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The Jordan University
Mathematics Department

Engineering Math. I (0301202)

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Section: 10 – 11

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(1) Determine a Suitable form for the particular solution if the method of undetermined coefficients is to be used for the D.E.

$$y''' - y'' + 2y = e^{-x} + xe^{x}\sin(x)$$

y_p :-

y_{p_1} : $e^{-x} \to \lambda e^{-x}$ تشابه
$\to \lambda x e^{-x}$

y_{p_2} :- $x e^{x} \sin x \to (ax+b)(\lambda_2 e^{x})(c \sin x + d\cos x)$

$y_{p_2} = (ax+b)(\lambda_2 e^{x}) \underbrace{(c\sin x + d\cos x)}_{\text{تشابه}} * x$

$y_p = y_{p_1} + y_{p_2} = \lambda x e^{-x} * (ax+b)(\lambda_2 e^{x})(c \sin x + d\cos x) * x$

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y_h :- $r^3 - r^2 + 2 = 0$

$(r+1)(r^2 - 2r + 2)$

$r = -1,\ 1-i,\ 1+i$

y_h :- $c_1 e^{-x} + c_2 e^{1x} * (\sin x + \cos x)$

(2) Find the general solution of

$$y^{(5)} + 4y''' + 5y'' = 0$$

y_h :- $r^5 + 4r^3 + 5r^2 = 0$

$r^2 (r^3 + 4r + 5) = 0$

$r = 0, 0,\ r = \frac{1 \pm \sqrt{19}\, i}{2}$

$y_h = c_1 e^{0} + c_2 e^{\frac{1}{2}x}\left(\sin \frac{\sqrt{19}}{2} x + \cos \frac{\sqrt{19}}{2}\right)$

$y_g = y_h = c_1 + c_2 e^{\frac{x}{2}}\left(\sin \frac{\sqrt{19}}{2} x + \cos \frac{\sqrt{19}}{2} x\right)$

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(3) Use Cramer's rule to find the value of x_2 for the following system of linear equations

$$x_1 + 2x_2 = 6$$
$$-3x_1 - 6x_3 = 3$$
$$x_1 - 2x_2 + 3x_3 = 2$$

solution

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 0 & -6 \\ 1 & -2 & 3 \end{pmatrix} \qquad u = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

$$\boxed{AX = u}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ -3 & 0 & -6 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

$$X_1 = \frac{|A_1|}{|A|} = \frac{-114}{-6} = 19$$

$$X_2 = \frac{|A_2|}{|A|} = \frac{39}{-6} = \frac{-13}{2}$$

$$X_3 = \frac{|A_3|}{|A|} = \frac{24}{-6} = -3$$

$$|A| = 1(-12) - 2(-3) + 0 = -6 \neq 0$$

$$|A_1| = \begin{pmatrix} 6 & 2 & 0 \\ 3 & 0 & -6 \\ 2 & -2 & 3 \end{pmatrix} = 6(-12) - 2(21) = -114$$

$$|A_2| = \begin{pmatrix} 1 & 6 & 0 \\ -3 & 3 & -6 \\ 1 & 2 & 3 \end{pmatrix} = 21 + 18 + 0 = 39$$

$$|A_3| = \begin{pmatrix} 1 & 2 & 6 \\ -3 & 0 & 3 \\ 1 & -2 & 2 \end{pmatrix} = 6 - 18 + 36 = 24$$

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(4) Find the general Solution of the following system

$$Y' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} Y$$

sol:-

Homoguinious system $F = 0$

eigen values, vectors :-

$$\begin{pmatrix} 3-\lambda & -4 \\ 1 & -1-\lambda \end{pmatrix} \Rightarrow (3-\lambda)(-1-\lambda) + 4 = 0$$

$$-3 + \lambda - 3\lambda + \lambda^2 + 4 = 0$$

$Au = \lambda u$ $\qquad \lambda^2 - 2\lambda + 1 = 0 \qquad \lambda = +1, \mp 1$

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} \longrightarrow \begin{matrix} 3x - 4y = -x & \dots (1) & \boxed{y = x} \\ x - y = -y & \dots (2) & \rightarrow \boxed{x = 0} \end{matrix}$$

Put $x = t \rightarrow y = t$

$\rightarrow \begin{pmatrix} t \\ t \end{pmatrix} \rightarrow t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ at $\underline{t=1}$

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$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is eigen vector $= E\lambda_1 = E\lambda_2$

$$Y_g = c_1 e^{\lambda_1 x} E\lambda_1 + c_2 e^{\lambda_1 x} E\lambda_1 + c_3 x e^{\lambda_1 x} E\lambda_1$$

$$= c_1 e^{-x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \cancel{c_2 e^{-x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_3 x e^{-x}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}.$$

$y_p = 0$ Because $F = 0$

(5) Find the general Solution of the following system using variation of parameters

$$Y' = \begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix} Y + \begin{pmatrix} e^t \\ t \end{pmatrix}$$

sol:- eigen values , vectors

$$\begin{pmatrix} 1-\lambda & -4 \\ -4 & 1-\lambda \end{pmatrix} = (1-\lambda)(1-\lambda)+16=0 \quad (1-\lambda)^2 = -16$$

$$1-\lambda = \pm 4i \qquad \boxed{\lambda = 1-4i \\ 1+4i}$$

vectors :-

$$\begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1-4i\begin{pmatrix} x \\ y \end{pmatrix}$$

$x-4y = (1-4i)x \rightarrow \boxed{4y = 4x}$

$-4x+y = (1+4i)y$

put $x = t \rightarrow y = it$

$$\begin{pmatrix} t \\ it \end{pmatrix} \rightarrow t\begin{pmatrix} 1 \\ i \end{pmatrix} \quad \text{at } t=1$$

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$$\begin{pmatrix} 1 \\ i \end{pmatrix} \text{ eigen vector} \Rightarrow \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_1} + i\underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B_2}$$

Y_h :- $Y_1 = e^{ax}\cos bx\, B_1 - e^{ax}\sin bx\, B_2$

$$= e^x\cos 4x\begin{pmatrix} 1 \\ 0 \end{pmatrix} - e^x\sin 4x\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$Y_2 = e^{ax}\sin bx\, B_1 + e^{ax}\cos bx\, B_2$ +3

$$= e^x\sin 4x\begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^x\cos 4x\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$Y_h = c_1 Y_1 + c_2 Y_2$

y_p :- $F = \begin{pmatrix} e^t \\ t \end{pmatrix} = e^t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

تابع خلف الورقة →

Q5// Φ :- $\Phi = \begin{pmatrix} Y_1 & Y_2 \end{pmatrix}$

$$\Phi = \begin{pmatrix} e^x \cos 4x & e^x \sin 4x \\ -e^x \sin 4x & e^x \cos 4x \end{pmatrix}$$

$$Y_1 = \begin{pmatrix} e^x \cos 4x \\ -e^x \sin 4x \end{pmatrix}$$

$$Y_2 = \begin{pmatrix} e^x \sin 4x \\ e^x \cos 4x \end{pmatrix}$$

$$\Phi^{-1} = \begin{pmatrix} e^x \cos 4x & -e^x \sin 4x \\ e^x \sin 4x & e^x \cos 4x \end{pmatrix}$$

flip the diagonal and change sign of the others ((2x2 matrix))

$$Y_p = \Phi \int \Phi^{-1} F \, dx$$

$$= \underbrace{\begin{pmatrix} e^x \cos 4x & e^x \sin 4x \\ -e^x \sin 4x & e^x \cos 4x \end{pmatrix}}_{\Phi} * \int \begin{pmatrix} e^x \cos 4x & -e^x \sin 4x \\ e^x \sin 4x & e^x \cos 4x \end{pmatrix} * \begin{pmatrix} e^x \\ x \end{pmatrix} dx$$

$$= \Phi * \int \begin{pmatrix} e^{2x} \cos 4x + x e^x \cos 4x - e^{2x} \sin 4x - x e^x \sin 4x \\ e^{2x} \sin 4x + x e^x \sin 4x + e^{2x} \cos 4x + x e^x \cos 4x \end{pmatrix} dx$$

$Y_g = Y_h + Y_p$ #