University of Jordan
Department of Mathematics
Second Exam

$17\frac{1}{2}$

Math. Engineering-II: 0301302 Date: 3/5/2012

Name: [redacted]

Number: ....

Q[1]) Use the divergence theorem to find the surface Integral $\iint_s \vec{F}.\hat{n}\,dA$, where $\vec{F} = y^2 i + x^2 j + z^2 k$ and S is the surface of the cone $z = \sqrt{x^2+y^2}$ that lies below the plane $z=2$. (6 points)

$\boxed{1st}$ step

$$\iint_s \bar{F}.\hat{n}\,dA = \int_0^{2\pi}\int_0^{2}\int_{r}^{2} 2z \; r\,dz\,dr\,d\theta \; - \iint_{s} F_j .\hat{n}\,dA$$

Div(F) = $0+0+\boxed{2z}$

$$= \int_0^{2\pi}\int_0^{2} \frac{\cancel{2}z^2}{\cancel{2}}r\Big]_{r}^{2} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{2} (4r - r^3)\,dr\,d\theta$$

dV = r dz dr dθ = dx dy

$$= \int_0^{2\pi} 2r^2 - \frac{r^4}{4}\Big]_0^2 d\theta$$

$$= \int_0^{2\pi} 8-4\,d\theta$$

$$= 8\pi$$

2nd Step

$S=2 \quad \nabla S = k$

$$\text{Now} \quad \iint F_s .\hat{n}\,dA = \int_0^{2\pi}\int_0^{2} 4r\,dr\,d\theta = \int_0^{2\pi} \frac{4r^2}{2}\Big]_0^2 d\theta$$

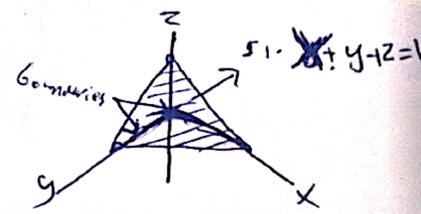
$$= 16\pi$$

$$\boxed{\text{Now} \quad \iint \bar{F}.\hat{n}.dA = 8\pi}$$

Q₂) Use stokes theorem to evaluate $\oint_C \vec{F}\cdot d\vec{r}$ where $F = xyi + yzj + zxk$ and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ oriented in the positive direction.

(6 points)

Now:-



$$\oint_C \bar{F}\cdot d\bar{r} = \iint (\nabla \times F)\cdot \hat{n}\, dA$$

$$\nabla \times \bar{F} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xy & yz & zx \end{vmatrix} = (0-y)\hat{i} - (z-0)\hat{j} + (0-x)\hat{k}$$

$$= -y\hat{i} - z\hat{j} - x\hat{k}$$

Now

$$\iint (-y\hat{i} - z\hat{j} - x\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})\, dx\, dy$$

because of the boundaries drawn

$$= \iint -y - z - x \, dx\, dy = \int_0^1\int_0^{1-y} -y - x \, dx\, dy$$

$$= \int_0^1 -yx - \frac{x^2}{2}\Big]_0^{1-y} dy$$

$$= \int_0^1 -y + y^2\, dy - \int_0^1 \frac{1}{2}(1 - 2y + y^2)\, dy$$

$$= -\frac{y^2}{2} + \frac{y^3}{3} - \frac{1}{2}\left(y - y^2 + \frac{y^3}{3}\right)\Big]_0^1 = -\frac{1}{2} + \frac{1}{3} - \frac{1}{2}\left(1 - 1 + \frac{1}{3}\right)$$

$$= \boxed{\frac{-2}{6}}$$

Q₃) (a) Let $f(x)=\begin{cases}2, & -2<x<0\\ 3, & x=0\\ x, & 0<x<2\end{cases}$. Show that the Fourier of $f(x)$ is given by

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$$f(x)=\frac{3}{2}-\frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos\left[(2n+1)\frac{\pi x}{2}\right]}{(2n+1)^2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\sin\frac{n\pi x}{2}.$$ (4 points)

$-2\frac{1}{2}$

(b) Show that $\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}$. (2 points)

(a) $a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{n\pi x}{2}\right)dx = \frac{1}{2}\left(\int_{-2}^{0} 2\cos\left(\frac{n\pi x}{2}\right)dx + \int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx\right)$

$= \frac{1}{2}\left(\frac{4}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Big]_{-2}^{0} + \left(\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Big]_0^2 - \int_0^2 \frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)dx\right)\right)$

$= \frac{1}{2}\left(+\frac{2}{n\pi}\sin(n\pi) + \frac{2}{n\pi}\sin(n\pi) + \frac{4}{n^2\pi^2}\cos(n\pi)\right)$

So $a_n = \frac{4}{2n^2\pi^2}\cos(n\pi) = \boxed{\frac{-4}{n^2\pi^2}}$,

Now $a_0 = \frac{1}{2(2)}\int_{-2}^{2} f(x)\,dx = \frac{1}{4}\left(\int_{-2}^{0} 2\,dx + \int_0^2 x\,dx\right)$

$= \frac{1}{4}\left(4 + \frac{x^2}{2}\Big]_0^2\right) = \frac{6}{4} = \boxed{\frac{3}{2}}$

Now $b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\left(\frac{n\pi x}{2}\right)dx = \frac{1}{2}\left(\int_{-2}^{0} 2\sin\left(\frac{n\pi x}{2}\right) + \int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx\right)$

$= \frac{1}{2}\left(\frac{-4}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big]_{-2}^{0} + \left(\frac{-2x}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big]_0^2 + \int_0^2 \frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)dx\right)$

$+ \text{zero})$

$= \frac{1}{2}\left(\frac{-2}{n\pi}\cos(-n\pi) - \frac{2}{n\pi}\cos(n\pi) + \text{zero}\right)$

$= \frac{-4}{2n\pi}\left(\cos(-n\pi) + \cos n\pi\right) = \boxed{\frac{-2}{\pi}}$

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Q_4) (a) Let $f(x)=\begin{cases}\cos x, & 0<x<\pi \\ 0 \quad , & x>\pi\end{cases}$. Show that $f(x)=\frac{2}{\pi}\int_0^\infty \frac{\cos \pi \cancel{w}x \sin \cancel{\pi x}w}{1-w^2}\,dw$. (4 points)

(b) Show that $\int_0^\infty \frac{\sin 2\pi w}{w^2-1}=\frac{\pi}{2}$.

(2 points)

$$A(w) = \frac{1}{\pi}\int_0^\infty f(x)\cos(wx)\,dx = \frac{1}{\pi}\left(\int_0^\pi \cos(x)\cos(wx)\,dx + \int_\pi^\infty \cancel{zero\,dx}\right)$$

$$= \frac{1}{\pi}\left(\int_0^\pi \cos(x)\cos(wx)\,dx\right)$$

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$$B(w) = \frac{1}{\pi}\int_0^\infty f(x)\sin(wx)\,dx = \frac{1}{\pi}\left(\int_0^\pi \cos(x)\sin(wx)\,dx + \cancel{zero}\right)$$

$$f(x) = \int_0^\infty A(w)\cos(wx) + B_w \sin(wx)\,dw$$

Q5) (a) Show that $F_C\{f''\} = -w^2 F_C\{f\} - \sqrt{\frac{2}{\pi}} f'(0)$.

(6 points)

(b) Use the formula in (a) to find the Fourier cosine transform of $f(x) = e^{-kx}, k > 0$.

$$F_c\{f''\} = \sqrt{\frac{2}{\pi}} \int_0^\infty f'' \cos(wx)\, dx$$

-4

$$= \sqrt{\frac{2}{\pi}} \lim_{z\to\infty} \int_0^z f'' \cos(wx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \lim_{x\to\infty} \left(\frac{f'}{w} \sin(wx) \Big]_0^x + \int_0^x \frac{\sin(wx)}{w} dx \right)$$

$$= \sqrt{\frac{2}{\pi}} \lim_{x\to\infty} \left(\frac{f'}{w} \sin wx - \frac{\cos(wx)}{w^2} \right)$$

$$\sqrt{\frac{2}{\pi}} f'(0)$$