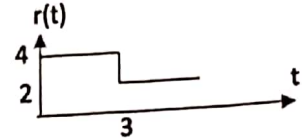


Problem-1 (5pts)

- Given the dynamical system $2\ddot{r}(t) - 6\dot{y}(t) = 8\dot{y}(t)$ find the following
 - Stability
 - Step response due to $r(t)=3$ (must solve and graph)
 - Steady state output when subjected to the following $r(t)$



$$4\ddot{y}(t) + 3\dot{y}(t) = r(t) \quad / \quad \mathcal{L} \text{ both sides}$$

$$\Rightarrow 4sY(s) + 3Y(s) = R(s)$$

$$\Rightarrow Y(s)[4s+3] = R(s)$$

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{1}{4s+3} = G(s)$$

[a] to ~~one~~ check the stability we need to find the Denominator.

$$4s+3=0$$

$$s = -\frac{3}{4} \Rightarrow \text{Pole } \left\{ -\frac{3}{4} \right\} \Rightarrow \text{it is Asymptotically stable}$$

[b] $Y(s) = G(s) * R(s) \Rightarrow \frac{3}{s} \cdot \frac{1}{4s+3} = \frac{3/3}{s(\frac{4}{3}s+1)}$

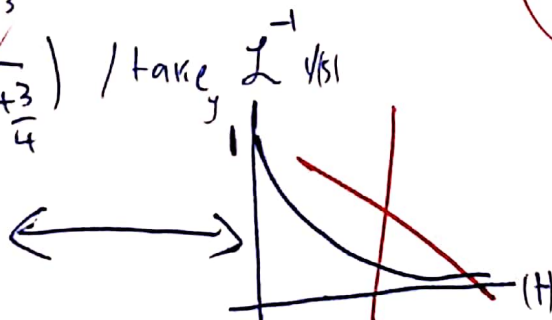
$$\frac{1}{s(\frac{4}{3}s+1)} = \frac{A}{s} + \frac{B}{\frac{4}{3}s+1}$$

$$A \Rightarrow \frac{1}{(\frac{4}{3}s+1)} \Big|_{s=0} = 1, \quad B = \frac{1}{s} \Big|_{s=-\frac{3}{4}} = -\frac{4}{3}$$

$$\Rightarrow Y(s) = \frac{1}{s} - \frac{4}{3} \left(\frac{1}{\frac{4}{3}s+1} \right)$$

$$Y(s) = \frac{1}{s} - 1 \left(\frac{1}{s+\frac{3}{4}} \right) \quad / \quad \text{take } \mathcal{L}^{-1} Y(s)$$

$$y(t) = 1 - e^{-\frac{3}{4}t}$$



[c] 1) when subjected to $r(t)=4$

$$Y(s) \Rightarrow \frac{4}{s} \cdot \frac{1}{4s+3}$$

$$Y_{s.s} = \lim_{s \rightarrow 0} s Y(s) = \frac{4}{3}$$

2) when subjected to $r(t)=2$

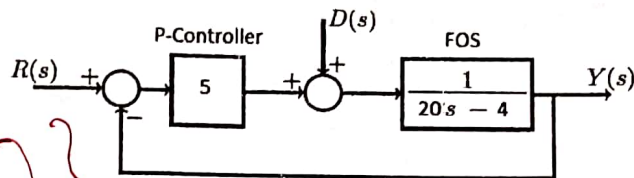
$$Y(s) = \frac{2}{s} \cdot \frac{1}{4s+3}$$

$$Y_{s.s} = \lim_{s \rightarrow 0} s Y(s) = \frac{2}{3}$$



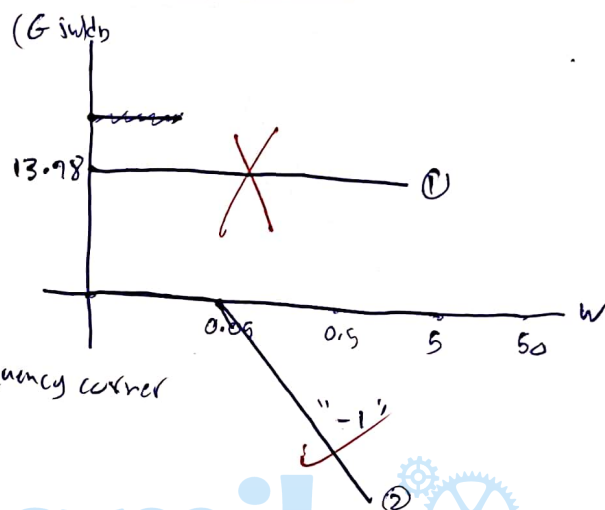
problem-2 (5pts)

- Given the block diagram
 - Draw the Bode plot of the output and disturbance
 - How does the system behave to disturbance below a certain frequency?
 - Why the controller was used in this application?



$$G(s)_{\text{sys}} = \frac{K_p G_s}{1 + K_p G_s H(s)} = \frac{5}{20s - 4} \cdot \frac{1}{1 + \frac{5}{20s - 4}}$$

$$\Rightarrow \frac{5}{20s + 1} = G(s) = \frac{5}{0.05s + 1} \quad \text{Frequency corner}$$

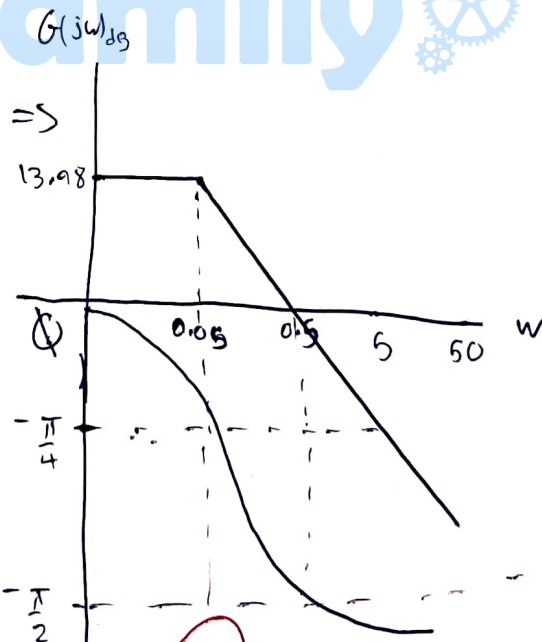


$$|G(s)|_{\text{db}} = 20 \log(5) - 20 \log |20s + 1|$$

$$|G(j\omega)| = 13.98 - 20 \log |20j\omega + 1|$$

$$= 13.98 - 20 \log \sqrt{1^2 + \left(\frac{\omega}{0.05}\right)^2}$$

- 2 It will be more stable and less disturbance at low frequency.



- 3 A. beat the disturbance.

B. to stabilize the unstable system.

C. enhance the performance.

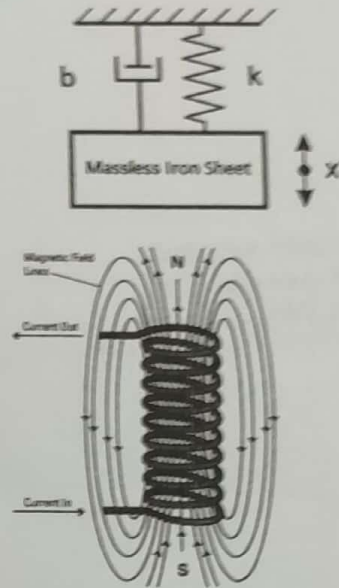
before use P.c $\Rightarrow$ Pole = $\left\{\frac{4}{20}\right\}$ so unstable.

After use P.c $\Rightarrow$ Pole = $\{-0.05\}$ so stable.

Problem-3 (5pts)

- Given the following dynamical electro-mechanical system Where a mass-less iron sheet is subjected to an electrical Magnetic field as shown. Assuming a proportional to current electromagnetic force is generated by the coil. Find

- Transfer function $G = \frac{X(s)}{V(s)}$ where V is the coil input voltage and X is the sheet vertical displacement
- Draw a block diagram of the DEs



a) $\Sigma T = m\ddot{x}$

$\Rightarrow \cancel{V_{in}} - C\dot{x} - Kx = 0$ mass less

$\Rightarrow V(s) = sX(s)C + X(s)K$

$\Rightarrow V(s) = X(s)[Cs + K]$

$\Rightarrow \frac{X(s)}{V(s)} = \frac{1}{Cs + K} = G(s) = \frac{1/K}{Ts + 1}$

