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Chapter Five

Interpolation and Curve Fitting

- Direct Interpolation Method
- Lagrangian Interpolation Method
- Newton's Divided Difference Method
- Neville's Interpolation Method
- Gergory Interpolation Method
- Hermite's Interpolation Method
- Least Square Interpolation Method/ Linear Curve Fitting
- Least Square Interpolation Method/ Parabolic Curve Fitting
- Least Square Interpolation Method/ Polynomial Curve Fitting
- Least Square Interpolation Method/ Linearization Technique
- Least Square Interpolation Method/ General-Type Curve Fitting

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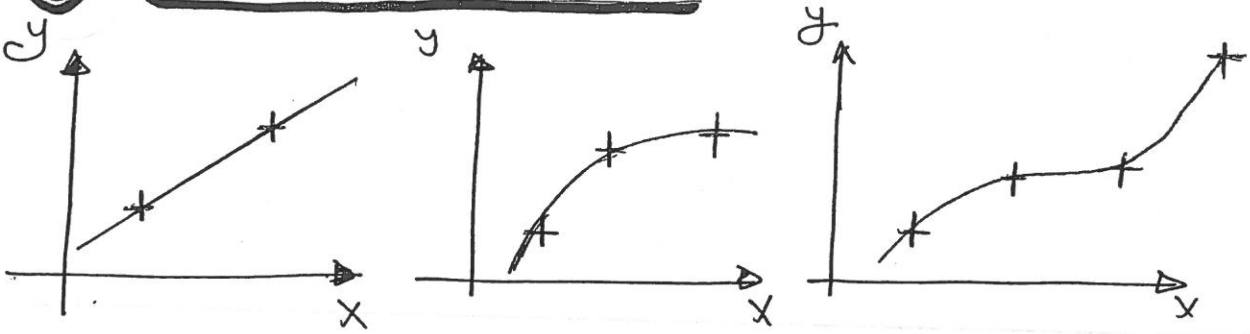
- Direct Method for Interpolation
- Lagrangian Interpolation
- Divided Difference
- Newton – Gergory Interpolation
- Hermite Polynomial Interpolation
- Least Square Method
- Linearization
- Neville's Interpolation

① Direct Interpolation



2 points	3 point	4 points
⇓ Linear	⇓ quadratic	⇓ Cubic
الدرجة الأولى	الدرجة الثانية	الدرجة الثالثة
$P_1(x) = a_0 + a_1 x$	$P_2(x) = a_0 + a_1 x + a_2 x^2$	$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

∴ درجة الاقتران تساوي عدد النقاط — ناقص واحد

Poly. degree= No. of data points - 1

Example Use the direct interpolation method

① to find the interpolating fuction $P(x)$

② Find the Value when $x = 4.2$?

③ when $x = 0$?

X	$y = f(x)$
3.2	22
2.7	17.8
1.0	14.2
4.8	38.3

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Sol.

(1)

عدد النقاط = ٤ نقاط (4 points)

∴ الاقتران من الدرجة الثالثة

$$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$P(3.2) = 22 \rightarrow a_0 + (3.2)a_1 + (3.2)^2 a_2 + (3.2)^3 a_3 = 22$ —— (1)

$P(2.7) = 17.8 \rightarrow a_0 + (2.7)a_1 + (2.7)^2 a_2 + (2.7)^3 a_3 = 17.8$ —— (2)

$P(1) = 14.2 \rightarrow a_0 + a_1 + a_2 + a_3 = 14.2$ —— (3)

$P(4.8) = 38.3 \rightarrow a_0 + (4.8)(a_1) + (4.8)^2 a_2 + (4.8)^3 a_3 = 38.3$ —— (4)

تكونت أربع معادلات بأربع مجاهيل هم

$$\begin{bmatrix} 1 & 3.2 & (3.2)^2 & (3.2)^3 \\ 1 & 2.7 & (2.7)^2 & (2.7)^3 \\ 1 & 1 & 1 & 1 \\ 1 & 4.8 & (4.8)^2 & (4.8)^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 17.8 \\ 14.2 \\ 38.3 \end{bmatrix}$$

Solve

$a_0 = 24.35$, $a_1 = -16.12$

$a_2 = 6.50$, $a_3 = -0.53$

the interpolating function is :

$$\boxed{P_3(X) = 24.35 - 16.12X + 6.5X^2 - 0.53X^3}$$

② Find the value when $X = 4.2$

$$\Rightarrow P(4.2) = 24.35 - 16.12(4.2) + 6.5(4.2)^2 - 0.53(4.2)^3$$

$$= 32.2$$

③ Find when $(X = 0)$:-

$$\Rightarrow P(0) = 24.35$$

* * * * * * *

Example Use Direct Method :

X	1	3	7
y	2.1	20	95

Sol. عدد النقاط = ٣ نقاط

درجة الاقتران = ٢ (الدرجة الثانية)

$$P_2(x) = a_0 + a_1 x + a_2 x^2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 7 & 49 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 20 \\ 95 \end{bmatrix}$$

find a_0, a_1, a_2

* * * * *

(2) Lagrangian Interpolation

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* Find the lagrangian poly. For the data points

x_0	x_1	x_2	x_3
f_0	f_1	f_2	f_3

عدد النقاط = ٤ ⇐ درجة الاقتران = ٣

$$P_3(X) = \frac{(X-X_1)(X-X_2)(X-X_3)}{(X_0-X_1)(X_0-X_2)(X_0-X_3)} \cdot f_0$$

$$+ \frac{(X-X_0)(X-X_2)(X-X_3)}{(X_1-X_0)(X_1-X_2)(X_1-X_3)} \cdot f_1$$

$$+ \frac{(X-X_0)(X-X_1)(X-X_3)}{(X_2-X_0)(X_2-X_1)(X_2-X_3)} \cdot f_2$$

$$+ \frac{(X-X_0)(X-X_1)(X-X_2)}{(X_3-X_0)(X_3-X_1)(X_3-X_2)} \cdot f_3$$

$$P_2(X) = \frac{(X-X_1)(X-X_2)}{(X_0-X_1)(X_0-X_2)} \cdot f_0$$

$$+ \frac{(X-X_0)(X-X_2)}{(X_1-X_0)(X_1-X_2)} \cdot f_1$$

$$+ \frac{(X-X_0)(X-X_1)}{(X_2-X_0)(X_2-X_1)} \cdot f_2$$

$$P_1(X) = \frac{X-X_1}{X_0-X_1} \cdot f_0 + \frac{X-X_0}{X_1-X_0} \cdot f_1$$

Example Use Lagrangian interpolation to solve for the value $X=5$

X	3.2	2.7	1	4.8
$f(X)$	22	17.8	14.2	38.3

Sol.

عدد النقاط = ٤ نقاط

إقتران درجة ثالثة

$$P_3(X) = \frac{(X-2.7)(X-1)(X-4.8)}{(3.2-2.7)(3.2-1)(3.2-4.8)}(22)$$

$$+ \frac{(X-3.2)(X-1)(X-4.8)}{(2.7-3.2)(2.7-1)(2.7-4.8)}(17.8)$$

$$+ \frac{(X-3.2)(X-2.7)(X-4.8)}{(1-3.2)(1-2.7)(1-4.8)}(14.2)$$

$$+ \frac{(X-3.2)(X-2.7)(X-1)}{(4.8-3.2)(4.8-2.7)(4.8-1)}(38.3)$$

عوّض مكان $X=5$

$$\Rightarrow P_3(5) = 40.25$$

Lagrangian Interpolation

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x	x_0	x_1	x_2	x_3
f	f_0	f_1	f_2	f_3

٤ نقاط
اقتران من الدرجة الثالثة

$$P(x) = \overbrace{\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}}^{L_0}\cdot f_0 + \overbrace{\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}}^{L_1}\cdot f_1 + \overbrace{\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}}^{L_2}\cdot f_2 + \overbrace{\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}}^{L_3}\cdot f_3$$

$$P(x) = \underbrace{L_0}_{\text{zero lagrange}} f_0 + \underbrace{L_1}_{\text{1st lagrange}} f_1 + \underbrace{L_2}_{\text{2nd lagrange}} f_2 + \underbrace{L_3}_{\text{3rd lagrange}} f_3$$

x	x_0	x_1	x_2
f	f_0	f_1	f_2

٣ نقاط
اقتران من الدرجة الثانية

$$P(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\cdot f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\cdot f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\cdot f_2$$

①

$$P(x) = \frac{(x-2.7)(x-1)(x-4.8)}{(3.2-2.7)(3.2-1)(3.2-4.8)} \cdot (22)$$

$$+ \frac{(x-3.2)(x-1)(x-4.8)}{(2.7-3.2)(2.7-1)(2.7-4.8)} \cdot (17.8)$$

$$+ \frac{(x-3.2)(x-2.7)(x-4.8)}{(1-3.2)(1-2.7)(1-4.8)} \cdot (14.2)$$

$$+ \frac{(x-3.2)(x-2.7)(x-1)}{(4.8-3.2)(4.8-2.7)(4.8-1)} \cdot (38.3)$$

② $x = 5$

$\Rightarrow P(5) = ? + ? + ? + ?$

$\Rightarrow P(5) = 40.2$

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سؤال آخر

Given:

ex] (0,0) (0.5, α) (1,3) (2,2)

if the Coeff Cubic Term 6

إذا كان معامل الحد التكعيبي في

معادلة لاجرانج Lagrange هو 7

أوجد قيمة α ؟

$\Rightarrow$ Find the value of α ??

الحل

x	0	0.5	1	2
$f(x)$	0	α	3	2

* The Coefficient of The Cubic Term is :-

$$\frac{\alpha}{(0.5-0)(0.5-1)(0.5-2)} + \frac{3}{(1-0)(1-0.5)(1-2)}$$

$$+ \frac{2}{(2-0)(2-0.5)(2-1)} = 6$$

$$\alpha = 4.25$$

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3 Newton's Method (Divided difference tables)

x	x_0	x_1	x_2	x_3	x_4
f	f_0	f_1	f_2	f_3	f_4

$$
\begin{aligned}
P(x) = a_0 &+ a_1(x-x_0) \\
&+ a_2(x-x_0)(x-x_1) \\
&+ a_3(x-x_0)(x-x_1)(x-x_2) \\
&+ a_4(x-x_0)(x-x_1)(x-x_2)(x-x_3) \\
&+ \cancel{a_5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}
\end{aligned}
$$

x	f	1st difference ∇_1	2nd diff ∇_2	3rd diff ∇_3	4th diff. ∇_4
x_0	f_0 (a_0)				
		$A=\dfrac{f_1-f_0}{x_1-x_0}$ (a_1)			
x_1	f_1		$E=\dfrac{B-A}{x_2-x_0}$ (a_2)		
		$B=\dfrac{f_2-f_1}{x_2-x_1}$		$H=\dfrac{F-E}{x_3-x_0}$ (a_3)	
x_2	f_2		$F=\dfrac{C-B}{x_3-x_1}$		$\dfrac{J-H}{x_4-x_0}$ (a_4)
		$C=\dfrac{f_3-f_2}{x_3-x_2}$		$J=\dfrac{G-F}{x_4-x_1}$	
x_3	f_3		$G=\dfrac{D-C}{x_4-x_2}$		
		$D=\dfrac{f_4-f_3}{x_4-x_3}$			
x_4	f_4				

** ميزة أن قيم الـ a هي عبارة عن رؤوس الأعمدة

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** Newton's polynomial Divided Difference

$$
\boxed{
\begin{aligned}
f(x) = {} & a_0 + a_1(x-x_0) \\
& + a_2(x-x_0)(x-x_1) \\
& + a_3(x-x_0)(x-x_1)(x-x_2) \\
& + a_4(x-x_0)(x-x_1)(x-x_2)(x-x_3) \\
& + a_5 \ldots \\
& + a_6 \ldots
\end{aligned}
}
$$

لمعرفة قيم المعاملات ⇐

$a_0, a_1, a_2, a_3, a_4, a_5, \ldots$

Table لا بد من تشكيل جدول

حيث تكون هذه المعاملات عبارة عن

رؤوس الأعمدة في هذا الجدول

X	f	Δ_1	Δ_2	Δ_3	Δ_4
X_0	$f[X_0]$ f_0				
		$f[X_0,X_1]$			
X_1	$f[X_1]$ f_1		$f[X_0,X_1,X_2]$		
		$f[X_1,X_2]$		$f[X_0,X_1,X_2,X_3]$	
X_2	$f[X_2]$ f_2		$f[X_1,X_2,X_3]$		$f[X_0,X_1,X_2,X_3,X_4]$
		$f[X_2,X_3]$		$f[X_1,X_2,X_3,X_4]$	
X_3	$f[X_3]$ f_3		$f[X_2,X_3,X_4]$		
		$f[X_3,X_4]$			
X_4	$f[X_4]$ f_4				

$\Rightarrow$ Find $f[X_1,X_2,X_3] = \dfrac{f[X_2,X_3]-f[X_1,X_2]}{X_3-X_1}$

ex)

① Use Newton (Divided difference) To solve

X	0	5	7	8	10
f	0	2	−1	−2	20

(0,0), (5,2), (7,−1), (8,−2), (10,20)

② Then find the value when $\underline{\underline{X = 6}}$

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Sol.

X	f	∇_1	∇_2	∇_3	∇_4
x_0 0	0 $f[x_0]$ → a_0				
		$\frac{2-0}{5-0}$ = 0.4 $f[x_0,x_1]$ → a_1			
x_1 5	2 $f[x_1]$		$\frac{-1.5-0.4}{7-0}$ = −0.271 $f[x_0,x_1,x_2]$ → a_2		
		$\frac{-1-2}{7-5}$ = −1.5 $f[x_1,x_2]$		$\frac{0.167-(-0.271)}{8-0}$ = 0.054 → a_3	
x_2 7	−1 $f[x_2]$		$\frac{-1-(-1.5)}{8-5}$ = 0.167 $f[x_1,x_2,x_3]$		$\frac{0.767-0.054}{10-0}$ = −0.07123 $f[x_0,x_1,x_2,x_3,x_4]$ → a_4
		$\frac{-2-(-1)}{8-7}$ = −1 $f[x_2,x_3]$		$\frac{4-0.167}{10-5}$ = 0.767 $f[x_1,x_2,x_3,x_4]$	
x_3 8	−2 $f[x_3]$		$\frac{11-(-1)}{10-7}$ = 4		
		$\frac{20-(-2)}{10-8}$ = 11			
x_4 10	20 $f[x_4]$				

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$a_0 = 0$

$a_1 = 0.4$

$a_2 = -0.271$

$a_3 = 0.054$

$a_4 = -0.7123$

$$
\begin{aligned}
P(x) = \; & 0 + 0.4(x-0) \\
& - 0.271(x-0)(x-5) \\
& + 0.054(x-0)(x-5)(x-7) \\
& - 0.7123(x-0)(x-5)(x-7)(x-8)
\end{aligned}
$$

$$
\Rightarrow \boxed{\begin{aligned} P(x) = \; & 0.4x - 0.271x(x-5) + 0.054x(x-5)(x-7) \\ & - 0.7123x(x-5)(x-7)(x-8) \end{aligned}}
$$

2

$x = 6 \Rightarrow$

$p(6) = 1.29736$

Σ

Find : $f[x_1, x_2, x_3] = 0.167$

Find : $\Delta_2 f(x_1, x_3) = 0.167$

Find : $f[x_0, x_1, x_2, x_3] = 0.054$

Find : $f[x_3, x_4] = 11$

لاحظوا

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{-1 - 2}{7 - 5} = -1.5$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{-2 - -1}{8 - 7} = -1$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{-1 - -1.5}{8 - 5} = 0.167$$

$$f[x_1, x_2, x_3] = \frac{\frac{f[x_3] - f[x_2]}{x_3 - x_2} - \frac{f[x_2] - f[x_1]}{x_2 - x_1}}{x_3 - x_1}$$

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ex) Use Newton Divided Diff

① To fill the Table $\boxed{2.}$

② Then find $f(3)$?

Given

	X	f	Δ_1	Δ_2	Δ_3	Δ_4
x0	0	$f[x_0]$ 30				
			$F[x_0,x_1]$ 6.25			
x1	1.6	$f[x_1]$ $\boxed{A}$		$F[x_0,x_1,x_2]$ −0.3213		
			$F[x_1,x_2]$ $\boxed{E}$ 4.7078		$F[x_0,x_1,x_2,x_3]$ 0.0203	
x2	4.8	$F[x_2]$ $\boxed{B}$		$F[x_1,x_2,x_3]$ $\boxed{H}$ 0.1406		$f[x_0,x_1,x_2,x_3,x_4]$ 0.0433
			$F[x_2,x_3]$ $\boxed{F}$		$f[x_1,x_2,x_3,x_4]$ $\boxed{M}$ 0.4533	
x3	8.9	$f[x_3]$ $\boxed{C}$		$F[x_2,x_3,x_4]$ $\boxed{K}$ 3.6671		
			$F[x_3,x_4]$ $\boxed{G}$			
x4	10	$f[x_4]$ $\boxed{D}$				

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$$0.0433 = \frac{M - 0.0203}{10 - 0}$$

$$\Rightarrow \quad M = 0.4533$$

$$0.0207 = \frac{H + 0.3213}{8.9 - 0}$$

$$\Rightarrow \quad H = -0.1406$$

$$0.4533 = \frac{K - -0.1406}{10 - 1.6}$$

$$\Rightarrow \quad K = 3.6671$$

$$-0.3213 = \frac{E - 6.25}{4.8 - 0}$$

$$E = 4.7078$$

$$-0.1406 = \frac{F - 4.7078}{8.9 - 1.6}$$

$$\Rightarrow \quad F = \text{---} \checkmark$$

$$\vdots \qquad \text{etc.}$$

$$f(x) = 30 + 6.25(x-0) - 0.3213(x-0)(x-1.6) + 0.0203(x-0)(x-1.6)(x-4.8) + 0.0433(x-0)(x-1.6)(x-4.8)(x-8.9)$$

$$\Rightarrow x = 3$$

$$\Rightarrow f(3) = \checkmark$$

ex) Given

x	f	Δ_1	Δ_2	Δ_3	Δ_4
0	□				
		□			
1.6	□		□		
		□		□	
4.8	□		□		0.0433
		□		0.4533	
8.9	□		3.6671		
		22.7272			
10	95				

① * Fill The Table ?

② * Then Find $f(5)$?

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* 4 Neville's Method

x	x_0	x_1	x_2	x_3	x_4
f	f_0	f_1	f_2	f_3	f_4

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X	f	$\Delta 1$	$\Delta 2$	$\Delta 3$	$\Delta 4$
x_0	f_0				
		A			
x_1	f_1		E		
		B		M	
x_2	f_2		F		K
		C		H	
x_3	f_3		G		
		D			
x_4	f_4				

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لاحظ ان ال K هو الجواب النهائي للمسألة

Final Answer

where :-

$$A = \frac{f_0(x-x_1) - f_1(x-x_0)}{x_0 - x_1}$$

$$B = \frac{f_1(x-x_2) - f_2(x-x_1)}{x_1 - x_2}$$

$$C = \frac{f_2(x-x_3) - f_3(x-x_2)}{x_2 - x_3}$$

$$D = \frac{f_3(x-x_4) - f_4(x-x_3)}{x_3 - x_4}$$

$$E = \frac{A(x-x_2) - B(x-x_0)}{x_0 - x_2}$$

$$F = \frac{B(x-x_3) - C(x-x_1)}{x_1 - x_3}$$

$$G = \frac{C(x-x_4) - D(x-x_2)}{x_2 - x_4}$$

$$M = \frac{E(x-x_3) - F(x-x_0)}{x_0 - x_3}$$

$$H = \frac{F(x-x_4) - G(x-x_1)}{x_1 - x_4}$$

$$K = \frac{M(x-x_4) - H(x-x_0)}{x_0 - x_4} \Rightarrow$$

Final Answer

 # Example: Use Neville's Interpolation To find the value for $x = 2.3$?

$\Rightarrow$ Find $f(2.3)$?

x	0	1	2	3
f(x)	1	5	7	9

(0,1)
(1,5)
(2,7)
(3,9)

Sol.

x	f	Δ_1	Δ_2	Δ_3
0	1			
		A 10.2		
1	5		D 7.21	
		B 7.6		F 7.509
2	7		E 7.6	
		C 7.6		
3	9			

$$A = \frac{1(2.3-1) - 5(2.3-0)}{0-1} = 10.2$$

$$B = \frac{5(2.3-2) - 7(2.3-1)}{1-2} = 7.60$$

$$C = \frac{7(2.3-3) - 9(2.3-2)}{2-3} = 7.60$$

$$D = \frac{10.2(2.3-2) - 7.6(2.3-0)}{0-2} = 7.21$$

$$E = \frac{7.6(2.3-3) - 7.6(2.3-1)}{1-3} = 7.60$$

$$F = \frac{7.21(2.3-3) - 7.6(2.3-0)}{0-3} = \underline{\underline{7.509}}$$

$$\therefore \; f(2.3) = 7.509$$

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(h = Constant)

(5) Newton Gergory Interpolation

نستخدم هذه الطريقة عندما تكون

الفروقات ما بين قيم الـ X

$$h \equiv (\Delta X = \text{Constant})$$ ثابتة ⇐

$$s = \frac{x - x_0}{\Delta x}$$

$$f(x) = a_0 + a_1 s + \frac{a_2}{2!} s(s-1) + \frac{a_3}{3!} s(s-1)(s-2) + \frac{a_4}{4!} s(s-1)(s-2)(s-3) + \frac{a_5}{5!} s(s-1)(s-2)(s-3)(s-4) + \cdots$$

* Ex: Use Gergory TO find $f(0.73)$?

x	f	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
0	$0 \to a_0$					
		$0.203 \to a_1$				
			$0.017 \to a_2$			
0.2	0.203			$0.024 \to a_3$		
		0.220			$0.020 \to a_4$	
			$0.261 - 0.220$ 0.041			$0.032 \to a_5$
0.4	0.423			0.044		
		0.261			0.052	
			0.085			
0.6	0.684			$0.181 - 0.85$ 0.096		
		$1.030 - 0.684$ 0.346				
			0.181			
0.8	1.030					
		0.527				
1	1.557					

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Just subtraction without need to divide over x.

(Centerd about x0= 0)

$a_0 = 0$

$a_1 = 0.203$

$a_2 = 0.017$

$a_3 = 0.024$

$a_4 = 0.020$

$a_5 = 0.032$

$$S = \frac{X - 0}{0.2} = \frac{X}{0.2}$$

$$\boxed{S = 5X}$$

$$f(x) = 0 + 0.203\,(5x)$$
$$+ \frac{0.017}{2}[5x][5x-1]$$
$$+ \frac{0.024}{6}[5x][5x-1][5x-2]$$
$$+ \frac{0.020}{24}[5x][5x-1][5x-2][5x-3]$$

sub. x=0.73

$$+ \frac{0.032}{120}[5x][5x-1][5x-2][5x-3][5x-4]$$

(Centerd about x0= 0.4)

$$\left.\begin{aligned} a_0 &= 0.423 \\ a_1 &= 0.261 \\ a_2 &= 0.085 \\ a_3 &= 0.096 \end{aligned}\right\} \quad \begin{aligned} S &= \frac{X - 0.4}{0.2} \\ S &= 5\left[X - 0.4\right] \\ &\boxed{S = 5X - 2} \end{aligned}$$

$$f(x) = 0.423 + 0.261\left[5X-2\right] + \frac{0.085}{2}\left[5X-2\right]\left[5X-3\right] + \frac{0.096}{6}\left[5X-2\right]\left[5X-3\right]\left[5X-4\right]$$

Now Sub. x=0.73 (more accurate)

⑥ "Hermite Interpolation"

* يتم استخدام Hermite عندما تكون صورة الاقتران معروفة وميل الاقتران (المشتقة الاولى) معروفة أيضاً

* لا بد عند بناء الجدول من مضاعفة عدد النقاط (Doubling points)

	X	x_0	x_1	x_2
Image	$f(x)$	f_0	f_1	f_2
Slope	$f'(x)$	f_0'	f_1'	f_2'

$$
\begin{aligned}
f(x) = \; & a_0 + a_1(x-x_0) \\
& + a_2(x-x_0)^2 \\
& + a_3(x-x_0)^2(x-x_1) \\
& + a_4(x-x_0)^2(x-x_1)^2 \\
& + a_5(x-x_0)^2(x-x_1)^2(x-x_2) \\
& + a_6(x-x_0)^2(x-x_1)^2(x-x_2)^2 \\
& + a_7(x-x_0)^2(x-x_1)^2(x-x_2)^2(x-x_3) \\
& + a_8 \ldots\ldots
\end{aligned}
$$

** حيث أن a_0 ، a_1 ، a_2 ، a_3 ، ...... هي عبارة عن رؤوس الأعمدة في الجدول.

X	f	f'	Δ_1	Δ_2	Δ_3	Δ_4
x_0	$f_0 \to a_0$					
		$f_0' \to a_1$				
			$\to a_2$			
x_0	f_0					
		$\frac{f_1-f_0}{x_1-x_0}$		$\to a_3$		
x_1	f_1				$\to a_4$	
		f_1'				$\to a_5$
x_1	f_1					
		$\frac{f_2-f_1}{x_2-x_1}$				
x_2	f_2					
		f_2'				
x_2	f_2					

** حيث يتم ايجاد القيم مكان هذه النقاط كما هو سابقاً

في طريقة Newton Divided Diff. تماماً

ex) Use Hermite Interpolation To Find $f(2.5)$?

x	1	3	4
$f(x)$	2	1	2
slope: $f'(x)$	1	-1	0

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X	f	f'	$\Delta 1$	$\Delta 2$	$\Delta 3$	$\Delta 4$
x_0 1	$\boxed{2}$ a_0					
		$\boxed{1}$ a_1				
x_0 1	2		$\frac{-0.5-1}{3-1}$ a_2 $\boxed{-0.75}$			
		-0.5		$\frac{-0.25+0.75}{3-1}$ a_3 $\boxed{0.25}$		
x_1 3	1		$\frac{-1+0.5}{3-1}$ -0.25		$\frac{0.75-0.25}{4-1}$ a_4 $\boxed{0.167}$	
		-1		$\frac{2+0.25}{4-1}$ 0.75		$\frac{-1.25-0.167}{4-1}$ a_5 $\boxed{-0.472}$
x_1 3	1		$\frac{1+1}{4-3}$ 2		$\frac{-3-0.75}{4-1}$ -1.25	
		$\frac{2-1}{4-3}$ 1		$\frac{-1-2}{4-3}$ -3		
x_2 4	2		$\frac{0-1}{4-3}$ -1			
		0				
x_2 4	2					

$a_0 = 2$

$a_1 = 1$

$a_2 = -0.75$

$a_3 = 0.25$

$a_4 = 0.167$

$a_5 = -0.472$

$X_0 = 1$

$X_1 = 3$

$X_2 = 4$

$$\boxed{\begin{aligned} f(x) = \; & 2 + (x-1) \\ & - 0.75(x-1)^2 \\ & + 0.25(x-1)^2(x-3) \\ & + 0.167(x-1)^2(x-3)^2 \\ & - 0.472(x-1)^2(x-3)^2(x-4) \end{aligned}}$$

Hermite Polynomial Function.

@ $x = 2.5 \Rightarrow$

$$f(2.5) = 2 + (2.5-1) - 0.75(2.5-1)^2 + \ldots\ldots$$

$$= \ldots\ldots$$

Ex) Using Hermite Poly :-

x	1	3	5
f	2	3	1
f'	1	-1	-2
f''	2	-3	1

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⇛ Find ① $f(4)$ ⇒ Solution ⇒ use

x	1	3	5
f	2	3	1
f'	1	-1	-2

② $f'(4)$ ⇒ Solution ⇒ use

x	1	3	5
f'	1	-1	-2
f''	2	-3	1

7 Least Square Curve Fitting

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هندسة ميكانيكية

1. Linear $y = AX + B$

2. parabola $y = Ax^2$

3. Any Polynomial of Any degree
$y = Ax^3$
$y = Ax^2 + BX + C$
$y = AX^3 + BX^2 + CX + D$

4. exponential $y = Ce^{AX}$
Exponential Model

5. Rational function $y = \frac{1}{AX+B}$, $y = \frac{C}{AX+B}$
Saturation-Growth Rate Eqn.

6. Trigonometric Function $y = A\sin x$
$y = A\sin x + B\cos x$

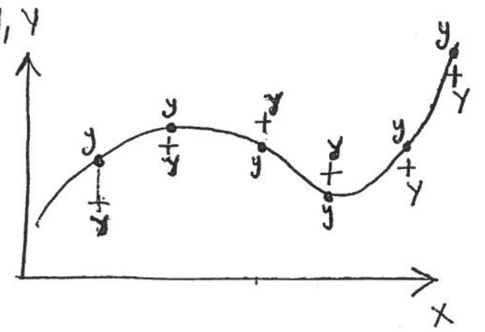
Error $\rightarrow$ $$E = \sum_{i=1}^{N} \left[y(x) - Y \right]^2$$

N ≡ عدد النقاط

$Y \equiv$ Exact point.
$y \equiv$ Approx point.

Minimizing Error $\rightarrow$

$$\frac{\partial E}{\partial A} = 0 \qquad \frac{\partial E}{\partial B} = 0 \qquad \frac{\partial E}{\partial C} = 0$$



حيث يتم حل هذه المعادلات الثلاثة باستخدام الآلة الحاسبة

أو باستخدام طريقة Cramer's Rule

ex) Find The Best line To fit the data points :- 215

(-1,10) (0,9) (1,7) (2,5) (3,4) (4,3) (5,0) (6,-1)

* No. of points → $N = 8$ points:

derive equations:

* $y = Ax + B$

* $E = \sum_{i=1}^{N} [Ax + B - y]^2$

* $\frac{\partial E}{\partial A} = \sum_{i=1}^{N} \cancel{2}[Ax + B - y][x] = 0$

→ $\boxed{A \sum_{i=1}^{N} x^2 + B \sum_{i=1}^{N} x = \sum_{i=1}^{N} yx}$ —①

* $\frac{\partial E}{\partial B} = \sum_{i=1}^{N} \cancel{2}[Ax + B - y][1] = 0$

→ $A \sum_{i=1}^{N} x + B \sum_{i=1}^{N} 1 = \sum y$

→ $A \sum_{i=1}^{N} x + BN = \sum y$ —②

$$\begin{bmatrix} \sum x^2 & \sum x \\ \sum x & N \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sum yx \\ \sum y \end{bmatrix}$$

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	X	y	x^2	YX
	-1	10	1	-10
	0	9	0	0
	1	7	1	7
	2	5	4	10
	3	4	9	12
	4	3	16	12
	5	0	25	0
	6	-1	36	-6
Sum	20	37	92	25

Formula:-
Using law :-

$$* \ A = \frac{8(25) - 20(37)}{8(92) - (20)^2}$$

$$\Rightarrow A = -1.67$$

$$* \ B = \frac{92(37) - 20(25)}{8(92) - (20)^2} = 8.64$$

$$y = -1.67x + 8.64$$

Equations :-

$$A(92) + B(20) = 25$$

$$\Rightarrow 92A + 20B = 25 \quad \text{——(1)}$$

Also

$$20A + 8B = 37 \quad \text{——(2)}$$

using Mode 5/1

$$\Rightarrow \begin{matrix} A = -1.67 \\ B = 8.64 \end{matrix} \Rightarrow y = -1.67x + 8.64$$

* Using Cramer's Rule :-

$$D = \begin{bmatrix} \sum x^2 & \sum x \\ \sum x & N \end{bmatrix} = N\sum x^2 - (\sum x)^2$$

$$D_1 = \begin{bmatrix} \sum yx & \sum x \\ \sum y & N \end{bmatrix} = N\sum yx - \sum x \sum y$$

$$D_2 = \begin{bmatrix} \sum x^2 & \sum yx \\ \sum x & \sum y \end{bmatrix} = \sum x^2 \sum y - \sum x \sum yx$$

$$A = \frac{N\sum yx - \sum x \sum y}{N\sum x^2 - (\sum x)^2}$$

$$y = Ax + B$$

$$B = \frac{\sum x^2 \sum y - \sum x \sum yx}{N\sum x^2 - (\sum x)^2}$$

OR: $B = \overline{y} - A\,\overline{x}$

where: $\overline{x} = \dfrac{\sum x}{N}$ and $\overline{y} = \dfrac{\sum y}{N}$

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** Find The error at $(x = 3)$

$y = -1.67(3) + 8.64 = 3.63$ [Approx. Value]

But $Y = 4$ [Exact Value From Table]

الخطأ المطلق

Absolute Error $= |Y - y| = |4 - 3.63| = 0.37$

الخطأ النسبي

Relative Error $= \dfrac{|Y - y|}{|Y|} = \dfrac{|4 - 3.63|}{|4|} = 0.0925 = 9.25\%$

نقطة غير موجودة في الجدول

** Find The Approximate value at $(x = 3.5)$?

$y = -1.67(3.5) + 8.64$

$\Rightarrow \quad y = 2.795$

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ex) Find The Best Parabola To fit the data points :-

X	-1	0	1	2	3
y	10	9	7	5	4

SoL

* No. of points **N = 5** points

* $y = Ax^2$ (Parabola)

derive equation:

* $$E = \sum_{i=1}^{N} [Ax^2 - y]^2$$

* $$\frac{\partial E}{\partial A} = \sum_{i=1}^{N} \cancel{2}[Ax^2 - y][x^2] = 0$$

$$\Rightarrow \quad A \sum_{i=1}^{N} x^4 = \sum_{i=1}^{N} yx^2$$

$$\boxed{A = \frac{\sum yx^2}{\sum x^4}}$$

X	y	x^2	x^4	yx^2
-1	10	1	1	10
0	9	0	0	0
1	7	1	1	7
2	5	4	16	20
3	4	9	81	36
*	*	*	99	73

sum

* $A = \frac{73}{99} = 0.737$

$y = Ax^2$

$y = 0.737\,x^2$

ex Find the Power Fit $f(x) = Kx^3$ 220

For $(2, 5.1)$ $(2.3, 7.5)$ $(2.6, 10.6)$ $(2.9, 14.4)$
$(3.2, 19)$

الحل

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* $N = 5$ points
* $y = Kx^3$

derive equation:

* $$E = \sum_{i=1}^{N} \left[Kx^3 - Y\right]^2$$

* $$\frac{\partial E}{\partial K} = \sum_{i=1}^{N} \cancel{2}\left[Kx^3 - Y\right]\left[x^3\right] = 0$$

* $$\Rightarrow \quad K\sum x^6 = \sum Yx^3$$

$$\Rightarrow \boxed{K = \frac{\sum Yx^3}{\sum x^6}}$$

x	y	x^3	x^6	Yx^3
⋮	⋮	⋮	⋮	⋮

ex) Find The power Fit For the quadratic polynomial $y = Ax^2 + BX + C$

For the data points :

(-1,10) (0,9) (1,7) (2,5) (3,8)

SOL

* $N = 5$ points

* $y = Ax^2 + BX + C$ [quadratic]

derive equations: * $E = \sum_{i=1}^{N} [Ax^2 + BX + C - Y]^2$

* $\frac{\partial E}{\partial A} = 0$

$$\Rightarrow A\sum X^4 + B\sum X^3 + C\sum X^2 = \sum YX^2 \quad — ①$$

* $\frac{\partial E}{\partial B} = 0$

$$\Rightarrow A\sum X^3 + B\sum X^2 + C\sum X = \sum YX \quad — ②$$

* $\frac{\partial E}{\partial C} = 0$

$$\Rightarrow A\sum X^2 + B\sum X + CN = \sum Y \quad — ③$$

$$\begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & N \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum Yx^2 \\ \sum YX \\ \sum Y \end{bmatrix}$$

x	y	x^2	x^3	x^4	yx	yx^2
−1	10	1	−1	1	−10	10
0	9	0	0	0	0	0
1	7	1	1	1	7	7
2	5	4	8	16	10	20
3	8	9	27	81	24	72
5	39	15	35	99	31	109

* $99A + 35B + 15C = 109$ —— (1)
* $35A + 15B + 5C = 31$ —— (2)
* $15A + 5B + 5C = 39$ —— (3)

$\rightarrow$ $A = 0.571$

$B = -1.943$

$C = 8.029$

$$y = 0.571\,x^2 - 1.943\,x + 8.029$$

Linearization

8 (Linearization) ⇛ least Square Method

$$y = Ce^{Ax} \rightarrow \ln y = \ln C + Ax$$

$$z = Ax + B \quad \text{(Linear)}$$

- Linearize:
  find A & C.

$$B = \ln C \Rightarrow \boxed{C = e^{B}}$$

$$\boxed{A = A}$$

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x	y	$z = \ln y$	x^2	zx
−1	6.62	1.89010	1	−1.89010
0	3.94	1.37118	0	0
1	2.17	0.77473	1	0.77473
2	1.35	0.3001	4	0.6002
3	0.89	−0.1165	9	−0.3496
Sum 5		4.21961	15	−0.8648

derive equations:

$$A \sum_{i=1}^{N} x^2 + B \sum_{i=1}^{N} x = \sum_{i=1}^{N} zx$$

$$\Rightarrow 15A + 5B = -0.8648 \quad \text{--- (1)}$$

$$A \sum_{i=1}^{N} x + BN = \sum z$$

$$5A + 5B = 4.21961 \quad \text{--- (2)}$$

$$\rightarrow \boxed{A = -0.508441}$$

$$B = 1.352363$$

$$\rightarrow C = e^{B} = e^{1.352363}$$

$$\rightarrow \boxed{C = 3.8666}$$

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$$y = C e^{AX}$$

$$y = 3.8666\, e^{-0.508441\, x}$$

- Linearize:
  find A & B.

$$\boxed{y = \frac{C}{Ax+B}}$$

$$\rightarrow \frac{1}{y} = \frac{Ax+B}{C}$$

$$\rightarrow \frac{1}{y} = \frac{A}{C}x + \frac{B}{C}$$

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$$\rightarrow \boxed{Z = K_1 x + K_2} \longrightarrow \text{Linear}$$

$$\rightarrow K_1 = \frac{A}{C} \rightarrow \boxed{A = K_1 C}$$

$$\text{and } K_2 = \frac{B}{C} \rightarrow \boxed{B = K_2 C}$$

ex)

X	y	$Z = \frac{1}{y}$	X^2	ZX
0	0.16667	6	0	0
2	0.07143	14	4	28
2.5	0.0625	16	6.25	40
4	0.04545	22	16	88
5	0.03846	26	25	130
Sum 13.5		84	51.25	286

derive equations:

$$K_1 \sum_{i=1}^{N} x^2 + K_2 \sum_{i=1}^{N} x = \sum_{i=1}^{N} z\,x$$

$$51.25\,K_1 + 13.5\,K_2 = 286 \quad \text{—(1)}$$

$$K_1 \sum_{i=1}^{N} x + K_2 N = \sum_{i=1}^{N} z$$

$$13.5\,K_1 + 5\,K_2 = 84 \quad \text{— (2)}$$

$$\Rightarrow \quad K_1 = 4$$

$$K_2 = 6$$

$$\Rightarrow \quad A = K_1 C \rightarrow A = (4)(1) \rightarrow A = 4$$

$$\Rightarrow \quad B = K_2 C \rightarrow B = (6)(1) \rightarrow B = 6$$

$$y = \frac{C}{Ax + B} = \frac{1}{4x + 6}$$

$$y = \frac{1}{4x + 6}$$

- Linearize:
  find A & C.

$$y = C\,10^{Ax}$$

$$\log y = \log C + Ax$$

$$z = Ax + B$$

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$$B = \log C \rightarrow \boxed{C = 10^{B}}$$

$$\rightarrow \boxed{A = A}$$

x	y	$z = \log y$	x^2	zx
⋮	⋮	⋮	⋮	⋮

- Linearize:
  find A & C.

$$y = C\,2^{Ax}$$

$$\ln y = \ln C + (Ax)\ln 2$$

$$z = K_1 x + K_2$$

* $K_2 = \ln C \rightarrow C = e^{K_2}$

* $K_1 = A \ln 2 \rightarrow A = \dfrac{K_1}{\ln 2}$

x	y	$z = \ln y$	x^2	xz
⋮	⋮	⋮	⋮	⋮

(Sinusoidal curve-fitting)

المثال $\quad y = A\sin x + B\cos x$ $\quad$ Given :-

x	y
⋮	⋮

derive equations:

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$$* \quad E = \sum_{i=1}^{N} \left[A\sin x + B\cos x - y\right]^2$$

$$* \quad \frac{\partial E}{\partial A} = \sum_{i=1}^{N} \cancel{2}\left[A\sin x + B\cos x - y\right]\left[\sin x\right] = 0$$

$$\rightarrow \boxed{A\sum \sin^2 x + B\sum \sin x\cos x = \sum y\sin x} \quad -①$$

$$* \quad \frac{\partial E}{\partial B} = \sum_{i=1}^{N} \cancel{2}\left[A\sin x + B\cos x - y\right]\left[\cos x\right] = 0$$

$$\rightarrow \boxed{A\sum \sin x\cos x + B\sum \cos^2 x = \sum y\cos x} \quad -②$$

x	y	$\sin x$	$\cos x$	$\sin^2 x$	$\cos^2 x$	$\sin x\cos x$	$y\sin x$	$y\cos x$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

"Change To Radians"

Dr. Hashem Alkhaldi

ex) Find the Best Curve (General-type) 229

$$y(x) = Ax^2 + \frac{B}{x} + C\ln x + D\cos x$$

Sol

$$E = \sum_{i=1}^{N}\left(Ax^2 + \frac{B}{x} + C\ln x + D\cos x - Y\right)^2$$

derive equations:

$$\frac{\partial E}{\partial A} = \sum \cancel{2}\left(Ax^2 + \frac{B}{x} + C\ln x + D\cos x - Y\right)\left(x^2\right) = 0$$

$$\Rightarrow A\sum x^4 + B\sum x + C\sum x^2\ln x + D\sum x^2\cos x = \sum Yx^2 \quad -①$$

$$\frac{\partial E}{\partial B} = \sum \cancel{2}\left(Ax^2 + \frac{B}{x} + C\ln x + D\cos x - Y\right)\left(\frac{1}{x}\right) = 0$$

$$\Rightarrow A\sum x + B\sum \frac{1}{x^2} + C\sum \frac{\ln x}{x} + D\sum \frac{\cos x}{x} = \sum \frac{Y}{x} \quad -②$$

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$$\frac{\partial E}{\partial C} = \sum_{i=1}^{N} \cancel{2}\left(Ax^2 + \frac{B}{x} + C\ln x + D\cos x - y\right)(\ln x) = 0$$

$$\rightarrow A\sum x^2 \ln x + B\sum \frac{\ln x}{x} + C\sum (\ln x)^2 + D\sum \cos x \ln x = \sum y \ln x \quad — (3)$$

$$\frac{\partial E}{\partial D} = \sum \cancel{2}\left(Ax^2 + \frac{B}{x} + C\ln x + D\cos x - y\right)(\cos x) = 0$$

$$\rightarrow A\sum x^2 \cos x + B\sum \frac{\cos x}{x} + C\sum \cos x \ln x + D\sum \cos^2 x = \sum y \cos x \quad — (4)$$

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$$\begin{bmatrix} \sum x^2 & \sum x & \sum x^2 \ln x & \sum x^2 \cos x \\ \sum x & \sum \frac{1}{x^2} & \sum \frac{\ln x}{x} & \sum \frac{\cos x}{x} \\ \sum x^2 \ln x & \sum \frac{\ln x}{x} & \sum (\ln x)^2 & \sum \cos x \ln x \\ \sum x^2 \cos x & \sum \frac{\cos x}{x} & \sum \cos x \ln x & \sum \cos^2 x \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} \sum y x^2 \\ \sum y/x \\ \sum y \ln x \\ \sum y \cos x \end{bmatrix}$$

Curve Fitting

Dr. Hashem Akhaldi

[1] linear Regression

$$y = ax + b$$

الخطأ
ERROR

$$e = \sum_{i=1}^{N} (y - Y)^2$$

القانون المهم

الاقتران

النقاط

$$e = \sum_{i=1}^{N} (ax + b - Y)^2$$

$$\frac{de}{da} = 0 \Rightarrow \sum 2(ax+b-Y)^1(x) = 0$$

$$\Rightarrow a\sum x^2 + b\sum x = \sum xY \quad \text{—— (1)}$$

$$\frac{de}{db} = 0 \Rightarrow \sum 2(ax+b-Y)^1(1) = 0$$

$$\Rightarrow a\sum x + bN = \sum Y \quad \text{—— (2)}$$

$$\begin{bmatrix} \sum x^2 & \sum x \\ \sum x & N \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum xY \\ \sum Y \end{bmatrix}$$

$$a\sum x^2 + b\sum x = \sum xy \quad \text{——} \ (1)$$

$$a\sum x + bN = \sum y \quad \text{——} \ (2)$$

$$\Rightarrow \begin{bmatrix} \sum x^2 & \sum x \\ \sum x & N \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum xy \\ \sum y \end{bmatrix}$$

OR:

$$bN + a\sum x = \sum y \quad \text{——} \ (1)$$

$$b\sum x + a\sum x^2 = \sum xy \quad \text{——} \ (2)$$

$$\Rightarrow \begin{bmatrix} N & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

OR:

$$a\sum x + bN = \sum y \quad \text{——} \ (1)$$

$$a\sum x^2 + b\sum x = \sum xy \quad \text{——} \ (2)$$

$$\begin{bmatrix} \sum x & N \\ \sum x^2 & \sum x \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

Cramer's Rule

Dr. Hashem Akhaldi

$$D = \begin{bmatrix} \sum x^2 & \sum x \\ \sum x & N \end{bmatrix} = N\sum x^2 - \left(\sum x\right)^2$$

$$D_1 = \begin{bmatrix} \sum xy & \sum x \\ \sum y & N \end{bmatrix} = N\sum xy - \sum x \sum y$$

$$D_2 = \begin{bmatrix} \sum x^2 & \sum xy \\ \sum x & \sum y \end{bmatrix} = \sum x^2 \sum y - \sum x \sum xy$$

$$a = \frac{D_1}{D} \Rightarrow a = \frac{N\sum xy - \sum x \sum y}{N\sum x^2 - \left(\sum x\right)^2}$$

$$b = \frac{D_2}{D} \Rightarrow b = \frac{\sum x^2 \sum y - \sum x \sum xy}{N\sum x^2 - \left(\sum x\right)^2}$$

OR $\quad b = \bar{y} - a\bar{x}$

where: $\quad \bar{x} = \frac{\sum x}{N} \qquad \bar{y} = \frac{\sum y}{N}$

Find the Best line to fit the

أفضل خط مستقيم

Data Points : (1,2) (2,4) (3,6) (4,7)
(5,9) (7,10)

1

$\boxed{N = 6}$ Number of points

2 then Find the Approx. image of X= 6.5

~~~~~~

Sol. :

	x	y	x^2	xy
	1	2	1	2
	2	4	4	8
	3	6	9	18
	4	7	16	28
	5	9	25	45
	7	10	49	70
SUM:	22	38	104	171
~~~~~~

$$a = \frac{(6)(171) - (22)(38)}{(6)(104) - (22)^2} = 1.357$$

$$b = \frac{(104)(38) - (22)(171)}{(6)(104) - (22)^2} = 1.357$$

أفضل خط مستقيم يمر من خلال النقاط السبعة هو

$$y = ax + b$$

The Best Line

$$y = 1.357X + 1.357$$

$$X = 6.5$$

$$y = 1.357(6.5) + 1.357$$

$$\Rightarrow y = 10.178$$

$$(6.5 , 10.178)$$

Relative

(3,6) أوجد مقدار الخطأ النسبي عند النقطة

$X = 3$

$y = 1.357(3) + 1.357$

$y = 5.428$ Approximate value

$$\text{ERROR} = \left| \frac{y_{True} - y_{App}}{y_{True}} \right|$$

$$= \left| \frac{6 - 5.428}{6} \right| * 100\%$$

$$= 0.0953$$

$$= 9.53\ \%$$

Dr. Hashem Akhaldi

2 polynomial Reg.

$$y = ax^2 + bx + c$$

Full Quadratic Regression

ERROR:
$$e = \sum_{i=1}^{N} \left(ax^2 + bx + c - y\right)^2$$

$$\frac{de}{da} = 0 \Rightarrow \sum \cancel{2}\left(ax^2 + bx + c - y\right)x^2 = 0$$

$$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum yx^2 \quad —(1)$$

$$\frac{de}{db} = 0 \Rightarrow \sum \cancel{2}\left(ax^2 + bx + c - y\right)x = 0$$

$$a\sum x^3 + b\sum x^2 + c\sum x = \sum yx \quad —(2)$$

$$\frac{de}{dc} = 0 \Rightarrow \sum \cancel{2}\left(ax^2 + bx + c - y\right)(1) = 0$$

$$a\sum x^2 + b\sum x + cN = \sum y \quad —(3)$$

$$\begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum yx^2 \\ \sum yx \\ \sum y \end{bmatrix}$$

ex)

$$y = ax^3 + bx^2 + cx + d$$

Full Cubic Regression

$$\begin{bmatrix} \sum x^6 & \sum x^5 & \sum x^4 & \sum x^3 \\ \sum x^5 & \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^4 & \sum x^3 & \sum x^2 & \sum x \\ \sum x^3 & \sum x^2 & \sum x & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \sum yx^3 \\ \sum yx^2 \\ \sum yx \\ \sum y \end{bmatrix}$$

****

ex)

$$y = ax^3 + bx \rightarrow \text{Find } a \text{ and } b?$$

$$e = \sum (ax^3 + bx - y)^2$$

$$\frac{de}{da} = 0 \rightarrow \sum 2(ax^3 + bx - y)(x^3) = 0$$

$$\rightarrow a\sum x^6 + b\sum x^4 = \sum yx^3 \quad -(1)$$

$$\frac{de}{db} = 0 \rightarrow \sum 2(ax^3 + bx - y)(x) = 0$$

$$a\sum x^4 + b\sum x^2 = \sum yx \quad -(2)$$

$$\begin{bmatrix} \sum x^6 & \sum x^4 \\ \sum x^4 & \sum x^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum yx^3 \\ \sum yx \end{bmatrix}$$

Cramer's Rule:

$$D = \begin{bmatrix} \sum x^6 & \sum x^4 \\ \sum x^4 & \sum x^2 \end{bmatrix} = \sum x^6 \sum x^2 - \left(\sum x^4\right)^2$$

$$D_1 = \begin{bmatrix} \sum yx^3 & \sum x^4 \\ \sum yx & \sum x^2 \end{bmatrix} = \sum x^2 \sum yx^3 - \sum x^4 \sum yx$$

$$D_2 = \begin{bmatrix} \sum x^6 & \sum yx^3 \\ \sum x^4 & \sum yx \end{bmatrix} = \sum x^6 \sum yx - \sum x^4 \sum yx^3$$

$$a = \frac{D_1}{D} \Rightarrow a = \frac{\sum x^2 \sum yx^3 - \sum x^4 \sum yx}{\sum x^6 \sum x^2 - \left(\sum x^4\right)^2}$$

$$b = \frac{D_2}{D} \Rightarrow b = \frac{\sum x^6 \sum yx - \sum x^4 \sum yx^3}{\sum x^6 \sum x^2 - \left(\sum x^4\right)^2}$$

ex) $y = KX^2$ Curve Fitting.

(Parabola)

$\Rightarrow$ Find K ?

Sol: $$e = \sum (Kx^2 - Y)^2$$

$$\frac{de}{dK} = 0 \Rightarrow \sum \cancel{2} (Kx^2 - Y)(x^2) = 0$$

$$\Rightarrow K \sum x^4 = \sum Y x^2 \quad \text{— (1)}$$

$$\Rightarrow K = \frac{\sum Y x^2}{\sum x^4}$$

****

ex) Find the best quadratic Function Pass through these data points

(0, 1) (1, 3) (2, 4) (3, 6) (4, 8) (5, 6) (8, -4)

Sol: $$y = ax^2 + bx + C$$

X	Y	X^2	X^3	X^4	YX	YX^2
0	1	0	0	0	0	0
1	3	1	0	1	3	3
2	4	4	8	16	8	16
3	6	9	27	81	18	54
4	8	16	64	256	32	128
5	6	25	125	625	30	150
8	-4	64	512	4096	-32	-256
23	24	119	736	5075	59	95

$N = 7$

$$\begin{bmatrix} 5075 & 736 & 119 \\ 736 & 119 & 23 \\ 119 & 23 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 95 \\ 59 \\ 24 \end{bmatrix}$$

$\rightarrow \quad a = -0.5 \quad b = 3.5 \quad c = 0.39$

The Best Curve $y = -0.5X^2 + 3.5X + 0.39$

*** Linear Multi-Variable Regression ***

$$z = ax + by + c$$

$$e = \sum (ax + by + c - z)^2$$

$$\frac{de}{da} = 0 \Rightarrow \sum 2(ax + by + c - z)(x) = 0$$

$$\Rightarrow a\sum x^2 + b\sum xy + c\sum x = \sum zx$$

$$\frac{de}{db} = 0 \Rightarrow \sum 2(ax + by + c - z)(y) = 0$$

$$a\sum xy + b\sum y^2 + c\sum y = \sum zy$$

$$\frac{de}{dc} = 0 \Rightarrow \sum 2(ax + by + c - z)(1) = 0$$

$$a\sum x + b\sum y + cN = \sum z$$

$$\begin{bmatrix} \sum x^2 & \sum xy & \sum x \\ \sum xy & \sum y^2 & \sum y \\ \sum x & \sum y & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum zx \\ \sum zy \\ \sum z \end{bmatrix}$$

given the 4 data points

(0, 1, 20) (1, 3, 24) (2, 4, 27) (3, 6, 30)

$$\boxed{N = 4}$$

x	y	z	x^2	y^2	xy	zx	zy
0	1	20					
1	3	24					
2	4	27					
3	6	30					

SUM ....

Exponential Reg.

$$y = \alpha e^{\beta X}$$

$$\ln y = \ln \alpha + \ln e^{\beta X}$$

$$\ln y = \ln \alpha + \beta X$$

$$\boxed{\ln y = \beta X + \ln \alpha}$$

$$y = a X + b$$

$$\overset{\beta}{\cancel{\alpha}} = \frac{N \sum X \ln y - \sum X \sum \ln y}{N \sum X^2 - \left(\sum X\right)^2}$$

$$\overset{\ln \alpha}{\cancel{b}} = \frac{\sum X^2 \sum \ln y - \sum X \sum X \ln y}{N \sum X^2 - \left(\sum X\right)^2}$$

X	y	ln y	X ln y	x^2
1	3	/	/	/
2	6	/	/	/
4	8			

.... etc.