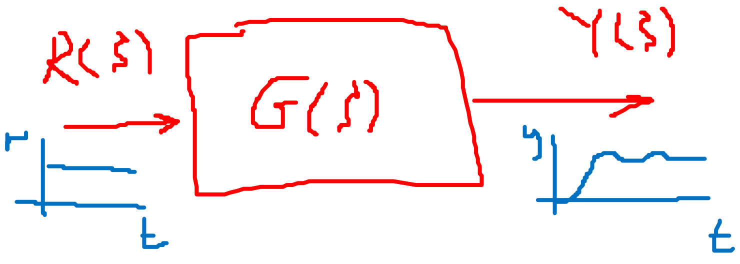
Frequency Domain Design

Frequency Response Function (FRF)

$s = \cancel{\sigma} + j\omega$, $\sigma \to 0$

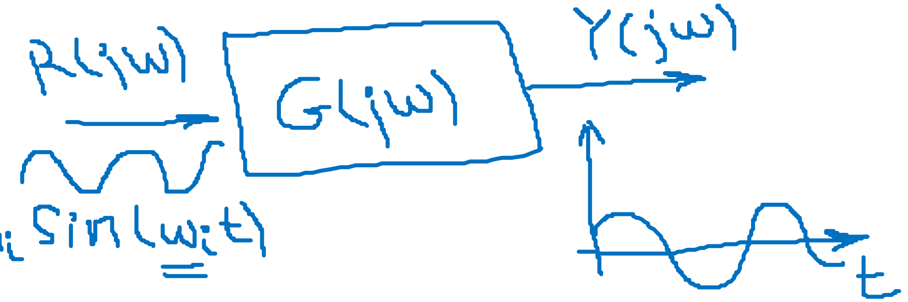
Stable -ve

Steady State



Time Domain

Resp. $e^{\sigma t} \xrightarrow{t \to \infty} 0$



$A_i \sin(\omega_i t)$

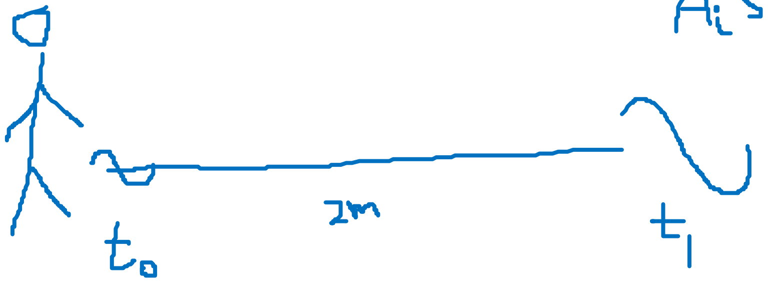
$A_o \sin(\omega_i t + \phi)$

Freq. Response

FRF

Graph

$\cancel{\omega}$ 0.001 ~ ∞ → $A_o(\omega)$, $\phi(\omega)$



Frequency Response Function

FOS $\quad G(s) = \frac{K_{dc}}{\tau s + 1}$ ✓

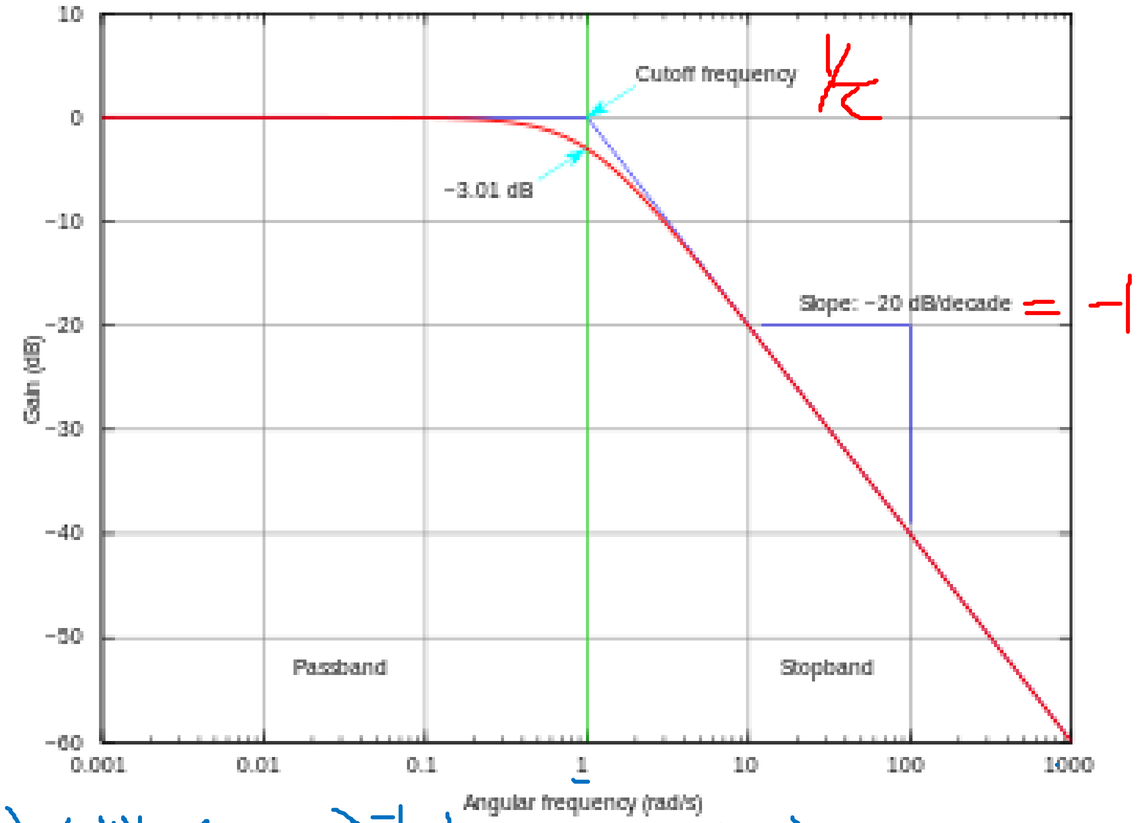
$$G(j\omega) = \frac{K_{dc}}{\tau \omega j + 1} = K_{dc}\left(\frac{\omega}{1/\tau} j + 1\right)^{-1}$$

def: Unit of Amplitude dB

$$dB \triangleq 20\log\frac{|output|}{|input|}$$

def: Units of Freq. ratio

dec $\quad \frac{\omega_2}{\omega_1} = 10$



$$|G(j\omega)|_{dB} = 20\log[\cdot] = 20\log|K_{dc}| + 20\log\left|\left(\frac{\omega}{1/\tau} j + 1\right)^{-1}\right| = A_0(\omega)$$

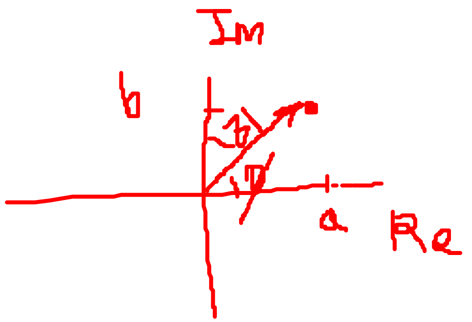
Curve 1

$$A_{01} = 20\log|K_{dc}|$$

$$\phi = 0$$

Rev. complex

$$Z = a + bj$$



$$|Z| = \sqrt{a^2 + b^2}$$

$$\angle Z = \tan^{-1}(b/a) = \phi$$

Curve 2

$$A_{02} = -20\log\left|\frac{\omega}{1/\tau}j + 1\right|$$

$$= -20\log\sqrt{1 + \left[\frac{\omega}{1/\tau}\right]^2} \quad \text{dB}$$

Cases

- $\omega \ll 1/\tau \longrightarrow A_{02} \sim$ Zero
- $\omega = 1/\tau \longrightarrow A_{02} = -20\log(2)^{\frac{1}{2}} \simeq -3\,\text{dB}$
- $\frac{\omega}{1/\tau} = 10$ 1dec $\longrightarrow A_{02} \cong -20\log(10) \simeq -20\,\text{dB/dec}$
  $= 100$ 2dec $\longrightarrow A_{02} \simeq -40\,\text{dB}/2\text{dec} = -20\,\text{dB/dec}$

Bode Plot of a Pole

$$\frac{K_{dc}}{\tau s + 1}$$

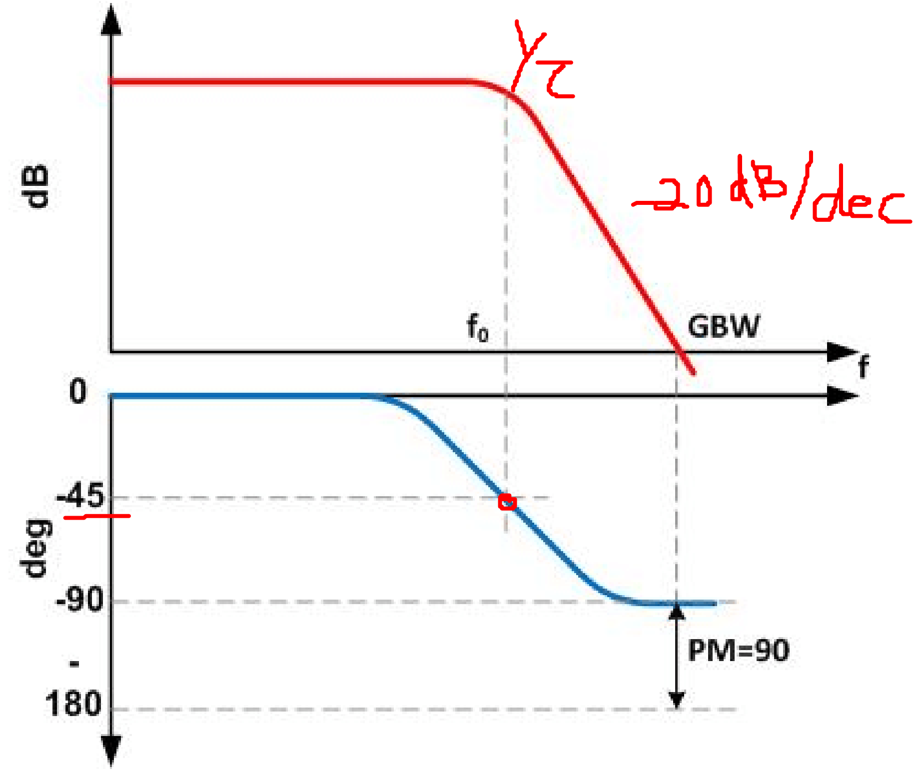
$$1 + j\frac{\omega}{1/\tau}$$

$$\phi = \tan^{-1}\left[\omega/\tfrac{1}{\tau} \,/\, 1\right]$$

$$= \tan^{-1}\left[\frac{\omega}{1/\tau}\right]$$

Cases

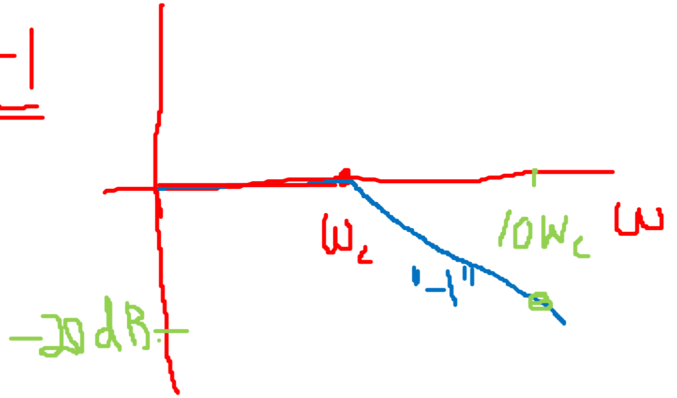
- $\rightarrow \omega << 1/\tau \rightarrow \phi \rightarrow 0$
- $\rightarrow \omega = 1/\tau \quad \phi = \frac{\pi}{4}$
- $\rightarrow \omega >> \frac{1}{\tau} \rightarrow \phi \rightarrow \pi/2$

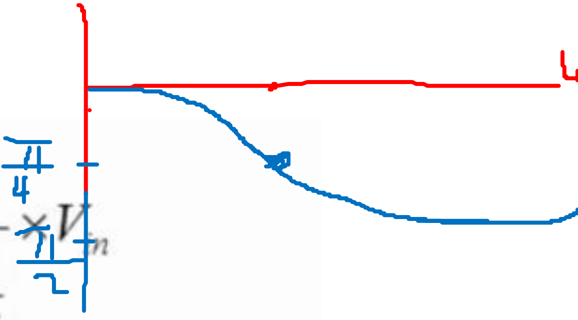


Application

$$G(j\omega) = \left(\frac{\omega}{\omega_c} j + 1\right)^{-1}$$

$$|G|_{dB} = 20\log|\cdot|$$



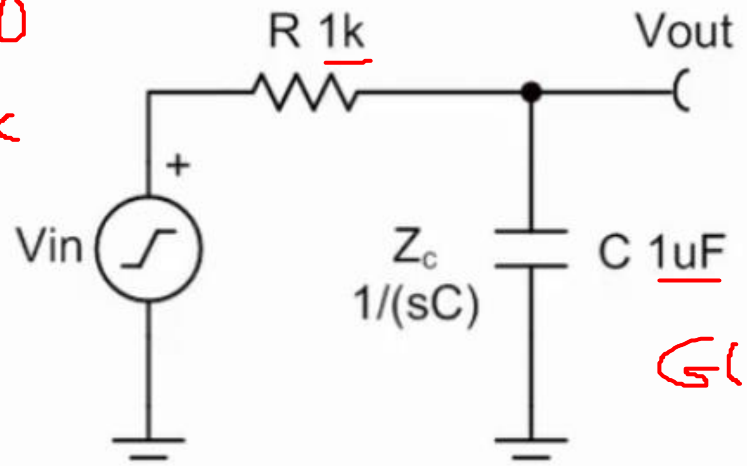


$\tau = RC$

$= 10^3\ 10^{-6}$

$= 10^{-3}$ sec

$\omega_c = 10^3$ Hz



$$V_{out} = \frac{Z_C}{R+Z_C}\times V_{in} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}\times V_{in}$$

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{1}{sRC+1} = \frac{1}{\frac{s}{\frac{1}{RC}}+1} = \frac{1}{\frac{s}{\omega_c}+1}$$

τ

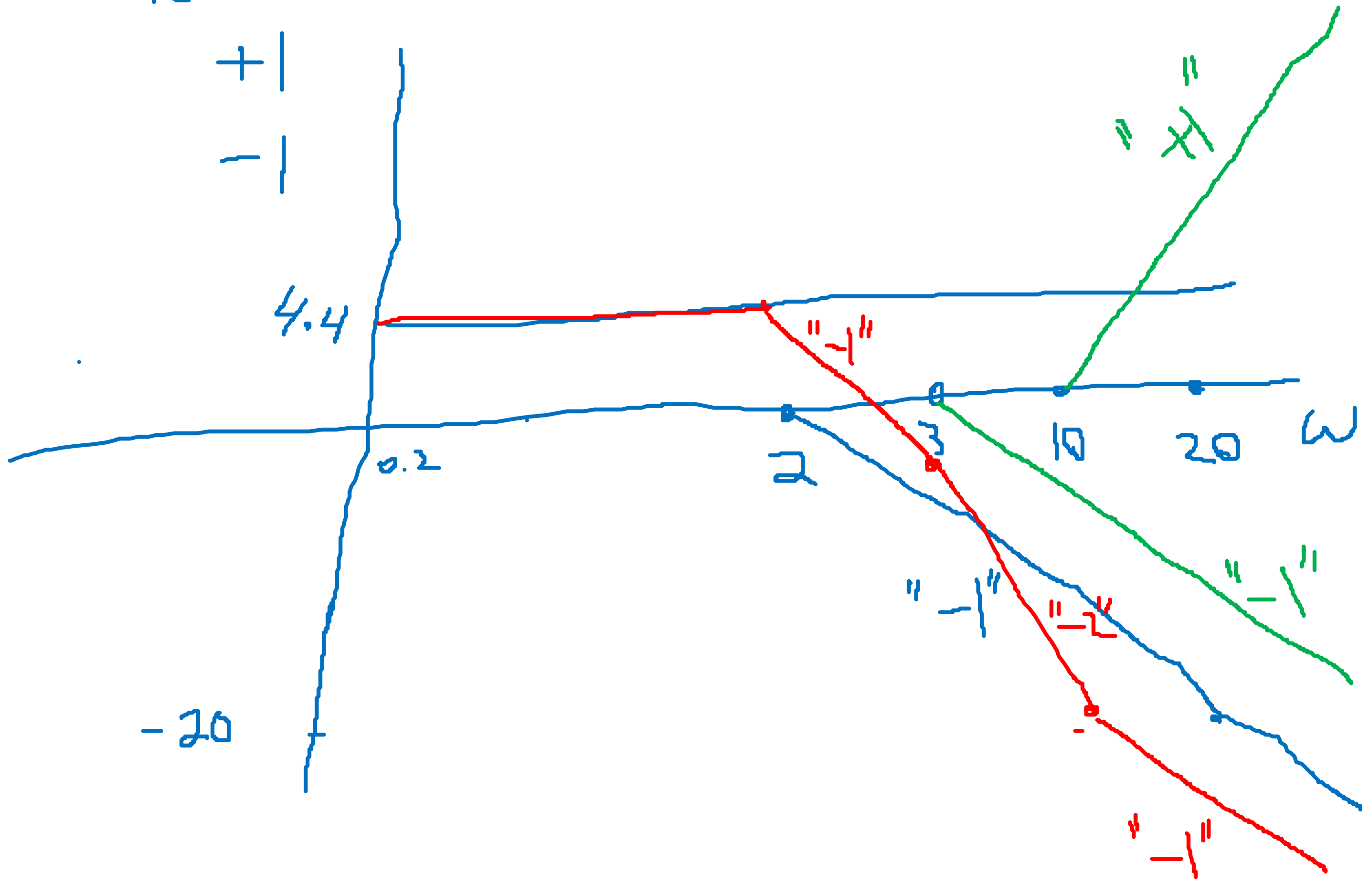
1 1

$$G(s) = \frac{s+10}{s^2+5s+6} = \frac{s+10}{(s+2)(s+3)} = \frac{10\left[\frac{1}{10}s+1\right]}{6\left(\frac{1}{2}s+1\right)\left(\frac{1}{3}s+1\right)}$$

$$G(j\omega) = \frac{5}{3}\left(\frac{\omega}{2}j+1\right)^{-1}\left(\frac{\omega}{3}j+1\right)^{-1}\left(\frac{\omega}{10}j+1\right)^{+1}$$

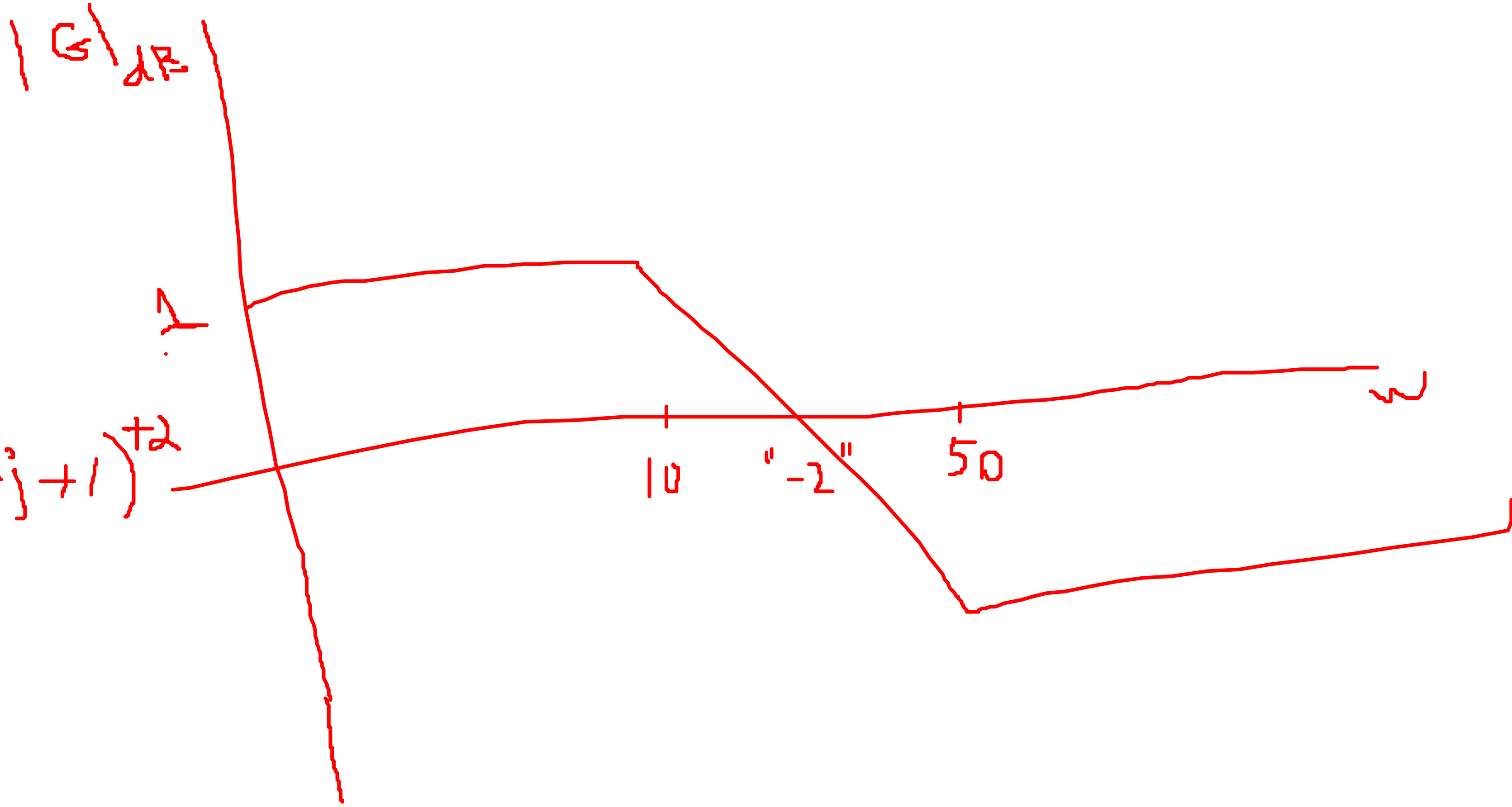
slope	0	-1	-1	$+1$
Total		-1	-2	-1

Constant $20\log\left|\frac{5}{3}\right| = 4.4$



$$G(s) = \frac{1.12\left(\frac{w}{50}j+1\right)^2}{\left(\frac{w}{10}j+1\right)^2}$$

$$G(jw) = 1.12\left(\frac{w}{10}j+1\right)^{-2}\left(\frac{w}{50}j+1\right)^{+2}$$

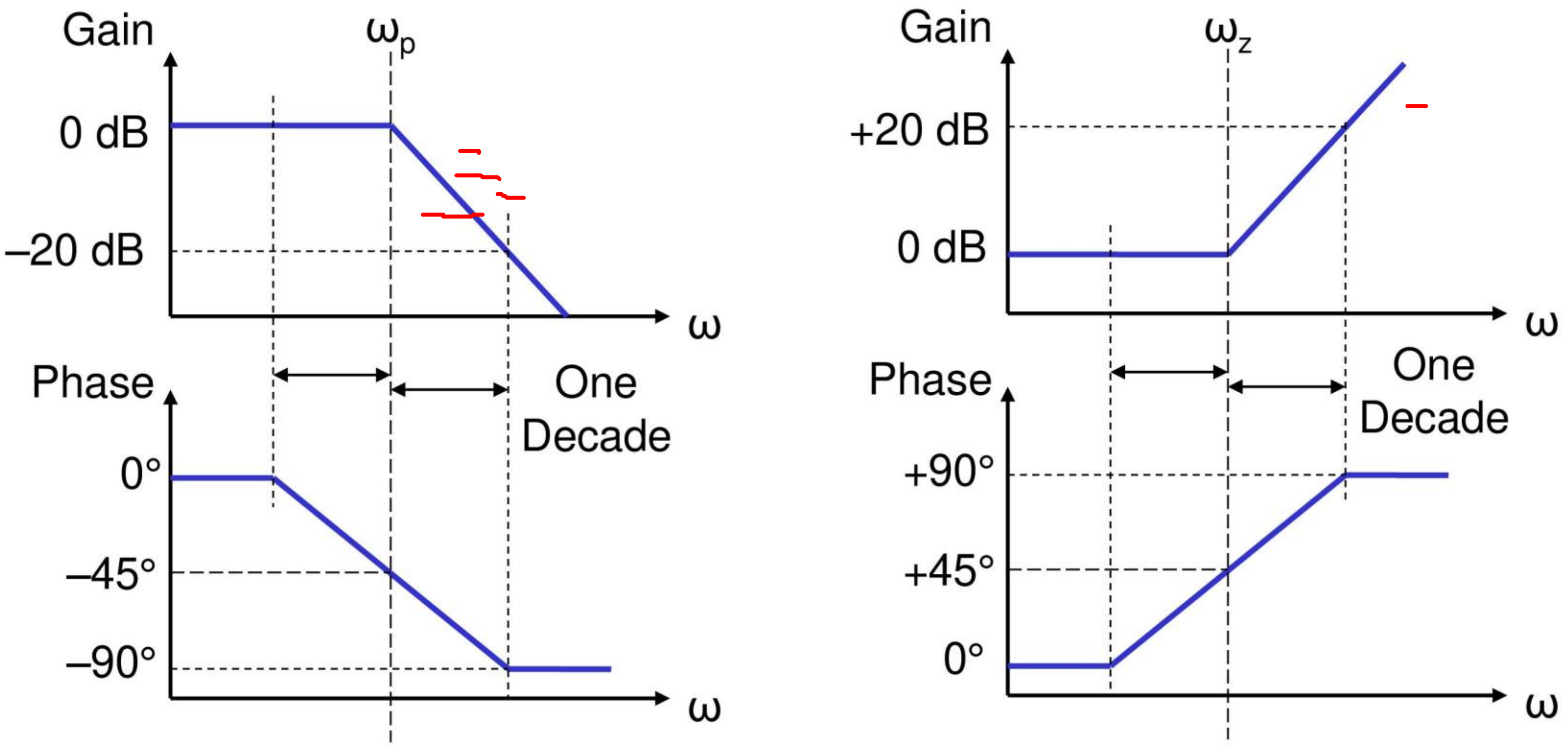


$$\frac{20}{20}\log|x| = \frac{1}{20}$$

$$x = 10^{1/20} = 1.12$$

Review

Single Pole & Zero Bode Plots



Pole at $\omega_p = 1/\tau$

Assume K=1
$20 \log_{10}(K) = 0$ dB

Zero at $\omega_z = 1/\tau$

How to Draw Bode Plot

- Keeping all the above points in mind, we are able to draw a Bode plot for any kind of control system. Now let us discuss the procedure of drawing a Bode plot:

- Substitute the $s = j\omega$ in the open loop transfer function $G(s) \times H(s)$.
- Find the corresponding corner frequencies and tabulate them.
- Now we are required one semi-log graph chooses a frequency range such that the plot should start with the frequency which is lower than the lowest corner frequency. Mark angular frequencies on the x-axis, mark slopes on the left hand side of the y-axis by marking a zero slope in the middle and on the right hand side mark phase angle by taking -180o in the middle.
- Calculate the gain factor and the type of order of the system.
- Now calculate slope corresponding to each factor.

How to Draw Bode Plot

For drawing the Bode magnitude plot:

- Mark the corner frequency on the semi-log graph paper.
- Tabulate these factors moving from top to bottom in the given sequence.
- Constant term K.
- Integral factor $\frac{1}{j\omega^n}$
- First order factor $\frac{1}{1+j\omega T}$ FOS
- First order factor (1+jωT).
- Second order or quadratic factor: $\left[\frac{1}{1+(2\zeta/\omega)}\times(j\omega)+\left(\frac{1}{\omega^2}\right)\times(j\omega)^2\right]$

- Now sketch the line with the help of the corresponding slope of the given factor. Change the slope at every corner frequency by adding the slope of the next factor. You will get the magnitude plot.
- Calculate the gain margin.

How to Draw Bode Plot

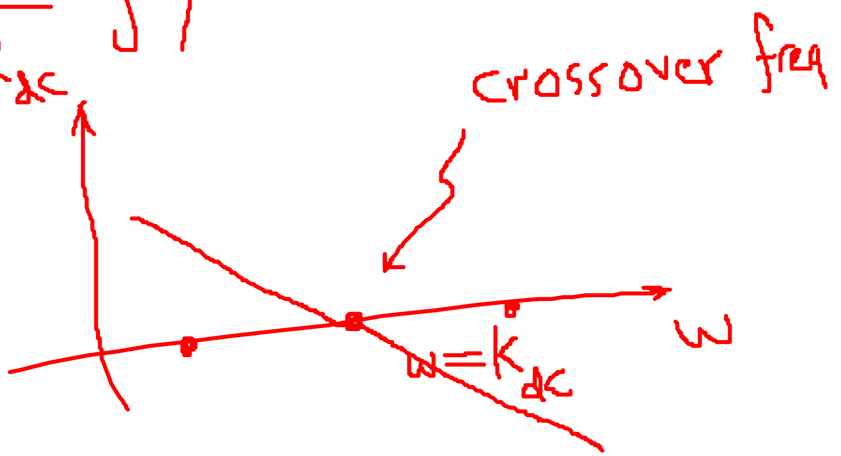
For drawing the Bode phase plot:

- Calculate the phase function adding all the phases of factors.
- Substitute various values to the above function in order to find out the phase at different points and plot a curve. You will get a phase curve.
- Calculate the phase margin.

Graphing K_{dc}/S

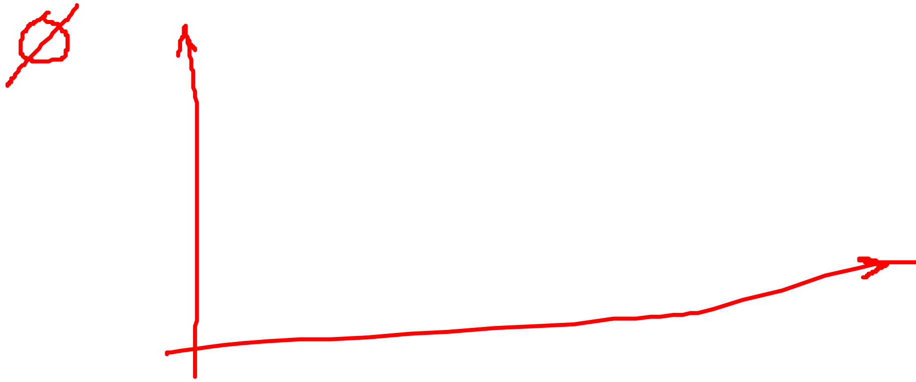
$$G(s) = \frac{K_{dc}}{s} = \frac{1}{\frac{1}{K_{dc}} s}, \quad G(j\omega) = \frac{1}{\frac{\omega}{K_{dc}} j} = \left(\frac{\omega}{K_{dc}} j\right)^{-1}$$

$$|G(j\omega)|_{dB} = 20 \log \left| \left(\frac{\omega}{K_{dc}} j \right)^{-1} \right| = -20 \log \left[\frac{\omega}{|K_{dc}|} \right]$$



Cases

- $\omega = K_{dc}$, $|G|_{dB} = 0$ dB
- $\omega = 10 K_{dc}$, $|G|_{dB} = -20$ dB // dec
- $\omega = 10^2 K_{dc}$, $|G|_{dB} = -40$ dB/dec $= -20$ dB/dec
- $\omega = 0.1 K_{dc}$, $|G|_{dB} = 20$ dB/-1 dec $= -20$ dB/dec



Second Order System

1. Over damp $\zeta > 1$

$$H = \frac{\omega_n^2}{(s+P_1)(s+P_2)}$$

2. Critical $\zeta = 1$

$$H = \frac{\omega_n^2}{(s+\omega_n)^2}$$

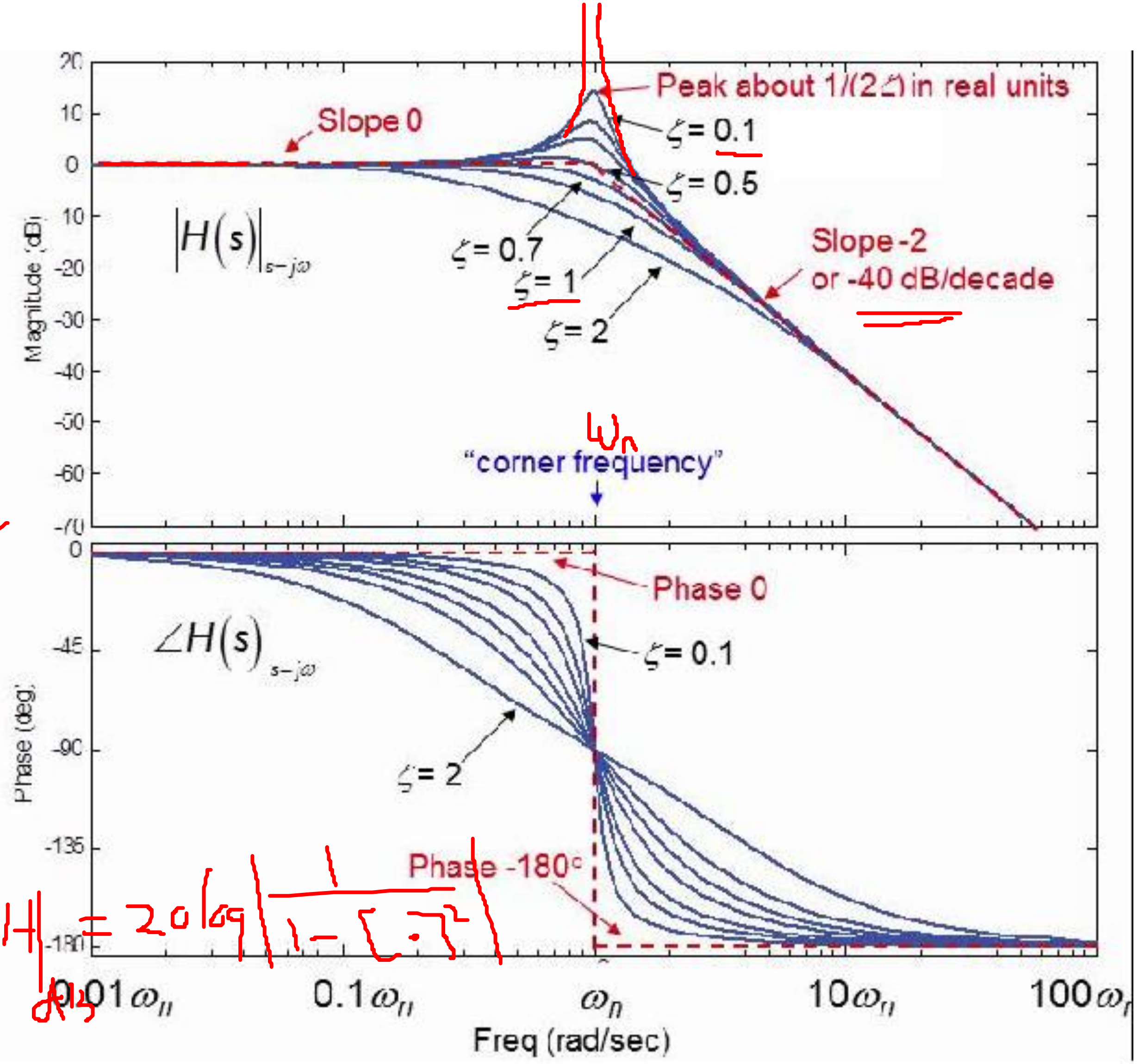
3- Underdamped $0 < \zeta < 1$

4. Undamped $\zeta = 0$

$$H(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2} = \frac{1}{1 - \left[\frac{\omega}{\omega_n}\right]^2} \rightarrow |H| = 20\log\left|\frac{1}{1-[\cdot]^2}\right| \text{ dB}$$

- Transfer function

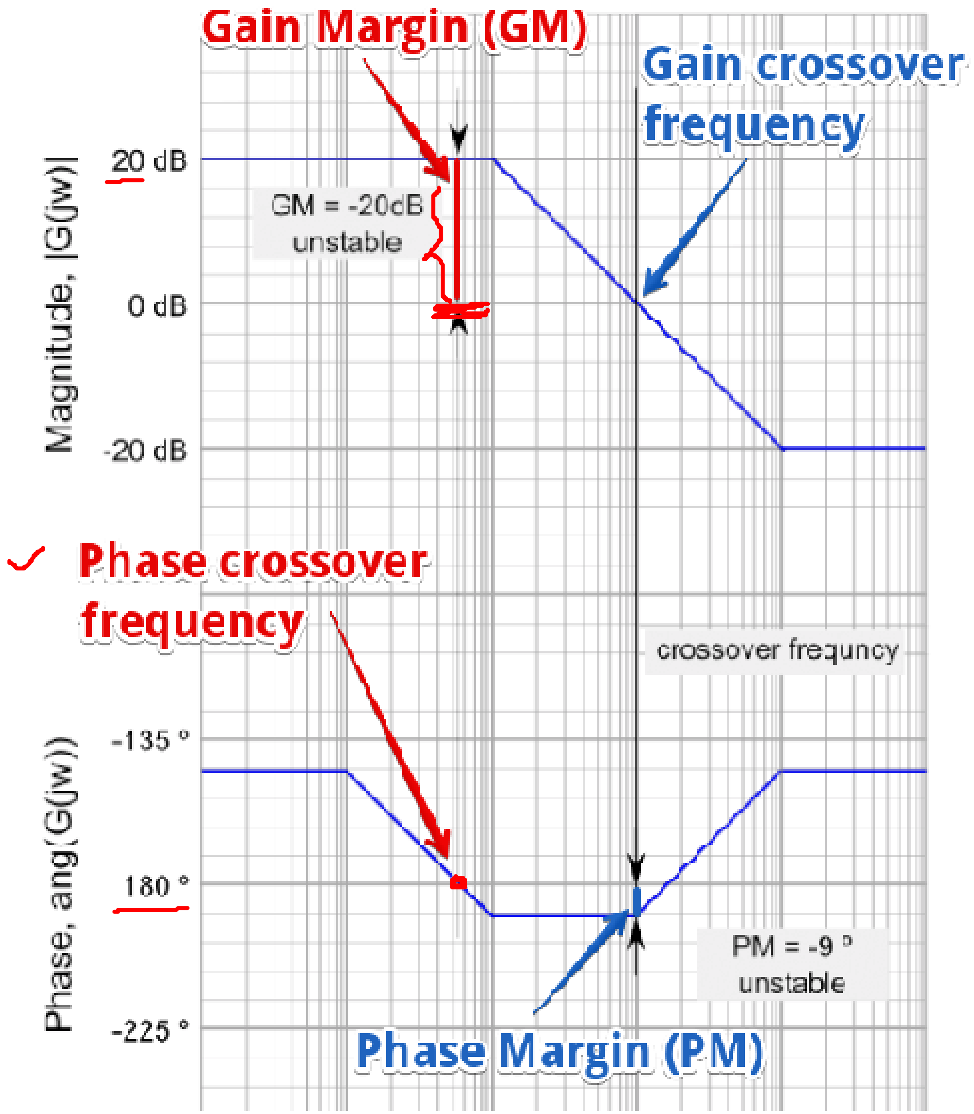
$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



Gain Margin

- The greater the Gain Margin (GM), the greater the stability of the system. The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.

- We can usually read the gain margin directly from the Bode plot (as shown in the diagram above). This is done by calculating the vertical distance between the magnitude curve (on the Bode magnitude plot) and the x-axis at the frequency where the Bode phase plot = 180°. This point is known as the phase crossover frequency.



Gain Margin

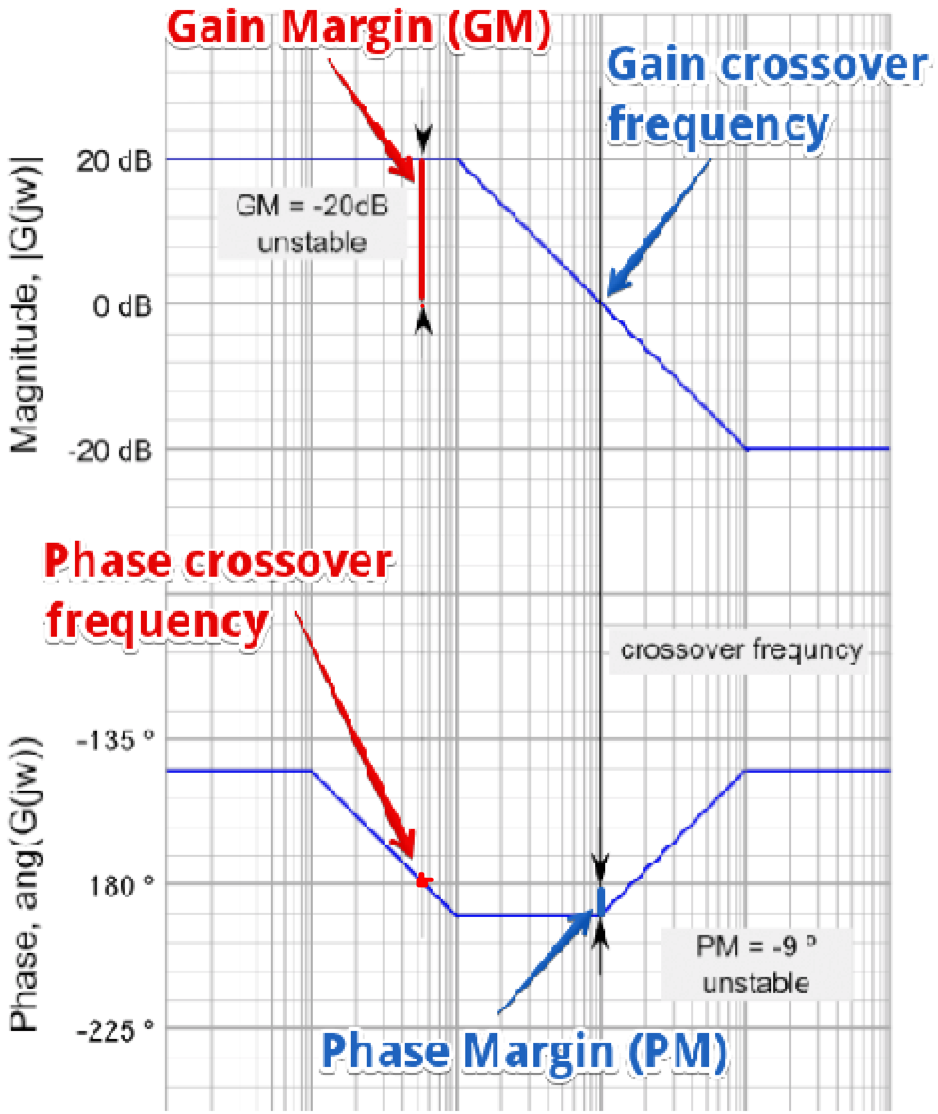
Gain Margin Formula

- The formula for Gain Margin (GM) can be expressed as:

$$GM = 0 - \overset{20}{G}\ dB = -20$$

- Where G is the gain. This is the magnitude (in dB) as read from the vertical axis of the magnitude plot at the phase crossover frequency.

- In our example shown in the graph above, the Gain (G) is 20. Hence using our formula for gain margin, the gain margin is equal to 0 – 20 dB = -20 dB (unstable).

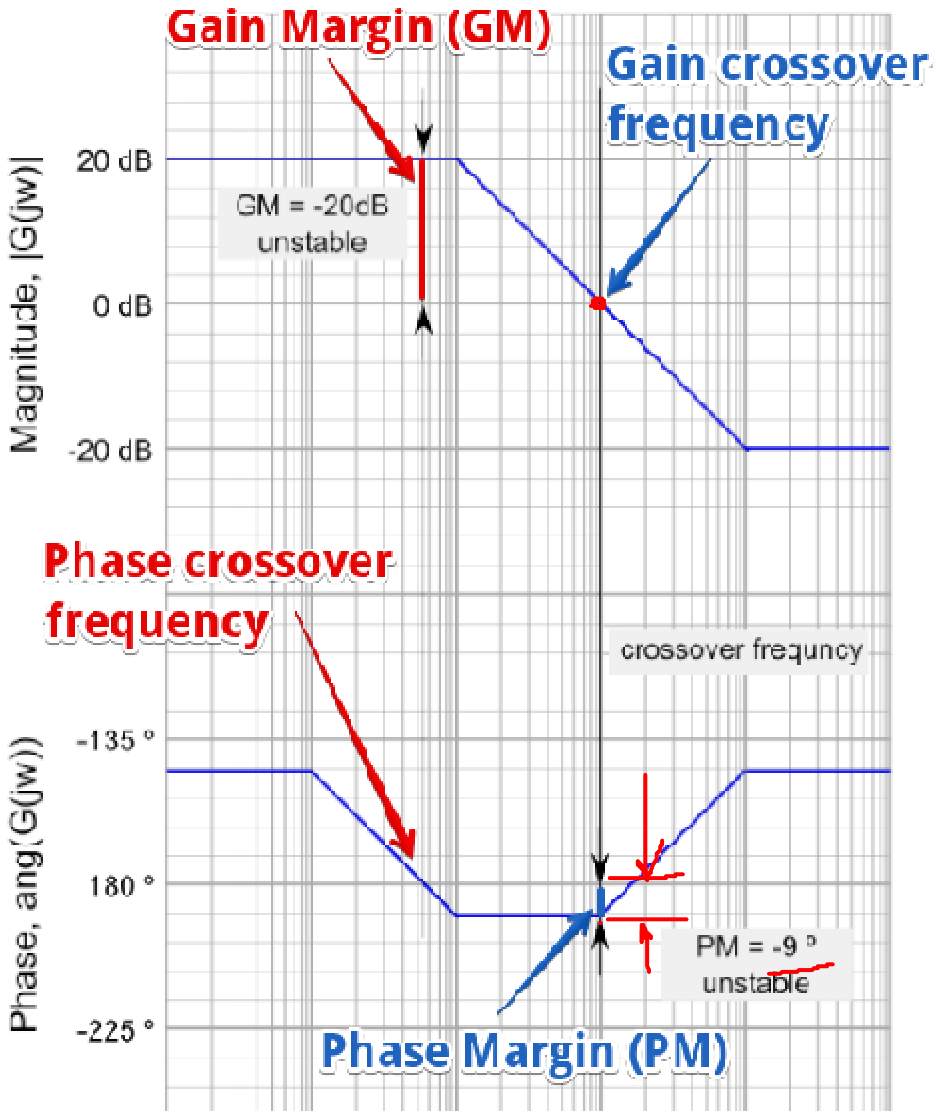


Phase Margin

Phase Margin

- The greater the Phase Margin (PM), the greater will be the stability of the system. The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.

- We can usually read the phase margin directly from the Bode plot (as shown in the diagram above). This is done by calculating the vertical distance between the phase curve (on the Bode phase plot) and the x-axis at the frequency where the Bode magnitude plot = 0 dB. This point is known as the gain crossover frequency.



Phase Margin

Phase Margin Formula

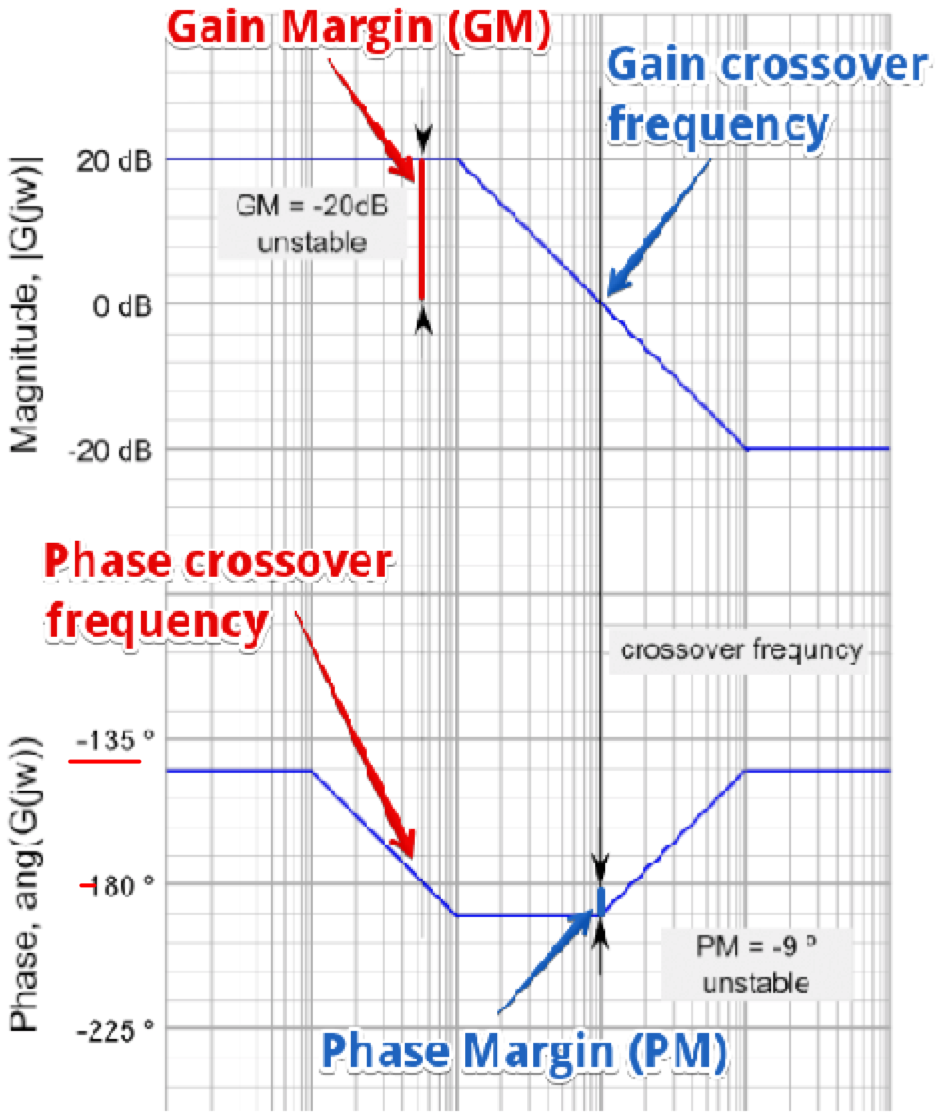
The formula for Phase Margin (PM) can be expressed as:

$$PM = \phi - (-180^{o})$$

Where phi is the phase lag (a number less than 0). This is the phase as read from the vertical axis of the phase plot at the gain crossover frequency.

- In our example shown in the graph above, the phase lag is -189°. Hence using our formula for phase margin, the phase margin is equal to -189° – (-180°) = -9° (unstable).

- As another example, if a system's open-loop gain crosses 0 dB at a frequency where the phase lag is -120°, then the phase lag -120°. Hence the phase margin of this feedback system is -120° – (-180°) = 60° (stable).



Bode Stability Criterion

- Stability conditions are given below:

- For a Stable System: Both the margins should be positive, or phase margin should be greater than the gain margin.
- For Marginal Stable System: Both the margins should be zero or phase margin should be equal to the gain margin.
- For Unstable System: If any of them is negative or phase margin should be less than the gain margin.

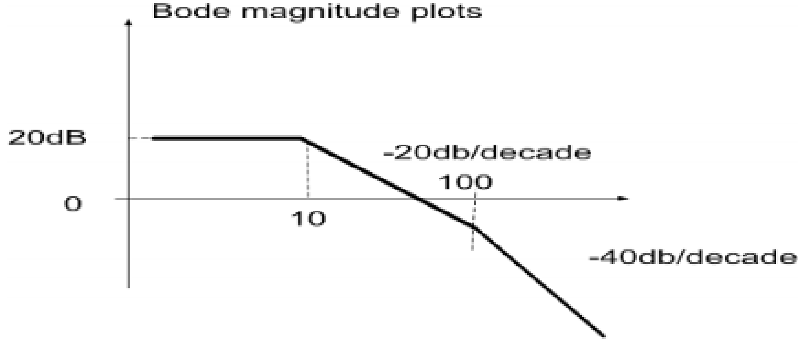
1) Sketch the Bode plots for the following transfer functions

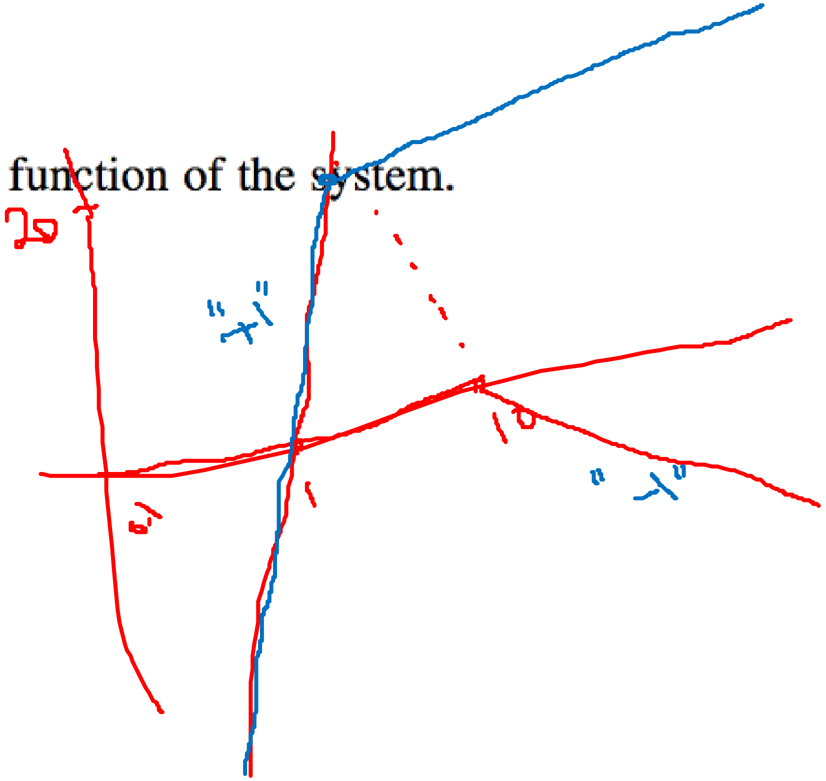
a) $H(s) = \frac{10s}{s+10}$ $= \frac{s}{\frac{1}{10}s+1}$, $H(j) = \left(\frac{\omega}{1}j\right)^{+1}\left(\frac{\omega}{10}j+1\right)^{-1}$

b) $H(s) = \frac{s(s+100)}{(s+2)(s+20)}$

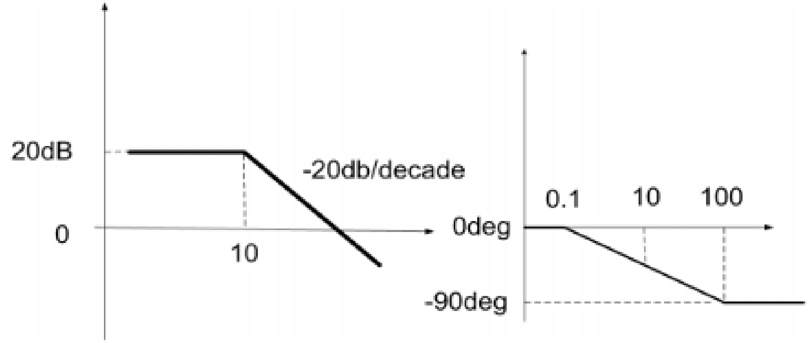
c) $H(s) = \frac{s}{(s+1)(s^2+14.14s+100)}$

2) A system has the following Bode magnitude plot. Find the transfer function of the system.



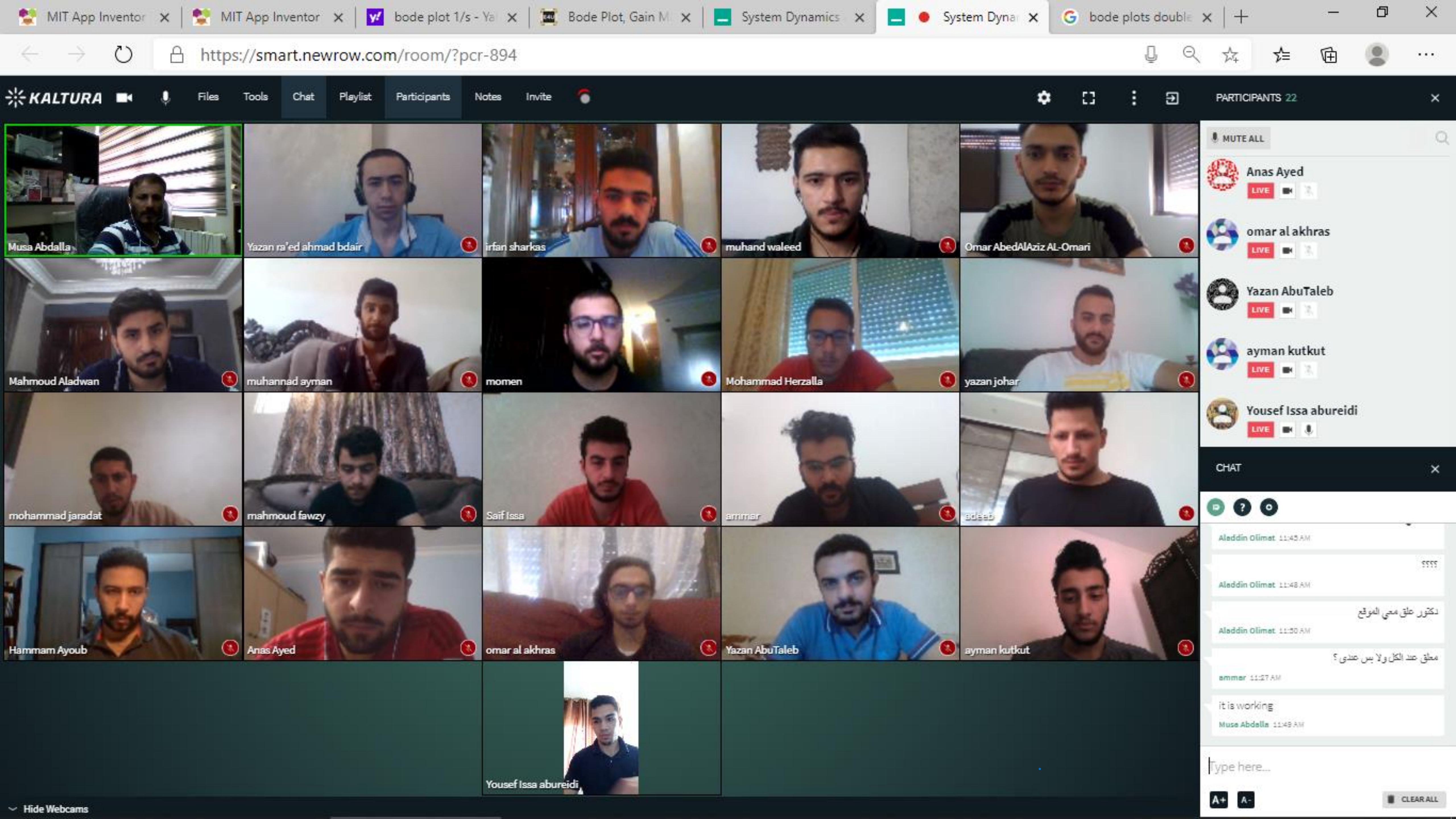


3) A system has the following Bode plots.



a) If the input to the system is $2\cos(5t)$, what is the steady state output?

b) If the input to the system is $2\cos(100t)$, what is the steady state output?



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bode plot 1/s - Ya
Bode Plot, Gain M
System Dynamics
System Dyna
bode plots double
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KALTURA
Files
Tools
Chat
Playlist
Participants
Notes
Invite
PARTICIPANTS 22
MUTE ALL
Anas Ayed
LIVE
omar al akhras
LIVE
Yazan AbuTaleb
LIVE
ayman kutkut
LIVE
Yousef Issa abureidi
LIVE
Musa Abdalla
Yazan ra'ed ahmad bdair
irfan sharkas
muhand waleed
Omar AbedAlAziz AL-Omari
Mahmoud Aladwan
muhannad ayman
momen
Mohammad Herzalla
yazan johar
mohammad jaradat
mahmoud fawzy
Saif Issa
ammar
adeeb
Hammam Ayoub
Anas Ayed
omar al akhras
Yazan AbuTaleb
ayman kutkut
Yousef Issa abureidi
Hide Webcams
CHAT
Aladdin Olimat 11:45 AM
؟؟؟؟
Aladdin Olimat 11:48 AM
دكتور علق معي الموقع
Aladdin Olimat 11:50 AM
معلق عند الكل ولا بس عندي ؟
ammar 11:27 AM
it is working
Musa Abdalla 11:49 AM
Type here...
A+
A-
CLEAR ALL