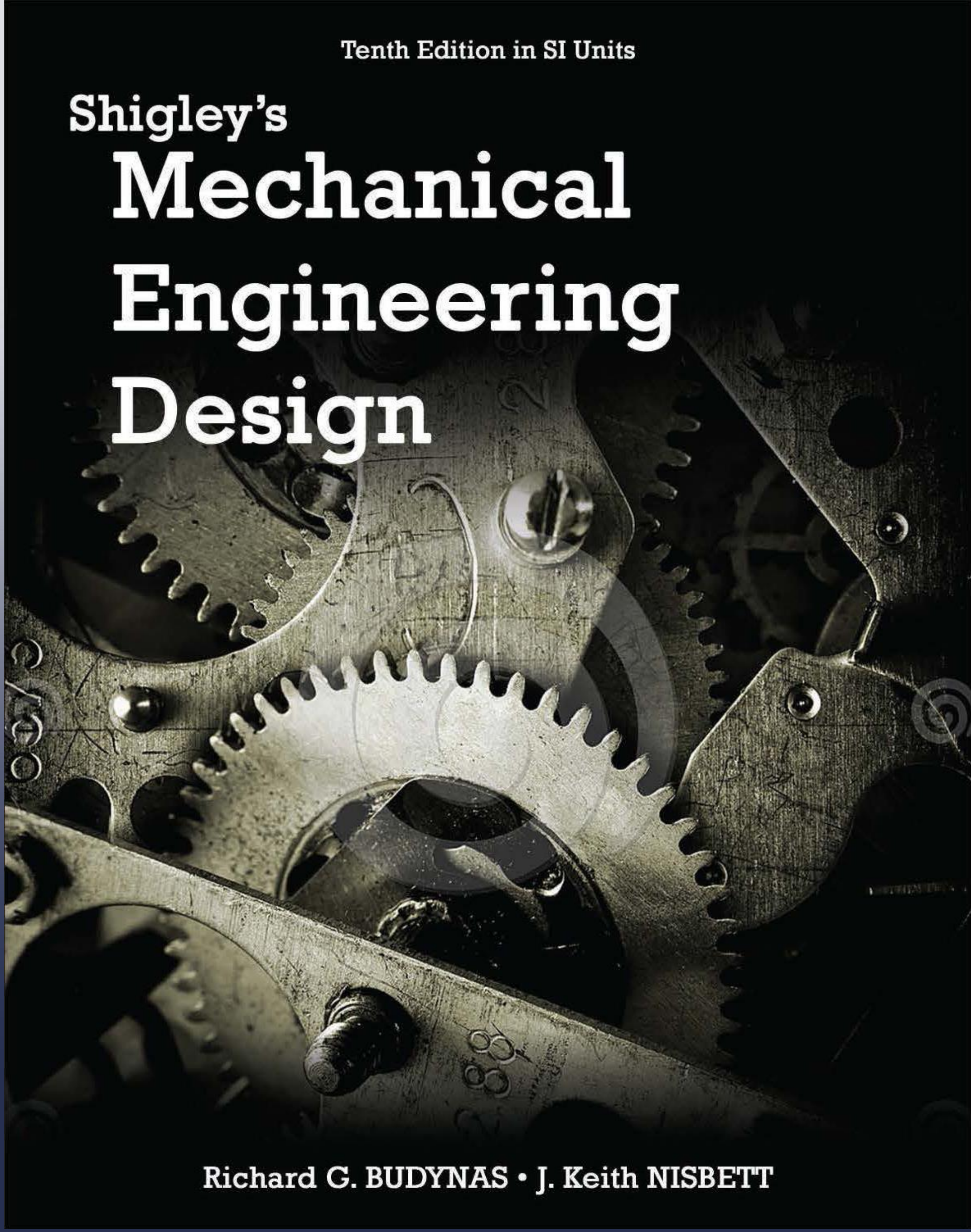
Lecture Slides

Chapter 11

Rolling-Contact Bearings



Chapter Outline

Nomenclature of a Ball Bearing

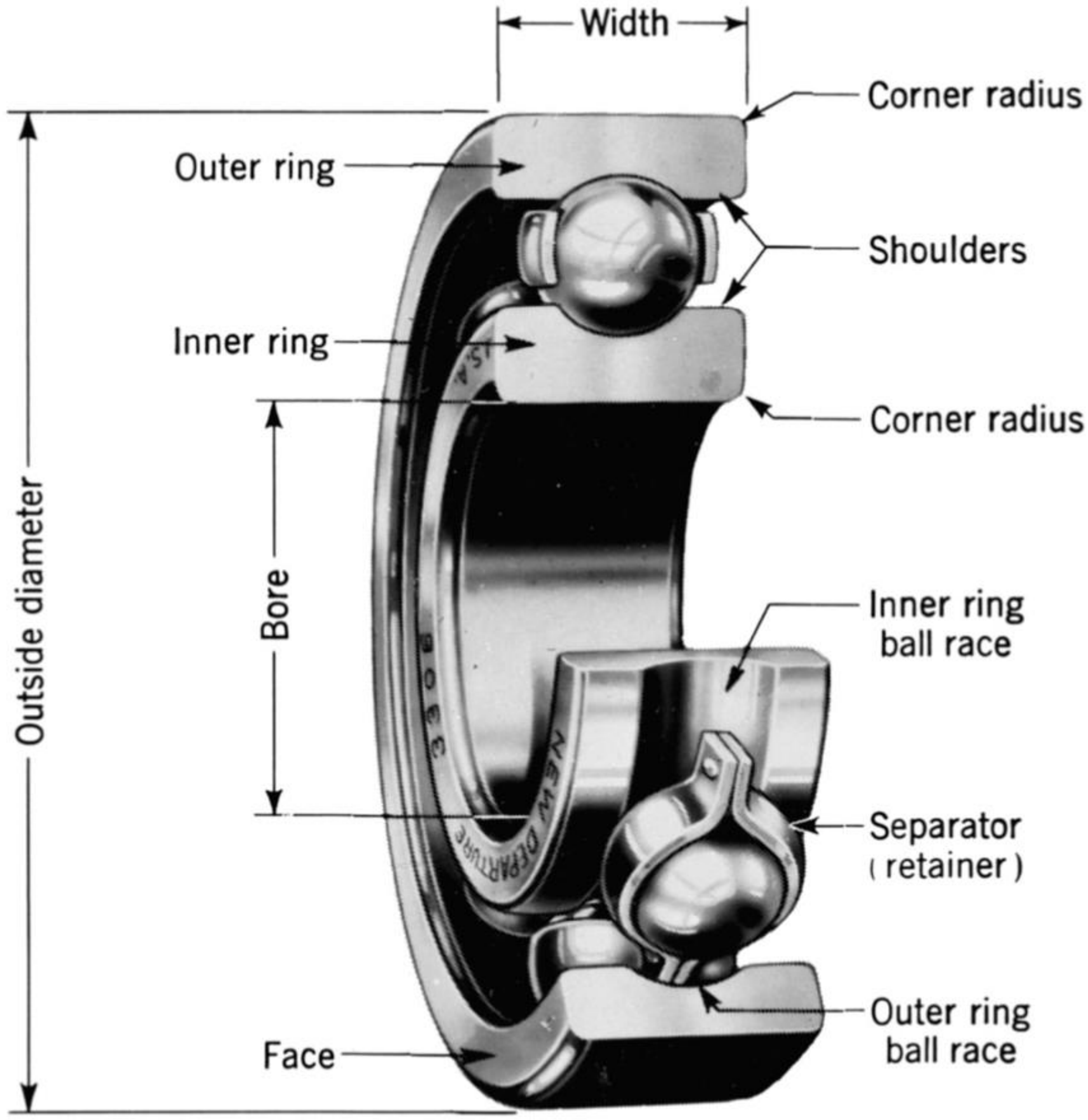


Fig. 11–1

Types of Ball Bearings

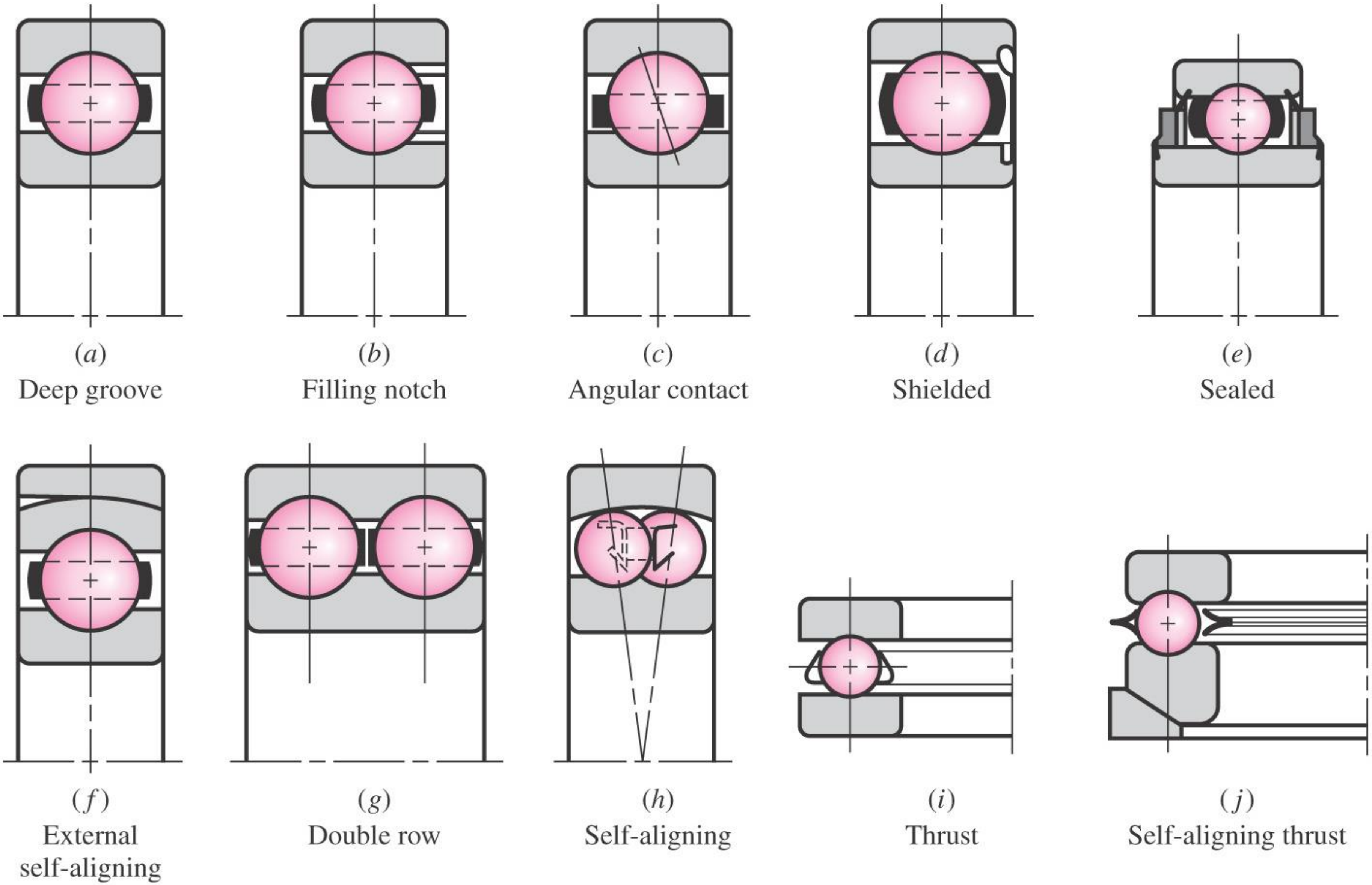
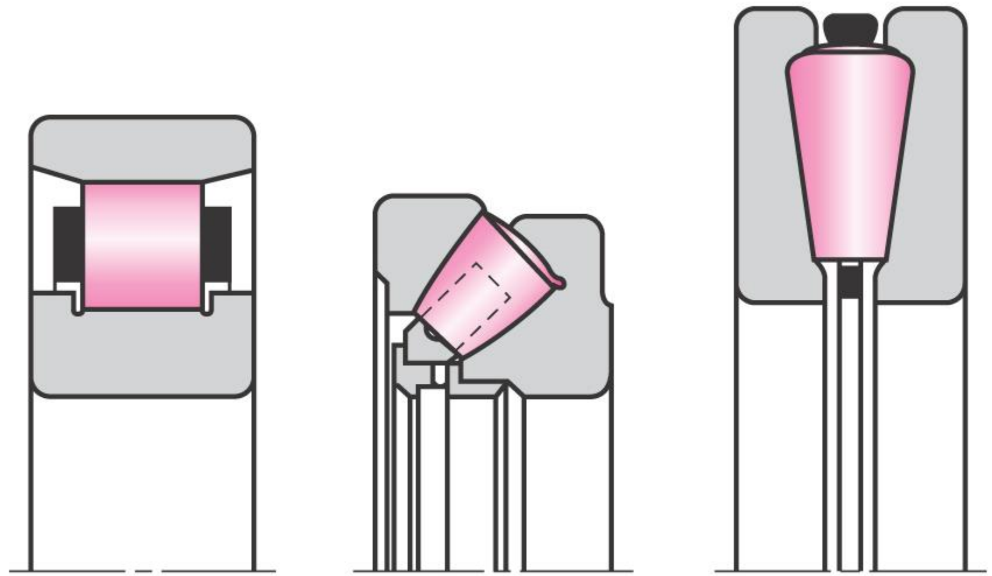


Fig. 11–2

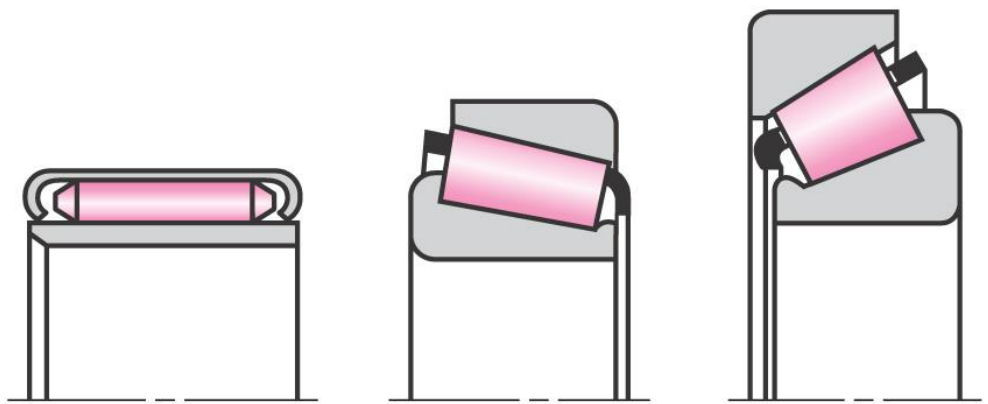
Types of Roller Bearings



(a) Straight Cylindrical

(b) Spherical Roller, thrust

(c) Tapered roller, thrust



(d) Needle

(e) Tapered roller

(f) Steep-angle tapered roller

Fig. 11–3

Bearing Life Definitions

- ***Bearing Failure:*** Spalling or pitting of an area of 0.01 in^2
- ***Life***: Number of revolutions (or hours @ given speed) required for failure.
  - For one bearing
- ***Rating Life***: *Life* required for 10% of sample to fail.
  - For a group of bearings
  - Also called *Minimum Life* or L_{10} *Life*
- ***Median Life***: Average life required for 50% of sample to fail.
  - For many groups of bearings
  - Also called *Average Life* or *Average Median Life*
  - *Median Life* is typically 4 or 5 times the L_{10} *Life*

Load Rating Definitions

- *Catalog Load Rating*, C_{10}: Constant radial load that causes 10% of a group of bearings to fail at the bearing manufacturer's rating life.
  - Depends on type, geometry, accuracy of fabrication, and material of bearing
  - Also called Basic Dynamic Load Rating, and Basic Dynamic Capacity
- *Basic Load Rating*, C: A catalog load rating based on a rating life of 10^6 revolutions of the inner ring.
  - The radial load that would be necessary to cause failure at such a low life is unrealistically high.
  - The Basic Load Rating is a reference value, not an actual load.

Load Rating Definitions

- *Static Load Rating, C_o*:
  Static radial load which corresponds to a permanent deformation of rolling element and race at the most heavily stressed contact of 0.0001*d*.
  - *d* = diameter of roller
  - Used to check for permanent deformation
  - Used in combining radial and thrust loads into an equivalent radial load
- *Equivalent Radial Load, F_e*:
  Constant stationary load applied to bearing with rotating inner ring which gives the same life as actual load and rotation conditions.

Load-Life Relationship

- Nominally identical groups of bearings are tested to the life-failure criterion at different loads.
- A plot of load vs. life on log-log scale is approximately linear.
- Using a regression equation to represent the line,

$$F L^{1/a} = \text{constant} \qquad (11\text{–}1)$$

  - a = 3 for ball bearings
  - a = 10/3 for roller bearings (cylindrical and tapered roller)

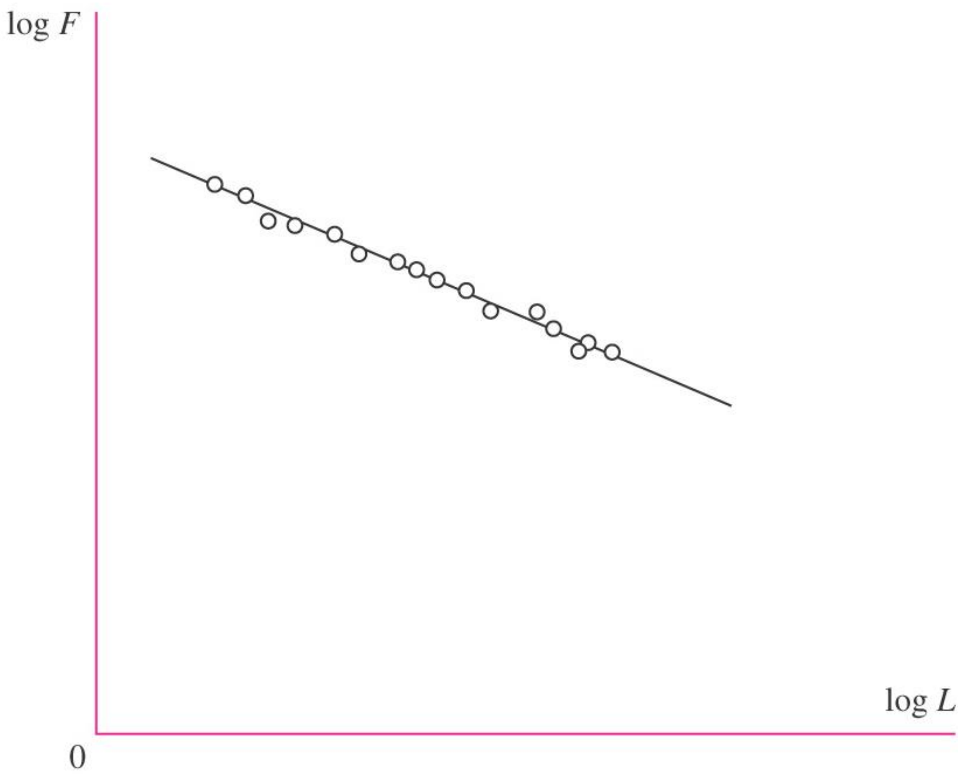


Fig. 11–4

Load-Life Relationship

- Applying Eq. (11–1) to two load-life conditions,

$$F_1 L_1^{1/a} = F_2 L_2^{1/a} \qquad (11\text{–}2)$$

- Denoting condition 1 with *R* for catalog rating conditions, and condition 2 with *D* for the desired design conditions,

$$F_R L_R^{1/a} = F_D L_D^{1/a} \qquad (a)$$

- The units of *L* are revolutions. If life $\mathcal{L}$ is given in hours at a given speed *n* in rev/min, applying a conversion of 60 min/h,

$$L = 60\,\mathcal{L} n \qquad (b)$$

- Solving Eq. (*a*) for F_R, which is just another notation for the catalog load rating,

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R}\right)^{1/a} = F_D \left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \qquad (11\text{–}3)$$

Load-Life Relationship

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R} \right)^{1/a} = F_D \left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60} \right)^{1/a} \qquad (11\text{–}3)$$

- The desired design load F_D and life L_D come from the problem statement.
- The rated life L_R will be stated by the specific bearing manufacturer. Many catalogs rate at $L_R = 10^6$ revolutions.
- The catalog load rating C_{10} is used to find a suitable bearing in the catalog.

Load-Life Relationship

- It is often convenient to define a dimensionless *multiple of rating life*

$$x_D = L_D / L_R$$

Example 11–1

Consider SKF, which rates its bearings for 1 million revolutions. If you desire a life of 5000 h at 1725 rev/min with a load of 400 lbf with a reliability of 90 percent, for which catalog rating would you search in an SKF catalog?

Solution

The rating life is $L_{10} = L_R = \mathcal{L}_R n_R 60 = 10^6$ revolutions. From Eq. (11–3),

$$C_{10} = F_D\left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} = 400\left[\frac{5000(1725)60}{10^6}\right]^{1/3} = 3211 \text{ lbf} = 14.3 \text{ kN}$$ Answer

Reliability vs. Life

- At constant load, the life measure distribution is right skewed.
- The Weibull distribution is a good candidate.
- Defining the life measure in dimensionless form as $x = L/L_{10}$, the reliability is expressed with a Weibull distribution as

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0} \right)^b \right] \qquad (11\text{–}4)$$

where R = reliability

x = life measure dimensionless variate, L/L_{10}

x_0 = guaranteed, or "minimum," value of x

θ = characteristic parameter. For rolling-contact bearings, this corresponds to the 63.2121 percentile value of x

b = shape parameter that controls the skewness. For rolling-contact bearings, $b \approx 1.5$

Reliability vs. Life

- From Eq. (1–8), $R = 1 - p$, where p is the probability of a value of x occurring between $-\infty$ and x.
- p is the integral of the probability distribution $f(x)$.
- From the derivative of Eq. (11–4), the Weibull probability density function is

$$f(x) = \begin{cases} \dfrac{b}{\theta - x_0}\left(\dfrac{x - x_0}{\theta - x_0}\right)^{b-1} \exp\left[-\left(\dfrac{x - x_0}{\theta - x_0}\right)^{b}\right] & x \geq x_0 \geq 0 \\ 0 & x < x_0 \end{cases} \tag{11–5}$$

Reliability vs. Life

- The mean and standard deviation of $f(x)$ are

$$\mu_x = x_0 + (\theta - x_0)\Gamma(1 + 1/b) \quad (11\text{–}6)$$

$$\hat{\sigma}_x = (\theta - x_0)\sqrt{\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)} \quad (11\text{–}7)$$

- Γ is the *gamma function*, and is tabulated in Table A–34

Reliability vs. Life

- Solving Eq. (11–4) for *x* yields

$$x = x_0 + (\theta - x_0)\left(\ln\frac{1}{R}\right)^{1/b} \qquad (11\text{–}8)$$

Example 11–2

Construct the distributional properties of a 02–30 mm deep-groove ball bearing if the Weibull parameters are $x_0 = 0.020$, $\theta = 4.459$, and $b = 1.483$.

Find the mean, median, 10th percentile life, standard deviation, and coefficient of variation.

Solution

From Eq. (11–6) and interpolating Table A–34, the mean dimensionless life is

$$\begin{aligned}\mu_x &= x_0 + (\theta - x_0)\Gamma(1 + 1/b)\\ &= 0.020 + (4.459 - 0.020)\Gamma(1 + 1/1.483)\\ &= 0.020 + 4.439\Gamma(1.67431) = 0.020 + 4.439(0.9040) = 4.033 \quad \text{Answer}\end{aligned}$$

This says that the average bearing life is $4.033\ L_{10}$.

The median dimensionless life corresponds to $R = 0.50$, or L_{50}, and from Eq. (11–8) is

$$x_{0.50} = x_0 + (\theta - x_0)\left(\ln\frac{1}{0.50}\right)^{1/b}$$

$$= 0.020 + (4.459 - 0.020)\left(\ln\frac{1}{0.50}\right)^{1/1.483} = 3.487 \quad \text{Answer}$$

or, $L = 3.487\ L_{10}$.

Example 11–2 (continued)

The 10th percentile value of the dimensionless life x is

$$x_{0.10} = 0.020 + (4.459 - 0.020)\left(\ln\frac{1}{0.90}\right)^{1/1.483} \approx 1 \qquad \text{(as it should be)}$$

Answer

The standard deviation of the dimensionless life, given by Eq. (11–7), is

$$\begin{aligned}\hat{\sigma}_x &= (\theta - x_0)\sqrt{\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)}\\ &= (4.459 - 0.020)\sqrt{\Gamma(1 + 2/1.483) - \Gamma^2(1 + 1/1.483)}\\ &= 4.439\sqrt{\Gamma(2.349) - \Gamma^2(1.674)} = 4.439\sqrt{1.2023 - 0.9040^2}\\ &= 2.755\end{aligned}$$

Answer

The coefficient of variation of the dimensionless life is

$$C_x = \frac{\hat{\sigma}_x}{\mu_x} = \frac{2.755}{4.033} = 0.683$$

Answer

Relating Load, Life, and Reliability

- Catalog information is at point *A*, at coordinates C_{10} and $x_{10}=L_{10}/L_{10}=1$, on the 0.90 reliability contour.
- The design information is at point *D*, at coordinates F_D and x_D, on the $R=R_D$ reliability contour.
- The designer must move from point *D* to point *A* via point *B*.

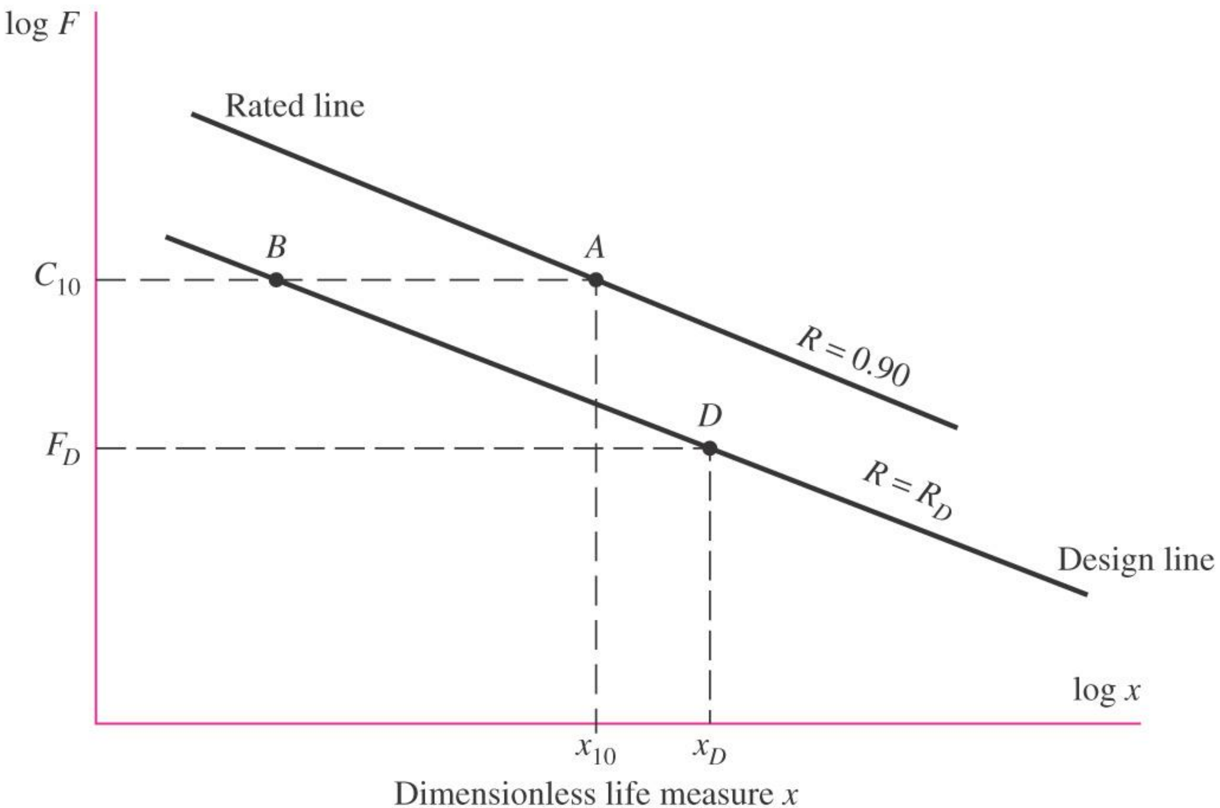


Fig. 11–5

Relating Load, Life, and Reliability

- Along a constant reliability contour (*BD*), Eq. (11–2) applies:

$$F_B x_B^{1/a} = F_D x_D^{1/a}$$

$$F_B = F_D \left(\frac{x_D}{x_B} \right)^{1/a} \qquad (a)$$

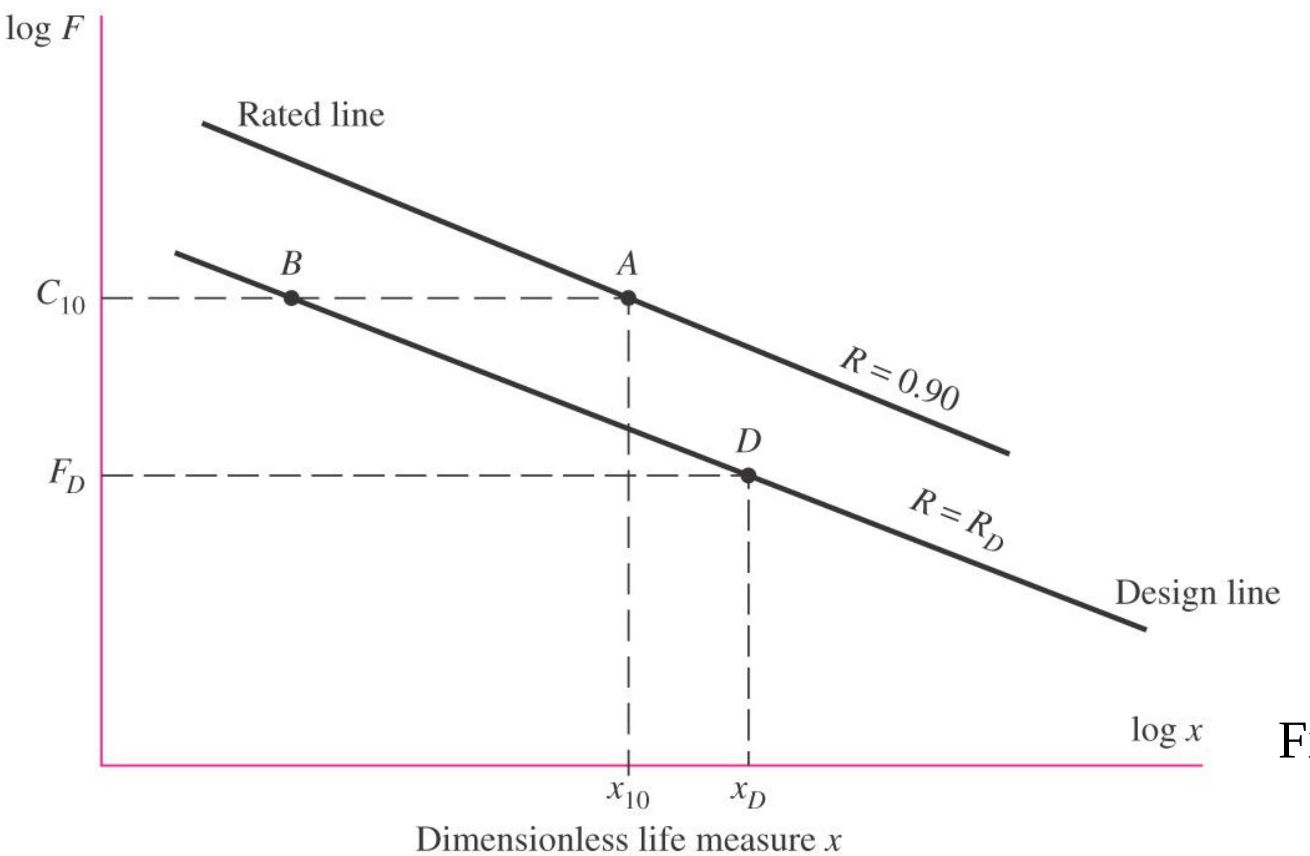


Fig. 11–5

Relating Load, Life, and Reliability

- Along a constant load line (*AB*), Eq. (11–4) applies:

$$R_D = \exp\left[-\left(\frac{x_B - x_0}{\theta - x_0}\right)^b\right]$$

- Solving for x_B,

$$x_B = x_0 + (\theta - x_0)\left(\ln\frac{1}{R_D}\right)^{1/b}$$

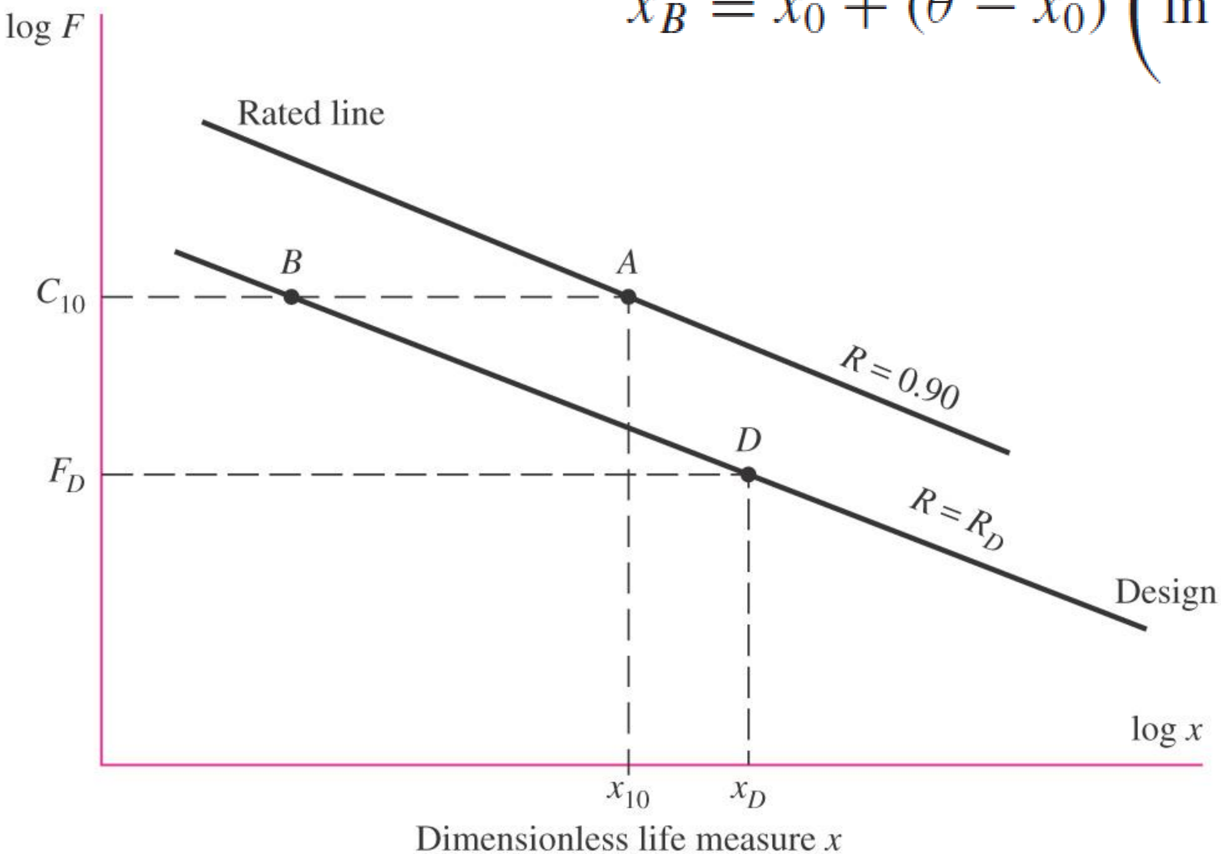


Fig. 11–5

Relating Load, Life, and Reliability

- Substituting x_B into Eq. (a),

$$F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}}\right]^{1/a}$$

- Noting that $F_B = C_{10}$, and including an application factor a_f,

$$C_{10} = a_f F_D\left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}}\right]^{1/a} \qquad (11\text{–}9)$$

- Note that when $R_D = 0.90$, the denominator equals one and the equation reduces to Eq. (11–3).

Weibull Parameters

- The Weibull parameters x_0, θ, and b are usually provided by the catalog.
- Typical values of Weibull parameters are given on p. 601 at the beginning of the end-of-chapter problems, and shown below.
- Manufacturer 1 parameters are common for tapered roller bearings
- Manufacturer 2 parameters are common for ball and straight roller bearings

		Weibull Parameters Rating Lives		
Manufacturer	Rating Life, Revolutions	x_0	θ	b
1	$90(10^6)$	0	4.48	1.5
2	$1(10^6)$	0.02	4.459	1.483

Relating Load, Life, and Reliability

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}} \right]^{1/a} \quad (11\text{–}9)$$

- Eq. (11–9) can be simplified slightly for calculator entry. Note that

$$\ln \frac{1}{R_D} = \ln \frac{1}{1 - p_f} = \ln(1 + p_f + \cdots) \approx p_f = 1 - R_D$$

where p_f is the probability for failure

- Thus Eq. (11–9) can be approximated by

$$C_{10} \approx a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \qquad R \geq 0.90 \qquad (11\text{–}10)$$

Example 11–3

The design load on a ball bearing is 1840 N and an application factor of 1.2 is appropriate. The speed of the shaft is to be 400 rev/min, the life to be 30 kh with a reliability of 0.99. What is the C_{10} catalog entry to be sought (or exceeded) when searching for a deep-groove bearing in a manufacturer's catalog on the basis of 10^6 revolutions for rating life? The Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$.

Solution

$$x_D = \frac{L_D}{L_R} = \frac{60\mathscr{L}_D n_D}{L_{10}} = \frac{60(30\,000)400}{10^6} = 720$$

Thus, the design life is 720 times the L_{10} life. For a ball bearing, $a = 3$. Then, from Eq. (11–10),

Answer

$$C_{10} = (1.2)(1.84)\left[\frac{720}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3} = 32.8\ \text{k}N$$

Combined Reliability of Multiple Bearings

- If the combined reliability of multiple bearings on a shaft, or in a gearbox, is desired, then the total reliability is equal to the product of the individual reliabilities.
- For two bearings on a shaft, $R = R_A R_B$
- If the bearings are to be identical, each bearing should have a reliability equal to the square root of the total desired reliability.
- If the bearings are not identical, their reliabilities need not be identical, so long as the total reliability is realized.

Dimension-Series Code

- ABMA standardized *dimension-series code* represents the relative size of the boundary dimensions of the bearing cross section for metric bearings.
- Two digit series number
- First digit designates the width series
- Second digit designates the diameter series
- Specific dimensions are tabulated in catalogs under a specific series

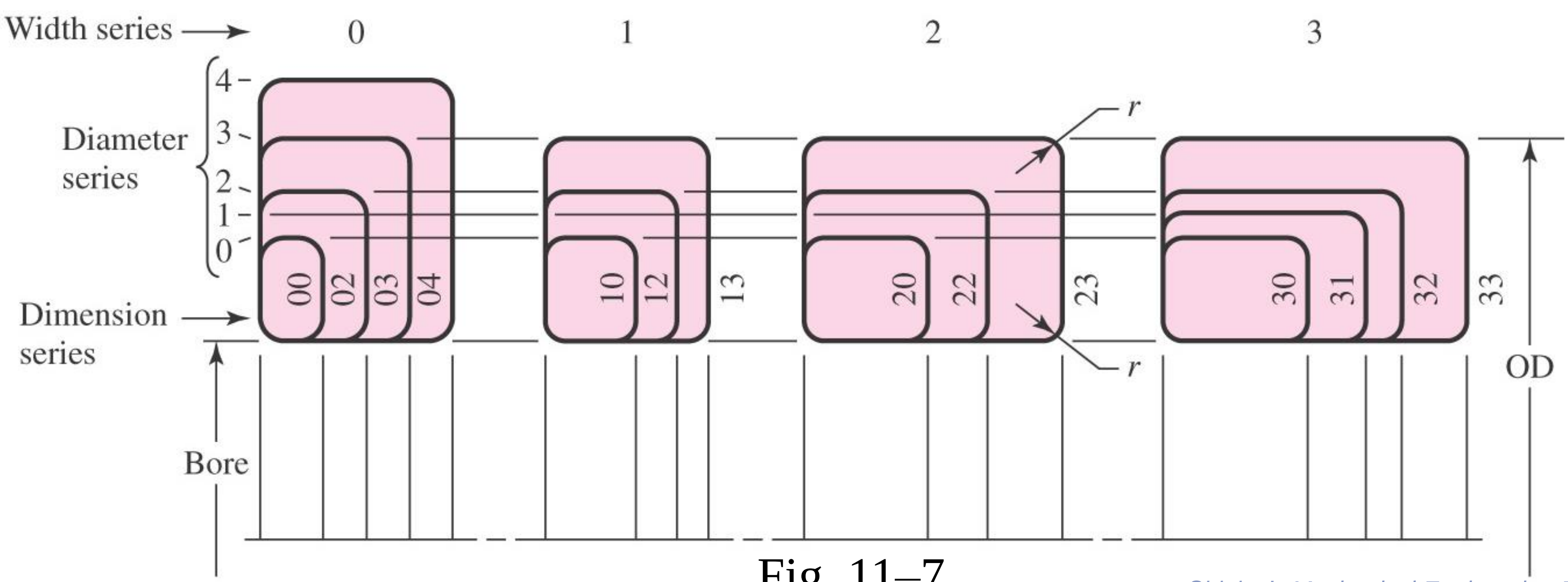


Fig. 11–7

Representative Catalog Data for Ball Bearings (Table 11–2)

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

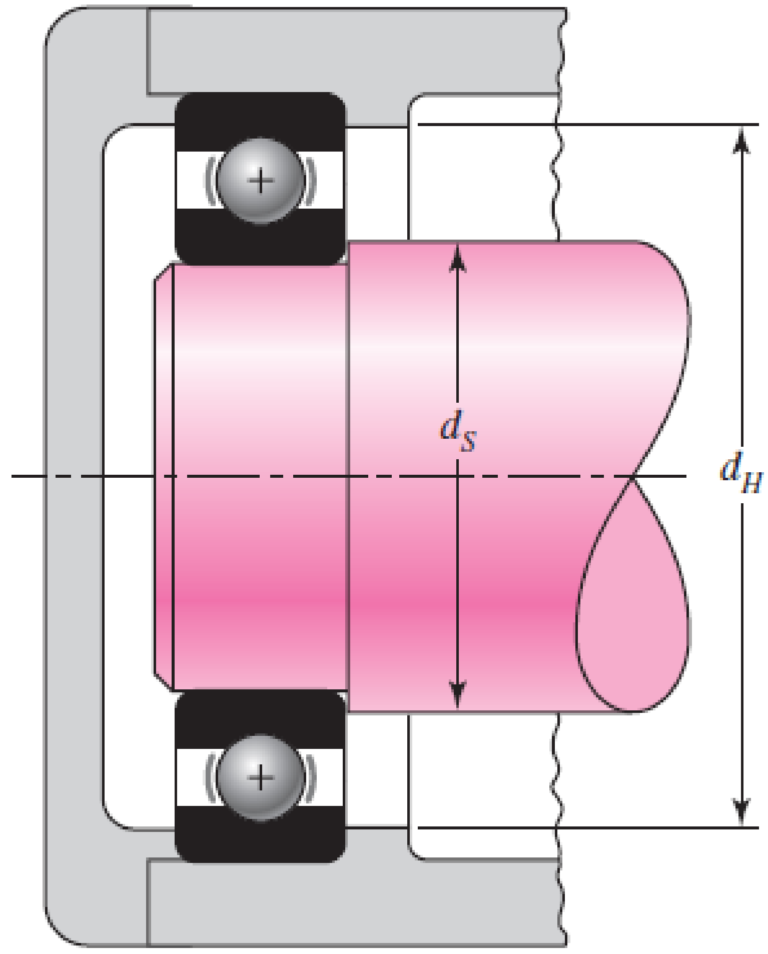
Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm		Load Ratings, kN			
						Deep Groove		Angular Contact	
				d_S	d_H	C_{10}	C_0	C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

Representative Catalog Data for Cylindrical Roller Bearings (Table 11–3)

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Figure 11–8

Shaft and housing shoulder diameters d_S and d_H should be adequate to ensure good bearing support.



Combined Radial and Thrust Loading

- When ball bearings carry both an axial thrust load F_a and a radial load F_r, an *equivalent radial load* F_e that does the same damage is used.
- A plot of $F_e/(VF_r)$ vs. $F_a/(VF_r)$ is obtained experimentally.
- V is a rotation factor to account for the difference in ball rotations for outer ring rotation vs. inner ring roation.
    - $V = 1$ for inner ring rotation
    - $V = 1.2$ for outer ring rotation

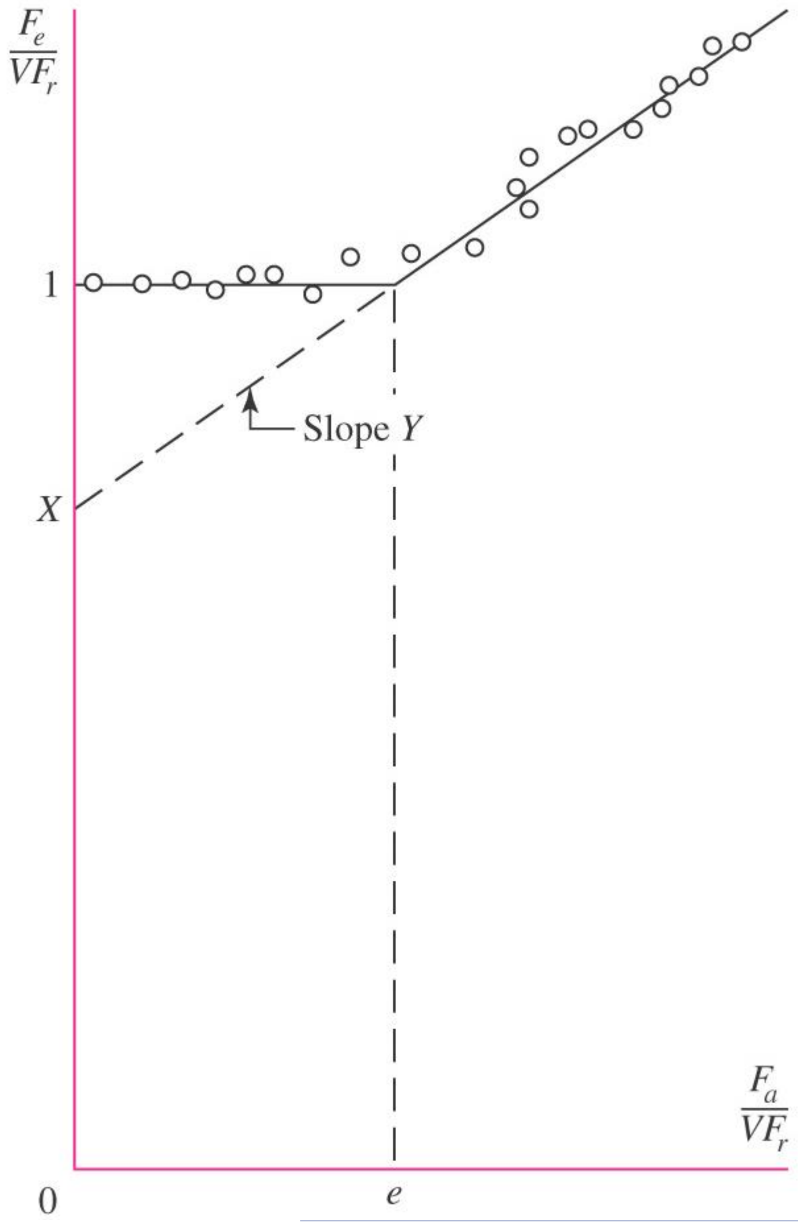


Fig. 11–6

Combined Radial and Thrust Loading

- The data can be approximated by two straight lines

$$\frac{F_e}{VF_r} = 1 \qquad \text{when } \frac{F_a}{VF_r} \le e$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \qquad \text{when } \frac{F_a}{VF_r} > e$$

- X is the ordinate intercept and Y is the slope
- Basically indicates that F_e equals F_r for smaller ratios of F_a/F_r, then begins to rise when F_a/F_r exceeds some amount e

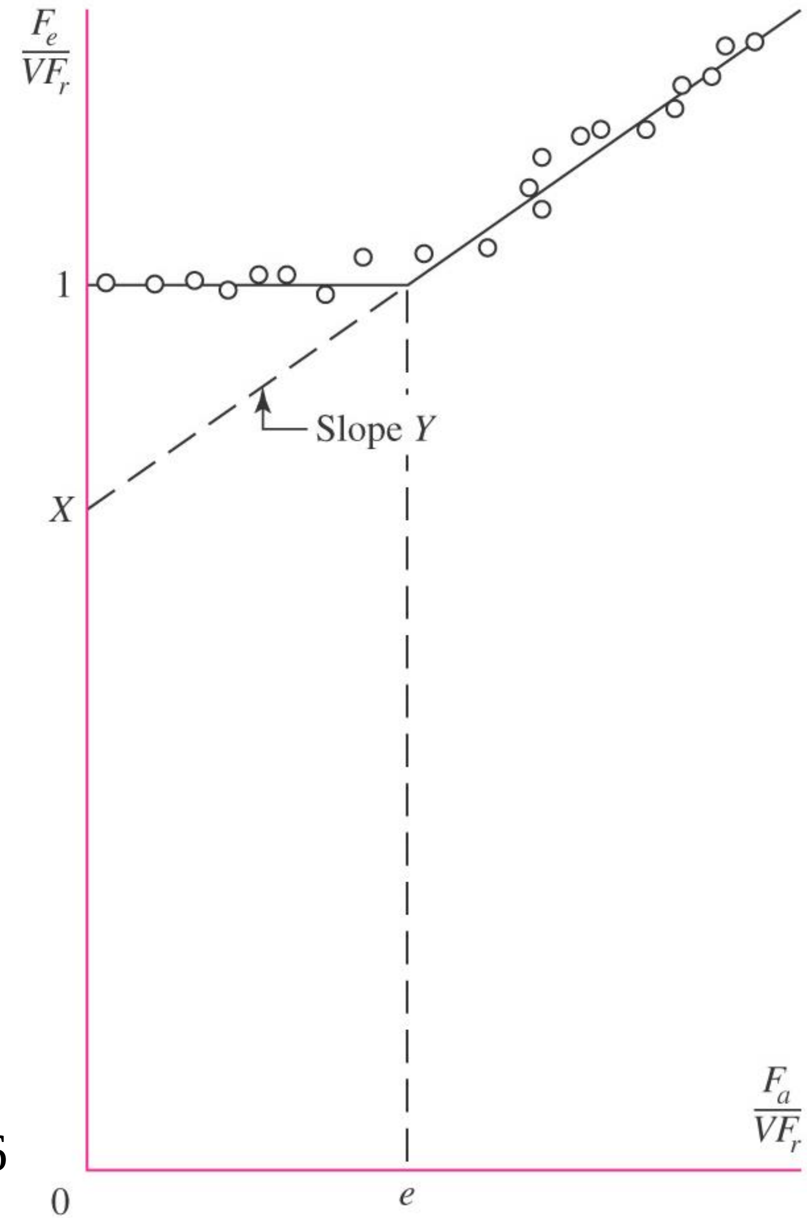


Fig. 11–6

Combined Radial and Thrust Loading

- It is common to express the two equations as a single equation

$$F_e = X_i V F_r + Y_i F_a \qquad (11\text{–}12)$$

where

$i = 1$ when $F_a/(VF_r) \leq e$

$i = 2$ when $F_a/(VF_r) > e$

- X and Y factors depend on geometry and construction of the specific bearing.

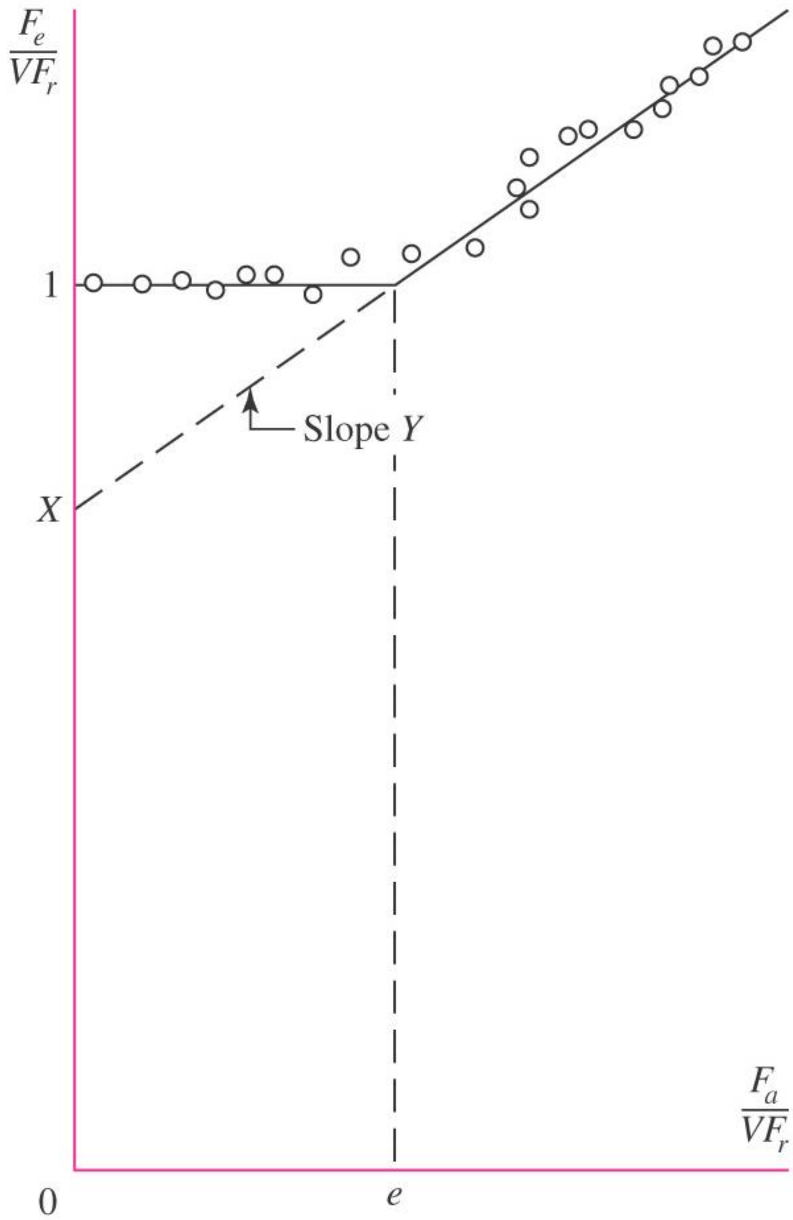


Fig. 11–6

Equivalent Radial Load Factors for Ball Bearings

$$F_e = X_i V F_r + Y_i F_a \qquad (11\text{–}12)$$

- X and Y for specific bearing obtained from bearing catalog.
- Table 11–1 gives representative values in a manner common to many catalogs.

Table 11–1

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

Equivalent Radial Load Factors for Ball Bearings

$$F_e = X_i V F_r + Y_i F_a \qquad (11\text{–}12)$$

Table 11–1

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

- X and Y are functions of e, which is a function of F_a/C_0.
- C_0 is the *basic static load rating*, which is tabulated in the catalog.

Bearing Life Recommendations (Table 11–4)

Type of Application	Life, kh
Instruments and apparatus for infrequent use	Up to 0.5
Aircraft engines	0.5–2
Machines for short or intermittent operation where service interruption is of minor importance	4–8
Machines for intermittent service where reliable operation is of great importance	8–14
Machines for 8-h service that are not always fully utilized	14–20
Machines for 8-h service that are fully utilized	20–30
Machines for continuous 24-h service	50–60
Machines for continuous 24-h service where reliability is of extreme importance	100–200

Recommended Load Application Factors (Table 11–5)

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

Example 11–4

An SKF 6210 angular-contact ball bearing has an axial load F_a of 400 lbf and a radial load F_r of 500 lbf applied with the outer ring stationary. The basic static load rating C_0 is 4450 lbf and the basic load rating C_{10} is 7900 lbf. Estimate the $\mathscr{L}_{10}$ life at a speed of 720 rev/min.

Solution

$V = 1$ and $F_a/C_0 = 400/4450 = 0.090$. Interpolate for e in Table 11–1:

F_a/C_0	e	
0.084	0.28	
0.090	e	from which $e = 0.285$
0.110	0.30	

Example 11–4 (continued)

$F_a/(VF_r) = 1780/[(1)2225] = 0.8 > 0.285$. Thus, interpolate for Y_2:

F_a/C_0	Y_2	
0.084	1.55	
0.090	Y_2	from which $Y_2 = 1.527$
0.110	1.45	

From Eq. (11–9),

$$F_e = X_2 V F_r + Y_2 F_a = 0.56(1)2225 + 1.527(1780) = 3964 \text{ kN}$$

With $L_D = L_{10}$ and $F_D = F_e$, solving Eq. (11–3) for L_{10} gives

Answer

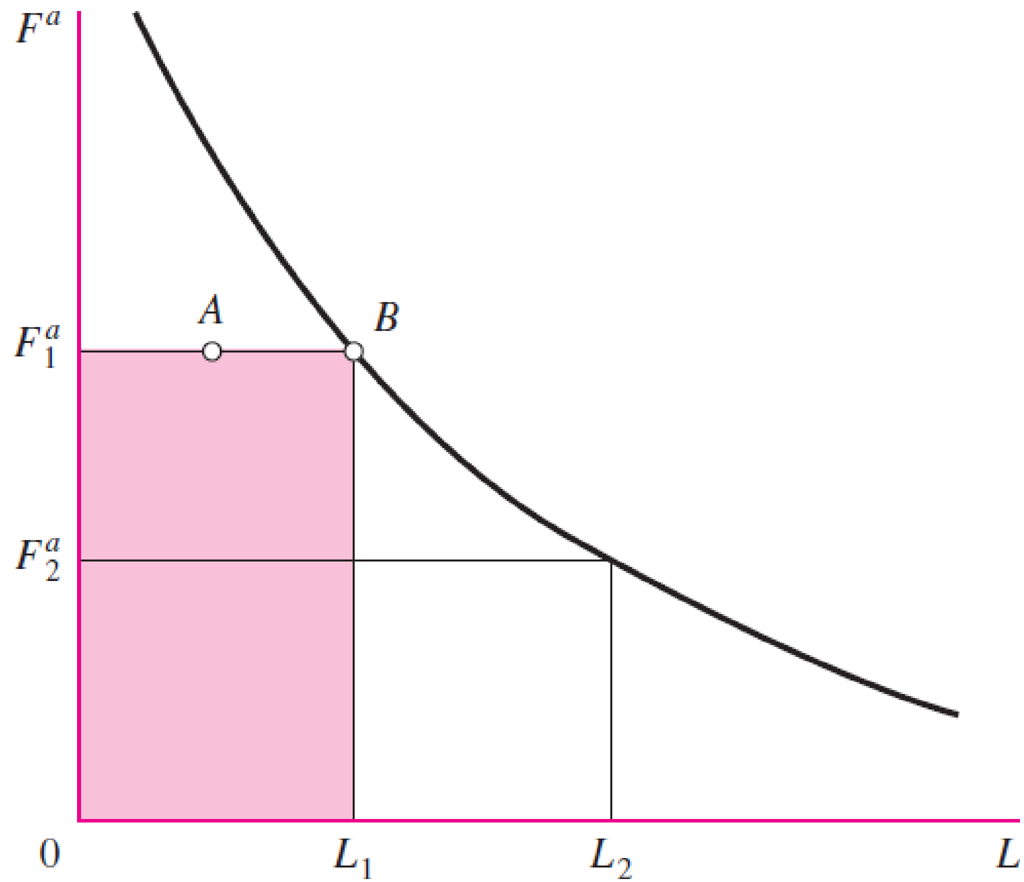
$$\mathcal{L}_{10} = \frac{60\mathcal{L}_R n_R}{60 n_D}\left(\frac{C_{10}}{F_e}\right)^a = \frac{10^6}{60(720)}\left(\frac{35\,150}{3964}\right)^3 = 161\,395 \text{ h}$$

Variable Loading

$$F^a L = \text{constant} = K \qquad (a)$$

Figure 11–9

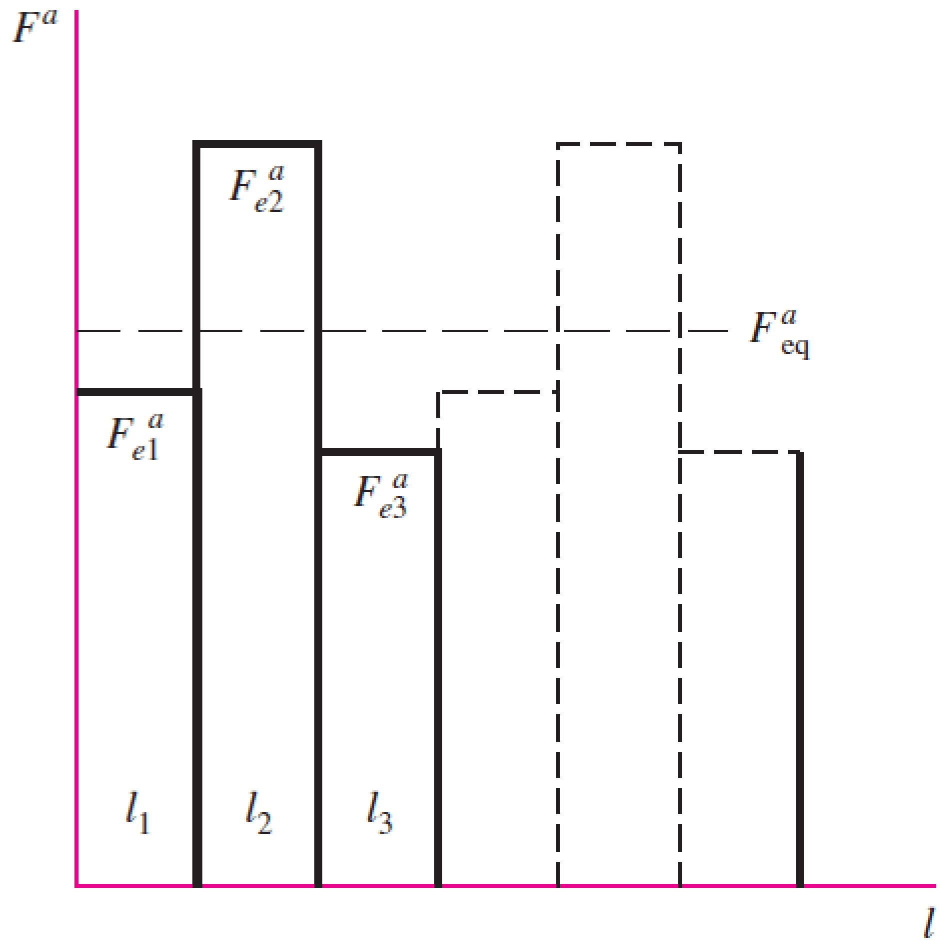
Plot of F^a as ordinate and L as abscissa for $F^a L = \text{constant}$. The linear damage hypothesis says that in the case of load F_1, the area under the curve from $L = 0$ to $L = L_A$ is a measure of the damage $D = F_1^a L_A$. The complete damage to failure is measured by $C_{10}^a L_B$.



Variable Loading with Piecewise Constant Loading

Figure 11–10

A three-part piecewise-continuous periodic loading cycle involving loads F_{e1}, F_{e2}, and F_{e3}. F_{eq} is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage D per period.



$$D = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 \qquad (b)$$

$$D = F_{eq}^a (l_1 + l_2 + l_3) \qquad (c)$$

Variable Loading with Piecewise Constant Loading

$$D = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 \qquad (b)$$

$$D = F_{eq}^a (l_1 + l_2 + l_3) \qquad (c)$$

$$F_{eq} = \left[\frac{F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3}{l_1 + l_2 + l_3} \right]^{1/a} = \left[\sum f_i F_{ei}^a \right]^{1/a} \qquad (11\text{–}13)$$

$$F_{eq} = \left[\frac{\sum n_i t_i F_{ei}^a}{\sum n_i t_i} \right]^{1/a} \qquad (11\text{–}14)$$

$$F_{eq} = \left[\sum f_i (a_{fi} F_{ei})^a \right]^{1/a} \qquad L_{eq} = \frac{K}{F_{eq}^a} \qquad (11\text{–}15)$$

Example 11–5

A ball bearing is run at four piecewise continuous steady loads as shown in the following table. Columns (1), (2), and (5) to (8) are given.

(1) Time Fraction	(2) Speed, rev/min	(3) Product, Column (1) × (2)	(4) Turns Fraction, (3)/Σ(3)	(5) F_{ri}, N	(6) F_{ai}, N	(7) F_{ei}, N	(8) a_{fi}	(9) $a_{fi} F_{ei}$, N
0.1	2000	200	0.077	2700	1350	3573	1.10	3930
0.1	3000	300	0.115	1350	1350	2817	1.25	3521
0.3	3000	900	0.346	3375	1350	3951	1.10	4346
0.5	2400	1200	0.462	1688	1350	3006	1.25	3758
		2600	1.000					

Columns 1 and 2 are multiplied to obtain column 3. The column 3 entry is divided by the sum of column 3, 2600, to give column 4. Columns 5, 6, and 7 are the radial, axial, and equivalent loads respectively. Column 8 is the appropriate application factor. Column 9 is the product of columns 7 and 8.

Example 11–5 (continued)

Solution

From Eq. (11–13), with $a = 3$, the equivalent radial load F_e is

$$F_e = [0.077(3930)^3 + 0.115(3521)^3 + 0.346(4346)^3 + 0.462(3758)^3]^{1/3} = 3971 \text{ N}$$

Answer

Variable Loading with Piecewise Constant Loading

$$F_{eq}^{a} L_{eq} = F_{e1}^{a} l_1 + F_{e2}^{a} l_2 + F_{e3}^{a} l_3$$

$$K = F_{e1}^{a} L_1 = F_{e2}^{a} L_2 = F_{e3}^{a} L_3$$

$$K = F_{e1}^{a} l_1 + F_{e2}^{a} l_2 + F_{e3}^{a} l_3 = \frac{K}{L_1} l_1 + \frac{K}{L_2} l_2 + \frac{K}{L_3} l_3 = K \sum \frac{l_i}{L_i}$$

$$\sum \frac{l_i}{L_i} = 1 \qquad (11\text{–}16)$$

Variable Loading with Periodic Variation

$$dD = F^a d\theta$$

$$D = \int dD = \int_0^{\phi} F^a d\theta = F_{eq}^a \phi$$

$$F_{eq} = \left[\frac{1}{\phi} \int_0^{\phi} F^a d\theta \right]^{1/a} \qquad L_{eq} = \frac{K}{F_{eq}^a} \qquad (11\text{–}17)$$

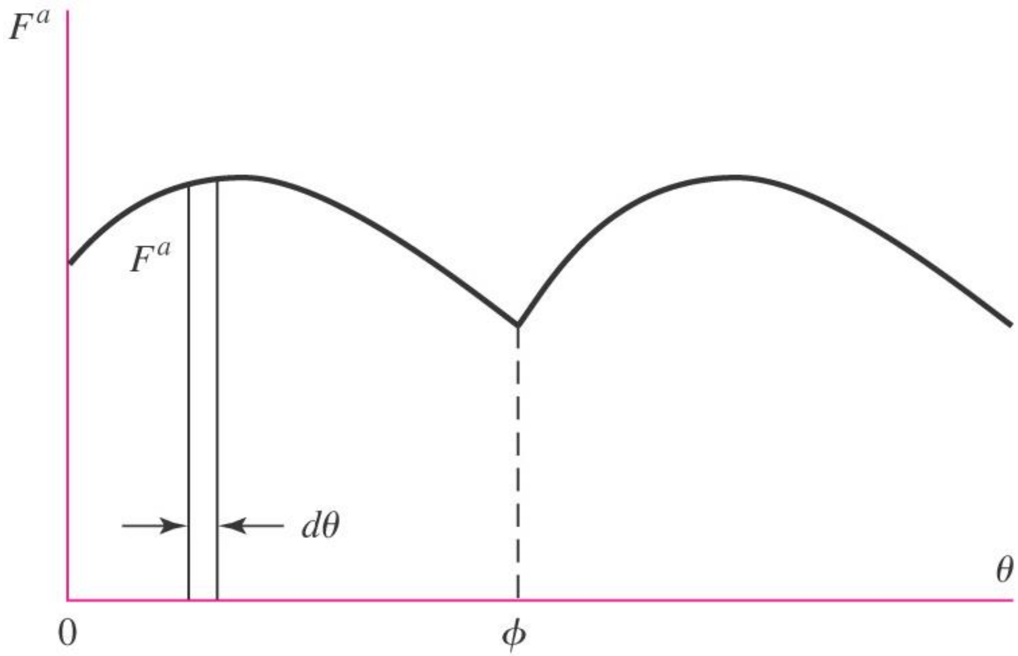


Fig. 11–11

Example 11–6

The operation of a particular rotary pump involves a power demand of $P = \overline{P} + A' \sin\theta$ where $\overline{P}$ is the average power. The bearings feel the same variation as $F = \overline{F} + A\sin\theta$. Develop an application factor a_f for this application of ball bearings.

Solution

From Eq. (11–17), with $a = 3$,

$$F_{eq} = \left(\frac{1}{2\pi}\int_0^{2\pi} F^a\,d\theta\right)^{1/a} = \left(\frac{1}{2\pi}\int_0^{2\pi} (\overline{F} + A\sin\theta)^3\,d\theta\right)^{1/3}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{2\pi} \overline{F}^3\,d\theta + 3\overline{F}^2 A\int_0^{2\pi} \sin\theta\,d\theta + 3\overline{F}A^2\int_0^{2\pi} \sin^2\theta\,d\theta + A^3\int_0^{2\pi} \sin^3\theta\,d\theta\right)\right]^{1/3}$$

$$F_{eq} = \left[\frac{1}{2\pi}(2\pi\overline{F}^3 + 0 + 3\pi\overline{F}A^2 + 0)\right]^{1/3} = \overline{F}\left[1 + \frac{3}{2}\left(\frac{A}{\overline{F}}\right)^2\right]^{1/3}$$

Example 11–6 (continued)

In terms of $\overline{F}$, the application factor is

Answer $$a_f = \left[1 + \frac{3}{2}\left(\frac{A}{\overline{F}}\right)^2\right]^{1/3}$$

We can present the result in tabular form:

$A/\overline{F}$	a_f
0	1
0.2	1.02
0.4	1.07
0.6	1.15
0.8	1.25
1.0	1.36

Example 11–7

Shown in Figure 11–12 is a gear-driven squeeze roll that mates with an idler roll. The roll is designed to exert a normal force of 5.25 N/mm of roll length and a pull of 4.2 N/mm on the material being processed. The roll speed is 300 rev/min, and a design life of 30 kh is desired. Use an application factor of 1.2, and select a pair of angular-contact 02-series ball bearings from Table 11–2 to be mounted at 0 and *A*. Use the same size bearings at both locations and a combined reliability of at least 0.92.

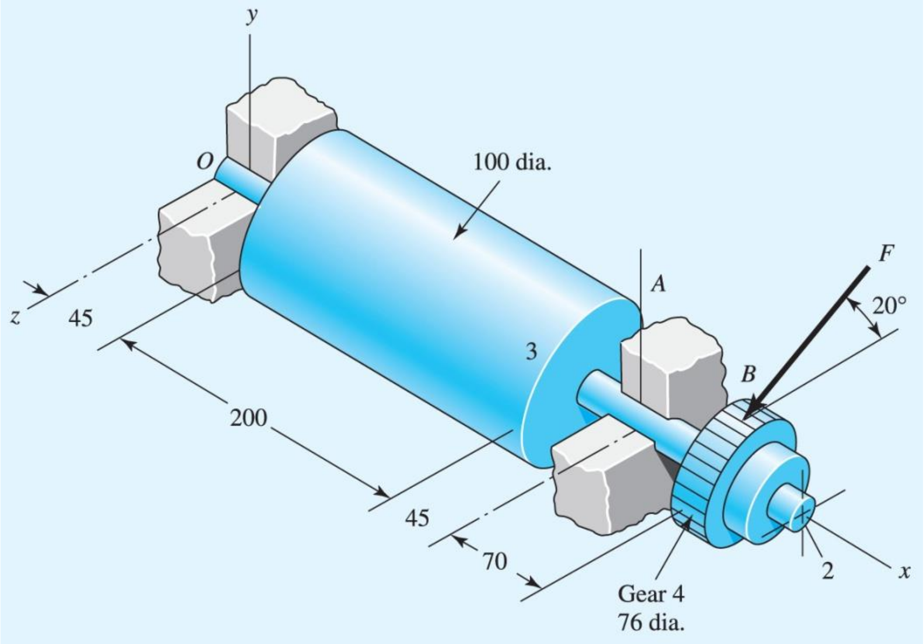
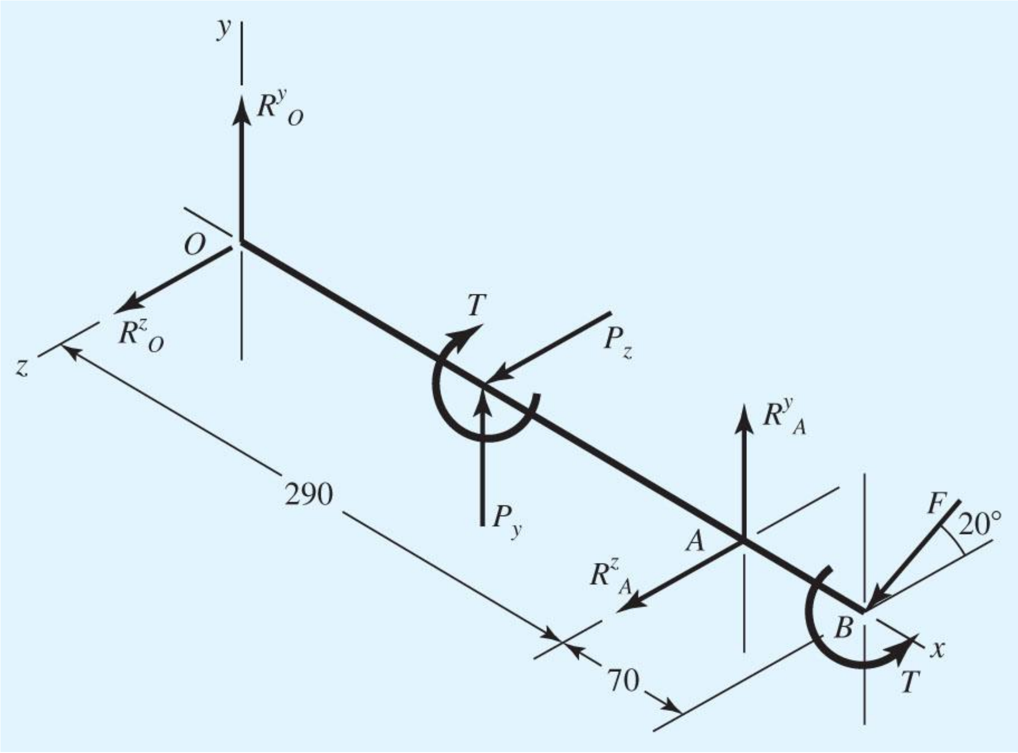


Fig. 11–12

Example 11–7



Example 11–7

Solution Assume concentrated forces as shown.

$$P_z = 200(4.2) = 840\ \text{N}$$

$$P_y = 200(5.25) = 1050\ \text{N}$$

$$T = 840(50) = 42\,000\ \text{N}\cdot\text{mm}$$

$$\sum T^x = -42\,000 + 38F\cos 20^\circ = 0$$

$$F = \frac{42\,000}{38(0.940)} = 1176.2\ \text{N}$$

$$\sum M_O^z = 145P_y + 290R_A^y - 360F\sin 20^\circ = 0;$$

thus

$$145(1050) + 290R_A^y - 360(1176.2)(0.342) = 0$$

$$R_A^y = -25.6\ \text{N}$$

$$\sum M_O^y = -145P_z - 290R_A^z - 360F\cos 20^\circ = 0;$$

Example 11–7

thus

$$-145(840) - 290R_A^z - 360(1176.2)(0.940) = 0$$

$$R_A^z = -1792 \text{ N};$$

$$R_A = [(-1792)^2 + (-25.6)^2]^{1/2} = 1792 \text{ N}$$

$$\sum F^z = R_O^z + P_z + R_A^z + F\cos 20^\circ = 0$$

$$R_O^z + 840 - 1792 + 1176.2(0.940) = 0$$

$$R_O^z = -153.3 \text{ N}$$

$$\sum F^y = R_O^y + P_y + R_A^y - F\sin 20^\circ = 0$$

$$R_O^y + 1050 - 25.6 - 1176.2(0.342) = 0$$

$$R_O^y = -622 \text{ N}$$

$$R_O = [(-153.3)^2 + (-622)^2]^{1/2} = 640.6 \text{ N}$$

Example 11–7 (continued)

So the reaction at A governs.

Reliability Goal: $\sqrt{0.92} = 0.96$

$$F_D = 1.2(1792) = 2150.4 \text{ N}$$

$$x_D = 30\,000(300)(60/10^6) = 540$$

$$C_{10} = 2150.4\left\{\frac{540}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}}\right\}^{1/3}$$

$$= 21.59 \text{ kN}$$

A 02–35 bearing will do.

Answer *Decision:* Specify an angular-contact 02–35 mm ball bearing for the locations at A and O. Check combined reliability.

Tapered Roller Bearings

- Straight roller bearings can carry large radial loads, but no axial load.
- Ball bearings can carry moderate radial loads, and small axial loads.
- Tapered roller bearings rely on roller tipped at an angle to allow them to carry large radial and large axial loads.
- Tapered roller bearings were popularized by the Timken Company.

Tapered Roller Bearings

- Two separable parts
  - Cone assembly
    - Cone (inner ring)
    - Rollers
    - Cage
  - Cup (outer ring)
- Rollers are tapered so virtual apex is on shaft centerline
- Taper allows for pure rolling of angled rollers
- Distance *a* locates the effective axial location for force analysis

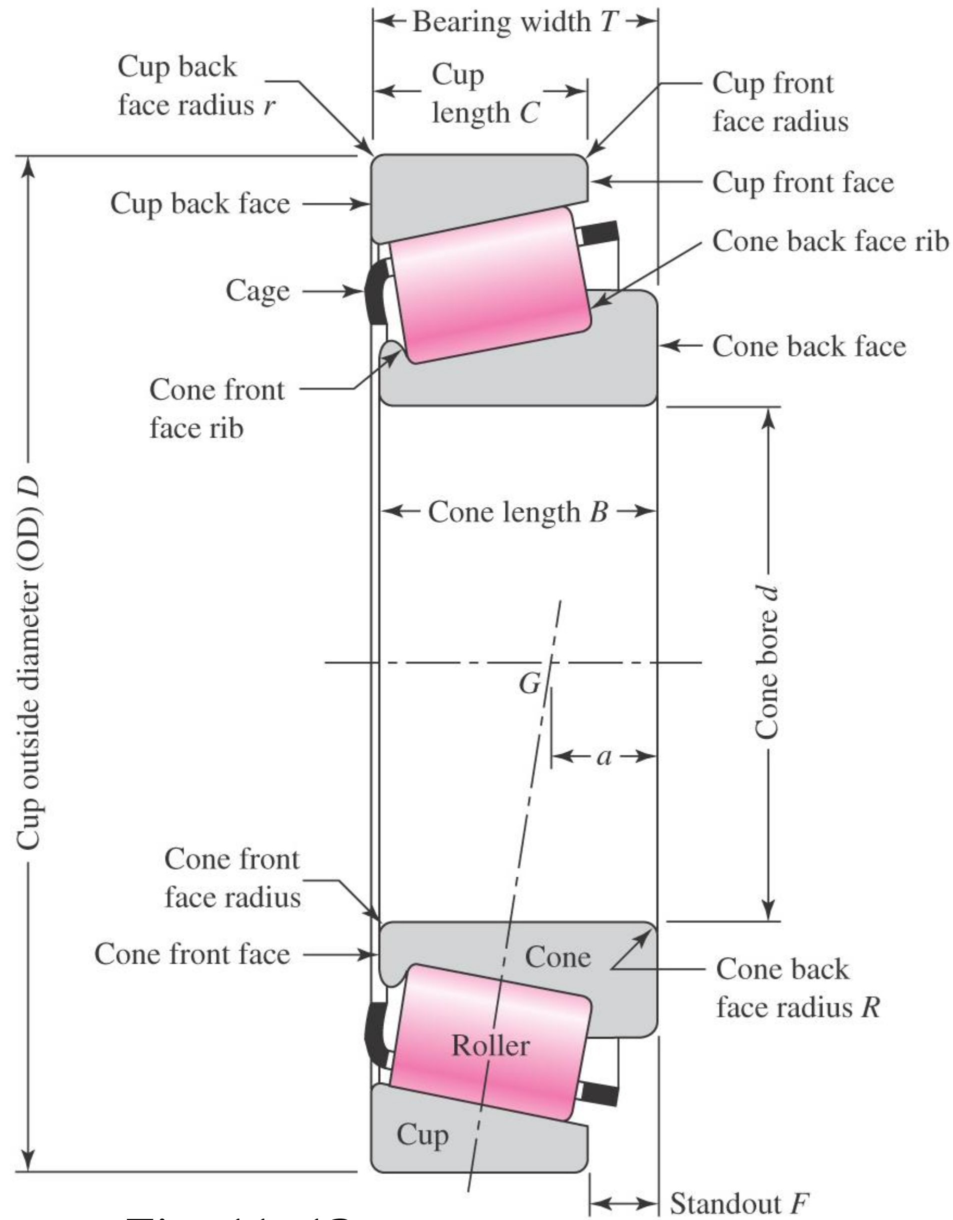


Fig. 11–13

Mounting Directions of Tapered Roller Bearings

- Mount pairs in opposite directions to counter the axial loads
- Can be mounted in *direct mounting* or *indirect mounting* configurations
- For the same effective spread a_e, direct mounting requires greater geometric spread a_g
- For the same geometric spread a_g, direct mounting provides smaller effect spread a_e

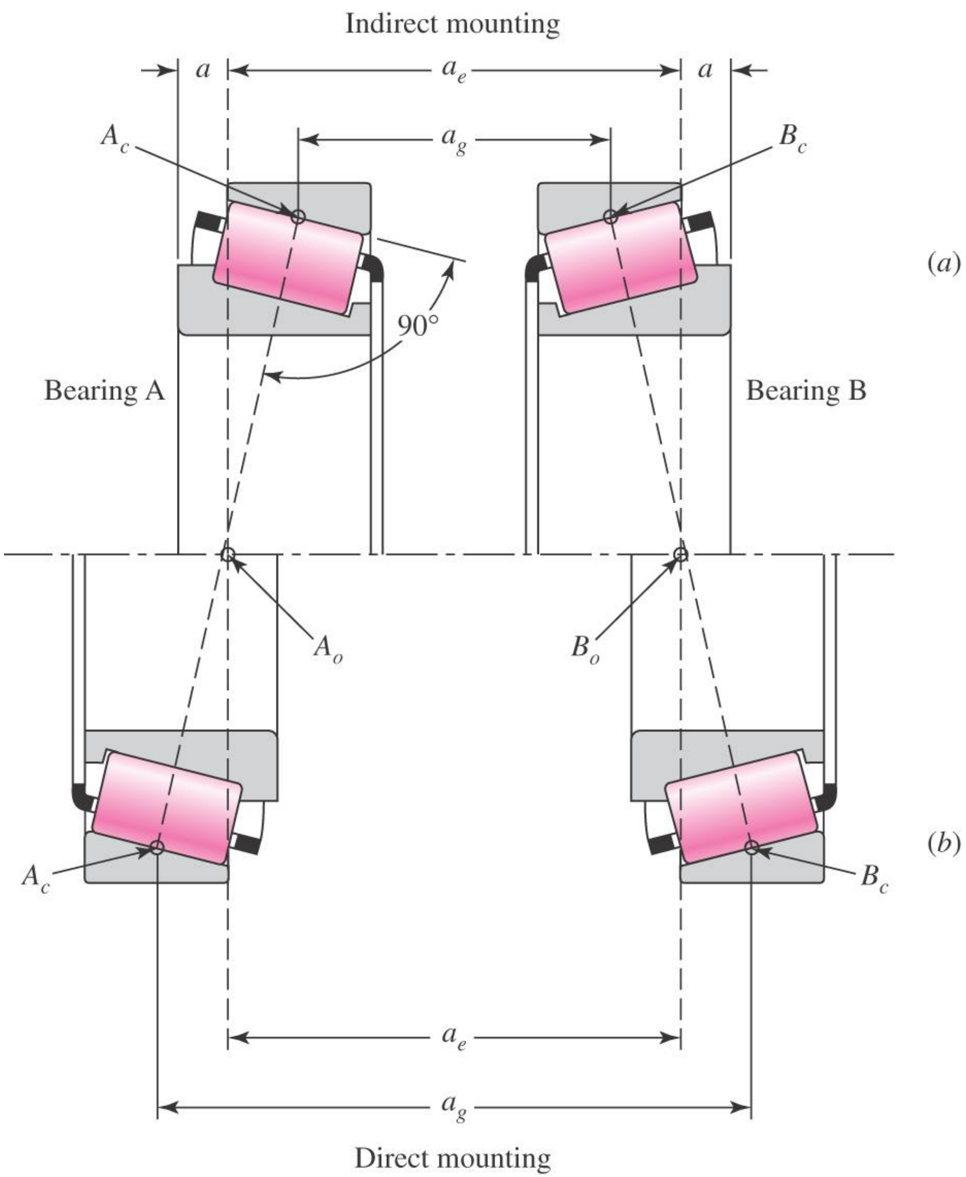
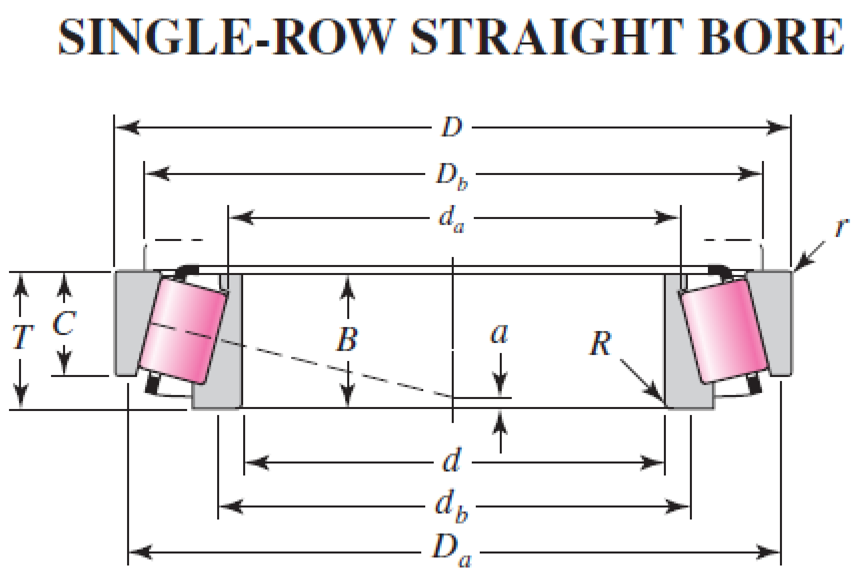


Fig. 11–14

Typical Catalog Data (Fig. 11–15)



Typical Catalog Data (Fig. 11–15)

									cone					cup			
bore	outside diameter	width	rating at 500 rpm for 3000 hours L_{10}		fac-tor	eff. load center	part numbers		max shaft fillet radius	width	backing shoulder diameters		max hous-ing fillet radius	width	backing shoulder diameters	
			one-row radial	thrust			cone	cup								
d	D	T	N lbf	N lbf	K	a②			R①	B	d_b	d_a	r①	C	D_b	D_a
25.000 0.9843	**52.000** 2.0472	**16.250** 0.6398	**8190** 1840	**5260** 1180	**1.56**	**−3.6** −0.14	**◆30205**	**◆30205**	**1.0** 0.04	**15.000** 0.5906	**30.5** 1.20	**29.0** 1.14	**1.0** 0.04	**13.000** 0.5118	**46.0** 1.81	**48.5** 1.91
25.000 0.9843	**52.000** 2.0472	**19.250** 0.7579	**9520** 2140	**9510** 2140	**1.00**	**−3.0** −0.12	**◆32205-B**	**◆32205-B**	**1.0** 0.04	**18.000** 0.7087	**34.0** 1.34	**31.0** 1.22	**1.0** 0.04	**15.000** 0.5906	**43.5** 1.71	**49.5** 1.95
25.000 0.9843	**52.000** 2.0472	**22.000** 0.8661	**13200** 2980	**7960** 1790	**1.66**	**−7.6** −0.30	**◆33205**	**◆33205**	**1.0** 0.04	**22.000** 0.8661	**34.0** 1.34	**30.5** 1.20	**1.0** 0.04	**18.000** 0.7087	**44.5** 1.75	**49.0** 1.93
25.000 0.9843	**62.000** 2.4409	**18.250** 0.7185	**13000** 2930	**6680** 1500	**1.95**	**−5.1** −0.20	**◆30305**	**◆30305**	**1.5** 0.06	**17.000** 0.6693	**32.5** 1.28	**30.0** 1.18	**1.5** 0.06	**15.000** 0.5906	**55.0** 2.17	**57.0** 2.24
25.000 0.9843	**62.000** 2.4409	**25.250** 0.9941	**17400** 3910	**8930** 2010	**1.95**	**−9.7** −0.38	**◆32305**	**◆32305**	**1.5** 0.06	**24.000** 0.9449	**35.0** 1.38	**31.5** 1.24	**1.5** 0.06	**20.000** 0.7874	**54.0** 2.13	**57.0** 2.24
25.159 0.9905	**50.005** 1.9687	**13.495** 0.5313	**6990** 1570	**4810** 1080	**1.45**	**−2.8** −0.11	**07096**	**07196**	**1.5** 0.06	**14.260** 0.5614	**31.5** 1.24	**29.5** 1.16	**1.0** 0.04	**9.525** 0.3750	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**50.005** 1.9687	**13.495** 0.5313	**6990** 1570	**4810** 1080	**1.45**	**−2.8** −0.11	**07100**	**07196**	**1.0** 0.04	**14.260** 0.5614	**30.5** 1.20	**29.5** 1.16	**1.0** 0.04	**9.525** 0.3750	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**50.005** 1.9687	**13.495** 0.5313	**6990** 1570	**4810** 1080	**1.45**	**−2.8** −0.11	**07100-S**	**07196**	**1.5** 0.06	**14.260** 0.5614	**31.5** 1.24	**29.5** 1.16	**1.0** 0.04	**9.525** 0.3750	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**50.292** 1.9800	**14.224** 0.5600	**7210** 1620	**4620** 1040	**1.56**	**−3.3** −0.13	**L44642**	**L44610**	**3.5** 0.14	**14.732** 0.5800	**36.0** 1.42	**29.5** 1.16	**1.3** 0.05	**10.668** 0.4200	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**50.292** 1.9800	**14.224** 0.5600	**7210** 1620	**4620** 1040	**1.56**	**−3.3** −0.13	**L44643**	**L44610**	**1.3** 0.05	**14.732** 0.5800	**31.5** 1.24	**29.5** 1.16	**1.3** 0.05	**10.668** 0.4200	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**51.994** 2.0470	**15.011** 0.5910	**6990** 1570	**4810** 1080	**1.45**	**−2.8** −0.11	**07100**	**07204**	**1.0** 0.04	**14.260** 0.5614	**30.5** 1.20	**29.5** 1.16	**1.3** 0.05	**12.700** 0.5000	**45.0** 1.77	**48.0** 1.89
25.400 1.0000	**56.896** 2.2400	**19.368** 0.7625	**10900** 2450	**5740** 1290	**1.90**	**−6.9** −0.27	**1780**	**1729**	**0.8** 0.03	**19.837** 0.7810	**30.5** 1.20	**30.0** 1.18	**1.3** 0.05	**15.875** 0.6250	**49.0** 1.93	**51.0** 2.01

Typical Catalog Data (Fig. 11–15 continued)

									cone				cup			
bore	outside diameter	width	rating at 500 rpm for 3000 hours L_{10}		fac-tor	eff. load center	part numbers		max shaft fillet radius	width	backing shoulder diameters		max hous-ing fillet radius	width	backing shoulder diameters	
			one-row radial	thrust			cone	cup								
d	D	T	N lbf	N lbf	K	a②			R①	B	d_b	d_a	r①	C	D_b	D_a
25.400 1.0000	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15102**	**15245**	**1.5** 0.06	**20.638** 0.8125	**34.0** 1.34	**31.5** 1.24	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15244**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**56.0** 2.20	**59.0** 2.32
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.5** 0.06	**15.875** 0.6250	**55.0** 2.17	**59.0** 2.32
25.400 1.0000	**64.292** 2.5312	**21.433** 0.8438	**14500** 3250	**13500** 3040	**1.07**	**−3.3** −0.13	**M86643**	**M86610**	**1.5** 0.06	**21.433** 0.8438	**38.0** 1.50	**36.5** 1.44	**1.5** 0.06	**16.670** 0.6563	**54.0** 2.13	**61.0** 2.40
25.400 1.0000	**65.088** 2.5625	**22.225** 0.8750	**13100** 2950	**16400** 3690	**0.80**	**−2.3** −0.09	**23100**	**23256**	**1.5** 0.06	**21.463** 0.8450	**39.0** 1.54	**34.5** 1.36	**1.5** 0.06	**15.875** 0.6250	**53.0** 2.09	**63.0** 2.48
25.400 1.0000	**66.421** 2.6150	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2687**	**2631**	**1.3** 0.05	**25.433** 1.0013	**33.5** 1.32	**31.5** 1.24	**1.3** 0.05	**19.050** 0.7500	**58.0** 2.28	**60.0** 2.36
25.400 1.0000	**68.262** 2.6875	**22.225** 0.8750	**15300** 3440	**10900** 2450	**1.40**	**−5.1** −0.20	**02473**	**02420**	**0.8** 0.03	**22.225** 0.8750	**34.5** 1.36	**33.5** 1.32	**1.5** 0.06	**17.462** 0.6875	**59.0** 2.32	**63.0** 2.48
25.400 1.0000	**72.233** 2.8438	**25.400** 1.0000	**18400** 4140	**17200** 3870	**1.07**	**−4.6** −0.18	**HM88630**	**HM88610**	**0.8** 0.03	**25.400** 1.0000	**39.5** 1.56	**39.5** 1.56	**2.3** 0.09	**19.842** 0.7812	**60.0** 2.36	**69.0** 2.72
25.400 1.0000	**72.626** 2.8593	**30.162** 1.1875	**22700** 5110	**13000** 2910	**1.76**	**−10.2** −0.40	**3189**	**3120**	**0.8** 0.03	**29.997** 1.1810	**35.5** 1.40	**35.0** 1.38	**3.3** 0.13	**23.812** 0.9375	**61.0** 2.40	**67.0** 2.64
26.157 1.0298	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15103**	**15245**	**0.8** 0.03	**20.638** 0.8125	**33.0** 1.30	**32.5** 1.28	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
26.162 1.0300	**63.100** 2.4843	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2682**	**2630**	**1.5** 0.06	**25.433** 1.0013	**34.5** 1.36	**32.0** 1.26	**0.8** 0.03	**19.050** 0.7500	**57.0** 2.24	**59.0** 2.32

Typical Catalog Data (Fig. 11–15 continued)

① These maximum fillet radii will be cleared by the bearing corners.

② Minus value indicates center is inside cone backface.

† For standard class **ONLY**, the maximum metric size is a whole mm value.

★ For "J" part tolerances—see metric tolerances, page 73, and fitting practice, page 65.

◆ ISO cone and cup combinations are designated with a common part number and should be purchased as an assembly. For ISO bearing tolerances—see metric tolerances, page 73, and fitting practice, page 65.

Induced Thrust Load

- A radial load induces a thrust reaction due to the roller angle.

$$F_i = \frac{0.47F_r}{K} \qquad (11\text{–}18)$$

- K is ratio of radial load rating to thrust load rating
- K is dependent on specific bearing, and is tabulated in catalog

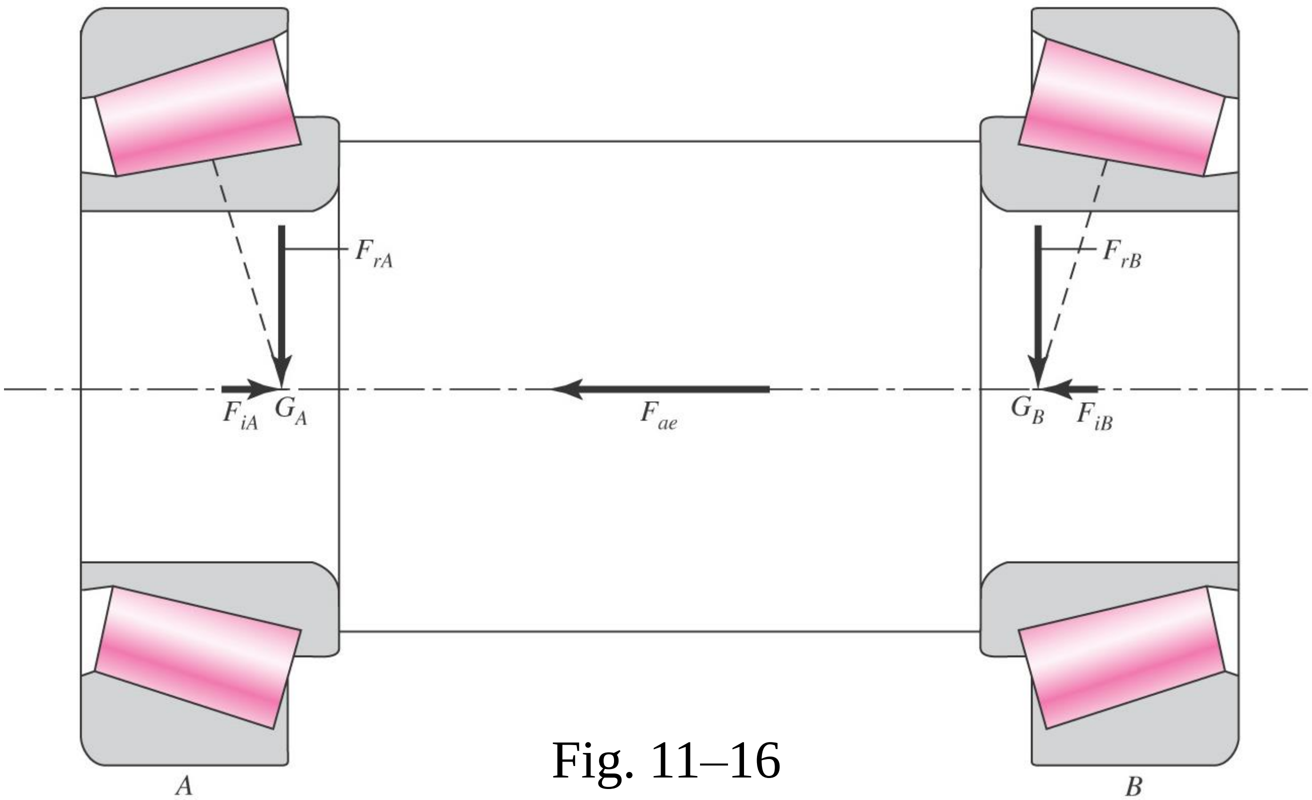


Fig. 11–16

Equivalent Radial Load

- The equivalent radial load for tapered roller bearings is found in similar form as before,

$$F_e = XVF_r + YF_a$$

- Timken recommends $X = 0.4$ and $Y = K$

$$F_e = 0.4F_r + KF_a$$

- F_a is the net axial load carried by the bearing, including induced thrust load from the other bearing and the external axial load carried by the bearing.
- Only one of the bearings will carry the external axial load

Determining Which Bearing Carries External Axial Load

- Regardless of mounting direction or shaft orientation, visually inspect to determine which bearing is being "squeezed"
- Label this bearing as *Bearing A*

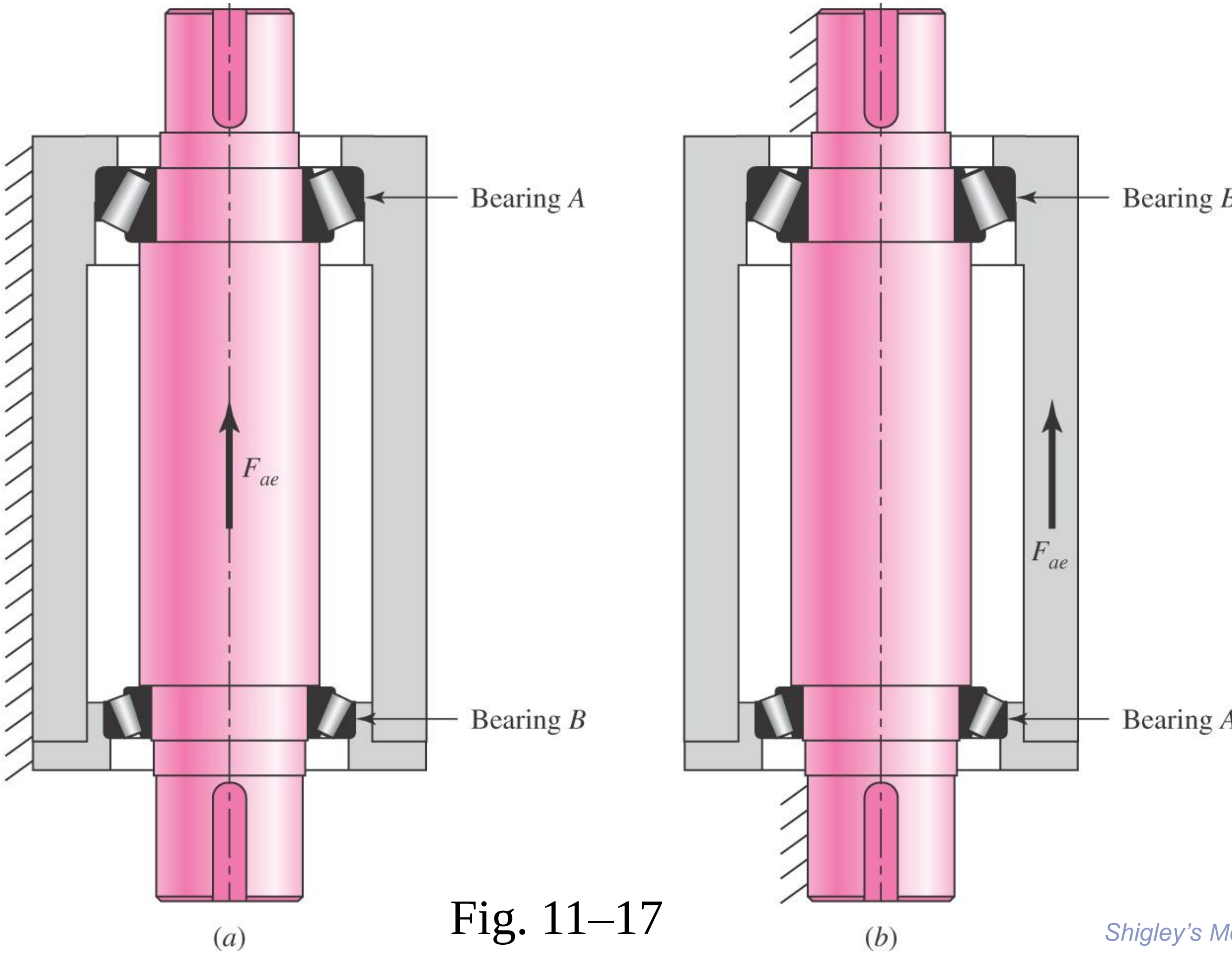


Fig. 11–17

Net Axial Load

- Generally, *Bearing A* (the squeezed bearing) carries the net axial load
- Occasionally the induced thrust from *Bearing A*, F_{iA}, is greater than the combination of the induced thrust from *Bearing B*, F_{iB}, and the external axial load F_{ae} , that is

$$F_{iA} > (F_{iB} + F_{ae})$$

- If this happens, then *Bearing B* actually carries the net axial load

Equivalent Radial Load

- Timken recommends using the full radial load for the bearing that is not carrying the net axial load.
- Equivalent radial load equation:

If $F_{iA} \leq (F_{iB} + F_{ae})$
$$\begin{cases} F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) & \text{(11–19a)} \\ F_{eB} = F_{rB} & \text{(11–19b)} \end{cases}$$

If $F_{iA} > (F_{iB} + F_{ae})$
$$\begin{cases} F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae}) & \text{(11–20a)} \\ F_{eA} = F_{rA} & \text{(11–20b)} \end{cases}$$

- If the equivalent radial load is less than the original radial load, then use the original radial load.

Example 11–8

The shaft depicted in Fig. 11–18*a* carries a helical gear with a tangential force of 3980 N, a radial force of 1770 N, and a thrust force of 1690 N at the pitch cylinder with directions shown. The pitch diameter of the gear is 200 mm. The shaft runs at a speed of 800 rev/min, and the span (effective spread) between the direct-mount bearings is 150 mm. The design life is to be 5000 h and an application factor of 1 is appropriate. If the reliability of the bearing set is to be 0.99, select suitable single-row tapered-roller Timken bearings.

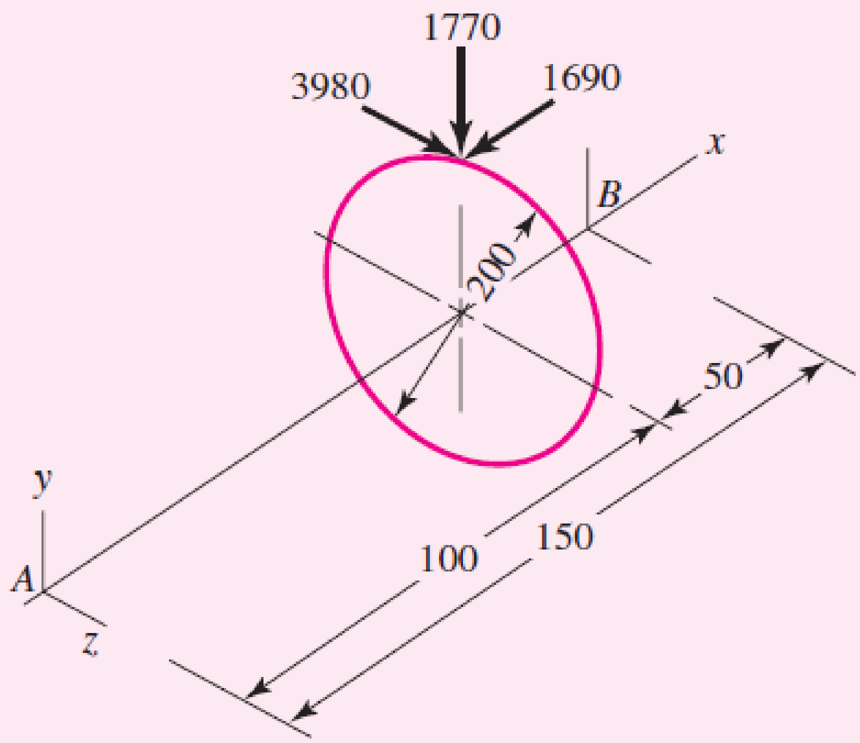


Fig. 11–18 (*a*)

Example 11–8 (continued)

The reactions in the xz plane from Fig. 11–18b are

$$R_{zA} = \frac{3980(50)}{150} = 1327\ \text{N}$$

$$R_{zB} = \frac{3980(100)}{150} = 2653\ \text{N}$$

The reactions in the xy plane from Fig. 11–18c are

$$R_{yA} = \frac{1770(50)}{150} + \frac{169\ 000}{150} = 1716.7 = 1717\ \text{N}$$

$$R_{yB} = \frac{1770(100)}{150} - \frac{169\ 000}{150} = 53.3\ \text{N}$$

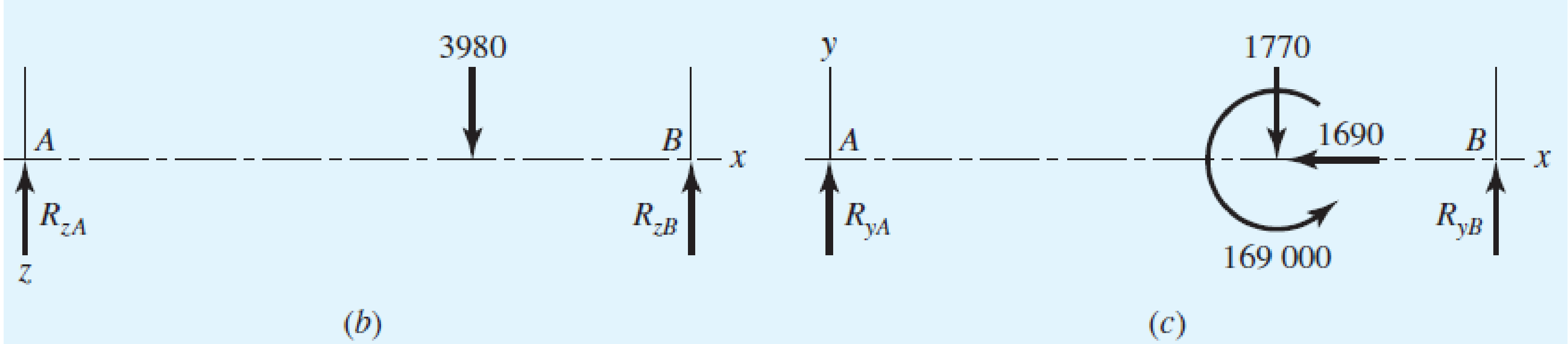


Fig. 11–18

Example 11–8 (continued)

The radial loads F_{rA} and F_{rB} are the vector additions of R_{yA} and R_{zA}, and R_{yB} and R_{zB}, respectively:

$$F_{rA} = \left(R_{zA}^2 + R_{yA}^2\right)^{1/2} = (1327^2 + 1717^2)^{1/2} = 2170\,\text{N}$$

$$F_{rB} = \left(R_{zB}^2 + R_{yB}^2\right)^{1/2} = (2653^2 + 53.3^2)^{1/2} = 2654\,\text{N}$$

Trial 1: With direct mounting of the bearings and application of the external thrust to the shaft, the squeezed bearing is bearing A as labeled in Fig. 11–18a. Using K of 1.5 as the initial guess for each bearing, the induced loads from the bearings are

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(2170)}{1.5} = 680\text{ N}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.5} = 832\text{ N}$$

Example 11–8 (continued)

Since F_{iA} is clearly less than $F_{iB} + F_{ae}$, bearing A carries the net thrust load, and Eq. (11–19) is applicable. Therefore, the dynamic equivalent loads are

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = 0.4(2170) + 1.5(832 + 1690) = 4651\text{ N}$$

$$F_{eB} = F_{rB} = 2654\text{ N}$$

The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{\mathscr{L}_D n_D 60}{L_R} = \frac{(5000)(800)(60)}{90(10^6)} = 2.67$$

Estimate R_D as $\sqrt{0.99} = 0.995$ for each bearing. For bearing A, from Eq. (11–10) the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4651)\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 11\,486\text{ N}$$

From Fig. 11–15, tentatively select type TS 15100 cone and 15245 cup, which will work: $K_A = 1.67$, $C_{10} = 12\,100$ N.

Example 11–8 (continued)

For bearing B, from Eq. (11–10), the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)2654\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 6554\ \text{N}$$

Tentatively select the bearing identical to bearing A, which will work: $K_B = 1.67$, $C_{10} = 12\ 100$ N.

Example 11–8 (continued)

Trial 2: Repeat the process with $K_A = K_B = 1.67$ from tentative bearing selection.

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(2170)}{1.67} = 611 \text{ N}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.67} = 747 \text{ N}$$

Since F_{iA} is still less than $F_{iB} + F_{ae}$, Eq. (11–19) is still applicable.

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = 0.4(2170) + 1.67(747 + 1690) = 4938 \text{ N}$$

$$F_{eB} = F_{rB} = 2654 \text{ N}$$

For bearing A, from Eq. (11–10) the corrected catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4938)\left[\frac{2.67}{(4.48)(1-0.995)^{2/3}}\right]^{3/10} = 12\,195 \text{ N}$$

Example 11–8 (continued)

Although this catalog entry exceeds slightly the tentative selection for bearing A, we will keep it since the reliability of bearing B exceeds 0.995. In the next section we will quantitatively show that the combined reliability of bearing A and B will exceed the reliability goal of 0.99.

For bearing B, $F_{eB} = F_{rB} = 2654$ N. From Eq. (11–10),

$$C_{10} = (1)2654\left[\frac{2.67}{(4.48)(1-0.995)^{2/3}}\right]^{3/10} = 6554\text{ N}$$

Select cone and cup 15100 and 15245, respectively, for both bearing A and B. Note from Fig. 11–14 the effective load center is located at $a = -5.8$ mm, that is, 5.8 mm into the cup from the back. Thus the shoulder-to-shoulder dimension should be $150 - 2(5.8) = 138.4$ mm. Note that in each iteration of Eq. (11–10) to find the catalog load rating, the bracketed portion of the equation is identical and need not be re-entered on a calculator each time.

Realized Bearing Reliability

- Eq. (11–9) was previously derived to determine a suitable catalog rated load for a given design situation and reliability goal.

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}} \right]^{1/a} \qquad (11\text{–}9)$$

- An actual bearing is selected from a catalog with a rating greater than C_{10}.
- Sometimes it is desirable to determine the realized reliability from the actual bearing (that was slightly higher capacity than needed).
- Solving Eq. (11–9) for the reliability,

$$R = \exp\left(- \left\{ \frac{x_D \left(\frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right\}^b \right) \qquad (11\text{–}21)$$

Realized Bearing Reliability

- Similarly for the alternate approximate equation, Eq. (11–10),

$$C_{10} \approx a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \qquad R \geq 0.90 \qquad (11\text{–}10)$$

$$R \approx 1 - \left\{ \frac{x_D \left(\dfrac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right\}^b \qquad R \geq 0.90 \qquad (11\text{–}22)$$

Example 11–9

In Ex. 11–3, the minimum required load rating for 99 percent reliability, at $x_D = L_D/L_{10} = 540$, is $C_{10} = 6696$ lbf = 29.8 kN. From Table 11–2 a 02-40 mm deep-groove ball bearing would satisfy the requirement. If the bore in the application had to be 70 mm or larger (selecting a 02-70 mm deep-groove ball bearing), what is the resulting reliability?

Solution

From Table 11–2, for a 02-70 mm deep-groove ball bearing, $C_{10} = 61.8$ kN = 13 888 lbf. Using Eq. (11–22), recalling from Ex. 11–3 that $a_f = 1.2$, $F_D = 413$ lbf, $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$, we can write

Answer

$$R \approx 1 - \left\{ \frac{\left[540\left[\dfrac{1.2(413)}{13\,888}\right]^3 - 0.02 \right]}{4.439} \right\}^{1.483} = 0.999\,963$$

which, as expected, is much higher than 0.99 from Ex. 11–3.

Realized Reliability for Tapered Roller Bearings

- Substituting typical Weibull parameters for tapered roller bearings into Eqs. (11–21) and (11–22) give realized reliability equations customized for tapered roller bearings.

$$R = \exp\left\{-\left[\frac{x_D}{\theta[C_{10}/(a_f F_D)]^a}\right]^b\right\}$$

$$= \exp\left\{-\left[\frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}}\right]^{3/2}\right\} \qquad (11\text{–}23)$$

$$R \approx 1 - \left\{\frac{x_D}{\theta[C_{10}/(a_f F_D)]^a}\right\}^b = 1 - \left\{\frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}}\right\}^{3/2} \qquad (11\text{–}24)$$

Example 11–10

In Ex. 11–8 bearings A and B (cone 15100 and cup 15245) have $C_{10} = 12\,100$ N. What is the reliability of the pair of bearings A and B?

Solution

The desired life x_D was $5000(800)60/[90(10^6)] = 2.67$ rating lives. Using Eq. (11–24) for bearing A, where from Ex. 11–8, $F_D = F_{eA} = 4938$ N, and $a_f = 1$, gives

$$R_A \approx 1 - \left\{ \frac{2.67}{4.48[12\,100/(1 \times 4938)]^{10/3}} \right\}^{3/2} = 0.994\,791$$

which is less than 0.995, as expected. Using Eq. (11–24) for bearing B with $F_D = F_{eB} = 2654$ N gives

$$R_B \approx 1 - \left\{ \frac{2.67}{4.48[12\,100/(1 \times 2654)]^{10/3}} \right\}^{3/2} = 0.999\,766$$

Example 11–10 (continued)

The reliability of the bearing pair is

Answer $$R = R_A R_B = 0.994\,791(0.999\,766) = 0.994\,558$$

which is greater than the overall reliability goal of 0.99. When two bearings are made identical for simplicity, or reducing the number of spares, or other stipulation, and the loading is not the same, both can be made smaller and still meet a reliability goal. If the loading is disparate, then the more heavily loaded bearing can be chosen for a reliability goal just slightly larger than the overall goal.

Example 11–11

Consider a constrained housing as depicted in Fig. 11–19 with two direct-mount tapered roller bearings resisting an external thrust F_{ae} of 8000 N. The shaft speed is 950 rev/min, the desired life is 10 000 h, the expected shaft diameter is approximately 1 in. The reliability goal is 0.95. The application factor is appropriately $a_f = 1$.

(*a*) Choose a suitable tapered roller bearing for *A*.

(*b*) Choose a suitable tapered roller bearing for *B*.

(*c*) Find the reliabilities R_A, R_B, and R.

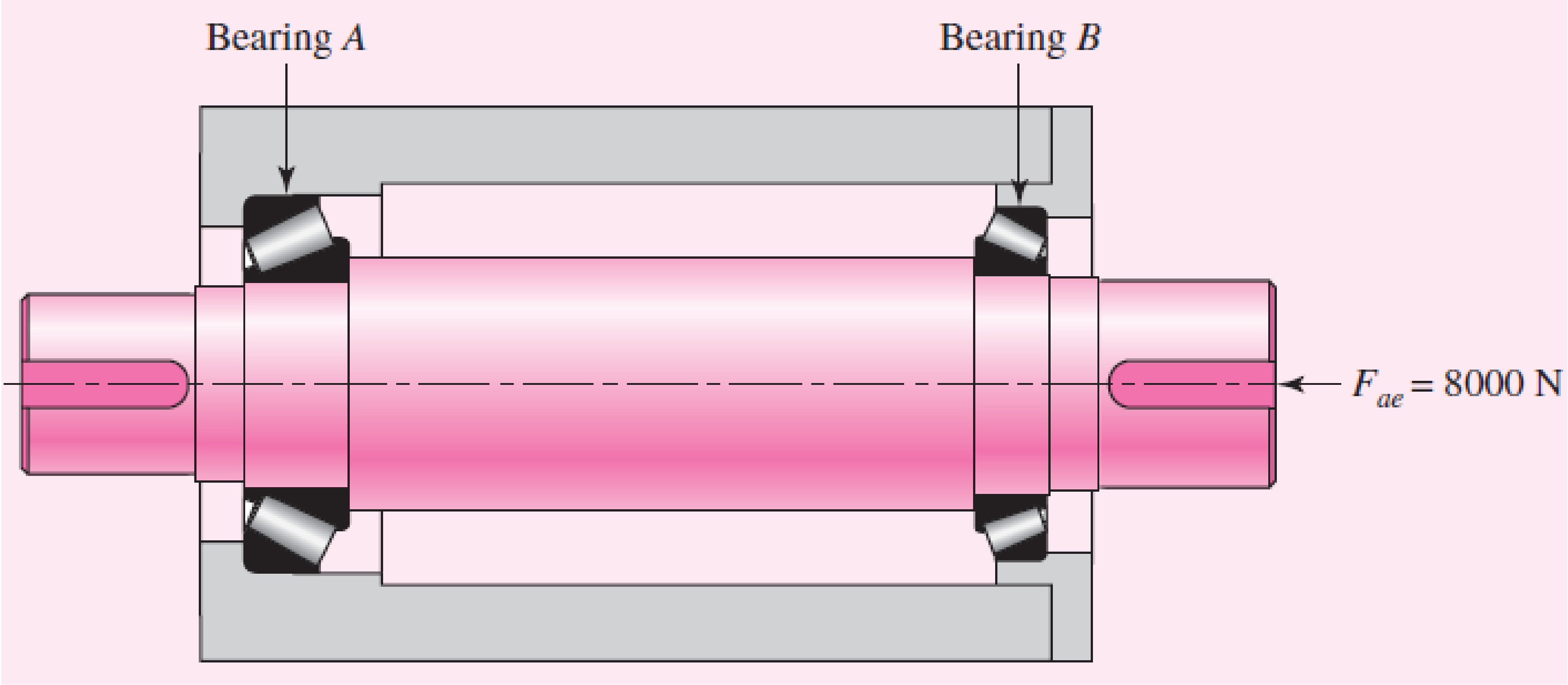


Fig. 11–19

Example 11–11 (continued)

Solution

(*a*) By inspection, note that the left bearing carries the axial load and is properly labeled as bearing *A*. The bearing reactions at *A* are

$$F_{rA} = F_{rB} = 0$$

$$F_{aA} = F_{ae} = 8000\text{ N}$$

Since bearing *B* is unloaded, we will start with $R = R_A = 0.95$.

With no radial loads, there are no induced thrust loads. Eq. (11–19) is applicable.

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = K_A F_{ae}$$

If we set $K_A = 1$, we can find C_{10} in the thrust column and avoid iteration:

$$F_{eA} = (1)8000 = 8000\text{ N}$$

$$F_{eB} = F_{rB} = 0$$

Example 11–11 (continued)

The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{\mathcal{L}_D n_D 60}{L_R} = \frac{(10\,000)(950)(60)}{90(10^6)} = 6.333$$

Then, from Eq. (11–10), for bearing A

$$C_{10} = a_f F_{eA} \left[\frac{x_D}{4.48(1 - R_D)^{2/3}} \right]^{3/10}$$

$$= (1)8000 \left[\frac{6.33}{4.48(1 - 0.95)^{2/3}} \right]^{3/10} = 16\,159 \text{ N}$$

Answer

Figure 11–15 presents one possibility in the 1-in bore (25.4-mm) size: cone, HM88630, cup HM88610 with a thrust rating $(C_{10})_a = 17\,200$ N.

Example 11–11 (continued)

(*b*) Bearing *B* experiences no load, and the cheapest bearing of this bore size will do, including a ball or roller bearing. Answer

(*c*) The actual reliability of bearing *A*, from Eq. (11–24), is

$$R_A \approx 1 - \left\{ \frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}} \right\}^{3/2}$$

$$\approx 1 - \left\{ \frac{6.333}{4.48[17\,200/(1 \times 8000)]^{10/3}} \right\}^{3/2} = 0.963$$ Answer

which is greater than 0.95, as one would expect. For bearing *B*,

$$F_D = F_{eB} = 0$$ Answer

$$R_B \approx 1 - \left[\frac{6.333}{0.85(17\,200/0)^{10/3}} \right]^{3/2} = 1 - 0 = 1$$

as one would expect. The combined reliability of bearings *A* and *B* as a pair is

$$R = R_A R_B = 0.963(1) = 0.963$$ Answer

which is greater than the reliability goal of 0.95, as one would expect.

Bearing Lubrication

- The purposes of bearing lubrication
  - To provide a film of lubricant between the sliding and rolling surfaces
  - To help distribute and dissipate heat
  - To prevent corrosion of the bearing surfaces
  - To protect the parts from the entrance of foreign matter

Bearing Lubrication

- Either oil or grease may be used, with each having advantages in certain situations.

Use Grease When	Use Oil When
1. The temperature is not over 200°F.	1. Speeds are high.
2. The speed is low.	2. Temperatures are high.
3. Unusual protection is required from the entrance of foreign matter.	3. Oiltight seals are readily employed.
4. Simple bearing enclosures are desired.	4. Bearing type is not suitable for grease lubrication.
5. Operation for long periods without attention is desired.	5. The bearing is lubricated from a central supply which is also used for other machine parts.

Some Common Bearing Mounting Configurations

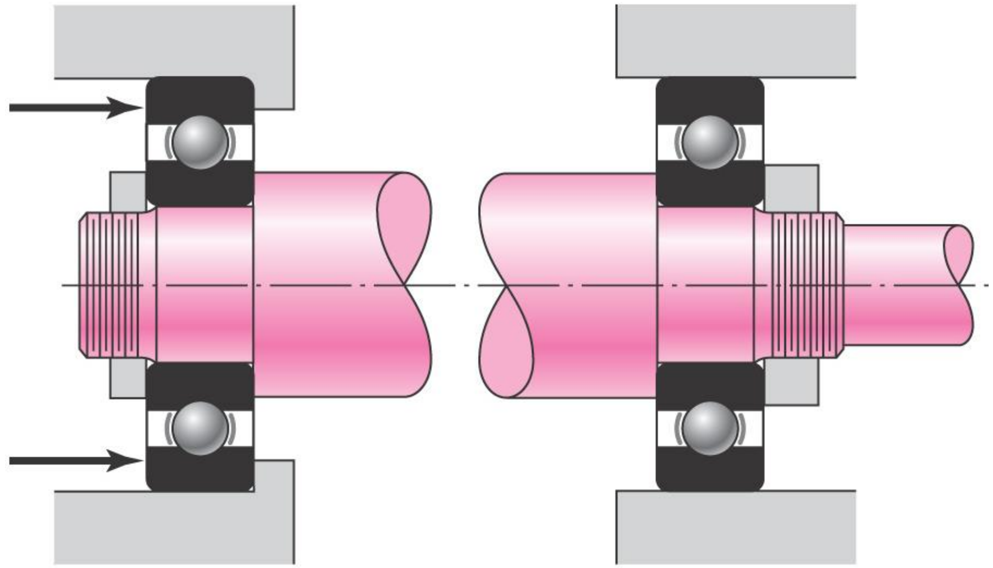


Fig. 11–20

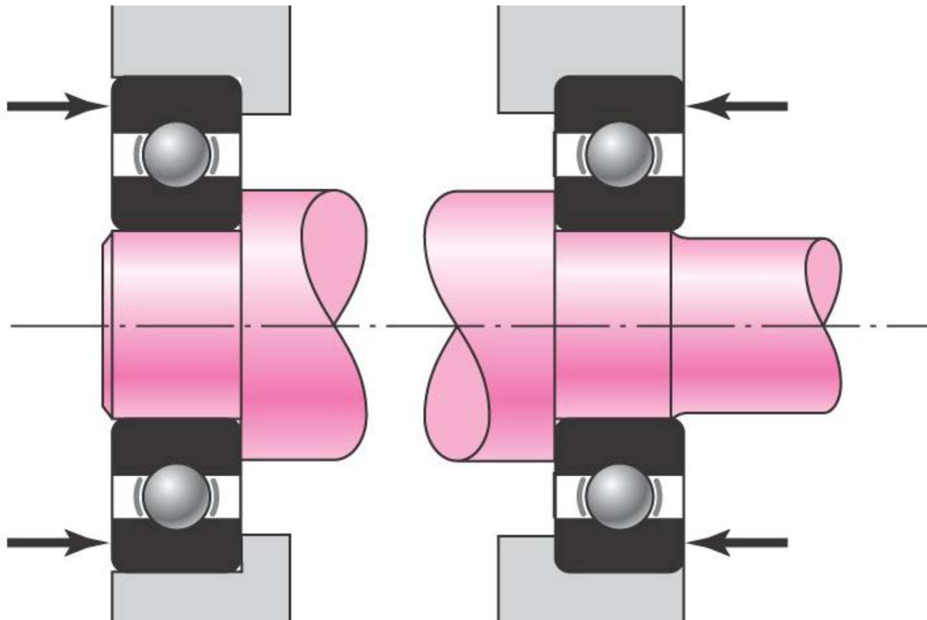


Fig. 11–21

Some Common Bearing Mounting Configurations

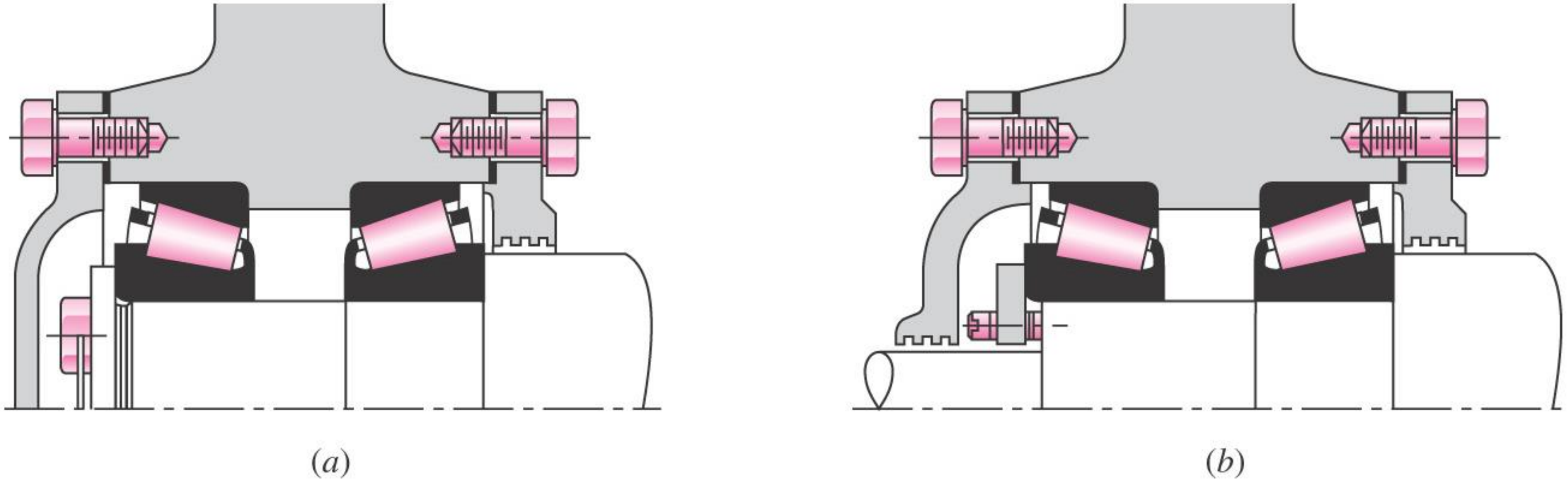


Fig. 11–22

Some Common Bearing Mounting Configurations

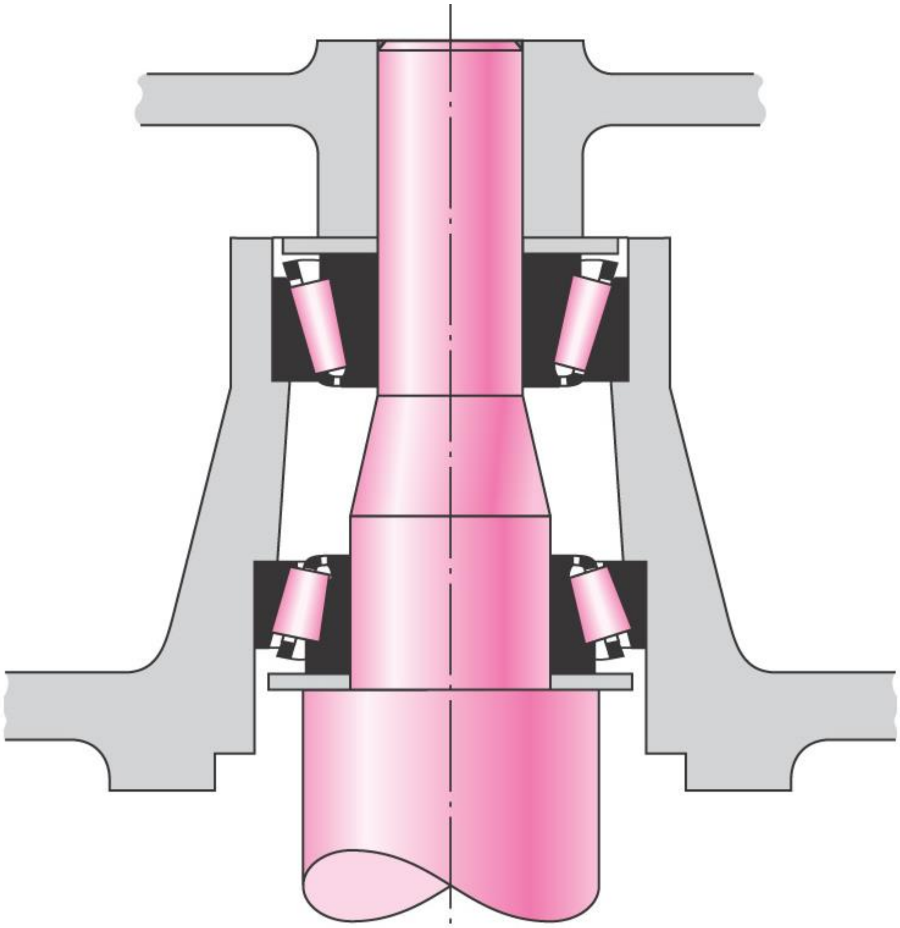
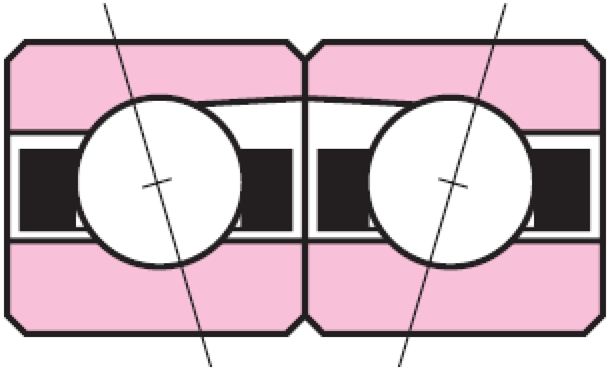


Fig. 11–23

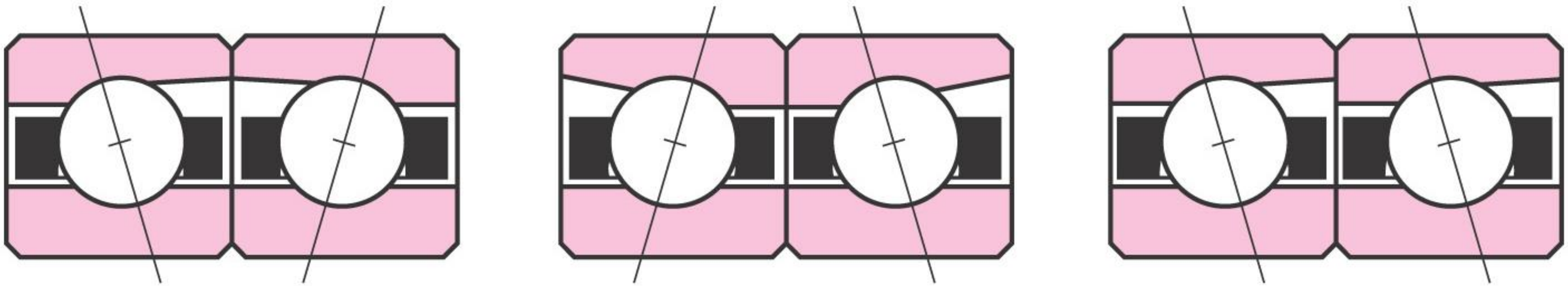
Duplexing

- When maximum stiffness and resistance to shaft misalignment is desired, pairs of angular-contact bearings can be used in an arrangement called *duplexing*.
- Duplex bearings have rings ground with an offset.
- When pairs are clamped together, a preload is established.



Duplexing Arrangements

- Three common duplexing arrangements:
  - (a) DF mounting – Face to face, good for radial and thrust loads from either direction
  - (b) DB mounting – Back to back, same as DF, but with greater alignment stiffness
  - (c) DT mounting – Tandem, good for thrust only in one direction



(a) (b) (c)

Fig. 11–24

Preferred Fits

- Rotating ring usually requires a press fit
- Stationary ring usually best with a push fit
- Allows stationary ring to creep, bringing new portions into the load-bearing zone to equalize wear

Preloading

- Object of preloading
  - Remove internal clearance
  - Increase fatigue life
  - Decrease shaft slope at bearing

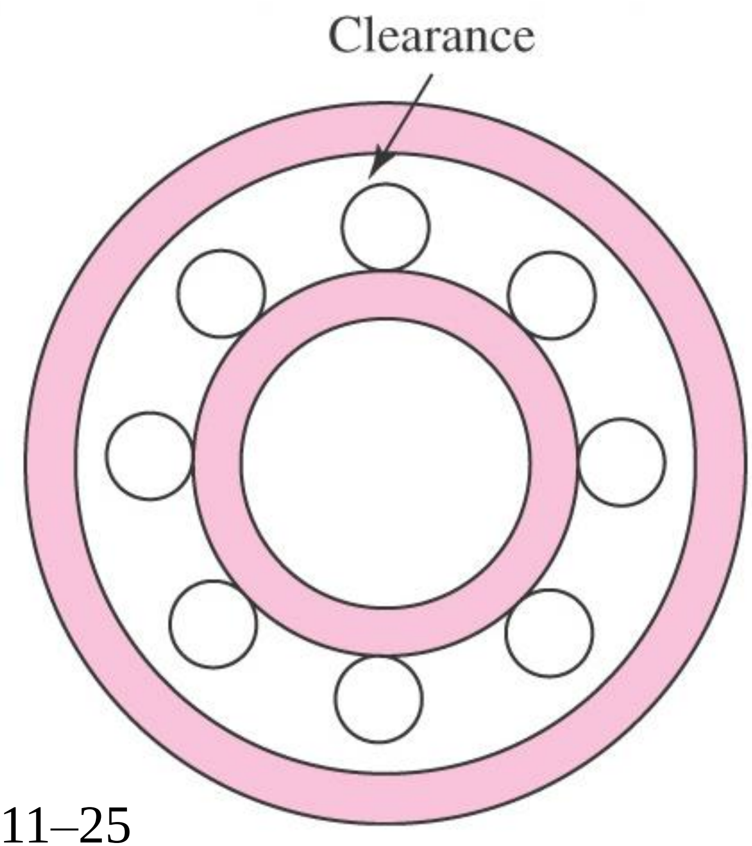


Fig. 11–25

Alignment

- Catalogs will specify alignment requirements for specific bearings
- Typical maximum ranges for shaft slopes at bearing locations

Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad

Enclosures

- Common shaft seals to exclude dirt and retain lubricant

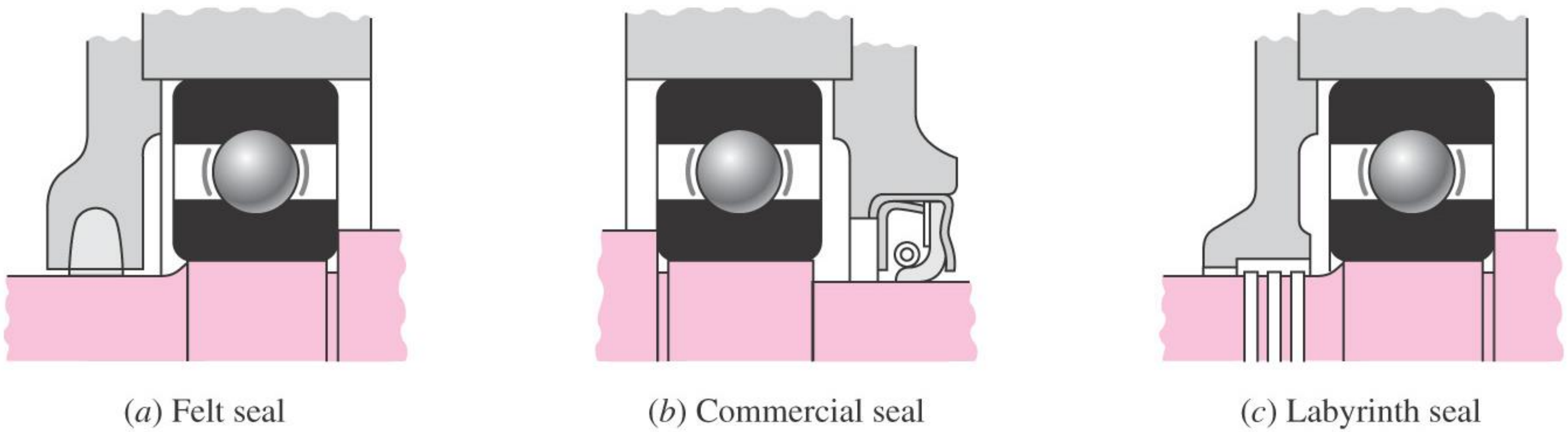


Fig. 11–26