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[1] (4 marks) Assume that $y = e^{2x}$ is a solution for the ODE

$y''' + y'' - 5y' - 2y = 0$. Find the general solution for this ODE.

$* \lambda^3 + \lambda^2 - 5\lambda - 2 = 0$

$*$ Factors of $\underline{\underline{2}}$: $(2, -2, 1, -1)$

$8 + 4 - 10 - 2 = 0$ $(\lambda = 2)$ is a Factor

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$(\lambda - 2) \,\big)\, \lambda^3 + \lambda^2 - 5\lambda - 2$

$* (\lambda - 2)(\lambda^2 + 3\lambda + 1) = 0$

$\# \frac{-b \pm \sqrt{b^2 - 4ac}}{2} = \frac{-3 \pm \sqrt{5}}{2}$

① $\lambda_1 = 2$

$\lambda_2 = \frac{-3 + \sqrt{5}}{2}$, $\lambda_3 = \frac{-3 - \sqrt{5}}{2}$

G.S

$y = c_1 e^{2x} + c_2 e^{\left(\frac{-3+\sqrt{5}}{2}\right)x} + c_3 e^{\left(\frac{-3-\sqrt{5}}{2}\right)x}$

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[2] (6 marks) Assume $\bar{y}' = \begin{bmatrix} 3 & 2 \\ -3 & -4 \end{bmatrix} \bar{y}$. Find the general solution for $\bar{y}$.

$* A = \begin{bmatrix} 3 & 2 \\ -3 & -4 \end{bmatrix}$, $(A - \lambda I) = \begin{bmatrix} 3-\lambda & 2 \\ -3 & -4-\lambda \end{bmatrix}$

$|A - \lambda I| = (3-\lambda)(-4-\lambda) + 6 = 0$

$= -12 - 3\lambda + 4\lambda + \lambda^2 + 6 = 0$

$= \lambda^2 + \lambda - 6 = 0$

$= (\lambda + 3)(\lambda - 2) = 0$

$\therefore \lambda + 3 = 0 \rightsquigarrow \boxed{\lambda_1 = -3}$

$\lambda - 2 = 0 \rightsquigarrow \boxed{\lambda_2 = 2}$

$*$ For $\lambda_1 = -3$:

$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ -3 & -1 & 0 \end{array}\right]$

$6x_1 + 2x_2 = 0$

$x_2 = \frac{-6x_1}{2} = -3x_1$

$x_1 = t \in \mathbb{R}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ -3t \end{bmatrix} = \underline{t} \underbrace{\begin{bmatrix} 1 \\ -3 \end{bmatrix}}_{x^{(1)}}$

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$*$ For $\lambda = 2$:

$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ -3 & -6 & 0 \end{array}\right]$

$x_1 + 2x_2 = 0$

$x_2 = \frac{-x_1}{2}$

$x_1 = t \in \mathbb{R}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ \frac{-t}{2} \end{bmatrix} = \underline{t} \underbrace{\begin{bmatrix} 1 \\ \frac{-1}{2} \end{bmatrix}}_{x^{(2)}}$

G.S:

$y = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 1 \\ \frac{-1}{2} \end{bmatrix} e^{2t}$

$y_1 = c_1 e^{-3t} + c_2 e^{2t}$

$y_2 = -3c_1 e^{-3t} + \frac{-1}{2} c_2 e^{2t}$

[3] (4 marks) Find a linear ODE with constant coefficients that has the general solution $y = \underbrace{c_1e^{x} + c_2e^{-x} + c_3xe^{-x}}_{y_h} + \underbrace{x}_{y_p}$.

* $r_1 = 1$
* $r_2 = r_3 = -1$

$\therefore (r-1)(r+1)(r+1) = 0$

$r^3 + r^2 - r - 1 = 0$

$y''' + y'' - y' - y = 0$

$y_p = x$

$y_p' = 1 \quad , y_p'' = 0 \quad , y_p''' = 0$

$0 + 0 - 1 - x = r(x)$

$r(x) = -(1+x)$

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* ODE:

$y''' + y'' - y' - y = -(1+x)$

[4](5 marks) Solve the system: (use gauss elimination).

x + y + z + w = 2

y + z + w = 1

x − y + w = 3

y − z − w = - 3

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$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & +1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -3 \end{bmatrix}$$

$$* A^* = (-1) \left[\begin{array}{cccc|c} \boxed{1} & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 & -3 \end{array}\right] = \begin{array}{c} \\ \\ (-1) \\ (+2) \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & \boxed{1} & 1 & 1 & 1 \\ 0 & -2 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 & -3 \end{array}\right]$$

$$(2)(-1) \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 & 2 \end{array}\right] /-2$$

$$(-2) \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & \boxed{1} & +1 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$\therefore x = 1$

$y = -3$

$z = 5$

$w = -1$

* لازم بهاد السؤال نعمل
Diagonal

اللي ما عمل diagonal خسر علامات السؤال.

$$y(p) = \begin{bmatrix} e^{2t} & e^{3t} \\ e^{2t} & -e^{3t} \end{bmatrix} \begin{bmatrix} \frac{3}{2}e^{t} - \frac{3}{2} \\ \frac{1}{2}t \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{3}{2}e^{2t} * e^{t} - \frac{3}{2}e^{2t}\right) + \left(\frac{1}{2}e^{3t}\, t\right) \\ \left(\frac{3}{2}e^{2t} * e^{t} - \frac{3}{2}e^{2t}\right) + \left(-e^{3t} * \frac{1}{2}t\right) \end{bmatrix}$$

$$\underline{\underline{y(p)}} = \begin{bmatrix} \frac{3}{2}e^{3t} - \frac{3}{2}e^{2t} + \frac{1}{2}te^{3t} \\ \frac{3}{2}e^{3t} - \frac{3}{2}e^{2t} - \frac{1}{2}te^{3t} \end{bmatrix}$$

$$\underline{\underline{y(h)}} = \left(c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} \right)$$

$$*y = y(p) + y(h)$$

[5] (6 marks) Assume that a 2×2 matrix A has eigenvalues 2 and 3 and corresponding eigenvectors $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\-1\end{bmatrix}$, respectively.

Find the general solution for the linear system: $\bar{y}' = A\bar{y} + \begin{bmatrix}2\\1\end{bmatrix}e^{3t}$

$$* \, y^{(h)} = c_1 \begin{bmatrix}1\\1\end{bmatrix}e^{2t} + c_2 \begin{bmatrix}1\\-1\end{bmatrix}e^{3t}$$

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$$* \, g(t) = \begin{bmatrix}2\\1\end{bmatrix}e^{3t} \,, \quad y_{(p)} = \begin{bmatrix}a_1\\a_2\end{bmatrix}e^{3t} + \underbrace{\begin{bmatrix}b_1\\b_2\end{bmatrix}t * e^{3t}}_{\text{لا}}$$

* Using the variation of Parameters:

$$* \, y_p = y(t) * u(t)$$

$$y(t) = \begin{bmatrix} e^{2t} & e^{3t} \\ e^{2t} & -e^{3t} \end{bmatrix} , \quad |y(t)| = \left(e^{2t} * -e^{+3t}\right) - \left(e^{3t} * e^{2t}\right) = \left(-e^{5t} - e^{5t}\right) = \boxed{-2e^{5t}}$$

$$y^{-1}(t) = \underbrace{\frac{1}{-2e^{5t}}}_{\left(\frac{-1}{2}e^{-5t}\right)} \begin{bmatrix} -e^{3t} & -e^{3t} \\ -e^{2t} & e^{2t} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{-2t} & e^{-2t} \\ e^{-3t} & -e^{-3t} \end{bmatrix}$$

$$* \, u'(t) = y^{-1}(t) \cdot g(t) = \frac{1}{2}\begin{bmatrix} e^{-2t} & e^{-2t} \\ e^{-3t} & -e^{-3t} \end{bmatrix} \cdot \begin{bmatrix} 2e^{3t} \\ e^{3t} \end{bmatrix}$$

$$u'(t) = \frac{1}{2}\begin{bmatrix} 2e^{3t} * e^{-2t} + e^{-2t} * e^{3t} \\ \underbrace{2e^{-3t} * e^{3t}}_{2} + \underbrace{(-e^{-3t})(e^{3t})}_{-1} \end{bmatrix} = \begin{bmatrix} e^{t} + \frac{1}{2}e^{t} \\ \textcircled{1} * \frac{1}{2} = \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}e^{t} \\ \frac{1}{2} \end{bmatrix}$$

$$u(t) = \int_0^t \begin{bmatrix} \frac{3}{2}e^{\tau} \\ \frac{1}{2} \end{bmatrix} d\tau = \begin{bmatrix} \frac{3}{2}e^{\tau} \\ \frac{1}{2}\tau \end{bmatrix}\Bigg|_0^t = \begin{bmatrix} \frac{3}{2}e^{t} - \frac{3}{2} \\ \frac{1}{2}t \end{bmatrix}$$

$$* \, y(P) = y(t) * u(t)$$

$$= \begin{bmatrix} e^{2t} & e^{3t} \\ e^{2t} & -e^{3t} \end{bmatrix} \begin{bmatrix} \underbrace{\frac{3}{2}e^{t} - \frac{3}{2}}_{\to \left(\frac{3e^{t}-3}{2}\right)} \\ \frac{1}{2}t \end{bmatrix}$$

$$\cancel{* \, y(P) = \begin{bmatrix} e^{2t} * \frac{3}{2}e^{t} + \left(-\frac{3}{2}\right)e^{3t} \\ e^{2t}\left(\frac{3}{2}\right)e^{t} + \end{bmatrix}}$$

خلف الصفحة

$\lambda^3+\lambda^2-5\lambda-2=0$ $\lambda=2$

(Q1)

~~1+1+5-2=~~

$-8+4+10-2$

$14-10=4$

~~1+1-5-2~~

$8+4-10-2$

$12 \quad -12=0$

$$\begin{array}{r|l} & \lambda^2+3\lambda+1 \\ \lambda-2 & \lambda^3+\lambda^2-5\lambda-2 \\ & \mp\lambda^3\pm 2\lambda^2 \\ \hline & 3\lambda^2-5\lambda-2 \\ & \mp 3\lambda^2\pm 6\lambda \\ \hline & \lambda-2 \\ & \mp\lambda\pm 2 \\ \hline & 0 \end{array}$$

$(\lambda-2)(\lambda^2+3\lambda+1)$

$\cancel{\lambda^3}+\cancel{3\lambda^2}+\lambda-\cancel{2\lambda^2}-6\lambda-2$

$\lambda^3+\lambda^2-5\lambda-2$

$(\lambda-2)(\lambda^2+3\lambda+1)$

$\frac{-3\pm\sqrt{9-4}}{2} = \frac{-3}{2}\pm\frac{\sqrt{5}}{2}$

$\left(\frac{-3+\sqrt{5}}{2}\right)$

$\left(\frac{-3-\sqrt{5}}{2}\right)$