Chapter 8

CAM DESIGN

TOPIC/PROBLEM MATRIX

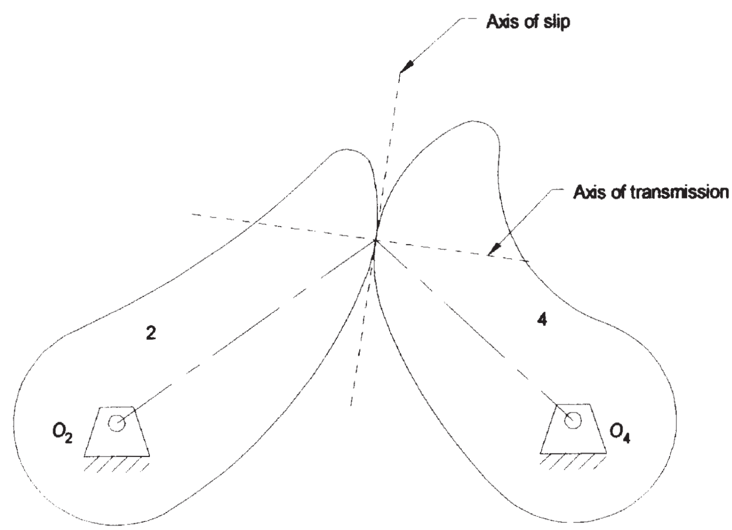
SECT	TOPIC	PROBLEMS
8.1	Cam Terminology	8-1, 8-3, 8-5
8.3	Double-Dwell Cam Design	8-55
	Simple Harmonic Motion (SHM)	8-26
	Cycloidal Displacement	8-23
	Modified Trapezoidal	8-7, 8-11, 8-21, 8-44
	Modified Sinusoidal	8-8, 8-10, 8-22, 8-45
	Polynomial Functions	8-24, 8-25, 8-33, 8-46
8.4	Single-Dwell Cam Design	8-9, 8-42, 8-47, 8-53
8.5	Critical Path Motion	8-17, 8-43, 8-48, 8-54
8.6	Sizing the Cam	
	Pressure Angle	8-2, 8-4, 8-6, 8-34
	Radius of Curvature	
	Roller Followers	8-18, 8-19, 8-20, 8-27, 8-28, 8-29, 8-30, 8-31, 8-32, 8-35, 8-36, 8-37, 8-38, 8-39, 8-40, 8-41
	Flat-Faced Followers	8-49, 8-50, 8-51, 8-52
	Roller & Flat-Faced Followers	8-12, 8-13, 8-14, 8-15, 8-16

PROBLEM 8-1

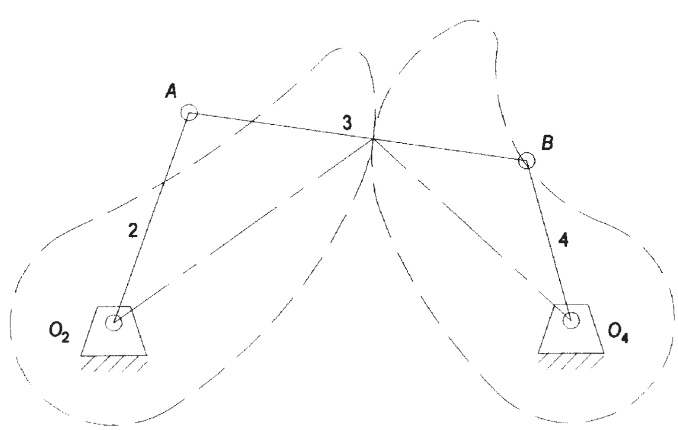
Statement: Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-1 and Mathcad file P0801.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

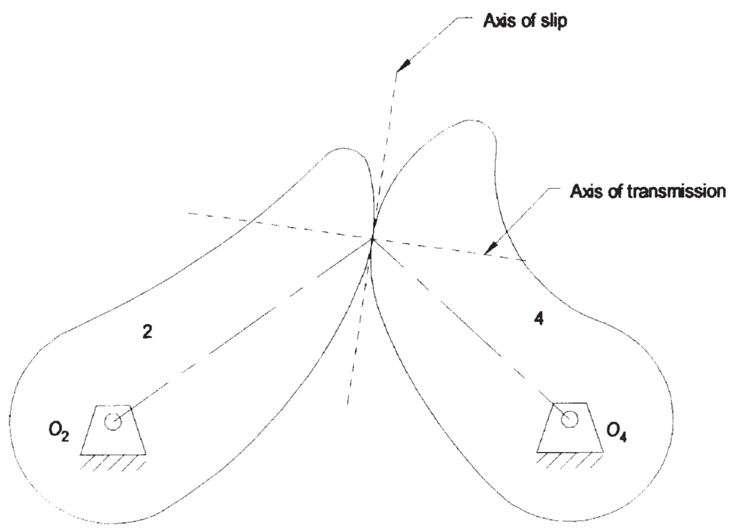


PROBLEM 8-2

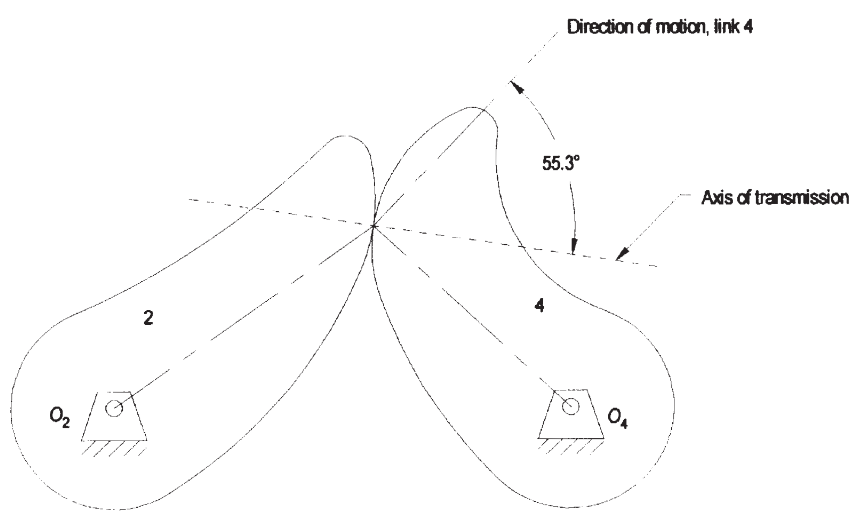
Statement: Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-1 and Mathcad file P0802.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.

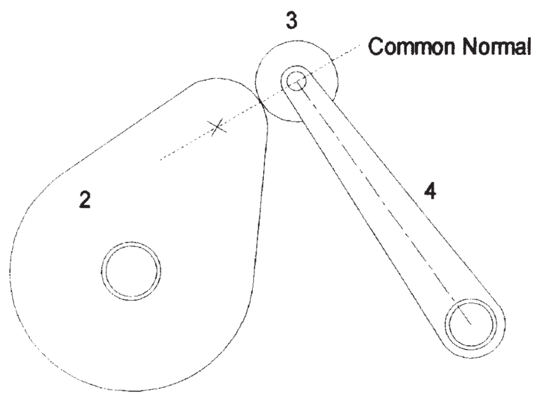


PROBLEM 8-3

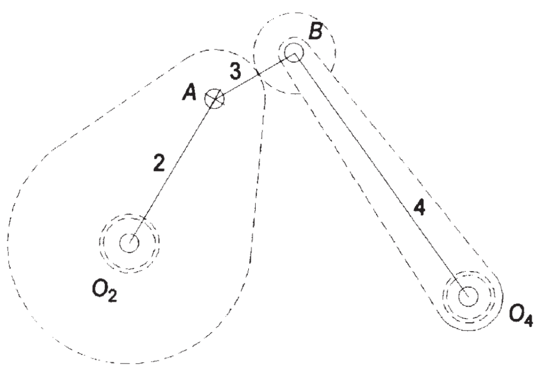
Statement: Figure P8-2 shows the cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-2 and Mathcad file P0803.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

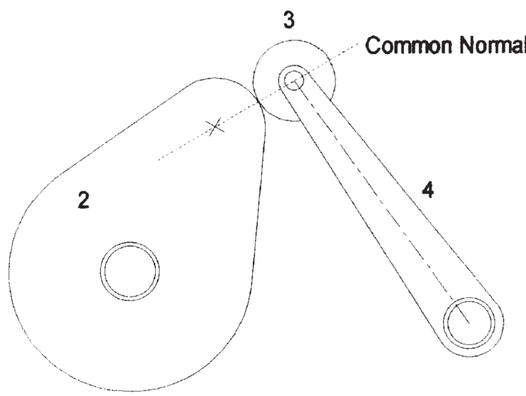


PROBLEM 8-4

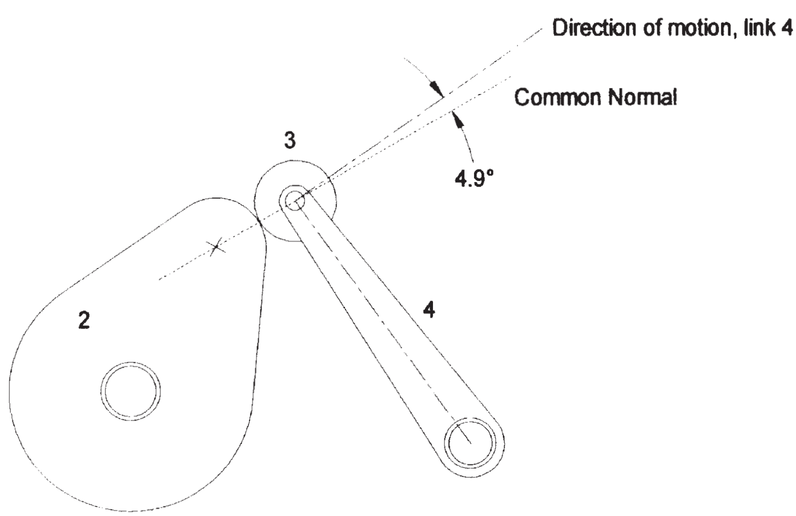
Statement: Figure P8-2 shows the cam and follower. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-2 and Mathcad file P0804.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.

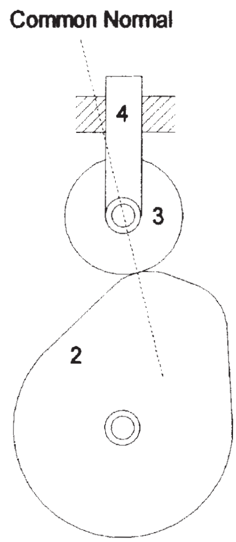


PROBLEM 8-5

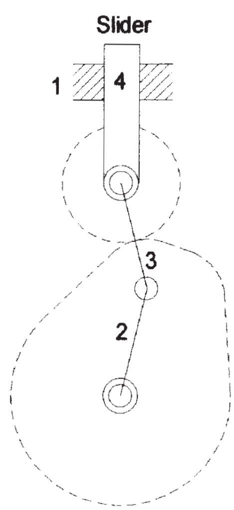
Statement: Figure P8-3 shows the cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-3 and Mathcad file P0805.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

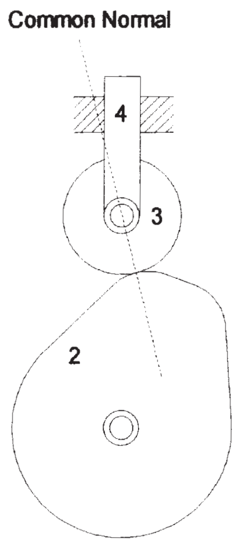


PROBLEM 8-6

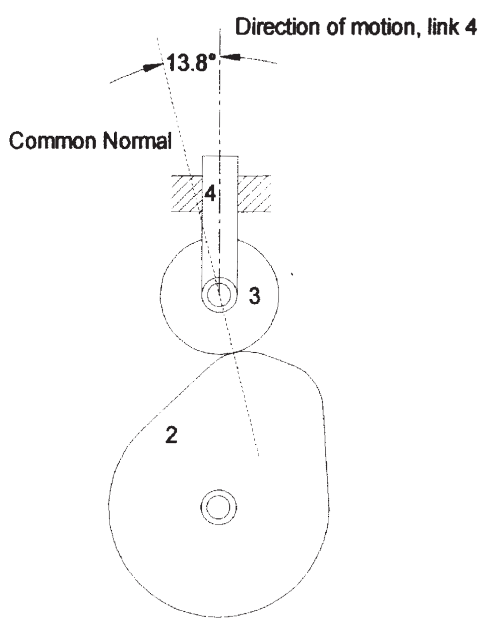
Statement: Figure P8-3 shows the cam and follower. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-3 and Mathcad file P0806.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.



PROBLEM 8-7

Statement: Design a double-dwell cam to move a follower from 0 to 2.5 in in 60 deg, dwell for 120 deg, fall 2.5 i in 30 deg and dwell for the remainder. The total cycle must take 4 sec. Choose suitable programs for rise and fall to minimize accelerations. Plot the *s v a j* diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot deg$	$\beta_2 := 120 \cdot deg$	$\beta_3 := 30 \cdot deg$	$\beta_4 := 150 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 2.5 \cdot in$	$h_4 := 0 \cdot in$

Cycle time: $\tau := 4 \cdot sec$

Solution: See Mathcad file P0807.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.571 \frac{rad}{sec}$$

2. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$$

$$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for 0 <= x <= b/2 where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left(\frac{\pi}{b} \cdot x\right)$$

for b/2 <= x <= (1 - d)/2

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for (1 - d)/2 <= x <= (1 + d)/2

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 <= \theta <= 360$ deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$

$$s_3(x) := h_3\left[1 - \left(R\left(x, 0, \frac{b}{2}\right)\cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := \frac{h_3}{\beta_3}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := \frac{h_3}{\beta_3^{\,2}}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := \frac{h_3}{\beta_3^{\,3}}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'''_5(x)\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$

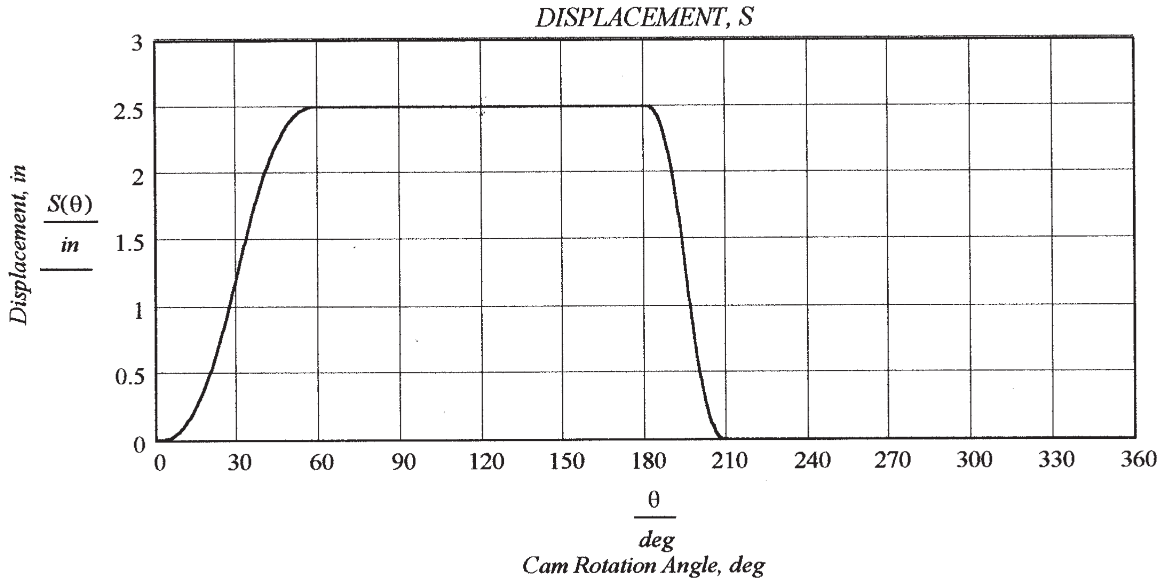
$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

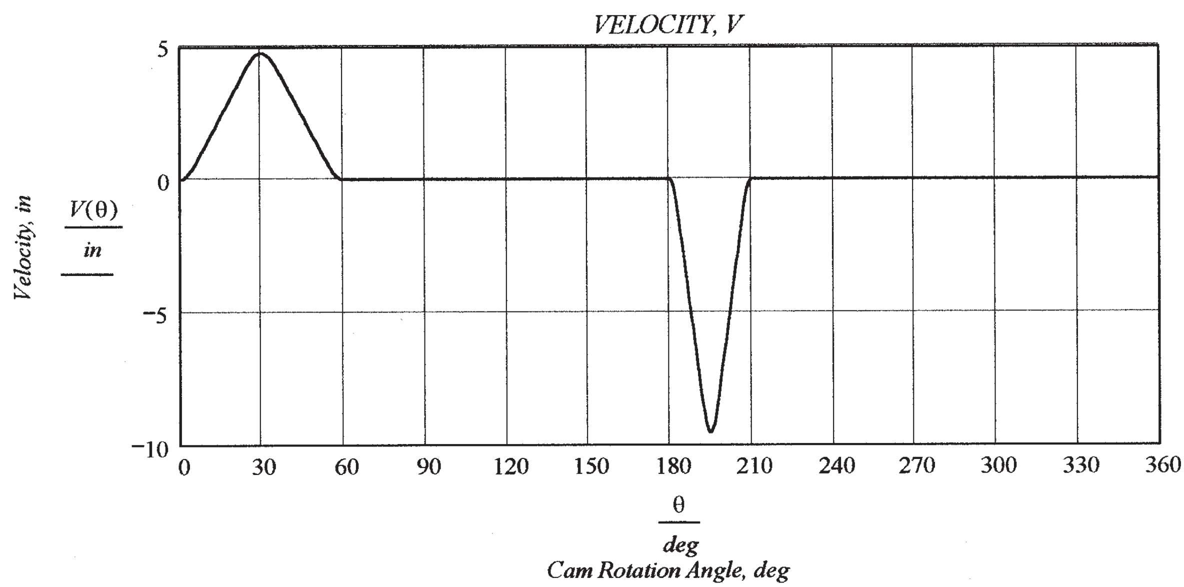
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \ldots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

$$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$$



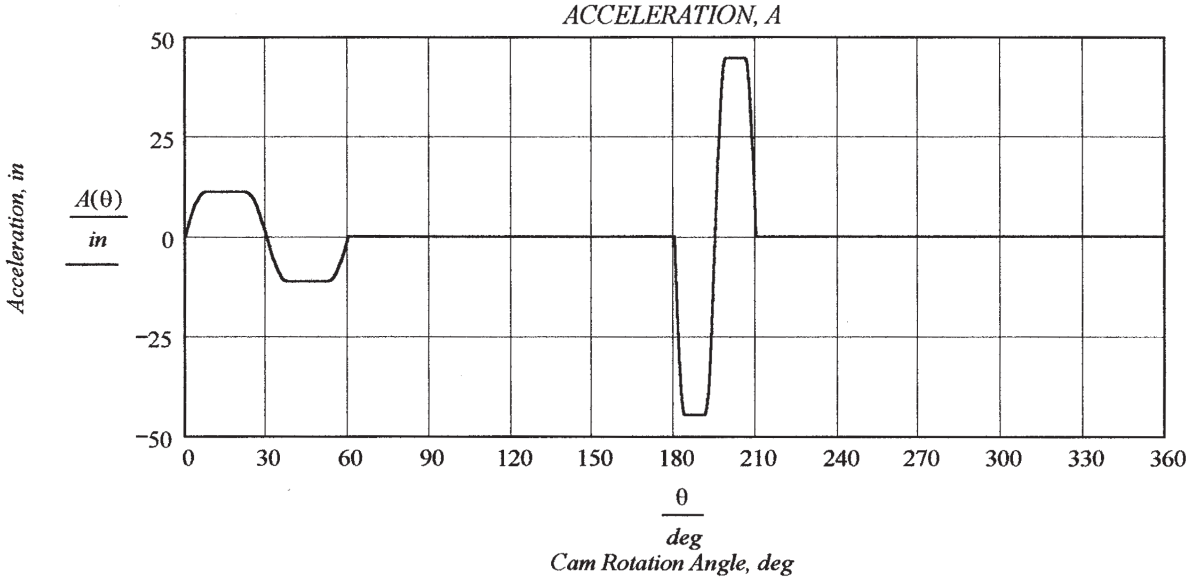
11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



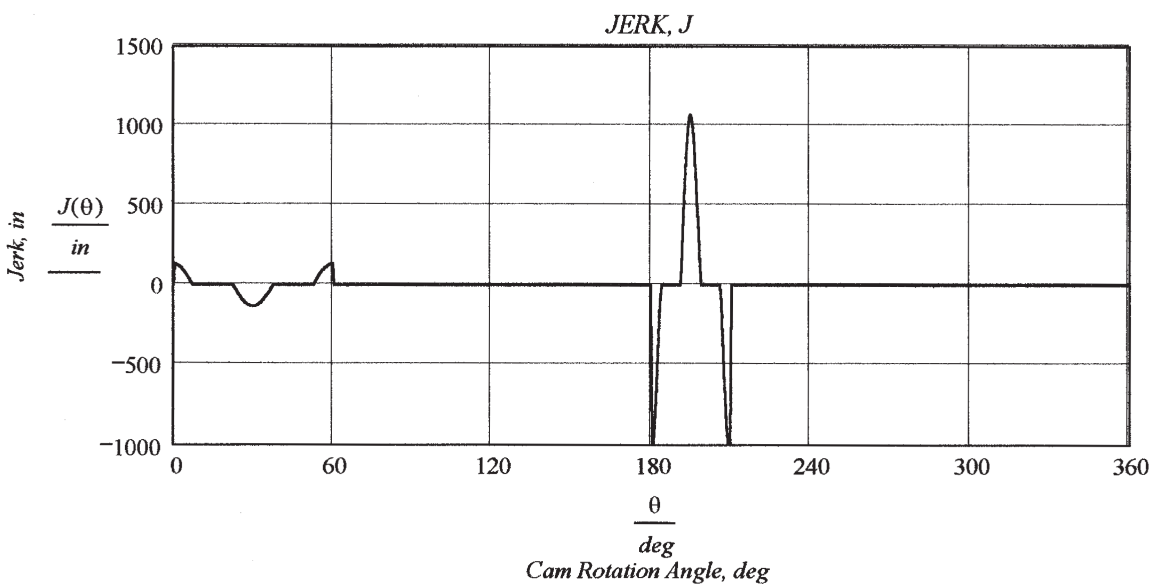
12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot j2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \ldots$$
$$+ R(\theta, \theta_2, \theta_3) \cdot j3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot j4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$



PROBLEM 8-8

Statement: Design a double-dwell cam to move a follower from 0 to 1.5 in in 45 deg, dwell for 150 deg, fall 1.5 in in 90 deg and dwell for the remainder. The total cycle must take 6 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 45\cdot deg$	$\beta_2 := 150\cdot deg$	$\beta_3 := 90\cdot deg$	$\beta_4 := 75\cdot deg$
$h_1 := 1.5\cdot in$	$h_2 := 0\cdot in$	$h_3 := 1.5\cdot in$	$h_4 := 0\cdot in$

Cycle time: $\tau := 6\cdot sec$

Solution: See Mathcad file P0808.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 1.047\,\frac{rad}{sec}$$

2. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.00 \qquad d := 0.75$$

$$C_a := 5.5280$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for 0 <= x <= $b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a\cdot\left[x\cdot\frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2\cdot sin\left(\frac{\pi}{b}\cdot x\right)\right] \qquad y'_1(x) := C_a\cdot\frac{b}{\pi}\cdot\left(1 - cos\left(\frac{\pi}{b}\cdot x\right)\right)$$

$$y''_1(x) := C_a\cdot sin\left(\frac{\pi}{b}\cdot x\right) \qquad y'''_1(x) := C_a\cdot\frac{\pi}{b}\cdot cos\left(\frac{\pi}{b}\cdot x\right)$$

for $b/2$ <= x <= $(1 - d)/2$

$$y_2(x) := C_a\cdot\left[\frac{x^2}{2} + b\cdot\left(\frac{1}{\pi} - \frac{1}{2}\right)\cdot x + b^2\cdot\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\right] \qquad y'_2(x) := C_a\cdot\left[x + b\cdot\left(\frac{1}{\pi} - \frac{1}{2}\right)\right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2$ <= x <= $(1 + d)/2$

$$y_3(x) := C_a\cdot\left[\left(\frac{b}{\pi} + \frac{c}{2}\right)\cdot x + \left(\frac{d}{\pi}\right)^2 + b^2\cdot\left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2\cdot cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]\right]$$

$$y'_3(x) := C_a\cdot\left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi}\cdot sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]\right]$$

$$y''_3(x) := C_a\cdot cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a\cdot\frac{\pi}{d}\cdot sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left(2 \cdot d^2 - b^2 \right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2 \right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x-1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for S, V, A, and J that applies over the range $0 <= \theta <= 360$ deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of S is the value of S at the end of the previous interval and the values of V, A, and J are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$

$$s_3(x) := h_3\left[1 - \left(R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \atop + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := \frac{h_3}{\beta_3}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \atop + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := \frac{h_3}{\beta_3^{\,2}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \atop + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := \frac{h_3}{\beta_3^{\,3}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \atop + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x)\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$. For this interval, the values of S, V, A, and J are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$

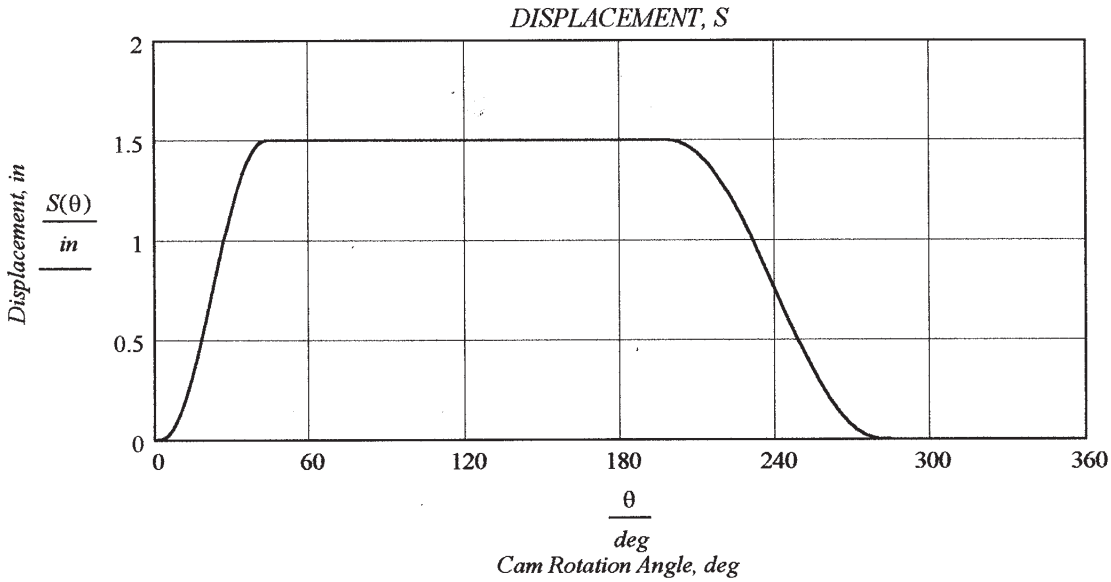
$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

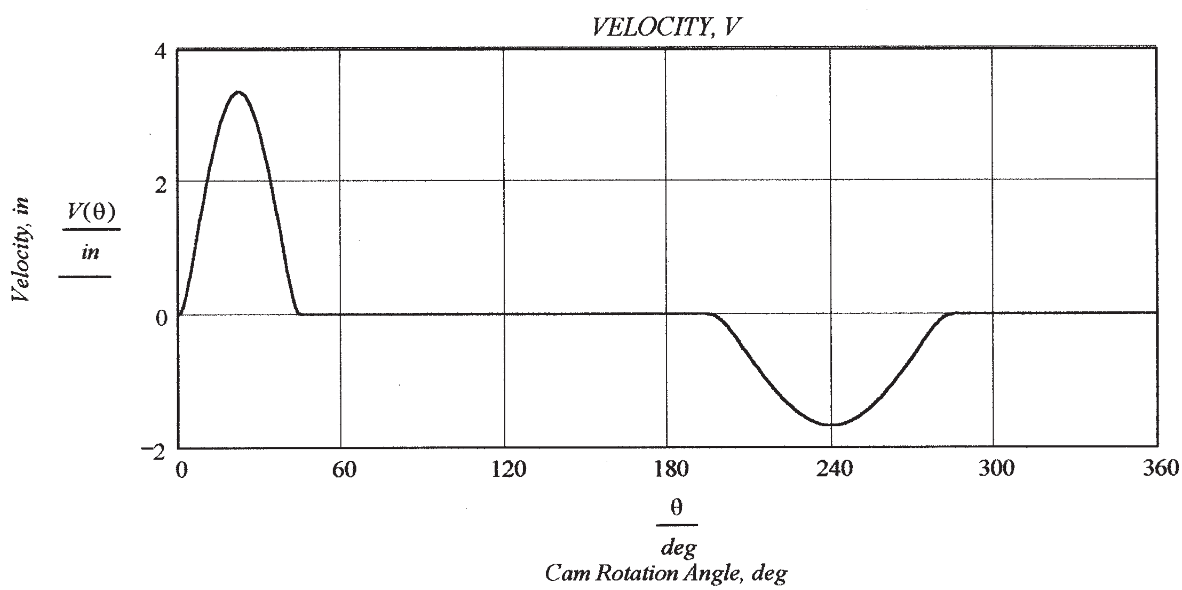
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$

$\theta := 0\cdot deg, 1\cdot deg .. 360\cdot deg$



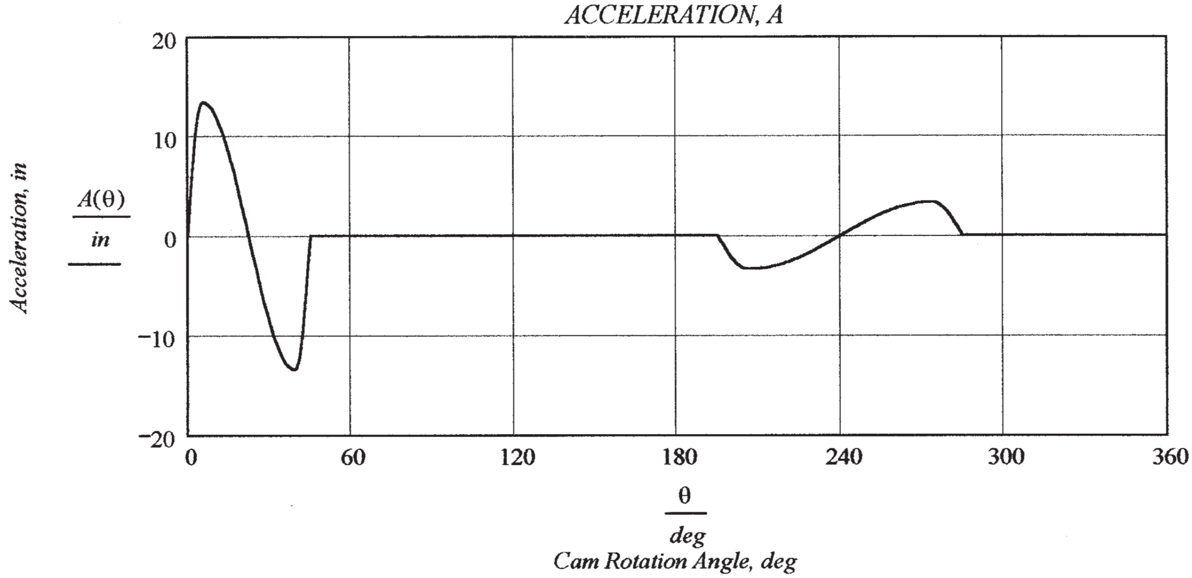
11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \cdots$$
$$+R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



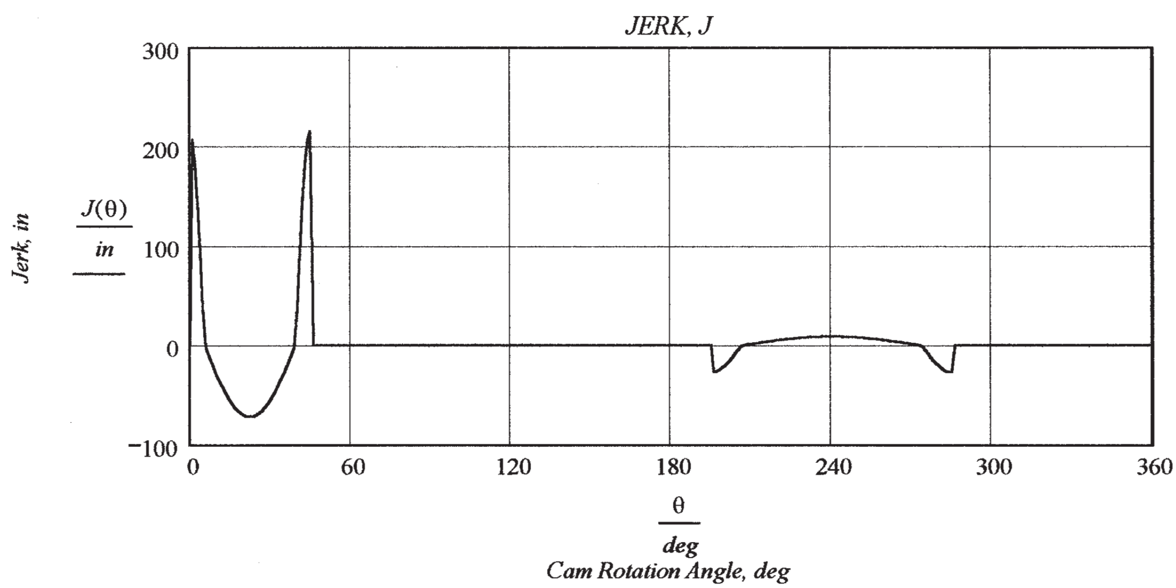
12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \cdots$$
$$+R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot j2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \cdots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot j3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot j4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$



PROBLEM 8-9

Statement: Design a single-dwell cam to move a follower from 0 to 2.0 in in 60 deg, fall 2.0 in in 90 deg and dwell for the remainder. The total cycle must take 2 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL
$\beta := 150 \cdot deg$	$\beta_3 := 210 \cdot deg$
$h := 2.0 \cdot in$	$h_3 := 0.0 \cdot in$

Cycle time: $\tau := 2 \cdot sec$

Solution: See Mathcad file P0809.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 3.142 \frac{rad}{sec}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

at $\theta = 0$: $s = 0$, $v = 0$, $a = 0$
$\theta = \beta_1$: $s = h$, $v = 0$
$\theta = \beta$: $s = 0$, $v = 0$, $a = 0$

This is a minimum set of 8 BCs. The v = 0 condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, *h*. Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 2.000\, in$

Rise interval: $\beta_1 := 60 \cdot deg$ $\qquad A := \frac{\beta_1}{\beta} \qquad A = 0.400$

Fall interval: $\beta_2 := 90 \cdot deg$

3. Use the 8 BCs and equation 8.23 to write 8 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 8 terms in the equation for *s* (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \qquad 0 := c_1 \qquad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

For $\theta = \beta$: $s = v = a = 0$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 & 6\cdot A^5 & 7\cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \qquad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1}\cdot H$$

$c_3 = 289.352\,in$ $\qquad c_4 = -1229.745\,in$ $\qquad c_5 = 1953.125\,in$

$c_6 = -1374.421\,in$ $\qquad c_7 = 361.690\,in$

5. Write the *SVAJ* equations for the rise/fall segment.

$$S(\theta) := c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5 + c_6\cdot\left(\frac{\theta}{\beta}\right)^6 + c_7\cdot\left(\frac{\theta}{\beta}\right)^7$$

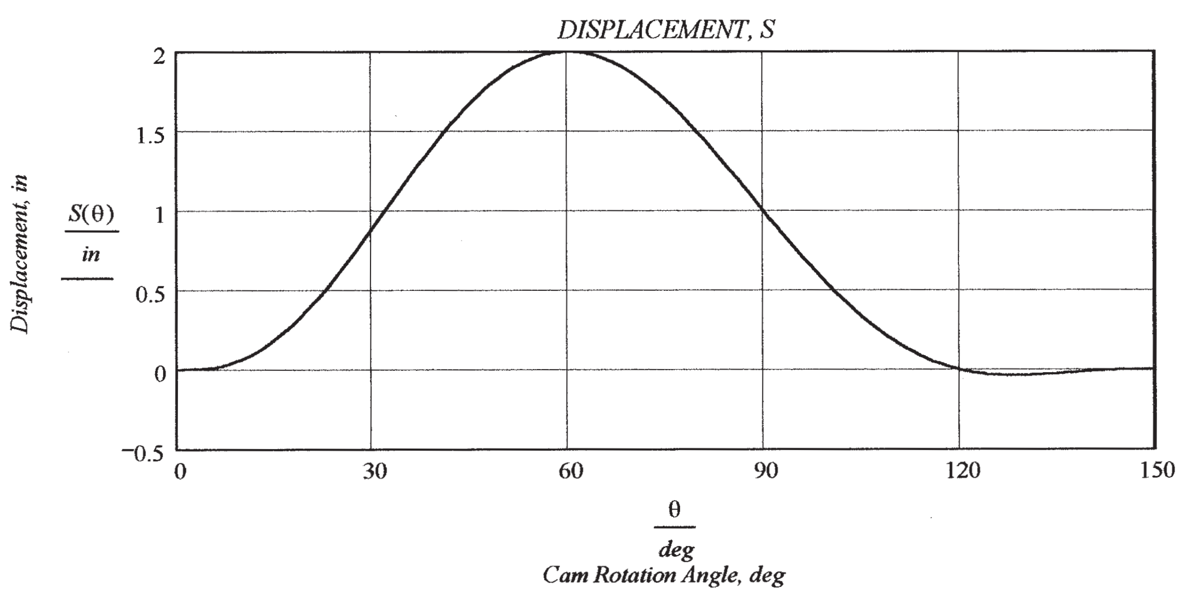
$$V(\theta) := \frac{1}{\beta}\cdot\left[3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4 + 6\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^5 + 7\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

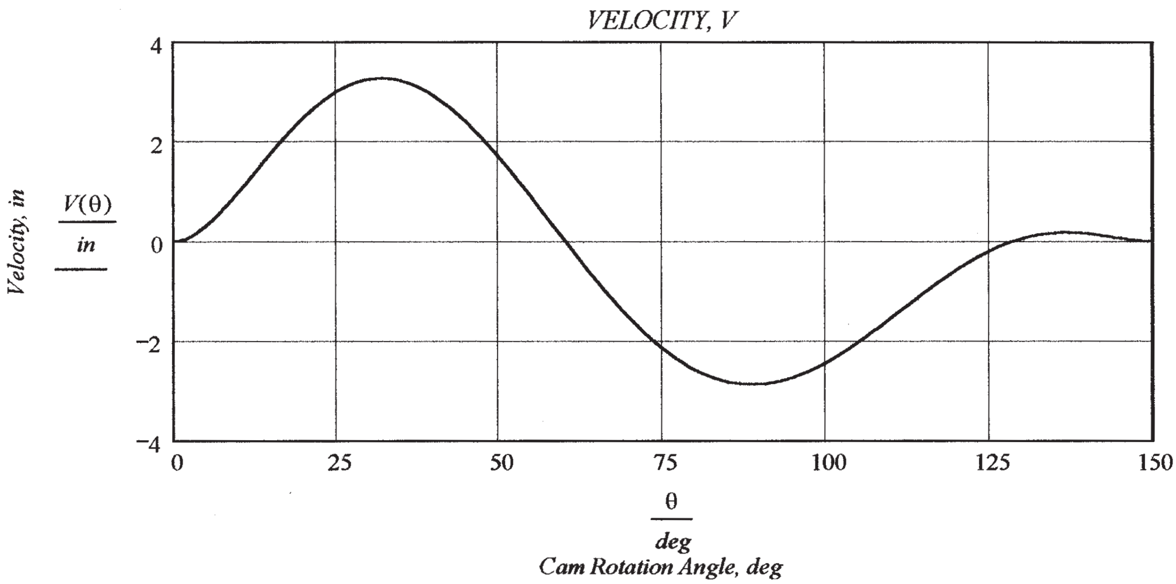
$$A(\theta) := \frac{1}{\beta^2}\cdot\left[6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3 + 30\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^4 + 42\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^5\right]$$

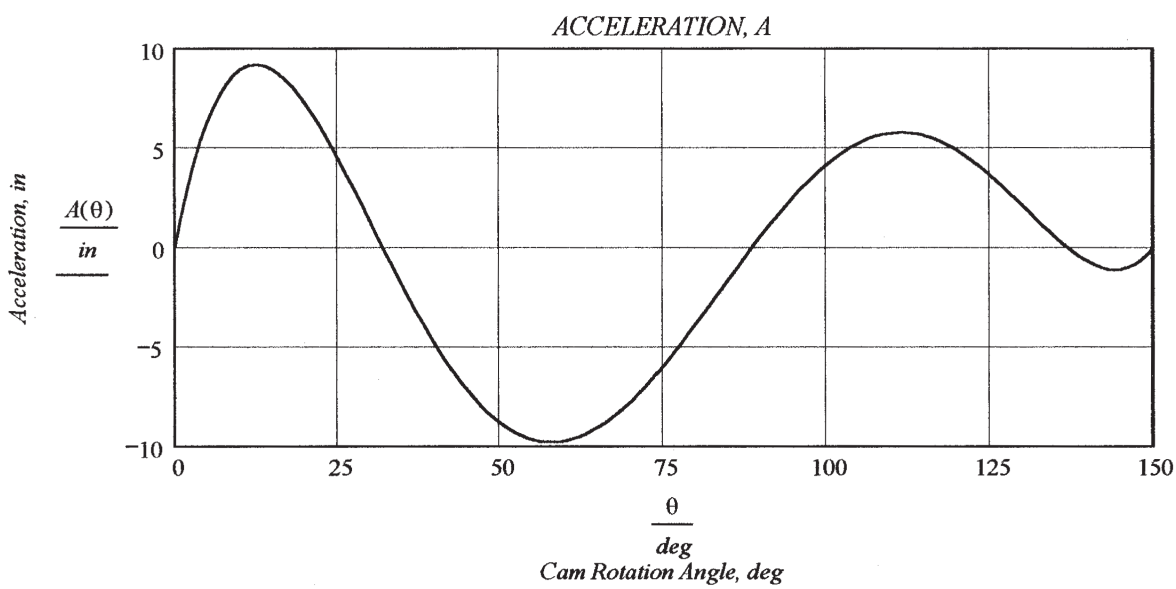
$$J(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2 + 120\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^3 + 210\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

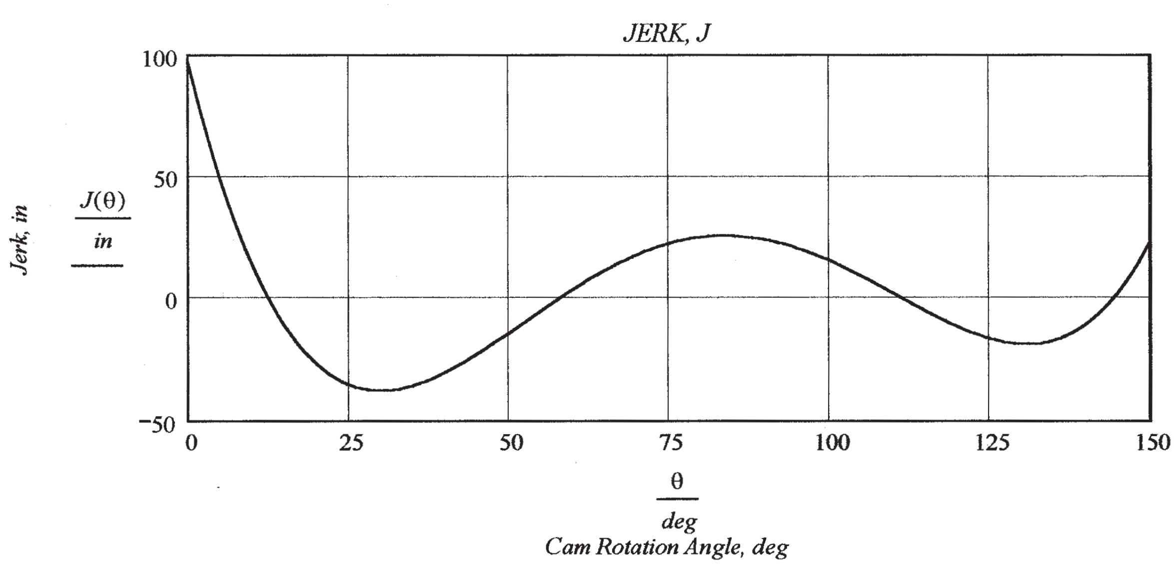
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$\theta := 0\cdot deg, 1\cdot deg .. \beta$




VELOCITY, V
Velocity, in
V(θ)
in
4
2
0
−2
−4
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
Acceleration, in
A(θ)
in
10
5
0
−5
−10
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg


JERK, J
Jerk, in
J(θ)
in
100
50
0
−50
0
25
50
75
100
125
150
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-10

Statement: Design a three-dwell cam to move a follower from 0 to 2.5 in in 40 deg, dwell for 100 deg, fall 1.5 in in 90 deg, dwell for 20 deg, fall 1.0 in in 30 deg, and dwell for the remainder. The total cycle must take 10 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0.0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0.0 \cdot in$
$\beta_5 := 30 \cdot deg$	$\beta_6 := 80 \cdot deg$		
$h_5 := 1.0 \cdot in$	$h_6 := 0.0 \cdot in$		

Cycle time: $\tau := 15 \cdot sec$

Solution: See Mathcad file P0810.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 0.419 \frac{rad}{sec}$$

2. From Table 8-3, the motion program with lowest velocity that does not have infinite jerk is the modified sinusoidal. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.00 \qquad d := 0.75$$

$$C_v := 1.7596 \qquad C_a := 5.5280 \qquad C_j := 69.466$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 <= x <= b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left(\frac{\pi}{b} \cdot x\right)$$

for $b/2 <= x <= (1 - d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2 <= x <= (1 + d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]$$

for (1 + *d*)/2 <= *x* <= 1 - *b*/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right)\cdot x + \left(2\cdot d^2 - b^2\right)\cdot\left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4}\right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for 1 - *b*/2 <= *x* <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi}\cdot x + \frac{2\cdot\left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{b}\cdot(x-1)\right]\right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b}\cdot(x-1)\right]\right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b}\cdot(x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b}\cdot(x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of six intervals (rise, dwell, fall, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, 0 <= θ <= β_1. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For 0 <= θ <= β_1 (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right)\cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right)\cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right)\cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right)\cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$ (dwell)

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$ (fall)

$$s_3(x) := h_3\left[1 - \left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x) \end{array}\right)\right] + h_1 - h_3$$

$$v_3(x) := -\frac{h_3}{\beta_3}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x) \end{array}\right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^{\,2}}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x) \end{array}\right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^{\,3}}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x) \end{array}\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$ (dwell)

$$s_4(x) := h_1 - h_3 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the local *svaj* equations for the fifth interval,

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4 + \beta_5$ (fall)

$$s_5(x) := h_5\left[1 - \left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x) \end{array}\right)\right]$$

$$v_5(x) := -\frac{h_5}{\beta_5}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x) \end{array}\right)$$

$$a_5(x) := \frac{h_5}{\beta_5^2}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x) \end{array}\right)$$

$$j_5(x) := \frac{h_5}{\beta_5^3}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x) \end{array}\right)$$

11. Write the local *svaj* equations for the sixth interval. For this interval, the values of S, V, A, and J are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$ (dwell)

$$s_6(x) := 0 \qquad v_6(x) := 0 \qquad a_6(x) := 0 \qquad j_6(x) := 0$$

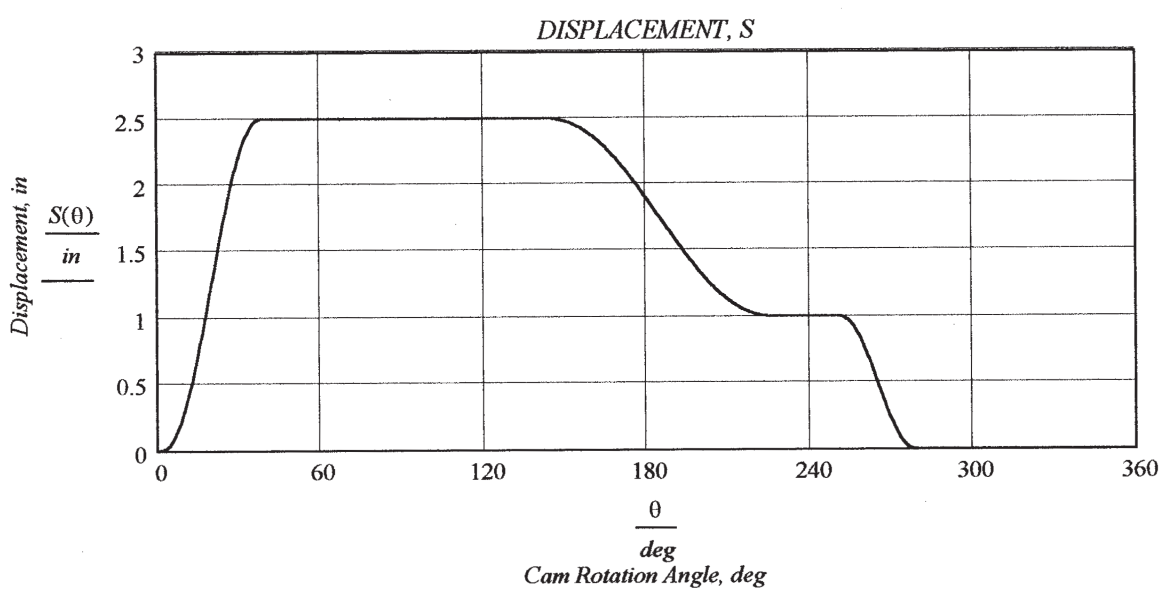
12. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

Let $\theta_1 := \beta_1$ $\qquad \theta_2 := \theta_1 + \beta_2$ $\qquad \theta_3 := \theta_2 + \beta_3$ $\qquad \theta_4 := \theta_3 + \beta_4$

$\theta_5 := \theta_4 + \beta_5$ $\qquad \theta_6 := \theta_5 + \beta_6$

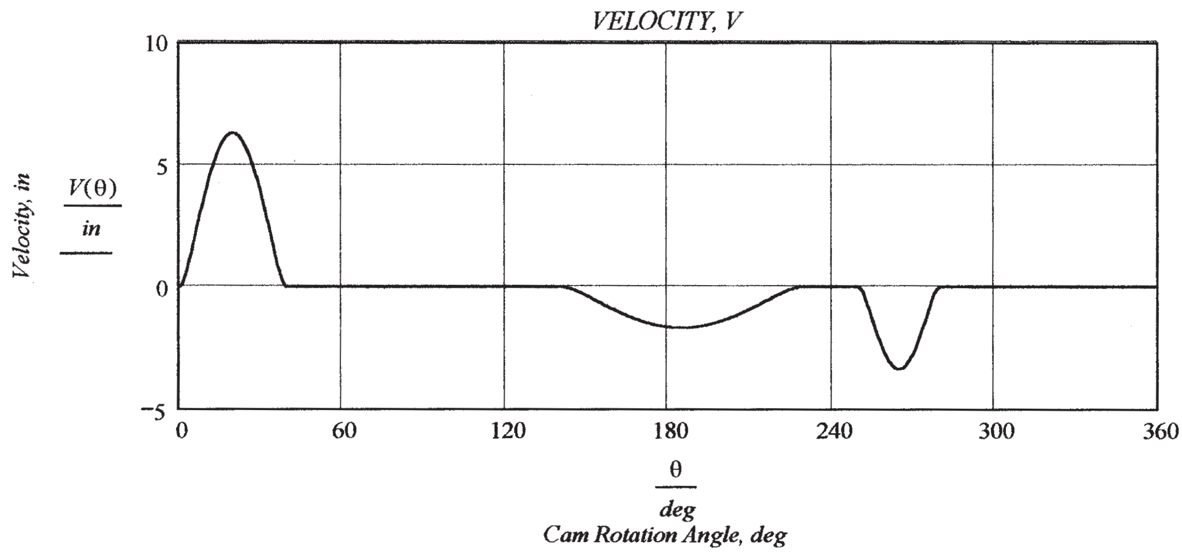
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \ldots$$
$$+R(\theta,\theta_4,\theta_5)\cdot s_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot s_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right)$$

$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$



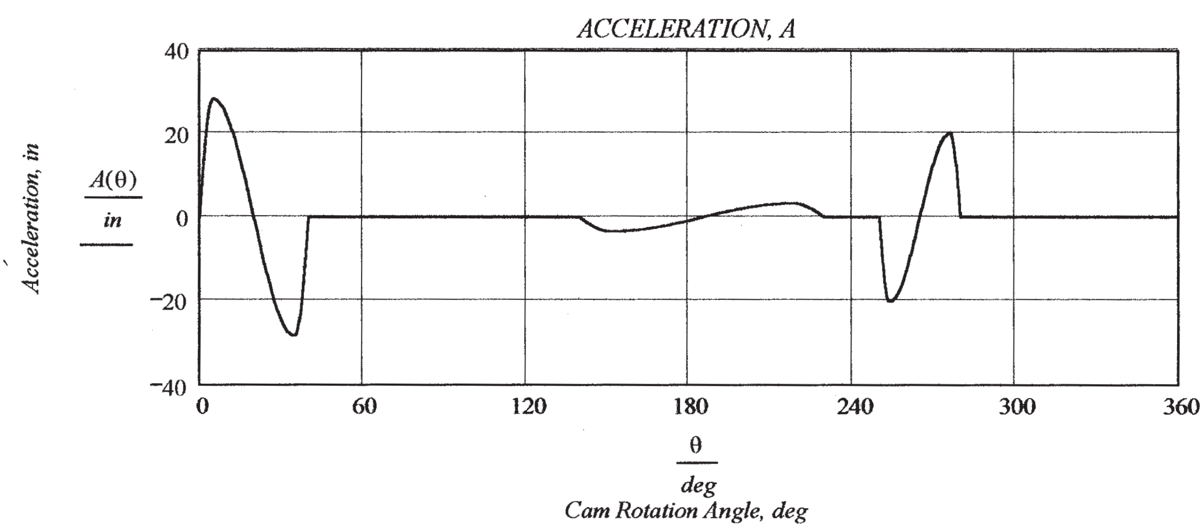
15. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \dots$$
$$+R(\theta,\theta_4,\theta_5)\cdot v_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot v_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right)$$



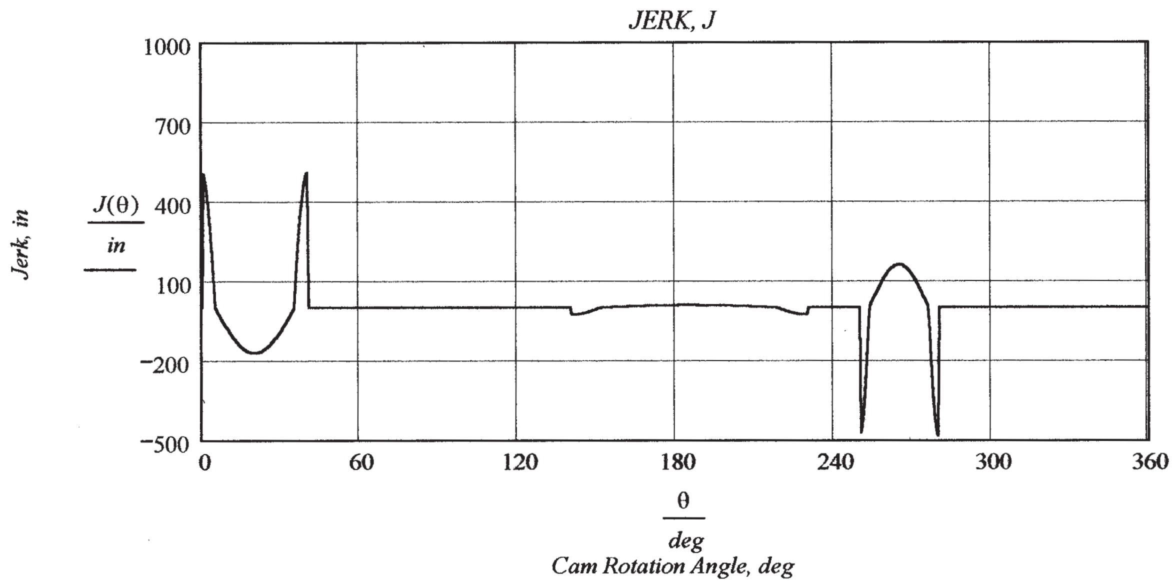
16. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \dots$$
$$+R(\theta,\theta_4,\theta_5)\cdot a_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot a_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right)$$



17. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot j2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot j3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot j4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \ldots$$
$$+R(\theta,\theta_4,\theta_5)\cdot j5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot j6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right)$$



PROBLEM 8-11

Statement: Design a four-dwell cam to move a follower from 0 to 2.5 in in 40 deg, dwell for 100 deg, fall 1.5 in in 90 deg, dwell for 20 deg, fall 0.5 in in 30 deg, dwell for 40 deg, fall 0.5 in in 30 deg, and dwell for the remainder. The total cycle must take 15 sec. Choose suitable programs for rise and fall to minimize accelerations. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0.0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0.0 \cdot in$
$\beta_5 := 30 \cdot deg$	$\beta_6 := 40 \cdot deg$	$\beta_7 := 30 \cdot deg$	$\beta_8 := 10 \cdot deg$
$h_5 := 0.5 \cdot in$	$h_6 := 0.0 \cdot in$	$h_7 := 0.5 \cdot in$	$h_8 := 0.0 \cdot in$

Cycle time: $\tau := 15 \cdot sec$

Solution: See Mathcad file P0811.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 0.419 \frac{rad}{sec}$$

2. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$$

$$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 <= x <= b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for $b/2 <= x <= (1 - d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2 <= x <= (1 + d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right)\cdot x + \left(2\cdot d^2 - b^2\right)\cdot\left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4}\right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi}\cdot x + \frac{2\cdot\left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{b}\cdot(x-1)\right]\right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b}\cdot(x-1)\right]\right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b}\cdot(x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b}\cdot(x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of eight intervals (rise, dwell, fall, dwell, fall, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 <= \theta <= 360$ deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x)\right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x)\right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x)\right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x)\right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$ (dwell)

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 .+ \beta_2 + \beta_3$ (fall)

$$s_3(x) := h_3\left[1 - \begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x) \end{pmatrix}\right] + h_1 - h_3$$

$$v_3(x) := -\frac{h_3}{\beta_3}\cdot\begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x) \end{pmatrix}$$

$$a_3(x) := -\frac{h_3}{\beta_3^{\,2}}\cdot\begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x) \end{pmatrix}$$

$$j_3(x) := -\frac{h_3}{\beta_3^{\,3}}\cdot\begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x) \end{pmatrix}$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4$ (dwell)

$$s_4(x) := h_1 - h_3 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the local *svaj* equations for the fifth interval,

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 <= \theta <= \beta_1 .+ \beta_2 + \beta_3 + \beta_4 + \beta_5$ (fall)

$$s_5(x) := h_5\left[1 - \begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x) \end{pmatrix}\right] + h_7$$

$$v_5(x) := -\frac{h_5}{\beta_5}\cdot\begin{pmatrix} R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x) \end{pmatrix}$$

$$a_5(x) := \frac{h_5}{\beta_5^2}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x) \end{array}\right)$$

$$j_5(x) := \frac{h_5}{\beta_5^3}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x) \end{array}\right)$$

11. **Write the local *svaj* equations for the sixth interval. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.**

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$ (dwell)

$$s_6(x) := h_5 \qquad v_6(x) := 0 \qquad a_6(x) := 0 \qquad j_6(x) := 0$$

12. **Write the local *svaj* equations for the seventh interval,**

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7$ (fall)

$$s_7(x) := h_7\left[1 - \left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x) \end{array}\right)\right]$$

$$v_7(x) := -\frac{h_7}{\beta_7}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x) \end{array}\right)$$

$$a_7(x) := -\frac{h_7}{\beta_7^2}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x) \end{array}\right)$$

$$j_7(x) := -\frac{h_7}{\beta_7^3}\cdot\left(\begin{array}{l} R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \\ + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x) \end{array}\right)$$

13. **Write the local *svaj* equations for the eighth interval. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.**

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8$ (dwell)

$$s_8(x) := 0 \qquad v_8(x) := 0 \qquad a_8(x) := 0 \qquad j_8(x) := 0$$

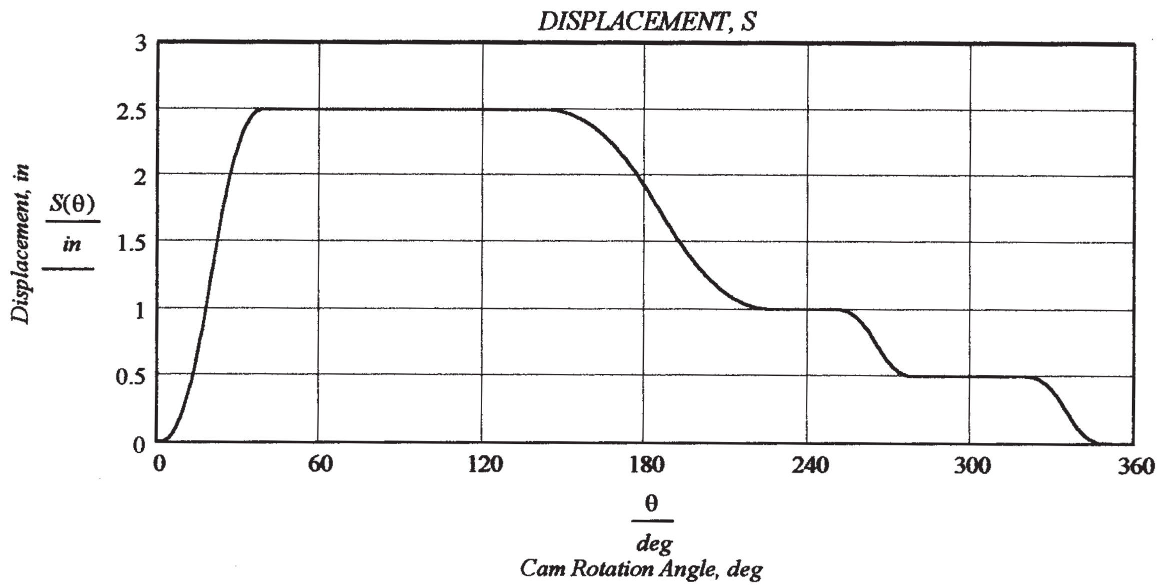
14. **Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.**

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

$\theta_5 := \theta_4 + \beta_5 \qquad \theta_6 := \theta_5 + \beta_6 \qquad \theta_7 := \theta_6 + \beta_7 \qquad \theta_8 := \theta_7 + \beta_8$

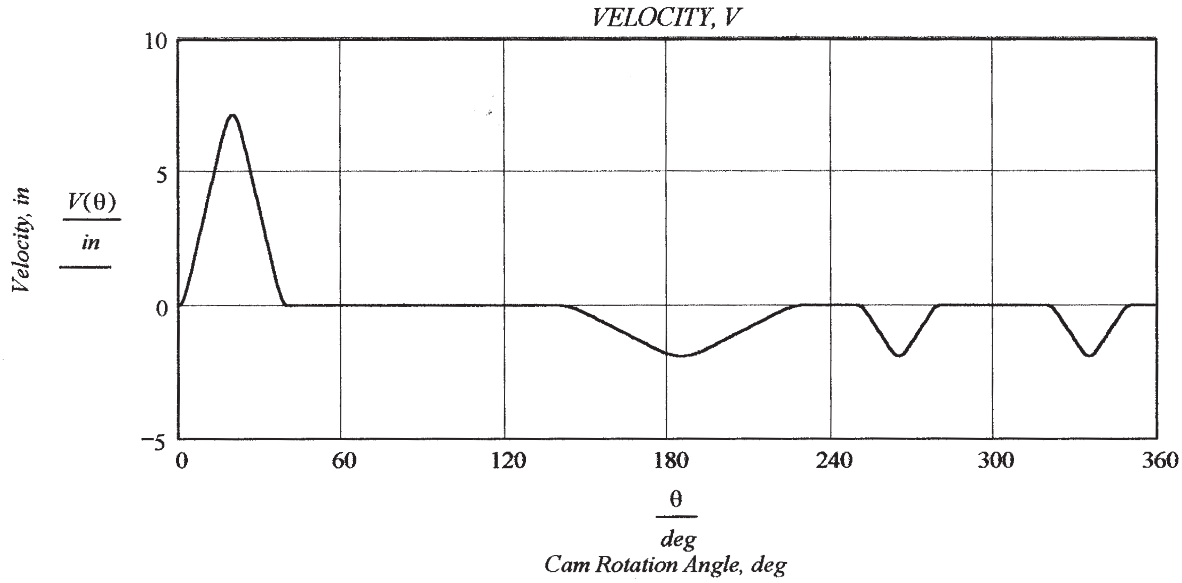
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \dots$$
$$+R(\theta,\theta_4,\theta_5)\cdot s_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot s_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right) \dots$$
$$+R(\theta,\theta_6,\theta_7)\cdot s_7\left(\frac{\theta-\theta_6}{\theta_7-\theta_6}\right) + R(\theta,\theta_7,\theta_8)\cdot s_8\left(\frac{\theta-\theta_7}{\theta_8-\theta_7}\right)$$

$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$



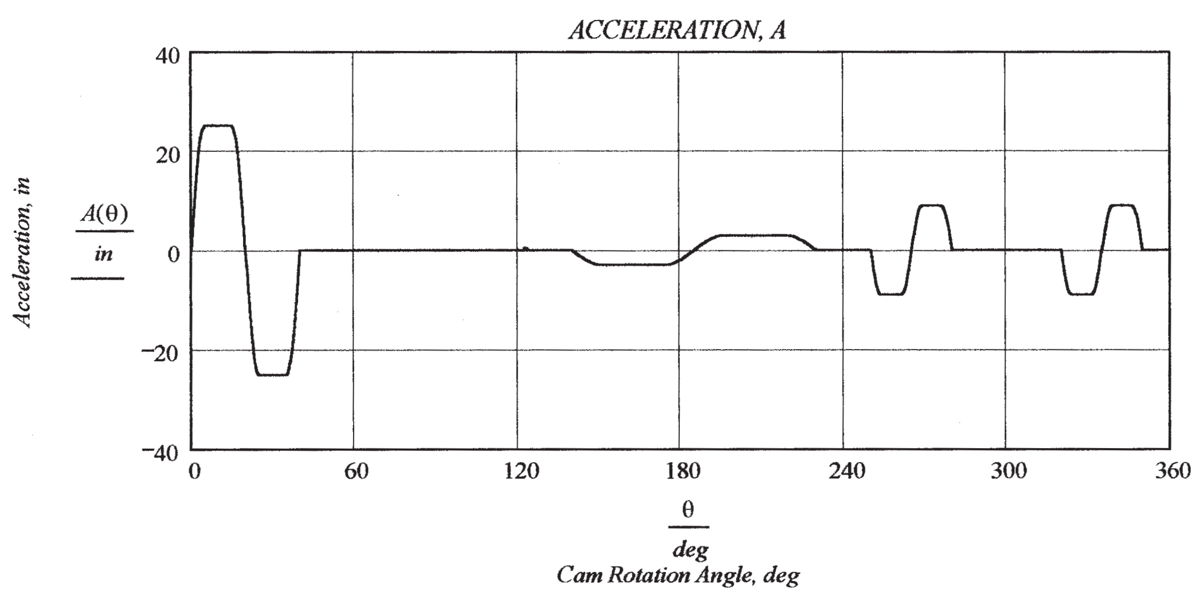
15. **Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.**

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \dots$$
$$+R(\theta,\theta_4,\theta_5)\cdot v_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot v_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right) \dots$$
$$+R(\theta,\theta_6,\theta_7)\cdot v_7\left(\frac{\theta-\theta_6}{\theta_7-\theta_6}\right) + R(\theta,\theta_7,\theta_8)\cdot v_8\left(\frac{\theta-\theta_7}{\theta_8-\theta_7}\right)$$



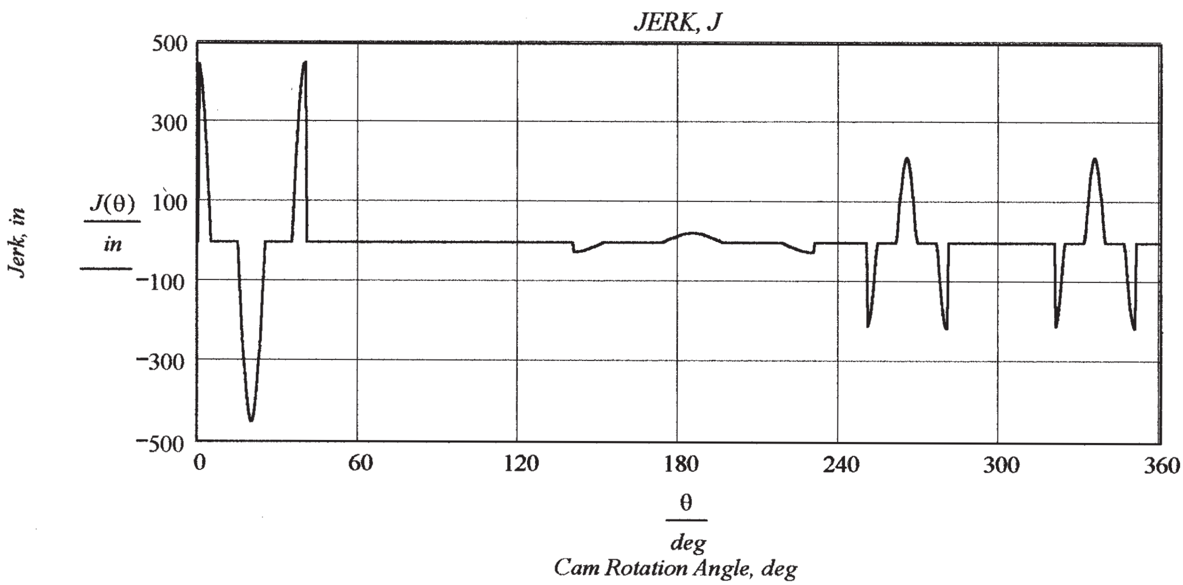
16. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \dots$$
$$+R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \dots$$
$$+R(\theta,\theta_4,\theta_5)\cdot a_5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot a_6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right) \dots$$
$$+R(\theta,\theta_6,\theta_7)\cdot a_7\left(\frac{\theta-\theta_6}{\theta_7-\theta_6}\right) + R(\theta,\theta_7,\theta_8)\cdot a_8\left(\frac{\theta-\theta_7}{\theta_8-\theta_7}\right)$$



17. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot j2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot j3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot j4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right) \ldots$$
$$+R(\theta,\theta_4,\theta_5)\cdot j5\left(\frac{\theta-\theta_4}{\theta_5-\theta_4}\right) + R(\theta,\theta_5,\theta_6)\cdot j6\left(\frac{\theta-\theta_5}{\theta_6-\theta_5}\right) \ldots$$
$$+R(\theta,\theta_6,\theta_7)\cdot j7\left(\frac{\theta-\theta_6}{\theta_7-\theta_6}\right) + R(\theta,\theta_7,\theta_8)\cdot j8\left(\frac{\theta-\theta_7}{\theta_8-\theta_7}\right)$$



PROBLEM 8-12

Statement: Size the cam from Problem 8-7 for a 1-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot deg$	$\beta_2 := 120 \cdot deg$	$\beta_3 := 30 \cdot deg$	$\beta_4 := 150 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 2.5 \cdot in$	$h_4 := 0 \cdot in$

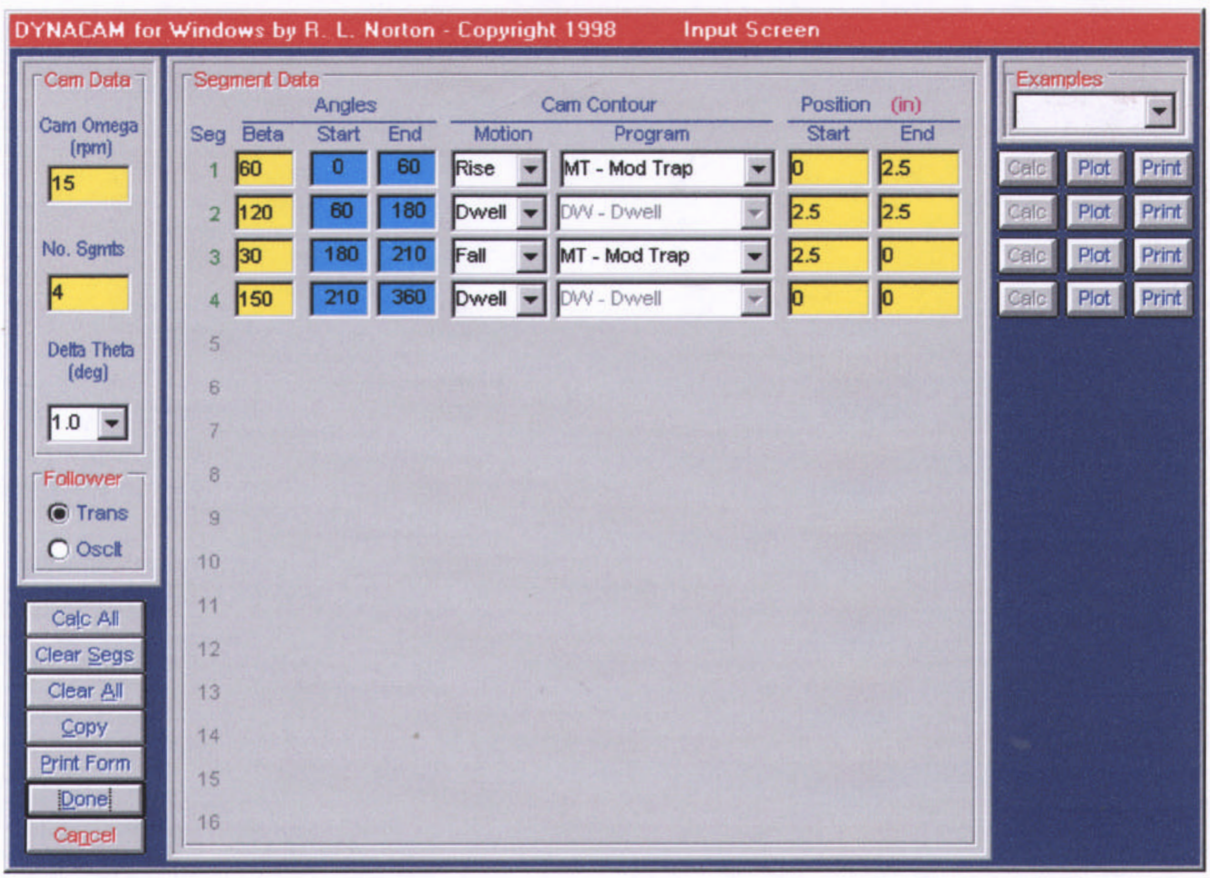
Cycle time: $\tau := 4 \cdot sec$

Solution: See Mathcad file P0812.

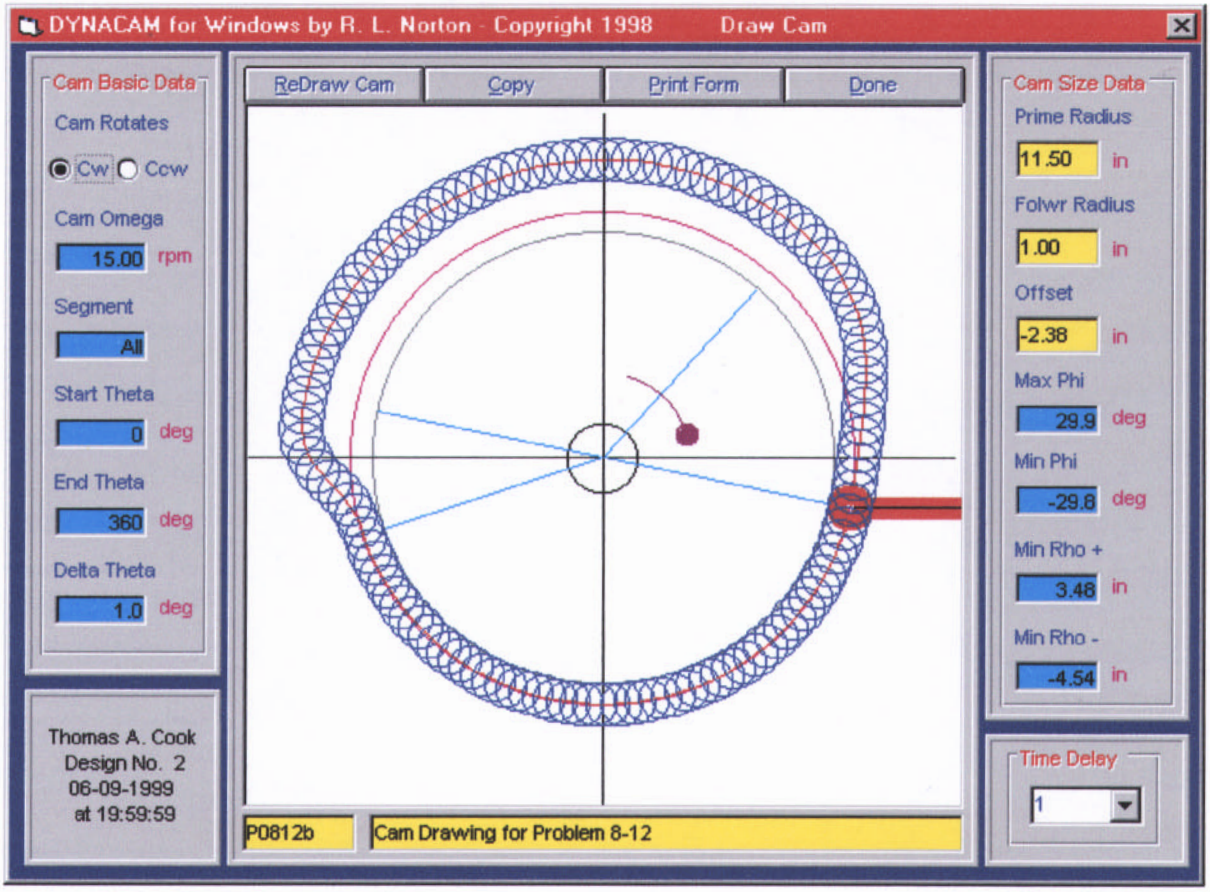
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.571 \frac{rad}{sec} \qquad \omega = 15.000\, rpm$$

2. Problem 8-7 used a four-segment cam with modified trapezoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.

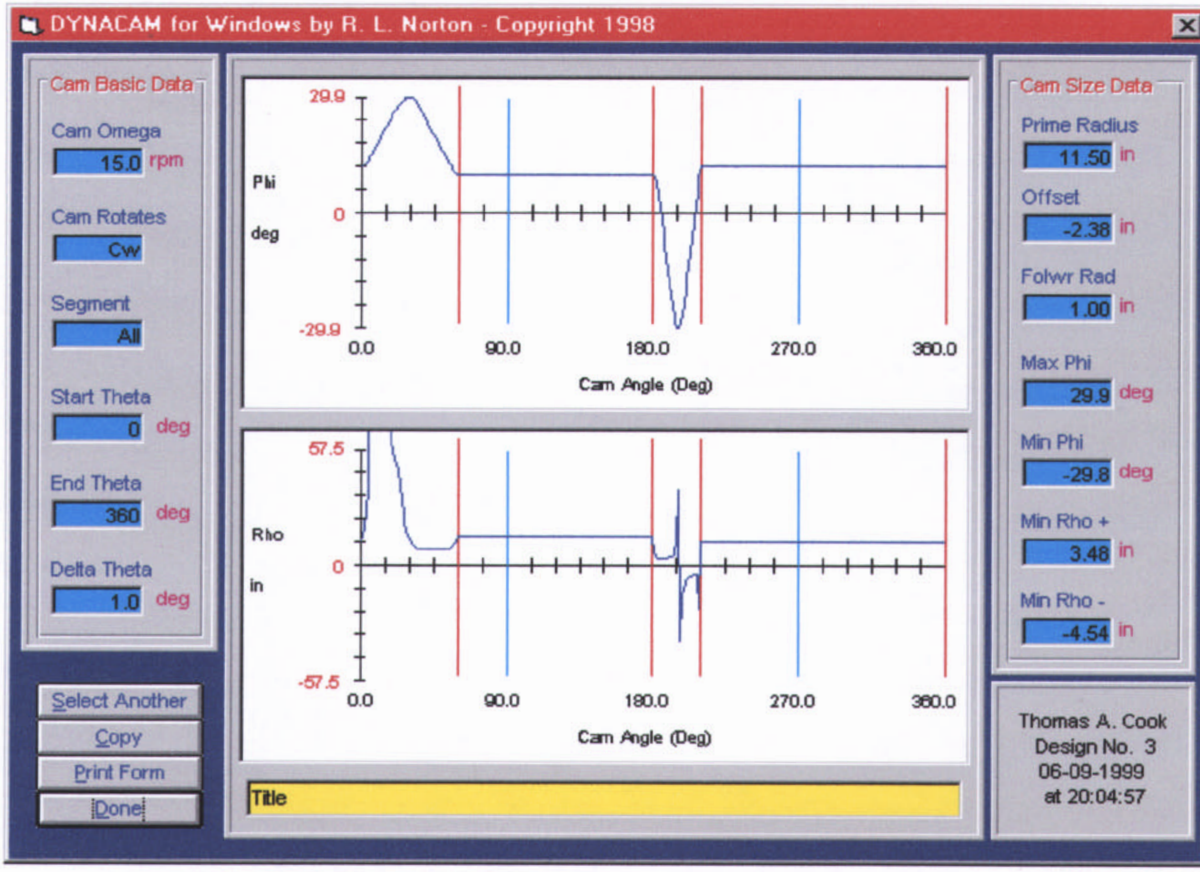


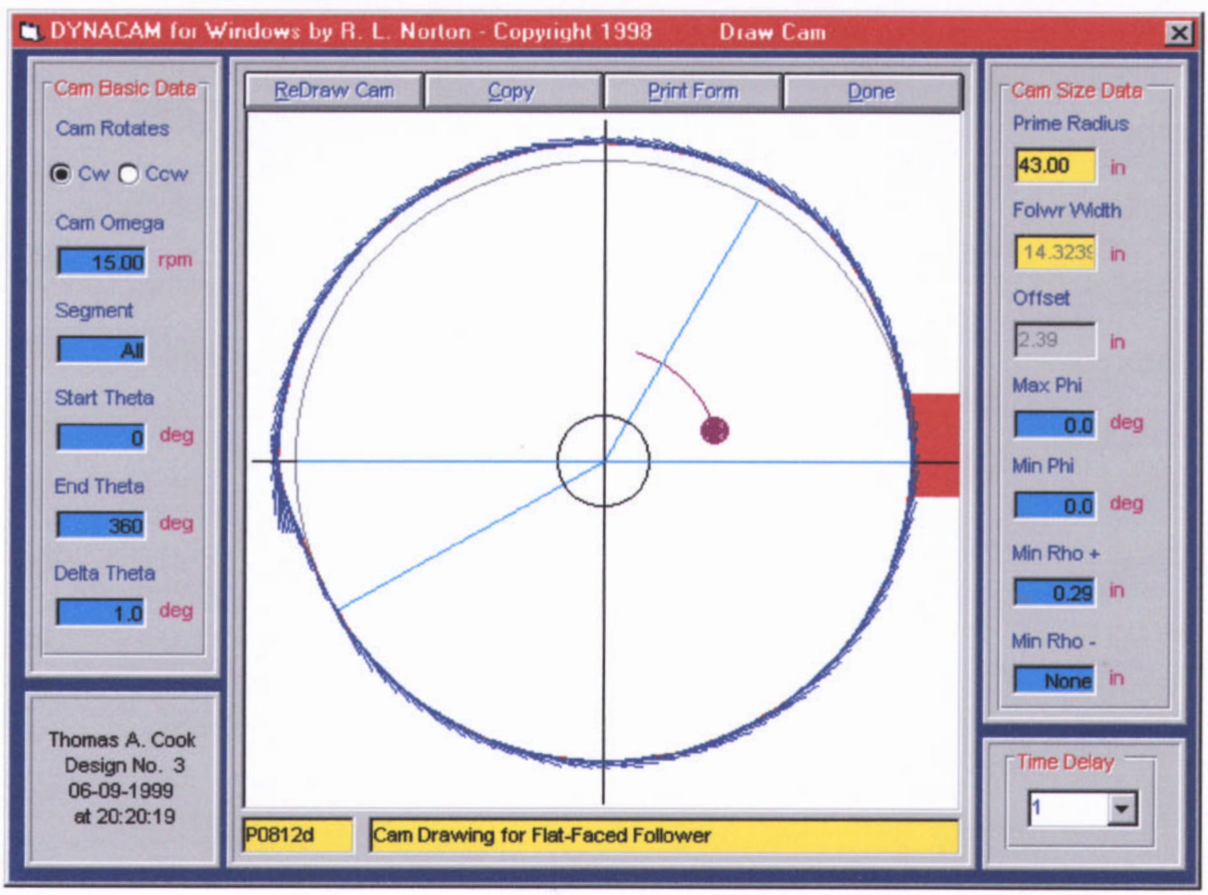
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

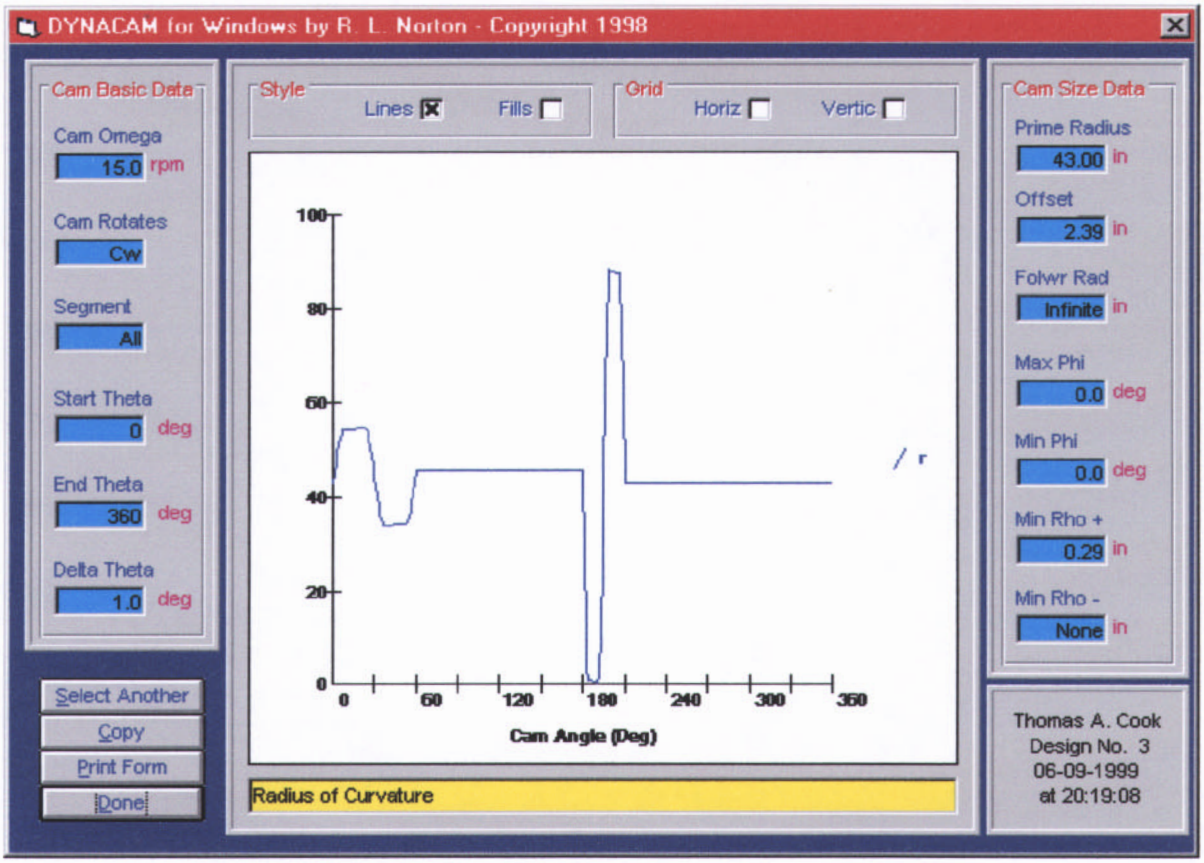
Prime circle radius	$R_p := 11.5 \cdot in$
Roller follower radius	$R_f := 1.00 \cdot in$
Follower eccentricity	$\varepsilon := -2.375 \cdot in$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.

6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 43 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
15.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
29.9
Phi
deg
0
-29.9
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
57.5
Rho
in
0
-57.5
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Title
Cam Size Data
Prime Radius
11.50 in
Offset
-2.38 in
Folwr Rad
1.00 in
Max Phi
29.9 deg
Min Phi
-29.8 deg
Min Rho +
3.48 in
Min Rho -
-4.54 in
Thomas A. Cook
Design No. 3
06-09-1999
at 20:04:57


DYNACAM for Windows by R. L. Norton - Copyright 1998 Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
15.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 3
06-09-1999
at 20:20:19
ReDraw Cam
Copy
Print Form
Done
P0812d
Cam Drawing for Flat-Faced Follower
Cam Size Data
Prime Radius
43.00 in
Folwr Width
14.3239 in
Offset
2.39 in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.29 in
Min Rho -
None in
Time Delay
1


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
15.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Style
Lines
Fills
Grid
Horiz
Vertic
100
80
60
40
20
0
0
60
120
180
240
300
360
Cam Angle (Deg)
Radius of Curvature
Cam Size Data
Prime Radius
43.00 in
Offset
2.39 in
Folwr Rad
Infinite in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.29 in
Min Rho -
None in
Thomas A. Cook
Design No. 3
06-09-1999
at 20:19:08

PROBLEM 8-13

Statement: Size the cam from Problem 8-8 for a 1.5-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 45 \cdot deg$	$\beta_2 := 150 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 75 \cdot deg$
$h_1 := 1.5 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0 \cdot in$

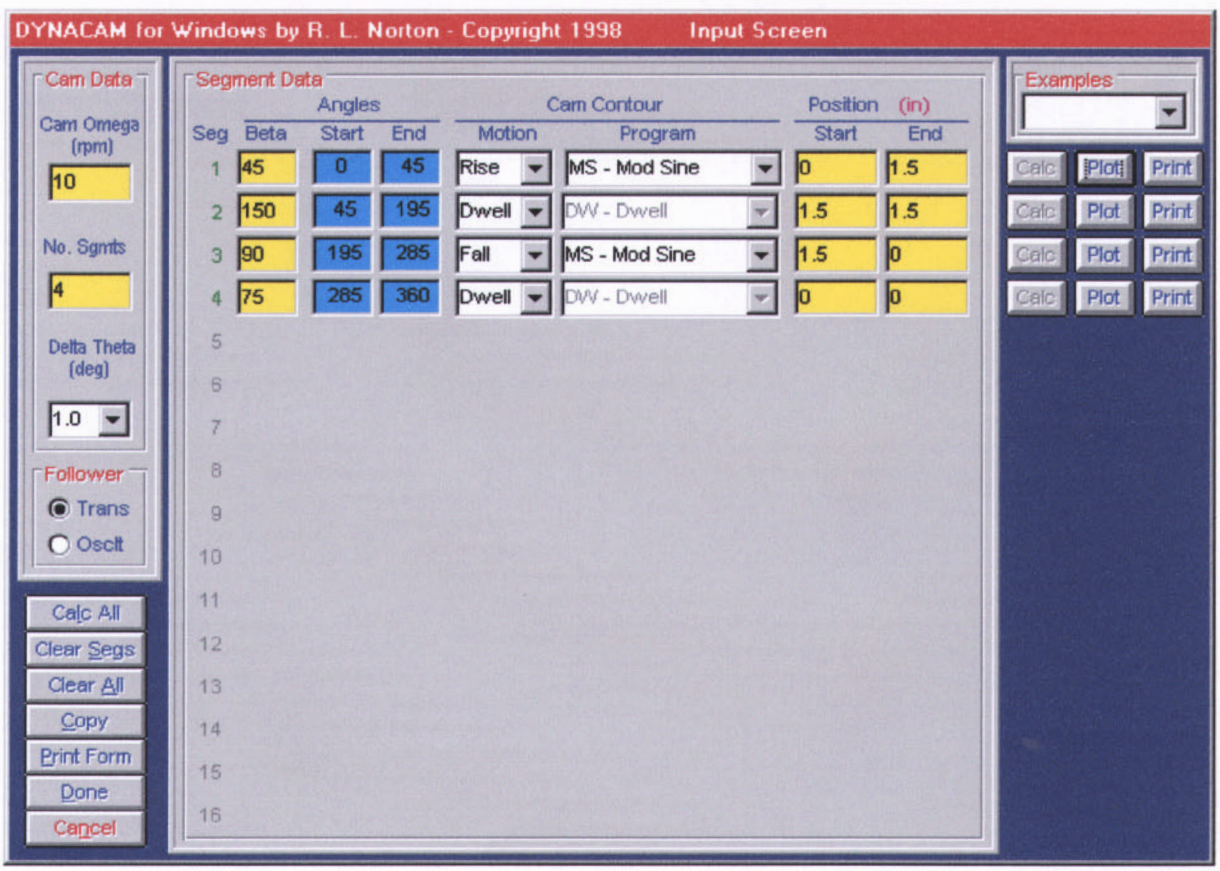
Cycle time: $\tau := 6 \cdot sec$

Solution: See Mathcad file P0813.

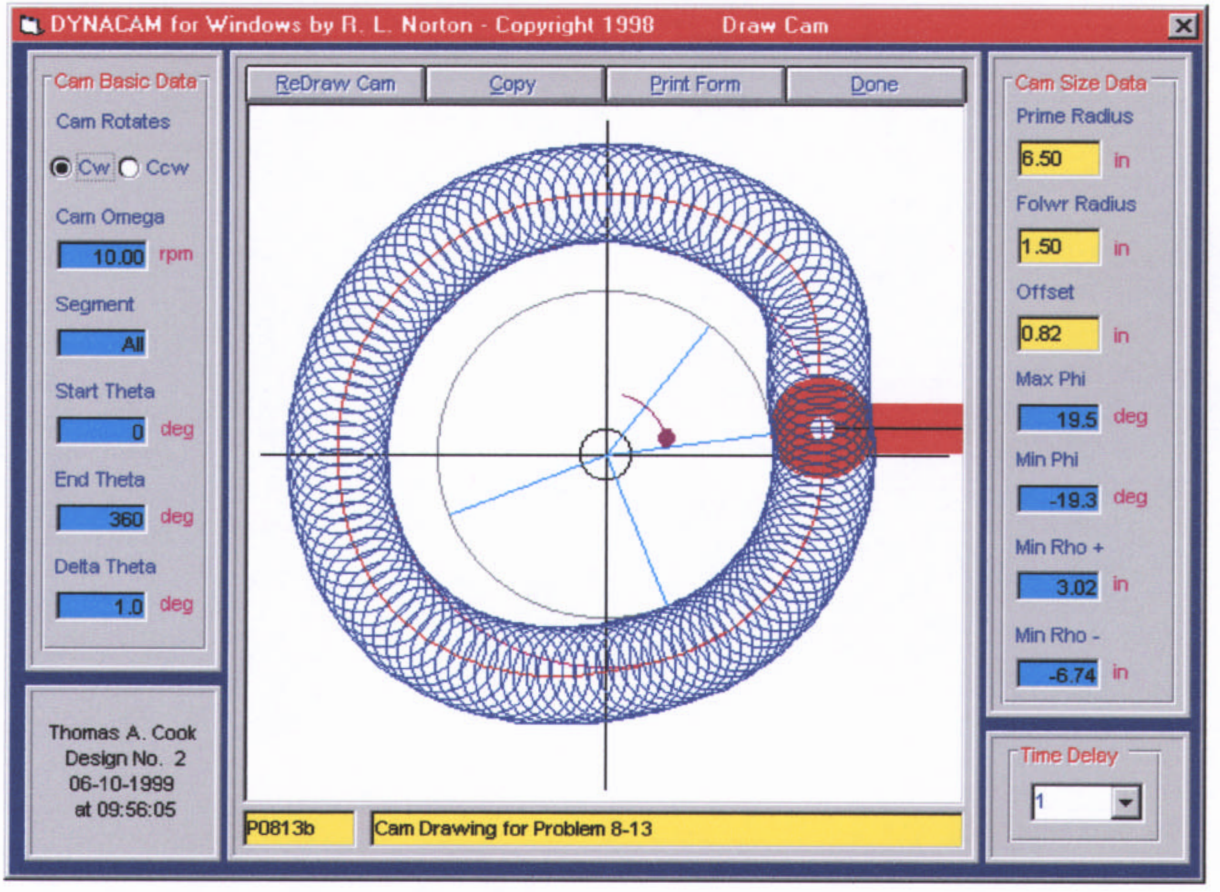
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.047 \frac{rad}{sec} \qquad \omega = 10.000\, rpm$$

2. Problem 8-8 used a four-segment cam with modified sinusoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.

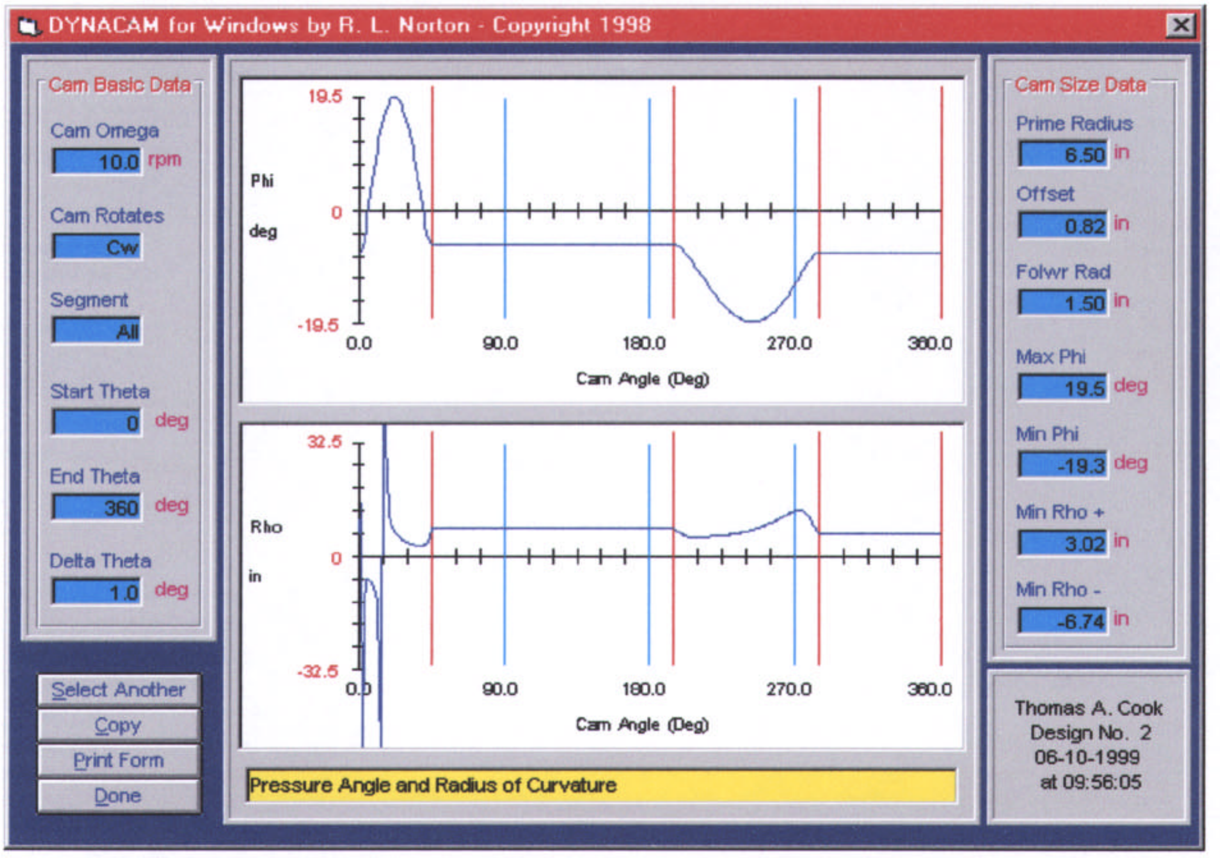


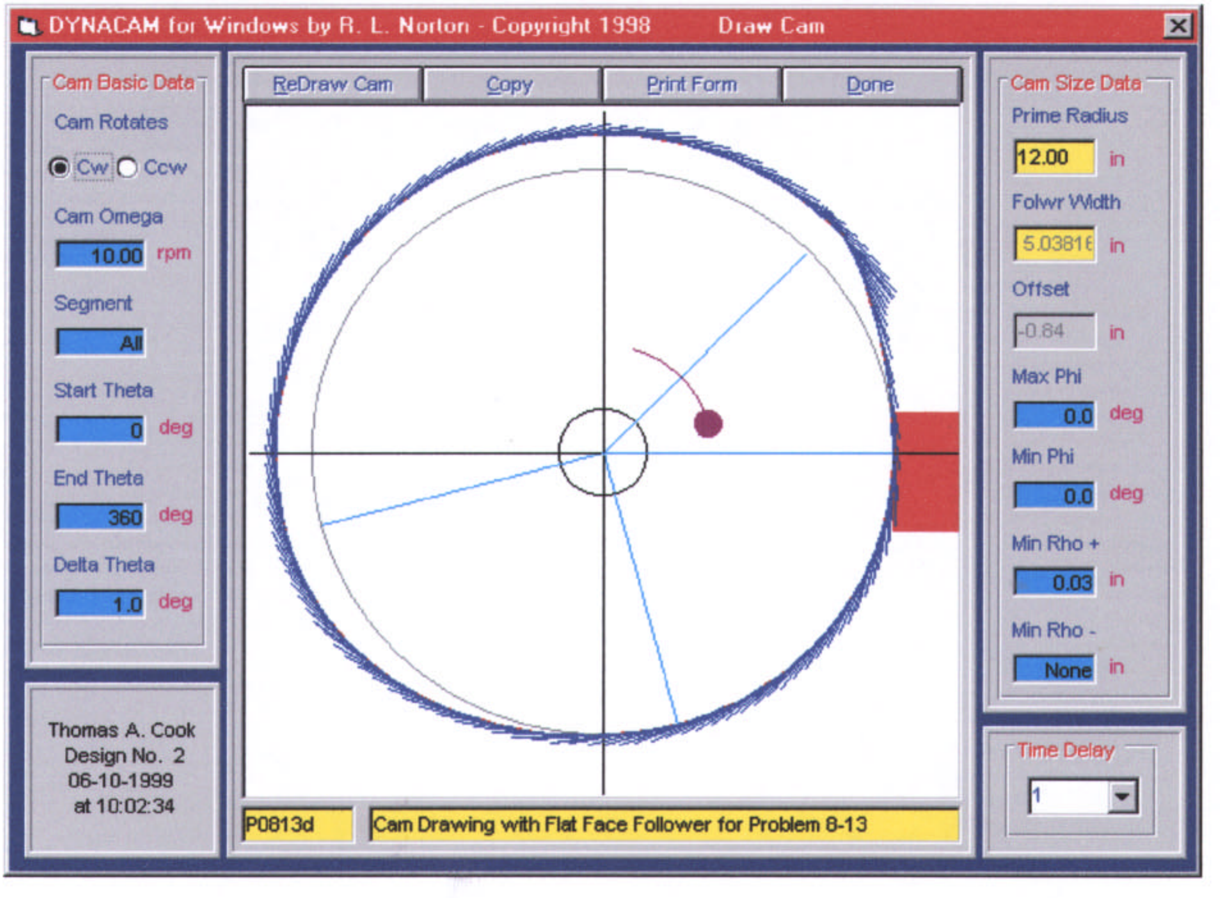
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The prime radius had to be increased above that which was necessary for the pressure angle constraint in order to provide a minimum radius of curvature that was twice the roller follower radius. The design has the following dimensions:

Prime circle radius	$R_p := 6.5 \cdot in$
Roller follower radius	$R_f := 1.50 \cdot in$
Follower eccentricity	$\varepsilon := 0.82 \cdot in$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.

6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 12 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
10.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
19.5
Phi
deg
0
-19.5
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
32.5
Rho
in
-32.5
Pressure Angle and Radius of Curvature
Cam Size Data
Prime Radius
6.50 in
Offset
0.82 in
Folwr Rad
1.50 in
Max Phi
19.5 deg
Min Phi
-19.3 deg
Min Rho +
3.02 in
Min Rho -
-6.74 in
Thomas A. Cook
Design No. 2
06-10-1999
at 09:56:05


DYNACAM for Windows by R. L. Norton - Copyright 1998 Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
10.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 2
06-10-1999
at 10:02:34
ReDraw Cam
Copy
Print Form
Done
P0813d
Cam Drawing with Flat Face Follower for Problem 8-13
Cam Size Data
Prime Radius
12.00 in
Folwr Width
5.03816 in
Offset
-0.84 in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.03 in
Min Rho -
None in
Time Delay
1

PROBLEM 8-14

Statement: Size the cam from Problem 8-9 for a 0.5-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

RISE/FALL	DWELL
$\beta := 150\cdot deg$	$\beta_2 := 150\cdot deg$
$h_1 := 2.0\cdot in$	$h_2 := 0.0\cdot in$

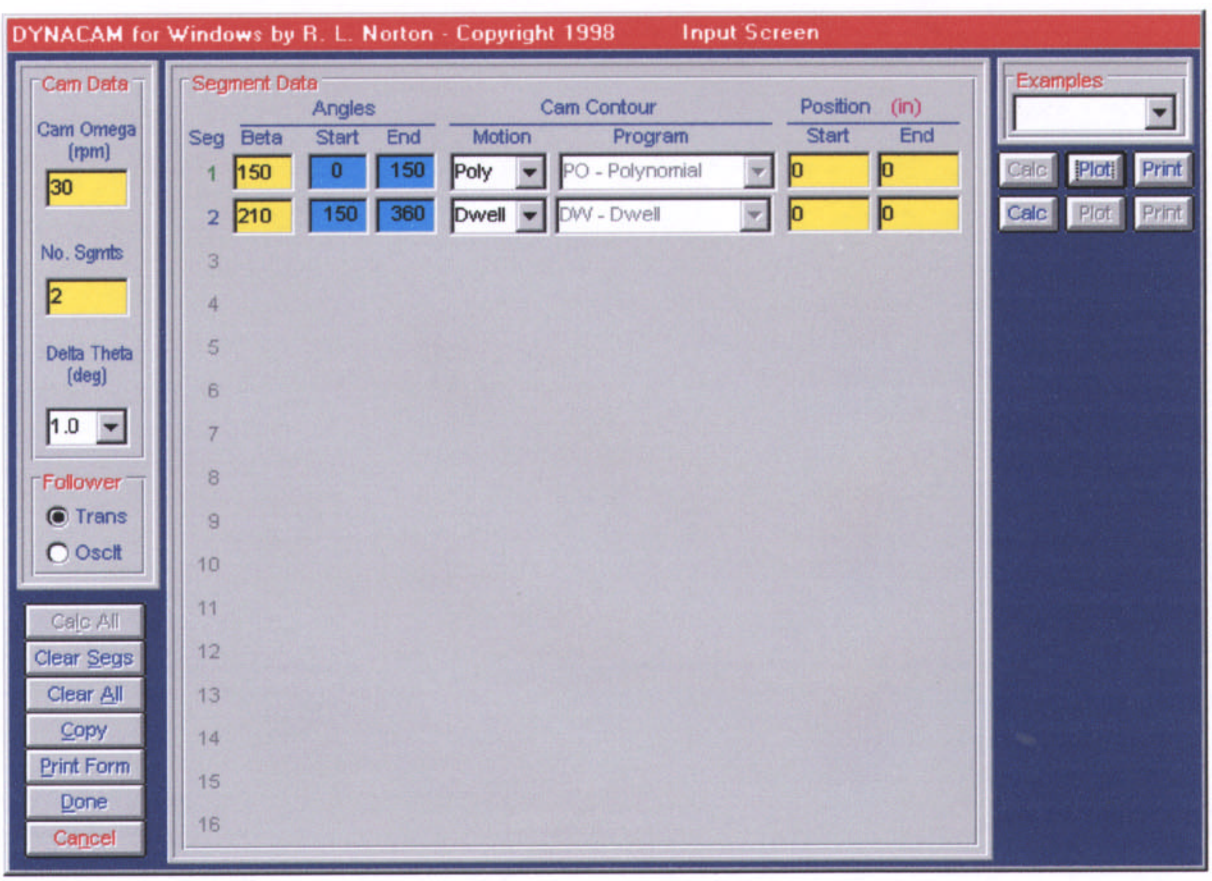
Cycle time: $\tau := 2\cdot sec$

Solution: See Mathcad file P0814.

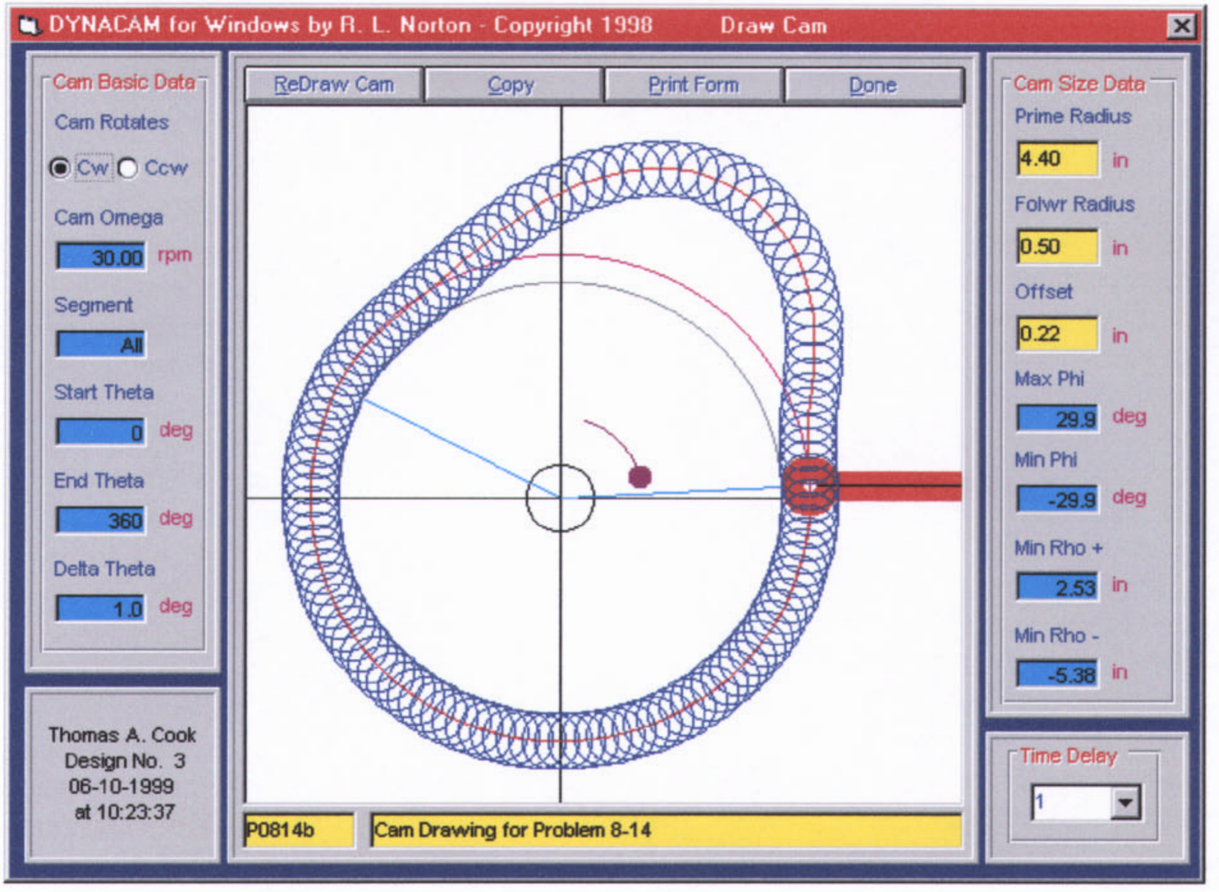
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 3.142\,\frac{rad}{sec} \qquad \omega = 30.000\,rpm$$

2. Problem 8-9 used a two-segment cam with one polynomial segment for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

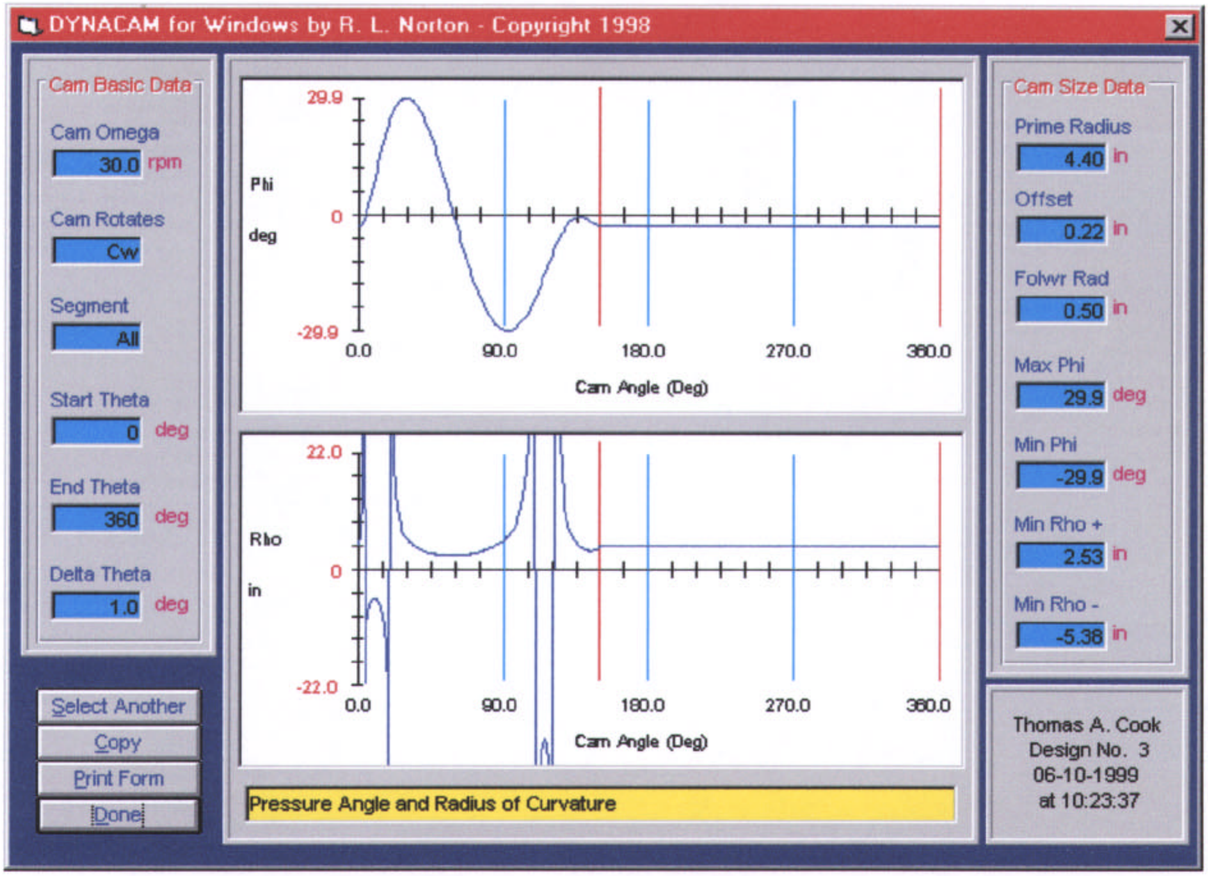
Prime circle radius $R_p := 4.40 \cdot in$

Roller follower radius $R_f := 0.50 \cdot in$

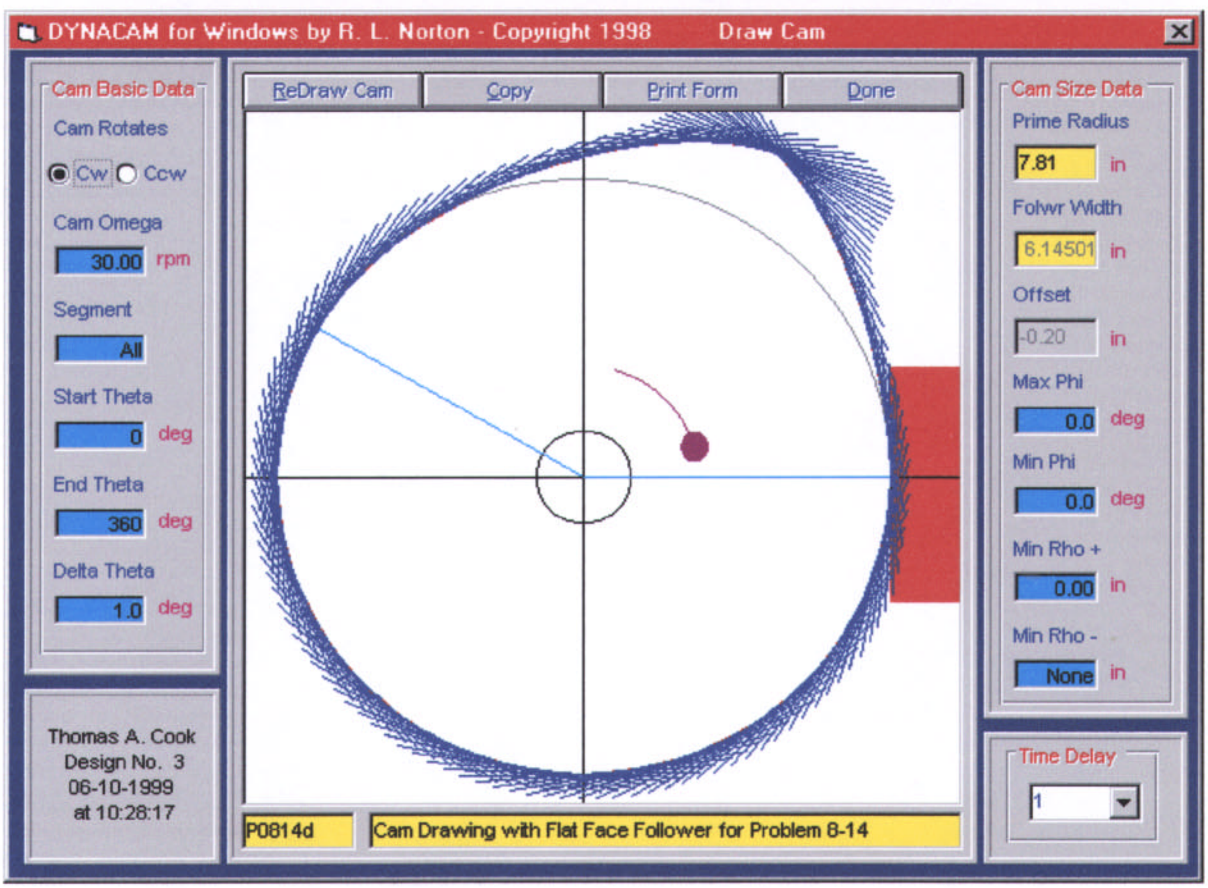
Follower eccentricity $\varepsilon := 0.22 \cdot in$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.

6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 7.81 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.



DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
30.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
29.9
Phi
deg
0
-29.9
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
22.0
Rho
in
0
-22.0
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Pressure Angle and Radius of Curvature
Cam Size Data
Prime Radius
4.40 in
Offset
0.22 in
Folwr Rad
0.50 in
Max Phi
29.9 deg
Min Phi
-29.9 deg
Min Rho +
2.53 in
Min Rho -
-5.38 in
Thomas A. Cook
Design No. 3
06-10-1999
at 10:23:37



DYNACAM for Windows by R. L. Norton - Copyright 1998 Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
30.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 3
06-10-1999
at 10:28:17
ReDraw Cam
Copy
Print Form
Done
P0814d
Cam Drawing with Flat Face Follower for Problem 8-14
Cam Size Data
Prime Radius
7.81 in
Folwr Width
6.14501 in
Offset
-0.20 in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.00 in
Min Rho -
None in
Time Delay
1

PROBLEM 8-15

Statement: Size the cam from Problem 8-10 for a 2.0-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0.0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0.0 \cdot in$
$\beta_5 := 30 \cdot deg$	$\beta_6 := 80 \cdot deg$		
$h_5 := 1.0 \cdot in$	$h_6 := 0.0 \cdot in$		

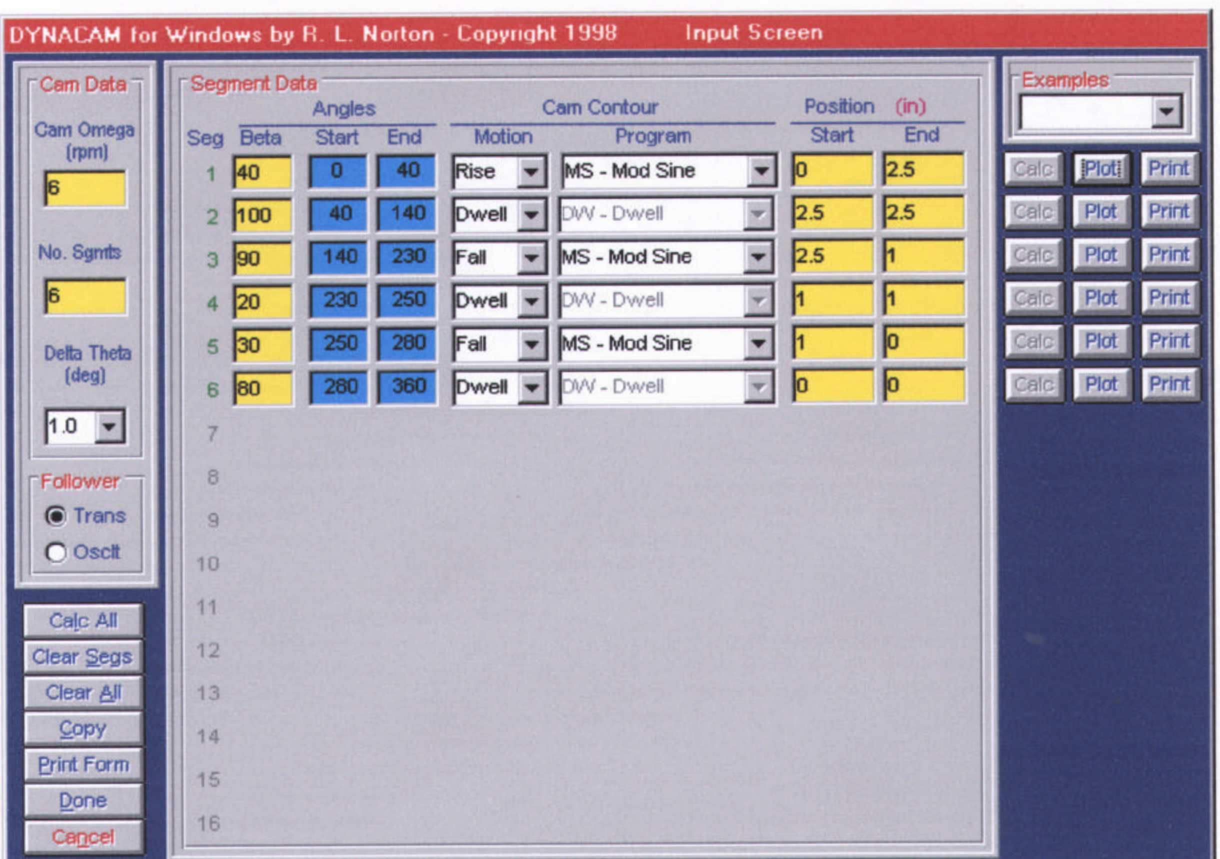
Cycle time: $\tau := 10 \cdot sec$

Solution: See Mathcad file P0815.

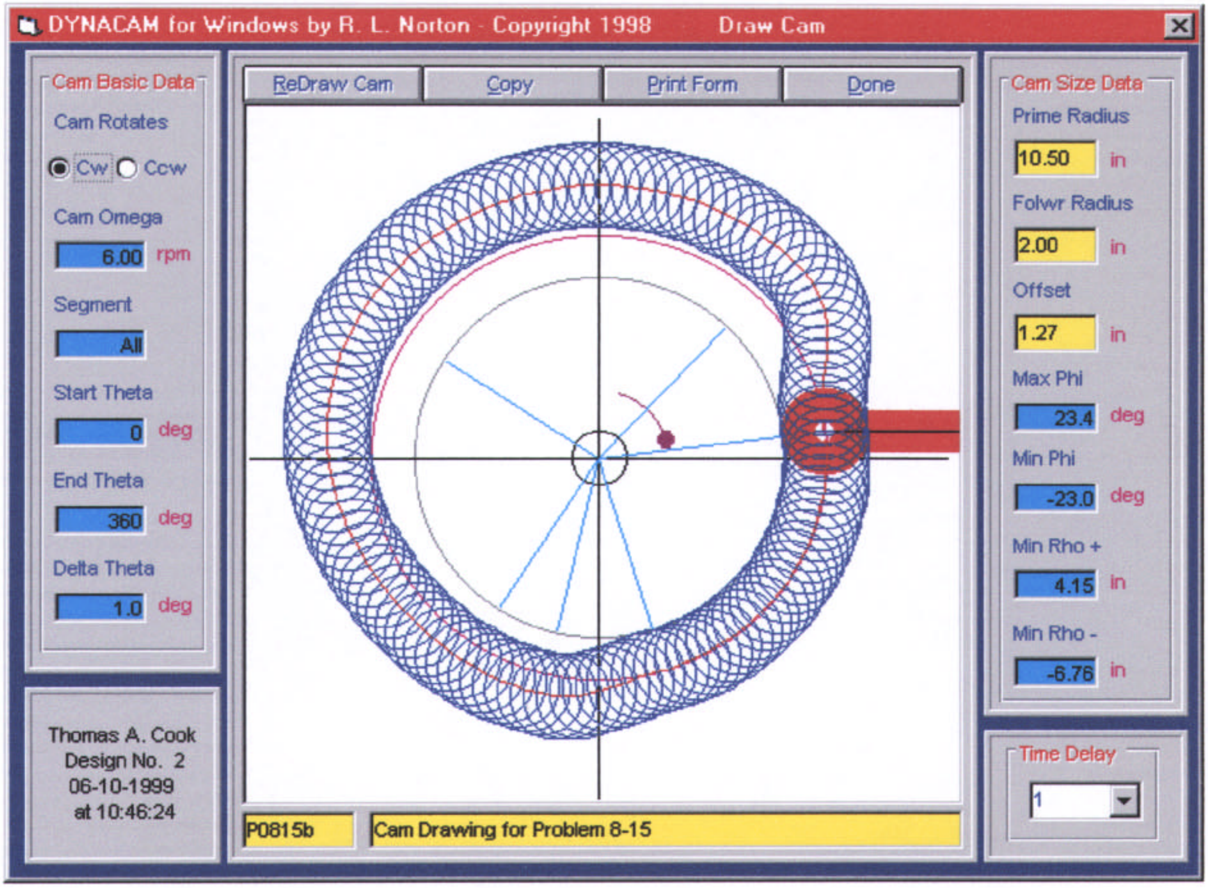
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 0.628 \frac{rad}{sec} \qquad \omega = 6.000\, rpm$$

2. Problem 8-10 used a six-segment cam with modified sinusoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The prime radius had to be increased above that which was necessary for the pressure angle constraint in order to provide a minimum radius of curvature that was twice the roller follower radius. The design has the following dimensions:

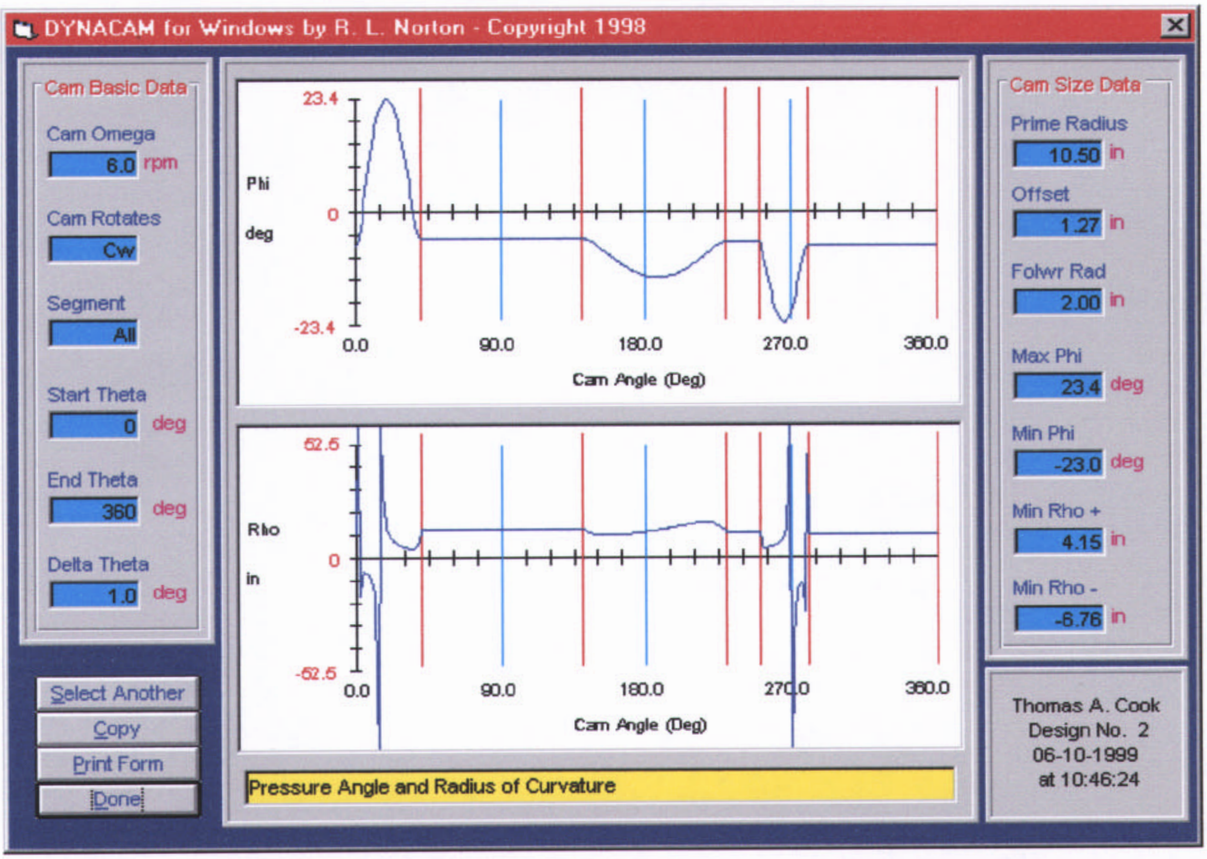
Prime circle radius $R_p := 10.50 \cdot in$

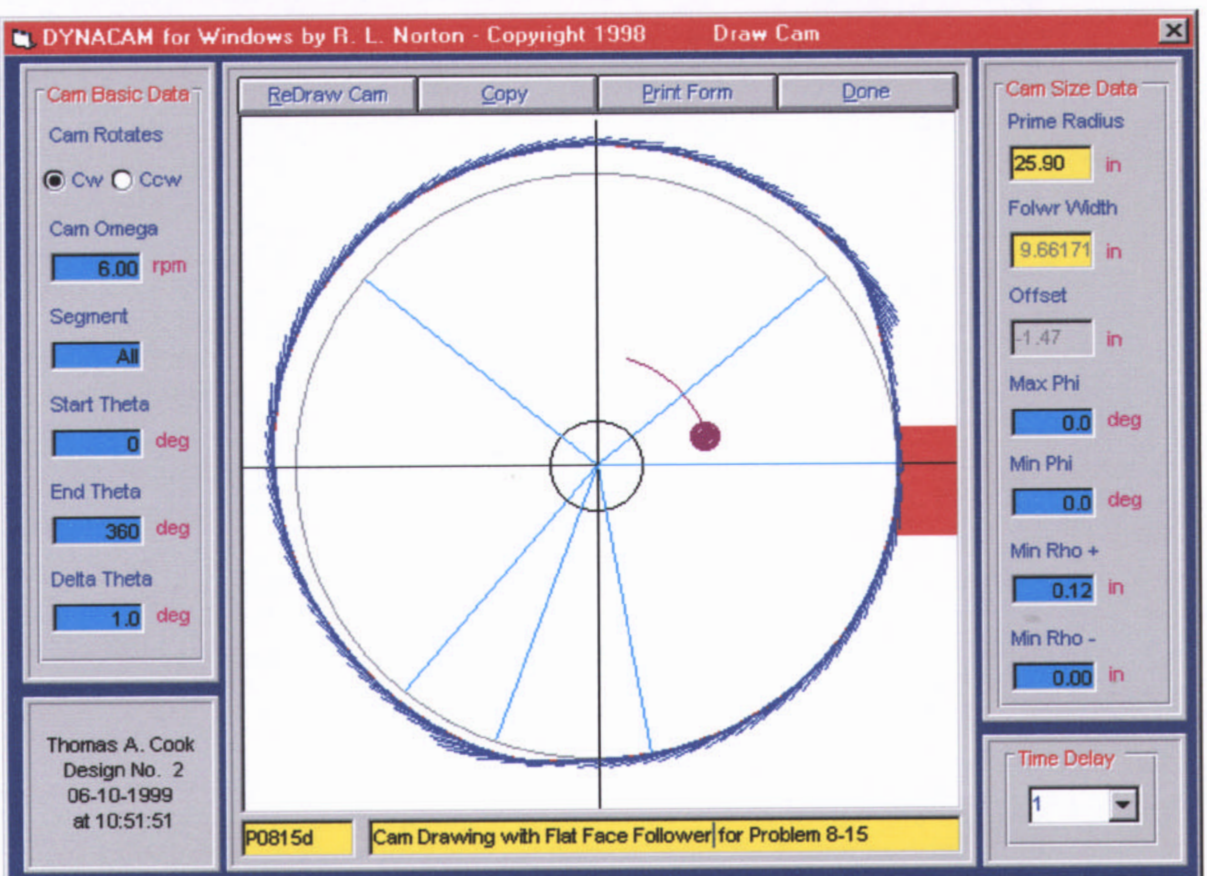
Roller follower radius $R_f := 2.00 \cdot in$

Follower eccentricity $\varepsilon := 1.27 \cdot in$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.

6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 25.9 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
6.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
23.4
Phi
deg
0
-23.4
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
52.5
Rho
in
0
-52.5
Pressure Angle and Radius of Curvature
Cam Size Data
Prime Radius
10.50 in
Offset
1.27 in
Folwr Rad
2.00 in
Max Phi
23.4 deg
Min Phi
-23.0 deg
Min Rho +
4.15 in
Min Rho -
-6.76 in
Thomas A. Cook
Design No. 2
06-10-1999
at 10:46:24


DYNACAM for Windows by R. L. Norton - Copyright 1998 Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
6.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 2
06-10-1999
at 10:51:51
ReDraw Cam
Copy
Print Form
Done
P0815d
Cam Drawing with Flat Face Follower for Problem 8-15
Cam Size Data
Prime Radius
25.90 in
Folwr Width
9.66171 in
Offset
-1.47 in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.12 in
Min Rho -
0.00 in
Time Delay
1

PROBLEM 8-16

Statement: Size the cam from Problem 8-11 for a 0.5-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40\cdot deg$	$\beta_2 := 100\cdot deg$	$\beta_3 := 90\cdot deg$	$\beta_4 := 20\cdot deg$
$h_1 := 2.5\cdot in$	$h_2 := 0.0\cdot in$	$h_3 := 1.5\cdot in$	$h_4 := 0.0\cdot in$
$\beta_5 := 30\cdot deg$	$\beta_6 := 40\cdot deg$	$\beta_7 := 30\cdot deg$	$\beta_8 := 10\cdot deg$
$h_5 := 0.5\cdot in$	$h_6 := 0.0\cdot in$	$h_7 := 0.5\cdot in$	$h_8 := 0.0\cdot in$

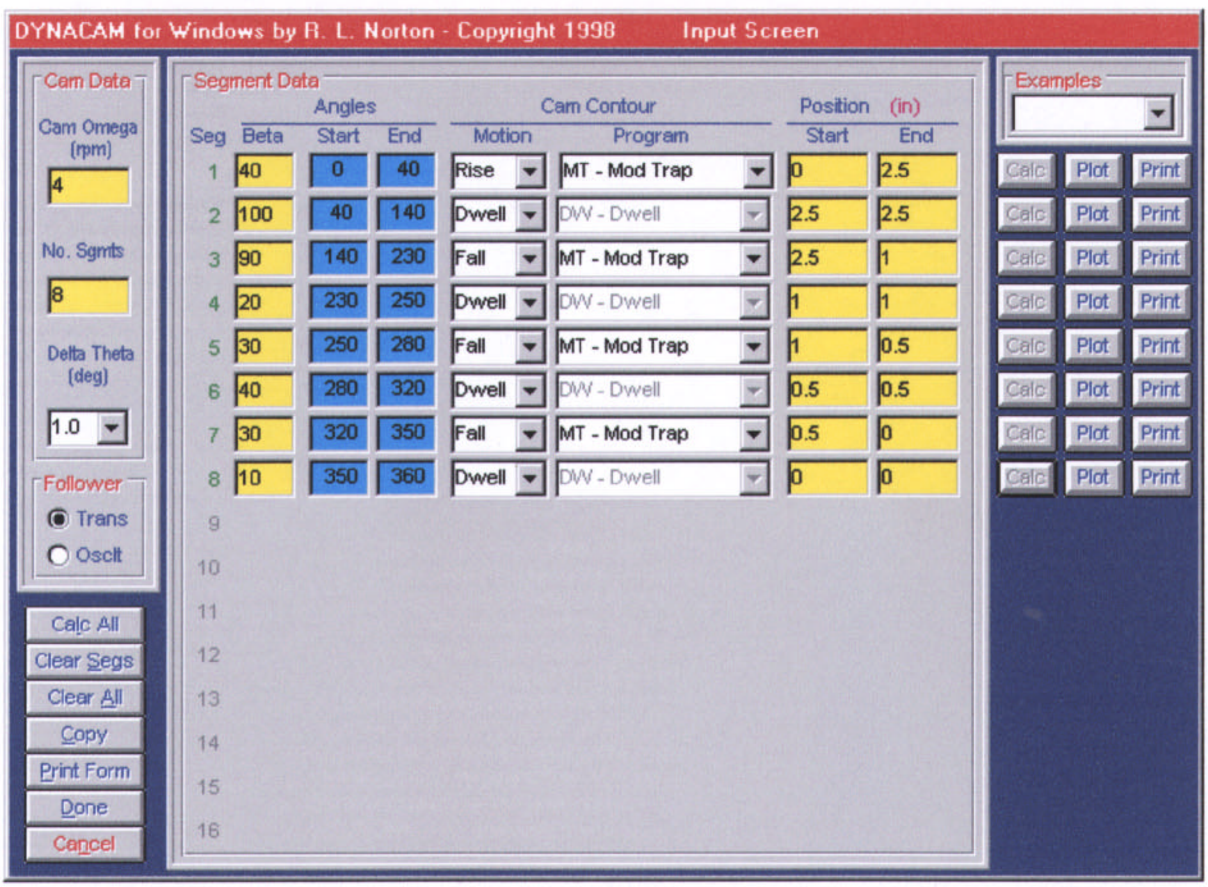
Cycle time: $\tau := 15\cdot sec$

Solution: See Mathcad file P0816.

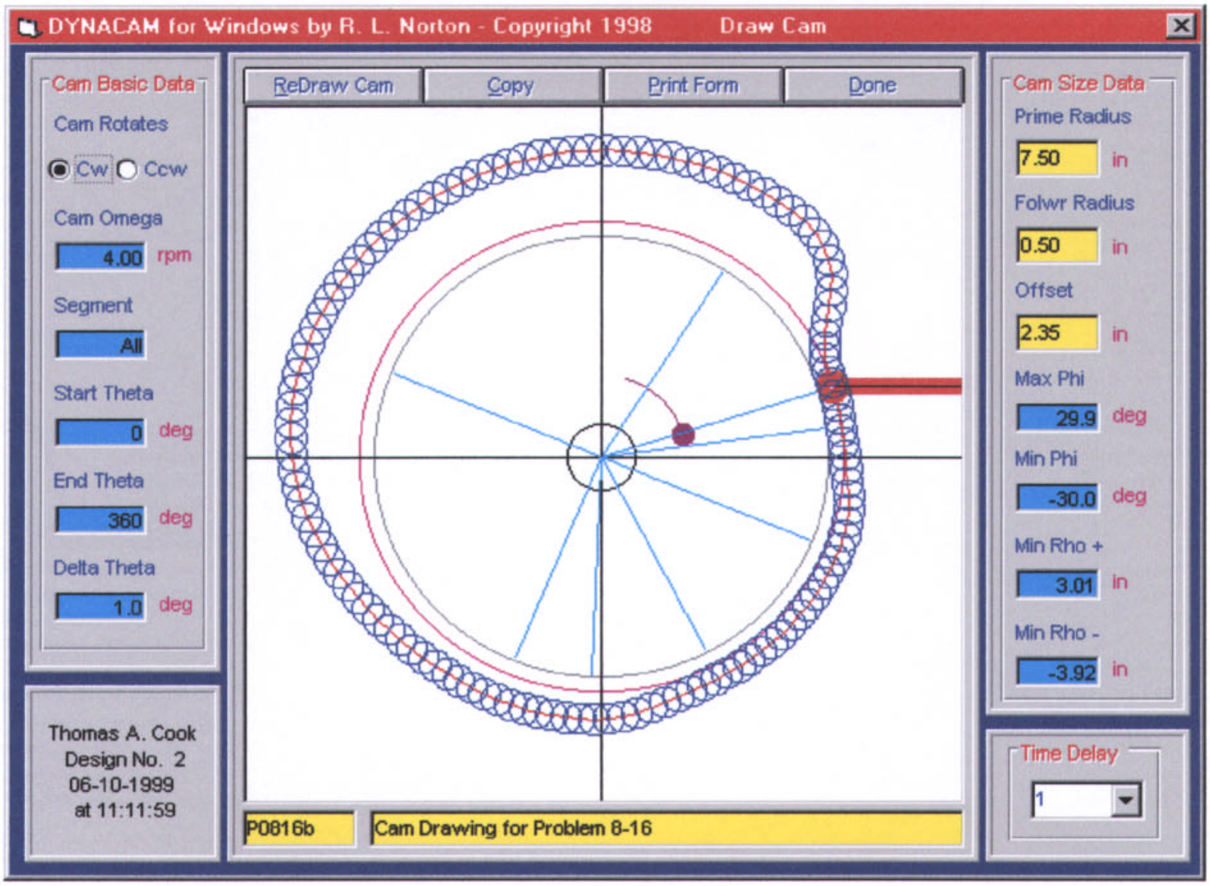
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 0.419\,\frac{rad}{sec} \qquad \omega = 4.000\,rpm$$

2. Problem 8-11 used an eight-segment cam with modified trapezoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

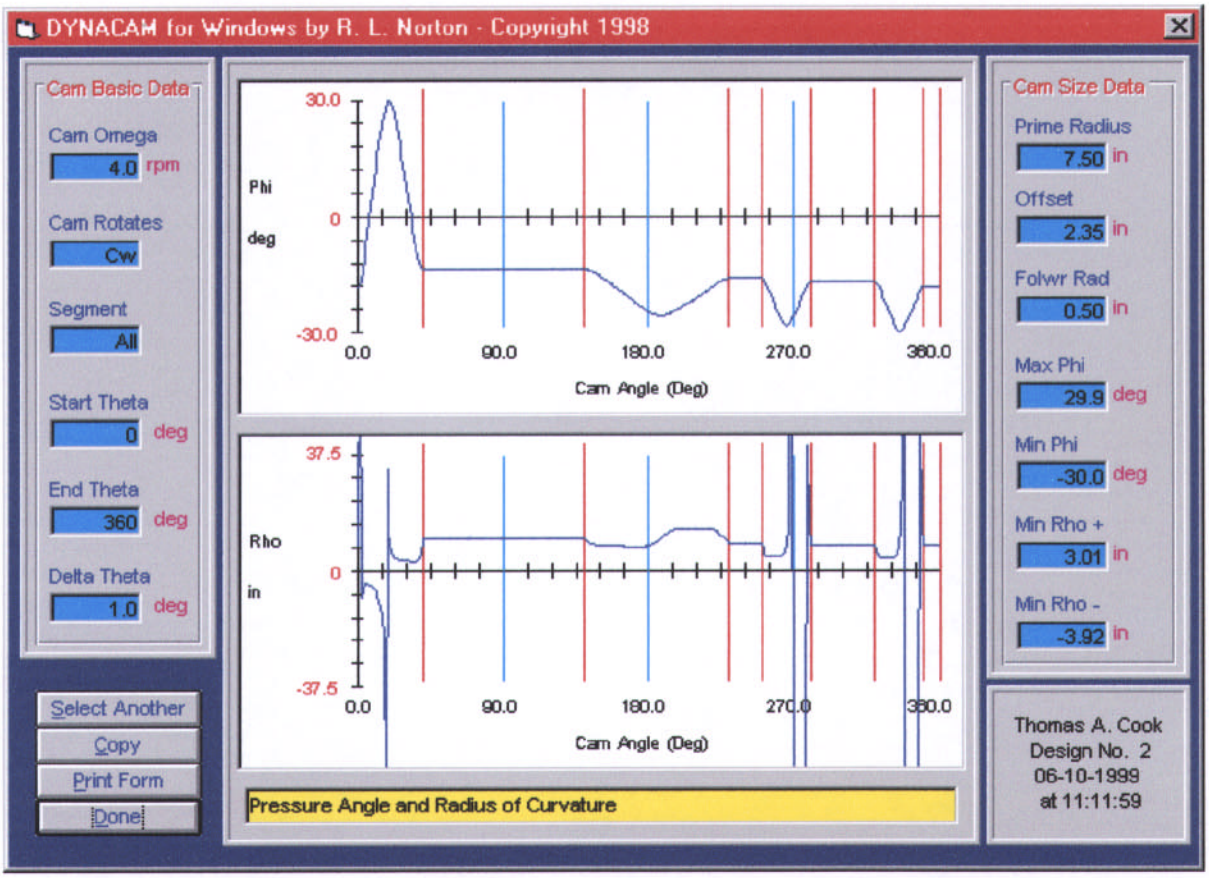
Prime circle radius $R_p := 7.50 \cdot in$

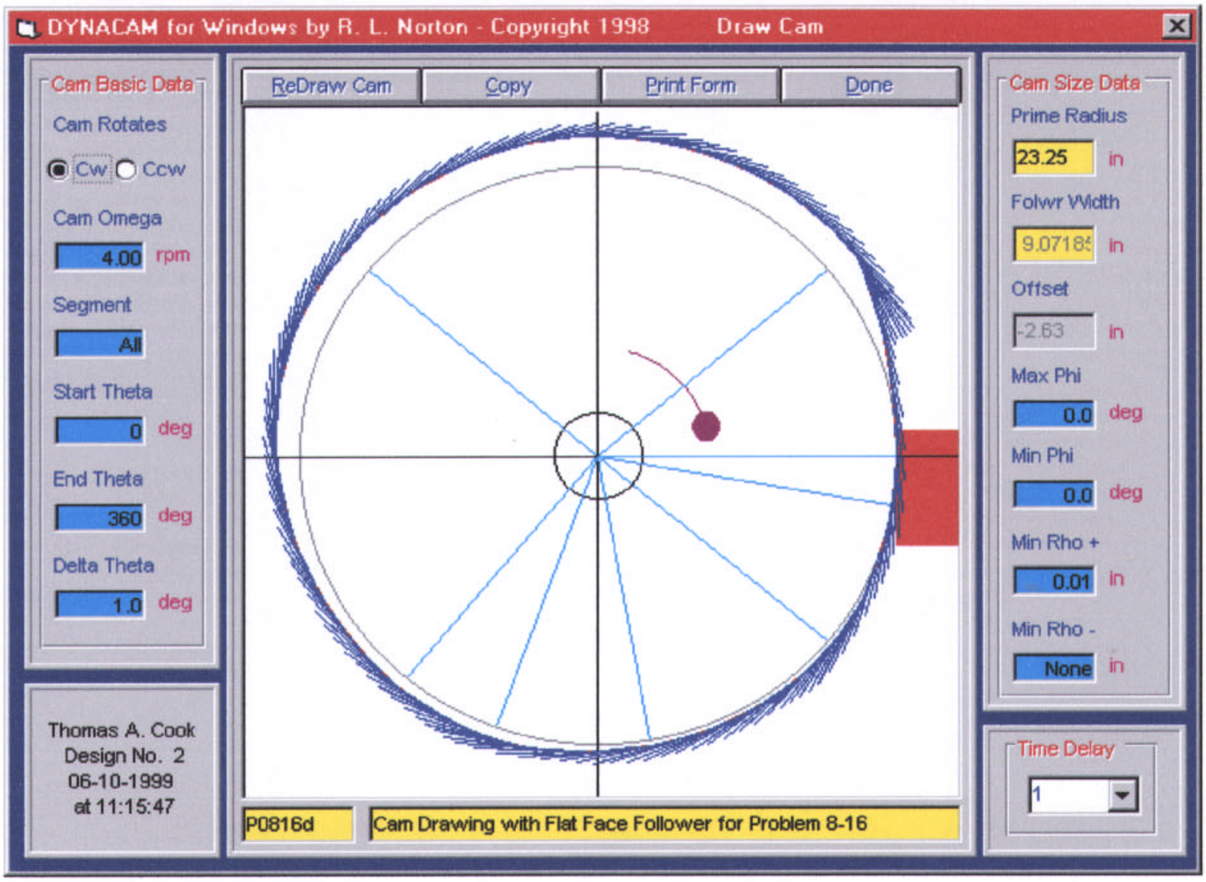
Roller follower radius $R_f := 0.50 \cdot in$

Follower eccentricity $\varepsilon := 2.35 \cdot in$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.

6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 23.25 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
4.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Phi
deg
30.0
0
-30.0
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Rho
in
37.5
0
-37.5
Pressure Angle and Radius of Curvature
Cam Size Data
Prime Radius
7.50 in
Offset
2.35 in
Folwr Rad
0.50 in
Max Phi
29.9 deg
Min Phi
-30.0 deg
Min Rho +
3.01 in
Min Rho -
-3.92 in
Thomas A. Cook
Design No. 2
06-10-1999
at 11:11:59


DYNACAM for Windows by R. L. Norton - Copyright 1998 Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
4.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 2
06-10-1999
at 11:15:47
ReDraw Cam
Copy
Print Form
Done
P0816d
Cam Drawing with Flat Face Follower for Problem 8-16
Cam Size Data
Prime Radius
23.25 in
Folwr Width
9.0718 in
Offset
-2.63 in
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
0.01 in
Min Rho -
None in
Time Delay
1

PROBLEM 8-17

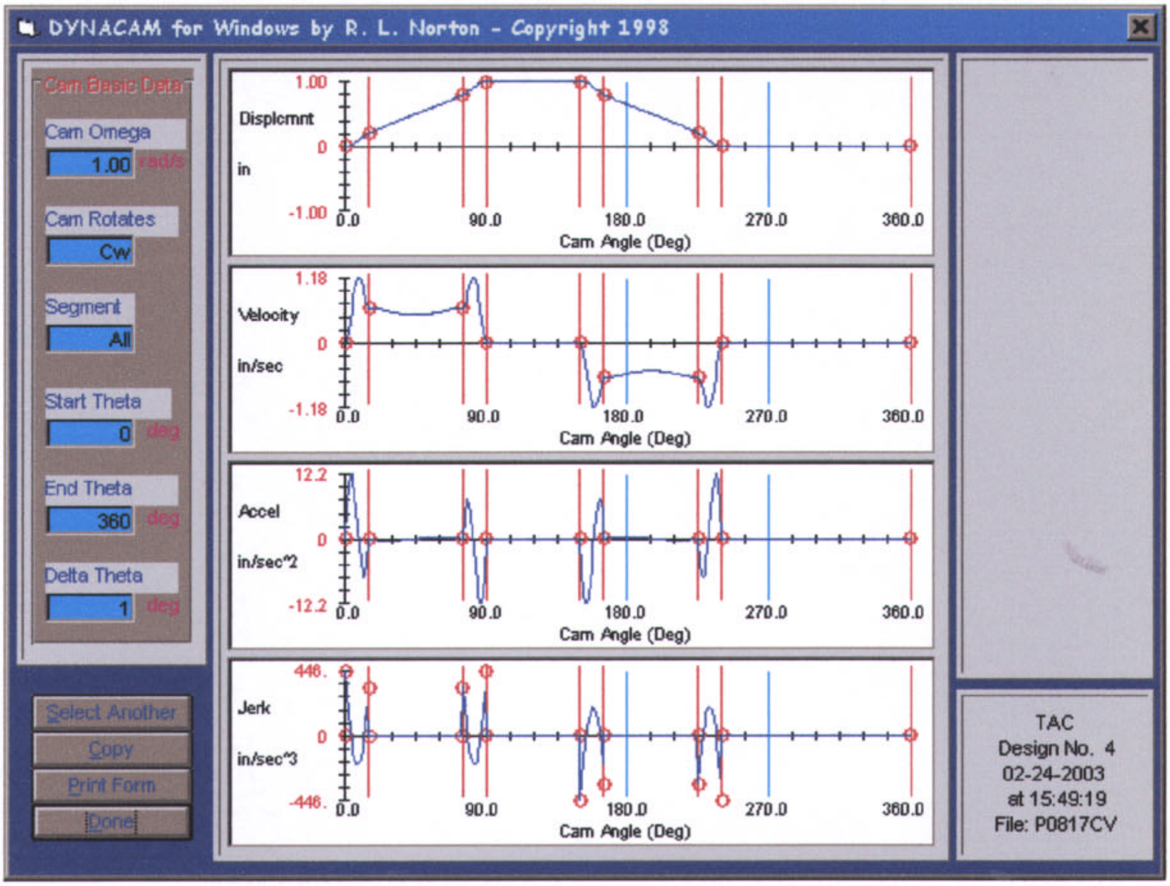
Statement: A high friction, high inertia load is to be driven. We wish to keep peak velocity low. Combine segments of polynomial displacements with a constant velocity segment on both rise and fall to reduce the maximum velocity below that obtainable with a full period modified sine acceleration alone (i.e., with no constant velocity portion). Compare the two designs and comment. Use an ω of one for comparison.

Given:

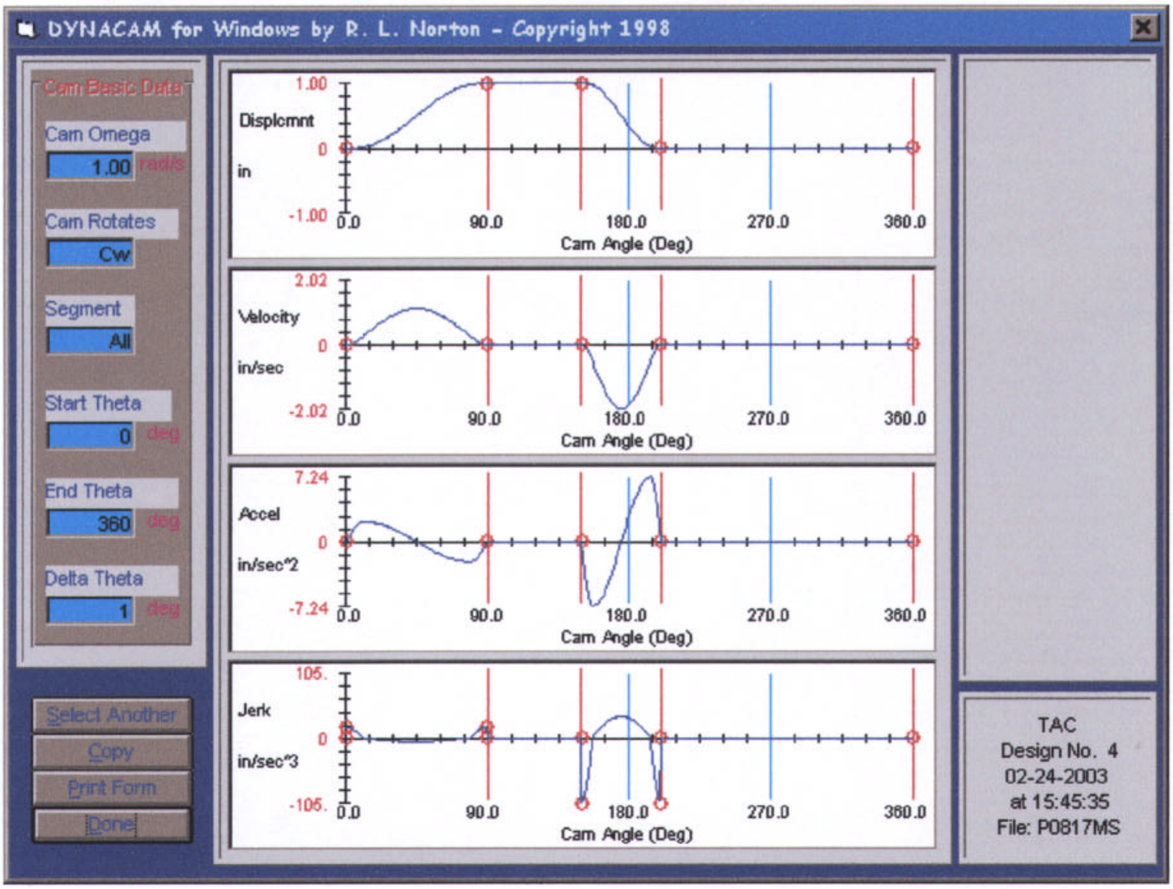
RISE	DWELL	FALL	DWELL
$\beta_1 := 90 \cdot deg$	$\beta_2 := 60 \cdot deg$	$\beta_3 := 50 \cdot deg$	$\beta_4 := 160 \cdot deg$
$h_1 := 1.0 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 1.0 \cdot in$	$h_4 := 0 \cdot in$

Solution: See Mathcad file P0817.

1. Use DYNACAM to get the data for comparison of these two cam profiles. Start with the CV-poly profile, which is shown below. In this design eight segments were used: three each for the rise and fall and one each for the two dwells. The (nearly) constant velocity segments were each 54 deg. The maximum acceleration and jerk are 12.2 and 446, respectively.



1. The DYNACAM output screen for the modified sine profile is shown below. The maximum acceleration and jerk are 74.4 and 105, respectively. Obviously, for the CV-poly design choices that were made, the modified sine profile results in lower inertia forces and a smoother follower motion.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.00 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Displcmnt
in
1.00
0
-1.00
Velocity
in/sec
2.02
-2.02
Accel
in/sec^2
7.24
-7.24
Jerk
in/sec^3
105.
-105.
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
TAC
Design No. 4
02-24-2003
at 15:45:35
File: P0817MS

PROBLEM 8-18

Statement: A constant velocity of 0.4 in/sec must be matched for 1.5 sec. Then the follower must return to your choice of starting point and dwell for 2 sec. The total cycle time is 6 sec. Design a cam for a follower radius of 0.75 in and a maximum pressure angle of 30 deg, absolute value.

Given:

Required constant velocity $V_{cv} := 0.4 \cdot in \cdot sec^{-1}$ $t_{cv} := 1.5 \cdot sec$

Dwell time $t_{dwell} := 2 \cdot sec$

Cycle time $t_{cycle} := 6 \cdot sec$

Follower radius $R_f := 0.75 \cdot in$

Solution: See Mathcad file P0818.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{t_{cycle}} \qquad \omega = 1.047 \frac{rad}{sec}$$

2. Use a four-segment polynomial cam as described below.

Segment	Function	Motion
1	Constant velocity	Rise
2	Polynomial (5 deg)	Fall
3	None	Dwell
4	Polynomial (5 deg)	Rise

3. Calculate the two known cam rotation interval widths.

Constant velocity segment $\beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi$ $\beta_1 = 90.000\, deg$

Dwell segment $\beta_3 := \frac{t_{dwell}}{t_{cycle}} \cdot 2 \cdot \pi$ $\beta_3 = 120.000\, deg$

4. This leaves 150 deg to allocate to segments 2 and 4. Tentatively, let

$\beta_2 := 100 \cdot deg$ $\beta_4 := 50 \cdot deg$

5. Calculate the lift for the constant velocity segment.

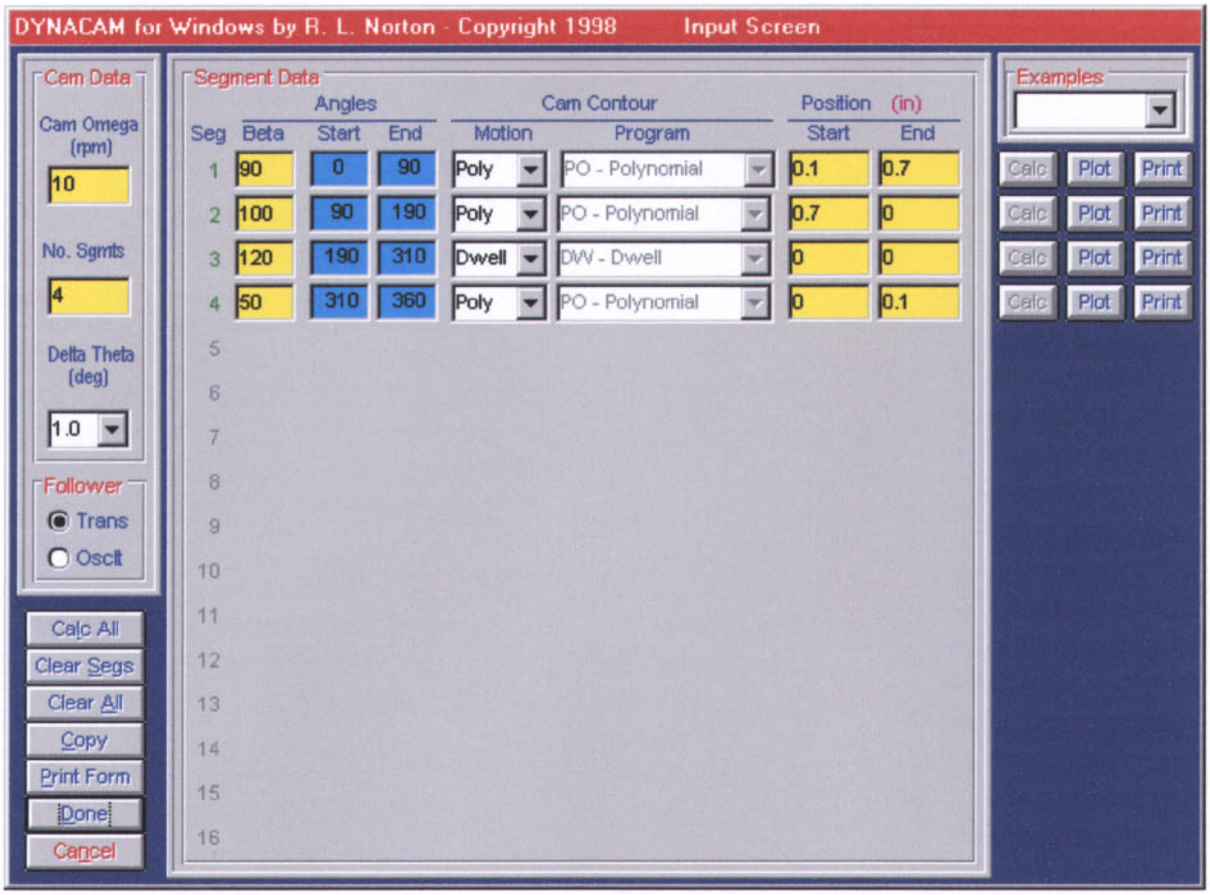
$L_1 := V_{cv} \cdot t_{cv}$ $L_1 = 0.600\, in$

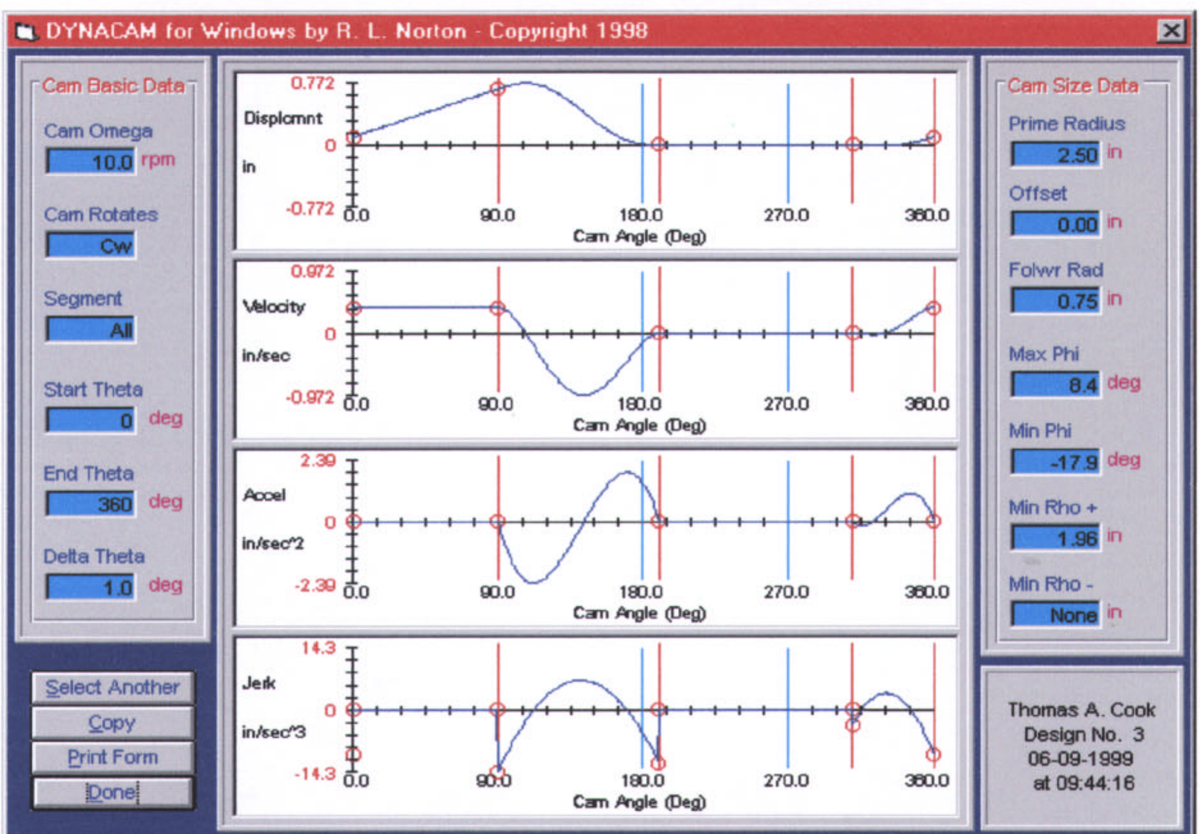
6. Make other initial design choices.

Lift of segment 4 (and starting position for segment 1) $L_4 := 0.1 \cdot in$

Prime circle radius $R_p := 2.50 \cdot in$

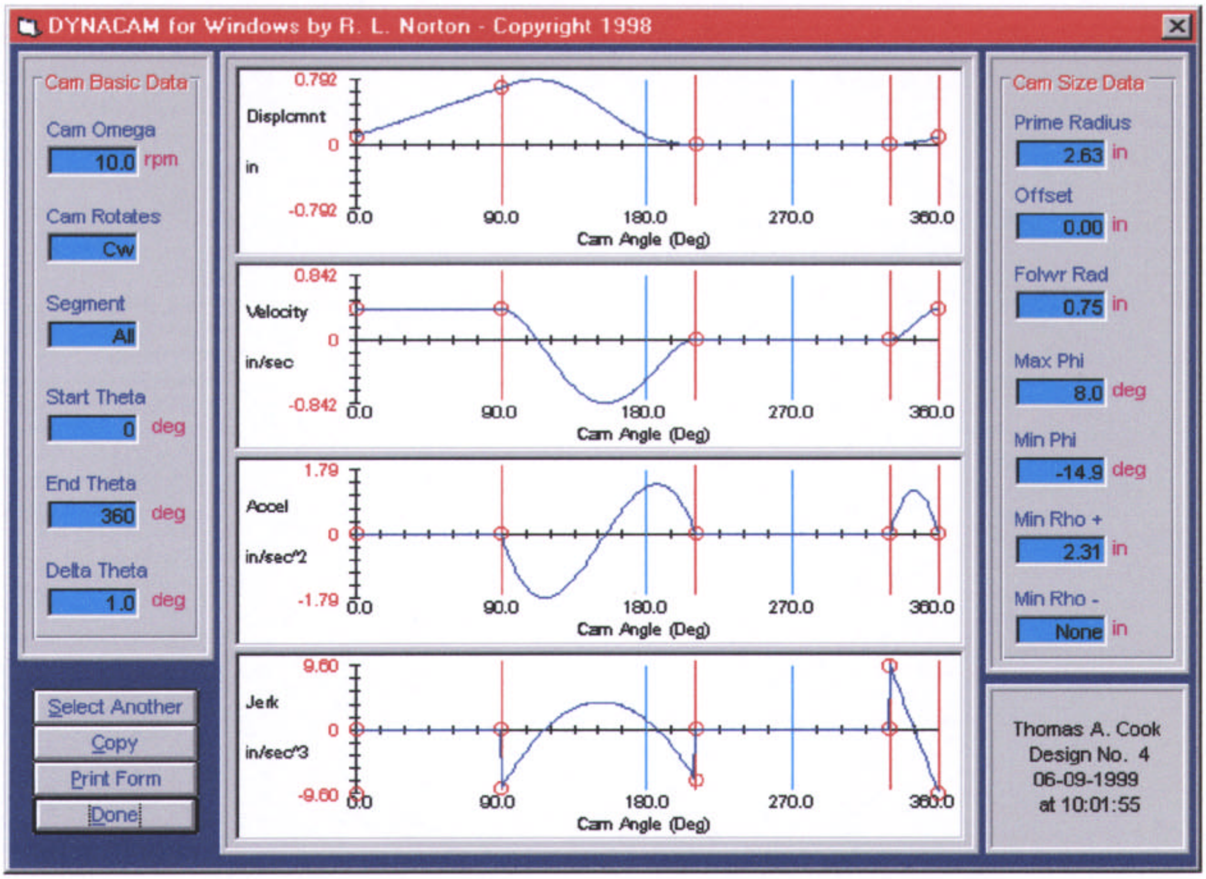
7. Enter the above data into program DYNACAM. The input screen and resulting *SVAJ* diagrams are shown on the next page.


DYNACAM for Windows by R. L. Norton - Copyright 1998 Input Screen
Cam Data
Cam Omega (rpm)
10
No. Sgmts
4
Delta Theta (deg)
1.0
Follower
Trans
Osclt
Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel
Segment Data
Angles
Cam Contour
Position (in)
Seg Beta Start End Motion Program Start End
1 90 0 90 Poly PO - Polynomial 0.1 0.7
2 100 90 190 Poly PO - Polynomial 0.7 0
3 120 190 310 Dwell DW - Dwell 0 0
4 50 310 360 Poly PO - Polynomial 0 0.1
Examples
Calc Plot Print


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
10.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Displcmnt in
0.772
-0.772
Velocity in/sec
0.972
-0.972
Accel in/sec^2
2.39
-2.39
Jerk in/sec^3
14.3
-14.3
0.0 90.0 180.0 270.0 360.0
Cam Angle (Deg)
Cam Size Data
Prime Radius
2.50 in
Offset
0.00 in
Folwr Rad
0.75 in
Max Phi
8.4 deg
Min Phi
-17.9 deg
Min Rho +
1.96 in
Min Rho -
None in
Thomas A. Cook
Design No. 3
06-09-1999
at 09:44:16

8. From *SVAJ* diagrams we see that all design goals are met but the peak acceleration in segment 2 is higher than the peak in segment 4. These can be more evenly balanced by increasing the segment 2 interval and decreasing the segment 4 interval. Iterate on these intervals until the peak accelerations are more nearly the same.

9. The second interval was changed to 120 deg and the fourth to 30 deg, and the prime circle radius was increased to 2.625 in (to maintain $\rho_{min}/R_f > 3$). The resulting peak accelerations are more in balance and are shown in the SVAJ diagram below. The maximum pressure angle is well within the required limit.



PROBLEM 8-19

Statement: A constant velocity of 0.25 in/sec must be matched for 3 sec. Then the follower must return to your choice of starting point and dwell for 3 sec. The total cycle time is 12 sec. Design a cam for a follower radius of 1.25 in and a maximum pressure angle of 35 deg, absolute value.

Given:

Required constant velocity $V_{cv} := 0.25 \cdot in \cdot sec^{-1}$ $t_{cv} := 3 \cdot sec$

Dwell time $t_{dwell} := 3 \cdot sec$

Cycle time $t_{cycle} := 12 \cdot sec$

Follower radius $R_f := 1.25 \cdot in$

Solution: See Mathcad file P0819.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{t_{cycle}} \qquad \omega = 0.524 \frac{rad}{sec}$$

2. Use a four-segment polynomial cam as described below.

Segment	Function	Motion
1	Constant velocity	Rise
2	Polynomial (5 deg)	Fall
3	None	Dwell
4	Polynomial (5 deg)	Rise

3. Calculate the two known cam rotation interval widths.

Constant velocity segment $\beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi$ $\beta_1 = 90.000\, deg$

Dwell segment $\beta_3 := \frac{t_{dwell}}{t_{cycle}} \cdot 2 \cdot \pi$ $\beta_3 = 90.000\, deg$

4. This leaves 180 deg to allocate to segments 2 and 4. Tentatively, let

$\beta_2 := 120 \cdot deg$ $\beta_4 := 60 \cdot deg$

5. Calculate the lift for the constant velocity segment.

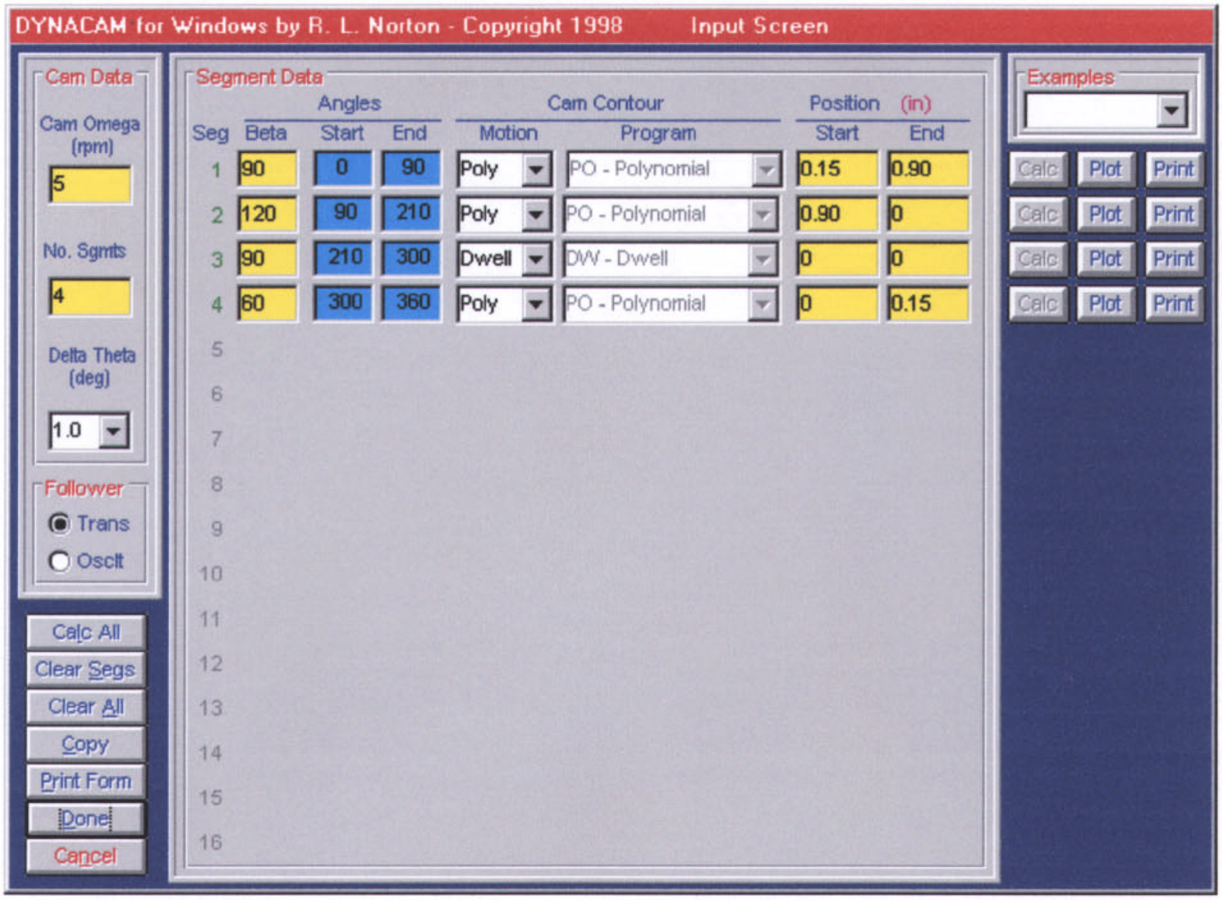
$L_1 := V_{cv} \cdot t_{cv}$ $L_1 = 0.750\, in$

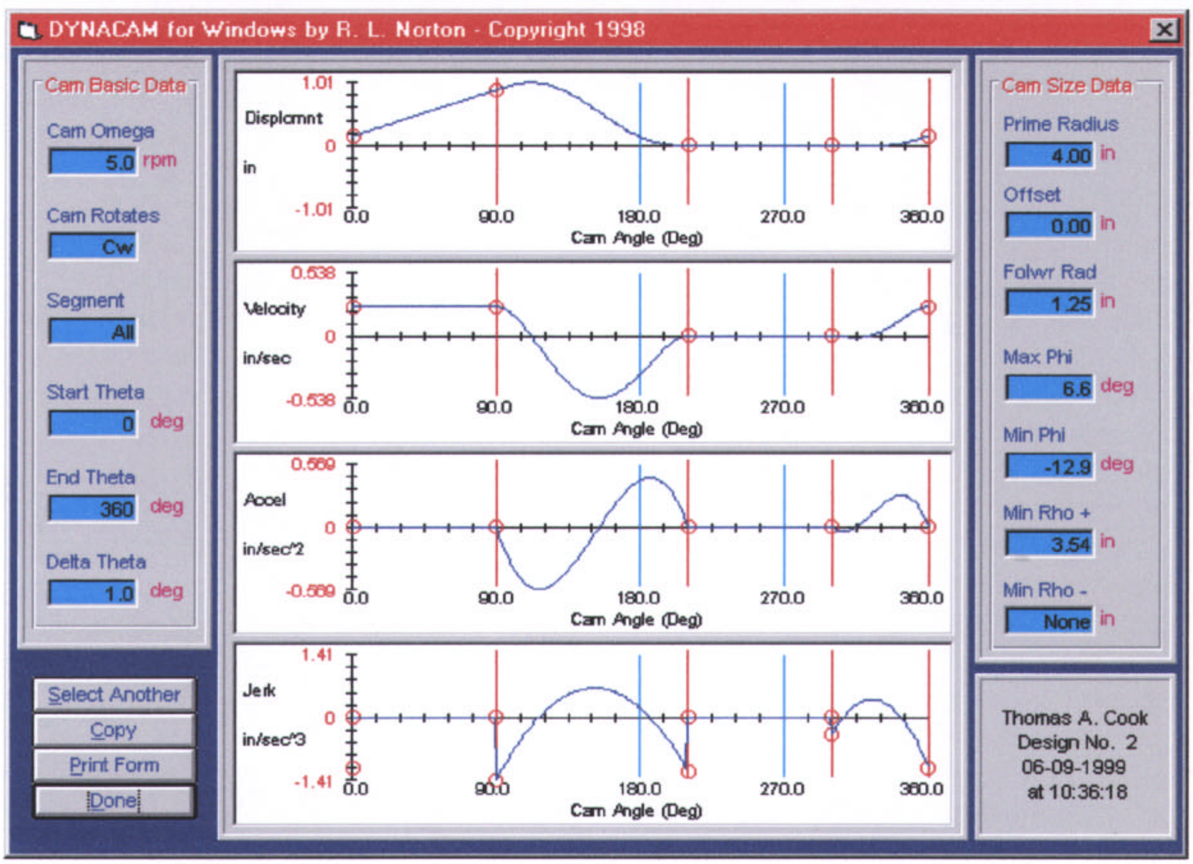
6. Make other initial design choices.

Lift of segment 4 (and starting position for segment 1) $L_4 := 0.15 \cdot in$

Prime circle radius $R_p := 4.00 \cdot in$

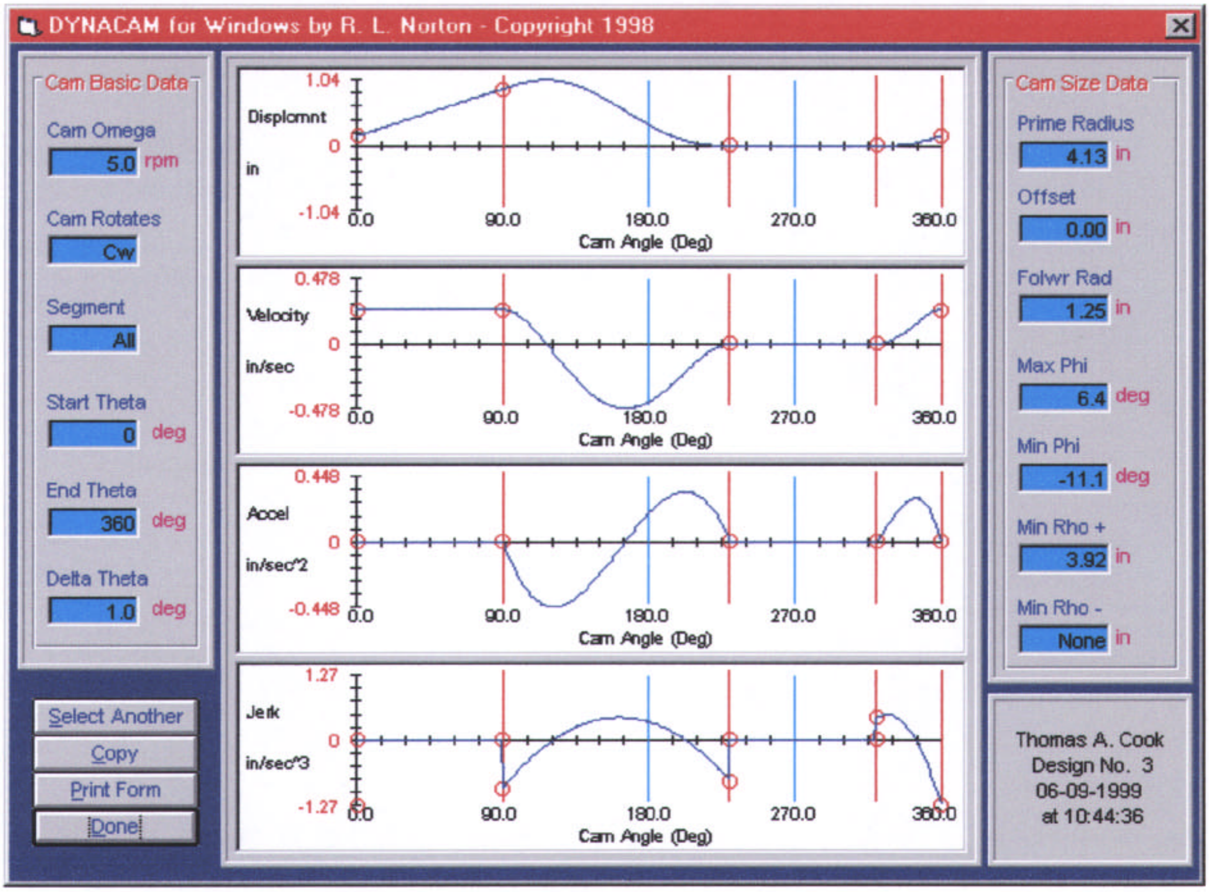
7. Enter the above data into program DYNACAM. The input screen and resulting *SVAJ* diagrams are shown on the next page.


DYNACAM for Windows by R. L. Norton - Copyright 1998 Input Screen
Cam Data
Cam Omega (rpm)
5
No. Sgmts
4
Delta Theta (deg)
1.0
Follower
Trans
Osclt
Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel
Segment Data
Angles
Cam Contour
Position (in)
Seg Beta Start End Motion Program Start End
1 90 0 90 Poly PO - Polynomial 0.15 0.90
2 120 90 210 Poly PO - Polynomial 0.90 0
3 90 210 300 Dwell DW - Dwell 0 0
4 60 300 360 Poly PO - Polynomial 0 0.15
Examples
Calc Plot Print


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
5.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Displcmnt
in
1.01
-1.01
Velocity
in/sec
0.538
-0.538
Accel
in/sec^2
0.569
-0.569
Jerk
in/sec^3
1.41
-1.41
Cam Angle (Deg)
0.0 90.0 180.0 270.0 360.0
Cam Size Data
Prime Radius
4.00 in
Offset
0.00 in
Folwr Rad
1.25 in
Max Phi
6.6 deg
Min Phi
-12.9 deg
Min Rho +
3.54 in
Min Rho -
None in
Thomas A. Cook
Design No. 2
06-09-1999
at 10:36:18

8. From *SVAJ* diagrams we see that all design goals are met but the peak acceleration in segment 2 is higher than the peak in segment 4. These can be more evenly balanced by increasing the segment 2 interval and decreasing the segment 4 interval. Iterate on these intervals until the peak accelerations are more nearly the same. Note also that ρ_{min} is not greater than 3 times R_f.

9. The second interval was changed to 140 deg and the fourth to 40 deg, and the prime circle radius was increased to 4.125 in (to maintain $\rho_{min}/R_f > 3$). The resulting peak accelerations are more in balance and are shown in the SVAJ diagram below. The maximum pressure angle is well within the required limit.



PROBLEM 8-20

Statement: A constant velocity of 2 in/sec must be matched for 1 sec. Then the follower must return to your choice of starting point. The total cycle time is 2.75 sec. Design a cam for a follower radius of 0.5 in and a maximum pressure angle of 25 deg, absolute value.

Given:

Required constant velocity $V_{cv} := 2.0 \cdot in \cdot sec^{-1}$ $\quad t_{cv} := 1 \cdot sec$

Cycle time $t_{cycle} := 2.75 \cdot sec$

Follower radius $R_f := 0.5 \cdot in$

Solution: See Mathcad file P0820.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{t_{cycle}} \qquad \omega = 2.285 \frac{rad}{sec}$$

2. Use a two-segment polynomial cam similar to that used in Example 8-10 and described below.

Segment	Function	Motion
1	Constant velocity	Rise
2	Polynomial (5 deg)	Fall

3. Calculate the known cam rotation interval width.

Constant velocity segment $$\beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi \qquad \beta_1 = 130.909\,deg$$

4. This leaves 229.091 deg for segments 2.

$$\beta_2 := 360 \cdot deg - \beta_1 \qquad \beta_2 = 229.091\,deg$$

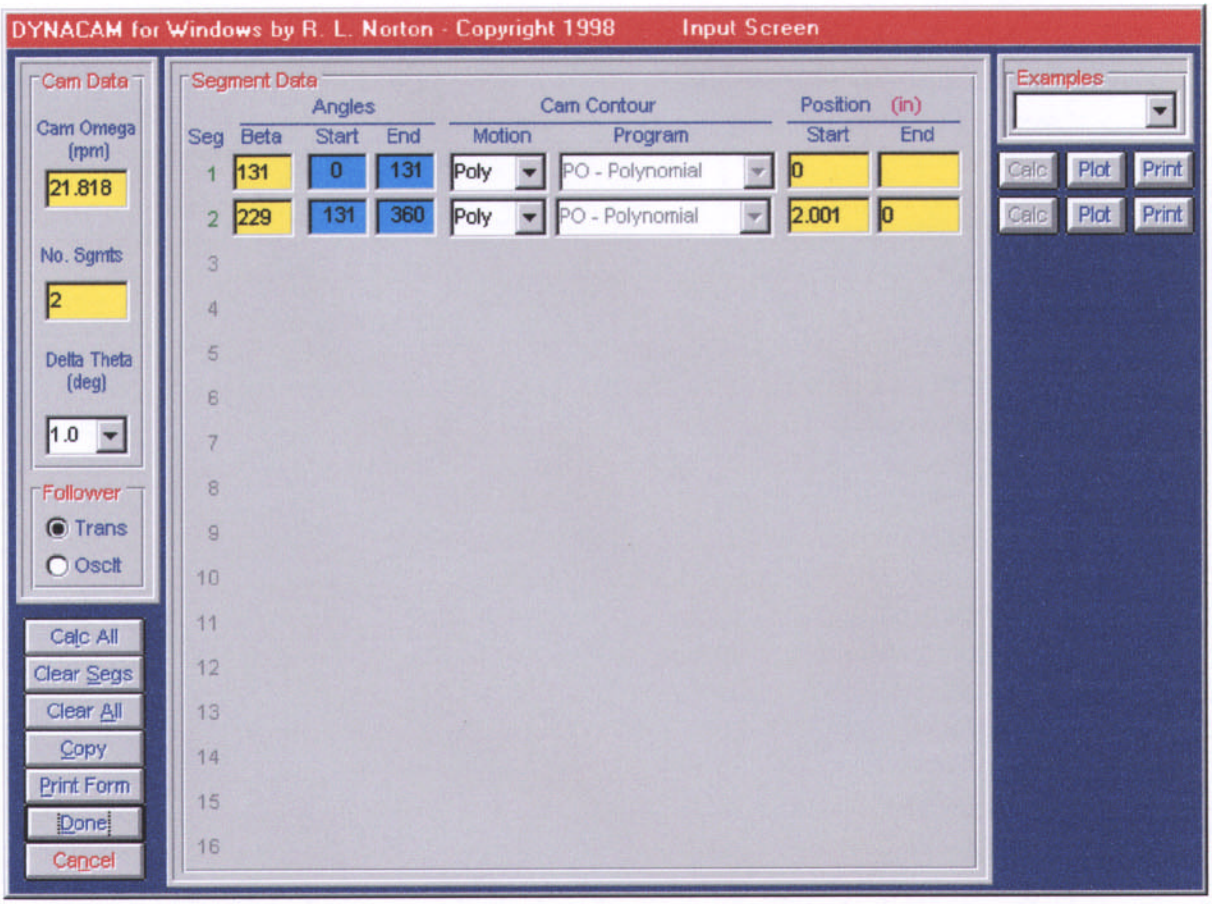
5. Calculate the lift for the constant velocity segment.

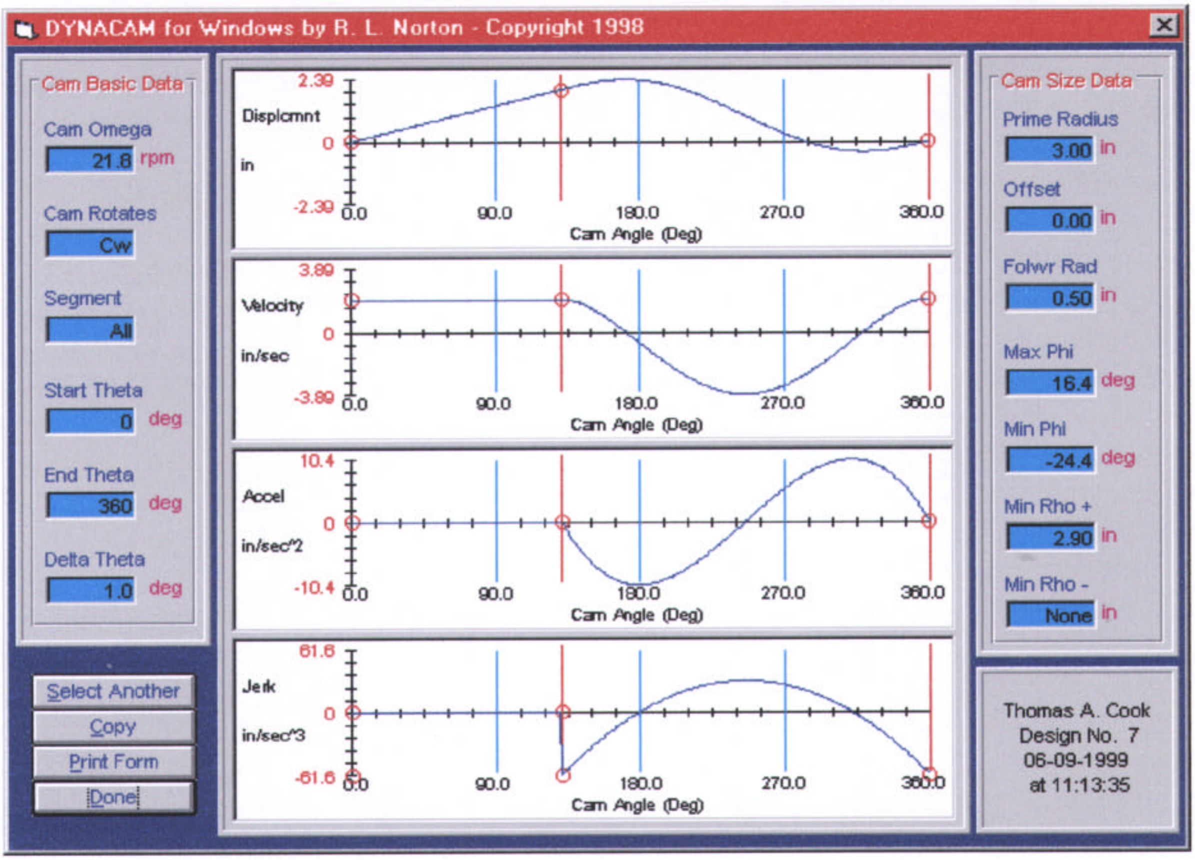
$$L_1 := V_{cv} \cdot t_{cv} \qquad L_1 = 2.000\,in$$

6. Make other initial design choices.

Prime circle radius $R_p := 4.00 \cdot in$

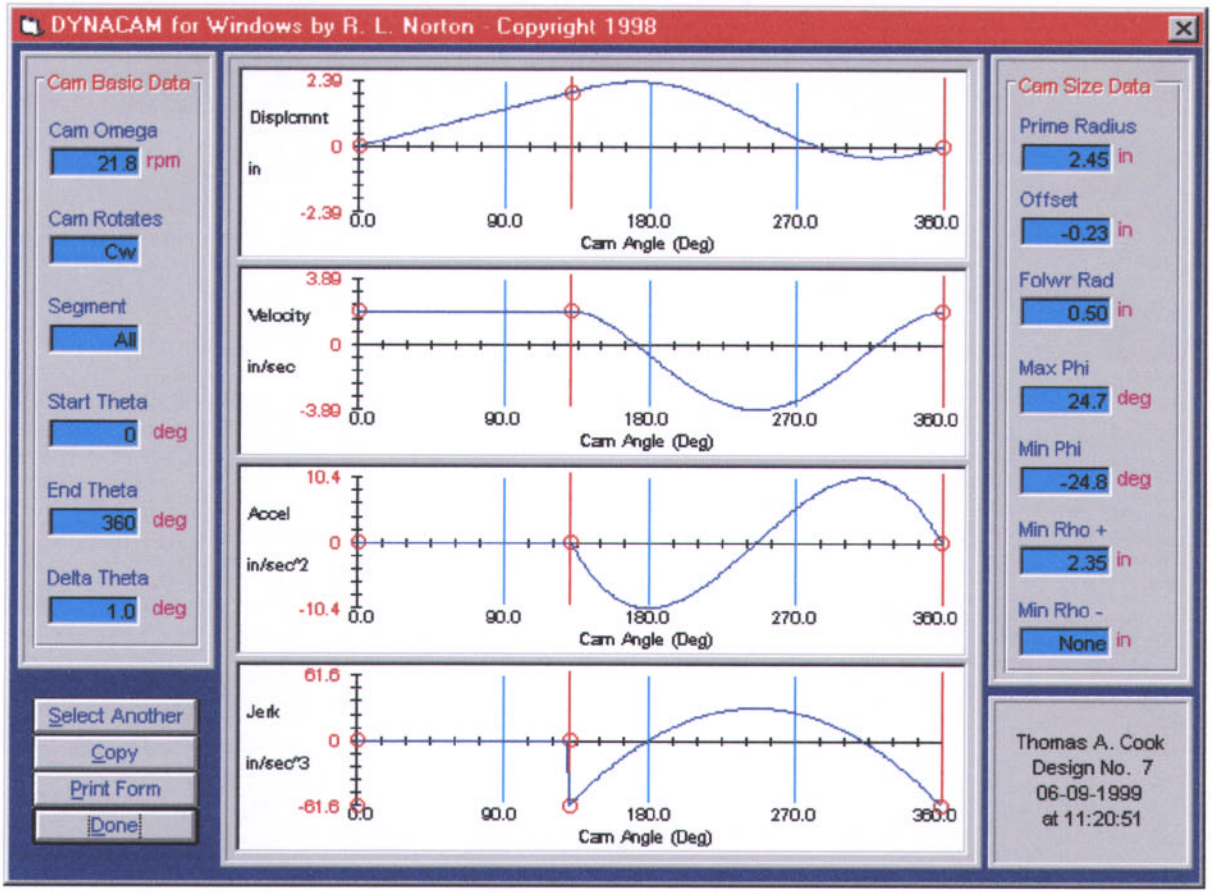
7. Enter the above data into program DYNACAM. The input screen and resulting *SVAJ* diagrams are shown on the next page.


DYNACAM for Windows by R. L. Norton - Copyright 1998 Input Screen
Cam Data
Cam Omega (rpm)
21.818
No. Sgmts
2
Delta Theta (deg)
1.0
Follower
Trans
Osclt
Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel
Segment Data
Seg
Angles
Beta Start End
Cam Contour
Motion Program
Position (in)
Start End
1 131 0 131 Poly PO - Polynomial 0
2 229 131 360 Poly PO - Polynomial 2.001 0
Examples
Calc Plot Print
Calc Plot Print


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega 21.8 rpm
Cam Rotates Cw
Segment All
Start Theta 0 deg
End Theta 360 deg
Delta Theta 1.0 deg
Select Another
Copy
Print Form
Done
Displcmnt in 2.39 0 -2.39
Velocity in/sec 3.89 0 -3.89
Accel in/sec^2 10.4 0 -10.4
Jerk in/sec^3 61.6 0 -61.6
0.0 90.0 180.0 270.0 360.0
Cam Angle (Deg)
Cam Size Data
Prime Radius 3.00 in
Offset 0.00 in
Folwr Rad 0.50 in
Max Phi 16.4 deg
Min Phi -24.4 deg
Min Rho + 2.90 in
Min Rho - None in
Thomas A. Cook
Design No. 7
06-09-1999
at 11:13:35

8. From *SVAJ* diagrams we see that all design goals are met but the maximum pressure angle is considerably lower (in absolute value) than the minimum pressure angle. These can be balanced by making the follower eccentric with respect to the cam centerline. This will also allow the prime radius to be reduced. Iterate on these parameters until the maximum and minimum pressure angles are nearly the same (numerically) and reduce the prime circle radius until the pressure angle is at, or below, the required limit.

9. The eccentricity was iterated to -0.23 in and the prime circle radius was reduced to 2.45 in. This resulted in a balanced maximum and minimum pressure angle that is just below the required limit. The minimum radius of curvature is almost 5 times the follower radius, which is ample. The *SVAJ* diagram for the final iteration is shown below.



PROBLEM 8-21

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a modified trapezoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter:

Lift: $h_1 := 20 \cdot mm$

Duration: $\beta_1 := 60 \cdot deg$

Solution: See Mathcad file P0821.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$

$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for 0 <= x <= b/2 where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for b/2 <= x <= (1 - d)/2

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for (1 - d)/2 <= x <= (1 + d)/2

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for (1 + d)/2 <= x <= 1 - b/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for 1 - b/2 <= x <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - cos\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y''_5(x) := C_a \cdot sin\left[\frac{\pi}{b} \cdot (x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left[\frac{\pi}{b} \cdot (x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S, V, A,* and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, 0 <= θ <= β_1. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For 0 <= θ <= β_1 (Rise)

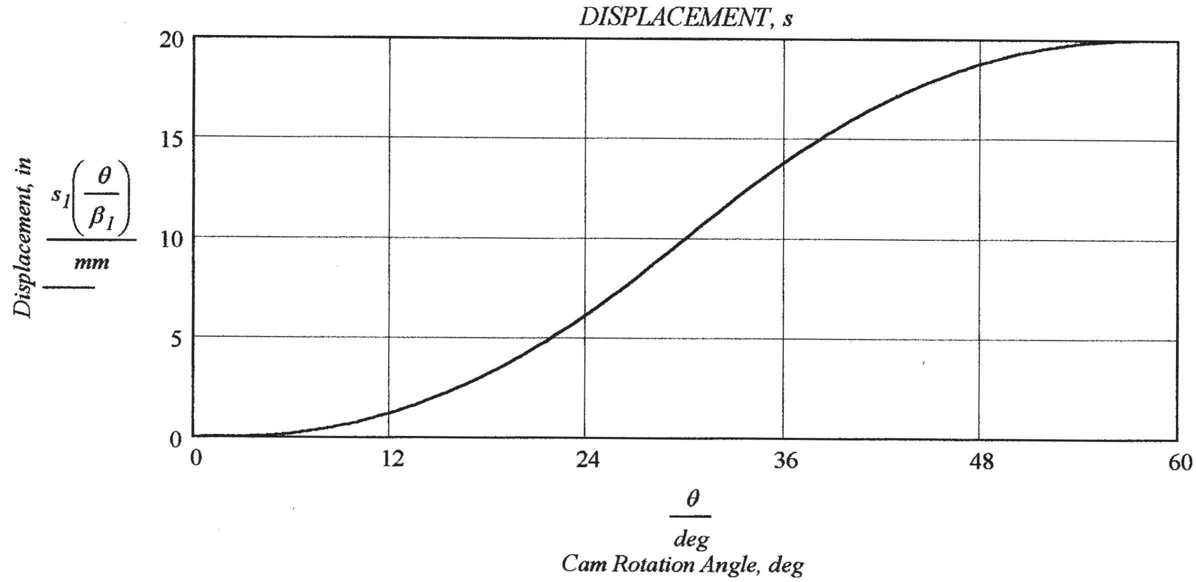
$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

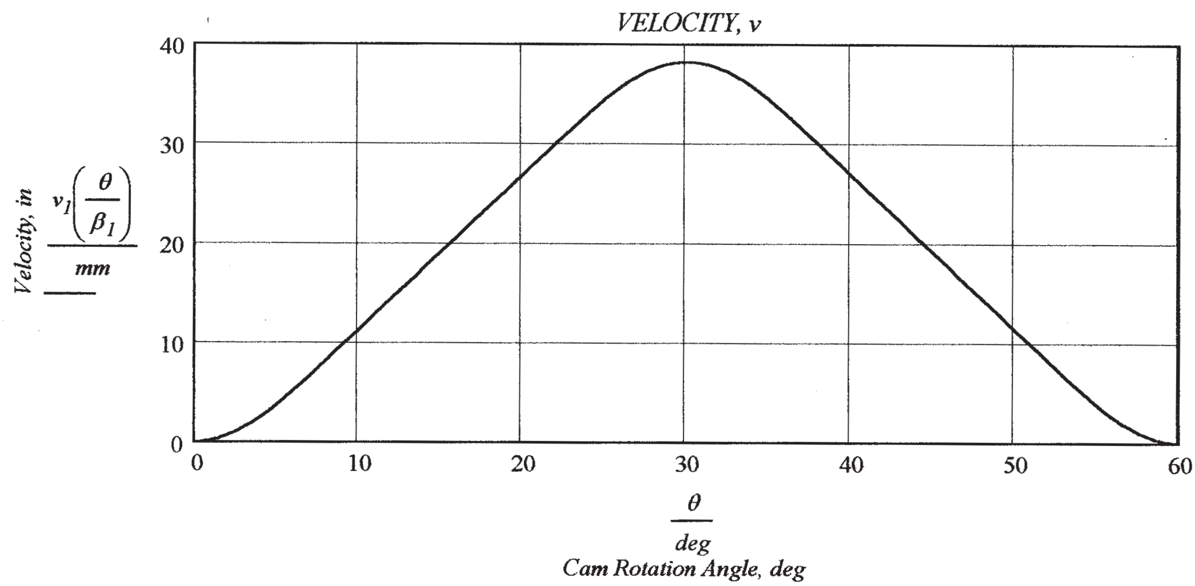
$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

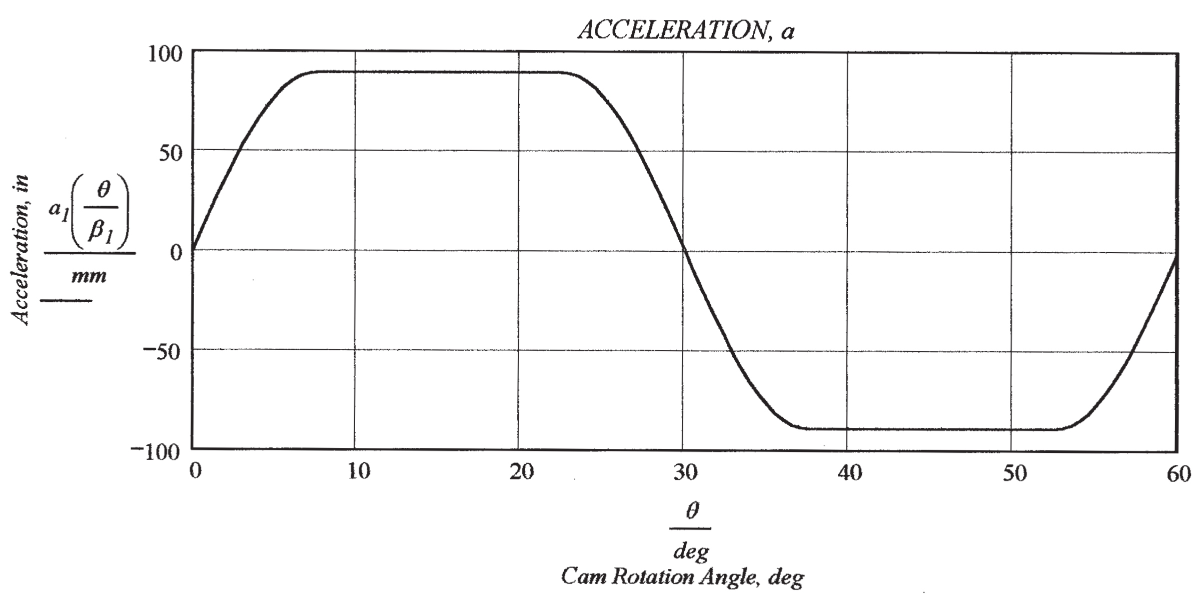
$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

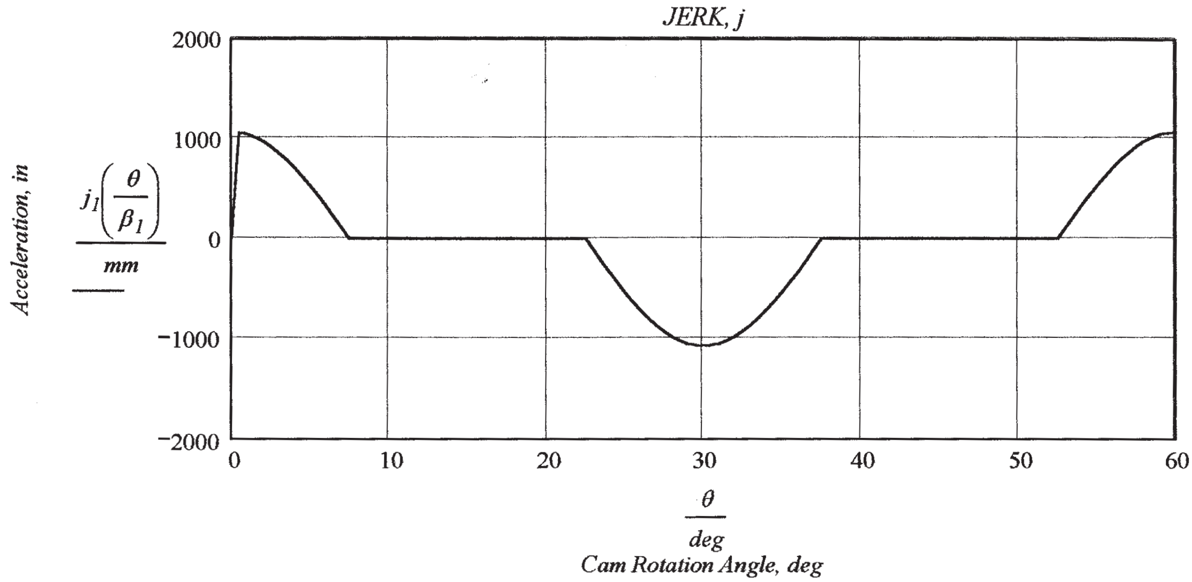
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta_1$


DISPLACEMENT, s
Displacement, in
$\frac{s_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
0
5
10
15
20
0
12
24
36
48
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


VELOCITY, v
Velocity, in
$\frac{v_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
0
10
20
30
40
0
10
20
30
40
50
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


ACCELERATION, a
Acceleration, in
$\frac{a_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
100
50
0
−50
−100
0
10
20
30
40
50
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


JERK, j
2000
1000
0
−1000
−2000
Acceleration, in
$j_1\left(\frac{\theta}{\beta_1}\right)$
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-22

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a modified sinusoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h_1 := 20 \cdot mm$

Duration: $\beta_1 := 60 \cdot deg$

Solution: See Mathcad file P0822.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$b := 0.25 \quad c := 0.00 \quad d := 0.75$

$C_v := 1.7596 \quad C_a := 5.5280 \quad C_j := 69.466$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for 0 <= *x* <= *b*/2 where, for these equations, *x* is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for *b*/2 <= *x* <= (1 - *d*)/2

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for (1 - *d*)/2 <= *x* <= (1 + *d*)/2

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for (1 + *d*)/2 <= *x* <= 1 - *b*/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for 1 - *b*/2 <= *x* <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, $0 \le \theta \le \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \le \theta \le \beta_1$ (Rise)

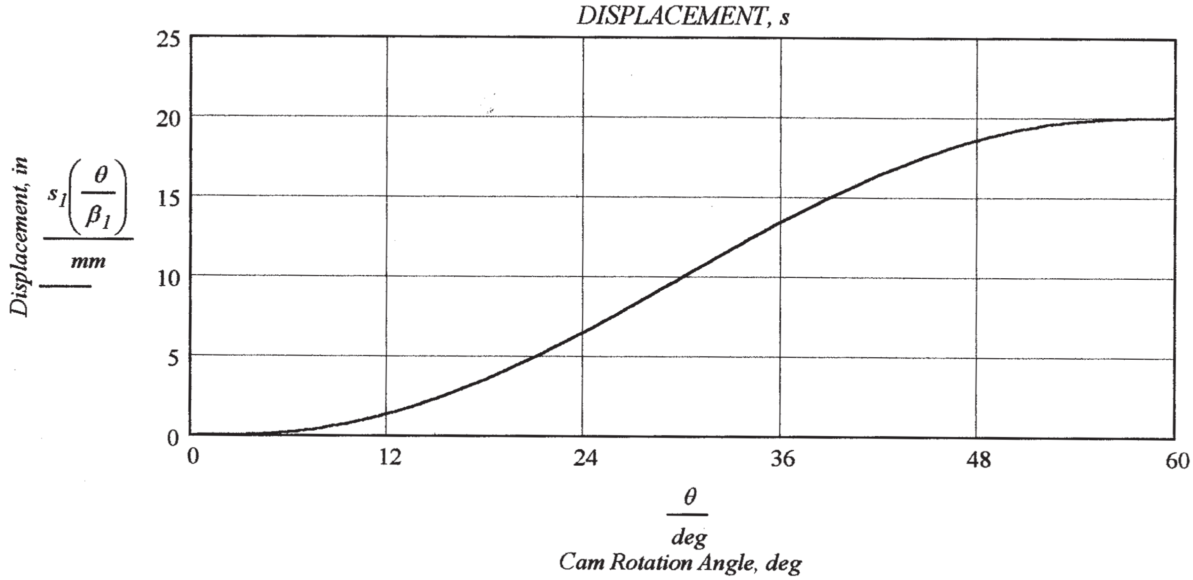
$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

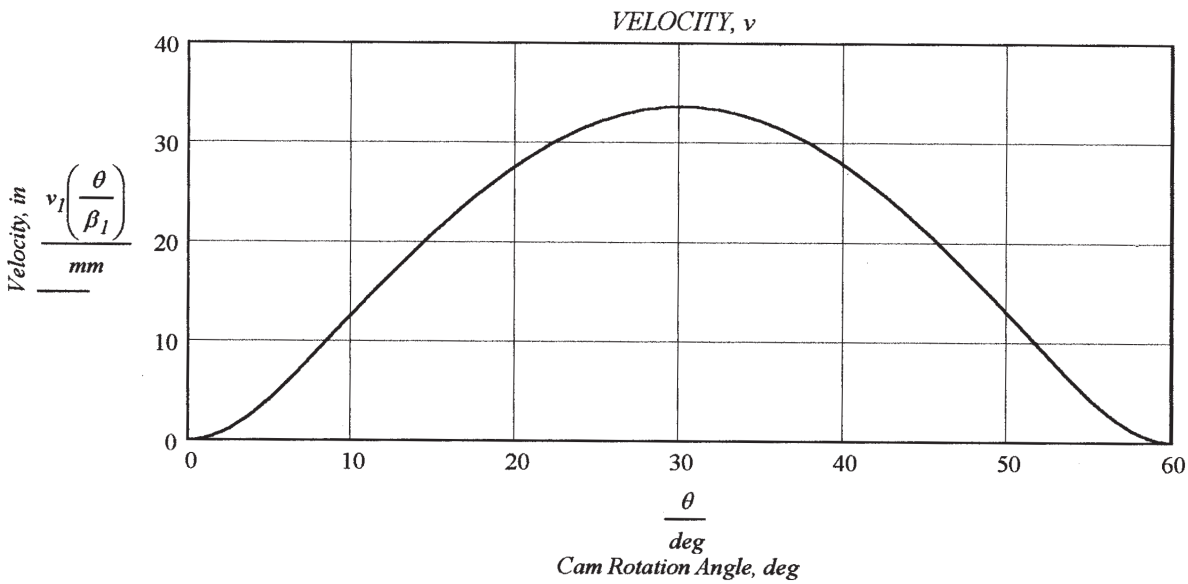
$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

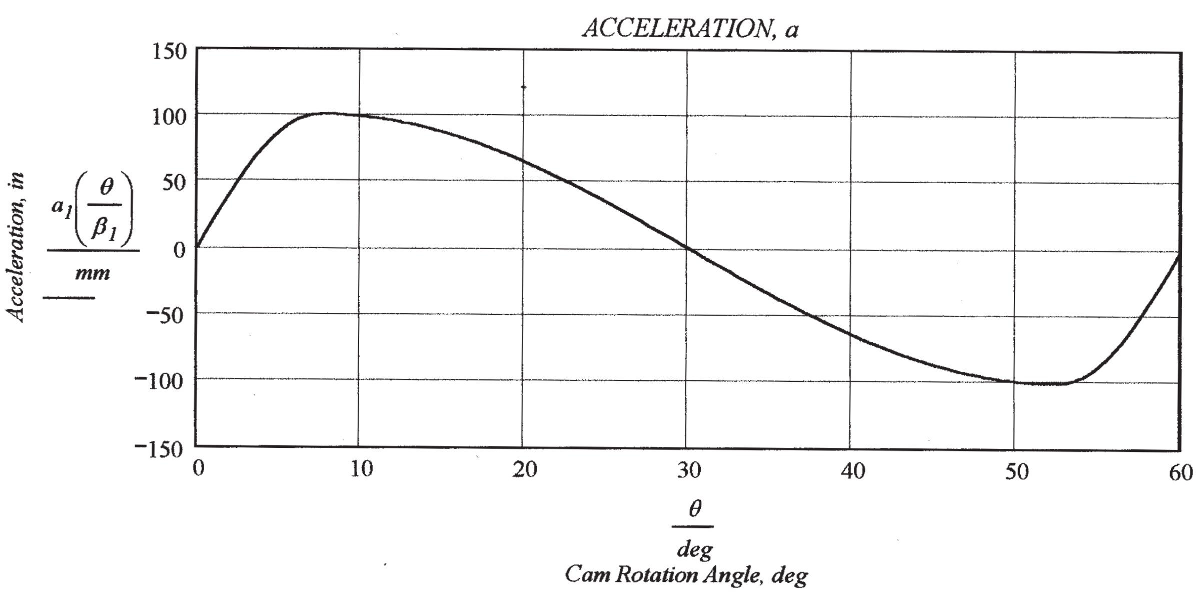
$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

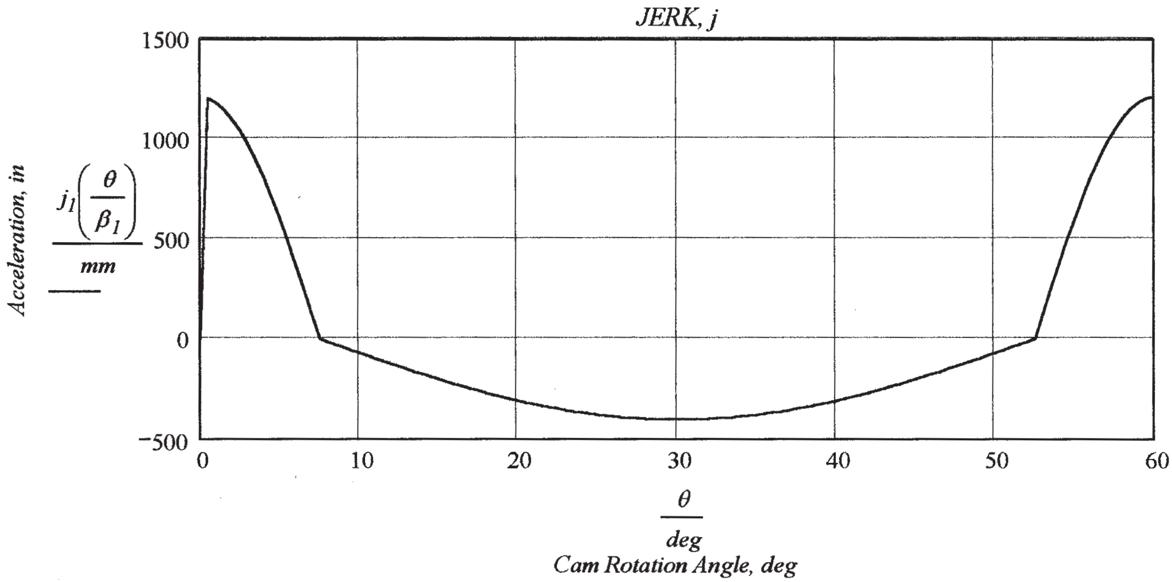
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta_1$


DISPLACEMENT, s
Displacement, in
$\frac{s_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
25
20
15
10
5
0
0
12
24
36
48
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


VELOCITY, v
Velocity, in
$\frac{v_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
40
30
20
10
0
0
10
20
30
40
50
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


ACCELERATION, a
Acceleration, in
$\frac{a_1\left(\frac{\theta}{\beta_1}\right)}{mm}$
150
100
50
0
−50
−100
−150
0
10
20
30
40
50
60
$\frac{\theta}{deg}$
Cam Rotation Angle, deg


JERK, j
1500
1000
500
0
−500
Acceleration, in
$j_1\left(\frac{\theta}{\beta_1}\right)$
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-23

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a cycloidal displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter:

Lift: $h := 20 \cdot mm$

Duration: $\beta := 60 \cdot deg$

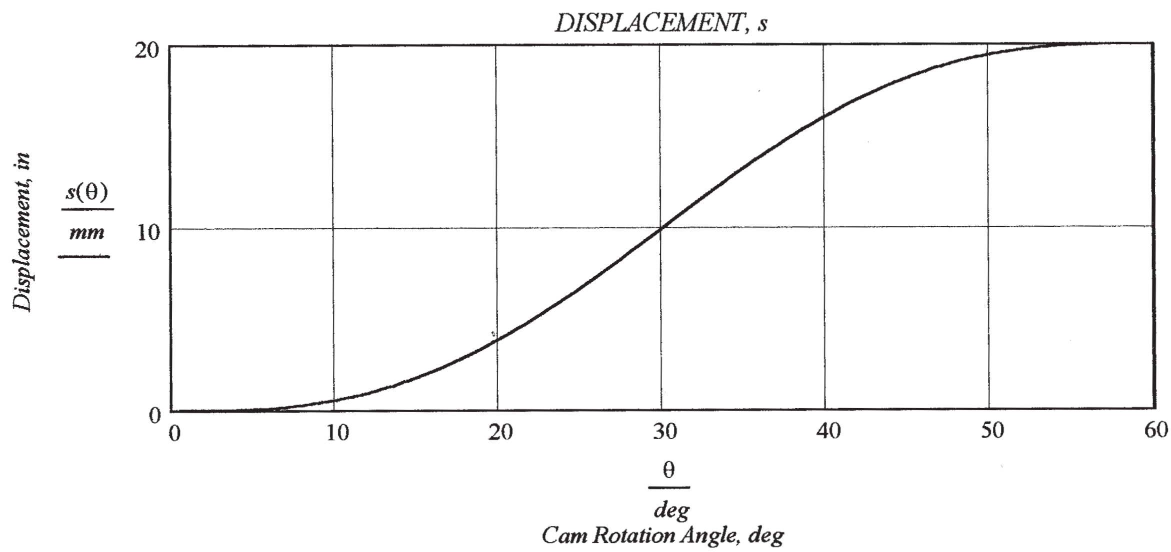
Solution: See Mathcad file P0823.

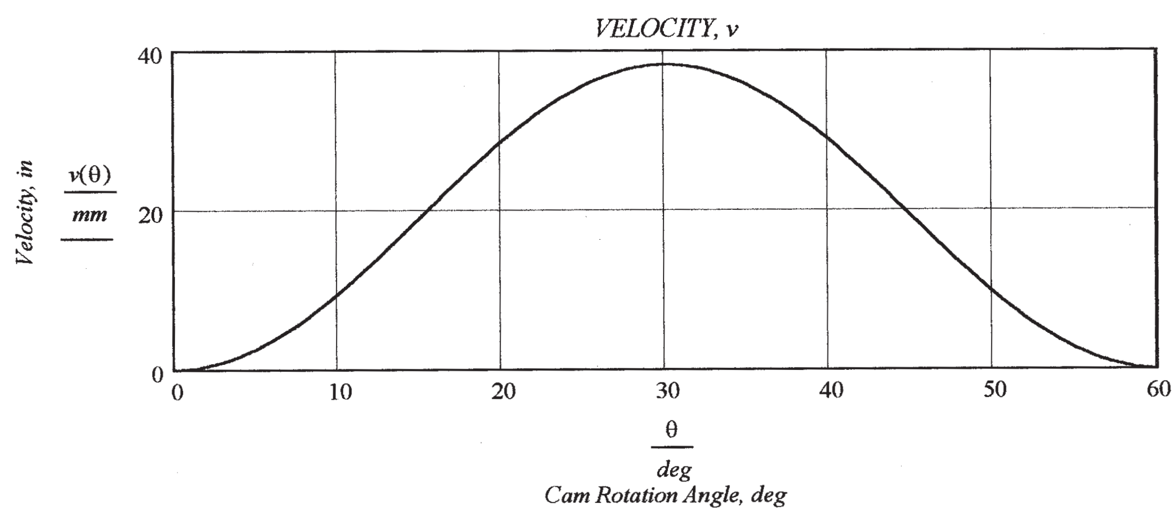
1. Cycloidal motion is defined in local coordinates by equations 8.12. They are:

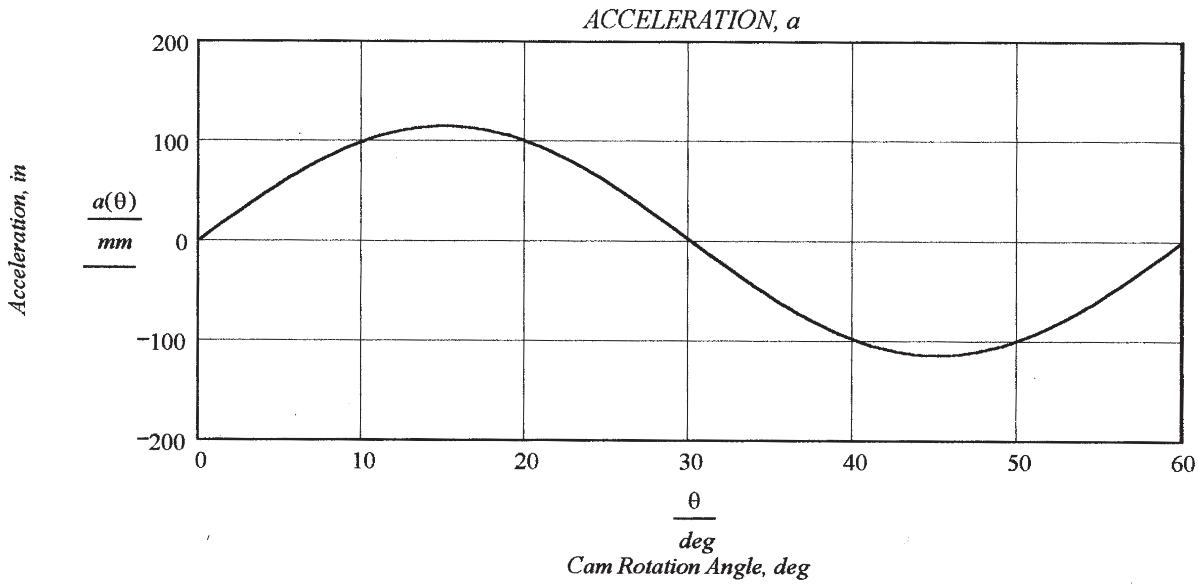
$$s(\theta) := h \cdot \left(\frac{\theta}{\beta} - \frac{1}{2 \cdot \pi} \cdot sin\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right) \qquad v(\theta) := \frac{h}{\beta} \cdot \left(1 - cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right)$$

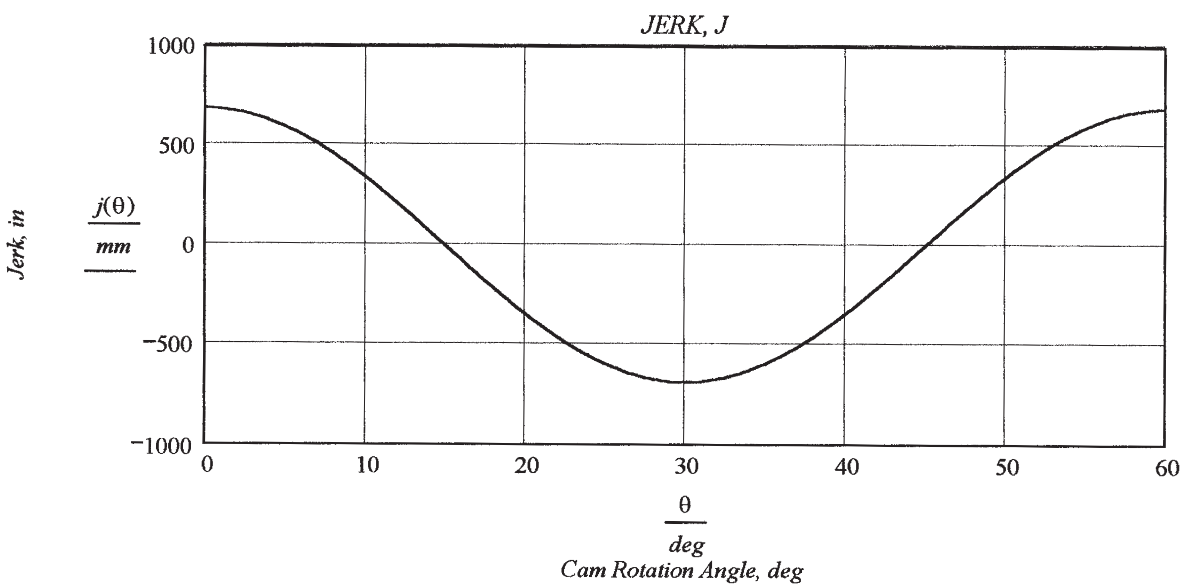
$$a(\theta) := 2 \cdot \pi \cdot \frac{h}{\beta^2} \cdot sin\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \qquad j(\theta) := 4 \cdot \pi^2 \cdot \frac{h}{\beta^3} \cdot cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right)$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$






ACCELERATION, a
200
100
0
−100
−200
Acceleration, in
a(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg


JERK, J
1000
500
0
−500
−1000
Jerk, in
j(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-24

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a 3-4-5 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot mm$

Duration: $\beta := 60 \cdot deg$

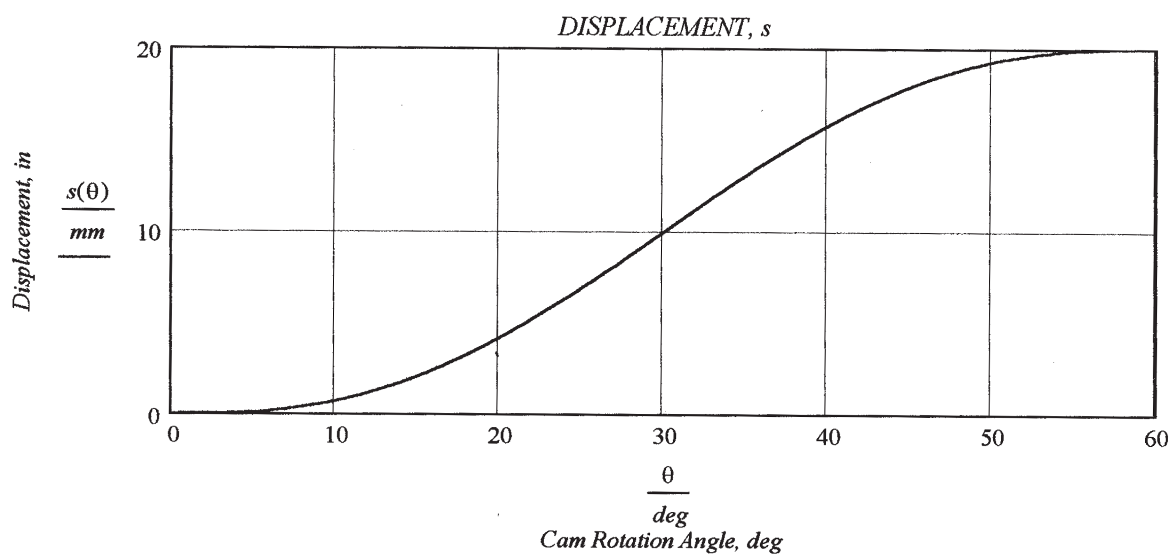
Solution: See Mathcad file P0824.

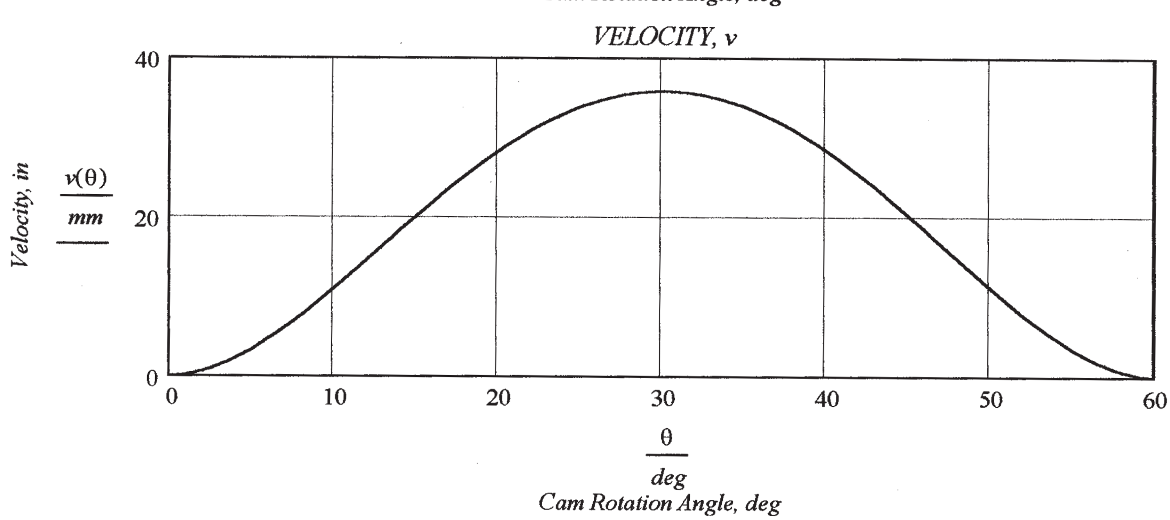
1. The 3-4-5 polynomial is defined in local coordinates by equations 8.24. They are:

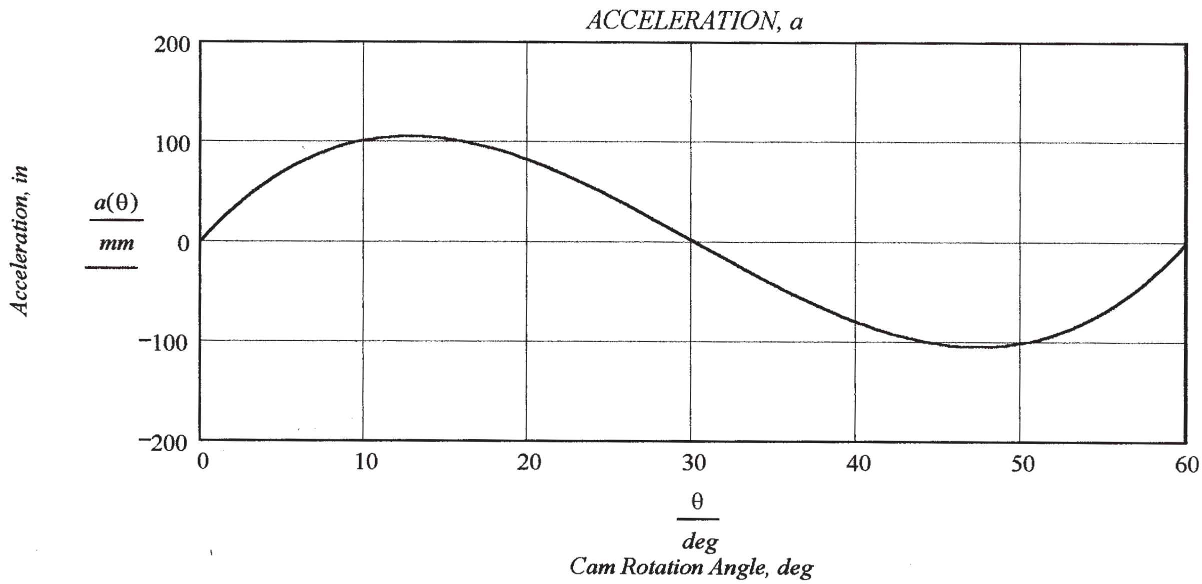
$$s(\theta) := h \cdot \left[10 \cdot \left(\frac{\theta}{\beta}\right)^3 - 15 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot \left(\frac{\theta}{\beta}\right)^5 \right] \qquad v(\theta) := \frac{h}{\beta} \cdot \left[30 \cdot \left(\frac{\theta}{\beta}\right)^2 - 60 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

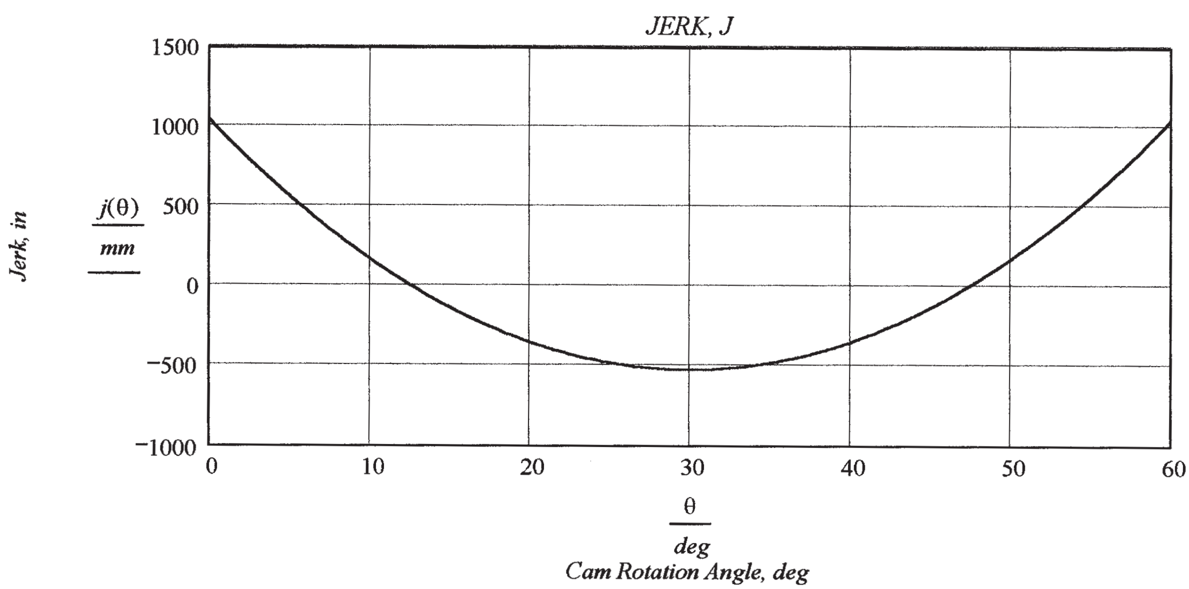
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[60 \cdot \left(\frac{\theta}{\beta}\right) - 180 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot \left(\frac{\theta}{\beta}\right)^3 \right] \qquad j(\theta) := \frac{h}{\beta^3} \cdot \left[60 - 360 \cdot \left(\frac{\theta}{\beta}\right) + 360 \cdot \left(\frac{\theta}{\beta}\right)^2 \right]$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$






ACCELERATION, a
200
100
0
−100
−200
Acceleration, in
a(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg


JERK, J
1500
1000
500
0
−500
−1000
Jerk, in
j(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-25

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a 4-5-6-7 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot mm$

Duration: $\beta := 60 \cdot deg$

Solution: See Mathcad file P0825.

1. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get *v*, *a*, and *j*.

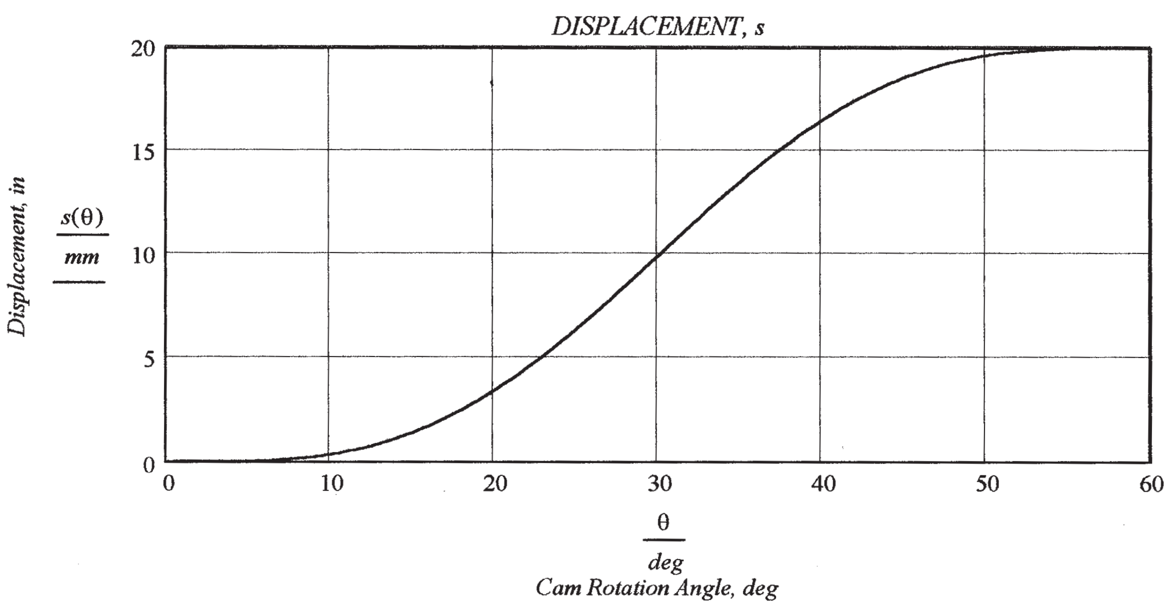
$$s(\theta) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta} \right)^4 - 84 \cdot \left(\frac{\theta}{\beta} \right)^5 + 70 \cdot \left(\frac{\theta}{\beta} \right)^6 - 20 \cdot \left(\frac{\theta}{\beta} \right)^7 \right]$$

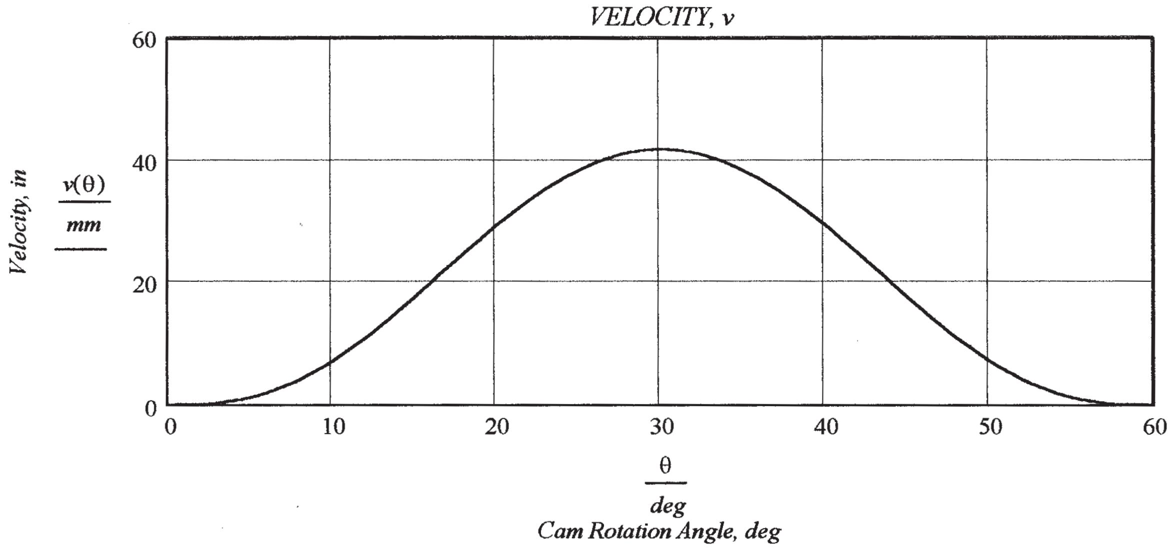
$$v(\theta) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta} \right)^3 - 420 \cdot \left(\frac{\theta}{\beta} \right)^4 + 420 \cdot \left(\frac{\theta}{\beta} \right)^5 - 140 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

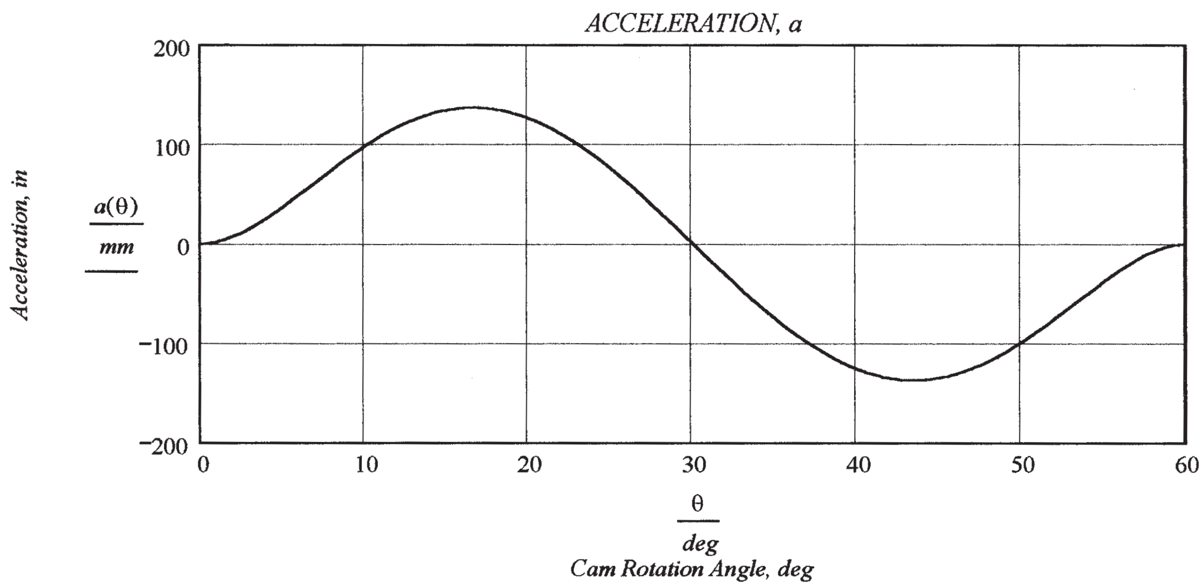
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta} \right)^2 - 1680 \cdot \left(\frac{\theta}{\beta} \right)^3 + 2100 \cdot \left(\frac{\theta}{\beta} \right)^4 - 840 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

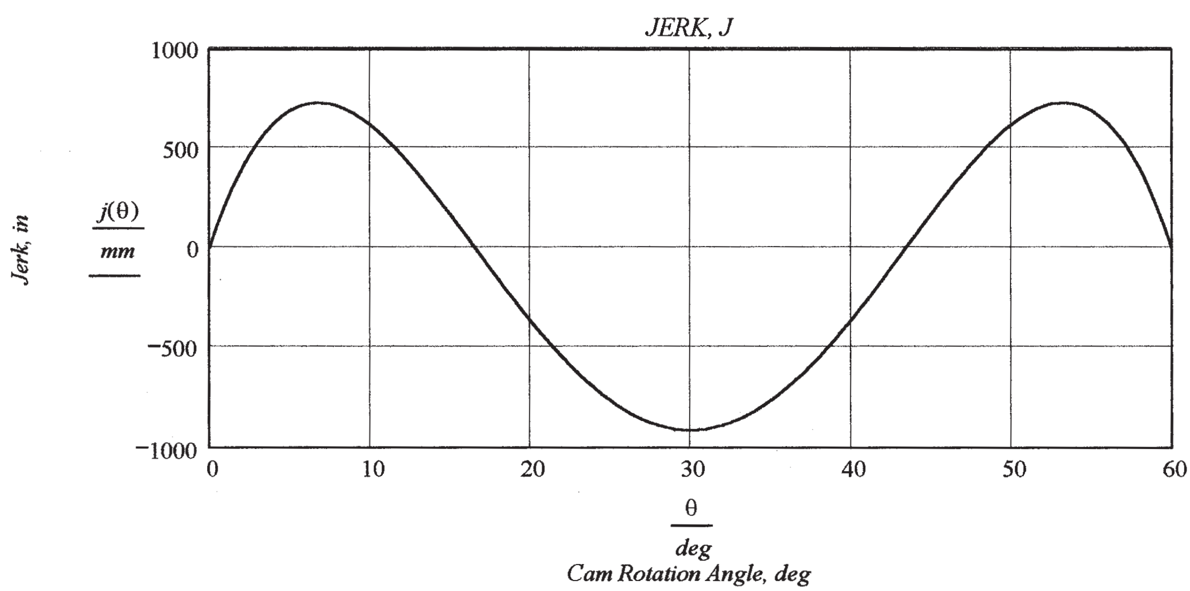
$$j(\theta) := \frac{h}{\beta^3} \cdot \left[840 \cdot \left(\frac{\theta}{\beta} \right) - 5040 \cdot \left(\frac{\theta}{\beta} \right)^2 + 8400 \cdot \left(\frac{\theta}{\beta} \right)^3 - 4200 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$




VELOCITY, v
60
40
20
0
Velocity, in
v(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg


ACCELERATION, a
200
100
0
−100
−200
Acceleration, in
a(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg


JERK, J
1000
500
0
−500
−1000
Jerk, in
j(θ)
mm
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-26

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a simple harmonic displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot mm$

Duration: $\beta := 60 \cdot deg$

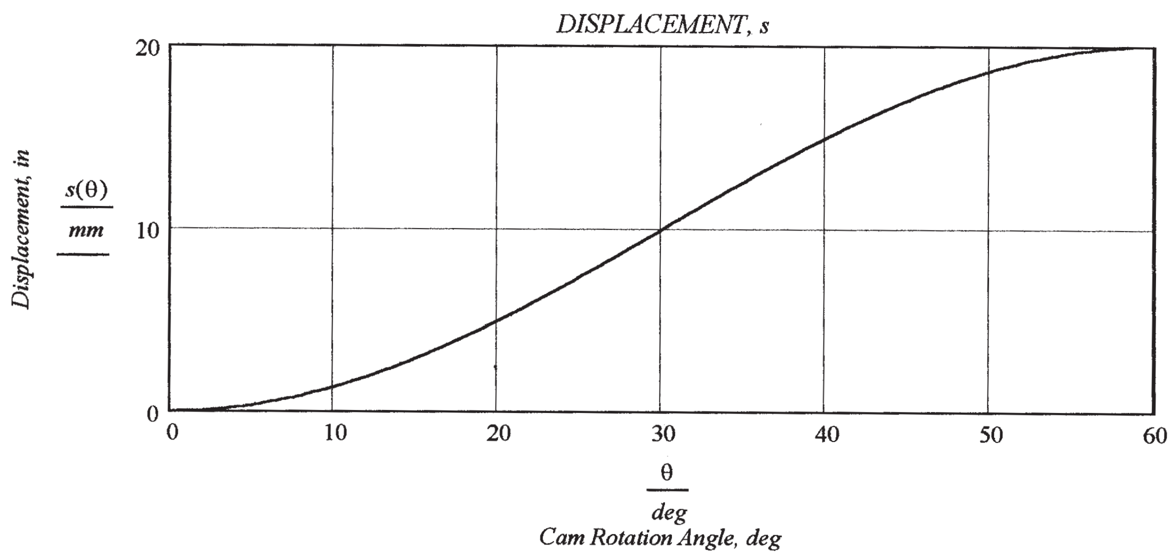
Solution: See Mathcad file P0826.

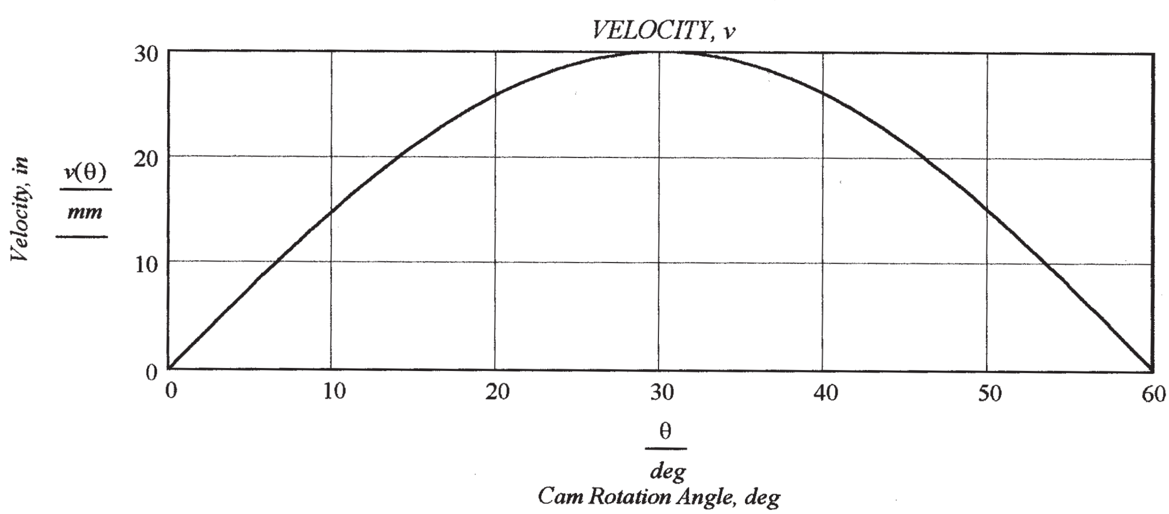
1. The simple harmonic motion (SHM) is defined in local coordinates by equations 8.6. They are:

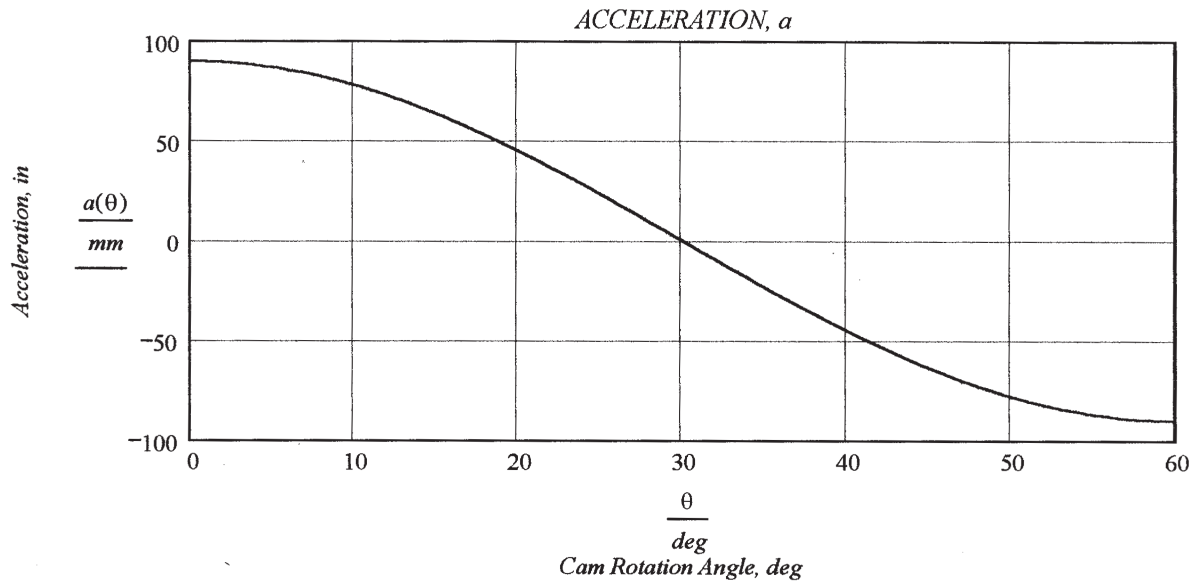
$$s(\theta) := \frac{h}{2} \cdot \left(1 - \cos\left(\pi \cdot \frac{\theta}{\beta}\right)\right) \qquad v(\theta) := \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

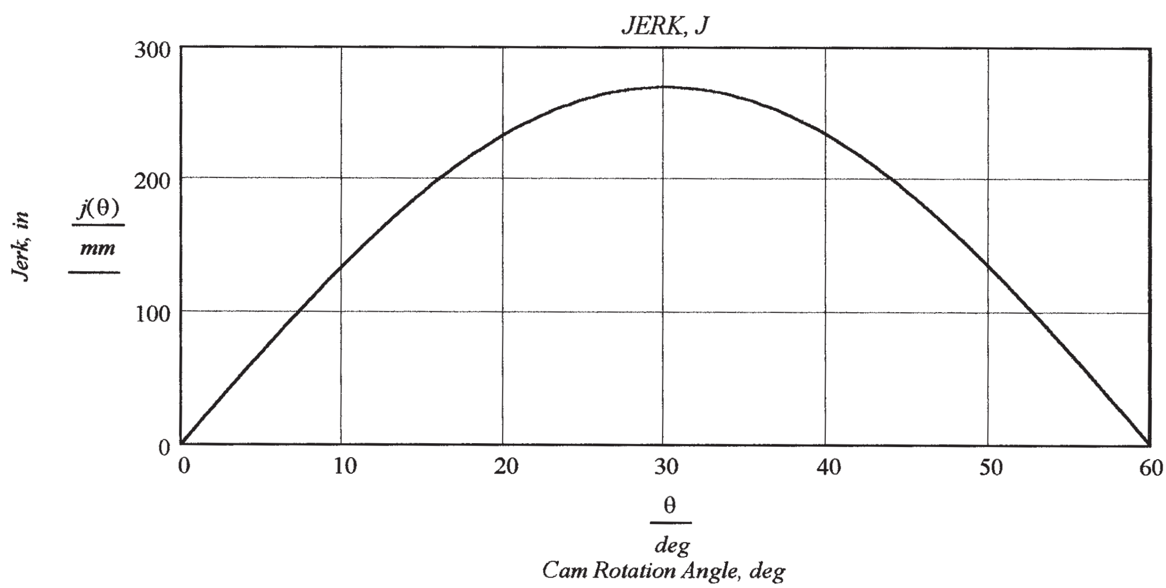
$$a(\theta) := \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \qquad j(\theta) := \frac{\pi^3}{\beta^3} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$






ACCELERATION, a
Acceleration, in
a(θ)
mm
100
50
0
−50
−100
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg


JERK, J
Jerk, in
j(θ)
mm
300
200
100
0
0
10
20
30
40
50
60
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-27

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a modified trapezoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h_1 := 20\,mm$ Eccentricity $\varepsilon := 4 \cdot mm$

Duration: $\beta_1 := 60 \cdot deg$ Prime circle radius $R_p := 50 \cdot mm$

Solution: See Mathcad file P0827.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$

$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for 0 <= x <= b/2 where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for b/2 <= x <= (1 - d)/2

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for (1 - d)/2 <= x <= (1 + d)/2

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for (1 + d)/2 <= x <= 1 - b/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for 1 - b/2 <= x <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right]\right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x-1)\right]\right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x, x1, x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

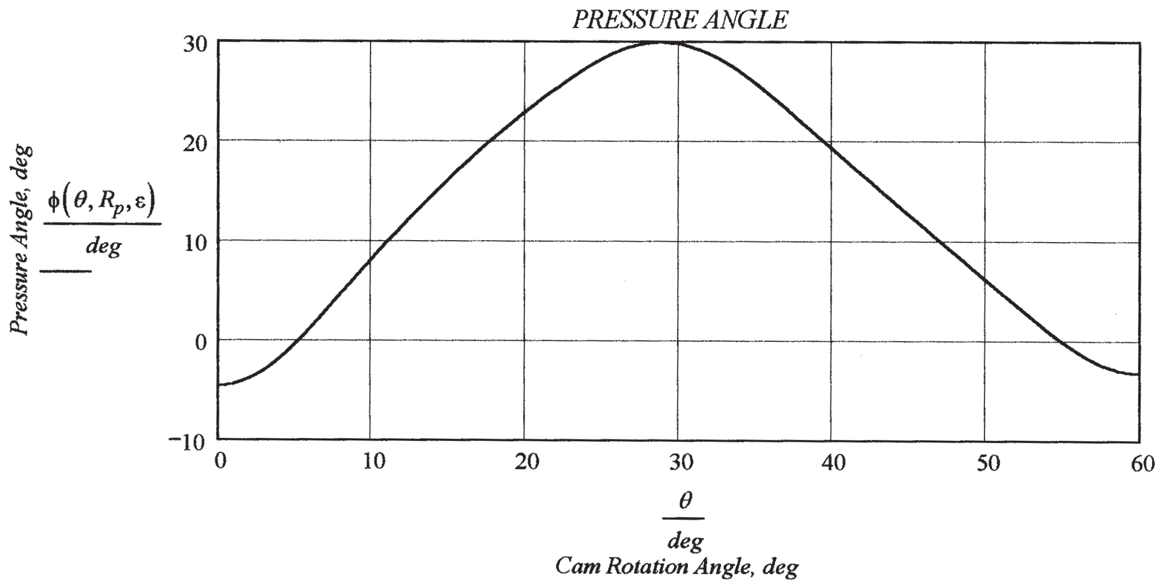
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

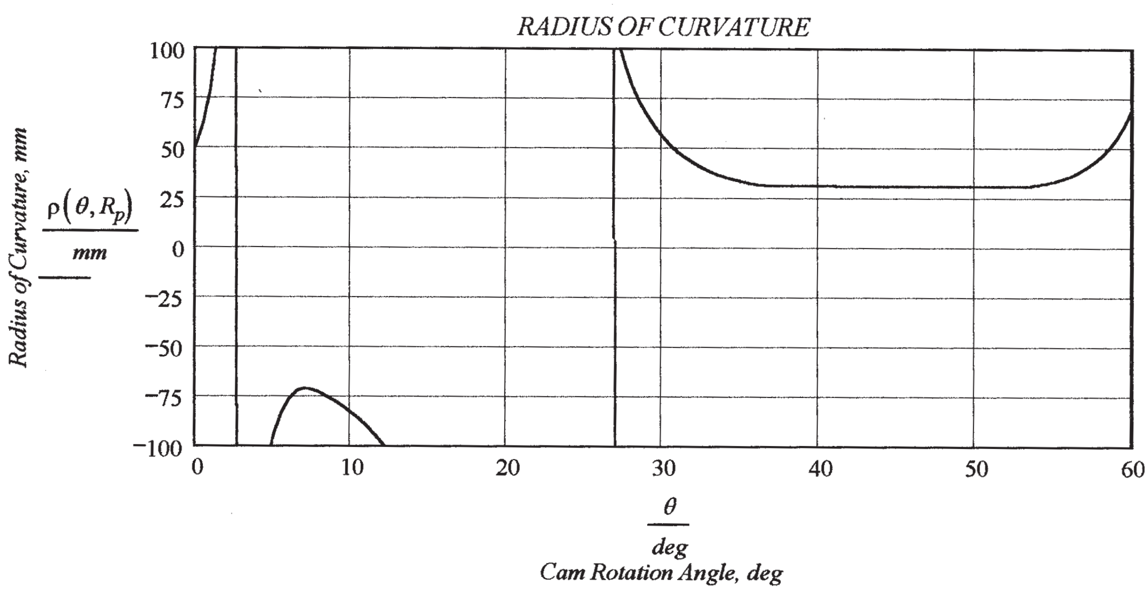
5. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi\left(\theta, R_p, \varepsilon\right) := atan\left(\frac{v_1\left(\frac{\theta}{\beta_1}\right) - \varepsilon}{s_1\left(\frac{\theta}{\beta_1}\right) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)^2 + v_1\left(\frac{\theta}{\beta_1}\right)^2\right]^{\frac{3}{2}}}{\left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)^2 + 2\cdot v_1\left(\frac{\theta}{\beta_1}\right)^2 - a_1\left(\frac{\theta}{\beta_1}\right)\cdot\left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)}$$

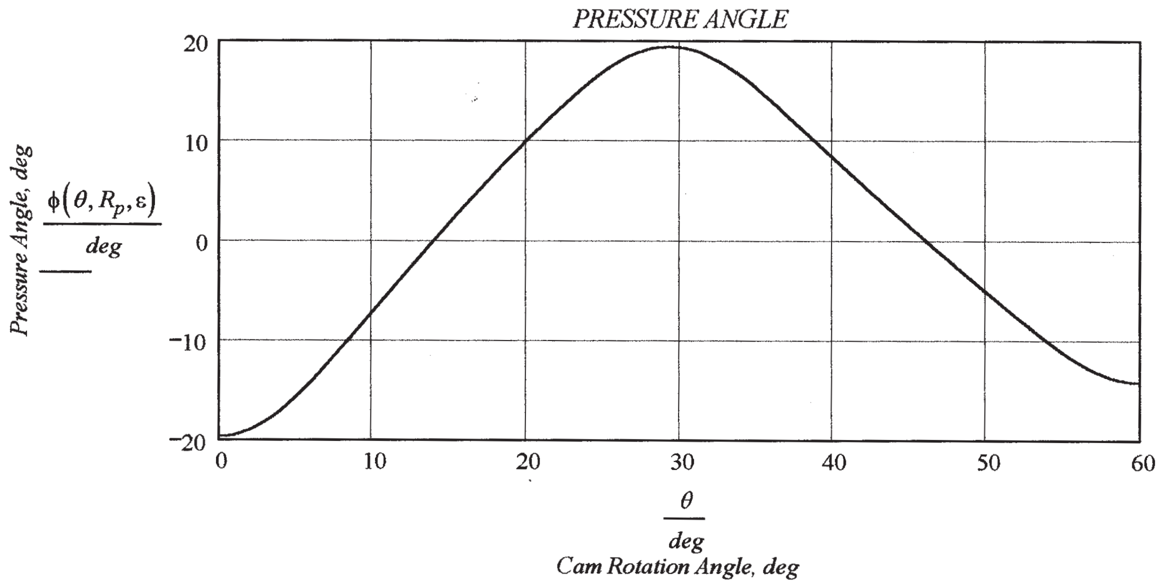
6. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0\cdot deg, 0.5\cdot deg .. \beta_1$





7. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 52\cdot mm$ $\qquad\qquad$ $\varepsilon := 17.5\cdot mm$



8. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$\rho_{min} := \rho(45 \cdot deg, R_p)$ $\qquad \rho_{min} = 32.123\, mm$

Using a multiple of 3, the maximum roller follower radius is $R_f := \dfrac{\rho_{min}}{3}$ $\qquad R_f = 10.7\, mm$

PROBLEM 8-28

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a modified sinusoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter: Lift: $h_1 := 20 \cdot mm$ Eccentricity $\varepsilon := 4 \cdot mm$

Duration: $\beta_1 := 60 \cdot deg$ Prime circle radius $R_p := 50 \cdot mm$

Solution: See Mathcad file P0828.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified sinusoidal equations are given in Table 8-2.

   $b := 0.25$ $c := 0.00$ $d := 0.75$

   $C_v := 1.7596$ $C_a := 5.5280$ $C_j := 69.466$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

   for 0 <= *x* <= *b*/2 where, for these equations, *x* is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

   for *b*/2 <= *x* <= (1 - *d*)/2

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

   for (1 - *d*)/2 <= *x* <= (1 + *d*)/2

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

   for (1 + *d*)/2 <= *x* <= 1 - *b*/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

   for 1 - *b*/2 <= *x* <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left[\frac{\pi}{b} \cdot (x-1)\right]\right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - cos\left[\frac{\pi}{b} \cdot (x-1)\right]\right]$$

$$y''_5(x) := C_a \cdot sin\left[\frac{\pi}{b} \cdot (x-1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left[\frac{\pi}{b} \cdot (x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x, x1, x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, 0 <= θ <= β_1. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For 0 <= θ <= β_1 (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

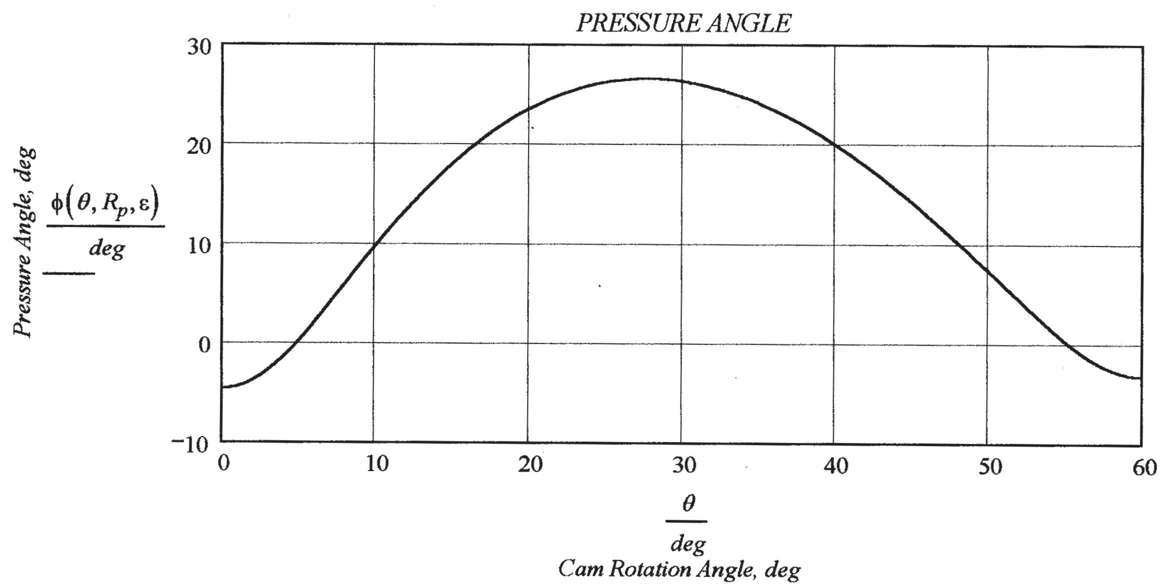
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

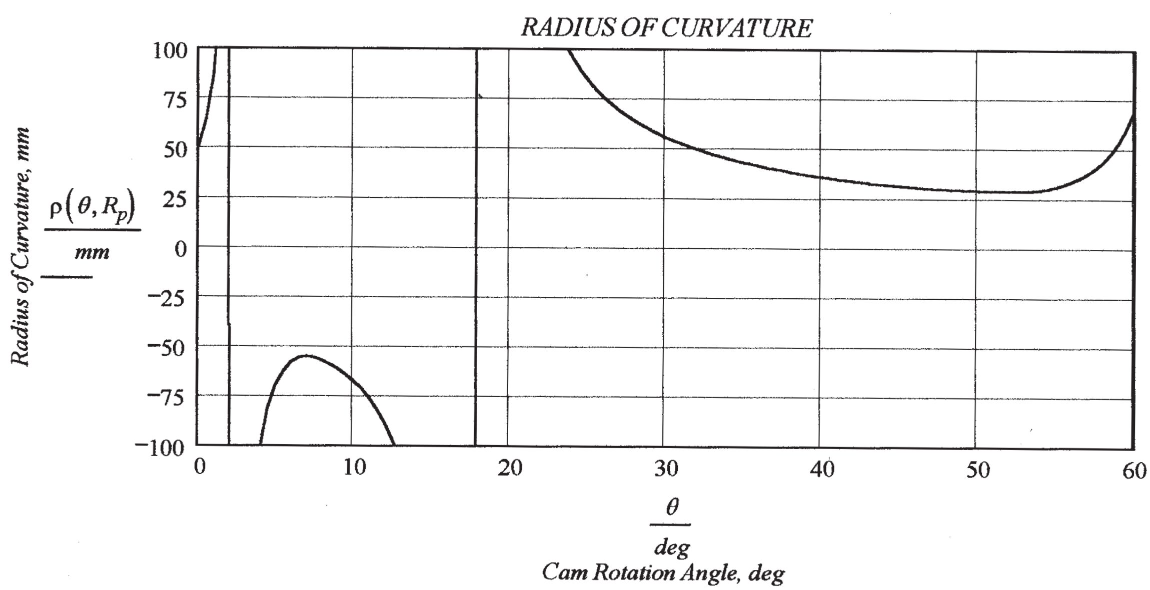
5. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi\left(\theta, R_p, \varepsilon\right) := atan\left(\frac{v_1\left(\frac{\theta}{\beta_1}\right) - \varepsilon}{s_1\left(\frac{\theta}{\beta_1}\right) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)^2 + v_1\left(\frac{\theta}{\beta_1}\right)^2\right]^{\frac{3}{2}}}{\left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)^2 + 2 \cdot v_1\left(\frac{\theta}{\beta_1}\right)^2 - a_1\left(\frac{\theta}{\beta_1}\right) \cdot \left(R_p + s_1\left(\frac{\theta}{\beta_1}\right)\right)}$$

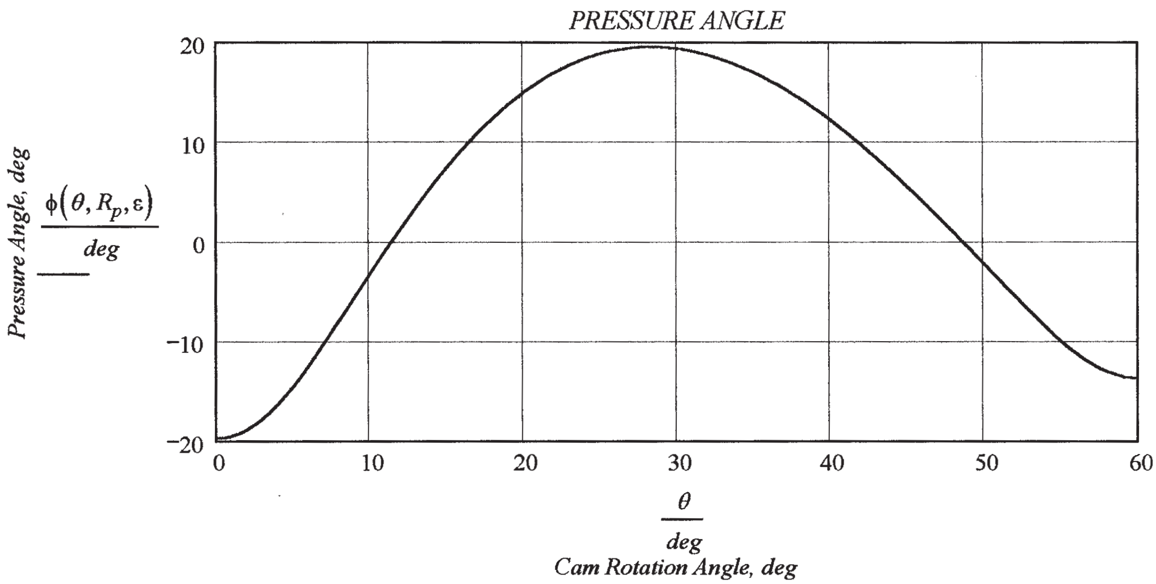
6. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta_1$





7. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 45 \cdot mm$ $\varepsilon := 15.2 \cdot mm$



8. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 52 deg.

$$\rho_{min} := \rho(52 \cdot deg, R_p) \qquad \rho_{min} = 25.562\ mm$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{min}}{3}$ $\qquad R_f = 8.5\ mm$

PROBLEM 8-29

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a cycloidal displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift:	$h := 20 \cdot mm$	Eccentricity	$\varepsilon := 4 \cdot mm$
Duration:	$\beta := 60 \cdot deg$	Prime circle radius	$R_p := 50 \cdot mm$

Solution: See Mathcad file P0829.

1. Cycloidal motion is defined in local coordinates by equations 8.12. They are:

$$s(\theta) := h \cdot \left(\frac{\theta}{\beta} - \frac{1}{2 \cdot \pi} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right) \qquad v(\theta) := \frac{h}{\beta} \cdot \left(1 - \cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right)$$

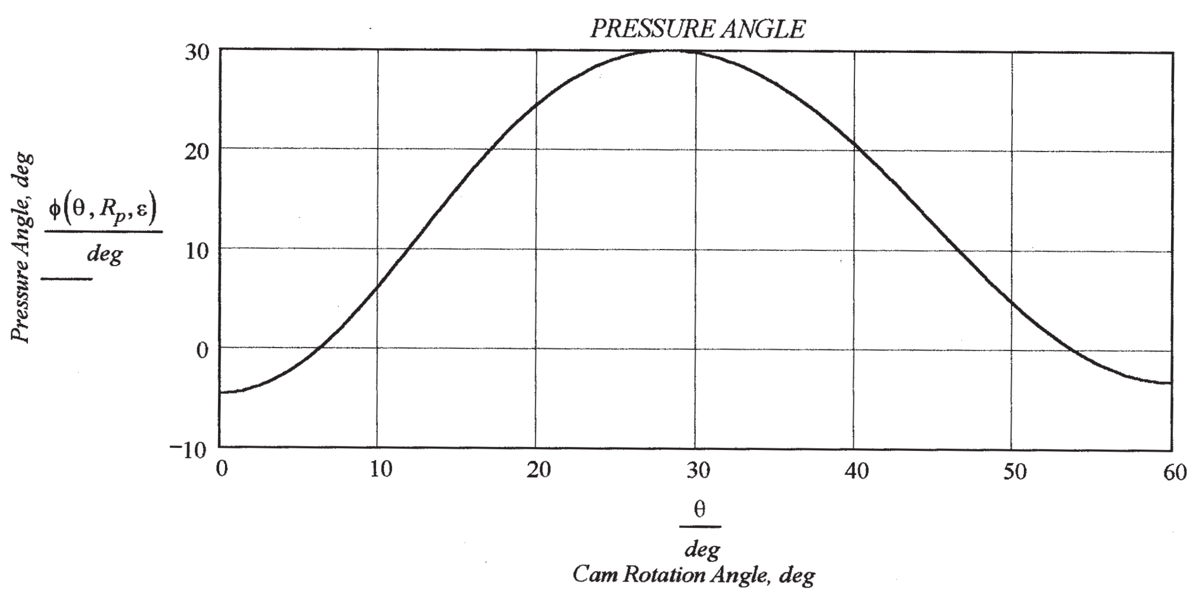
$$a(\theta) := 2 \cdot \pi \cdot \frac{h}{\beta^2} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \qquad j(\theta) := 4 \cdot \pi^2 \cdot \frac{h}{\beta^3} \cdot \cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right)$$

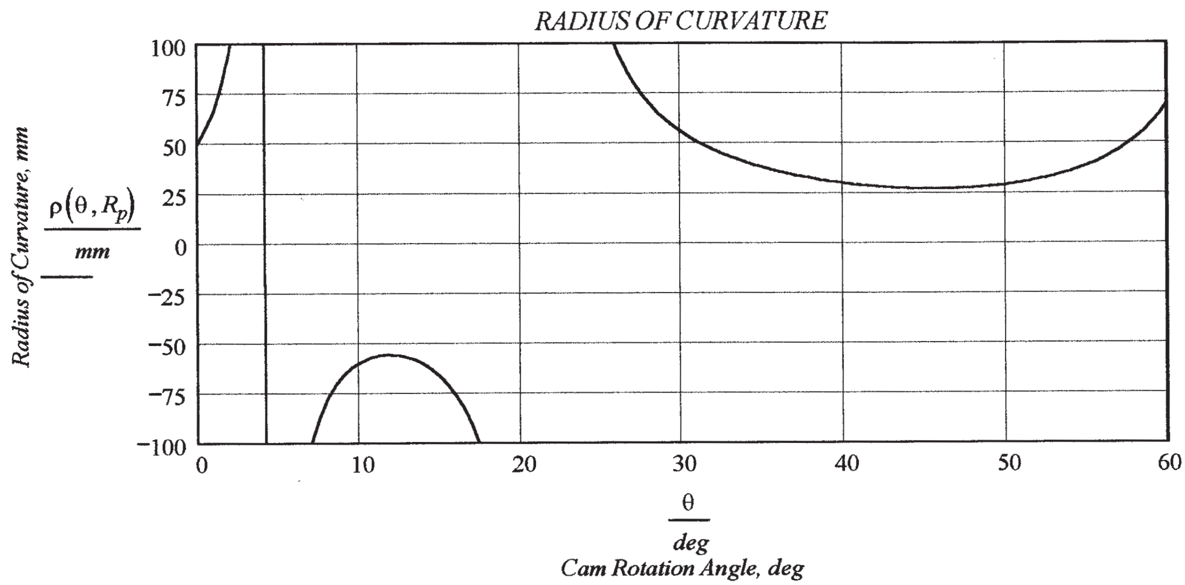
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := atan\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

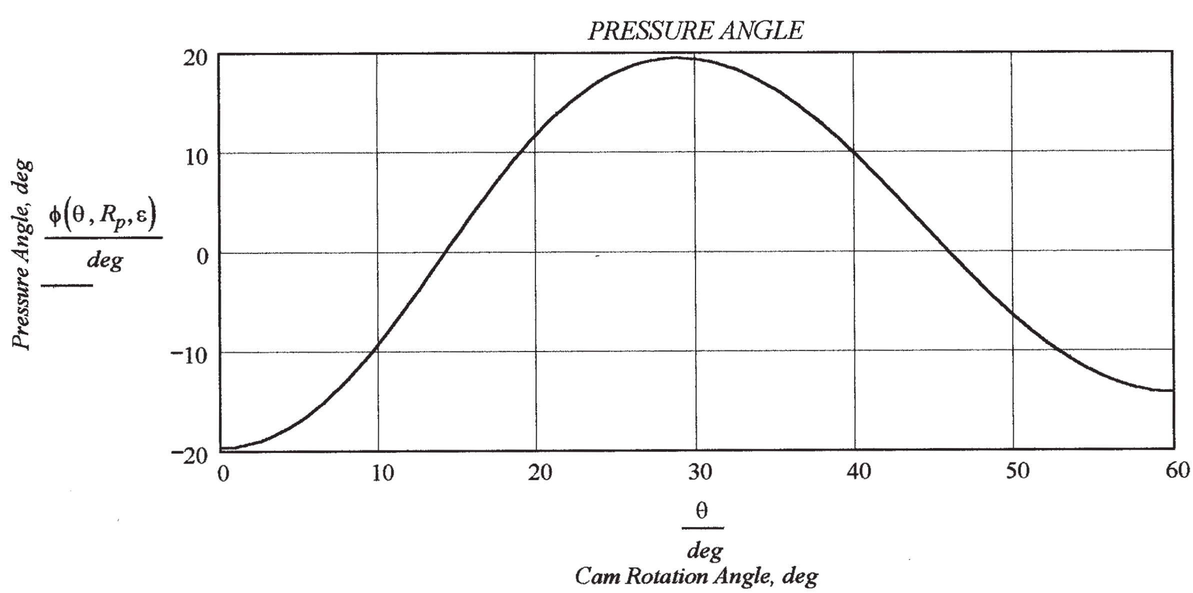
3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$





4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 52 \cdot mm$ $\qquad$ $\varepsilon := 17.5 \cdot mm$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$\rho_{min} := \rho(45 \cdot deg, R_p)$ $\qquad$ $\rho_{min} = 28.093\ mm$

Using a multiple of 3, the maximum roller follower radius is $R_f := \dfrac{\rho_{min}}{3}$ $\qquad$ $R_f = 9.4\ mm$

PROBLEM 8-30

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a 3-4-5 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift:	$h := 20 \cdot mm$	Eccentricity	$\varepsilon := 4 \cdot mm$
Duration:	$\beta := 60 \cdot deg$	Prime circle radius	$R_p := 50 \cdot mm$

Solution: See Mathcad file P0830.

1. The 3-4-5 polynomial is defined in local coordinates by equations 8.24. They are:

$$s(\theta) := h \cdot \left[10 \cdot \left(\frac{\theta}{\beta}\right)^3 - 15 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot \left(\frac{\theta}{\beta}\right)^5 \right] \qquad v(\theta) := \frac{h}{\beta} \cdot \left[30 \cdot \left(\frac{\theta}{\beta}\right)^2 - 60 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

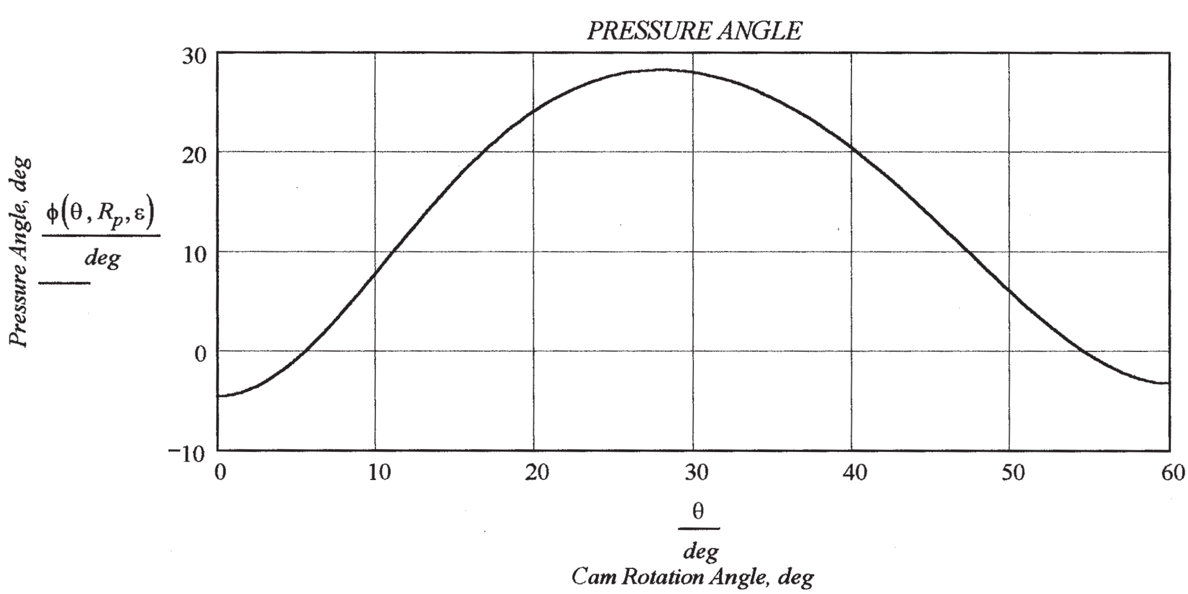
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[60 \cdot \left(\frac{\theta}{\beta}\right) - 180 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot \left(\frac{\theta}{\beta}\right)^3 \right] \qquad j(\theta) := \frac{h}{\beta^3} \cdot \left[60 - 360 \cdot \left(\frac{\theta}{\beta}\right) + 360 \cdot \left(\frac{\theta}{\beta}\right)^2 \right]$$

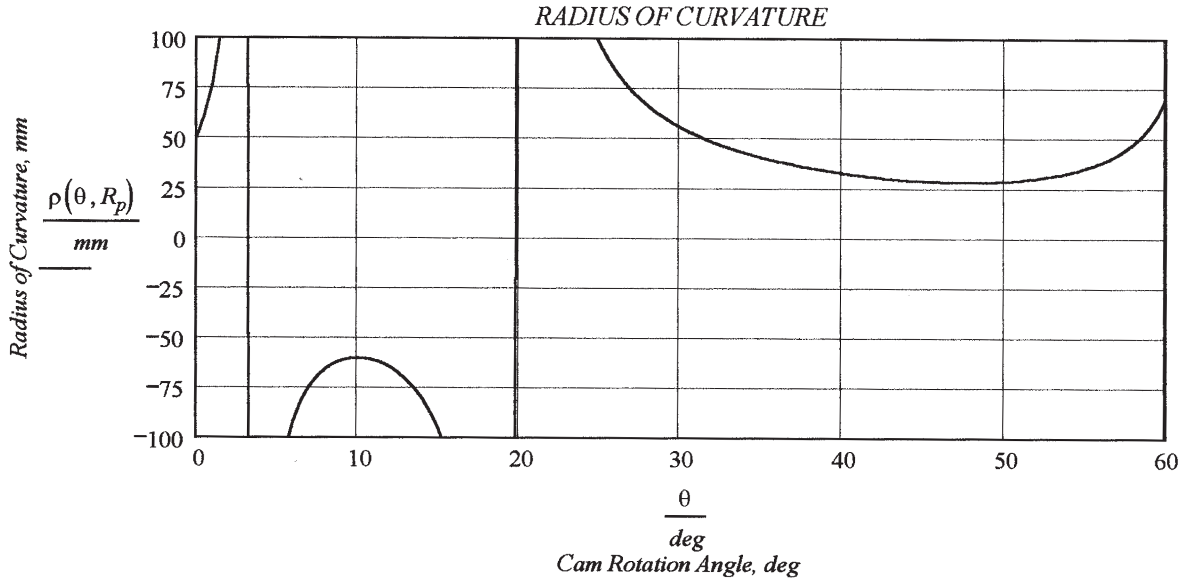
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := atan\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

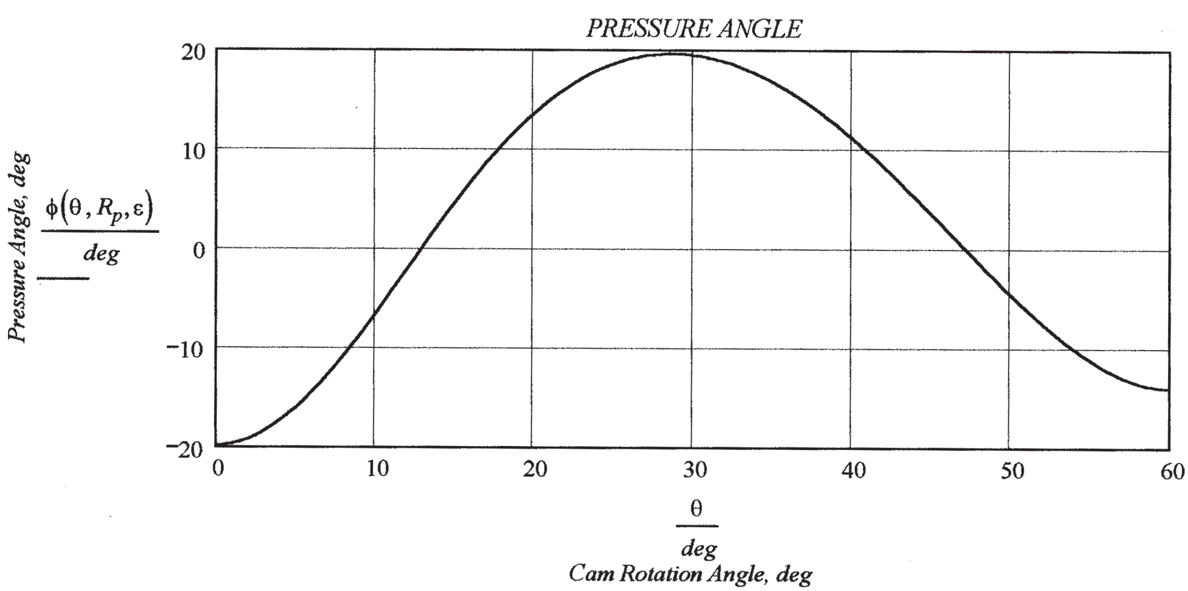
3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$





4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 48 \cdot mm$ $\qquad\qquad$ $\varepsilon := 16.3 \cdot mm$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 48 deg.

$\rho_{min} := \rho(48 \cdot deg, R_p)$ $\qquad\qquad$ $\rho_{min} = 26.885\ mm$

Using a multiple of 3, the maximum roller follower radius is $R_f := \dfrac{\rho_{min}}{3}$ $\qquad$ $R_f = 9.0\ mm$

PROBLEM 8-31

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a 4-5-6-7 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot mm$ Eccentricity $\varepsilon := 4 \cdot mm$

Duration: $\beta := 60 \cdot deg$ Prime circle radius $R_p := 50 \cdot mm$

Solution: See Mathcad file P0831.

1. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get v, a, and j.

$$s(\theta) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta}\right)^4 - 84 \cdot \left(\frac{\theta}{\beta}\right)^5 + 70 \cdot \left(\frac{\theta}{\beta}\right)^6 - 20 \cdot \left(\frac{\theta}{\beta}\right)^7 \right]$$

$$v(\theta) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta}\right)^3 - 420 \cdot \left(\frac{\theta}{\beta}\right)^4 + 420 \cdot \left(\frac{\theta}{\beta}\right)^5 - 140 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

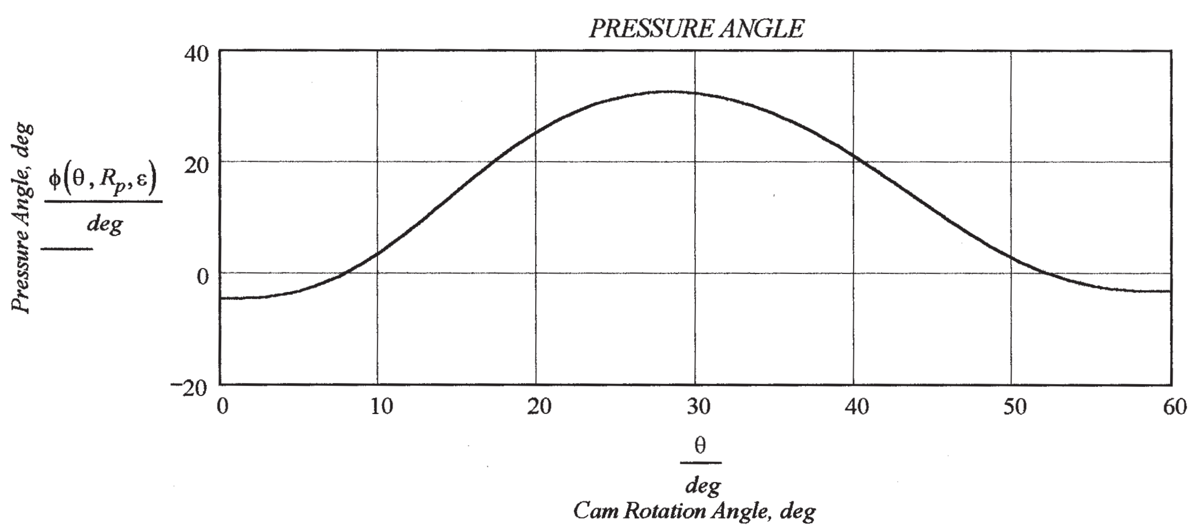
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta}\right)^2 - 1680 \cdot \left(\frac{\theta}{\beta}\right)^3 + 2100 \cdot \left(\frac{\theta}{\beta}\right)^4 - 840 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

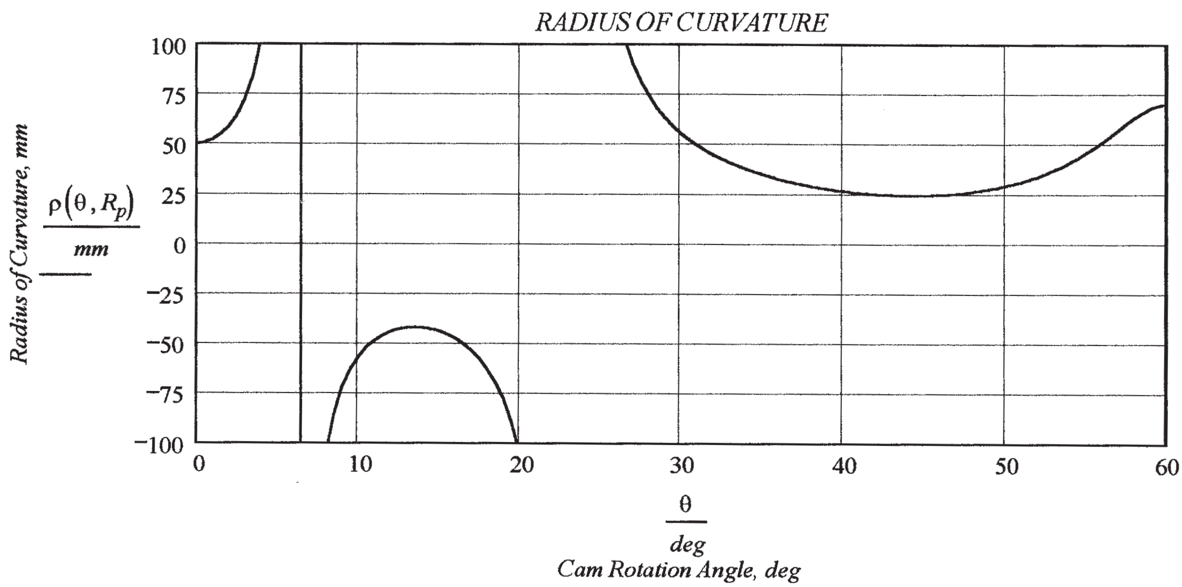
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := atan\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^{\,2} - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s(\theta)\right)^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{\left(R_p + s(\theta)\right)^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot \left(R_p + s(\theta)\right)}$$

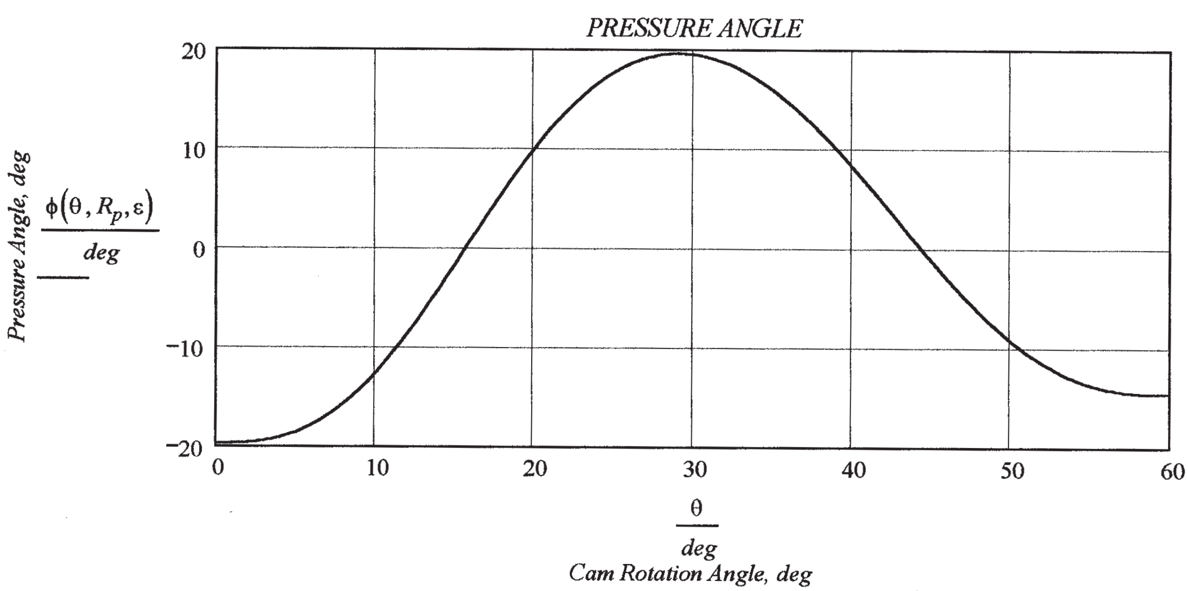
3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$





4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 57 \cdot mm$ $\qquad\qquad$ $\varepsilon := 19.2 \cdot mm$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$\rho_{min} := \rho(45 \cdot deg, R_p)$ $\qquad\qquad$ $\rho_{min} = 28.317\, mm$

Using a multiple of 3, the maximum roller follower radius is $R_f := \dfrac{\rho_{min}}{3}$ $\qquad$ $R_f = 9.4\, mm$