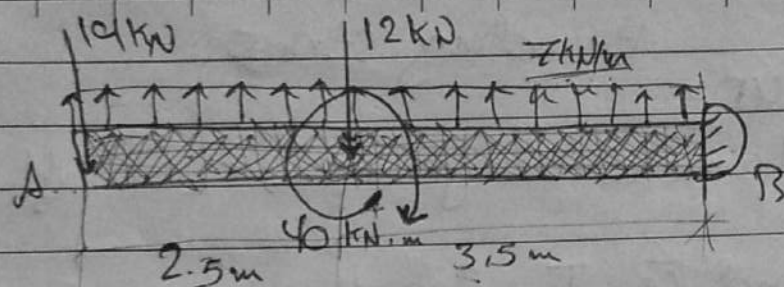
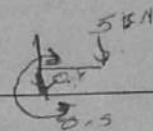


ملخص واسئلة مقترحة

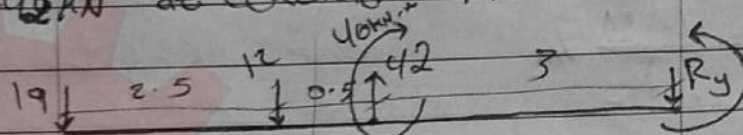
STRENGTH

اعداد يزن العقرباوي

(1)



$7 \times 6 = 42 \text{ kN}$ at centroid of force



$$\sum F_y = 0,$$

$$19 + 12 + 42 + R_y = 0 \Rightarrow R_y = 11 \text{ kN}$$

$$\sum M_B = 0,$$

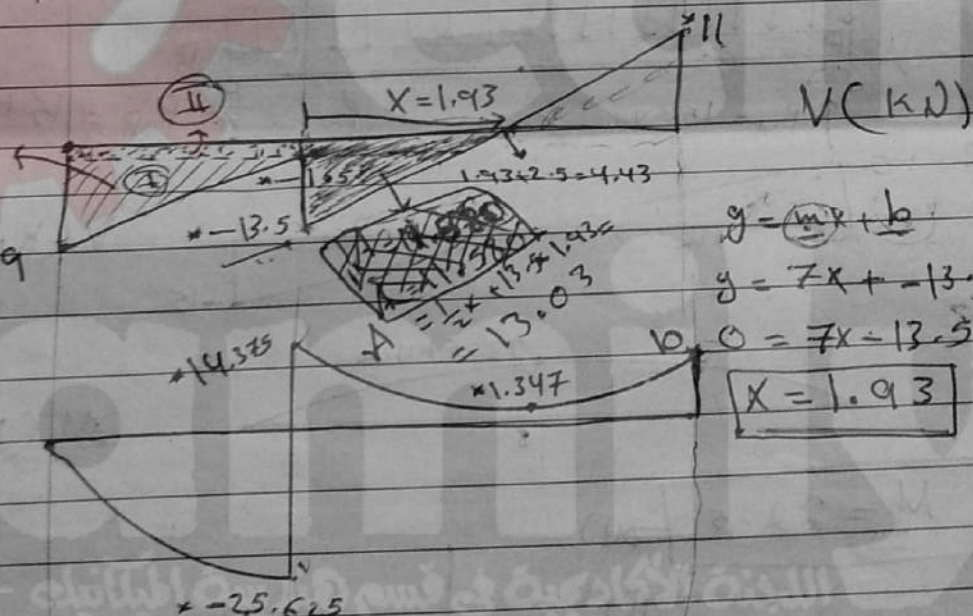
$$M_B = 42 \times 3 - 12 \times 3.5 - 19 \times 6 - 40 = 10 \text{ kN.m}$$

$$A = \text{I} + \text{II}$$

$$= \frac{1}{2} (2.5) \times (19 - 1.5)$$

$$+ 1.5 \times 2.5 \times 19$$

$$= 25.625$$



$$y = mx + b$$

$$y = 7x + 13.5$$

$$0 = 7x - 13.5$$

$$x = 1.93$$

(2)

272

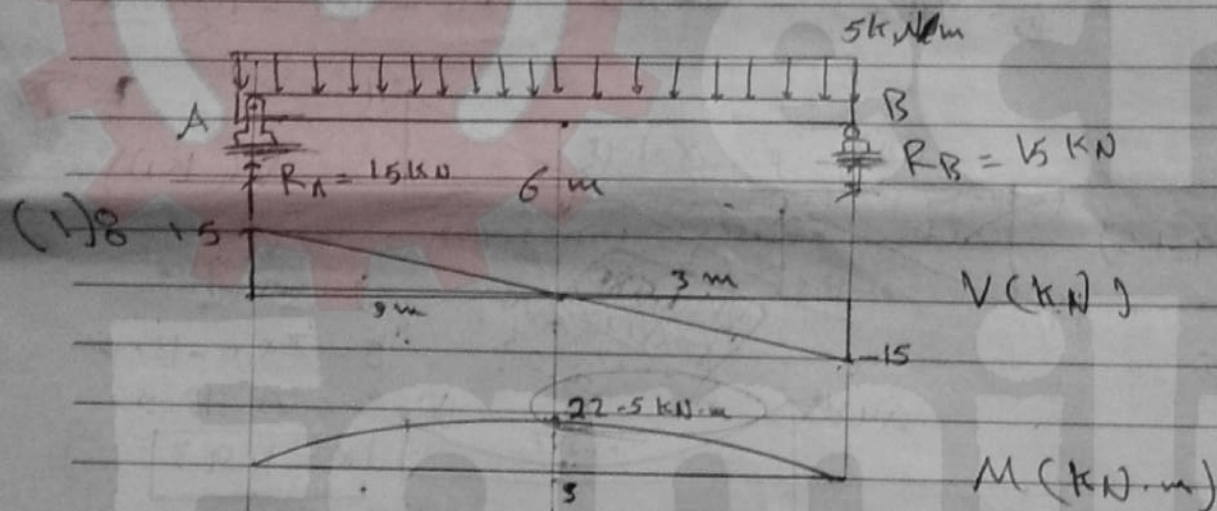
Bending stress

Procedure

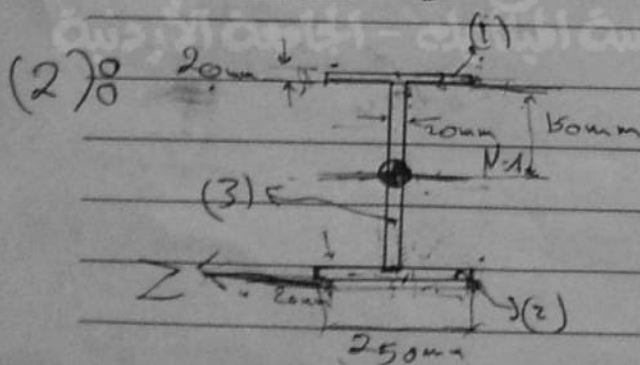
1 - Moment Diagram \rightarrow read $\frac{M}{I}$ 2 - Find \bar{y} , I for the cross section

3 - $\sigma_x = \frac{My}{I_z}$

determine the absolute maximum bending stress in the beam ~~and draw the shear stress distribution at this location~~



$$M = 22.5$$



Part	A (m ²)	\bar{y} (m)	$A\bar{y}$ (m ³)
(1)	5×10^{-3}	330×10^{-3}	1.65×10^{-1}
(2)	5×10^{-3}	10×10^{-3}	5×10^{-5}
(3)	6×10^{-3}	170×10^{-3}	1.02×10^{-1}
Σ	16×10^{-3}		2.72×10^{-1}

(cont.)

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2.72}{16} = 0.17 = 170 \text{ mm} \quad (\text{can be skipped})$$

(3)

Part	$A(m^2)$	\bar{y}	$A\bar{y}$	d_{cc} (m)	$A d_{cc}^2$ (m ²)	\bar{I}_c (m ⁴)	$\bar{I} = \bar{I}_c + A d_{cc}^2$ (m ⁴)
(1)	5×10^{-3}			0.16	1.28×10^{-4}	1.66×10^{-7}	1.28166×10^{-4}
(2)	5×10^{-3}			0.16	1.28×10^{-4}	1.66×10^{-7}	1.28166×10^{-4}
(3)	6×10^{-3}			0.0	0.0	4.5×10^{-5}	4.5×10^{-5}
Σ							$301.3 \times 10^{-6} \text{ m}^4$

* d_{cc} is distance from the centroid of the part (c) to the centroid of the whole cross-section (C)

* $\bar{I} = \bar{I}_c + A d_{cc}^2$

$\bar{y} = 170 \text{ mm}$ and $\bar{I} = 301.3 \times 10^{-6} \text{ m}^4 \Rightarrow (2)$

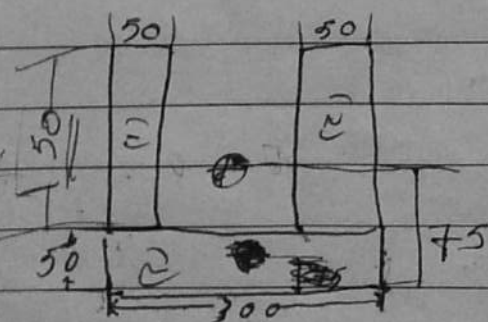
(3): $\sigma_{max} = \frac{M_{max} y}{I}$

* Here y is distance from the centroid of the whole cross section (C) to the point we want to find σ at.

* Since we want σ maximum y will be 170 mm to maximize σ

$\sigma_{max} = \frac{22.5 \times 10^3 \times 170 \times 10^{-3}}{301.3 \times 10^{-6}} \approx 12.7 \text{ MPa} \Rightarrow (3)$

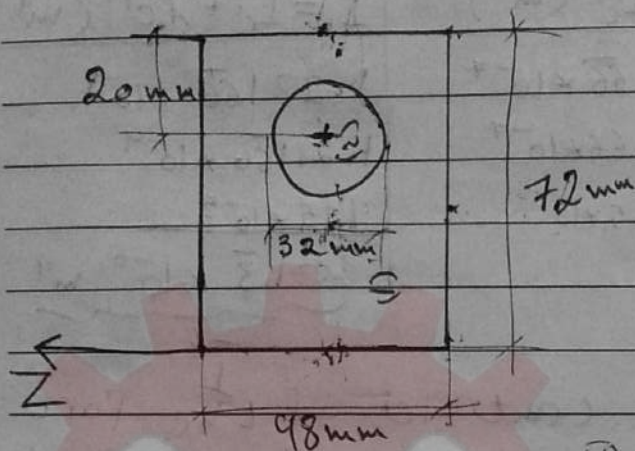
Finished



Part	A	\bar{y}	$A\bar{y}$
(1)	7500	125	937500
(2)	7500	125	937500
(3)	15000	25	375000
Σ	30000		2250000

$\bar{y} = \frac{2250000}{30000} = 75 \text{ mm}$

(4)



Find the moment capacity of this beam if the max. stress is 150 MPa tension and 100 MPa in compression and draw the ~~stress~~ stress distribution.

$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{48 \times 72 \times \frac{72}{2} - \frac{\pi}{4} \times (32)^2 \times (52)}{48 \times 72 - \frac{\pi}{4} \times (32)^2}$$

$$= \boxed{31.15 \text{ mm}}$$

I: ~~b // axis~~

b // axis about which the rotation will occur

$$I = \frac{48 \times 10^{-3} \times (72 \times 10^{-3})^3}{12} + (36 \times 10^{-3} - 31.15 \times 10^{-3})^2 \times 48 \times 10^{-3} \times 72 \times 10^{-3}$$

$$- \frac{\pi}{64} \times (32 \times 10^{-3})^4 - (40.853 \times 10^{-3} - 20 \times 10^{-3})^2 \times \frac{\pi}{4} \times (32 \times 10^{-3})^2$$

$$= 1.1733 \times 10^{-6} \text{ m}^4$$

$$\sigma = -\frac{M y}{I} \Rightarrow M = \frac{\sigma I}{y}$$

top: $y = 40.85 \text{ mm}$, $\sigma = 150 \text{ MPa}$
 $\Rightarrow M = 4.8 \text{ kN.m}$

bottom: $y = 31.15 \text{ mm}$, $\sigma = 100 \text{ MPa}$

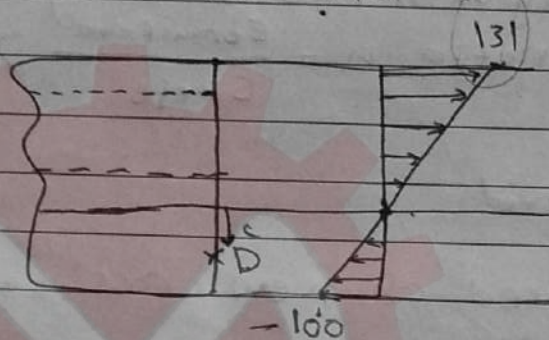
$$\Rightarrow M = \boxed{3.77 \text{ kN.m}}$$

(5)

So 3.77 controls

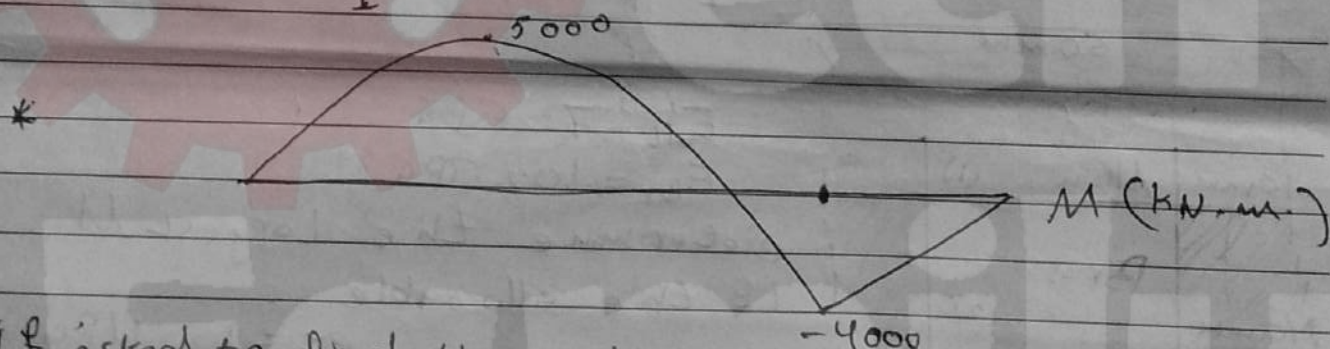
$$\sigma_{comp} = 100 \text{ MPa}$$

$$\sigma_{tens} = \frac{3.77 \times 10^3 \times 40.853 \times 10^3}{1.173 \times 10^{-6}} = 131 \text{ MPa}$$



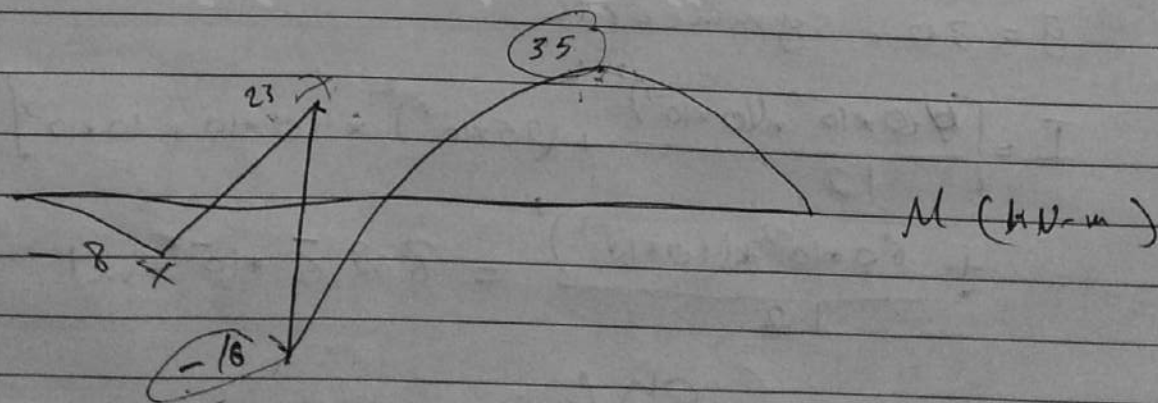
* ask at another point D for example

$$\sigma = \frac{Mc}{I}$$

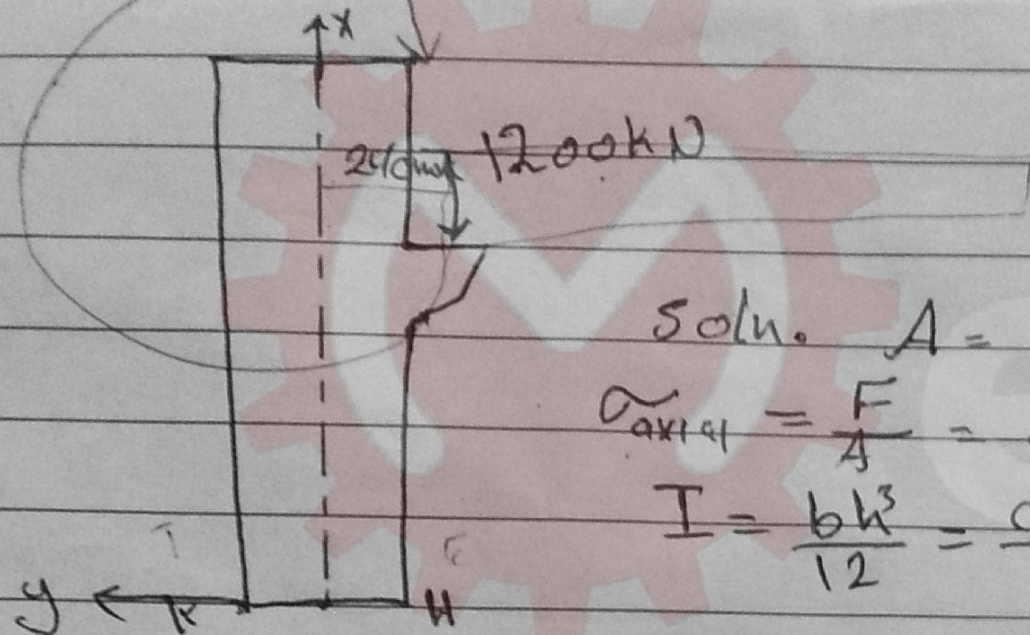


* if asked to find the maximum or you have to check both the 4000 and the 5000 Moments.

DONT Think!!



Eccentric Axial

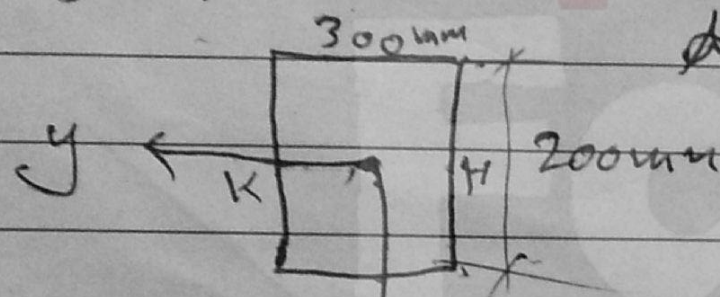
Find σ_K , σ_H 

Soln. $A = 0.06 \text{ m}^2$

$$\sigma_{\text{axial}} = \frac{F}{A} = 20 \text{ MPa} \quad [C]$$

$$I = \frac{bh^3}{12} = \frac{0.2 \times (0.3)^3}{12} = 4.5 \times 10^{-4} \text{ m}^4$$

$$M = 1200 \times 10^3 \times 240 \times 10^{-3} = 288 \text{ kNm}$$



$$\sigma_{\text{Bend}} = \frac{My}{I} = \frac{288 \times 10^3 \times 150 \times 10^{-3}}{4.5 \times 10^{-4}}$$

$$= 96 \text{ MPa}$$

$$\sigma_K = 96 - 20 = 76 \text{ MPa}$$

$$\sigma_H = 20 + 96 = 116 \text{ MPa}$$

Five Apple

MPa

Comments:

1- if two conditions were given:

(1) $\sigma = 150 \text{ MPa}$ [1]

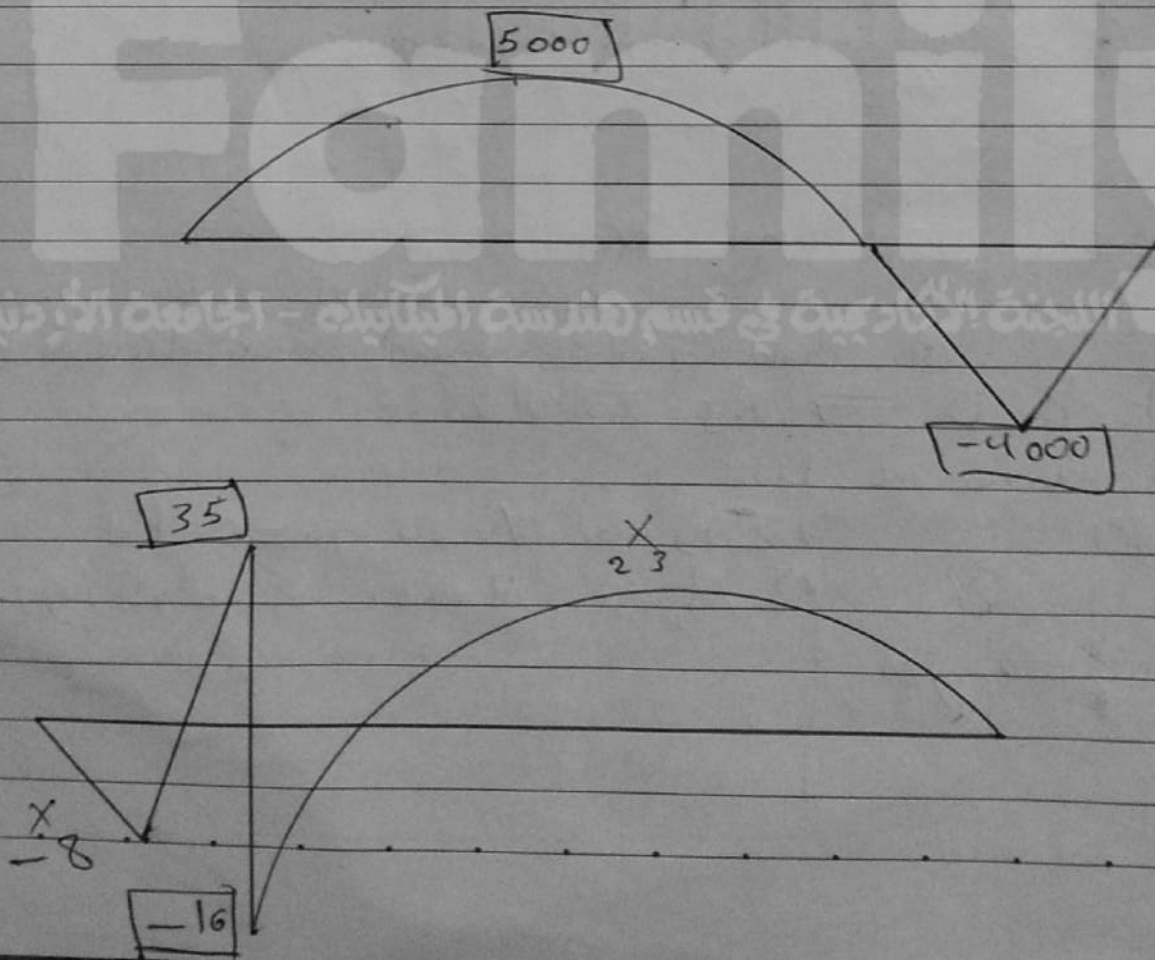
(2) $\sigma = 100 \text{ MPa}$ [C]

we solve for both of them then we take the safer (more safe) value

F, M, σ, T, d_i - we take smallest
 D, d_o , thickness - we take largest

2- if asked not for maximum but for specified point then find the Moment at that point using equations (not graphs.)

3- if asked for σ_{\max} then check for all extreme values that is (Don't think)



No transverse shear

Procedure

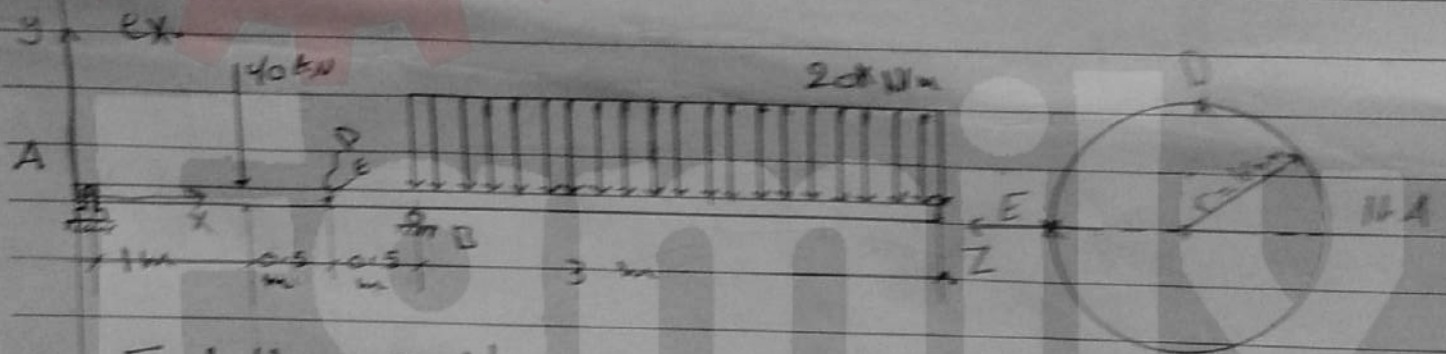
1. Shear force diagram \rightarrow read V
2. Find \bar{y}, I, Q, t
3. $\tau = \frac{VQ}{It}$

$$Q = A' \bar{y}'$$

A' is the Area from point of interest and Away from the centroidal axis.

\bar{y}' is the distance from the centroidal axis to the centroid of (A')

t is thickness (Parallel to the centroidal axis)



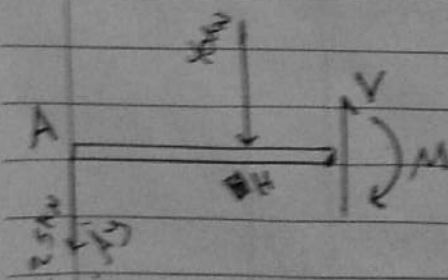
Find the ① Normal stress at Point D, E

② Shear stress at Point D, E

③ Draw normal stress and shear stress distribution

$$F.B.D \Rightarrow A_y = 25 \text{ kN} \downarrow$$

Section at D, E



$$\sum F_y = 0, V = 25 + 40 = 65 \text{ kN}$$

$$\sum M = 0, M = (25 \times 0.5) + (40 \times 1) = 57.5$$

No. _____

$$I = \frac{\pi}{2} r^4 = 1.02944 \times 10^{-7} \text{ m}^4$$

$$\bar{y} = 0.016 \text{ m}$$

Bending:

Point D:-

$$* y = c = 0.016$$

$$* \sigma = \frac{57.5 \times 10^3 \times 0.016}{1.02944 \times 10^{-7}} = -8.93 \text{ GPa}$$

[T]

Point E

$$* y = 0 \text{ (Neutral axis)}$$

$$* \sigma = 0$$

Shear:

Point D:-

$$* Q = 0$$

$$* \tau = 0$$

Point E:-

$$Q = A' y'$$

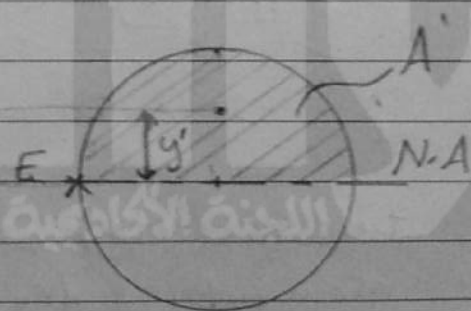
$$= \frac{\pi r^2}{2} \times \frac{4r}{5\pi}$$

$$= 2.731 \times 10^{-6} \text{ m}^3$$

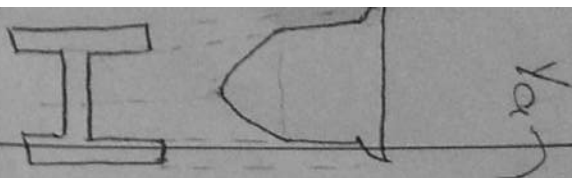
$$t = 0.016 \times 2 = 0.032$$

$$\tau_E = \frac{VQ}{It} = \frac{65 \times 10^3 \times 2.731 \times 10^{-6}}{1.02944 \times 10^{-7} \times 0.032}$$

$$= 5.38 \text{ MPa}$$



** note for an I-beam

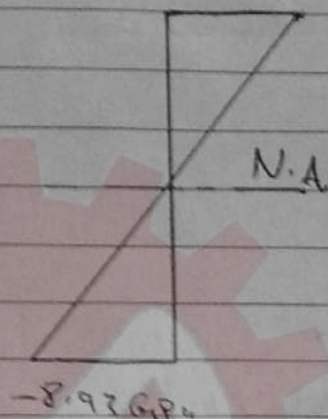


No.

③

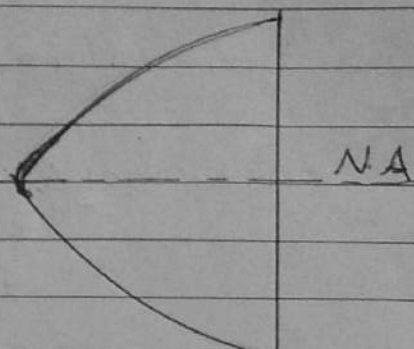
Bending

8.93 GPa



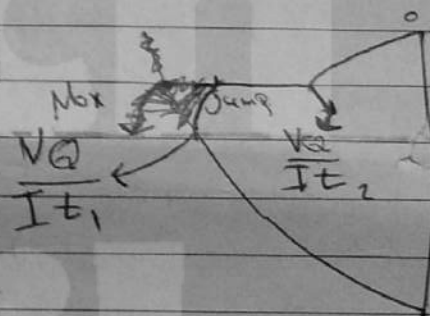
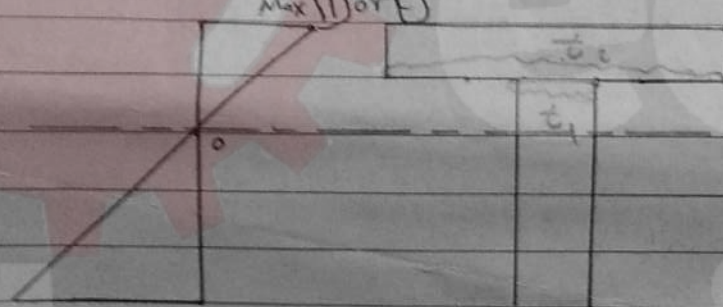
Shear

5.38 MPa



IF the cross section was a T shape

Max σ or ϵ



Max σ or ϵ

** Q : Selection :-

the Property Q is the moment of Area that is from the Point of interest and Away from the N.A, when the Point of interest at the surface of the beam Q is said to be zero and no T presents

However, care should be practiced

when choosing the surface which makes Q=0 since it must be either facing or opposing

** the max. shear stress value occurs at the N.A when Q is the moment of the Area above or under N.A. Also when the thickness suddenly changes a jump on the shear distribution will happen we calculate τ for both thicknesses then we draw the diagram.

the shear force and perpendicular to it when the shear force and the surface are parallel to each other $Q \neq 0$ and its value should be calculated, see M9.7 in mechanics

* Shear flow.

Key equations.

$$q = \frac{VQ}{I} = \frac{nF}{s} \quad \text{usually (2)}$$

q is a measure of the force per unit length of a beam.

$q \leftarrow V \leftarrow Q \leftarrow I \leftarrow$ no. of nails $F \leftarrow$ force
 $F \leftarrow$ shear force by each nail $s \leftarrow$ spacing

ex: a simply supported beam carries a load of 10 kN at the centre of 4m span.

Provided that each nail can transmit 0.5 kN calculate

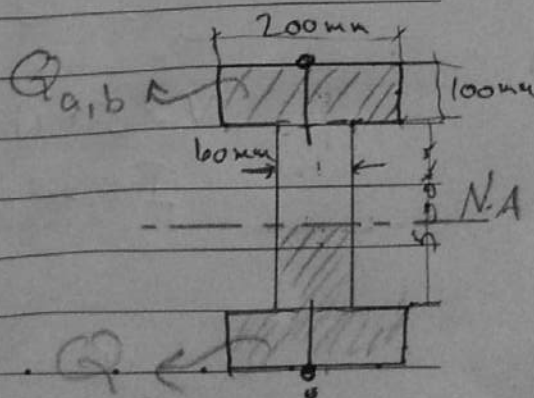
- the horizontal force transferred from each flange to the web
- the maximum spacing s
- maximum horizontal shear stress

soln. a) calculate $q = \frac{VQ}{I}$ then multiply by length 4m

b)

$$q = \frac{VQ}{I} = \frac{nF}{s}$$

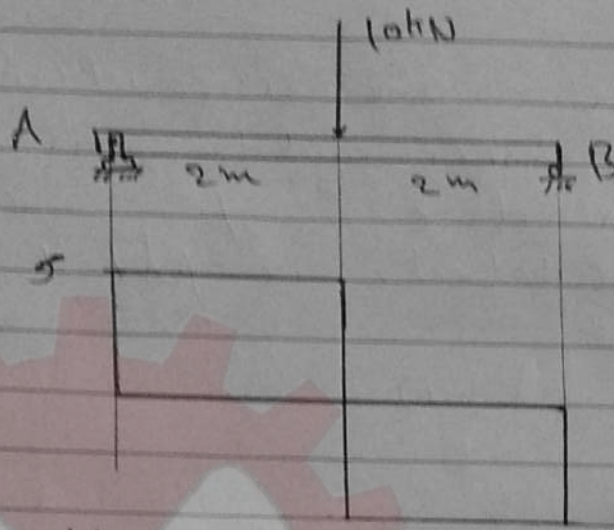
cont \rightarrow



(14)

17

No.



$$R_A = R_B = 5 \text{ kN}$$

** Q is always the area that is to slide if there was no nails
V (kN)

So $V = 5 \text{ kN}$, $Q = \underbrace{0.1 \times 0.2 \times 0.3}_{A'} \underbrace{\quad}_{y'}$
 $= 6 \times 10^{-3}$

$$I = 2 \times \left[\underbrace{0.2 \times (0.1)^3 \times \frac{1}{12}}_{\frac{b h^3}{12}} + \underbrace{0.2 \times 0.1 \times (0.3)^2}_A \right]$$

$$+ 0.1 \times (0.5)^3 \times \frac{1}{12}$$

$$= 4.675 \times 10^{-3} \text{ m}^4$$

$$n = 1$$

$$F = 0.5 \text{ kN}$$

$$\frac{VQ}{I} = \frac{nF}{s} \Rightarrow s = \frac{nFI}{VQ}$$

$$77.91 \times 10^{-3} \text{ m} \approx 77 \text{ mm}$$

round to a less number

c) $\tau = \frac{VQ}{It}$ where

$$V = 5 \text{ kN}$$

$$Q = 0.1 \times 0.25 \times 0.125 + 0.1 \times 0.25 \times 0.125$$

I is same

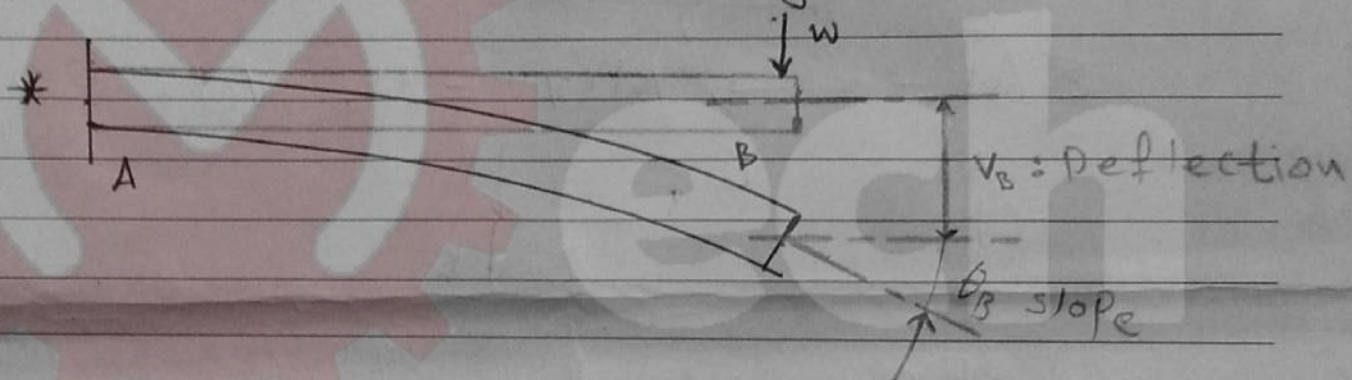
$$t = 0.1$$

No. Deflection

I) integration method

Procedure

1. obtain an eqn. for the beam's internal Moment
2. $EI \frac{d^2 v}{dx^2} = M$ $\xrightarrow{\text{integrate}}$ slope eqn $\xrightarrow{\text{integrate}}$ deflection
3. Apply boundary conditions to evaluate the constants of integration C_1 and C_2



* boundary conditions are point where the slope and/or the deflection is known

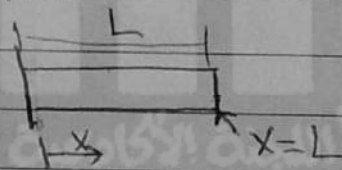
see M 10.1 in mecmovies

* Fixed support

$$v(0) = 0, \theta(0) = 0$$

$$M(L) = 0, V(L) = 0$$

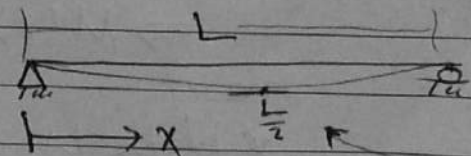
shear force



* Simply supported

$$v(0) = 0, v(L) = 0$$

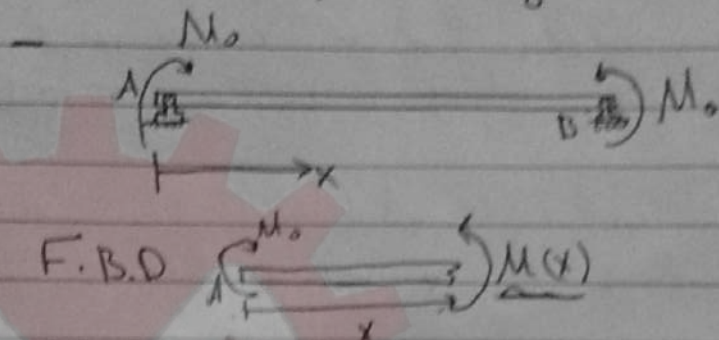
$$M(0) = 0, M(L) = 0$$



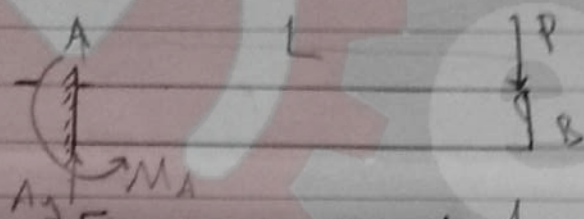
- if the load is uniform or concentrated at the centre then an extra condition is $\theta(\frac{L}{2}) = 0$

* obtaining an eqn. for the internal Moment $M(x)$

to obtain $M(x)$ you have to Draw a F.B.D



$$\sum M = 0 \Rightarrow M(x) = M_0 \text{ very simple}$$

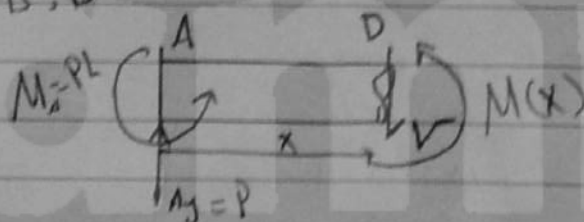


First we Find A_y, M_A

$$* \sum F_y = 0 \rightarrow A_y = P \uparrow$$

$$* \sum M_A = 0 \rightarrow M_A = PL \curvearrowright$$

F.B.D



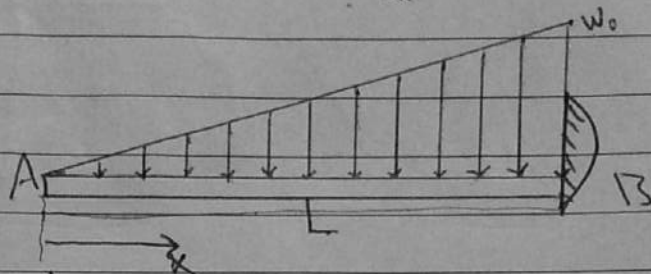
$$\sum M_D = 0 = M_A - P \cdot x + M(x) - A_y \cdot x$$

$$\Rightarrow M(x) = Px - PL$$

* Note that the F.B.D starts from $x=0$ and goes to the cut point so All forces and Moments in the region ($x=0 \rightarrow \text{cut}$) must be known.

$$* \theta = \frac{dV}{dx} \text{ (always remember it)}$$

ex. Find the elastic curve and V_{max} and θ_{max}



1 obtain $M(x)$:-

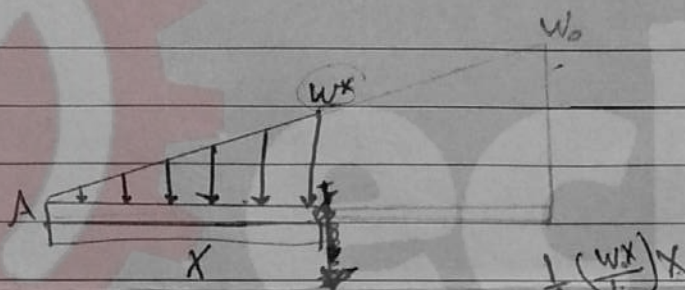
F.B.D

in our region

w^* is unknown

but $\frac{w_0}{L} = \frac{w^*}{x}$

$\Rightarrow w^* = \frac{w_0 x}{L}$



* $\sum M_D = 0$

$M = \frac{1}{2} \left(\frac{w_0 x}{L} \right) \cdot x \cdot \frac{x}{3}$

$= -\frac{w_0 x^3}{6L}$

the \ominus because M is clockwise

$EI \frac{d^2 v}{dx^2} = -\frac{w_0}{6L} x^3$

$EI \frac{dv}{dx} = -\frac{w_0}{24L} x^4 + C_1$

$EI v = -\frac{w_0}{120L} x^5 + C_1 x + C_2$

boundary cond.

$\theta|_{x=L} = \frac{dv}{dx}(L) = 0 \rightarrow C_1 = \frac{w_0 L^3}{24}$

$V(L) = 0 \rightarrow C_2 = \frac{-w_0 L^4}{30}$

$V=0, x=L$

$V = \frac{w_0}{120EI} (-x^5 + 5L^4 x - 4L^5)$

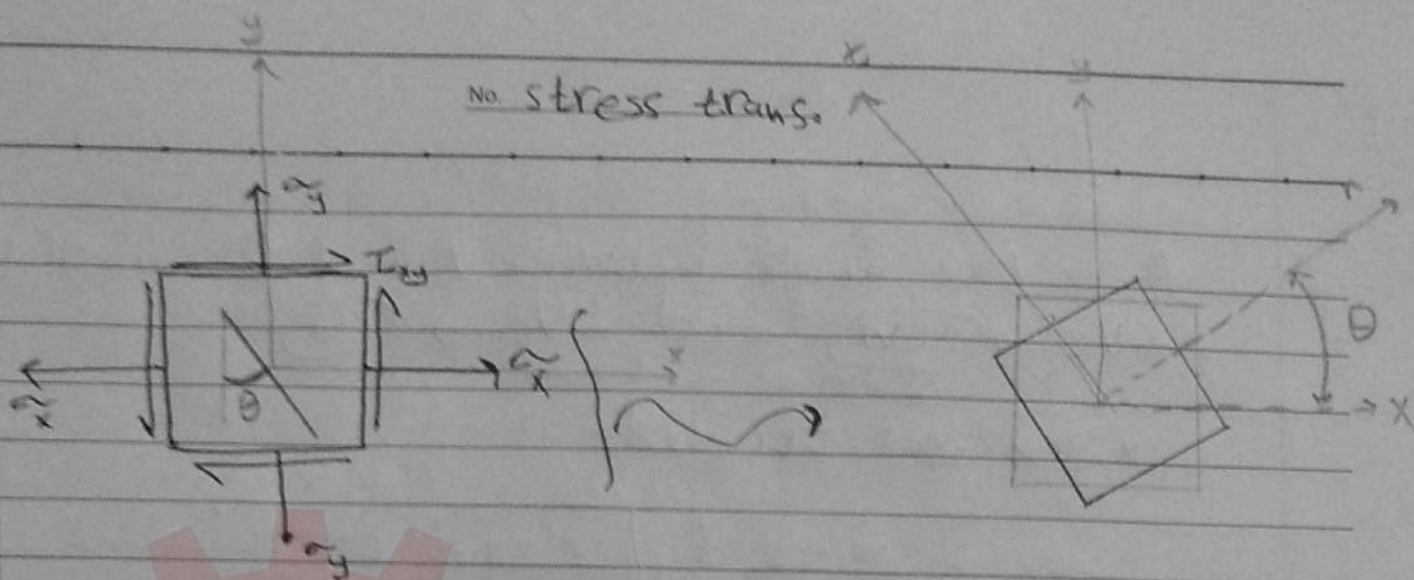
$V_{max} = V(0) = \frac{-w_0 L^4}{30EI}$

also $\theta = \frac{w_0}{24EI} (L^4 - x^4)$

$\theta_{max} = \theta(0)$

$= \frac{dV}{dx}(0) = \frac{w_0 L^3}{24EI}$

No. stress trans.



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

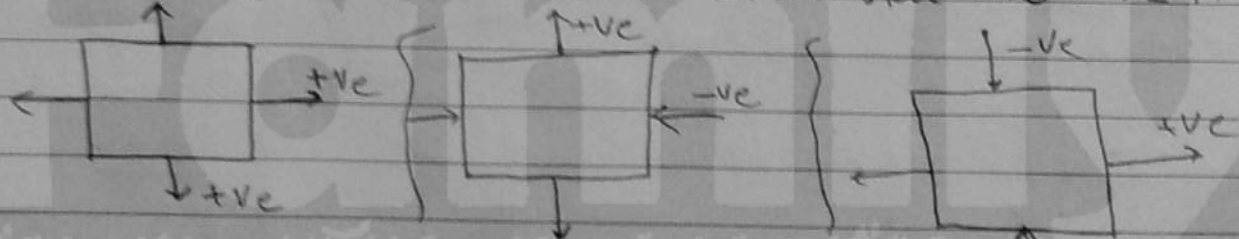
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

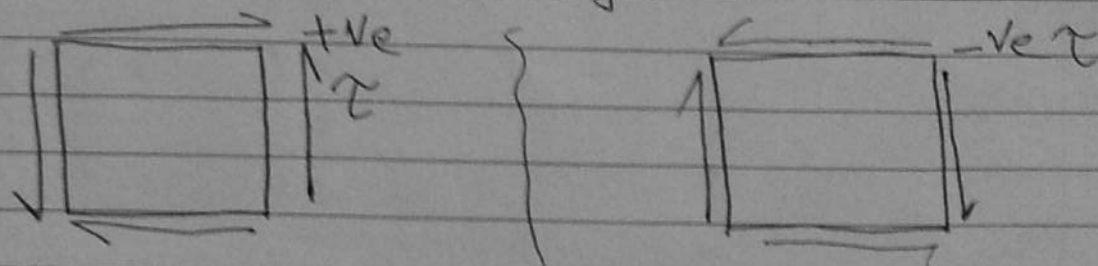
$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$$

Notes (very very important)

σ_x, σ_y are Positive in tension and -ve in comp.



τ_{xy} is Positive when acting upward at the right



θ is the angle between x and t and it is
 +ve \rightarrow (ccw) and -ve \rightarrow (cw)

* Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{nb} = 0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \theta_p: \theta \rightarrow \text{Principal stress}$$

* Max. and min. shear stresses

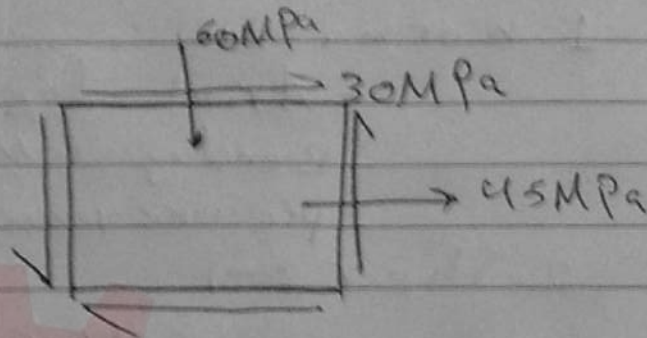
$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} \quad \text{OR} \quad \theta_s = \theta_p + 45^\circ$$

$$\sigma_n = \sigma_t = \sigma_{av} = \frac{\sigma_x + \sigma_y}{2}$$

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ex. Find the Principal stresses and their orientation and the maximum shear stresses and their orientation.



① $\sigma_x = 45 \text{ MPa}$, $\sigma_y = -60 \text{ MPa}$, $\tau_{xy} = 30 \text{ MPa}$

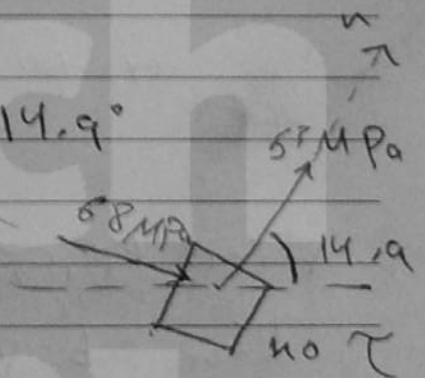
$$\sigma_1 = -7.5 + 60.5 = 53 \text{ MPa}$$

$$\sigma_2 = -7.5 - 60.5 = -68 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2 \times 30}{45 - 60}$$

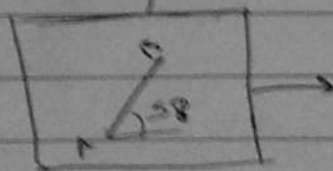
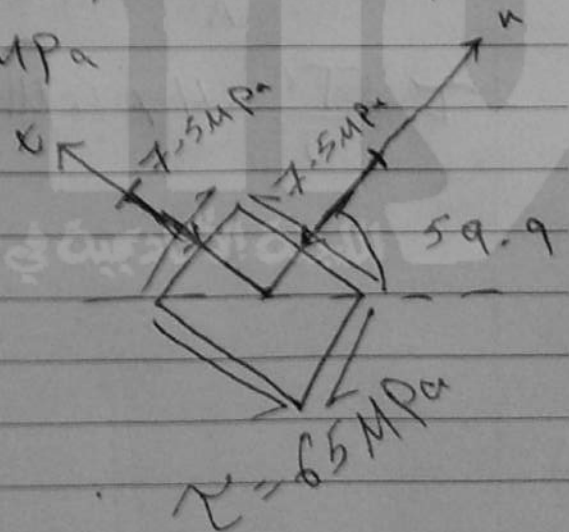
$$\Rightarrow \theta_p = 14.9^\circ$$

$$\tau_{\text{max}} = 0$$



$$\tau_{\text{max}} = +60.5 \quad \theta_s = \theta_p + 45 = 59.9$$

$$\sigma_1 = \sigma_2 = \sigma_{av} = -7.5 \text{ MPa}$$



$\sigma_x = 400$ $\tau_{(400)} = 550$ $\sigma_y = ??$

Mohr's circle

First we have to determine $\sigma_x, \sigma_y, \tau_{xy}$

then we locate Points A, B, C where

$$A(\sigma_x, \tau_{xy})$$

$$B(\sigma_y, -\tau_{xy})$$

$$C(\sigma_{av}, 0)$$

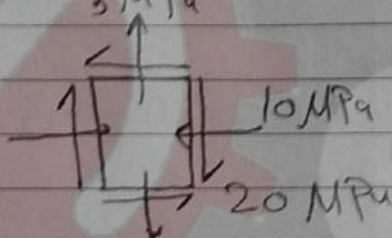
Diep

Diep

Diep

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2}$$

ex.



$$\sigma_x = -10 \text{ MPa}, \sigma_y = +5 \text{ MPa}, \tau_{xy} = -20 \text{ MPa}$$

$$\rightarrow \sigma_{av} = \frac{-10 + 5}{2} = -2.5$$

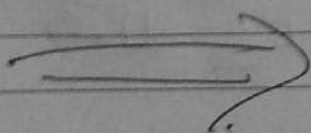
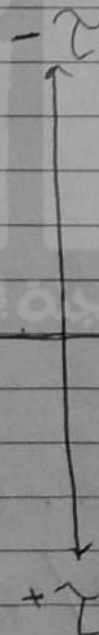
$$A(-10, -20)$$

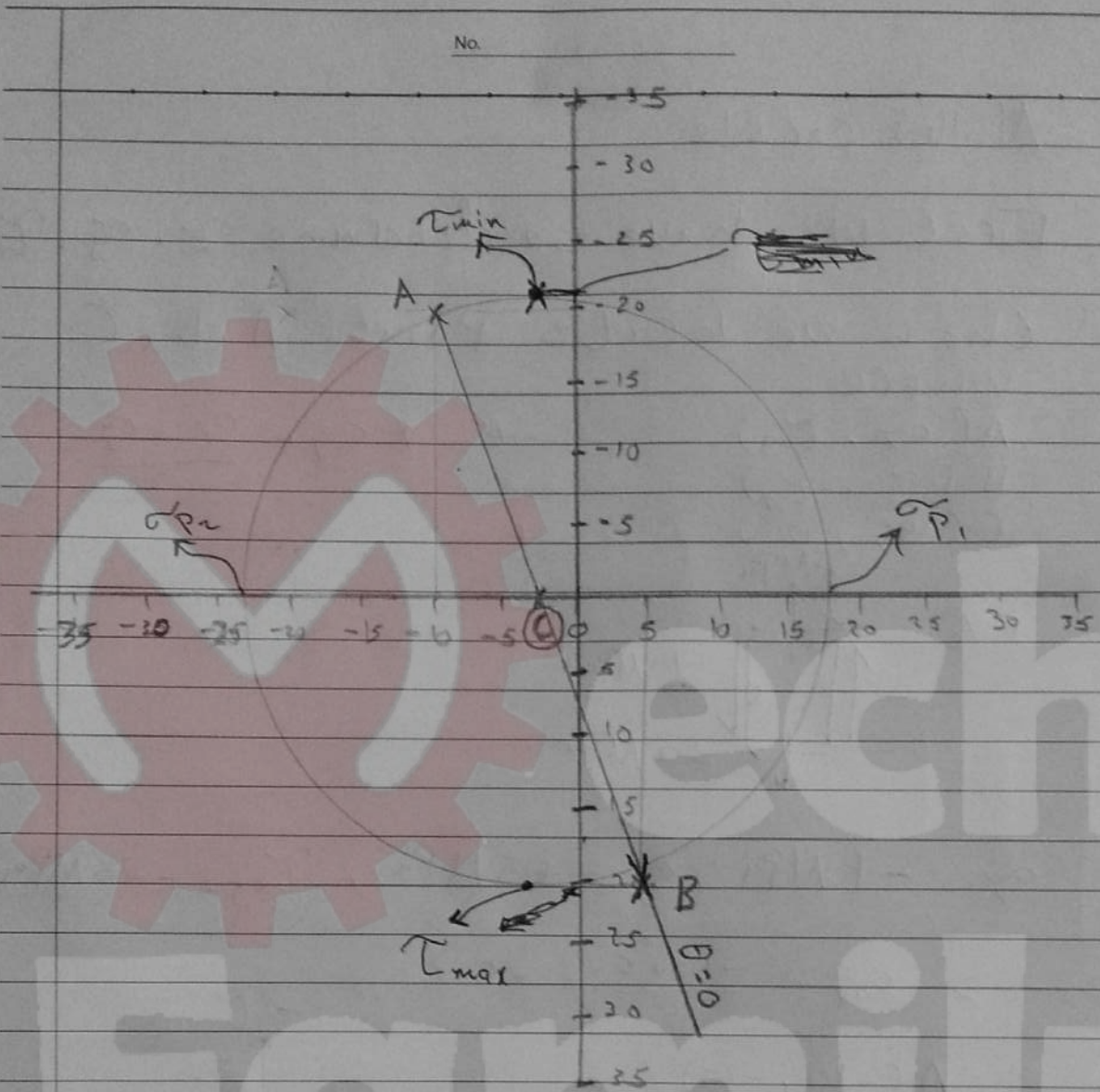
$$B(5, 20)$$

$$C(-2.5, 0)$$

then establish a coordinate system with σ as x)

τ as y





الرسم يتم بالخطوة ويتم أخذ Scale مناسب
في هذه الرسمة scale ← كل اسم 5 MPa

ثم نعين النقطتين A, B, C نقطة مع مراعاة
أن قيم σ سالبة في الأعلى والموجبة في الأسفل
ثم نركز القوس في (C) ونرسم دائرة ثم نحدد A, B
ثم نصل A, B, C بنقطتين

ثم نجد طول الخط CB ليكون $\sigma_{P1} = \sigma_{av} + CB$
وكذلك $\tau_{max} = CB$
 $\sigma_{P2} = \sigma_{av} - CB$

ثم نقيس الزاوية بين الخط \overline{AB} و \overline{CD} و θ
 1- $\angle (a) = 2\theta_p$ نقيسها على \overline{CD} لنجد θ_p

* من هنا الزاوية وضع ذلك الزاوية على \overline{CD} ونقيس
 منه أي أن زاوية الخط \overline{AB} في P هي θ_p ونراها في \overline{CD}
 نقيس $(+ve CW)$ و $(-ve CCW)$

$$\theta_s = \theta_p + 45^\circ$$

Combined loading types of loads

1. Normal (s-)

- Axial $\left(\frac{P}{A}\right) = \sigma$
- Bending $\left(\frac{My}{I}\right) = \sigma$
- Pressure vessels. \rightarrow later

2. Shear (s-)

- shear stress $\left(\frac{VQ}{It}\right) = \tau$
- torsional $\left(\frac{Tr}{J}\right) = \tau$

Add normal together and shear together

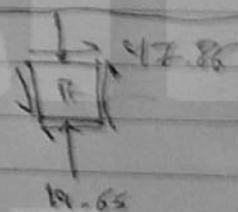
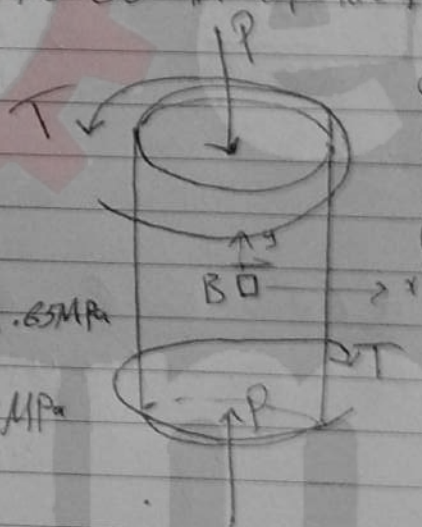
ex. to shaft $d_o = 114$, $d_i = 102$, $T = 5 \text{ kNm}$, $P = 90 \text{ kN}$

Find σ_1, σ_2

τ_{\max} at B

$$- \sigma_{\text{axial}} = \frac{P}{A} = 19.65 \text{ MPa}$$

$$- \tau = \frac{Tr}{J} = 47.86 \text{ MPa}$$



$$\sigma_1 = 89.08$$

$$\sigma_2 = -58.68$$

$$\tau = 0$$

$$\tau_{\max} = 47.86$$

$$\sigma_n = \sigma_b = \sigma_v = 9.82$$

الجامعة الإسلامية في غزة - الجامعة الإسلامية

ملاحظات هامة

يجب دراسة ال pressure vessels من دوستة الدكتور
هاشم الخالدي وحفظ القوانين الثلاث فيها

هام جدا وملزم: يجب دراسة جميع اسئلة الكومبايند
لودينغ الموجودة على الميك موفيز دون استثناء لأن
اسئلة الإمتحان ستكون شبيهة جدا

يجب حل مجموعة من الأسئلة على مور سيركل