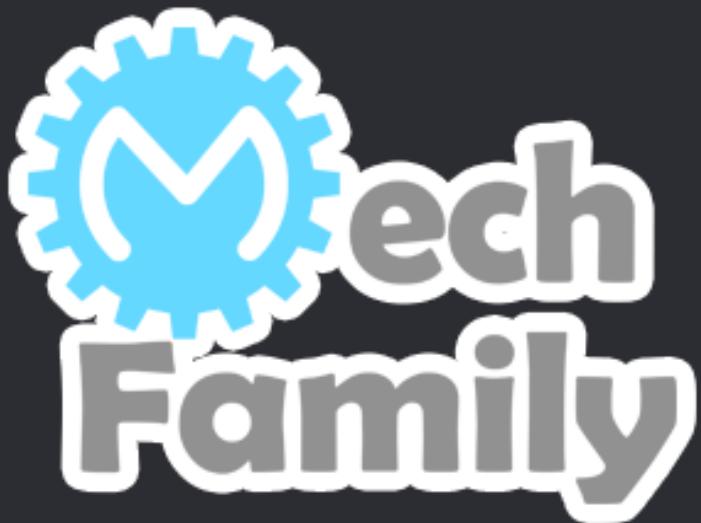


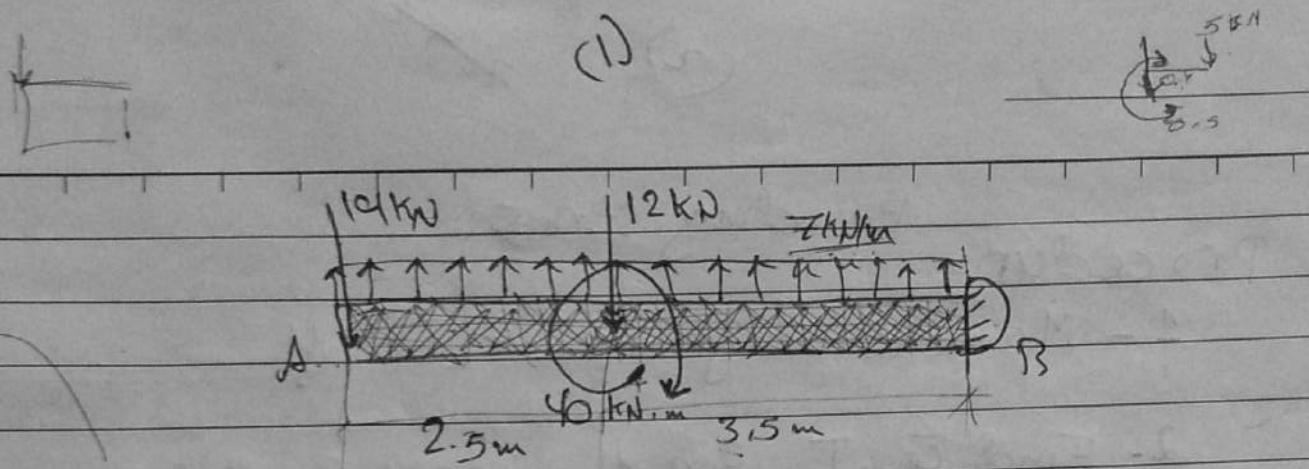
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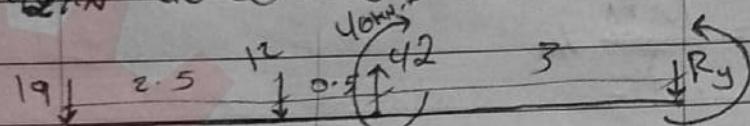
طاجیکستان‌میں اسلامی قدرتی

STRENGTH

عربی میں اسلامی قدرتی



$$7 \times 6 = 42 \text{ kN at centroid of force}$$

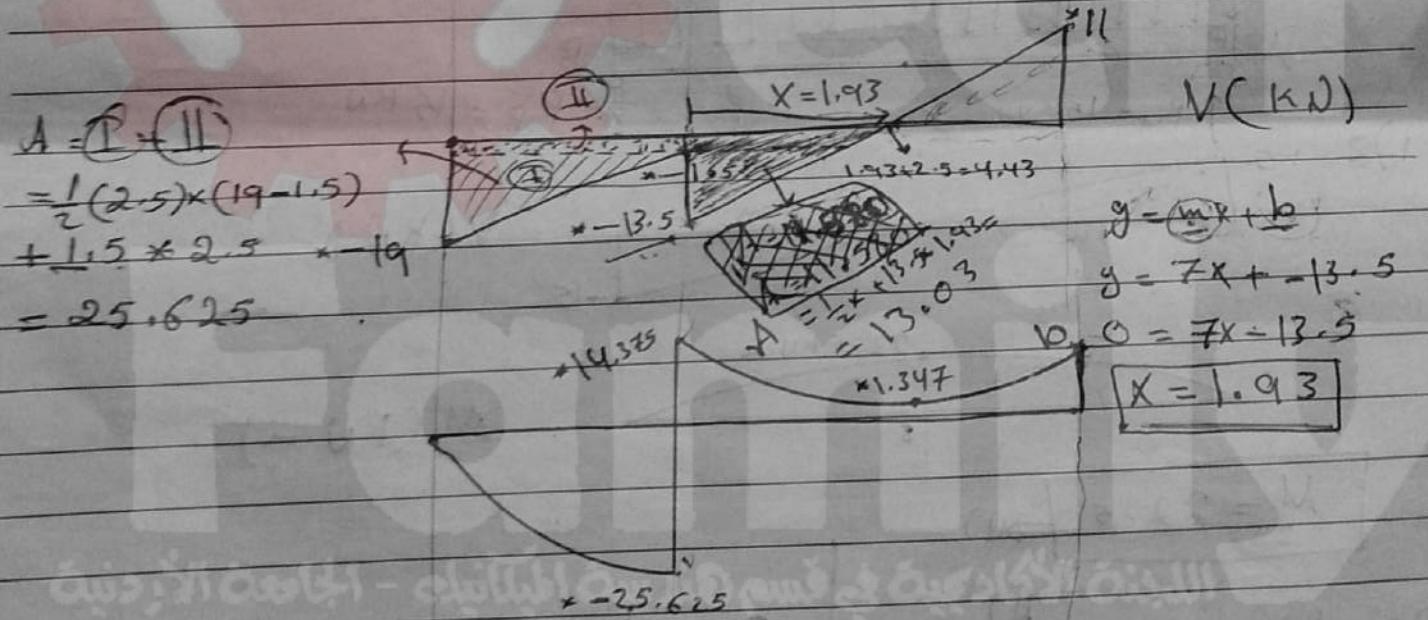


$$\sum F_y = 0,$$

$$19 + 12 + 42 + R_y = 0 \Rightarrow R_y = 11 \text{ kN}$$

$$\sum M_B = 0,$$

$$M_B = 42 \times 3 - 12 \times 3.5 - 19 \times 6 + 40 = 10 \text{ kNm}$$



(2)

2/2

Bending stress

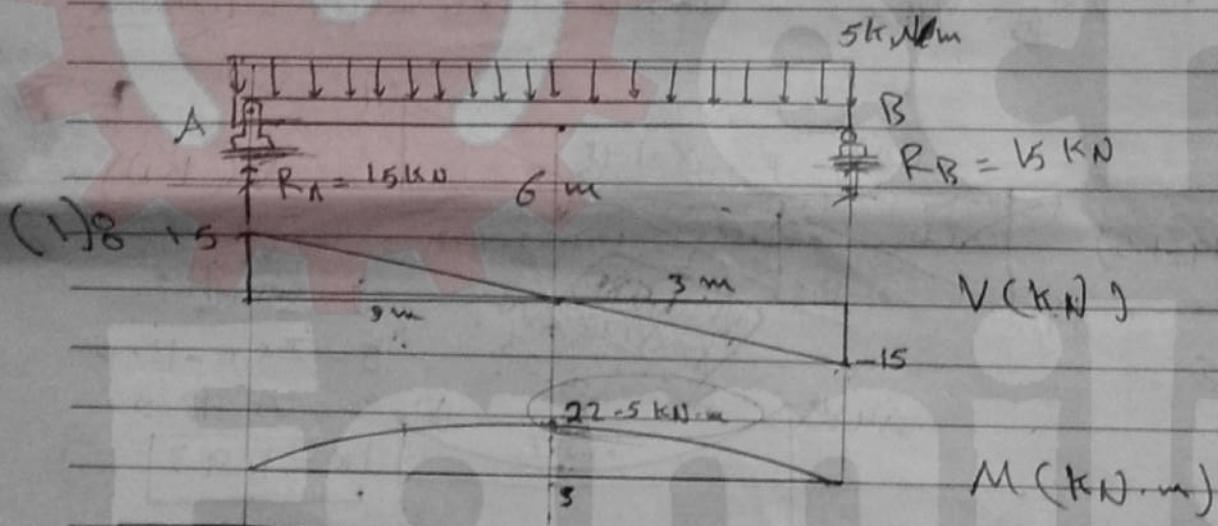
procedure

1 - Moment Diagram \rightarrow read M

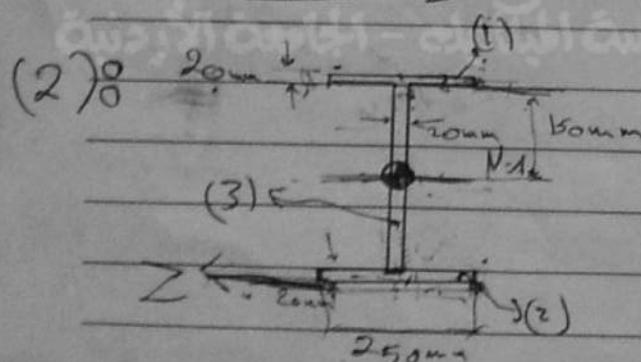
2 - Find \bar{y} , \bar{I}_z for the cross section

3 - $\sigma_x = \frac{My}{I_z}$

determine the absolute maximum bending stress
in the beam ~~and draw the stress~~
~~distribution at this location~~



$$M_{\text{max}} = 22.5$$



Part	$A(\text{cm}^2)$	$\bar{y}(\text{cm})$	$A\bar{y}(\text{cm}^3)$
(1)	5×10^3	330×10^3	1.65×10^6
(2)	5×10^3	10×10^3	5×10^5
(3)	6×10^3	170×10^3	1.02×10^6
Σ		16×10^3	2.72×10^6

Cont.

$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = \frac{2.72}{16} = 0.17 = 170 \text{ mm}$$

(Can be skipped)

(3)

Part	$A(\text{m}^2)$	\bar{y}	$A\bar{y}$	d_{cc} (m)	Ad_{cc}^2	$\bar{I}_c' (\text{m}^4)$	$\bar{I} = \bar{I}_c' + Ad^2 (\text{m}^4)$
(1)	5×10^{-3}	5	25	0.16	1.28×10^{-4}	1.66×10^{-7}	1.28166×10^{-4}
(2)	5×10^{-3}	5	25	0.16	1.28×10^{-4}	1.66×10^{-7}	1.28166×10^{-4}
(3)	6×10^{-3}	10	60	0.0	0.0	4.5×10^{-5}	4.5×10^{-5}
Σ							$301.3 \times 10^{-6} \text{ m}^4$

* d_{cc} is distance from the centroid of the part (c) to the centroid of the whole cross-section (C)

$$* \bar{I} = \bar{I}_c' + Ad^2$$

$$\bar{y} = 170 \text{ mm} \text{ and } \bar{I} = 301.3 \times 10^{-6} \text{ m}^4 \Rightarrow (2)$$

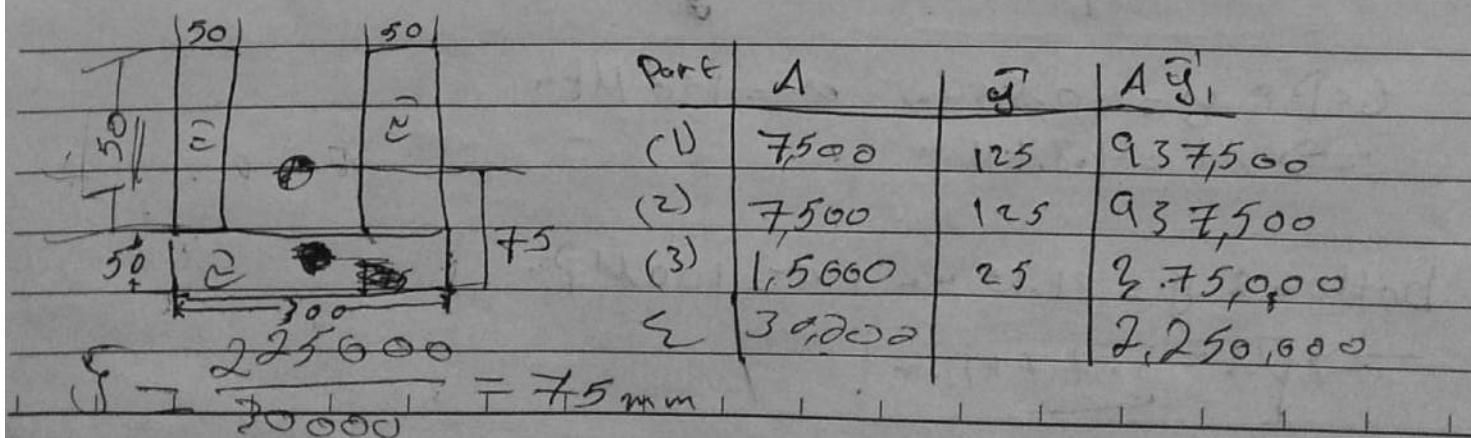
$$(3) \sigma_{\max} = \frac{M_{max} y}{I}$$

* here y is distance from the centroid of the whole cross section (C) to the point we want to find σ at.

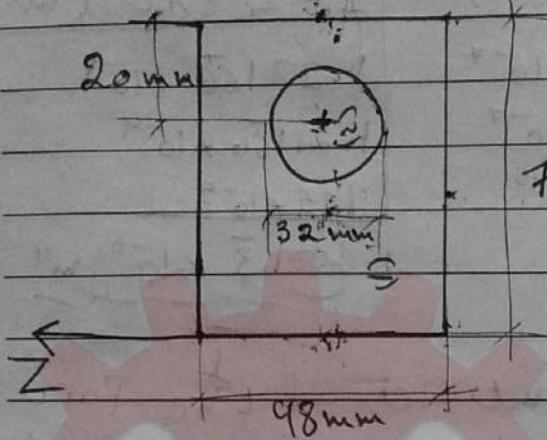
* Since we want σ_{\max} y will be 170mm to maximize σ

$$\sigma_{\max} = \frac{22.5 \times 10^3 \times 170 \times 10^{-3}}{301.3 \times 10^{-6}} \approx 12.7 \text{ MPa} \Rightarrow \boxed{12.7}$$

finished ~~12.7~~



(u)



Find the moment capacity of this beam if the max. stress is 150 MPa tension and 100 MPa in compression and draw the shear stress distribution.

$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{48 \times 72 \times 26 - \frac{\pi}{4} \times (32)^2 \times 52}{48 \times 72 - \frac{\pi}{4} \times (32)^2}$$

$$= 31.15 \text{ mm}$$

I : ~~b // axis~~

b // axis about which the rotation will occur

$$I = \frac{48 \times 10^{-3} \times (72 \times 10^{-3})^3}{12} + (36 \times 10^{-3} - 31.15 \times 10^{-3})^2 \times 48 \times 10^{-3} \times 72 \times 10^{-5}$$

$$- \frac{\pi}{64} \times (32 \times 10^{-3})^4 - (40.853 \times 10^{-3} - 20 \times 10^{-3})^2 \times \frac{\pi}{4} \times (32 \times 10^{-3})^2$$

$$= 1.1733 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{My}{I} \Rightarrow M = \frac{\sigma I}{y}$$

$$\text{top } \therefore y = 40.85 \text{ mm} \quad \sigma = 150 \text{ MPa}$$

$$\Rightarrow M = 4.9 \text{ kN.m}$$

$$\text{bottom } \therefore y = 31.15 \text{ mm}, \sigma = 100 \text{ MPa}$$

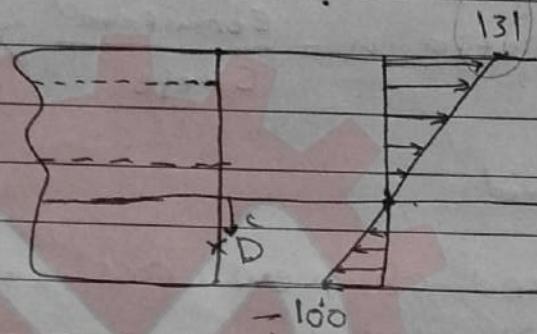
$$\Rightarrow M = 3.77 \text{ kN.m}$$

(5)

So 3.77 controls

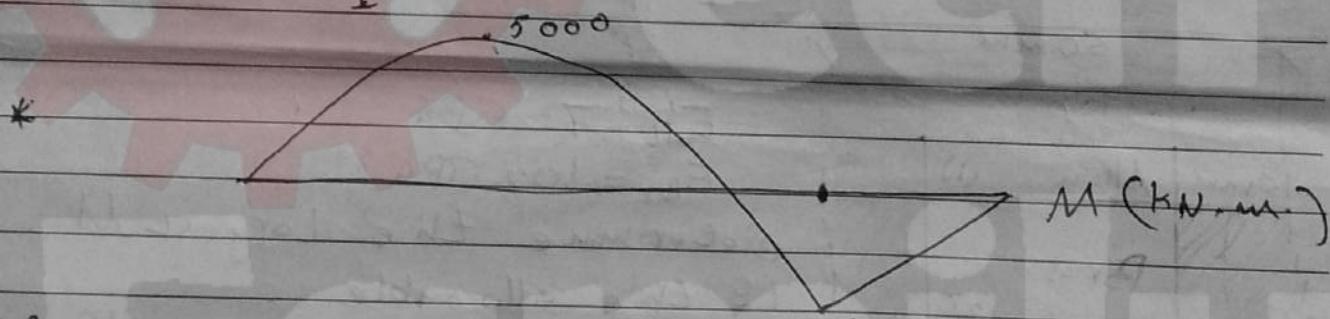
$$\sigma_{comp} = 100 \text{ MPa}$$

$$\sigma_{tens} = \frac{3.77 \times 10^3 \times 40.853 \times 10^3}{1.1733 \times 10^{-6}} = 131 \text{ MPa}$$



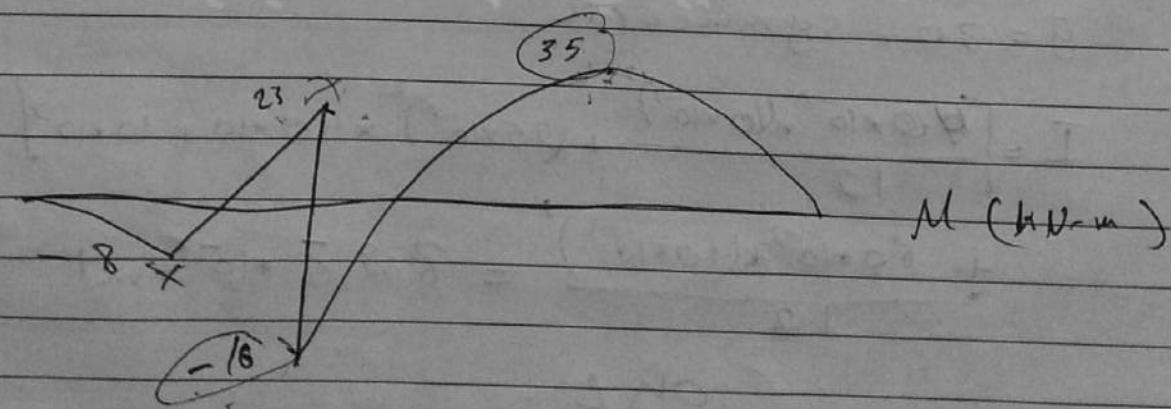
* ask at another point D for example

$$\sigma = \frac{Mc}{I}$$

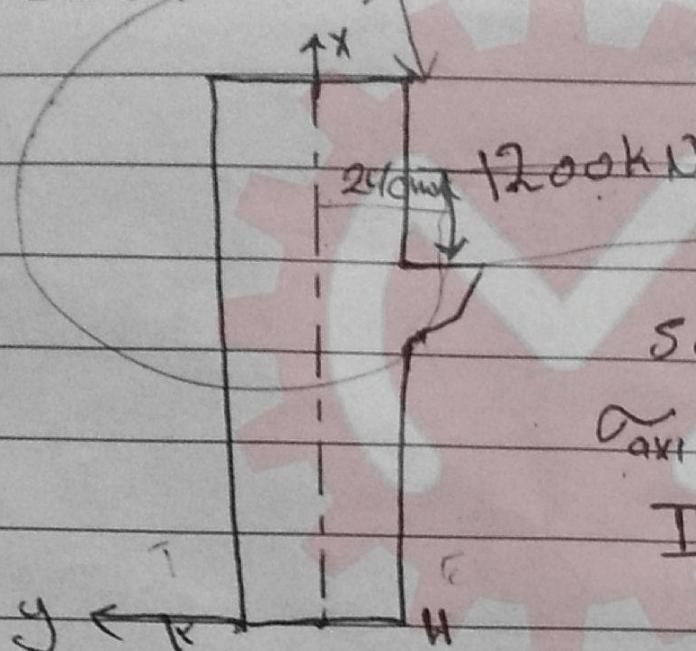


if asked to find the maximum σ you have to check both the -4000 and the 5000 moments.

DON'T Think!!



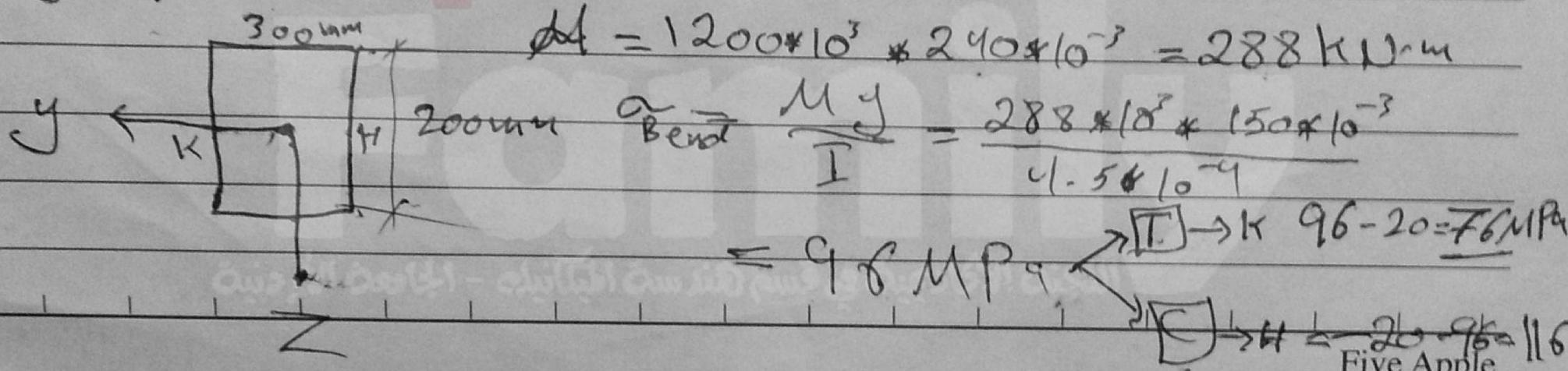
Eccentric Axial

Find σ_K , σ_H 

$$\text{Soln. } A = 0.06 \text{ m}^2$$

$$\sigma_{\text{Axial}} = \frac{F}{A} = 20 \text{ MPa} \quad \boxed{C}$$

$$I = \frac{bh^3}{12} = \frac{0.2 \times (0.3)^3}{12} = 4.5 \times 10^{-4} \text{ m}^4$$



$$P = 1200 \times 10^3 \times 240 \times 10^{-3} = 288 \text{ kN-m}$$

$$\sigma_{\text{Bend}} = \frac{M y}{I} = \frac{288 \times 10^3 \times 150 \times 10^{-3}}{4.5 \times 10^{-4}}$$

$$= 96 \text{ MPa} \quad \boxed{T} \rightarrow K \quad 96 - 20 = \underline{76 \text{ MPa}}$$

$$\boxed{H} \rightarrow H = \frac{20 \times 96}{116} = 16 \text{ MPa}$$

Comments:

1- if two conditions were given:

(1) $\sigma = 150 \text{ MPa}$

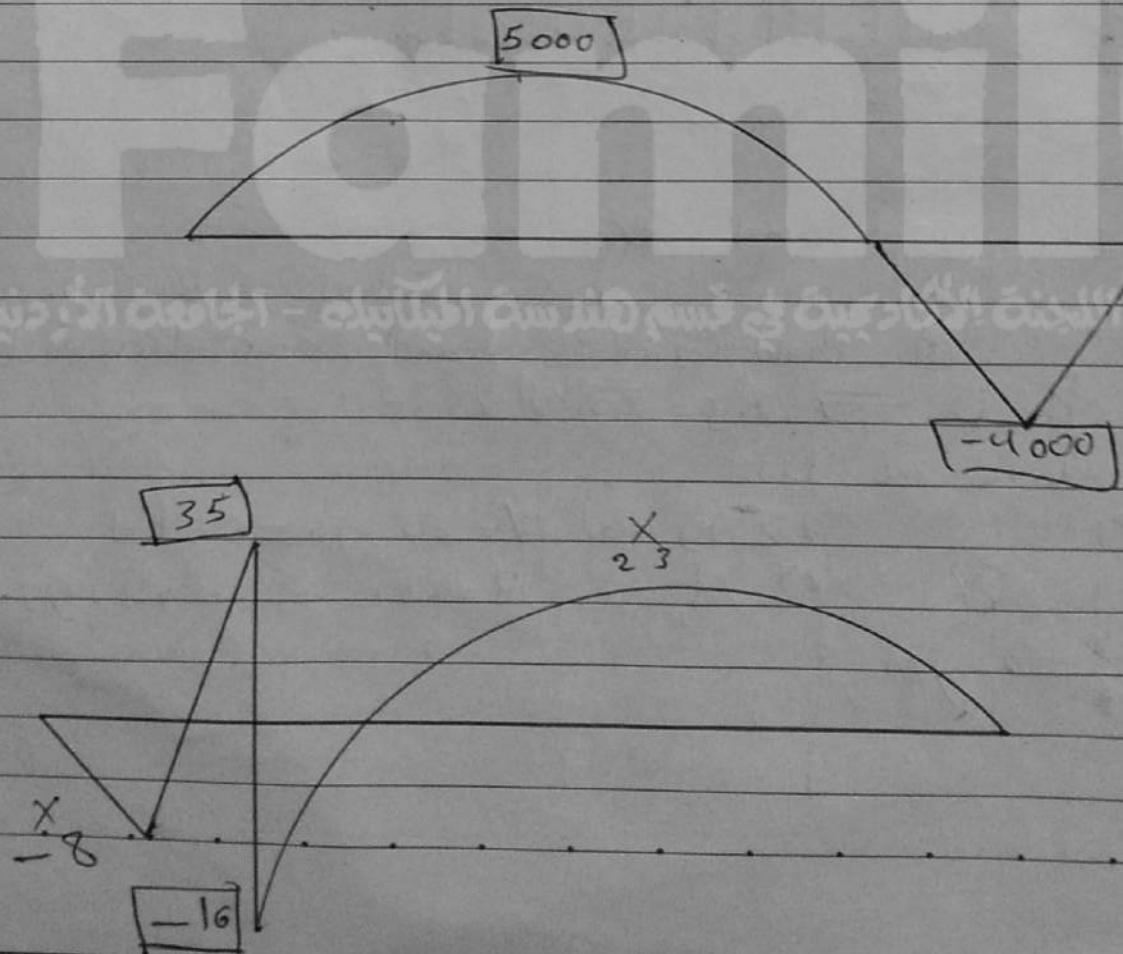
(2) $\sigma = 100 \text{ MPa}$

we solve for both of them then we take the safest (more safe) value

F, M, σ, T, d_i - we take smallest
 $D, d, \text{ thickness}$ we take largest

2- if asked not for maximum but for specified point then find the Moment at that point using equations (not graphs.)

3- if asked for σ_{\max} then check for all extreme values that is. (Don't think)



no transverse shear

Procedure

1 - shear force diagram \rightarrow find V 2 - Find g, I, Q, t

$$3 - \tau = \frac{VQ}{It}$$

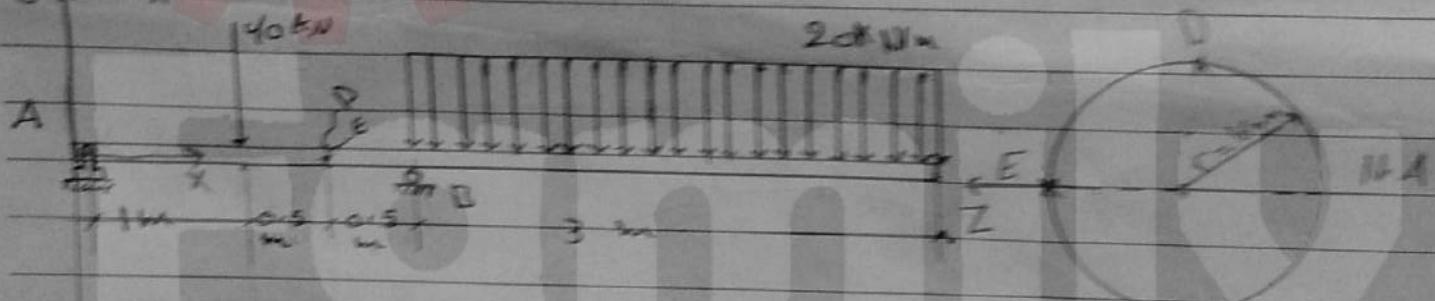
$$* G = A' \bar{J}'$$

A' is the Area from point of interest and
away from the centroidal axis.

\bar{J}' is the distance from the centroidal axis
 to the centroid of (A')

t : thickness (Parallel to the centroidal axis)

3. ex.



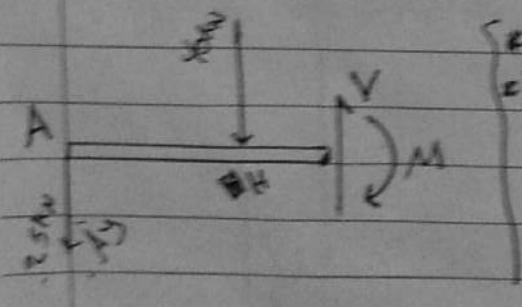
Find the ① Normal stress at point D, E

② Shear stress at Point D, E

③ Draw normal stress and shear stress distributions

$$F.B.D \Rightarrow A_g = 25 \text{ kN} \downarrow$$

Section at D, E



$$\left\{ \begin{array}{l} \sum F_y = 0, V = 25 + 40 = 65 \text{ kN} \\ \sum M_A = 0, M = (65 \times 0.5) + (25 \times 1) = 57.5 \end{array} \right.$$

$$\left\{ \begin{array}{l} \sum F_y = 0, V = 25 + 40 = 65 \text{ kN} \\ \sum M_A = 0, M = (65 \times 0.5) + (25 \times 1) = 57.5 \end{array} \right.$$

$$I = \frac{\pi}{2} r^4 = 1.02944 \times 10^{-7} \text{ m}^4$$

$$\bar{y} = 0.016 \text{ mm}$$

Bending:-

Point D₃:-

$$* y = c = 0.016,$$

$$* \sigma = \frac{57.5 \times 10^3 \times 0.016}{1.02944 \times 10^{-7}}$$

-7.93 GPa

T

Point E₃:-

$$* y = 0, (\text{Neutral axis})$$

$$* \sigma = 0,$$

Shear:-

Point D₃:-

$$* Q = 0,$$

$$* q = 0,$$

Point E₃:-

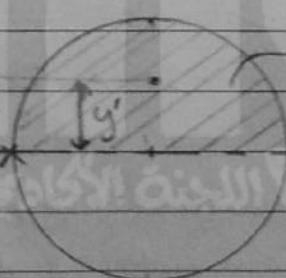
$$Q = A' y' \\ = \frac{\pi r^2}{2} \times \frac{4r}{3\pi}$$

$$= 2.731 \times 10^{-6} \text{ m}^3$$

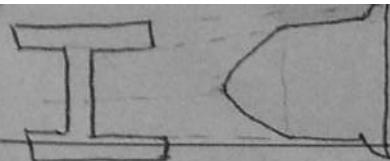
$$t = 0.016 \times 2 = 0.032$$

$$\tau_E = \frac{\sqrt{Q}}{It} = \frac{65 \times 10^3 \times 2.731 \times 10^{-6}}{1.02944 \times 10^{-7} \times 0.032}$$

$$= 5.38 \text{ MPa}$$



** note for an I-beam



V_Q

No. _____

③

Bending

8.93 GPa

Shear

N.A.

5.38 MPa

N.A.

-8.93 GPa

IF the cross section was a T shape

Max \bar{I} or \bar{E}

Max \bar{I} or \bar{C}

$\frac{VQ}{It_1}$

$\frac{VQ}{It_2}$

** Q Selection :-

the property Q is the moment of area that is from the point of interest and away from the N.A., when the point of interest at the surface of the beam Q is said to be zero and no T presents.

However, care should be practised

when choosing the surface which makes $Q=0$ since it must be either facing or opposing

the max. shear stress value occurs at the N.A. where Q is the moment of the area above or under N.A. Also when the thickness suddenly changes a jump on the shear distribution will happen we calculate T for both thicknesses then we draw the diagrams.

the shear force and perpendicular to it when the shear force and the surface are parallel to each other $Q \neq 0$ and its value should be calculated, see M9.7 in mechanics

* Shear flow.

Key equations.

$$q = \frac{VQ}{I} = \frac{nF}{s} \xrightarrow{\text{usually}} (2)$$

q is a measure of the force per unit length of a beam.

q , V , Q , I , n no. of nails, F force
 F is shear force by each nail, s spacing

ex: a simply supported beam carries a load of 10kN at the centre of 4m span.

Provided that each nail can transmit 0.5kN calculate

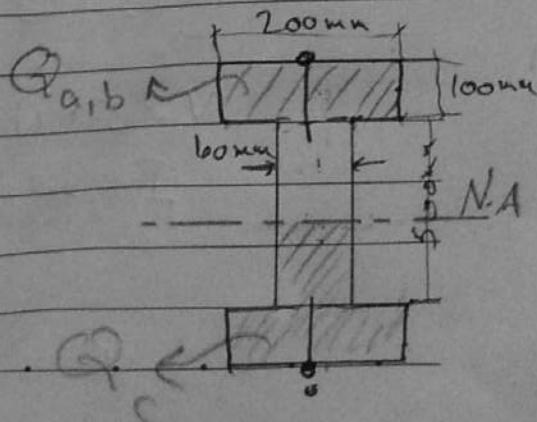
- the horizontal force transferred from each flange to the web
- the maximum spacing s
- maximum horizontal shear stress

sln. a) calculate $q = \frac{VQ}{I}$ then multiply by length 4m

b)

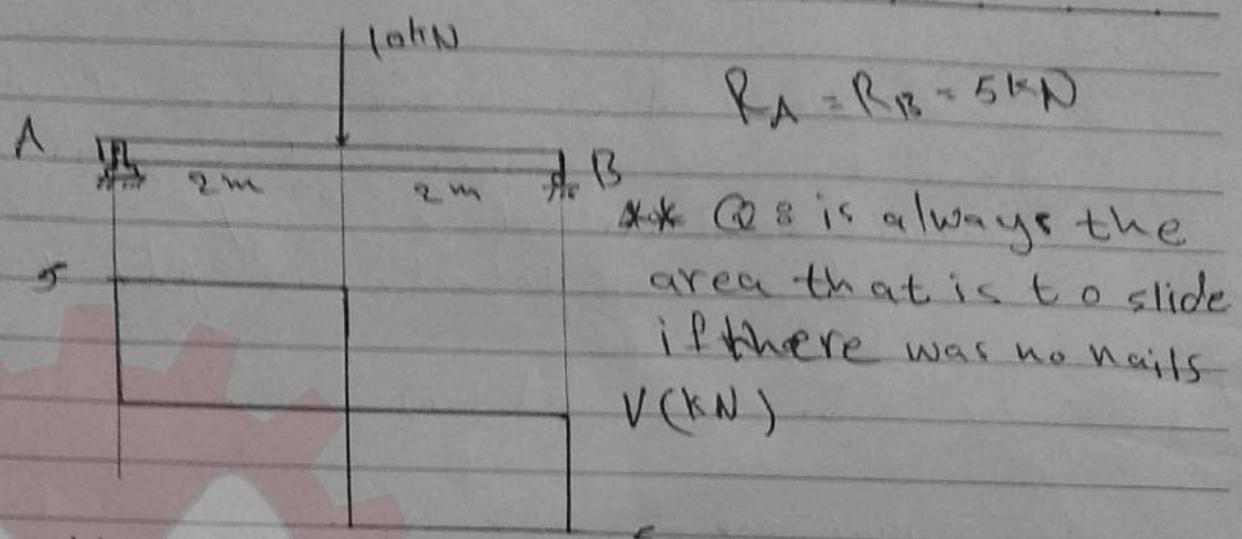
$$q = \frac{VQ}{I} = \frac{nF}{s}$$

cont \rightarrow





No.



$$R_A = R_B = 5 \text{ kN}$$

** Q_s is always the area that is to slide if there was no nails

$$V(\text{kN})$$

$$\text{So } V = 5 \text{ kN}, Q = \underbrace{0.1 * 0.2 * 0.3}_{\text{A}'} \underbrace{g}_{\text{g}}$$

$$= 6 * 10^{-3}$$

$$I = 2 * \left[\underbrace{0.2 * (0.1)^3 * \frac{1}{12}}_{\frac{b h^3}{12}} + \underbrace{0.2 * 0.1 * (0.3)^2}_{A} \right]$$

$$+ 0.1 * (0.5)^3 * \frac{1}{12}$$

$$= 4.675 * 10^{-3} \text{ m}^4$$

$$n = 1$$

$$F = 0.5 \text{ kN}$$

$$\frac{VQ}{I} = \frac{nF}{s} \Rightarrow s = \frac{nFI}{VQ}$$

$$77.91 * 10^{-3} \text{ m} \approx 77 \text{ mm}$$

round to a less number

$$c) Q_t = \frac{VQ}{It} \text{ when}$$

$$V = 5 \text{ kN}$$

I is same

$$Q = 0.1 * 0.25 * 0.125 + 0.1 * 0.0625$$

$$t = 0.1$$

No. Deflection

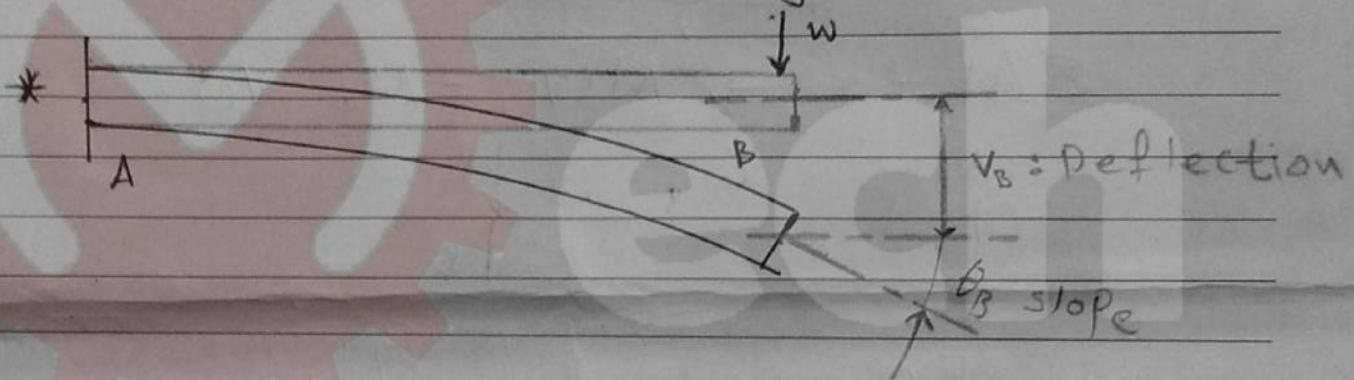
I) integration method

Procedure

1 - obtain an eqn. for the beam's internal Moment

2 - $EI \frac{d^2v}{dx^2} = M$ $\xrightarrow{\text{integrate}}$ slope eqn $\xrightarrow{\text{integrate}}$ deflection

3 - Apply boundary conditions to evaluate the constants of integration C_1 and C_2



* boundary conditions are point where the slope and/or the deflection is known

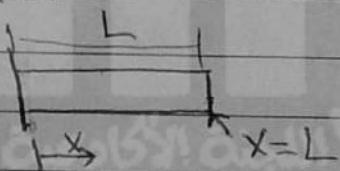
see M 10.1 in mechanics movies

* Fixed support

$$v(0) = 0, \theta(0) = 0$$

$$M(L) = 0, V(L) = 0$$

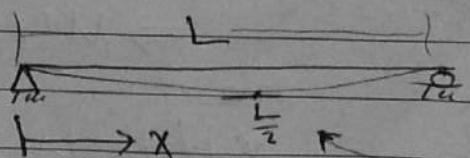
shear force



* simply supported

$$v(0) = 0, v(L) = 0$$

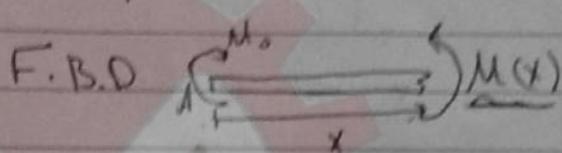
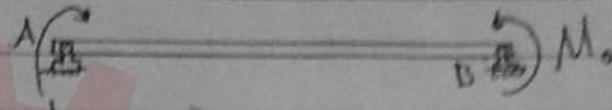
$$M(0) = 0, M(L) = 0$$



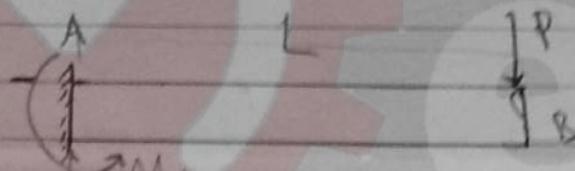
- if the load is uniform or concentrated at the centre then a new condition is $\theta(\frac{L}{2}) = 0$

* obtaining an eqn. for the internal Moment $M(x)$

- to obtain $M(x)$ you have to draw a F.B.D



$$\sum M = 0 \rightarrow M(x) = M_o \text{ Very simple}$$

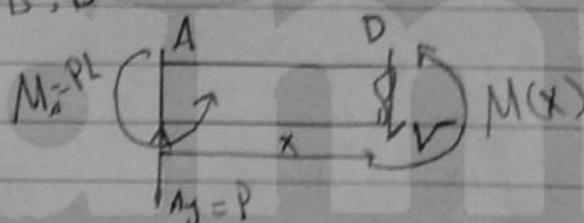


* First we find A_y, M_A

$$* \sum F_y = 0 \rightarrow A_y = P \uparrow$$

$$* \sum M_A = 0 \rightarrow M_A = P L$$

F.B.D



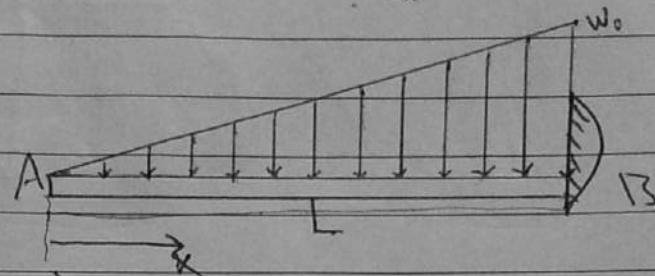
$$\sum M_D = 0 = M_A - M(x) + M(x) - A_y \cdot x$$

$$\Rightarrow M(x) = P x - P L$$

* Note that the F.B.D starts from $x=0$ and goes to the cut point so all forces and moments in the region ($x=0 \rightarrow \text{cut}$) must be known.

$$* \theta = \frac{d\psi}{dx} \text{ (always remember it)}$$

Ex. Find the elastic curve and V_{max} and Θ_{max}



1 obtain $M(x)$:-

F.B.D

in our region

w^* is unknown

$$\text{but } \frac{w_0}{L} = \frac{w^*}{x}$$

$$\Rightarrow w^* = \frac{w_0 x}{L}$$

$$\star \sum M_D = 0$$

$$M(x) = \frac{1}{2} \left(\frac{w_0 x}{L} \right) x \cdot \frac{x}{3}$$

$$= \frac{w_0 x^3}{6L} \quad \text{the } \ominus \text{ because } M \text{ is clockwise}$$

$$EI \frac{d^2v}{dx^2} = -\frac{w_0}{6L} x^3$$

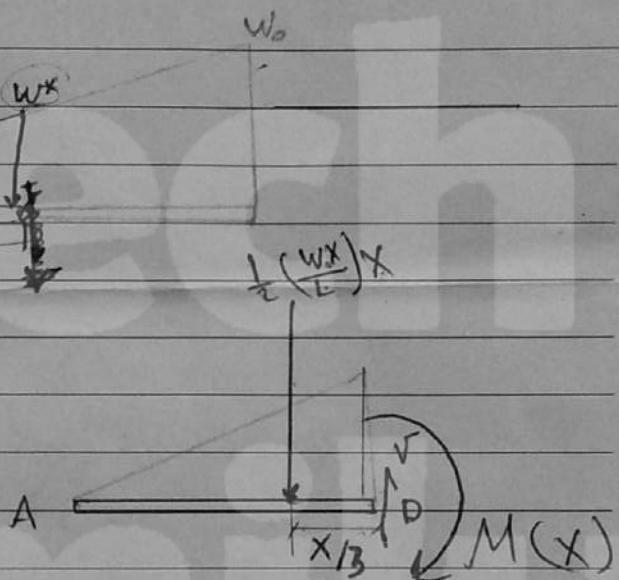
$$EI \frac{dv}{dx} = -\frac{w_0}{24L} x^4 + C_1$$

$$EI v = -\frac{w_0}{120L} x^5 + C_1 x + C_2$$

boundary cond.

$$\theta(L) = \frac{dv}{dx}(L) = 0 \rightarrow C_1 = \frac{w_0 L^3}{24}$$

$$v(L) = 0 \rightarrow C_2 = -\frac{w_0 L^4}{30}$$



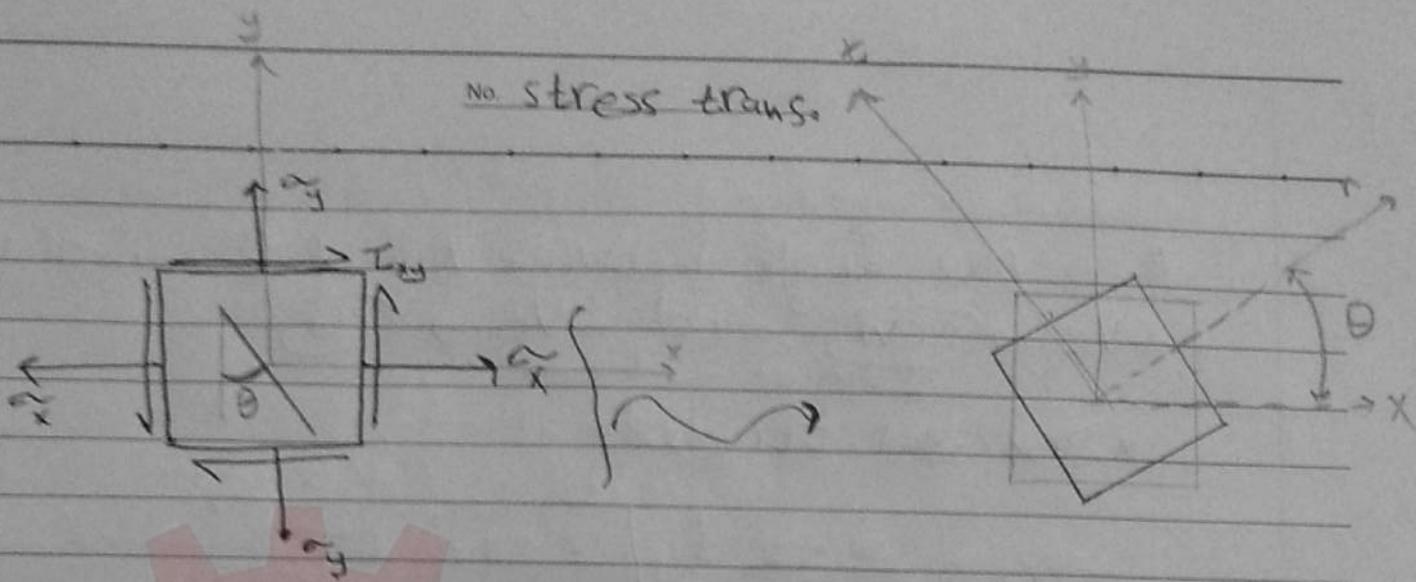
$$v = \frac{w_0}{120EI} (-x^5 + 5L^4 x - 4L^5)$$

$$V_{max} = v(0) = -\frac{w_0 L^4}{30 EI}$$

$$\text{also } \theta = \frac{w_0}{24EI} (L^4 - x^4)$$

$$\theta_{max} = \theta(0)$$

$$= \frac{dV(0)}{dx} = \frac{w_0 L^3}{24 EI}$$



$$* \sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

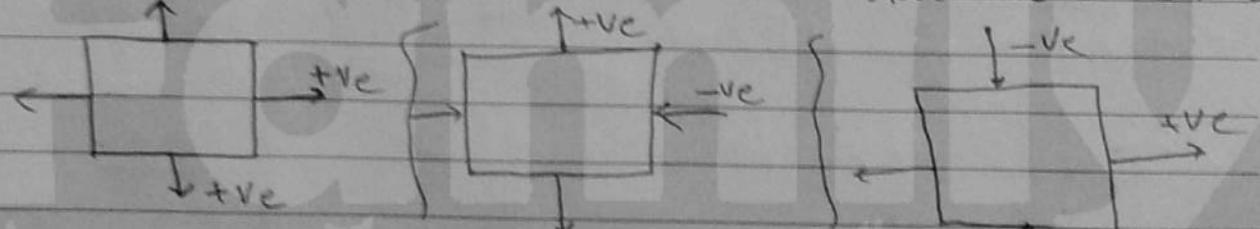
$$* \sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$* \tau'_{xy} = - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

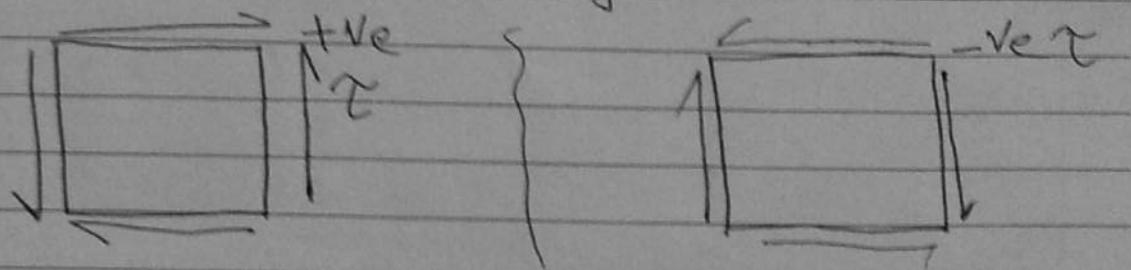
$$* \sigma_x + \sigma_y = \sigma'_x + \sigma'_y$$

Notes (Very very important)

- σ_x, σ_y are Positive in tension and -ve in comp.



τ_{xy} is positive when acting upward at the right



θ is the angle between \underline{x} and \underline{t} and it is
 +Ve \rightarrow (CCW) and -Ve \rightarrow (CW)

* Principal Stresses

$$-\alpha_{1,2} = \frac{\alpha_x + \alpha_y}{2} \pm \frac{1}{2} \sqrt{\left(\frac{\alpha_x - \alpha_y}{2}\right)^2 + \tau_{xy}^2}$$

$$-\tau_{n_b} = 0$$

$$-\tan 2\theta_p = \frac{2\tau_{xy}}{\alpha_x - \alpha_y} \quad \text{Op: } \theta \rightarrow \text{Principal stress}$$

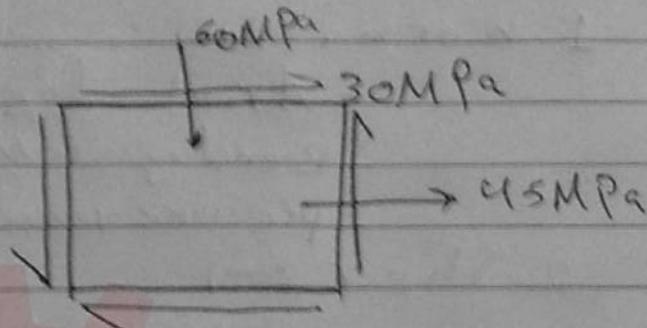
* Max. and min. shear stresses

$$-\tau_{\max, \min} = \pm \sqrt{\left(\frac{\alpha_x - \alpha_y}{2}\right)^2 + \tau_{xy}^2}$$

$$-\tan 2\theta_s = \frac{\alpha_y - \alpha_x}{2\tau_{xy}} \quad \text{or} \quad \theta_s = \theta_p + 45^\circ$$

$$-\alpha_n = \alpha_t = \alpha_{av} = \frac{\alpha_x + \alpha_y}{2}$$

Ex. Find the Principal stresses and their orientation and the maximum shear stresses and their orientation.



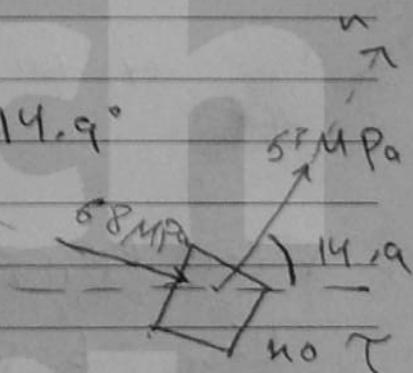
Given: $\sigma_x = 60 \text{ MPa}$, $\sigma_y = 45 \text{ MPa}$, $\sigma_z = 30 \text{ MPa}$

$$\tilde{\sigma}_1 = -7.5 + 60.5 = 53 \text{ MPa}$$

$$\tilde{\sigma}_2 = -7.5 - 60.5 = -68 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2 \times 30}{45 - 60} \Rightarrow \theta_p = 14.9^\circ$$

$$\tilde{\tau}_{xy} = 0$$



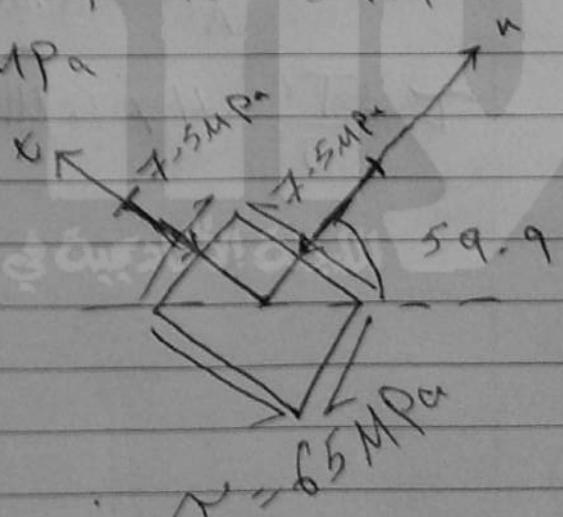
$$\sigma_{\max} = \pm 60.5 \quad \sigma_s = \sigma_p + 45 = 59.9$$

$$\tilde{\sigma}_3 = \tilde{\sigma}_1 = \tilde{\sigma}_2 = -7.5 \text{ MPa}$$

Chaitanya - Chaitanya



$$\sigma_x = 400 \quad \tau_{xy} = 550 \quad \tilde{\sigma}_y = ??$$



Mohr's circle.

First we have to determine $\sigma_x, \sigma_y, \tau_{xy}$

then we locate Points A, B, C
where

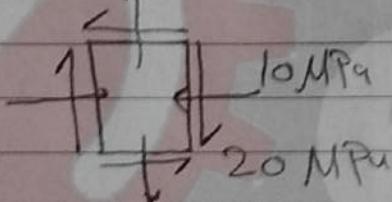
$$A(\sigma_x, \tau_{xy})$$

$$B(\sigma_y, -\tau_{xy})$$

$$C(\sigma_{av}, 0)$$

5 MPa

ex.



$$\sigma_x = -10 \text{ MPa}, \sigma_y = +5 \text{ MPa}, \tau_{xy} = -20 \text{ MPa}$$

$$\rightarrow \sigma_{av} = \frac{-10+5}{2} = -2.5$$

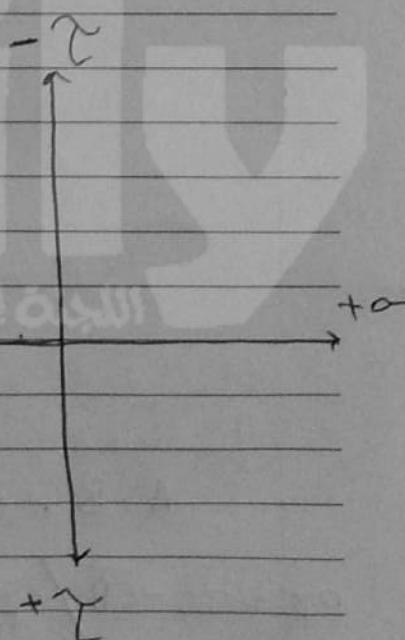
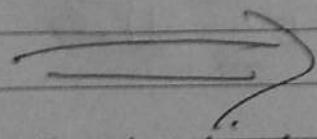
$$A(-10, -20)$$

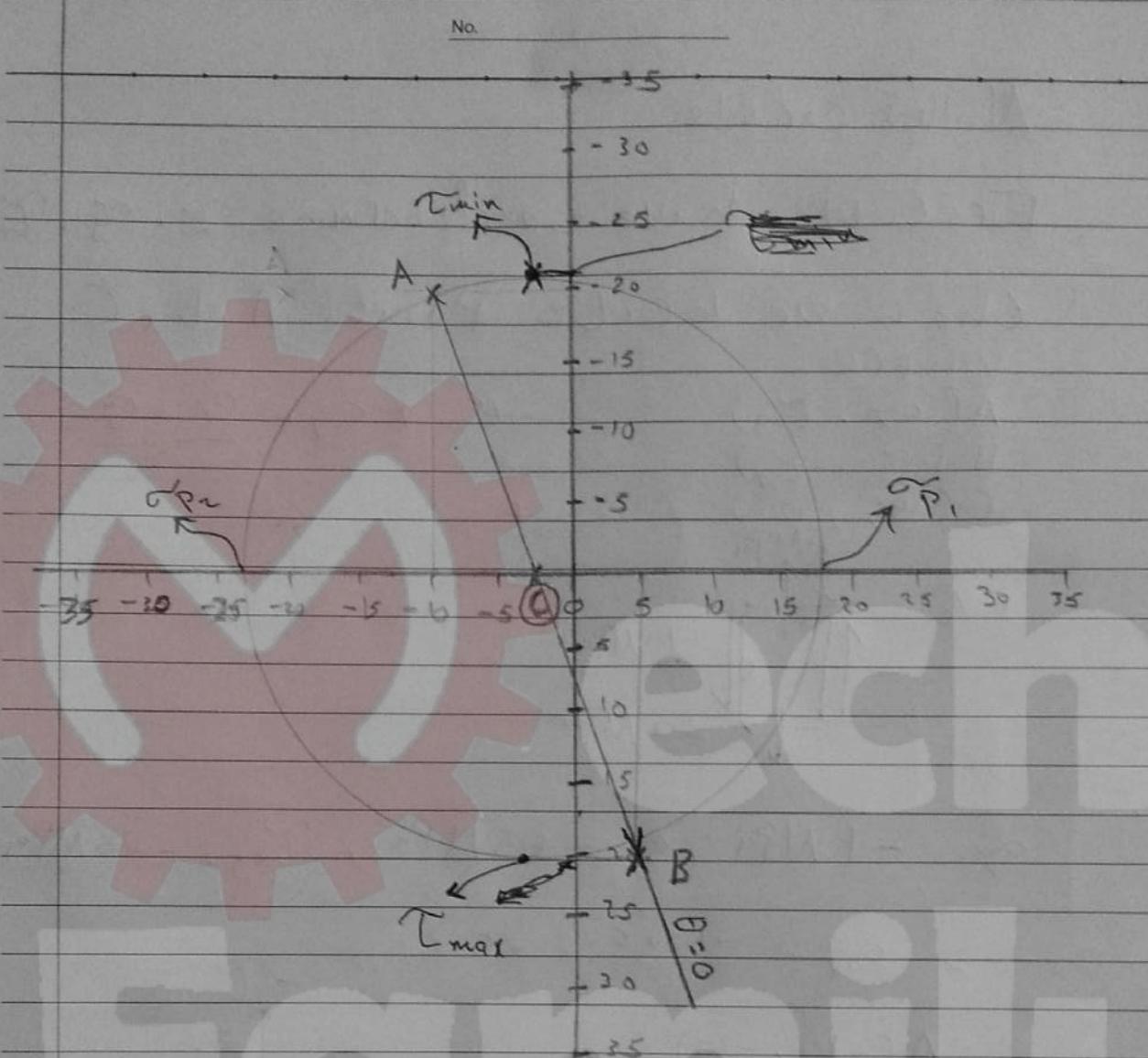
$$B(5, 20)$$

$$C(-2.5, 0)$$

then establish a coordinate system
with σ_x as x)

σ_y as y





الخط يتواءل على المسطرة ويمثل المسطرة
 $5MPa = 100 \text{ scale}$ في ميدان

لذلك نصل إلى معادلة A, B, C بحسب المسطرة
 أن قيم θ هي المطلقة في الأعلى والمنقصة في الأسفل

لذلك نذكر الزوايا في (C) ونرسم دائرة ثم نجد

$\sqrt{p_2} = \sqrt{p_1} - \sqrt{CB} / \sqrt{p_1} = \sqrt{p_1} + \sqrt{CB}$ ليكون \sqrt{CB} يمثل الخط
 ثم \sqrt{CB} هو الخط θ وعند $\theta = \theta_{\max}$

نوع \overline{CB} يُسمى حسب الميل المماس α فنعتبر الميل α $\theta_p = \alpha$

نوع \overline{CB} يُسمى حسب الميل المماس α $\theta_p = \alpha$ $\theta_p = \alpha$ $\theta_p = \alpha$

$$\theta_s = \theta_p + 45^\circ$$

الكلمة الأكاديمية في قسم الهندسة الميكانيكية - الجامعات الأهلية

No. _____

Combined loading types of loads

1- Normal σ -

- Axial $(\frac{P}{A}) = \sigma$

- Bending $(\frac{M_y}{I}) = \sigma$

- Pressure vessels. later

2- Shear τ -

- shear stress $(\frac{VQ}{It}) = \tau$

- torsional $(\frac{T_r}{J}) = \tau$

Add normal together and shear together

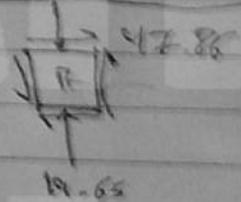
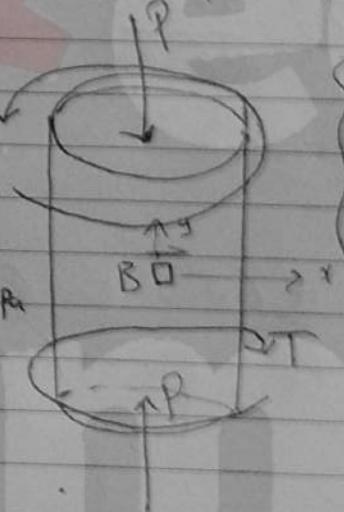
Ex to shaft $D_o = 114$ $D_i = 102$, $T = 5 \text{ kN.m}$ $P = 40 \text{ kN}$

Find σ_1, σ_2

τ_{\max} at B

$$\sigma_{\text{axial}} = \frac{P}{A} = 19.65 \text{ MPa}$$

$$\tau = \frac{T_r}{J} = 47.86 \text{ MPa}$$



$$\sigma_1 = 39.05$$

$$\sigma_2 = 58.68$$

$$\tau = 0$$

$$\tau_{\max} = 47.86$$

$$\sigma_n = \sigma_0 = \sigma_1 = 39.05$$

الاجمالي للحادي عشر قسم في الميكانيكا

ملاحظات هامة

يجب دراسته الـ pressure vessels من دوستة الدكتور هاشم الخالدي وحفظ القوانين الثلاث فيها

هام جداً وملزم: يجب دراسة جميع أسئلة الكومبايند لودينغ الموجودة على الميك موفيز دون استثناء لأن أسئلة الامتحان ستكون شبيهة جداً جداً

يجب حل مجموعة من الأسئلة على مور سيركل