Chapter 5

ANALYTICAL LINKAGE SYNTHESIS

TOPIC/PROBLEM MATRIX

SECT	TOPIC	PROBLEMS
5.3	Two-Position Motion Generation by Analytical Synthesis	5-1, 5-2, 5-8, 5-9, 5-12, 5-13, 5-16, 5-17, 5-21, 5-22, 5-23
5.6	Three-Position Motion Generation by Analytical Synthesis	5-3, 5-10, 5-14, 5-18, 5-24, 5-25, 5-27, 5-28, 5-31, 5-32, 5-34, 5-37, 5-38, 5-39, 5-41, 5-42,

PROBLEM 5-1

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of output rocker motion with no quick-return. (See Problem 3-3).

Given: Coordinates of the points C_1, D_1, C_2, and D_2 with respect to C_1:

$C_{1x} := 0.0$ $D_{1x} := 1.721$ $C_{2x} := 2.656$ $D_{2x} := 5.065$

$C_{1y} := 0.0$ $D_{1y} := -1.750$ $C_{2y} := -0.751$ $D_{2y} := -0.281$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Assumptions: Use the pivot point between links 3 and 4 (C_1 and C_2)as the precision points P_1 and P_2. Define position vectors in the global frame whose origin is at C_1.

Solution: See solution to Problem 3-3 and Mathcad file P0501.

1. Note that this is a two-position function generation (FG) problem because the output is specified as an angular displacement of the rocker, link 4. See Section 5.13 which details the 3-position FG solution. See also Section 5.3 in which the equations for the two-position motion generation problem are derived. These are really the same problem and have the same solution. The method of Section 5.3 will be used here.

2. Two solution methods are derived in Section 5.3 and are presented in equations 5.7 and 5.8 for the left dyad and equations 5.11 and 5.12 for the right dyad. The first method (equations 5.7 and 5.11) looks like the better one to use in this case since it allows us to choose the link's angular positions and excursions and solve for the lengths of links 2 and 3 (w and z). Unfortunately, this method fails in this problem because of the requirement for a non-quick-return Grashof linkage, which requires the angular displacement of link 3 in going from position 1 to position 2 to be zero ($\alpha_2 = 0$) causing a divide-by-zero error in equations 5.7d.

3. Method 2 (equations 5.8 and 5.12) requires the choosing of two angles and a length for each dyad.

4. To obtain the same solution as was done graphically in Problem 3-4, we need to know the location of the fixed pivot O_4 with respect to the given CD. While we could take the results from Problem 3-3 and use them here to establish the location of O_4, that won't be done. Instead, we will use the point C as the joint between links 3 and 4 as well as the precision point P.

5. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} C_{2x} \\ C_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 2.656$$

$$P_{21y} = -0.751$$

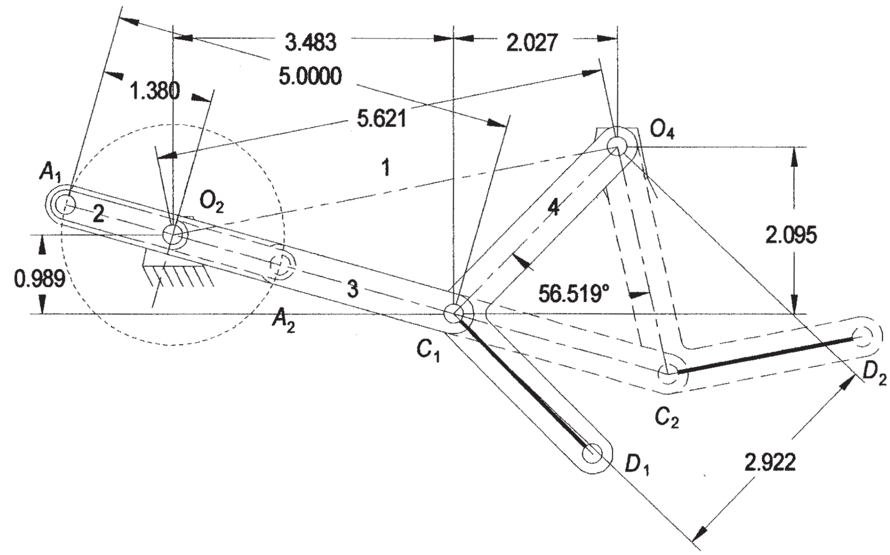
$$p_{21} := \sqrt{\left(P_{21x}{}^2 + P_{21y}{}^2\right)} \qquad p_{21} = 2.760$$

6. From the trigonometric relationships given in Figure 5-1, determine δ_2. From the requirement for a non-quick-return, $\alpha_2 := 0$.

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -15.789\,deg$$

7. From a graphical solution (see figure below), determine the values necessary for input to equations 5.8.

$z := 5.000$ $\beta_2 := 180 \cdot deg$ $\phi := \delta_2$



8. Solve for the **WZ** dyad using equations 5.8.

$Z_{1x} := z \cdot cos(\phi)$ $Z_{1x} = 4.811$ $Z_{1y} := z \cdot sin(\phi)$ $Z_{1y} = -1.360$

$A := cos(\beta_2) - 1$ $A = -2.000$ $D := sin(\alpha_2)$ $D = 0.000$

$B := sin(\beta_2)$ $B = 0.000$ $E := p_{21} \cdot cos(\delta_2)$ $E = 2.656$

$C := cos(\alpha_2) - 1$ $C = 0.000$ $F := p_{21} \cdot sin(\delta_2)$ $F = -0.751$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = -1.328$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = 0.376$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 1.380$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 164.211\,deg$$

This is the expected value of w (one half of p_{21}) based on the design choices made in the graphical solution and the assumptions made in this problem.

9. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$s := 0$ $\gamma_2 := 56.519 \cdot deg$ $\psi := 0 \cdot deg$

10. Solve for the **US** dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = 0.000$ $S_{1y} := s \cdot sin(\psi)$ $S_{1y} = 0.000$

$A := \cos(\gamma_2) - 1$ $\quad A = -0.448$ $\quad D := \sin(\alpha_2)$ $\quad D = 0.000$

$B := \sin(\gamma_2)$ $\quad B = 0.834$ $\quad E := p_{21}\cdot\cos(\delta_2)$ $\quad E = 2.656$

$C := \cos(\alpha_2) - 1$ $\quad C = 0.000$ $\quad F := p_{21}\cdot\sin(\delta_2)$ $\quad F = -0.751$

$$U_{1x} := \frac{A\cdot(-C\cdot S_{1x} + D\cdot S_{1y} + E) + B\cdot(-C\cdot S_{1y} - D\cdot S_{1x} + F)}{-2\cdot A} \qquad U_{1x} = -2.027$$

$$U_{1y} := \frac{A\cdot(-C\cdot S_{1y} - D\cdot S_{1x} + F) + B\cdot(C\cdot S_{1x} - D\cdot S_{1y} - E)}{-2\cdot A} \qquad U_{1y} = -2.095$$

$$u := \sqrt{U_{1x}^{\ 2} + U_{1y}^{\ 2}} \qquad u = 2.915$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = 225.952\ deg$$

This is the expected value of u based on the design choices made in the graphical solution and the assumptions made in this problem.

11. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z\cdot\cos(\phi) - s\cdot\cos(\psi)$ $\qquad V_{1x} = 4.811$

$V_{1y} := z\cdot\sin(\phi) - s\cdot\sin(\psi)$ $\qquad V_{1y} = -1.360$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\qquad \theta_3 = -15.789\ deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $\qquad v = 5.000$

Link 1: $G_{1x} := w\cdot\cos(\theta) + v\cdot\cos(\theta_3) - u\cdot\cos(\sigma)$ $\qquad G_{1x} = 5.510$

$G_{1y} := w\cdot\sin(\theta) + v\cdot\sin(\theta_3) - u\cdot\sin(\sigma)$ $\qquad G_{1y} = 1.110$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\qquad \theta_1 = 11.391\ deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $\qquad g = 5.621$

12. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad \theta_{2i} = 152.820\ deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\qquad \theta_{2f} = 332.820\ deg$

13. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\qquad \delta_p := \phi - \theta_3$

$r_p = 5.000$ $\qquad \delta_p = 0.000\ deg$

which is correct for the assumption that the precision point is at C.

14. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(C_{1x}, C_{1y})$ $\rho_1 = 180.000\,deg$

$R_1 := \sqrt{\left(C_{1x}^{\,2} + C_{1y}^{\,2}\right)}$ $R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $O_{2x} = -3.483$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $O_{2y} = 0.985$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = 2.027$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = 2.095$

15. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\theta_{rot} = 11.391\,deg$

16. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(w, g, u, v) = \text{"Grashof"}$

17. **DESIGN SUMMARY**

Link 2:	$w = 1.380$	$\theta = 164.211\,deg$
Link 3:	$v = 5.000$	$\theta_3 = -15.789\,deg$
Link 4:	$u = 2.915$	$\sigma = 225.952\,deg$
Link 1:	$g = 5.621$	$\theta_1 = 11.391\,deg$
Coupler:	$r_p = 5.000$	$\delta_p = 0.000\,deg$
Crank angles:		
	$\theta_{2i} = 152.820\,deg$	
	$\theta_{2f} = 332.820\,deg$	

PROBLEM 5-2

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of coupler motion.

Given: Coordinates of the points A_1, B_1, A_2, and B_2 with respect to A_1:

$A_{1x} := 0.0$ $B_{1x} := 1.721$ $A_{2x} := 2.656$ $B_{2x} := 5.065$

$A_{1y} := 0.0$ $B_{1y} := -1.750$ $A_{2y} := -0.751$ $B_{2y} := -0.281$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Assumptions: Use the points A_1 and A_2 as the precision points P_1 and P_2. Define position vectors in the global frame whose origin is at A_1.

Solution: See solution to Problem 3-4 and Mathcad file P0502.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. See Section 5.3 in which the equations for the two-position motion generation problem are derived.
2. Two solution methods are derived in Section 5.3 and are presented in equations 5.7 and 5.8 for the left dyad and equations 5.11 and 5.12 for the right dyad.
3. Method 1 (equations 5.7 and 5.11) requires the choosing of three angles for each dyad. Method 2 (equations 5.8 and 5.12) requires the choosing of two angles and a length for each dyad. Method 1 is used in this solution.
4. In order to obtain the same solution as was done graphically in Problem 3-4, the necessary assumed values were taken from that solution as shown below.
5. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 2.656$$

$$P_{21y} = -0.751$$

$$p21 := \sqrt{\left(P_{21x}{}^2 + P_{21y}{}^2\right)} \qquad p21 = 2.760$$

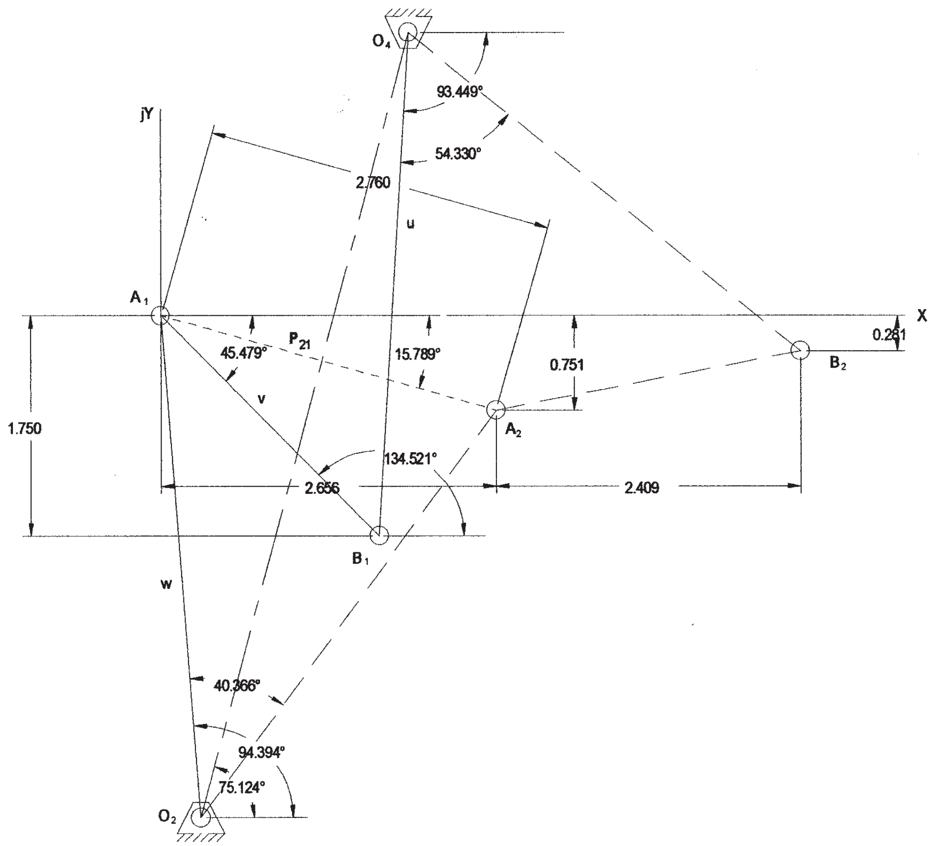
6. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := atan2\left[\left(A_{2x} - B_{2x}\right),\left(A_{2y} - B_{2y}\right)\right] \ldots \\ + -atan2\left[\left(A_{1x} - B_{1x}\right),\left(A_{1y} - B_{1y}\right)\right] \qquad \alpha_2 = 56.519\,deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = -15.789\,deg$$

7. From the graphical solution (see figure below), determine the values necessary for input to equations 5.7.

$$\theta := 94.394\cdot deg \qquad \beta_2 := -40.366\cdot deg \qquad \phi := -45.479\cdot deg$$



8. Solve for the **WZ** dyad using equations 5.7.

$$A := \cos(\theta)\cdot(\cos(\beta_2) - 1) - \sin(\theta)\cdot\sin(\beta_2)$$

$$B := \cos(\phi)\cdot(\cos(\alpha_2) - 1) - \sin(\phi)\cdot\sin(\alpha_2) \qquad C := p_{21}\cdot\cos(\delta_2)$$

$$D := \sin(\theta)\cdot(\cos(\beta_2) - 1) + \cos(\theta)\cdot\sin(\beta_2)$$

$$E := \sin(\phi)\cdot(\cos(\alpha_2) - 1) + \cos(\phi)\cdot\sin(\alpha_2) \qquad F := p_{21}\cdot\sin(\delta_2)$$

$$w := \frac{C\cdot E - B\cdot F}{(A\cdot E - B\cdot D)} \qquad z := \frac{A\cdot F - C\cdot D}{(A\cdot E - B\cdot D)}$$

$$w = 4.000 \qquad z = 0.000$$

These are the expected values of w and z based on the design choices made in the graphical solution and the assumptions made in this problem.

9. From the graphical solution (see figure above), determine the values necessary for input to equations 5.11.

$$\sigma := -93.449\cdot deg \qquad \gamma_2 := 54.330\cdot deg \qquad \psi := 134.521\cdot deg$$

10. Solve for the **US** dyad using equations 5.11.

$$A' := \cos(\sigma)\cdot(\cos(\gamma_2) - 1) - \sin(\sigma)\cdot\sin(\gamma_2)$$

$$B' := \cos(\psi)\cdot(\cos(\alpha_2) - 1) - \sin(\psi)\cdot\sin(\alpha_2) \qquad C := p_{21}\cdot\cos(\delta_2)$$

$$D' := sin(\sigma)\cdot(cos(\gamma_2) - 1) + cos(\sigma)\cdot sin(\gamma_2)$$

$$E' := sin(\psi)\cdot(cos(\alpha_2) - 1) + cos(\psi)\cdot sin(\alpha_2) \qquad F := p_{21}\cdot sin(\delta_2)$$

$$u := \frac{C\cdot E' - B'\cdot F}{(A'\cdot E' - B'\cdot D')} \qquad s := \frac{A'\cdot F - C\cdot D'}{(A'\cdot E' - B'\cdot D')}$$

$$u = 4.000 \qquad s = 2.455$$

These are the expected values of u and s based on the design choices made in the graphical solution and the assumptions made in this problem.

11. Solve for the links 3 and 1 using the vector definitions of **V** and **G**.

Link 3:

$$V_{1x} := z\cdot cos(\phi) - s\cdot cos(\psi) \qquad V_{1x} = 1.721$$

$$V_{1y} := z\cdot sin(\phi) - s\cdot sin(\psi) \qquad V_{1y} = -1.750$$

$$\theta_3 := atan2(V_{1x}, V_{1y}) \qquad \theta_3 = -45.479\, deg$$

$$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)} \qquad v = 2.455$$

Link 1:

$$G_{1x} := w\cdot cos(\theta) + v\cdot cos(\theta_3) - u\cdot cos(\sigma) \qquad G_{1x} = 1.655$$

$$G_{1y} := w\cdot sin(\theta) + v\cdot sin(\theta_3) - u\cdot sin(\sigma) \qquad G_{1y} = 6.231$$

$$\theta_1 := atan2(G_{1x}, G_{1y}) \qquad \theta_1 = 75.123\, deg$$

$$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)} \qquad g = 6.447$$

12. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \qquad \theta_{2i} = 19.271\, deg$$

$$\theta_{2f} := \theta_{2i} + \beta_2 \qquad \theta_{2f} = -21.095\, deg$$

13. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \qquad \delta_p := \phi - \theta_3$$

$$r_p = 0.000 \qquad \delta_p = 0.000\, deg$$

which is correct for the assumption that the precision point is at A.

14. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := atan2(A_{1x}, A_{1y}) \qquad \rho_1 = 180.000\, deg$$

$$R_1 := \sqrt{\left(A_{1x}^2 + A_{1y}^2\right)} \qquad R_1 = 0.000$$

$$O_{2x} := R_1\cdot cos(\rho_1) - z\cdot cos(\phi) - w\cdot cos(\theta) \qquad O_{2x} = 0.306$$

$$O_{2y} := R_1\cdot sin(\rho_1) - z\cdot sin(\phi) - w\cdot sin(\theta) \qquad O_{2y} = -3.988$$

$$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = 1.962$$

$$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = 2.243$$

15. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 75.123\,deg$$

16. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(v, g, w, u) = \text{"non-Grashof"}$$

17. **DESIGN SUMMARY**

Link 2:	$w = 4.000$	$\theta = 94.394\,deg$
Link 3:	$v = 2.455$	$\theta_3 = -45.479\,deg$
Link 4:	$u = 4.000$	$\sigma = -93.449\,deg$
Link 1:	$g = 6.447$	$\theta_1 = 75.123\,deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\,deg$
Crank angles:		
	$\theta_{2i} = 19.271\,deg$	
	$\theta_{2f} = -21.095\,deg$	

PROBLEM 5-3

Statement: Design a fourbar mechanism to give the three positions of coupler motion with no quick return shown in Figure P3-2. (See Problem 3-5). Ignore the fixed pivot points in the Figure.

Given: Coordinates of points *A* and *B* with respect to point A_1:

$A_{1x} := 0.0 \qquad A_{1y} := 0.0 \qquad B_{1x} := 0.741 \qquad B_{1y} := -2.383$

$A_{2x} := 2.019 \qquad A_{2y} := -1.905 \qquad B_{2x} := 4.428 \qquad B_{2y} := -2.557$

$A_{3x} := 3.933 \qquad A_{3y} := -1.035 \qquad B_{3x} := 6.304 \qquad B_{3y} := -0.256$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Assumptions: Let points A_1, A_2, and A_3 be the precision points P_1, P_2, and P_3, respectively.

Solution: See Figure P3-2 Mathcad file P0503.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{A_{2x}^2 + A_{2y}^2} \qquad p_{21} = 2.776 \qquad \delta_2 := atan2(A_{2x}, A_{2y}) \qquad \delta_2 = -43.336\,deg$$

$$p_{31} := \sqrt{A_{3x}^2 + A_{3y}^2} \qquad p_{31} = 4.067 \qquad \delta_3 := atan2(A_{3x}, A_{3y}) \qquad \delta_3 = -14.744\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\theta_{P1} := atan2[(A_{1x} - B_{1x}),(A_{1y} - B_{1y})] \qquad \theta_{P1} = 107.273\,deg$$

$$\theta_{P2} := atan2[(A_{2x} - B_{2x}),(A_{2y} - B_{2y})] \qquad \theta_{P2} = 164.856\,deg$$

$$\theta_{P3} := atan2[(A_{3x} - B_{3x}),(A_{3y} - B_{3y})] \qquad \theta_{P3} = 198.188\,deg$$

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = 57.582\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = 90.915\,deg$$

3. The free choices for this linkage are (from the graphical solution to Problem 3-5):

$$\beta_2 := 78.375\cdot deg \qquad \beta_3 := 135.560\cdot deg \qquad \gamma_2 := 59.771\cdot deg \qquad \gamma_3 := 107.023\cdot deg$$

4. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21}\cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$G := \sin(\beta_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = -2.178$ $\quad W_{1y} = -0.286$ $\quad Z_{1x} = 0.000$ $\quad Z_{1y} = 0.000$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 187.477\,deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 174.344\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}, \quad w = 2.197$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^2 + Z_{1y}^2\right)}, \quad z = 0.000$

5. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -1.995$ $\quad U_{1y} = -3.121$ $\quad S_{1x} = -0.741$ $\quad S_{1y} = 2.383$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 237.409\,deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 107.267\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^2 + U_{1y}^2\right)}, \quad u = 3.704$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^2 + S_{1y}^2\right)}, \quad s = 2.495$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\quad V_{1x} = 0.741$

$V_{1y} := Z_{1y} - S_{1y}$ $\quad V_{1y} = -2.383$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = -72.734\ deg$

$v := \sqrt{\left(V_{1x}{}^2 + V_{1y}{}^2\right)}$ $\quad v = 2.495$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $\quad G_{1x} = 0.558$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $\quad G_{1y} = 0.452$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = 39.009\ deg$

$g := \sqrt{\left(G_{1x}{}^2 + G_{1y}{}^2\right)}$ $\quad g = 0.718$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\quad \theta_{2i} = 148.468\ deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\quad \theta_{2f} = 284.028\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\quad \delta_p := \phi - \theta_3$

$r_p = 0.000$ $\quad \delta_p = 247.078\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z \cdot cos(\phi) - w \cdot cos(\theta)$ $\quad O_{2x} = 2.178$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $\quad O_{2y} = 0.286$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\quad O_{4x} = 2.736$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\quad O_{4y} = 0.738$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\quad \theta_{rot} = 39.009\ deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(g, u, w, v) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

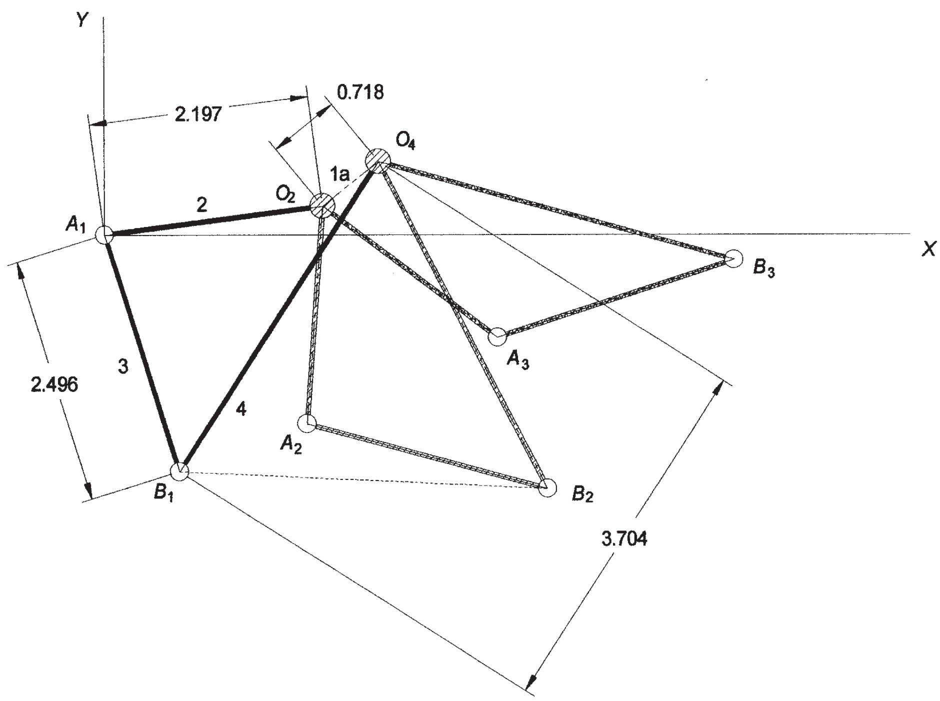
Link 2:	$w = 2.197$	$\theta = 187.477\ deg$
Link 3:	$v = 2.495$	$\theta_3 = -72.734\ deg$
Link 4:	$u = 3.704$	$\sigma = 237.409\ deg$
Link 1:	$g = 0.718$	$\theta_1 = 39.009\ deg$
Coupler:	$r_p = 0.000$	$\delta_p = 247.078\ deg$

Crank angles:

$$\theta_{2i} = 148.468\ deg$$

$$\theta_{2f} = 284.028\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



This is the same result as that found in Problem 3-5.

PROBLEM 5-4

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 (see Problem 3-6). Use analytical synthesis and design it for the fixed pivots shown.

Given: Link end points (with respect to A_1):

$A_{1x} := 0.0$ $B_{1x} := 0.741$ $A_{2x} := 2.019$ $B_{2x} := 4.428$ $A_{3x} := 3.933$ $B_{3x} := 6.304$

$A_{1y} := 0.0$ $B_{1y} := -2.383$ $A_{2y} := -1.905$ $B_{2y} := -2.557$ $A_{3y} := -1.035$ $B_{3y} := -0.256$

Fixed pivot points (with respect to A_1):

$O_{2x} := 0.995$ $O_{2y} := -5.086$ $O_{4x} := 5.298$ $O_{4y} := -5.086$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P3-2 and Mathcad file P0504.

1. Determine the angle changes between precision points from the body angles given.

$\theta_{P1} := atan2(A_{1x} - B_{1x}, A_{1y} - B_{1y})$ $\theta_{P1} = 107.273\ deg$

$\theta_{P2} := atan2(A_{2x} - B_{2x}, A_{2y} - B_{2y})$ $\theta_{P2} = 164.856\ deg$

$\theta_{P3} := atan2(A_{3x} - B_{3x}, A_{3y} - B_{3y})$ $\theta_{P3} = 198.188\ deg$

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\alpha_2 = 57.582\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\alpha_3 = 90.915\ deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$P_{21x} := A_{2x}$ $P_{21x} = 2.019$ $P_{31x} := A_{3x}$ $P_{31x} = 3.933$

$P_{21y} := A_{2y}$ $P_{21y} = -1.905$ $P_{31y} := A_{3y}$ $P_{31y} = -1.035$

$R_{1x} := -O_{2x}$ $R_{1x} = -0.995$ $R_{1y} := -O_{2y}$ $R_{1y} = 5.086$

$R_{2x} := R_{1x} + P_{21x}$ $R_{2x} = 1.024$

$R_{2y} := R_{1y} + P_{21y}$ $R_{2y} = 3.181$

$R_{3x} := R_{1x} + P_{31x}$ $R_{3x} = 2.938$

$R_{3y} := R_{1y} + P_{31y}$ $R_{3y} = 4.051$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $R_1 = 5.182$

$$R_2 := \sqrt{R_{2x}^{\;2} + R_{2y}^{\;2}} \qquad R_2 = 3.342$$

$$R_3 := \sqrt{R_{3x}^{\;2} + R_{3y}^{\;2}} \qquad R_3 = 5.004$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 101.069\, deg$$

$$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 72.156\, deg$$

$$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 54.048\, deg$$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = 1.352$$

$$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = 3.679$$

$$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3) \qquad C_3 = -8.007$$

$$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3) \qquad C_4 = 5.127$$

$$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2) \qquad C_5 = -5.851$$

$$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2) \qquad C_6 = 1.294$$

$$A_1 := -C_3^{\;2} - C_4^{\;2} \qquad A_1 = -90.406$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \qquad A_2 = 19.633$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \qquad A_3 = -53.487$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \qquad A_4 = -22.524$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \qquad A_5 = -19.633$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \qquad A_6 = -29.689$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \qquad K_1 = 1.146 \times 10^3$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \qquad K_2 = 1.788 \times 10^3$$

$$K_3 := \frac{A_1^{\;2} - A_2^{\;2} - A_3^{\;2} - A_4^{\;2} - A_6^{\;2}}{2} \qquad K_3 = 1.769 \times 10^3$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^{\;2} + K_2^{\;2} - K_3^{\;2}}}{K_1 + K_3}\right) \qquad \beta_{31} = 90.915\, deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^{\;2} + K_2^{\;2} - K_3^{\;2}}}{K_1 + K_3}\right) \qquad \beta_{32} = 23.770\, deg$$

The first value is the same as α_3, so use the second value $\qquad \beta_3 := \beta_{32}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 16.790\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 16.790\,deg$$

Since both values are the same, $\beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x} \qquad R_{1x} = -5.298 \qquad R_{1y} := -O_{4y} \qquad R_{1y} = 5.086$

$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = -3.279$

$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = 3.181$

$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = -1.365$

$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = 4.051$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 7.344$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 4.568$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 4.275$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 136.170\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 135.869\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 108.621\,deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = -1.023$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = 4.349$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3) \qquad C_3 = -3.636$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3) \qquad C_4 = 9.430$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2) \qquad C_5 = -3.855$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2) \qquad C_6 = 4.927$

$A_1 := -C_3^2 - C_4^2 \qquad A_1 = -102.135$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \qquad A_2 = 18.434$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \qquad A_3 = -60.472$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = -25.460$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -18.434$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -37.287$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 1.785 \times 10^3$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 2.227 \times 10^3$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 2.198 \times 10^3$$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 90.915\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 11.643\,deg$$

The first value is the same as α_3, so use the second value $\qquad \gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 11.069\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = 11.069\,deg$$

Since both angles are the same, $\qquad \gamma_2 := \gamma_{21}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.776$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -43.336\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 4.067$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = -14.744\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$

$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W1x = 3.594$ $W1y = 7.810$ $Z1x = -4.589$ $Z1y = -2.724$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 8.597$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -2.400$ $U1y = 7.549$ $S1x = -2.898$ $S1y = -2.463$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 7.921$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = -1.691$

$V1y := Z1y - S1y$ $V1y = -0.261$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 1.711$

$G1x := W1x + V1x - U1x$ $G1x = 4.303$

$G1y := W1y + V1y - U1y$ $G1y = 1.137 \times 10^{-13}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 4.303$

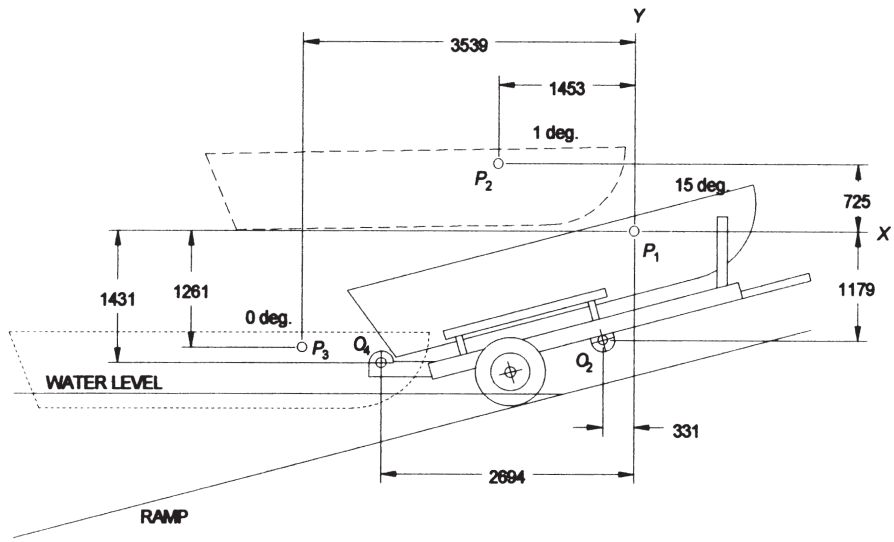
PROBLEM 5-5

Statement: See Project P3-8. Define three positions of the boat and analytically synthesize a linkage to move through them.

Assumptions: Launch ramp angle is 15 deg to the horizontal.

Solution: See Project P3-8 and Mathcad file P0505.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than is stated by deciding on three positions for the boat to move through. The figure below shows one such set of positions (dimensions are in mm).



2. From the figure, the design choices are:

$P_{21x} := -1453$ $\quad P_{21y} := 725$ $\quad P_{31x} := -3539$ $\quad P_{31y} := -1261$

$O_{2x} := -331$ $\quad O_{2y} := -1179$ $\quad O_{4x} := -2694$ $\quad O_{4y} := -1431$

Body angles: $\theta_{P1} := 15 \cdot deg$ $\quad \theta_{P2} := 1 \cdot deg$ $\quad \theta_{P3} := 0 \cdot deg$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.

4. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = -14.000\, deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -15.000\, deg$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 331.000$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 1.179 \times 10^3$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = -1.122 \times 10^3$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 1.904 \times 10^3$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -3.208 \times 10^3$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = -82.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 1.225 \times 10^3$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 2.210 \times 10^3$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 3.209 \times 10^3$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 74.318\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 120.510\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = -178.536\ deg$

7. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = -2.542 \times 10^3$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = -1.433 \times 10^3$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\qquad C_3 = 3.833 \times 10^3$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\qquad C_4 = -1.135 \times 10^3$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\qquad C_5 = 1.728 \times 10^3$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\qquad C_6 = 840.097$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -1.598 \times 10^7$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 5.182 \times 10^6$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -5.671 \times 10^6$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = -2.607 \times 10^6$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -5.182 \times 10^6$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -1.137 \times 10^7$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 5.096 \times 10^{13}$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 7.370 \times 10^{13}$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 3.015 \times 10^{13}$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = 125.675\ deg$$

$$\beta_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = -15.000\,deg$$

The second value is the same as α_3, so use the first value $\beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5\cdot sin(\beta_3) + A_3\cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 39.836\,deg$$

$$\beta_{22} := asin\left(\frac{A_3\cdot sin(\beta_3) + A_2\cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 39.836\,deg$$

Since both values are the same, $\beta_2 := \beta_{21}$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x} \qquad R_{1x} = 2.694 \times 10^3 \qquad R_{1y} := -O_{4y} \qquad R_{1y} = 1.431 \times 10^3$

$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = 1.241 \times 10^3$

$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = 2.156 \times 10^3$

$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = -845.000$

$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = 170.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 3.050 \times 10^3$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 2.488 \times 10^3$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 861.931$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 27.976\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 60.075\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 168.625\,deg$

10. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3\cdot cos(\alpha_2 + \zeta_3) - R_2\cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = -2.536 \times 10^3$

$C_2 := R_3\cdot sin(\alpha_2 + \zeta_3) - R_2\cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = -1.392 \times 10^3$

$C_3 := R_1\cdot cos(\alpha_3 + \zeta_1) - R_3\cdot cos(\zeta_3) \qquad C_3 = 3.818 \times 10^3$

$C_4 := -R_1\cdot sin(\alpha_3 + \zeta_1) + R_3\cdot sin(\zeta_3) \qquad C_4 = -514.981$

$C_5 := R_1\cdot cos(\alpha_2 + \zeta_1) - R_2\cdot cos(\zeta_2) \qquad C_5 = 1.719 \times 10^3$

$C_6 := -R_1\cdot sin(\alpha_2 + \zeta_1) + R_2\cdot sin(\zeta_2) \qquad C_6 = 1.419 \times 10^3$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $\qquad A_1 = -1.484 \times 10^7$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 6.303 \times 10^6$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -5.832 \times 10^6$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = -4.008 \times 10^6$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -6.303 \times 10^6$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -1.040 \times 10^7$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 3.537 \times 10^{13}$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 8.891 \times 10^{13}$

$K_3 := \dfrac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2}$ $\qquad K_3 = 1.115 \times 10^{13}$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \gamma_{31} = 151.615\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \gamma_{32} = -15.000\,deg$$

The second value is the same as α_3, so use the first value $\qquad \gamma_3 := \gamma_{31}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 56.167\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = 56.167\,deg$$

Since both angles are the same, $\qquad \gamma_2 := \gamma_{21}$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$p_{21} := \sqrt{P_{21x}^{\,2} + P_{21y}^{\,2}}$ $\qquad p_{21} = 1.624 \times 10^3$

$\delta_2 := atan2(P_{21x}, P_{21y})$ $\qquad \delta_2 = 153.482\,deg$

$p_{31} := \sqrt{P_{31x}^{\,2} + P_{31y}^{\,2}}$ $\qquad p_{31} = 3.757 \times 10^3$

$\delta_3 := atan2(P_{31x}, P_{31y})$ $\qquad \delta_3 = -160.388\,deg$

12. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$W1x = 1.331 \times 10^3 \qquad W1y = 1.653 \times 10^3 \qquad Z1x = -1.000 \times 10^3 \qquad Z1y = -474.187$$

14. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $\qquad w = 2122.473$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$$U1x = 1.690 \times 10^3 \qquad U1y = 951.273 \qquad S1x = 1.004 \times 10^3 \qquad S1y = 479.727$$

17. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $\qquad u = 1939.291$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \qquad V1x = -2.004 \times 10^3$$

$$V1y := Z1y - S1y \qquad V1y = -953.914$$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $\qquad v = 2219.601$

$G1x := W1x + V1x - U1x$ $\qquad G1x = -2.363 \times 10^3$

$G1y := W1y + V1y - U1y$ $\qquad G1y = -252.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $\qquad g = 2376.399$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $\qquad O2x = -331.000$

$O2y := -Z1y - W1y$ $\qquad O2y = -1179.000$

$O4x := -S1x - U1x$ $\qquad O4x = -2694.000$

$O4y := -S1y - U1y$ $\qquad O4y = -1431.000$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $\quad z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $\qquad z = 1106.834$ $\qquad rp := z$

Angle *BAP* (δ_p) $\quad s := \sqrt{S1x^2 + S1y^2}$ $\qquad s = 1112.770$

$\psi := atan2(S1x, S1y)$ $\qquad \psi = 25.538\,deg$

$\phi := atan2(Z1x, Z1y)$ $\qquad \phi = -154.633\,deg$

$\theta_3 := atan2\left(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi)\right)$

$\theta_3 = -154.547\,deg$

$\delta_p := \phi - \theta_3$ $\qquad \delta_p = -0.086\,deg$

21. **DESIGN SUMMARY**

Link 1: $\quad g = 2376.4$

Link 2: $\quad w = 2122.5$

Link 3: $\quad v = 2219.6$

Link 4: $\quad u = 1939.3$

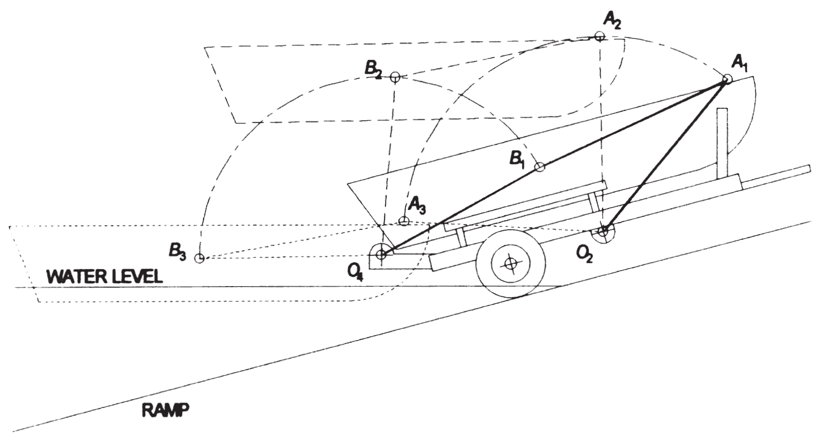
Coupler point: $\quad rp = 1106.8$ $\qquad \delta_p = -0.086\,deg$

22. **VERIFICATION**: The calculated values of *g* (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).

23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none, but is close to toggle at position 1. This could be used as a locking feature.

24. The transmission angles need to be checked also. This design has poor transmission angles, especially in and near positions 1 and 3. Unfortunately, this is where a large overturning moment is created by the mass of the boat. A large mechanical advantage input device will need to be used here, such as a hydraulic cylinder or

geared drive. Note that the drive mechanism must also resist being overdriven (back driven) by the load as the boat descends from its high point onto the trailer.

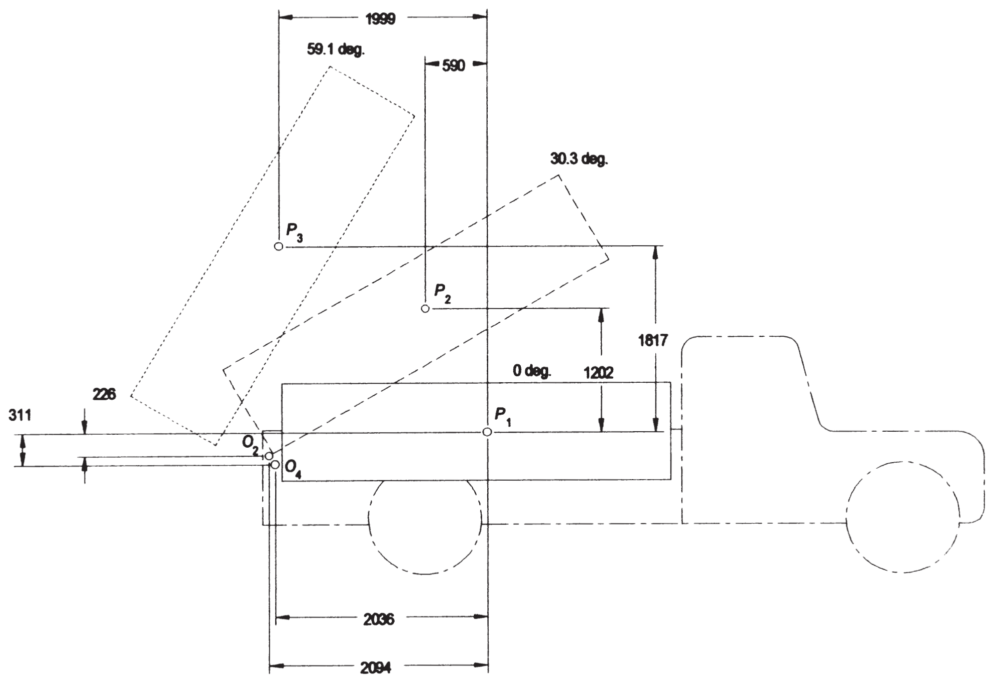


PROBLEM 5-6

Statement: See Project P3-20. Define three positions of the dumpster and analytically synthesize a linkage to move through them. The fixed pivots must be located on the existing truck.

Solution: See Project P3-20 and Mathcad file P0506.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than it is stated by deciding on three positions for the dumpster box to move through. The figure below shows one such set of positions (dimensions are in mm).



2. From the figure, the design choices are:

$P_{21x} := -590$ $\quad P_{21y} := 1202$ $\quad P_{31x} := -1999$ $\quad P_{31y} := 1817$

$O_{2x} := -2094$ $\quad O_{2y} := -226$ $\quad O_{4x} := -2036$ $\quad O_{4y} := -311$

Body angles: $\quad \theta_{P1} := 0 \cdot deg$ $\quad \theta_{P2} := 30.3 \cdot deg$ $\quad \theta_{P3} := 59.1 \cdot deg$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.

4. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = 30.300\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = 59.100\,deg$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 2.094 \times 10^3$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 226.000$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 1.504 \times 10^3$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 1.428 \times 10^3$

$$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = 95.000$$

$$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = 2.043 \times 10^3$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 2.106 \times 10^3$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 2.074 \times 10^3$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 2.045 \times 10^3$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 6.160\, deg$$

$$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 43.515\, deg$$

$$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 87.338\, deg$$

7. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \qquad C_1 = -495.777$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \qquad C_2 = -212.019$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \qquad C_3 = 786.433$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \qquad C_4 = 130.152$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \qquad C_5 = 189.927$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \qquad C_6 = 176.392$$

$$A_1 := -C_3^2 - C_4^2 \qquad A_1 = -6.354 \times 10^5$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \qquad A_2 = 1.140 \times 10^5$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \qquad A_3 = -1.723 \times 10^5$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \qquad A_4 = -2.313 \times 10^5$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \qquad A_5 = -1.140 \times 10^5$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \qquad A_6 = -3.623 \times 10^5$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \qquad K_1 = 3.607 \times 10^{10}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \qquad K_2 = 8.115 \times 10^{10}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 8.816 \times 10^{10}$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = 72.976\, deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = 59.100\,deg$$

The second value is the same as α_3, so use the first value $\qquad \beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 34.802\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 34.802\,deg$$

Since both values are the same, $\qquad \beta_2 := \beta_{21}$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x} \qquad R_{1x} = 2.036 \times 10^3 \qquad R_{1y} := -O_{4y} \qquad R_{1y} = 311.000$

$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = 1.446 \times 10^3$

$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = 1.513 \times 10^3$

$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = 37.000$

$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = 2.128 \times 10^3$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 2.060 \times 10^3$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 2.093 \times 10^3$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 2.128 \times 10^3$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 8.685\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 46.297\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 89.004\,deg$

10. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = -486.018$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = -161.777$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3) \qquad C_3 = 741.712$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3) \qquad C_4 = 221.269$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2) \qquad C_5 = 154.965$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2) \qquad C_6 = 217.266$

$$A_1 := -C_3^2 - C_4^2 \qquad A_1 = -5.991 \times 10^5$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \qquad A_2 = 1.269 \times 10^5$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \qquad A_3 = -1.630 \times 10^5$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \qquad A_4 = -2.275 \times 10^5$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \qquad A_5 = -1.269 \times 10^5$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \qquad A_6 = -3.247 \times 10^5$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \qquad K_1 = 2.406 \times 10^{10}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \qquad K_2 = 7.828 \times 10^{10}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 7.953 \times 10^{10}$$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 86.724\, deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 59.100\, deg$$

The second value is the same as α_3, so use the first value $\gamma_3 := \gamma_{31}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 39.743\, deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = 39.743\, deg$$

Since both angles are the same, $\gamma_2 := \gamma_{21}$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 1338.994$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 116.144\, deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 2701.387$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 137.731\, deg$$

12. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$W1x = 3.194 \times 10^3 \qquad W1y = 829.763 \qquad Z1x = -1.100 \times 10^3 \qquad Z1y = -603.763$$

14. The length of link 2 is:

$$w := \sqrt{\left(W1x^2 + W1y^2\right)} \qquad w = 3299.543$$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$$U1x = 1.621 \times 10^3 \qquad U1y = 13.492 \qquad S1x = 415.016 \qquad S1y = 297.508$$

17. The length of link 4 is:

$$u := \sqrt{\left(U1x^2 + U1y^2\right)} \qquad u = 1621.040$$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \qquad V1x = -1.515 \times 10^3$$

$V1y := Z1y - S1y$ $\quad V1y = -901.272$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $\quad v = 1762.404$

$G1x := W1x + V1x - U1x$ $\quad G1x = 58.000$

$G1y := W1y + V1y - U1y$ $\quad G1y = -85.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $\quad g = 102.903$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $\quad O2x = -2094.000$

$O2y := -Z1y - W1y$ $\quad O2y = -226.000$

$O4x := -S1x - U1x$ $\quad O4x = -2036.000$

$O4y := -S1y - U1y$ $\quad O4y = -311.000$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $\quad z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $\quad z = 1254.370$ $\quad rp := z$

Angle *BAP* (δ_p) $\quad s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 510.637$

$\psi := atan2(S1x, S1y)$ $\quad \psi = 35.635\,deg$

$\phi := atan2(Z1x, Z1y)$ $\quad \phi = -151.228\,deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = -149.244\,deg$

$\delta_p := \phi - \theta_3$ $\quad \delta_p = -1.984\,deg$

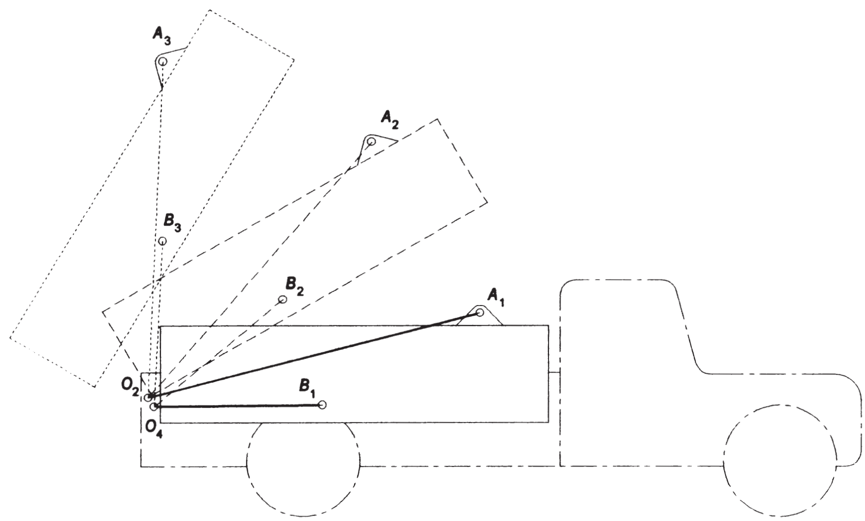
21. **DESIGN SUMMARY**

Link 1:	$g = 102.9$	
Link 2:	$w = 3299.5$	
Link 3:	$v = 1762.4$	
Link 4:	$u = 1621.0$	
Coupler point:	$rp = 1254.4$	$\delta_p = -1.984\,deg$

22. **VERIFICATION**: The calculated values of *g* (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).

23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none, but is close to toggle at position 3.

24. The transmission angles need to be checked also. A large mechanical advantage input device will need to be used here, such as a hydraulic cylinder. Note that the drive mechanism must also resist being overdriven (back driven) by the load as the dumpster descends from its high point onto the truck.

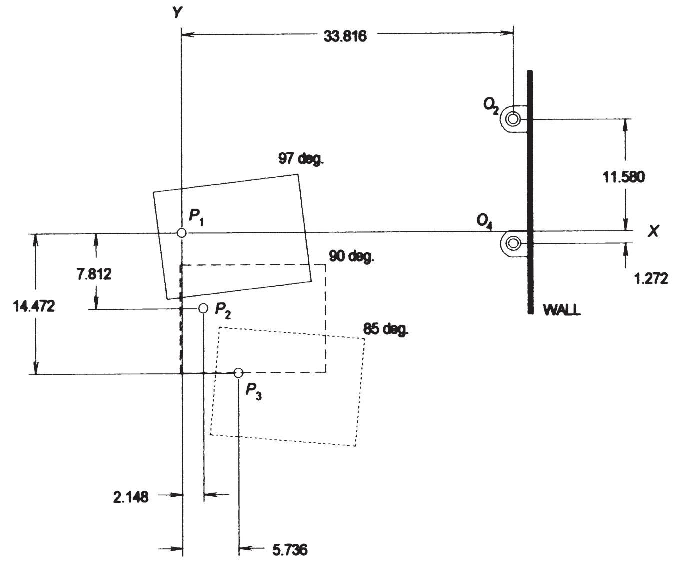


PROBLEM 5-7

Statement: See Project P3-7. Define three positions of the computer monitor and analytically synthesize a linkage to move through them. The fixed pivots must be located on the floor or wall.

Solution: See Project P3-7 and Mathcad file P0507.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than it is stated by deciding on three positions for the computer monitor to move through. The figure below shows one such set of positions (dimensions are in inches).



2. From the figure, the design choices are:

$P_{21x} := 2.148$ $\quad P_{21y} := -7.812$ $\quad P_{31x} := 5.736$ $\quad P_{31y} := -14.472$

$O_{2x} := 33.816$ $\quad O_{2y} := 11.580$ $\quad O_{4x} := 33.816$ $\quad O_{4y} := -1.272$

Body angles: $\theta_{P1} := 97 \cdot deg$ $\quad \theta_{P2} := 90 \cdot deg$ $\quad \theta_{P3} := 85 \cdot deg$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.

4. Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -7.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -12.000\,deg$$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$R_{1x} := -O_{2x} \qquad R_{1x} = -33.816 \qquad R_{1y} := -O_{2y} \qquad R_{1y} = -11.580$$

$$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = -31.668$$

$$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = -19.392$$

$$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = -28.080$$

$$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = -26.052$$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 35.744$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 37.134$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 38.304$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\quad \zeta_1 = -161.097\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\quad \zeta_2 = -148.519\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\quad \zeta_3 = -137.146\,deg$

7. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 3.962$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = -10.052$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = -7.405$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -21.756$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -3.307$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -12.019$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -528.143$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 17.049$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -285.981$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -11.771$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -17.049$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -248.020$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 7.073 \times 10^4$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = 7.595 \times 10^3$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\quad K_3 = 6.760 \times 10^4$

$\beta_{31} := 2 \cdot atan\left(\dfrac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\quad \beta_{31} = 24.258\,deg$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = -12.000\,deg$$

The second value is the same as α_3, so use the first value $\qquad \beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 12.435\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 12.435\,deg$$

Since both values are the same, $\qquad \beta_2 := \beta_{21}$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x} \qquad R_{1x} = -33.816 \qquad R_{1y} := -O_{4y} \qquad R_{1y} = 1.272$

$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = -31.668$

$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = -6.540$

$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = -28.080$

$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = -13.200$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 33.840$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 32.336$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 31.028$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 177.846\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = -168.331\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = -154.822\,deg$

10. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = 2.856$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = -9.867$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3) \qquad C_3 = -4.733$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3) \qquad C_4 = -21.475$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2) \qquad C_5 = -1.741$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2) \qquad C_6 = -11.924$

$A_1 := -C_3^2 - C_4^2$ $A_1 = -483.571$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = 19.043$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -264.299$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = -14.646$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = -19.043$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -225.402$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 5.929 \times 10^4$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 8.163 \times 10^3$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = 5.630 \times 10^4$

$\gamma_{31} := 2 \cdot atan\left(\dfrac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\gamma_{31} = 27.678\, deg$

$\gamma_{32} := 2 \cdot atan\left(\dfrac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\gamma_{32} = -12.000\, deg$

The second value is the same as α_3, so use the first value $\gamma_3 := \gamma_{31}$

$\gamma_{21} := acos\left(\dfrac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right)$ $\gamma_{21} = 14.435\, deg$

$\gamma_{22} := asin\left(\dfrac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right)$ $\gamma_{22} = 14.435\, deg$

Since both angles are the same, $\gamma_2 := \gamma_{21}$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$ $p_{21} = 8.102$

$\delta_2 := atan2(P_{21x}, P_{21y})$ $\delta_2 = -74.626\, deg$

$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$ $p_{31} = 15.567$

$\delta_3 := atan2(P_{31x}, P_{31y})$ $\delta_3 = -68.379\, deg$

12. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F := \cos(\beta_3) - 1$

$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$W1x = -36.030 \qquad W1y = -8.098 \qquad Z1x = 2.214 \qquad Z1y = -3.482$

14. The length of link 2 is:

$$w := \sqrt{\left(W1x^2 + W1y^2\right)} \qquad w = 36.929$$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$U1x = -32.294 \qquad U1y = -2.592 \qquad S1x = -1.522 \qquad S1y = 3.864$

17. The length of link 4 is:

$$u := \sqrt{\left(U1x^2 + U1y^2\right)} \qquad u = 32.398$$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x \qquad V1x = 3.736$

$V1y := Z1y - S1y \qquad V1y = -7.346$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 8.241$

$G1x := W1x + V1x - U1x$ $G1x = 0.000$

$G1y := W1y + V1y - U1y$ $G1y = -12.852$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 12.852$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = 33.816$

$O2y := -Z1y - W1y$ $O2y = 11.580$

$O4x := -S1x - U1x$ $O4x = 33.816$

$O4y := -S1y - U1y$ $O4y = -1.272$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 4.126$ $rp := z$

Angle *BAP* (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 4.153$

$\psi := atan2(S1x, S1y)$ $\psi = 111.494\,deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = -57.551\,deg$

$\theta_3 := atan2\left(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi)\right)$

$\theta_3 = -63.046\,deg$

$\delta_p := \phi - \theta_3$ $\delta_p = 5.495\,deg$

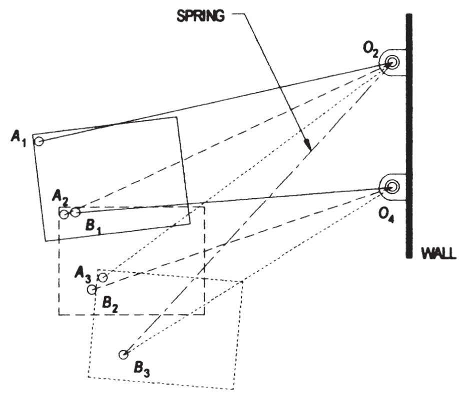
21. **DESIGN SUMMARY**

Link 1:	$g = 12.852$	
Link 2:	$w = 36.929$	
Link 3:	$v = 8.241$	
Link 4:	$u = 32.398$	
Coupler point:	$rp = 4.126$	$\delta_p = 5.495\,deg$

22. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).

23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none.

24. The transmission angles need to be checked also. A means to support the weight of the monitor must be provided. The figure below shows a spring placed between links to provide the balancing moment. Further design and analysis needs to be done to optimize the spring placement in order to compensate for its change in force with deflection and the change in moment arm as the linkage moves.



PROBLEM 5-8

Statement: Design a linkage to carry the body in Figure P5-1 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := -1.236 \quad P_{2y} := 2.138$

Angles made by the body in positions 1 and 2:

$\theta_{P1} := 210 \cdot deg \quad \theta_{P2} := 147.5 \cdot deg$

Free choices for the **WZ** dyad :

$z := 1.075 \quad \beta_2 := -27.0 \cdot deg \quad \phi := 204.4 \cdot deg$

Free choices for the **US** dyad :

$s := 1.240 \quad \gamma_2 := -40.0 \cdot deg \quad \psi := 74.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-1 and Mathcad file P0508.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. Because of the data given in the hint, the second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \quad P_{21x} = -1.236$$

$$P_{21y} = 2.138$$

$$p21 := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p21 = 2.470$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -62.500\ deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 120.033\ deg$$

4. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \quad Z_{1x} = -0.979 \quad Z_{1y} := z \cdot sin(\phi) \quad Z_{1y} = -0.444$$

$A := \cos(\beta_2) - 1$ $\quad A = -0.109$ $\quad D := \sin(\alpha_2)$ $\quad D = -0.887$

$B := \sin(\beta_2)$ $\quad B = -0.454$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad E = -1.236$

$C := \cos(\alpha_2) - 1$ $\quad C = -0.538$ $\quad F := p_{21} \cdot \sin(\delta_2)$ $\quad F = 2.138$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = -1.462$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = -3.367$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 3.670$$

$$\theta := \operatorname{atan2}(W_{1x}, W_{1y}) \qquad \theta = 246.528 \; deg$$

5. Solve for the US dyad using equations 5.12.

$S_{1x} := s \cdot \cos(\psi)$ $\quad S_{1x} = 0.342$ $\quad S_{1y} := s \cdot \sin(\psi)$ $\quad S_{1y} = 1.192$

$A := \cos(\gamma_2) - 1$ $\quad A = -0.234$ $\quad D := \sin(\alpha_2)$ $\quad D = -0.887$

$B := \sin(\gamma_2)$ $\quad B = -0.643$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad E = -1.236$

$C := \cos(\alpha_2) - 1$ $\quad C = -0.538$ $\quad F := p_{21} \cdot \sin(\delta_2)$ $\quad F = 2.138$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = -3.180$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = -4.439$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 5.461$$

$$\sigma := \operatorname{atan2}(U_{1x}, U_{1y}) \qquad \sigma = 234.381 \; deg$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$ $\qquad V_{1x} = -1.321$

$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$ $\qquad V_{1y} = -1.636$

$\theta_3 := \operatorname{atan2}(V_{1x}, V_{1y})$ $\qquad \theta_3 = 231.086 \; deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $\qquad v = 2.103$

Link 1: $G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$ $\qquad G_{1x} = 0.398$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $\qquad G_{1y} = -0.564$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\qquad \theta_1 = -54.796\ deg$

$g := \sqrt{\left(G_{1x}{}^2 + G_{1y}{}^2\right)}$ $\qquad g = 0.690$

7. Determine the initial and final values of the input crank with respect to the vector G.

$\theta_{2i} := \theta - \theta_1$ $\qquad \theta_{2i} = 301.323\ deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\qquad \theta_{2f} = 274.323\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\qquad \delta_p := \phi - \theta_3$

$r_p = 1.075$ $\qquad \delta_p = -26.686\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\qquad \rho_1 = 180.000\ deg$

$R_1 := \sqrt{\left(P_{1x}{}^2 + P_{1y}{}^2\right)}$ $\qquad R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\qquad O_{2x} = 2.441$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\qquad O_{2y} = 3.811$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad O_{4x} = 2.838$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad O_{4y} = 3.247$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\qquad \theta_{rot} = -54.796\ deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

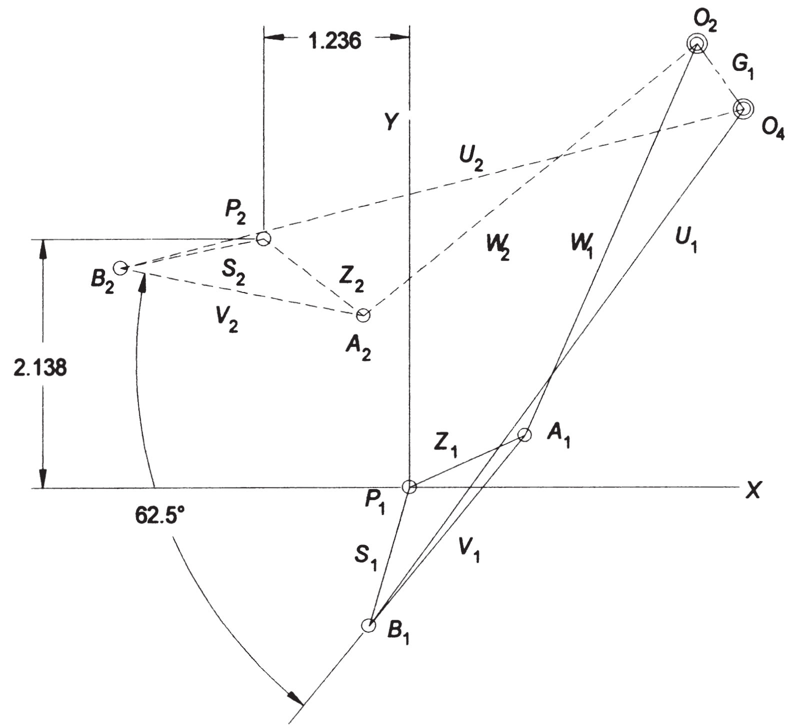
$Condition(g, w, u, v) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

Link 2:	$w = 3.670$	$\theta = 246.528\ deg$
Link 3:	$v = 2.103$	$\theta_3 = 231.086\ deg$
Link 4:	$u = 5.461$	$\sigma = 234.381\ deg$

Link 1:	$g = 0.690$	$\theta_1 = -54.796\ deg$
Coupler:	$r_p = 1.075$	$\delta_p = -26.686\ deg$
Crank angles:		
	$\theta_{2i} = 301.323\ deg$	
	$\theta_{2f} = 274.323\ deg$	

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-9

Statement: Design a linkage to carry the body in Figure P5-1 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Hint: First try a rough graphical solution to create realistic values for free choices.

Given: Coordinates of the points P_2 and P_3 with respect to P_1:

$$P_{2x} := -1.236 \quad P_{2y} := 2.138 \qquad P_{3x} := -2.500 \qquad P_{3y} := 2.931$$

Angles made by the body in positions 1 and 2:

$$\theta_{P2} := 147.5 \cdot deg \qquad \theta_{P3} := 110.2 \cdot deg$$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-1 and Mathcad file P0509.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = -1.264$$

$$P_{21y} = 0.793$$

$$p_{21} := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p_{21} = 1.492$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P3} - \theta_{P2} \qquad \alpha_2 = -37.300\,deg$$

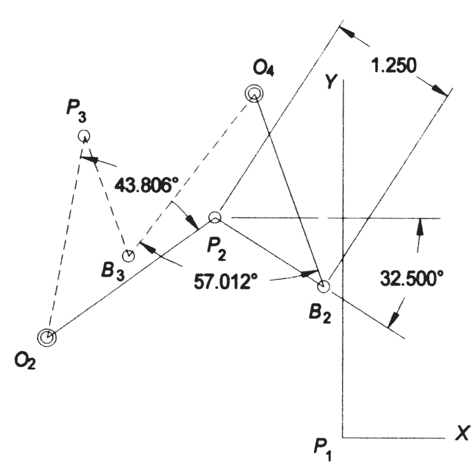
$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 147.897\,deg$$

4. From a graphical solution (see figure at right), determine the values necessary for input to equations 5.8.

$$z := 0.0$$

$$\beta_2 := 43.806 \cdot deg$$

$$\phi := -32.500 \cdot deg$$



5. Solve for the **WZ** dyad using equations 5.8.

$Z_{1x} := z \cdot cos(\phi)$ $\quad Z_{1x} = 0.000$ $\quad Z_{1y} := z \cdot sin(\phi)$ $\quad Z_{1y} = 0.000$

$A := cos(\beta_2) - 1$ $\quad A = -0.278$ $\quad D := sin(\alpha_2)$ $\quad D = -0.606$

$B := sin(\beta_2)$ $\quad B = 0.692$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad E = -1.264$

$C := cos(\alpha_2) - 1$ $\quad C = -0.205$ $\quad F := p_{21} \cdot sin(\delta_2)$ $\quad F = 0.793$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = 1.618$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = 1.175$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 2.000$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 35.994\,deg$$

5. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$s := 1.250$ $\quad \gamma_2 := -57.012 \cdot deg$ $\quad \psi := 147.5 \cdot deg$

6. Solve for the **US** dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $\quad S_{1x} = -1.054$ $\quad S_{1y} := s \cdot sin(\psi)$ $\quad S_{1y} = 0.672$

$A := cos(\gamma_2) - 1$ $\quad A = -0.456$ $\quad D := sin(\alpha_2)$ $\quad D = -0.606$

$B := sin(\gamma_2)$ $\quad B = -0.839$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad E = -1.264$

$C := cos(\alpha_2) - 1$ $\quad C = -0.205$ $\quad F := p_{21} \cdot sin(\delta_2)$ $\quad F = 0.793$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = 0.675$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = -1.883$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 2.000$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = -70.278\,deg$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $\quad V_{1x} = 1.054$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $\quad V_{1y} = -0.672$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = -32.500\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 1.250$

Link 1: $G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $G_{1x} = 1.997$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $G_{1y} = 2.386$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 50.070\,deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $g = 3.112$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = -14.077\,deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\theta_{2f} = 29.729\,deg$

9. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\,deg$

which is correct for the assumption that the precision point is at *C*.

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{2x}, P_{2y})$ $\rho_1 = 120.033\,deg$

$R_1 := \sqrt{\left(P_{2x}^2 + P_{2y}^2\right)}$ $R_1 = 2.470$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $O_{2x} = -2.854$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $O_{2y} = 0.963$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = -0.857$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = 3.349$

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\theta_{rot} = 50.070\,deg$

12. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return } \text{"Grashof" } \textit{if } SL < PQ \\ \textit{return } \text{"Special Grashof" } \textit{if } SL = PQ \\ \textit{return } \text{"non-Grashof" } \textit{otherwise} \end{array} \right.$$

$Condition(v, g, u, w) = $ "non-Grashof"

13. **DESIGN SUMMARY**

Link 2:	$w = 2.000$	$\theta = 35.994\ deg$
Link 3:	$v = 1.250$	$\theta_3 = -32.500\ deg$
Link 4:	$u = 2.000$	$\sigma = -70.278\ deg$
Link 1:	$g = 3.112$	$\theta_1 = 50.070\ deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\ deg$
Crank angles:		
	$\theta_{2i} = -14.077\ deg$	
	$\theta_{2f} = 29.729\ deg$	

PROBLEM 5-10

Statement: Design a linkage to carry the body in Figure P5-1 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -1.236$ $\quad P_{2y} := 2.138$

$P_{3x} := -2.500$ $\quad P_{3y} := 2.931$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 210 \cdot deg$ $\quad \theta_{P2} := 147.5 \cdot deg$ $\quad \theta_{P3} := 110.2 \cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := 30.0 \cdot deg$ $\quad \beta_3 := 60.0 \cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -10.0 \cdot deg$ $\quad \gamma_3 := 25.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \land y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \land y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-1 and Mathcad file P0510.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 2.470 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 120.033\, deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 3.852 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 130.463\, deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -62.500\, deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -99.800\, deg$$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = 2.920$ $\quad W_{1y} = 1.720$ $\quad Z_{1x} = -0.756$ $\quad Z_{1y} = -0.442$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 30.493\, deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 210.303\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $\quad w = 3.389$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^{\ 2} + Z_{1y}^{\ 2}\right)}$, $\quad z = 0.876$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $\quad B' := sin(\gamma_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -1.009$ $\quad U_{1y} = 2.693$ $\quad S_{1x} = -0.792$ $\quad S_{1y} = -2.418$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 110.545\, deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 251.875\, deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $\quad u = 2.875$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^{\ 2} + S_{1y}^{\ 2}\right)}$, $\quad s = 2.544$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\quad V_{1x} = 0.036$

$V_{1y} := Z_{1y} - S_{1y}$ $\quad V_{1y} = 1.976$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 88.968\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 1.977$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 3.965$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 1.003$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 14.202\,deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $g = 4.090$

6. **Determine the initial and final values of the input crank with respect to the vector G.**

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 16.291\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 76.291\,deg$

7. **Define the coupler point with respect to point *A* and the vector V.**

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.876$ $\delta_p = 121.335\,deg$

8. **Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.**

$O_{2x} := -z \cdot cos(\phi) - w \cdot cos(\theta)$ $O_{2x} = -2.164$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $O_{2y} = -1.278$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = 1.801$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = -0.274$

9. **Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.**

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\theta_{rot} = 14.202\,deg$

10. **Determine the Grashof condition.**

$Condition(S, L, P, Q) :=$
```
SL ← S + L
PQ ← P + Q
return "Grashof"  if SL < PQ
return "Special Grashof"  if SL = PQ
return "non-Grashof"  otherwise
```

$Condition(v, g, u, v) =$ "non-Grashof"

11. **DESIGN SUMMARY**

Link 2: $w = 3.389$ $\theta = 30.493\,deg$

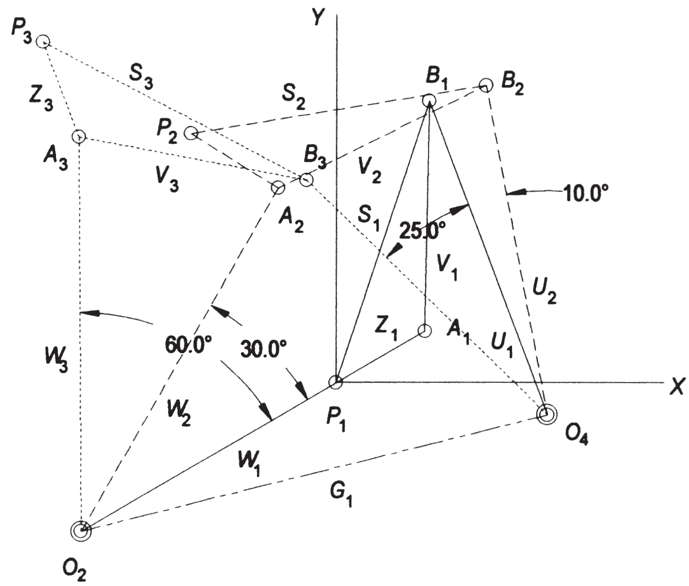
Link 3:	$v = 1.977$	$\theta_3 = 88.968\ deg$
Link 4:	$u = 2.875$	$\sigma = 110.545\ deg$
Link 1:	$g = 4.090$	$\theta_1 = 14.202\ deg$
Coupler:	$r_p = 0.876$	$\delta_p = 121.335\ deg$

Crank angles:

$$\theta_{2i} = 16.291\ deg$$

$$\theta_{2f} = 76.291\ deg$$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-11

Statement: Design a linkage to carry the body in Figure P5-1 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := -1.236$ $\quad P_{21y} := 2.138$ $\quad P_{31x} := -2.500$ $\quad P_{31y} := 2.931$

$O_{2x} := -2.164$ $\quad O_{2y} := -1.260$ $\quad O_{4x} := 2.190$ $\quad O_{4y} := -1.260$

Body angles: $\theta_{P1} := 210 \cdot deg$ $\quad \theta_{P2} := 147.5 \cdot deg$ $\quad \theta_{P3} := 110.2 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-1 and Mathcad file P0511.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = -62.500\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -99.800\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 2.164$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 1.260$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 0.928$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 3.398$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -0.336$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 4.191$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 2.504$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 3.522$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 4.204$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 30.210\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 74.725\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 94.584\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = 0.372$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 3.726$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = 1.209$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = 6.538$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = 1.189$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 4.736$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $A_1 = -44.206$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = -2.046$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -32.399$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = 6.937$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = 2.046$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -23.911$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 760.497$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = -273.669$

$K_3 := \dfrac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2}$ $K_3 = 140.232$

$\beta_{31} := 2 \cdot atan\left(\dfrac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right)$ $\beta_{31} = 60.217\,deg$

$\beta_{32} := 2 \cdot atan\left(\dfrac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right)$ $\beta_{32} = -99.800\,deg$

The second value is the same as α_3, so use the first value $\beta_3 := \beta_{31}$

$\beta_{21} := acos\left(\dfrac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right)$ $\beta_{21} = 30.143\,deg$

$\beta_{22} := asin\left(\dfrac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right)$ $\beta_{22} = 30.143\,deg$

Since both values are the same, $\beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $R_{1x} = -2.190$ $R_{1y} := -O_{4y}$ $R_{1y} = 1.260$

$R_{2x} := R_{1x} + P_{21x}$ $R_{2x} = -3.426$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 3.398$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -4.690$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 4.191$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 2.527$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 4.825$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 6.290$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 150.086\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 135.235\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 138.216\,deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = -2.380$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = 3.298$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\qquad C_3 = 6.304$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\qquad C_4 = 2.247$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\qquad C_5 = 3.532$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\qquad C_6 = 0.874$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -44.796$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = -2.431$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -24.233$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = 15.441$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = 2.431$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -22.414$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 505.612$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = -428.679$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\qquad K_3 = 336.363$

$$\gamma_{31} := 2\cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 19.215\,deg$$

$$\gamma_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -99.800\,deg$$

The second value is the same as α_3, so use the first value $\gamma_3 := \gamma_{31}$

$$\gamma_{21} := acos\left(\frac{A_5\cdot sin(\gamma_3) + A_3\cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 6.628\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3\cdot sin(\gamma_3) + A_2\cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -6.628\,deg$$

Since γ_2 is not in the first quadrant , $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.470$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 120.033\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 3.852$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 130.463\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21}\cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31}\cdot cos(\delta_3) \qquad M := p_{21}\cdot sin(\delta_2) \qquad N := p_{31}\cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1}\cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W1x = 2.915 \qquad W1y = 1.702 \qquad Z1x = -0.751 \qquad Z1y = -0.442$$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 3.376$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -1.371$ $U1y = 3.634$ $S1x = -0.819$ $S1y = -2.374$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 3.884$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 0.068$

$V1y := Z1y - S1y$ $V1y = 1.932$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 1.933$

$G1x := W1x + V1x - U1x$ $G1x = 4.354$

$G1y := W1y + V1y - U1y$ $G1y = -4.885 \times 10^{-15}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 4.354$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -2.164$

$O2y := -Z1y - W1y$ $O2y = -1.260$

$O4x := -S1x - U1x$ $O4x = 2.190$

$O4y := -S1y - U1y$ $O4y = -1.260$

These check with Figure P5-1.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 0.871$ $rp := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\qquad s = 2.511$

$\psi := atan2(S1x, S1y)$ $\qquad \psi = 250.963\ deg$

$\phi := atan2(Z1x, Z1y)$ $\qquad \phi = 210.445\ deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 87.994\ deg$

$\delta_p := \phi - \theta_3$ $\qquad \delta_p = 122.451\ deg$

18. **DESIGN SUMMARY**

Link 1: $g = 4.354$

Link 2: $w = 3.376$

Link 3: $v = 1.933$

Link 4: $u = 3.884$

Coupler point: $rp = 0.871$ $\qquad \delta_p = 122.451\ deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-12

Statement: Design a linkage to carry the body in Figure P5-2 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 1.903$ $\quad P_{2y} := 1.347$

Angles made by the body in positions 1 and 2:

$\theta_{P1} := 101.0 \cdot deg$ $\quad \theta_{P2} := 62.0 \cdot deg$

Free choices for the **WZ** dyad :

$z := 2.000$ $\quad \beta_2 := 30.0 \cdot deg$ $\quad \phi := 150.0 \cdot deg$

Free choices for the **US** dyad :

$s := 3.000$ $\quad \gamma_2 := 40.0 \cdot deg$ $\quad \psi := -50.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array} \right.$$

Solution: See Figure P5-2 and Mathcad file P0512.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. Because of the data given in the hint, the second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 1.903$$

$$P_{21y} = 1.347$$

$$p21 := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p21 = 2.331$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -39.000\,deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 35.292\,deg$$

4. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = -1.732 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 1.000$$

$A := \cos(\beta_2) - 1$ $A = -0.134$ $D := \sin(\alpha_2)$ $D = -0.629$

$B := \sin(\beta_2)$ $B = 0.500$ $E := p_{21} \cdot \cos(\delta_2)$ $E = 1.903$

$C := \cos(\alpha_2) - 1$ $C = -0.223$ $F := p_{21} \cdot \sin(\delta_2)$ $F = 1.347$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = 0.452$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = -1.896$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 1.949$$

$$\theta := \operatorname{atan2}(W_{1x}, W_{1y}) \qquad \theta = -76.607\,deg$$

5. Solve for the US dyad using equations 5.12.

$S_{1x} := s \cdot \cos(\psi)$ $S_{1x} = 1.928$ $S_{1y} := s \cdot \sin(\psi)$ $S_{1y} = -2.298$

$A := \cos(\gamma_2) - 1$ $A = -0.234$ $D := \sin(\alpha_2)$ $D = -0.629$

$B := \sin(\gamma_2)$ $B = 0.643$ $E := p_{21} \cdot \cos(\delta_2)$ $E = 1.903$

$C := \cos(\alpha_2) - 1$ $C = -0.223$ $F := p_{21} \cdot \sin(\delta_2)$ $F = 1.347$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = 0.924$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = -6.216$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 6.284$$

$$\sigma := \operatorname{atan2}(U_{1x}, U_{1y}) \qquad \sigma = -81.540\,deg$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$ $V_{1x} = -3.660$

$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$ $V_{1y} = 3.298$

$\theta_3 := \operatorname{atan2}(V_{1x}, V_{1y})$ $\theta_3 = 137.980\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 4.927$

Link 1: $G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$ $G_{1x} = -4.133$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $\qquad G_{1y} = 7.617$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\qquad \theta_1 = 118.485\ deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $\qquad g = 8.667$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad \theta_{2i} = -195.092\ deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\qquad \theta_{2f} = -165.092\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\qquad \delta_p := \phi - \theta_3$

$r_p = 2.000$ $\qquad \delta_p = 12.020\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\qquad \rho_1 = 180.000\ deg$

$R_1 := \sqrt{\left(P_{1x}^2 + P_{1y}^2\right)}$ $\qquad R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\qquad O_{2x} = 1.281$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\qquad O_{2y} = 0.896$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad O_{4x} = -2.853$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad O_{4y} = 8.514$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\qquad \theta_{rot} = 118.485\ deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(w, g, u, v) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

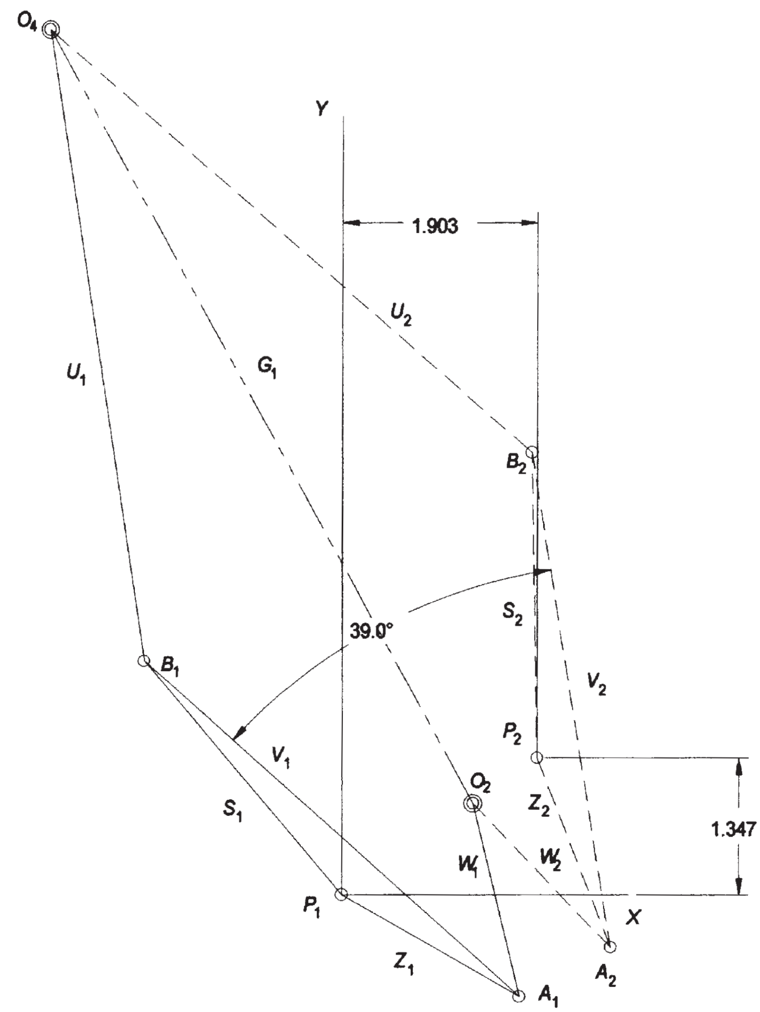
Link 2:	$w = 1.949$	$\theta = -76.607\ deg$
Link 3:	$v = 4.927$	$\theta_3 = 137.980\ deg$
Link 4:	$u = 6.284$	$\sigma = -81.540\ deg$

Link 1:	$g = 8.667$	$\theta_1 = 118.485\ deg$
Coupler:	$r_p = 2.000$	$\delta_p = 12.020\ deg$
Crank angles:		

$$\theta_{2i} = -195.092\ deg$$

$$\theta_{2f} = -165.092\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-13

Statement: Design a linkage to carry the body in Figure P5-2 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Hint: First try a rough graphical solution to create realistic values for free choices.

Given: Coordinates of the points P_2 and P_3 with respect to P_1:

$P_{2x} := 1.903$ $\quad P_{2y} := 1.347$ $\quad P_{3x} := 1.389$ $\quad P_{3y} := 1.830$

Angles made by the body in positions 1 and 2:

$\theta_{P2} := 62.0 \cdot deg$ $\quad \theta_{P3} := 39.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-2 and Mathcad file P0513.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = -0.514$$

$$P_{21y} = 0.483$$

$$p_{21} := \sqrt{\left(P_{21x}{}^2 + P_{21y}{}^2\right)} \qquad p_{21} = 0.705$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P3} - \theta_{P2} \qquad \alpha_2 = -23.000\,deg$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 136.781\,deg$$

4. From a graphical solution (see figure next page), determine the values necessary for input to equations 5.8.

$z := 3$ $\quad \beta_2 := 45.0 \cdot deg$ $\quad \phi := 100 \cdot deg$

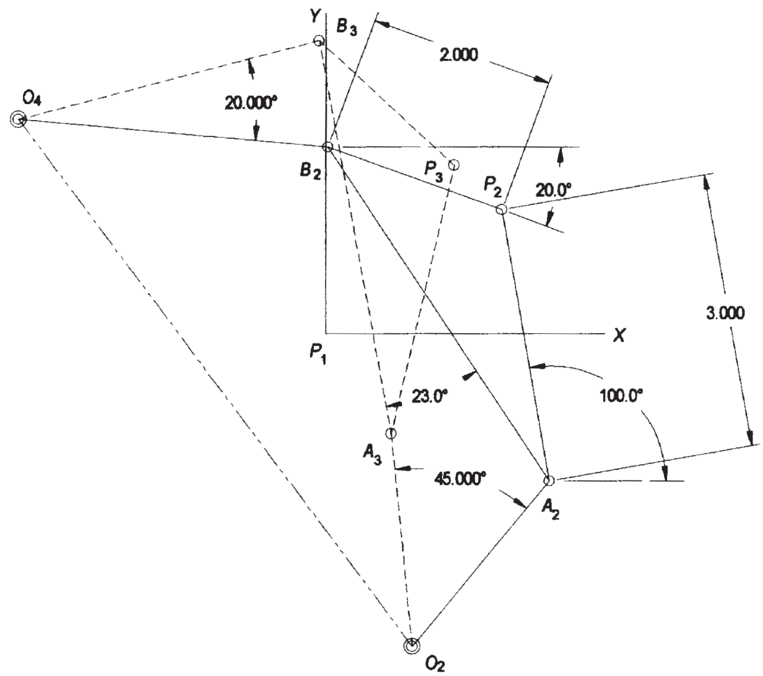
5. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = -0.521 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 2.954$$

$$A := cos(\beta_2) - 1 \qquad A = -0.293 \qquad D := sin(\alpha_2) \qquad D = -0.391$$

$$B := sin(\beta_2) \qquad B = 0.707 \qquad E := p_{21} \cdot cos(\delta_2) \qquad E = -0.514$$

$$C := cos(\alpha_2) - 1 \qquad C = -0.079 \qquad F := p_{21} \cdot sin(\delta_2) \qquad F = 0.483$$



$$W_{1x} := \frac{A \cdot \left(-C \cdot Z_{1x} + D \cdot Z_{1y} + E\right) + B \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right)}{-2 \cdot A} \qquad W_{1x} = 1.476$$

$$W_{1y} := \frac{A \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right) + B \cdot \left(C \cdot Z_{1x} - D \cdot Z_{1y} - E\right)}{-2 \cdot A} \qquad W_{1y} = 1.807$$

$$w := \sqrt{W_{1x}^{\ 2} + W_{1y}^{\ 2}} \qquad w = 2.333$$

$$\theta := atan2\left(W_{1x}, W_{1y}\right) \qquad \theta = 50.759\,deg$$

5. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$s := 2.000$ $\qquad \gamma_2 := 20.0 \cdot deg$ $\qquad \psi := -20.0 \cdot deg$

6. Solve for the US dyad using equations 5.12.

$S_{1x} := s \cdot \cos(\psi)$ $\qquad S_{1x} = 1.879$ $\qquad S_{1y} := s \cdot \sin(\psi)$ $\qquad S_{1y} = -0.684$

$A := \cos(\gamma_2) - 1$ $\qquad A = -0.060$ $\qquad D := \sin(\alpha_2)$ $\qquad D = -0.391$

$B := \sin(\gamma_2)$ $\qquad B = 0.342$ $\qquad E := p_{21} \cdot \cos(\delta_2)$ $\qquad E = -0.514$

$C := \cos(\alpha_2) - 1$ $\qquad C = -0.079$ $\qquad F := p_{21} \cdot \sin(\delta_2)$ $\qquad F = 0.483$

$$U_{1x} := \frac{A \cdot \left(-C \cdot S_{1x} + D \cdot S_{1y} + E\right) + B \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right)}{-2 \cdot A} \qquad U_{1x} = 3.346$$

$$U_{1y} := \frac{A \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right) + B \cdot \left(C \cdot S_{1x} - D \cdot S_{1y} - E\right)}{-2 \cdot A} \qquad U_{1y} = -0.306$$

$u := \sqrt{U_{1x}^2 + U_{1y}^2}$ $\quad u = 3.360$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = -5.216\,deg$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $\quad V_{1x} = -2.400$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $\quad V_{1y} = 3.638$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = 123.413\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $\quad v = 4.359$

Link 1: $G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $\quad G_{1x} = -4.271$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $\quad G_{1y} = 5.751$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = 126.601\,deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $\quad g = 7.163$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\quad \theta_{2i} = -75.842\,deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\quad \theta_{2f} = -30.842\,deg$

9. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\quad \delta_p := \phi - \theta_3$

$r_p = 3.000$ $\quad \delta_p = -23.413\,deg$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{2x}, P_{2y})$ $\quad \rho_1 = 35.292\,deg$

$R_1 := \sqrt{\left(P_{2x}^2 + P_{2y}^2\right)}$ $\quad R_1 = 2.331$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\quad O_{2x} = 0.948$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\quad O_{2y} = -3.414$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\quad O_{4x} = -3.323$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\quad O_{4y} = 2.337$

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\quad \theta_{rot} = 126.601\,deg$

12. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array} \right.$$

$$Condition(w,g,u,v) = \text{"non-Grashof"}$$

13. **DESIGN SUMMARY**

Link 2:	$w = 2.333$	$\theta = 50.759\,deg$
Link 3:	$v = 4.359$	$\theta_3 = 123.413\,deg$
Link 4:	$u = 3.360$	$\sigma = -5.216\,deg$
Link 1:	$g = 7.163$	$\theta_1 = 126.601\,deg$
Coupler:	$r_p = 3.000$	$\delta_p = -23.413\,deg$
Crank angles:		

$$\theta_{2i} = -75.842\,deg$$

$$\theta_{2f} = -30.842\,deg$$

PROBLEM 5-14

Statement: Design a linkage to carry the body in Figure P5-2 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 1.903$ $\quad P_{2y} := 1.347$

$P_{3x} := 1.389$ $\quad P_{3y} := 1.830$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 101\cdot deg$ $\quad \theta_{P2} := 62.0\cdot deg$ $\quad \theta_{P3} := 39.0\cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := 40.0\cdot deg$ $\quad \beta_3 := 75.0\cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := 0.0\cdot deg$ $\quad \gamma_3 := 30.0\cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-2 and Mathcad file P0514.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^{\,2} + P_{2y}^{\,2}} \qquad p_{21} = 2.331 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 35.292\,deg$$

$$p_{31} := \sqrt{P_{3x}^{\,2} + P_{3y}^{\,2}} \qquad p_{31} = 2.297 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 52.801\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -39.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -62.000\,deg$$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21}\cdot\cos(\delta_2) \qquad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31}\cdot\cos(\delta_3) \qquad M := p_{21}\cdot\sin(\delta_2) \qquad N := p_{31}\cdot\sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = 3.110$ $\quad W_{1y} = -1.061$ $\quad Z_{1x} = 0.297$ $\quad Z_{1y} = 3.201$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = -18.843\,deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 84.698\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\;2} + W_{1y}^{\;2}\right)}$, $\quad w = 3.286$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^{\;2} + Z_{1y}^{\;2}\right)}$, $\quad z = 3.215$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = 1.658$ $\quad U_{1y} = 3.361$ $\quad S_{1x} = -2.853$ $\quad S_{1y} = 2.013$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 63.740\,deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 144.792\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\;2} + U_{1y}^{\;2}\right)}$, $\quad u = 3.748$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^{\;2} + S_{1y}^{\;2}\right)}$, $\quad s = 3.492$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\qquad V_{1x} = 3.150$

$V_{1y} := Z_{1y} - S_{1y}$ $\qquad V_{1y} = 1.188$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\qquad \theta_3 = 20.657\,deg$

$v := \sqrt{\left(V_{1x}^{\;2} + V_{1y}^{\;2}\right)}$ $\qquad v = 3.367$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $\qquad G_{1x} = 4.602$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $\qquad G_{1y} = -3.234$

$\theta_1 := atan2\left(G_{1x}, G_{1y}\right)$ $\qquad \theta_1 = -35.099\,deg$

$g := \sqrt{\left(G_{1x}^{\;2} + G_{1y}^{\;2}\right)}$ $\qquad g = 5.625$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad \theta_{2i} = 16.256\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\qquad \theta_{2f} = 91.256\,deg$

7. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\qquad \delta_p := \phi - \theta_3$

$r_p = 3.215$ $\qquad \delta_p = 64.041\,deg$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot cos(\phi) - w\cdot cos(\theta)$ $\qquad O_{2x} = -3.407$

$O_{2y} := -z\cdot sin(\phi) - w\cdot sin(\theta)$ $\qquad O_{2y} = -2.140$

$O_{4x} := -s\cdot cos(\psi) - u\cdot cos(\sigma)$ $\qquad O_{4x} = 1.195$

$O_{4y} := -s\cdot sin(\psi) - u\cdot sin(\sigma)$ $\qquad O_{4y} = -5.374$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\qquad \theta_{rot} = -35.099\,deg$

10. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(w, g, u, v) = \text{"non-Grashof"}$

11. **DESIGN SUMMARY**

Link 2:	$w = 3.286$	$\theta = -18.843\,deg$
Link 3:	$v = 3.367$	$\theta_3 = 20.657\,deg$
Link 4:	$u = 3.748$	$\sigma = 63.740\,deg$

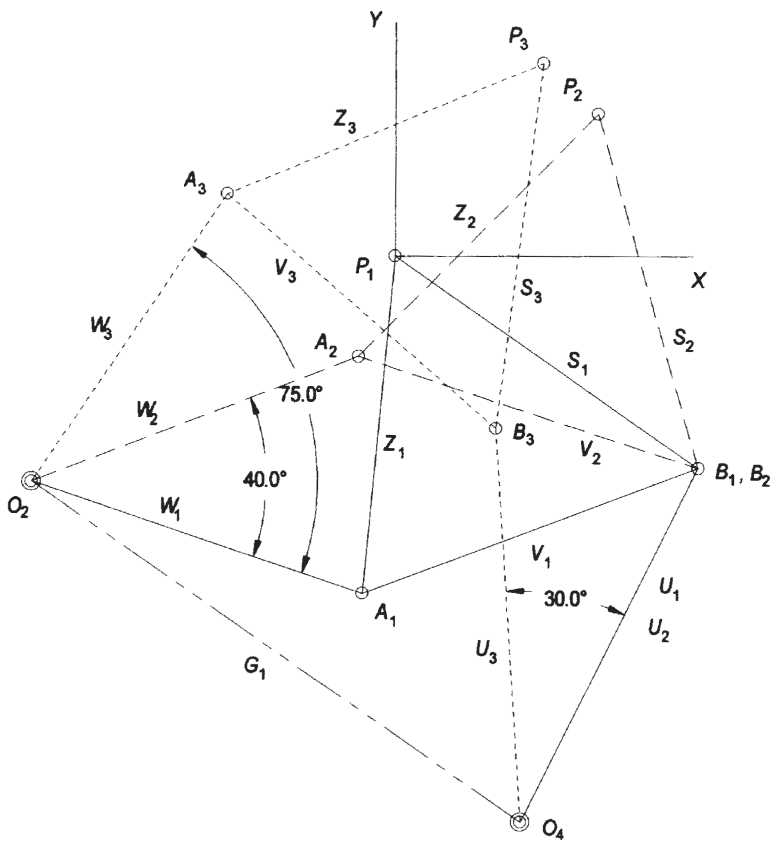
Link 1:	$g = 5.625$	$\theta_1 = -35.099\,deg$
Coupler:	$r_p = 3.215$	$\delta_p = 64.041\,deg$

Crank angles:

$$\theta_{2i} = 16.256\,deg$$

$$\theta_{2f} = 91.256\,deg$$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-15

Statement: Design a linkage to carry the body in Figure P5-2 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := 1.903$ $\quad P_{21y} := 1.347$ $\quad P_{31x} := 1.389$ $\quad P_{31y} := 1.830$

$O_{2x} := -0.884$ $\quad O_{2y} := -1.251$ $\quad O_{4x} := 3.062$ $\quad O_{4y} := -1.251$

Body angles: $\theta_{P1} := 101 \cdot deg$ $\quad \theta_{P2} := 62 \cdot deg$ $\quad \theta_{P3} := 39 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-2 and Mathcad file P0515.

1. Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -39.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -62.000\,deg$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$R_{1x} := -O_{2x} \qquad R_{1x} = 0.884 \qquad R_{1y} := -O_{2y} \qquad R_{1y} = 1.251$$

$$R_{2x} := R_{1x} + P_{21x} \qquad R_{2x} = 2.787$$

$$R_{2y} := R_{1y} + P_{21y} \qquad R_{2y} = 2.598$$

$$R_{3x} := R_{1x} + P_{31x} \qquad R_{3x} = 2.273$$

$$R_{3y} := R_{1y} + P_{31y} \qquad R_{3y} = 3.081$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \qquad R_1 = 1.532$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \qquad R_2 = 3.810$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \qquad R_3 = 3.829$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 54.754\,deg$$

$$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 42.990\,deg$$

$$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 53.582\,deg$$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 0.103$

$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2)$ $\quad C_2 = 2.205$

$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3)$ $\quad C_3 = -0.753$

$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3)$ $\quad C_4 = 3.274$

$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2)$ $\quad C_5 = -1.313$

$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$ $\quad C_6 = 2.182$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -11.288$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 2.654$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -8.134$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -1.324$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -2.654$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -7.297$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 55.842$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = 30.136$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = -0.392$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = 118.708\, deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = -62.000\, deg$$

The second value is the same as α_3, so use the first value $\quad \beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 59.564\, deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 59.564\, deg$$

Since both values are the same, $\quad \beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = -3.062$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 1.251$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = -1.159$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 2.598$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -1.673$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 3.081$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 3.308$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 2.845$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 3.506$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 157.777\, deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 114.042\, deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 118.502\, deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = -1.111$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = 1.204$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\qquad C_3 = 1.340$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\qquad C_4 = -0.210$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\qquad C_5 = -0.433$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\qquad C_6 = -0.301$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -1.840$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = -0.495$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = 0.517$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = 1.847$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = 0.495$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -1.236$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = -1.553$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 0.344$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = -1.033$$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = -62.000\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 36.991\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 73.415\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -73.415\,deg$$

Since γ_2 is not in the first quadrant, $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.331$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 35.292\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 2.297$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 52.801\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W1x = 1.262 \qquad W1y = -1.109 \qquad Z1x = -0.378 \qquad Z1y = 2.360$$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $\qquad w = 1.680$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21}\cdot\cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31}\cdot\cos(\delta_3) \qquad M := p_{21}\cdot\sin(\delta_2) \qquad N := p_{31}\cdot\sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1}\cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -0.326 \qquad U1y = 0.830 \qquad S1x = -2.736 \qquad S1y = 0.421$$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $\qquad u = 0.892$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \qquad V1x = 2.359$$

$$V1y := Z1y - S1y \qquad V1y = 1.939$$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $\qquad v = 3.054$

$$G1x := W1x + V1x - U1x \qquad G1x = 3.946$$

$$G1y := W1y + V1y - U1y \qquad G1y = 0.000$$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $\qquad g = 3.946$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$$O2x := -Z1x - W1x \qquad O2x = -0.884$$

$$O2y := -Z1y - W1y \qquad O2y = -1.251$$

$$O4x := -S1x - U1x \qquad O4x = 3.062$$

$$O4y := -S1y - U1y \qquad O4y = -1.251$$

These check with Figure P5-2.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $\quad z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $\qquad z = 2.390 \qquad rp := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 2.769$

$\psi := atan2(S1x, S1y)$ $\quad \psi = 171.262\,deg$

$\phi := atan2(Z1x, Z1y)$ $\quad \phi = 99.095\,deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 39.430\,deg$

$\delta_p := \phi - \theta_3$ $\quad \delta_p = 59.666\,deg$

18. **DESIGN SUMMARY**

Link 1: $g = 3.946$

Link 2: $w = 1.680$

Link 3: $v = 3.054$

Link 4: $u = 0.892$

Coupler point: $rp = 2.390$ $\quad \delta_p = 59.666\,deg$

19. **VERIFICATION**: The calculated values of *g* (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-16

Statement: Design a linkage to carry the body in Figure P5-3 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0 \qquad P_{1y} := 0.0 \qquad P_{2x} := -0.907 \qquad P_{2y} := 0.0$

Angles made by the body in positions 1 and 2:

$\theta_{P1} := 111.8 \cdot deg \qquad \theta_{P2} := 191.1 \cdot deg$

Free choices for the **WZ** dyad :

$z := 1.500 \qquad \beta_2 := 44.0 \cdot deg \qquad \phi := -50.0 \cdot deg$

Free choices for the **US** dyad :

$s := 2.500 \qquad \gamma_2 := -55.0 \cdot deg \qquad \psi := 20.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array} \right.$$

Solution: See Figure P5-3 and Mathcad file P0516.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = -0.907$$

$$P_{21y} = 0.000$$

$$p_{21} := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p_{21} = 0.907$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = 79.300\, deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 180.000\, deg$$

4. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = 0.964 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = -1.149$$

$$A := cos(\beta_2) - 1 \qquad A = -0.281 \qquad D := sin(\alpha_2) \qquad D = 0.983$$

$B := sin(\beta_2)$ $B = 0.695$ $E := p_{21} \cdot cos(\delta_2)$ $E = -0.907$

$C := cos(\alpha_2) - 1$ $C = -0.814$ $F := p_{21} \cdot sin(\delta_2)$ $F = 0.000$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = -1.705$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = 2.490$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 3.018$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 124.405 \, deg$$

5. Solve for the US dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = 2.349$ $S_{1y} := s \cdot sin(\psi)$ $S_{1y} = 0.855$

$A := cos(\gamma_2) - 1$ $A = -0.426$ $D := sin(\alpha_2)$ $D = 0.983$

$B := sin(\gamma_2)$ $B = -0.819$ $E := p_{21} \cdot cos(\delta_2)$ $E = -0.907$

$C := cos(\alpha_2) - 1$ $C = -0.814$ $F := p_{21} \cdot sin(\delta_2)$ $F = 0.000$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = 0.625$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = 2.579$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 2.654$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = 76.373 \, deg$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $V_{1x} = -1.385$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $V_{1y} = -2.004$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 235.352 \, deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 2.436$

Link 1: $G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $G_{1x} = -3.715$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $G_{1y} = -2.094$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\qquad$ $\theta_1 = 209.404\,deg$

$g := \sqrt{\left(G_{1x}^{\;2} + G_{1y}^{\;2}\right)}$ $\qquad$ $g = 4.265$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad$ $\theta_{2i} = -84.999\,deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\qquad$ $\theta_{2f} = -40.999\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\qquad$ $\delta_p := \phi - \theta_3$

$r_p = 1.500$ $\qquad$ $\delta_p = -285.352\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\qquad$ $\rho_1 = 180.000\,deg$

$R_1 := \sqrt{\left(P_{1x}^{\;2} + P_{1y}^{\;2}\right)}$ $\qquad$ $R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\qquad$ $O_{2x} = 0.741$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\qquad$ $O_{2y} = -1.341$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad$ $O_{4x} = -2.975$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad$ $O_{4y} = -3.434$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\qquad$ $\theta_{rot} = 209.404\,deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left|\begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textbf{return } \text{"Grashof"} \ \ if\ SL < PQ \\ \textbf{return } \text{"Special Grashof"} \ \ if\ SL = PQ \\ \textbf{return } \text{"non-Grashof"} \ \ otherwise \end{array}\right.$$

$Condition(v, g, u, w) = \text{"non-Grashof"}$

12. **DESIGN SUMMARY**

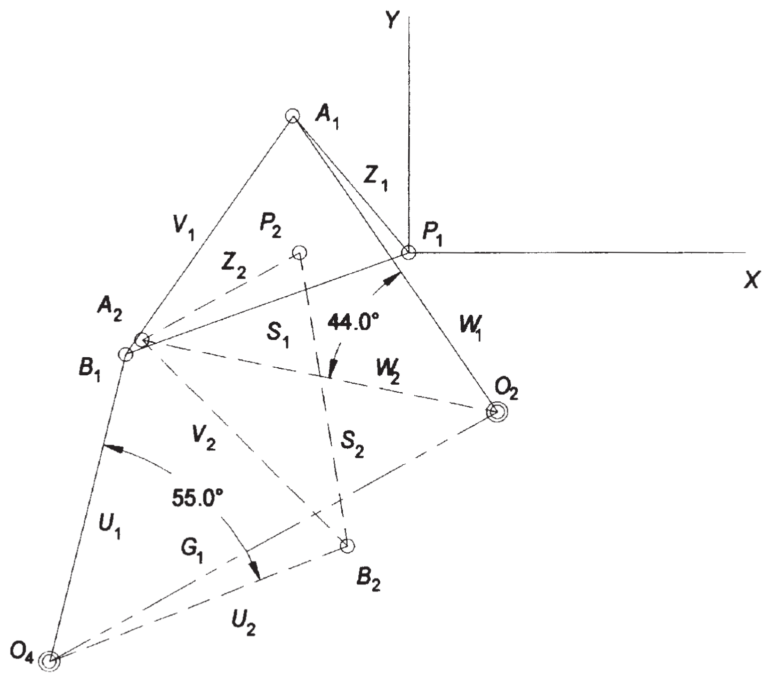
Link 2:	$w = 3.018$	$\theta = 124.405\,deg$
Link 3:	$v = 2.436$	$\theta_3 = 235.352\,deg$
Link 4:	$u = 2.654$	$\sigma = 76.373\,deg$
Link 1:	$g = 4.265$	$\theta_1 = 209.404\,deg$
Coupler:	$r_p = 1.500$	$\delta_p = -285.352\,deg$

Crank angles:

$$\theta_{2i} = -84.999\,deg$$

$$\theta_{2f} = -40.999\,deg$$

13. **Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.**



PROBLEM 5-17

Statement: Design a linkage to carry the body in Figure P5-3 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_2 and P_3 with respect to P_1:

$P_{2x} := -0.907 \quad P_{2y} := 0.0 \qquad P_{3x} := -1.447 \qquad P_{3y} := 0.0$

Angles made by the body in positions 1 and 2:

$\theta_{P2} := 191.1 \cdot deg \qquad \theta_{P3} := 237.4 \cdot deg$

Free choices for the **WZ** dyad :

$z := 2.000 \qquad \beta_2 := 66.0 \cdot deg \qquad \phi := -60.0 \cdot deg$

Free choices for the **US** dyad :

$s := 3.000 \qquad \gamma_2 := -44.0 \cdot deg \qquad \psi := 30.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-3 and Mathcad file P0517.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = -0.540$$

$$P_{21y} = 0.000$$

$$p21 := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p21 = 0.540$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P3} - \theta_{P2} \qquad \alpha_2 = 46.300\ deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 180.000\ deg$$

4. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = 1.000 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = -1.732$$

$$A := cos(\beta_2) - 1 \qquad A = -0.593 \qquad D := sin(\alpha_2) \qquad D = 0.723$$

$B := \sin(\beta_2)$ $\quad B = 0.914$ $\quad E := p_{21}\cdot\cos(\delta_2)$ $\quad E = -0.540$

$C := \cos(\alpha_2) - 1$ $\quad C = -0.309$ $\quad F := p_{21}\cdot\sin(\delta_2)$ $\quad F = 0.000$

$$W_{1x} := \frac{A\cdot(-C\cdot Z_{1x} + D\cdot Z_{1y} + E) + B\cdot(-C\cdot Z_{1y} - D\cdot Z_{1x} + F)}{-2\cdot A} \qquad W_{1x} = -0.227$$

$$W_{1y} := \frac{A\cdot(-C\cdot Z_{1y} - D\cdot Z_{1x} + F) + B\cdot(C\cdot Z_{1x} - D\cdot Z_{1y} - E)}{-2\cdot A} \qquad W_{1y} = 1.771$$

$$w := \sqrt{W_{1x}^{\;2} + W_{1y}^{\;2}} \qquad w = 1.786$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 97.314\,deg$$

5. Solve for the US dyad using equations 5.12.

$S_{1x} := s\cdot\cos(\psi)$ $\quad S_{1x} = 2.598$ $\quad S_{1y} := s\cdot\sin(\psi)$ $\quad S_{1y} = 1.500$

$A := \cos(\gamma_2) - 1$ $\quad A = -0.281$ $\quad D := \sin(\alpha_2)$ $\quad D = 0.723$

$B := \sin(\gamma_2)$ $\quad B = -0.695$ $\quad E := p_{21}\cdot\cos(\delta_2)$ $\quad E = -0.540$

$C := \cos(\alpha_2) - 1$ $\quad C = -0.309$ $\quad F := p_{21}\cdot\sin(\delta_2)$ $\quad F = 0.000$

$$U_{1x} := \frac{A\cdot(-C\cdot S_{1x} + D\cdot S_{1y} + E) + B\cdot(-C\cdot S_{1y} - D\cdot S_{1x} + F)}{-2\cdot A} \qquad U_{1x} = 1.077$$

$$U_{1y} := \frac{A\cdot(-C\cdot S_{1y} - D\cdot S_{1x} + F) + B\cdot(C\cdot S_{1x} - D\cdot S_{1y} - E)}{-2\cdot A} \qquad U_{1y} = 2.375$$

$$u := \sqrt{U_{1x}^{\;2} + U_{1y}^{\;2}} \qquad u = 2.608$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = 65.609\,deg$$

6. Solve for links 3 and 1 using the vector definitions of V and G.

Link 3: $V_{1x} := z\cdot\cos(\phi) - s\cdot\cos(\psi)$ $\quad V_{1x} = -1.598$

$V_{1y} := z\cdot\sin(\phi) - s\cdot\sin(\psi)$ $\quad V_{1y} = -3.232$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = 243.690\,deg$

$v := \sqrt{\left(V_{1x}^{\;2} + V_{1y}^{\;2}\right)}$ $\quad v = 3.606$

Link 1: $G_{1x} := w\cdot\cos(\theta) + v\cdot\cos(\theta_3) - u\cdot\cos(\sigma)$ $\quad G_{1x} = -2.902$

$G_{1y} := w\cdot\sin(\theta) + v\cdot\sin(\theta_3) - u\cdot\sin(\sigma)$ $\quad G_{1y} = -3.836$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = 232.889\,deg$

$$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)} \qquad g = 4.810$$

7. **Determine the initial and final values of the input crank with respect to the vector G.**

$$\theta_{2i} := \theta - \theta_1 \qquad \theta_{2i} = -135.575\ deg$$

$$\theta_{2f} := \theta_{2i} + \beta_2 \qquad \theta_{2f} = -69.575\ deg$$

8. **Define the coupler point with respect to point A and the vector V.**

$$r_p := z \qquad \delta_p := \phi - \theta_3$$

$$r_p = 2.000 \qquad \delta_p = -303.690\ deg$$

9. **Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.**

$$\rho_1 := atan2\left(P_{2x}, P_{2y}\right) \qquad \rho_1 = 180.000\ deg$$

$$R_1 := \sqrt{\left(P_{2x}^{\ 2} + P_{2y}^{\ 2}\right)} \qquad R_1 = 0.907$$

$$O_{2x} := R_1 \cdot cos\left(\rho_1\right) - z \cdot cos(\phi) - w \cdot cos(\theta) \qquad O_{2x} = -1.680$$

$$O_{2y} := R_1 \cdot sin\left(\rho_1\right) - z \cdot sin(\phi) - w \cdot sin(\theta) \qquad O_{2y} = -0.039$$

$$O_{4x} := R_1 \cdot cos\left(\rho_1\right) - s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = -4.582$$

$$O_{4y} := R_1 \cdot sin\left(\rho_1\right) - s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = -3.875$$

10. **Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.**

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 232.889\ deg$$

11. **Determine the Grashof condition.**

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return}\ \text{"Grashof"}\ \ \textit{if}\ SL < PQ \\ \textit{return}\ \text{"Special Grashof"}\ \ \textit{if}\ SL = PQ \\ \textit{return}\ \text{"non-Grashof"}\ \ \textit{otherwise} \end{array} \right.$$

$$Condition(w, g, u, v) = \text{"non-Grashof"}$$

12. **DESIGN SUMMARY**

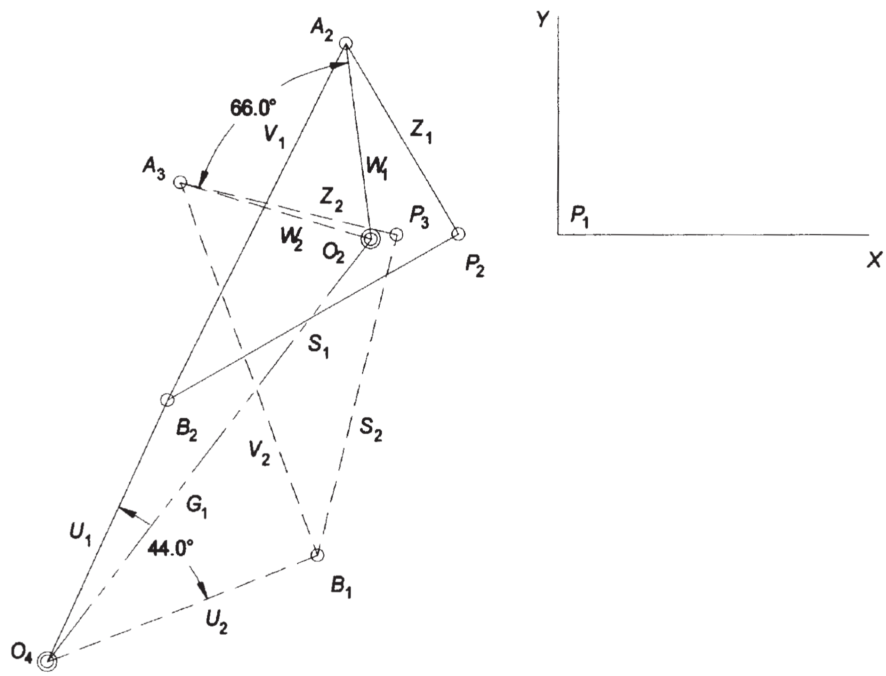
Link 2:	$w = 1.786$	$\theta = 97.314\ deg$
Link 3:	$v = 3.606$	$\theta_3 = 243.690\ deg$
Link 4:	$u = 2.608$	$\sigma = 65.609\ deg$
Link 1:	$g = 4.810$	$\theta_1 = 232.889\ deg$
Coupler:	$r_p = 2.000$	$\delta_p = -303.690\ deg$

Crank angles:

$$\theta_{2i} = -135.575\ deg$$

$$\theta_{2f} = -69.575\ deg$$

13. **Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.**



PROBLEM 5-18

Statement: Design a linkage to carry the body in Figure P5-3 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -0.907$ $\quad P_{2y} := 0.0$

$P_{3x} := -1.447$ $\quad P_{3y} := 0.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 111.8 \cdot deg$ $\quad \theta_{P2} := 191.1 \cdot deg$ $\quad \theta_{P3} := 237.4 \cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := 40.0 \cdot deg$ $\quad \beta_3 := 80.0 \cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := 20.0 \cdot deg$ $\quad \gamma_3 := 50.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\dfrac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\dfrac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P5-3 and Mathcad file P0518.

1. Determine the magnitudes and orientation of the position difference vectors.

$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2}$ $\quad p_{21} = 0.907$ $\quad \delta_2 := atan2(P_{2x}, P_{2y})$ $\quad \delta_2 = 180.000\, deg$

$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2}$ $\quad p_{31} = 1.447$ $\quad \delta_3 := atan2(P_{3x}, P_{3y})$ $\quad \delta_3 = 180.000\, deg$

2. Determine the angle changes of the coupler between precision points.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = 79.300\, deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = 125.600\, deg$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$A := \cos(\beta_2) - 1$ $\quad B := \sin(\beta_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F := \cos(\beta_3) - 1$

$G := \sin(\beta_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = 1.696$ $\quad W_{1y} = -0.038$ $\quad Z_{1x} = -0.396$ $\quad Z_{1y} = 0.872$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = -1.280\, deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 114.406\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $\quad w = 1.696$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^{\ 2} + Z_{1y}^{\ 2}\right)}$, $\quad z = 0.958$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = 1.483$ $\quad U_{1y} = 0.409$ $\quad S_{1x} = 0.048$ $\quad S_{1y} = 0.650$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 15.412\, deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 85.790\, deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $\quad u = 1.538$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^{\ 2} + S_{1y}^{\ 2}\right)}$, $\quad s = 0.652$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\qquad V_{1x} = -0.444$

$V_{1y} := Z_{1y} - S_{1y}$ $\qquad V_{1y} = 0.222$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\qquad \theta_3 = 153.426\, deg$

$$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)} \qquad v = 0.496$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $\qquad G_{1x} = -0.230$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y} \qquad G_{1y} = -0.225$$

$$\theta_1 := atan2\left(G_{1x}, G_{1y}\right) \qquad \theta_1 = 224.300\,deg$$

$$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)} \qquad g = 0.322$$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \qquad \theta_{2i} = -225.581\,deg$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \qquad \theta_{2f} = -145.581\,deg$$

7. Define the coupler point with respect to point *A* and the vector **V**.

$$r_p := z \qquad \delta_p := \phi - \theta_3$$

$$r_p = 0.958 \qquad \delta_p = -39.020\,deg$$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot cos(\phi) - w \cdot cos(\theta) \qquad O_{2x} = -1.300$$

$$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta) \qquad O_{2y} = -0.834$$

$$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = -1.530$$

$$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = -1.059$$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 224.300\,deg$$

10. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(v, w, u, g) = \text{"non-Grashof"}$$

11. **DESIGN SUMMARY**

Link 2:	$w = 1.696$	$\theta = -1.280\,deg$
Link 3:	$v = 0.496$	$\theta_3 = 153.426\,deg$
Link 4:	$u = 1.538$	$\sigma = 15.412\,deg$

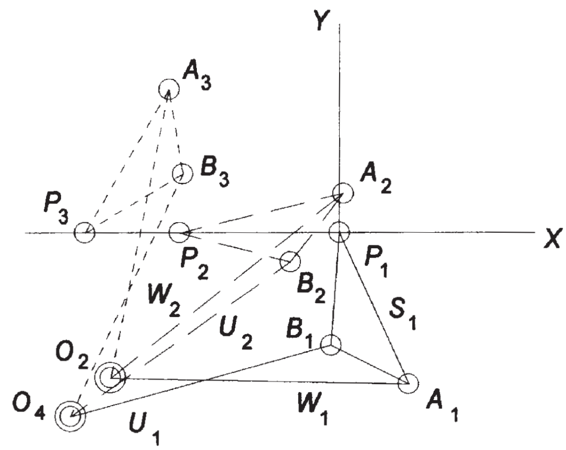
Link 1:	$g = 0.322$	$\theta_1 = 224.300\ deg$
Coupler:	$r_p = 0.958$	$\delta_p = -39.020\ deg$

Crank angles:

$\theta_{2i} = -225.581\ deg$

$\theta_{2f} = -145.581\ deg$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-19

Statement: Design a linkage to carry the body in Figure P5-3 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := -0.907$ $\quad P_{21y} := 0.0$ $\quad P_{31x} := -1.447$ $\quad P_{31y} := 0.0$

$O_{2x} := -1.788$ $\quad O_{2y} := -1.994$ $\quad O_{4x} := 0.212$ $\quad O_{4y} := -1.994$

Body angles: $\theta_{P1} := 111.8 \cdot deg$ $\quad \theta_{P2} := 191.1 \cdot deg$ $\quad \theta_{P3} := 237.4 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-3 and Mathcad file P0519.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = 79.300\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = 125.600\ deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 1.788$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 1.994$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 0.881$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 1.994$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 0.341$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 1.994$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 2.678$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 2.180$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 2.023$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 48.118\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 66.163\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 80.296\ deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 0.238$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = 1.150$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = -3.003$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = 1.701$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -2.508$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -0.133$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -11.912$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 4.666$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -7.307$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -3.048$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -4.666$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -2.671$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 5.293$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = 34.731$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 25.158$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = 125.600\, deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = 37.070\, deg$$

The first value is the same as α_3, so use the second value $\quad \beta_3 := \beta_{32}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 18.241\, deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 18.241\, deg$$

Since both values are the same, $\quad \beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = -0.212$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 1.994$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = -1.119$

$R_{2y} := R_{1y} + P_{21y}$ $R_{2y} = 1.994$

$R_{3x} := R_{1x} + P_{31x}$ $R_{3x} = -1.659$

$R_{3y} := R_{1y} + P_{31y}$ $R_{3y} = 1.994$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $R_1 = 2.005$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $R_2 = 2.287$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $R_3 = 2.594$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\zeta_1 = 96.069\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\zeta_2 = 119.300\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\zeta_3 = 129.760\ deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = -1.297$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 0.811$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = 0.161$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = 3.327$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -0.880$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 1.832$

$A_1 := -C_3^2 - C_4^2$ $A_1 = -11.096$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = 3.222$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -5.954$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = -4.186$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = -3.222$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -2.906$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 3.816$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 34.288$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = 25.658$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 125.600\, deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 41.699\, deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 31.159\, deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = 31.159\, deg$$

Since both values are the same, $\gamma_2 := \gamma_{21}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 0.907$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 180.000\, deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 1.447$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 180.000\, deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W1x = 1.943 \qquad W1y = 1.529 \qquad Z1x = -0.155 \qquad Z1y = 0.465$$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 2.472$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21}\cdot\cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31}\cdot\cos(\delta_3) \qquad M := p_{21}\cdot\sin(\delta_2) \qquad N := p_{31}\cdot\sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1}\cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -0.395$ $U1y = 2.460$ $S1x = 0.183$ $S1y = -0.466$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 2.491$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = -0.338$

$V1y := Z1y - S1y$ $V1y = 0.931$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 0.991$

$G1x := W1x + V1x - U1x$ $G1x = 2.000$

$G1y := W1y + V1y - U1y$ $G1y = -4.885 \times 10^{-15}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 2.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -1.788$

$O2y := -Z1y - W1y$ $O2y = -1.994$

$O4x := -S1x - U1x$ $O4x = 0.212$

$O4y := -S1y - U1y$ $O4y = -1.994$

These check with Figure P5-3.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 0.491$ $rp := z$

Angle *BAP* (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 0.501$

$\psi := atan2(S1x, S1y)$ $\psi = -68.558\,deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = 108.446\,deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 109.959\,deg$

$\delta_p := \phi - \theta_3$ $\delta_p = -1.513\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 2.000$	
Link 2:	$w = 2.472$	
Link 3:	$v = 0.991$	
Link 4:	$u = 2.491$	
Coupler point:	$rp = 0.491$	$\delta_p = -1.513\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-20

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-19 using an equation solver or any program language.

Given:

$P_{21x} := -0.907$	$P_{21y} := 0.0$	$P_{31x} := -1.447$	$P_{31y} := 0.0$
$O_{2x} := -1.788$	$O_{2y} := -1.994$	$O_{4x} := 0.212$	$O_{4y} := -1.994$
Body angles:	$\theta_{P1} := 111.8 \cdot deg$	$\theta_{P2} := 191.1 \cdot deg$	$\theta_{P3} := 237.4 \cdot deg$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 18.241 deg$ and $\gamma_2 := 31.159 deg$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := 79.3 deg$

Solution: See Figure P5-3 and Mathcad file P0520.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 0.907$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 180.000\, deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 1.447$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 180.000\, deg$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 125.6 \cdot deg \qquad \beta_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F(\beta_3) := \cos(\beta_3) - 1$$

$$G(\beta_3) := \sin(\beta_3) \qquad H(\alpha_3) := \cos(\alpha_3) - 1 \qquad K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1 \qquad W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$Z1x(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_3$ $\qquad Z1y(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_4$

3. Check this against the solutions in Problem 5-19:

$W1x(\alpha_3, 37.070 \cdot deg) = 1.943$ $\qquad W1y(\alpha_3, 37.070 \cdot deg) = 1.529$

$Z1x(\alpha_3, 37.070 \cdot deg) = -0.155$ $\qquad Z1y(\alpha_3, 37.070 \cdot deg) = 0.465$

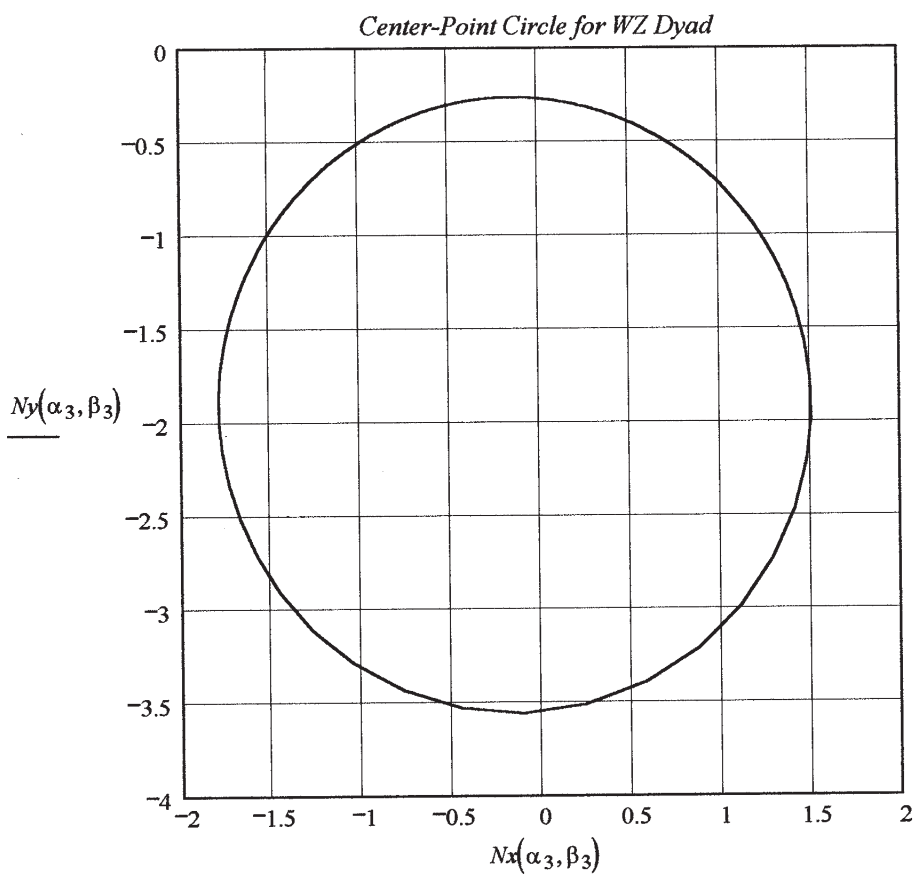
These are the same as the values calculated in Problem 5-19.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3,\beta_3) := -W1x(\alpha_3,\beta_3) - Z1x(\alpha_3,\beta_3)$$

$$Ny(\alpha_3,\beta_3) := -W1y(\alpha_3,\beta_3) - Z1y(\alpha_3,\beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.

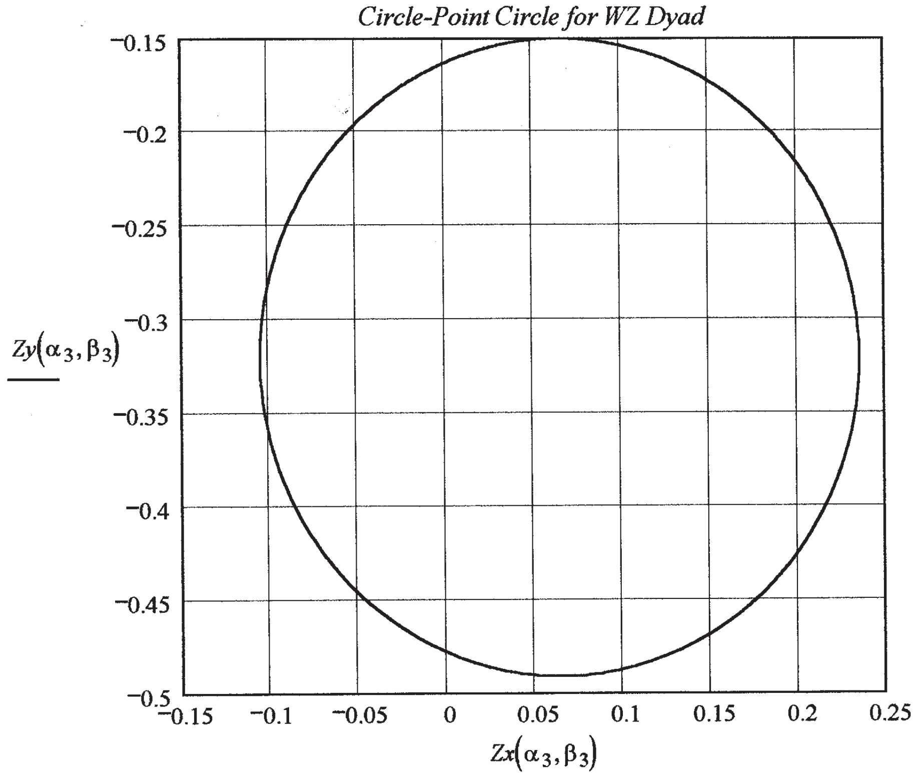


4. Form the vector **Z**, whose tip describes the center-point circle for the **WZ** dyad.

$\beta_3 := 37.070 \cdot deg$ $\qquad \alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$Zx(\alpha_3,\beta_3) := -Z1x(\alpha_3,\beta_3)$ $\qquad Zy(\alpha_3,\beta_3) := -Z1y(\alpha_3,\beta_3)$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. **Evaluate terms in the US coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.**

$\alpha_3 := 125.6 \cdot deg$ $\qquad \gamma_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$A := \cos(\gamma_2) - 1 \qquad B := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F(\gamma_3) := \cos(\gamma_3) - 1$

$G(\gamma_3) := \sin(\gamma_3) \qquad H(\alpha_3) := \cos(\alpha_3) - 1 \qquad K(\alpha_3) := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1 \qquad U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$

$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3 \qquad S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$

7. **Check this against the solutions in Problem 5-19:**

$U1x(\alpha_3, 41.699 \cdot deg) = -0.395 \qquad U1y(\alpha_3, 41.699 \cdot deg) = 2.460$

$S1x(\alpha_3, 41.699 \cdot deg) = 0.183$ $\qquad$ $S1y(\alpha_3, 41.699 \cdot deg) = -0.466$

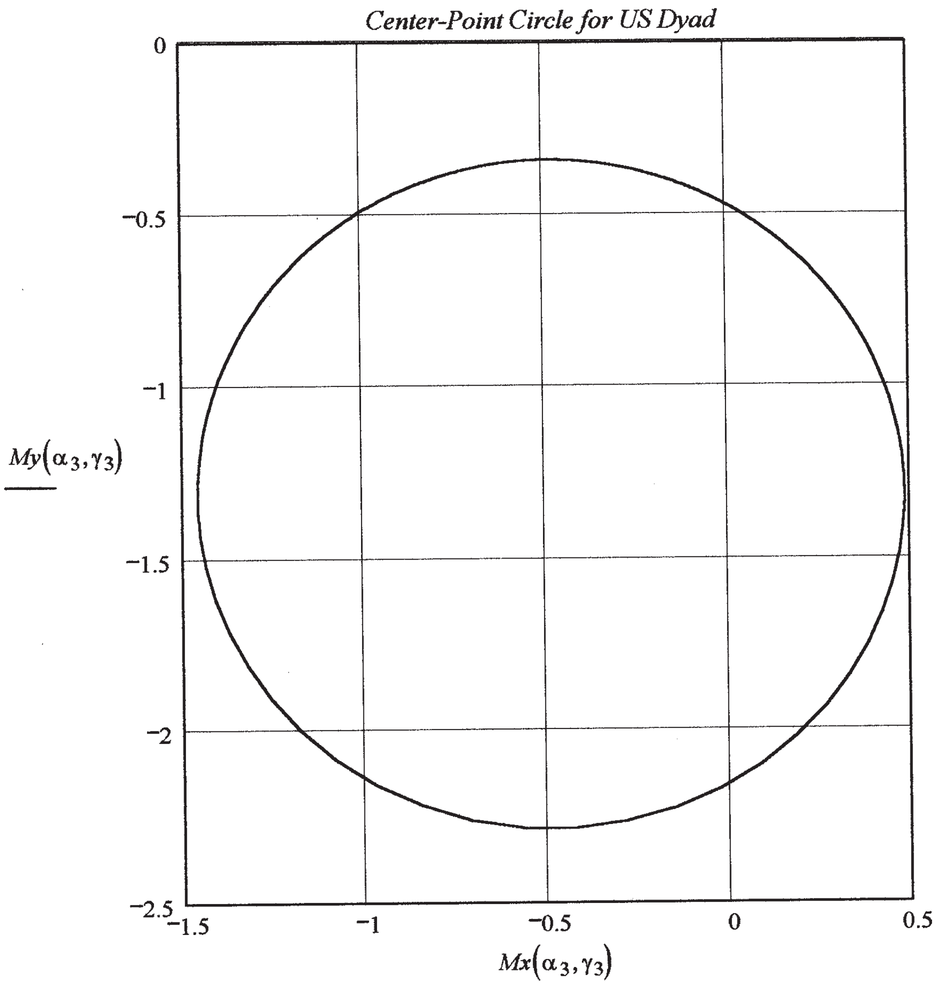
These are the same as the values calculated in Problem 5-19.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Form the vector **S**, whose tip describes the center-point circle for the **US** dyad.

$\gamma_3 := 41.699 \cdot deg$ $\qquad$ $\alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$ $\qquad$ $Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$

11. Plot the circle-point circle for the **WZ** dyad (see next page).

$S1x(\alpha_3, 41.699 \cdot deg) = 0.183$ $\qquad$ $S1y(\alpha_3, 41.699 \cdot deg) = -0.466$

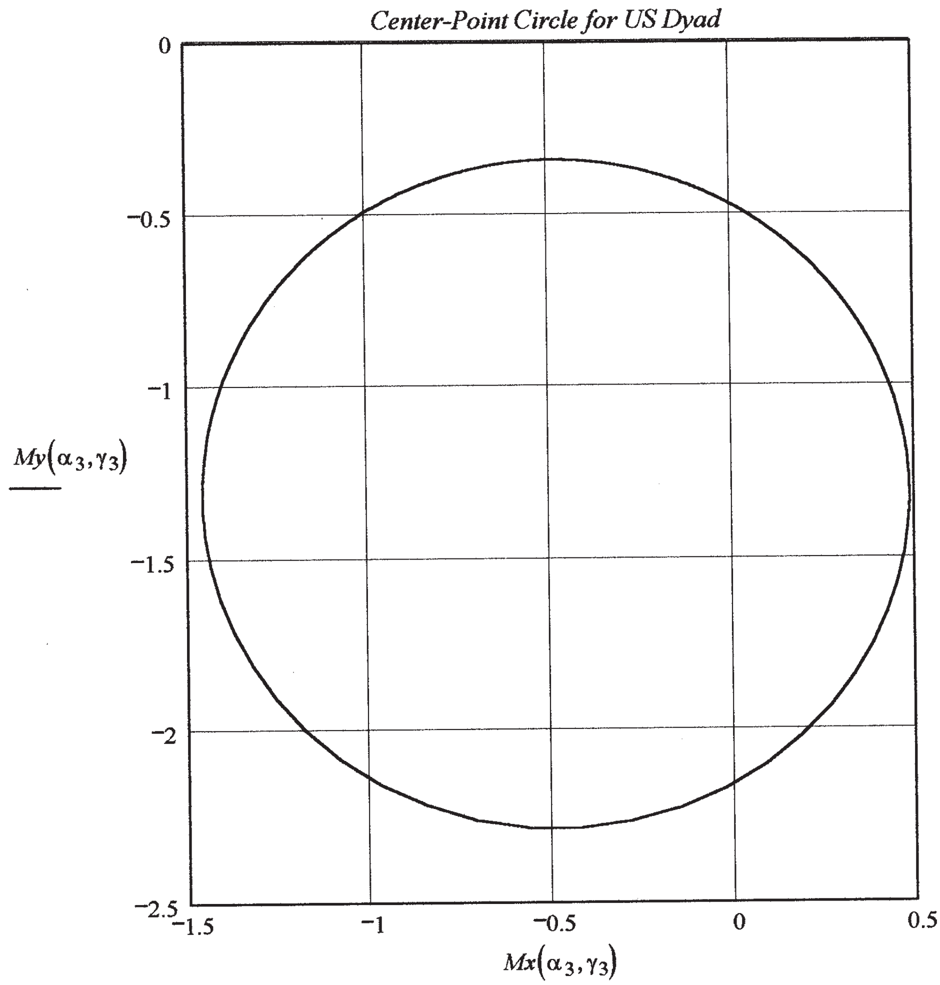
These are the same as the values calculated in Problem 5-19.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.

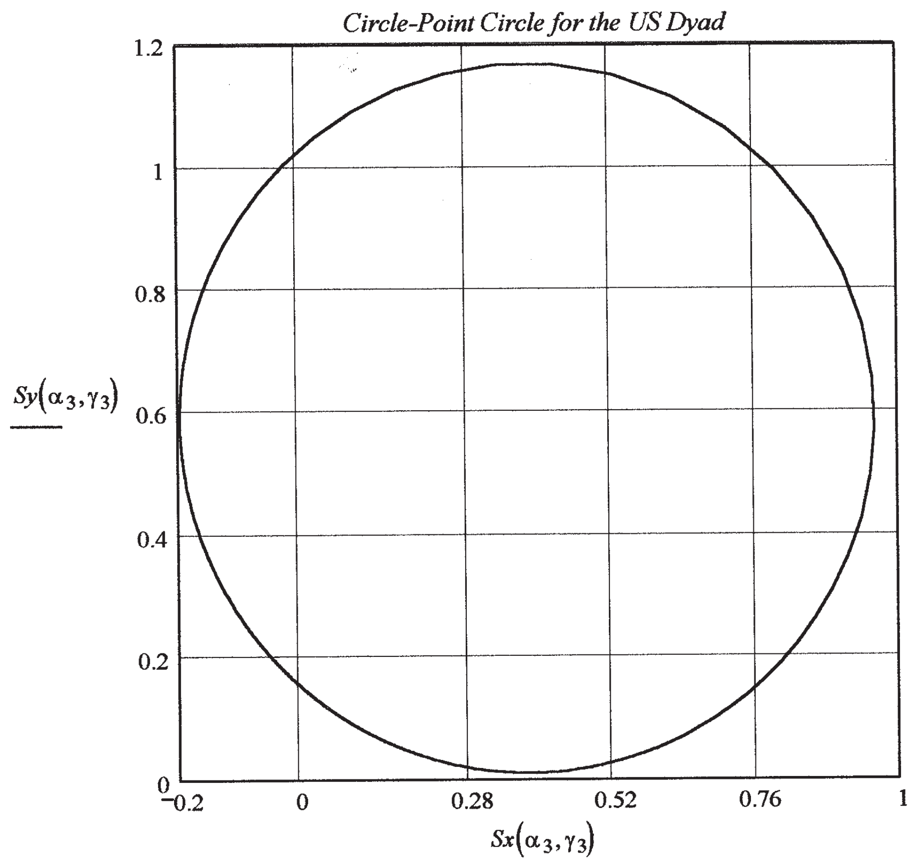


10. Form the vector **S**, whose tip describes the center-point circle for the **US** dyad.

$\gamma_3 := 41.699 \cdot deg$ $\qquad$ $\alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$ $\qquad$ $Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$

11. Plot the circle-point circle for the **WZ** dyad (see next page).


Circle-Point Circle for the US Dyad
1.2
1
0.8
0.6
0.4
0.2
0
$Sy(\alpha_3, \gamma_3)$
−0.2
0
0.28
0.52
0.76
1
$Sx(\alpha_3, \gamma_3)$

PROBLEM 5-21

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 1 to 2 without regard for the fixed pivots shown. Use points *A* and *B* for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 184.0$ $\quad P_{2y} := -17.0$

Angles made by the body in positions 1 and 2:

$\theta_{P1} := 90.0 \cdot deg$ $\quad \theta_{P2} := 45.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := 17.0$ $\quad A_{1y} := -43.0$ $\quad B_{1x} := 69.0$ $\quad B_{1y} := -43.0$

Free choice for the **WZ** dyad : $\beta_2 := -44.0 deg$

Free choice for the **US** dyad : $\gamma_2 := -55.0 deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-4 and Mathcad file P0521.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 184.000$$

$$P_{21y} = -17.000$$

$$p_{21} := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p_{21} = 184.784$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -45.000\,deg$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -5.279\,deg$$

4. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{\left(P_{1x} - A_{1x}\right)^2 + \left(P_{1y} - A_{1y}\right)^2} \qquad z = 46.239$$

$$s := \sqrt{\left(P_{1x} - B_{1x}\right)^2 + \left(P_{1y} - B_{1y}\right)^2} \qquad s = 81.302$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \qquad v = 52.000$$

$$\phi := acos\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) \qquad \phi = 111.571\ deg$$

$$\psi := \pi - acos\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \qquad \psi = 148.069\ deg$$

5. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = -17.000 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 43.000$$

$$A := cos(\beta_2) - 1 \qquad A = -0.281 \qquad D := sin(\alpha_2) \qquad D = -0.707$$

$$B := sin(\beta_2) \qquad B = -0.695 \qquad E := p_{21} \cdot cos(\delta_2) \qquad E = 184.000$$

$$C := cos(\alpha_2) - 1 \qquad C = -0.293 \qquad F := p_{21} \cdot sin(\delta_2) \qquad F = -17.000$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = -53.979$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = 192.131$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 199.570$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 105.693\ deg$$

6. Solve for the **US** dyad using equations 5.12.

$$S_{1x} := s \cdot cos(\psi) \qquad S_{1x} = -69.000 \qquad S_{1y} := s \cdot sin(\psi) \qquad S_{1y} = 43.000$$

$$A := cos(\gamma_2) - 1 \qquad A = -0.426 \qquad D := sin(\alpha_2) \qquad D = -0.707$$

$$B := sin(\gamma_2) \qquad B = -0.819 \qquad E := p_{21} \cdot cos(\delta_2) \qquad E = 184.000$$

$$C := cos(\alpha_2) - 1 \qquad C = -0.293 \qquad F := p_{21} \cdot sin(\delta_2) \qquad F = -17.000$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = -15.598$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = 154.713$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 155.497$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = 95.757\ deg$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $\quad V_{1x} = 52.000$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $\quad V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = 0.000\, deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $\quad v = 52.000$

Link 1: $G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $\quad G_{1x} = 13.619$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $\quad G_{1y} = 37.418$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = 70.000\, deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $\quad g = 39.819$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\quad \theta_{2i} = 35.692\, deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\quad \theta_{2f} = -8.308\, deg$

9. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\quad \delta_p := \phi - \theta_3$

$r_p = 46.239$ $\quad \delta_p = 111.571\, deg$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\quad \rho_1 = 180.000\, deg$

$R_1 := \sqrt{\left(P_{1x}^2 + P_{1y}^2\right)}$ $\quad R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\quad O_{2x} = 70.979$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\quad O_{2y} = -235.131$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\quad O_{4x} = 84.598$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\quad O_{4y} = -197.713$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\quad \theta_{rot} = 70.000\, deg$

12. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array} \right.$$

$$Condition(v, w, u, g) = \text{"non-Grashof"}$$

13. **DESIGN SUMMARY**

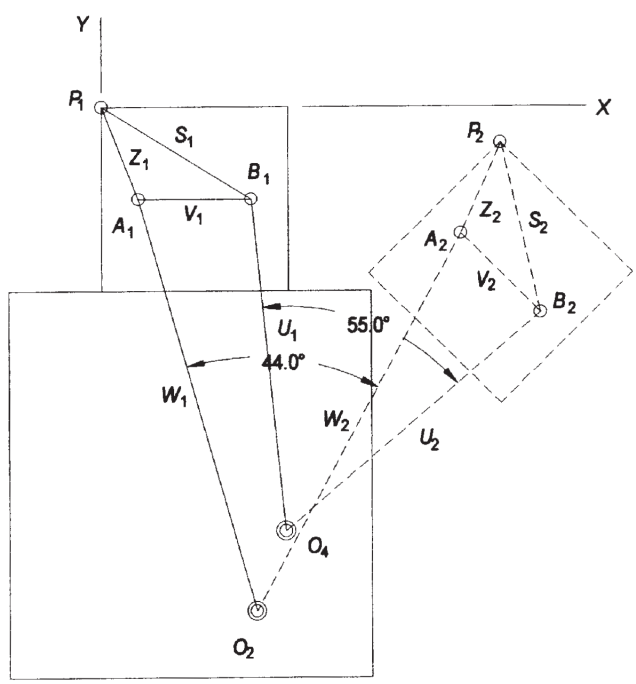
Link 2:	$w = 199.570$	$\theta = 105.693\,deg$
Link 3:	$v = 52.000$	$\theta_3 = 0.000\,deg$
Link 4:	$u = 155.497$	$\sigma = 95.757\,deg$
Link 1:	$g = 39.819$	$\theta_1 = 70.000\,deg$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571\,deg$

Crank angles:

$\theta_{2i} = 35.692\,deg$

$\theta_{2f} = -8.308\,deg$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-22

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 1 to 3 without regard for the fixed pivots shown. Use points *A* and *B* for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 and P_3 with respect to P_1:

$P_{1x} := 0.0 \qquad P_{1y} := 0.0 \qquad P_{3x} := 211.0 \qquad P_{3y} := -180.0$

Angles made by the body in positions 1 and 3:

$\theta_{P1} := 90.0 \cdot deg \qquad \theta_{P3} := 0.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := 17.0 \qquad A_{1y} := -43.0 \qquad B_{1x} := 69.0 \qquad B_{1y} := -43.0$

Free choice for the **WZ** dyad : $\beta_2 := -70.0 deg$

Free choice for the **US** dyad : $\gamma_2 := -95.0 deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi & if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi & if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P5-4 and Mathcad file P0522.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 211.000$$

$$P_{21y} = -180.000$$

$$p21 := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p21 = 277.346$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P3} - \theta_{P1} \qquad \alpha_2 = -90.000\, deg$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -40.467\, deg$$

4. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{\left(P_{1x} - A_{1x}\right)^2 + \left(P_{1y} - A_{1y}\right)^2} \qquad z = 46.239$$

$$s := \sqrt{\left(P_{1x} - B_{1x}\right)^2 + \left(P_{1y} - B_{1y}\right)^2} \qquad s = 81.302$$

$$v := \sqrt{\left(A_{1x} - B_{1x}\right)^2 + \left(A_{1y} - B_{1y}\right)^2} \qquad v = 52.000$$

$$\phi := acos\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) \qquad \phi = 111.571\ deg$$

$$\psi := \pi - acos\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \qquad \psi = 148.069\ deg$$

5. Solve for the **WZ** dyad using equations 5.8.

$Z_{1x} := z \cdot cos(\phi)$ $\quad Z_{1x} = -17.000$ $\quad Z_{1y} := z \cdot sin(\phi)$ $\quad Z_{1y} = 43.000$

$A := cos(\beta_2) - 1$ $\quad A = -0.658$ $\quad D := sin(\alpha_2)$ $\quad D = -1.000$

$B := sin(\beta_2)$ $\quad B = -0.940$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad E = 211.000$

$C := cos(\alpha_2) - 1$ $\quad C = -1.000$ $\quad F := p_{21} \cdot sin(\delta_2)$ $\quad F = -180.000$

$$W_{1x} := \frac{A \cdot \left(-C \cdot Z_{1x} + D \cdot Z_{1y} + E\right) + B \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right)}{-2 \cdot A} \qquad W_{1x} = 34.467$$

$$W_{1y} := \frac{A \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right) + B \cdot \left(C \cdot Z_{1x} - D \cdot Z_{1y} - E\right)}{-2 \cdot A} \qquad W_{1y} = 184.825$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 188.012$$

$$\theta := atan2\left(W_{1x}, W_{1y}\right) \qquad \theta = 79.436\ deg$$

6. Solve for the **US** dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $\quad S_{1x} = -69.000$ $\quad S_{1y} := s \cdot sin(\psi)$ $\quad S_{1y} = 43.000$

$A := cos(\gamma_2) - 1$ $\quad A = -1.087$ $\quad D := sin(\alpha_2)$ $\quad D = -1.000$

$B := sin(\gamma_2)$ $\quad B = -0.996$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad E = 211.000$

$C := cos(\alpha_2) - 1$ $\quad C = -1.000$ $\quad F := p_{21} \cdot sin(\delta_2)$ $\quad F = -180.000$

$$U_{1x} := \frac{A \cdot \left(-C \cdot S_{1x} + D \cdot S_{1y} + E\right) + B \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right)}{-2 \cdot A} \qquad U_{1x} = 44.882$$

$$U_{1y} := \frac{A \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right) + B \cdot \left(C \cdot S_{1x} - D \cdot S_{1y} - E\right)}{-2 \cdot A} \qquad U_{1y} = 148.358$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 154.999$$

$$\sigma := atan2\left(U_{1x}, U_{1y}\right) \qquad \sigma = 73.168\ deg$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3:

$V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $\quad V_{1x} = 52.000$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $\quad V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = 0.000\, deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $\quad v = 52.000$

Link 1:

$G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $\quad G_{1x} = 41.585$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $\quad G_{1y} = 36.467$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = 41.248\, deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $\quad g = 55.310$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\quad \theta_{2i} = 38.188\, deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\quad \theta_{2f} = -31.812\, deg$

9. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\quad \delta_p := \phi - \theta_3$

$r_p = 46.239$ $\quad \delta_p = 111.571\, deg$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\quad \rho_1 = 180.000\, deg$

$R_1 := \sqrt{\left(P_{1x}^2 + P_{1y}^2\right)}$ $\quad R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $\quad O_{2x} = -17.467$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $\quad O_{2y} = -227.825$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\quad O_{4x} = 24.118$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\quad O_{4y} = -191.358$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\quad \theta_{rot} = 41.248\, deg$

12. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(v,w,u,g) = \text{"non-Grashof"}$$

13. **DESIGN SUMMARY**

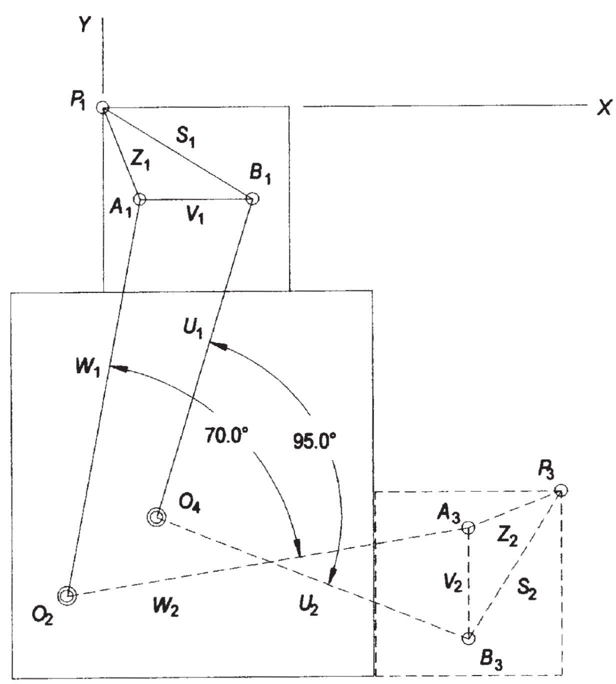
Link 2:	$w = 188.012$	$\theta = 79.436\ deg$
Link 3:	$v = 52.000$	$\theta_3 = 0.000\ deg$
Link 4:	$u = 154.999$	$\sigma = 73.168\ deg$
Link 1:	$g = 55.310$	$\theta_1 = 41.248\ deg$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571\ deg$

Crank angles:

$$\theta_{2i} = 38.188\ deg$$

$$\theta_{2f} = -31.812\ deg$$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-23

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 2 to 3 without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_2 and P_3 with respect to P_1:

$$P_{2x} := 184.0 \qquad P_{2y} := -17.0 \qquad P_{3x} := 211.0 \qquad P_{3y} := -180.0$$

Angles made by the body in positions 1 and 3:

$$\theta_{P2} := 45.0 \cdot deg \qquad \theta_{P3} := 0.0 \cdot deg$$

Coordinates of the points A_1 and B_1 with respect to P_1:

$$A_{1x} := 17.0 \qquad A_{1y} := -43.0 \qquad B_{1x} := 69.0 \qquad B_{1y} := -43.0$$

Free choice for the **WZ** dyad: $\beta_2 := -60.0 deg$

Free choice for the **US** dyad: $\gamma_2 := -45.0 deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \land y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \land y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-4 and Mathcad file P0523.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 27.000$$

$$P_{21y} = -163.000$$

$$p21 := \sqrt{\left(P_{21x}^{\ 2} + P_{21y}^{\ 2}\right)} \qquad p21 = 165.221$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P3} - \theta_{P2} \qquad \alpha_2 = -45.000\, deg$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -80.595\, deg$$

4. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(0.0 - A_{1x})^2 + (0.0 - A_{1y})^2} \qquad z = 46.239$$

$$s := \sqrt{(0.0 - B_{1x})^2 + (0.0 - B_{1y})^2} \qquad s = 81.302$$

$$v := \sqrt{\left(A_{1x} - B_{1x}\right)^2 + \left(A_{1y} - B_{1y}\right)^2} \qquad v = 52.000$$

$$\phi := acos\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) - 45 \cdot deg \qquad \phi = 66.571\ deg$$

$$\psi := \pi - acos\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) - 45 \cdot deg \qquad \psi = 103.069\ deg$$

5. Solve for the **WZ** dyad using equations 5.8.

$Z_{1x} := z \cdot cos(\phi)$ $Z_{1x} = 18.385$ $Z_{1y} := z \cdot sin(\phi)$ $Z_{1y} = 42.426$

$A := cos(\beta_2) - 1$ $A = -0.500$ $D := sin(\alpha_2)$ $D = -0.707$

$B := sin(\beta_2)$ $B = -0.866$ $E := p_{21} \cdot cos(\delta_2)$ $E = 27.000$

$C := cos(\alpha_2) - 1$ $C = -0.293$ $F := p_{21} \cdot sin(\delta_2)$ $F = -163.000$

$$W_{1x} := \frac{A \cdot \left(-C \cdot Z_{1x} + D \cdot Z_{1y} + E\right) + B \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right)}{-2 \cdot A} \qquad W_{1x} = 117.950$$

$$W_{1y} := \frac{A \cdot \left(-C \cdot Z_{1y} - D \cdot Z_{1x} + F\right) + B \cdot \left(C \cdot Z_{1x} - D \cdot Z_{1y} - E\right)}{-2 \cdot A} \qquad W_{1y} = 70.852$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 137.594$$

$$\theta := atan2\left(W_{1x}, W_{1y}\right) \qquad \theta = 30.993\ deg$$

6. Solve for the **US** dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = -18.385$ $S_{1y} := s \cdot sin(\psi)$ $S_{1y} = 79.196$

$A := cos(\gamma_2) - 1$ $A = -0.293$ $D := sin(\alpha_2)$ $D = -0.707$

$B := sin(\gamma_2)$ $B = -0.707$ $E := p_{21} \cdot cos(\delta_2)$ $E = 27.000$

$C := cos(\alpha_2) - 1$ $C = -0.293$ $F := p_{21} \cdot sin(\delta_2)$ $F = -163.000$

$$U_{1x} := \frac{A \cdot \left(-C \cdot S_{1x} + D \cdot S_{1y} + E\right) + B \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right)}{-2 \cdot A} \qquad U_{1x} = 201.643$$

$$U_{1y} := \frac{A \cdot \left(-C \cdot S_{1y} - D \cdot S_{1x} + F\right) + B \cdot \left(C \cdot S_{1x} - D \cdot S_{1y} - E\right)}{-2 \cdot A} \qquad U_{1y} = 34.896$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 204.640$$

$$\sigma := atan2\left(U_{1x}, U_{1y}\right) \qquad \sigma = 9.818\ deg$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$ $\qquad V_{1x} = 36.770$

$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$ $\qquad V_{1y} = -36.770$

$\theta_3 := \mathrm{atan2}(V_{1x}, V_{1y})$ $\qquad \theta_3 = -45.000\ deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $\qquad v = 52.000$

Link 1: $G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$ $\qquad G_{1x} = -46.924$

$G_{1y} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma)$ $\qquad G_{1y} = -0.813$

$\theta_1 := \mathrm{atan2}(G_{1x}, G_{1y})$ $\qquad \theta_1 = 180.993\ deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $\qquad g = 46.931$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad \theta_{2i} = -150.000\ deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\qquad \theta_{2f} = -210.000\ deg$

9. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\qquad \delta_p := \phi - \theta_3$

$r_p = 46.239$ $\qquad \delta_p = 111.571\ deg$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := \mathrm{atan2}(P_{2x}, P_{2y})$ $\qquad \rho_1 = -5.279\ deg$

$R_1 := \sqrt{\left(P_{2x}^{\ 2} + P_{2y}^{\ 2}\right)}$ $\qquad R_1 = 184.784$

$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$ $\qquad O_{2x} = 47.665$

$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$ $\qquad O_{2y} = -130.278$

$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$ $\qquad O_{4x} = 0.742$

$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$ $\qquad O_{4y} = -131.092$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := \mathrm{atan2}\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\qquad \theta_{rot} = 180.993\ deg$

12. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array} \right.$$

$$Condition(v,u,w,g) = \text{"non-Grashof"}$$

13. **DESIGN SUMMARY**

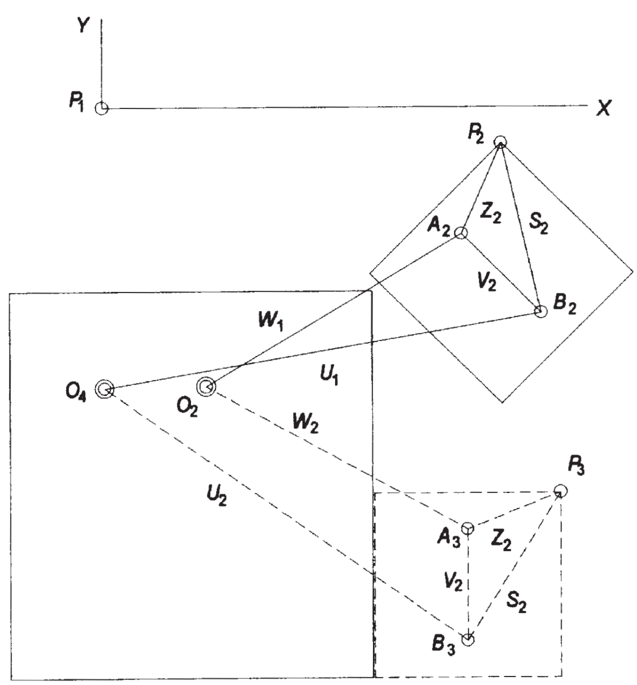
Link 2:	$w = 137.594$	$\theta = 30.993\ deg$
Link 3:	$v = 52.000$	$\theta_3 = -45.000\ deg$
Link 4:	$u = 204.640$	$\sigma = 9.818\ deg$
Link 1:	$g = 46.931$	$\theta_1 = 180.993\ deg$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571\ deg$

Crank angles:

$\theta_{2i} = -150.000\ deg$

$\theta_{2f} = -210.000\ deg$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-24

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 184.0$ $\quad P_{2y} := -17.0$

$P_{3x} := 211.0$ $\quad P_{3y} := -180.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 90.0 \cdot deg$ $\quad \theta_{P2} := 45.0 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := -80.0 \cdot deg$ $\quad \beta_3 := -160.0 \cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -80.0 \cdot deg$ $\quad \gamma_3 := -170.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5 \cdot \pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P5-4 and Mathcad file P0524.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 184.784 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = -5.279\, deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 277.346 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = -40.467\, deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -45.000\, deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -90.000\, deg$$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = -51.854$ $\quad W_{1y} = 109.176$ $\quad Z_{1x} = -43.555$ $\quad Z_{1y} = 29.523$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 115.406\,deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 145.869\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}{}^2 + W_{1y}{}^2\right)}$, $\quad w = 120.864$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}{}^2 + Z_{1y}{}^2\right)}$, $\quad z = 52.618$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $\quad B' := sin(\gamma_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -35.056$ $\quad U_{1y} = 94.023$ $\quad S_{1x} = -62.812$ $\quad S_{1y} = 62.282$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 110.448\,deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 135.242\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}{}^2 + U_{1y}{}^2\right)}$, $\quad u = 100.345$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}{}^2 + S_{1y}{}^2\right)}$, $\quad s = 88.456$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\quad V_{1x} = 19.256$

$V_{1y} := Z_{1y} - S_{1y}$ $\quad V_{1y} = -32.759$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = -59.552\,deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $\quad v = 38.000$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $\quad G_{1x} = 2.458$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $\quad G_{1y} = -17.606$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\quad \theta_1 = -82.052\,deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $\quad g = 17.777$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\quad \theta_{2i} = 197.458\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\quad \theta_{2f} = 37.458\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\quad \delta_p := \phi - \theta_3$

$r_p = 52.618$ $\quad \delta_p = 205.422\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z \cdot cos(\phi) - w \cdot cos(\theta)$ $\quad O_{2x} = 95.410$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $\quad O_{2y} = -138.699$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\quad O_{4x} = 97.868$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\quad O_{4y} = -156.305$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\quad \theta_{rot} = -82.052\,deg$

11. Determine the Grashof condition.

$Condition(S, L, P, Q) :=$
| $SL \leftarrow S + L$
| $PQ \leftarrow P + Q$
| *return* "Grashof" *if* $SL < PQ$
| *return* "Special Grashof" *if* $SL = PQ$
| *return* "non-Grashof" *otherwise*

$Condition(g, u, v, w) =$ "Grashof"

12. **DESIGN SUMMARY**

Link 2:	$w = 120.864$	$\theta = 115.406\,deg$
Link 3:	$v = 38.000$	$\theta_3 = -59.552\,deg$
Link 4:	$u = 100.345$	$\sigma = 110.448\,deg$

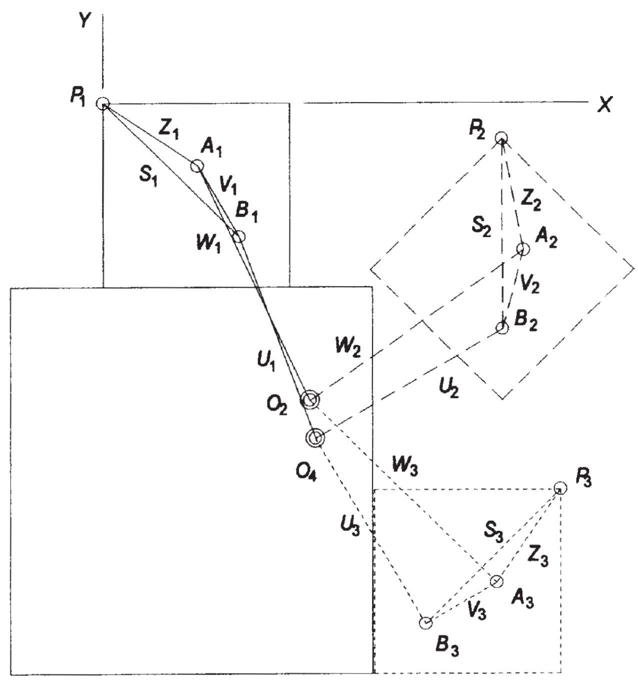
Link 1:	$g = 17.777$	$\theta_1 = -82.052\ deg$
Coupler:	$r_p = 52.618$	$\delta_p = 205.422\ deg$

Crank angles:

$$\theta_{2i} = 197.458\ deg$$

$$\theta_{2f} = 37.458\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-25

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points *A* and *B* for your attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 184.0$ $\quad P_{2y} := -17.0$

$P_{3x} := 211.0$ $\quad P_{3y} := -180.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 90.0 \cdot deg$ $\quad \theta_{P2} := 45.0 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := 17.0$ $\quad A_{1y} := -43.0$ $\quad B_{1x} := 69.0$ $\quad B_{1y} := -43.0$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P5-4 and Mathcad file P0525.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 184.784 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = -5.279\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 277.346 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = -40.467\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -45.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -90.000\,deg$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \qquad z = 46.239$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \qquad s = 81.302$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \qquad v = 52.000$$

$$\phi := acos\left(\frac{v^2 + z^2 - s^2}{2\cdot v\cdot z}\right) \qquad \phi = 111.571\,deg$$

$$\psi := \pi - acos\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \qquad \psi = 148.069\,deg$$

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = -17.000$$

$$Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 43.000$$

$$S_{1x} := s \cdot cos(\psi) \qquad S_{1x} = -69.000$$

$$S_{1y} := s \cdot sin(\psi) \qquad S_{1y} = 43.000$$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -50 \quad W_{1y} := 200 \quad \beta_2 := -80 \cdot deg \quad \beta_3 := -160 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) \ldots + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) \ldots + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) \ldots + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) \ldots + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -86.887\,deg \qquad \beta_3 = -165.399\,deg$$

The components of the **W** vector are:

$$W_{1x} = -65.636 \qquad W_{1y} = 86.672 \qquad \theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 127.136\,deg$$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $\quad w = 108.720$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points A and B are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -30 \quad U_{1y} := 100 \quad \gamma_2 := -80 \cdot deg \quad \gamma_3 := -160 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) \ldots + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) \ldots + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y}\cdot(\cos(\gamma_2)-1)+U_{1x}\cdot\sin(\gamma_2)\ ...\ +S_{1y}\cdot(\cos(\alpha_2)-1)+S_{1x}\cdot\sin(\alpha_2) = p_{21}\cdot\sin(\delta_2)$$

$$U_{1y}\cdot(\cos(\gamma_3)-1)+U_{1x}\cdot\sin(\gamma_3)\ ...\ +S_{1y}\cdot(\cos(\alpha_3)-1)+S_{1x}\cdot\sin(\alpha_3) = p_{31}\cdot\sin(\delta_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = -76.700\ deg$ $\gamma_3 = -161.878\ deg$

The components of the **U** vector are:

$U_{1x} = -33.074$ $U_{1y} = 110.894$ $\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 106.607\ deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}{}^2 + U_{1y}{}^2\right)}$, $u = 115.721$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 52.000$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 0.000\ deg$

$v := \sqrt{\left(V_{1x}{}^2 + V_{1y}{}^2\right)}$ $v = 52.000$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 19.438$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -24.222$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = -51.253\ deg$

$g := \sqrt{\left(G_{1x}{}^2 + G_{1y}{}^2\right)}$ $g = 31.057$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 178.389\ deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 12.990\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 46.239$ $\delta_p = 111.571\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta)$ $O_{2x} = 82.636$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \qquad O_{2y} = -129.672$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \qquad O_{4x} = 102.074$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \qquad O_{4y} = -153.894$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = -51.253\,deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(g, u, v, w) = \text{"Grashof"}$$

12. **DESIGN SUMMARY**

Link 2:	$w = 108.720$	$\theta = 127.136\,deg$
Link 3:	$v = 52.000$	$\theta_3 = 0.000\,deg$
Link 4:	$u = 115.721$	$\sigma = 106.607\,deg$
Link 1:	$g = 31.057$	$\theta_1 = -51.253\,deg$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571\,deg$

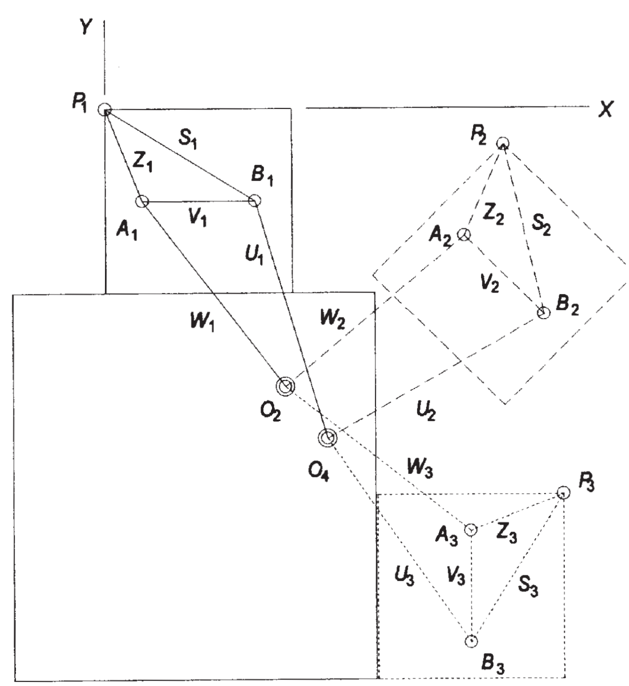
Crank angles:

$$\theta_{2i} = 178.389\,deg$$

$$\theta_{2f} = 12.990\,deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2.



PROBLEM 5-26

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given:

$P_{21x} := 184.0$ $\quad P_{21y} := -17.0$ $\quad P_{31x} := 211.0$ $\quad P_{31y} := -180.0$

$O_{2x} := 86.0$ $\quad O_{2y} := -132.0$ $\quad O_{4x} := 104.0$ $\quad O_{4y} := -155.0$

Body angles: $\theta_{P1} := 90\cdot deg$ $\quad \theta_{P2} := 45\cdot deg$ $\quad \theta_{P3} := 0\cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-4 and Mathcad file P0526.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = -45.000\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -90.000\ deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = -86.000$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 132.000$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 98.000$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 115.000$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 125.000$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = -48.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 157.544$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 151.093$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 133.899$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 123.085\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 49.563\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = -21.007\ deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = -60.553$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = -24.329$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = 7.000$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -134.000$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -65.473$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -39.149$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $\quad A_1 = -1.800 \times 10^4$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = -9.047 \times 10^3$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -4.788 \times 10^3$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = 7.944 \times 10^3$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = 9.047 \times 10^3$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -3.684 \times 10^3$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = -5.423 \times 10^7$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -7.136 \times 10^7$

$$K_3 := \frac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2} \qquad K_3 = 7.136 \times 10^7$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{31} = -90.000\,deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{32} = -164.466\,deg$$

The first value is the same as α_3, so use the second value $\quad \beta_3 := \beta_{32}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 85.240\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = -85.240\,deg$$

Since β_2 is not in the first quadrant, $\quad \beta_2 := \beta_{22}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = -104.000$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 155.000$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 80.000$

$R_{2y} := R_{1y} + P_{21y}$ $\quad R_{2y} = 138.000$

$R_{3x} := R_{1x} + P_{31x}$ $\quad R_{3x} = 107.000$

$R_{3y} := R_{1y} + P_{31y}$ $\quad R_{3y} = -25.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 186.657$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 159.512$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 109.882$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\quad \zeta_1 = 123.860\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\quad \zeta_2 = 59.899\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\quad \zeta_3 = -13.151\ deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = -80.017$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = -13.338$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = 48.000$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -129.000$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -43.938$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -45.141$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -1.894 \times 10^4$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = -7.835 \times 10^3$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -3.714 \times 10^3$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = 9.682 \times 10^3$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = 7.835 \times 10^3$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -5.561 \times 10^3$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = -5.520 \times 10^7$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -7.953 \times 10^7$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\quad K_3 = 7.953 \times 10^7$

$$\gamma_{31} := 2\cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = -90.000\,deg$$

$$\gamma_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -159.525\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5\cdot sin(\gamma_3) + A_3\cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 75.253\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3\cdot sin(\gamma_3) + A_2\cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -75.253\,deg$$

Since γ_2 is not in the first quadrant , $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 184.784$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -5.279\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 277.346$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = -40.467\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A := cos(\beta_2) - 1$ $\quad B := sin(\beta_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21}\cdot cos(\delta_2)$ $\quad F := cos(\beta_3) - 1$

$G := sin(\beta_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31}\cdot cos(\delta_3)$ $\quad M := p_{21}\cdot sin(\delta_2)$ $\quad N := p_{31}\cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1}\cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W1x = -62.394$ $\quad W1y = 91.663$ $\quad Z1x = -23.606$ $\quad Z1y = 40.337$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 110.884$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -29.920$ $U1y = 116.933$ $S1x = -74.080$ $S1y = 38.067$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 120.700$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 50.474$

$V1y := Z1y - S1y$ $V1y = 2.270$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 50.525$

$G1x := W1x + V1x - U1x$ $G1x = 18.000$

$G1y := W1y + V1y - U1y$ $G1y = -23.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 29.206$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = 86.000$

$O2y := -Z1y - W1y$ $O2y = -132.000$

$O4x := -S1x - U1x$ $O4x = 104.000$

$O4y := -S1y - U1y$ $O4y = -155.000$

These check with Figure P5-4.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 46.736$ $rp := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\qquad s = 83.288$

$\psi := atan2(S1x, S1y)$ $\qquad \psi = 152.803\,deg$

$\phi := atan2(Z1x, Z1y)$ $\qquad \phi = 120.337\,deg$

$\theta_3 := atan2(z\cdot cos(\phi) - s\cdot cos(\psi), z\cdot sin(\phi) - s\cdot sin(\psi))$

$\theta_3 = 2.575\,deg$

$\delta_p := \phi - \theta_3$ $\qquad \delta_p = 117.762\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 29.206$
Link 2:	$w = 110.884$
Link 3:	$v = 50.525$
Link 4:	$u = 120.700$
Coupler point:	$rp = 46.736$ $\qquad \delta_p = 117.762\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-27

Statement: Design a fourbar linkage to carry the box in Figure P5-5 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 421.0$ $\quad P_{2y} := 963.0$

$P_{3x} := 184.0$ $\quad P_{3y} := 1400.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 0.0\cdot deg$ $\quad \theta_{P2} := 27.0\cdot deg$ $\quad \theta_{P3} := 88.0\cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := -50.0\cdot deg$ $\quad \beta_3 := -100.0\cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -50.0\cdot deg$ $\quad \gamma_3 := -80.0\cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-5 and Mathcad file P0527.

1. Determine the magnitudes and orientation of the position difference vectors.

$p_{21} := \sqrt{P_{2x}^{\ 2} + P_{2y}^{\ 2}}$ $\quad p_{21} = 1.051 \times 10^3$ $\quad \delta_2 := atan2(P_{2x}, P_{2y})$ $\quad \delta_2 = 66.386\,deg$

$p_{31} := \sqrt{P_{3x}^{\ 2} + P_{3y}^{\ 2}}$ $\quad p_{31} = 1.412 \times 10^3$ $\quad \delta_3 := atan2(P_{3x}, P_{3y})$ $\quad \delta_3 = 82.513\,deg$

2. Determine the angle changes of the coupler between precision points.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = 27.000\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = 88.000\,deg$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$A := cos(\beta_2) - 1$ $\quad B := sin(\beta_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21}\cdot cos(\delta_2)$ $\quad F := cos(\beta_3) - 1$

$G := sin(\beta_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = -784.602$ $\quad W_{1y} = 362.803$ $\quad Z_{1x} = 1.092 \times 10^3$ $\quad Z_{1y} = 39.947$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 155.184\,deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 2.094\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}{}^2 + W_{1y}{}^2\right)}$, $\quad w = 864.423$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}{}^2 + Z_{1y}{}^2\right)}$, $\quad z = 1093.069$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $\quad B' := sin(\gamma_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -924.539$ $\quad U_{1y} = 281.738$ $\quad S_{1x} = 802.719$ $\quad S_{1y} = 82.798$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 163.052\,deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 5.889\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}{}^2 + U_{1y}{}^2\right)}$, $\quad u = 966.514$

The length of vector S is: $s := \sqrt{\left(S_{1x}{}^2 + S_{1y}{}^2\right)}$, $\quad s = 806.978$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\quad V_{1x} = 289.619$

$V_{1y} := Z_{1y} - S_{1y}$ $\quad V_{1y} = -42.852$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = -8.416\,deg$

$v := \sqrt{\left(V_{1x}^{\;2} + V_{1y}^{\;2}\right)}$ $v = 292.772$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 429.556$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 38.214$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 5.084\,deg$

$g := \sqrt{\left(G_{1x}^{\;2} + G_{1y}^{\;2}\right)}$ $g = 431.252$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 150.100\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 50.100\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 1093.069$ $\delta_p = 10.511\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot cos(\phi) - w\cdot cos(\theta)$ $O_{2x} = -307.736$

$O_{2y} := -z\cdot sin(\phi) - w\cdot sin(\theta)$ $O_{2y} = -402.750$

$O_{4x} := -s\cdot cos(\psi) - u\cdot cos(\sigma)$ $O_{4x} = 121.820$

$O_{4y} := -s\cdot sin(\psi) - u\cdot sin(\sigma)$ $O_{4y} = -364.536$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\theta_{rot} = 5.084\,deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(v, u, g, w) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

Link 2:	$w = 864.423$	$\theta = 155.184\,deg$
Link 3:	$v = 292.772$	$\theta_3 = -8.416\,deg$

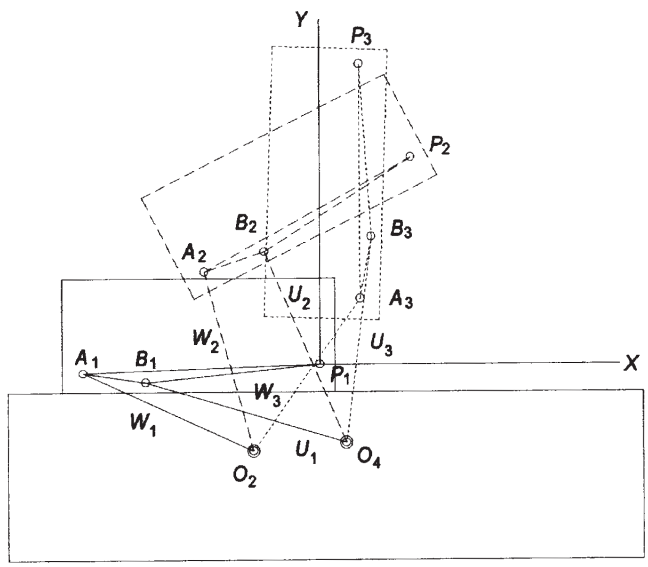
Link 4:	$u = 966.514$	$\sigma = 163.052\ deg$
Link 1:	$g = 431.252$	$\theta_1 = 5.084\ deg$
Coupler:	$r_p = 1093.069$	$\delta_p = 10.511\ deg$

Crank angles:

$$\theta_{2i} = 150.100\ deg$$

$$\theta_{2f} = 50.100\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-28

Statement: Design a fourbar linkage to carry the box in Figure P5-5 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $P_{1y} := 0.0$ $P_{2x} := 421.0$ $P_{2y} := 963.0$

$P_{3x} := 184.0$ $P_{3y} := 1400.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 0.0 \cdot deg$ $\theta_{P2} := 27.0 \cdot deg$ $\theta_{P3} := 88.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := -1080$ $A_{1y} := 0.0$ $B_{1x} := -740$ $B_{1y} := -60$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P5-5 and Mathcad file P0528.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 1.051 \times 10^3 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 66.386\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 1.412 \times 10^3 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 82.513\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = 27.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = 88.000\,deg$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \qquad z = 1080$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \qquad s = 742.428$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \qquad v = 345.254$$

$$\phi := 0 \qquad \phi = 0.000\,deg$$

$\psi := acos\left(\frac{z^2 + s^2 - v^2}{2 \cdot z \cdot s}\right)$ $\psi = 4.635\,deg$

$Z_{1x} := z \cdot cos(\phi)$ $Z_{1x} = 1080$

$Z_{1y} := z \cdot sin(\phi)$ $Z_{1y} = 0.000$

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = 740.000$

$S_{1y} := s \cdot sin(\psi)$ $S_{1y} = 60.000$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -50$ $W_{1y} := 200$ $\beta_2 := -80 \cdot deg$ $\beta_3 := -160 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) \ldots + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) \ldots + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) \ldots + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) \ldots + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = -54.243\,deg$ $\beta_3 = -107.466\,deg$

The components of the **W** vector are:

$W_{1x} = -730.785$ $W_{1y} = 289.533$ $\theta := atan2(W_{1x}, W_{1y})$ $\theta = 158.387\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $w = 786.051$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points A and B are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -30$ $U_{1y} := 100$ $\gamma_2 := -80 \cdot deg$ $\gamma_3 := -160 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) \ldots + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) \ldots + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y}\cdot\left(\cos\left(\gamma_2\right)-1\right)+U_{1x}\cdot\sin\left(\gamma_2\right)\ ...\ +S_{1y}\cdot\left(\cos\left(\alpha_2\right)-1\right)+S_{1x}\cdot\sin\left(\alpha_2\right) = p_{21}\cdot\sin\left(\delta_2\right)$$

$$U_{1y}\cdot\left(\cos\left(\gamma_3\right)-1\right)+U_{1x}\cdot\sin\left(\gamma_3\right)\ ...\ +S_{1y}\cdot\left(\cos\left(\alpha_3\right)-1\right)+S_{1x}\cdot\sin\left(\alpha_3\right) = p_{31}\cdot\sin\left(\delta_3\right)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find\left(U_{1x}, U_{1y}, \gamma_2, \gamma_3\right)$$

$\gamma_2 = -51.378\,deg$ $\gamma_3 = -77.949\,deg$

The components of the **U** vector are:

$U_{1x} = -923.018$ $U_{1y} = 232.957$ $\sigma := atan2\left(U_{1x}, U_{1y}\right)$ $\sigma = 165.835\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $u = 951.962$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 340.000$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = -60.000$

$\theta_3 := atan2\left(V_{1x}, V_{1y}\right)$ $\theta_3 = -10.008\,deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $v = 345.254$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 532.233$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -3.424$

$\theta_1 := atan2\left(G_{1x}, G_{1y}\right)$ $\theta_1 = -0.369\,deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $g = 532.244$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 158.755\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 51.289\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 1080.000$ $\delta_p = 10.008\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos\left(\phi\right) - w\cdot\cos\left(\theta\right)$ $O_{2x} = -349.215$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $\qquad O_{2y} = -289.533$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad O_{4x} = 183.018$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad O_{4y} = -292.957$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right),\left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = -0.369\,deg$$

11. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return } \text{"Grashof"} \;\; \textit{if } SL < PQ \\ \textit{return } \text{"Special Grashof"} \;\; \textit{if } SL = PQ \\ \textit{return } \text{"non-Grashof"} \;\; \textit{otherwise} \end{array} \right.$$

$Condition(v,u,g,w) = $ "Grashof"

12. **DESIGN SUMMARY**

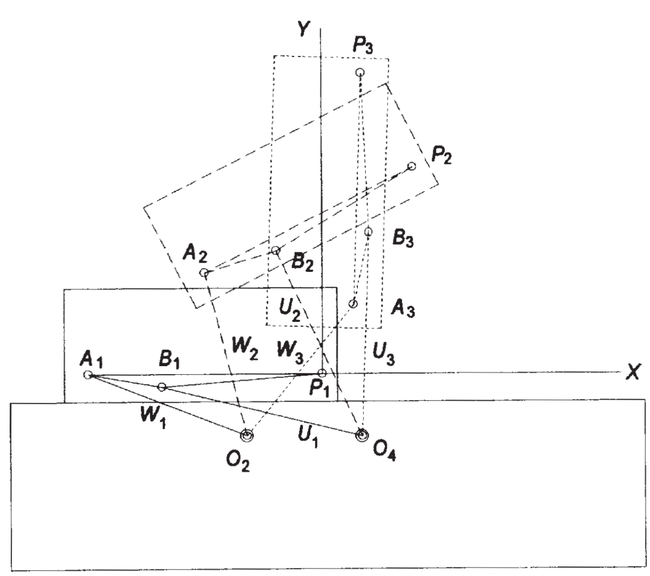
Link 2:	$w = 786.051$	$\theta = 158.387\,deg$
Link 3:	$v = 345.254$	$\theta_3 = -10.008\,deg$
Link 4:	$u = 951.962$	$\sigma = 165.835\,deg$
Link 1:	$g = 532.244$	$\theta_1 = -0.369\,deg$
Coupler:	$r_p = 1080.000$	$\delta_p = 10.008\,deg$

Crank angles:

$\theta_{2i} = 158.755\,deg$

$\theta_{2f} = 51.289\,deg$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-29

Statement: Design a linkage to carry the object in Figure P5-5 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := 421.0$ $\quad P_{21y} := 963.0$ $\quad P_{31x} := 184.0$ $\quad P_{31y} := 1400.0$

$O_{2x} := -362.0$ $\quad O_{2y} := -291.0$ $\quad O_{4x} := 182.0$ $\quad O_{4y} := -291.0$

Body angles: $\theta_{P1} := 0 \cdot deg$ $\quad \theta_{P2} := 27 \cdot deg$ $\quad \theta_{P3} := 88 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-5 and Mathcad file P0529.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = 27.000\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = 88.000\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 362.000$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 291.000$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 783.000$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 1.254 \times 10^3$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 546.000$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 1.691 \times 10^3$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 464.462$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 1.478 \times 10^3$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 1.777 \times 10^3$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 38.795\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 58.019\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 72.105\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 944.701$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = 928.284$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = -824.189$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = 1.319 \times 10^3$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -592.567$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = 830.373$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -2.419 \times 10^6$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 9.725 \times 10^4$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -1.584 \times 10^6$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = 4.810 \times 10^5$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -9.725 \times 10^4$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -2.003 \times 10^6$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 3.219 \times 10^{12}$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -5.670 \times 10^{11}$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = -4.543 \times 10^{11}$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = 88.000\,deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = -107.980\,deg$$

The first value is the same as α_3, so use the second value $\quad \beta_3 := \beta_{32}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 54.008\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = -54.008\,deg$$

Since $\beta 2$ is not in the first quadrant, $\quad \beta_2 := \beta_{22}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = -182.000$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 291.000$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 239.000$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 1.254 \times 10^3$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 2.000$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 1.691 \times 10^3$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 343.227$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 1.277 \times 10^3$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 1.691 \times 10^3$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 122.023\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 79.209\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 89.932\ deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = 478.979$

$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = 1.225 \times 10^3$

$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3)$ $\qquad C_3 = -299.174$

$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3)$ $\qquad C_4 = 1.863 \times 10^3$

$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2)$ $\qquad C_5 = -533.274$

$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$ $\qquad C_6 = 1.077 \times 10^3$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -3.559 \times 10^6$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 6.710 \times 10^5$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -2.166 \times 10^6$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = 5.257 \times 10^5$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -6.710 \times 10^5$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -2.425 \times 10^6$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 5.606 \times 10^{12}$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 4.884 \times 10^{11}$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\qquad K_3 = 6.838 \times 10^{11}$

$$\gamma_{31} := 2\cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 88.000\,deg$$

$$\gamma_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -78.042\,deg$$

The first value is the same as α_3, so use the second value $\qquad \gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5\cdot sin(\gamma_3) + A_3\cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 51.463\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3\cdot sin(\gamma_3) + A_2\cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -51.463\,deg$$

Since γ_2 is not in the first quadrant, $\qquad \gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 1051.004$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 66.386\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 1412.040$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 82.513\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21}\cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31}\cdot cos(\delta_3) \qquad M := p_{21}\cdot sin(\delta_2) \qquad N := p_{31}\cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1}\cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W1x = -728.089 \qquad W1y = 294.291 \qquad Z1x = 1.090 \times 10^3 \qquad Z1y = -3.291$$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 785.316$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \qquad B' := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21}\cdot\cos(\delta_2) \qquad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \qquad H := \cos(\alpha_3) - 1 \qquad K := \sin(\alpha_3)$$

$$L := p_{31}\cdot\cos(\delta_3) \qquad M := p_{21}\cdot\sin(\delta_2) \qquad N := p_{31}\cdot\sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1}\cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -921.699$ $U1y = 231.572$ $S1x = 739.699$ $S1y = 59.428$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 950.344$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 350.390$

$V1y := Z1y - S1y$ $V1y = -62.718$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 355.959$

$G1x := W1x + V1x - U1x$ $G1x = 544.000$

$G1y := W1y + V1y - U1y$ $G1y = -1.108 \times 10^{-12}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 544.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -362.000$

$O2y := -Z1y - W1y$ $O2y = -291.000$

$O4x := -S1x - U1x$ $O4x = 182.000$

$O4y := -S1y - U1y$ $O4y = -291.000$

These check with Figure P5-5.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 1.090 \times 10^3$ $rp := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\quad$ $s = 742.082$

$\psi := atan2(S1x, S1y)$ $\quad$ $\psi = 4.593\,deg$

$\phi := atan2(Z1x, Z1y)$ $\quad$ $\phi = -0.173\,deg$

$\theta_3 := atan2(z\cdot cos(\phi) - s\cdot cos(\psi), z\cdot sin(\phi) - s\cdot sin(\psi))$

$\theta_3 = -10.148\,deg$

$\delta_p := \phi - \theta_3$ $\quad$ $\delta_p = 9.975\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 544.000$	
Link 2:	$w = 785.316$	
Link 3:	$v = 355.959$	
Link 4:	$u = 950.344$	
Coupler point:	$rp = 1090.094$	$\delta_p = 9.975\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-30

Statement: To the linkage solution from Problem 5-29, add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given: Solution to Problem 5-29:

Length of link 2	$w := 785.316$	
Angle of link 2 in first position	$\theta := 157.992 \cdot deg$	
Rotation angles for link 2	$\beta_2 := -54.008 \cdot deg$	$\beta_3 := -107.980 \cdot deg$
Coordinates of O_2	$O_{2x} := -362.0$	$O_{2y} := -291.0$

Solution: See Figure P5-5 and Mathcad file P0530.

1. Link 2 of the solution to Problem will become the output link for the driver dyad. Select a point on link 2 of the Problem 5-29 solution and label it C. Let the distance O_2C be $c := 200$ The solution that follows is an analytic analog of the graphic solution method of Example 3-4 (p. 87)

2. Determine the coordinates of the points C_1 and C_3.

Guess: $C_{1x} := -547$ $C_{1y} := -216$ $C_{3x} := -233$ $C_{3y} := -137$

Given

$$(C_{1x} - O_{2x})^2 + (C_{1y} - O_{2y})^2 = c^2$$

$$C_{1y} = (C_{1x} - O_{2x}) \cdot tan(\theta) + O_{2y}$$

$$(C_{3x} - O_{2x})^2 + (C_{3y} - O_{2y})^2 = c^2$$

$$C_{3y} = (C_{3x} - O_{2x}) \cdot tan(\theta + \beta_3) + O_{2y}$$

$$\begin{pmatrix} C_{1x} \\ C_{1y} \\ C_{3x} \\ C_{3y} \end{pmatrix} := Find(C_{1x}, C_{1y}, C_{3x}, C_{3y})$$

$C_{1x} = -547.426$ $C_{1y} = -216.053$

$C_{3x} = -233.475$ $C_{3y} = -137.764$

3. Select a suitable length for the driver dyad coupler, say $v_d := 850$.

4. Determine the coordinates of the joint between the driving crank and the dyad coupler (point D).

Guess: $D_{1x} := -1372$ $D_{1y} := -421$

Given

$$(D_{1x} - C_{1x})^2 + (D_{1y} - C_{1y})^2 = v_d^2$$

$$D_{1y} = (D_{1x} - C_{1x}) \cdot \left(\frac{C_{3y} - C_{1y}}{C_{3x} - C_{1x}} \right) + C_{1y}$$

$$\begin{pmatrix} D_{1x} \\ D_{1y} \end{pmatrix} := Find(D_{1x}, D_{1y})$$

$$D_{1x} = -1372.170 \qquad D_{1y} = -421.715$$

5. Determine the length of the driving crank.

$$w_d := \frac{1}{2}\cdot\sqrt{(C_{3x} - C_{1x})^2 + (C_{3y} - C_{1y})^2} \qquad w_d = 161.783$$

6. Determine the coordinates of the driver dyad fixed pivot.

Guess: $O_{6x} := -1215$ $\quad O_{6y} := -382$

Given $(O_{6x} - D_{1x})^2 + (O_{6y} - D_{1y})^2 = w_d^{\,2}$

$$O_{6y} = (O_{6x} - D_{1x})\cdot\left(\frac{C_{3y} - C_{1y}}{C_{3x} - C_{1x}}\right) + D_{1y}$$

$$\begin{pmatrix} O_{6x} \\ O_{6y} \end{pmatrix} := Find(O_{6x}, O_{6y})$$

$$O_{6x} = -1215.195 \qquad O_{6y} = -382.571$$

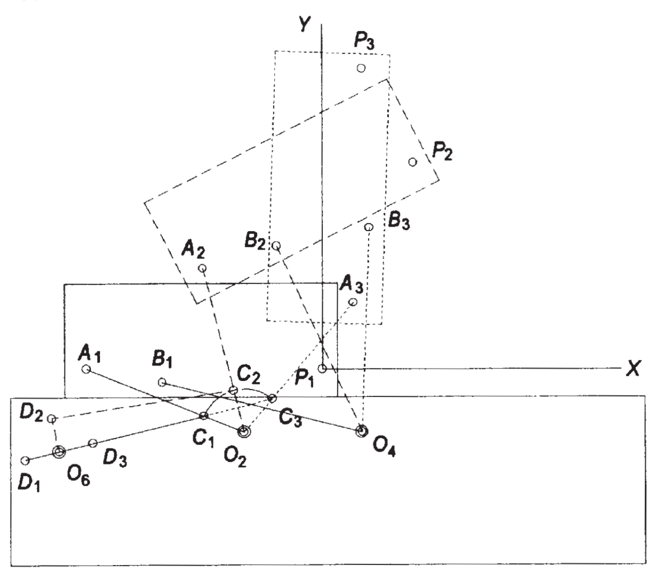
7. Determine the Grashof condition.

$w_d = 161.783 \quad v_d = 850.000 \quad c = 200.000 \quad g_d := \sqrt{(O_{2x} - O_{6x})^2 \ldots + (O_{2y} - O_{6y})^2} \quad g_d = 858.095$

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return} \text{ "Grashof" } \textit{if } SL \le PQ \\ \textit{return} \text{ "non-Grashof" } \textit{otherwise} \end{array} \right.$$

$Condition(w_d, g_d, c, v_d) = $ "Grashof"

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.



PROBLEM 5-31

Statement: Design a fourbar linkage to carry the box in Figure P5-6 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 130.0$ $\quad P_{2y} := -29.0$

$P_{3x} := 148.0$ $\quad P_{3y} := -187.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 90.0 \cdot deg$ $\quad \theta_{P2} := 65.0 \cdot deg$ $\quad \theta_{P3} := -11.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := -69.0$ $\quad A_{1y} := -43.0$ $\quad B_{1x} := -17.0$ $\quad B_{1y} := -43.0$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P5-6 and Mathcad file P0531.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 133.195 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = -12.575\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 238.481 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = -51.640\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -25.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -101.000\,deg$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \qquad z = 81.302$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \qquad s = 46.239$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \qquad v = 52.000$$

$$\phi := acos\left(\frac{v^2 + z^2 - s^2}{2\cdot v\cdot z}\right) \qquad \phi = 31.931\,deg$$

$$\psi := \pi - acos\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \qquad \psi = 68.429\,deg$$

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = 69.000$$

$$Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 43.000$$

$$S_{1x} := s \cdot cos(\psi) \qquad S_{1x} = 17.000$$

$$S_{1y} := s \cdot sin(\psi) \qquad S_{1y} = 43.000$$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -50$ $\quad W_{1y} := 200$ $\quad \beta_2 := -80 \cdot deg$ $\quad \beta_3 := -160 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) \ldots + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) \ldots + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) \ldots + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) \ldots + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -52.277\,deg \qquad \beta_3 = -96.147\,deg$$

The components of the **W** vector are:

$$W_{1x} = -63.415 \qquad W_{1y} = 118.432 \qquad \theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 118.167\,deg$$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $\quad w = 134.341$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points *A* and *B* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -30$ $\quad U_{1y} := 100$ $\quad \gamma_2 := -80 \cdot deg$ $\quad \gamma_3 := -160 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) \ldots + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) \ldots + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y}\cdot(\cos(\gamma_2)-1)+U_{1x}\cdot\sin(\gamma_2)\ldots = p_{21}\cdot\sin(\delta_2)$$
$$+S_{1y}\cdot(\cos(\alpha_2)-1)+S_{1x}\cdot\sin(\alpha_2)$$

$$U_{1y}\cdot(\cos(\gamma_3)-1)+U_{1x}\cdot\sin(\gamma_3)\ldots = p_{31}\cdot\sin(\delta_3)$$
$$+S_{1y}\cdot(\cos(\alpha_3)-1)+S_{1x}\cdot\sin(\alpha_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = -79.044\,deg$ $\qquad$ $\gamma_3 = -147.982\,deg$

The components of the **U** vector are:

$U_{1x} = -45.930$ $\qquad$ $U_{1y} = 77.634$ $\qquad$ $\sigma := atan2(U_{1x}, U_{1y})$ $\qquad$ $\sigma = 120.609\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^2 + U_{1y}^2\right)}$, $\quad u = 90.203$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\qquad$ $V_{1x} = 52.000$

$V_{1y} := Z_{1y} - S_{1y}$ $\qquad$ $V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\qquad$ $\theta_3 = 0.000\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $\qquad$ $v = 52.000$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $\qquad$ $G_{1x} = 34.515$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $\qquad$ $G_{1y} = 40.798$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\qquad$ $\theta_1 = 49.768\,deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $\qquad$ $g = 53.439$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\qquad$ $\theta_{2i} = 68.399\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\qquad$ $\theta_{2f} = -27.748\,deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\qquad$ $\delta_p := \phi - \theta_3$

$r_p = 81.302$ $\qquad$ $\delta_p = 31.931\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta)$ $\qquad$ $O_{2x} = -5.585$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \qquad O_{2y} = -161.432$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \qquad O_{4x} = 28.930$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \qquad O_{4y} = -120.634$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 49.768\,deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return} \text{ "Grashof" } \textit{if } SL < PQ \\ \textit{return} \text{ "Special Grashof" } \textit{if } SL = PQ \\ \textit{return} \text{ "non-Grashof" } \textit{otherwise} \end{array} \right.$$

$$Condition(v, w, g, u) = \text{"non-Grashof"}$$

12. **DESIGN SUMMARY**

Link 2:	$w = 134.341$	$\theta = 118.167\,deg$
Link 3:	$v = 52.000$	$\theta_3 = 0.000\,deg$
Link 4:	$u = 90.203$	$\sigma = 120.609\,deg$
Link 1:	$g = 53.439$	$\theta_1 = 49.768\,deg$
Coupler:	$r_p = 81.302$	$\delta_p = 31.931\,deg$

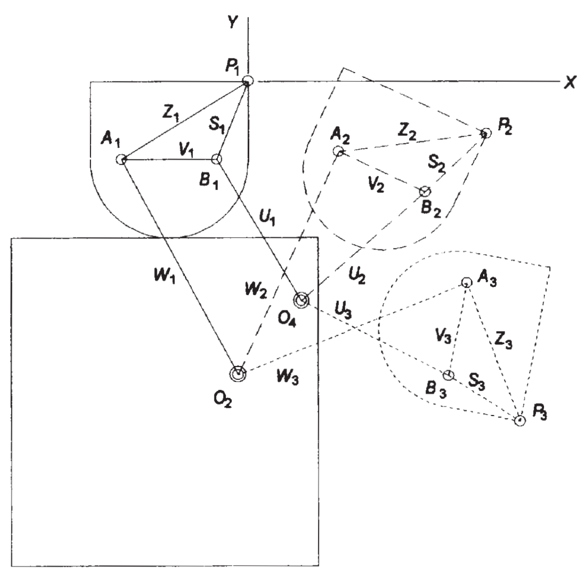
Crank angles:

$$\theta_{2i} = 68.399\,deg$$

$$\theta_{2f} = -27.748\,deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2.



PROBLEM 5-32

Statement: Design a fourbar linkage to carry the box in Figure P5-6 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3. The fixed pivots should be on the base.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $P_{1y} := 0.0$ $P_{2x} := 130.0$ $P_{2y} := -29.0$

$P_{3x} := 148.0$ $P_{3y} := -187.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 90.0 \cdot deg$ $\theta_{P2} := 65.0 \cdot deg$ $\theta_{P3} := -11.0 \cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := -52.0 \cdot deg$ $\beta_3 := -95.0 \cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -76.0 \cdot deg$ $\gamma_3 := -145.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Figure P5-6 and Mathcad file P0532.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^{\,2} + P_{2y}^{\,2}} \qquad p_{21} = 133.195 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = -12.575\,deg$$

$$p_{31} := \sqrt{P_{3x}^{\,2} + P_{3y}^{\,2}} \qquad p_{31} = 238.481 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = -51.640\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -25.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -101.000\,deg$$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = -63.498$ $W_{1y} = 118.196$ $Z_{1x} = 69.575$ $Z_{1y} = 44.896$

$\theta := atan2(W_{1x}, W_{1y})$ $\theta = 118.246\,deg$ $\phi := atan2(Z_{1x}, Z_{1y})$ $\phi = 32.834\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $w = 134.173$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^{\ 2} + Z_{1y}^{\ 2}\right)}$, $z = 82.803$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $B' := sin(\gamma_2)$ $C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $E := p_{21} \cdot cos(\delta_2)$ $F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $H := cos(\alpha_3) - 1$ $K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -46.325$ $U_{1y} = 82.956$ $S_{1x} = 17.754$ $S_{1y} = 37.985$

$\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 119.180\,deg$ $\psi := atan2(S_{1x}, S_{1y})$ $\psi = 64.949\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $u = 95.014$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^{\ 2} + S_{1y}^{\ 2}\right)}$, $s = 41.930$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 51.820$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 6.910$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 7.596\,deg$

$v := \sqrt{\left(V_{1x}^{\,2} + V_{1y}^{\,2}\right)}$ $v = 52.279$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 34.647$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 42.151$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 50.580\,deg$

$g := \sqrt{\left(G_{1x}^{\,2} + G_{1y}^{\,2}\right)}$ $g = 54.563$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 67.666\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = -27.334\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 82.803$ $\delta_p = 25.238\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot cos(\phi) - w\cdot cos(\theta)$ $O_{2x} = -6.076$

$O_{2y} := -z\cdot sin(\phi) - w\cdot sin(\theta)$ $O_{2y} = -163.092$

$O_{4x} := -s\cdot cos(\psi) - u\cdot cos(\sigma)$ $O_{4x} = 28.571$

$O_{4y} := -s\cdot sin(\psi) - u\cdot sin(\sigma)$ $O_{4y} = -120.941$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\theta_{rot} = 50.580\,deg$

11. Determine the Grashof condition.

```
Condition(S,L,P,Q) := | SL ← S + L
                      | PQ ← P + Q
                      | return "Grashof" if SL < PQ
                      | return "Special Grashof" if SL = PQ
                      | return "non-Grashof" otherwise
```

$Condition(v, w, g, u) =$ "non-Grashof"

12. **DESIGN SUMMARY**

Link 2:	$w = 134.173$	$\theta = 118.246\,deg$
Link 3:	$v = 52.279$	$\theta_3 = 7.596\,deg$

Link 4:	$u = 95.014$	$\sigma = 119.180\ deg$
Link 1:	$g = 54.563$	$\theta_1 = 50.580\ deg$
Coupler:	$r_p = 82.803$	$\delta_p = 25.238\ deg$

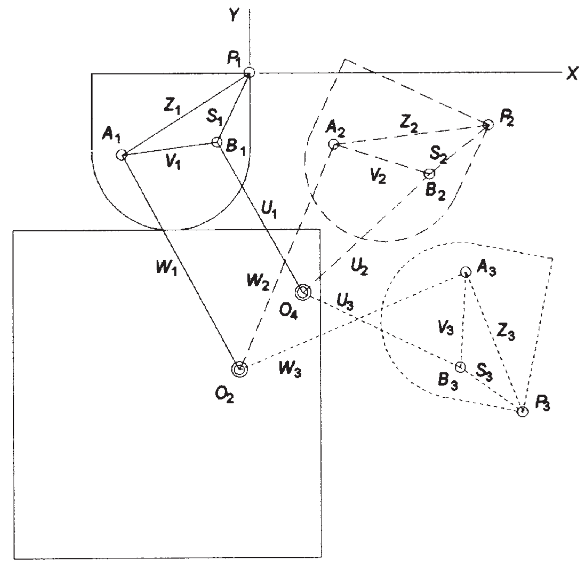
Crank angles:

$$\theta_{2i} = 67.666\ deg$$

$$\theta_{2f} = -27.334\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2.



PROBLEM 5-33

Statement: Design a linkage to carry the object in Figure P5-6 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given:

$P_{21x} := 130.0$ $\quad P_{21y} := -29.0$ $\quad P_{31x} := 148.0$ $\quad P_{31y} := -187.0$

$O_{2x} := -6.2$ $\quad O_{2y} := -164.0$ $\quad O_{4x} := 28.0$ $\quad O_{4y} := -121.0$

Body angles: $\theta_{P1} := 90 \cdot deg$ $\quad \theta_{P2} := 65 \cdot deg$ $\quad \theta_{P3} := -11 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-6 and Mathcad file P0533.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = -25.000\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = -101.000\ deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\quad R_{1x} = 6.200$ $\quad R_{1y} := -O_{2y}$ $\quad R_{1y} = 164.000$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 136.200$

$R_{2y} := R_{1y} + P_{21y}$ $\quad R_{2y} = 135.000$

$R_{3x} := R_{1x} + P_{31x}$ $\quad R_{3x} = 154.200$

$R_{3y} := R_{1y} + P_{31y}$ $\quad R_{3y} = -23.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 164.117$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 191.769$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 155.906$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\quad \zeta_1 = 87.835\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\quad \zeta_2 = 44.746\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\quad \zeta_3 = -8.484\ deg$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \qquad C_1 = 23.501$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \qquad C_2 = 73.444$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \qquad C_3 = 5.604$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \qquad C_4 = 14.379$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \qquad C_5 = -61.271$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \qquad C_6 = -11.014$$

$$A_1 := -C_3^{\,2} - C_4^{\,2} \qquad A_1 = -238.152$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \qquad A_2 = 819.286$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \qquad A_3 = 501.727$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \qquad A_4 = 749.483$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \qquad A_5 = -819.286$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \qquad A_6 = -924.339$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \qquad K_1 = 1.503 \times 10^5$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \qquad K_2 = 1.133 \times 10^6$$

$$K_3 := \frac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2} \qquad K_3 = -1.141 \times 10^6$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{31} = -101.000\,deg$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{32} = -94.106\,deg$$

The first value is the same as α_3, so use the second value $\qquad \beta_3 := \beta_{32}$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 53.072\,deg$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = -53.072\,deg$$

Since β_2 is not in the first quadrant, $\qquad \beta_2 := \beta_{22}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$$R_{1x} := -O_{4x} \qquad R_{1x} = -28.000 \qquad R_{1y} := -O_{4y} \qquad R_{1y} = 121.000$$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 102.000$

$R_{2y} := R_{1y} + P_{21y}$ $\quad R_{2y} = 92.000$

$R_{3x} := R_{1x} + P_{31x}$ $\quad R_{3x} = 120.000$

$R_{3y} := R_{1y} + P_{31y}$ $\quad R_{3y} = -66.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 124.197$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 137.361$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 136.953$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\quad \zeta_1 = 103.029\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\quad \zeta_2 = 42.049\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\quad \zeta_3 = -28.811\,deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 10.017$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = 7.150$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = 4.120$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -70.398$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -76.240$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -29.497$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -4.973 \times 10^3$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = -5.489 \times 10^3$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -1.762 \times 10^3$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -675.715$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = 5.489 \times 10^3$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = 544.601$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 2.749 \times 10^6$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = 4.180 \times 10^6$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\quad K_3 = -4.628 \times 10^6$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = -145.661\ deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -101.000\ deg$$

The second value is the same as α_3, so use the first value $\qquad \gamma_3 := \gamma_{31}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 77.265\ deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -77.265\ deg$$

Since γ_2 is not in the first quadrant , $\qquad \gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 133.195$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -12.575\ deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 238.481$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = -51.640\ deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W1x = -61.917 \qquad W1y = 112.415 \qquad Z1x = 68.117 \qquad Z1y = 51.585$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 128.339$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -46.374$ $U1y = 80.382$ $S1x = 18.374$ $S1y = 40.618$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 92.800$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 49.743$

$V1y := Z1y - S1y$ $V1y = 10.967$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 50.938$

$G1x := W1x + V1x - U1x$ $G1x = 34.200$

$G1y := W1y + V1y - U1y$ $G1y = 43.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 54.942$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -6.200$

$O2y := -Z1y - W1y$ $O2y = -164.000$

$O4x := -S1x - U1x$ $O4x = 28.000$

$O4y := -S1y - U1y$ $O4y = -121.000$

These check with Figure P5-6.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 85.446$ $rp := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 44.581$

$\psi := atan2(S1x, S1y)$ $\quad \psi = 65.660\,deg$

$\phi := atan2(Z1x, Z1y)$ $\quad \phi = 37.136\,deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 12.433\,deg$

$\delta_p := \phi - \theta_3$ $\quad \delta_p = 24.704\,deg$

18. **DESIGN SUMMARY**

Link 1: $g = 54.942$

Link 2: $w = 128.339$

Link 3: $v = 50.938$

Link 4: $u = 92.800$

Coupler point: $rp = 85.446$ $\quad \delta_p = 24.704\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-34

Statement: Design a fourbar linkage to carry the bolt in Figure P5-7 from positions 1 to 2 to 3 without regard for the fixed pivots shown. The bolt is fed into the gripper in the *z* direction (into the paper). The gripper grabs the bolt, and your linkage moves it to position 3 to be inserted into the hole. A second degree of freedom within the gripper assembly (not shown) pushes the bolt into the hole. The moving pivots should be on, or close to, the gripper assembly, and the fixed pivots should be on the base. Use the free choices given below.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 99.0$ $\quad P_{2y} := 13.0$

$P_{3x} := 111.3$ $\quad P_{3y} := -151.8$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 272.3 \cdot deg$ $\quad \theta_{P2} := 301.7 \cdot deg$ $\quad \theta_{P3} := 270.0 \cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := 70 \cdot deg$ $\quad \beta_3 := 140 \cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -5 \cdot deg$ $\quad \gamma_3 := -49 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array} \right.$$

Solution: See Figure P5-7 and Mathcad file P0534.

1. Determine the magnitudes and orientation of the position difference vectors.

$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2}$ $\quad p_{21} = 99.850$ $\quad \delta_2 := atan2(P_{2x}, P_{2y})$ $\quad \delta_2 = 7.481\ deg$

$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2}$ $\quad p_{31} = 188.231$ $\quad \delta_3 := atan2(P_{3x}, P_{3y})$ $\quad \delta_3 = -53.751\ deg$

2. Determine the angle changes of the coupler between precision points.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = 29.400\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = -2.300\ deg$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$A := \cos(\beta_2) - 1$ $\quad B := \sin(\beta_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F := \cos(\beta_3) - 1$

$G := \sin(\beta_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = -86.624$ $W_{1y} = 50.030$ $Z_{1x} = 198.147$ $Z_{1y} = -233.314$

$\theta := atan2(W_{1x}, W_{1y})$ $\theta = 149.991\, deg$ $\phi := atan2(Z_{1x}, Z_{1y})$ $\phi = -49.660\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $w = 100.033$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^{\ 2} + Z_{1y}^{\ 2}\right)}$, $z = 306.100$

4. Evaluate terms in the US coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $B' := sin(\gamma_2)$ $C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $E := p_{21} \cdot cos(\delta_2)$ $F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $H := cos(\alpha_3) - 1$ $K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = 107.545$ $U_{1y} = 205.365$ $S_{1x} = 3.375$ $S_{1y} = -166.927$

$\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 62.360\, deg$ $\psi := atan2(S_{1x}, S_{1y})$ $\psi = -88.842\, deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $u = 231.821$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^{\ 2} + S_{1y}^{\ 2}\right)}$, $s = 166.961$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 194.772$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = -66.387$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = -18.821\ deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 205.775$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 0.602$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -221.722$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = -89.844\ deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $g = 221.723$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 239.836\ deg$

$\theta_{2f} := \theta_{2i} + \beta_3 - 2\cdot\pi$ $\theta_{2f} = 19.836\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 306.100$ $\delta_p = -30.838\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot cos(\phi) - w\cdot cos(\theta)$ $O_{2x} = -111.523$

$O_{2y} := -z\cdot sin(\phi) - w\cdot sin(\theta)$ $O_{2y} = 183.284$

$O_{4x} := -s\cdot cos(\psi) - u\cdot cos(\sigma)$ $O_{4x} = -110.920$

$O_{4y} := -s\cdot sin(\psi) - u\cdot sin(\sigma)$ $O_{4y} = -38.438$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\theta_{rot} = -89.844\ deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return}\ \text{"Grashof"}\ \ \textit{if}\ SL < PQ \\ \textit{return}\ \text{"Special Grashof"}\ \ \textit{if}\ SL = PQ \\ \textit{return}\ \text{"non-Grashof"}\ \ \textit{otherwise} \end{array} \right.$$

$Condition(w, u, g, v) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

Link 2:	$w = 100.033$	$\theta = 149.991\ deg$
Link 3:	$v = 205.775$	$\theta_3 = -18.821\ deg$

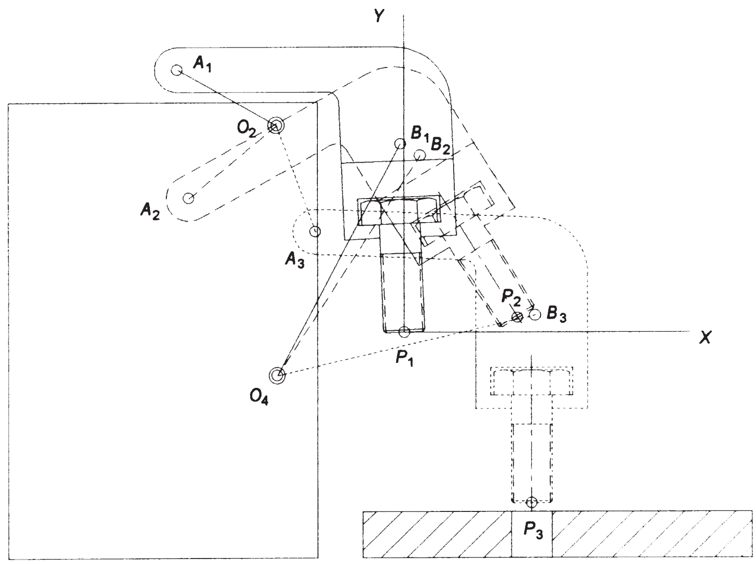
Link 4:	$u = 231.821$	$\sigma = 62.360\ deg$
Link 1:	$g = 221.723$	$\theta_1 = -89.844\ deg$
Coupler:	$r_p = 306.100$	$\delta_p = -30.838\ deg$

Crank angles:

$$\theta_{2i} = 239.836\ deg$$

$$\theta_{2f} = 19.836\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-35

Statement: Design a linkage to carry the bolt in Figure P5-7 from positions 1 to 2 to 3 using the fixed pivots shown. See Problem 5-34 for more details.

Given:

$P_{21x} := 99.0$ $\quad P_{21y} := 13.0$ $\quad P_{31x} := 111.3$ $\quad P_{31y} := -151.8$

$O_{2x} := -111.5$ $\quad O_{2y} := 183.2$ $\quad O_{4x} := -111.5$ $\quad O_{4y} := -38.8$

Body angles: $\theta_{P1} := 272.3 \cdot deg$ $\quad \theta_{P2} := 301.7 \cdot deg$ $\quad \theta_{P3} := 270 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-7 and Mathcad file P0535.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = 29.400\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -2.300\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 111.500$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = -183.200$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 210.500$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = -170.200$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 222.800$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = -335.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 214.463$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 270.700$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 402.324$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = -58.674\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = -38.957\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = -56.373\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 155.059$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = -3.973$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = -118.742$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -147.473$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -23.426$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -65.329$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $\quad A_1 = -3.585 \times 10^4$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 4.303 \times 10^3$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -1.242 \times 10^4$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -2.240 \times 10^4$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -4.303 \times 10^3$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -1.900 \times 10^4$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 1.395 \times 10^8$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = 3.598 \times 10^8$

$$K_3 := \frac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2} \qquad K_3 = 1.250 \times 10^8$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{31} = 139.911\, deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{32} = -2.300\, deg$$

The second value is the same as α_3, so use the first value $\quad \beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 69.984\, deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 69.984\, deg$$

Since both angles are the same, $\quad \beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = 111.500$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 38.800$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 210.500$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 51.800$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 222.800$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = -113.000$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 118.058$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 216.780$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 249.818$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 19.187\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 13.825\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = -26.893\,deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = 37.169$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = -32.384$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\qquad C_3 = -109.833$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\qquad C_4 = -147.294$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\qquad C_5 = -132.407$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\qquad C_6 = -36.739$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -3.376 \times 10^4$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = -1.547 \times 10^4$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -1.995 \times 10^4$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = -1.918 \times 10^3$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = 1.547 \times 10^4$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -8.852 \times 10^3$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 2.063 \times 10^8$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = -9.865 \times 10^7$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\qquad K_3 = 2.101 \times 10^8$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = -2.300\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -48.814\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 4.951\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -4.951\,deg$$

Since γ_2 is not in the first quadrant , $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 99.850$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 7.481\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 188.231$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = -53.751\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W_{1x} = -86.684 \qquad W_{1y} = 49.977 \qquad Z_{1x} = 198.184 \qquad Z_{1y} = -233.177$$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 100.059$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = 108.268$ $U1y = 205.938$ $S1x = 3.232$ $S1y = -167.138$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 232.664$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 194.953$

$V1y := Z1y - S1y$ $V1y = -66.039$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 205.834$

$G1x := W1x + V1x - U1x$ $G1x = -1.705 \times 10^{-13}$

$G1y := W1y + V1y - U1y$ $G1y = -222.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 222.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -111.500$

$O2y := -Z1y - W1y$ $O2y = 183.200$

$O4x := -S1x - U1x$ $O4x = -111.500$

$O4y := -S1y - U1y$ $O4y = -38.800$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 306.020$ $r_P := z$

Angle *BAP* (δ_p)

$s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 167.169$

$\psi := atan2(S1x, S1y)$ $\quad \psi = -88.892\, deg$

$\phi := atan2(Z1x, Z1y)$ $\quad \phi = -49.638\, deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = -18.714\, deg$

$\delta_p := \phi - \theta_3$ $\quad \delta_p = -30.924\, deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 222.000$	
Link 2:	$w = 100.059$	
Link 3:	$v = 205.834$	
Link 4:	$u = 232.664$	
Coupler point:	$rp = 306.020$	$\delta_p = -30.924\, deg$

19. **VERIFICATION**: The calculated values of *g* (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-36

Statement: To the linkage solution from Problem 5-35, add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given: Solution to Problem 5-35:

Length of link 4	$u := 232.664$	
Angle of link 4 in first position	$\sigma := 62.268 \cdot deg$	
Rotation angles for link 2	$\gamma_2 := -4.951 \cdot deg$	$\gamma_3 := -48.814 \cdot deg$
Coordinates of O_4	$O_{4x} := -111.5$	$O_{4y} := -38.8$

Solution: See Figure P5-7 and Mathcad file P0536.

1. Link 4 of the solution to Problem will become the output link for the driver dyad. Select a point on link 4 of the Problem 5-35 solution and label it C. Let the distance O_4C be $c := 60$ The solution that follows is an analytic analog of the graphic solution method of Example 3-4 (p. 87)

2. Determine the coordinates of the points C_1 and C_3.

Guess: $C_{1x} := -83$ $\quad C_{1y} := 14$ $\quad C_{3x} := -53$ $\quad C_{3y} := -25$

Given

$$(C_{1x} - O_{4x})^2 + (C_{1y} - O_{4y})^2 = c^2$$

$$C_{1y} = (C_{1x} - O_{4x}) \cdot tan(\sigma) + O_{4y}$$

$$(C_{3x} - O_{4x})^2 + (C_{3y} - O_{4y})^2 = c^2$$

$$C_{3y} = (C_{3x} - O_{4x}) \cdot tan(\sigma + \gamma_3) + O_{4y}$$

$$\begin{pmatrix} C_{1x} \\ C_{1y} \\ C_{3x} \\ C_{3y} \end{pmatrix} := Find(C_{1x}, C_{1y}, C_{3x}, C_{3y})$$

$C_{1x} = -83.580$ $\quad C_{1y} = 14.308$

$C_{3x} = -53.147$ $\quad C_{3y} = -24.840$

3. Select a suitable length for the driver dyad coupler, say $v_d := 200$.

4. Determine the coordinates of the joint between the driving crank and the dyad coupler (point D).

Guess: $D_{1x} := -206$ $\quad D_{1y} := 172$

Given

$$(D_{1x} - C_{1x})^2 + (D_{1y} - C_{1y})^2 = v_d^2$$

$$D_{1y} = (D_{1x} - C_{1x}) \cdot \left(\frac{C_{3y} - C_{1y}}{C_{3x} - C_{1x}} \right) + C_{1y}$$

$$\begin{pmatrix} D_{1x} \\ D_{1y} \end{pmatrix} := Find(D_{1x}, D_{1y})$$

$D_{1x} = -206.329$ $\qquad D_{1y} = 172.208$

5. Determine the length of the driving crank.

$$w_d := \frac{1}{2}\cdot\sqrt{(C_{3x} - C_{1x})^2 + (C_{3y} - C_{1y})^2} \qquad w_d = 24.793$$

6. Determine the coordinates of the driver dyad fixed pivot.

Guess: $O_{6x} := -190$ $\qquad O_{6y} := 150$

Given $\quad (O_{6x} - D_{1x})^2 + (O_{6y} - D_{1y})^2 = w_d^2$

$$O_{6y} = (O_{6x} - D_{1x})\cdot\left(\frac{C_{3y} - C_{1y}}{C_{3x} - C_{1x}}\right) + D_{1y}$$

$$\begin{pmatrix} O_{6x} \\ O_{6y} \end{pmatrix} := Find(O_{6x}, O_{6y})$$

$O_{6x} = -191.113$ $\qquad O_{6y} = 152.634$

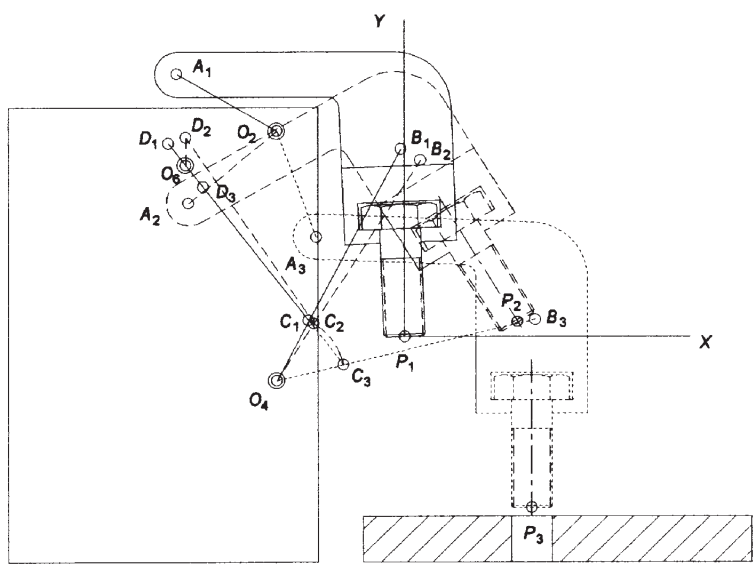
7. Determine the Grashof condition.

$w_d = 24.793$ $\qquad v_d = 200.000$ $\qquad c = 60.000$ $\qquad g_d := \sqrt{(O_{4x} - O_{6x})^2 \ldots + (O_{4y} - O_{6y})^2}$ $\qquad g_d = 207.329$

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return} \text{ "Grashof" } \textit{if } SL \le PQ \\ \textit{return} \text{ "non-Grashof" } \textit{otherwise} \end{array} \right.$$

$Condition(w_d, g_d, c, v_d) = $ "Grashof"

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.



PROBLEM 5-37

Statement: Figure P5-8 shows an off-loading mechanism for paper rolls. The V-link is rotated through 90 deg by an air-driven fourbar slider-crank linkage. Design a pin-jointed fourbar linkage to replace the existing off-loading station and perform essentially the same function. Choose three positions of the roll including its two end positions and synthesize a substitute mechanism. Use a link similar to the existing V-link as one of your links. Add a driver dyad to limit its motion to the range desired.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 450$ $\quad P_{2y} := -140$

$P_{3x} := 1000.0$ $\quad P_{3y} := -1000.0$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 90.0 \cdot deg$ $\quad \theta_{P2} := 65.0 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Coordinates of the points A_1 and B_1 with respect to P_1:

$A_{1x} := -400.0$ $\quad A_{1y} := -1035.0$ $\quad B_{1x} := 35.0$ $\quad B_{1y} := -600.0$

Two argument inverse tangent

$$atan2(x,y) := \left| \begin{array}{l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array} \right.$$

Solution: See Figure P5-8 and Mathcad file P0537.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 471.275 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = -17.281\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 1.414 \times 10^3 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = -45.000\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -25.000\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -90.000\,deg$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \qquad z = 1109.606$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \qquad s = 601.020$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \qquad v = 615.183$$

$$\phi := atan2(A_{1x}, A_{1y}) - \pi \qquad \phi = 68.870\,deg$$

$\psi := \pi + atan2(B_{1x}, B_{1y})$ $\psi = 93.338\,deg$

$Z_{1x} := z \cdot \cos(\phi)$ $Z_{1x} = 400.000$

$Z_{1y} := z \cdot \sin(\phi)$ $Z_{1y} = 1035.000$

$S_{1x} := s \cdot \cos(\psi)$ $S_{1x} = -35.000$

$S_{1y} := s \cdot \sin(\psi)$ $S_{1y} = 600.000$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -50$ $W_{1y} := 200$ $\beta_2 := -80 \cdot deg$ $\beta_3 := -160 \cdot deg$

Given

$$W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) \ldots = p_{21} \cdot \cos(\delta_2)$$
$$+ Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2)$$

$$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) \ldots = p_{31} \cdot \cos(\delta_3)$$
$$+ Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3)$$

$$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) \ldots = p_{21} \cdot \sin(\delta_2)$$
$$+ Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2)$$

$$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) \ldots = p_{31} \cdot \sin(\delta_3)$$
$$+ Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = -11.094\,deg$ $\beta_3 = -47.757\,deg$

The components of the **W** vector are:

$W_{1x} = -673.809$ $W_{1y} = 194.748$ $\theta := atan2(W_{1x}, W_{1y})$ $\theta = 163.879\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $w = 701.388$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points A and B are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -30$ $U_{1y} := 100$ $\gamma_2 := -80 \cdot deg$ $\gamma_3 := -160 \cdot deg$

Given

$$U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) \ldots = p_{21} \cdot \cos(\delta_2)$$
$$+ S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2)$$

$$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) \ldots = p_{31} \cdot \cos(\delta_3)$$
$$+ S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3)$$

$$U_{1y}\cdot(\cos(\gamma_2)-1)+U_{1x}\cdot\sin(\gamma_2)\ \dots\ +S_{1y}\cdot(\cos(\alpha_2)-1)+S_{1x}\cdot\sin(\alpha_2) = p_{21}\cdot\sin(\delta_2)$$

$$U_{1y}\cdot(\cos(\gamma_3)-1)+U_{1x}\cdot\sin(\gamma_3)\ \dots\ +S_{1y}\cdot(\cos(\alpha_3)-1)+S_{1x}\cdot\sin(\alpha_3) = p_{31}\cdot\sin(\delta_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = -25.881\ deg$ $\gamma_3 = -71.807\ deg$

The components of the **U** vector are:

$U_{1x} = 117.928$ $U_{1y} = 469.583$ $\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 75.903\ deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $u = 484.164$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 435.000$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 435.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 45.000\ deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $v = 615.183$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = -356.736$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 160.165$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 155.821\ deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $g = 391.042$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 8.058\ deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = -39.699\ deg$

8. Define the coupler point with respect to point *A* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 1109.606$ $\delta_p = 23.870\ deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta)$ $O_{2x} = 273.809$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $\qquad O_{2y} = -1229.748$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad O_{4x} = -82.928$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad O_{4y} = -1069.583$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\qquad \theta_{rot} = 155.821\,deg$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(g, w, u, v) = \text{"Grashof"}$

12. **DESIGN SUMMARY**

Link 2:	$w = 701.388$	$\theta = 163.879\,deg$
Link 3:	$v = 615.183$	$\theta_3 = 45.000\,deg$
Link 4:	$u = 484.164$	$\sigma = 75.903\,deg$
Link 1:	$g = 391.042$	$\theta_1 = 155.821\,deg$
Coupler:	$r_p = 1109.606$	$\delta_p = 23.870\,deg$

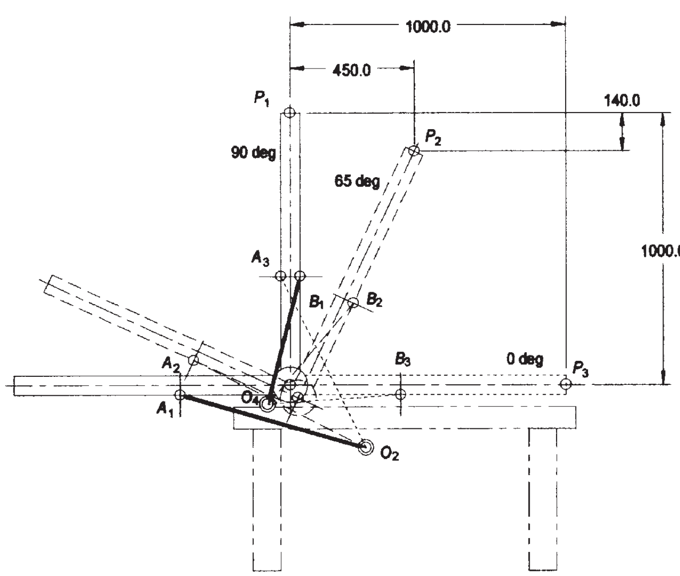
Crank angles:

$\theta_{2i} = 8.058\,deg$

$\theta_{2f} = -39.699\,deg$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2.



PROBLEM 5-38

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points C and D for your attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $P_{1y} := 0.0$ $P_{2x} := 4.500$ $P_{2y} := 1.900$

$P_{3x} := 7.600$ $P_{3y} := 1.000$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 33.70 \cdot deg$ $\theta_{P2} := 14.60 \cdot deg$ $\theta_{P3} := 0.0 \cdot deg$

Coordinates of the points C_1 and D_1 with respect to P_1:

$C_{1x} := 0.0$ $C_{1y} := 0.0$ $D_{1x} := 3.744$ $D_{1y} := 2.497$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-9 and Mathcad file P0538.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 4.885 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 22.891\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 7.666 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 7.496\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -19.100\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -33.700\,deg$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \qquad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} \qquad s = 4.500$$

$$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} \qquad v = 4.500$$

$$\phi := \theta_{P1} \qquad \phi = 33.700\,deg$$

$$\psi := \theta_{P1} + \pi \qquad \psi = 213.700\,deg$$

$Z_{1x} := z \cdot cos(\phi)$ $\quad Z_{1x} = 0.000$

$Z_{1y} := z \cdot sin(\phi)$ $\quad Z_{1y} = 0.000$

$S_{1x} := -s \cdot cos(\theta_{P1})$ $\quad S_{1x} = -3.744$

$S_{1y} := -s \cdot sin(\theta_{P1})$ $\quad S_{1y} = -2.497$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-57.

Guess: $W_{1x} := -4$ $\quad W_{1y} := 4$ $\quad \beta_2 := -50 \cdot deg$ $\quad \beta_3 := -80 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = -47.370\, deg$ $\quad \beta_3 = -78.160\, deg$

The components of the **W** vector are:

$W_{1x} = -4.416$ $\quad W_{1y} = 4.179$ $\quad \theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 136.576\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $\quad w = 6.080$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -4$ $\quad U_{1y} := 4$ $\quad \gamma_2 := -44 \cdot deg$ $\quad \gamma_3 := -76 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y} \cdot (cos(\gamma_2) - 1) + U_{1x} \cdot sin(\gamma_2) + S_{1y} \cdot (cos(\alpha_2) - 1) + S_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$U_{1y}\cdot\left(\cos\left(\gamma_3\right)-1\right)+U_{1x}\cdot\sin\left(\gamma_3\right)\ ... = p_{31}\cdot\sin\left(\delta_3\right)$$
$$+S_{1y}\cdot\left(\cos\left(\alpha_3\right)-1\right)+S_{1x}\cdot\sin\left(\alpha_3\right)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find\left(U_{1x}, U_{1y}, \gamma_2, \gamma_3\right)$$

$\gamma_2 = -43.852\,deg$ $\gamma_3 = -76.170\,deg$

The components of the **U** vector are:

$U_{1x} = -3.223$ $U_{1y} = 6.080$ $\sigma := atan2\left(U_{1x}, U_{1y}\right)$ $\sigma = 117.929\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\ 2} + U_{1y}^{\ 2}\right)}$, $u = 6.881$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 3.744$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 2.497$

$\theta_3 := atan2\left(V_{1x}, V_{1y}\right)$ $\theta_3 = 33.700\,deg$

$v := \sqrt{\left(V_{1x}^{\ 2} + V_{1y}^{\ 2}\right)}$ $v = 4.500$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 2.551$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 0.596$

$\theta_1 := atan2\left(G_{1x}, G_{1y}\right)$ $\theta_1 = 13.155\,deg$

$g := \sqrt{\left(G_{1x}^{\ 2} + G_{1y}^{\ 2}\right)}$ $g = 2.620$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 123.420\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 45.261\,deg$

8. Define the coupler point with respect to point C and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos\left(\phi\right) - w\cdot\cos\left(\theta\right)$ $O_{2x} = 4.416$

$O_{2y} := -z\cdot\sin\left(\phi\right) - w\cdot\sin\left(\theta\right)$ $O_{2y} = -4.179$

$$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = 6.967$$

$$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = -3.583$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 13.155\ deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array} \right.$$

$$Condition(g, u, v, w) = \text{"Grashof"}$$

12. **DESIGN SUMMARY**

Link 2:	$w = 6.080$	$\theta = 136.576\ deg$
Link 3:	$v = 4.500$	$\theta_3 = 33.700\ deg$
Link 4:	$u = 6.881$	$\sigma = 117.929\ deg$
Link 1:	$g = 2.620$	$\theta_1 = 13.155\ deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\ deg$

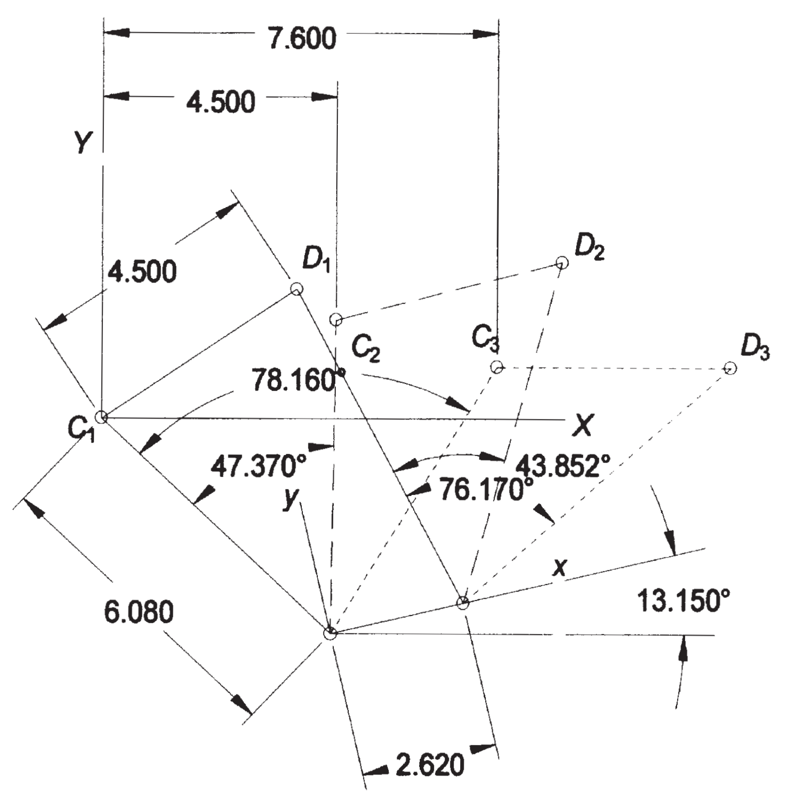
Crank angles:

$$\theta_{2i} = 123.420\ deg$$

$$\theta_{2f} = 45.261\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.



PROBLEM 5-39

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 4.500$ $\quad P_{2y} := 1.900$

$P_{3x} := 7.600$ $\quad P_{3y} := 1.000$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 33.70 \cdot deg$ $\quad \theta_{P2} := 14.60 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1:

$C_{1x} := 0.0$ $\quad C_{1y} := 0.0$ $\quad E_{1x} := 3.744$ $\quad E_{1y} := 0.000$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P5-9 and Mathcad file P0539.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 4.885 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 22.891\ deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 7.666 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 7.496\ deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -19.100\ deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -33.700\ deg$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \qquad z = 0.000$$

$$s := \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} \qquad s = 3.744$$

$$v := \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} \qquad v = 3.744$$

$$\phi := atan2(E_{1x} - C_{1x}, E_{1y} - C_{1y}) \qquad \phi = 0.000\ deg$$

$$\psi := \phi + \pi \qquad \psi = 180.000\ deg$$

$Z_{1x} := z \cdot \cos(\phi)$ $\qquad Z_{1x} = 0.000$

$Z_{1y} := z \cdot \sin(\phi)$ $\qquad Z_{1y} = 0.000$

$S_{1x} := s \cdot \cos(\psi)$ $\qquad S_{1x} = -3.744$

$S_{1y} := s \cdot \sin(\psi)$ $\qquad S_{1y} = 0.000$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-57.

Guess: $\quad W_{1x} := -4 \quad W_{1y} := 4 \quad \beta_2 := -50 \cdot deg \quad \beta_3 := -80 \cdot deg$

Given

$$W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) \ldots = p_{21} \cdot \cos(\delta_2)$$
$$+ Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2)$$

$$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) \ldots = p_{31} \cdot \cos(\delta_3)$$
$$+ Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3)$$

$$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) \ldots = p_{21} \cdot \sin(\delta_2)$$
$$+ Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2)$$

$$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) \ldots = p_{31} \cdot \sin(\delta_3)$$
$$+ Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = -47.370\, deg \qquad \beta_3 = -78.160\, deg$

The components of the **W** vector are:

$W_{1x} = -4.416 \qquad W_{1y} = 4.179 \qquad \theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 136.576\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\ 2} + W_{1y}^{\ 2}\right)}$, $\quad w = 6.080$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

Guess: $\quad U_{1x} := -4 \quad U_{1y} := 4 \quad \gamma_2 := -44 \cdot deg \quad \gamma_3 := -76 \cdot deg$

Given

$$U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) \ldots = p_{21} \cdot \cos(\delta_2)$$
$$+ S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2)$$

$$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) \ldots = p_{31} \cdot \cos(\delta_3)$$
$$+ S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3)$$

$$U_{1y} \cdot (\cos(\gamma_2) - 1) + U_{1x} \cdot \sin(\gamma_2) \ldots = p_{21} \cdot \sin(\delta_2)$$
$$+ S_{1y} \cdot (\cos(\alpha_2) - 1) + S_{1x} \cdot \sin(\alpha_2)$$

$$U_{1y} \cdot (\cos(\gamma_3) - 1) + U_{1x} \cdot \sin(\gamma_3) \ldots = p_{31} \cdot \sin(\delta_3)$$
$$+ S_{1y} \cdot (\cos(\alpha_3) - 1) + S_{1x} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = -48.844\, deg$ $\gamma_3 = -84.280\, deg$

The components of the **U** vector are:

$U_{1x} = -2.890$ $U_{1y} = 4.391$ $\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 123.354\, deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^2 + U_{1y}^2\right)}$, $u = 5.256$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 3.744$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 0.000\, deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 3.744$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 2.218$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -0.211$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = -5.444\, deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $g = 2.228$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 142.019\, deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 63.860\, deg$

8. Define the coupler point with respect to point *C* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\, deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$ $O_{2x} = 4.416$

$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$ $O_{2y} = -4.179$

$$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = 6.634$$

$$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = -4.391$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = -5.444\ deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(g, u, v, w) = \text{"Grashof"}$$

12. **DESIGN SUMMARY**

Link 2:	$w = 6.080$	$\theta = 136.576\ deg$
Link 3:	$v = 3.744$	$\theta_3 = 0.000\ deg$
Link 4:	$u = 5.256$	$\sigma = 123.354\ deg$
Link 1:	$g = 2.228$	$\theta_1 = -5.444\ deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\ deg$

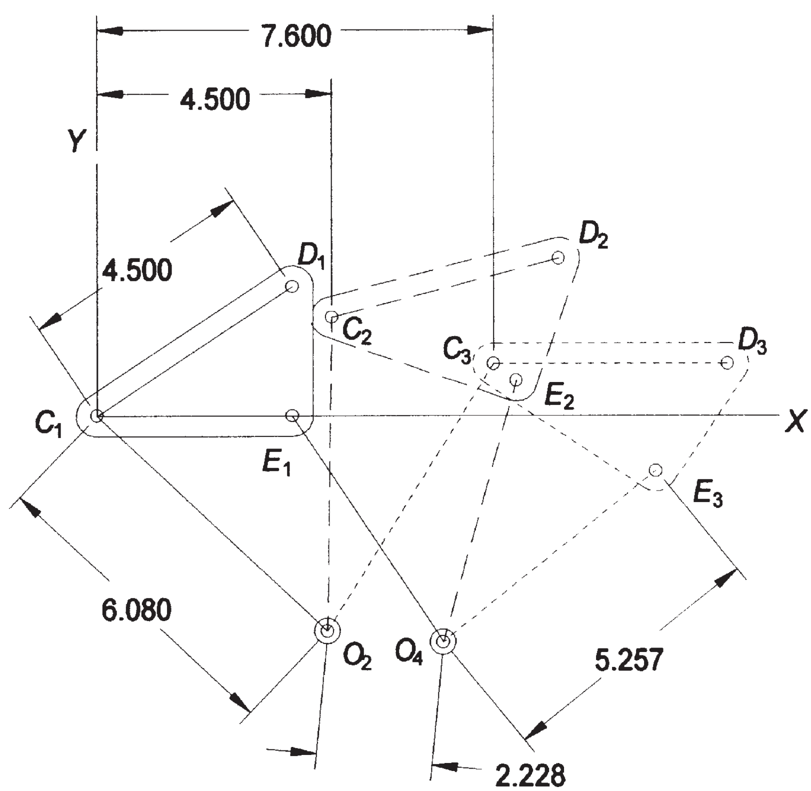
Crank angles:

$$\theta_{2i} = 142.019\ deg$$

$$\theta_{2f} = 63.860\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.



PROBLEM 5-40

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given:

$P_{21x} := 4.500$ $\quad P_{21y} := 1.900$ $\quad P_{31x} := 7.600$ $\quad P_{31y} := 1.000$

$O_{2x} := 2.900$ $\quad O_{2y} := -5.100$ $\quad O_{4x} := 5.900$ $\quad O_{4y} := -5.100$

Body angles: $\theta_{P1} := 33.70 \cdot deg$ $\quad \theta_{P2} := 14.60 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{vmatrix}$$

Solution: See Figure P5-9 and Mathcad file P0540.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = -19.100\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -33.700\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = -2.900$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 5.100$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 1.600$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 7.000$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 4.700$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 6.100$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 5.867$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 7.181$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 7.701$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 119.624\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 77.125\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 52.386\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 1.222$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = -0.710$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = -4.283$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = 0.248$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = -2.672$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = 1.232$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -18.405$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = -4.613$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -11.748$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = 3.343$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = 4.613$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -5.059$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 44.009$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -62.605$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \qquad K_3 = 71.350$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{31} = -33.700\ deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \beta_{32} = -76.089\ deg$$

The first value is the same as α_3, so use the second value $\quad \beta_3 := \beta_{32}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 47.808\ deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = -47.808\ deg$$

Use the negative value, $\quad \beta_2 := \beta_{22}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\quad R_{1x} = -5.900$ $\quad R_{1y} := -O_{4y}$ $\quad R_{1y} = 5.100$

$R_{2x} := R_{1x} + P_{21x}$ $R_{2x} = -1.400$

$R_{2y} := R_{1y} + P_{21y}$ $R_{2y} = 7.000$

$R_{3x} := R_{1x} + P_{31x}$ $R_{3x} = 1.700$

$R_{3y} := R_{1y} + P_{31y}$ $R_{3y} = 6.100$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $R_1 = 7.799$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $R_2 = 7.139$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $R_3 = 6.332$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\zeta_1 = 139.160\, deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\zeta_2 = 101.310\, deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\zeta_3 = 74.427\, deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = 0.883$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = -1.393$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = -3.779$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = -1.417$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -2.506$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 0.250$

$A_1 := -C_3^2 - C_4^2$ $A_1 = -16.286$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = -4.496$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -9.117$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = 4.011$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = 4.496$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -5.310$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 30.381$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = -60.441$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = 58.811$

$$\gamma_{31} := 2\cdot atan\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \gamma_{31} = -33.700\,deg$$

$$\gamma_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \gamma_{32} = -92.928\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5\cdot sin(\gamma_3) + A_3\cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 55.029\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3\cdot sin(\gamma_3) + A_2\cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -55.029\,deg$$

Use the negative value, $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the *X* axis.

$$p_{21} := \sqrt{P_{21x}^{\,2} + P_{21y}^{\,2}} \qquad p_{21} = 4.885$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 22.891\,deg$$

$$p_{31} := \sqrt{P_{31x}^{\,2} + P_{31y}^{\,2}} \qquad p_{31} = 7.666$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 7.496\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21}\cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31}\cdot cos(\delta_3) \qquad M := p_{21}\cdot sin(\delta_2) \qquad N := p_{31}\cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1}\cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W1x = -5.043$ $\quad W1y = 3.126$ $\quad Z1x = 2.143$ $\quad Z1y = 1.974$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $\quad w = 5.933$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -2.773$ $\quad U1y = 2.998$ $\quad S1x = -3.127$ $\quad S1y = 2.102$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $\quad u = 4.083$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $\quad V1x = 5.271$

$V1y := Z1y - S1y$ $\quad V1y = -0.128$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $\quad v = 5.272$

$G1x := W1x + V1x - U1x$ $\quad G1x = 3.000$

$G1y := W1y + V1y - U1y$ $\quad G1y = -1.688 \times 10^{-14}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $\quad g = 3.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $\quad O2x = 2.900$

$O2y := -Z1y - W1y$ $\quad O2y = -5.100$

$O4x := -S1x - U1x$ $\quad O4x = 5.900$

$O4y := -S1y - U1y$ $\quad O4y = -5.100$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 2.914$ $rp := z$

Angle *BAP* (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 3.768$

$\psi := atan2(S1x, S1y)$ $\psi = 146.092\,deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = 42.651\,deg$

$\theta_3 := atan2\left(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi)\right)$

$\theta_3 = -1.392\,deg$

$\delta_p := \phi - \theta_3$ $\delta_p = 44.042\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 3.000$	
Link 2:	$w = 5.933$	
Link 3:	$v = 5.272$	
Link 4:	$u = 4.083$	
Coupler point:	$rp = 2.914$	$\delta_p = 44.042\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-41

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points C and D for your attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -0.743$ $\quad P_{2y} := 1.514$

$P_{3x} := -1.750$ $\quad P_{3y} := 2.228$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 62.59 \cdot deg$ $\quad \theta_{P2} := 68.25 \cdot deg$ $\quad \theta_{P3} := 90.0 \cdot deg$

Coordinates of the points C_1 and D_1 with respect to P_1:

$C_{1x} := 0.0$ $\quad C_{1y} := 0.0$ $\quad D_{1x} := 1.036$ $\quad D_{1y} := 1.998$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5 \cdot \pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P5-10 and Mathcad file P0541.

1. Determine the magnitudes and orientation of the position difference vectors.

$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2}$ $\quad p_{21} = 1.686$ $\quad \delta_2 := atan2(P_{2x}, P_{2y})$ $\quad \delta_2 = 116.140\,deg$

$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2}$ $\quad p_{31} = 2.833$ $\quad \delta_3 := atan2(P_{3x}, P_{3y})$ $\quad \delta_3 = 128.148\,deg$

2. Determine the angle changes of the coupler between precision points.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = 5.660\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = 27.410\,deg$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2}$ $\quad z = 0.000$

$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2}$ $\quad s = 2.251$

$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2}$ $\quad v = 2.251$

$\phi := \theta_{P1}$ $\quad \phi = 62.590\,deg$

$\psi := \theta_{P1} + \pi$ $\quad \psi = 242.590\,deg$

$Z_{1x} := z \cdot cos(\phi)$ $\qquad Z_{1x} = 0.000$

$Z_{1y} := z \cdot sin(\phi)$ $\qquad Z_{1y} = 0.000$

$S_{1x} := -s \cdot cos(\theta_{P1})$ $\qquad S_{1x} = -1.036$

$S_{1y} := -s \cdot sin(\theta_{P1})$ $\qquad S_{1y} = -1.998$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points *C* and *D* are to be used as pivots, *z* and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-61.

Guess: $\quad W_{1x} := 3 \quad W_{1y} := 0.5 \quad \beta_2 := 33.3 \cdot deg \quad \beta_3 := 57.0 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) \ldots + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) \ldots + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) \ldots + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) \ldots + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = 33.028\, deg \qquad \beta_3 = 57.045\, deg$

The components of the **W** vector are:

$W_{1x} = 2.925 \qquad W_{1y} = 0.496 \qquad \theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 9.626\, deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}{}^2 + W_{1y}{}^2\right)}$, $\quad w = 2.967$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points *C* and *D* are to be used as pivots, *s* and ψ are known from the calculations above.

Guess: $\quad U_{1x} := 3.2 \quad U_{1y} := 0.8 \quad \gamma_2 := 32.3 \cdot deg \quad \gamma_3 := 68.4 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) \ldots + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) \ldots + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y} \cdot (cos(\gamma_2) - 1) + U_{1x} \cdot sin(\gamma_2) \ldots + S_{1y} \cdot (cos(\alpha_2) - 1) + S_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$U_{1y}\cdot(\cos(\gamma_3)-1)+U_{1x}\cdot\sin(\gamma_3)\ ...\ +S_{1y}\cdot(\cos(\alpha_3)-1)+S_{1x}\cdot\sin(\alpha_3) = p_{31}\cdot\sin(\delta_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = 32.559\,deg$ $\gamma_3 = 68.259\,deg$

The components of the **U** vector are:

$U_{1x} = 3.223$ $U_{1y} = 0.815$ $\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 14.189\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^2 + U_{1y}^2\right)}$, $u = 3.324$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 1.036$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 1.998$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 62.590\,deg$

$v := \sqrt{\left(V_{1x}^2 + V_{1y}^2\right)}$ $v = 2.251$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 0.738$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 1.679$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 66.275\,deg$

$g := \sqrt{\left(G_{1x}^2 + G_{1y}^2\right)}$ $g = 1.834$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = -56.650\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 0.395\,deg$

8. Define the coupler point with respect to point *C* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta)$ $O_{2x} = -2.925$

$O_{2y} := -z\cdot\sin(\phi) - w\cdot\sin(\theta)$ $O_{2y} = -0.496$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = -2.187$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = 1.183$

10. **Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.**

$\theta_{rot} := atan2[(O_{4x} - O_{2x}),(O_{4y} - O_{2y})]$ $\theta_{rot} = 66.275\,deg$

11. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(g,u,v,w) =$ "Grashof"

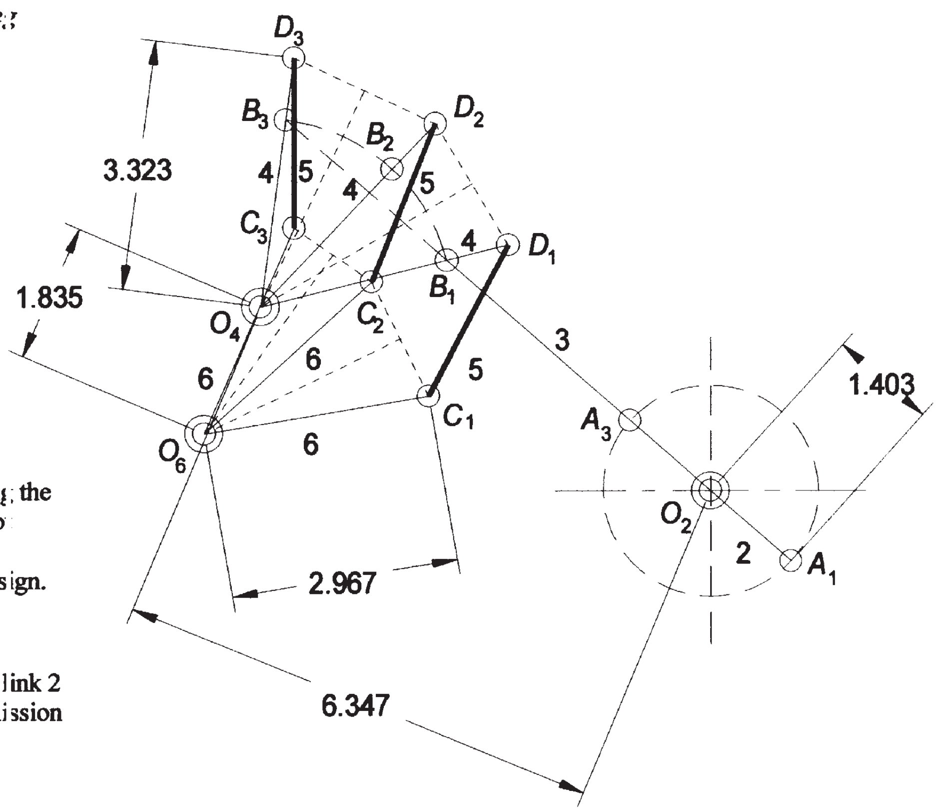
12. **DESIGN SUMMARY**

Link 2:	$w = 2.967$	$\theta = 9.626\,deg$
Link 3:	$v = 2.251$	$\theta_3 = 62.590\,deg$
Link 4:	$u = 3.324$	$\sigma = 14.189\,deg$
Link 1:	$g = 1.834$	$\theta_1 = 66.275\,deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\,deg$

Crank angles:

$\theta_{2i} = -56.650\,deg$

$\theta_{2f} = 0.395\,deg$



13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.

PROBLEM 5-42

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $P_{1y} := 0.0$ $P_{2x} := -0.743$ $P_{2y} := 1.514$

$P_{3x} := -1.750$ $P_{3y} := 2.228$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 62.59 \cdot deg$ $\theta_{P2} := 68.25 \cdot deg$ $\theta_{P3} := 90.0 \cdot deg$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1:

$C_{1x} := 0.0$ $C_{1y} := 0.0$ $E_{1x} := 1.036$ $E_{1y} := 0.000$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-10 and Mathcad file P0542.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 1.686 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 116.140\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 2.833 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 128.148\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = 5.660\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = 27.410\,deg$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \qquad z = 0.000$$

$$s := \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} \qquad s = 1.036$$

$$v := \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} \qquad v = 1.036$$

$$\phi := atan2(E_{1x} - C_{1x}, E_{1y} - C_{1y}) \qquad \phi = 0.000\,deg$$

$$\psi := \phi + \pi \qquad \psi = 180.000\,deg$$

$Z_{1x} := z \cdot \cos(\phi)$ $\quad Z_{1x} = 0.000$

$Z_{1y} := z \cdot \sin(\phi)$ $\quad Z_{1y} = 0.000$

$S_{1x} := s \cdot \cos(\psi)$ $\quad S_{1x} = -1.036$

$S_{1y} := s \cdot \sin(\psi)$ $\quad S_{1y} = 0.000$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-61.

Guess: $W_{1x} := 3$ $\quad W_{1y} := 0.5$ $\quad \beta_2 := 33.3 \cdot deg$ $\quad \beta_3 := 57.0 \cdot deg$

Given

$$W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) + Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2) = p_{21} \cdot \cos(\delta_2)$$

$$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) + Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3) = p_{31} \cdot \cos(\delta_3)$$

$$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) + Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2) = p_{21} \cdot \sin(\delta_2)$$

$$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) + Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3) = p_{31} \cdot \sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = 33.028\,deg$ $\quad \beta_3 = 57.045\,deg$

The components of the **W** vector are:

$W_{1x} = 2.925$ $\quad W_{1y} = 0.496$ $\quad \theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 9.626\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\,2} + W_{1y}^{\,2}\right)}$, $\quad w = 2.967$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := 3.2$ $\quad U_{1y} := 0.8$ $\quad \gamma_2 := 32.3 \cdot deg$ $\quad \gamma_3 := 68.4 \cdot deg$

Given

$$U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) + S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2) = p_{21} \cdot \cos(\delta_2)$$

$$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) + S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3) = p_{31} \cdot \cos(\delta_3)$$

$$U_{1y} \cdot (\cos(\gamma_2) - 1) + U_{1x} \cdot \sin(\gamma_2) + S_{1y} \cdot (\cos(\alpha_2) - 1) + S_{1x} \cdot \sin(\alpha_2) = p_{21} \cdot \sin(\delta_2)$$

$$U_{1y}\cdot(\cos(\gamma_3) - 1) + U_{1x}\cdot\sin(\gamma_3) \ldots = p_{31}\cdot\sin(\delta_3)$$
$$+S_{1y}\cdot(\cos(\alpha_3) - 1) + S_{1x}\cdot\sin(\alpha_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \mathit{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = 22.323\,deg$ $\gamma_3 = 41.857\,deg$

The components of the **U** vector are:

$U_{1x} = 4.470$ $U_{1y} = 1.088$ $\sigma := atan2(U_{1x}, U_{1y})$ $\sigma = 13.676\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}{}^2 + U_{1y}{}^2\right)}$, $u = 4.600$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 1.036$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 0.000$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 0.000\,deg$

$v := \sqrt{\left(V_{1x}{}^2 + V_{1y}{}^2\right)}$ $v = 1.036$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = -0.509$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -0.591$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 229.301\,deg$

$g := \sqrt{\left(G_{1x}{}^2 + G_{1y}{}^2\right)}$ $g = 0.780$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = -219.675\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = -162.630\,deg$

8. Define the coupler point with respect to point *C* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta)$ $O_{2x} = -2.925$

$O_{2y} := -z\cdot\sin(\phi) - w\cdot\sin(\theta)$ $O_{2y} = -0.496$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $\qquad$ $O_{4x} = -3.434$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $\qquad$ $O_{4y} = -1.088$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right),\left(O_{4y} - O_{2y}\right)\right]$ $\qquad$ $\theta_{rot} = 229.301\ deg$

11. Determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(g,u,v,w) = \text{"non-Grashof"}$

12. **DESIGN SUMMARY**

Link 2:	$w = 2.967$	$\theta = 9.626\ deg$
Link 3:	$v = 1.036$	$\theta_3 = 0.000\ deg$
Link 4:	$u = 4.600$	$\sigma = 13.676\ deg$
Link 1:	$g = 0.780$	$\theta_1 = 229.301\ deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\ deg$

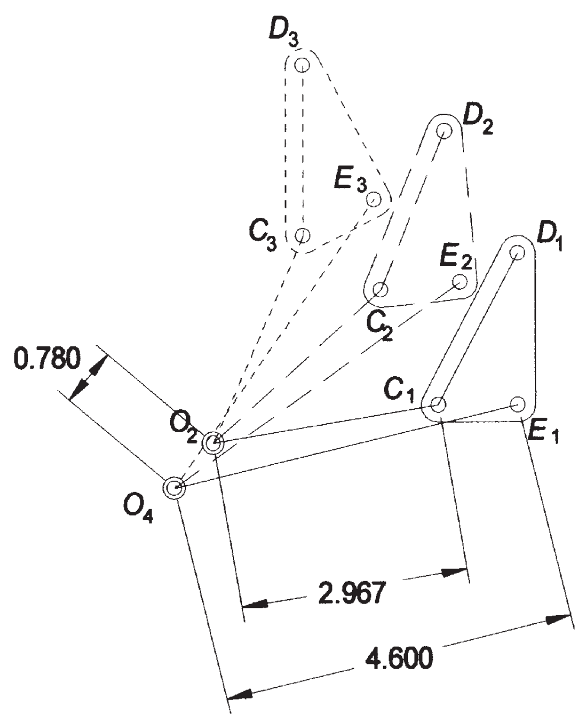
Crank angles:

$\theta_{2i} = -219.675\ deg$

$\theta_{2f} = -162.630\ deg$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.



PROBLEM 5-43

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given:

$P_{21x} := -0.743$ $\quad P_{21y} := 1.514$ $\quad P_{31x} := -1.750$ $\quad P_{31y} := 2.228$

$O_{2x} := -3.100$ $\quad O_{2y} := -1.200$ $\quad O_{4x} := -0.100$ $\quad O_{4y} := -1.200$

Body angles: $\quad \theta_{P1} := 62.59 \cdot deg$ $\quad \theta_{P2} := 68.25 \cdot deg$ $\quad \theta_{P3} := 90.0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Figure P5-10 and Mathcad file P0543.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = 5.660\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = 27.410\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 3.100$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 1.200$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 2.357$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 2.714$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 1.350$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 3.428$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 3.324$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 3.595$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 3.684$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 21.161\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 49.027\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 68.505\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = 0.162$

$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = 0.050$

$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3)$ $\qquad C_3 = 0.850$

$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3)$ $\qquad C_4 = 0.936$

$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2)$ $\qquad C_5 = 0.610$

$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$ $\qquad C_6 = 1.214$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -1.597$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 0.461$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -1.654$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = 0.194$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -0.461$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = 0.091$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = -0.061$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = -0.364$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\qquad K_3 = -0.221$

$\beta_{31} := 2 \cdot \operatorname{atan}\left(\dfrac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\qquad \beta_{31} = 27.410\,deg$

$\beta_{32} := 2 \cdot \operatorname{atan}\left(\dfrac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\qquad \beta_{32} = 133.549\,deg$

The first value is the same as α_3 so use the second value $\qquad \beta_3 := \beta_{32}$

$\beta_{21} := \operatorname{acos}\left(\dfrac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$ $\qquad \beta_{21} = 124.137\,deg$

$\beta_{22} := \operatorname{asin}\left(\dfrac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$ $\qquad \beta_{22} = 55.863\,deg$

Use the first value, $\qquad \beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\qquad R_{1x} = 0.100$ $\qquad R_{1y} := -O_{4y}$ $\qquad R_{1y} = 1.200$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = -0.643$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 2.714$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -1.650$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 3.428$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 1.204$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 2.789$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 3.804$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 85.236\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 103.329\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 115.703\ deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = -0.160$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = 1.135$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\qquad C_3 = 1.186$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\qquad C_4 = 2.317$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\qquad C_5 = 0.624$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\qquad C_6 = 1.510$

$A_1 := -C_3^2 - C_4^2$ $\qquad A_1 = -6.774$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 0.345$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = -4.239$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = 0.977$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -0.345$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -2.820$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = 12.289$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = -3.165$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\qquad K_3 = 9.452$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 27.410\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -56.299\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 43.866\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -43.866\,deg$$

Use the negative value , $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 1.686$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 116.140\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 2.833$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 128.148\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W1x = 0.621$ $W1y = -0.489$ $Z1x = 2.479$ $Z1y = 1.689$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 0.790$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $B' := \sin(\gamma_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21}\cdot\cos(\delta_2)$ $F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31}\cdot\cos(\delta_3)$ $M := p_{21}\cdot\sin(\delta_2)$ $N := p_{31}\cdot\sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1}\cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -1.459$ $U1y = -1.294$ $S1x = 1.559$ $S1y = 2.494$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 1.950$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 0.920$

$V1y := Z1y - S1y$ $V1y = -0.805$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 1.222$

$G1x := W1x + V1x - U1x$ $G1x = 3.000$

$G1y := W1y + V1y - U1y$ $G1y = 0.000$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 3.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -3.100$

$O2y := -Z1y - W1y$ $O2y = -1.200$

$O4x := -S1x - U1x$ $O4x = -0.100$

$O4y := -S1y - U1y$ $O4y = -1.200$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $\quad z := \sqrt{\left(Z1x^2 + Z1y^2\right)} \quad z = 3.000 \quad rp := z$

Angle BAP (δ_p) $\quad s := \sqrt{S1x^2 + S1y^2} \quad s = 2.941$

$\psi := atan2(S1x, S1y) \quad \psi = 57.983\,deg$

$\phi := atan2(Z1x, Z1y) \quad \phi = 34.270\,deg$

$\theta_3 := atan2\left(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi)\right)$

$\theta_3 = -41.182\,deg$

$\delta_p := \phi - \theta_3 \quad \delta_p = 75.452\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 3.000$	
Link 2:	$w = 0.790$	
Link 3:	$v = 1.222$	
Link 4:	$u = 1.950$	
Coupler point:	$rp = 3.000$	$\delta_p = 75.452\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. However, this mechanism as designed has a branch defect. The three positions can only be reached by changing the links from a crossed circuit to an open circuit.

PROBLEM 5-44

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points *C* and *D* for your attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -2.332$ $\quad P_{2y} := -0.311$

$P_{3x} := -2.751$ $\quad P_{3y} := -2.015$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 45.0 \cdot deg$ $\quad \theta_{P2} := 24.14 \cdot deg$ $\quad \theta_{P3} := 86.84 \cdot deg$

Coordinates of the points C_1 and D_1 with respect to P_1:

$C_{1x} := 0.0$ $\quad C_{1y} := 0.0$ $\quad D_{1x} := 1.591$ $\quad D_{1y} := 1.591$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-11 and Mathcad file P0544.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 2.353 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 187.596\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 3.410 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 216.221\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -20.860\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = 41.840\,deg$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \qquad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} \qquad s = 2.250$$

$$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} \qquad v = 2.250$$

$$\phi := \theta_{P1} \qquad \phi = 45.000\,deg$$

$$\psi := \theta_{P1} + \pi \qquad \psi = 225.000\,deg$$

$Z_{1x} := z \cdot \cos(\phi)$ $\qquad Z_{1x} = 0.000$

$Z_{1y} := z \cdot \sin(\phi)$ $\qquad Z_{1y} = 0.000$

$S_{1x} := -s \cdot \cos(\theta_{P1})$ $\qquad S_{1x} = -1.591$

$S_{1y} := -s \cdot \sin(\theta_{P1})$ $\qquad S_{1y} = -1.591$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-65.

Guess: $\quad W_{1x} := -1.75 \quad W_{1y} := -0.47 \qquad \beta_2 := 81 \cdot deg \qquad \beta_3 := 138 \cdot deg$

Given

$$W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) \ldots + Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2) = p_{21} \cdot \cos(\delta_2)$$

$$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) \ldots + Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3) = p_{31} \cdot \cos(\delta_3)$$

$$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) \ldots + Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2) = p_{21} \cdot \sin(\delta_2)$$

$$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) \ldots + Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3) = p_{31} \cdot \sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = 79.928\,deg \qquad \beta_3 = 137.178\,deg$

The components of the **W** vector are:

$W_{1x} = 0.980 \qquad W_{1y} = 1.547 \qquad \theta := atan2(W_{1x}, W_{1y}) \qquad \theta = 57.632\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^{\,2} + W_{1y}^{\,2}\right)}, \quad w = 1.831$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

Guess: $\quad U_{1x} := -0.1 \quad U_{1y} := -7.0 \qquad \gamma_2 := -17.5 \cdot deg \qquad \gamma_3 := -37 \cdot deg$

Given

$$U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) \ldots + S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2) = p_{21} \cdot \cos(\delta_2)$$

$$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) \ldots + S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3) = p_{31} \cdot \cos(\delta_3)$$

$$U_{1y} \cdot (\cos(\gamma_2) - 1) + U_{1x} \cdot \sin(\gamma_2) \ldots + S_{1y} \cdot (\cos(\alpha_2) - 1) + S_{1x} \cdot \sin(\alpha_2) = p_{21} \cdot \sin(\delta_2)$$

$$U_{1y}\cdot\left(\cos\left(\gamma_3\right)-1\right)+U_{1x}\cdot\sin\left(\gamma_3\right)\ ... \quad = p_{31}\cdot\sin\left(\delta_3\right)$$
$$+S_{1y}\cdot\left(\cos\left(\alpha_3\right)-1\right)+S_{1x}\cdot\sin\left(\alpha_3\right)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find\left(U_{1x}, U_{1y}, \gamma_2, \gamma_3\right)$$

$\gamma_2 = -17.457\,deg \qquad \gamma_3 = -37.139\,deg$

The components of the **U** vector are:

$U_{1x} = 4.132 \qquad U_{1y} = -5.598 \qquad \sigma := atan2\left(U_{1x}, U_{1y}\right) \qquad \sigma = -53.567\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^{\,2} + U_{1y}^{\,2}\right)}$, $\quad u = 6.958$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x} \qquad V_{1x} = 1.591$

$V_{1y} := Z_{1y} - S_{1y} \qquad V_{1y} = 1.591$

$\theta_3 := atan2\left(V_{1x}, V_{1y}\right) \qquad \theta_3 = 45.000\,deg$

$v := \sqrt{\left(V_{1x}^{\,2} + V_{1y}^{\,2}\right)} \qquad v = 2.250$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x} \qquad G_{1x} = -1.561$

$G_{1y} := W_{1y} + V_{1y} - U_{1y} \qquad G_{1y} = 8.736$

$\theta_1 := atan2\left(G_{1x}, G_{1y}\right) \qquad \theta_1 = 100.130\,deg$

$g := \sqrt{\left(G_{1x}^{\,2} + G_{1y}^{\,2}\right)} \qquad g = 8.874$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1 \qquad \theta_{2i} = -42.498\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3 \qquad \theta_{2f} = 94.681\,deg$

8. Define the coupler point with respect to point *C* and the vector **V**.

$r_p := z \qquad \delta_p := \phi - \theta_3$

$r_p = 0.000 \qquad \delta_p = 0.000\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos(\phi) - w\cdot\cos(\theta) \qquad O_{2x} = -0.980$

$O_{2y} := -z\cdot\sin(\phi) - w\cdot\sin(\theta) \qquad O_{2y} = -1.547$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \qquad O_{4x} = -2.541$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \qquad O_{4y} = 7.189$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 100.130\, deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } \; if \; SL < PQ \\ return \text{ "Special Grashof" } \; if \; SL = PQ \\ return \text{ "non-Grashof" } \; otherwise \end{array} \right.$$

$Condition(w, g, u, v) =$ "non-Grashof"

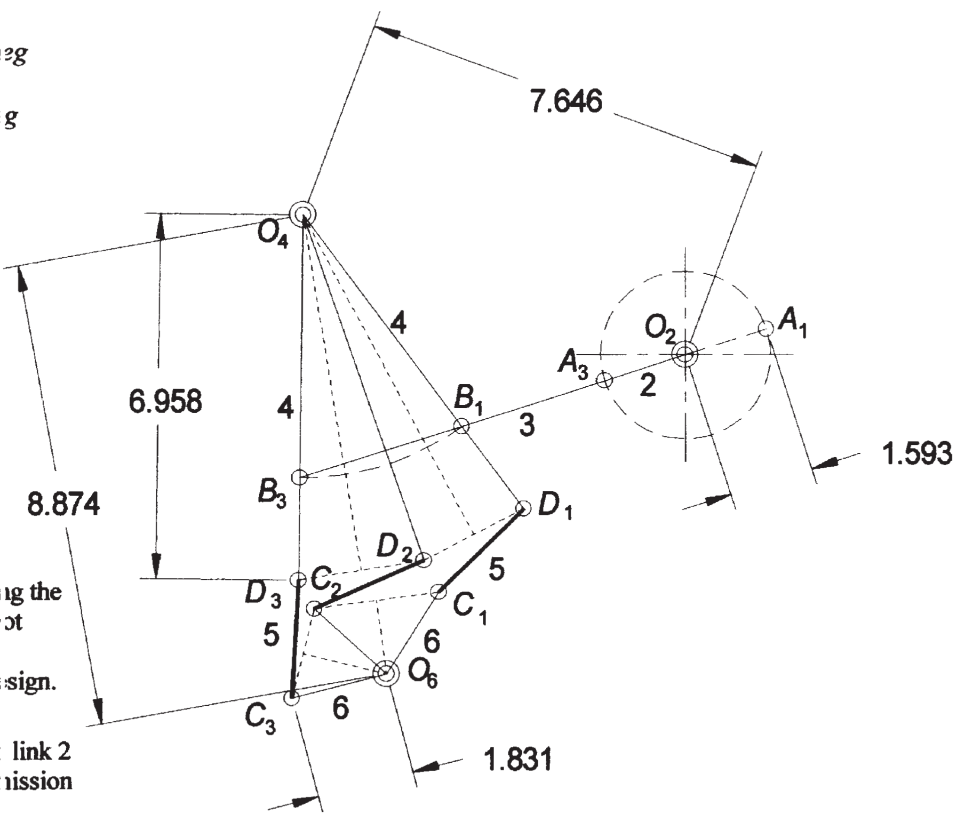
12. **DESIGN SUMMARY**

Link 2:	$w = 1.831$	$\theta = 57.632\, deg$
Link 3:	$v = 2.250$	$\theta_3 = 45.000\, deg$
Link 4:	$u = 6.958$	$\sigma = -53.567\, deg$
Link 1:	$g = 8.874$	$\theta_1 = 100.130\, deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\, deg$

Crank angles:

$\theta_{2i} = -42.498\, deg$

$\theta_{2f} = 94.681\, deg$



13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.

PROBLEM 5-45

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -2.332$ $\quad P_{2y} := -0.311$

$P_{3x} := -2.751$ $\quad P_{3y} := -2.015$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 45.0 \cdot deg$ $\quad \theta_{P2} := 24.14 \cdot deg$ $\quad \theta_{P3} := 86.84 \cdot deg$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1:

$C_{1x} := 0.0$ $\quad C_{1y} := 0.0$ $\quad E_{1x} := 1.591$ $\quad E_{1y} := 0.000$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P5-11 and Mathcad file P0545.

1. Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \qquad p_{21} = 2.353 \qquad \delta_2 := atan2(P_{2x}, P_{2y}) \qquad \delta_2 = 187.596\,deg$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \qquad p_{31} = 3.410 \qquad \delta_3 := atan2(P_{3x}, P_{3y}) \qquad \delta_3 = 216.221\,deg$$

2. Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -20.860\,deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = 41.840\,deg$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z, s, ϕ, and ψ.

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \qquad z = 0.000$$

$$s := \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} \qquad s = 1.591$$

$$v := \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} \qquad v = 1.591$$

$$\phi := atan2(E_{1x} - C_{1x}, E_{1y} - C_{1y}) \qquad \phi = 0.000\,deg$$

$$\psi := \phi + \pi \qquad \psi = 180.000\,deg$$

$Z_{1x} := z \cdot cos(\phi)$ $Z_{1x} = 0.000$

$Z_{1y} := z \cdot sin(\phi)$ $Z_{1y} = 0.000$

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = -1.591$

$S_{1y} := s \cdot sin(\psi)$ $S_{1y} = 0.000$

4. Use equations 5.24 to solve for w, θ, β_2, and β_3. Since the points *C* and *D* are to be used as pivots, *z* and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-65.

Guess: $W_{1x} := -1.75$ $W_{1y} := -0.47$ $\beta_2 := 81 \cdot deg$ $\beta_3 := 138 \cdot deg$

Given

$$W_{1x} \cdot (cos(\beta_2) - 1) - W_{1y} \cdot sin(\beta_2) \ldots + Z_{1x} \cdot (cos(\alpha_2) - 1) - Z_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$W_{1x} \cdot (cos(\beta_3) - 1) - W_{1y} \cdot sin(\beta_3) \ldots + Z_{1x} \cdot (cos(\alpha_3) - 1) - Z_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$W_{1y} \cdot (cos(\beta_2) - 1) + W_{1x} \cdot sin(\beta_2) \ldots + Z_{1y} \cdot (cos(\alpha_2) - 1) + Z_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$W_{1y} \cdot (cos(\beta_3) - 1) + W_{1x} \cdot sin(\beta_3) \ldots + Z_{1y} \cdot (cos(\alpha_3) - 1) + Z_{1x} \cdot sin(\alpha_3) = p_{31} \cdot sin(\delta_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := Find(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = 79.928\,deg$ $\beta_3 = 137.178\,deg$

The components of the **W** vector are:

$W_{1x} = 0.980$ $W_{1y} = 1.547$ $\theta := atan2(W_{1x}, W_{1y})$ $\theta = 57.632\,deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $w = 1.831$

5. Use equations 5.28 to solve for u, σ, γ_2, and γ_3. Since the points *C* and *D* are to be used as pivots, *s* and ψ are known from the calculations above.

Guess: $U_{1x} := 1$ $U_{1y} := -5.0$ $\gamma_2 := -17.5 \cdot deg$ $\gamma_3 := -37 \cdot deg$

Given

$$U_{1x} \cdot (cos(\gamma_2) - 1) - U_{1y} \cdot sin(\gamma_2) \ldots + S_{1x} \cdot (cos(\alpha_2) - 1) - S_{1y} \cdot sin(\alpha_2) = p_{21} \cdot cos(\delta_2)$$

$$U_{1x} \cdot (cos(\gamma_3) - 1) - U_{1y} \cdot sin(\gamma_3) \ldots + S_{1x} \cdot (cos(\alpha_3) - 1) - S_{1y} \cdot sin(\alpha_3) = p_{31} \cdot cos(\delta_3)$$

$$U_{1y} \cdot (cos(\gamma_2) - 1) + U_{1x} \cdot sin(\gamma_2) \ldots + S_{1y} \cdot (cos(\alpha_2) - 1) + S_{1x} \cdot sin(\alpha_2) = p_{21} \cdot sin(\delta_2)$$

$$U_{1y}\cdot\left(\cos\left(\gamma_3\right)-1\right)+U_{1x}\cdot\sin\left(\gamma_3\right)\ \ldots\ = p_{31}\cdot\sin\left(\delta_3\right)$$
$$+S_{1y}\cdot\left(\cos\left(\alpha_3\right)-1\right)+S_{1x}\cdot\sin\left(\alpha_3\right)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := Find\left(U_{1x}, U_{1y}, \gamma_2, \gamma_3\right)$$

$\gamma_2 = -21.548\,deg$ $\gamma_3 = -27.543\,deg$

The components of the **U** vector are:

$U_{1x} = 3.524$ $U_{1y} = -5.963$ $\sigma := atan2\left(U_{1x}, U_{1y}\right)$ $\sigma = -59.417\,deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}{}^2 + U_{1y}{}^2\right)}$, $u = 6.926$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 1.591$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 0.000$

$\theta_3 := atan2\left(V_{1x}, V_{1y}\right)$ $\theta_3 = 0.000\,deg$

$v := \sqrt{\left(V_{1x}{}^2 + V_{1y}{}^2\right)}$ $v = 1.591$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = -0.952$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 7.510$

$\theta_1 := atan2\left(G_{1x}, G_{1y}\right)$ $\theta_1 = 97.228\,deg$

$g := \sqrt{\left(G_{1x}{}^2 + G_{1y}{}^2\right)}$ $g = 7.570$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = -39.596\,deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 97.582\,deg$

8. Define the coupler point with respect to point *C* and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z\cdot\cos\left(\phi\right) - w\cdot\cos\left(\theta\right)$ $O_{2x} = -0.980$

$O_{2y} := -z\cdot\sin\left(\phi\right) - w\cdot\sin\left(\theta\right)$ $O_{2y} = -1.547$

$$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma) \qquad O_{4x} = -1.933$$

$$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma) \qquad O_{4y} = 5.963$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right] \qquad \theta_{rot} = 97.228\,deg$$

11. Determine the Grashof condition.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(v, g, u, w) = \text{"non-Grashof"}$$

12. **DESIGN SUMMARY**

Link 2:	$w = 1.831$	$\theta = 57.632\,deg$
Link 3:	$v = 1.591$	$\theta_3 = 0.000\,deg$
Link 4:	$u = 6.926$	$\sigma = -59.417\,deg$
Link 1:	$g = 7.570$	$\theta_1 = 97.228\,deg$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000\,deg$

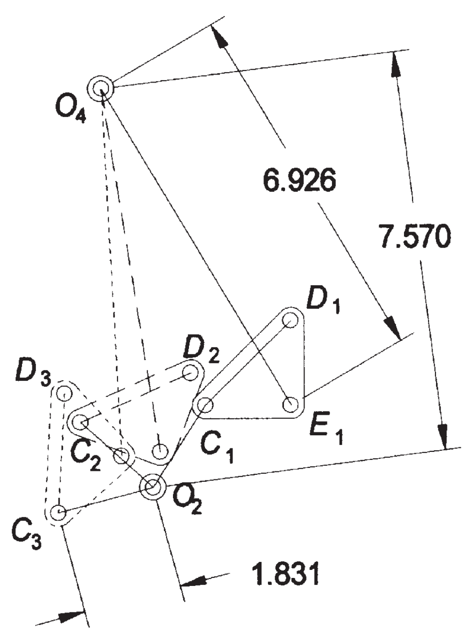
Crank angles:

$$\theta_{2i} = -39.596\,deg$$

$$\theta_{2f} = 97.582\,deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. Add a driver dyad to link 2 and check the transmission angles.



PROBLEM 5-46

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given:

$P_{21x} := -2.332$ $P_{21y} := -0.311$ $P_{31x} := -2.751$ $P_{31y} := -2.015$

$O_{2x} := -3.679$ $O_{2y} := -3.282$ $O_{4x} := 0.321$ $O_{4y} := -3.282$

Body angles: $\theta_{P1} := 45.0 \cdot deg$ $\theta_{P2} := 24.14 \cdot deg$ $\theta_{P3} := 86.84 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{vmatrix}$$

Solution: See Figure P5-11 and Mathcad file P0546.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\alpha_2 = -20.860\,deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\alpha_3 = 41.840\,deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $R_{1x} = 3.679$ $R_{1y} := -O_{2y}$ $R_{1y} = 3.282$

$R_{2x} := R_{1x} + P_{21x}$ $R_{2x} = 1.347$

$R_{2y} := R_{1y} + P_{21y}$ $R_{2y} = 2.971$

$R_{3x} := R_{1x} + P_{31x}$ $R_{3x} = 0.928$

$R_{3y} := R_{1y} + P_{31y}$ $R_{3y} = 1.267$

$R_1 := \sqrt{R_{1x}^{\,2} + R_{1y}^{\,2}}$ $R_1 = 4.930$

$R_2 := \sqrt{R_{2x}^{\,2} + R_{2y}^{\,2}}$ $R_2 = 3.262$

$R_3 := \sqrt{R_{3x}^{\,2} + R_{3y}^{\,2}}$ $R_3 = 1.571$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\zeta_1 = 41.736\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\zeta_2 = 65.611\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\zeta_3 = 53.780\,deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = 2.297$

$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2)$ $\qquad C_2 = -2.258$

$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3)$ $\qquad C_3 = -0.376$

$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3)$ $\qquad C_4 = -3.632$

$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2)$ $\qquad C_5 = 3.260$

$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$ $\qquad C_6 = 1.214$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $\qquad A_1 = -13.335$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\qquad A_2 = 11.382$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\qquad A_3 = 5.637$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\qquad A_4 = -7.492$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\qquad A_5 = -11.382$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\qquad A_6 = -9.068$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\qquad K_1 = -136.387$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\qquad K_2 = 60.979$

$$K_3 := \frac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2} \qquad K_3 = -60.933$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{31} = -90.019\,deg$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{32} = 41.840\,deg$$

The second value is the same as α_3 so use the first value $\qquad \beta_3 := \beta_{31}$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 99.989\,deg$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 80.011\,deg$$

Use the first value, $\qquad \beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\qquad R_{1x} = -0.321$ $\qquad R_{1y} := -O_{4y}$ $\qquad R_{1y} = 3.282$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = -2.653$

$R_{2y} := R_{1y} + P_{21y}$ $\quad R_{2y} = 2.971$

$R_{3x} := R_{1x} + P_{31x}$ $\quad R_{3x} = -3.072$

$R_{3y} := R_{1y} + P_{31y}$ $\quad R_{3y} = 1.267$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 3.298$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 3.983$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 3.323$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\quad \zeta_1 = 95.586\,deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\quad \zeta_2 = 131.764\,deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\quad \zeta_3 = 157.587\,deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\quad C_1 = 1.539$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $\quad C_2 = 1.834$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $\quad C_3 = 0.644$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $\quad C_4 = -0.964$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $\quad C_5 = 3.522$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $\quad C_6 = -0.210$

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -1.343$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = 3.260$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -2.469$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = -0.303$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = -3.260$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = 2.758$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = -7.799$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -8.243$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $\quad K_3 = -11.309$

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 41.840\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 51.335\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 8.321\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = 8.321\,deg$$

Both are the same so use the first value , $\gamma_2 := \gamma_{21}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.353$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 187.596\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 3.410$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 216.221\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A := cos(\beta_2) - 1$ $\qquad B := sin(\beta_2)$ $\qquad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\qquad E := p_{21} \cdot cos(\delta_2)$ $\qquad F := cos(\beta_3) - 1$

$G := sin(\beta_3)$ $\qquad H := cos(\alpha_3) - 1$ $\qquad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\qquad M := p_{21} \cdot sin(\delta_2)$ $\qquad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the **W** and **Z** vectors are:

$W_{1x} = 1.732$ $\qquad W_{1y} = 1.000$ $\qquad Z_{1x} = 1.947$ $\qquad Z_{1y} = 2.282$

11. The length of link 2 is: $w := \sqrt{\left(W1x^2 + W1y^2\right)}$ $w = 2.000$

12. Evaluate terms in the US coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $B' := sin(\gamma_2)$ $C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $E := p_{21} \cdot cos(\delta_2)$ $F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $H := cos(\alpha_3) - 1$ $K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -1.178$ $U1y = 6.903$ $S1x = 0.857$ $S1y = -3.621$

14. The length of link 4 is: $u := \sqrt{\left(U1x^2 + U1y^2\right)}$ $u = 7.002$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $V1x = 1.090$

$V1y := Z1y - S1y$ $V1y = 5.903$

The length of link 3 is: $v := \sqrt{\left(V1x^2 + V1y^2\right)}$ $v = 6.002$

$G1x := W1x + V1x - U1x$ $G1x = 4.000$

$G1y := W1y + V1y - U1y$ $G1y = 1.510 \times 10^{-14}$

The length of link 1 is: $g := \sqrt{\left(G1x^2 + G1y^2\right)}$ $g = 4.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = -3.679$

$O2y := -Z1y - W1y$ $O2y = -3.282$

$O4x := -S1x - U1x$ $O4x = 0.321$

$O4y := -S1y - U1y$ $O4y = -3.282$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := \sqrt{\left(Z1x^2 + Z1y^2\right)}$ $z = 3.000$ $rp := z$

Angle *BAP* (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 3.721$

$\psi := atan2(S1x, S1y)$ $\psi = -76.683\,deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = 49.525\,deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 79.535\,deg$

$\delta_p := \phi - \theta_3$ $\delta_p = -30.010\,deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 4.000$	
Link 2:	$w = 2.000$	
Link 3:	$v = 6.002$	
Link 4:	$u = 7.002$	
Coupler point:	$rp = 3.000$	$\delta_p = -30.010\,deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-47

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-40 using an equation solver or any program language.

Given:

$P_{21x} := 4.500$ $\quad P_{21y} := 1.900$ $\quad P_{31x} := 7.600$ $\quad P_{31y} := 1.000$

$O_{2x} := 2.900$ $\quad O_{2y} := -5.100$ $\quad O_{4x} := 5.900$ $\quad O_{4y} := -5.100$

Body angles: $\theta_{P1} := 33.70 \cdot deg$ $\quad \theta_{P2} := 14.60 \cdot deg$ $\quad \theta_{P3} := 0.0 \cdot deg$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := -47.808 deg$ and $\gamma_2 := -55.029 deg$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -19.100 deg$

Solution: See Figure P5-9 and Mathcad file P0547.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the *X* axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 4.885$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 22.891\, deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 7.666$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 7.496\, deg$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := -33.700 \cdot deg \qquad \beta_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F(\beta_3) := cos(\beta_3) - 1$$

$$G(\beta_3) := sin(\beta_3) \qquad H(\alpha_3) := cos(\alpha_3) - 1 \qquad K(\alpha_3) := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1 \qquad W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3 \qquad Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Problem 5-40:

$$W1x(\alpha_3, -76.089 \cdot deg) = -5.043 \qquad W1y(\alpha_3, -76.089 \cdot deg) = 3.126$$

$$Z1x(\alpha_3, -76.089 \cdot deg) = 2.143 \qquad Z1y(\alpha_3, -76.089 \cdot deg) = 1.974$$

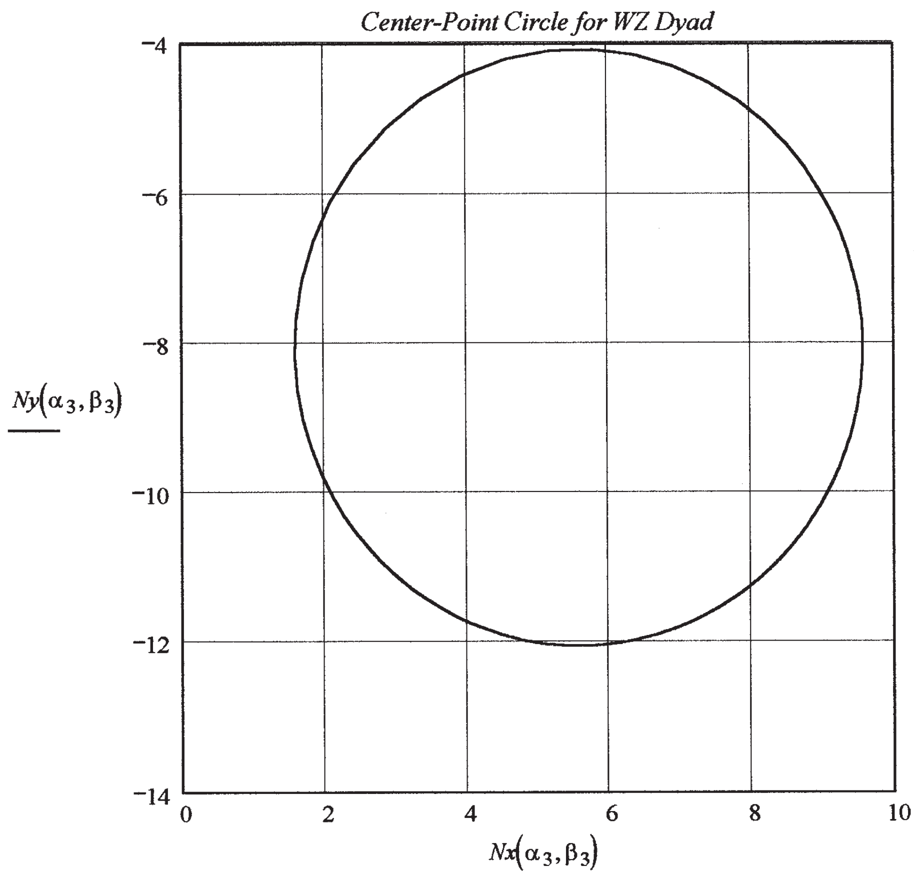
These are the same as the values calculated in Problem 5-40.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.

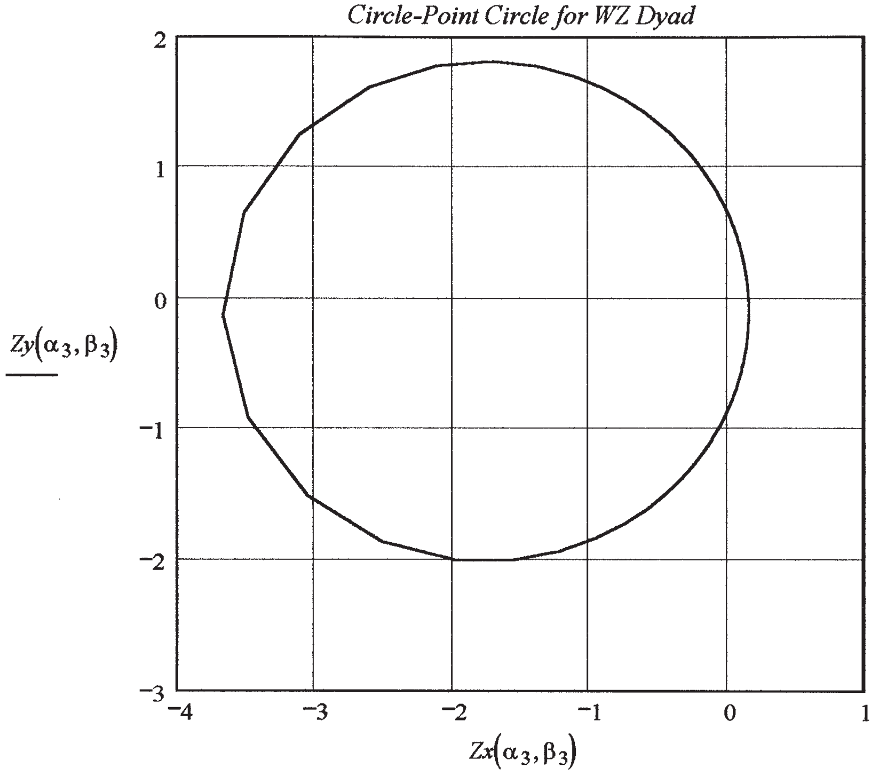


4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

$$\beta_3 := -76.089 \cdot deg \qquad \alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3) \qquad Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$\alpha_3 := -33.700 \cdot deg \qquad \gamma_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$A := \cos(\gamma_2) - 1 \qquad B := \sin(\gamma_2) \qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2) \qquad E := p_{21} \cdot \cos(\delta_2) \qquad F(\gamma_3) := \cos(\gamma_3) - 1$

$G(\gamma_3) := \sin(\gamma_3) \qquad H(\alpha_3) := \cos(\alpha_3) - 1 \qquad K(\alpha_3) := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3) \qquad M := p_{21} \cdot \sin(\delta_2) \qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1 \qquad U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$

$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3 \qquad S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$

7. Check this against the solutions in Problem 5-40:

$U1x(\alpha_3, -92.928 \cdot deg) = -2.773 \qquad U1y(\alpha_3, -92.928 \cdot deg) = 2.998$

$S1x(\alpha_3, -92.928 \cdot deg) = -3.128 \qquad S1y(\alpha_3, -92.928 \cdot deg) = 2.102$

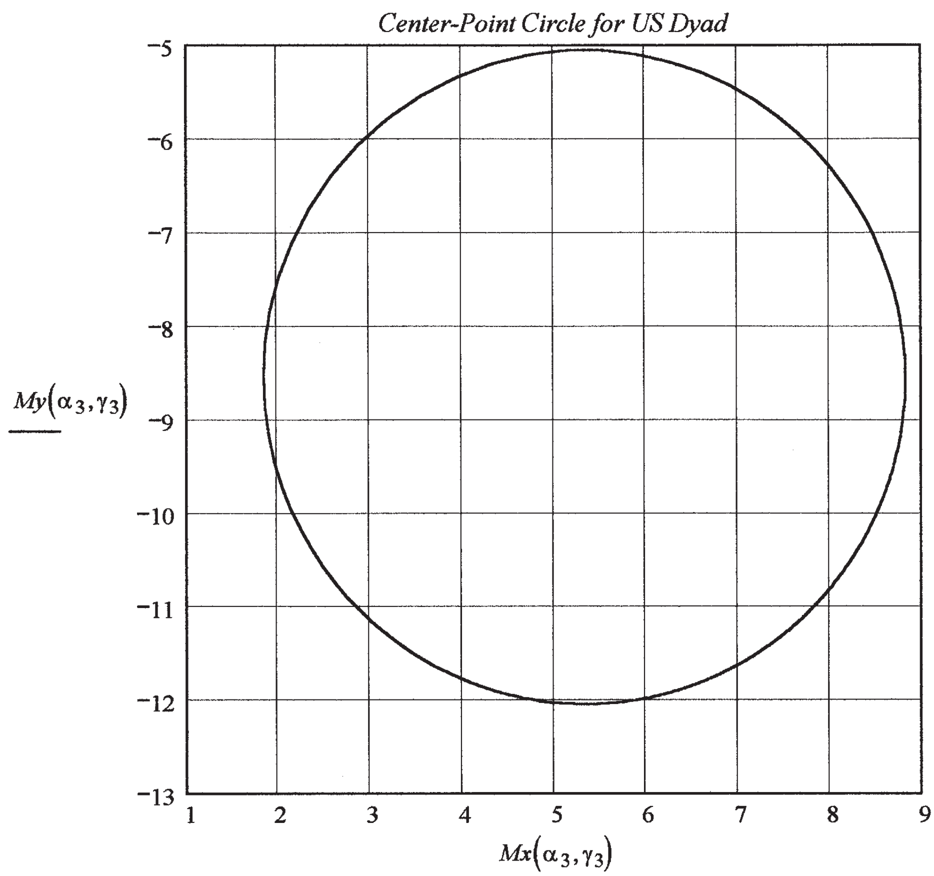
These are the same as the values calculated in Problem 5-40.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3,\gamma_3) := -U1x(\alpha_3,\gamma_3) - S1x(\alpha_3,\gamma_3)$$

$$My(\alpha_3,\gamma_3) := -U1y(\alpha_3,\gamma_3) - S1y(\alpha_3,\gamma_3)$$

9. Plot the center-point circle for the **US** dyad.

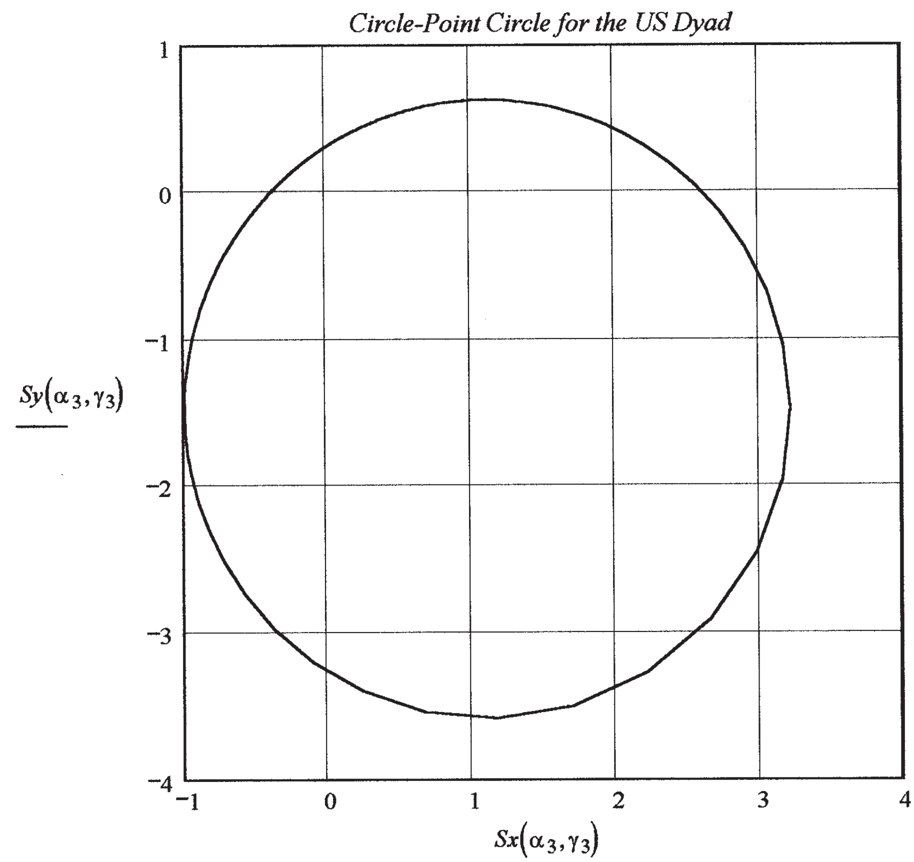


10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

$$\gamma_3 := -92.928 \cdot deg \qquad \alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$Sx(\alpha_3,\gamma_3) := -S1x(\alpha_3,\gamma_3) \qquad Sy(\alpha_3,\gamma_3) := -S1y(\alpha_3,\gamma_3)$$

11. Plot the circle-point circle for the **WZ** dyad (see next page).


Circle-Point Circle for the US Dyad
1
0
−1
−2
−3
−4
$Sy(\alpha_3, \gamma_3)$
−1
0
1
2
3
4
$Sx(\alpha_3, \gamma_3)$

PROBLEM 5-48

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-43 using an equation solver or any program language.

Given:

$P_{21x} := -0.743$ $\quad P_{21y} := 1.514$ $\quad P_{31x} := -1.750$ $\quad P_{31y} := 2.228$

$O_{2x} := -3.100$ $\quad O_{2y} := -1.200$ $\quad O_{4x} := -0.100$ $\quad O_{4y} := -1.200$

Body angles: $\theta_{P1} := 62.59 \cdot deg$ $\quad \theta_{P2} := 68.25 \cdot deg$ $\quad \theta_{P3} := 90.0 \cdot deg$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 124.137 deg$ and $\gamma_2 := -43.866 deg$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := 5.660 deg$

Solution: See Figure P5-10 and Mathcad file P0548.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 1.686$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 116.140\ deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 2.833$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 128.148\ deg$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$\alpha_3 := 27.410 \cdot deg$ $\qquad \beta_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$A := \cos(\beta_2) - 1$ $\qquad B := \sin(\beta_2)$ $\qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\qquad E := p_{21} \cdot \cos(\delta_2)$ $\qquad F(\beta_3) := \cos(\beta_3) - 1$

$G(\beta_3) := \sin(\beta_3)$ $\qquad H(\alpha_3) := \cos(\alpha_3) - 1$ $\qquad K(\alpha_3) := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\qquad M := p_{21} \cdot \sin(\delta_2)$ $\qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1$ $\qquad W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$

$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$ $\qquad Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$

3. Check this against the solutions in Problem 5-43:

$W1x(\alpha_3, 133.549 \cdot deg) = 0.621$ $\qquad$ $W1y(\alpha_3, 133.549 \cdot deg) = -0.489$

$Z1x(\alpha_3, 133.549 \cdot deg) = 2.479$ $\qquad$ $Z1y(\alpha_3, 133.549 \cdot deg) = 1.689$

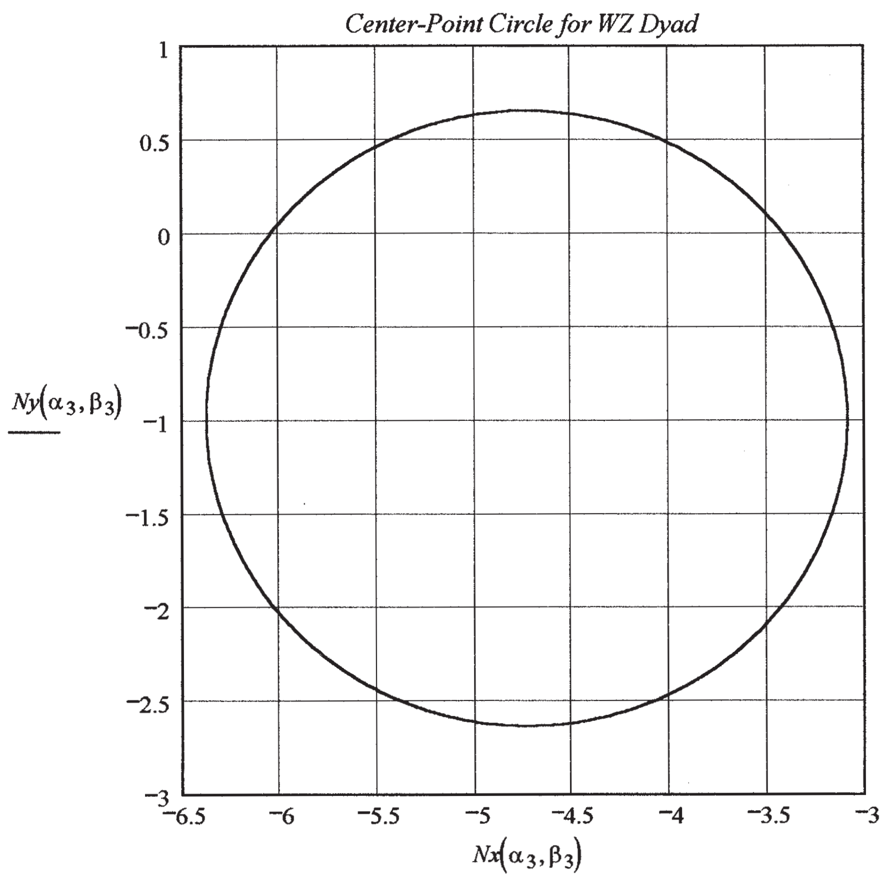
These are the same as the values calculated in Problem 5-43.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.

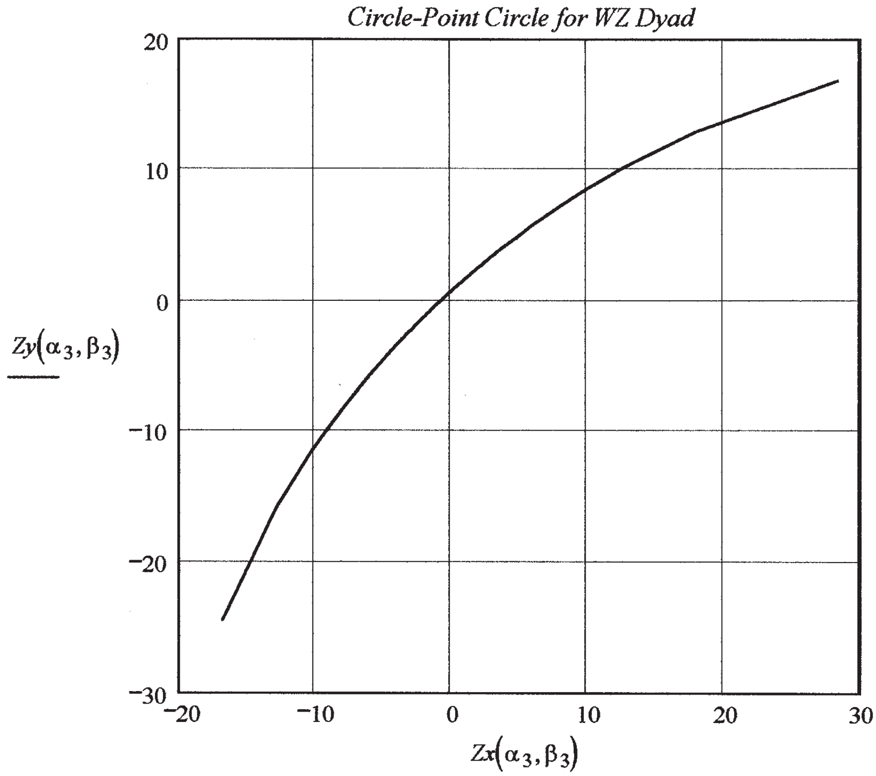


4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

$\beta_3 := 133.549 \cdot deg$ $\qquad$ $\alpha_3 := 8 \cdot deg, 9 \cdot deg .. 364 \cdot deg$

$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$ $\qquad$ $Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$

5. Plot the circle-point arc for the **WZ** dyad (see next page).



6. Evaluate terms in the US coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$\alpha_3 := 27.410 \cdot deg$ $\qquad \gamma_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$A := \cos(\gamma_2) - 1$ $\qquad B := \sin(\gamma_2)$ $\qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\qquad E := p_{21} \cdot \cos(\delta_2)$ $\qquad F(\gamma_3) := \cos(\gamma_3) - 1$

$G(\gamma_3) := \sin(\gamma_3)$ $\qquad H(\alpha_3) := \cos(\alpha_3) - 1$ $\qquad K(\alpha_3) := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\qquad M := p_{21} \cdot \sin(\delta_2)$ $\qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$ $\qquad U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$

$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$ $\qquad S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$

7. Check this against the solutions in Problem 5-43:

$U1x(\alpha_3, -56.299 \cdot deg) = -1.459$ $\qquad U1y(\alpha_3, -56.299 \cdot deg) = -1.294$

$S1x(\alpha_3, -56.299 \cdot deg) = 1.559$ $\qquad S1y(\alpha_3, -56.299 \cdot deg) = 2.494$

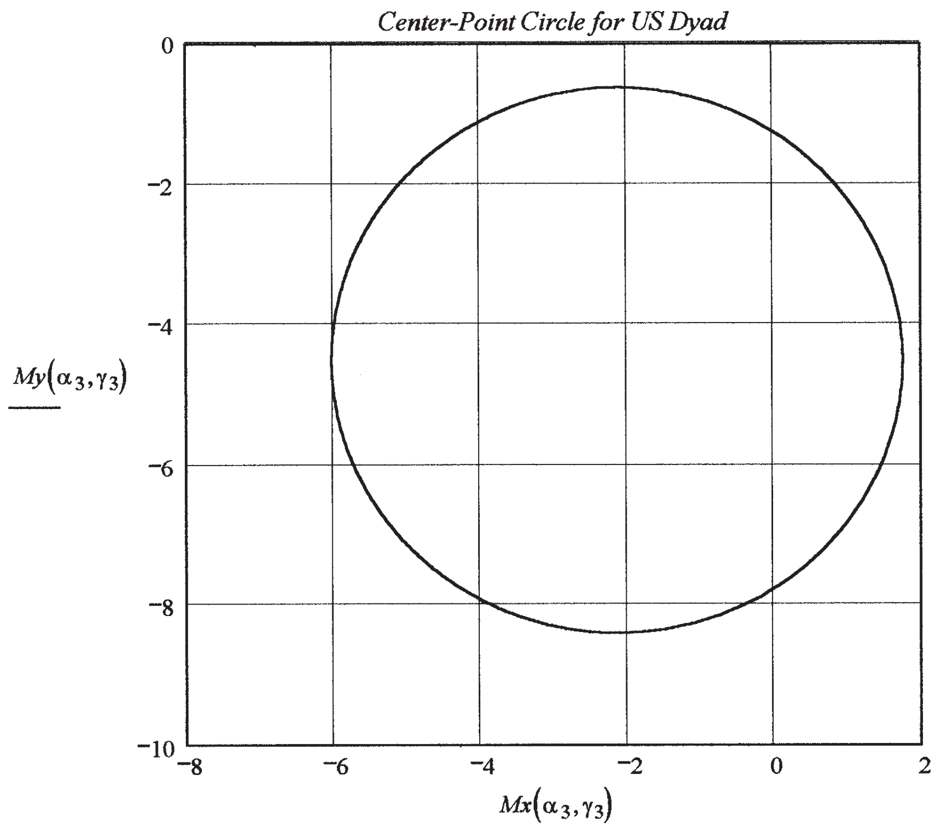
These are the same as the values calculated in Problem 5-43.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.

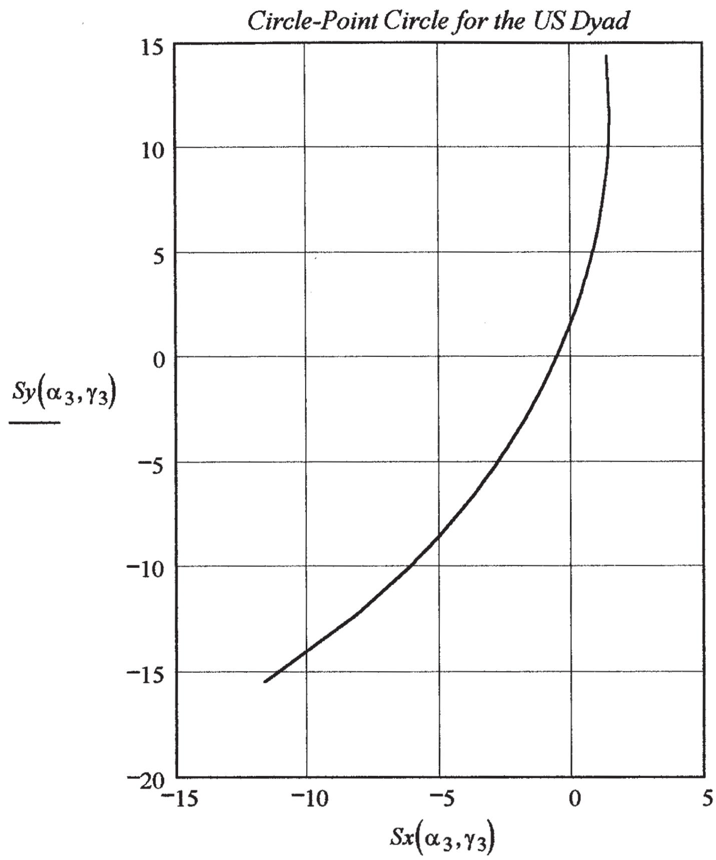


10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

$\gamma_3 := -56.299 \cdot deg$ $\alpha_3 := 10 \cdot deg, 11 \cdot deg .. 363 \cdot deg$

$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$ $Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$

11. Plot the circle-point arc for the **WZ** dyad (see next page).


Circle-Point Circle for the US Dyad
15
10
5
0
−5
−10
−15
−20
$Sy(\alpha_3, \gamma_3)$
−15
−10
−5
0
5
$Sx(\alpha_3, \gamma_3)$

PROBLEM 5-49

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-46 using an equation solver or any program language.

Given:

$P21_x := -2.332$ $P21_y := -0.311$ $P31_x := -2.751$ $P31_y := -2.015$

$O_{2x} := -3.679$ $O_{2y} := -3.282$ $O_{4x} := 0.321$ $O_{4y} := -3.282$

Body angles: $\theta_{P1} := 45.0\cdot deg$ $\theta_{P2} := 24.14\cdot deg$ $\theta_{P3} := 86.84\cdot deg$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 99.989deg$ and $\gamma_2 := 8.321deg$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -20.860deg$

Solution: See Figure P5-11 and Mathcad file P0549.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P21_x^{\,2} + P21_y^{\,2}} \qquad p_{21} = 2.353$$

$$\delta_2 := atan2(P21_x, P21_y) \qquad \delta_2 = -172.404\,deg$$

$$p_{31} := \sqrt{P31_x^{\,2} + P31_y^{\,2}} \qquad p_{31} = 3.410$$

$$\delta_3 := atan2(P31_x, P31_y) \qquad \delta_3 = -143.779\,deg$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 41.840\cdot deg \qquad \beta_3 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$$

$$A := \cos(\beta_2) - 1 \qquad B := \sin(\beta_2) \qquad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \qquad E := p_{21}\cdot\cos(\delta_2) \qquad F(\beta_3) := \cos(\beta_3) - 1$$

$$G(\beta_3) := \sin(\beta_3) \qquad H(\alpha_3) := \cos(\alpha_3) - 1 \qquad K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31}\cdot\cos(\delta_3) \qquad M := p_{21}\cdot\sin(\delta_2) \qquad N := p_{31}\cdot\sin(\delta_3)$$

$$AA(\alpha_3,\beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3,\beta_3) := AA(\alpha_3,\beta_3)^{-1}\cdot CC$$

$$W1x(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_1 \qquad W1y(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_2$$

$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$ $\qquad$ $Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$

3. Check this against the solutions in Problem 5-46:

$W1x(\alpha_3, -90.019 \cdot deg) = 1.732$ $\qquad$ $W1y(\alpha_3, -90.019 \cdot deg) = 1.000$

$Z1x(\alpha_3, -90.019 \cdot deg) = 1.947$ $\qquad$ $Z1y(\alpha_3, -90.019 \cdot deg) = 2.282$

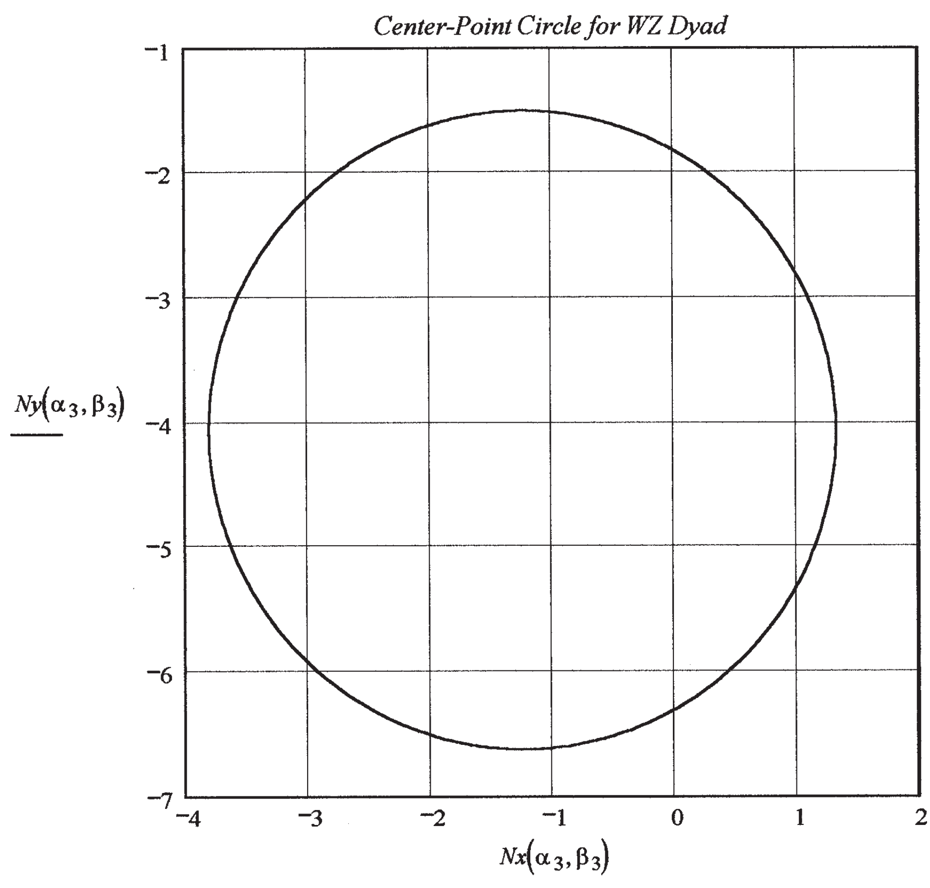
These are the same as the values calculated in Problem 5-46.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.

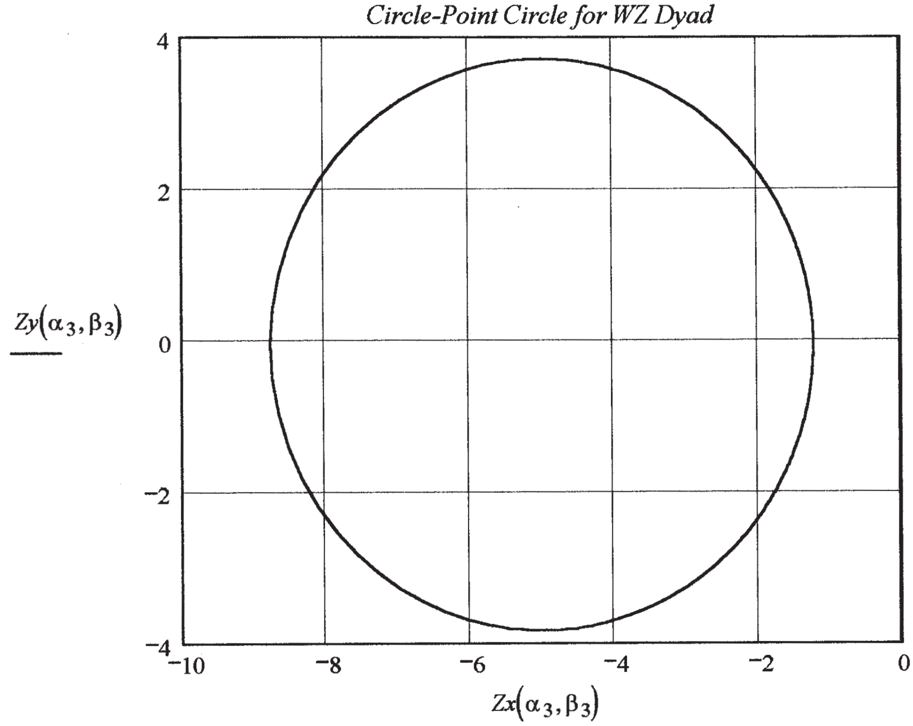


4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

$\beta_3 := -90.019 \cdot deg$ $\qquad$ $\alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$ $\qquad$ $Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. Evaluate terms in the US coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$\alpha_3 := 41.840 \cdot deg \qquad \gamma_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$A := cos(\gamma_2) - 1 \qquad B := sin(\gamma_2) \qquad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F(\gamma_3) := cos(\gamma_3) - 1$

$G(\gamma_3) := sin(\gamma_3) \qquad H(\alpha_3) := cos(\alpha_3) - 1 \qquad K(\alpha_3) := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$

$$AA(\alpha_3,\gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3,\gamma_3) := AA(\alpha_3,\gamma_3)^{-1} \cdot CC$$

$U1x(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_1 \qquad U1y(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_2$

$S1x(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_3 \qquad S1y(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_4$

7. Check this against the solutions in Problem 5-46:

$U1x(\alpha_3, 51.335 \cdot deg) = -1.178 \qquad U1y(\alpha_3, 51.335 \cdot deg) = 6.903$

$S1x(\alpha_3, 51.335 \cdot deg) = 0.857$ $\qquad$ $S1y(\alpha_3, 51.335 \cdot deg) = -3.621$

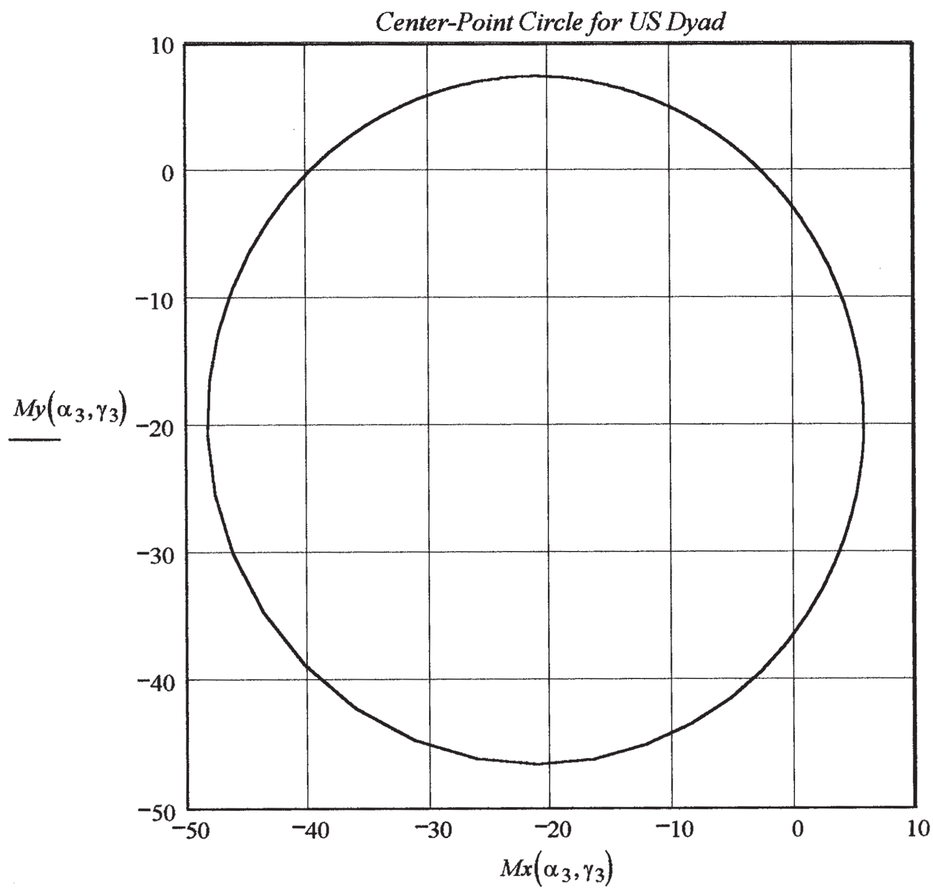
These are the same as the values calculated in Problem 5-46.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.

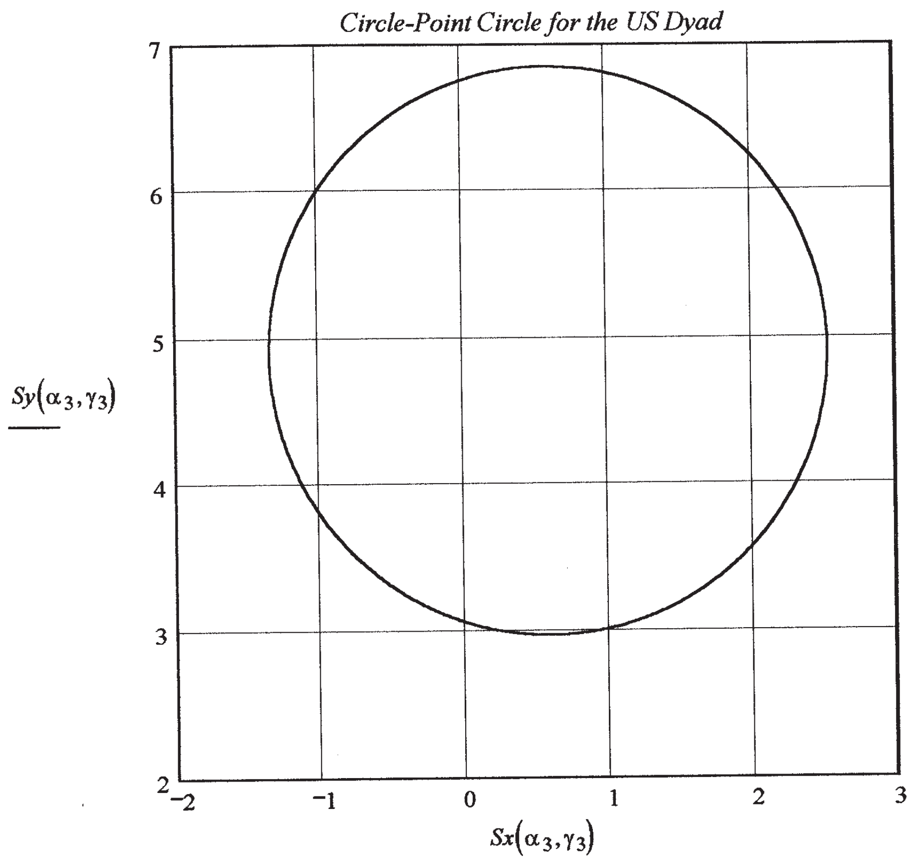


10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

$\gamma_3 := 51.335 \cdot deg$ $\qquad$ $\alpha_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$ $\qquad$ $Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$

11. Plot the circle-point circle for the **WZ** dyad (see next page).


Circle-Point Circle for the US Dyad
7
6
5
4
3
2
Sy(α3, γ3)
−2
−1
0
1
2
3
Sx(α3, γ3)

PROBLEM 5-50

Statement: In Example 5-2 the precision points and rotation angles are specified while the input and output rotation angles β and γ are free choices. Using the choices given for β_2 and γ_2, determine the radii and center coordinates of the center-point circles for O_2 and O_4. Plot those circles (or portions of them) and show that the choices of β3 and γ3 give a solution that falss on the center-point circles.

Given:

$P_{21x} := 2.394$ $\quad P_{21y} := -1.449$ $\quad P_{31x} := 3.761$ $\quad P_{31y} := -1.103$

$O_{2x} := -1.234$ $\quad O_{2y} := -7.772$ $\quad O_{4x} := 2.737$ $\quad O_{4y} := 0.338$

Body angles: $\theta_{P1} := 38.565 \cdot deg$ $\quad \theta_{P2} := -6.435 \cdot deg$ $\quad \theta_{P3} := 47.865 \cdot deg$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 342.3\,deg$ and $\gamma_2 := 30.9\,deg$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -45.0\,deg$

Solution: See Figure 5-5 and Mathcad file P0550.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.798$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = -31.185\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 3.919$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = -16.345\,deg$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad. Since this is a non-Grashof linkage, the range of β_3 is limited to approximately -56 deg < β_3 < -3 deg.

$\alpha_3 := 9.3 \cdot deg$ $\qquad \beta_3 := -56 \cdot deg, -55 \cdot deg .. -3 \cdot deg$

$A := \cos(\beta_2) - 1$ $\qquad B := \sin(\beta_2)$ $\qquad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\qquad E := p_{21} \cdot \cos(\delta_2)$ $\qquad F(\beta_3) := \cos(\beta_3) - 1$

$G(\beta_3) := \sin(\beta_3)$ $\qquad H(\alpha_3) := \cos(\alpha_3) - 1$ $\qquad K(\alpha_3) := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\qquad M := p_{21} \cdot \sin(\delta_2)$ $\qquad N := p_{31} \cdot \sin(\delta_3)$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$W1x(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_1$ $\quad$ $W1y(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_2$

$Z1x(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_3$ $\quad$ $Z1y(\alpha_3,\beta_3) := DD(\alpha_3,\beta_3)_4$

3. Check this against the solutions in Example 5-2:

$W1x(\alpha_3, 324.8\cdot deg) = 0.055$ $\quad$ $W1y(\alpha_3, 324.8\cdot deg) = 6.832$

$Z1x(\alpha_3, 324.8\cdot deg) = 1.179$ $\quad$ $Z1y(\alpha_3, 324.8\cdot deg) = 0.940$

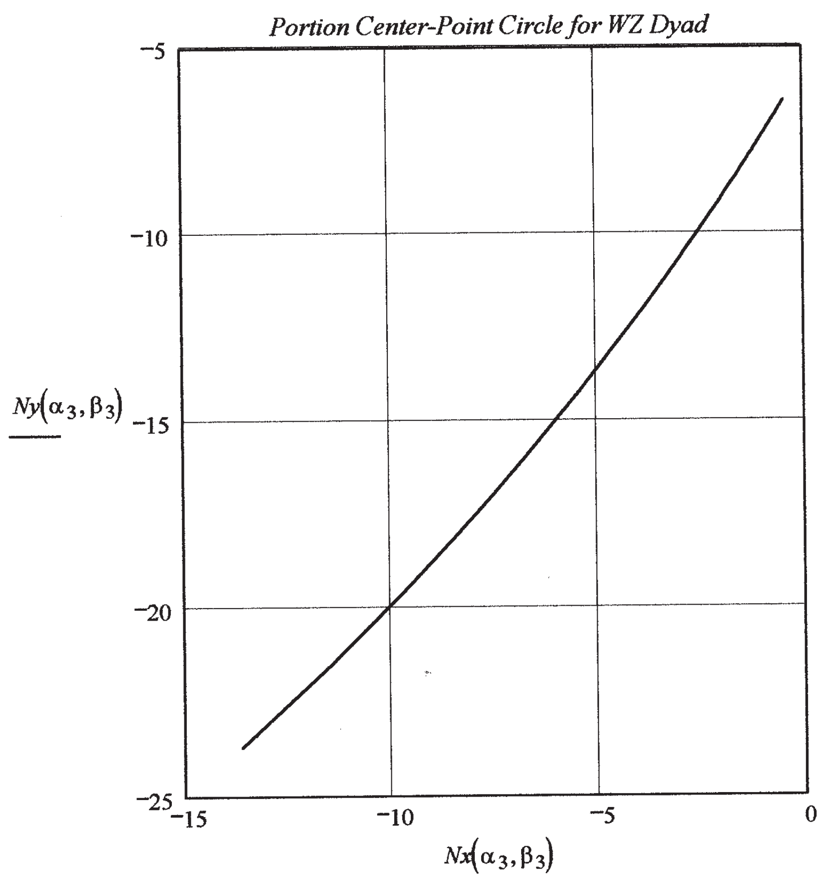
These are the same as the values calculated in Example 5-2.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3,\beta_3) := -W1x(\alpha_3,\beta_3) - Z1x(\alpha_3,\beta_3)$$

$$Ny(\alpha_3,\beta_3) := -W1y(\alpha_3,\beta_3) - Z1y(\alpha_3,\beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Find the center and radius of this arc by taking three points on it and, from them, determine the radius and center point of the circle.

Three points:

$x_1 := Nx(\alpha_3, -56\cdot deg)$ $\quad$ $x_1 = -0.470$

$y_1 := Ny(\alpha_3, -56\cdot deg)$ $\quad$ $y_1 = -6.430$

$x_2 := Nx(\alpha_3, -10\cdot deg)$ $\quad$ $x_2 = -5.506$

$y_2 := Ny(\alpha_3, -10\cdot deg)$ $\quad$ $y_2 = -14.302$

$$x_3 := Nx(\alpha_3, -3 \cdot deg) \qquad x_3 = -13.631$$

$$y_3 := Ny(\alpha_3, -3 \cdot deg) \qquad y_3 = -23.695$$

Form four ratios:

$$a := \frac{y_2^{\,2} - y_1^{\,2} + x_2^{\,2} - x_1^{\,2}}{2 \cdot (y_2 - y_1)} \qquad a = -12.278$$

$$b := \frac{x_2 - x_1}{y_2 - y_1} \qquad b = 0.640$$

$$c := \frac{y_3^{\,2} - y_2^{\,2} + x_3^{\,2} - x_2^{\,2}}{2 \cdot (y_2 - y_1)} \qquad c = -32.547$$

$$d := \frac{x_3 - x_2}{y_3 - y_2} \qquad d = 0.865$$

Circle center x coordinate: $h := \frac{a - c}{b - d}$ $\qquad h = -89.999$

Circle center y coordinate: $k := a - b \cdot h$ $\qquad k = 45.303$

Center-point circle radius: $R := \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$ $\qquad R = 103.40$

4. Show that the point determined by $\beta_3 = 324.8$ deg falls on this circle.

Coordinates of this point:

$$x_{b3} := Nx(\alpha_3, 324.8 \cdot deg) \qquad x_{b3} = -1.234$$

$$y_{b3} := Ny(\alpha_3, 324.8 \cdot deg) \qquad y_{b3} = -7.772$$

Substitute into radius equation:

$$R := \sqrt{(x_{b3} - h)^2 + (y_{b3} - k)^2} \qquad R = 103.42$$

Since the radius is the same, this point does fall on the center-point circle for the **WZ** dyad.

6. Evaluate terms in the US coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 9.3 \cdot deg \qquad \gamma_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$A := cos(\gamma_2) - 1 \qquad B := sin(\gamma_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21} \cdot cos(\delta_2) \qquad F(\gamma_3) := cos(\gamma_3) - 1$$

$$G(\gamma_3) := sin(\gamma_3) \qquad H(\alpha_3) := cos(\alpha_3) - 1 \qquad K(\alpha_3) := sin(\alpha_3)$$

$$L := p_{31} \cdot cos(\delta_3) \qquad M := p_{21} \cdot sin(\delta_2) \qquad N := p_{31} \cdot sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_1 \qquad U1y(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_2$$

$$S1x(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_3 \qquad S1y(\alpha_3,\gamma_3) := DD(\alpha_3,\gamma_3)_4$$

7. Check this against the solutions in Example 5-2:

$$U1x(\alpha_3, 80.6\cdot deg) = -2.628 \qquad U1y(\alpha_3, 80.6\cdot deg) = -1.825$$

$$S1x(\alpha_3, 80.6\cdot deg) = -0.109 \qquad S1y(\alpha_3, 80.6\cdot deg) = 1.487$$

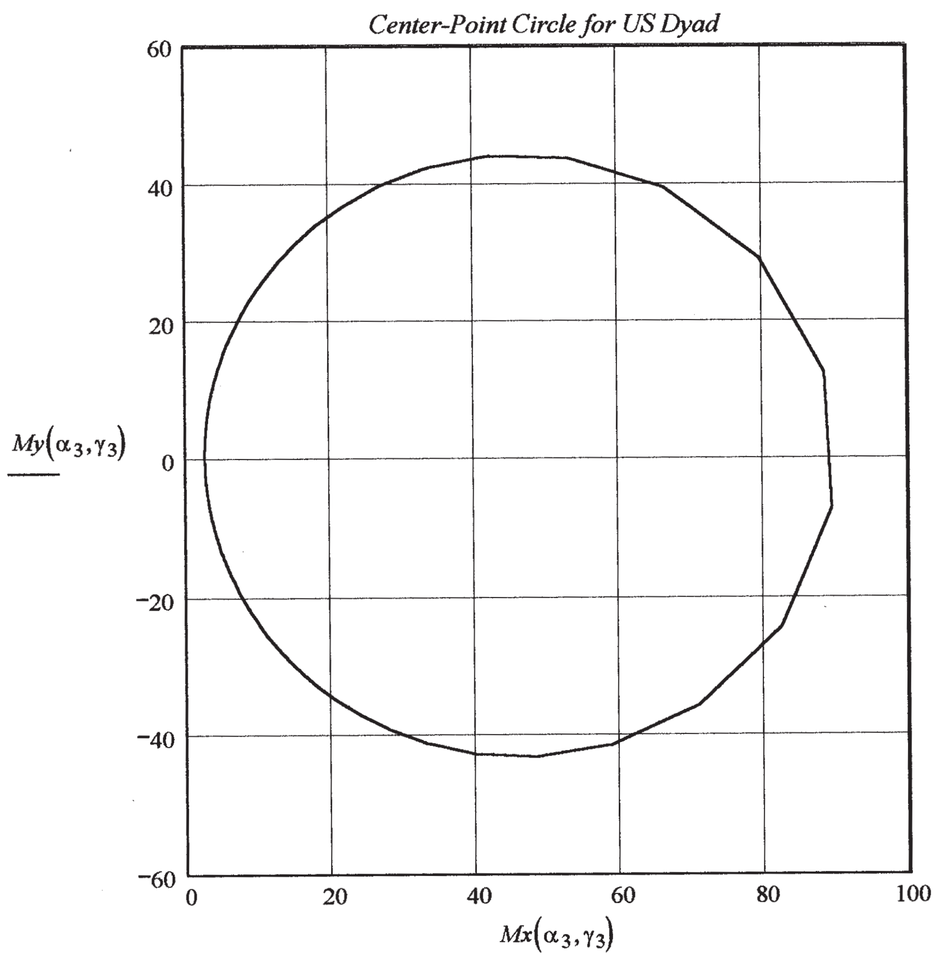
These are the same as the values calculated in Example 5-2.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3,\gamma_3) := -U1x(\alpha_3,\gamma_3) - S1x(\alpha_3,\gamma_3)$$

$$My(\alpha_3,\gamma_3) := -U1y(\alpha_3,\gamma_3) - S1y(\alpha_3,\gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Find the center and radius of this circle by taking three points on it and, from them, determine the radius and center point of the circle.

Three points:

$x_1 := Mx(\alpha_3, 0\cdot deg)$ $\quad x_1 = 42.228$

$y_1 := My(\alpha_3, 0\cdot deg)$ $\quad y_1 = 44.088$

$x_2 := Mx(\alpha_3, 90\cdot deg)$ $\quad x_2 = 2.744$

$y_2 := My(\alpha_3, 90\cdot deg)$ $\quad y_2 = -0.214$

$x_3 := Mx(\alpha_3, 350\cdot deg)$ $\quad x_3 = 39.862$

$y_3 := My(\alpha_3, 350\cdot deg)$ $\quad y_3 = -42.592$

Form four ratios:

$$a := \frac{y_2^{\,2} - y_1^{\,2} + x_2^{\,2} - x_1^{\,2}}{2\cdot(y_2 - y_1)} \qquad a = 41.977$$

$$b := \frac{x_2 - x_1}{y_2 - y_1} \qquad b = 0.891$$

$$c := \frac{y_3^{\,2} - y_2^{\,2} + x_3^{\,2} - x_2^{\,2}}{2\cdot(y_2 - y_1)} \qquad c = -38.321$$

$$d := \frac{x_3 - x_2}{y_3 - y_2} \qquad d = -0.876$$

Circle center *x* coordinate: $h := \dfrac{a - c}{b - d}$ $\quad h = 45.441$

Circle center *y* coordinate: $k := a - b\cdot h$ $\quad k = 1.479$

Center-point circle radius: $R := \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$ $\quad R = 42.73$

11. Show that the point determined by $\gamma_3 = 80.6$ deg falls on this circle.

Coordinates of this point:

$x_{g3} := Mx(\alpha_3, 80.6\cdot deg)$ $\quad x_{g3} = 2.738$

$y_{g3} := My(\alpha_3, 80.6\cdot deg)$ $\quad y_{g3} = 0.338$

Substitute into radius equation:

$$R := \sqrt{(x_{g3} - h)^2 + (y_{g3} - k)^2} \qquad R = 42.72$$

Since the radius is the same, this point does fall on the center=point circle for the US dyad.