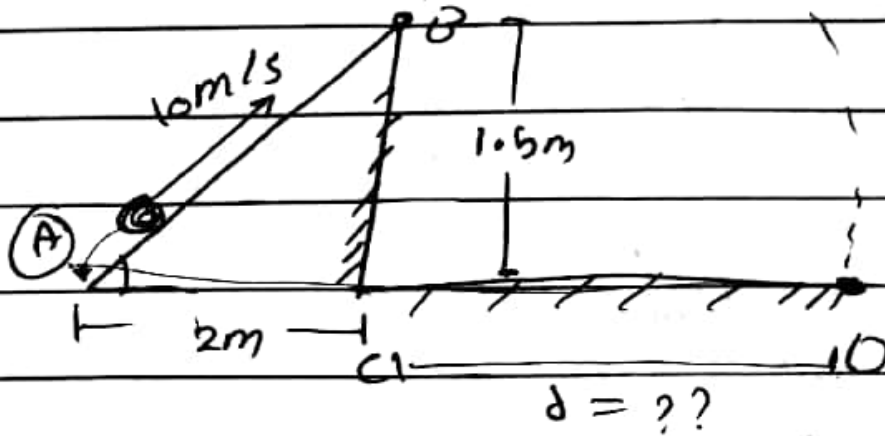
66 67

$$V_B = \text{m/s}$$

72

14-75

$m = 2 \text{ kg}$



$v_D = ??$

$v_D = 10 \text{ m/s}$

D, A is Ref is

$T = T$

$T_A = \frac{1}{2} (10)^2 = 100$ $V = V$

$T_B = \frac{1}{2} (v_B)^2 = v_B$

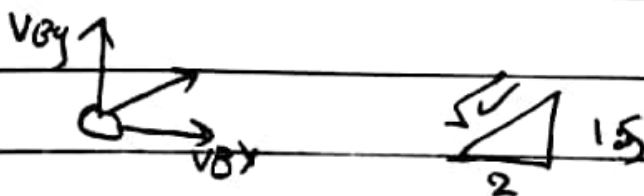
$v_{Ag} = 0$

$v_{Bg} = 2g(1.5) = 3g$

76 77
88 79

Ex 14-4

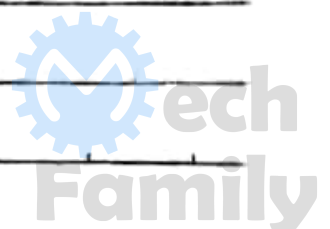
$v_B = \sqrt{2g(1.5)} = 8.4 \text{ m/s}$



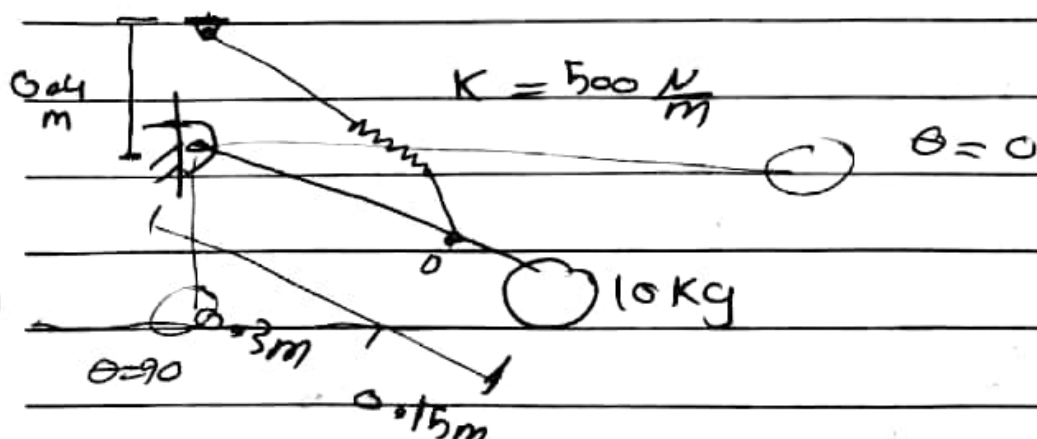
$1.5 = 0 + 8.4(\sin \theta) - \frac{g}{2} t^2$

$t = \sqrt{\dots}$

$d = 8.4 \cos \theta (t) = \dots$



14-96



at $\theta = 0$ start from rest

Tension in the spring = 100 N

$v = ??$ at $\theta = 90^\circ$

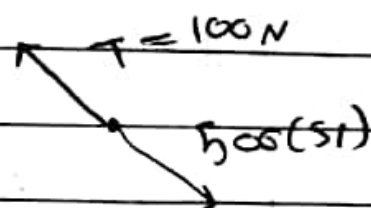
$$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$$

$$V_{1g} = 10g(0.3 + 0.15) \checkmark$$

$$V_{2g} = 0 \checkmark$$

$$V_{1s} = \frac{500}{2} s_1^2$$

$$V_{2s} = \frac{500}{2} s_2^2$$



$$100 - 500 s_1 = 0$$

$$s_1 = 0.2 \text{ m}$$

$$\text{at } \theta = 0 \rightarrow L = 0.5 \text{ m}$$

$$s_1 = L - L_0 ; L_0 = L - s_1 = 0.5 - 0.2$$

$$\text{free length} = 0.3 \text{ m}$$

can stretch

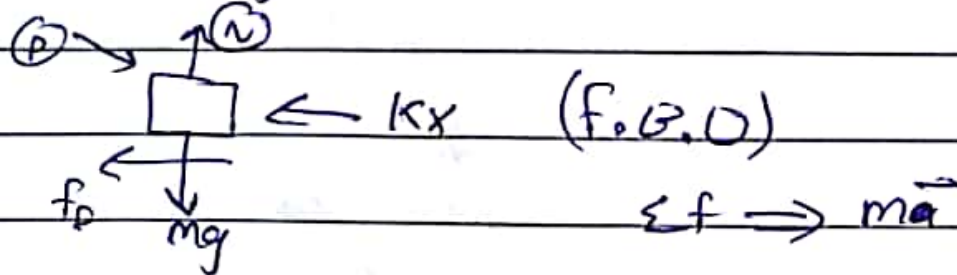
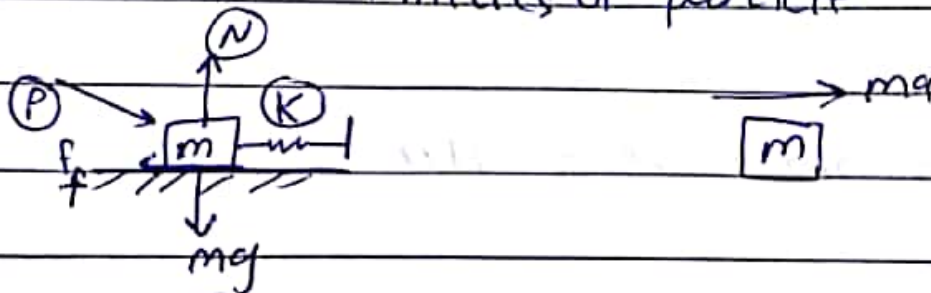
at $\theta = 90^\circ$

$$L_2 = 0.4 + 0.3$$

$$s_2 = L_2 - l_0 = 0.7 - 0.3 = 0.4 \text{ m}$$

ch:15 "Impulse and Momentum principle"

Kinetics of particle



$$\int_{t_1}^{t_2} \sum \vec{F} \cdot dt = \int_{v_1}^{v_2} m \frac{d\vec{v}}{dt} dt$$

$$= m[v_2 - v_1]$$

$$m\vec{v}_1 + \int_{t_1}^{t_2} \sum \vec{F} \cdot dt = m\vec{v}_2$$

→ Impulse

* Momentum ; $\vec{L} = m\vec{v}$

* Impulse ; $I = \int_{t_1}^{t_2} \sum \vec{F} \cdot dt$

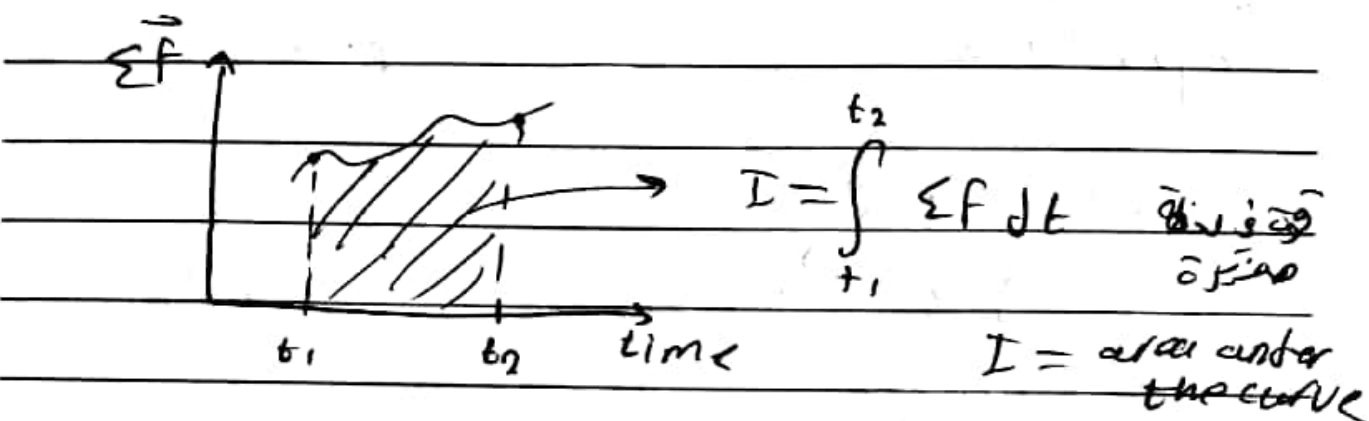
ch 13 $\left[\vec{L}_1 + \vec{I}_{1 \rightarrow 2} = \vec{L}_2 \right]$ vector $(\sum \vec{F})$, v, t

ch 14 $\left[T_1 + U_{1 \rightarrow 2} = T_2 \right] \rightarrow \left(\sum f \right) v, d$ scalar $\left[\sum \vec{F} = m\vec{a} \right]$ - ch 13

↑ $\left[\sum \vec{F} = m\vec{a} \right]$ vector

$$m v_{1x} + \int_{t_1}^{t_2} \Sigma F_x dt = m v_{2x}$$

$$m v_{1y} + \int_{t_1}^{t_2} \Sigma F_y dt = m v_{2y}$$



$$\Sigma \vec{F} = m\vec{a} = \frac{d}{dt} (m\vec{v}) = \frac{d}{dt} (\vec{I})$$

$$\int \Sigma F dt = \int d(mv)$$

Internal force

sys. of particle

(1st) $m_1 v_{1i} + I_{1 \rightarrow 2} = m_1 v_{1f}$

2nd $m_2 v_{2i} + I_{2 \rightarrow 1} = m_2 v_{2f}$

...

nth $m_n v_{ni} + I_{n \rightarrow 1} = m_n v_{nf}$

$\sum_{i=1}^n m_i v_{i1} + I_{1 \rightarrow 2} = \sum_{i=1}^n m_i v_{i2}$

Internal force only

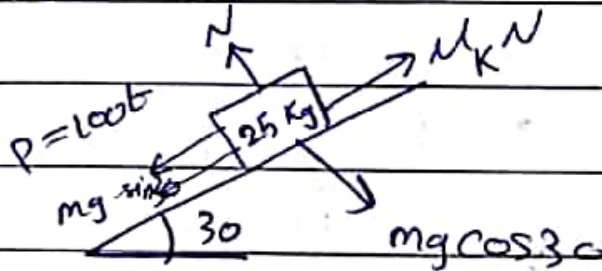
if $(F_{ext} = 0)$ Impulse $_{1 \rightarrow 2} = 0$

$$\sum_{i=1}^n m_i v_{i1} = \sum_{i=1}^n m_i v_{i2}$$

Conservation of Linear Momentum

Impulse $\rightarrow$ function of time

Exp: 15-2



$\rightarrow$ function of time
[] correct

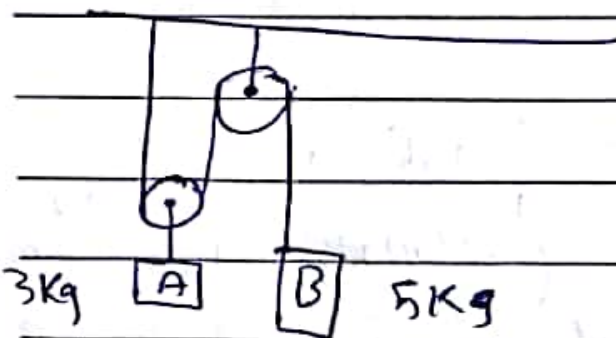
$$\mu_k = 0.3$$

$$v_1 = 1 \text{ m/s}$$

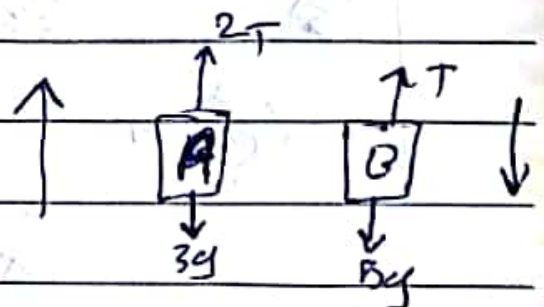
$$25(1) + \int_0^2 [1000t + 25g \sin 30 - 3(25)g \cos 30] dt = 25v_2$$

$$v_2 = \sqrt{\quad} \text{ m/s}$$

Exp



$v_B = ??$ after 6s



* particle A

$$0 + \int_0^6 (2T - 3g) dt = m_A v_A \quad \text{--- (1)}$$

particle B $0 + \int_0^6 (5g - T) dt = m_B v_B \quad \text{--- (2)}$

$$\textcircled{1} \rightarrow \uparrow (2T - 3g) 6 = 3V_A$$

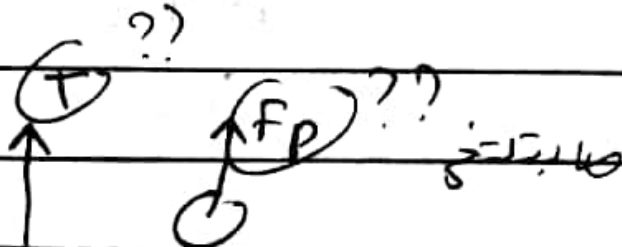
$$\textcircled{2} \rightarrow \downarrow 2 \times [5g - T] \times 6 = 5V_B$$

$$-18g + 60g = 3V_A + 10V_B$$

$$2y_A + y_B = L$$

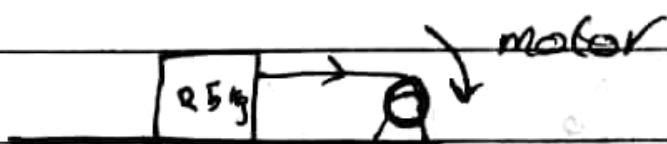
$$\textcircled{\text{Ave}} -2V_A + V_B = 0$$

$$2V_A = V_B$$



F15-3

$$F = 20t^2$$



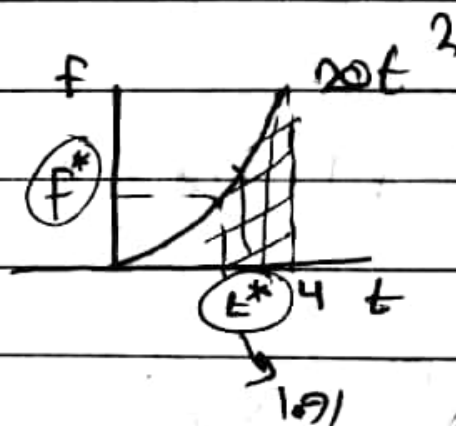
$$\mu_s = 0.3$$

$$\mu_k = 0.25$$

$$v = ?? \text{ at } t = 4s$$

soln

1) check for motion



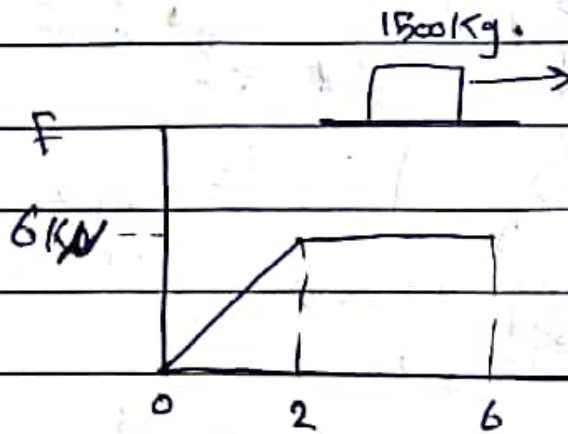
$$F^* = 20t^{*2} \text{ ? } \mu_s N$$

$$20t^{*2} = 0.3(25g)$$

$$t^* = \sqrt{\frac{0.3(25)g}{20}} \quad t^* = 1.91 \text{ s}$$

$$0 + \int_{1.91}^4 (20t^2 - \mu_k(Mg)) dt = \cancel{25} \text{ (V)} \quad \text{25 V}$$

F 15-4



$$v = ?? \text{ at } t = 6s$$

state 1 $\rightarrow$ at rest

State 2 $\rightarrow$ at $t = 6s$

$$0 + I_{1 \rightarrow 2} = mv_2$$

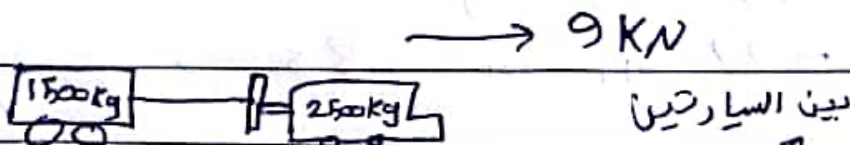
$$\text{KN} \quad 10^3 \text{ N}$$

$$I_{1 \rightarrow 2} = \frac{1}{2}(2)(6 \times 10^3) + 4(6 \times 10^3)$$

$= \text{N} \cdot \text{s}$

$$v_2 = 20 \text{ m/s}$$

F 15-5



$$v = ?? \text{ after } 20 \text{ s}$$

$$f_{\text{coupling}} = ??$$

$$I = \int f dt = f(\Delta t)$$

$$0 + \int 9 \times 10^3 \cdot 20 = (2500 + 1500) v \quad v = 45 \text{ m/s}$$

Subject _____ Day 15.6
 15.8
 15.14
 15.18

* $\Sigma (F_{coupling}) \Delta t = 1500(45)$

* reaction force

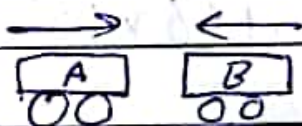
$F_{coupling} = \frac{1500 \times 45}{20}$

$= 3.375 \text{ KN}$

Diagram showing a block B with forces $F_{coupling}$ and f_T acting on it. A car is shown with forces f_T and f_s acting on it. A note indicates $f_T > f_s$ leads to $f_T < f_s$ Eq.

Diagram showing a block A with forces $F_{coupling}$ and 9×10^3 acting on it.

Exp: 15-4



$m_A = 15000 \text{ Kg}$

$m_B = 12000 \text{ Kg}$

$v_A = 1.5 \text{ m/s}$

$v_B = 0.75 \text{ m/s}$

→ collide and
 couple together

a) speed after coupling

b) $F_{average} = ??$ if $\Delta t = 0.8 \text{ s}$

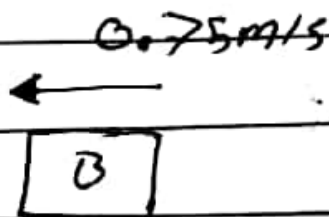
for th sy. [conservation of linear momentum]

+ve

$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = (m_A + m_B) \vec{v}_2$

$$15 \times 10^3 (1.5) - 12000 \times 0.75 = (27 \times 10^3) V_2$$

$$V_2 = 0.5 \text{ m/s}$$



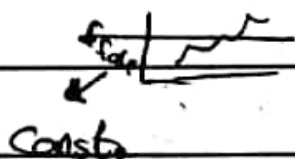
$$\xrightarrow{0.5}$$

$$m_B = \text{Impulse} + \boxed{B} \quad 12000$$

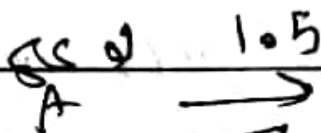
⊕ قوتنا

$$= 0.75 (12 \times 10^3) + f_{\text{average}} (0.8)$$

$$12 \times 10^3 (0.5)$$



$$f_{\text{ave}} = \checkmark \quad \checkmark$$



$$\boxed{A} + \text{Impulse} = \boxed{A}$$

$$15 \times 10^3 (1.5) - f_{\text{ave}} (0.8) = 15 \times 10^3 (0.5)$$

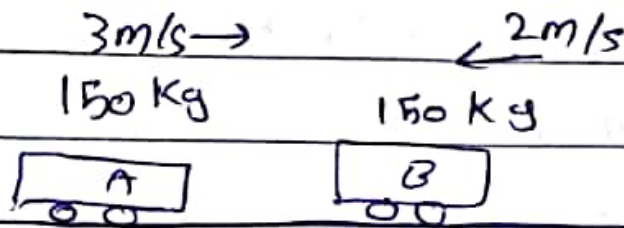
$$f_{\text{ave}} = \checkmark$$

⊗ Energy quantity

$\Delta T = \text{Energy lost during the Impact}$

$$\Delta T = 0.5 [12 \times 10^3 (0.75)^2 + 15 \times 10^3 (1.5)^2 - 27 \times 10^3$$

Exp 15-5



$$V_{A2} = V_{B2} \quad \text{No Energy lost ??}$$

No applied force $\rightarrow$ conservation of $\vec{L}$

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$

(conservation of momentum)

$$3 - 2 = v_{A2} + v_{B2}$$

$\rightarrow (+)$
 $v_{A2} \quad v_{B2}$

$$\frac{m}{2} [3^2 + 2^2] = \frac{m}{2} [v_{A2}^2 + v_{B2}^2] \quad \text{--- (2)}$$

$$1 = v_{A2} + v_{B2} \quad \text{--- (1)} \quad \rightarrow v_{A2} = -2 \text{ m/s} = 2 \text{ m/s} \leftarrow$$

$$13 = v_{A2}^2 + v_{B2}^2 \quad \text{--- (2)} \quad v_{B2} = 3 \text{ m/s} = 3 \text{ m/s} \rightarrow$$

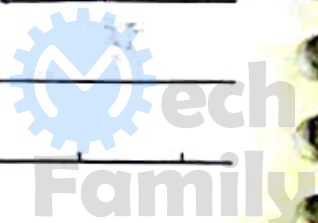
coefficient of (Restitution) e

Elastic $e = 1$

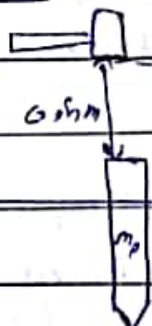
$$\text{Elastic } e = \frac{v_{A2} - v_{B2}}{v_{B1} - v_{A1}} \quad \text{--- (3)}$$

plastic $e = 0$

$$\text{plastic } e = \frac{v_{A2} - v_{B2}}{v_{B1} - v_{A1}}$$



Exp: 15-6

from rest  hammer (300 Kg)

0.5m

②

Ref (g)

 Sand

loose
no
resistance

pile

800Kg

* Spring and mg force are non-impulsive

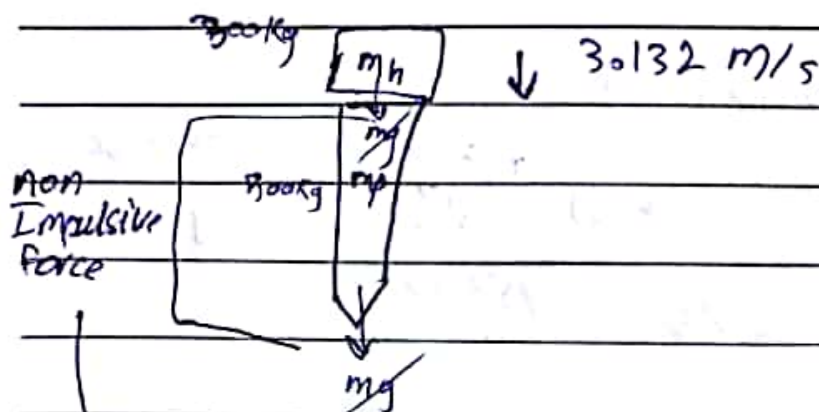
$$T_1 = 0, V_{1g} = 300g(0.5)$$

$$T_2 = ??, V_{2g} = 0$$

$$T_1 + V_{1g} = T_2 + V_{2g}$$

$$150g = \frac{300}{2} V^2 \quad V_2 = m/s \downarrow$$

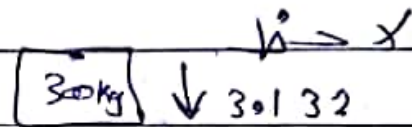
50



→ conservation of linear Momentum of the system

$$300(3.132) + 800(0) = (300 + 800) V_3$$

$$V_3 = \frac{300(3.132)}{1100} = 0.854 m/s \downarrow$$



Impulse
from the
pile to
the
hammer

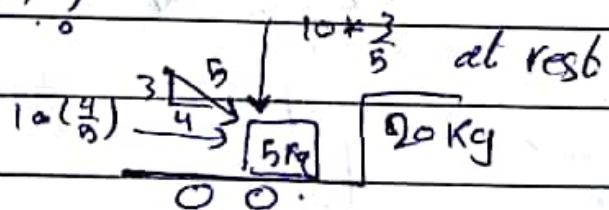
$$300(3.132) + \text{Impulse} = 300(0.854)$$

300 kg $\downarrow$ 0.854

$$\text{Impulse} = -683 \text{ N/s}$$

F 15-8

→ final common velo. ?



* direction

$$20(0) + 5(10)\left(\frac{4}{5}\right) = (20 + 5)V$$

$$\frac{40}{25} = V \quad V = 1.6 \text{ m/s}$$

Impulse

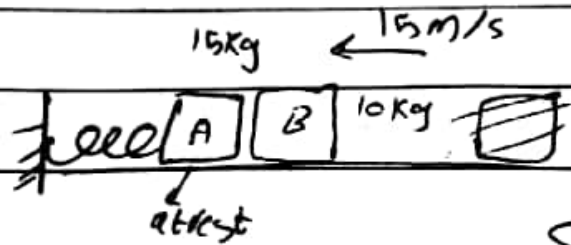
$$6(5) + \text{Impulse} = 0$$

cart

cart

Impulse

f 15-11

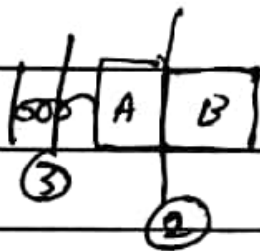


they couple together

conservation of linear momentum

$$15(0) + 10(15) = (15+10) V_2$$

$$V_2 = 6 \text{ m/s} \leftarrow$$



→ conservation of Energy

$$\frac{1}{2} (15+10) (6)^2 + 0 + 0 = 0 + 0 + \frac{1}{2} (10 \times 10^3) x^2$$

kinetic

spring

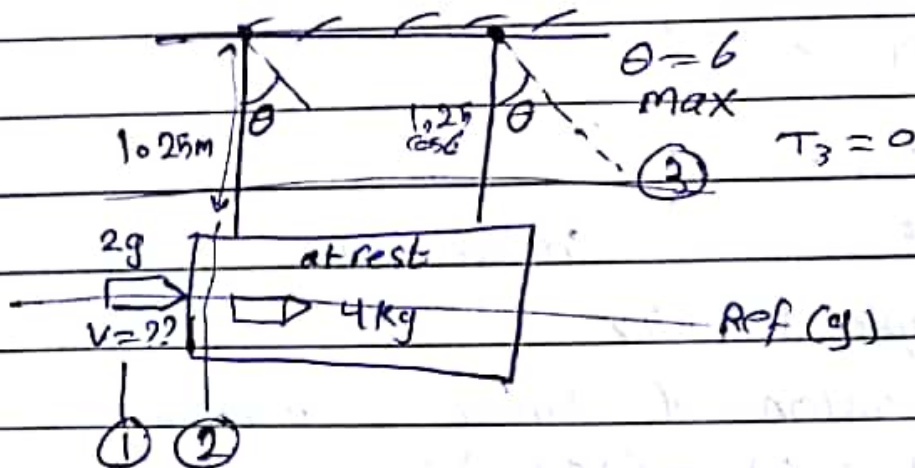
$$x = m$$

Ex 15-43

15-49

15-4

* 15-39



$$T_2 = \frac{1}{2} (4 + 2 \times 10^{-3}) v_2^2$$

$$v_{2g} = 0$$

$$v_{3g} = (4 + 2 \times 10^{-3}) g * (1.25 - 1.25 \cos 6)$$

$$= v$$

$$T_2 + v_{2g} = T_3 + v_{3g}$$

$$v_2 = 0.36 \text{ m/s}$$

$$2 \times 10^{-3} (v_1) + 4(0) = (4 + 2 \times 10^{-3}) v_2$$

$$v_1 = 720 \text{ m/s (initial speed)}$$

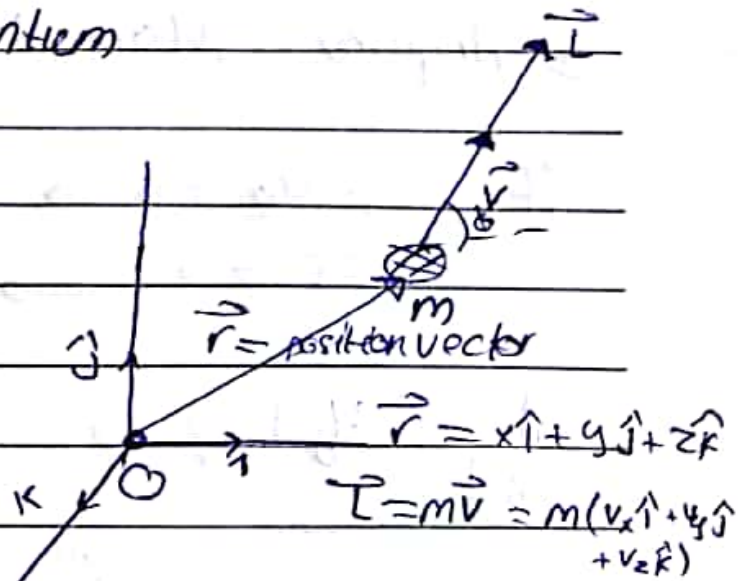
15-5 Angular Momentum

moment of ($\vec{L}$)

$$\vec{L} = m\vec{v}$$

$$\vec{H}_O = \vec{r} \times m\vec{v}$$

Angular Momentum



$$H = |\vec{r}| |m\vec{v}| \sin\phi \quad \text{or}$$

$\hat{i}$	$\hat{j}$	$\hat{k}$
x	y	z

$$\frac{d}{dt}(H_O) = \dot{H}_O = \vec{r} \times \frac{d}{dt}(m\vec{v})$$

$$+ \left| \frac{d\vec{r}}{dt} \times m\vec{v} \right|$$

$$\dot{H}_O = \vec{r} \times \frac{d}{dt}(m\vec{v})$$

Subject _____ Day _____ Date _____

Exp: 15-13

$$m = 1500 \text{ kg}$$

$$F = 150 t^2$$

$$v_2 = ?? \text{ at } t = 5 \text{ s}$$

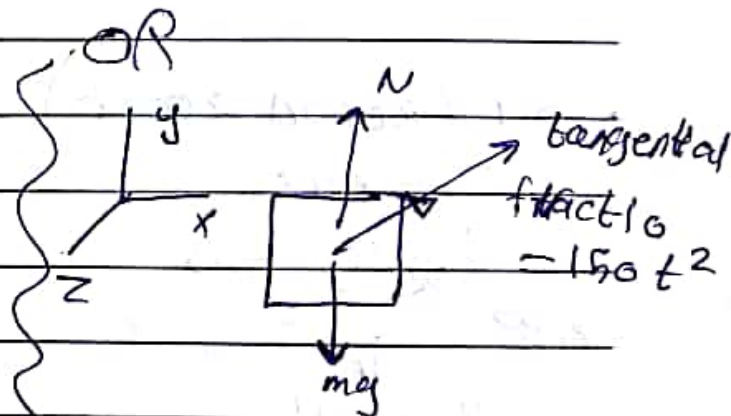
$$v_1 = 5 \text{ m/s}$$

soln:

$$\Sigma F = ma_t$$

$$150 t^2 = m \frac{dv}{dt}$$

$$\int_0^5 150 t^2 dt = \int_0^{v_2} m dv$$



$$\Sigma M_{Oy} = 100 (150 t^2)$$

$$\vec{H}_O = (\vec{r}_1 \times m \vec{v}_1)$$

const curve

$$H_{O,y} + \left(\int_0^5 \Sigma M_O dt \right)_y = H_{O,y}$$

$$100 \times 1500 \times 5 + \int_0^5 15000 t^2 dt = 100 \times 1500 \times v_2$$

$$v_2 = 10 \text{ m/s}$$

* Angular Momentum :

$$\text{if } \Sigma M_o = 0 \rightarrow \vec{H}_o = \text{constant}$$

$$\Sigma F = 0 \rightarrow \vec{I} = \text{const}$$

$$\frac{d}{dt} (H_o) \neq 0$$

Impulse

$$\text{* Linear } m \vec{v}_1 + \int \Sigma \vec{F} dt = m \vec{v}_2$$

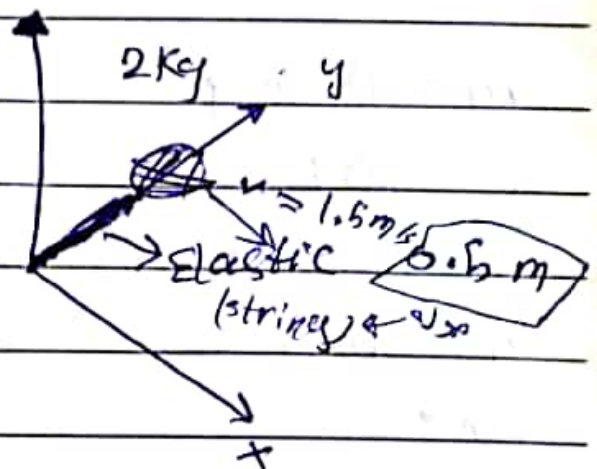
Angular Impulse

$$\text{* Angular } H_{o1} + \int \Sigma M_o dt = H_{o2}$$

$$\vec{H}_{o1} = \vec{H}_{o2}$$

Exp **15-15**

* rate at which the cord is being stretched



→ (rotation about z-axis)

$$k = 20 \text{ N/m}$$

$$L_{SM} = 0$$

$$H_{O_1} = H_{O_2}$$

$$\vec{r}_1 \times m_1 \vec{v}_1 = \vec{r}_2 \times m_2 \vec{v}_2$$



$$0.5 * 2 * 1.5 = (0.5 + 0.2) (2) (v_2)$$

$$v_{2L} = 1.07 \text{ m/s}$$

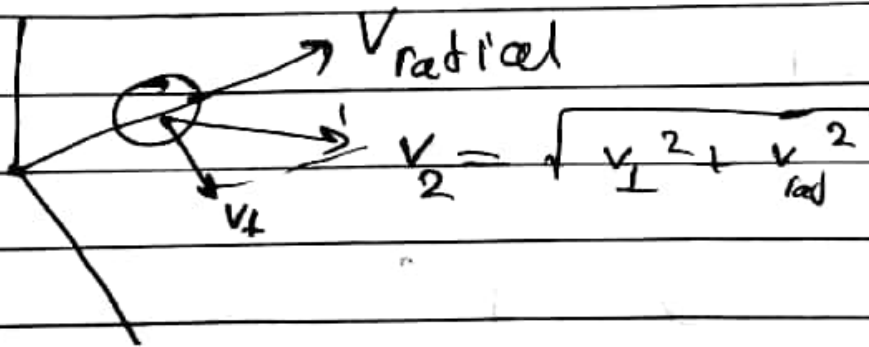
conservation of energy

$$T_1 + \cancel{v_1 s} + \cancel{v_1 g} = T_2 + \cancel{v_2 s} + \cancel{v_2 g}$$

$$\frac{2}{2} (1.5)^2 = \frac{1}{2} (2) (v_2)^2 + \frac{1}{2} (20) (0.2)^2$$

total

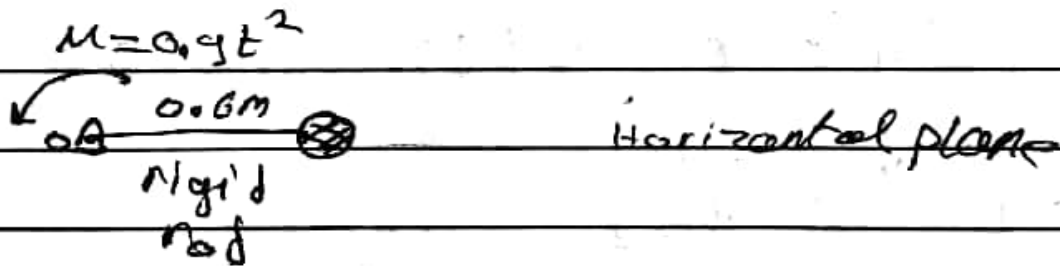
$$v_2 = 1.15$$



$$V_2^2 = V_t^2 + V_{rad}^2$$

1.07

F15-23



→ from rest (Horizontal)

$$V = ?? \text{ at } t = 0.5$$

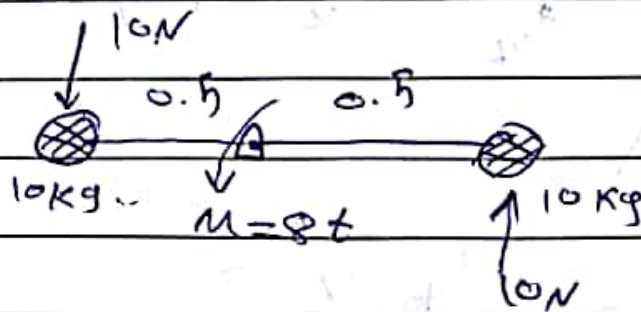
rest $H_{01} = 0$

$$H_{02} = \vec{r} \times m \vec{v}_2 = 0.6 (2) V_2$$

$$\int \tau dt = \int 0.9t^2 dt \rightarrow H_{01} + \int_0^5 \tau dt = H_{02}$$

Subject _____ Day _____ Date _____

F15-24



rest $H_1 = 0$ $V_2 = ??$ $t = 4s$

$$H_{02} = 2[0.5(10)(V_2)]$$

$$\int_0^4 [8t + 0.5(10) \times 2] dt = \text{Impulse}$$

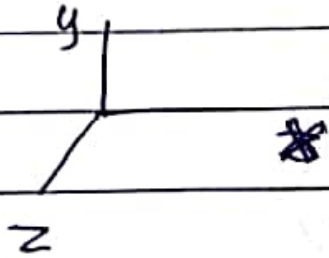
$$0 + \quad = H_{02}$$

$$V_2 = \quad m/s$$

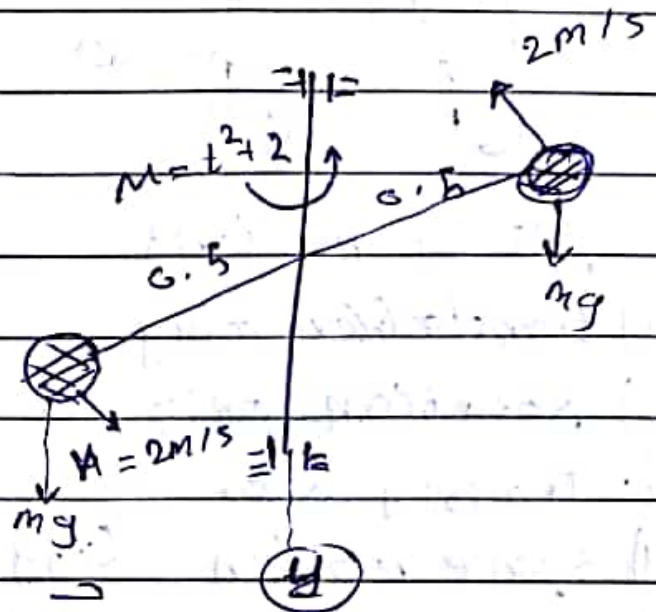
Subject _____ Day _____ Date _____

rigid link
نقطة التماس

15-100



$v_2 = ?$ at $t = 3.5$

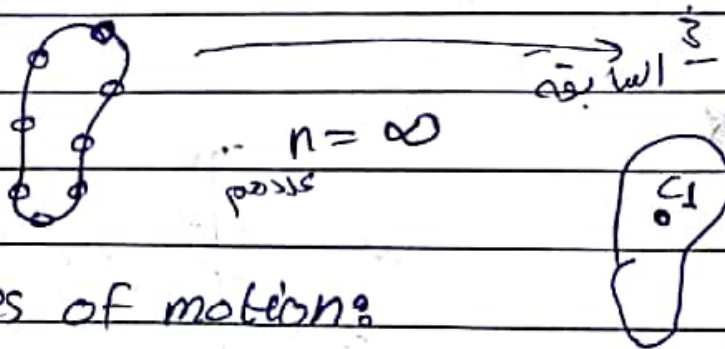


$$\vec{H}_{O,y} + \int \sum M_{O,y} dt = \vec{H}_{O,y}$$

$$2(0.5)(10)(2) + \int_0^3 (t^2 + 2) dt = 2(0.5)(10)(\frac{1}{2})$$

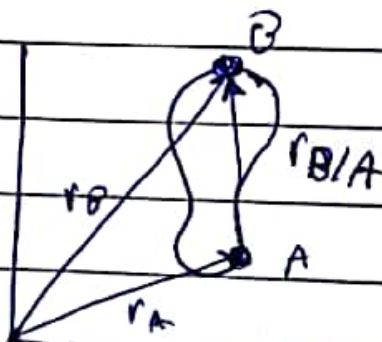
$$v_2 = 1 \text{ m/s}$$

* Rigid Body Motion → Kinematics ch 16
→ kinetics ch. 17, 18



types of motion:

- 1) translation only
- 2) Rotation only (about fixed axis)
- 3) Trans. + Rot.
- 4) space motion (x, y, z, Rot about x, y, z)
- 5) Rotation about point (x, y, z)



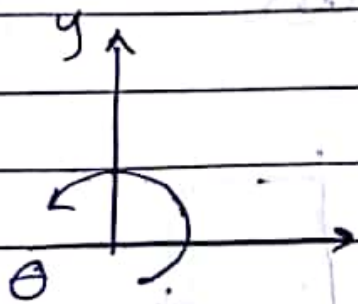
translation $\rightarrow \frac{dC}{dt}$

$$r_B = r_A + r_{B/A}$$

$$v_B = v_A + \frac{d}{dt}(r_{B/A}) \Rightarrow v_C = v_D = \dots = v_H$$

$$a_B = a_A = \dots = a_C = a_D = \dots = a_H$$

disp 9

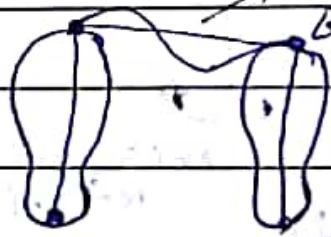


في دراسة space

curvilinear

Rectilinear

كيف يدي احد وصف الحركة



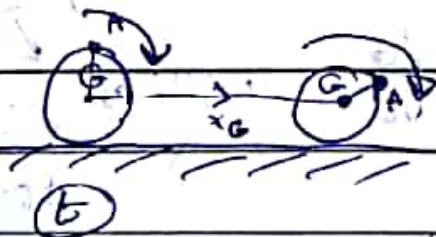
الخطين

time(t) (t+Δt)

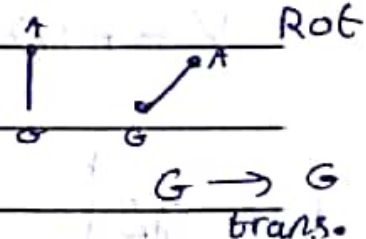
AB // A'B

translating

Rolling motion



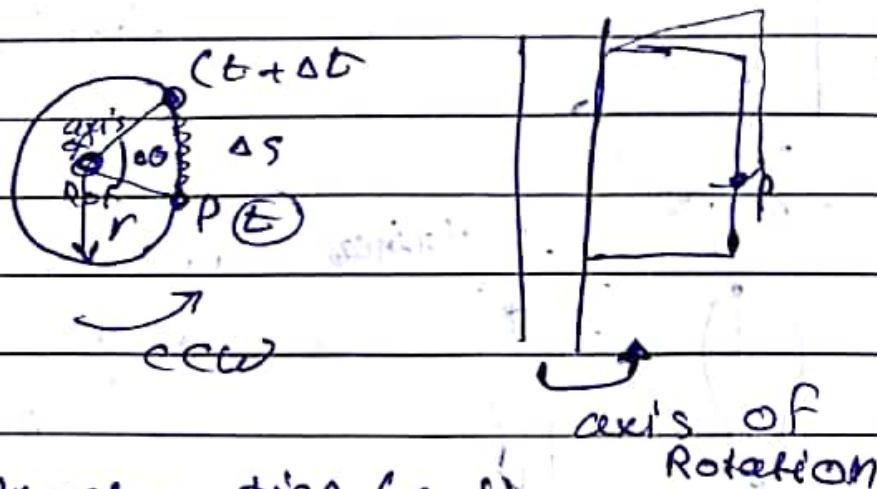
Rot.



Trans. + Rot. $\Rightarrow$ (General plane motion)

G.P.M = trans. + Rot.

⊗ Rotation about fixed axis



$\Delta\theta = \text{Angular disp (rad)}$

$2\pi \text{ (rad)} = 360^\circ = 1 \text{ revolution}$

$$\Delta s = (\Delta\theta) r \quad \boxed{ds = r d\theta}$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

parameter

θ, ω, α

$$v = r\omega \quad \omega = \text{angular speed} = \frac{d\theta}{dt}$$

angular acc.

$$\alpha = \frac{d\omega}{dt} \left(\frac{\text{rad}}{\text{s}^2} \right)$$

$$\boxed{\alpha = \frac{dv}{dt}}$$

$\alpha = \text{const}$ → Rotation ($\alpha = \text{const}$)

$$\frac{d\omega}{dt} = \alpha = \text{const}$$

$$\int_{\omega_0}^{\omega} d\omega = \int \alpha dt \rightarrow \omega = \omega_0 + \alpha t \rightarrow v = v_0 + \alpha r t$$

$$* \int_{\theta_0}^{\theta} \frac{d\theta}{dt} = \omega = \int_0^t (\omega_0 + \alpha t) dt$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad (2) \quad x = x_0 + v_0 t + \frac{1}{2} a t^2$$

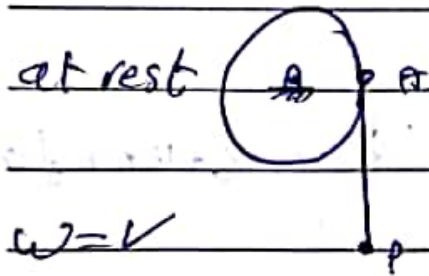
$$* \alpha = \frac{d\omega}{dt}, \quad \omega = \frac{d\theta}{dt}$$

$$\uparrow \quad dt = \frac{d\theta}{\omega}$$

$$\alpha = \omega \frac{d\omega}{d\theta} \quad \boxed{\omega \frac{d\omega}{d\theta}}$$

$$\int \alpha d\theta = \int \omega d\omega \rightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad (3)$$

Exp 8 16-1



$$w = v$$

$$\theta = v \quad a = 4t = f(t)$$

$$a = 4t = \frac{dv}{dt}$$

$$\int_{v_0}^v dv = \int_0^t 4t dt$$

$$v - v_0 = 2t^2$$

$$v = v_0 + 2t^2$$

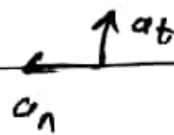
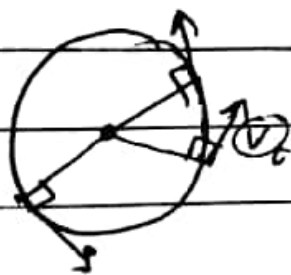
$$\boxed{v_p = 2t^2}$$

$$v_A = v_p = 2t^2 = r\omega$$

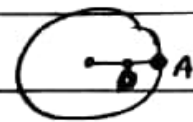
$$\omega = \text{function of time} = \frac{2t^2}{r}$$

$$\omega = \frac{2}{r} t^2 = \frac{d\theta}{dt}$$

$$\int_{\theta_0}^{\theta} d\theta = \frac{2}{r} \int t^2 dt$$



دور از مرکز

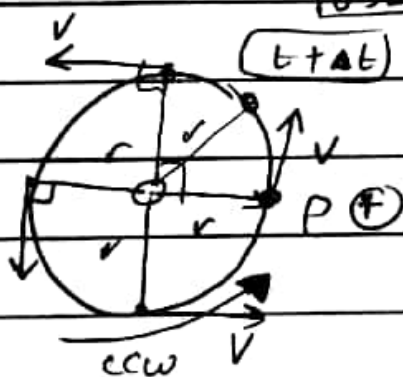


$$\omega_A = \omega_P$$

$$v_A > v_P$$

⊕ Rotation about fixed axis :-

$$ds = r d\theta$$



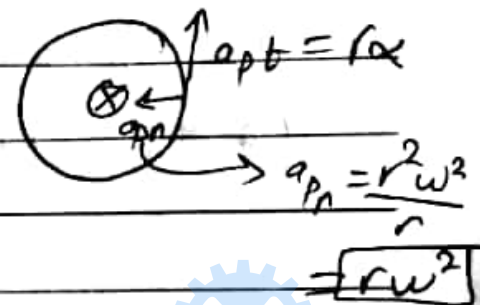
$$\theta, \omega, \alpha$$

موتور
البرق

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$v = r \frac{d\theta}{dt} = r \omega$$

$$a_t = \frac{d^2 s}{dt^2} = r \alpha$$

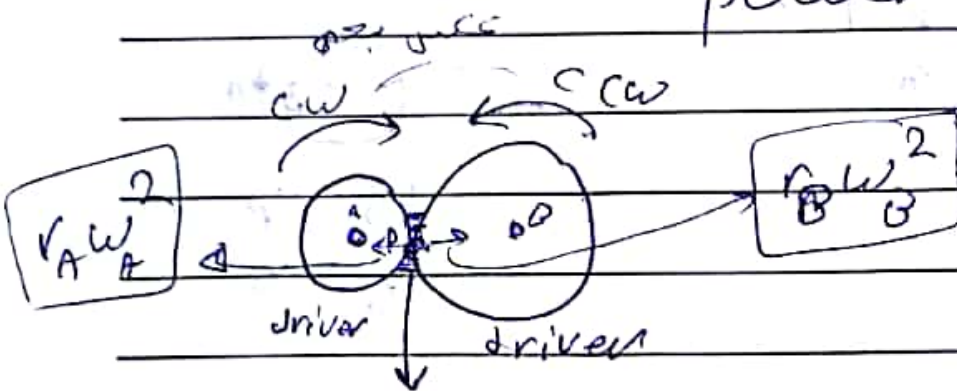


$$\vec{a}_n = \vec{\omega} \times [\underbrace{\vec{\omega} \times \vec{r}}_{\vec{v}}]$$



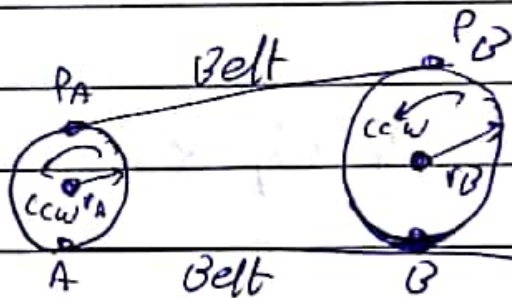
⊗ Gears pulleys

power Transmission



$$r_A \omega_A = v_p = r_B \omega_B$$

$$r_A \propto \omega_A = r_B \propto \omega_B$$



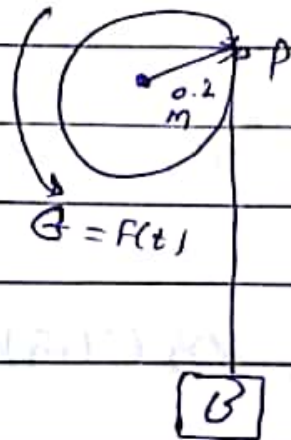
$$v_{P_A} = v_{P_B}$$

$$r_A \omega_A = r_B \omega_B$$

$$r_A \propto \omega_A = r_B \propto \omega_B$$

F16-4

$$\Theta = 15t + 0.5t^3$$



$$\omega = \frac{d\Theta}{dt} = 15 + 1.5t^2$$

$t = 3s$

$$= 15 + 1.5(3)^2 = 19.5 \text{ rad/s}$$

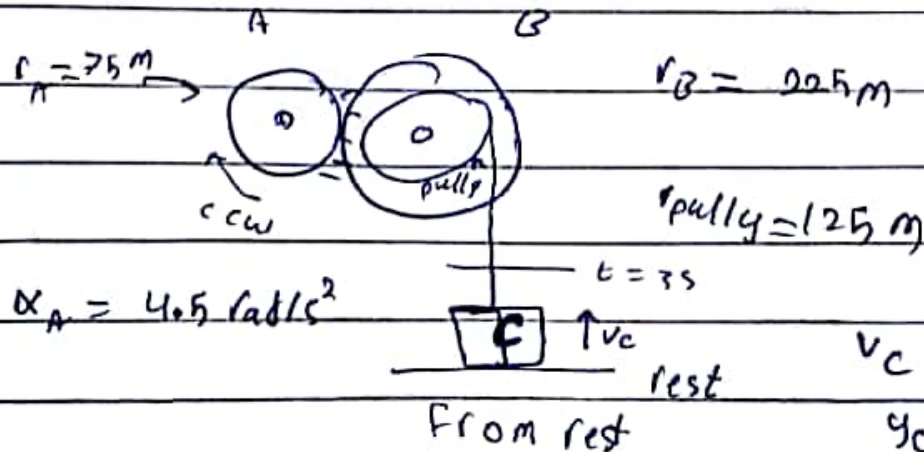
$$\begin{pmatrix} v_B \\ a_B \end{pmatrix}$$

$$\alpha = \frac{d\omega}{dt} = 3t = 9 \text{ rad/s}^2 \quad \text{at } t = 3s$$

$$v_B = v_p - (0.2)(28.5) = 1.5 \text{ m/s}$$

$$a_B = a_{pt} = (0.2)(9) = 1.8 \text{ m/s}^2 \quad \text{at } t = 3s$$

F16-6



$$\alpha_A = 4.5 \text{ rad/s}^2$$

$$t = 3s$$

$$v_C = ?$$

$$y_C = ?$$

Tech Family

Subject _____ Day _____ Date _____

$$v_c = 0 + a_c (3)$$

16-13

$$a_c = a_{pt} = (0.125) \alpha_B$$

$$\textcircled{*} \quad 25(4.5) = 225 \alpha_B$$

$$\alpha_B = 1.5 \text{ rad/s}^2 \quad \uparrow \text{ccw}$$

$$a_{pt} = 0.125(1.5) = a_c$$

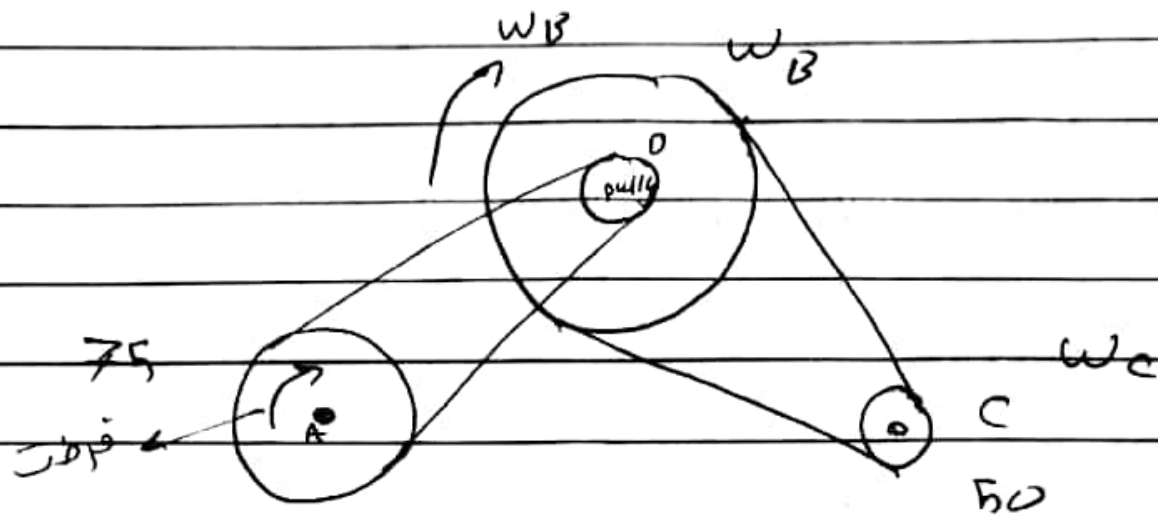
$$v_c = a_c \cdot (3) = \boxed{0.45 \text{ m/s}}$$

$$\text{then } y_c = 0 + \frac{1}{2} a_c (3)^2$$

16-20

16-11

Subject _____ Day _____ Date _____



$$\omega_A = 60 \text{ rad/s}$$

$$60(75) = 100 \omega_B \rightarrow \omega_B = 45 \text{ rad/s} \quad \downarrow \text{cw}$$

$$100(45) = 50 \omega_C$$

$$\omega_C = 90 \text{ rad/s} \quad \downarrow \text{cw}$$

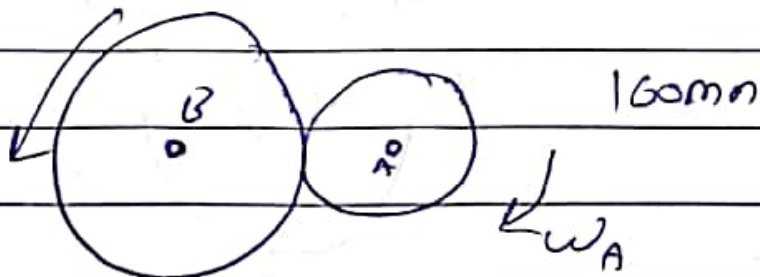
16.4 July 20

Subject _____ Day _____ Date _____

16-18

$$\alpha_A = 2t^3$$

$$\omega_{A_0} = 15 \text{ rad/s}$$



$$\omega_B = ?? \text{ at } t = \underline{\underline{3 \text{ s}}}$$

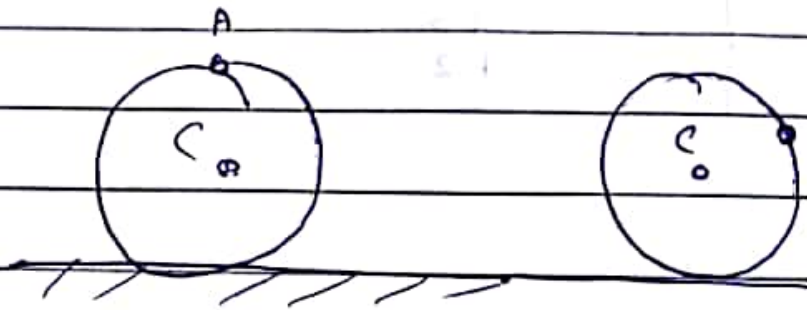
$$175 \alpha_B = 100 \alpha_A$$

$$\alpha_B = \frac{100}{175} (2t^3)$$

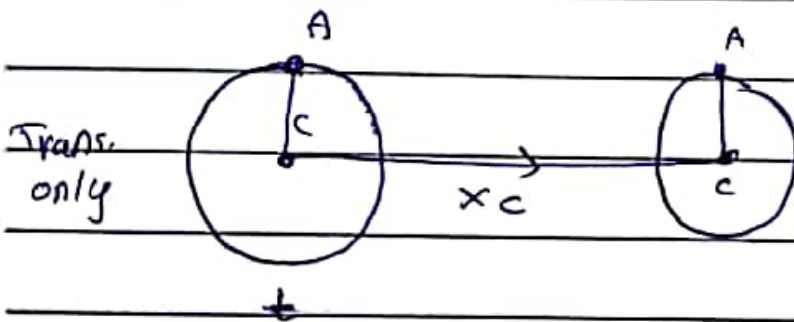
$$\int_{\omega_{B_0}}^{\omega_B} d\omega_B = \int_0^3 \frac{200}{175} t^3 dt$$

$$100(15) = 175(\omega_{B_0})$$

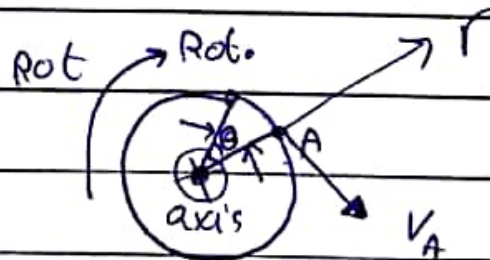
G.p.M



G.p.M = Trans. + Rot.



$$V_A|_{\text{trans}} = V_C$$

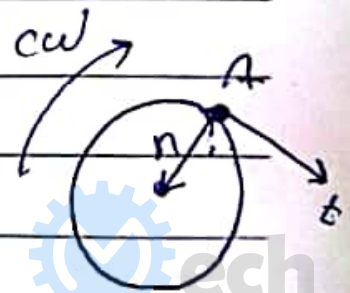


$$\omega = \frac{d\theta}{dt}$$

$$V_A|_{\text{rot}} = \vec{\omega} \times \vec{r}$$

$$* \vec{V}_A = \vec{V}_C + \vec{\omega} \times \vec{r}$$

$$\vec{a}_A = \vec{a}_C + \underbrace{\vec{\alpha} \times \vec{r}}_{\text{trans.}} + \underbrace{\vec{\omega} \times [\vec{\omega} \times \vec{r}]}_{\text{normal}}$$



الاجابة
في صفحة 1

Subject _____ Day _____ Date _____

Σx 16.6

$\approx$

7

8

Σx 16.13

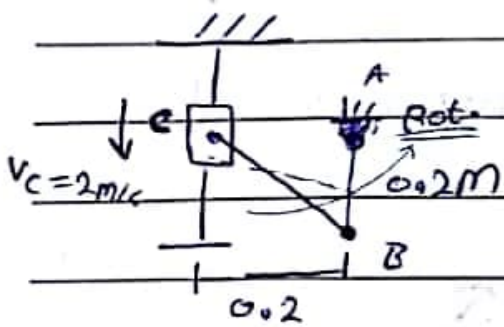
14

15

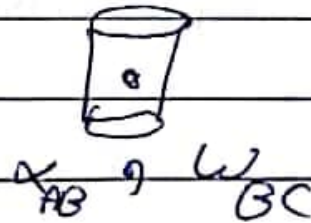
16

17

Exp: 16-8 + 16-16
v a

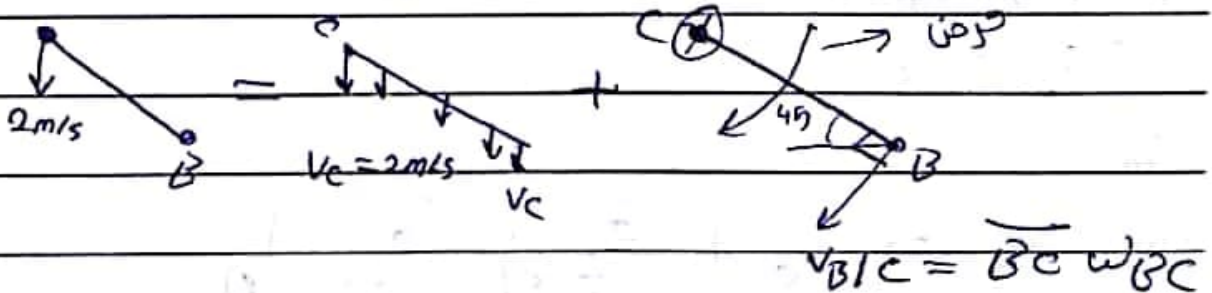


$$BC = 0.2\sqrt{2}$$



G.P.M { scalar }
 { vector }

G.P.M = Trans. + Rot.
 w.r.t c w.r.t c



vector $\vec{v}_B = \vec{v}_c + \vec{v}_{B/c}$

$$0.2\omega_{AB} \leftarrow = 2 \downarrow + 0.2\sqrt{2} \omega_{BC} \rightarrow \swarrow 45^\circ$$

vertical, $0 = 2 - 0.2\sqrt{2} \frac{\cos 45^\circ}{\sqrt{2}} \omega_{BC} \Rightarrow \omega_{BC} = -10 \text{ rad/s}$

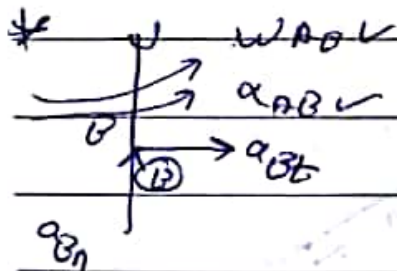
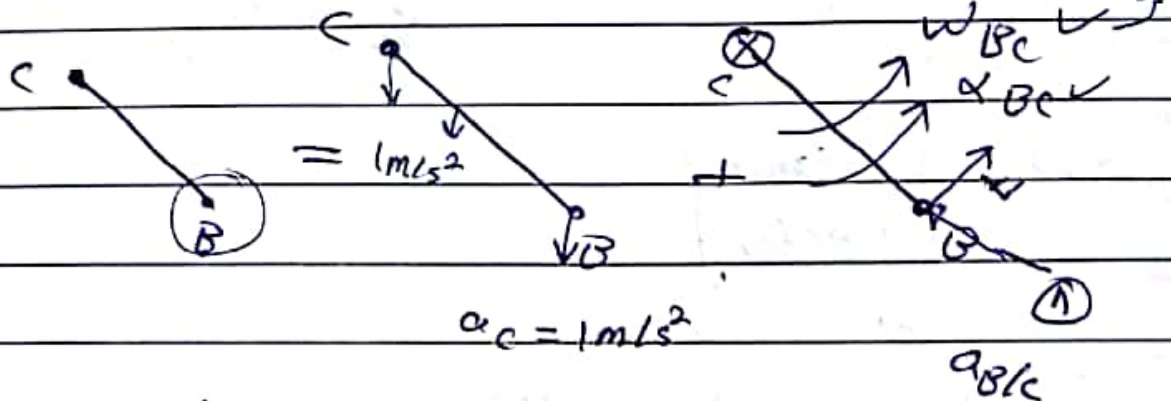


$$v_B = 0.2 \omega_{AB}$$

= 10 rad/s
CCW

Horizontal, $-0.2\omega_{AB} = 0 - 0.2\sqrt{2} \left(\frac{1}{\sqrt{2}}\right)(-10)$

$\omega_{AB} = -10 \text{ rad/s} = 10 \text{ rad/s} \nearrow \text{ccw}$



$\vec{a}_B = \vec{a}_C + \vec{a}_{B/C} + \vec{a}_{B/C}$

$\vec{a}_{B_t} + \vec{a}_{B_n} = \vec{a}_C + \vec{BC} \alpha_{B/C} \nearrow 45^\circ + \vec{BC} \omega_{B/C} \searrow 45^\circ$

$0.2 \alpha_{AB} \rightarrow + 10^2 (0.2) \uparrow = 1 \downarrow + 0.2 \sqrt{2} \alpha_{B/C} \nearrow 45^\circ + 0.2 \sqrt{2} (10)^2 \searrow 45^\circ$

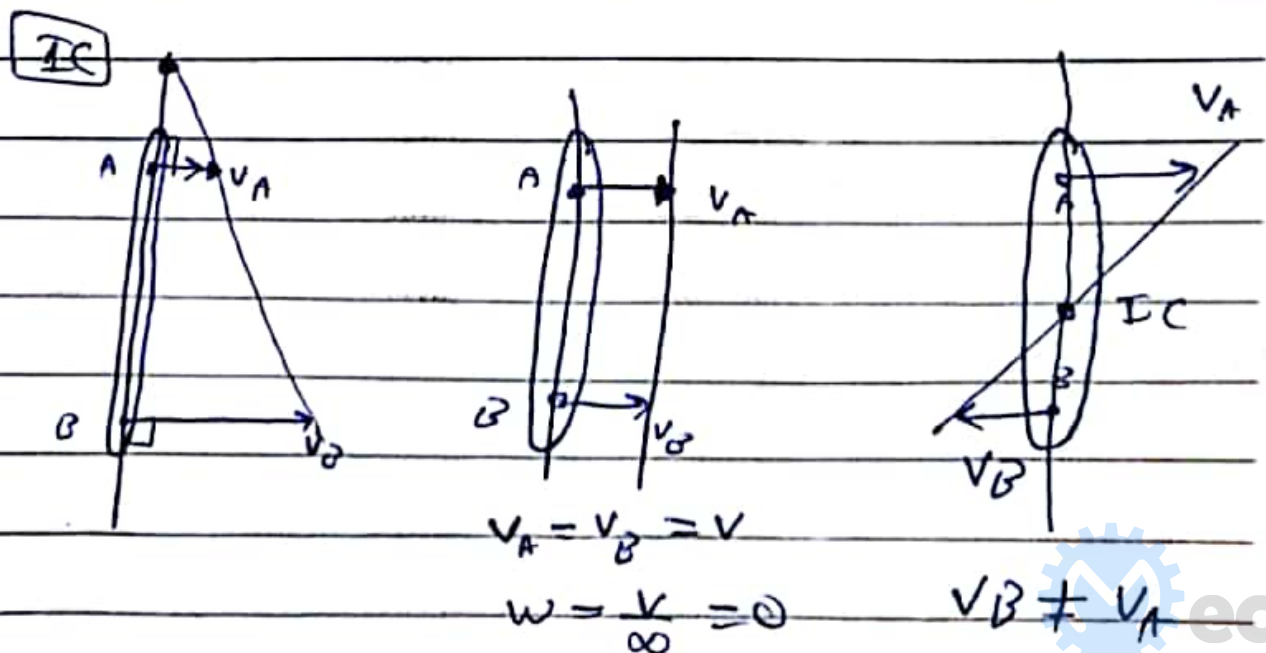
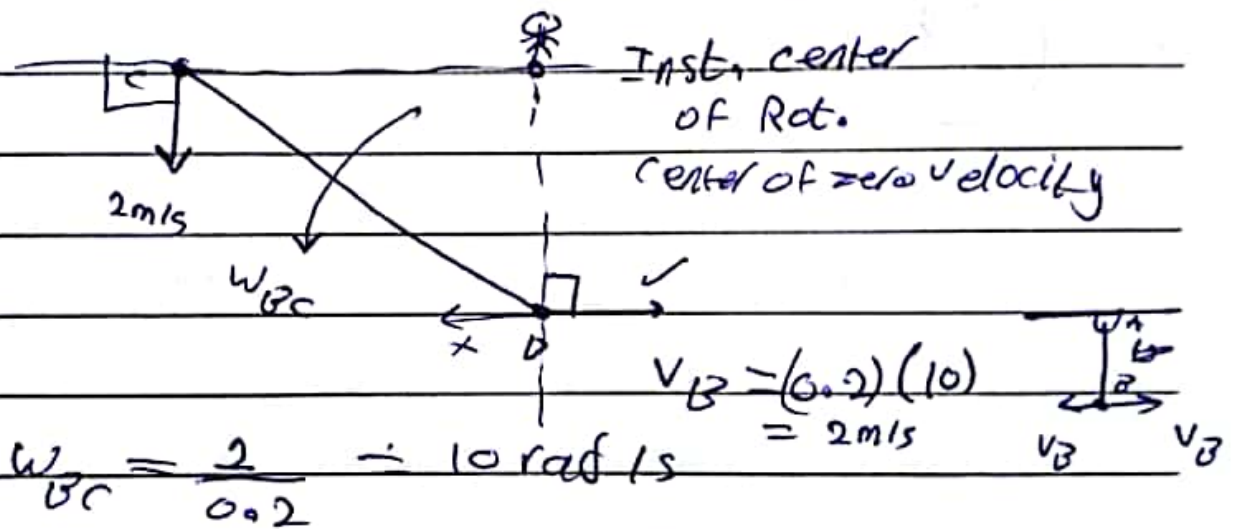
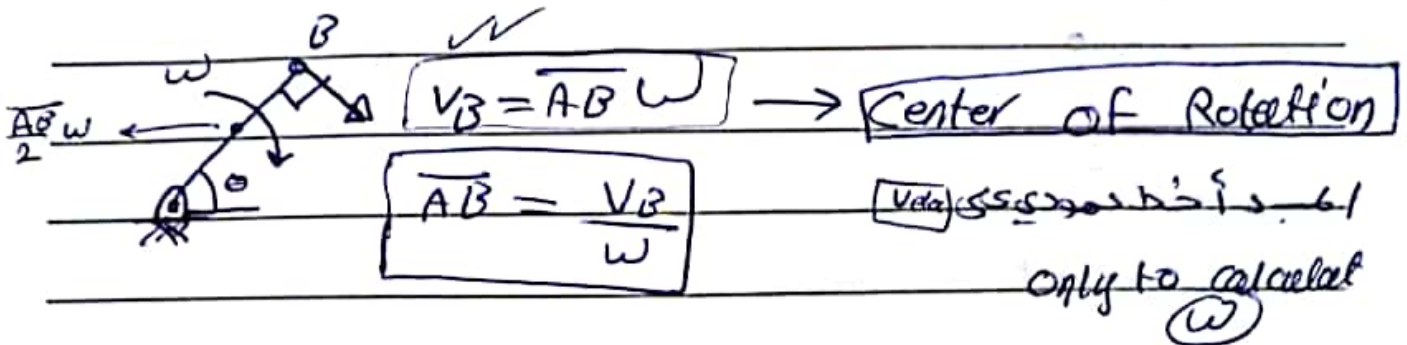
$0 + 20 = -1 + 0.2 \alpha_{B/C} + 20 \rightarrow \alpha_{B/C} = \sqrt{\text{rad/s}^2}$

$0.2 \alpha_{AB} + 0 = 0 + 0.2 \alpha_{B/C} - 20 \rightarrow \alpha_{AB} = \sqrt{\text{rad/s}^2}$

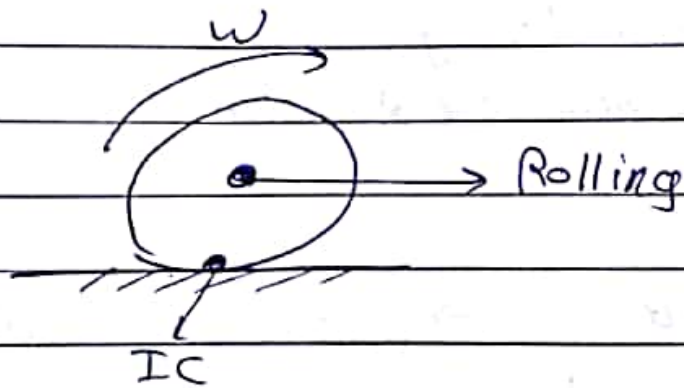
بنظر
الاستاذ

* Center of Zero Velocity

Instantaneous center of Rotation

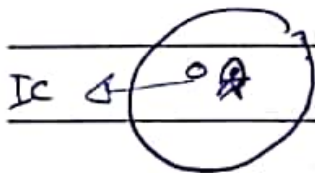


Subject _____ Day _____ Date _____

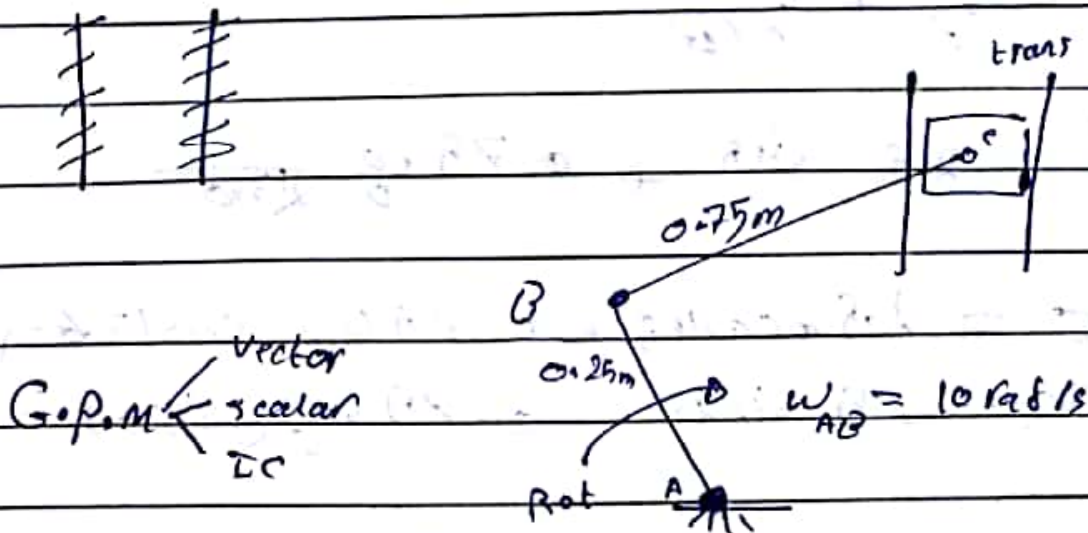


✓
समान
दिशा में
चल रहा है
कोण
वृद्धि कर रहा है

but

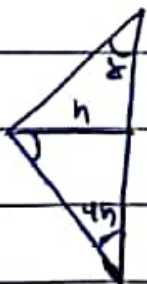
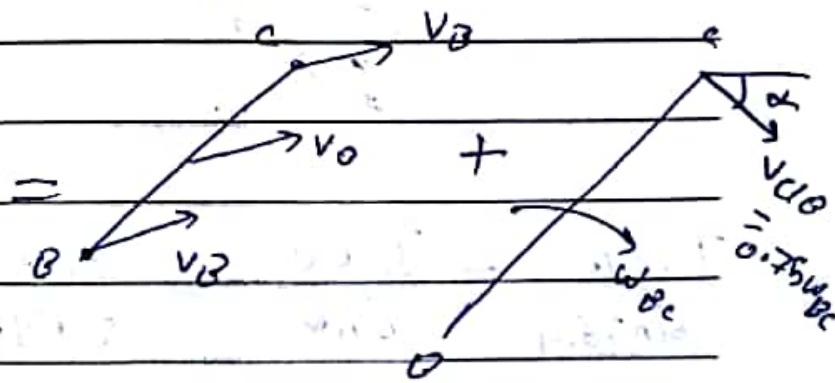
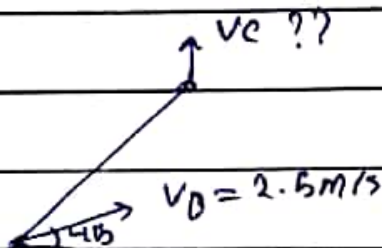
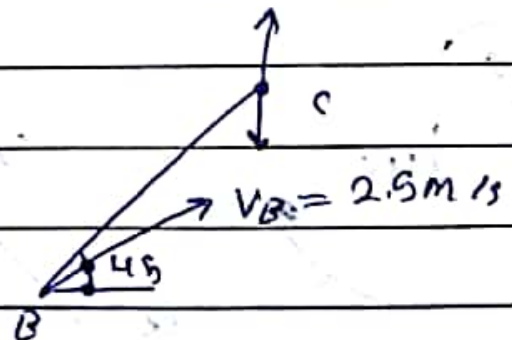
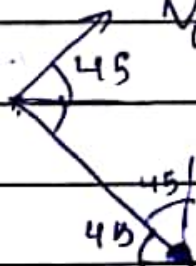


Exp 9 16-12



GoP.M ← vector
scalar
LC

$$N_B = (10)(0.25)$$



$$\sin 45 = \frac{h}{0.25}$$

$$\sin \theta = \frac{h}{0.75}$$

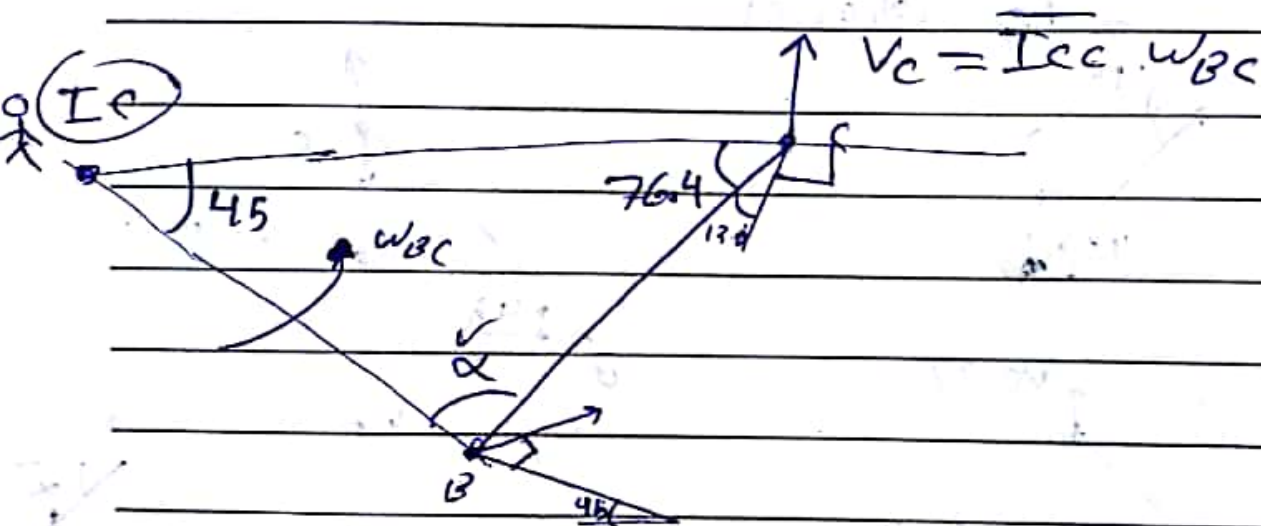
$$\theta = \sin^{-1} \left[\frac{0.25 \times \sin 45}{0.75} \right]$$

$$V_c = V_B + V_{c/B}$$

$$V_c \uparrow = 2.5 \angle 45^\circ + 0.75 \omega_{BC} \angle 130.6^\circ$$

$$H: 0 = 2.5 \cos 45^\circ + 0.75 \omega_{BC} \cos 130.6^\circ = \boxed{4.05 \text{ rad/s}}$$

$$V: V_c = 2.5 \sin 45^\circ - 0.75 \omega_{BC} \sin 130.6^\circ$$



$$\frac{I_{CB}}{\sin 76.4^\circ} = \frac{I_{CC}}{\sin \alpha} = \frac{0.75}{\sin 45^\circ}$$

$$\omega_{BC} = \frac{2.5}{I_{CB}} = \sqrt{\text{rad/s}}$$

Subject _____

Day _____

Date

16-13 14-15

17-18

①

90

89

85

86

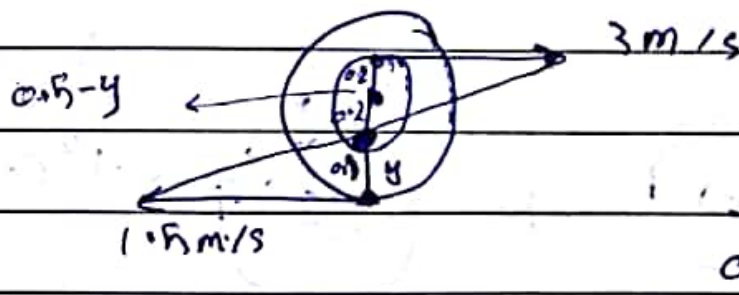
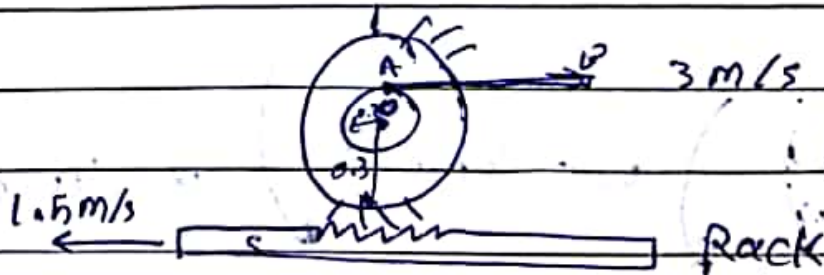
88

EX/6-10

16-11

16-12

F16-6

 $\omega = ??, v_o = ?$ 

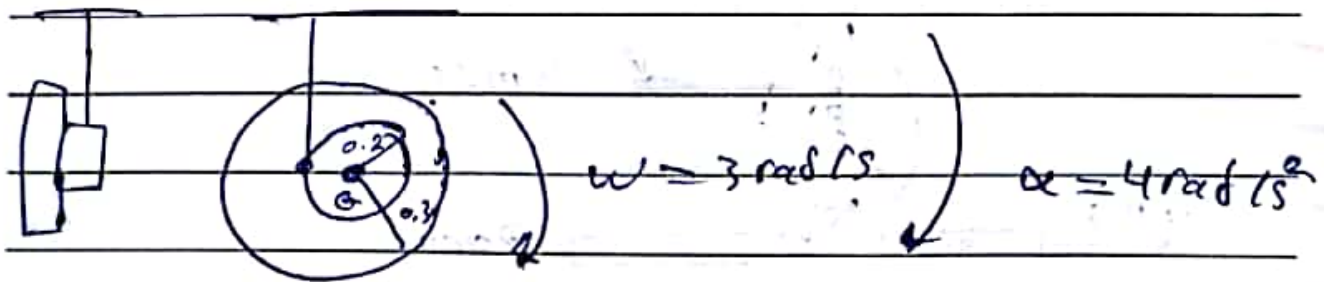
$$\frac{3}{0.3 - y} = \frac{1.5}{y}$$

$$\omega_{\text{gear}} = \frac{1.5}{y} = \omega \text{ rad/s}$$

$$v_o = \omega_{\text{gear}} \times (0.3 - y) = \omega \text{ m/s}$$

Subject 16-12 Day 16- Date 19 20 21 22 23 24

Σ xp 16-15



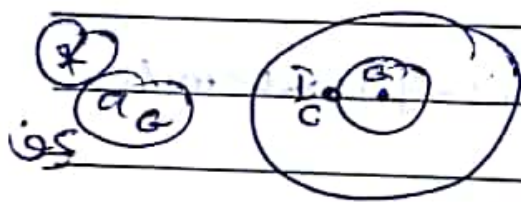
تأثير التسارع الزاوي

$$a_B = a_G + a_{B/G}$$

$$a_B = a_G + \omega^2 r$$

$$a_B = 0 + (3)^2 (0.3)$$

$$a_B = 2.7 \text{ m/s}^2$$



$$y_G = 2\pi r$$

$$y_G = \theta r$$

$$v_G = \omega r$$

$$a_G = \alpha r$$

$$4 (0.2) = 0.8 \text{ m/s}^2 \downarrow$$

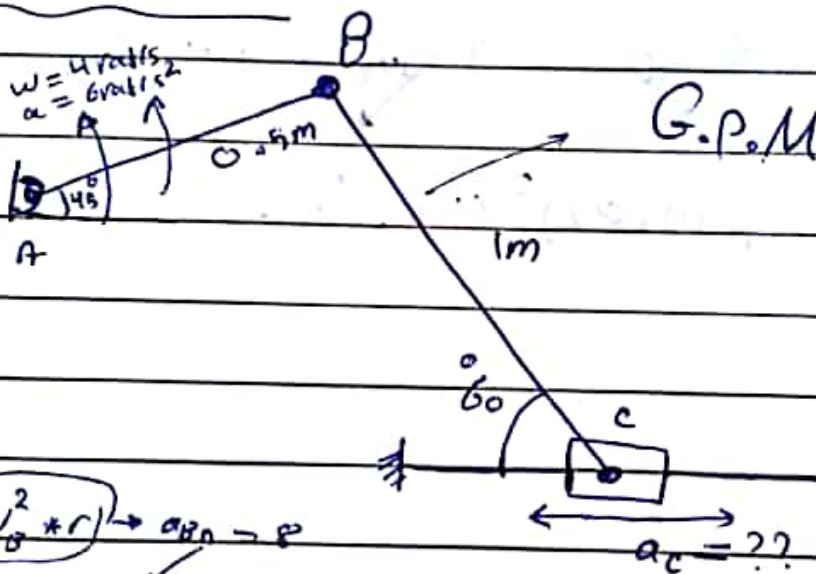
$$\vec{a}_{B_H} + \vec{a}_{B_V} = 0.8 \downarrow + 1.2 \rightarrow + 2.7 \downarrow$$

$$\downarrow$$

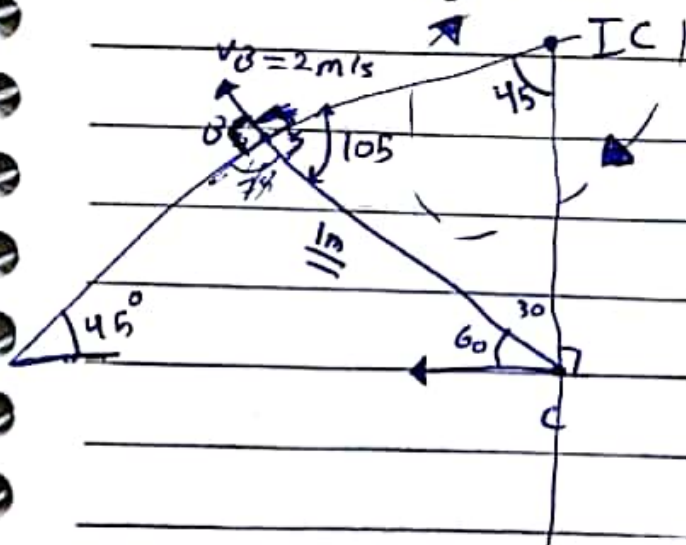
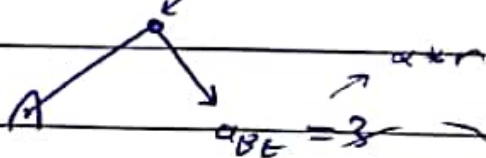
$$1.2 \rightarrow + 3.5 \downarrow$$

المسألة (20 و 21)

* 16-167



$$w^2 * r \rightarrow a_{Bn} = 8$$

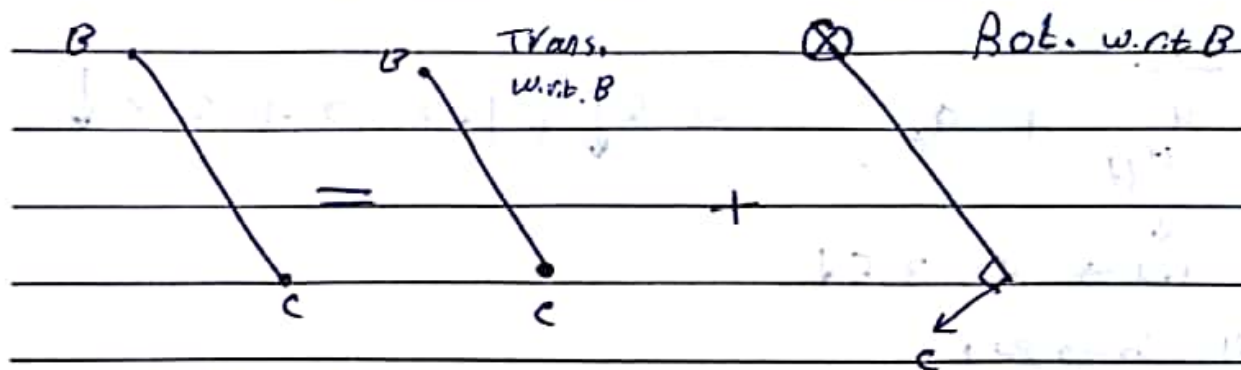


$$\frac{IC_C}{\sin 105} = \frac{1}{\sin 45} = \frac{IC_B}{\sin 30}$$

$$w_{BC} = \frac{2}{BIC} = 2.82$$

$$v_C = w_{BC} * IC_C$$

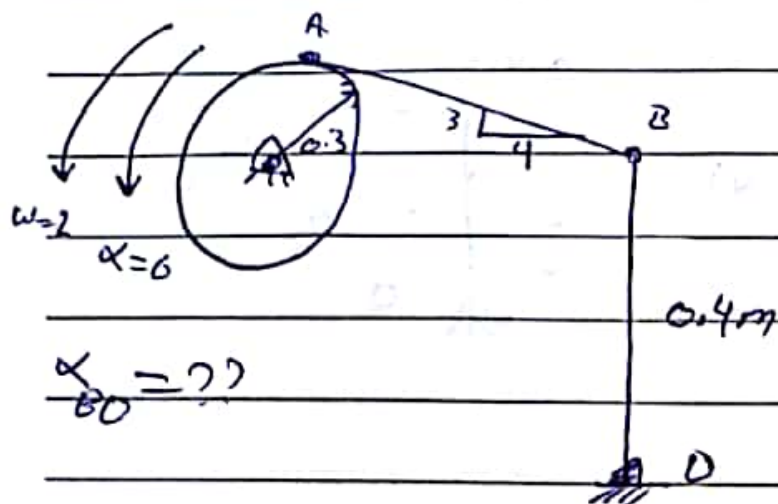
Subject _____ Day _____ Date _____



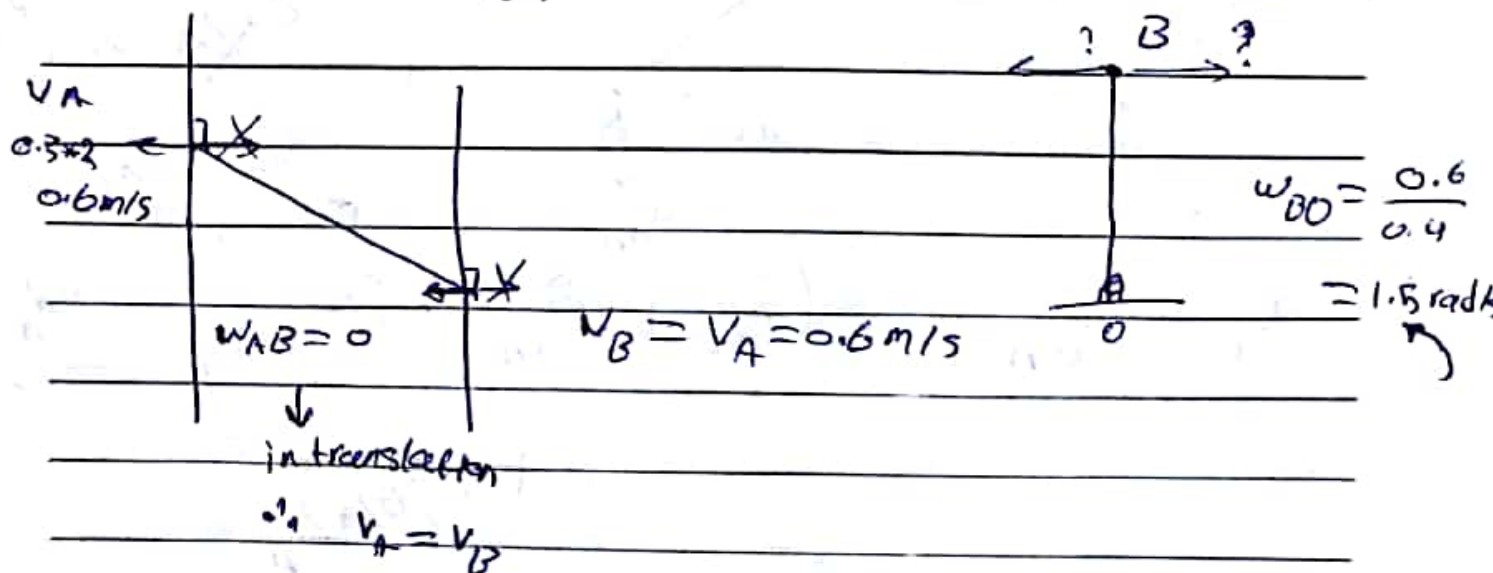
$$\vec{a}_c = \vec{a}_B + \vec{a}_{c/B}_t + \vec{a}_{c/B}_n$$

$$\vec{a}_c \rightarrow = 8 \angle 45^\circ + 3 \angle 45^\circ + 1 \angle 30^\circ + (2.82)^2 (1) \angle 60^\circ$$

16-108



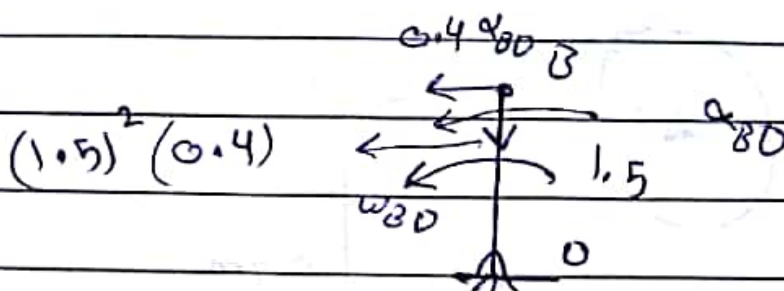
→ velocity ← Is vector
 scalar



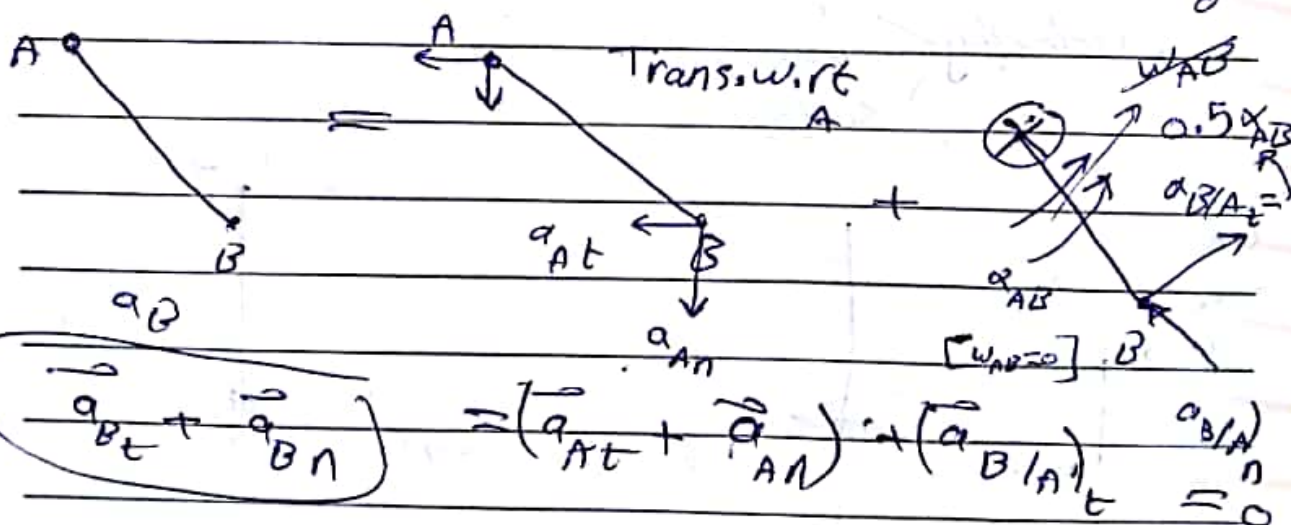
$$a_{At} = 1.8 \text{ m/s}^2$$



$$a_{An} = 1.2 \text{ m/s}^2$$

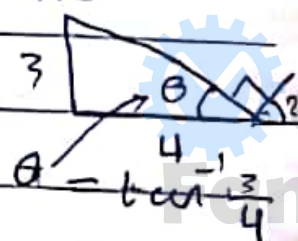


G.P.M



$$+ (\vec{a}_{B/A})$$

$$0.4 \vec{a}_{BD} \leftarrow + (1.5)^2 \cdot 0.4 \downarrow = 1.8 \leftarrow + 1.2 \downarrow + 0 + 0.5 \vec{a}_{AB}$$



$$V, \quad 0 - 1.5^2 (0.4) = 0 - 1.2 + 0 + 0.5 \alpha \sin \theta_{AB}$$

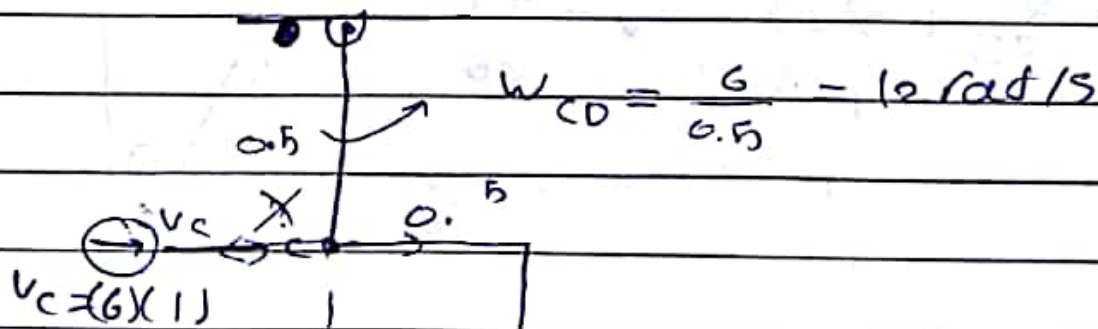
$$\rightarrow \alpha_{AB} = \checkmark$$

$$H, \quad -0.4 \times 80 + 0 = -1.8 + 0 + 0.5 \alpha \cos(90^\circ)$$

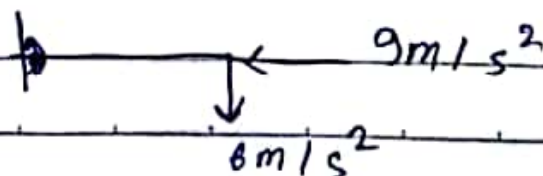
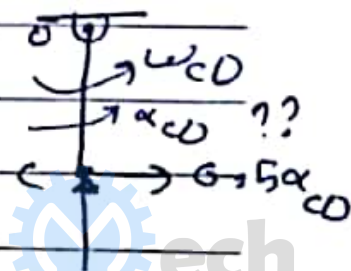
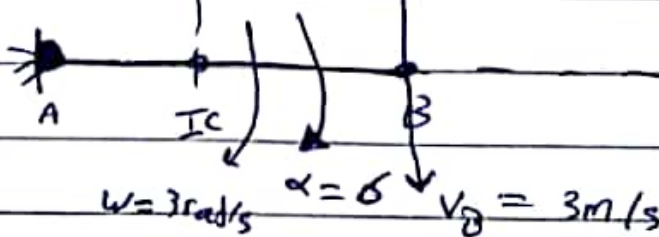
$$\alpha_{BD} = \checkmark$$

16-115

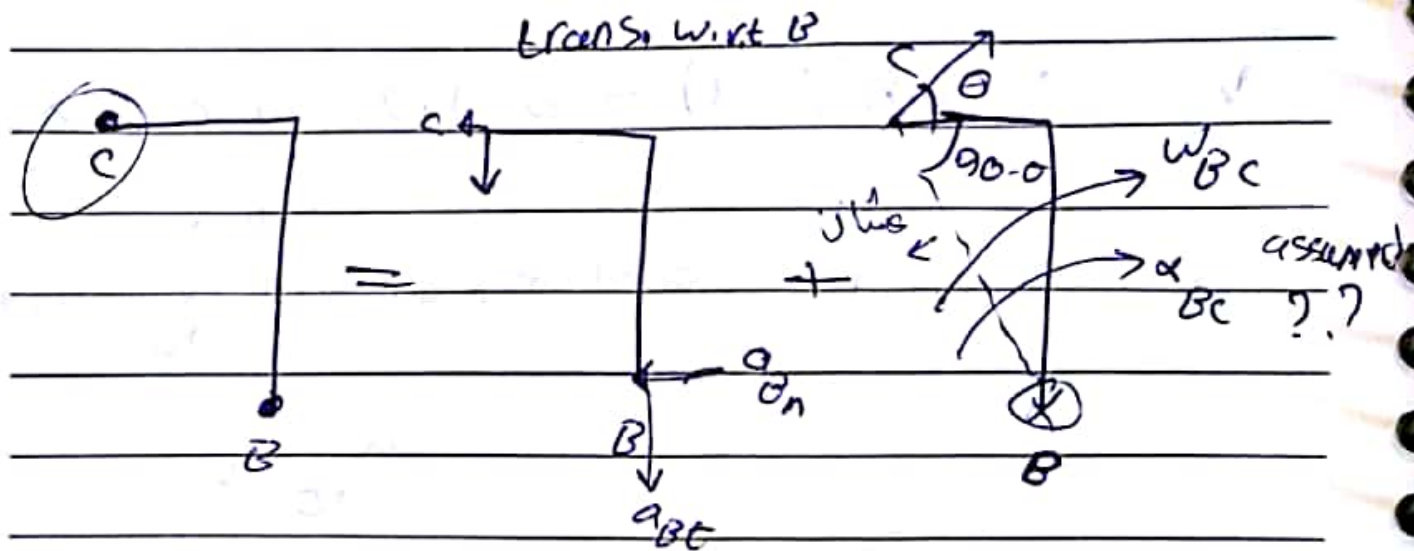
$$\alpha_{CD} = ???$$



$$\omega_{BC} = \frac{3}{(1-0.5)} = 6 \text{ rad/s}$$



$$12(0.5)^2 = 72$$

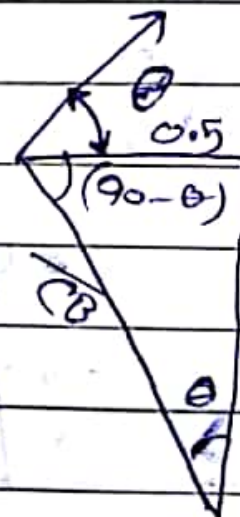


$$\vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$$

$$0.5 a_{C/B} + 7.2 \uparrow = 6 \downarrow + 9 \leftarrow$$

$$+ BC (a_{BC}^{??}) \Delta \theta$$

$$+ (6)^2 (BC) \sqrt{90^\circ - \theta}$$



$$\theta = \tan^{-1} \frac{0.5}{1}$$

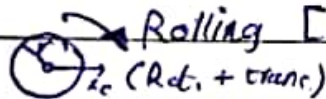
* 16-4 Absolute Motion analysis

→ parametric analysis (x, y, α)

* Relation between the linear motion and the angular motion
 (θ, ω, α)

Rolling Disk/cylinder

Exp. 16-4 Rolling [without slipping]



→ $x_C = 2\pi r \rightarrow$ for one Rev. $= 2\pi \text{ rad} = 360^\circ$

Linear motion

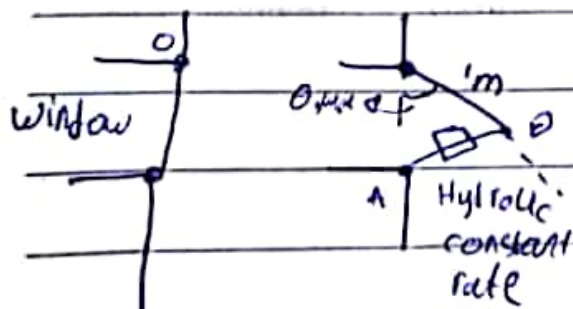
$$x_C = r\theta^*$$

$$v_C = \frac{dx_C}{dt} = r \frac{d\theta}{dt} = r\omega^*$$

$$a_C = \frac{dv_C}{dt} = r \frac{d\omega}{dt} = r\alpha^*$$

* angular parameter.

Exp: 16-5



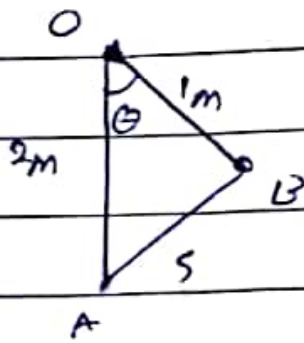
$\omega = ?$

$\alpha = ??$

When $\theta = 30^\circ$

constant rate $= 0.5 \text{ m/s}$

Subject _____ Day _____ Date _____



$$s^2 = (2)^2 + 1^2 - 2(1)(2) \cos \theta$$

 $\frac{ds}{dt}$

initial

$$2s \frac{ds}{dt} = 0 - 4(-\sin \theta) \frac{d\theta}{dt} \quad \omega$$

$$2(s)(0.5) = 4 \sin 30^\circ \omega \quad \omega = v$$

$$s = \sqrt{5 - 4 \cos(30^\circ)}$$

$\theta = 30^\circ$

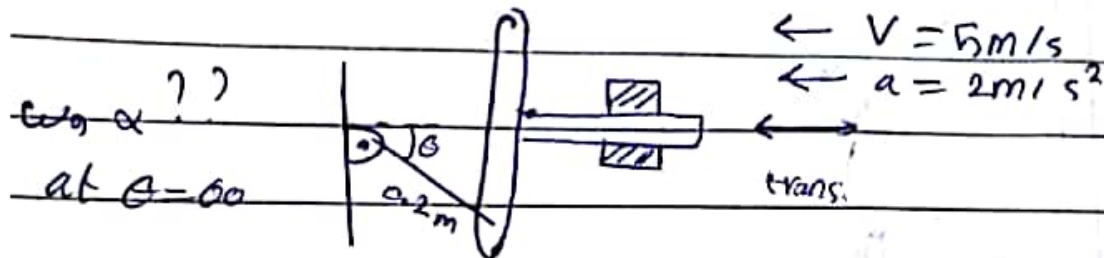
$$2s \frac{ds}{dt} = 4 \sin(\theta) \omega$$

(0.5)

$$2 \left[s(0) + \frac{ds}{dt} \frac{ds}{dt} \right] = 4 \left[\sin \theta (\alpha) + \omega \cos \theta \frac{d\theta}{dt} \right]$$

0.5 0.5

16-40



* الحل
بايجاد
السرعة
الزاوية
الزاوية

$$X = 0.2 \cos \theta$$

$$v = \frac{dx}{dt} = -0.2 (-\sin \theta) \frac{d\theta}{dt}$$

$$5 = +0.2 (\sin \theta) \omega$$

$$\omega = \frac{5}{0.2 \sin 60} = \checkmark$$

$$v = 0.2 \omega \sin \theta$$

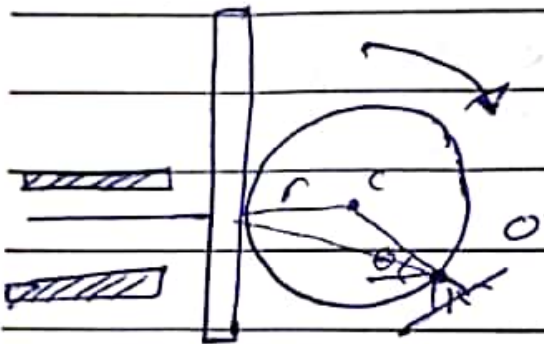
$$a = 0.2 \left[\omega \cos \theta \frac{d\theta}{dt} + \sin \theta \frac{d\omega}{dt} \right]$$

$$2 = 0.2 \left[\omega^2 \cos 60 + \sin 60 \alpha \right]$$

$$\alpha = \checkmark \text{ rad/s}^2$$

Subject _____ Day _____ Date _____

16-47



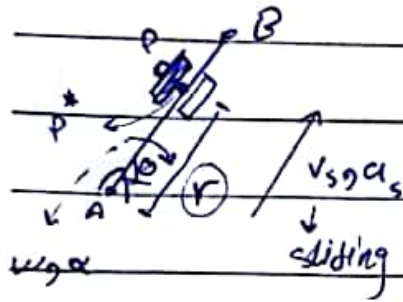
$$x = ? \quad \leftarrow r \quad \rightarrow r \cos \theta$$
$$v = ?$$

$$x = r + r \cos \theta$$

$$v = \frac{dx}{dt} = -r \sin \theta \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} = -r [\omega^2 \cos \theta + \sin \theta \alpha]$$

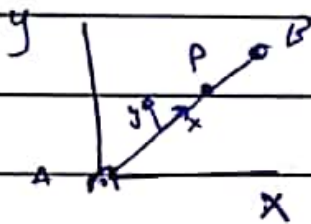
Subject _____ Day _____ Date _____



→ velocity, $\vec{V}_P = \vec{V}_P^* + \vec{V}_s$

$$\vec{V}_P = \vec{\omega} \times \vec{r} + \vec{V}_s$$

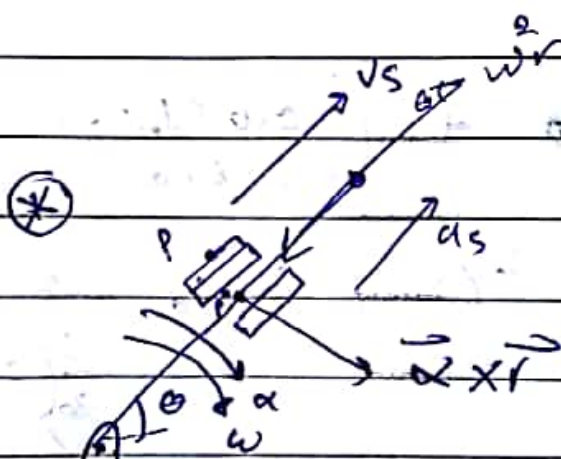
$$\vec{V}_P = \omega r \nabla_{90^\circ} + v_s \nabla_{\theta}$$



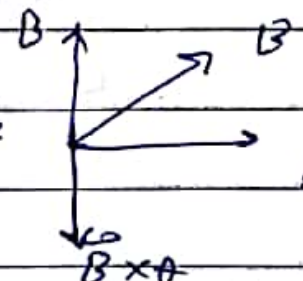
$$\vec{a}_P = \vec{a}_{Pt} + \vec{a}_{Pn} + \vec{a}_s + \text{Complementary Acc}$$

Coriolis acceleration

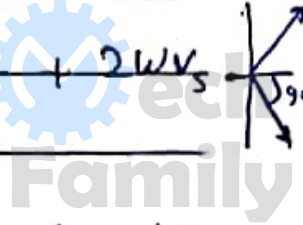
$$2 \vec{\omega} \times \vec{V}_s$$



Rel. Acc. (3) + (2) = 3

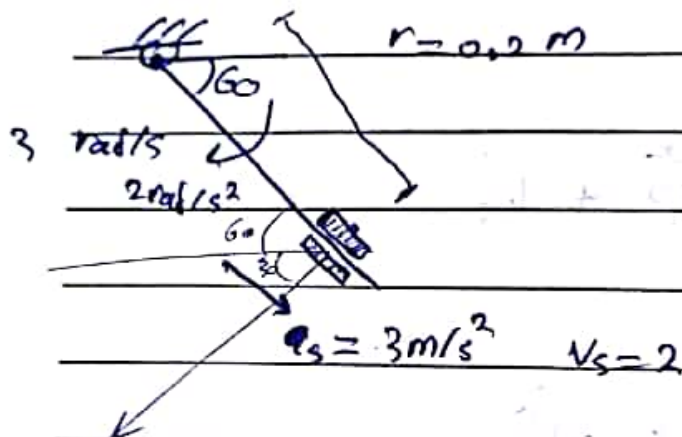


$$\vec{a}_P = \alpha r \nabla_{90^\circ - \theta} + \omega^2 r \nabla_{\theta} + a_s \nabla_{\theta} + 2\omega v_s \nabla_{90^\circ - \theta}$$



Subject _____ Day _____ Date _____

Exp 16-18



* $\vec{v}_c = \vec{v}_c^* + \vec{v}_s$

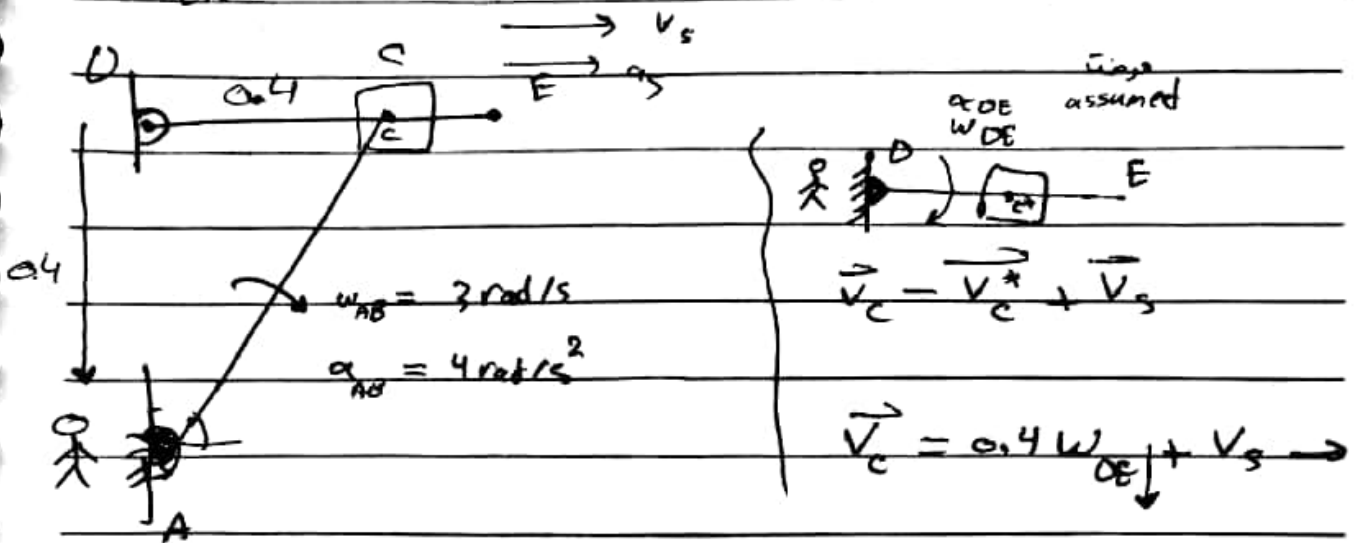
$\vec{v}_c = \vec{v}_c^* + \vec{v}_s$

$\vec{v}_c = 0.6 \angle 30^\circ + 2 \angle 60^\circ$

$\vec{v}_c = (-0.6 \cos 30^\circ + 2 \cos 60^\circ) \hat{i} + (-0.6 \sin 30^\circ - 2 \sin 60^\circ) \hat{j}$

$\vec{a}_c = 0.4 \angle 30^\circ + 1.8 \angle 60^\circ$, $3 \angle 60^\circ$ (2/3/2)

Ex 16-19



$$* \vec{v}_c = 0.4 \sqrt{2} \times 3 \Rightarrow 1.2 \sqrt{2}$$

$$0.4 \sqrt{2} (3) = \vec{v}_c = 0.4 w_{DE} + v_s$$

$$H: 1.2 \sqrt{2} \times \frac{1}{\sqrt{2}} = 0 + v_s \Rightarrow v_s = 1.2 \text{ m/s}$$

$$V: 1.2 \sqrt{2} \times \frac{1}{\sqrt{2}} = 0.4 w_{DE} \Rightarrow w_{DE} = 3 \text{ rad/s}$$

clock

$$\vec{a}_c = \vec{a}_{c_t} + \vec{a}_{c_n} + \vec{a}_s + 2 w_{DE} \times \vec{v}_s$$

as viewed from D

slipping occurs

$$\vec{a}_c = 0.4 \alpha_{DE} + (3)^2 (0.4) + a_s + 7.2 \downarrow$$

as viewed from A

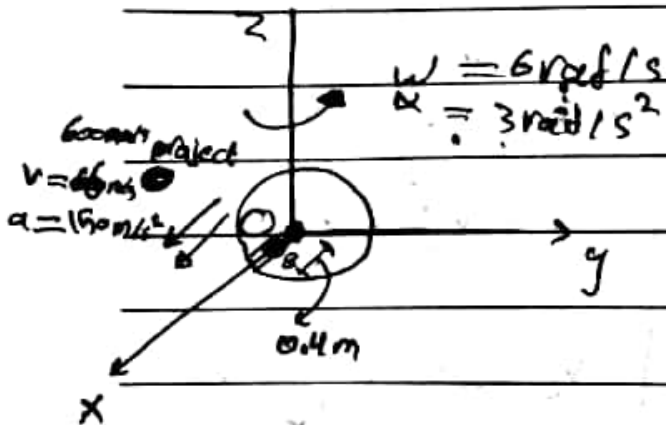
$$\vec{a}_c = 4 (0.4) \sqrt{2} \downarrow 45^\circ + (3)^2 0.4 \sqrt{2} \downarrow 45^\circ$$

$$H: a_s = \downarrow$$

$$V: \alpha_{DE} = \downarrow$$

Subject _____ Day _____ Date _____

16- (129)

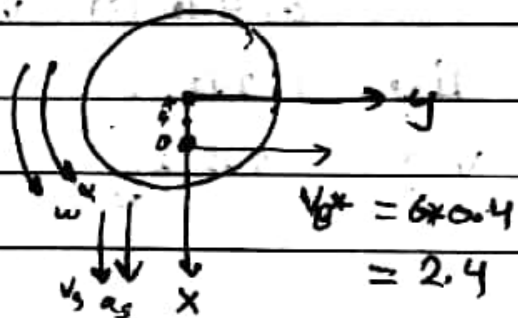


$$v_B =$$

$$a_B =$$

Top view

$$V_B = V_B^* + V_s$$



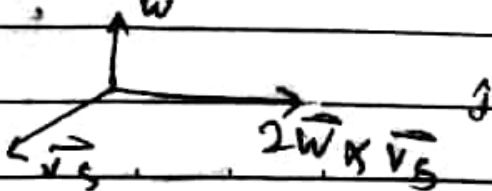
$$V_B = 2.4 \hat{j} + 0.6 \hat{i}$$

$$V_B^*$$

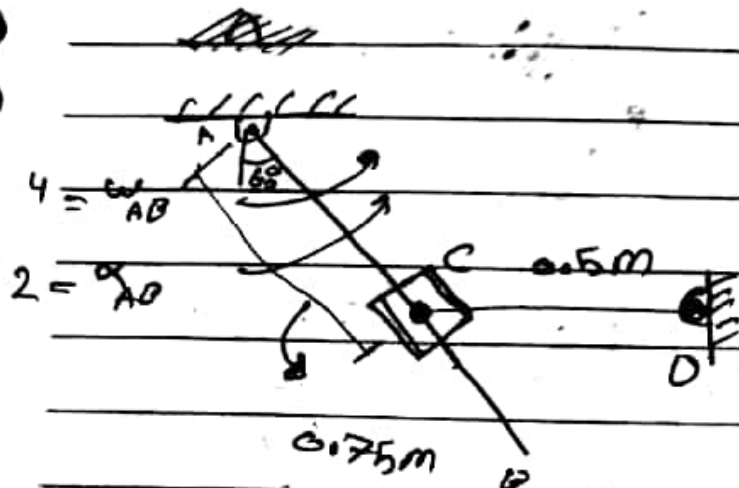
$$* \vec{a}_B = (\vec{a}_{B_t} + \vec{a}_{B_n}) + \vec{a}_s + 2\vec{\omega} \times \vec{V}_s$$

$$= 1.2 \hat{j} (-6^2)(0.4) \hat{i} + 0.15 \hat{i} + 2(6)(0.6) \hat{j}$$

$$= (\dots) \hat{i} + (\dots) \hat{j}$$

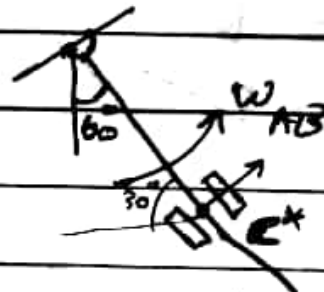
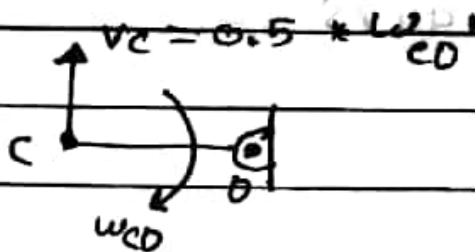


141



ω_{CD}

α_{CD}



$$v_C = \vec{v}_A + \vec{v}_s$$

$$v_C = 4 * 0.75 \angle 60^\circ + v_s \angle 30^\circ$$

Equation

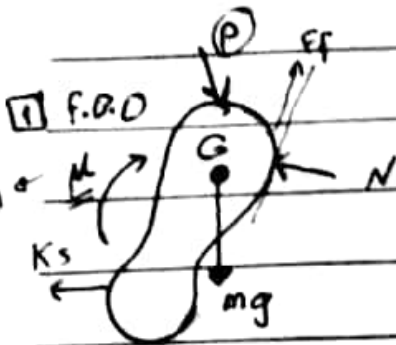
$$0.5 \omega_{CD} \uparrow = 3 \angle 60^\circ + v_s \angle 30^\circ$$

$$H, 0 = 3 \cos 60^\circ + v_s \cos 30^\circ \rightarrow v_s = -\frac{3 \cos 60^\circ}{\cos 30^\circ} = -\frac{3 * 0.5}{0.866} = -1.732$$

$$V, 0.5 \omega_{CD} = 3 \sin 60^\circ - (-1.732) \sin 30^\circ \quad \left\{ v_s = \sqrt{3} \right\}$$

$$\omega_{CD} = 6.9 \text{ rad/s}$$

ch 17: Kinetics of Rigid bodies



size

2 Kinetic Diagram:

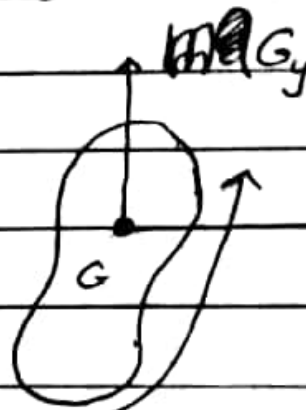
3 EOM

$$+ve \rightarrow \sum F_x = m a_{Gx}$$

$$+ve \uparrow \sum F_y = m a_{Gy}$$

$$\sum F = mg$$

$$+ve \Rightarrow \sum M_G = I_G \alpha$$



planer motion x, y, θ Rot

mass moment of Inertia about G

$$I_G = I_G + m d^2$$

المoment حول مركز الدوران Rot. $I_G = I_G + m d^2$



$$\Sigma M_P = ?? = \bar{I} \alpha + \text{moment of } (m \vec{a}_{Gx})$$

also ΣM_G + moment of $(m \vec{a}_{Gy})$

$$\boxed{\bar{I} \alpha + 0 + 0}$$

$$\Sigma M_P = \Sigma M_{\text{effective}} = \bar{I} \alpha + \text{moment of } (m \vec{a}_G)$$

Disk cylinder sphere

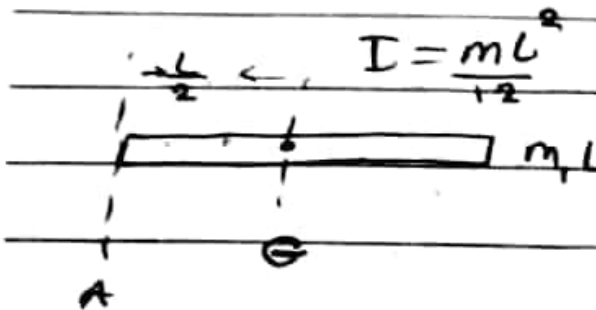
$$\bar{I} = \frac{m L^3}{12}$$

$$\frac{m R^2}{2}$$

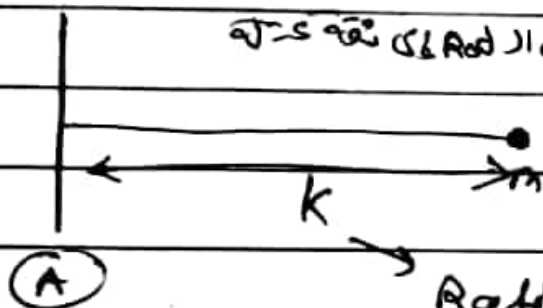
$$\frac{m R^2}{2}$$

$$I_P = \bar{I} + m(d)^2$$

* Radius of Gyration:-



parallel axis theorem $\rightarrow I_A = \frac{1}{12} ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + 3\frac{ml^2}{12} = \frac{ml^2}{3}$



असमान वस्तु (Rod) को $\rightarrow$ *

$$I_A = mK^2$$

Radius of gyration

$$I_A = I_A \rightarrow \frac{mL^2}{3} = mk^2$$

किसी वस्तु का $\rightarrow$ $K = \frac{L}{\sqrt{3}}$

Exp 17-5

Translation

$$m = 200 \text{ kg}$$

$$a = ??$$

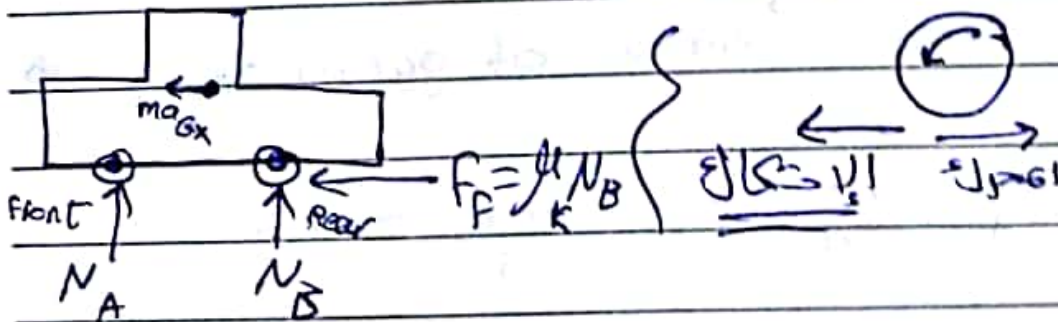


rear wheels

always slipping

→ front → free to Rot.

$$\mu_k = 0.25$$



$$1) \leftarrow +ve \quad \mu_k N_B = m a_{Gx} \quad \text{--- (1)}$$

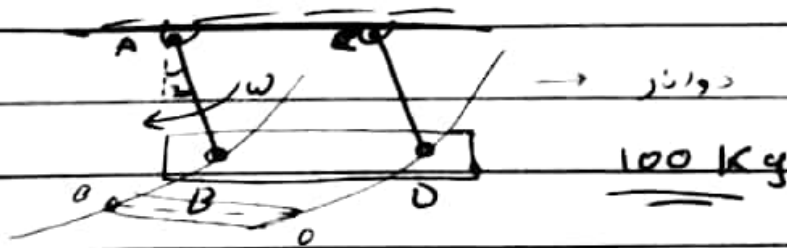
$$2) \uparrow +ve \quad N_A + N_B - mg = 0 \quad \text{--- (2)}$$

$$3) \curvearrowright +ve \quad \sum M_A = \sum M_{eff} = I \alpha + \text{moment of } m a_{Gx}$$

$$(1.25 + 0.75) N_B - mg(1.25) = 0.3(m a_{Gx})$$

$$\boxed{a_{Gx} = \checkmark}$$

Ex 17-7



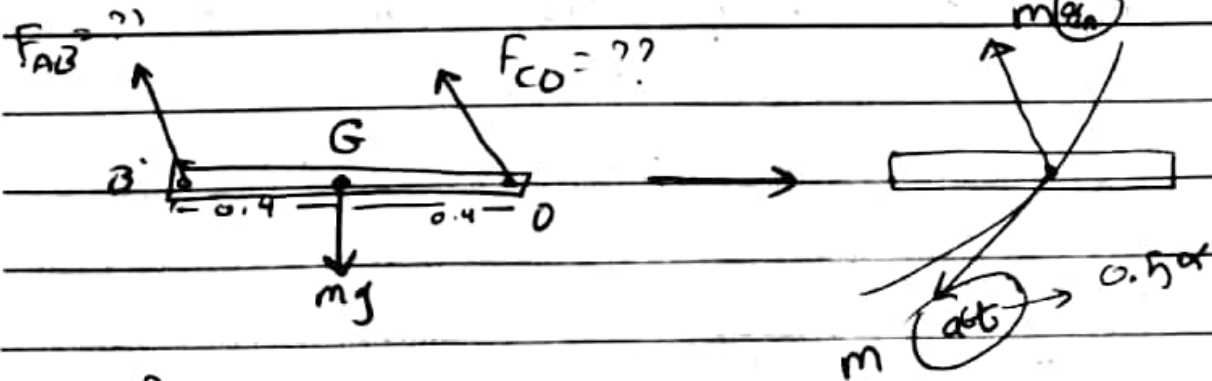
$F_{AB}, F_{CD} ??$

$$\overline{AB} - \overline{CD} = 0.5 \text{ m}$$

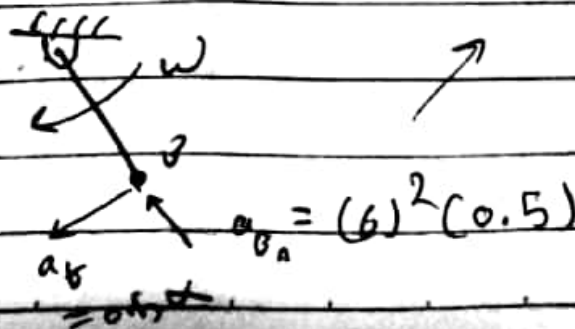
$$W = 6 \text{ rad/s}$$

translation $\leftarrow$ curvilinear motion

$$\text{curve } \rho = 0.5 \text{ m}$$



$$\sum F = ma$$



Subject _____ Day _____ Date _____

(17,)

2

3

4

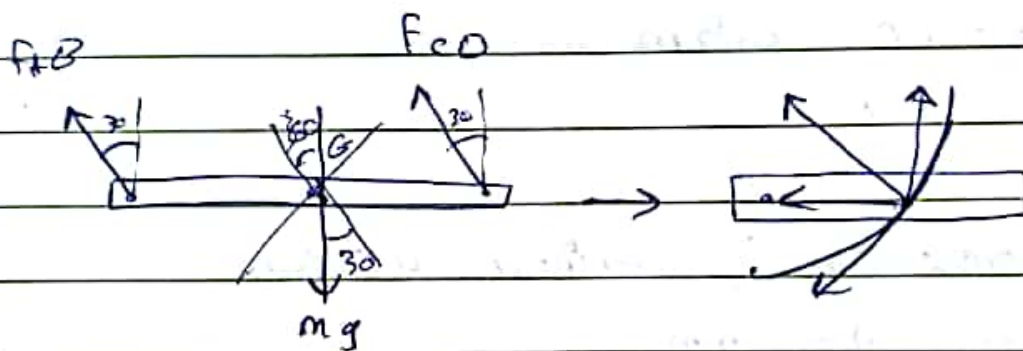
5

6

$$\sum f_t = ma_t$$

$$\sum f_n = ma_{gn}$$

$$\sum M_p = \sum M_{eff}$$



$$mg \sin 30 = ma_t = 4.905 \text{ m/s}^2$$

$$f_{AB} + f_{CD} - mg \cos 30 = 100(18) \quad (2)$$

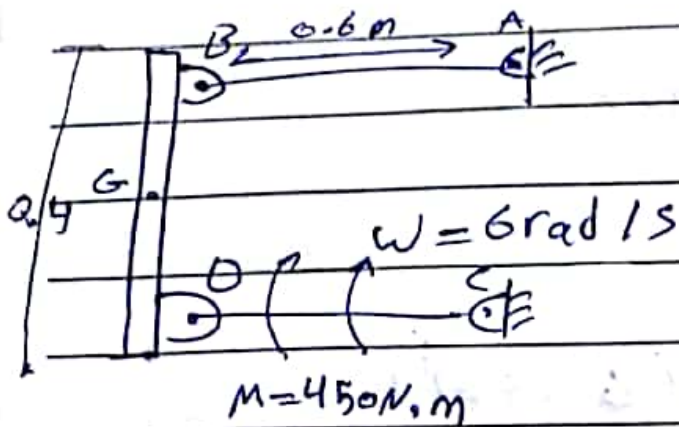
$$\sum M_B = \sum M_{eff}$$

$$0.4 mg - 0.8 f_{CD} \cos 30 = -(0.4)(100)18 \cos 30 + 0.4(100)a_t \cos 30$$



Family

f17-6



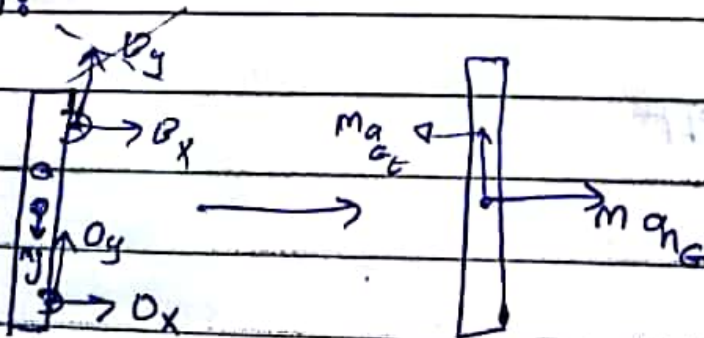
$$F_{AB} = ??$$

$$D_x, D_y = ?$$

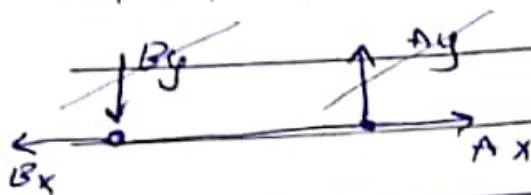
$$\alpha = ??$$

$$m = 5 \text{ kg}$$

Soln:

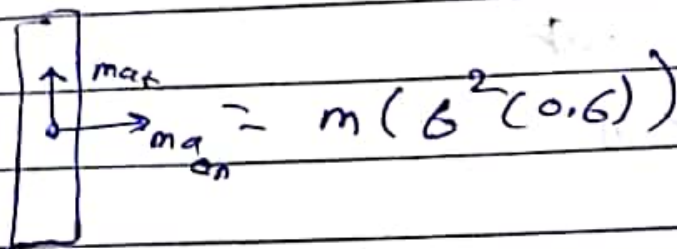


$\sum M_C = I_C \alpha + \text{moment of } (m a)$
 $\sum M_C = 0 \quad 450 - D_y(0.6) = 0$
 $D_y = 750 \text{ N}$



Two force member

$$B_x = \checkmark$$



* Rotation about fixed Axis?

1) F.B.D

2) Kinetic

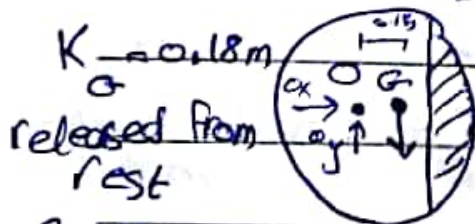
$$3) \sum \vec{F} = m\vec{a}$$

$$\sum M = \sum M_{eff}$$

Ex

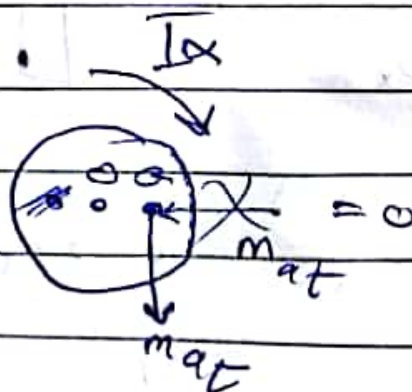
17.8

2.5 kg



$$\omega = 0$$

$$\alpha = \omega^2 r = 0$$



$$+ve \leftarrow \sum F_x = m a_{Gx}$$

$$-0_x = 0 \rightarrow \boxed{0_x = 0} - 0$$

$$+ve \uparrow \sum F_y = m a_{Gy}$$

$$0_y = mg = -m a_{Gy} \quad (2)$$

$$+ve \curvearrowright \sum M_O = \sum M_{eff} = \bar{I} \alpha + \text{moment of } (m)$$

$$mg(0.15) = \bar{I} \alpha + (0.15)(0.15\alpha) \quad (25)$$

$$\alpha = \checkmark$$

$$a_{Gt} = 0.15 \alpha$$

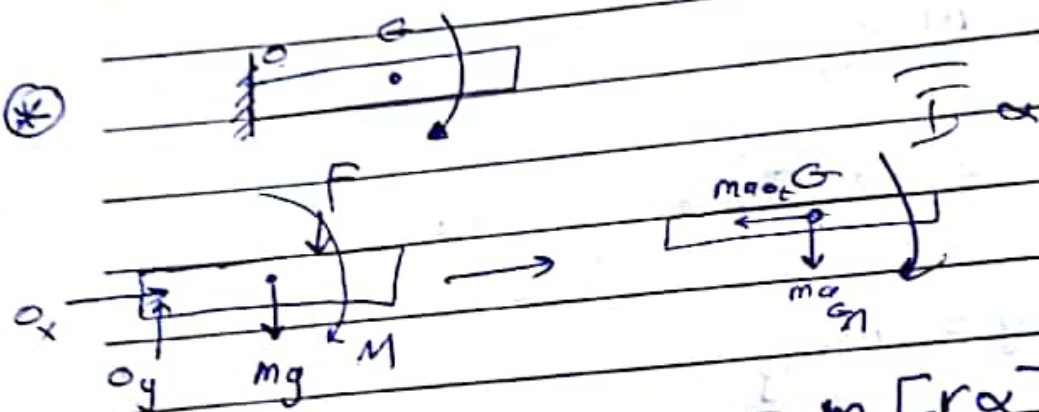
$$\bar{I} =$$

Subject _____

Day _____

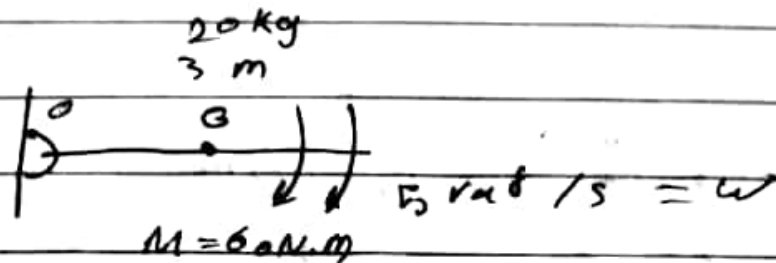
Date _____

⑧

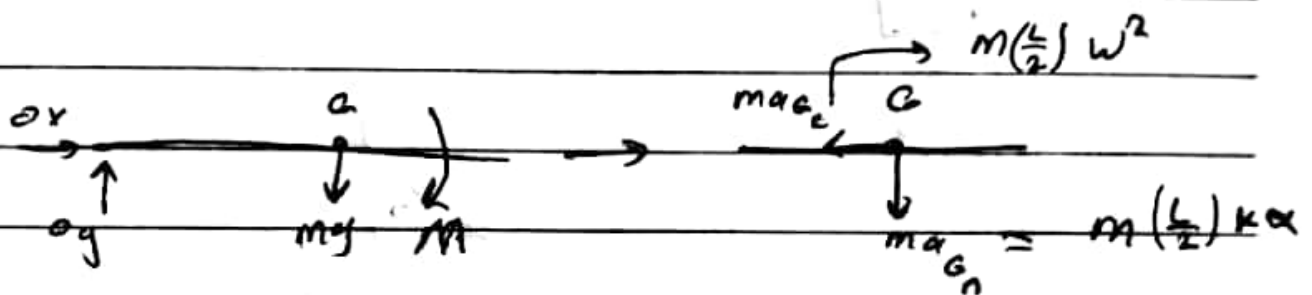


$$ma_{Gt} = m[r\alpha]$$

Exp 17-9:



soln



$$\tau_x = -20 * 1.5 * (5)^2 = \checkmark \text{ N}$$

??

??

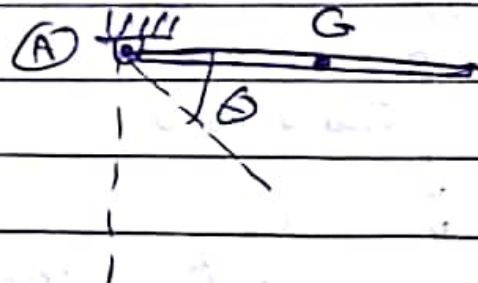
$$0_y - mg = -20(1.5)\alpha \rightarrow 0_y = \checkmark$$

$$\sum M_o = \sum M_{eff} = I_o \alpha$$

$$\begin{aligned} * I_o &= \frac{1}{2} m L^2 + m \left(\frac{L}{2}\right)^2 \\ &= \frac{m L^2}{3} = \frac{20(3)^2}{3} \\ &= 60 \text{ kg.m}^2 \end{aligned}$$

$$20 g \left(\frac{L}{2}\right) + 60 = 60 \alpha \rightarrow \alpha = 1 \text{ rad/s}^2$$

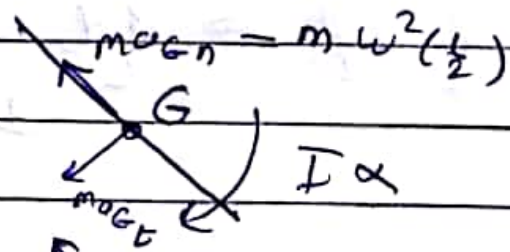
Exp 17-11



released from rest
when $\theta = 0$

A_y, A_x ?
at $\theta = 90^\circ$

solving



$$F_A = \sqrt{A_t^2 + A_n^2}$$

$$\sum F_t = m a_{Gt}$$

tangential
 mg

$$-A_t + mg \cos \theta = m \left(\frac{L}{2} \right) \alpha \quad (1)$$

$$\sum F_n = m a_{Gn}$$

$$A_n - mg \sin \theta = m \left(\frac{L}{2} \right) \omega^2 \quad (2)$$

$$\sum M_A = I_A \alpha = m \frac{L^2}{3} \alpha \rightarrow \alpha = \frac{3mg \cos \theta}{L}$$

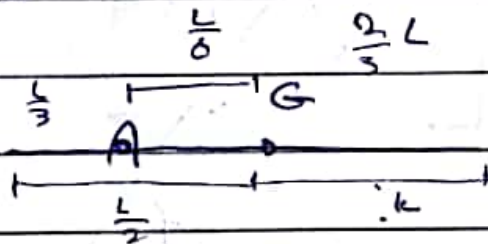
$$\alpha = \frac{3g}{2L} \cos \theta = \omega \frac{d\omega}{d\theta}$$

$$\int_0^{\omega} \omega d\omega = \frac{3g}{2L} \int_0^{90} \cos \theta d\theta$$

$$\frac{\omega^2}{2} = \frac{3g}{L} \rightarrow \omega^2 = \frac{2 \times 3g}{L}$$

(2) $\frac{1}{2} L \omega^2$

17-59



m

$\theta = 0 \rightarrow$ From rest

(θ_x, θ_y) at $\theta = 90^\circ$

$$I_G = \frac{1}{12} m L^2 + m \left(\frac{L}{6} \right)^2$$

$$= m L^2 \left(\frac{3+1}{36} \right)$$

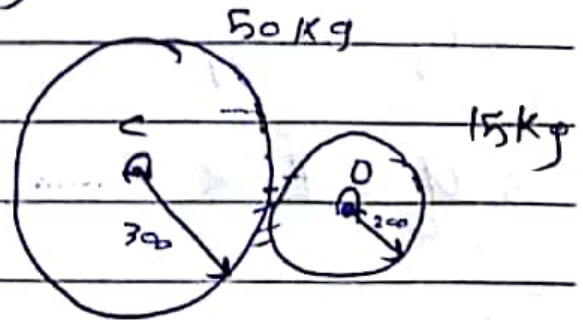
$\omega \rightarrow$ ←

17-65

$$M = 200(1 - e^{-0.2t})$$

$$K_C = 0.25 \text{ m}$$

$$K_D = 0.15 \text{ m}$$



$$\omega_0 = 0, \omega = ? \text{ at } t = 3 \text{ s}$$

$$\text{the) } \Sigma M_C = I_C \alpha$$

$$200(1 - e^{-0.2t}) = 50(0.25)^2 \alpha$$

$$\alpha = \frac{200(1 - e^{-0.2t})}{50(0.25)^2} = f(t) = \frac{dw}{dt}$$

$$\int_0^{\omega} dw = \int_0^3 f(t) \cdot dt$$

$$* \omega = \int_0^3 \frac{200(1 - e^{-0.2t})}{50(0.25)^2} dt$$

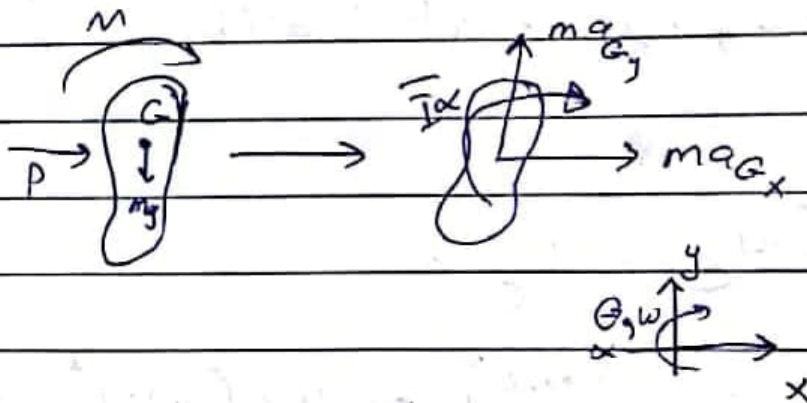
* General plane motion

$$\sum F_x = m a_{Gx}$$

$$\sum F_y = m a_{Gy}$$

$$\sum M_G = \bar{I} \alpha \quad \boxed{G.P.M}$$

$$\sum M_P = \sum M_{\text{effective}}$$



Exp 12-13

$$\sum F_x = m a_{Gx} \quad (1)$$

$$f_p = m a_{Gx} \quad (1)$$

$$a_G = ??$$

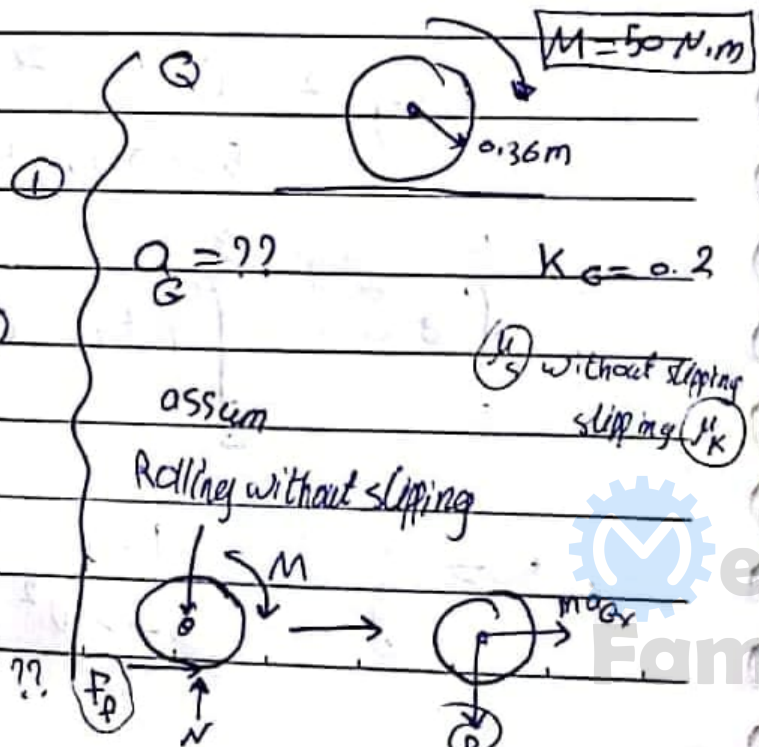
$$K_G = 0.2$$

assum

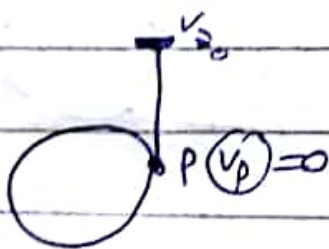
Rolling without slipping

(1) without slipping
(2) slipping μ_k

ملاحظات



Exp: 17-14



$$\sum f_y = m a_{Gy} \quad - (1)$$

$$100 + T - mg = m a_{Gy}$$

$$\sum M_P = \sum M_{eff}$$

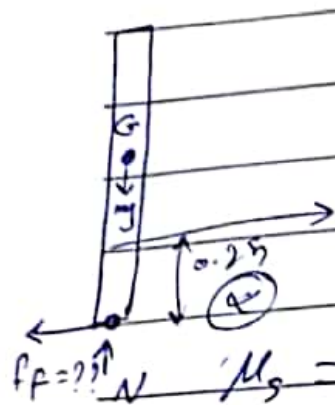
$$100(0.5 + 0.2) - mg(0.5) =$$

$$I \alpha + m a_{Gy} (0.5)$$

$T = \checkmark$

0.5α

* Exp 17-14

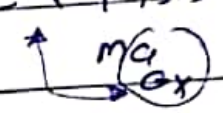


$\mu_s = 0.3$
 $\mu_k = 0.25$

$$400 - f_f = m a_{Gx} \quad (1)$$

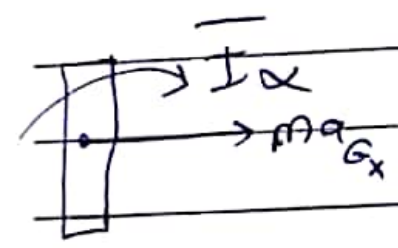
$$N - mg = 0 \quad (2)$$

$$400(0.5) = I \alpha + 1.5$$

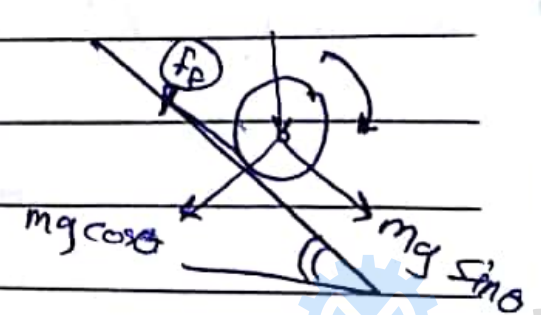
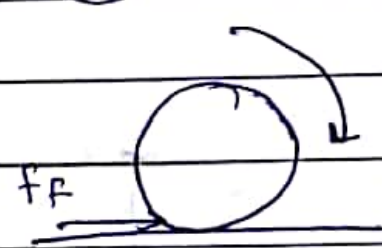


assume no slipping

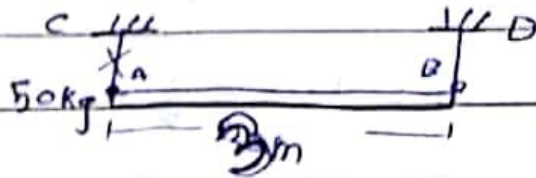
$$a_{Gx} = 1.5 \alpha$$



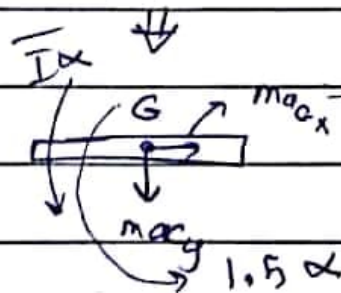
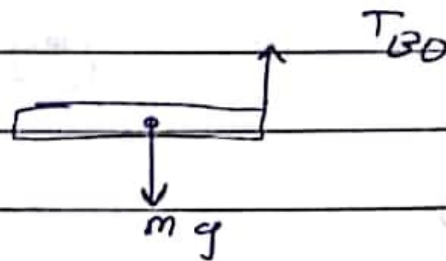
(*)



Ex 17-15



$$T_{BD} = ??$$



$$0 = ma_{Gx} \quad a_{Gx} = 0 \quad (1)$$

$$T_{BD} - mg = -m(a_{Gy}) \quad (2)$$

$$\sum M_B = \sum M_{eff}$$

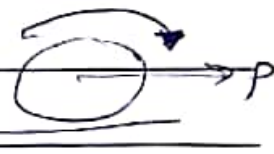
$$mg(1.5) = I\alpha + 1.5 ma_{Gy}$$

$$= \frac{1}{2} m l^2 \alpha + 1.5 m (1.5 \alpha) \rightarrow \alpha = \text{rad/s}$$

$$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}^t + \vec{a}_{G/B}^n$$

$$a_{Gy} \downarrow + a_{Gx} \rightarrow = 0 + 1.5 \alpha \downarrow + \omega^2 (1.5) \rightarrow$$

Link is rotating apply eqn of force



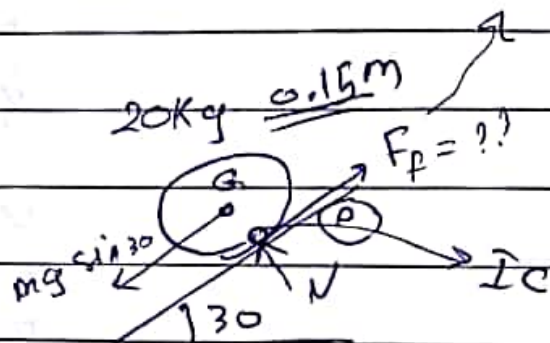
F 17-16

(mg) $\downarrow$ $\vec{r}_{G/B}$ & $\vec{v}_G$

w/o slipping

$$\alpha = ??$$

$$a_G = ??$$



$$a_G = r \alpha$$

* w/o $r \alpha = s_G$

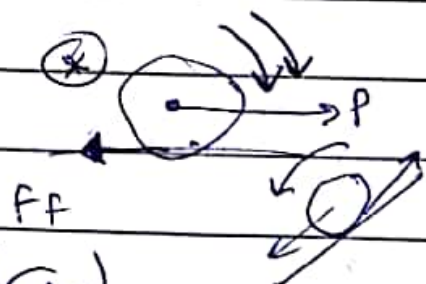
$$a_G = r \alpha$$

$$\sum M_P = \sum M_{eff}$$

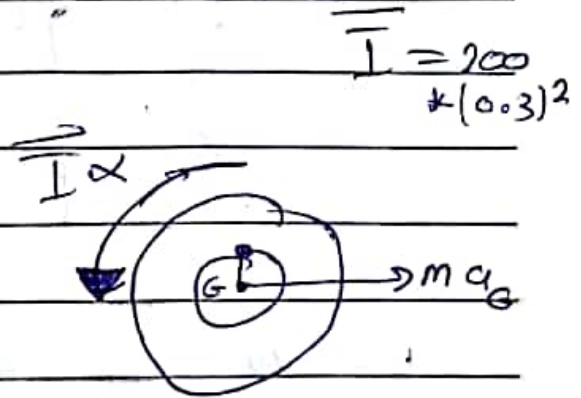
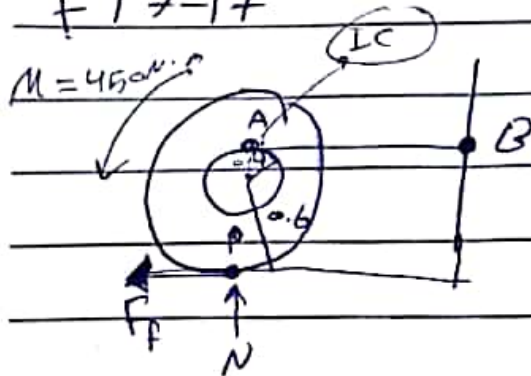
$$mg \sin 30 \cdot r = I \alpha + r m (r \alpha)$$

$$\alpha = \omega$$

$$a_G = r \alpha$$



f 17-17



200 Kg

$\mu_k = 0.3$

$\mu_k = 0.2$

$a_G = ??$ $T = ??$

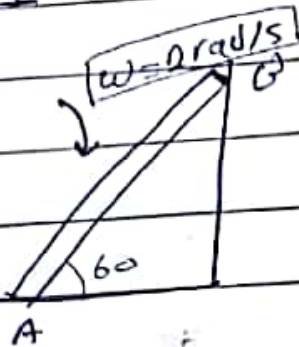
$$T - F_f = m \cdot a_G \quad \text{--- (1)}$$

$$N - mg = 0$$

$$\sum M_A = \sum M_{eff}$$

$$450 - \mu_k mg(0.6 + 0.4) = I \alpha + 0.4(m) a_G$$

17-94



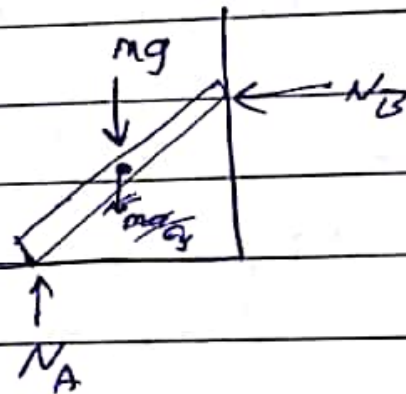
$$L = 3\text{m}$$

$$m = 12\text{kg}$$

$$\alpha = ??$$

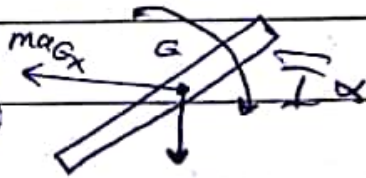
$$N_A = ??$$

$$N_B = ??$$

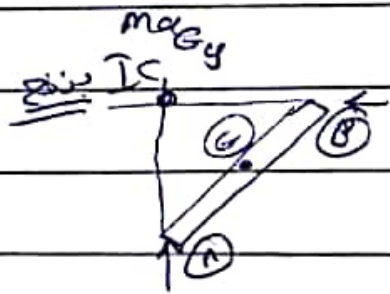


$$\leftarrow +ve \quad N_B = ma_x \quad (1)$$

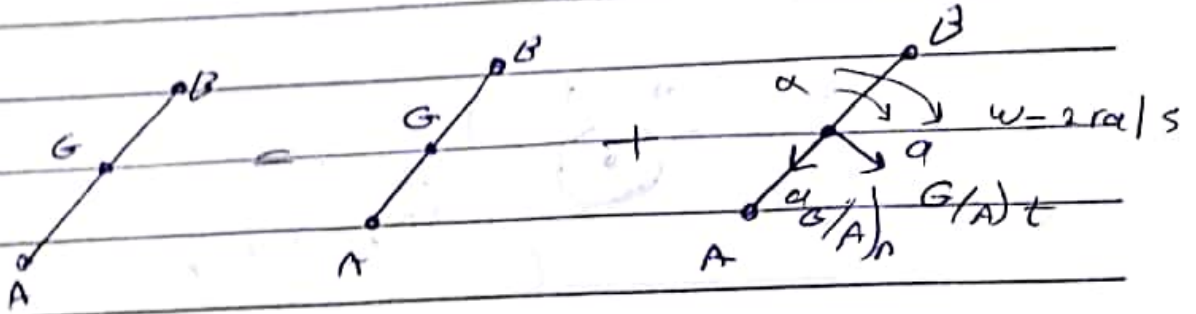
$$+ve \downarrow \quad mg - N_A = ma_y \quad (2)$$



$$+ve \quad \sum M_{IC} = \sum M_{eff}$$



$$mg(1.5 \cos 60) =$$



$$\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}t + \vec{a}_{G/A}n$$

$$\leftarrow a_{Gx} + a_{Gy} \downarrow = a_A \leftarrow + (2)^2 \cdot 1.5 \text{ m/s}^2$$

$$+ 1.5 \times \sqrt{3} \text{ m/s}^2$$

$$a_B = a_A + a_{B/A}t + a_{B/A}n$$

$$a_B \downarrow = a_A \leftarrow + 3 \times \sqrt{3} \text{ m/s}^2 + (2)^2 (3) \text{ m/s}^2$$



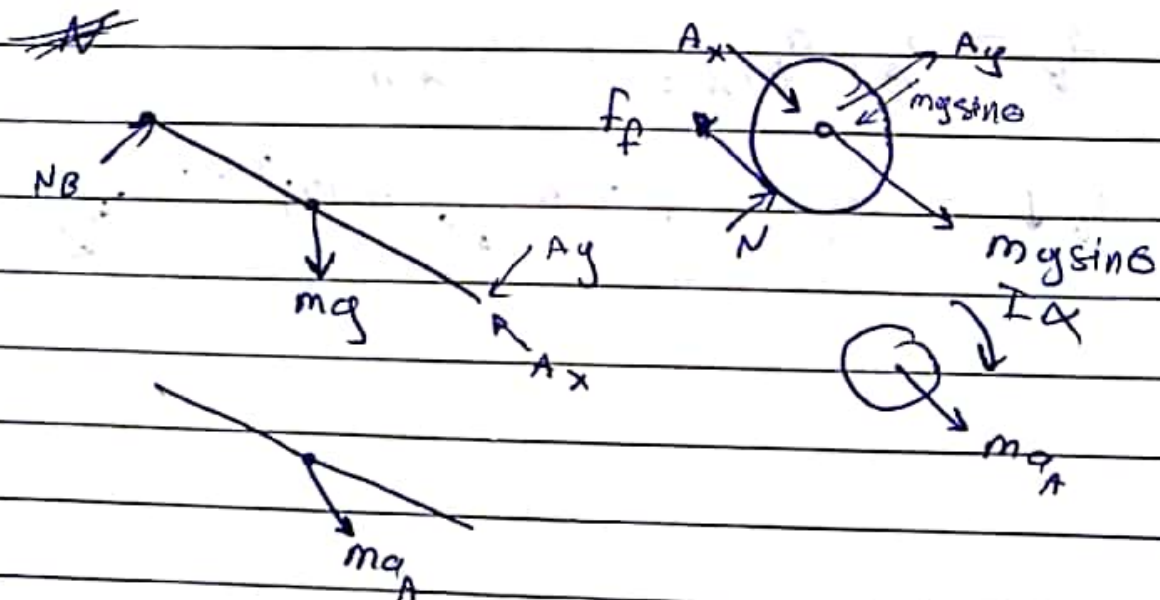
$$\alpha = ??$$

$$\mu_s = 0.6$$

$$\mu_k = 0.4$$

w/o slipping

$$a_A = a_G = r\alpha = 0.3\alpha$$

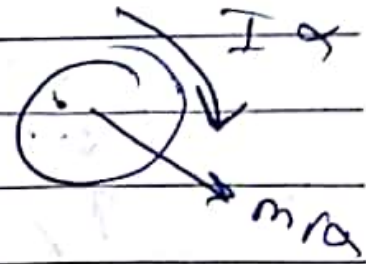
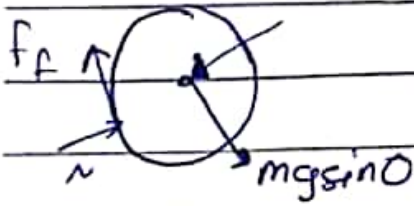


Subject _____

Day _____

Rolling
w/o slipping
Date 10/10/20

(97)



$$mg \sin \theta - f_f = m(r\alpha)$$

$$N - mg \cos \theta = 0$$

$$\sum \tau_c = \sum \tau_{eff}$$

$$mg \sin \theta r = \frac{1}{2} m I \alpha + r(m r \alpha)$$

$$\alpha = \alpha$$

$$a_A = r \alpha$$

$$f_f = \mu N$$

(و) ١.٥١

$$\mu_s N \rightarrow \mu_k N$$

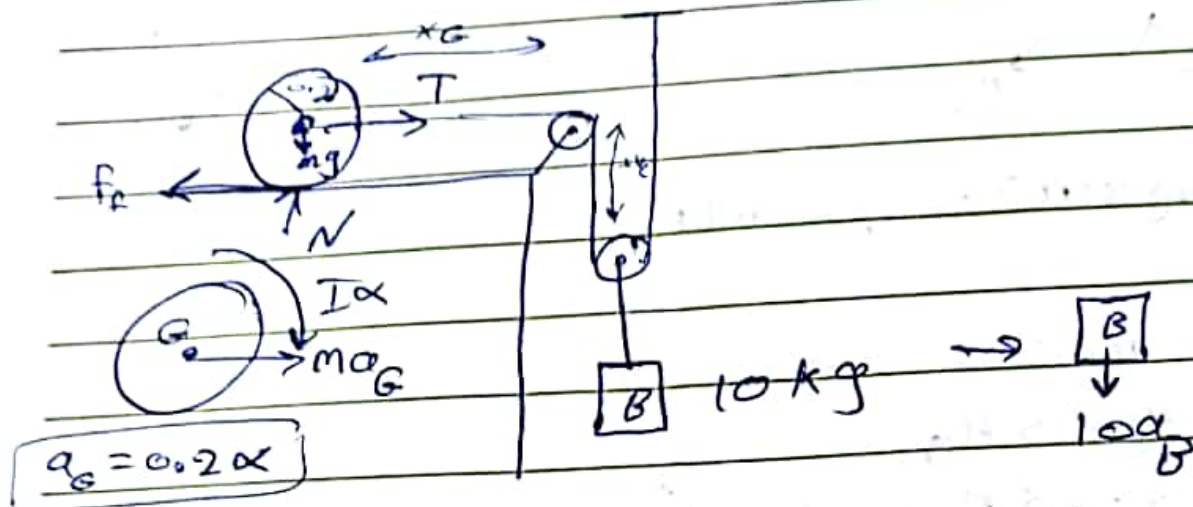
$$mg \sin \theta - \mu_k mg \cos \theta = m a_A$$

$$(mg \sin \theta) r = I \alpha + r m a_A$$

$$\sum \tau \quad (f_f) r = I \alpha$$

w/o (12)

Subject _____ Day _____ Date _____

17-112

α
 a_B
 T

$$\oplus \downarrow 10g - 2T = 10a_B - (1)$$

$$-N + mg = 0 \Rightarrow N = mg - (2)$$

$$T - F_p = m(0.2\alpha) - (3)$$

$$\oplus \downarrow \Sigma M_p = \Sigma M_{\text{eff}}$$

$$T(0.2) = \frac{1}{2} m r \alpha^2 + r(m r \alpha) - (4)$$

$$\# \quad x_G + 2y_B = L$$

$$-a_G + 2a_B = 0$$

$$a_G = 2a_B = \boxed{0.2\alpha}$$

$$\boxed{a_B = 0.1\alpha}$$

⑥

From (1) $T = \frac{10g}{2} - \frac{10}{2} \alpha$

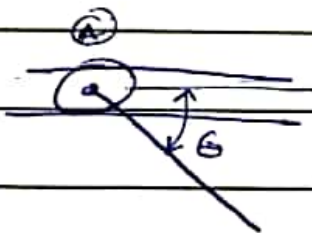
$T = 5g - 0.5 \alpha$

$\alpha = \checkmark$

$a_D = 0.1 \alpha \checkmark$

$T = \checkmark$

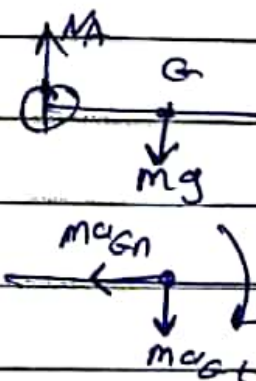
F-17-18



$\sum f_x = m a_{Gn}$

$0 = m a_{Gn} \quad (1)$

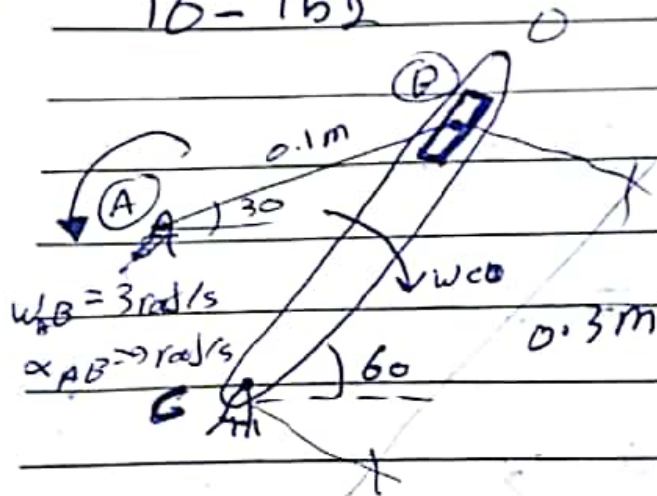
$mg - N_A = m a_{Ge} \quad (2)$



$+\vee \sum M_A = \sum M_{eff}$

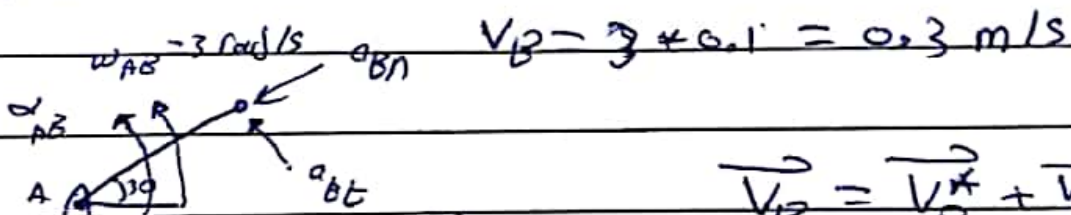
$mg \frac{L}{2} = \frac{1}{12} m L^2 \alpha + \frac{L}{2} * m a_{Ge} \quad \frac{L}{2} \alpha$

16-152



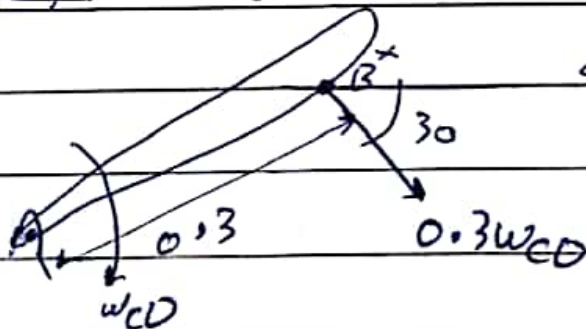
① Velocity analysis.

② acc



$$\vec{V}_B = \vec{V}_B^* + \vec{V}_{\text{sliding}}$$

$$0.3 \vec{\omega}_{AB} =$$



$$\vec{V}_B = \vec{V}_B^* + \vec{V}_{\text{sliding}}$$

$$0.3 \vec{\omega}_{AB} = 0.3 \omega_{CD} \vec{e}_{30} + V_s \vec{e}_{60}$$

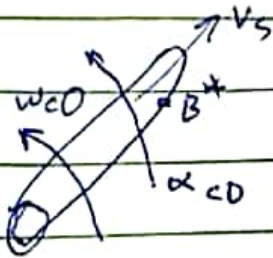
$$0.3 \sin 60 = -0.3 \omega_{CD} \sin 30 + V_s \sin 60$$

$$-0.3 \cos 60 = 0.3 \omega_{CD} \cos 30 + V_s \cos 60$$

$$\omega_{CD} = V$$

$$(-ve)$$

$$V_s = V$$



$$\vec{a}_B = \vec{a}_B^* + \vec{a}_{\text{sliding}} + 2\vec{\omega}_{CD} \times \vec{V}_s$$

$$\vec{a}_{B_t} + \vec{a}_{B_n} = \vec{a}_{B_t}^* + \vec{a}_{B_n}^* + a_s \Delta 60$$

$$+ 2(\omega_{CD})(V_s)$$

$$0.9 \Delta 60 + 3^2 (\omega_{CD})^2 = 0.3 \Delta 60 + a_s \Delta 60$$

$$+ (\omega_{CD})^2 (0.3) + 2\omega_{CD} V_s$$

ch. 18 "Energy methods"



conservative
force

path is conservative

$$T_{\text{kinetic}} = \underbrace{\frac{1}{2} m V_G^2}_{\text{trans}} + \underbrace{\frac{1}{2} \bar{I} \omega^2}_{\text{Rot.}}$$

$$T_1 + U_{1 \rightarrow 2} = T_2$$

$$\int \rho d\vec{r} + \left[\frac{-K}{2} [x_2^2 - x_1^2] \right] + \left[(\mu_k N) \times \Delta s \right]$$

$$- mg [y_{G_2} - y_{G_1}] + \int \vec{M} \cdot d\vec{\theta}$$



(*) conservation of energy [when $U_{1 \rightarrow 2} = 0$]

$$\begin{aligned} U_{1 \rightarrow 2} &= 0 \\ U_{1 \rightarrow 2} &= 0 \\ U_{1 \rightarrow 2} &= 0 \end{aligned}$$

kinetic potential

$$\begin{aligned} V_g &= mgy_G \\ V_s &= \frac{1}{2} Kx^2 \end{aligned}$$

$$\left[\frac{1}{2} m V_{G_1}^2 + \frac{1}{2} \bar{I} \omega_1^2 \right] + V_{s_1} + V_{g_1} =$$

$$\left[\frac{1}{2} m V_{G_2}^2 + \frac{1}{2} \bar{I} \omega_2^2 \right] + V_{s_2} + V_{g_2}$$

kinetic

potential

Subject _____

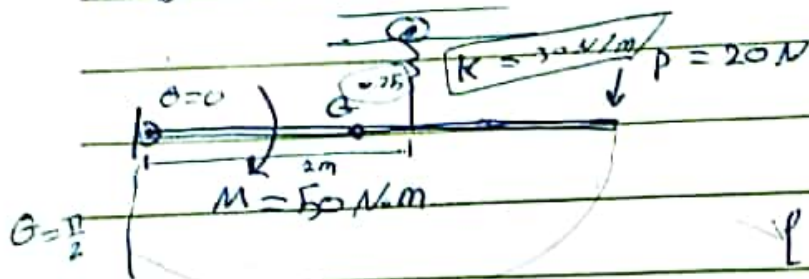
Day _____

Date

 $\frac{2}{3}$
4

Ex: 18-1

$$(*) L_2 = 2 + 0.75$$



$$x_2 = L_2 - L_0$$

$$L = 30 \text{ m}$$

$$L_0 = 0.5 \text{ m}$$

$$m = 10 \text{ kg}$$

$$u_{1 \rightarrow 2})_{\text{total}} = ??$$

Type of motion $\rightarrow$ Rotation

$$u_{1 \rightarrow 2})_g = -mg \left[y_{g2} - y_{g1} \right] = -mg [-1.5]$$

$$u_{1 \rightarrow 2})_p = 80 \left(\frac{2\pi (3)}{4} \right)$$

$$u_{1 \rightarrow 2})_s = -\frac{k}{2} [2.25^2 - 0.25^2]$$

$$(*) T_2 = \frac{1}{2} [m v_g^2 + I \omega_2^2]$$

statp

②

$$(1.5 \times \omega_2)^2$$

$$\omega_2 = \omega$$

Subject _____

Day _____

Date _____

1
2
3
4

* Exp 18-5:

10 Kg

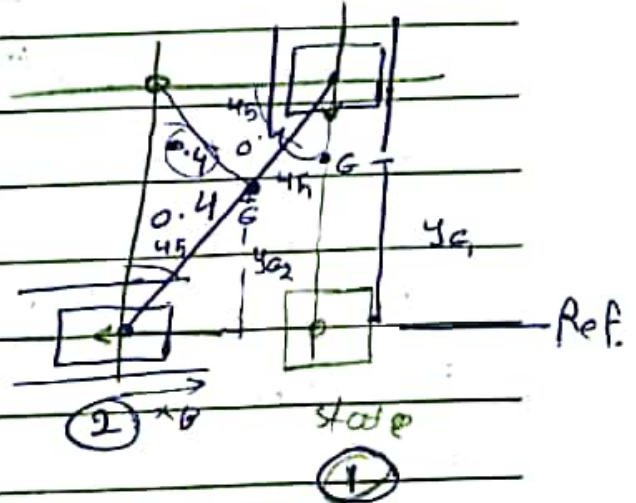
at rest when

$$\theta = 0$$

$$W = ?$$

when $\theta = 45^\circ$

$$P = 50\text{ N}$$



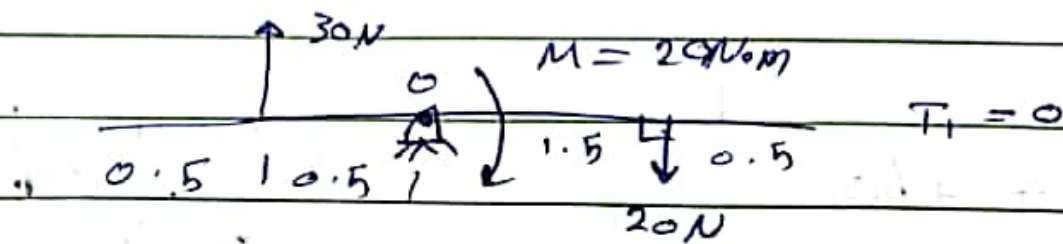
$$T_2 = \frac{1}{2} m v_{G_2}^2 + \frac{1}{2} I \omega_2^2 = \frac{1}{2} [m(0.4)^2 + I] \omega_2^2$$

$$v_{G_2} = 0.4 \omega_2$$

$$U_{1 \rightarrow 2)_{\text{ext}}} = \int \vec{P} \cdot d\vec{r} = 50 \Delta x_P = 50 \times 0.8 \cos 45^\circ$$

$$U_{1 \rightarrow 2)_{\text{g}}} = -mg[y_{G_2} - y_{G_1}] = -mg[0.4 \times \sin 45^\circ - 0.4]$$

F 18 5



30 Kg

from rest

$$\omega = ??$$

$$\theta = 4 \text{ rev}$$

$$* T_2 = \frac{1}{2} I \omega_2^2 + \frac{1}{2} m v_{G_2}^2$$

$$T_2 = \frac{1}{2} [I + m (0.5)^2] \omega_2^2$$

$$\theta = 4 \times 2\pi$$

$$U_{1 \rightarrow 2} = 20 \times (4) (2\pi)$$

$$U_{1 \rightarrow 2} = 20 \times 4 (2\pi) \times 1.5$$

$$U_{1 \rightarrow 2} = 30 \times 4 \times 2\pi \times 0.5$$

$$\omega_2 =$$

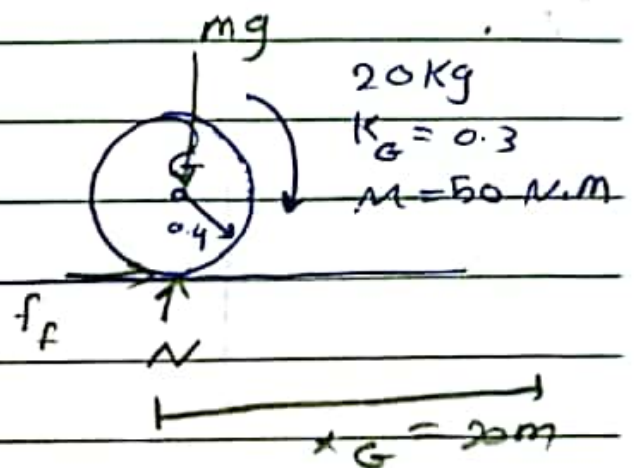
منه

F-18-6

Rolls w/o slipping

$w = ??$

$x_G = 20\text{ m}$



$$(*) \quad x_G = 20\text{ m} = \theta (0.4) \quad \rightarrow \quad u_{1 \rightarrow 2} = 0$$

$$\theta = \frac{20}{0.4} = 50\text{ rad}$$

$$u_{1 \rightarrow 2} = 0 \quad \text{نفس المكان}$$

$$0 + 50 \theta = \frac{1}{2} \times 20 (0.3)^2 \omega_2^2 + \frac{1}{2} (20) v_G^2$$

$(0.4 \omega_2)^2$

(19.8)

The system starts from rest.
Find the velocity of the 100kg mass after the pulley has turned 5 rev.

Solution :-

Note → it's better to apply work & Energy principle on the system not on each body not to deal with more unknowns like (T), (Internal Forces).

$$T_1 + U_{1 \rightarrow 2} = T_2 \quad , \quad T_1 = 0 \text{ (starts from rest)}$$

$$T_2 = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$
$$= \frac{1}{2} (100) V^2 + \frac{1}{2} (25) (0.15)^2 \omega^2$$

$$(I = m k^2 = 25 (0.15)^2)$$

$$* \text{ From kinematics : } V = 0.1125 \omega$$

$$\omega = V / 0.1125$$

$$r_A \theta_A = r_B \theta_B$$

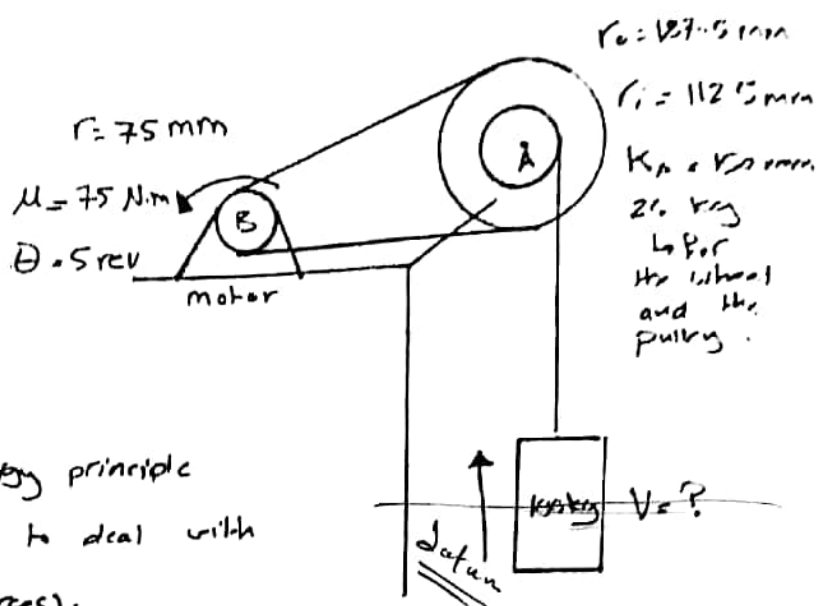
$$* \text{ to find } y_2 : \theta_{\text{wheel}} = \frac{5(2\pi)(75)}{187.5}$$
$$= 12.57$$

$$y_2 = \theta_{\text{wheel}} (0.1125)$$
$$= 1.414$$

$$U_{1 \rightarrow 2} = (U_{1 \rightarrow 2})_p + (U_{1 \rightarrow 2})_g(\text{mass})$$
$$= \int \vec{M} \cdot d\vec{\theta} - mg(y_2 - y_1)$$
$$= 75(5 \times 2\pi) - 100(9.81)(y_2 - 0)$$

now you have an eqn with one variable (V).

$$V = \quad \text{m/s}$$

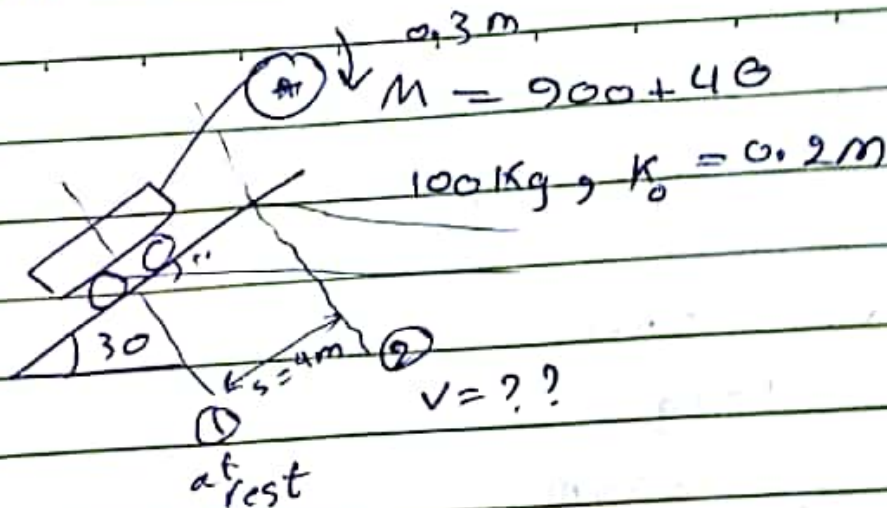


⊕ (1) 100 kg

Subject _____

Day _____

Date _____



$$* y_2 = 4 \sin 30^\circ$$

$$T_1 = 0$$

at rest

$$T_2 = \frac{1}{2} \left[300 V^2 + 100 (0.2)^2 \left(\frac{V}{0.3} \right)^2 \right]$$

$$u_{1 \rightarrow 2)g} = -300g [4 \sin 30^\circ - 0]$$

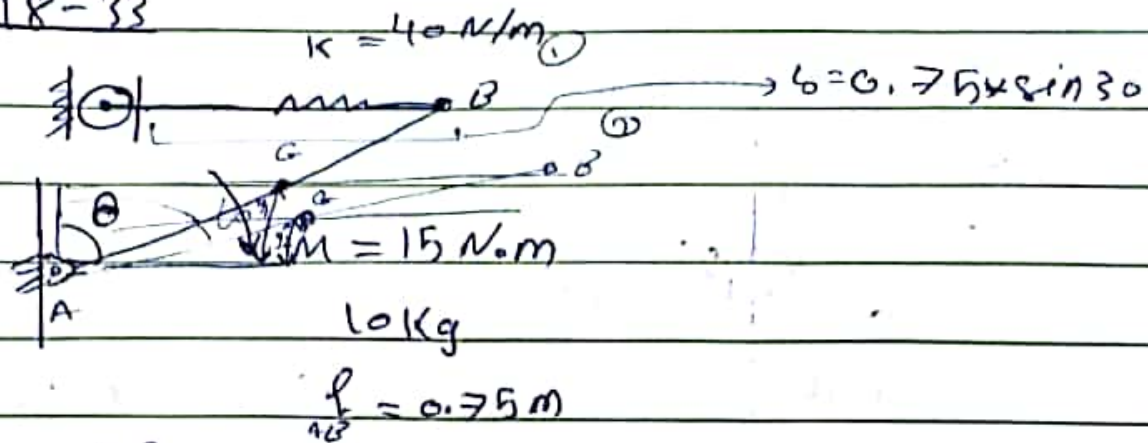
$$u_{1 \rightarrow 2)M} = \int_0^{\theta_2} (900 + 4\theta) d\theta$$

$$\theta_2 \rightarrow s = 4\text{ m} = \theta_2 (0.3)$$

$$T_1 + u_{1 \rightarrow 2)g} + u_{1 \rightarrow 2)M} = T_2$$

$$V = m/s$$

18-33



released from
 rest
 when $\theta = 30^\circ$ unstretched
 $\omega = ?$ when $\theta = 60^\circ$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} I_A \omega^2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m v_G^2$$

$$T_2 = \frac{1}{2} \left(\frac{m l^2}{3} \right) \omega^2 \Rightarrow \omega = v$$

$$u_{1 \rightarrow 2) s} = -\frac{k}{2} [0.75 \times \sin 60 - 0.75 \times \sin 30]^2 - 0]$$

$$u_{1 \rightarrow 2) g} = -mg [0.75 \times \cos 60 - 0.75 \times \cos 30]$$

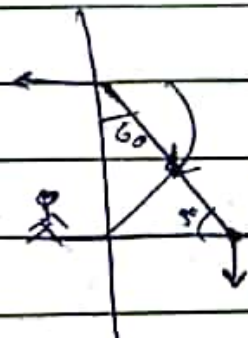
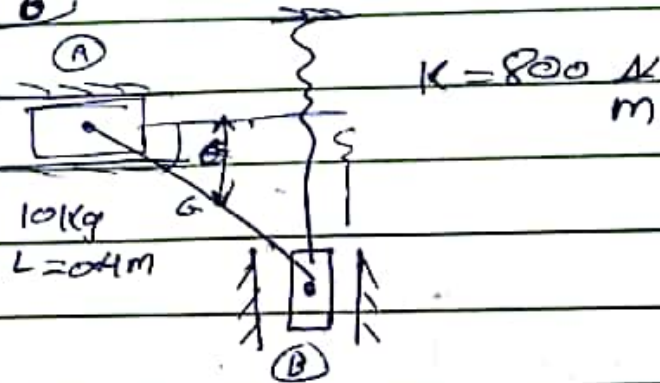
$$u_{1 \rightarrow 2) M} = 15 \left((60 - 30) \times \frac{\pi}{180} \right) = v$$

W/P

Ex ②

Subject _____ Day _____ Date _____

18-6



$$a_{\theta} = 0 \rightarrow s_1 = 0$$

[W] at $\theta = 30^\circ$

$$L = 0.4 \text{ m}$$

isb
Friction
external

$$T_1 + V_1g + V_1s = T_2 + V_2g + V_2s$$

$$V_2g = mg y_{G_2} = 10g (-0.2 * \sin 30) = -10$$

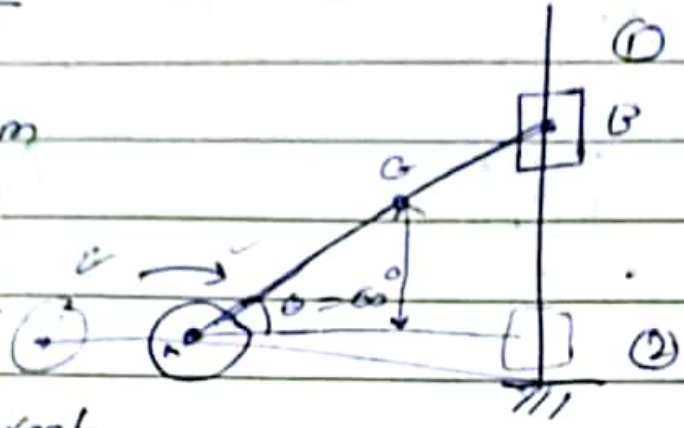
$$V_2s = \frac{1}{2} K s_2^2 = \frac{800}{2} * (0.4 * \sin 30)^2$$

$$T_2 = \frac{1}{2} [I \omega^2 + m v^2] = \frac{1}{2} [\frac{10 (0.4)^2}{12} + 10 (I_{CG} \omega)^2]$$

G.P.M

Ex 18-8

$L = 0.6 \text{ m}$
rod
rod 5 Kg
disk 10 Kg



$\theta = 60^\circ$ at rest

W when $\theta = 0$

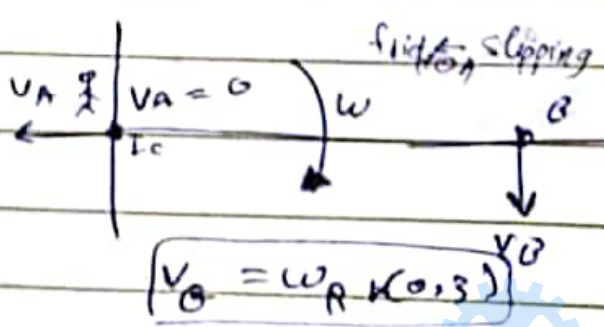
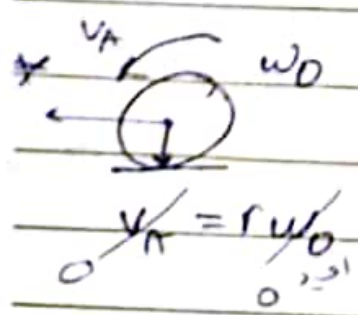
mass negligible

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \left[\frac{1}{2} I_O \omega_O^2 + m \frac{v_A^2}{2} \right] + \frac{1}{2} \left[\frac{1}{2} I_P \omega_R^2 + m_R \frac{v_C^2}{2} \right]$$

$$v_{2g} = 0$$

$$v_{1g} = 5 * g (0.3 \sin 60) + 0 + 0$$



when $\omega = ??$ $\theta = 45^\circ$