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The University of Jordan

DEPARTMENT OF MATHEMATICS

Eng. Math I

FIRST EXAM / Summer 2018

Date: 01/07/2018

الاسم

وقت المحاضرة: (..................)

Q1 [6 pts]. Solve the initial val

$y(\ln x - \ln y)dx - xdy = 0, \quad y(1) = 1. \quad (x > 0)$

$M = y\ln\left(\frac{x}{y}\right)$

$\frac{\partial M}{\partial y} = y * \frac{\frac{-x}{y^2}}{\frac{x}{y}} + \ln\left(\frac{x}{y}\right) = \cancel{y} * -\frac{1}{\cancel{y}} + \ln\left(\frac{x}{y}\right) = \ln\left(\frac{x}{y}\right) - 1$

$\frac{\partial N}{\partial x} = -1$ So Not exact

$R(x) = \frac{M_y - N_x}{N} = \frac{\ln\left(\frac{x}{y}\right) - 1 + 1}{-x} = \frac{-\ln\left(\frac{x}{y}\right)}{x}$

$R(y) = \frac{N_x - M_y}{M} = \frac{\cancel{-1} - \ln\left(\cancel{\frac{x}{y}}\right) \cancel{+1}}{y \ln\left(\cancel{\frac{x}{y}}\right)} = -\frac{1}{y}$

and then $\mu(y) = e^{\int R(y)\,dy} = e^{\int -\frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$

$(\ln x - \ln y)dx - \left(\frac{x}{y}\right)dy = 0$

$u = \int M\, dy + k(x) = \int -\frac{x}{y}dy + k(x)$

$u = -x\ln|y| + k(x) \rightarrow \frac{\partial u}{\partial x} = -\ln|y| + \frac{dk}{dx}$

$\ln x \cancel{- \ln y} = \cancel{-\ln y} + \frac{dk}{dx} \rightarrow \int dk = \int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv}$

$k = \overset{u}{(\ln x)} \overset{v}{(x)} - \int \overset{v}{x} * \overset{du}{\frac{1}{x}} dx \Rightarrow k = x\ln x - x$ by parts

the solution $\rightarrow -x\ln|y| + x\ln x - x = C$

$-(1)\ln(1) + (1)\ln(1) - (1) = C$

$\boxed{C = -1}$ so $\boxed{-x\ln y + x\ln x - x = -1}$

$y = x_1 e^{\frac{1-x}{x}}$

Q2 [6 pts]. Find all solutions to:

$$(x+ye^{y})y'+(\cos x+\ln y)y=0.$$

$$(x+ye^{y})\frac{dy}{dx} = -(\cos x+\ln y)y$$

$$\frac{1}{y}*(x+ye^{y})dy+\frac{1}{y}(\cos x+\ln y)y\,dx=0$$

$$\left(\frac{x}{y}+e^{y}\right)dy+(\cos x+\ln y)\,dx=0$$

$N \qquad M$

$$\frac{\partial M}{\partial y}=\frac{1}{y} \quad \text{and} \quad \frac{\partial N}{\partial x}=\frac{1}{y}$$

so exact and then

$$U=\int N\,dy+k(x) \rightarrow U=\int \frac{x}{y}+e^{y}\,dy+k(x)$$

$$U=x\ln|y|+e^{y}+k(x) \rightarrow \frac{\partial U}{\partial x}=\ln|y|+\frac{dk}{dx}$$

$$\cos x+\cancel{\ln y}=\cancel{\ln y}+\frac{dk}{dx} \rightarrow \int dk=\int \cos x\,dx$$

$k=\sin x$ and then

$$\boxed{x\ln y+e^{y}+\sin x=C}$$

Q3 [4 pts]. Find the form of the particular solution to:

$$y''+5y'+6y=x^{2}e^{-3x}. \qquad \textbf{(DO NOT SOLVE)}$$

$$y_h \rightarrow \lambda^2+5\lambda+6=0 \rightarrow (\lambda+3)(\lambda+2)$$

$$\lambda=-3 \quad \lambda=-2$$

$$\text{so } y_h=c_1e^{-3x}+c_2e^{-2x}$$

$$y_p=(Ax^2+Bx+c)e^{-3x}*x$$

$$\boxed{y_p=(Ax^3e^{-3x}+Bx^2e^{-3x}+cxe^{-3x})}$$

Q4 [6 pts]. Solve the differential equation:

$$y'' + (y')^2 = 2e^{-y}.$$

$\boxed{y' = t}$

$y'' = \frac{dt}{dy} * \frac{dy}{dx} \rightarrow y'' = t'y' \rightarrow \boxed{y'' = t't}$

$\frac{t't}{t} + \frac{t^2}{t} = \frac{2e^{-y}}{t}$

$t' + t = (2e^{-y})t^{-1}$ and then $\boxed{u = t^2}$ $\boxed{u' = 2t\,t'}$

$2tt' + 2tt = (2e^{-y})2tt^{-1}$

$u' + \overset{p(y)}{2}u = (4e^{-y})$ linear

and then $p(y) = 2 \rightarrow e^{\int p(y)dy} = e^{\int 2dy} = e^{2y}$

$e^{2y} \cdot u = \int e^{2y} * 4e^{-y} dy \rightarrow e^{2y} \cdot u = 4e^{y} + c$

$u = 4e^{-y} + ce^{-2y}$

$t^2 = 4e^{-y} + ce^{-2y} \rightarrow t = (4e^{-y} + ce^{-2y})^{\frac{1}{2}}$

$\frac{dy}{dx} = (4e^{-y} + ce^{-2y})^{\frac{1}{2}} \rightarrow (4e^{-y} + ce^{-2y})^{\frac{-1}{2}} dy = dx$

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Q5 [8 pts]. Find a general solution to:

$$xy'' - 2y' + \frac{2}{x}y = x^2. \quad (x > 0)$$

$$x^2y'' - 2xy' + 2y = x^3 \qquad y = x^m$$

y_h $m(m-1) - 2m + 2 = 0$

$m^2 - 3m + 2 = 0$

$(m-1)(m-2) \rightarrow m_{1,2} = 1, 2$

$$\boxed{y_h = c_1x + c_2x^2}$$

$$y'' - \frac{2}{x}y' + \frac{2}{x^2} = x \rightarrow r(x)$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1' = (x\,2x) - (x^2)$$

$$\boxed{W(y_1, y_2) = x^2}$$

and then

$$y_p = -y_1\int \frac{y_2 r}{W}dx + y_2\int \frac{y_1 r}{W}dx$$

$$y_p = -x\int \frac{x^2 x}{x^2}dx + x^2\int \frac{x\,x}{x^2}dx$$

$$y_p = -x\int x\,dx + x^2\int dx = -\frac{x^3}{2} + x^3$$

$$\boxed{y_{general} = c_1x + c_2x^2 - \frac{1}{2}x^3 + x^3}$$