18

The University of Jordan | **Department of Mathematics**

Student's Name:

Student's Number: [redacted]

Instructor's Name:

Class Time: 9:30 – 11:00

Second Exam ◇ Engineering Mathematics II (0331302) ◇ Summer 2017

Note: This exam is composed of 4 questions. You have 60 minutes to finish

Question 1: Use Stokes' theorem to evaluate

$f(z, x, y)$

$$\oint_C z\,dx + x\,dy + y\,dz$$

where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counter-clockwise as viewed from above.

$z = 2 - y$

$r(u,v) = [u, v, 2 - v]$

$N = r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$= 0\hat{i} + 1\hat{j} + \dots$

$= (0, 1, 1)$

$\sqrt{1^2 + 1^2} = \sqrt{2}$

$\hat{n} = \left(\frac{0}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\hat{n} = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$r(u) = [2 - v, u, v]$

$r'(u) = [$

$\cancel{\oint_C f \cdot dr} = \int \cancel{f(r(u)) \cdot r'(u)}\, du$

$= \iint_S \text{curl} f \cdot \hat{n}\, dA$

$= \int_0^{2\pi}\int_0^{2-y} [1, 1, 1] \cdot \left[0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

$= \int\int_0 \left[0 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] dv\, du$

$= \int_0^{2\pi} \frac{2-u}{2\sqrt{2}}$

$= \frac{2 - u^2}{4\sqrt{2}} \Big|_0^{2\pi}$

$= \frac{2\pi^2}{\sqrt{2}}$

1.5

Question 2:

$e^{ax} \cdot \sin x = \frac{e^{ax}}{a^2+b^2}(a \sin bx + b \cos bx)$

(a) Find the Fourier sine transform of $f(x) = e^{-|x|}$.

$\cancel{f_s(\omega) = \{}$

$$f_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-|x|} \cdot \sin \omega x$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{(-1)^2 + 1^2} (\sin x + \cos x) \right]_0^\infty$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{2} \right]$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{e^{-x}}{2}$$

1

(b) Use the result in [illegible] t (a) t[illegible]te [illegible] ∞ [illegible] nm [illegible] dx [illegible] 1)

$\int \frac{x \sin}{\pi^2 + \omega^2}$

$\frac{\pi}{2} e^{-3\sqrt{\pi}}$

$= \int \frac{x \sin 3x}{\pi^2 + x^2}$

$$= \int_0^\infty \frac{x \sin mx}{1 + x^2} dx$$

$$= \frac{1}{2} \cdot e^{-m\sqrt{1}}$$

$$= \frac{1}{2} \cdot e^{-m}$$

Question 3:

(a) Find the Fourier cosine transform of $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1, \\ 0 & \text{for } x > 1. \end{cases}$

1.5

$$f_c(w) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos wx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^1 1 \cdot \cos wx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \cdot \left[\frac{\sin wx}{w} \right]_0^1$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{w}$$

3

(b) Use the result ... part (... al ... e $\int_0$ si ... d

$$f(x) = \int_0^\infty \frac{\sin x}{x} \cos wx = \frac{\pi}{2} f(x) \begin{cases} \frac{\pi}{2} * 1, & x < 0 \\ \frac{\pi}{2} * \frac{1}{2}, & x = 0 \\ \frac{\pi}{2} * 1 & x > 0 \end{cases}$$

$$x = 0 > \int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2} f(x) = \frac{\pi}{2} f(1) = \frac{\pi}{2}$$

Question 4: Solve the PDE

$$
\begin{aligned}
&u_{tt}(x,t) = c^2 u_{xx}(x,t) && \text{for all } 0 < x < 1 \text{ and } t > 0,\\
\text{B.C} \quad & u(0,t) = u(1,t) = 0 && \text{for all } t > 0,\\
\text{I.C} \quad & u(x,0) = \sin(5\pi x) + 2\sin(7\pi x) && \text{for all } 0 < x < 1,\\
& u_t(x,0) = 0 && \text{for all } 0 < x < 1.
\end{aligned}
$$

5

$u_{tt} = c^2 u_{xx}$

$u(x,t) = F(x)\,G(t)$

$\dfrac{G''F}{FG} = \dfrac{c^2 F'' G}{FG} \quad \div FG$

$\dfrac{G''}{G} = c^2 \dfrac{F''}{F} = k$

$F'' - kF = 0$

$G'' - kc^2 G = 0$

$u(0,t) = F(0)\,G(t) = \ldots$

$u(1,t) = F(1)\ldots$

$= f(1)\ldots$

$f'' - kf = 0$

If $k = 0$

$F'' = 0 \qquad f(x) = Ax + B$

$f(0) = B = 0$

$f(1) = A = 0$

$f(x) = 0$ trivial solution

If $k = p^2 > 0$

$F'' - p^2 F = 0$

$r^2 - p^2 = 0$

$r = \pm p$

$f(x) = A e^{px} + B e^{-px}$

$f(0) = A + B = 0 \Rightarrow A = -B$

$f(x) = -B e^{px} + B e^{-px}$

$= B(e^{px} - e^{-px})$

$f(1) = B = 0 \qquad f(x)$ is trivial solution

If $k = -p^2 < 0$

$F'' + p^2 F = 0$

$r^2 + p^2 = 0 \Rightarrow r = \pm i p$

$f(x) = A\cos px + B\sin px$

$f(0) = A = 0$

$f(1) = B\sin px \qquad px = n\pi, \quad p = \dfrac{n\pi x}{L}$

$f_n(x) = \sin\dfrac{n\pi x}{L}$

Step II $\quad G'' + p^2 c^2 G = 0 \qquad$ let $\lambda_n = \dfrac{n^2\pi^2 x^2}{L^2}$

$G'' + \dfrac{n^2\pi^2 x^2}{L^2} G = 0$

$G'' + \lambda_n^2 G = 0 \Rightarrow r^2 + \lambda_n^2 = 0 \Rightarrow r = \pm\lambda_n i$

$\ldots \cos\lambda_n t + B_n^* \sin\lambda_n t$

$u(x,t) = [B_n \cos\lambda_n t + B_n^* \sin\lambda_n t]\cdot\sin\dfrac{n\pi x}{L}$

I.C

$u(x,0) = [B_n\cos\lambda_n t]\cdot\sin\dfrac{n\pi x}{L}$

$= B_n \sin\dfrac{n\pi x}{L}$

$\ldots \sin\dfrac{n\pi x}{1}$

~~$u(x,0) = \sin\frac{n\pi x}{L}$~~

$u(x,0) = \sin(n\pi x) + 2\sin(n\pi x) + 3\sin(n\ldots$

$= \sin(5\pi x) + 2\sin(7\pi x) + \ldots$

$u_t(x,0) = [-B_n\lambda_n\sin\lambda_n t + B_n^*\lambda_n\cos\lambda_n t]\cdot\sin\dfrac{n\pi x}{L}$

$= [B_n^*\lambda_n]\cdot\sin\dfrac{n\pi x}{L}$

$\lambda_n B_n^* = \dfrac{2}{L}\displaystyle\int_0^L f(x)\cdot\sin\dfrac{n\pi x}{L}$

$B_n^* = \dfrac{2}{cn\pi L}\displaystyle\int_0^L f(x)\cdot\sin\dfrac{n\pi x}{L}$

$= \dfrac{2}{cn\pi L}\displaystyle\int_0^L \ldots \sin\dfrac{n\pi x}{L} = 0$

4