Chapter Two

Roots of Nonlinear Equations of Single Variable

- Bisection Method
- False Positioning Method
- Newton's Method
- Secant Method
- Modified Secant Method
- Fixed Point Iteration Method
- Muller Method

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1 (Bisection)

طريقة التنصيف

$$\boxed{X_3 = \frac{X_1 + X_2}{2}}$$

التخمين المبدئي
initial guess

$[X_1 , X_2]$

صورة الـ X_1 → $f(x_1)$

الصوره → $f(x_2)$

** الشرط الاساسي لاستخدام Bisection هو أن تكون اشارة $f(x_1)$ مخالفه لاشارة $f(x_2)$

$$\Longrightarrow \quad f(x_1) \cdot f(x_2) < 0 \text{ negative}$$

** اذا كانت $f(x_1) \cdot f(x_2) > 0 \Rightarrow$ Stop

** شرط الوصول للحل النهائي (شرط التوقف Stop) هو:

$$f(x_r) \leq \varepsilon$$

** عدد الاشواط المتوقعه للوصول للحل النهائي هو (Number of iterations)

$$n \overset{\text{int.}}{=} \left(\frac{\ln(X_2 - X_1) - \ln \varepsilon}{\ln 2} \right)$$

عدد صحيح

Example Use Bisection To solve (to find the Root)

$$f(x) = e^{x} - 3x$$

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Using $[1,2]$, $\varepsilon = 0.07$

الحل

$$n = \text{int}\left[\frac{\ln(2-1) - \ln(0.07)}{\ln 2}\right]$$

$$n = \text{int}(3.8) \approx 4$$

بحاجه الى ثلاثه
أو أربعه أو خمسه
اشواط
لحل المسأله

الشوط الاول
$n=1$

$$X_1 = 1 \xrightarrow{\text{الصوره}} f(1) = -0.2817$$

$$X_2 = 2 \xrightarrow{\text{الصوره}} f(2) = 1.3891 \Rightarrow f(x_1)\cdot f(x_2) < 0 \text{ Continue}$$

$$X_3 = \frac{1+2}{2} = 1.5 \xrightarrow{\text{الصوره}} f(1.5) = -0.01831$$

* غالباً لا يوجد داعي لحساب قيمة الخطأ عند الشوط الاول *

$f(1)\cdot f(1.5) > 0$

الشوط الثاني
$n=2$

$$X_1 = 1.5 \xrightarrow{\text{الصوره}} f(1.5) = \cancel{-0.2817}\ -0.01831$$

$$X_2 = 2 \xrightarrow{\text{الصوره}} f(2) = \cancel{-0.01831}\ 1.3891 \Rightarrow f(x_1)\cdot f(x_2) < 0 \text{ Continue}$$

$$X_3 = \frac{1+1.5}{2} = 1.75 \xrightarrow{\text{الصوره}} f(1.75) = 0.5046$$

$|f(1.75)| \not> \varepsilon$ Cont.

الدور الثالث $n=3$

$$X_1 = 1.5 \longrightarrow f(1.5) = -0.01831$$

$$X_2 = 1.75 \longrightarrow f(1.75) = 0.5046$$

$$X_3 = \frac{1.5+1.75}{2} = 1.625 \longrightarrow f(1.625) = 0.2034$$

error الخطأ $|f(1.625)| > \varepsilon$ Cont.

$f(1.5) \cdot f(1.625) < 0$

الدور الرابع $n=4$

$$X_1 = 1.5 \longrightarrow f(1.5) = -0.01831$$

$$X_2 = 1.625 \longrightarrow f(1.625) = 0.2034$$

$$X_3 = \frac{1.5+1.625}{2} = 1.5625 \longrightarrow f(1.5625) = 0.0832$$

error $|f(1.5625)| > \varepsilon$ Cont.

$f(1.5) \cdot f(1.5625) < 0$

الدور الخامس $n=5$

$$X_1 = 1.5 \longrightarrow f(1.5) = -0.01831$$

$$X_2 = 1.5625 \longrightarrow f(1.5625) = 0.0832$$

$$X_3 = \frac{1.5+1.5625}{2} = 1.5313 \longrightarrow f(1.5313) = 0.0302$$

الخطأ $|f(1.5313)| < \varepsilon$ Stop.

الحل النهائي للسؤال هو :

$$X_r = 1.5313$$

Root.

ex)

Solve using Bisection

$$y(x) = \sin x - 2x + 1$$

$$[0, 1] \quad , \quad \varepsilon = 0.03$$

الحل

$$n = \text{int.} \frac{\ln(1-0) - \ln(0.03)}{\ln 2} = 5.059$$

(عدد الدوران)

الدورة الأولى
$n=1$

$X_1 = 0 \longrightarrow f(0) = 1$

$X_2 = 1 \longrightarrow f(1) = -0.1585$

$X_3 = \frac{0+1}{2} = 0.5 \longrightarrow f(0.5) = 0.4794$

الدورة الثانية
$n=2$

$X_1 = 0.5 \longrightarrow f(0.5) = 0.4794$

$X_2 = 1 \longrightarrow f(1) = -0.1585$

$X_3 = \frac{1+0.5}{2} = 0.75 \longrightarrow f(0.75) = 0.1816$

$|f(0.75)| > \varepsilon$ Cont.

$n=3$

$X_1 = 0.75 \longrightarrow f(0.75) = 0.1816$

$X_2 = 1 \longrightarrow f(1) = -0.1585$

$X_3 = \frac{1+0.75}{2} = 0.875 \longrightarrow f(0.875) = 0.0175$

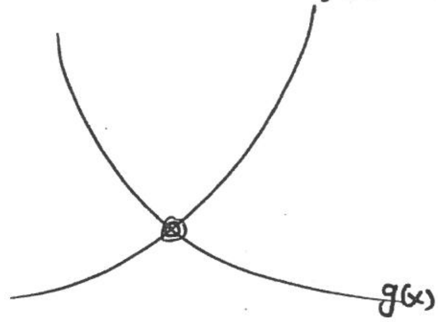
$|f(0.875)| < \varepsilon$ Stop.

الحل النهائي هو
($X_r = 0.875$)

ex

اوجد نقطة التقاطع ما بين اقترانين

point of intersection



استخدم الخطأ

$\varepsilon = 0.004$

الحل

لإيجاد نقطة التقاطع بين اقترانين

$\Rightarrow \quad \cos 3x = x^2 - 2$

$\Rightarrow \quad f(x) = \cos 3x - x^2 + 2$

اختيار خاطئ

$[0 \, , \, 1]$

العدد 3 ، العدد $+0.0100075$

اختيار صحيح

$[0 \, , \, 2]$

العدد موجب 3 ، العدد سالب -1.03983

المحاولة الأولى

$X_1 = 0 \longrightarrow f(0) = 3$

$X_2 = 2 \longrightarrow f(2) = -1.03983$

$X_3 = \frac{0+2}{2} = 1 \longrightarrow f(1) = 0.0100075$

$\underline{n=2}$

$X_1 = 1 \longrightarrow f(1) = 0.0100075$

$X_2 = 2 \longrightarrow f(2) = -1.03983$

$X_3 = \frac{1+2}{2} = 1.5 \longrightarrow f(1.5) = -0.4608$

$|f(1.5)| > \varepsilon$ Cont.

$f(1) = 0.0100075$

$X_1 = 1 \longrightarrow$

$\underline{n=3}$

$f(1.5) = -0.4608$

$X_2 = 1.5$

$\longrightarrow f(1.25) = -0.3831$

$X_3 = \frac{1+1.5}{2} = 1.25$

$X_1 = 1 \longrightarrow$

$\underline{n=4}$

$X_2 = 1.25 \longrightarrow$

$X_3 = \frac{1+1.25}{2} = \longrightarrow$

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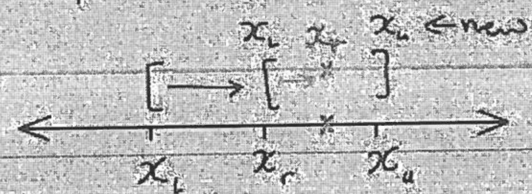
2

False positioning

$$X_{root} = \frac{x_u \cdot f(x_L) - x_L f(x_u)}{f(x_L) - f(x_u)} \quad [x_L , x_u]$$

note * Best way to study = table

Iteration	X_L	X_u	X_r
	x_{L_0}	X_{u0}	$\dfrac{x_u f(x_{L0}) - x_L f(x_n)}{f(x_L) - f(x_u)}$



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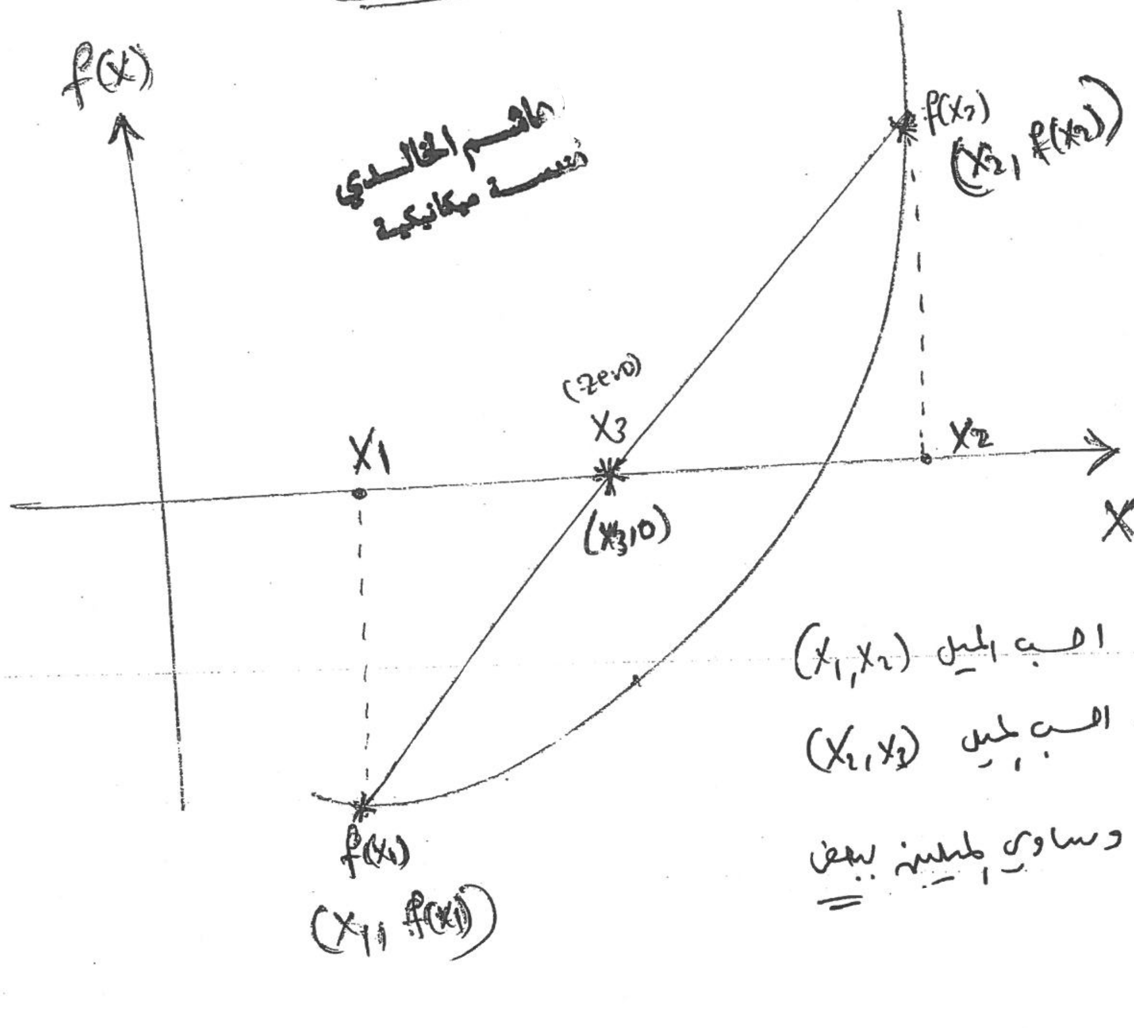
(False Position Method)

وهي نفس خطوات الطريقه ال Bisection تماماً

مع الاختلاف الوحيد وهو في ايجاد ال $\underline{X_3}$ حسب القانون التالي

$$X_3 = X_2 - \left[\frac{X_2 - X_1}{f(X_2) - f(X_1)}\right] \cdot f(X_2)$$



* احسب الميل (X_1, X_2)
* احسب الميل (X_2, X_3)
* وساوي الميلين ببعض

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\overset{\text{zero}}{\cancel{f(x_3)}} - f(x_2)}{x_3 - x_2}$$

$$x_3 - x_2 = \frac{(x_2 - x_1)(-f(x_2))}{f(x_2) - f(x_1)}$$

$$\Rightarrow \boxed{x_3 = x_2 - \left[\frac{x_2 - x_1}{f(x_2) - f(x_1)}\right] \cdot f(x_2)}$$

ex) use False Method.

To Solve

$$f(x) = e^x - 3x \quad , \quad [1,2] \quad , \quad \varepsilon = 0.08$$

الدور الأول
$n = 1$

$$X_1 = 1 \longrightarrow f(1) = -0.2817$$

$$X_2 = 2 \longrightarrow f(2) = 1.3891$$

$$X_3 = 2 - \left[\frac{2-1}{1.3891 - -0.2817}\right]\left[1.3891\right]$$

$$X_3 = 1.1686 \longrightarrow f(1.1686) = -0.2883$$

(لا داعي لحساب الخطأ في الدور الأول)

الدور الثاني
$n = 2$

$$X_1 = 1.1686 \longrightarrow f(1.1686) = -0.2883$$

$$X_2 = 2 \longrightarrow f(2) = 1.3891$$

$$X_3 = 2 - \left[\frac{2 - 1.1686}{1.3891 - -0.2883}\right]\left[1.3891\right]$$

$$= 1.3115 \longrightarrow f(1.3115) = -0.2228$$

إذن $|f(1.3115)| > \varepsilon \rightarrow$ Cont.

الدور الثالث
$n = 3$

$$X_1 = 1.3115 \longrightarrow f(1.3115) = -0.2228$$

$$X_2 = 2 \longrightarrow f(2) = 1.3891$$

$$X_3 = 2 - \left[\frac{2 - 1.3115}{1.3891 - -0.2228}\right]\left[1.3891\right] = 1.4067 \longrightarrow f(1.4067) = -0.1176$$

إذن $|f(1.4067)| > \varepsilon$ Cont.

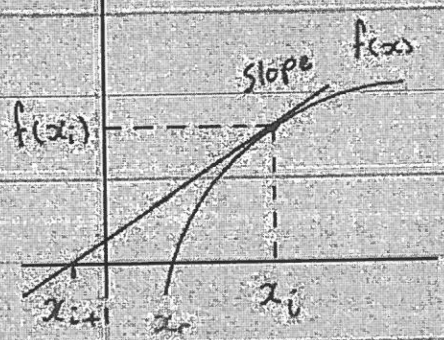
3 Newton Raphson method

Root locating approximately method, when it's difficult to find using algebraic ways.

* note: i = iteration

$x_i = x_0 \quad i = 0$

usually given



$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$\text{Slope} = f'(x_i)$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}} \qquad f(x_i) = f'(x_i)\,(x_i - x_{i+1})$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

given, function

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

derivative

$i = 0 \quad x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} \qquad i = 1 \quad x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

* disadvantage: 1 root, $f'(x)$ might be hard to find.

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Solution:

1. Find fuction and x from problem
2. State first derivative
3. State rule $x_{i+1} = x_i - \frac{f(x_i)}{f/(x_i)}$
4. i =0
5. Absolute relative % of true or approximate error

$$\varepsilon_t = \left| \frac{\text{true V} - \text{Appx V}}{\text{true val.}} \right| \times 100$$

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Newton's Method

$$X_n = X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})}$$

حيث نعتمد نقطة واحدة X_0 للبدء بحل السؤال

الشوط الرابع $$X_4 = X_3 - \frac{f(X_3)}{f'(X_3)}$$

الشوط التاسع $$X_9 = X_8 - \frac{f(X_8)}{f'(X_8)}$$

الشوط السادس عشر $$X_{16} = X_{15} - \frac{f(X_{15})}{f'(X_{15})}$$

الشوط الثاني $$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

ex/ use Newton-Raphson Method

To solve $f(x) = \cos 3x - x^2 + 3$

using $x_0 = 1$, $\varepsilon = 1 \times 10^{-3}$

$= 0.001$

الحل

$f(x) = \cos 3x - x^2 + 3$ الاقتران

$f'(x) = -3 \sin 3x - 2x$ المشتقة

الدورة الأولى $n = 1$

$x_0 = 1 \Rightarrow f(1) = 1.010008$

$\rightarrow f'(1) = -2.4234$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1.010008}{-2.4234} = \boxed{1.4148}$$

1.4168

الخطأ $|f(x_1)| = |f(1.4148)| = \boxed{\cancel{5.0017}} > \varepsilon$ Cont.

0.5469

الدورة الثانية $n = 2$

$x_1 = 1.4148 \longrightarrow f(1.4148) = 5.0017$

$f'(1.4148) = -0.1522$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.4148 - \frac{5.0017}{-0.1522} = 1.4289$$

الخطأ $|f(x_2)| = |f(1.4289)| = 0.545372 > \varepsilon$ Cont.

الخطوة الثالثة
$n=3$

$X_2 = 1.4284 \longrightarrow f(1.4284) = 0.5454$

$\longrightarrow f'(1.4284) = -0.1264$

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)} = 1.4284 - \frac{0.5454}{-0.1264} = 1.4284$$

Check.

اذن $|f(X_3)| = |f(1.4284)| = 0.545$

$\Rightarrow$ Root آخر جواب

$X_r = 1.4284.$

Find the Root of the Function

$$f(x) = 2^{\sin x} \longrightarrow f'(x) = \cos x \cdot 2^{\sin x} \cdot \ln(2)$$

$$f(x) = e^{\cos x} \longrightarrow f'(x) = -\sin x \; e^{\cos x} \cdot \cancel{\ln e}^{\,1}$$

$$f(x) = (x^2+3)(\sin x - 5) \longrightarrow f'(x) = (x^2+3)(\cos x) + (\sin x - 5)(2x)$$

$$f(x) = \ln(x^3 + 8x^2 + 1) \longrightarrow f'(x) = \frac{3x^2 + 16x}{x^3 + 8x^2 + 1}$$

$$f(x) = \ln(\sin 8x + \sqrt{2x+1}) \to f'(x) = \frac{8\cos 8x + \frac{2}{2\sqrt{2x+1}}}{\sin 8x + \sqrt{2x+1}}$$

$$f(x) = \sqrt{3\sin x + x^3} \longrightarrow f'(x) = \frac{3\cos x + 3x^2}{2\sqrt{3\sin x + x^3}}$$

$$f(x) = \frac{x+1}{x-1} \to f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$$

$$f(x) = \frac{x^3 + \sin 6x}{\sqrt{x} - \ln x}$$

$$f'(x) = \frac{(\sqrt{x} - \ln x)(3x^2 + 6\cos 6x) - (x^3 + \sin 6x)\left(\frac{1}{2\sqrt{x}} - \frac{1}{x}\right)}{\left[\sqrt{x} - \ln x\right]^2}$$

$$f(x) = x^x$$

$$y = x^x \Rightarrow \ln y = \ln x^x$$

$$\Rightarrow \ln y = x \ln x$$

$$\Rightarrow \frac{1}{y} y' = \cancel{x} \frac{1}{\cancel{x}} + \ln x$$

$$\Rightarrow \frac{1}{y} y' = 1 + \ln x$$

$$\Rightarrow y' = [1 + \ln x] y$$

$$\Rightarrow y' = [1 + \ln x] x^x$$

$$f'(x) = x^x [1 + \ln x]$$

$$y = \left[f(x)\right]^{g(x)}$$

$$\ln y = g(x) \ln f(x)$$

$$\frac{1}{y}\, y' = g(x) \cdot \frac{f(x)}{f'(x)} + g'(x) \ln f(x)$$

القاعدة العامة

$$\boxed{y' = \left[\frac{g(x)\, f(x)}{f'(x)} + g'(x) \cdot \ln f(x)\right] \cdot \left[f(x)\right]^{g(x)}}$$

$$f(x) = (x^2+3)^{\sin x}.$$

$$f'(x) = \left[\frac{(\sin x)\,(x^2+3)}{2x} + \cos x \cdot \ln(x^2+3)\right] (x^2+3)^{\sin x}.$$

$$y = x^{\sin x} \longrightarrow y' = \left[x \sin x + \cos x \ln x\right] x^{\sin x}.$$

ex) Using Newton To find :- $\sqrt{38}$?

الحل

$$X = \sqrt{38}$$

$$\Rightarrow X^2 = 38$$

$$\Rightarrow f(x) = X^2 - 38 \qquad X_0 = 6$$

المشتقة

$$f'(x) = 2X$$

الخطوة الاولى

$$X_0 = 6 \longrightarrow \begin{aligned} f(6) &= -2 \\ f'(6) &= 12 \end{aligned}$$

$$X_1 = 6 - \frac{-2}{12} = 6.16667$$

الخطوة الثانية

$$X_1 = 6.16667 \longrightarrow \begin{aligned} f(6.16667) &= 0.027819 \\ f'(6.16667) &= 12.3333 \end{aligned}$$

$$X_2 = 6.16667 - \frac{0.027819}{12.3333} = 6.16441$$

ex) Find $\sqrt[3]{613}$? Newton.

الحل

$$X = \sqrt[3]{613}$$

$$x^3 = 613$$

$$f(x) = x^3 - 613$$

$$X_0 = 8$$

المشتقة $f'(x) = 3x^2$

١. الشرط الأول

٢. الشرط الثاني

ex)

Find By Newton $\ln(37)$?

الحل

$$X = \ln(37)$$

$$\Rightarrow e^{X} = 37$$

$$f(x) = e^{x} - 37 \qquad X_0 = 3$$

المشتقة $$f'(x) = e^{x}$$

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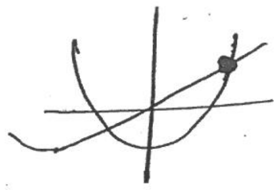
ex Find the point of intersection between the two functions

أوجد نقط التقاطع بين الاقترانين

$$y_1 = 3\sin 4X + X$$

$$y_2 = X^2 - 16$$



Using Newton Method.

الحل

$$y_1 = y_2$$

$$\Rightarrow \quad 3\sin 4X + X = X^2 - 16$$

$$\Rightarrow \quad 3\sin 4X + X - X^2 + 16 = 0$$

$$\Rightarrow \quad f(X) = 3\sin 4X - X^2 + X + 16$$

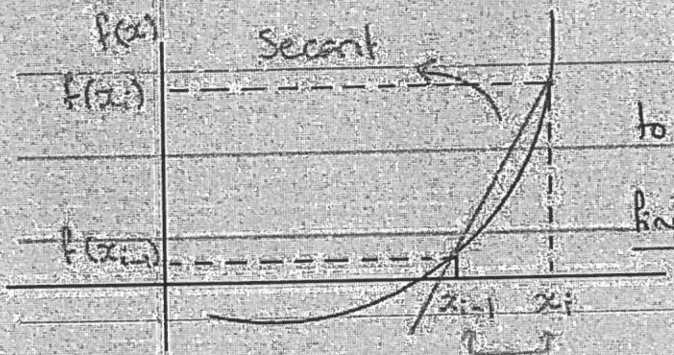
جذر هذا الاقتران هو عبارة عن نقطة التقاطع

$$f'(X) = 12\cos 4X - 2X + 1$$

$$X_{i+1} = X_i - \frac{3\sin 4X_i - X_i^2 + X_i + 16}{12\cos 4X_i - 2X_i + 1}$$

Initial guess $\Rightarrow X_0 = 0$

4 Secant method



given in problem statement

a method used in order to find a root without finding the first derivative

why? Advantage

$$f'(x_i) \approx \frac{\text{Raise}}{\text{Run}} = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$$x_{i+1} = \frac{x_i - f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

(Secant Method)

وهي نفس الطريقة Newton مع اختلاف لقانون (slope)

$$X_{n+1} = X_n - \left[\frac{X_n - X_{n-1}}{f(X_n) - f(X_{n-1})} \right] \cdot f(X_n)$$

حيث يلزم نقطتين للبدء بهذه الطريقة (X_0, X_1)

السؤال الرابع $n=4$

$$X_5 = X_4 - \left[\frac{X_4 - X_3}{f(X_4) - f(X_3)} \right] \cdot f(X_4)$$

السؤال العاشر

$$X_{11} = X_{10} - \left[\frac{X_{10} - X_9}{f(X_{10}) - f(X_9)} \right] \cdot f(X_{10})$$

ex) Using Secant Method To Find Root

$$f(x) = x^3 - 6x^2 + 11x - 6.1$$

using $\left.\begin{array}{l} X_0 = 2.5 \\ X_1 = 3.5 \end{array}\right\} \rightarrow$ using $[2.5, 3.5]$

and $\varepsilon = 0.1\,\% \Rightarrow 0.001 = 1 * 10^{-3}$

الحل

الخطوة الاولى

$X_0 = 2.5 \longrightarrow f(2.5) = -0.475$

$X_1 = 3.5 \longrightarrow f(3.5) = 1.775$

$$X_2 = 3.5 - \left[\frac{3.5 - 2.5}{1.775 - (-0.475)}\right]\left[1.775\right] = 2.711$$

$\longrightarrow f(2.711) = -0.4515$

error $|-0.4515| > \varepsilon$ Cont.

الخطوة الثانية

$n = 2$

$X_1 = 3.5 \longrightarrow f(3.5) = 1.775$

$X_2 = 2.711 \longrightarrow f(2.711) = -0.4515$

$$X_3 = 2.711 - \left[\frac{2.711 - 3.5}{-0.4515 - 1.775}\right]\left[-0.4515\right] = 2.871$$

$\longrightarrow f(2.871) = -0.3101$

error $|-0.3101| > \varepsilon$ Cont.

التكرار الثاني

$n=3$

$$X_2 = 2.711 \longrightarrow f(2.711) = -0.4515$$

$$X_3 = 2.871 \longrightarrow f(2.871) = -0.3101$$

$$X_4 = 2.871 - \left[\frac{2.871 - 2.711}{-0.3101 - {}^{-}0.4515}\right]\left[-0.3101\right] = 3.22$$

Check.

$$\longrightarrow f(3.22) = 3.2216 \;!!$$

* error = ...

التكرار الرابع

$$X_3 = 2.871 \longrightarrow f(2.871) = -0.3101$$

$$X_4 = 3.22 \longrightarrow f(3.22) = ---$$

$$X_5 = ----$$

Example:-

Use Newton's method to locate the root

$f(x) = x^3 - x - 1$ (4 iterations $i = 3$)

① State function $f(x) = x^3 - x - 1$

② State 1st derivitive $f'(x) = 3x^2 - 1$

③ $x_0 = 1$ stated from experience from bracketing

$f(1) = 1^3 - 1 - 1 = -1$ $\quad f'(1) = 2$

④ State function $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

⑤ iterations $i = 0$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1 - \dfrac{-1}{2} = \dfrac{3}{2} = 1.5$

$f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875$ $\quad f'(x) = 5.75$

$i = 1$ $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1.5 - \dfrac{0.875}{5.750} = 1.348$

$f(1.348) = 0.10057$ $\quad f'(1.348) = 4.44969$

$i = 2$ $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} \Rightarrow 1.348 - \dfrac{0.10057}{4.44969} = 1.32520$

$f(1.32520) = 0.002057$ $\quad f'(1.32520) = 4.268465$

$i = 3$ $x_4 = 1.32520 - \dfrac{0.002057}{4.268465} = 1.32472$

True value = 1.324717957244 75

*note: Used when derivative is easy, otherwise use secant

$\boxed{\text{Example 2:}}$

Use Newton's method to find $f(x) = x^2+3x+2$ $x_0=2$

① State function $f(x) = x^2+3x+2$ and ② $f'(x) = 2x+3$

③ State $x_0 = 2$

④ State Rule $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

⑤ Iterations

$i=0$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - \dfrac{12}{7} = 0.2857$

$f(x) = 2.924$ $f'(x) = 3.5714$

$i=1$ $x_2 = 0.2857 - \dfrac{2.924}{3.5714} = -0.533$

$f(x) = 0.685$ $f'(x) = 1.934$

$i=2$ $x_3 = -0.533 - \dfrac{0.685}{1.934} = -0.8872$

$f(x) = 0.1255$ $f'(x) = 1.2256$

$i=3$ $x_4 = -0.8872 - \dfrac{0.1255}{1.2256} = -0.98963$

$x_{true} = -2$ & -1

Secant Method

$$\boxed{X_{i+1} = X_i - \frac{[X_i - X_{i-1}]\, f(x_i)}{f(x_i) - f(x_{i-1})}}$$

ex) Use Secant Method to find the root

$$f(x) = x^3 - x^2 - 10x - 8$$

using $X_0 = 3$ $\quad X_1 = 6$

أو

$[3, 6]$ $\quad , \varepsilon = 0.05$

الحل

خطوة أولى

$X_0 = 3 \longrightarrow f(3) = -20$

$X_1 = 6 \longrightarrow f(6) = 112$

$$X_2 = 6 - \frac{6 - 3}{112 - -20}(112) = 4.303$$

$\text{Error} = f(4.303) = 10.1277 > \varepsilon$ $\quad$ Cont.

خطوة ثانية

$X_1 = 6 \longrightarrow f(6) = 112$

$X_2 = 4.303 \longrightarrow f(4.303) = 10.1277$

$$X_3 = 4.303 - \frac{4.303 - 6}{10.1277 - 112}(10.1277) = 4.134$$

$*$ $Error = 4.2265 > \varepsilon$ Cont.

دورة ثالثة

$X_2 = 4.303 \longrightarrow f(4.303) = 10.1277$

$X_3 = 4.1342 \longrightarrow f(4.1342) = 4.2265$

$$X_4 = 4.1342 - \frac{4.1342 - 4.303}{4.2265 - 10.1277}(4.2265)$$

$$= 4.0133$$

$*$ $Error = 0.401 > \varepsilon$ Cont.

دورة رابعة

$X_3 = 4.1342 \longrightarrow f(4.1342) = 4.2265$

$X_4 = 4.0133 \longrightarrow f(4.0133) = 0.401$

$$X_5 = 4.0133 - \frac{4.0133 - 4.1342}{0.401 - 4.2265}(0.401)$$

$$= 4.001$$

$*$ $Error = 0.03 < \varepsilon$ Stop !!

$$X = 4.001$$

الجواب النهائي **Root.**

$\boxed{\text{Example 3}}$: Secant method

$f(x) = -\frac{1}{3}x^3 - 2x + 5$

① State function $f(0) = 5$ $f(2) = -5/3$

② State starting points $x_{-1} = 0$ $x_0 = 2$

③ State approximation rule.

$$f(x) = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

④ Iteration

*usually starting at 0

$i = 0$ $x_1 = 2 - \frac{f(2)(0-2)}{f(0) - f(2)} = 2 - \frac{-5/3(2)}{5 - (-5/3)} = 1.5$

$i = 1$ $x_2 = 1.5 - \frac{f(1.5)\cdot(2-1.5)}{f(2) - f(1.5)} = 1.5 - \frac{0.875(0.5)}{\frac{-5}{3} - 0.875} = 1.67213$

$i = 2$ $x_3 = 1.67213 - \frac{f(1.67213) - (1.5 - 1.67213)}{f(1.5) - f(1.67213)} = 1.693667$

* Will get question of convergence / divergence

2 iterations, which conv. or div. according to error.

$i = 3$ $x_4 = 1.6937 - \frac{-0.006766(1.67213 - 1.6937)}{0.0973 - (-0.006766)} = 1.69227$

Converging according to approximation error

Example 4 Secant method

① $f(x) = 2x^2 - x + 1$

② $x_{-1} = 0 \quad x_0 = 1$

$i=0 \quad x_1 = x_0 - \dfrac{f(x_0)(x_{-1} - x_0)}{f(x_{-1}) - f(x_0)} = 1 - \dfrac{f(1)(0-1)}{f(0) - f(1)} = -1$

$i=1 \quad x_2 = -1 - \dfrac{f(-1)(1-(-1))}{f(1) - f(-1)} = -1 - \dfrac{4(1+1)}{2-4} = -1 - \dfrac{8}{-2} = 3$

5

Modified Secant method 24/12/2013

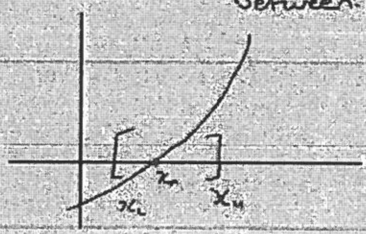
$$f'(x) \simeq \frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}$$

Secant

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$ ← open method

$$x_r = x_u - \left(\frac{f(x_u)(x_L - x_u)}{f(x_L) - f(x_u)} \right)$$ ← bracketing

↳ we know where the root is between.



modified Secant method

advantage: multiple Roots.

① One starting point x_i

$$f'(x_i) \simeq \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i}$$

$$\boxed{\text{Rule} \Rightarrow x_{i+1} = x_i - \frac{\delta x_i\, f(x_i)}{f(x_i + \delta x_i) - f(x_i)}}$$

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المحسنة
Modified Secant Method

$$X_{i+1} = X_i - \left[\frac{\delta x_i}{f(x_i + \delta x_i) - f(x_i)} \right] f(x_i)$$

مهم

$$\delta x_i = \varepsilon\, x_i$$

ex use Modifed secant Method to solve

$$f(x) = x^3 - 6x^2 + 11x - 6.1$$

using $x_0 = 3.5$
and $\varepsilon = 0.01$

الحل

تعويض القوانين

$$x_0 = 3.5 \longrightarrow f(x_0) = 1.775$$

$$\delta x_0 = (0.01)(3.5) = 0.035$$

$$(x_0 + \delta x_0) = 3.535 \longrightarrow f(x_0 + \delta x_0) = 1.981$$

$$X_1 = X_0 - \left[\frac{\delta x_0}{f(x_0 + \delta x_0) - f(x_0)} \right] f(x_0)$$

$$= 3.5 - \left[\frac{0.035}{1.981 - 1.775} \right] [1.775]$$

$$= 3.199$$

الخطأ $E = \left| \frac{X_1 - X_0}{X_1} \right| * 100\% = \left| \frac{3.199 - 3.5}{3.199} \right| * 100\% = 9.4\%$

$$= 0.094 > \varepsilon$$

دورة أولى

$$X_1 = 3.199 \longrightarrow f(x_1) = 0.4246$$

$$\delta x_1 = (0.01)(3.199) = 0.03199$$

$$(x_1 + \delta x_1) = 3.23099 \longrightarrow f(x_1 + \delta x_1) = 0.5344$$

$$X_2 = X_1 - \left[\frac{\delta x_1}{f(x_1+\delta x_1) - f(x_1)}\right] f(x_1)$$

$$\Rightarrow X_2 = 3.199 - \left[\frac{0.03199}{0.5344 - 0.4246}\right]\left[0.4246\right]$$

$$\Rightarrow X_2 = 3.0753$$

$$E = \left|\frac{X_2 - X_1}{X_2}\right| * 100 = \left|\frac{3.0753 - 3.199}{3.0755}\right| * 100\%$$

$$= 4.02\% = 0.0402 > \varepsilon$$

دورة ثانية

$$X_2 = 3.0753 \longrightarrow f(x_2) = 0.068$$

$$\delta x_2 = (0.01)(3.0753) = 0.030753$$

$$(x_2 + \delta x_2) = 3.1061 \longrightarrow f(x_2 + \delta x_2) = 0.1469$$

$$X_3 = 3.0753 - \left[\frac{0.030753}{0.1469 - 0.068}\right]\left[0.068\right] = 3.048$$

$$E = 0.895\% = 0.00895 < \varepsilon \text{ stop}$$

$\boxed{\text{Example:}}$ $x_i = 5 \quad \delta = 0.1$

$$x_{i+1} = x_i - \frac{\delta\, x_i\, f(x_i)}{f(x_i + \delta x_i) - f(x_i)}$$

$$= 5 - \frac{(0.1)(5)\, f(5)}{f(\overset{5.5}{5 + (0.1)(5)}) - f(5)}$$

* Condition: must be continuous

$$\varepsilon_a = \left| \frac{\text{new value} - \text{old value}}{\text{new value}} \right| \times 100$$

Ex. $x^{3.5} = 80 \qquad x_0 = 3.5 \qquad \delta = 0.01$

$\hookrightarrow f(x) = x^{3.5} - 80$

① state function

② state x_0, δ

$$x_1 = x_0 - \frac{\delta\, x_0\, f(x_0)}{f(x_0 + \delta x_0) - f(x_0)}$$

$$x_1 = 3.5 - \frac{(0.01)(3.5)(f(3.5))}{f((3.5) + (0.01)(3.5)) - f(3.5)}$$

$x_i + \delta x_i = 3.5 + 0.035$

$x_i = 3.5 \qquad \delta x_i = 0.035 \qquad f(x_i) = 0.2$

$f(3.5) = 0.217 \qquad f(x_i + \delta x_i) = f(3.5 + 0.035)$

$f(3.535) = (3.535)^{3.5} - 80 = 4.536$

$$x_2 = 3.498 - \frac{(0.03498)(0.05147)}{2.88847 - 0.05147} = 3.49737$$

$$\Rightarrow \quad X_3 = \frac{\delta(X_2)\, f(X_2)}{f(X_2 + \delta X_2) - f(X_2)} = 3.497357278$$

[6] Fixed point iteration

$$f(x) = 0$$

$$\Rightarrow g(x) - x = 0$$

$$\Rightarrow x_{n+1} = g(x_n)$$

ex Solve $f(x) = -x^2 + e^{2x} + 4$

using $x_0 = 0$ $\varepsilon = 0.03$

الحل

$$-x^2 + e^{2x} + 4 = 0$$

$$x^2 = 4 + e^{2x}$$

$$x = \sqrt{4 + e^{2x}}$$

$$X_{n+1} = \sqrt{4 + e^{2x_n}}$$

الخطوة الأولى

$x_0 = 0$

$$X_1 = \sqrt{4 + e^{2(0)}} = \sqrt{5} = 2.2360$$

$$X_1 = 2.2360$$

$E = |X_1 - X_0|$

التكرار الثاني $X_2 = \sqrt{4 + e^{2(2.2360)}}$

$$\boxed{X_2 = 9.5672}$$

$E = |X_2 - X_1| =$

التكرار الثالث $X_3 = \sqrt{4 + e^{2(9.5672)}}$

$$\boxed{X_3 = ----}$$

$E = ---$

وهكذا

=

الخطأ يزيد

error increasing !!

Divergent !!

* Try another choice

$$\boxed{X = \frac{1}{2} \ln (X^2+4)} \quad X_0 = 0$$

التكرار الأول $X_1 = 0.69315 \longrightarrow E = 0.69315$

التكرار الثاني $X_2 = 0.74986 \longrightarrow E = 0.05671$

التكرار الثالث $X_3 = 0.7589 \longrightarrow E = 0.009052 < \varepsilon$ stop

Convergent *

7 Müller Method

نستخدم نقطة عند الشوط الأول

given (x_1 , x_2) $\Rightarrow$ $x_0 = \dfrac{x_1 + x_2}{2}$

رتبها تصاعدياً

1

$$x_2 < x_0 < x_1$$

الصغرى   الوسطى   الكبرى

2

$$h_1 = x_1 - x_0$$
$$h_2 = x_0 - x_2$$
$$\gamma = \frac{h_2}{h_1}$$

3

$$\alpha = \frac{f(x_2) + \gamma f(x_1) - (1+\gamma) f(x_0)}{\gamma(1+\gamma)\, h_1^2}$$

$$b = \frac{f(x_1) - f(x_0) - a h_1^2}{h_1}$$

$$c = f(x_0)$$

4 الجذر Root

$$x_r = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

اذا كانت (b) موجبة نستخدم الموجب
اذا كانت (b) سالبة نستخدم السالب

$\boxed{5}$

Case ① if $X_r > X_0 \Rightarrow$

X_0	X_r	X_1

Case ② if $X_r < X_0 \Rightarrow$

X_0	X_r	X_2

حيث يتم الدخول بهذه القيم ⇐
إلى الشوط الثاني

ويتم إعادة ترتيب هذه القيم
الثلاثة من جديد تصاعدياً

وإعادة تسميتها من جديد
بحيث أن الكبيرة X_1
والوسط X_0
والصغرى X_2

ex Use Müller To Solve

$$f(x) = x^3 + 4x^2 - 10$$

initial guess $[1, 2]$

استخدام ثلاثة اشواط
Do Three loops
(أو $\varepsilon = 1 \times 10^{-3}$)

الحل

١ الشوط الأول

$X_1 = 2$ $\Rightarrow$ $X_0 = \frac{2+1}{2} = 1.5$

$X_2 = 1$

① $1 < 1.5 < 2$

$\downarrow X_2$ $\quad \downarrow X_0$ $\quad \downarrow X_1$

② $h_1 = 2 - 1.5 = 0.5$ $\Rightarrow$ $\gamma = \frac{0.5}{0.5} = 1$

$h_2 = 1.5 - 1 = 0.5$

③ $f(X_2) = f(1) = -5$

$f(X_0) = f(1.5) = 2.375$

$f(X_1) = f(2) = 14$

عوض

$a = 8.5$

$b = 19$

$c = 2.375$

$$X_r = 1.5 - \frac{2(2.375)}{19 + \sqrt{19^2 - 4(8.5)(2.375)}}$$

$$\Rightarrow X_r = 1.3671$$

$f(X_r) = ----$

error $=$ $> \varepsilon$

استمر
Cont.

Case # 2 $\Rightarrow$ $X_r < X_0$

رتب تصاعدياً

إعادة التسمية من جديد

أول إشي

x_0	x_r	x_2
1.5	1.3671	1
X_1	X_0	X_2

1. $X_1 = 1.5$ $\quad X_2 = 1$ $\quad X_0 = 1.3671$

2. $h_1 = 1.5 - 1.3671 = 0.1329$ $\quad \Rightarrow \gamma = \frac{0.3671}{0.1329} = 2.7622$

   $h_2 = 1.3671 - 1 = 0.3671$

3. $f(X_2) = f(1) = -5$

   $f(X_0) = f(1.3671) = 0.03088$

   $f(X_1) = f(1.5) = 2.375$

عوض $a = 7.8671$

$b = 16.5924$

$c = 0.03088$

$\boxed{4}$ Root $X_r = 1.3652$

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<u>error</u> $f(X_r)$ ----- $\nless \varepsilon$

stop

Proof: ~~Proof Müller:~~

$$f(x) = a(x-x_0)^2 + b(x-x_0) + c$$

$h_1 = x_1 - x_0$

$h_2 = x_0 - x_2$

let:- $\gamma = \frac{h_2}{h_1}$

If $x = x_1$

$$* \ f(x_1) = a(x_1-x_0)^2 + b(x_1-x_0) + c$$

$$\boxed{f(x_1) = a h_1^2 + b h_1 + c} \quad \text{———} \ (1)$$

If $x = x_2$

$$* \ f(x_2) = a(x_2-x_0)^2 + b(x_2-x_0) + c$$

$$\boxed{f(x_2) = a h_2^2 - b h_2 + c} \quad \text{———} \ (2)$$

$$f(x_0) = a(0) + b(0) + c$$

$$\boxed{f(x_0) = c} \quad \text{———} \ (3)$$

معادلة رقم 1 ومعادلة رقم 2

$$-\frac{h_2^2}{h_1^2} \Big/ \quad a h_1^2 + b h_1 = f(x_1) - c \qquad -(1)$$

$$a h_2^2 - b h_2 = f(x_2) - c \qquad -(2)$$

$$-a h_2^2 - b\frac{h_2^2}{h_1} = -\left[f(x_1) - c\right]\frac{h_2^2}{h_1^2}$$

الأولى $\Rightarrow \quad -a \cancel{h_2^2} - b\,\gamma\, h_2 = -\gamma^2\left[f(x_1) - c\right]$

$$= -\gamma^2 f(x_1) + \gamma^2 c$$

الثانية $\quad +\cancel{a} h_2^2 - b h_2 = f(x_2) - c$

$$-b h_2\left[1 + \gamma\right] = -\gamma^2 f(x_1) + f(x_2) + c(\gamma^2 - 1)$$

$$b = \frac{\gamma^2 f(x_1) - f(x_2) - c(\gamma^2 - 1)}{h_2\left[1 + \gamma\right]}$$

$$X_0 - X_r = \left(\frac{-b \mp \sqrt{b^2-4ac}}{2a}\right) * \left(\frac{-b - \sqrt{b^2-4ac}}{-b - \sqrt{b^2-4ac}}\right)$$

$$X_0 - X_r = \frac{\cancel{b^2} - \cancel{b^2} + \overset{2}{\cancel{4}}ac}{\cancel{2a}\left(-b - \sqrt{b^2-4ac}\right)}$$

$$X_0 - X_r = \frac{2c}{-b - \sqrt{b^2-4ac}}$$

$$\Rightarrow \quad X_r = X_0 - \frac{2c}{-b \mp \sqrt{b^2-4ac}}$$

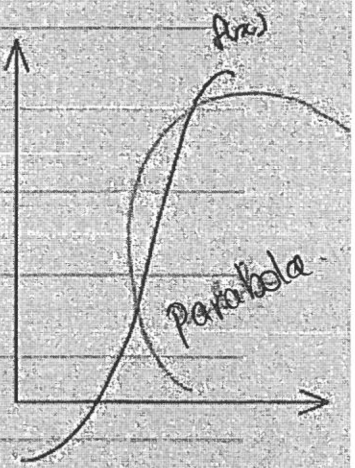
Müller Method

$$\cancel{x_3 = x_2 - \frac{\delta (x_2) f(x_2)}{f(x_2+\delta x_2) - f(x_2)} = 3.497351218}$$

الإثبات

Proof :-

Müller's Numerical Method



- a numeral root locating method
- Newton Raphson method
  Projected a line onto x-axis
- Parabolic - a curve drawn using a second degree polynomial.

$f_2(x) = ax^2 + bx + c$ a, b, c - constant

* To find the roots, we need 3 intersecting points.
  ∴ we need 3 starting points (initial guesses)

$f(x) \longrightarrow$ true function - being approximated

$f_2(x) \longrightarrow$ Approximating function - the tool.

$(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))$

$f_2(x) = a(x - x_2)^2 + b(x - x_2) + c$

$f_2(x_0) = a(x_0 - x_2)^2 + b(x_0 - x_2) + c$

$$f_2(x_2) = a(\cancel{x_2 - x_2})^2 + b(\cancel{x_2 - x_2}) + c = f(x_2)$$

$$\therefore \quad c = f(x_2)$$

$$a(x_1 - x_2)^2 + b(x_1 + x_2)^2 + f(x_2) = f(x_1)$$

$$\frac{(x_1 - x_2)}{(x_1 - x_2)}\left[a(x_1 - x_2) + b\right) = \frac{f(x_1) - f(x_2)}{(x_1 - x_2)}$$

$$b = \frac{f(x_1) - f(x_2)}{(x_1 - x_2)} + a(x_1 - x_2)$$

$$f(x_0) = a(x_0 - x_2)^2 + \left[\frac{f(x_1) - f(x_2)}{x_1 - x_2} - a(x_0 - x_2)\right](x_0 - x_2) + f(x_2)$$

$$\left[\frac{f(x_1) - f(x_2)}{x_1 - x_2} - a(x_0 - x_2)\right] = \frac{f(x_0) - f(x_2)}{(x_0 - x_2)}$$

$$h_0 = x_1 - x_0 \qquad h_1 = x_2 - x_1$$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} \qquad \delta_1 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0} \qquad b = a h_1 + \delta_1 \qquad c = f(x_2)$$

$$x_3 - x_2 = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}} \Rightarrow x_3 = x_2 - \frac{2c}{(b \pm \sqrt{b^2 - 4ac})}$$

when larger $x_3 = x_2$ is closer

Müller's Method

Step 1

$$x_0 \rightarrow f(x_0)$$
$$x_1 \rightarrow f(x_1)$$
$$x_2 \rightarrow f(x_2)$$

Step 2

$$h_0 = x_1 - x_0$$
$$h_1 = x_2 - x_1$$

Step 3

$$\delta_0 = \frac{f(x_1) - f(x_0)}{h_0}$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{h_1}$$

Step 4

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0}$$

$$b = a h_1 + \delta_1$$

$$c = f(x_2)$$

Step 5

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

Same sign as b

Modified Newton-Raphson Method
(Second-derivative)

Multiple Roots (Modified Newton-Raphson)

$$x_{i+1} = x_i - \frac{f(x_i)\, f'(x_i)}{[f'(x_i)]^2 - f(x_i)\, f''(x_i)}$$

Modified Newton-Raphson Method
(Second-derivative)

Newton's Method

Suppose that $f \in C^2[a, b]$. Let $p_0 \in [a, b]$ be an approximation to p such that $f'(p_0) \neq 0$ and $|p - p_0|$ is "small." Consider the first Taylor polynomial for $f(x)$ expanded about p_0 and evaluated at $x = p$.

$$f(p) = f(p_0) + (p - p_0)f'(p_0) + \frac{(p - p_0)^2}{2} f''(\xi(p)),$$

where $\xi(p)$ lies between p and p_0. Since $f(p) = 0$, this equation gives

$$0 = f(p_0) + (p - p_0)f'(p_0) + \frac{(p - p_0)^2}{2} f''(\xi(p)).$$

Newton's method is derived by assuming that since $|p - p_0|$ is small, the term involving $(p - p_0)^2$ is much smaller, so

$$0 \approx f(p_0) + (p - p_0)f'(p_0).$$

Solving for p gives

$$p \approx p_0 - \frac{f(p_0)}{f'(p_0)} \equiv p_1.$$

This sets the stage for Newton's method, which starts with an initial approximation p_0 and generates the sequence $\{p_n\}_{n=0}^{\infty}$, by

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}, \quad \text{for } n \geq 1.$$

Newton-Raphson Method
(First-derivative)

Ex8 Factorize

$$F(x) = (x^3-8)(x^2-5x+6)(x^2-4)$$
$$= (x-2)(x^2+2x+4)(x-2)(x-3)(x-2)(x+2)$$
$$= (x-2)^3(x+2)(x-3)(x^2+2x+4)$$

Def 8
if $F(x) = (x-a)^m g(x)$, where $g(a) \neq 0$
then the Root $x=a$ is of multiplicity (m)

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$$F(x) = (x-2)^3 g(x) \text{ , where } g(x) = (x+2)(x-3)(x^2+2x+4)$$

Multiple Root

IF $F(x) = (x-a)^m g(x)$ where $g(x) \neq 0$

$$F'(x) = m(x-a)^{m-1} g(x) + (x-a)^m g'(x)$$

مشتقة الضرب = مشتقة الاول x الثاني + الاول x مشتقة الثاني

Let $H(x) = \dfrac{F(x)}{F'(x)}$ that $F'(a) = 0$

$$* \; H(x) = \frac{F(x)}{F'(x)} = \frac{(x-a)^m g(x)}{(x-a)^{m-1} * m * g(x) + (x-a)^m g'(x)}$$

عامل $(x-a)^{m-1}$ مشترك بنطلع

$$= \frac{\cancel{(x-a)^{m}} \; g(x)}{\cancel{(x-a)^{m-1}} \left(m g(x) + (x-a) g'(x) \right)}$$

$$= \frac{(x-a)\, g(x)}{m g(x) + (x-a) g'(x)}$$

* Apply Newton Raphson Formula on Hx

$$x_{i+1} = x_i - \frac{Hx}{H'x} = x_i - \frac{F(x) / F'(x)}{\dfrac{F'(x_i)^2 - F(x_i) F''(x_i)}{F'(x_i)^2}}$$

Modified Newton Raphson $x_{i+1} = x_i - \dfrac{F(x_i)\, F'(x_i)}{F'(x_i)^2 - F(x_i) F''(x_i)}$

Ex8 Use N.R Method & Modified N.R Method to locate the Root of $F(x) = x^3 + 4x^2 - 10$

do three iteration with $X_0 = 1.5$

- N.R. Method $\rightarrow x_{i+1} = x_i - \dfrac{x_i^3 + 4x_i^2 - 10}{3x_i^2 + 8x_i}$

- M. N.R. Method $\rightarrow x_{i+1} = x_i - \dfrac{(x_i^3 + 4x_i^2 - 10)(3x_i^2 + 8x_i)}{(3x_i^3 + 8x_i)^2 - (x_i^3 + 4x_i^2 - 10)(6x_i - 8)}$

i	N.R	M.N.R
1	1.458333	1.411764706
2	1.436607143	1.414211438
3	1.425497619	1.414213562