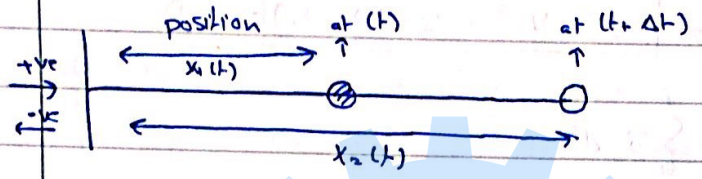
Dynamics of particles
- → Kinematics (motion parameters (velocity, acc., Displacement))
- → Kinetics (Relation between the cause of motion with motion parameters.

* Kinematics:

→ Reference :-



→ Displacement :-

$\vec{d} = x_2 - x_1$ (unit → (m))

→ velocity :-

$\vec{V}_{avg} = \frac{\Delta x}{\Delta t} = \Delta V$

$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ (unit → m/s)

→ Acceleration:

$\vec{a}_{avg} = \frac{\Delta V}{\Delta t}$

$\vec{a} = \frac{dV}{dt} = \frac{d^2x}{dt^2}$ (unit → m/s^2)

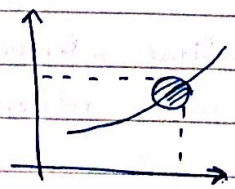
⊛ Types of motion:

1) Rectilinear motion (1D)

2) Curvilinear motion (2D)

→ either I seperate mt. (x & y)

or I deal with curvilinear coordinates (tangent & normal)



* Rectilinear motion :-

→ uniform motion ($\vec{v}$ = constant , $\vec{a} = 0$)

→ uniform acceleration ($\vec{a}$ = constant)

$v = \frac{dx}{dt}$ $\qquad a = \frac{dv}{dt}$

$dt = dx/v$

$\therefore a = \frac{dv}{(dx/v)} \rightarrow \boxed{a\,dx = v\,dv}$

Integrate → $\int_{x_1}^{x_2} a\,dx = \int_{v_1}^{v_2} v\,dv \rightarrow \boxed{v^2 = v_1^2 + 2a(x_2 - x_1)} \rightarrow$ ①

$\vec{a} = \frac{dv}{dt} \rightarrow dv = \vec{a}\,dt \rightarrow \int_{v_1}^{v_2} dv = \int_{0}^{t} \vec{a}\,dt$

$\rightarrow v_2 - v_1 = \vec{a}t \rightarrow \boxed{v_2 = v_1 + \vec{a}t} \rightarrow$ ②

from ② $\quad v = \frac{dx}{dt}$

$\frac{dx}{dt} = v + at \rightarrow dx = (v + at)\,dt \rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} (v + at)\,dt$

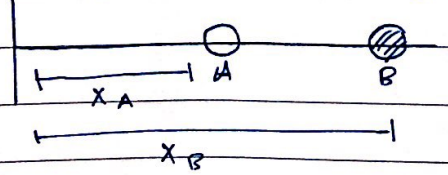
$\rightarrow \boxed{\Delta x = v_1 t + \frac{1}{2} a t^2} \rightarrow$ ③

* Variable acceleration :-

[1] Function of time	[2] Function of position
$a = f(t)$	$a = g(x) = v\frac{dv}{dx}$
$f(t) = \frac{dv}{dt} \rightarrow \int_{v_1}^{v_2} dv = \int_{0}^{t} f(t)\,dt$	$\int_{v_1}^{v} v\,dv = \int_{x_1}^{x} g(t)\,dx$

→ There could be more than one particle on the rectilinear line of motion

$V_A = V_{1A} + a_A t \qquad V_B = V_{1B} + a_B t$

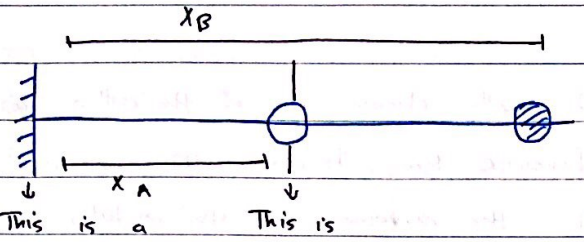


we can study the motion of one particle with reference to another

→ in this case the reference would be moving (not fixed)

→ Reference could be → ① Fixed reference (abs parameters)

② moving reference (relative parameters)



note → The motion of these particles is independent motion (each one moves by it's own with no relation to the other

→ position of (B) with relative to (A)

$X_{B/A} = X_B - X_A$
$X_{A/B} = X_A - X_B$ } equal in magnitude But opposite directions.

$\frac{d}{dt}(X_{B/A}) = V_B - V_A$ → $V_{B/A} = V_B - V_A$

$\frac{d}{dt}(V_{B/A}) = a_B - a_A$
$a_{B/A} = a_B - a_A$

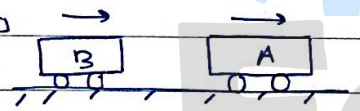
motion of bodies could be → Independent (like all the previous examples) / → Dependent motion.

→ Dependent motion :-

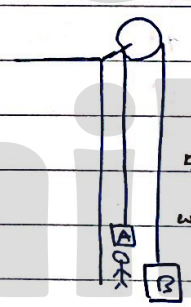
- one of the motions depends on the other

examples on dependent motion:

1)



2)

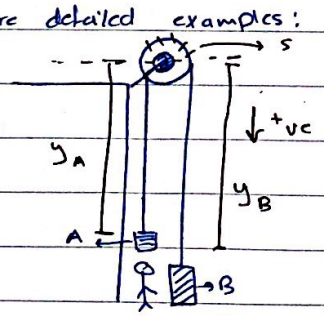


This is called a (constrained motion)

→ if (A) is pulled with 5 m/s rate, then B will move with 5 m/s rate also (the relation between the two particles)

→ more detailed examples:

1)



→ choosing the reference : I can choose any reference I want as long as it's fixed because the speed wouldn't change no matter where I put it.

→ in this example the reference is the center of the pulley

→ cable length is constant = L

$L = y_A + y_B + S$, $L^* = y_A + y_B$

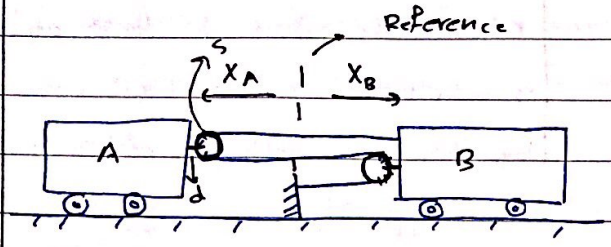
$L^* = y_A + y_B$

$0 = V_A + V_B$ → (it makes sence because both velocitys will be equal in magnitude but opposite in direction).

$0 = a_A + a_B$

$V_A = -V_B$

[2]



→ I can't choose any of the pulleys as a reference here because they move with the movement of the particles.

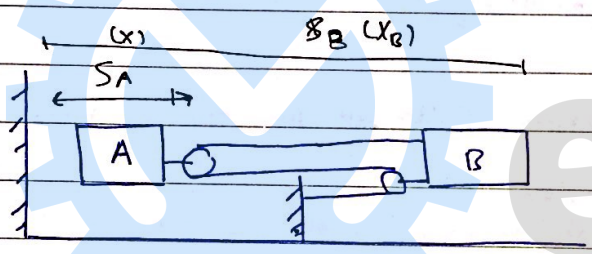
→ constant (half the motion of the pulley)

$$L = (S_A + S_B) + X_B + 2(X_B - d) + 2(-X_A) - d$$

$$L^* = 3X_B - 2X_A$$

$$0 = 3V_B - 2V_A$$

[3]



$$L = 2(S_B - S_A) + (S_B - D) + \text{const.}$$

$$3S_B - 2S_A = L^*$$

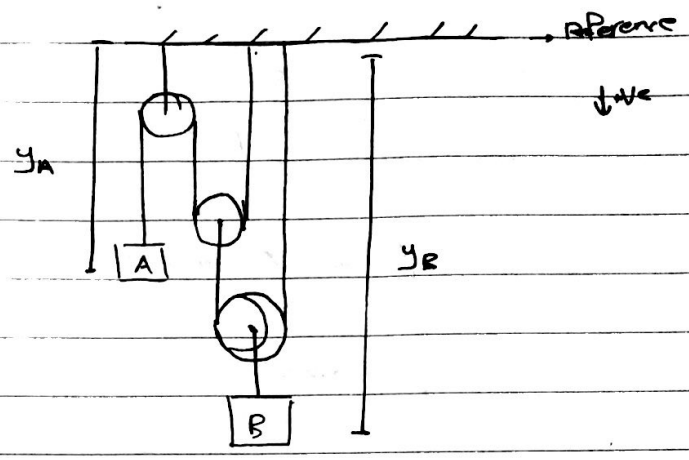
$$3V_B - 2V_A = L^*$$

Exp 12.22

$V_A = ??$

$L_1 = y_A + 2y_c$

$0 = V_A + 2V_c$



$L_2 = (y_B - y_c) + y_B$

$0 = 2V_B - V_c$

$V_c = 2V_B$

$0 = V_A + 2V_B \rightarrow V_A = -4V_B$

$= -4(-6) = 24$ m/s

F 12.40

$V_A = ??$

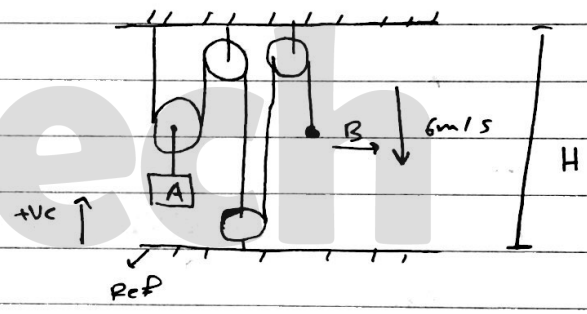
$L = (H - y_A) + (H - y_A) + 2H + (H - y_B)$

$L = -2y_A - y_B$

$0 = -2V_A - V_B$

$0 = 2V_A + V_B$

$V_A = \frac{-V_B}{2} = -\frac{(-6)}{2} = 3$ m/s ↑ up



F 12.43

$L = (S_A - S_P) - 3S_A$

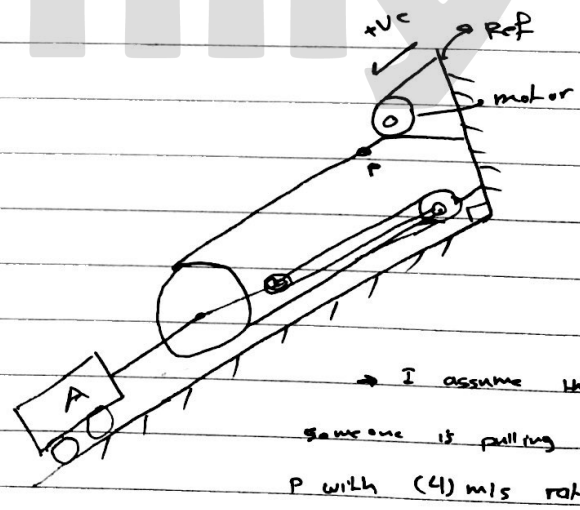
$0 = 4V_A - V_P$

$4V_A = V_P$

$4V_A = -4$

$V_A = -1$ m/s

$= 1$ m/s ↗

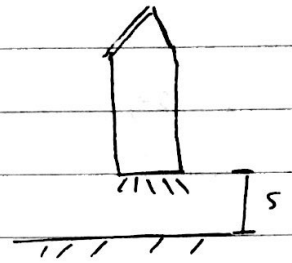


→ I assume that someone is pulling the point P with (4) m/s rate (the motor's speed)

(12-13)

$a = 6 + 0.02s$

$v\big|_{s=2000\,m} = ??$, $t = ??$



$v_1 = 0$, $s_1 = 0$, $t_1 = 0$

$a = v\dfrac{dv}{ds} = 6 + 0.2s$

$\int_0^v v\,dv = \int_0^{2000} (6 + 0.2s)\,ds$

$\frac{1}{2}v^2 = 6s + 0.1s^2 \Big]_0^{2000}$

$v = \sqrt{2(6s + 0.1s^2)}$

$\dot{} = ds/dt$

$\int_0^t dt = \int_0^{2000} \dfrac{ds}{\sqrt{2(6s + 0.1s^2)}}$

note → not a constant acc. so we can't use the (3) known eqns, we have to use the basic definition of acceleration.

(12-26)

$d = ??$

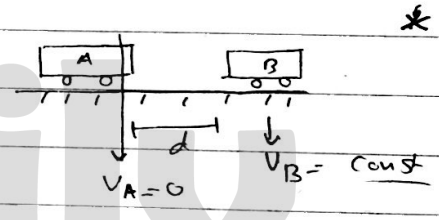
$\cancel{x_B = x_A}$

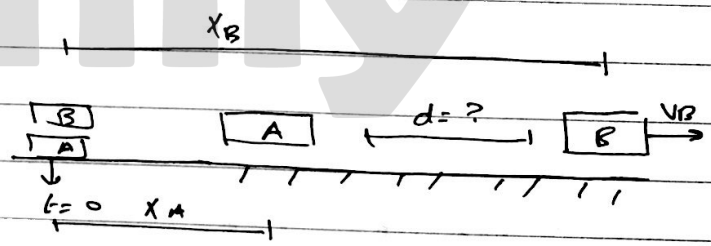
$\cancel{v_B(t) = (v_A t - \frac{a_A t^2}{2}) + d}$

$x_B = x_A + d$

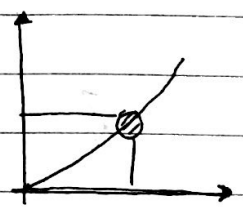
$v_B(t) = (v_A t - \frac{a_A}{2} t^2) + d$

→ I find (t) then I use $v = \frac{x}{t}$ to find x_B





* Curvilinear motion :-

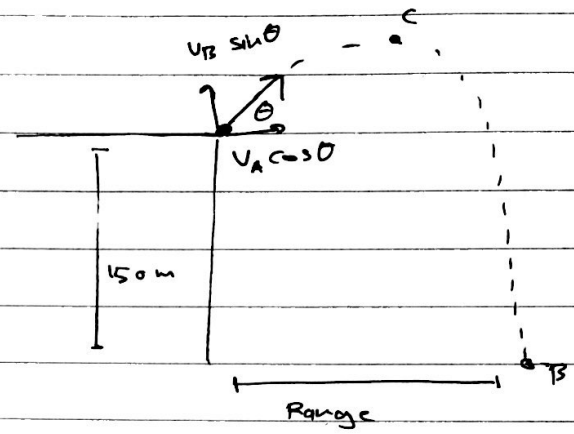


* Example :-

Gravitational force only

$a_y = g$ $\theta = 30°$

$a_x = 0$



① Range = $V \cos\theta \, t$

$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$

$-150 = 0 + (20 \sin 30) t - \frac{g}{2} t^2$

$t_1 = -$ Vc ✗

$t = +$ Vc ✓

Range $= (20 \cos 30) t = \ldots$ m

② $V_{By} = V_{Ay} + a_y t$

$= 20 \sin 30 + g t$

$=$ ✓ m/s

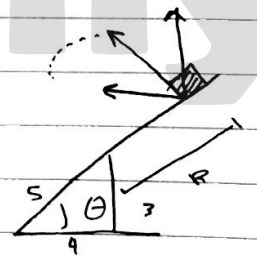
$V_B = \sqrt{(V_{Bx})^2 + (V_{By})^2}$

③ maximum height $\rightarrow V_y = 0$

$V_{cy}^2 = V_{Ay}^2 + 2 a_y [y - y_0]$

$0 = (20 \sin 30)^2 - 2g [h - 0]$ ✓

* Lecture 3 :-



F 12-24

$x = R \cos\theta$, $y = R \sin\theta$

$R \cos\theta = R(4/5) = 20 (3/5) t$

$y = -R(3/5) = 20(4/5) - \frac{g t^2}{2}$

$t = \frac{4R}{60} = \frac{R}{15}$, $-R(3/5) = 20 (4/5)(R/15)$

$R =$ ✓ m

Q 12-101 : $V_A = ?$, $V_B = ??$

$80 = V_A \cos 25 \, (t)$

$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$

$0 = (60 + 4) + (V_A \sin 25) t - \frac{g t^2}{2}$

$t = 80 / (V_A \cos 25)$

$V_A =$ ✓ m/s

$t =$ ✓ s

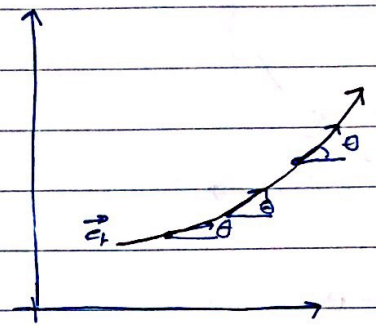
* projectile :-

- curvilinear coordinate

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{v} = \underbrace{v}_{\text{speed}}\, \vec{e}_t$

$\vec{a} = v \frac{d}{dt}(\vec{e}_t) + \frac{dv}{dt}\vec{e}_t$ → tangential (if it was a constant this would equal zero)



$\frac{d}{dt}(\vec{e}_t) = \frac{d}{dl}(\vec{e}_t)\ \frac{d\theta}{ds}\ \frac{ds}{dl}$ → v

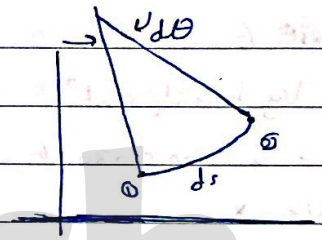
$\vec{e}_t$ ∴ tangential unit vector

$|\vec{e}_t| = 1$ → turns counter clockwise.

v = speed = ds/dt

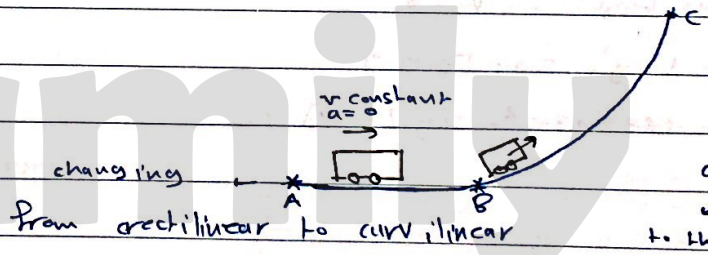
ρ → Radius of curvature

$\frac{d}{dt}(\vec{e}_t) = \frac{1}{\rho} v\, \vec{e}_n$



$\vec{a} = \underbrace{\frac{v^2}{\rho}\vec{e}_n}_{\text{normal}} + \underbrace{\frac{dv}{dt}\vec{e}_t}_{\text{tangential}}$

* state (1) → constant speed



$a_t = 0$

$a_n = \frac{v^2}{\rho}$ → to the center.

* state (2) → constant acceleration

$a = \swarrow$ (a→b)

$a_t = a$ (B→C)

$a_n = \frac{v^2}{\rho}$ (variable) → at the certain instant he asks for.

$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$

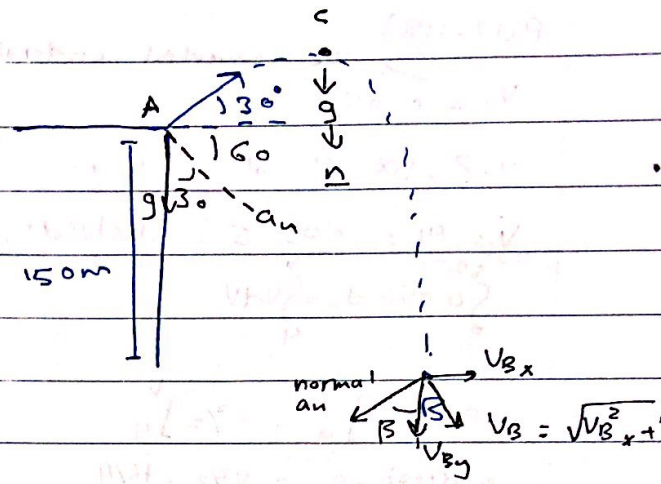
* state (3) → a = variable = f (t) or g(x) (A→B)

a_t = f(t) or g(x) (B→C)

a_n = variable) (B→C)

note → in curvilinear motion $\vec{a}_n$ could never be zero.

Example :-



① ρ_A, ρ_B, ρ_C at a

$a_n = g\cos 30 = V_A^2/\rho_A$

$\rho_A = \dfrac{20^2}{g\cos 30}$

② at c

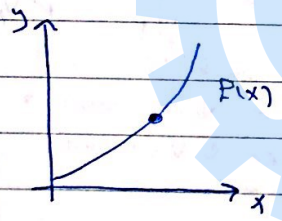
$a_n = g = V_c^2/\rho_c = (20\cos 30)/\rho_c$

$\rho_c =$ ✓

$g\cos\beta = V_B^2/\rho_B$

$\tan^{-1}\dfrac{V_{Bx}}{V_{By}} = \gamma \rightarrow \beta = 90 - \gamma$

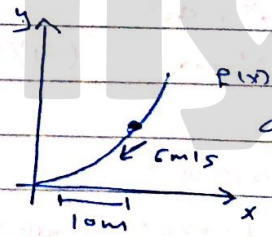
note :-



ρ at any point $= \dfrac{[1+y'^2]^{3/2}}{|y''|}$

Question :- (12-14)

Radius of curvature ??



$f(x) = y = \frac{1}{20}x^2$

$a_t = 2\,m/s^2$

$y' = 2x/20 = x/10$

$y'' = 1/10$

$\rho = (1 + (\frac{10}{10})^2)^{3/2} / |1/10| =$ ✓ m

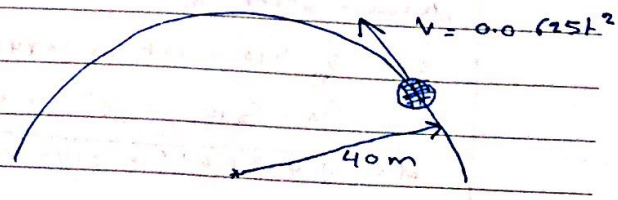
$a_n = 6^2/\rho =$ ✓

$a_t =$ ✓

$|\vec{a}| = \sqrt{(a_t)^2 + (a_n)^2}$

F (12-27)

$|\vec{a}|$ = ?? at $t = 10\,s$



$|\vec{a}| = \sqrt{(a_t)^2 + (a_n)^2}$

$V|_{t=10} = 0.0625(10)^2 = 6.25\,m/s$

$a_n = V^2/\rho = (6.25)^2/40\ m/s^2$

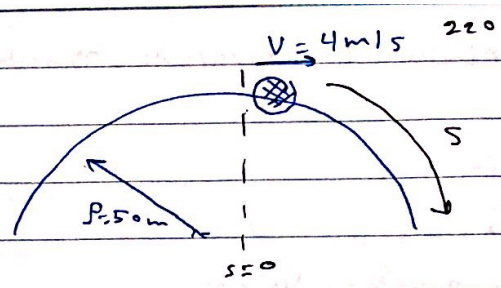
$a_t = dv/dt = 0.1250t$

$a_t|_{t=10} = (0.125)(10) = 1.25\ m/s^2$

$|\vec{a}| = \sqrt{(1.25)^2 + (a_n)^2} =$ m/s^2

✓

P(12-118)



$V_0 = 0.05S$ the tangential acceleration

$V = ?$, $|a| = ?$ at $S = 10$ m

$\dot{V} = a_t = 0.05\,S = V\,dV/ds$

$\int_0^{S0} 0.05S\,ds = \int_4^V V\,dV$

$\frac{0.05S^2}{2}\Big]_0^{10} = V^2/2\Big]_4^V$

$\frac{0.05(10)}{2} - 0 = V^2/2 - 16/4$

$V = \checkmark$ m/s

$|\vec{a}| = \sqrt{a_n^2 + a_t^2}$

$a_n = V^2/\rho = \checkmark/50 = $ m/s²

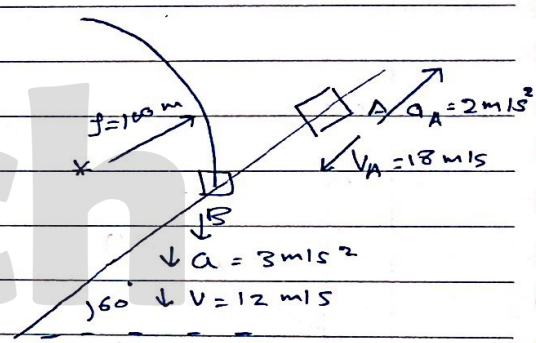
$a_t\big|_{s=10} = (0.05)(10) = 0.5$ m/s²

$|\vec{a}| = \checkmark$ m/s²

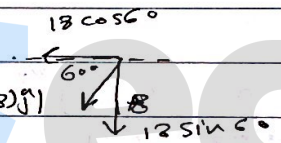
section 10 (Exp 12-27) :-

determine → $\vec{V}_{B/A} = ??$

$\vec{a}_{B/A} = ??$



$V_{B/A} = V_B - V_A$



$-12\hat{j} - (18\cos 60\,\hat{i} + \sin 60(18)\hat{j})$

$= (18\cos 60\,\hat{i}) + (12 + 18\sin 60)\hat{j}$

$\theta = \tan^{-1}$ ____ $= \checkmark$

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

$\vec{a}_B = \sqrt{a_n^2 + a_t^2}$

$a_n = (12)^2/100$ (V^2/ρ) it is moving on a curve

$a_t = 3$

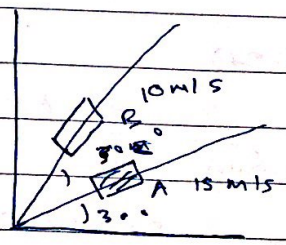
$\vec{a}_B = (12)^2/100\,\hat{i} - 3\hat{j}$

$\vec{a}_A = 2(\cos 60\,\hat{j} + \sin 60\,\hat{j})$

$\vec{a}_{B/A} = (-1.44 - 2\cos 60)\hat{i} + (-3 - 2\sin 60)\hat{j}$

F (12-47)

distance after 4s



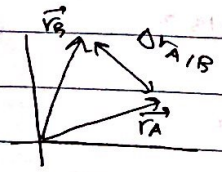
$S_B = S_{10} = 10t \rightarrow 10 \times 4 = 40$ m

$S_A = S_{15} = 15t \rightarrow 15(4) = 60$ m

$d = \sqrt{40^2 + 60^2 - 2(40 \times 60)\cos 30} = \checkmark$ m

OR $|\Delta \vec{r}_{A/B}| =$

$|\Delta \vec{V}_{A/B}| * t$



I can use relative motion

$r_{A/B} = r_A - r_B$

CHAPTER 13 - Kinetics of particles

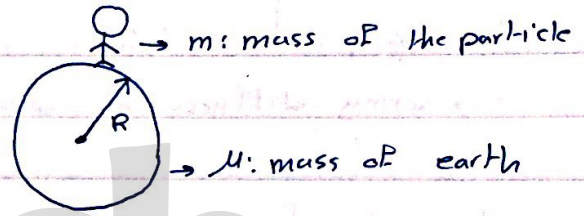
13.1 \ 13.2

Kinetics → A branch of dynamics that deals with the relationship between the change in motion and the forces that <u>cause</u> that change

---

Types of forces
- → Gravitational force ($\vec{w} = m\vec{g}$) $g = 9.81\ m/s^2$
- → Contact forces
  - → Directed force
  - → Friction force / spring force
- → Electrical force

→ Newton's law of gravitational attraction :-



$$F_G = G\frac{m_1 m_2}{R^2}$$

$$F_{G(particle - earth)} = \frac{GMm}{R^2} = w = mg$$

$$\therefore g = \frac{GM}{R^2}$$

→ Newton's first law of motion :

As long as $\Sigma F = 0$ the body is
- → At rest (v & $a = 0$)
- → constant velocity ($a = 0$, $v = c$)

→ Newton's second law of motion (the basis of kinetics)

If $\Sigma F \neq 0$, the particle will have an acceleration in the direction of ΣF proportional to it's magnitude

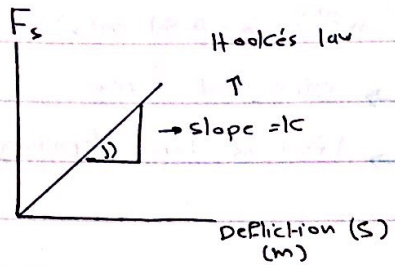
$$\boxed{\Sigma F = m\vec{a}}$$ → "The equation of motion"

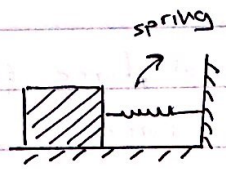
Forces we will be dealing with :

1. Directed force
2. Friction force $F_F = \mu N$
3. spring force $F_s = kS$
4. Normal force
5. weight. (mg)

Spring Force (F_s)

→ it is considerd a "restoring force" the spring tries to restore it's original length.



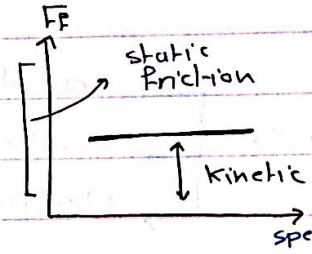


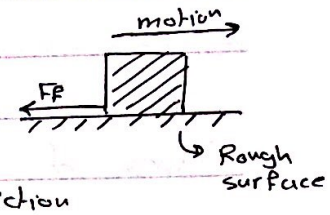
$F_s = k s$ | $s = l - l_0$

k → spring stiffness or spring constant

Friction Force (F_f)

→ it is carsed by rough surfaces and related to the normal force.





$F_f \propto N$

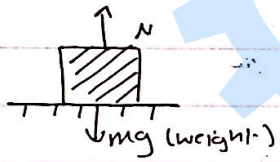
$F_f = \mu N$, μ: coefficient of friction

$\mu \rightarrow \mu_s$ → static friction

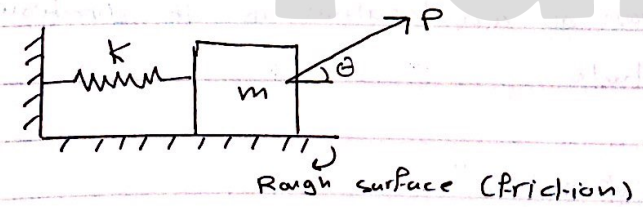
$\mu \rightarrow \mu_k$ → kinetic friction

→ Normal force

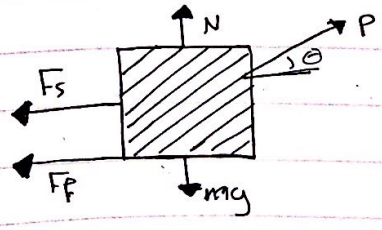
→ the force from the surface on the particle



* We will use two diagrams → Free body diagram / → kinetic diagram.

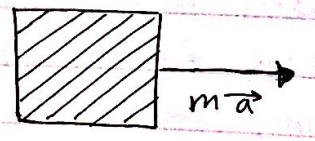


F.B.D (all forces on the particle)



" This diagram shows all magnitudes and directions of the applied forces"

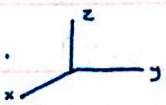
Kinetic Diagram (only resultant force that causes the acceleration)



$\Sigma \vec{F} = m\vec{a}$

" This diagram shows the magnitude and direction of the Resultant force "

13.4 → Equations of motion : Rectangular coordinates.



↳ we will be using rectangular coordinates with problems including rectilinear motion.

---

* When you solve a problem :-

*→ it is very important to draw a free body diagram and a kinetic diagram.

*→ After you apply the equations of motion : $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$ you may need to use kinematics.

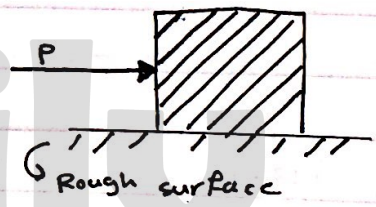
*→ Friction Force always acts in the opposite direction of motion.

*→ If the direction of motion is unknown assume it is in the positive direction of the coordinate axis.

*→ Don't forget to check if the particle is going to move in case you have a rough surface ($F_{applied} > F_{FS}$) →

Example :-

$m = 10$ kg , $\mu_s = 0.4$, $\mu_k = 0.3$, $\vec{W} = 100$ N (weight)



→ Find the acceleration when :

① P = 35 N ② P = 40 N ③ P = 45 N

solution :

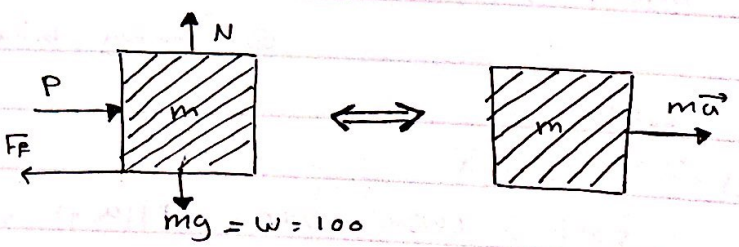
[1] P = 35 N - $N = W = 100$ N

$F_{FS} = \mu_s N$

$= (0.4)(100) = 40$ N

40 N > 35 N so the particle will not move.

→ no motion $\vec{a} = 0$



- F.B.D -

- Kinetic Diagram -

[2] P = 40 N $F_{FS} = 40$

$\Sigma F_x = m a_x$

$P - F_{FS} = m a_x \rightarrow 40 - 40 = 0$

$\vec{a} = 0$ (no motion)

[3] P = 45 $45 > 40$ (F_{FS})

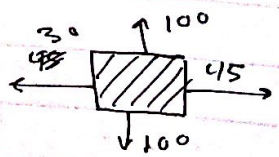
$F_{Fk} = \mu_k N = (0.3)(100) = 30$ N

$\Sigma F_x = m a_x$

$45 - 30 = 10 a_x$

$15 = 10 a_x$

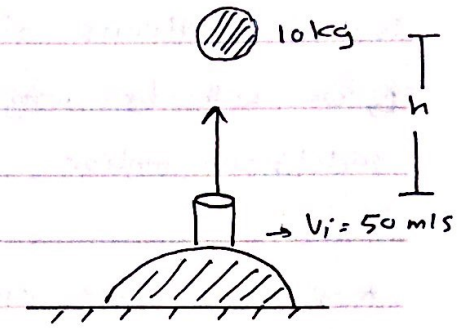
$\vec{a}_x = 1.5$ m/s²



Example (13-2)

A projectile is fired with an initial velocity (50 m/s). $m = 10\,kg$

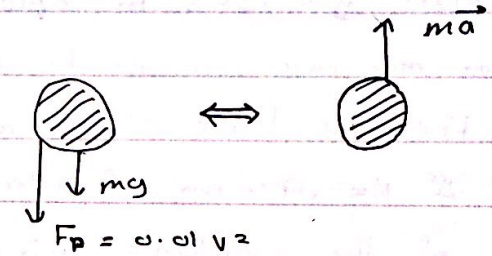
Determine the maximum height if the air resistance is measured as $F_D = (0.01 v^2)\,N$



Solution :

$h_{max} \longrightarrow V = 0$

→ Air resistance is in the opposite direction of motion.



$+\uparrow \sum F_y = m\vec{a}_y$

$-mg - F_D = 10\,\vec{a}_y$

$-10(9.81) - 0.01 V^2 = 10\,a_y$

$a_y = -9.81 - 0.01 V^2$

$V\dfrac{dv}{dy} = -9.81 - 0.01 V^2 \longrightarrow \int_0^h dy = \int_{50}^{0} -\left(\dfrac{v\,dv}{0.001 V^2 + 9.81}\right)$

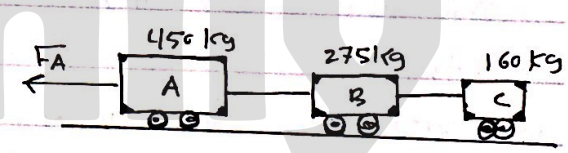
$= -500 \ln (V^2 + 9810) \Big|_{50}^{0}$

$h = 114\,m$

Example (13-3)

If $F_A = 200t$ and the truck starts from rest, Determine : ① V at $t = 2$ seconds.

② $F_{coupling}$ between A & B at $t = 2s$



Solution :-

[1] $\xleftarrow{+ve} \sum F = m\vec{a}$

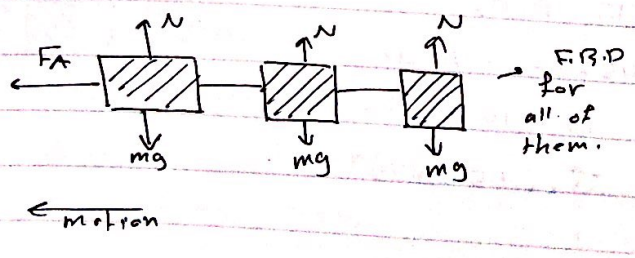
$200t = (450 + 275 + 160)\,a$

$200t = (450 + 275 + 160)\dfrac{dv}{dt}$

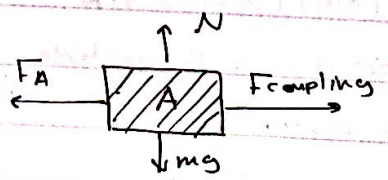
$\int_0^2 200t\,dt = \int_0^v 825\,dv$

$100t^2 \Big|_0^2 = 825\,v \Big|_0^v$

$400 = 825\,v \longrightarrow v = 0.484$ m/s



[2]



$\sum F = m\,a$

$(200t - F_{coupling}) = 450\dfrac{dv}{dt}$

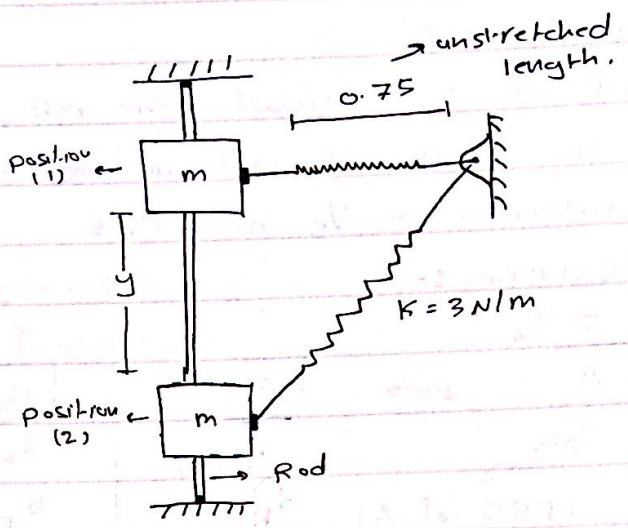
$(200t - F_{coupling})\,dt$

Example (13-4)

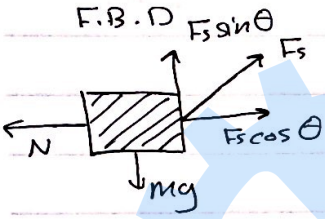
$m = 2kg$, $k = 3\ N/m$

→ The particle moves along the rod a distance (y) and causes the spring to stretch.

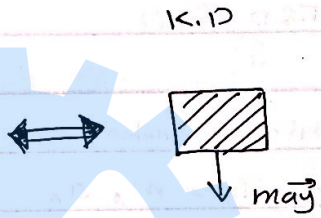
Determine → The acceleration and the normal force of the rod on the particle. when $\underline{y = 1m}$



F.B.D



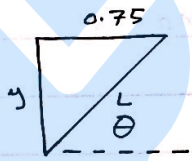
K.D



$F_s = ks$

$s = L - L_0$

we obtain (L) using geometry



$L = \sqrt{(y)^2 + (0.75)^2} \xrightarrow{y=1} L = 1.25$

$s = 1.25 - 0.75 = 0.5$

$F_s = 3(0.5) = 1.5\ N$

→ $\Sigma F_x = m\vec{a}_x = 0$ (no motion along the x)

$F_s \cos\theta - N = 0$

$1.5\left(\frac{0.75}{1.25}\right) - N = 0 \rightarrow N = 0.9\ N$

$\Sigma F_y = m\vec{a}_y$

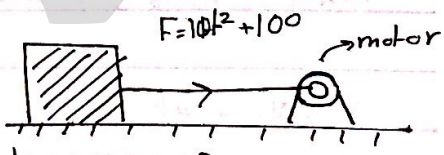
$mg - F_s \sin\theta = 2a_y$

$19.62 - (1.5)\left(\frac{1}{1.25}\right) = 2a_y \rightarrow a_y = 9.21\ m/s^2 \downarrow$

Example (13-2)

$\mu_s = 0.3$, $\mu_k = 0.25$, $m = 25kg$

Determine the particles velocity when $(t = 4s)$

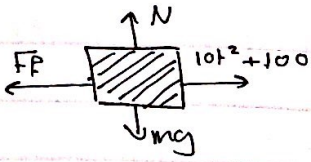


→ check for motion (applied force must be more than F_s)

$F_s^* = \mu_s N = (0.3)(25)(9.81) = 73.575$

$73.575 \overset{?}{=} 10t^2 + 100$ (when $t = 0$ $F = 100$)

∴ there is motion in the direction of the applied force.



$\Sigma F_x = m\vec{a}_x$

$10t^2 + 100 - 61.31 = 25\ dv/dt$

$\int_0^v dv = \int_0^{4s} 0.4t^2 + 1.5475 \rightarrow v = 14.7\ m/s$

$F_f = \mu_k N$

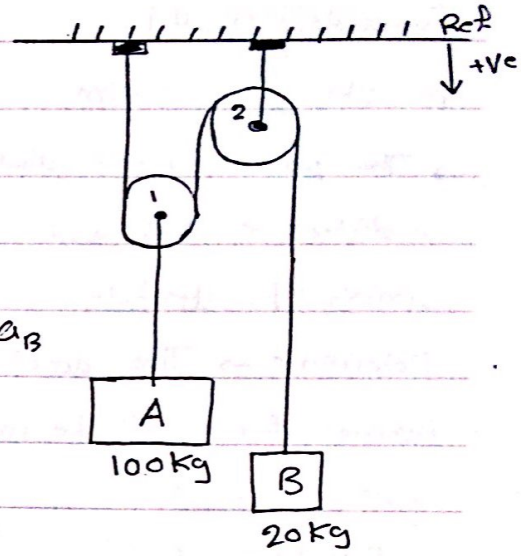
$= (0.25)(\cancel{73.57})$ 245.25

$= 61.31\ N$

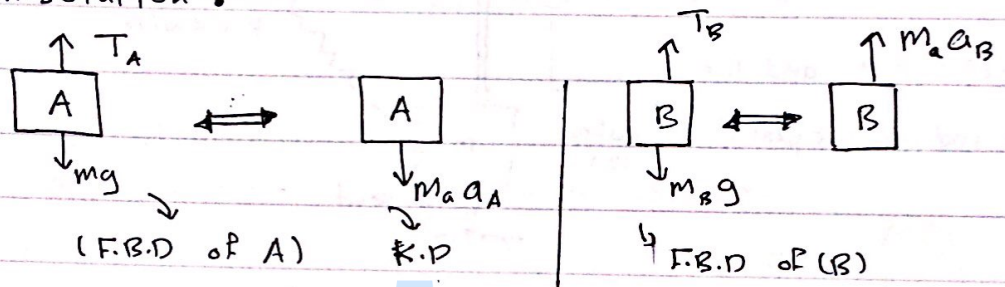
Example (13-5)

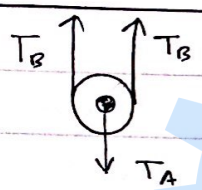
→ Block (A) is released from rest, masses of the pulleys and cards are neglected

Determine → V_B at $t = 2\,s$



*Solution :-





F.B.D of the pulley (1)

$\sum F = m\vec{a}$

$m = 0$

$2T_B - T_A = 0$

$2T_B = T_A$

Equations of motion :-

$m_A g - T_A = m_A a_A$

$T_B - m_B g = m_B a_B$

Kinematics :

$2y_A + y_B = L$

$2a_A + a_B = 0 \rightarrow 2|a_A| = |a_B|$ as a magnitude

↳ using this result & $2T_B = T_A$:-

$m_A g - 2T_B = m_A a_A$

$2T_B - 2m_B g = 2m_B [2a_A]$

$m_A g - 2m_B g = m_A a_A + 4 m_B a_A \rightarrow a_A = 3.27\ m/s^2$

↳ $2a_A = -a_B \rightarrow a_B = -6.54\ m/s^2$

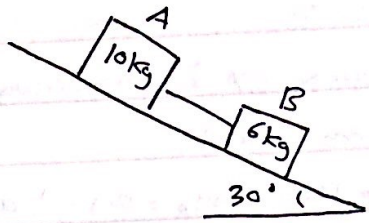
$V = V_0 + at$

$V = 0 - (6.54)(2) = -13.1\ m/s$

Problem (13-5)

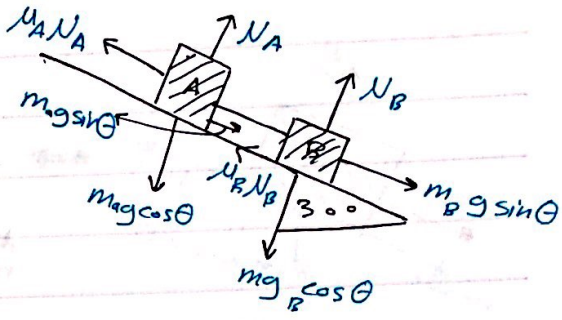
$\mu_A = 0.1$, $\mu_B = 0.3$

Determine → the force developed in the link.



solution :-

to find acceleration : equ of motion for the system



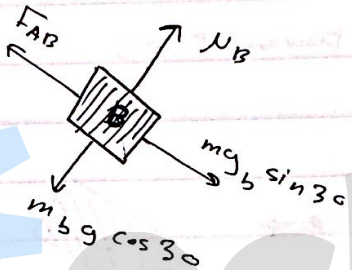
$-\mu_A N_A - \mu_B N_B + (m_A + m_B) g \sin 30 = (m_A + m_B) a$

$-\mu_A (m_A g \cos 30) - \mu_B m_B g \cos 30 + (m_A + m_B) g \sin 30 = \circlearrowleft$

$a = 3.42 \ m/s^2$

$m g \sin 30 - F_{AB} - \mu_B N_B = m_B a$

$F_{AB} = 6.37 \ N$

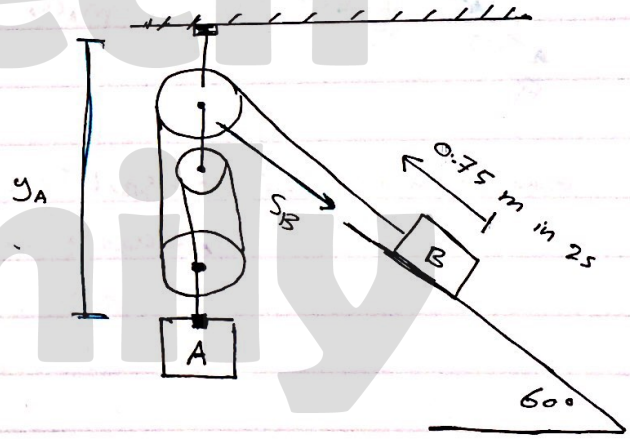


Question :-

$m_B = 5 kg$

if B moves (0.75 m) in 2s

find m_A



→ solution :-

$S = V.t + \frac{1}{2} a t^2$

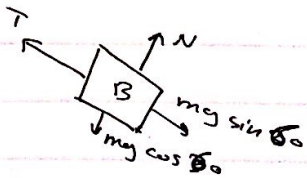
$0.75 = 0 + \frac{a_B}{2} (2)^2$

$a_B = 0.375 \ m/s^2$

$3 y_A + S_B = L$

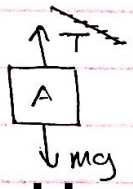
$3 a_A + a_B = 0$

$|a_A| = \frac{|a_B|}{3} = \frac{0.375}{3} = 0.125 \ m/s^2$



+ve $\sum F_B = ma$

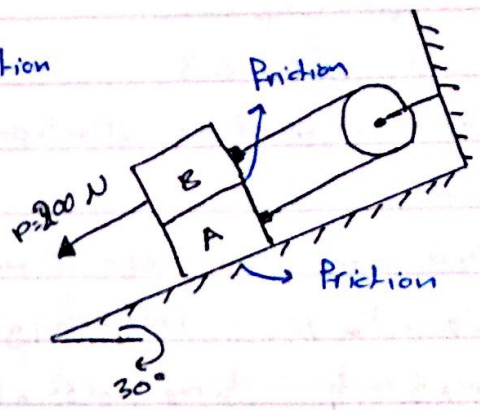
$T - m_B g \sin 60 = m_B a_B \rightarrow T - 5(9.81) \sin 60 = 5(0.375) \rightarrow T = 44.35 \ N$

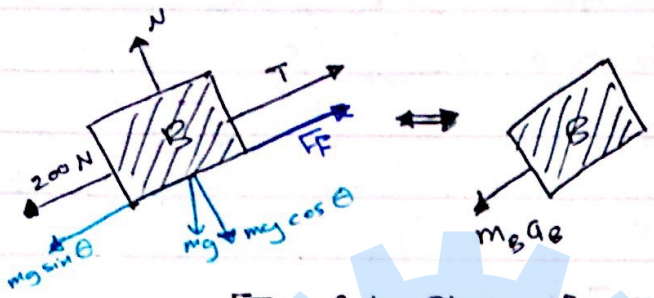


Example :-

→ surfaces A & B are rough (There is friction between them)

$m_A = 10kg$, $m_B = 20kg$ \ $a_A, a_B = ??$

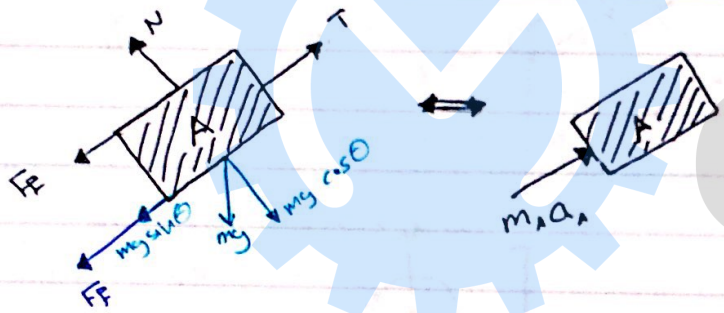




[Free Body Diagram of B]

→ to check for motion calculate P^* → the force needed to move the block.

if $P^* < P$ it will move.



using kinematics → $S_A + S_B = L$

$a_A + a_B = 0$ → $|a_A| = |a_B|$

13.5 → Equations of motion : Normal & Tangential coordinates

↳ we use them when the particle is moving along a curved path.

* Equations of motion :-

$\Sigma F_t = m a_t$ $\quad a_t = dv/dt$ $\quad \rho = \dfrac{[1+(\frac{dy}{dx})^2]^{3/2}}{|d^2y/dx^2|}$

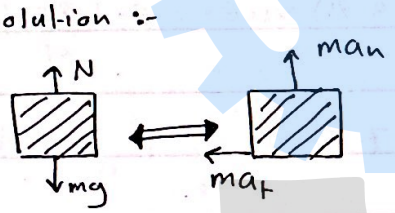
$\Sigma F_n = m a_n$ $\quad a_n = v^2/\rho$

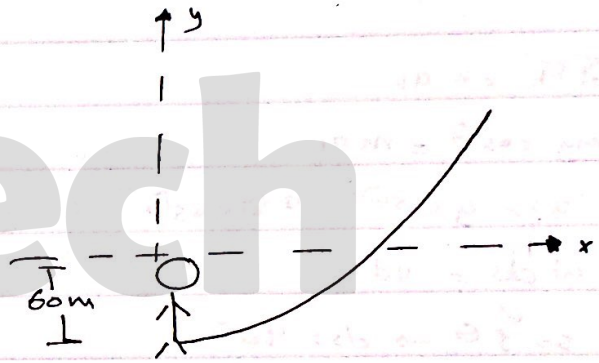
Example (13-8)

$m = 70kg$, $V_A = 20$ m/s

Determine : N, a_A

Solution :-





$\Sigma F_n = m a_n = \dfrac{mV^2}{\rho}$

$N - mg = \dfrac{mV_A^2}{\rho} \rightarrow N =$

$\Sigma F_t = m a_t$

$0 = m a_t \rightarrow a_t = 0$

$a = \sqrt{(a_t)^2 + (a_n)^2} \rightarrow \dfrac{V^2}{\rho}$

$= \sqrt{0 +} \quad =$

$y = \dfrac{x^2}{60} - 60$

$y' = 2x/60$

$y'' = 1/30$

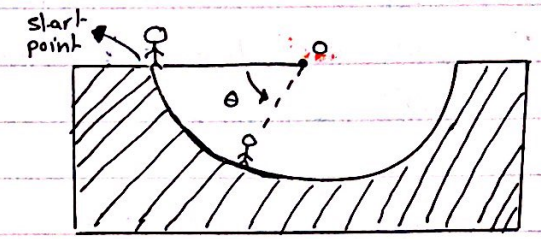
$\rho_A = \dfrac{[1+y'^2]^{3/2}}{|y''|} =$

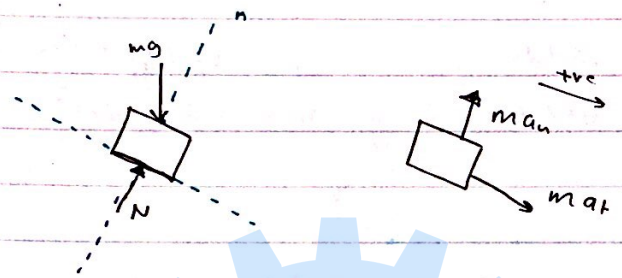
Example (13-9)

The particle starts from rest

$m = 60\ kg$, $\rho = 4\ m$

Determine → N when $\theta = 60°$





$\Sigma F_t = m a_t$

$mg \cos\theta = m a_t$

$a_t = g \cos\theta = 9.81 \cos\theta$

$a_t ds = v dv$

$s = \rho\theta \rightarrow ds = 4 d\theta$

$\int_0^{60} (9.81 \cos\theta)(4 d\theta) = \int_0^v v dv$

$39.24 \sin\theta \Big|_0^{60} = v^2/2$

$v = 8.24\ m/s$

$\Sigma F_n = m a_n$

$N - mg \sin\theta = m a_n = \frac{mv^2}{\rho}$

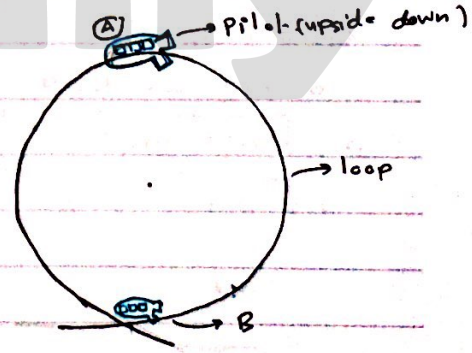
$N - (60)(9.81) \sin 60° = 60 \frac{(8.24)^2}{4}$

$N = 527.7\ N$

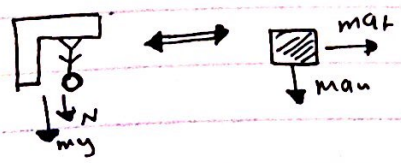
F (13-9)

$\rho = 120\ m$ / $m =$

Determine → $N = ?$ at A & B



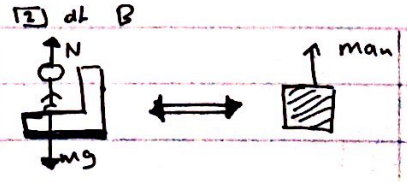
1 at A



$mg + N = \frac{mv^2}{\rho} \rightarrow N = m\left[\frac{36^2}{120} - g\right]$

$N = 69.3$

2 at B



$\Sigma F_n = m a_n$

$N - mg = \frac{mv^2}{\rho}$

$N = m\left(g + \frac{v^2}{\rho}\right) = 1442$

note → The normal force in this problem represents "apparent weight"

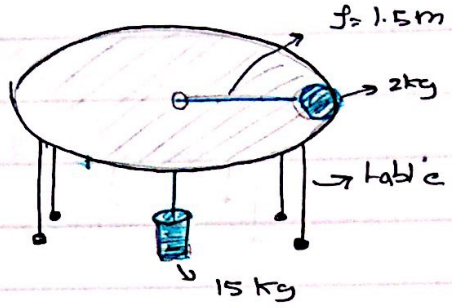
*at rest

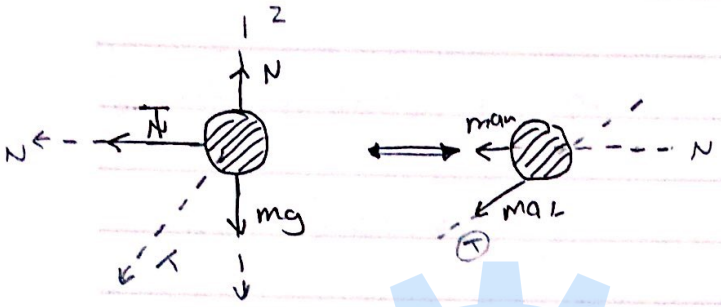
$N = mg$

P(13.5$)

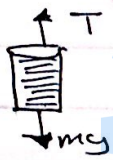
$m_B = 2kg$, the block travels along a circular path of radius $r = 1.5m$

Determine → The speed of the block





$$\Sigma F_n = ma_n = \frac{mV^2}{\rho} \longrightarrow T = \frac{mV^2}{\rho}$$



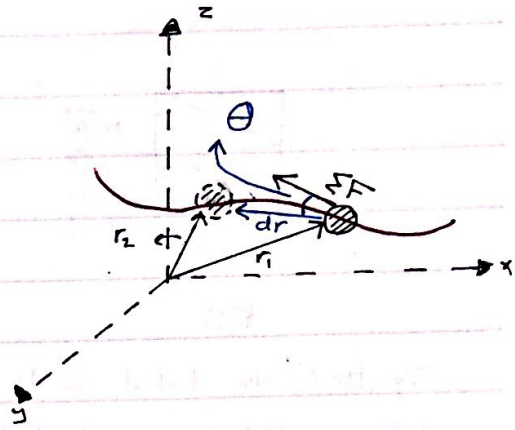
$$\rightarrow mg = T$$

$$(9.81)(15) = T = 147.15\ N$$

$$T = \frac{mV^2}{\rho}$$

$$147.15 = \frac{2(V^2)}{1.5} \longrightarrow V = 10.5\ m/s$$

CHAPTER 14 - Kinetics of particles, work & Energy principles.



→ when $\Sigma\vec{F} \neq 0$, the resultant force will cause a displacement in it's direction. (dr)

$d\vec{r} = \vec{r}_2 - \vec{r}_1$, $\vec{r}$: position vector.

→ a force (F) will only do work (U) on a particle if the particle goes under a displacement <u>in the direction of that force.</u>

↳ a condition.

dU → work done by $\Sigma\vec{F}$

$dU = |\Sigma\vec{F}||d\vec{r}|\cos\theta$, θ → between $\Sigma\vec{F}$ & $d\vec{r}$

↳ This is dot product

$\int dU = \int \Sigma\vec{F} \cdot d\vec{r}$

$U = \int_{r_1}^{r_2} \Sigma\vec{F} \cdot d\vec{r}$ → This represents the work done by $\Sigma\vec{F}$.

---

* Notes :-

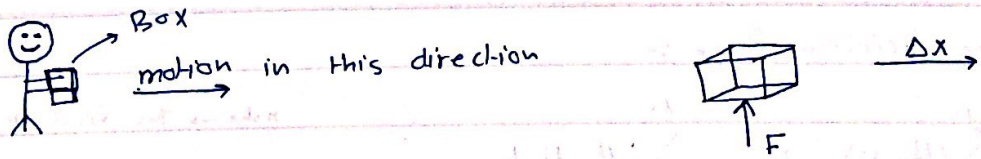
$dU = F\,dr\cos\theta$ $\qquad U = \int_{r_1}^{r_2} \Sigma\vec{F} \cdot d\vec{r}$

→ if $0 < \theta < 90$ work is positive 

→ if $90 < \theta < 180$ work is negative 

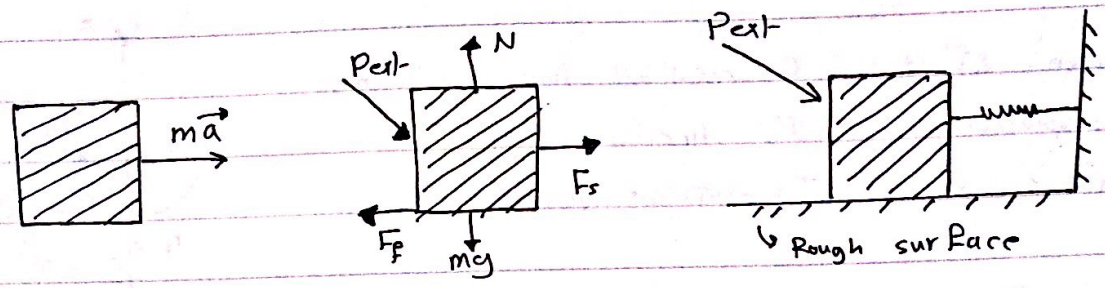
→ if $\theta = 90$ work $= 0$ (no displacement in the direction of force)

* for more understanding :-



↳ in this case the force exerted on the box he is holding doesn't do any work ($F \perp \Delta x$) → $\theta = 90$ → $U = 0$

→ work done by several forces :-



F.D F.B.D

To find the total work done by all these forces :- (when the particle goes under displacement)

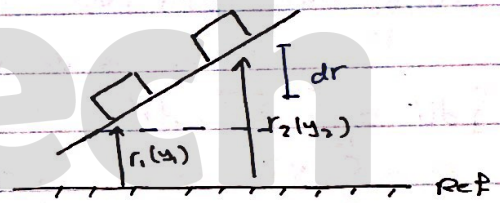
$$U_{1\to2} = (U_{1\to2})_g + (U_{1\to2})_{F_s} + (U_{1\to2})_{F_f} + (U_{1\to2})_{P_{ext}}$$

1 work done by gravity (weight) :-

$$(U_{1\to2})_g = \int_{y_1}^{y_2} mg \cdot d\vec{y}$$ dot product

$$= \int_{y_1}^{y_2} \underbrace{mg}_{\text{constants}}\, dy\, (-1)$$ $\cos\theta$

$$(U_{1\to2})_g = mg\,(y_2 - y_1)$$



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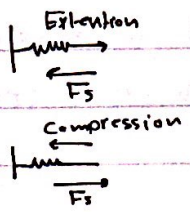
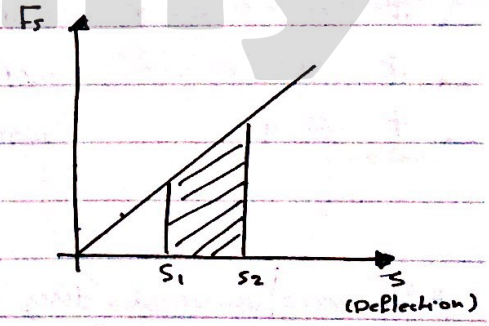
2 work done by the spring :-

$$(U_{1\to2})_{F_s} = \int_{s_1}^{s_2} F_s \cdot ds$$

$$= \int_{s_1}^{s_2} ks\,(-1)\,ds$$ $\cos\theta$

$$= \frac{-k}{2}(s_2^2 - s_1^2)$$





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3 work done by friction force :-

$F_f = \mu_k N$

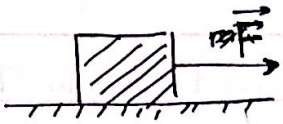
$$(U_{1\to2})_f = \int_{x_1}^{x_2} (\mu_k N) \cdot dx = \int_{x_1}^{x_2} -\mu_k N\, dx$$

$$= -\mu_k N (x_2 - x_1)$$

note → The friction force is always in the opposite direction of motion (displacement)

work done by external force

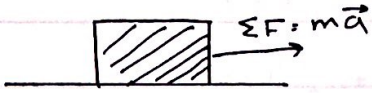
$$(U_{1\to2})_{P_{ext}} = \int_{r_1}^{r_2} \vec{P}\cdot dr$$



↳ The force must be as a function of position (s)

→ Principle of work & energy.

$\Sigma F = m\vec{a}$ $\qquad \vec{a} = v\frac{dv}{ds}$



$\Sigma F = m v \frac{dv}{ds}$

$$\int_{s_1}^{s_2} \Sigma F\, ds = \int_{v_1}^{v_2} m v\, dv \qquad \int_{s_1}^{s_2} F\cdot ds \to \text{Definition of work } (U_{1\to2})$$

→ Total work

$$\Sigma U_{1\to2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \qquad T = \frac{1}{2} m v^2 \text{ (kinetic energy)}$$

$$\Sigma U_{1\to2} = T_2 - T_1$$

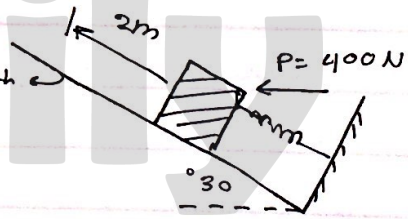
$$T_1 + \Sigma U_{1\to2} = T_2 \longrightarrow \text{principle of work \& energy.}$$

Example (14-1)

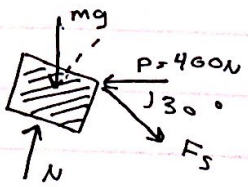
$K = 30\ N/m$, the spring is originally stretched $0.5\ m$

$m = 10\ kg$

Determine → the total work when the block is pushed $2m$.



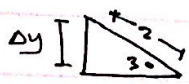
solution



$$U_{tot} = U_g + U_p + U_s$$

$$(U_{1\to2})_s = -\frac{k}{2}(S_2^2 - S_1^2) = -\frac{k}{2}((2.5)^2 - (0.5)^2) = -90\ J$$

$$(U_{1\to2})_g = -mg(y_2 - y_1) = -10(9.81)(2\sin 30) = -98.1\ J$$



$$(U_{1\to2})_{P_{ext}} = \int \vec{P}\cdot d\vec{r} = 400\cos 30\,(2) = 692.82\ J$$

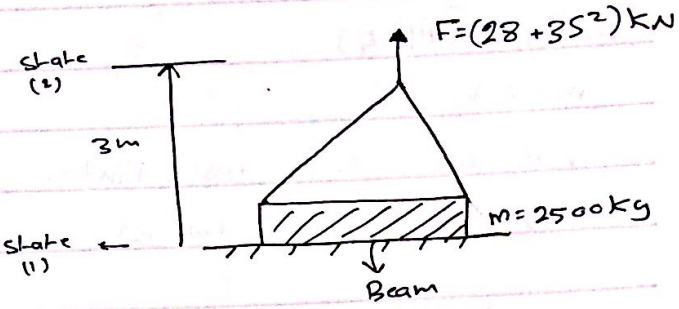
$$U_{tot} = 692.82 - 98.1 - 90 = 504.7\ J$$

Example (14-3)

→ The force (F) lifts the beam up

→ it starts from rest ($V_1 = 0$)

Determine → V & t when $s = 3$



Solution :-

[1] V at $s = 3$

$$T_1 + \Sigma U_{1\to2} = T_2$$

$$\frac{1}{2} m \cancel{V_1^2}^{\,0} + U_{tot} = \frac{1}{2} m V_2^2$$

$$U_g + U_F = \frac{1}{2} m V_2^2$$

$$-mgs + \int_0^s (28 + 3s^2)\,ds = \frac{1}{2} m V^2 \quad (\times 10^3)$$

$$-24525 + 28000s + \frac{3000s^3}{3} = \frac{1}{2}(2500)V^2$$

$$V = (2.78s + 0.8s^3)^{\frac{1}{2}}$$

→ (it's better to write it as a function of (s) first because you will need it to obtain t)

$$V\big|_{s=3} = 5.47 \text{ m/s}$$

[2] t at $s = 3$

$$V = (2.78s + 0.8s^3)^{\frac{1}{2}}$$

$$\frac{ds}{dt} = (2.78s + 0.8s^3)^{\frac{1}{2}} \rightarrow \int_0^t dt = \int_0^3 (2.78 + 0.8s^3)^{-\frac{1}{2}}\,ds$$

$$\boxed{t = 1.79 \text{ s}}$$

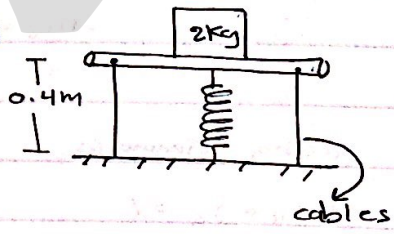
Example (14-4)

spring $L_0 = 1m$, $k = 200$

at 1 → The cords keep the spring compressed 0.6m $(1 - 0.6 = 0.4)$

at 2 → The mass is pushed down then released from rest. Determine → maximum height. (0.1m)

[1]



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\cancel{\frac{1}{2} m V_1^2}^{\,0} + (U_s + U_g) = \cancel{\frac{1}{2} m V_2^2}^{\,0}$$

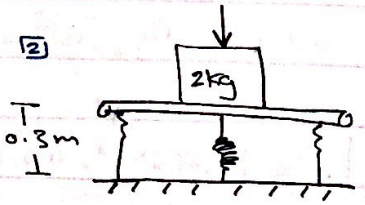
$$\left(-\frac{1}{2} k (S_2^2 - S_1^2)\right) - mg\Delta y = 0$$

$$-\frac{1}{2}(200)\left((0.6)^2 - (0.7)^2\right) - 2(9.81)(h - 0.3) = 0$$

maximum height

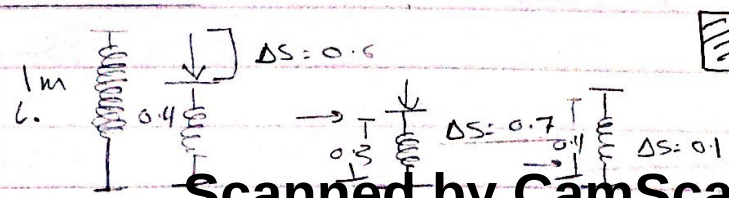
$$h = 0.963 \text{ m}$$

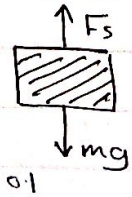
[2]



$S_1 = (0.1 + 0.6) = 0.7$

$S_2 = 0.6$



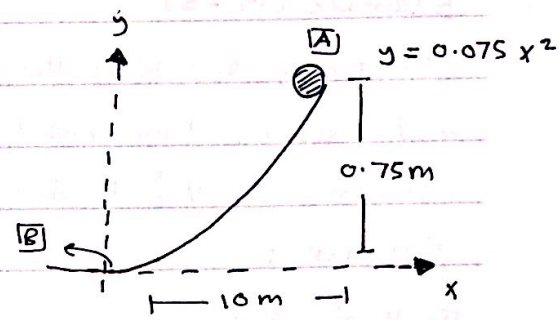


Example (14-5)

$m = 40\ kg$.

→ it starts from rest, Find :-

$V_B = ?$, $N = ?$ (at B)

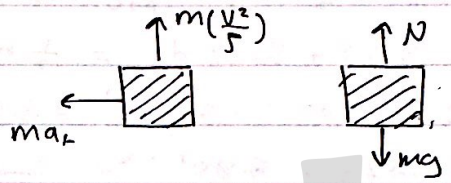


$\cancel{T_1}^{0} + \Sigma U_{A\to B} = T_2$

$U_g = \frac{1}{2} m V^2$

$-\cancel{m}g\,(0 - 7.5) = \frac{1}{2}\cancel{m}V_B^2 \rightarrow V = 12.1\ m/s$

$$\rho = \left.\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{|d^2y/dx^2|}\right|_{x=0} = 6.667\ m$$



$\uparrow \Sigma F_n = m a_n \rightarrow N_B - mg = m\frac{V^2}{\rho}$

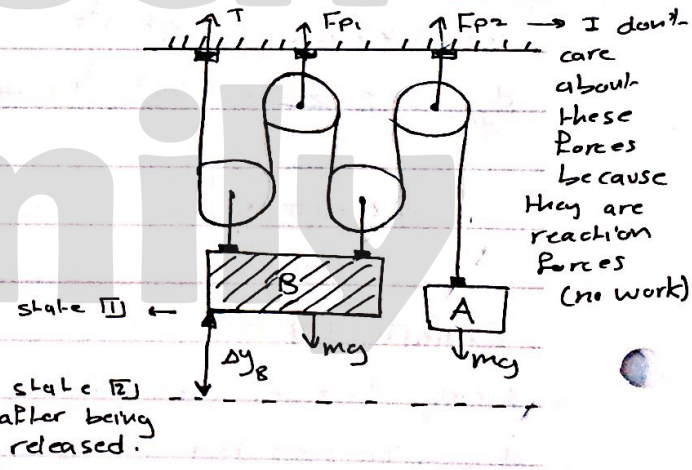
$N_B - 40(9.81) = \frac{40(12.1)^2}{6.667} \rightarrow N_B = 1.28\ kN$

Example (14-6)

$m_A = 10\ kg$, $m_B = 100\ kg$

$V_B = 2\ m/s$, the system is released from rest. $(V_1 = 0)$

For the system :-



$\cancel{T_{1\,sys}}^{0} + U_{1\to 2\,sys} = T_{2\,sys}$

$U_{1\to 2\,sys} = \frac{1}{2}[m_A V_A^2 + m_B V_B^2]$

$U_{1\to 2\,sys} = -m_B g\,[\Delta y_B - 0] - m_A g\,[\Delta y_A - 0]$

using kinematics

$4y_B + y_A = L$

$4V_B + V_A = 0 \rightarrow |4V_B| = |V_A| \rightarrow |V_A| = 8\ m/s$

$4\Delta y_B + \Delta y_A = 0 \rightarrow |4\Delta y_B| = |\Delta y_A| \rightarrow \Delta y_A = -4\Delta y_B$

$\hookrightarrow \frac{1}{2}[(10)(8)^2 + (100)(2)^2] = -100(9.81)(\Delta y_B) - 10(-4\Delta y_B)(9.81)$

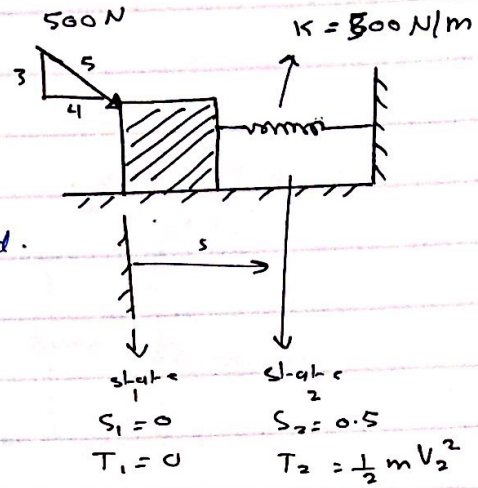
$\Delta y_B = 0.883\ m \downarrow$

F(14-1)

$m = 10\,kg$, $k = 500\,N/m$

Determine the velocity when $s = 0.5$.

* when $s=0$, the spring is not compressed.



$\cancel{T_1}^{\,0} + U_{1\to2} = T_2$

$U_{1\to2} = (U_{1\to2})_p + (U_{1\to2})_s$

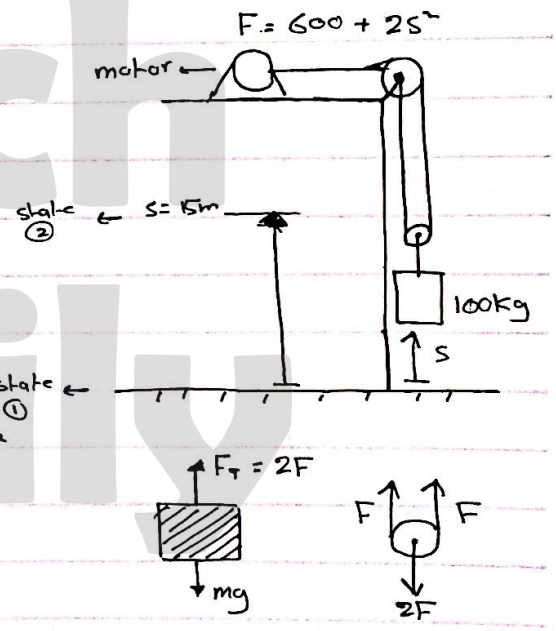
$= 500\left(\frac{4}{5}\right)(0.5) - \frac{k}{2}\left((0.5)^2 - 0\right)$

$137.5 = \frac{1}{2}(10)\,V_2^2$

$V_2 = 5.24$ m/s

F(14-3)

The crate is initially at rest, Determine the speed when $s = 15$ m.



$T_1 = 0$ $\quad T_2 = \frac{1}{2}(100)\,V^2$

$U_{1\to2} = U_{1\to2_T} + U_{1\to2_g}$

$= \int_0^{15} 2(600 + 2s^2)\,ds - mg(15)$

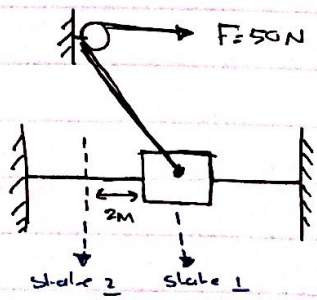
$= 1200s + \frac{4s^3}{3}\Big|_0^{15} - (100)(9.81)(15) = \frac{1}{2}(100)V^2$

$V = 12.5$ m/s

P(14-6)

There is no friction, $m = 6\,kg$, V at state 2?

starts from rest



$T_1 = 0$ $\quad T_2 = \frac{1}{2} m V^2 = \frac{1}{2}(6)\,V^2$

$U_{1\to2} = F(\Delta x)\cos\theta = (50)(2)(1)$

$= 100$

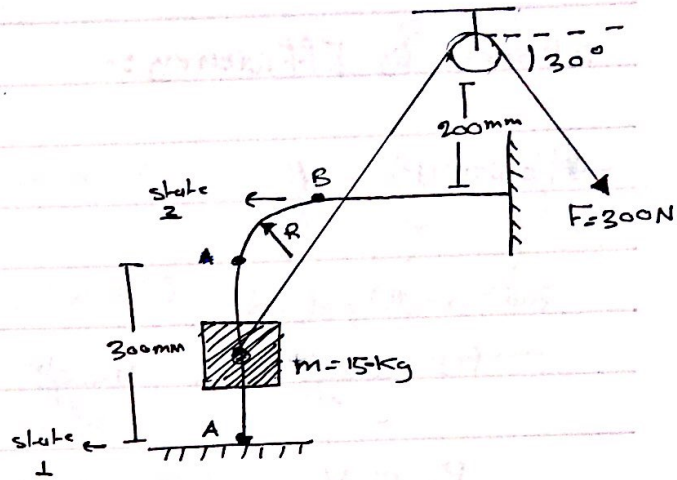
$\xrightarrow{F}$ $\xrightarrow{\Delta x}$ ↳ the increase in length.

$\cancel{T_1}^{\,0} + U_{1\to2} = T_2$

$100 = 3V^2 \rightarrow V = \sqrt{\frac{100}{3}} = 5.8$ m/s

$R = 200$ mm , no friction , starts from rest

Determine the speed (v) at point B



solution :-

$T_A^{\,0} + U_{A\to B} = T_B$

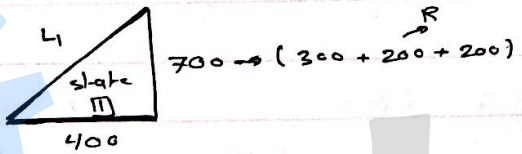
$U_{A\to B} = (U_{A\to B})_g + (U_{A\to B})_F$

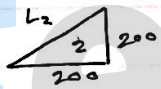
$= -mg(0.5-0) + 300(L_1 - L_2)$

→ we obtain L_1, L_2 using geometry in states 1 & 2

$L_1 = 306.2$

$L_2 = 282.8$





↳ $(-15)(9.81)(0.5) + 300(L_1 - L_2)$

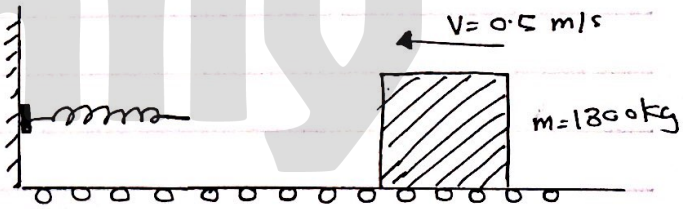
$= -73.6 + 300(523.4) = 449.8$ Joule.

Two springs

$k_A = 5$ kN/m

$k_B = ?$, For the crate to stop at $v=0$

$s = 0.3$ m.



solution

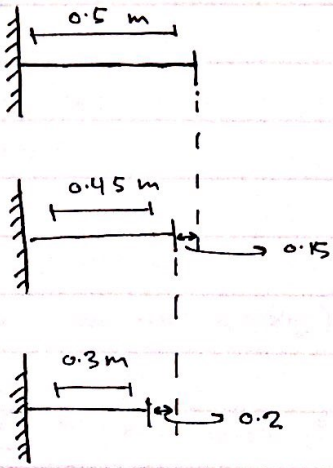
$T_1 = \frac{1}{2}(1800)(0.5)^2 = 225$

$T_2 = 0$

$T_1 + U_{1\to 2} = \cancel{T_2}^{\,0}$

$U_{1\to 2} = (U_{1\to 2})_{SA} + (U_{1\to 2})_{SB}$

$= \frac{-k_A}{2}(0.2)^2 - \frac{k_B}{2}(0.15)^2$



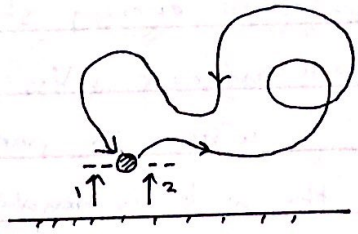
↳ $225 =$

* Conservation of energy :-

→ Conservative Forces :-

The force is considered to be conservative if it's work is independent of the path.

Example → gravitational force (g), spring force (F_s)



$(U_{1\to2})_g = -mg\,(y_2 - y_1) = 0$

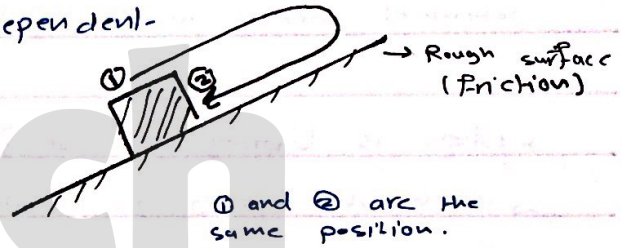
$y_1 = y_2$

→ notice that although the particle moved through a long path but it's work depends only on it's initial and final position (y_1 & y_2)

→ Non-conservative Forces.

The work of such forces is path-dependent.

Example → Frictional force.



In this case the ~~total~~ work $(U_{1\to2})_{F_f}$ caused by the friction force is not zero.

---

* conservation of energy principle:

$T_1 + U_{1\to2} = T_2$

$T_1 + (U_{1\to2})_g + (U_{1\to2})_s + (U_{1\to2})_{ext} = T_2$

→ If $F_{ext} = 0$

$T_1 + [-mgy_2 + mgy_1] + [-\frac{k}{2}S_2^2 + \frac{k}{2}S_1^2] = T_2$

$$\underbrace{T_1 + mgy_1 + \frac{k}{2}S_1^2}_{\text{state 1}} = \underbrace{T_2 + mgy_2 + \frac{k}{2}S_2^2}_{\text{state 2}}$$

$V_g = mgy$ (Potential energy due to gravity)

$V_s = \frac{k}{2}S^2$ (Potential energy stored in the spring)

$\boxed{T_1 + V_{g_1} + V_{s_1} = T_2 + V_{g_2} + V_{s_2} = \text{constant}}$ → Conservation of energy principle.

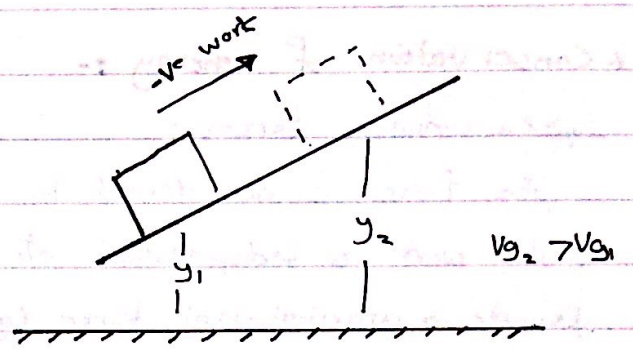
→ This principle is a special case of (work-energy) principle.

→ we can use this principle only when (F_{ext} & $F_f = 0$)

→ Gravitational potential energy :-

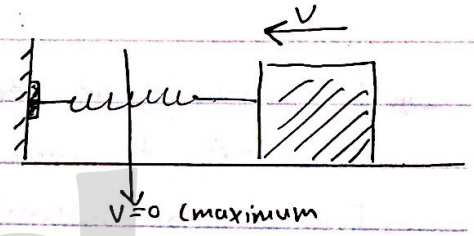
→ when (y) is increasing upward potential energy stored in the particle will increase ($V_g = mgy$)

→ when the particle moves upward the work is negative but V_g is positive



→ energy stored in a spring :

→ when the mass gets to maximum compression that the spring could take ($V=0$) all the kinetic energy ($\frac{1}{2}mV^2$) the mass had transforms to potential energy in the spring ($\frac{1}{2}KS^2$).



* when is U_{spring} positive ?

→ when the spring is compressed then released it will be the cause of motion and F_s will be the same direction as Δx so U is +ve

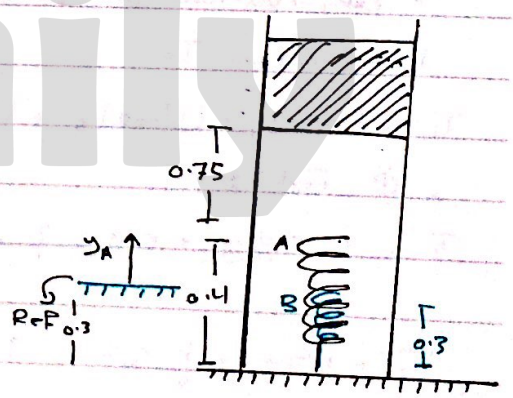
---

Example (14-9)

$K_A = 12\ kN/m$, $K_B = 15\ kN/m$, $m = 100\ kg$

→ The mass is released from rest, There is no friction or external forces.

Determine → The maximum displacement of (A) needed to stop the downward motion.



solution :-

$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$

$0 + 100\,g\,(y_A + 0.75) + 0 = 0 + \frac{K_A}{2} y_A^2 + \frac{K_B}{2}(y_A - 0.1)^2$ $\qquad 0.4 - 0.3 = 0.1$

$98.1(y_A + 0.75) = \frac{12{,}000}{2} y_A^2 + \frac{15{,}000}{2}(y_A - 0.1)^2$

$y_A =$

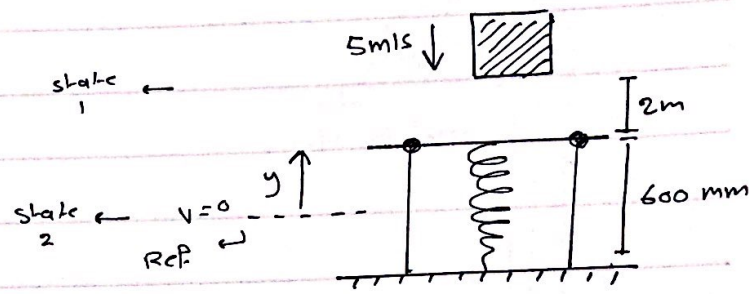
$48.1\, y_A + 73.57 = 6000$

P(14-79)

$L_0 = 750$ mm (it has initial deflection) 150 mm

→ no friction, no external forces.

$m = 40$ kg, $K = 25$ kN/m



Solution :-

$T_1 = \frac{1}{2} m v^2 = \frac{1}{2}(40)(5)^2 = 500$ J

$T_2 = 0$

$V_{1g} = mg(2+y)$ $\qquad V_{1s} = \frac{K}{2}(0.15)^2$

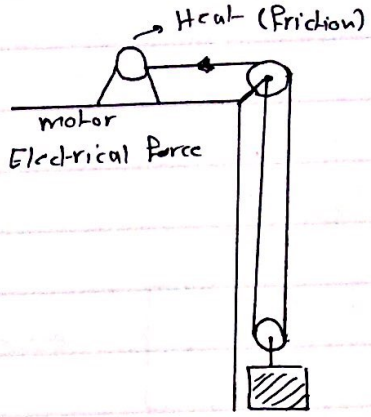
$V_{2g} = 0$ $\qquad V_{2s} = \frac{K}{2}(0.15+y)^2$

$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$

$500 + mg(2+y) + \frac{K}{2}(0.15)^2 = 0 + 0 + \frac{K}{2}(0.15+y)^2$

$y =$ ✓

* Power & Efficiency :-



→ Power (P) $= \frac{dU}{dt}$

But → $dU = F.dr$

$P = \frac{dU}{dt} = F.\left(\frac{dr}{dt}\right) \rightarrow V$

$P = F.V$ → power is a scalar

units of Power → $P = \frac{J}{s} = N.m/s = W$ (watt)

→ Efficiency (ϵ)

$\epsilon = \frac{\text{Power output}}{\text{Power input}}$

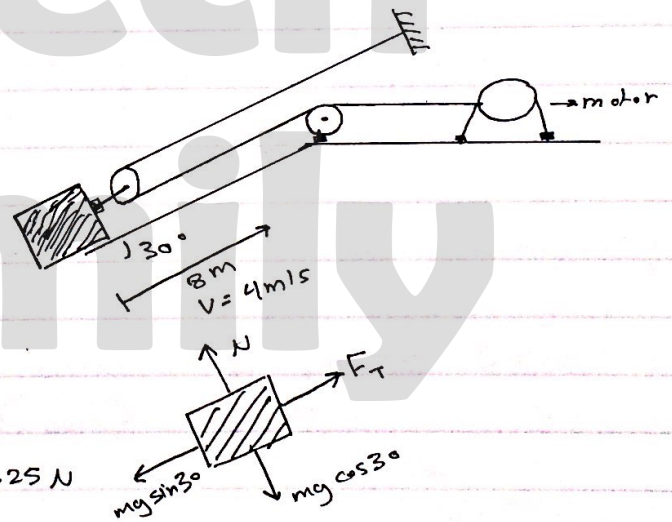
note → power output will be less than power input (because of energy lost by friction), so power efficiency is always less than 1

13th edition ← P(14-51)

$m = 50\,kg$, $V = 4\,m/s$

$\epsilon = 0.74$

Determine → power input



$\Sigma F = m\vec{a}$

$V^2 = V_0^2 + 2a\Delta S$

$16 = 0 + 16a \rightarrow a = 1\,m/s^2$

↳ $F_T - mg\sin 30 = 50$

$F_T = 50 + (50)(9.81)(\sin 30) = 295.25\,N$

$P = F.V = FV\cos\theta$

$= (295.25)(4) = 1181\,W$

$\epsilon = \frac{\text{Power output}}{\text{Power input}}$

$0.74 = \frac{1181}{\text{power input}}$ → power input = ~~1595.94 watt~~ = 1595.94 watt

↳ note that the power input is more than the power output because frictional forces always develope in machines → (motor)

CHAPTER (15)

sections → 15.1, 15.2, 15.3, 15.5, 15.6, 15.7

suggested → * All Fundementals

* 40, 39, 23, 6, 8

CHAPTER (16)

sections → 16.1, 16.2, 16.3, 16.4, 16.5, 16.6, 16.7, 16.8

suggested → * All Fundementals

* 1, 2, 3, 4, 16, 41, 42, 43, 47, 61, 62, 67, 82, 86, 90, 104

105, 106, 108, 111, 113, 131, 136, 141, 143, 152

CHAPTER (17)

sections → 17.1, 17.2, 17.3, 17.4, 17.5

suggested → * All Fundementals

* 57, 59, 61, 63, 65, 72, 75, 96, 100, 103, 105, 108, 44, 112

CHAPTER (18)

sections → 18.1, 18.2, 18.3, 18.4, 18.5

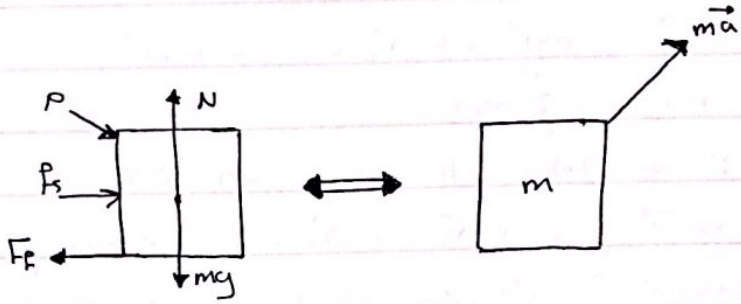
suggested → * All Fundementals

* 2, 4, 5, 6, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20, 21, 25, 26

28, 33, 47, 48, 52, 53, 60

CHAPTER "15" - Impulse & momentum principle
Kinetics of particles.

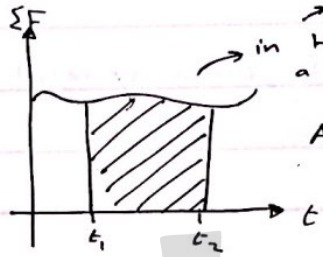
$\Sigma \vec{F} = m\vec{a}$, newton's second law



$\Sigma \vec{F} = m \frac{dv}{dt}$

$\int_{t_1}^{t_2} (\Sigma \vec{F})\, dt = \int_{v_1}^{v_2} m\, dv$

$m\vec{V_1} + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = m\vec{V_2}$



→ (variable force) in this case F is a function of (t)

Area = $Imp_{1\to2}$

Impulse $\equiv \int \Sigma F\, dt$

Linear momentum ($\vec{L}$) = $m\vec{V}$

$\vec{L_1} + \vec{Imp}_{1\to2} = \vec{L_2}$ (F, m, V, t)

$T_1 + U_{1\to2} = T_2$ (F, m, V, d)

$\Sigma \vec{F} = m\vec{a}$ (F, m, a)

→ Kinetics of particles

---

* For a system of particles :-

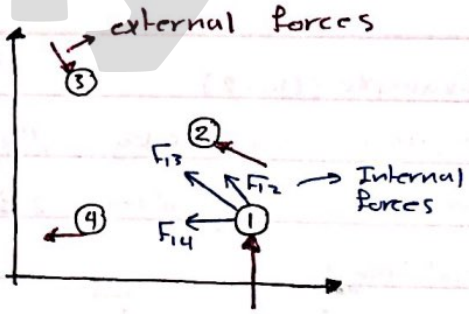
$L_{1,1} + \int (\Sigma F_{ext} + \Sigma F_{int})\, dt = L_{2,1}$ → number of the particle

$\vdots$

$L_{1,n} + \int (\Sigma F_{ext} + \Sigma F_{int})\, dt = L_{2,n}$

from particle ① to particle ⓝ

+



$\sum_{i=1}^{n} \vec{L_{1,i}} + \int (\vec{F_{ext}})_{sys}\, dt = \sum_{i=1}^{n} \vec{L_{2,i}}$

→ notice that when you add them all up all internal forces cancel each other out.

* When $\Sigma \vec{F}_{ext} = 0$, we will have :-

$\sum_{i=1}^{n} L_{1,i} = \sum_{i=1}^{n} L_{2,i}$ (conservation of linear momentum)

$\cancel{m_1 V_{1,1} + m_2 V_{1,2} \ldots}$

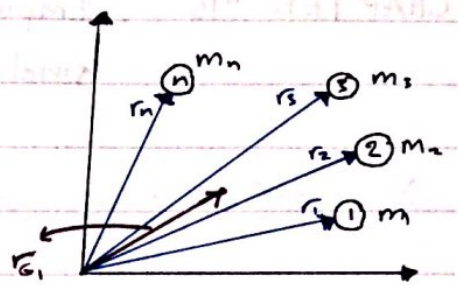
$m_1 V_{1,1} + m_2 V_{1,2} + \ldots + m_n V_{1,n} = m_1 V_{2,1} + m_2 V_{2,2} + \ldots + m_n V_{2,n}$

$$\sum_{i=1}^{n} m_i = m_1 + m_2 + \dots + m_n$$

$= (m)$ system

$$m\vec{r}_G = m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_n\vec{r}_n$$

$$m\vec{r}_G = \sum_{i=1}^{n} m_i r_i$$



If we take the time derivative :-

$$\underbrace{m}_{m(\text{system})}(\vec{V})_G = m_1\vec{V}_1 + m_2\vec{V}_2 + \dots + m_n\vec{V}_n = \sum_{i=1}^{n} m_i V_i$$

velocity of the mass center of the system.

conservation of linear moment becomes:

$$m\vec{V}_{G_1} = m\vec{V}_{G_2}$$

---

* Notes :-

→ It is important to determine the direction of the initial & final velocities of the particles.

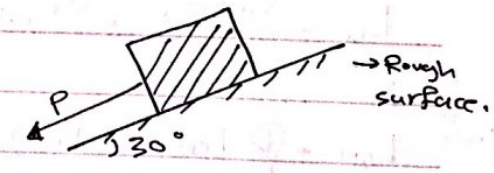
→ each force on the free body diagram of the particle will create an impulse even if they do no work.

→ To determine the impulse
- → $F = \text{const}$ $\quad Imp = F(t_2 - t_1)$
- → $F = F(t)$ $\quad Imp = \int_{t_1}^{t_2} F(t)\,dt$
- → F (Graph) $\quad Imp =$ area under the curve.

Example (15-2)

$P = 100t$, $m = 25\,kg$, $\mu_k = 0.3$, $V_1 = 1\,m/s$

Determine → V_2 after 2 seconds



Solution :-

$$L_1 + \int \Sigma F\,dt = \vec{L}_2$$

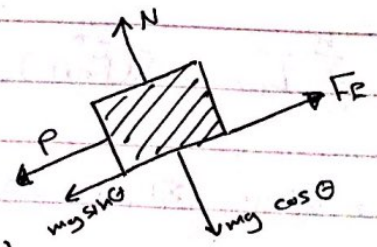
$$m_1 V_1 + \int \Sigma F\,dt = m_2 V_2$$

$$25(1) + \int_0^2 (mg\sin 30 + 100t - \mu_k \cdot mg\cos 30)\,dt = mV_2$$

$$= 25 + 25(9.81)\sin 30\,(2-0) + \frac{100t^2}{2}\Big|_0^2 - (0.3)(25)(9.81)\cos 30\,(2-0)$$

$$342.8 = 25V_2$$

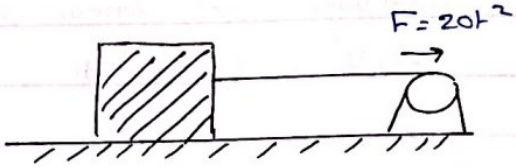
$$V_2 = 13.71\ m/s$$



F (15-3)

$\mu_s = 0.3$, $\mu_k = 0.25$, $m = 25\,kg$

$V = ?$ at $t = 4\,sec$



Solution :-

→ we have (μ_s) → check for motion & determine at what value of (t) will the particle start to move.

$F^* = \mu_s N$ (force needed to move the particle).

$20t^{2*} = 0.3(25)(9.81)$

$t^* = 1.92$ sec

now, we apply principle of Impulse & momentum.

$$m_1 V_1 + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = m V_2$$

$$0 + \int_{1.92}^{4} (20t^2 - \mu_k mg)\, dt = 25 V_2$$

$$\frac{20t^3}{3}\Big|_{1.92}^{4} - (0.25)(25)(9.81)(4 - 1.92) = 25 V_2$$

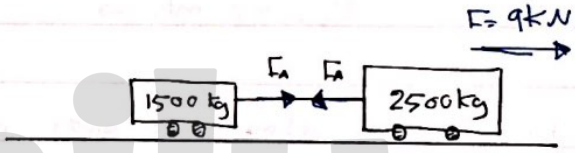
$$V_2 = 10.1 \text{ m/s}$$

F (15-5)

starts from rest, determine;

1) V at $t = 20$ sec

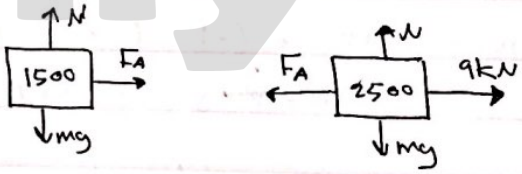
2) $F_{coupling}$



Rolling motion (no friction)

solution :-

[1] $m V_1 + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = m V_2$ → (for the system)



$$0 + \int_0^{20} (9 * 10^3)\, dt = \overset{m(sys)}{(2500 + 1500)} V_2$$

$V = 45$ m/s

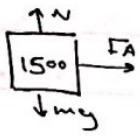
[2] we apply (Imp, L) principle on one of the particles (not the system)

$$m V_1 + \int_0^{20} \Sigma F\, dt = m V_2$$

$$0 + \int_0^{20} F_A\, dt = 1500(45)$$

$$20 F_A = 1500(45)$$

$$F_A = 3.3 \text{ kN}$$



note → you should get the same result if you use the (2500) particle

note that → when we applied it on the system (F_A) was cancelled out because it is an internal force

* Conservation of linear momentum :-

$$\sum_{i=1}^{n} m_i \vec{V_i} + \int (\Sigma F_{ext})\, dt = \sum_{i=1}^{n} m_i V_{i2}$$

if $(\Sigma F_{ext})_{sys} = 0 \rightarrow$ we will have conservation of linear momentum

$$\sum_{i=1}^{n} m_i \vec{V_{i1}} = \sum_{i=1}^{n} m_i V_{i2}$$

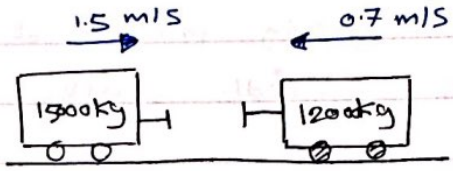
Example (15.4)

Determine :-

(1) speed just after coupling.

(2) $F_{coupling}$ if $\Delta t = 0.8s$

→ no external forces (conservation of L)



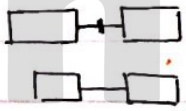
[1] $\sum_{i=1}^{n} m_i \vec{V_{i1}} = \sum_{i=1}^{n} m_i V_{i2}$

$m_1 V_{1,1} + m_2 V_{1,2} = m V_2$ → $(m_1 + m_2)$

$\xrightarrow{+ve}$ $(15000)(1.5) - 12000(0.7) = (15000 + 12000) V_2$

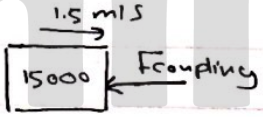
$V_2 = 0.5$ m/s →

Just b4 coupling



after coupling (it's like they became one particle).

[2] to obtain ($F_{coupling}$) we apply principle of linear momentum to one of them



$\xrightarrow{+ve}$ $m_1 V_1 + \int \Sigma F\, dt = m_2 V_2$

$(15000)(1.5) + (-F)(0.8) = (15000)(0.5)$

$F_{coupling} = 18.8$ kN

Note :-

In this chapter there are two types of forces we'll deal with:

[1] Impulsive forces → forces which are large, act for a short period of time & they produce significant change in momentum

example → forces between particles that collide are impulsive

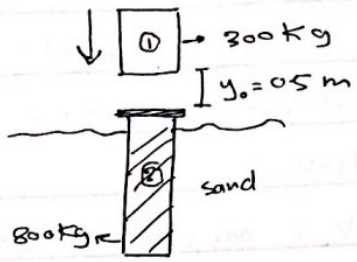
→ we consider them in any principle we use

[2] Non-Impulsive forces → forces that cause negligible impulse because are very small compaired to other forces.

example → weight in some cases.

→ we neglect them

Example (15-6)



If 1 starts from rest, 1 doesn't rebound.

Determine the impulse of 2 on 1

→ In this examples the weighs of 1 & 2 and the resistance force of the sand are all non-impulsive so we neglict them from calculations.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m_1 \cancel{V_{1,1}^2}^{\,0} + m_1 g y_0 = \frac{1}{2} m_1 (V_{1,2})^2 + m_1 g y_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300) V_{1,2}^2 + 0$$

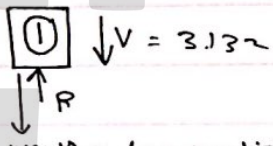
$$V_{1,2} = 3.132 \text{ m/s}$$ (Just b4 collision)

$$\sum_{i=1}^{n} m_i V_{i_1} = \sum_{i=1}^{n} m_i V_{i_2}$$

$$m_1 V_{1,1} + m_2 V_{2,1} = m_1 V_{1,2} + m_2 V_{2,2}$$

$$(300)(3.132) + 0 = 300 V_2 + 800 V_2$$ → Because 1 doesn't rebound velocities will be equal

$$V_2 = 0.8542 \text{ m/s}$$ (after collision)



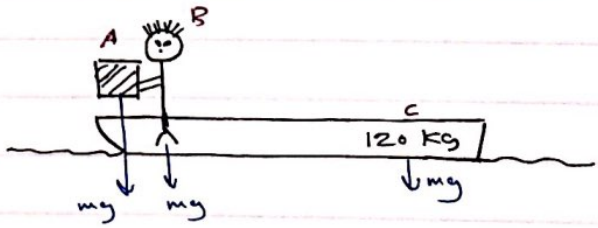
$mg \simeq 0$ (we neglict it non-impulsive) too-small.

$$m_1 V_1 + \int_{t_1}^{t_2} R\,dt = m V_2$$

$$300(3.132) - \underbrace{\int R\,dt}_{\text{Impulse}} = 300(0.8542)$$

$$\int R\,dt = 683 \text{ N.S} \quad \#$$

Example (15-7)



$m_1 = 80$ kg (man) | $V_{Box} = 4$ m/s →

$m = 20$ kg (Box)

→ Determine → how far the boat will move in 3 sec.

- all weighs are non-impulsive forces.

solution: cons. of linear momentum

$$m_A V_{A_1} + m_B V_{B_1} + m_C V_{C_1} = m_A V_{A_2} + (m_B + m_C) V$$

$$0 + 0 + 0 = 20(V_{Box}) + (120 + 80)\vec{V}_{boat}$$

$$0 = -20 V_{Box} + 200 V_{Boat}$$

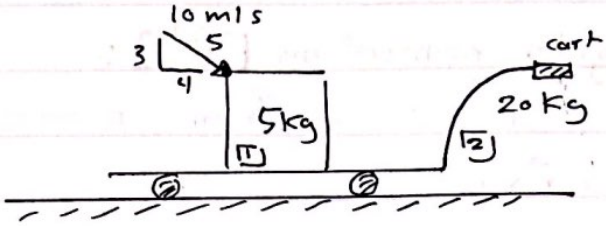
$$\vec{V}_{Box/Boat} = \vec{V}_{Box} - \vec{V}_{Boat}$$

$$-4 = \vec{V}_{Box} - \vec{V}_{Boat} \rightarrow -4 = 10 V_{Boat} - V_{Boat}$$

$V_{Box} = 10 V_{Boat}$

$$V_{Boat} = 0.3636 \text{ m/s} \rightarrow V = \frac{x}{t} \rightarrow x = (0.36)(3) = 1.09 \text{ m}$$

F(15-8)



the cart is initially at rest.

Determine → the common velocity of the cart & package after impact.

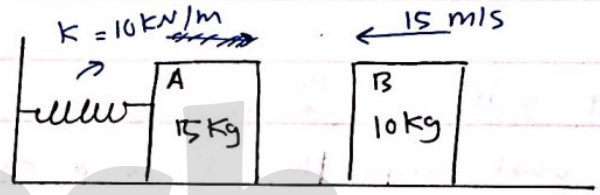
solution :-

$m_1 V_{1,1} + m_2 V_{2,1} = (m_1 + m_2) V_2$

$\xrightarrow{+ve}$ $5(10)(\frac{4}{5}) + 0 = (5 + 20) V_2$

$V_2 = 1.6$ m/s

F(15-11)



spring is initially un compressed.

A is initially at rest.

Determine → maximum compression of the spring.

1) stage #1

$m_A V_{A1} + m_B V_{B1} = (m_A + m_B) V_2$

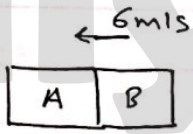
$15(0) + 10(15) = (15 + 10) V_2$

$V_2 = 6$ m/s

2) stage #2 - conservation of energy:

$T_2 + \cancel{V_2}^{0} = \cancel{T_3}^{0} + V_3$ max. compr.

$V_3 = 0$



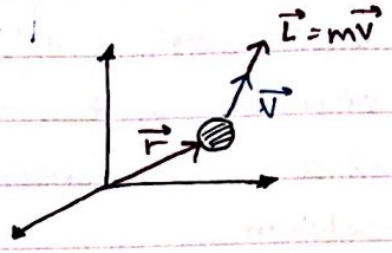
$\frac{1}{2} m V^2 = \frac{1}{2} K S^2$

$\frac{1}{2}(15+10)(6)^2 + 0 = 0 + \frac{1}{2}(10{,}000) X_{max}$ — after impact)

$X_{max} = 0.3$ m

* Angular momentum $[\vec{H}_o]$:-

$\vec{H}_o \equiv$ moment of linear moment about point o



$\vec{H}_o = \vec{r} \times m\vec{V}$

$\frac{d}{dt}(\vec{H}_o) = \dot{\vec{H}}_o$ 1'st derivative.

$\dot{\vec{H}}_o = \frac{d}{dt}(\vec{r}) \times m\vec{V} + \vec{r} \times \frac{d}{dt}(m\vec{V})$

$\theta = 0$ ← 0

$= \vec{V} \times m\vec{V} + \vec{r} \times \Sigma\vec{F}$ → moment

$\dot{H}_o = 0 + \Sigma\vec{M}_o$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$m\vec{V} = m[V_x\hat{i} + V_y\hat{j} + V_z\hat{k}]$

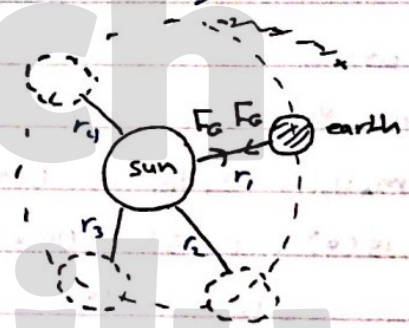
$\Sigma F = m\vec{a}$

$\Sigma\vec{F} = m\frac{dV}{dt} = \frac{d(m\vec{V})}{dt}$

If $\Sigma\vec{M}_o = 0$ then $\dot{H}_o = 0$

which means that H_o is a constant (conservation of linear momentum).

$\vec{r} \times m\vec{V} =$ constant



$F_G = \frac{G\mu m}{r^2}$

$\vec{r}_1 \times m\vec{V}_1 = \vec{r}_2 \times m\vec{V}_2 = \vec{r}_n\, m\vec{V}_n =$ constant

If $\Sigma\vec{M}_o \neq 0$

$\frac{d}{dt}(\vec{H}_o) = \Sigma\vec{M}_o$

$\int_{H_{o1}}^{H_{o2}} d(\vec{H}_o) = \int_{t_1}^{t_2} \Sigma M_o\, dt$

$\vec{H}_{o2} - \vec{H}_{o1} = \int_{t_1}^{t_2} M_o\, dt$

$H_{o1} + \int_{t_1}^{t_2} \Sigma M_o\, dt = \vec{H}_{o2}$ → Principle of angular Impulse & momentum.

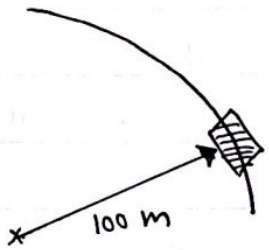
If $\int_{t_1}^{t_2} \Sigma M_o\, dt = 0$ then $\underbrace{\vec{H}_{o1} = \vec{H}_{o2}}_{\text{conservation of angular momentum.}}$

Example (15-13)

$m = 1500$ kg , Friction force of wheels on road $F = 150t^2$

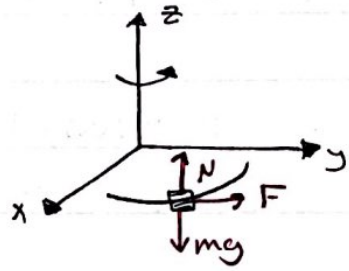
$V_1 = 5$ m/s

Determine → V at $t = 5$ seconds



→ solution :-

$$\vec{H}_{o_1} + \int_{t_1}^{t_2} \Sigma M_o \, dt = \vec{H}_{o_2}$$



$$H_{o_{1z}} + \int_{t_1}^{t_2} \Sigma M_z \, dt = H_{z_2}$$

$$\vec{r}_1 \times m\vec{V}_1 + \int_{t_1}^{t_2} \Sigma M \, dt = \vec{r}_2 \times m\vec{V}_2$$

$$(100)(1500)(5) + \int_0^5 (100 \times 150t^2)\, dt = 100(1500)V_2$$

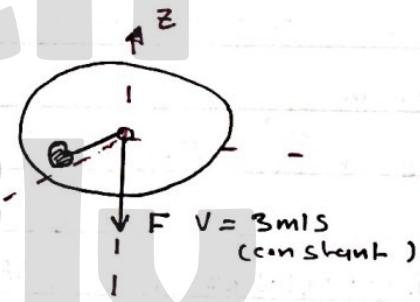
$$V_2 = \left(750000 + \frac{150000t^3}{3}\Big|_0^5\right) / 150000$$

$$V_2 = 9.17 \text{ m/s}$$

Example (15-14)

$r_1 = 0.875$ m , $m = 0.8$ kg

$V_1 = 2$ m/s



(a) $V = ?$ at $r = 0.3$ m

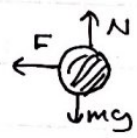
(b) work done by the force F

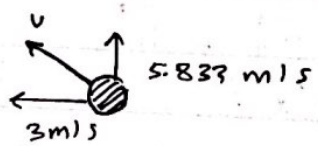
(a) note that → F on the ball passes through z axis & weight + N are parallel to it - so moments of all these forces is zero ($\Sigma M_{o_z} = 0$) - conservation of angular momentum.

$$\vec{H}_{o_1} = \vec{H}_{o_2}$$

$$\vec{r}_1 \times m\vec{V}_1 = r_2 \times m\vec{V}_2$$

$$(0.875)(2) = (0.3)V_2 \rightarrow V_2 = 5.833 \text{ m/s}$$





$$V_2 = \sqrt{(3)^2 + (5.833)^2}$$

$$= 6.559 \text{ m/s}$$

(b) $T_1 + U_{1\to2} = T_2$

$$U_{1\to2} = T_2 - T_1 = \frac{1}{2}m_2V_2^2 - \frac{1}{2}m_1V_1^2$$

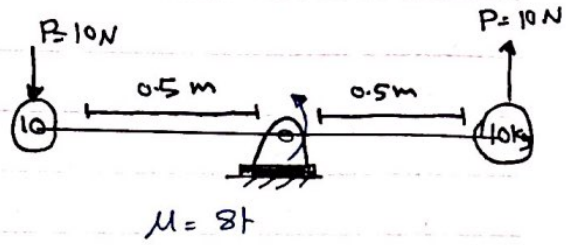
$$= \frac{1}{2}(0.8)(6.559^2 - 2^2)$$

$$= 57.56 \text{ Joule}$$

F (15-24)

Horizontal plane, it starts from rest.

Determine the speed at $t = 4$ sec.



Solution:

$\vec{H}_{o_1} + \int M_o \, dt = \vec{H}_{o_2}$

$+\circlearrowleft \quad \vec{r} \times m\vec{V}_1 + \int M_o \, dt = \vec{r} \times m\vec{V}_2$

$0 + \int_0^4 (8t + 10(0.5)(2)) \, dt = (0.5)(10)(V_2) + 2$

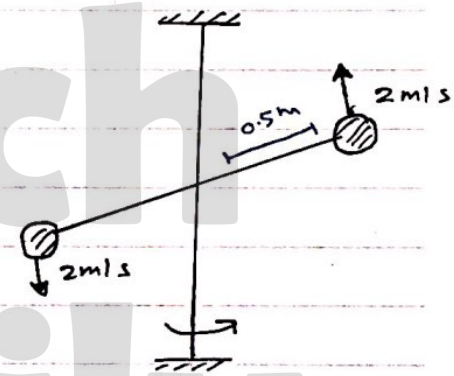
$V_2 =$ ✓ m/s

P (15-100)

$M = t^2 + 2$, $m = 10$ kg

Initial speed (2 m/s)

V of each ball at $t = 3$ sec



Solution:

$H_{o_{1z}} + \int_{t_1}^{t_2} \Sigma M_o \, dt = H_{o_{2z}}$

$2\,[(0.5)(10)(2)] + \int_0^3 (t^2 + 2) \, dt = 2\,[(0.5)(10)(V_2)]$

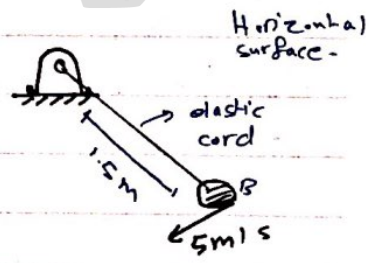
$V_2 =$

P (15-107)

$m = 5$ kg, Determine :- ① Rate of shortening at $r = 1.2$ m

$L_o = 0.5$ m (unstretched) ② speed of the disk

$K = 200$ N/m



② $\vec{H}_{o_1} = \vec{H}_{o_2}$

$\vec{r_1} \times m\vec{V_1} = \vec{r_2} \times m\vec{V_2}$

$1.5\,(m)(5) = 1.2\,(m)\,V_2$

$V_2 = 6.25$ m/s → this is only one component.

① $T_1 + V_1 = T_2 + V_2$

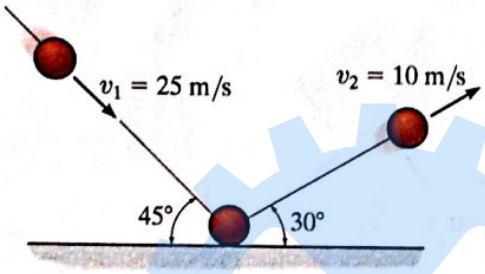
$\frac{1}{2} m (5)^2 + \frac{1}{2} K (1.5 - 0.5)^2 = \frac{1}{2}(5)(\text{speed})^2 + \frac{1}{2}(200)(0.7)^2$

speed = ✓ m/s

speed $\doteq \sqrt{V_2^2 + V_{rate}^2}$ → speed $^2 = (6.25)^2 + V_{rate}^2$ → V_{rate} = ✓ m/s

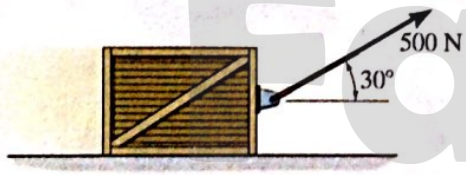
FUNDAMENTAL PROBLEMS

F15–1. The 0.5-kg ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.



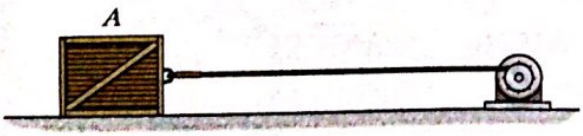
Prob. F15–1

F15–2. If the coefficient of kinetic friction between the 75-kg crate and the ground is $\mu_k = 0.2$, determine the speed of the crate when $t = 4$ s. The crate starts from rest and is towed by the 500-N force.



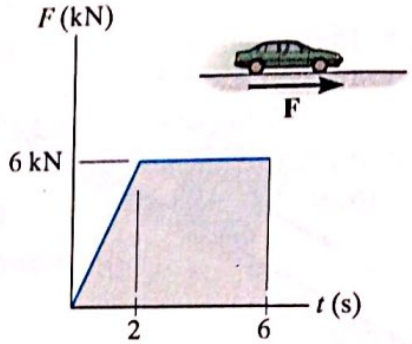
Prob. F15–2

F15–3. The motor exerts a force of $F = (20t^2)$ N on the cable, where t is in seconds. Determine the speed of the 25-kg crate when $t = 4$ s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.



Prob. F15–3

F15–4. The wheels of the 1.5-Mg car generate the traction force **F** described by the graph. If the car starts from rest, determine its speed when $t = 6$ s.

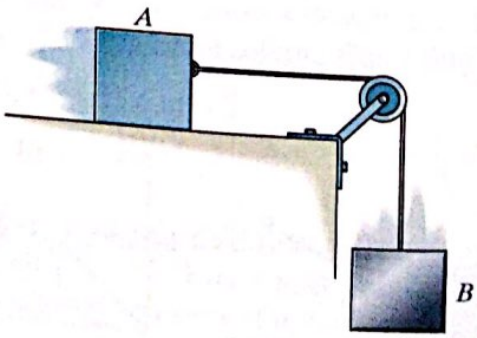


Prob. F15–4

F15–5. The 2.5-Mg four-wheel-drive SUV tows the 1.5-Mg trailer. The traction force developed at the wheels is $F_D = 9$ kN. Determine the speed of the truck in 20 s, starting from rest. Also, determine the tension developed in the coupling, A, between the SUV and the trailer. Neglect the mass of the wheels.

Prob. F15–5

F15–6. The 10-kg block A attains a velocity of 1 m/s in 5 seconds, starting from rest. Determine the tension in the cord and the coefficient of kinetic friction between block A and the horizontal plane. Neglect the weight of the pulley. Block B has a mass of 8 kg.



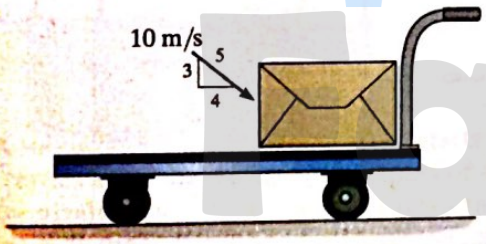
Prob. F15–6

FUNDAMENTAL PROBLEMS

F15–7. The freight cars A and B have a mass of 20 Mg and 15 Mg, respectively. Determine the velocity of A after collision if the cars collide and rebound, such that B moves to the right with a speed of 2 m/s. If A and B are in contact for 0.5 s, find the average impulsive force which acts between them.

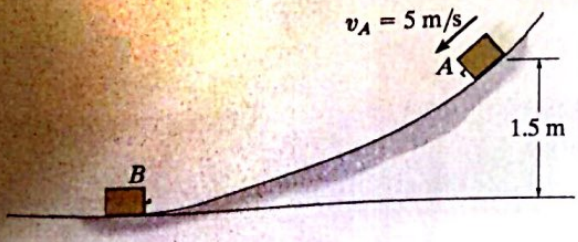
Prob. F15–7

F15–8. The cart and package have a mass of 20 kg and 5 kg, respectively. If the cart has a smooth surface and it is initially at rest, while the velocity of the package is as shown, determine the final common velocity of the cart and package after the impact.



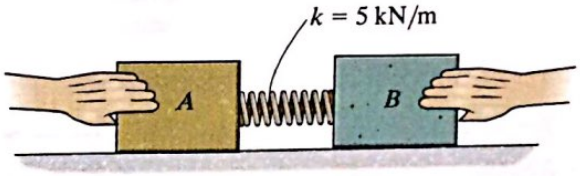
Prob. F15–8

F15–9. The 5-kg block A has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with the stationary block B of mass 8 kg. If the two blocks couple together after collision, determine their common velocity immediately after collision.



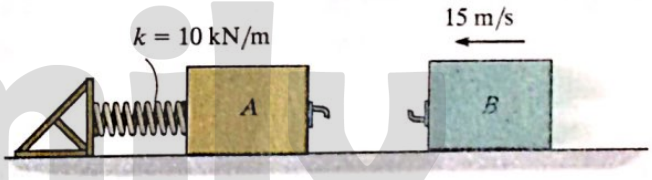
Prob. F15–9

F15–10. The spring is fixed to block A and block B is pressed against the spring. If the spring is compressed $s = 200$ mm and then the blocks are released, determine their velocity at the instant block B loses contact with the spring. The masses of blocks A and B are 10 kg and 15 kg, respectively.



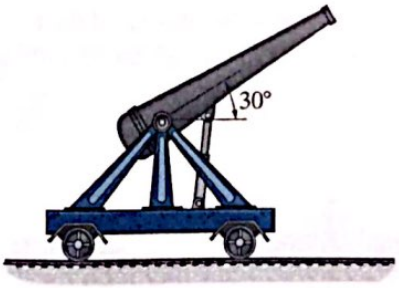
Prob. F15–10

F15–11. Blocks A and B have a mass of 15 kg and 10 kg, respectively. If A is stationary and B has a velocity of 15 m/s just before collision, and the blocks couple together after impact, determine the maximum compression of the spring.



Prob. F15–11

F15–12. The cannon and support without a projectile have a mass of 250 kg. If a 20-kg projectile is fired from the cannon with a velocity of 400 m/s, measured *relative* to the cannon, determine the speed of the projectile as it leaves the barrel of the cannon. Neglect rolling resistance.



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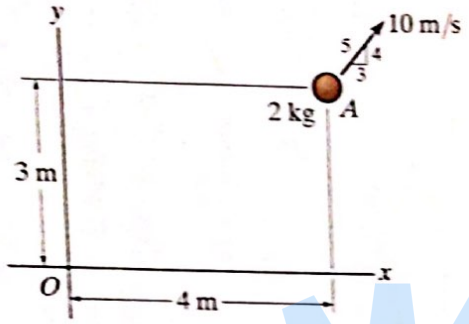
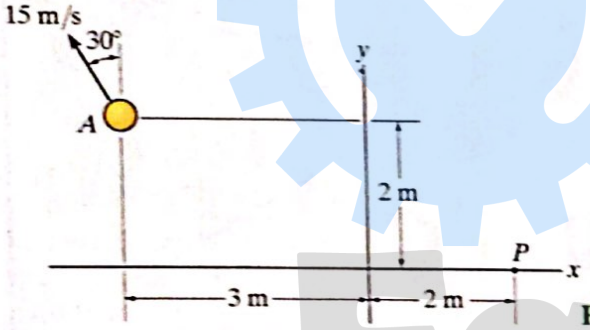
Prob. F15–12

F15–19. The 2-kg particle A has the velocity shown. Determine its angular momentum $\mathbf{H}_O$ about point O.



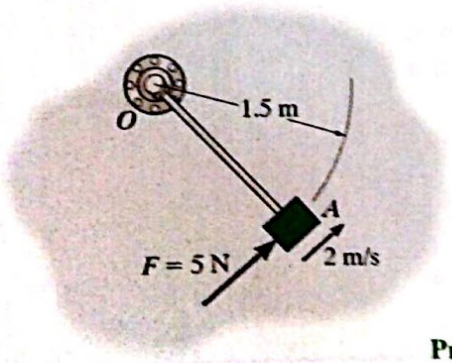
Prob. F15–19

F15–20. The 2-kg particle A has the velocity shown. Determine its angular momentum $\mathbf{H}_P$ about point P.



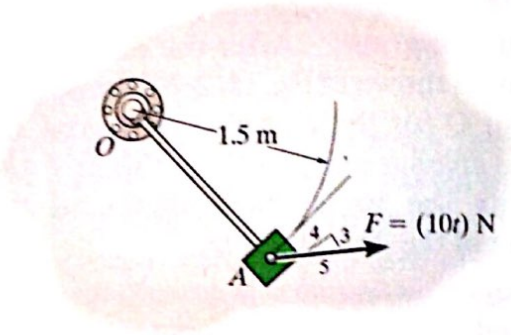
Prob. F15–20

F15–21. Initially the 5-kg block is moving with a constant speed of 2 m/s around the circular path centered at O on the smooth horizontal plane. If a constant tangential force $F = 5$ N is applied to the block, determine its speed when $t = 3$ s. Neglect the size of the block.



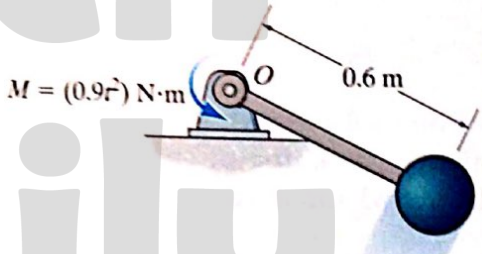
Prob. F15–21

F15–22. The 5-kg block is moving around the circular path centered at O on the smooth horizontal plane when it is subjected to the force $F = (10t)$ N, where t is in seconds. If the block starts from rest, determine its speed when $t = 4$ s. Neglect the size of the block. The force maintains the same constant angle tangent to the path.



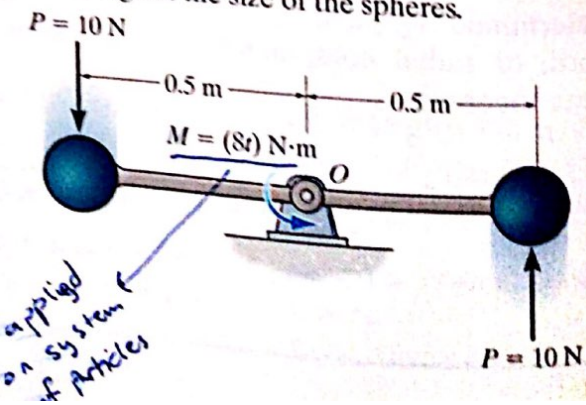
Prob. F15–22

F15–23. The 2-kg sphere is attached to the light rigid rod, which rotates in the *horizontal plane* centered at O. If the system is subjected to a couple moment $M = (0.9t^2)$ N·m, where t is in seconds, determine the speed of the sphere at the instant $t = 5$ s starting from rest.



Prob. F15–23

F15–24. Two identical 10-kg spheres are attached to the light rigid rod, which rotates in the horizontal plane centered at pin O. If the spheres are subjected to tangential forces of $P = 10$ N, and the rod is subjected to a couple moment $M = (8t)$ N·m, where t is in seconds, determine the speed of the spheres at the instant $t = 4$ s. The system starts from rest. Neglect the size of the spheres.



applied on system of Particles

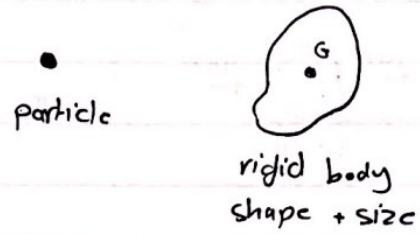
Prob. F15–24

CHAPTER (16) . Kinematics of a rigid body.

C.m → center of mass

C.G → center of gravity

C → center of geometry.



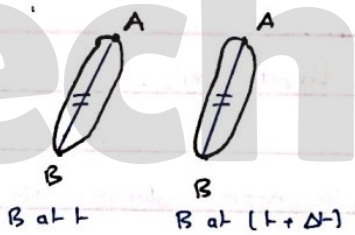
Types of motion (Rigid body) :-

1) Translation
2) Rotation about a fixed axis.
3) General plane motion (translation + rotation)
4) Rotation about a point.
5) 3-D motion "Trans, x,y,z + Rot about x,y,z"

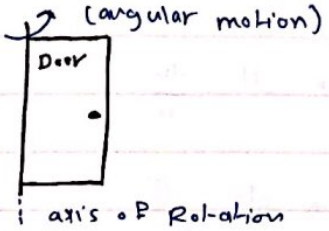
1 Translation :

→ If the rigid body moved from one position to another and the orientation of the line (AB) is the same I know that it is translation motion.

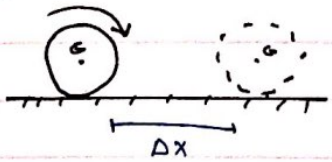
↳ Rectilinear or curvilinear.



2 Rotation about a fixed axis :-

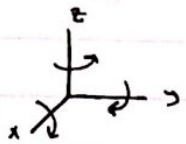


3 General plane motion :-



Rotation about G & Translation

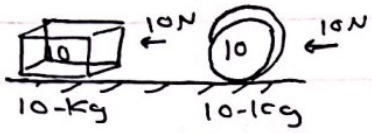
4 Rotation about a point :-



(Ball & socket)

5 3-D Motion

note → Even if two (Rigid bodies) have the same mass, it's not necessary that they have the same kinetics (unlike particles)

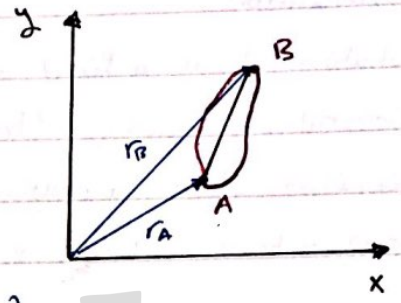


→ They are not the same.

---

[1] Translation :-

$$\vec{V_B} = \vec{V_A} + \frac{d}{dt}[\vec{r}_{B/A}]$$



$$\vec{V_B} = \vec{V_A} = \vec{V_C} = \vec{V_D} \dots = \vec{V_n}$$

$$\vec{a_B} = \vec{a_A} = \vec{a_C} \dots = \vec{a_n}$$

→ During translation all particles "points" have the same (displacement, velocity & acceleration)

---

[2] Rotation about a fixed axis :-

$\Delta\theta$ ≡ angular displacement during the time interval Δt.

$\Delta\omega = \frac{\Delta\theta}{\Delta t}$, $\Delta\omega$: avg angular velocity.

$\omega = d\theta/dt$ (in rad) → $(360° = 2\pi \equiv 1 \text{ Rev})$

α ≡ Angular acceleration $= \frac{d\omega}{dt}$ (rad/s^2)

$\frac{d\omega}{dt} = \alpha = \text{const}$

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha \, dt$$

$$\omega - \omega_0 = \alpha t \rightarrow \omega = \omega_0 + \alpha t \quad - ①$$

$$\int_{\theta_0}^{\theta} d\theta = \int_0^t (\omega + \alpha t)\, dt$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \quad - ②$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta_2 - \theta_0) \quad - ③$$

$\alpha = \frac{d\omega}{dt} \rightarrow dt = \frac{d\omega}{\alpha}$

$\omega = \frac{d\theta}{dt} \rightarrow dt = \frac{d\theta}{\omega}$

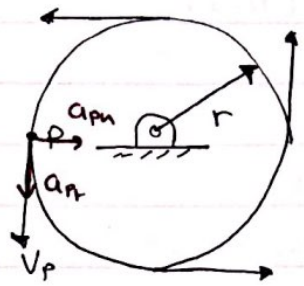
$\frac{d\omega}{\alpha} = \frac{d\theta}{\omega} \rightarrow \omega\, d\omega = \alpha\, d\theta$

* Rotation about a fixed axis :-

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

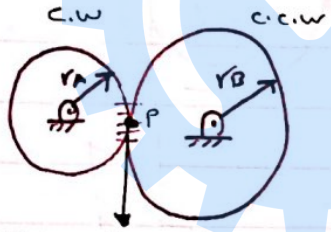


$V_p = \omega r$

$a_{pt} = \alpha r$ $\quad a_{pn} = \frac{V_p^2}{\rho} = \frac{(r_p \omega_p)^2}{r} = \omega_p^2 r_p$

$a_p = \sqrt{(a_{pt})^2 + (a_{pn})^2}$

* Gears :-



note → only tangential acceleration is the same.

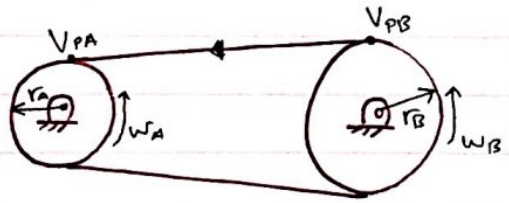
$\omega_A r_B = V_p = \omega_B r_B$

(Tangential direction)

$a_{pt} = \alpha_A r_A$

$a_{pn} = r_A \omega_A^2$] $\vec{a}_p = \sqrt{a_{pt}^2 + a_{pn}^2}$

* Pulleys

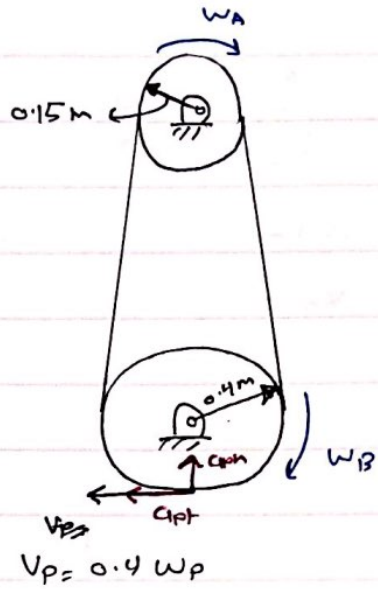


$\omega_A r_A = \omega_B r_B$

$\alpha_A r_A = \alpha_B r_B$

Example (16-2)

$|V_P| = ?$, $|a_P| = ?$



solution:

$$a_{Pt} = (0.4)\left(\frac{0.15(2)}{0.4}\right) = \checkmark \text{ m/s}^2$$

$$a_{Pn} = \omega_B^2 (0.4) = \checkmark \text{ m/s}^2$$

$$|a_P| = \sqrt{a_{Pt}^2 + a_{Pn}^2}$$

$\omega_0 = 0$

$\alpha_A = 2 \text{ rad/s}^2$

$\theta = 2 \text{ rev} = 2(2\pi)$

$0.15\,\omega_A = 0.4\,\omega_B$

$0.15\,\alpha_A = 0.4\,\alpha_B$

$\omega_A^2 = 0^2 + 2(2)(2 \times 2\pi)$

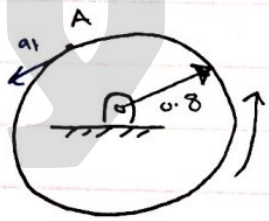
$\omega_B = \frac{0.15}{0.4}\,\omega_B = \checkmark \text{ rad/s}$

P(16-8)

$\alpha = 12 + 3t^2$ → we have to use the basic def. of acceleration

$\omega_0 = 12 \text{ rad/s}$

a_t at $t = 2\text{sec}$?



solution:

$V_A = 0.8\,\omega$

$$\int_{12}^{\omega} d\omega = \int_0^2 (12 + 3t^2)\,dt$$

$$\omega - 12 = 12(2) + \frac{3}{3}(2)^3$$

$\omega = 44 \text{ rad/s}$

$V_A = (0.8)(44) = \checkmark \text{ m/s}$

$= 35.2 \text{ m/s}$

$a_{At} = (12 + 3(2)^2)(0.8) = 19.2 \text{ m/s}^2$

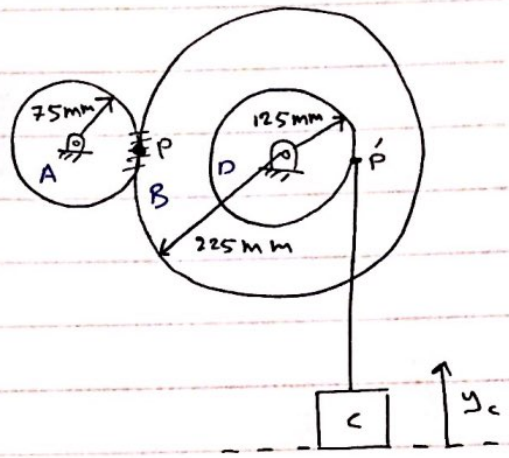
$a_{An} = (44)^2 (0.8)$

F(16-6)

$\alpha_A = 4.5 \text{ rad/s}^2$

starts from rest.

Determine → velocity of (C) & the distance it travels when $t = 3$ sec.



Solution :-

$a_c = a_{pt} = (0.125)\,\alpha_B$

$(75)(4.5) = 22.5\,\alpha_B$

$\alpha_B = 15 \text{ rad/s}^2$

$a_c = 0.1875 \text{ m/s}^2 \uparrow$

$V_c = 0.5625 \text{ m/s}$

~~$\omega_A = 0$~~

~~$\alpha_A = 4.5 \text{ rad/s}$~~

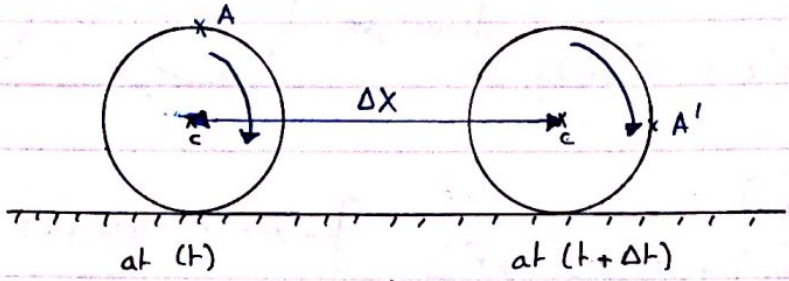
$V_c = 0 + a_c(3)$

$y_c = V_c t + \frac{1}{2} a_c t^2$

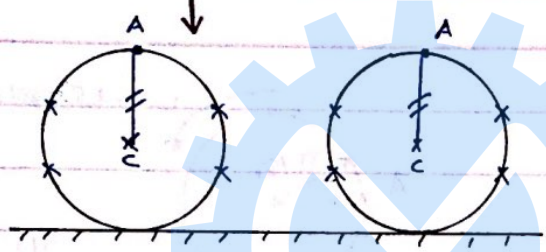
$y_c = 0.843 \text{ m}$ #

* General plane motion :-

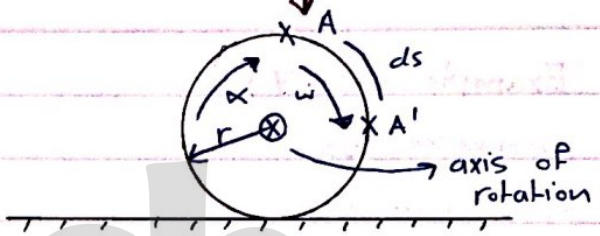
→ when a body is in general plane motion, it undergoes two types of motion :- [1] Translation [2] Rotation



(Rolling motion)



(Translation only)

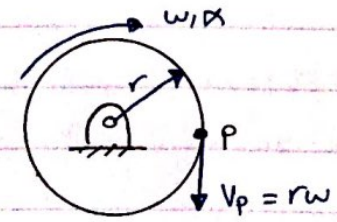


(Rotation only)

→ In this part of this subject it will be required to know how to find (V, a) of any point on a rigid body subjected to G.P.M and (α, ω) to that rigid body.

→ In problem solving we will always separate the problem into two parts : [1] Translation <u>only</u> [2] Rotation <u>only</u>

* Rotation about a fixed axis (rivision) :-



$V_p = r\omega$ $\qquad \vec{V_p} = \vec{\omega} \times \vec{r}$

$a_{pt} = \alpha r$ $\qquad \vec{a_{pt}} = \vec{\alpha} \times \vec{r}$

$a_{pn} = \omega^2 r$ $\qquad \vec{a_{pn}} = \vec{\omega} \times \vec{V_p}$

$= \vec{\omega} \times (\vec{\omega} \times \vec{r})$

note → (ω) as a vector is found by the right hand rule (fingers with the direction of (ω) and the thumb is the direction of $\vec{\omega}$)

* Instantaneous center of zero velocity:

→ This method is used when we want to find (w) of a body.

→ To use this method:

- you have to know how to locate I.C
- you should have (mostly) two points with two velocities which you know their direction.
- after you locate I.C (which is a point that has zero velocity at that instant) apply the equation

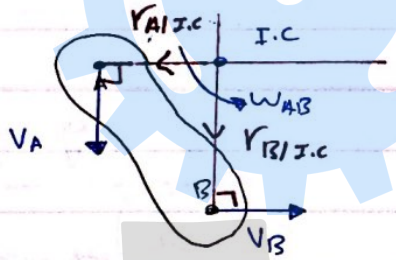
$V_A = \cancel{V_{I.C}}^{0} + V_{A/I.C}$

$V_A = \omega \, r_{A/I.C}$

---

* Cases :-

[1] Two non parallel velocities:-

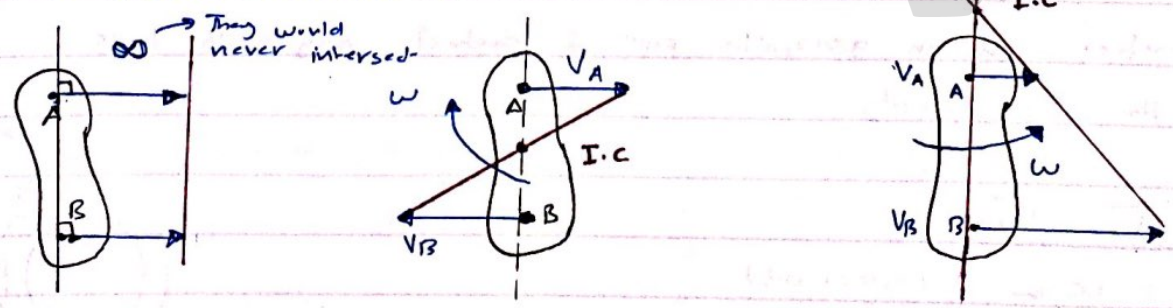


→ Draw two lines ⊥ to V_A & V_B

→ The intersection of these lines is the (I.C)

→ you will have to use geometry to find $r_{A/I.C}$ or $r_{B/I.C}$

[2] Two parallel velocities :-



If you have two velocities equal in mag and parallel then:

$\omega = \frac{V}{\infty} = 0$

Translation only.

- the body will not rotate.

* Velocity & acceleration analysis :-

→ we can use three methods to find V or a :-

1. scalar analysis
2. Vector analysis
3. Instantaneous center of zero velocity (I.C)

scalar :

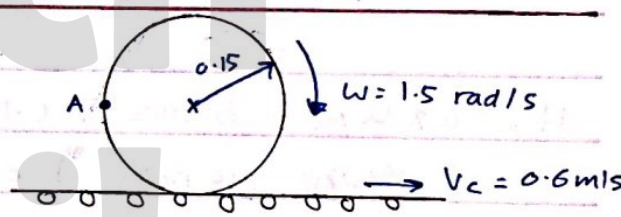
* $\vec{V_A} = \vec{V_c} + \vec{V_{A/c}}$
* $\vec{a_A} = \vec{a_c} + [(\vec{a_{A/c}})_t + (\vec{a_{A/c}})_n]$

Vector :-

* $\vec{V_A} = \vec{V_c} + [\vec{\omega} \times \vec{r_{A/c}}]$
* $\vec{a_A} = \vec{a_c} + \vec{\alpha} \times \vec{r_{A/c}} + \vec{\omega} \times [\vec{\omega} \times \vec{r_{A/c}}]$

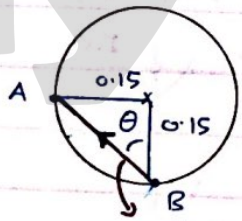
Example (16-7)

Determine (V_A).



Solution :-

↳ we choose (B) as the second point because it has a known velocity.



$r_{A/B}$

$\tan\theta = \frac{0.15}{0.15} = 1$

$\theta = 45°$

$r_{A/B} = \frac{0.15}{\cos 45} = \sqrt{2}\,(0.15)$

$\vec{V_A} = \vec{V_B} + \vec{V_{A/B}}$

$\underset{\rightarrow}{V_{Ax}} + V_{Ay}\uparrow = \underset{\rightarrow}{V_B} + \omega r_{A/B} \angle 45°$

$\underset{\rightarrow}{V_{Ax}} + V_{Ay}\uparrow = \underset{\rightarrow}{0.6} + 1.5(\sqrt{2})(0.15) \angle 45°$

↑V ; $V_{Ay} = (1.5)(\sqrt{2})(0.15)\sin 45$ → we take all vertical components.

$V_{Ay} = 0.225$ m/s

→H ; $V_{Ax} = 0.6 + (1.5)(\sqrt{2})(0.15)\cos 45)$

$V_{Ax} = 0.825$ m/s

The direction of $V_{A/B}$ is ⊥ to $r_{A/B}$.

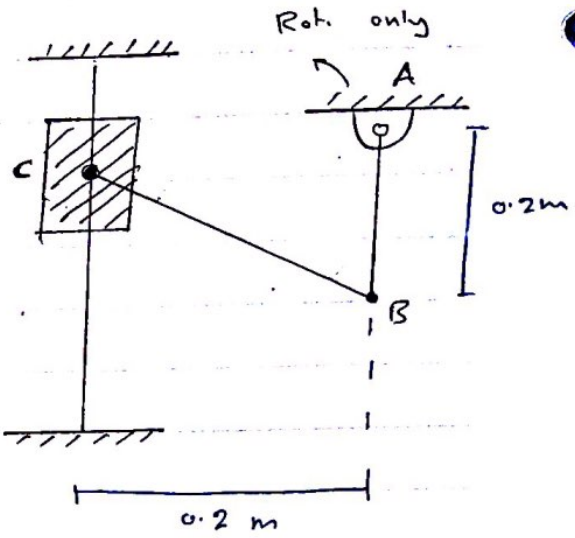
$V_A = \sqrt{(V_{Ax})^2 + (V_{Ay})^2} = \sqrt{(0.225)^2 + (0.825)^2} = 0.855$ m/s

Example (16 - 8 & 16)

Determine a_B

$V_c = 2\,m/s \downarrow$

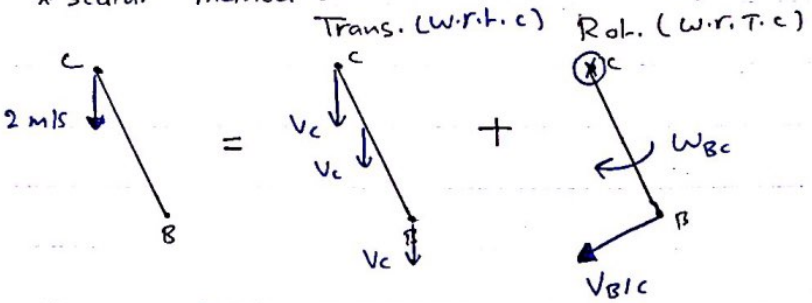
$a_c = 1\,m/s^2 \downarrow$



Solution :-

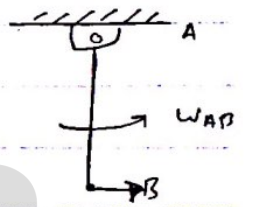
* scalar method :-

with respect to



$\vec{V_B} = \vec{V_c} + \vec{V_{B/c}}$

$0.2\,\omega_{AB} = 2\downarrow + (\sqrt{2})(0.2)\,\omega_{BC}$ (45°)



$V_B = 0.2\,\omega_{AB}$

$a_B = ?$

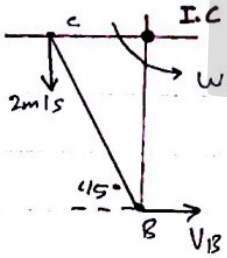
V: $0 = -2 - 0.2\sqrt{2}\,\omega_{BC}\sin 45°$

$\omega_{BC} = -10\ rad/s = \underline{10\ rad/s}$ (opposite to the direction we assumed)

H: $0.2\,\omega_{AB} = 0 - 0.2\sqrt{2}(-10)\cos 45$

$\underline{\omega_{AB} = 10\ rad/s}$ c.c.w

[2] now to find V_B we'll use I.C :-



$\omega_{BC} = 2$

$V_B = \omega_{BC}(0.2)\sin 45$

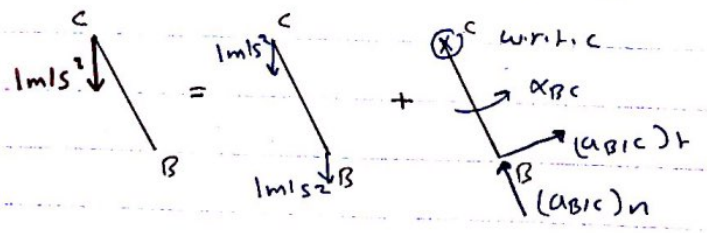
* To determine the location of I.C you draw two lines ⊥ to the velocities (V_B & V_A) → the point of intersection of these two lines is (I.C) which has a velocity of zero so the eqn becomes:

$V_B = \cancel{V_{IC}}^{0} + V_{B/IC}$

$V_B = \omega\, r_{B/IC}$

(the same for V_A)

* acceleration analysis:

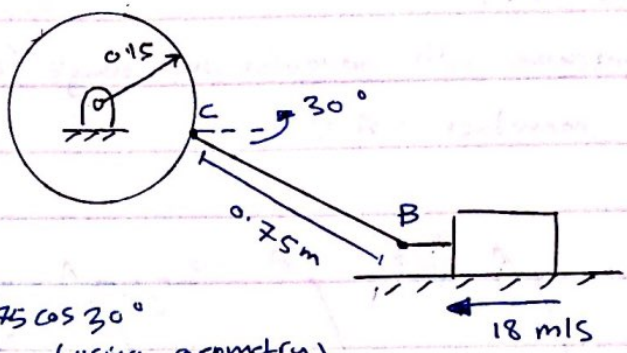


$\vec{a}_B = 1\downarrow + (\sqrt{2})(0.2)\,\alpha_{BC}\ \angle 45° + \sqrt{2}(0.2)(10)^2$ (45°)

$0.2\,\alpha_{AB} + (10)^2(0.2)\uparrow = 1\downarrow + 0.2\sqrt{2}\,\alpha_{BC}\ \angle 45 + 20\sqrt{2}$ (45°)

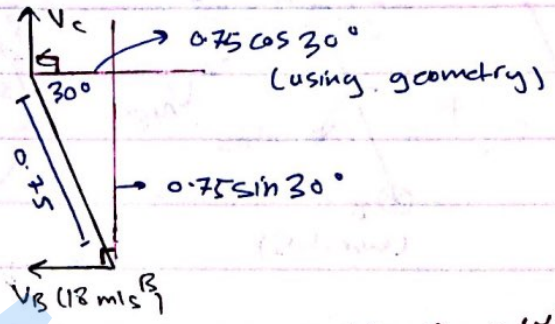
F(16-11)

Determine → The angular velocity and acceleration at the instant shown.



Solution :-

* using I.C :-



$V_B = \omega_{BC}\, r_{B/IC}$

$\omega_{BC} = \dfrac{18}{0.75 \sin 30} = 48$ rad/s

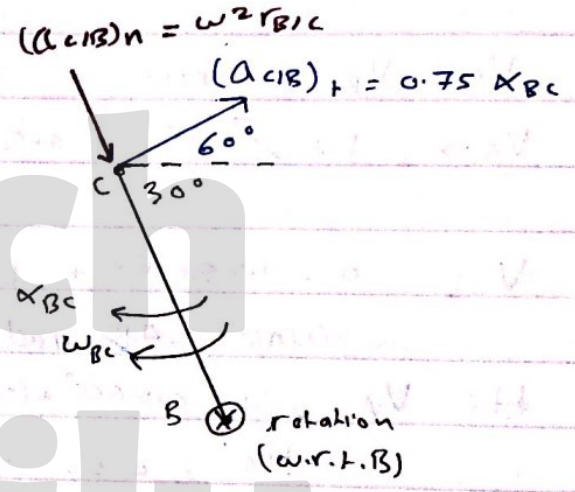
c.w

$V_C = \omega_{BC}\, r_{C/I.C}$

$= (48)(0.75)\cos 30 = 31.2$ m/s ↑ upward

also $V_C = \omega_{disk}\, r_{disk}$

$\omega_{disk} = \dfrac{31.2}{0.15}$ rad/s



* acceleration analysis: (scalar method)

$\vec{a}_C = \vec{a}_B + (\vec{a}_{C/B})_t + (\vec{a}_{C/B})_n$ constant speed

$0.15\,\alpha_D \uparrow + (208)^2(0.15) \leftarrow = 0 + 0.75\,\alpha_{BC} \angle 60° + 48^2(0.75) \angle 30°$

H; $0 - 6489.6 = 0.75\cos 60\,(\alpha_{BC}) + 48^2(0.75)\cos 30$

$\alpha_{BC} = \quad$ rad/s²

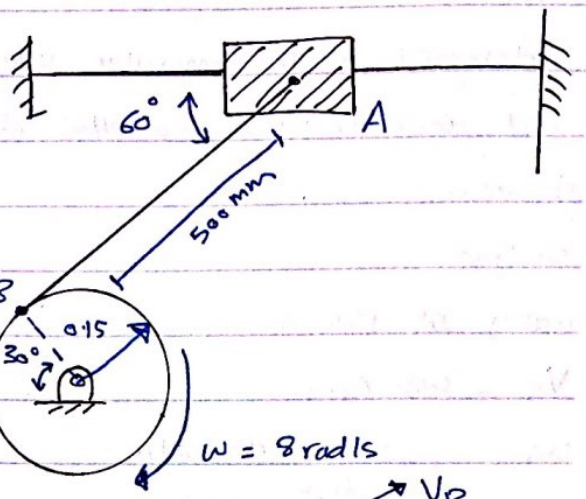
V; $0.15\,\alpha_D + 0 = (0.75)(\quad)\sin 60 - 48^2(0.75)\sin 30$

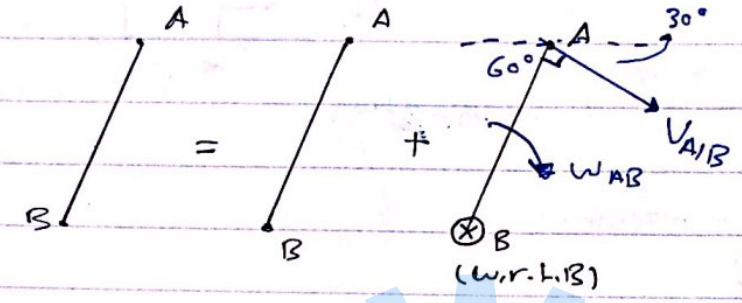
$\alpha_D = \quad$ rad/s²

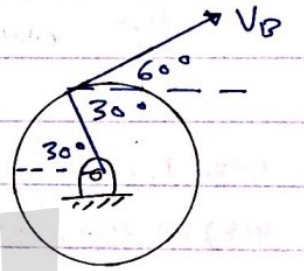
(D → Disk)

P(16-57)

Determine the angular velocity (ω) of member AB.







$V_A = V_B + V_{A/B}$

$V_A \rightarrow = 1.2 \angle 60° + 0.5\,\omega_{AB} \angle 30°$

$V_B = \omega r$

$= (8)(0.15) = 1.2$ m/s

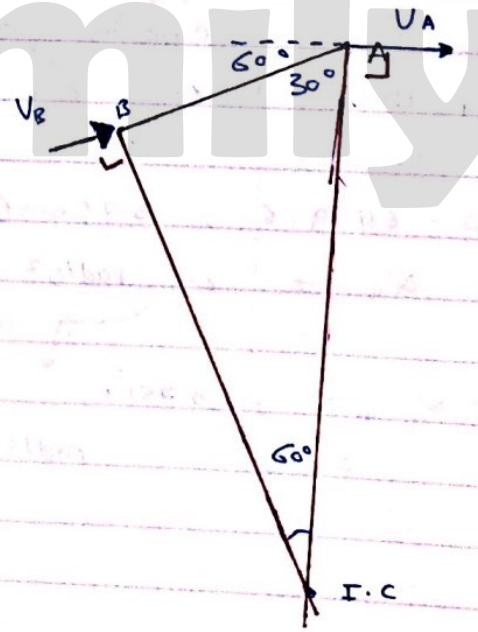
V: $0 = 1.2 \sin 60 + (-0.5)\,\omega_{AB} \sin 30$

$\omega_{AB} = 4.15$ rad/s

H: $V_A = 1.2 \cos 60° + (-0.5\,\omega_{AB} \cos 30°)$

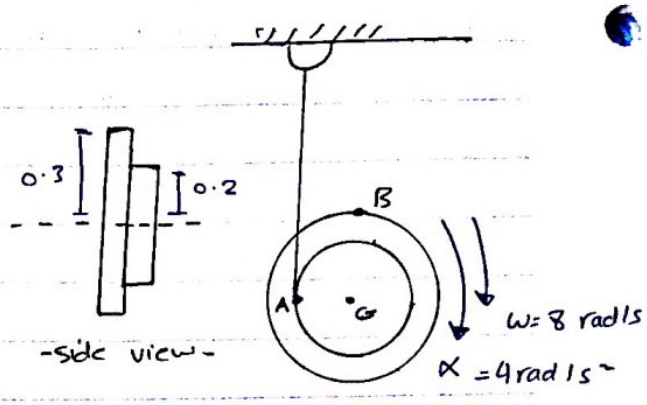
$V_A = 0.33$ m/s

If you want to use I.C:



Example (16-15)

Determine the acceleration of point B (a_B).



Solution :-

(G.P.M)

$\vec{a}_B = \vec{a}_G + (\vec{a}_{B/G})_t + (\vec{a}_{B/G})_n$

to find a_G: (I use point A)

$\vec{a}_G = \vec{a}_A + (\vec{a}_{G/A})_t + (\vec{a}_{G/A})_n$

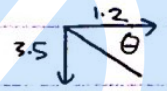
V: $a_G \downarrow = 0 + 4(0.2)$

$a_G = 0.8 \text{ m/s}^2 \downarrow$

$\hookrightarrow \vec{a}_B = 0.8\downarrow + 4(0.3) \rightarrow + (3)^2(0.3)\downarrow$

$|\vec{a}_B| = \sqrt{(1.2)^2 + (3.5)^2} = 3.7 \text{ m/s}^2$

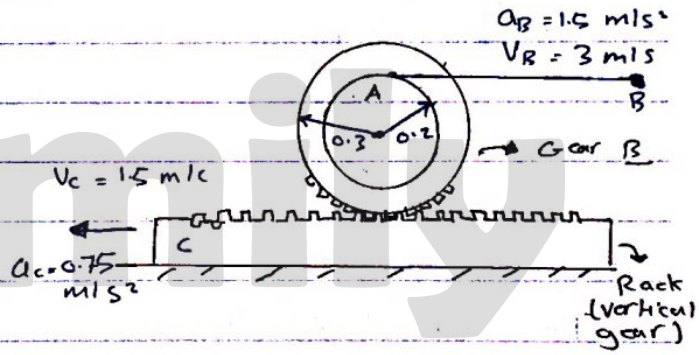
$\theta = \tan^{-1}\left(\frac{3.5}{1.2}\right)$



F (16-22)

Gear is attached to the pulley

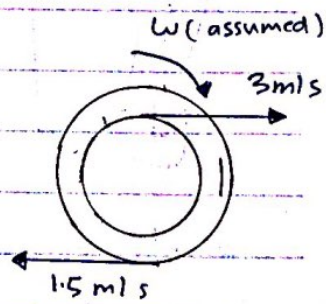
Determine → the angular acceleration of the gear at this instant.



Solution:

G. P. M

→ Remember: ~~at~~ in gears, at the point of contact they both have the same velocity.



* $\vec{V}_A = \vec{V}_c + \vec{V}_{A/c}$

$3 \rightarrow = 1.5 \leftarrow + \omega(0.2 + 0.3)$

$3 = -1.5 + 0.5\,\omega \rightarrow \omega = \frac{4.5}{0.5} = 9 \text{ rad/s} \circlearrowleft \omega$

* $\vec{a}_A = \vec{a}_c + (\vec{a}_{A/c})_t + (\vec{a}_{A/c})_n$

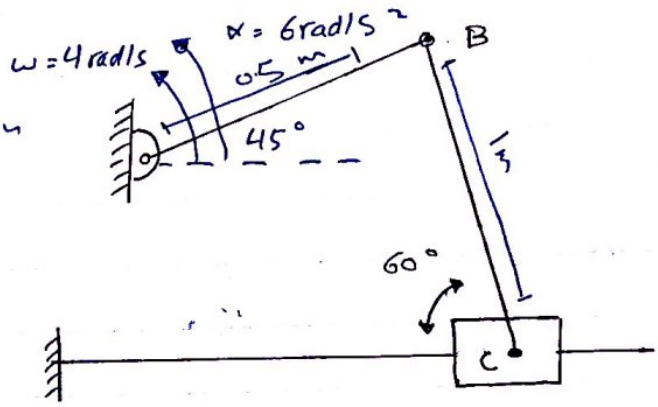
H ; $1.5 \rightarrow = 0.75 \leftarrow + 0.5\,\alpha \rightarrow$

$1.5 = -0.75 + 0.5\,\alpha$

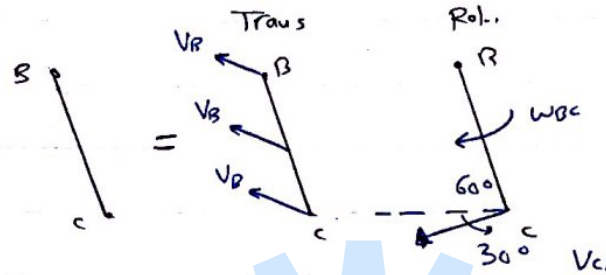
$\alpha = \frac{2.25}{0.5} = 4.5 \text{ rad/s}^2 \circlearrowleft$ c.w

Problem (107)

Determine → The velocity & acceleration of c (a_c & V_c)



→ solution:-



$V_{c/B} = \omega_{BC}$

$\vec{V_c} = \vec{V_B} + \vec{V_{c/B}}$

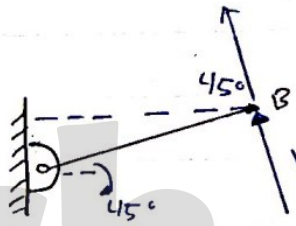
$V_c \leftarrow = 2 \angle 45 + \omega_{BC} (1) \angle 30$

V; $0 = 2\sin 45 - \omega_{BC} \sin 30$

$\omega_{BC} = 2.8$ rad/s (c.w)

H; $-V_c = -2\cos 45 - (2.8)\cos 30$

$V_c = 3.8$ m/s ←



$V_B = 4(0.5)$
$= 2$ m/s

$\vec{a}_c = (\vec{a}_B)_t + (\vec{a}_B)_n + (\vec{a}_{c/B})_n + (\vec{a}_{c/B})_t$

$a_c \leftarrow = 3 \angle 45 + 8 \angle 45 + (2.8)^2(1) \angle 60 + \alpha(1) \angle 30$

+ve↑ V; $0 = 3\sin 45 - 8\sin 45 + (2.8)^2 \sin 60 + (-\alpha)\sin 30$

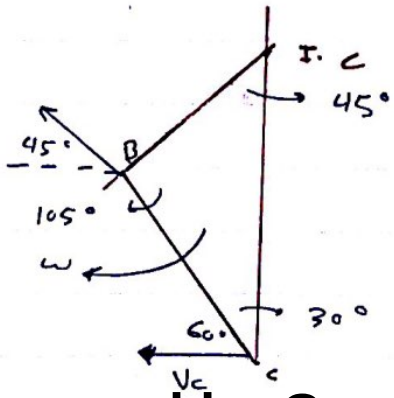
$\alpha_{BC} = 6.5$ rad/s²

+ve← H; $a_c = 3\cos 45 + 8\cos 45 + (2.8)^2 \cos 60 + \alpha_{BC}\cos 30$

$a_c =$ m/s²

* If you want to use I.C:

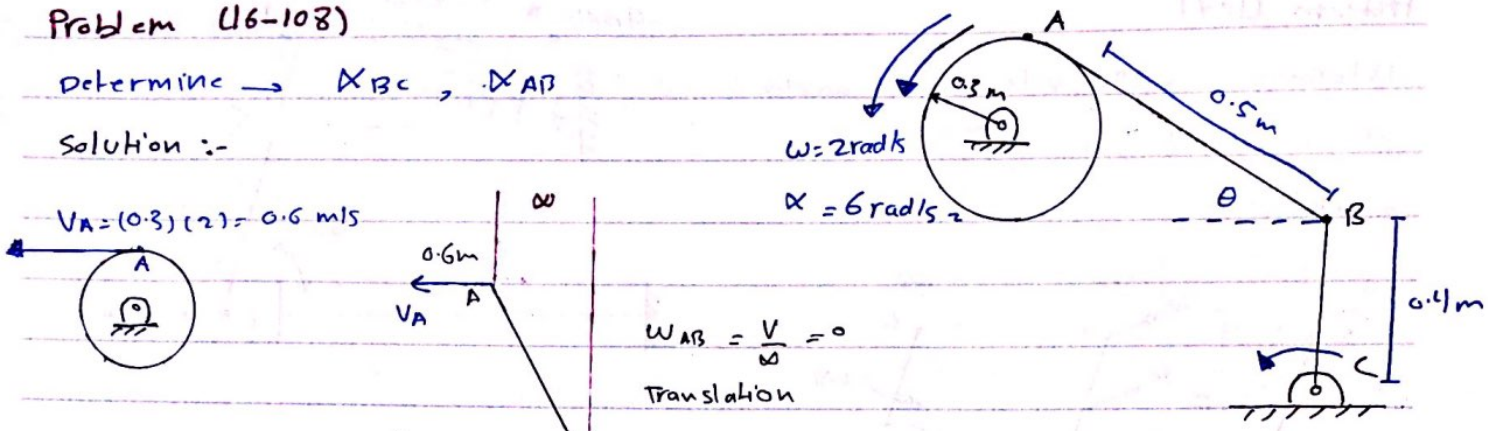
$V_B = \omega_{BC}\, r_{B/IC}$



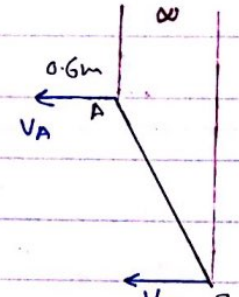
Problem (16-108)

Determine → α_{BC} , α_{AB}

Solution :-



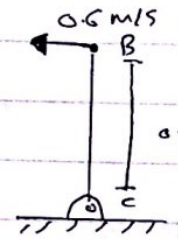
$V_A = (0.3)(2) = 0.6$ m/s

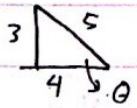


$\omega_{AB} = \frac{V}{\infty} = 0$

Translation

$V_B = V_A = 0.6$ m/s

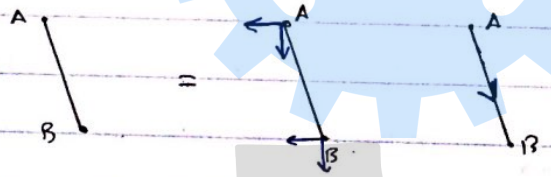


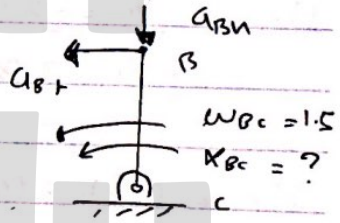


$V_B = \omega_{BC}\, r$

$\omega_{BC} = \frac{0.6}{0.4} = 1.5$ rad/s ↺

acceleration :- G.P.M





$(a_{Bn}) = \omega^2 r$

$= (1.5)^2 (0.4)$

$(a_B)_t = \alpha r$

$= \alpha (0.4)$

$\vec{a}_B = (\vec{a}_A)_t + (\vec{a}_A)_n + (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n$

$(1.5)^2(0.4) \downarrow + 0.4\,\alpha_{BC} \leftarrow = 1.8 \leftarrow + 1.2 \downarrow + 0.5\,\alpha_{AB} \angle 90-\theta$

V ; $0.9 \downarrow + 0 = 0 - 1.2 + 0.5\,\alpha_{AB} \sin(53)$

$\alpha_{AB} = 0.75$ rad/s²

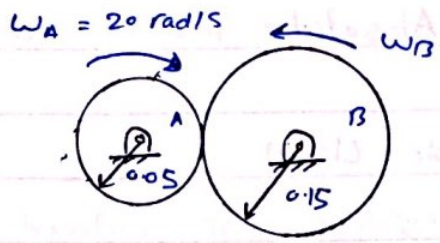
H ; $-0.4\,\alpha_{BC} = -1.8 + 0.5 \cos 53\,(0.75)$

$\alpha_{BC} = 3.93$ rad/s²

Problem

$\alpha_A = 4t^3$

find ω_B at $t = 2$ sec.



Solution :-

$r_A \alpha_A = r_B \alpha_B$

$\alpha_B = \frac{r_A}{r_B}(4t^3) = 3(4t) = 12t^3$

$r_A \omega_A = r_B \omega_B$

$\omega_B = \frac{r_A}{r_B}\omega_A$

$\alpha_B = 12t^3 = \frac{d\omega}{dt}$

$\int_{\omega_i}^{\omega} \frac{d\omega}{dt} = \int_0^2 12t^3 dt$

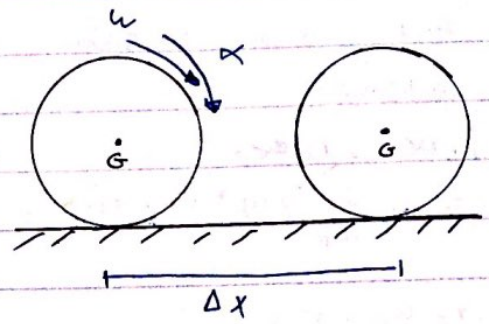
$\omega_B = 3t^4 \Big|_0^2 = 3(16) - 0 = 48$ rad/s

16.4 (Absolute motion analysis)

Example (16-4)

The cylinder rolls without slipping

Determine → velocity & acceleration of center (G)



Solution

$1 \text{ rev} = 2\pi \text{ rad} = 360°$

$x = 2\pi r \big|_{1 \text{ rev}}$

for angle θ

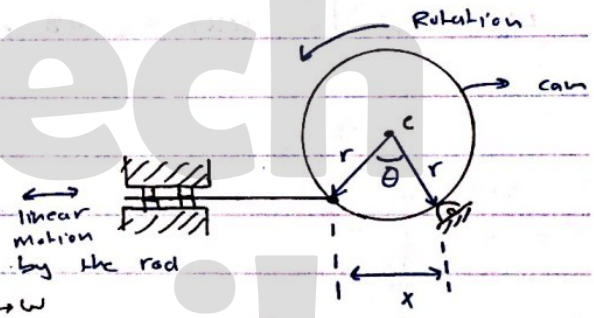
$x = r\theta \longrightarrow V_G = \frac{dx}{dt} = \frac{r\,d\theta}{dt} = r\omega$

→ By taking the 2nd derivative:

$a_G = \frac{d^2x}{dt^2} = r\alpha$ #

Example (16-3)

Determine → velocity & acceleration of the rod when the cam is in the arbitrary position θ



solution:

$V = \frac{dx}{dt} = \frac{d(2r\cos\theta)}{dt} = -2r\sin\theta \frac{d\theta}{dt} \rightarrow \omega$

$= -2r\omega\sin\theta$

$x = 2r\cos\theta$

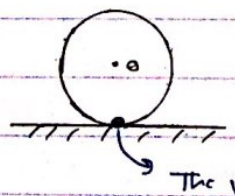
$a = \frac{dv}{dt} = -2r\left[\omega(\cos\theta)\frac{d\theta}{dt} + \sin\theta\frac{d\omega}{dt}\right]$

$= -2r[\omega^2\cos\theta + \alpha\sin\theta]$ #

---

note :-

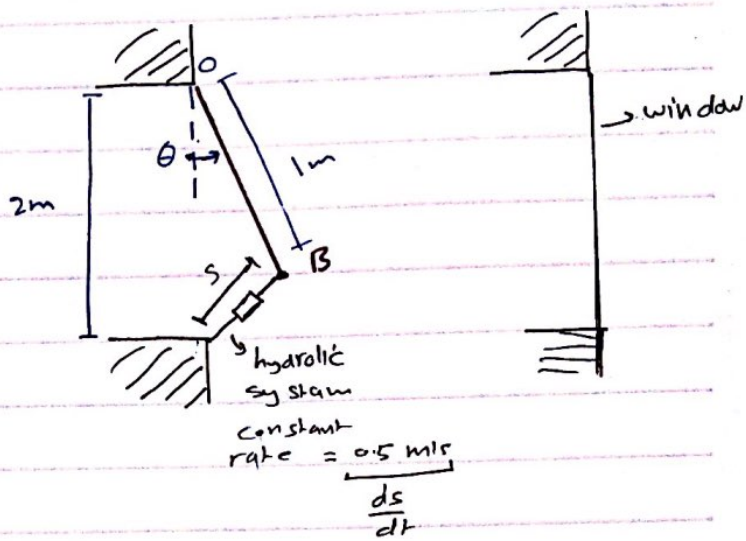
when a body rolls without slipping : (it will stay on the ground)



The velocity of this point is <u>zero</u>

Example (16-5)

Determine → ω, α when $\theta = 30°$



Solution:

→ using law of cosines:

$$s^2 = (2)^2 + (1)^2 - 2(1)(2)\cos\theta$$

by $\frac{d}{dt}$

$$2s\frac{ds}{dt} = 0 - 4(-\sin\theta)\frac{d\theta}{dt}$$

$$2s(0.5) = 4\omega\sin\theta$$

at $\theta = 30°$

$$\omega = \frac{(1.239)(0.5)}{2\sin 30} = 0.62 \text{ rad/s}$$

To obtain α (I take the second derivative)

by $\frac{d}{dt}$

$$2s\overbrace{\frac{ds}{dt}}^{\text{constant}} = 4\omega\sin\theta$$

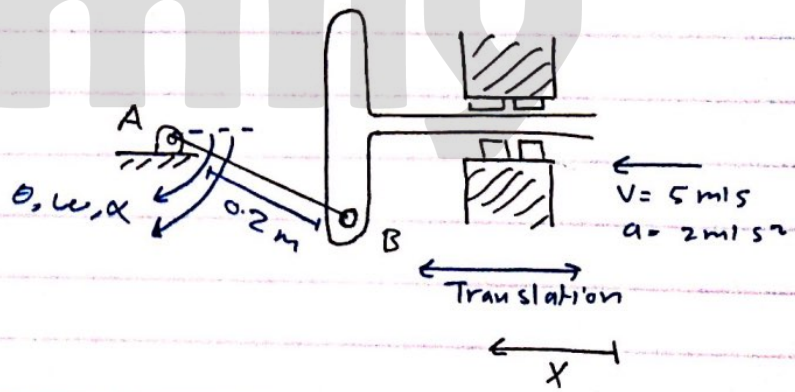
$$2s(0) + 2\left(\frac{ds}{dt}\right)^2 = 4\omega\cos\theta\left(\frac{d\theta}{dt}\right)^{\to\omega} + 4\sin\theta\left(\frac{d\omega}{dt}\right)^{\to\alpha}$$

$$2(0.5)^2 = 4(0.62)^2\cos\overset{\to 30°}{\theta} + 4\sin\overset{\to 30°}{\theta}\,\alpha$$

$$\alpha = -0.415 \text{ rad/s}^2$$

Problem (16-40)

Determine → θ, ω, α at $\theta = 60°$



Solution:-

$$X = 0.2 - 0.2\cos\theta$$

$$V = \frac{dx}{dt}$$

$$= -0.2[-\sin\theta]\frac{d\theta}{dt}^{\to\omega}$$

$$= 0.2\omega\sin\theta$$

$$\omega = \frac{5}{0.2\sin 60} = 28.9 \text{ rad/s}$$

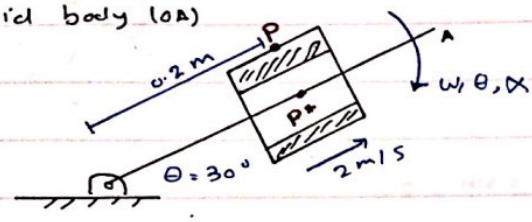
$$a = \frac{dv}{dt} = 0.2\left(\sin\theta\frac{d\omega}{dt} + \omega\cos\theta\frac{d\theta}{dt}^{\to\omega}\right)$$

$$2 = 0.2\left[(\sin 60)(\alpha) + \overset{\to 28.9}{\omega^2}\cos 60\right]$$

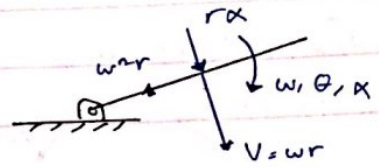
$$\alpha = \checkmark \text{ rad/s}^2$$

(16.8) - Relative motion analysis using translating axes.

→ The collar is translating or the rigid body (OA) which is rotating.



Rotation only



→ P is a point on the collar, the velocity of (P) results from:

1) The Rotational motion of the rigid body (OA)

2) The translational motion on (OA)

so :-

* $V_P = V_{P\,Rot} + V_{P\,trans}$

→ P* → is a coinident point that has the same position as (P) BUT on the rigid body. so :- $V_{P\,Rot} = V_{P^*}$

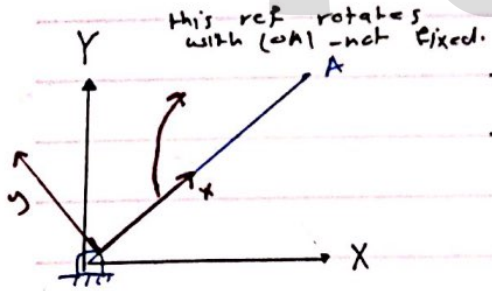
* $V_P = V_{P^*} + V_{sliding}$ → same as trans. (scalar)

→ If we assume that the collar is moving upwards with V = 2 m/s and ω = 5 rad/s, then we have : (for the instant shown (θ = 30°))

$V_P = V_{P^*} + V_{sliding}$

$= (0.2)(5) \angle 60° + 2 \angle 30°$

* $V_P = (\vec{\omega} \times \vec{r}) + \vec{V}_{P/\text{Rot sliding axis}}$ (vector)



→ The rigid body (OA) is rotating with ref to (YX)

→ The collar is translating with ref to (xy)

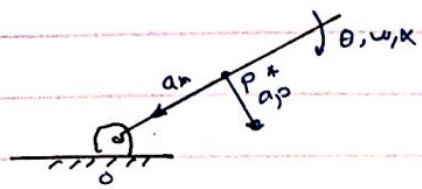
↳ so the (2 m/s) mentioned b4 is measured with ref to a rotating axis.

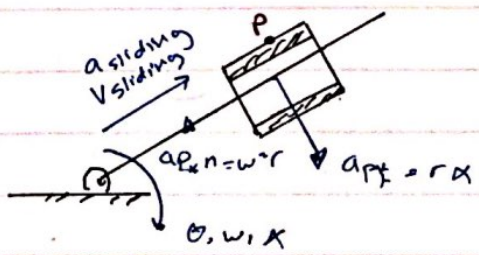
acceleration :-

$\vec{a}_P = \vec{a}_{P\,t} + \vec{a}_{P\,n} + a_{P/\text{rotating axis}} + 2\vec{\omega} \times V_{sliding}$

→ Coriolis acceleration

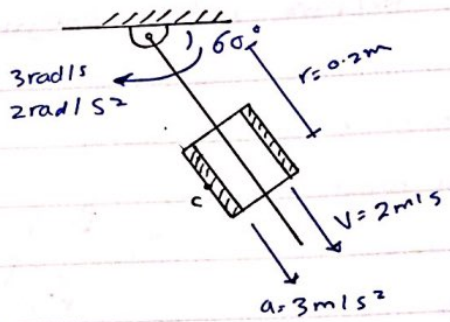
↓ this component is a result of taking the derivative ($\frac{dv}{dt}$) with respect to a (non-fixed) axis





Example (16-B) - 1

V_{collar} , a_{collar} ?



solution:-

$\vec{V}_c = \vec{V}_{c*} + \vec{V}_{c/Rot\ axis}$

$= (0.2)(3)\angle 30° + 2\angle 60°$

$\vec{V}_c = (-0.6\cos 30 + 2\cos 60)\rightarrow + (-0.6\sin - 2\sin 60)\uparrow$

$V_c = \sqrt{(\ )^2 + (\ )^2}$

$\vec{a}_c = \vec{a}_{c*t} + \vec{a}_{c*n} + \vec{a}_{c/tran.axis} + 2\vec{\omega} \times V_{c/Rot\ axis}$

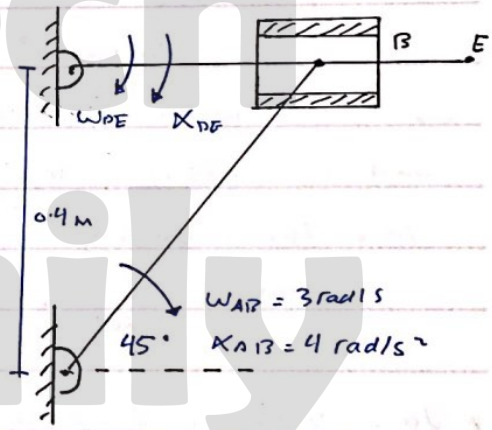
$= 0.4\angle 30° + 1.8\angle 60° + 3\angle 60° + 2(3)(2)\angle 30°$

$\vec{a}_c = \swarrow$ m/s²

Example (16.19, 20)

Find → ① ω_{DE}

② α_{DE}



solution:-

1) $\vec{V}_c = V_{c*} + \vec{V}_{collar/DE}$

$\sqrt{2}(0.4)(3)\angle 45° = 0.4\,\omega_{DE}\downarrow + V_{sliding}\rightarrow$

H; $1.69\cos 45 = 0 + V_{sliding}$

$V_{sliding} = 1.19$ m/s →

V: $-1.69\sin 45 = -0.4\,\omega_{DE} + 0$

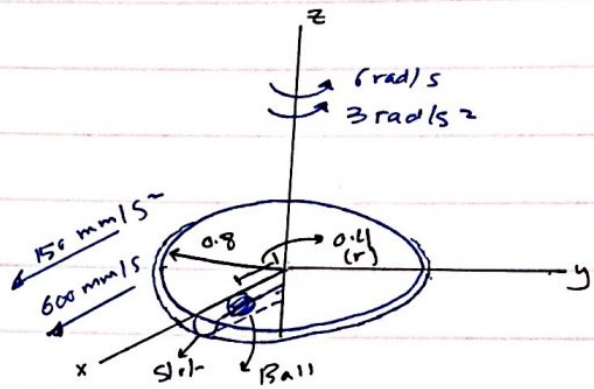
$\omega_{DE} = 3$ rad/s ↻ c.w

2) $\vec{a}_c = \vec{a}_{c*t} + \vec{a}_{c*n} + \vec{a}_{sliding} + 2\vec{\omega} \times \vec{V}_{sliding}$

$0.65(4)\angle 45° + 0.56(3)^2\angle 45° = 0.4\,\alpha_{DE}\downarrow + 0.4(3)^2 + \vec{a}_{sliding} + (2)(3)(1.19)\downarrow$

P(16-129)

V_{Ball}, a_{Ball} ?



Solution:-

$\vec{V}_B = \vec{V}_{sliding} + \vec{V}_{B*}$

$= (0.6)\hat{i} + r(6)\hat{j}$

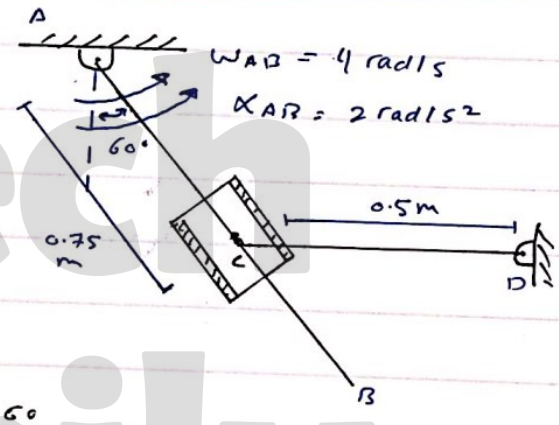
$\vec{a}_B = \vec{a}_{B*t} + \vec{a}_{B*n} + \vec{a}_{sliding} + 2\vec{\omega} \times \vec{V}_{sliding}$

$= (0.4)(3)\hat{j} + 0.4(6)^2(-\hat{i}) + 0.15\hat{i} + 2(6\hat{k} \times 0.6\hat{i})$

$\vec{a}_B = (1.2\hat{j} + 7.2\hat{j}) + (0.15\hat{i} - 3.6(0.4)\hat{i})$

P(16-141)

ω_{CD}, α_{CD} ?



Solution:-

$\vec{V}_C = \vec{V}_{C*} + \vec{V}_{sliding}$

$0.5\,\omega_{CD} \downarrow = V_{sliding} \searrow 30° + 4(0.75) \nearrow 60°$

H: $0 = V_{sliding}(\cos 30) + 4(0.75)\cos 30 + 3\cos 60$

$V_{sliding} = -1.7$ m/s

V: $-0.5\,\omega_{CD} = 1.7 \sin 30 + 3 \sin 60$

$\omega_{CD} = -6.89$ rad/s $\circlearrowleft = 6.89 \circlearrowright$ c.w

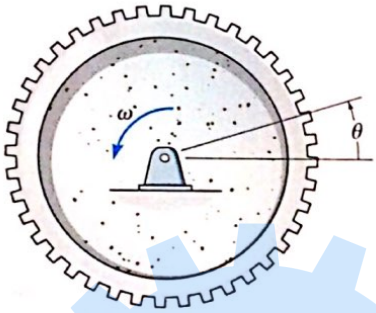
$\vec{a}_C = \vec{a}_{C*t} + \vec{a}_{C*n} + \vec{a}_{sliding} + 2\vec{\omega}_{AB} \times \vec{V}_{sliding}$

$0.5\,\alpha_{CD} \downarrow + 0.5\,\omega^2_{CD} \rightarrow = 0.75(2) \nearrow 60° + \omega^2(0.75) \nwarrow 30° + a_{sliding} \searrow 30°$

$+ 2(6.89)(1.7) \nearrow 60°$

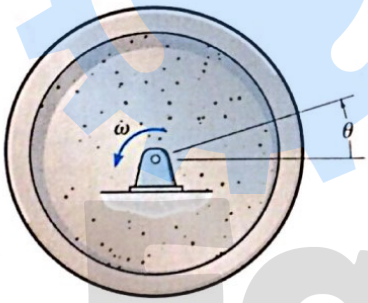
suggested: 131, 136, 141, 143, 152

F16–1. When the gear rotates 20 revolutions, it achieves an angular velocity of $\omega = 30\ \text{rad/s}$, starting from rest. Determine its constant angular acceleration and the time required.



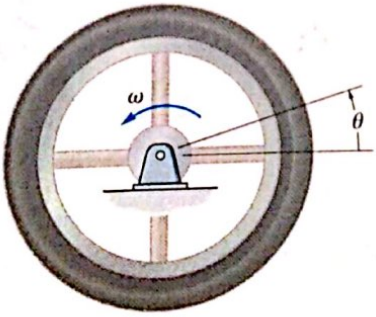
Prob. F16–1

F16–2. The flywheel rotates with an angular velocity of $\omega = (0.005\theta^2)\ \text{rad/s}$, where θ is in radians. Determine the angular acceleration when it has rotated 20 revolutions.



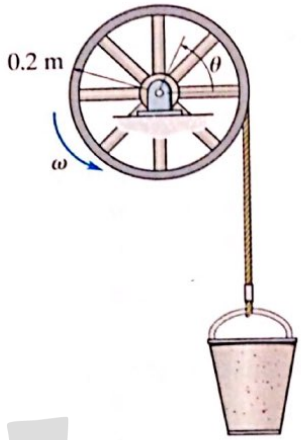
Prob. F16–2

F16–3. The flywheel rotates with an angular velocity of $\omega = (4\,\theta^{1/2})\ \text{rad/s}$, where θ is in radians. Determine the time it takes to achieve an angular velocity of $\omega = 150\ \text{rad/s}$. When $t = 0$, $\theta = 1$ rad.



Prob. F16–3

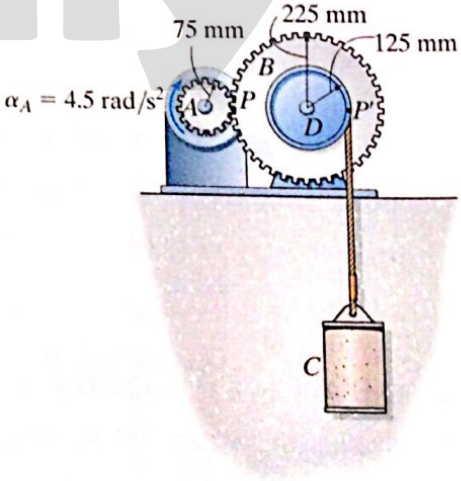
F16–4. The bucket is hoisted by the rope that wraps around a drum wheel. If the angular displacement of the wheel is $\theta = (0.5t^3 + 15t)$ rad, where t is in seconds, determine the velocity and acceleration of the bucket when $t = 3$ s.



Prob. F16–4

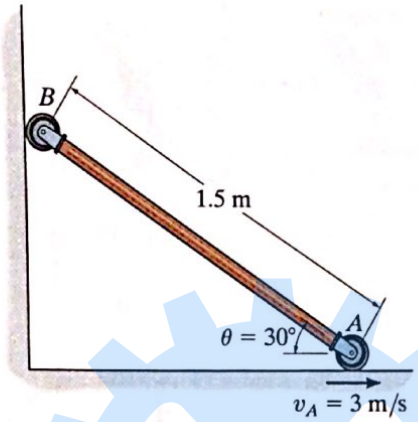
F16–5. A wheel has an angular acceleration of $\alpha = (0.5\,\theta)\ \text{rad/s}^2$, where θ is in radians. Determine the magnitude of the velocity and acceleration of a point P located on its rim after the wheel has rotated 2 revolutions. The wheel has a radius of 0.2 m and starts at $\omega_0 = 2\ \text{rad/s}$.

F16–6. For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5\ \text{rad/s}^2$, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pulley D which is rigidly attached to gear B.



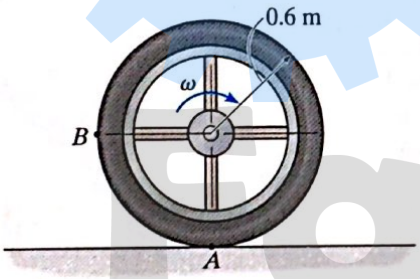
Prob. F16–6

F16–7. If roller A moves to the right with a constant velocity of $v_A = 3$ m/s, determine the angular velocity of the link and the velocity of roller B at the instant $\theta = 30°$.



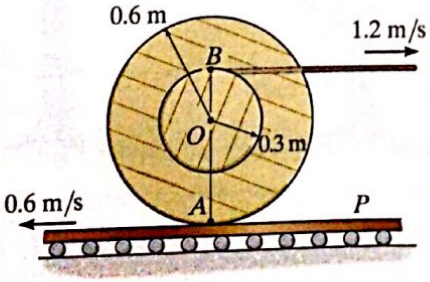
Prob. F16–7

F16–8. The wheel rolls without slipping with an angular velocity of $\omega = 10$ rad/s. Determine the magnitude of the velocity of point B at the instant shown.



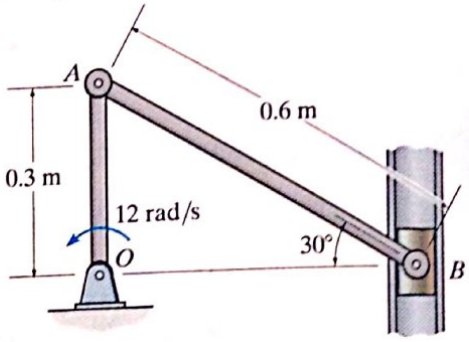
Prob. F16–8

F16–9. Determine the angular velocity of the spool. The cable wraps around the inner core, and the spool does not slip on the platform P.



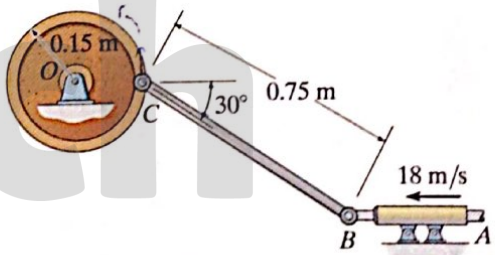
Prob. F16–9

F16–10. If crank OA rotates with an angular velocity of $\omega = 12$ rad/s, determine the velocity of piston B and the angular velocity of rod AB at the instant shown.



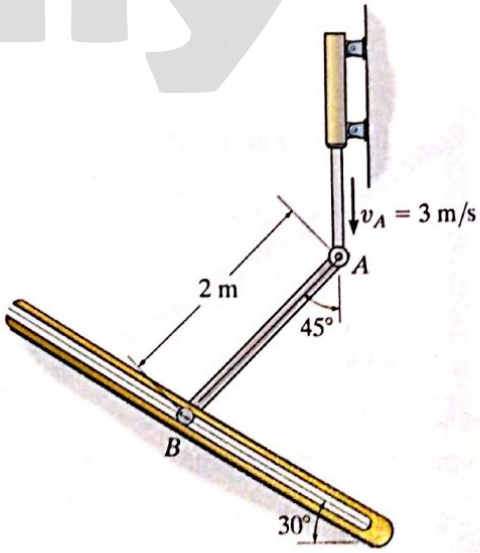
Prob. F16–10

F16–11. If rod AB slides along the horizontal slot with a velocity of 18 m/s, determine the angular velocity of link BC at the instant shown.



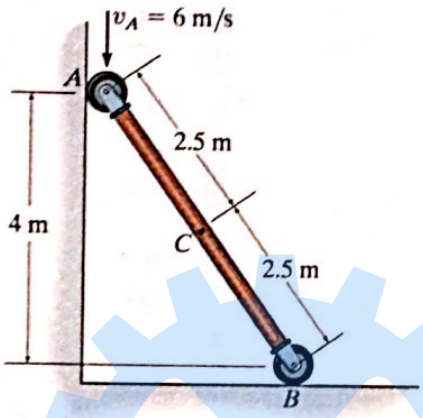
Prob. F16–11

F16–12. End A of the link has a velocity of $v_A = 3$ m/s. Determine the velocity of the peg at B at this instant. The peg is constrained to move along the slot.



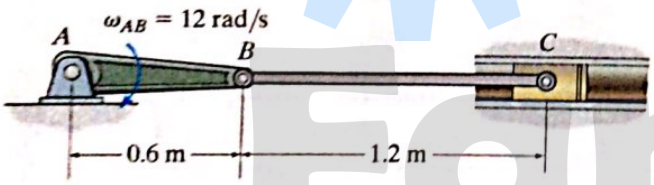
Prob. F16–12

F16–13. Determine the angular velocity of the rod and the velocity of point C at the instant shown.



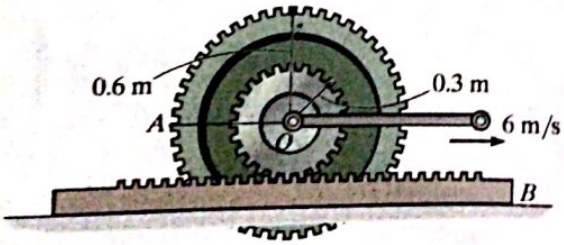
Prob. F16–13

F16–14. Determine the angular velocity of link BC and velocity of the piston C at the instant shown.



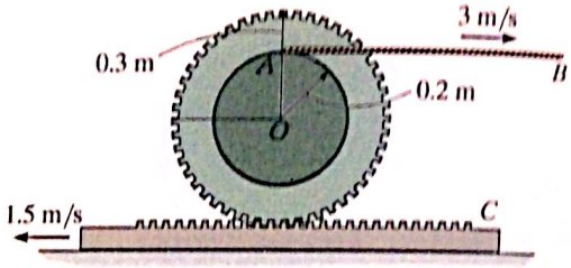
Prob. F16–14

F16–15. If the center O of the wheel is moving with a speed of $v_O = 6$ m/s, determine the velocity of point A on the wheel. The gear rack B is fixed.



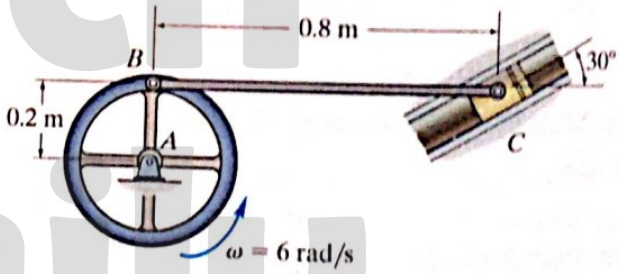
Prob. F16–15

F16–16. If cable AB is unwound with a speed of 3 m/s, the gear rack C has a speed of 1.5 m/s, determine angular velocity of the gear and the velocity of its center



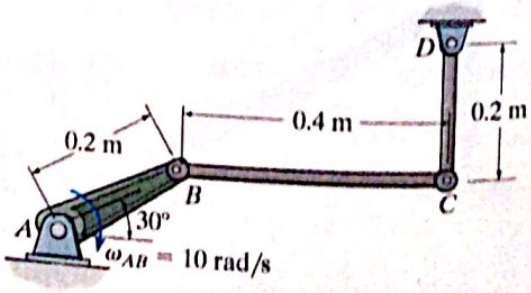
Prob. F16–16

F16–17. Determine the angular velocity of link BC and the velocity of the piston C at the instant shown.



Prob. F16–17

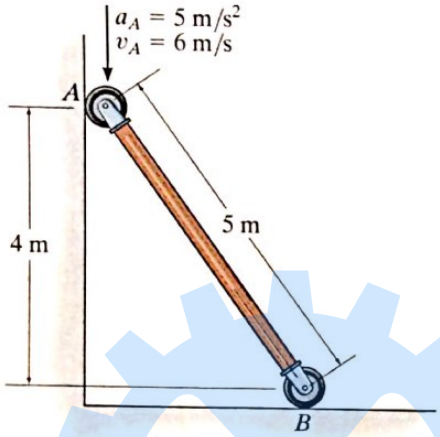
F16–18. Determine the angular velocity of links BC and CD at the instant shown.



Prob. F16–18

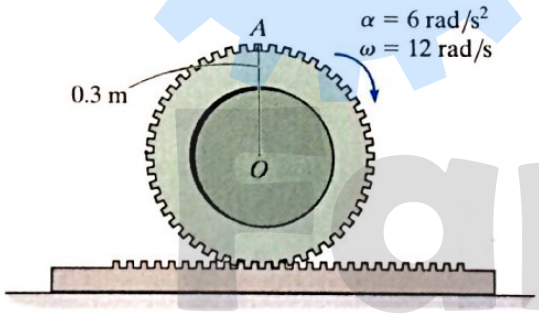
FUNDAMENTAL PROBLEMS

F16–19. At the instant shown, end A of the rod has the velocity and acceleration shown. Determine the angular acceleration of the rod and acceleration of end B of the rod.



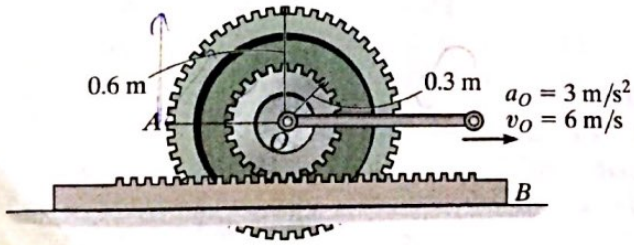
Prob. F16–19

F16–20. The gear rolls on the fixed rack with an angular velocity of $\omega = 12\ \text{rad/s}$ and angular acceleration of $\alpha = 6\ \text{rad/s}^2$. Determine the acceleration of point A.



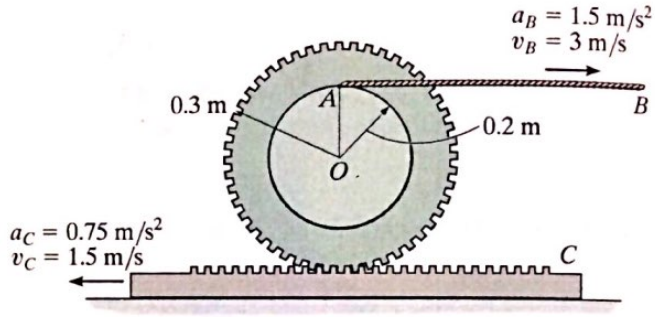
Prob. F16–20

F16–21. The gear rolls on the fixed rack B. At the instant shown, the center O of the gear moves with a velocity of $v_O = 6\ \text{m/s}$ and acceleration of $a_O = 3\ \text{m/s}^2$. Determine the angular acceleration of the gear and acceleration of point A at this instant.



Prob. F16–21

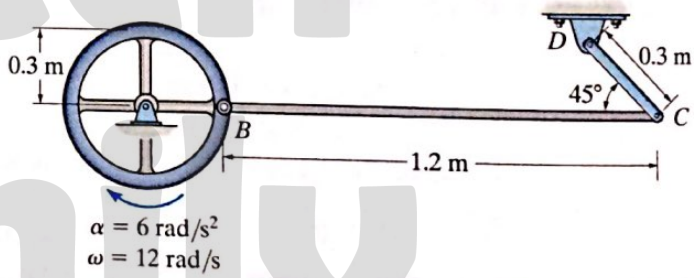
F16–22. At the instant shown, cable AB has a velocity of 3 m/s and acceleration of $1.5\ \text{m/s}^2$, while the gear rack has a velocity of 1.5 m/s and acceleration of $0.75\ \text{m/s}^2$. Determine the angular acceleration of the gear at this instant.



Prob. F16–22

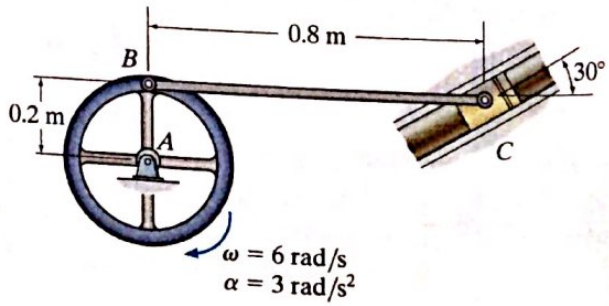
Check

* **F16–23.** At the instant shown, the wheel rotates with an angular velocity of $\omega = 12\ \text{rad/s}$ and an angular acceleration of $\alpha = 6\ \text{rad/s}^2$. Determine the angular acceleration of link BC at the instant shown.



Prob. F16–23

F16–24. At the instant shown, wheel A rotates with an angular velocity of $\omega = 6\ \text{rad/s}$ and an angular acceleration of $\alpha = 3\ \text{rad/s}^2$. Determine the angular acceleration of link BC and the acceleration of piston C.



Prob. F16–24

CHAPTER (17) _ Planar kinetics of a rigid body.

→ The difference between the motion of a particle and the motion of a rigid body is that the rigid body can rotate as it's moving through space as a result we now have two equations of motion :-

* $\sum \vec{F} = m\vec{a}$
* $\sum M_o = I_G \alpha$

I_G → mass moment of inertia :-

* what is (I)?

(I) is a measure of the body's resistance to rotate, just like the mass is a measure of the body's resistance to translate, it depends on both shape and mass.

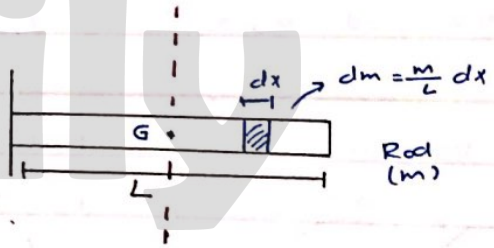
* Calculating I :-

I is equal to the integral of the second moment about an axis of all elements of mass (dm) which compose the body.

Example:

Determine (I_G)

$$I_G = \int x^2\, dm = \int x^2 \left(\frac{m}{L} dx\right) = \int_{-L/2}^{L/2} \left(\frac{m}{L}\right) x^2 dx$$



→ $I_{G_{rod}} = \frac{1}{12} mL^2$

where I_G → mass moment of inertia about an axis ⊥ to the plane and passes through (G).

(I)

I_G (G is mass center)

we will use three relations:

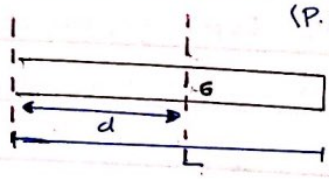
→ $I_{G\,rod} = (1/12)\, m L^2$

→ $I_{G\,sphere} = (2/5)\, mr^2$

→ $I_{G\,disk} = mr^2/2$

I_B (B is any other point)

we will use "parallel axis theorem", as follows: (P.A.T)



$$I_B = I_G + m d_{GB}^2$$

* Radius of Gyration :- (K)

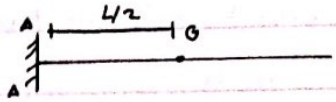
→ Related to mass moment of inertia :-

when (K) and (m) are known you can find (I) using :

$I = mk^2$

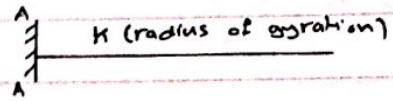
$I_A = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$ → P.A.T

$= \frac{mL^2}{3}$



But $I_A = mk^2$

$\cancel{m}k^2 = \frac{\cancel{m}L^2}{3} \rightarrow k = \frac{L}{\sqrt{3}}$



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* In this chapter we will deal with 3 types of motion:

(Translation, rotation, G.P.M)

* when you solve a problem:

1→ Identification of motion (Trans., Rot., G.P.M)

2→ Draw a F.B.D and K.D (kinetic diagram)

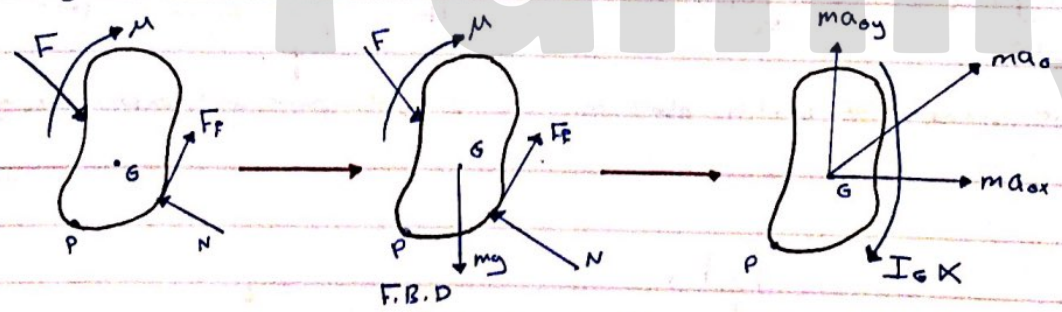
3→ Apply equations of motion :- $\Sigma \vec{F} = m\vec{a}$

$(\Sigma M_G)_z = I_G \alpha$

4→ Find a, α

---

* Diagrams → F.B.D + K.D



F.B.D
All forces and moments

---

* when you apply ($\Sigma M_O = I_O \alpha$), point (O) could be :

1→ G (center of gravity) → $\Sigma M_G = I_G \alpha$

2→ any other point (P) → $\Sigma M_P = I_G \alpha$ + moment of $(m\vec{a}_G)$ = $\Sigma M_{effective}$

moment of ma_{Gx} and ma_{Gy}

---

* Translation only → $\alpha = 0$ $a = \leftarrow$

* Rotation only → $\alpha = \curvearrowleft$ $a = 0$

* G.P.M → $\alpha = \leftarrow$ $a = \leftarrow$

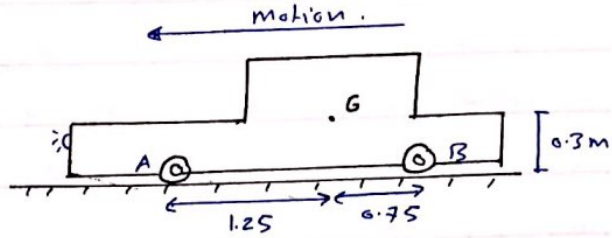
Example (17-5)

$m = 2000$ kg , $\mu_k = 0.25$

Rear wheels are slipping

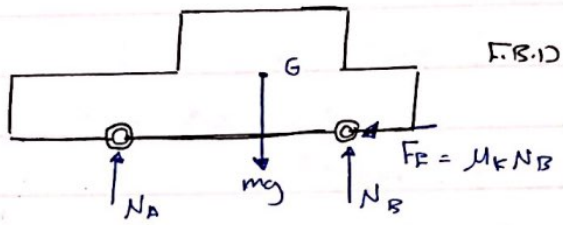
Front wheels free to roll

Determine → $a = ?$



Solution :

Translation only

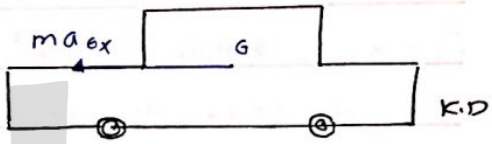


$\overset{+ve}{\leftarrow} \Sigma F_x = m a_{Gx}$; $\mu_k N_B = m a_{Gx}$

↳ $N_B = \dfrac{m a_{Gx}}{\mu_k}$

$\overset{+ve}{\uparrow} \Sigma F_y = 0$; $(N_A + N_B - mg) = 0$

$\overset{+ve}{\circlearrowleft} \Sigma M_A = \Sigma M_{eff}$



$0 + 0.3\, m a_{Gx} + N_B((1.25) + 0.75) - mg(1.25) = 0.3\, m a_{Gx}$

$$N_B = \frac{(2000)(9.81)(1.25)}{(1.25) + (0.75)} = 12.27 \text{ kN}$$

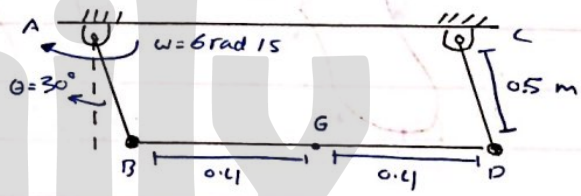
$a_{Gx} = N_B \mu_k / m = (12700)(0.25)/2000 = 1.587 \text{ m/s}^2$

Example (17.7)

AB, CD → negligible mass

$m_{BD} = 100$ kg. F_{AB} & $F_{CD} = ?$

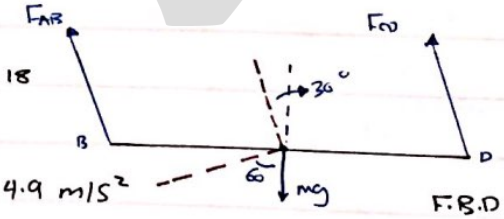
solution :-



→ curvilinear translation (a_t & a_n)

$a_n = \omega^2 r$

$= 6^2(0.5) = 18$



$\overset{+ve}{\swarrow} \Sigma F_t = m a_t$; $mg \cos 60 = m a_t$ → $a_t = g \cos 60$

↳ $a_t = 4.9 \text{ m/s}^2$

$(+)\nwarrow \Sigma F_n = m a_n$; $F_{AB} + F_{CD} - mg \sin 60 = m a_n$

$\Sigma M_P = \Sigma M_{eff}$

ΣM_B = moment of ma + $I\alpha$ ($\alpha = 0$)

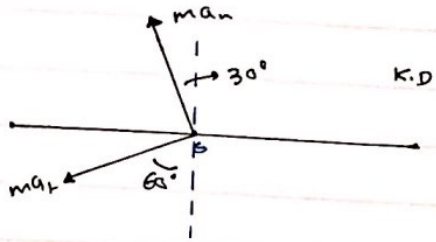
$mg(0.4) - F_{CD}(\cos 30)(0.8) = \cos(60)(m a_t)(0.4) - m a_n \cos 30 (0.4)$

$$-F_{CD} = \frac{(100)(4.9)(0.4)\cos 60 - (100)(18)(0.4)\cos 30 - 100(9.81)(0.4)}{(0.8)\cos 30}$$

$F_{CD} = 13.24$ kN

↳ $F_{AB} + 1324.9 - (100)(9.81)\sin 60 = (100)(18)$

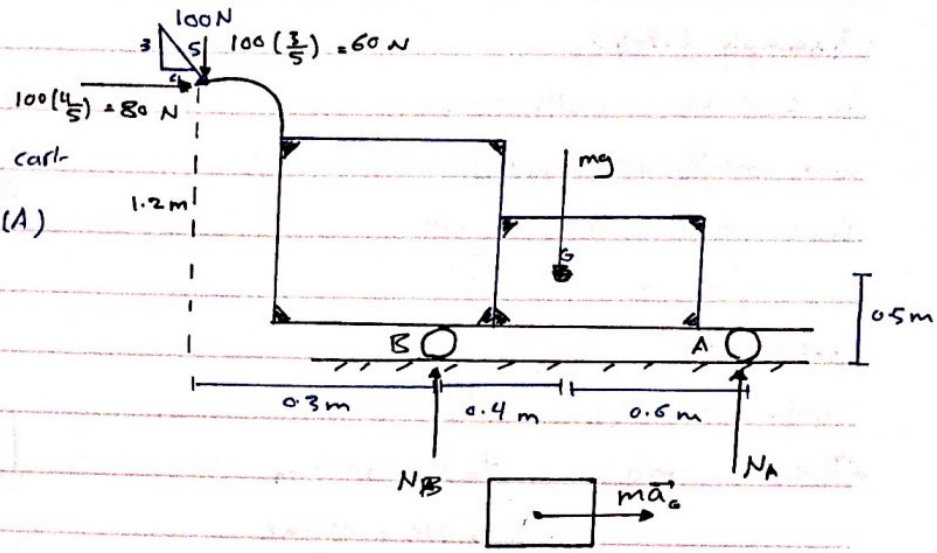
$F_{AB} = 13.24$ kN



F (17-1)

$m = 100\ kg$

Determine → acceleration of the cart and the normal reactions at (A) and (B).



solution :-

$\Sigma \vec{F}_x = m\vec{a}_x$; $80 = 100\,\vec{a}_x$

$a_{Gx} = 0.8\ m/s^2$

$\Sigma \vec{F}_y = m\cancel{a_y}^{\,0}$

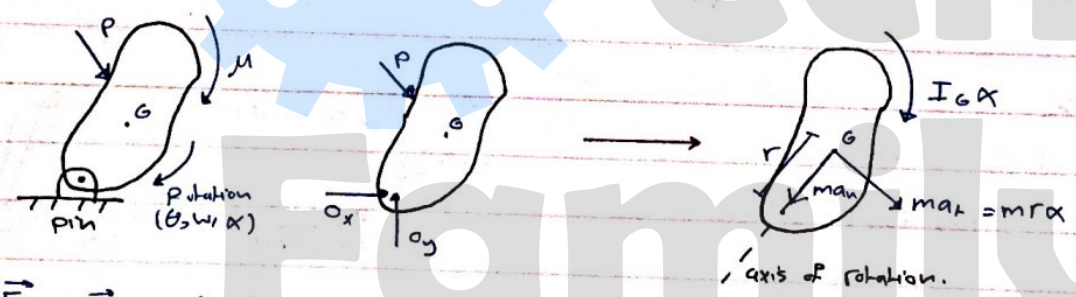
$N_B + N_A - mg - 60 = 0 \rightarrow N_B + N_A = 1041$

$+\circlearrowleft \Sigma M_B = \Sigma M_{eff} = I\cancel{\alpha}^{\,0} +$ moment of $m\vec{a}_G$

$N_A(0.6 + 0.4) - mg(0.4) + 60(0.3) - 80(1.2) = 0 + ma_G(0.5)$

~~$N_A = 4$~~ $N_A = \_\_\ N \rightarrow N_B = \_\_$

Rotation :-



$\Sigma \vec{F} = m\vec{a}$ (y & x or n & t)

$\Sigma M_P = \Sigma M_{eff}$
arbitrary point (P)

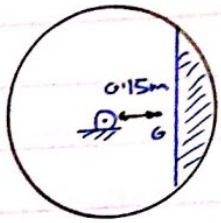
$\Sigma M_o = \Sigma M_{eff} = I_G\alpha +$ moment of (ma_t, ma_n)

$\Sigma M_o = I_G\alpha + 0 + r(m\overset{a_t}{(r\alpha)})$

$\Sigma M_o = (I_G + mr^2)\,\alpha$ → parallel axis theorem.

$= I_o\,\alpha$

Example (17-8)



$m = 25$ kg

radius of gyration ← $K_G = 0.18$ m , Released from rest

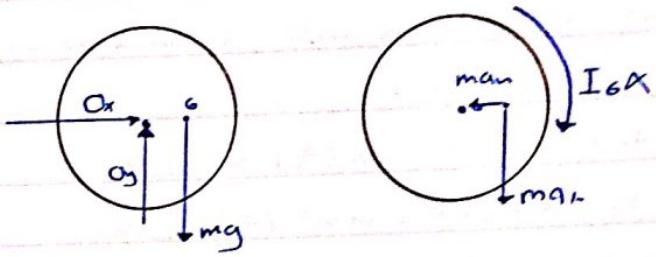
$O_x, O_y = ?$

solution :-

$\Sigma F_x = m a_x$; $O_x = 0$

$\Sigma F_y = m a_y$

+ve↑ $O_y - mg = -m(0.15)\alpha = -m a_L$



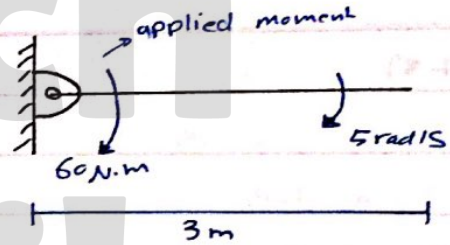
$\Sigma M_o = I_o \alpha$

$(0.15)(mg) = (mk_G^2 + m(0.15)^2)\alpha$ $\qquad I_o = I_G + md^2$

$\alpha = \dfrac{(25)(9.81)(0.15)}{1.3725} = 26.8$ rad/s² $\qquad = mk_G^2 + m(0.15)^2 = 1.3725$ kg·m²

↳ $O_y = 144.75$ N

Example (17-9)



$m = 20$ kg

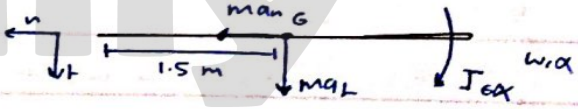
Determine → α, O_x, O_y .

solution :- _Rotation_

+ve ← $\Sigma F_n = m a_n$; $-O_x = m(\omega^2 r) = (20)(25)(1.5)$

$= 750$ N

+ve↓ $\Sigma F_t = m a_t$; $-O_y + mg = m(r\alpha)$

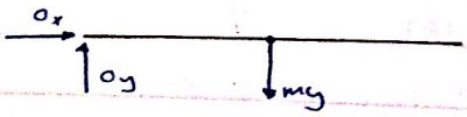


+↻ $\Sigma M_o = I_o \alpha$

$mg(\frac{L}{2}) + 60 = \left[\frac{mL^2}{12} + m(\frac{L}{2})^2\right]\alpha$

$\alpha = 5.9$ rad/s²

↳ $O_y = mg - m(r\alpha) \simeq 19$ N



Example (17-10)

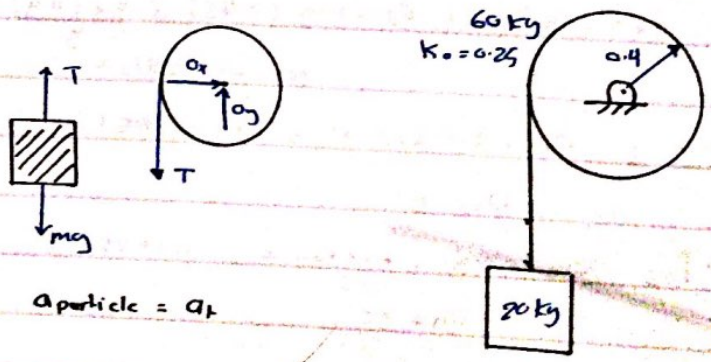
Released from rest , $\alpha = ?$

$\Sigma M = 0$; $T(0.4) = I_o \alpha$

$20g - T = 20a \longrightarrow a = 0.4\alpha$ (a_t)

↳ $20g - \left(\frac{I_o \alpha}{0.4}\right) = 20(0.4)\alpha$

$\alpha = 11.3$ rad/s²

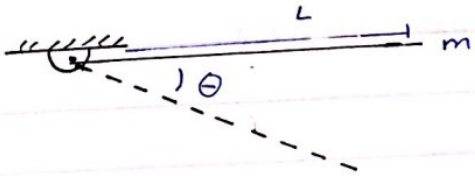


$a_{particle} = a_t$

$I_o = mk_o^2$

$= 60(0.25)^2 = 3.75$

Example (17-11)

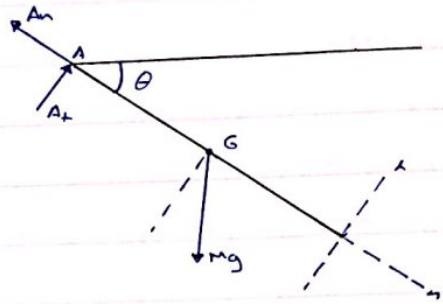


starts from rest ($\theta = 0$)

Determine → Reactions at A when $\theta = 90°$

→ here (α) is not a constant so we have to use the basic definition of (α).

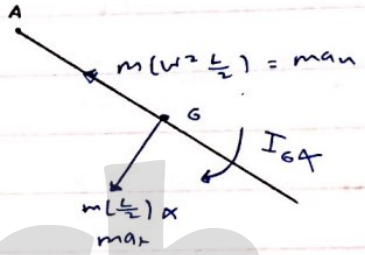
→ we have to draw a F.B.D at an arbitrary position not when $\theta = 90°$.



$+\swarrow$; $-A_t + mg\cos\theta = m\left(\frac{L}{2}\right)\alpha$ — (1)

$+\nwarrow$; $A_n - mg\sin\theta = m\omega^2\left(\frac{L}{2}\right)$ — (2)

$$\Sigma M_A = I_A \alpha$$

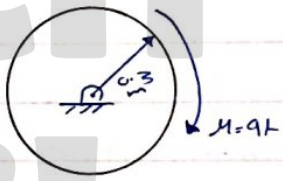


$$mg\cos\theta\left(\frac{L}{2}\right) = \left(\frac{mL^2}{3}\right)\alpha$$

$$mg\cos\theta\left(\frac{L}{2}\right) = \frac{mL^2}{3}\left(\omega\frac{d\omega}{d\theta}\right)$$

$$\int_0^{\pi/2} \frac{3}{2L} g\cos\theta = \int_0^{\omega \text{ at } \theta = 90°} \omega\, d\omega$$

F(17-8)



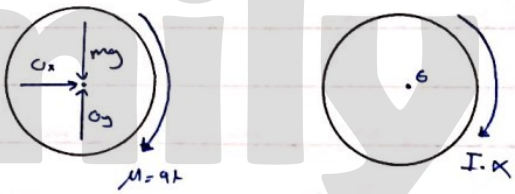
$m = 50$ kg , $\omega_0 = 0$

$\omega = ?$ at $t = 4$s

$$I_o = \frac{mr^2}{2}$$

$$\Sigma M_o = I_o \alpha$$

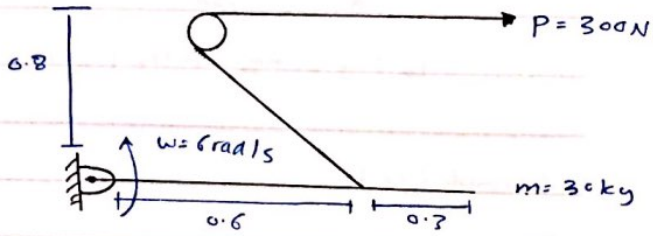
$$9t = \left(\frac{50 \times 0.3^2}{2}\right)\frac{d\omega}{dt}$$



$$\int_0^4 9t\, dt = \int_0^{\omega} I_o\, d\omega \rightarrow \omega = \leftarrow \text{ rad/s}$$

F(17-12)

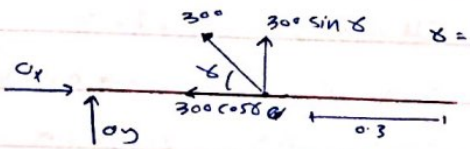
Determine → O_x, O_y, α



$\circlearrowleft + \Sigma M_o = I_o \alpha$

$$300(\sin\gamma)(0.6) - mg(0.45) = \underbrace{\frac{30(0.9)^2}{3}}_{\frac{mL^2}{3}}$$

$$\alpha = \leftarrow \text{ rad/s}^2$$



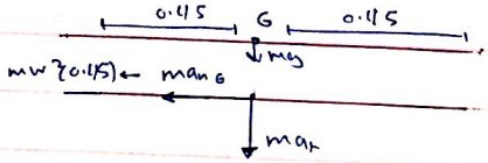
$\gamma = \tan^{-1}\left(\frac{0.8}{0.6}\right)$

$\xrightarrow{+ve}$; $O_x - 300\cos\gamma = -m(0.45)(\omega^2)$

$O_x = \leftarrow$

$+ve\uparrow$; $O_y - mg + 300\sin\gamma = m(0.45)\alpha$

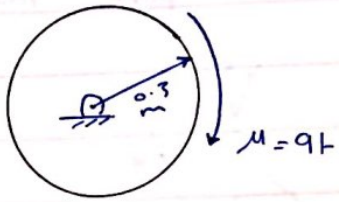
$O_y = \leftarrow$



F (17-8)

$m = 50\ kg$, $\omega_0 = 0$

$\omega = ?$ at $t = 4\ s$

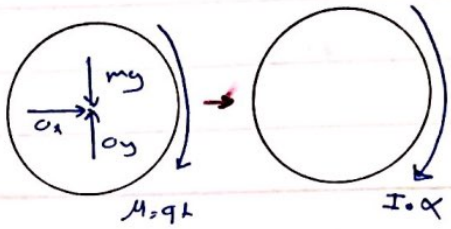


Solution :

$\cancel{\omega \times \mu}$ $\Sigma M_o = I_o \alpha$

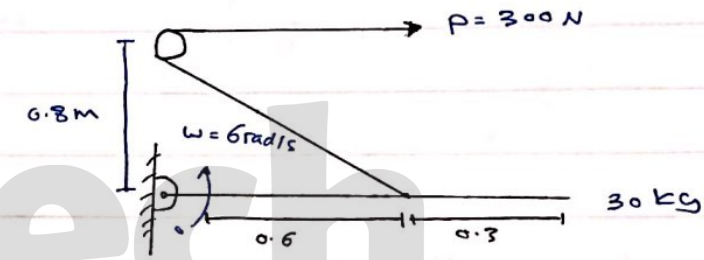
$9t = \left(\frac{50 * 0.3^2}{2}\right) \frac{d\omega}{dt}$ $\qquad I_o = \frac{mr^2}{2}$

$\int_0^4 9t\, dt = \int_0^\omega I_o\, d\omega \rightarrow \omega = \checkmark\ rad/s$



F (17-12)

O_x , O_y , α ?

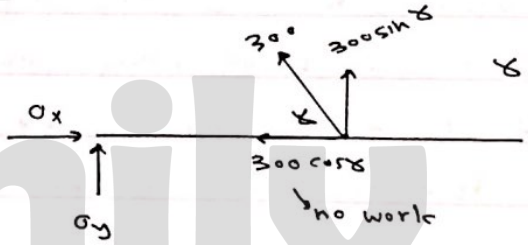


+ve ↺ $\Sigma M_o = I_o \alpha$

$300 (\sin\gamma)(0.6) - mg(0.45) = \frac{30(0.9)^2}{3}$ $\rightarrow \frac{mL^2}{3}$

$\alpha = \checkmark\ rad/s^2$

$\gamma = \tan^{-1}\left(\frac{0.8}{0.6}\right)$

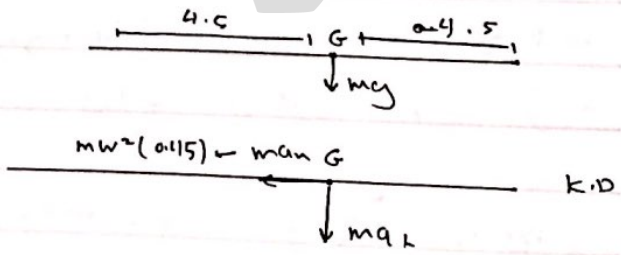


+ve → ; $O_x - 300 \cos\gamma = -m(0.45)(\omega^2)$

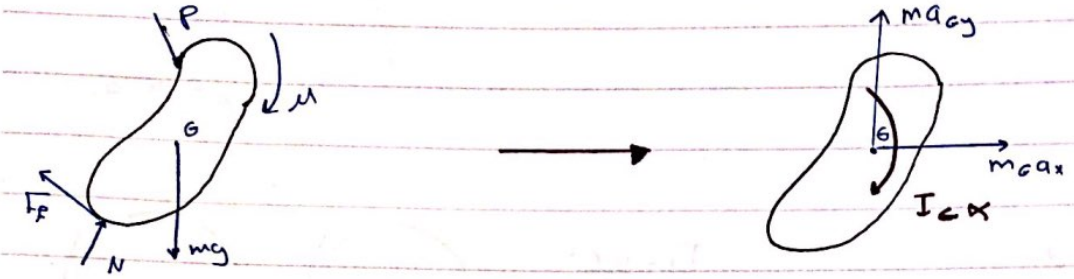
$O_x = \checkmark$

+ve ↑ ; $O_y - mg + 300 \sin\gamma = m(0.45)\alpha$

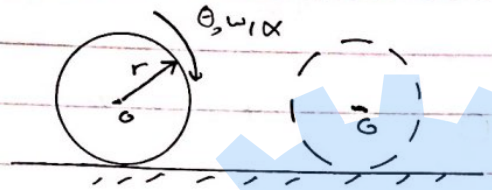
$O_y = \checkmark$



General plane motion:-



to obtain a_x, α, a_y we will use kinematic analysis (sometimes)



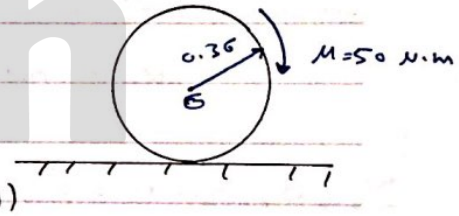
Rolling (G.P.M)

Rolling motion
- with slipping → $F_f = \mu_k N$
- without slipping → F_f = unknown.

Both have friction

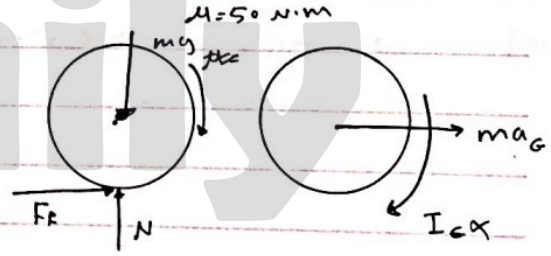
Example (17-15)

$m = 25$ kg, $k_G = 0.2$, $\mu_s = 0.3$, $\mu_k = 0.25$



I don't know if it's rolling with or without slipping but it's easier to assume that it is (rolling w/o slipping)

if $F_f < \mu_s N$ ✓ if not ✗



$\xrightarrow{+ve}$ $\Sigma F_x = ma_{Gx}$; $F_f = ma_G$ — [1]

$+ve\uparrow$ $\Sigma F_y = ma_{Gy} = 0$; $N - mg = 0$ — [2]

$+ve\circlearrowleft$ $\Sigma M_A = \Sigma M_{eff}$ — [3]

$50 = I_G\alpha + (0.36)(ma_G)$

from kinematics:

$x_G = r\theta \rightarrow v_G = r\omega \rightarrow a_G = r\alpha$

$50 = ((25)(0.2)^2\alpha + (0.36)(25)(0.36\alpha))$

$\alpha = 11.79$ rad/s², $F_f = 106.1$ N $> \overset{\mu_s N}{77.6}$ assumption not valid.

point of contact can be considered as I.C

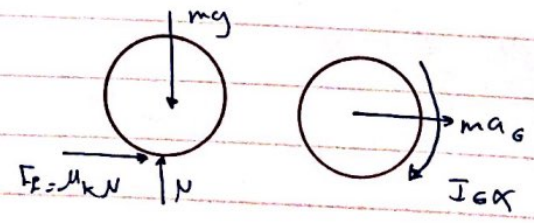
→ I have (3) unknowns so I go to kinematics

$F_f > \mu_s N$ → slipping occurs !!

now: $(0.25)(mg) = ma_G$; $a_G = ✓$ m/s²

$50 = I_G\alpha + (0.36)(ma_G)$

$\alpha = 27.93$ rad/s²



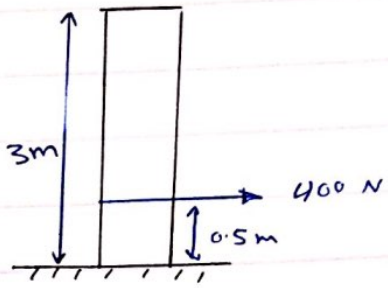
Example (17-14)

$m = 100\ kg$

$\mu_s = 0.3$

$\mu_k = 0.25$, From rest

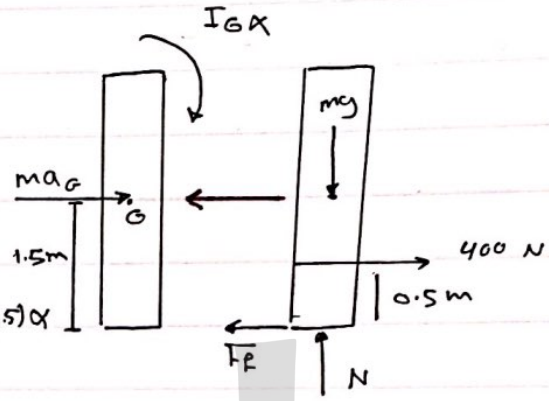
$\alpha = ?$



solution :- G.P.M

assume (Rolling without slipping)

$a_G = 1.5\alpha$



$400 - F_f = 100\, a_G \rightarrow F_f = 300\ N$

$N_A - mg = 0$

$\Sigma M_A = \Sigma M_{eff}$; $(400)(0.5) = \frac{1}{12} mL^2\alpha + 1.5m(1.5)\alpha$

$\alpha = 0.667\ rad/s^2$

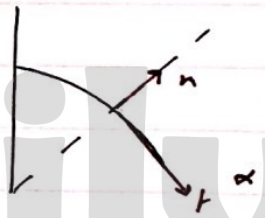
$300\ N > (0.3)(981) = 294$

sol $\rightarrow$ assumption not valid (slipping occurgs) $(F_f > \mu_s N)$

$400 - 0.25(100g) = ma_G$

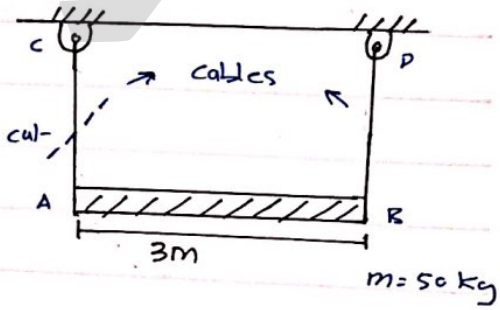
$a_G = 1.55\ m/s^2$

$\alpha = -0.428\ rad/s^2 = 0.428$ (c.c.w)



Example (17-15)

Find T_{BD}, α after the cut.

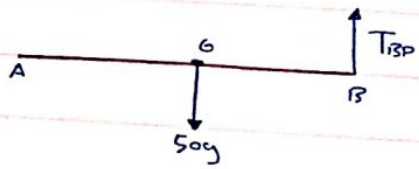


$\xrightarrow{+ve}$ $\Sigma F_x = ma_{Gx}$; $0 = ma_{Gx} \rightarrow a_{Gx} = 0$ — [1]

$+ve\downarrow$ $\Sigma F_y = ma_{Gy}$; $50g - T_{BD} = ma_{Gy}$ — [2]

$+ve\circlearrowleft$ $\Sigma M_B = \Sigma M_{eff}$;

$(1.5)(50g) = I_{Gy} + (1.5)(ma_{Gy})$ — [3]



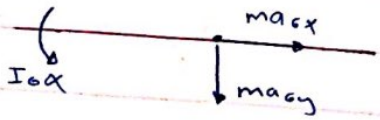
* Kinematic analysis:

$\vec{a}_G = \vec{a}_B + (\vec{a}_{G/B})_t + (\vec{a}_{G/B})_n$ $\quad 0 (\omega_0 = 0)$

$a_{Gy}\downarrow = \underset{\rightarrow}{a_{Bx}} + \alpha(1.5)\downarrow + \cancel{\omega^2}(1.5)$

$a_{Bx} = 0$

$a_{Gy} = 1.5\alpha$ — [4]

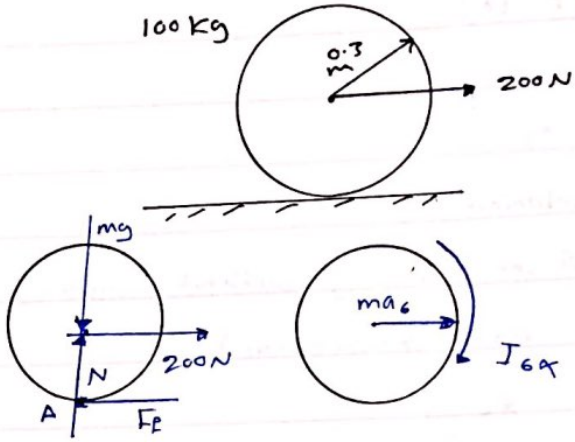


F (17-14)

Rolls without slipping

Determine → a_G, α

solution:-



$a_G = 0.3\alpha$

$\Sigma F_x = ma_{Gx}$

$200 - F_f = ma_A$

$\circlearrowleft + \Sigma M_A = \Sigma M_{eff}$

$200(0.3) = I_G\alpha + 0.3(m)(0.3\alpha)$

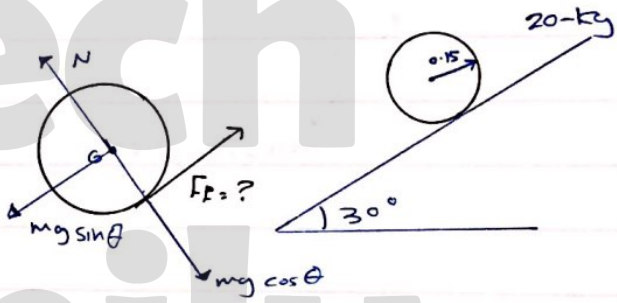
$\alpha =$ rad/s²

F (17-16)

Rolls without slipping

$\alpha = ?$

solution:-



$\Sigma M_A = \Sigma M_{eff}$

$= I_G\alpha + 0.15\, m a_G$

$mg\sin\theta\,(r) = I_G\alpha + (0.15)^2\alpha\, m$

$\alpha =$ rad/s², $a_G =$ m/s²

~~Example~~ (17-17)

$\alpha = ?$, $T_{cable} = ?$, $a_G = ?$

$I_G = 200(0.3)^2$

$a_G = \vec{a}_A + (\vec{a}_{G/A})_t + (\vec{a}_{G/A})_n$

$\underset{\leftarrow}{a_G} = 0 + \underset{\rightarrow}{0.4\alpha} + 0$

$|a_G| = |0.4\alpha|$

$T - \mu_k(mg) = m(0.4)\alpha$

$+\text{ve}\circlearrowleft \Sigma M_C = \Sigma M_{eff}$

$450 - T(0.6 + 0.4) = I_G\alpha - m(0.4\alpha)(0.$

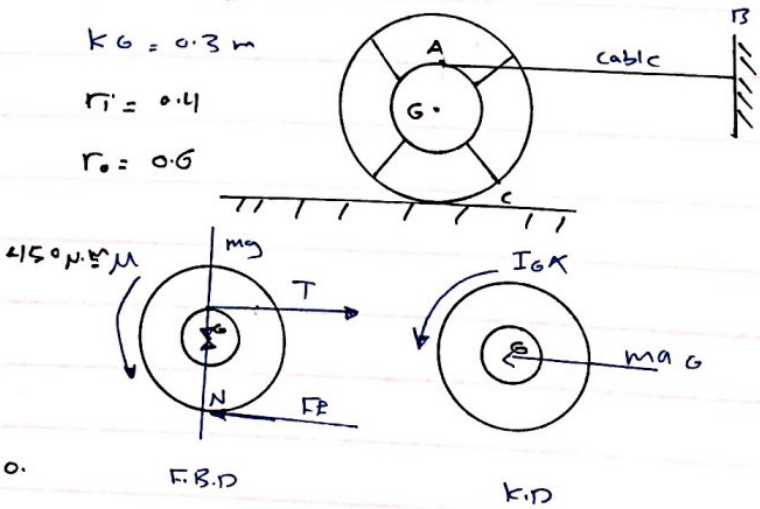
$\alpha =$, $a_G =$

$m = 200$ kg

$k_G = 0.3$ m

$r_i = 0.4$

$r_o = 0.6$



$\mu_s = 0.25$

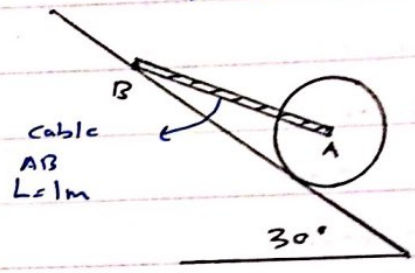
$\mu_k = 0.2$

P(17_96)

Released from rest , $\mu_s = 0.6$, $\mu_k = 0.4$

$\alpha_D = ?$

no friction at B

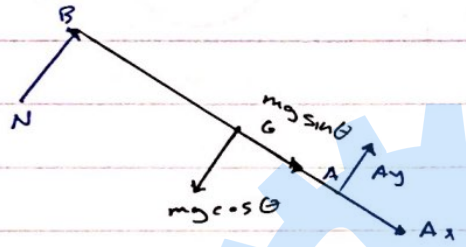


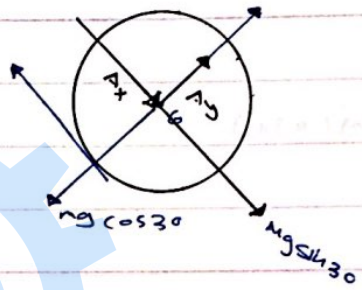
solution:-

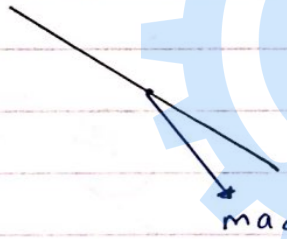
Assume rolling without slipping

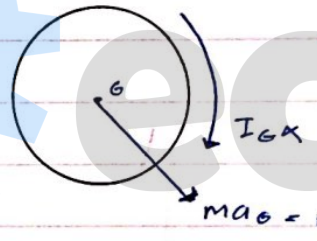
trans. (Rectilinear)

→ point of contact is considered as a pin.





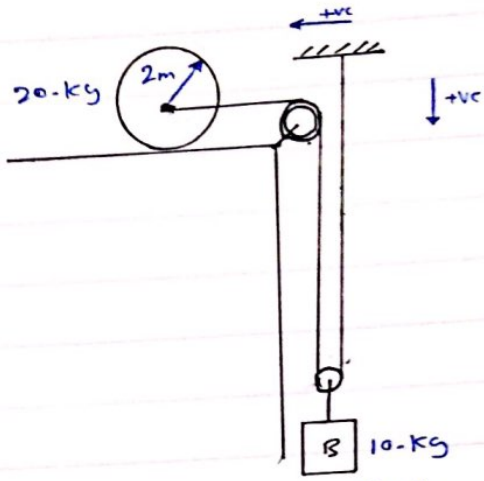




P (17 - 112)

Rolls without slipping

determine → α, a_B, T ?



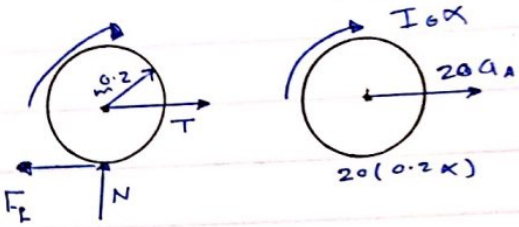
$a_A = 0.2\alpha$

cable length $= L \rightarrow S_A + 2y_B = L$

$a_A + 2a_B = 0 \rightarrow |a_A| = |2a_B|$

$a_B = \frac{1}{2} a_A = \frac{1}{2}(0.2)\alpha = 0.1\alpha$

$10g - 2T = 10(0.1)\alpha$



$+\circlearrowleft \Sigma M = \Sigma M_{eff}$

$T(0.2) = \left(\frac{1}{2}(20)(0.2)^2\right)\alpha + 0.2(20)(0.2)\alpha$

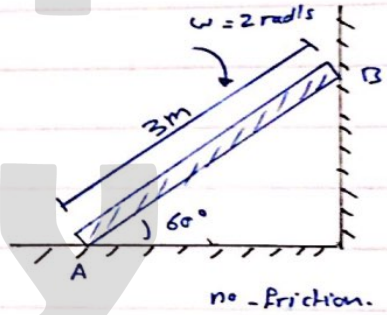
$\alpha =$ rad/s^2 , $a_B =$ m/s^2

$T =$ N

P(17-91)

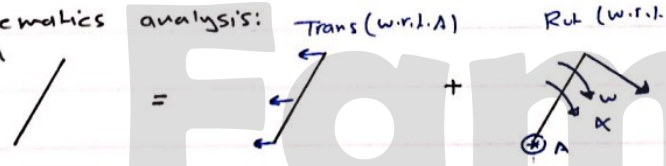
$m = 12$-kg

Determine → α, N_A, N_B



kinematics analysis:

G.P.M



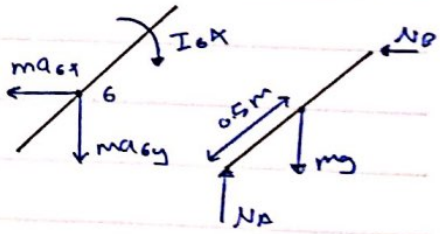
$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n$

$a_B \downarrow = \underline{a_A} + 3\alpha \angle 30° + (2)^2(3) \angle 60°$

This formulation is true but I have to use G in this Q so X

$\vec{a}_G = \vec{a}_A + (\vec{a}_{G/A})_t + (\vec{a}_{G/A})_n$

$a_{Gx} \leftarrow + a_{Gy} \downarrow = a_A \leftarrow + (1.5)\alpha \angle 30° + (1.5)(2)^2 \angle 60°$



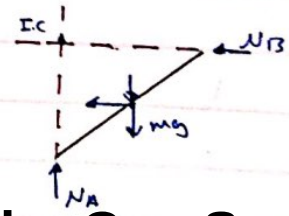
$\Sigma M_{I.C} = \Sigma M_{eff}$

$= I_G\alpha + ma_{Gx}(1.5\sin 60) + ma_{Gy}(1.5)\cos 60$

$\alpha =$ rad/s^2

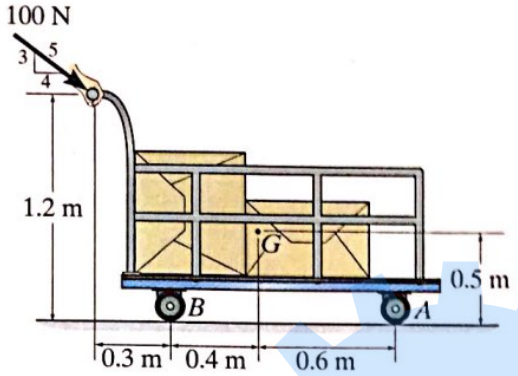
$N_A =$ N

$N_B =$ N



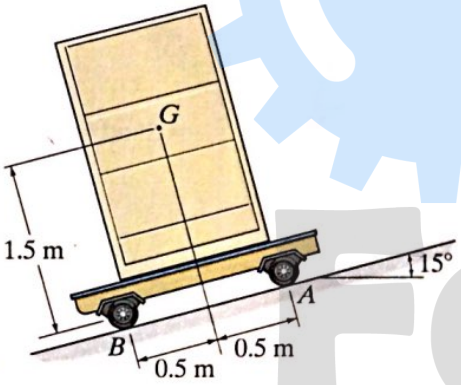
FUNDAMENTAL PROBLEMS

F17–1. The cart and its load have a total mass of 100 kg. Determine the acceleration of the cart and the normal reactions on the pair of wheels at A and B. Neglect the mass of the wheels.



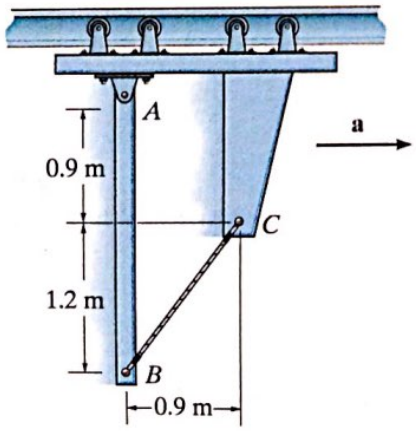
Prob. F17–1

F17–2. If the 80-kg cabinet is allowed to roll down the inclined plane, determine the acceleration of the cabinet and the normal reactions on the pair of rollers at A and B that have negligible mass.



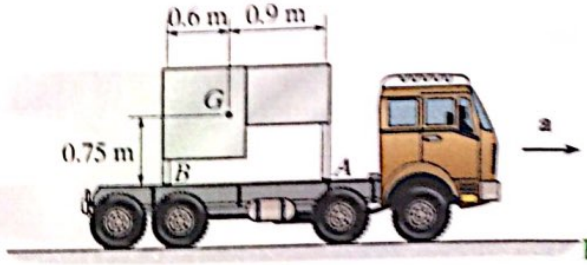
Prob. F17–2

F17–3. The 10-kg link AB is pinned to a moving frame at A and held in a vertical position by means of a string BC which can support a maximum tension of 50 N. Determine the maximum acceleration of the frame without breaking the string. What are the corresponding components of reaction at the pin A?



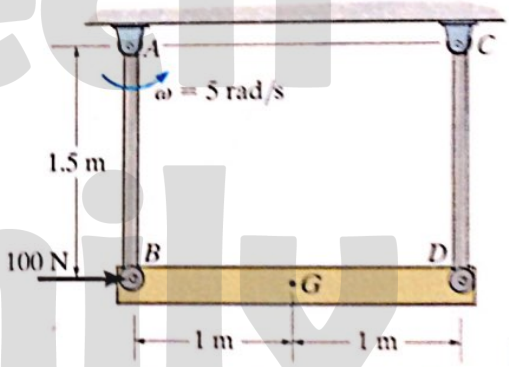
Prob. F17–3

F17–4. Determine the maximum acceleration of the truck without causing the assembly to move relative to the truck. Also what is the corresponding normal reaction on legs A and B? The 100-kg table has a mass center at G and the coefficient of static friction between the legs of the table and the bed of the truck is $\mu_s = 0.2$.



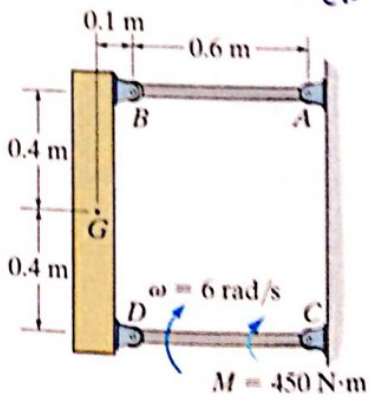
Prob. F17–4

F17–5. At the instant shown both rods of negligible mass swing with a counterclockwise angular velocity of $\omega = 5$ rad/s, while the 50-kg bar is subjected to the 100-N horizontal force. Determine the tension developed in the rods and the angular acceleration of the rods at this instant.



Prob. F17–5

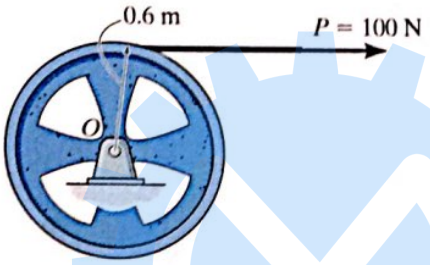
F17–6. At the instant shown, link CD rotates with an angular velocity of $\omega = 6$ rad/s. If it is subjected to a couple moment $M = 450$ N·m, determine the force developed in link AB, the horizontal and vertical component of reaction on pin D, and the angular acceleration of link CD at this instant. The block has a mass of 50 kg and center of mass at G. Neglect the mass of links AB and CD.



Prob. F17–6

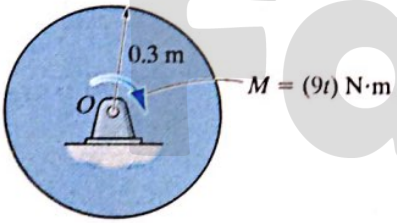
FUNDAMENTAL PROBLEMS

F17–7. The 100-kg wheel has a radius of gyration about its center O of $k_O = 500$ mm. If the wheel starts from rest, determine its angular velocity in $t = 3$ s.



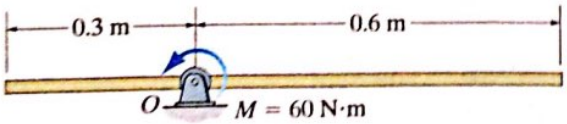
Prob. F17–7

F17–8. The 50-kg disk is subjected to the couple moment of $M = (9t)$ N·m, where t is in seconds. Determine the angular velocity of the disk when $t = 4$ s starting from rest.



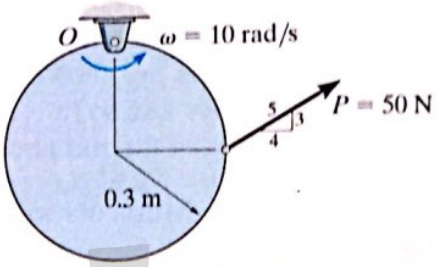
Prob. F17–8

F17–9. At the instant shown, the uniform 30-kg slender rod has a counterclockwise angular velocity of $\omega = 6$ rad/s. Determine the tangential and normal components of reaction of pin O on the rod and the angular acceleration of the rod at this instant.



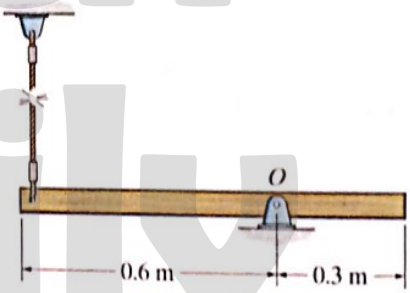
Prob. F17–9

F17–10. At the instant shown, the 30-kg disk has a counterclockwise angular velocity of $\omega = 10$ rad/s. Determine the tangential and normal components of reaction of the pin O on the disk and the angular acceleration of the disk at this instant.



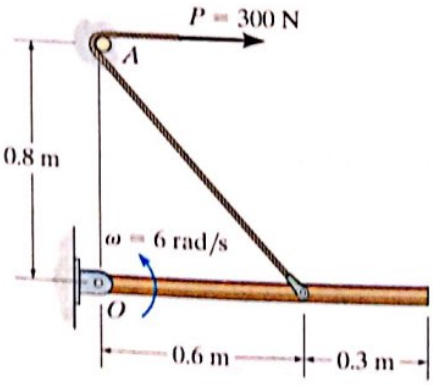
Prob. F17–10

F17–11. The uniform slender rod has a mass of 15 kg. Determine the horizontal and vertical components of reaction at the pin O, and the angular acceleration of the rod just after the cord is cut.



Prob. F17–11

F17–12. The uniform 30-kg slender rod is being pulled by the cord that passes over the small smooth peg at A. If the rod has a counterclockwise angular velocity of $\omega = 6$ rad/s at the instant shown, determine the tangential and normal components of reaction at the pin O and the angular acceleration of the rod.

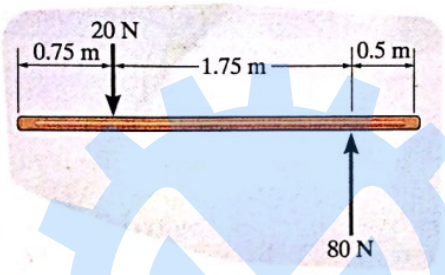


Prob. F17–12

* الاحتكاك إذا عندك موضح بيكون حسب قاعدة الكتاب أما إذا ما في عكس الحركة *

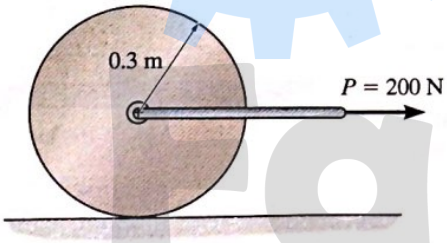
FUNDAMENTAL PROBLEMS

F17–13. The uniform 60-kg slender bar is initially at rest on a smooth *horizontal plane* when the forces are applied. Determine the acceleration of the bar's mass center and the angular acceleration of the bar at this instant.



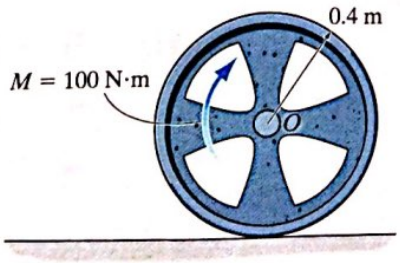
Prob. F17–13

F17–14. The 100-kg cylinder rolls without slipping on the horizontal plane. Determine the acceleration of its mass center and its angular acceleration.



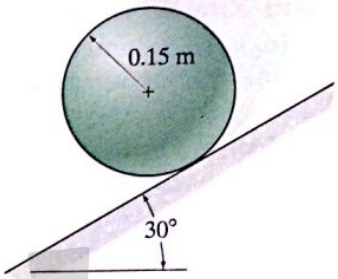
Prob. F17–14

F17–15. The 20-kg wheel has a radius of gyration about its center O of $k_O = 300$ mm. When the wheel is subjected to the couple moment, it slips as it rolls. Determine the angular acceleration of the wheel and the acceleration of the wheel's center O. The coefficient of kinetic friction between the wheel and the plane is $\mu_k = 0.5$.



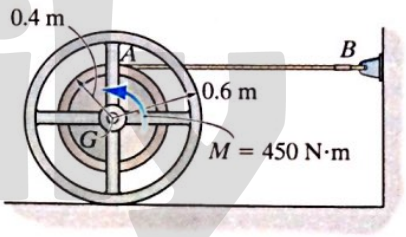
Prob. F17–15

F17–16. The 20-kg sphere rolls down the inclined plane without slipping. Determine the angular acceleration of the sphere and the acceleration of its mass center.



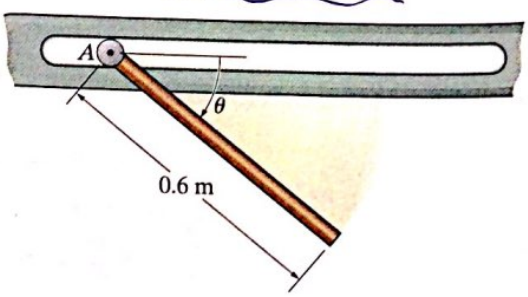
Prob. F17–16

* **F17–17.** The 200-kg spool has a radius of gyration about its mass center of $k_G = 300$ mm. If the couple moment is applied to the spool and the coefficient of kinetic friction between the spool and the ground is $\mu_k = 0.2$, determine the angular acceleration of the spool, the acceleration of G and the tension in the cable.



Prob. F17–17

* **F17–18.** The 12-kg slender rod is pinned to a small roller A that slides freely along the slot. If the rod is released from rest at $\theta = 0°$, determine the angular acceleration of the rod and the acceleration of the roller immediately after the release.



Prob. F17–18

CHAPTER - 18 - work & Energy.

→ From chapter 14 :-

$T_1 + U_{1\to2} = T_2$

$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$] → For particles

$T_1 = T_{trans.} + T_{rot}$

$= \frac{1}{2} m (V_G)^2 + \underline{\frac{1}{2} I_G \omega^2}$

$U_{1\to2} = (U_{1\to2})_{Pext} + (U_{1\to2})_{Ff} + (U_{1\to2})_{spring} + (U_{1\to2})_{gravity} + (U_{1\to2})_{moment}$

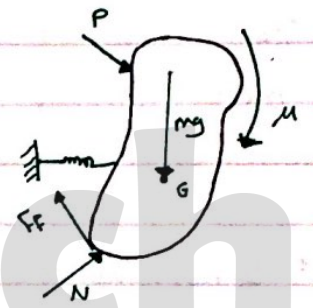
→ $(U_{1\to2})_{Pext} = \int \vec{F} \cdot d\vec{r}$

→ $(U_{1\to2})_{Ff} = -F_f (\Delta S)$

→ $(U_{1\to2})_{spring} = -\frac{k}{2}[S_2^2 - S_1^2]$

→ $(U_{1\to2})_{gravity} = -mg[y_{G2} - y_{G1}]$

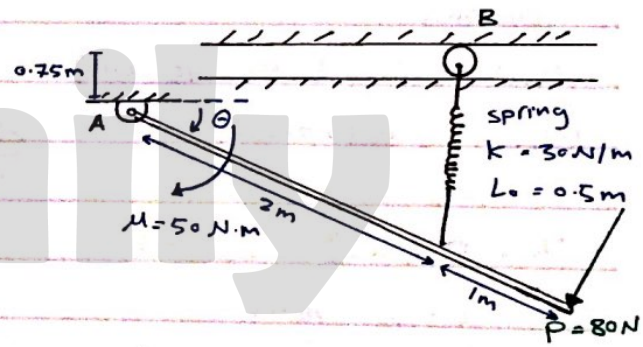
→ $(U_{1\to2})_{moment} = \int \vec{M} \cdot d\vec{\theta}$



Example (18-1) :-

$m = 10\,kg$, $U_{1\to2} = ?$

State ; from $\theta = 0$ to $\theta = 90°$



solution :-

$(U_{1\to2})_g = -mg(y_{G2} - y_{G1})$

$= -10g(-1.5 - 0) = 147.15$

$(U_{1\to2})_s = -\frac{k}{2}(S_2^2 - S_1^2)$

$= -\frac{k}{2}(2.25^2 - 0.25^2)$

$S_1 = 0.75 - 0.5 = 0.25\ m$

$S_2 = (2 + 0.75 - L_0)$

$= 2.25\ m$

$(U_{1\to2})_M = \int \vec{M} \cdot d\vec{\theta}$ → M is constant

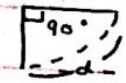
$= M(\theta_2 - \theta_1) = 50(\frac{\pi}{2} - 0)$

$(U_{1\to2})_P = \int \vec{P} \cdot d\vec{r} = \frac{80(2\pi * 3)}{4}$

→ the distance it travelled is :

$\frac{1}{4}$ (the circle's circumference)

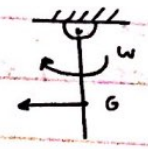
$= \frac{1}{4}(2\pi r) = \frac{1}{4}(2\pi * 3)$



From the door ($\omega_2 = ??$)

$T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega_2^2 = \frac{1}{2}[m((1.5)\omega)^2) + I_G \omega^2]$

$= \frac{1}{2}[\underbrace{I_G + m(1.5)^2}_{I_A\ (P.A.T)}]\omega^2 = \frac{1}{2} I_A \omega^2$



pure rotation

$V_G = 1.5\omega$

For rotation only

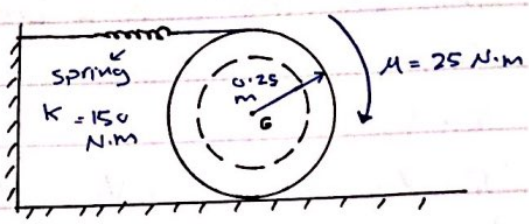
$T = \frac{1}{2} I_O \omega^2$

Example (18-3)

Rolls without slipping, from rest, $m = 20\,kg$

$K_G = 0.2\,m$, $\omega = ?$, $X_G = 0.18\,m$

Disk with inner pulley



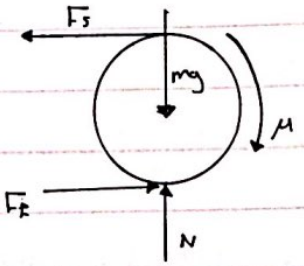
Solution :- → spring initially unstretched

$T_1 = 0$

$T_2 = \frac{1}{2}(m V_G^2 + I_G \omega_2^2)$ G.P.μ

$(U_{1\to2})_\mu = 25(\Delta\theta)$ → constant μ

$= 25\left(\frac{0.18}{0.25}\right) = 18$



from kinematics:

$V_G = 0.25\,\omega_2$ (point of contact is considered as I.C)

$x = r\theta$

$\theta = \frac{x}{r} = \frac{0.18}{0.25}$

$(U_{1\to2})_s = -\frac{k}{2}(0.36^2 - 0) = -9.72$ $\quad s_1 = 0$

↳ $T_1 + U_{1\to2} = T_2$ $\quad s_2 = (0.18)(2) = 0.36$

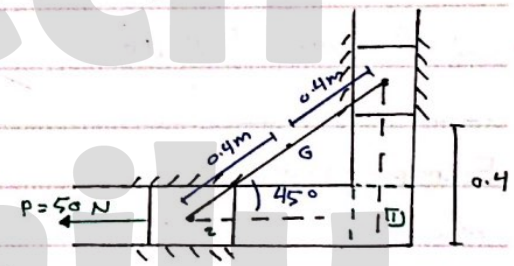
$\omega = \leftarrow$ rad/s →

note → even if you have μ_s & μ_k (DO NOT) use them (without slipping !!)
↳ unknown and it won't do work

Example (18-5)

$\cancel{(U_{1\to2})_g = -mg}$ at rest when $\theta = 0$ (vertical position), $m = 10\,kg$

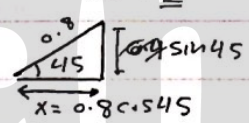
Determine → (ω) when $\theta = 45°$



solution :-

state 2

$(U_{1\to2})_g = -mg\,[0.4\sin 45 - 0.4]$

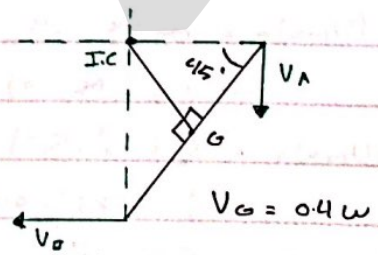


$(U_{1\to2})_P = 50(0.8\cos 45)$

$T_2 = \frac{1}{2}(m V_G^2 + I_G \omega^2)$ $\quad V_G = 0.4\omega$

$\omega = \curvearrowleft$ rad/s

to relate V_G with ω:



extra :-

* (If) A & B had masses of $m_A = 5\,kg$, $m_B = 7\,kg$:-

→ T_2 = same as b4 + $\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$V_A = (0.8\cos 45)\omega$, $V_B = (0.8)\sin 45(\omega)$

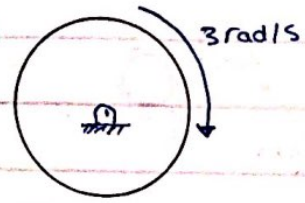
$(U_{1\to2})_g$ = same + 0 + 5g(0.8 − 0.8 sin 45)

P(18-1)

m = 100 kg

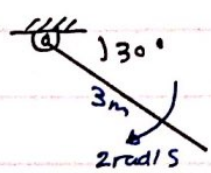
Determine (T) for each of the following:

1)



$T = \frac{mL^2}{2}$

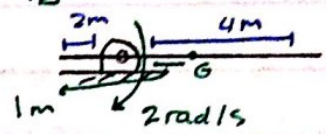
2)



$T = \frac{1}{2} I_o \omega^2$

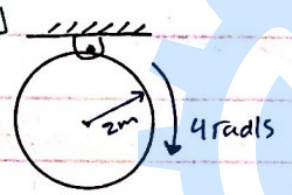
$I_o = \frac{1}{12} mL^2 + (m(1.5))^2$

3)



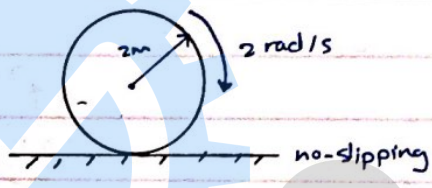
$T = \frac{1}{2}\left[\frac{m(6)^2}{12} + m(1)^2\right](2)^2$

4)



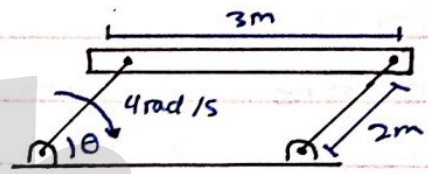
$I_o = \frac{1}{2} m(2)^2 + m(2)^2$

$T = \frac{1}{2} I_o \omega^2$



$T = \frac{1}{2} m V_G^2 + I_G \omega^2$

$I_G = \frac{mR^2}{2}$

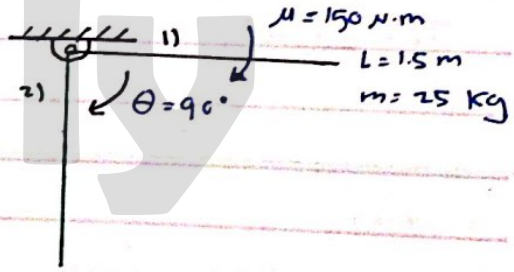


$T = \frac{1}{2} m V_G^2$

$V_G = V_B = (2)(4)$

F(18-2).

$\omega = ??$



solution:-

$T_1 = 0$

$T_2 = \frac{1}{2} I_o \omega^2 \rightarrow T_1 + U_{1\to2}\, g + U_{1\to2}\, \mu = T_2$

$I_o = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3}$

$(U_{1\to2})_\mu = 150\left(\frac{\pi}{2}\right)$

$\frac{1}{2}(1.5) \rightarrow$ position of G

$(U_{1\to2})_g = -mg(y_{G2} - y_{G1}) \rightarrow y_{G2} = -0.75$

$-25(9.81)(-0.75 - 0) = \checkmark +ve$

$\omega = \checkmark$ rad/s

F(18-3)

$m = 50$ kg

Determine → ω ?

solution :-

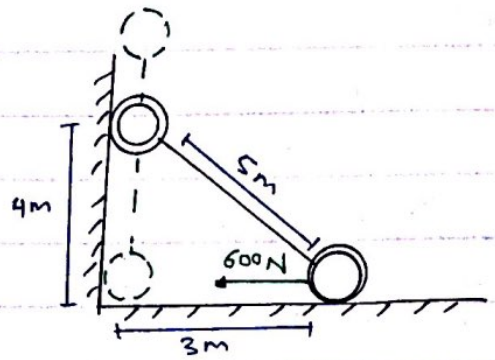
$T_2 = \frac{1}{2} I_G \omega^2 + \frac{1}{2} m V_G^2$

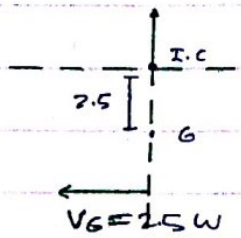
$(U_{1\to2})_g = -mg\,(2.5 - 2)$

$= \leftarrow$ -ve

$(U_{1\to2})_p = 600(3) = \leftarrow$

$\omega = \leftarrow$ rad/s





F(18-5)

$m = 30$ kg $\quad T_1 = 0$ (from rest)

$\omega = ??$ $\quad \theta = 4$ rev

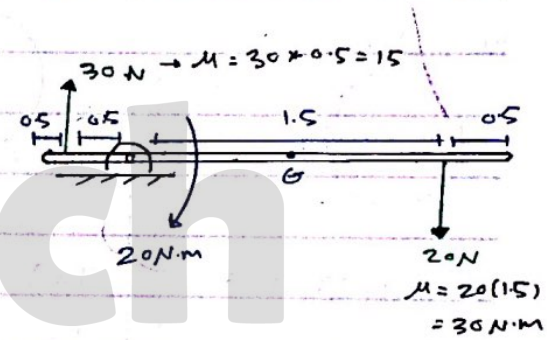
solution :

$(U_{1\to2})_g = 0$

$(U_{1\to2})_\mu = 20(4)(2\pi)$

$(U_{1\to2})_{forces} = (30+15)(4)(2\pi)$

$T_2 = \frac{I_o \omega^2}{2}$



$I_o = \frac{1}{12}(30)(3)^2 + (30)(0.5)^2$

$= \leftarrow$ kg.m²

F(18-6)

Rolling without slipping , $m = 20$ kg

$k_G = 0.3$, $x_G = 20$ m

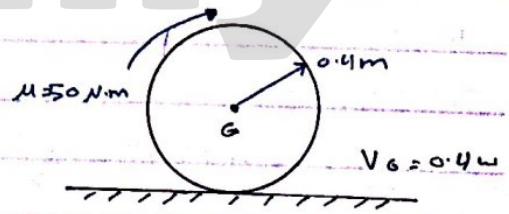
Determine (ω).

solution :-

$(U_{1\to2})_\mu = 50\theta = \leftarrow$

$T_2 = \frac{1}{2}(I_G \omega^2 + m V_G^2) = \frac{1}{2}(I_G + m(0.4)^2)\omega^2$

$\omega = \leftarrow$ rad/s



$x = r\theta$

$20 = 0.4\,\theta$

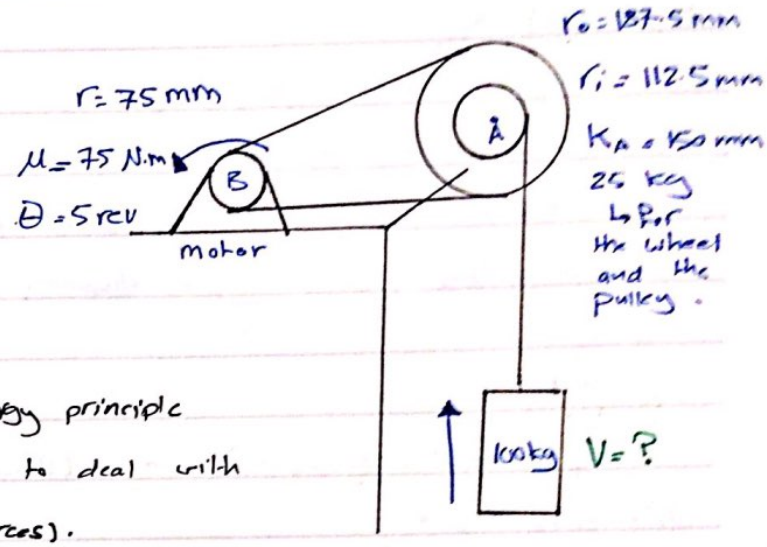
$\theta = 20/0.4 = \leftarrow$ rad

$I_G = m k_G^2 = (20)(0.3)^2$

P(18-8)

The system starts from rest.

→ Find the velocity of the (100kg) mass after the pulley has turned 5 rev.



Solution :-

note → it's better to apply work & Energy principle on the system not on each body not to deal with more unknowns like (T), (Internal Forces).

* $T_1 + U_{1\to2} = T_2$, $T_1 = 0$ (starts from rest.)

* $T_2 = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$ $\quad (I = mk^2 = 25(0.15)^2)$

$= \frac{1}{2}(100) V^2 + \frac{1}{2}(25)(0.15)^2 \omega^2$

* from kinematics : $V = 0.1125\omega$

$\omega = V / 0.1125$

* $U_{1\to2} = (U_{1\to2})_M + (U_{1\to2})_{g(mass)}$

$= \int \vec{M} \cdot d\theta - mg(y_{G_2} - y_{G_1})$

$= 75(5 * 2\pi) - 100(9.81)(y_2 - 0)$

$r_A \theta_A = r_B \theta_B$

* to find y_2 : $\theta_{wheel} = \frac{5(2\pi)(75)}{187.5} = 12.57$

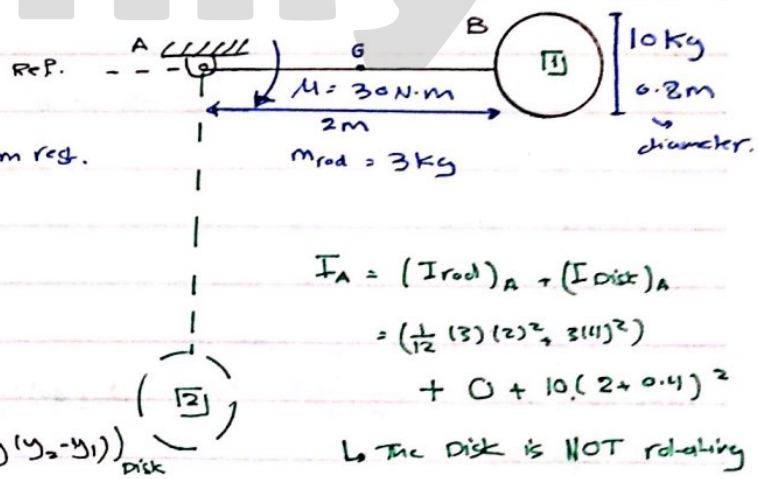
$y_2 = \theta_{wheel}(0.1125) = 1.414$

now you have an eqn with one variable (V).

V = ✓ m/s

P (18-9)

Determine the angular velocity when it rotates clockwise 90°. Released from rest.



Solution :-

$T_1 = 0$

$T_2 = \frac{1}{2} I_A \omega^2$

$U_{1\to2} = (U_{1\to2})_M + (U_{1\to2})_g$

$= \int M d\theta + (mg(y_2 - y_1))_{rod} + (mg(y_2 - y_1))_{Disk}$

$= 30(\frac{\pi}{2}) + 3g(-1 - 0) - 10g(-2.4 - 0) = +ve$

$T_1 + U_{1\to2} = T_2$

solve for ω :

ω = 3.16 rad/s

$I_A = (I_{rod})_A + (I_{Disk})_A$

$= (\frac{1}{12}(3)(2)^2 + 3(1)^2)$

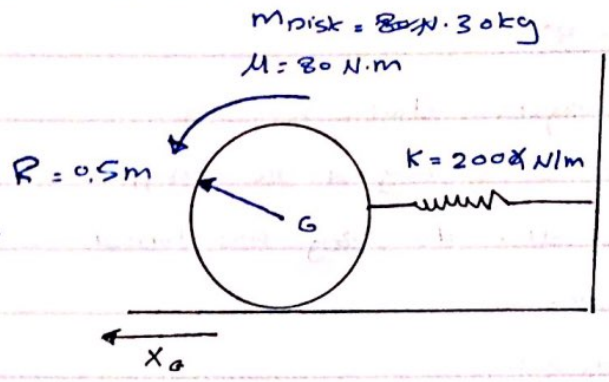
$+ 0 + 10(2 + 0.4)^2$

↳ The Disk is NOT rotating so we deal with it as a concentrated mass. I don't use $(\frac{mR^2}{2})$ in this case.

P (18-19)

The spring is initially unstretched

Determine → how far the center of mass of the disk travels along the plane before it momentarily stops. Rolls without slipping.



Solution :-

$T_1 = 0$

$T_2 = \frac{1}{2}(30)V_2^2 + \frac{1}{2} I_G \omega_2^2 = 0$ (Because it will stop so V_2 & $\omega_2 = 0$)

$\cancel{T_1}^{0} + U_{1\to2} = \cancel{T_2}^{0}$

$U_{1\to2} = 0$

$(U_{1\to2})_M + (U_{1\to2})_{spring} = 0$

$\int M\,d\theta - \frac{K}{2}(S_2^2 - S_1^2) = 0$ → $S_1 = 0$, $S_2 = X_G$, $\theta = \frac{X_G}{0.5}$

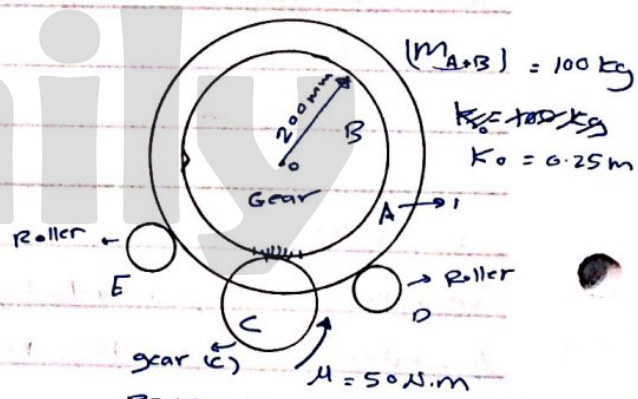
$80\,\theta - \frac{200}{2}(S_2^2 - 0) = 0$

$80\left(\frac{X_G}{0.5}\right) - \frac{200}{2}(X_G^2) = 0$

Answer → $X_G = 1.60\ m$

P (18-20)

→ Determine the angular velocity of (A) after (C) has rotated (10 rev) from rest.



$R = 150mm$

$m = 30\ kg$

$K_c = 0.125\ m$

gears:

$\omega_B r_B = \omega_c r_c$

$(200)\,\omega_B = 150\,\omega_c$

$\omega_B = \frac{150}{200}\,\omega_c$

or $\omega_c = \frac{200}{150}\,\omega_B$

$(U_{1\to2})_M = \int M\,d\theta$

$= 50(10)(2\pi)$

$= \leftarrow$

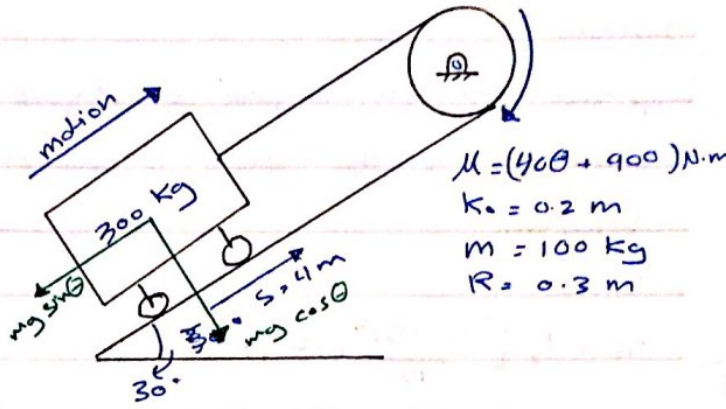
now you have one unknown (ω_c), solve the equation.

$\omega_c = \leftarrow$ rad/s

P(18-24)

→ Determine the speed (V) of the car after it travels s = 4m.

Starts from rest.



Solution:-

$T_1 = 0$

$T_2 = \frac{1}{2}(300)V^2 + \frac{1}{2}\left[100(0.2)^2\left(\frac{V}{0.3}\right)^2\right]$

$U_{1\to2} = \int_0^{(4/0.3)} (900 + 40\theta)\, d\theta - 300g(4)\sin 30$

→ $V = \omega(0.3)$

$\omega = \frac{V}{0.3}$

→

$T_1 + U_{1\to2} = T_2$ (solve from V)

Answer → $V_c = 7.49$ m/s

---

* Conservation of Energy :-

$T_1 + \underbrace{(U_{1\to2})_g + (U_{1\to2})_s}_{\text{conservative forces}} + \underbrace{(U_{1\to2})_M + (U_{1\to2})_F + (U_{1\to2})_{FE}}_{\text{non-conservative forces}} = T_2$

If F_E & F_s & $M = 0$:

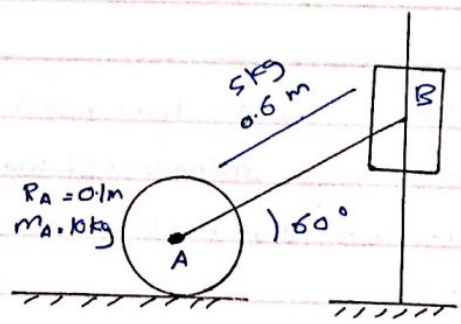
$T_1 + (V_{1g} - V_{2g}) + (V_{1s} - V_{2s}) = T_2$

$T_1 + V_{1g} + V_{1s} = T_2 + V_{2g} + V_{2s}$

Example (18-8) :-

→ Determine the angular velocity when $\theta = 0°$

Rolls without slipping & realeased from rest.



Solution:-

$T_1 = 0$

$T_2 = \frac{1}{2}m_r(V_{G_2})^2 + \frac{1}{2}I_G(\omega_r)_2^2 + \frac{1}{2}m_d\cancel{(V_A)_2^2} + \frac{1}{2}\cancel{I_A(\omega_d)_2^2}$

$= \frac{1}{2}(5)(0.3)(\omega_r)_2^2 + \frac{1}{2}\left[\frac{1}{12}(5)(0.6)^2\right]\omega_{r_2}^2 + 0 + 0$

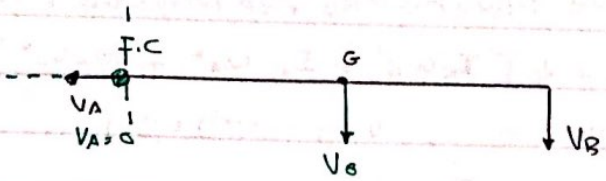
$= (0.3)(\omega_r)_2^2$

$V_{1\,system} = 0 + 5g(0.3\sin 60)$

$V_{2\,system} = 0 + 0$

$T_1 + V_1 = T_2 + V_2$

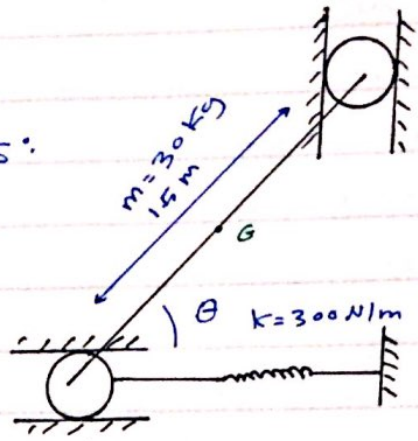
$(\omega_r)_2 = 6.52$ rad/s



$V_G = 0.3\omega$

F (18-11)

→ Determine the angular velocity of the rod when $\theta = 0$; The spring is unstretched at $\theta = 45°$.



Solution:-

$T_1 = 0$

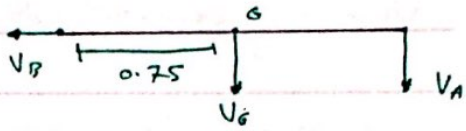
$T_2 = \frac{1}{2}\left[\frac{30(1.5)^2}{12}\omega^2 + 30 v_G^2\right]$

$V_{1g} = (0.75)\sin(45)(30g)$; $V_{1s} = 0$

$V_{2g} = 0$; $V_{2s} = \frac{k}{2}(1.5 - 1.5\cos 45)^2$

$T_1 + V_1 = T_2 + V_2$ solve for (ω)

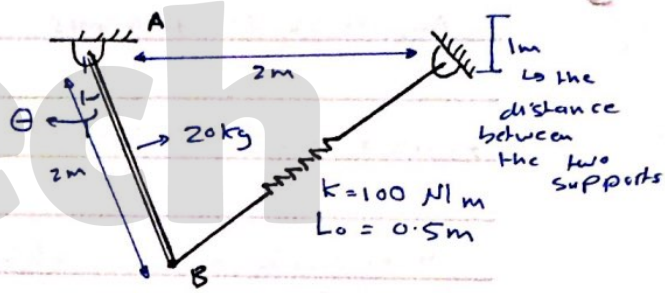
→ $V_G = 0.75\,\omega$



Ans → $\omega_2 = 3.36$ rad/s

F (18-12)

→ Determine ω at $\theta = 90°$ if the rod is released from rest when $\theta = 0$.



Solution:-

$T_1 = 0$

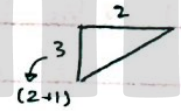
$T_2 = \frac{1}{12} I_A \omega^2$; $I_A = \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3}$

$V_{1g} = -mg(1) = -20(9.81)(1)$

$V_{2g} = 0$

$V_{1s} = \frac{k}{2}(S_1)^2$; $S_1 = \sqrt{(3)^2 + (2)^2} - L_0$ (0.5)

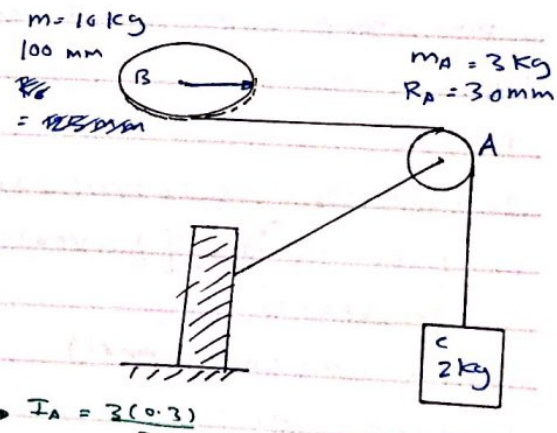
$V_{2s} = \frac{k}{2}(S_2)^2$; $S_2 = 1 - L_0 = 1 - 0.5$



Answer → $\omega_2 = 4.53$ rad/s

P (18-42) (starts from rest)

Determine → The distance (y) that (C) must move to cause (B) to have $\omega = 6$ rad/s.



Answer:-

$T_1 = 0$

~~$T_2 = \frac{1}{2}[10(0.125)^2\omega_A^2 + 30(0.45)^2\omega_B^2 + 50 v^2$~~

$T_2 = \frac{1}{2}[I_B\omega_B^2 + I_A\omega_A^2 + m_c v_c^2]$

$V_{1g} = 0$, $V_{2g} = -mg(y_c)$

$T_1 + V_1 = T_2 + V_2$

Answer → $y_c = 78.0$ mm.

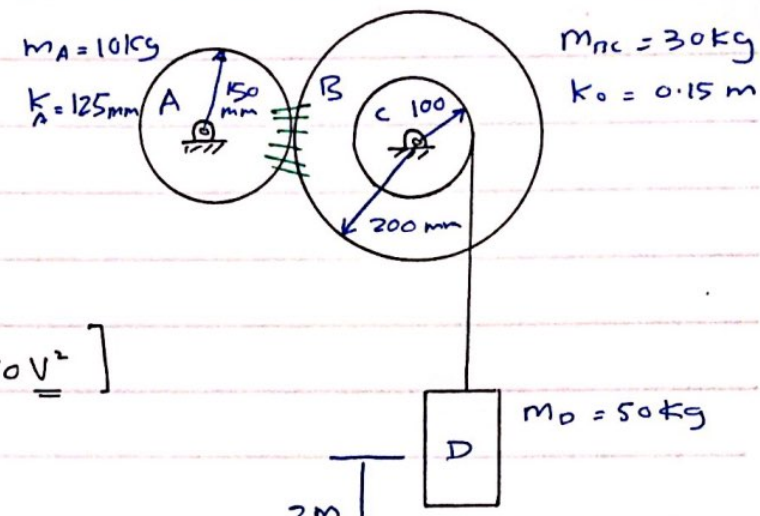
→ $I_A = \frac{3(0.3)}{2}$

$I_B = \frac{10(0.1)^2}{2}$

P(18-45)

Two gears.

Determine (V) of (D) after it moves a distance of 2m, starting from rest.



Solution:-

$T_1 = 0$

$T_2 = \frac{1}{2}\left[10(0.125)^2 \omega_A^2 + 30(0.15)^2 \omega_B^2 + 50 \underline{\underline{V}}^2\right]$

$V_{1g} = 0$

$V_{2g} = -mg(y_2 - y_1) = -50g(-2)$

$V = \checkmark$ m/s.

$V = 0.15\,\omega_B$

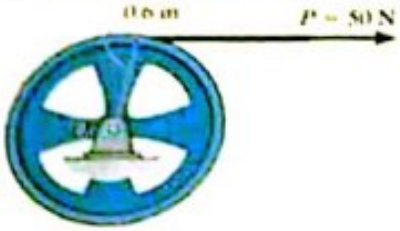
$\omega_A(0.15) = \omega_B(0.2)$

$\omega_A = \left(\frac{0.2}{0.15}\right)\left(\frac{V}{0.1}\right)$

note → you can also calculate the acceleration since it's constant and you have (y, V_1, V_2)

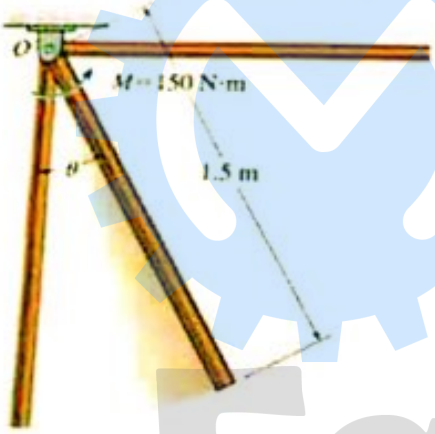
FUNDAMENTAL PROBLEMS

F18–1. The 50-kg wheel has a radius of gyration about its mass center G of $k_G = 400$ mm. Determine its angular velocity after it has rotated 20 revolutions starting from rest.



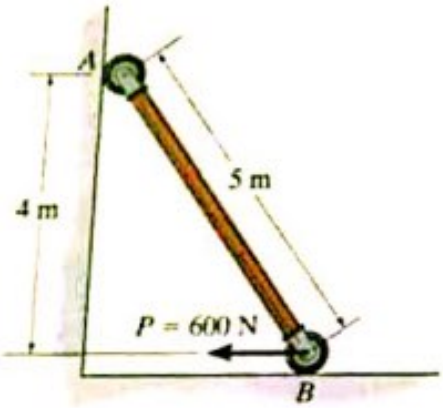
Prob. F18–1

F18–2. The uniform 25-kg slender rod is subjected to a couple moment of $M = 150$ N·m. If the rod is at rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$.



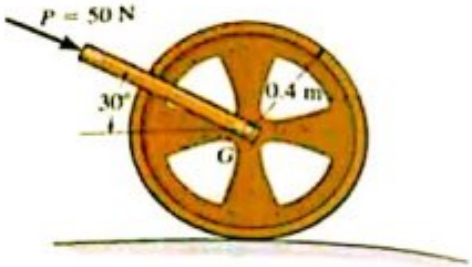
Prob. F18–2

F18–3. The uniform 50-kg slender rod is at rest in the position shown when $P = 600$ N is applied. Determine the angular velocity of the rod when the rod reaches the vertical position.



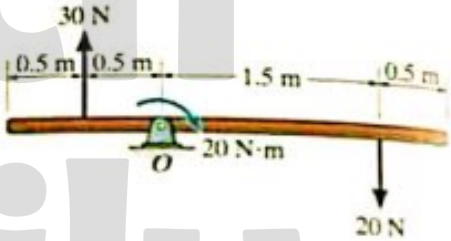
Prob. F18–3

F18–4. The 50-kg wheel is subjected to a force of 50 N. If the wheel starts from rest and rolls without slipping, determine its angular velocity after it has rotated 10 revolutions. The radius of gyration of the wheel about its mass center G is $k_G = 0.3$ m.



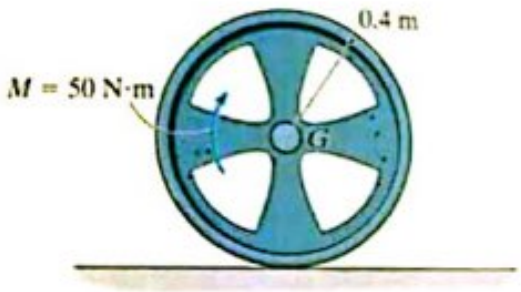
Prob. F18–4

F18–5. If the uniform 30-kg slender rod starts from rest at the position shown, determine its angular velocity after it has rotated 4 revolutions. The forces remain perpendicular to the rod.



Prob. F18–5

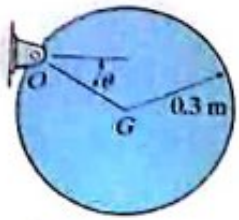
F18–6. The 20-kg wheel has a radius of gyration about its center G of $k_G = 300$ mm. When it is subjected to a couple moment of $M = 50$ N·m, it rolls without slipping. Determine the angular velocity of the wheel after its mass center G has traveled through a distance of $s_G = 20$ m, starting from rest.



Prob. F18–6

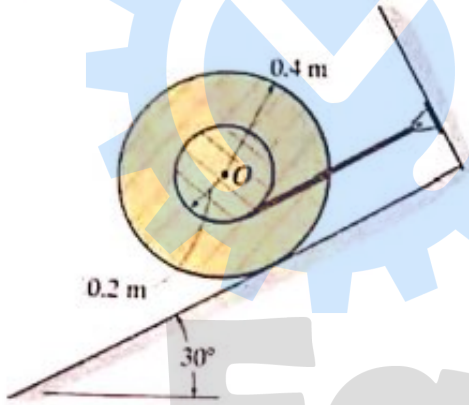
FUNDAMENTAL PROBLEMS

F18–7. If the 30-kg disk is released from rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$.



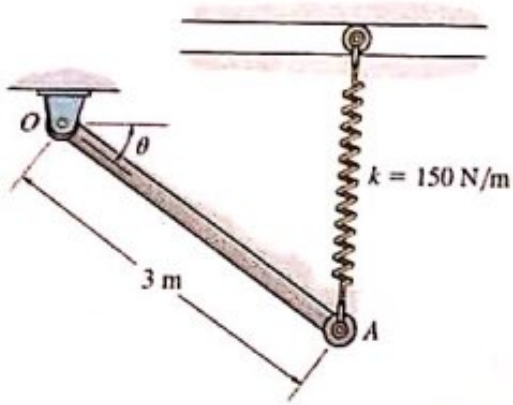
Prob. F18–7

F18–8. The 50-kg reel has a radius of gyration about its center O of $k_O = 300$ mm. If it is released from rest, determine its angular velocity when its center O has traveled 6 m down the smooth inclined plane.



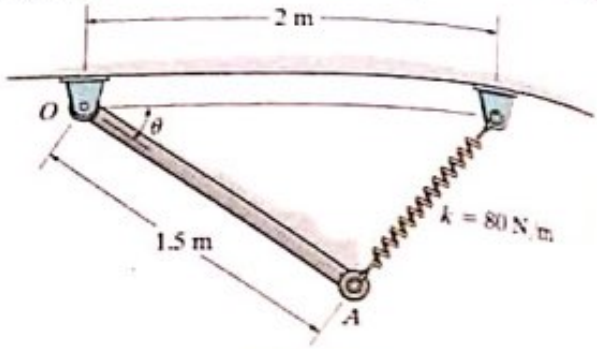
Prob. F18–8

F18–9. The 60-kg rod OA is released from rest when $\theta = 0°$. Determine its angular velocity when $\theta = 45°$. The spring remains vertical during the motion and is unstretched when $\theta = 0°$.



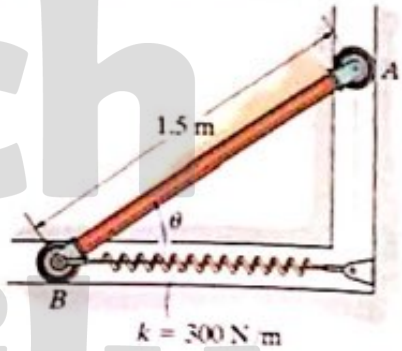
Prob. F18–9

F18–10. The 30-kg rod is released from rest when $\theta = 0°$. Determine the angular velocity of the rod when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.



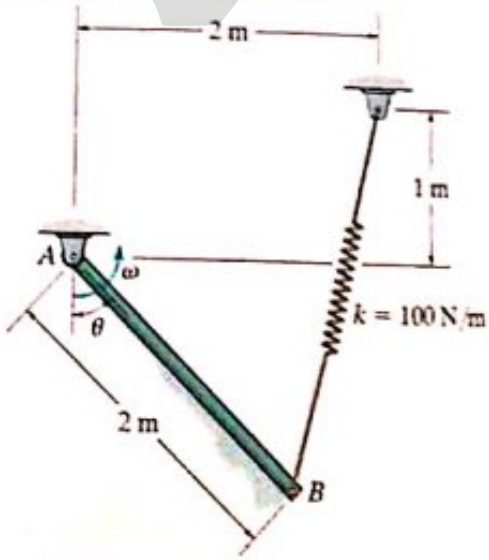
Prob. F18–10

F18–11. The 30-kg rod is released from rest when $\theta = 45°$. Determine the angular velocity of the rod when $\theta = 0°$. The spring is unstretched when $\theta = 45°$.



Prob. F18–11

F18–12. The 20-kg rod is released from rest when $\theta = 0°$. Determine its angular velocity when $\theta = 90°$. The spring has an unstretched length of 0.5 m.



Prob. F18–12