25.5

The University of Jordan
Department of Mathematics

Eng. Math.(II)

First exam
21/10/2018

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Q1.(6 points) Let $\vec{F}(x,y,z)$ be a vector field and $f(x,y,z)$ be a scalar field in space.
Which of the following expressions are defined?
Is the result a scalar or vector field?
Complete the table by checking (✓) the entry that indicates the correct answer.

Formula	Defined	Not defined	Scalar	Vector
$curl(grad(div(\vec{F})))$	✓			✓
$curl(div(grad(f)))$		✓		
$grad(div(curl(\vec{F})))$	✓			✓
$grad(curl(div(\vec{F})))$		✓		
$div(curl(grad(f)))$	✓		✓	
$div(grad(curl(f)))$		✓		

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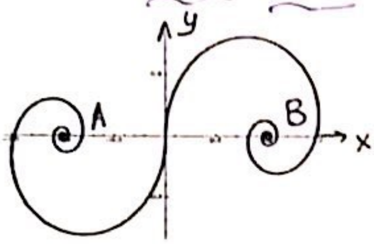
Q2.(6 points) **Find the line integral for the vector field** $\vec{F}(x,y) = (15x^4 - 2xy^2)\hat{i} + (e^y - 2x^2y)\hat{j}$ **along**

the ornamental curve $\vec{r}(t) = \left(\frac{t}{|t|}(1 - \frac{1}{1+t^2}\cos(4t))\right)\hat{i} + \left(\frac{1}{1+t^2}\sin(4t)\right)\hat{j}$

from A(-1,0) to B(1,0).



$$\text{curl}\,\vec{F} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (15x^4 - 2xy^2) & (e^y - 2x^2y) & (0) \end{vmatrix}$$

$$i(0) - j(0) + \hat{k}(-4xy + 4xy) \boxed{= 0}$$

so $\vec{F}$ is conservative field

$$\Rightarrow F_1 = \frac{df}{dx} = 15x^4 - 2xy^2 \Rightarrow f = 3x^5 - x^2y^2 +$$

$$\frac{\partial f}{\partial y} = F_2 = e^y - 2x^2y = 0 - 2x^2y + g'(y)$$

$$\boxed{g(y) = e^y}$$

$$\frac{\partial f}{\partial z} = F_3 = 0 \Rightarrow h(z) = C$$

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$$\Rightarrow f = 3x^5 - x^2y^2 + e^y + C$$

$$\Rightarrow f(-1,0) - f(1,0)$$

$$= (-3 - 0 + 1 + \cancel{C}) - (3 - 0 + 1 +$$

$$\boxed{= -6}$$

$\divideontimes$

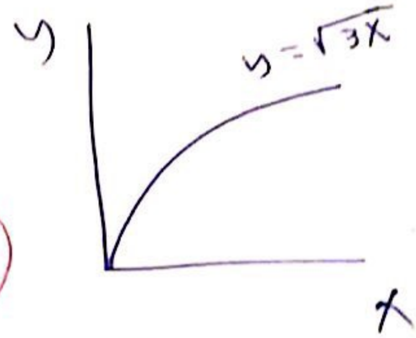
$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -yz & x & xy^2z \end{vmatrix}$$

$i(2xyz - 0) + j(y^2z - y) + k(1 - z)$

Q3.(6 points) **Evaluate** $\int_C \vec{F}\cdot d\vec{r}$ **, where** $\vec{F}(x,y,z) = [-yz,\ x,\ xy^2z]$

and C: $y^2 = 3x$, $z = 1$

from the point $(3,3,1)$ **to** $(1,\sqrt{3},1)$.



let $x = x$, $z = 1$
$y = \sqrt{3x}$

$r(x,y) = [x, \sqrt{3x}, 1]$

(2)

$Fr(x,y) = [-\sqrt{3x},\ x,\ 3x^2]$

$r'(x,y) = [1,\ \frac{\sqrt{3}}{2\sqrt{x}},\ 0]$

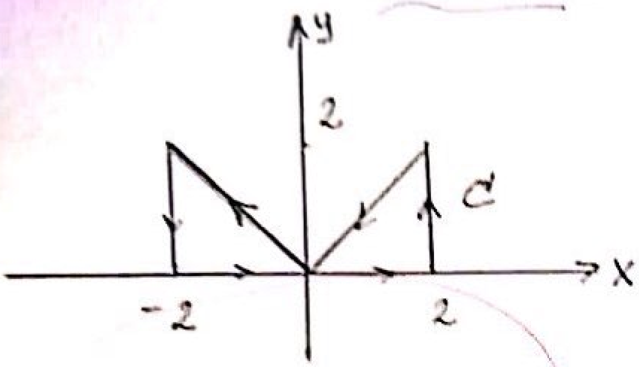
$\int\int Fr(x,y)\cdot r'(x,y)\,dx\,dy$

$\int_3^{\sqrt{3}}\int_3^1 -\sqrt{3x} + \frac{\sqrt{3}\,x}{2\sqrt{x}}\,dx\,dy$

$\int_3^{\sqrt{3}} -\sqrt{3}\times x^{\frac{3}{2}}\times\frac{2}{3} + \frac{\sqrt{3}}{2}\,\frac{x^{3}}{2}\times\frac{2}{3}\Big|_3^1\,dy$

Q4.(6 points)Evaluate $\oint_C \vec{F} \cdot d\vec{r}$, **where**

$\vec{F}(x,y,z) = [\underbrace{\cos x + 5y}_{F_1},\ \underbrace{8x + e^y + \sin(y^2+3)}_{F_2}]$ **and**
C is shown in the figure



$\frac{dF_2}{dx} = 8$

$\frac{dF_1}{dy} = 5$

Using Stokes Th.

$$\iint_R \frac{dF_2}{dx} - \frac{dF_1}{dy} \, dR$$

$$= \iint_R (8-5) \, dR$$

$$3 \iint_R dR \Rightarrow \text{Area of Rigon}$$

$$3 * [2 * (\tfrac{1}{2} * 2 * 2] = \boxed{12}$$

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Q5. (6 points) Evaluate $\iint_S (z + x^2 y)\,dA$, where

S is the cone $z = \sqrt{x^2 + y^2}$ for $0 \le z \le 2$.

$2\pi \ge u \ge 0$

$2 \ge v \ge 0$

~~let x = x , y = y , z =~~

$r(u,v) = [u\cos v, u\sin v, u]$

~~$N = \{r_u \times r_v\} =$~~

$\vec{r}_u = [\cos v, \sin v, 1]$

$\vec{r}_v = [-u\sin v, u\cos v\ 0]$

≈ 5.7

$$N = |r_u \times r_v| = \begin{vmatrix} i & j & \hat{k} \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$= i(-u\cos v) - j(u\sin v) + k(u\cos v^2 + u\sin^2 v$

$= [-u\cos v, -u\sin v, u]$

$\Rightarrow \iint_S F(r_{u,v})\,|r_u \times r_v|\,ds$

$|r_u \times r_v| = \sqrt{u^2\cos^2 v + u^2\sin^2 v}$

$= \sqrt{2}\,u$

~~$\int_0^2\int_0^{2\pi} [u\cos v, u\sin v, u] \cdot \sqrt{2}\,u\ du\,dv$~~

$\Rightarrow \int_0^{2}\int_0^{2\pi} (u + u^3\cos v\sin v) \cdot \sqrt{2}\,u\ du\,dv$

$= \iint \sqrt{2}\,u^2 + \sqrt{2}\,u^4\cos v\sin v\ du\,dv$

$u^2(\cos^2 v + \dots)$

خلف الورقة