Chapter 11

# DYNAMIC FORCE ANALYSIS

**TOPIC/PROBLEM MATRIX**

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**PROBLEM 11-2**

***Statement:*** Draw free body diagrams of the links in the sixbar linkage shown in Figure 4-12 and write the dynamic equations to solve for all forces plus the driving torque. Assemble the symbolic equations in matrix form for solution.

***Solution:*** No solution is provided to this algebraic exercise.

## PROBLEM 11-3a

***Statement:*** Table P11-1 shows kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure P11-1. The point locations are defined as described in the text. For row *a* in the table, solve for forces and torques at the position shown. Also, compute the shaking force and the shaking torque. Consider the coefficient of friction μ between slider and ground to be zero.

***Units:*** $blob := lbf \cdot sec^2 \cdot in^{-1}$

***Given:*** Link lengths:

Link 2 ($O_2$ to $A$) $a := 4.00 \cdot in$ Link 3 ($A$ to $B$) $b := 12.00 \cdot in$

Offset $c := 0.00 \cdot in$ Friction: $\mu := 0$

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\theta_3 := 166.40 \cdot deg$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.060 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $T_3 := 20 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 203.96 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 213.69 \cdot deg$

$\alpha_3 := -2.40 \cdot rad \cdot sec^{-2}$ $a_{G3} := 371.08 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 200.84 \cdot deg$

$a_{G4} := 357.17 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 180.0 \cdot deg$

***Solution:*** See Figure P11-1, Table P11-1, and Mathcad file P1103a.

1. Calculate the *x* and *y* components of the position vectors.

$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg)$ $R_{12x} = -1.414\ in$

$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg)$ $R_{12y} = -1.414\ in$

$R_{32x} := R_{CG2} \cdot cos(\theta_2)$ $R_{32x} = 1.414\ in$

$R_{32y} := R_{CG2} \cdot sin(\theta_2)$ $R_{32y} = 1.414\ in$

$R_{23x} := R_{CG3} \cdot cos(\theta_3)$ $R_{23x} = -4.860\ in$

$R_{23y} := R_{CG3} \cdot sin(\theta_3)$ $R_{23y} = 1.176\ in$

$R_{43x} := (b - R_{CG3}) \cdot cos(\theta_3 + 180 \cdot deg)$ $R_{43x} = 6.804\ in$

$R_{43y} := (b - R_{CG3}) \cdot sin(\theta_3 + 180 \cdot deg)$ $R_{43y} = -1.646\ in$

$R_{P3x} := R_{P3} \cdot cos(\theta_3 + 180 \cdot deg + \delta_{RP3})$ $R_{P3x} = 0.000\ in$

$R_{P3y} := R_{P3} \cdot sin(\theta_3 + 180 \cdot deg + \delta_{RP3})$ $R_{P3y} = 0.000\ in$

2. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2}) \qquad a_{G2x} = -169.705\ in \cdot sec^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2}) \qquad a_{G2y} = -113.136\ in \cdot sec^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3}) \qquad a_{G3x} = -346.803\ in \cdot sec^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3}) \qquad a_{G3y} = -132.015\ in \cdot sec^{-2}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4}) \qquad a_{G4x} = -357.170\ in \cdot sec^{-2}$$

3. Calculate the $x$ and $y$ components of the external force at $P$ on link 3 in the CGS.

$$F_{P3x} := F_{P3} \cdot \cos(\delta_{FP3}) \qquad F_{P3x} = 0.000\ lbf$$

$$F_{P3y} := F_{P3} \cdot \sin(\delta_{FP3}) \qquad F_{P3y} = 0.000\ lbf$$

4. Substitute these given and calculated values into the matrix equation 11.10g, modified for this problem. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ (m_3 \cdot a_{G3x} - F_{P3x}) \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{P3y}) \cdot lbf^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{P3x} \cdot F_{P3y} + R_{P3y} \cdot F_{P3x} - T_3) \cdot lbf^{-1} \cdot in^{-1} \\ m_4 \cdot a_{G4x} \cdot lbf^{-1} \\ 0 \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot lbf$ $\quad F_{12x} = -28.7\,lbf$ $\quad F_{12y} := R_2 \cdot lbf$ $\quad F_{12y} = 5.87\,lbf$

$F_{32x} := R_3 \cdot lbf$ $\quad F_{32x} = 28.4\,lbf$ $\quad F_{32y} := R_4 \cdot lbf$ $\quad F_{32y} = -6.10\,lbf$

$F_{43x} := R_5 \cdot lbf$ $\quad F_{43x} = 21.4\,lbf$ $\quad F_{43y} := R_6 \cdot lbf$ $\quad F_{43y} = -8.74\,lbf$

$F_{14y} := R_7 \cdot lbf$ $\quad F_{14y} = -8.74\,lbf$

$T_{12} := R_8 \cdot lbf \cdot in$ $\quad T_{12} = 99.6\,lbf \cdot in$

5. Calculate the shaking force and shaking torque using equations 11.15.

$\mathbf{F_{21}} := -F_{12x} - j \cdot F_{12y}$ $\quad \mathbf{F_{41}} := -j \cdot F_{14y}$

$\mathbf{F_s} := \mathbf{F_{21}} + \mathbf{F_{41}}$ $\quad \mathbf{F_s} = 28.706 + 2.867j\,lbf$

Magnitude: $F_s := |\mathbf{F_s}|$ $\quad F_s = 28.848\,lbf$

Angle: $\theta_{Fs} := arg(\mathbf{F_s})$ $\quad \theta_{Fs} = 5.703\,deg$

$\mathbf{T_s} := -T_{12}$ $\quad \mathbf{T_s} = -99.6\,lbf \cdot in$

## PROBLEM 11-4a

**Statement:** Tables P11-1 and P11-2 show kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure P11-1. The point locations are defined as described in the text. For row *a* in the table, solve for the input torque on link 2 using the method of virtual work at the position shown. Consider the coefficient of friction μ between slider and ground to be zero.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$

**Given:** Link lengths:

Link 2 ($O_2$ to $A$) $a := 4.00 \cdot in$ Link 3 ($A$ to $B$) $b := 12.00 \cdot in$

Offset $c := 0.00 \cdot in$ Friction: $\mu := 0$

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\theta_3 := 166.40 \cdot deg$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.060 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $T_3 := 20 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 203.96 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 213.69 \cdot deg$

$\alpha_3 := -2.40 \cdot rad \cdot sec^{-2}$ $a_{G3} := 371.08 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 200.84 \cdot deg$

$a_{G4} := 357.17 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 180.0 \cdot deg$

Velocities: $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $v_{G2} := 20.0 \cdot in \cdot sec^{-1}$ $\theta_{VG2} := 135.0 \cdot deg$

$\omega_3 := -2.43 \cdot rad \cdot sec^{-1}$ $v_{G3} := 35.24 \cdot in \cdot sec^{-1}$ $\theta_{VG3} := 152.09 \cdot deg$

$v_{G4} := 35.14 \cdot in \cdot sec^{-1}$ $\theta_{VG4} := 180.0 \cdot deg$

$v_{P3} := 35.24 \cdot in \cdot sec^{-1}$ $\theta_{VP3} := 152.04 \cdot deg$

**Solution:** See Figure P11-1, Table P11-1, Table P11-2, and Mathcad file P1103a.

1. Calculate the *x* and *y* components of the velocity vectors.

$v_{G2x} := v_{G2} \cdot cos(\theta_{VG2})$ $v_{G2x} = -14.142\ in \cdot sec^{-1}$

$v_{G2y} := v_{G2} \cdot sin(\theta_{VG2})$ $v_{G2y} = 14.142\ in \cdot sec^{-1}$

$v_{G3x} := v_{G3} \cdot cos(\theta_{VG3})$ $v_{G3x} = -31.141\ in \cdot sec^{-1}$

$v_{G3y} := v_{G3} \cdot sin(\theta_{VG3})$ $v_{G3y} = 16.495\ in \cdot sec^{-1}$

$v_{G4x} := v_{G4} \cdot cos(\theta_{VG4})$ $v_{G4x} = -35.140\ in \cdot sec^{-1}$

$v_{P3x} := v_{P3} \cdot cos(\theta_{VP3})$ $v_{P3x} = -31.127\ in \cdot sec^{-1}$

$v_{P3y} := v_{P3} \cdot \sin(\theta_{VP3})$ $\qquad v_{P3y} = 16.522\ in \cdot sec^{-1}$

2. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2})$ $\qquad a_{G2x} = -169.705\ in \cdot sec^{-2}$

$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2})$ $\qquad a_{G2y} = -113.136\ in \cdot sec^{-2}$

$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3})$ $\qquad a_{G3x} = -346.803\ in \cdot sec^{-2}$

$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3})$ $\qquad a_{G3y} = -132.015\ in \cdot sec^{-2}$

$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4})$ $\qquad a_{G4x} = -357.170\ in \cdot sec^{-2}$

3. Calculate the *x* and *y* components of the external force at *P* in the CGS.

$F_{P3x} := F_{P3} \cdot \cos(\delta_{FP3})$ $\qquad F_{P3x} = 0.000\ lbf$

$F_{P3y} := F_{P3} \cdot \sin(\delta_{FP3})$ $\qquad F_{P3y} = 0.000\ lbf$

4. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \begin{bmatrix} m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \ldots \\ + m_4 \cdot (a_{G4x} \cdot v_{G4x}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3) \ldots \\ + -(F_{P3x} \cdot v_{P3x} + F_{P3y} \cdot v_{P3y}) - T_3 \cdot \omega_3 \end{bmatrix}$$

$T_{12} = 99.687\ lbf \cdot in$

## PROBLEM 11-5a

**Statement:** Table P11-3 shows kinematic and geometric data for several pin-jointed fourbar linkages of the type and orientation shown in Figure P11-2. All have $\theta_1 = 0$. The point locations are defined as described in the text. For row *a* in the table, solve for forces and torques at the position shown. Also, compute the shaking force and the shaking torque. Work in any units system you prefer.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$

**Given:** Link lengths:

Link 2 ($O_2$ to $A$) $a := 4.00 \cdot in$ Link 3 ($A$ to $B$) $b := 12.00 \cdot in$

Link 4 ($B$ to $O_4$) $c := 8.00 \cdot in$ Link 3 ($O_2$ to $O_4$) $d := 15.00 \cdot in$

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\omega_2 := 20 \cdot rad \cdot sec^{-1}$

Other link angles: $\theta_3 := 24.97 \cdot deg$ $\omega_3 := -5.62 \cdot rad \cdot sec^{-1}$ $\theta_4 := 99.30 \cdot deg$ $\omega_4 := 3.56 \cdot rad \cdot sec^{-}$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Rocker point: $R_{P4} := 8.0 \cdot in$ $\delta_{RP4} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.100 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$ $I_{G4} := 0.50 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

$R_{CG4} := 4.00 \cdot in$ $\delta_4 := 30 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $F_{P4} := 40 \cdot lbf$ $\delta_{FP4} := -30 \cdot deg$

$T_3 := -15 \cdot lbf \cdot in$ $T_4 := 25 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 801.00 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 222.14 \cdot deg$

$\alpha_3 := 75.29 \cdot rad \cdot sec^{-2}$ $a_{G3} := 1691.49 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 208.24 \cdot deg$

$\alpha_4 := 244.43 \cdot rad \cdot sec^{-2}$ $a_{G4} := 979.02 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 222.27 \cdot deg$

**Solution:** See Figure P11-2, Table P11-3 and Mathcad file P1105a.

1. Calculate the *x* and *y* components of the position vectors.

$R_{12x} := R_{CG2} \cdot cos(\theta_2 + \delta_2 + 180 \cdot deg)$ $R_{12x} = -1.414\,in$

$R_{12y} := R_{CG2} \cdot sin(\theta_2 + \delta_2 + 180 \cdot deg)$ $R_{12y} = -1.414\,in$

$R_{32} := \sqrt{(R_{CG2} \cdot sin(\delta_2))^2 + (a - R_{CG2} \cdot cos(\delta_2))^2}$ $R_{32} = 2.000\,in$

$\delta_{32} := atan2(a - R_{CG2} \cdot cos(\delta_2), R_{CG2} \cdot sin(\delta_2))$ $\delta_{32} = 0.000\,deg$

$R_{32x} := R_{32} \cdot cos(\theta_2 - \delta_{32})$ $R_{32x} = 1.414\,in$

$R_{32y} := R_{32} \cdot sin(\theta_2 - \delta_{32})$ $R_{32y} = 1.414\,in$

$R_{23x} := R_{CG3} \cdot cos(\theta_3 + \delta_3 + 180 \cdot deg)$ $\qquad R_{23x} = -4.533\ in$

$R_{23y} := R_{CG3} \cdot sin(\theta_3 + \delta_3 + 180 \cdot deg)$ $\qquad R_{23y} = -2.111\ in$

$R_{43} := \sqrt{(R_{CG3} \cdot sin(\delta_3))^2 + (b - R_{CG3} \cdot cos(\delta_3))^2}$ $\qquad R_{43} = 7.000\ in$

$\delta_{43} := atan2(b - R_{CG3} \cdot cos(\delta_3), R_{CG3} \cdot sin(\delta_3))$ $\qquad \delta_{43} = 0.000\ deg$

$R_{43x} := R_{43} \cdot cos(\theta_3 - \delta_{43})$ $\qquad R_{43x} = 6.346\ in$

$R_{43y} := R_{43} \cdot sin(\theta_3 - \delta_{43})$ $\qquad R_{43y} = 2.955\ in$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + \delta_4 + 180 \cdot deg)$ $\qquad R_{14x} = 2.534\ in$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + \delta_4 + 180 \cdot deg)$ $\qquad R_{14y} = -3.095\ in$

$R_{34} := \sqrt{(R_{CG4} \cdot sin(\delta_4))^2 + (c - R_{CG4} \cdot cos(\delta_4))^2}$ $\qquad R_{34} = 4.957\ in$

$\delta_{34} := atan2(c - R_{CG4} \cdot cos(\delta_4), R_{CG4} \cdot sin(\delta_4))$ $\qquad \delta_{34} = 23.794\ deg$

$R_{34x} := R_{34} \cdot cos(\theta_4 - \delta_{34})$ $\qquad R_{34x} = 1.241\ in$

$R_{34y} := R_{34} \cdot sin(\theta_4 - \delta_{34})$ $\qquad R_{34y} = 4.799\ in$

$R_{P3x} := R_{P3} \cdot cos(\theta_3 + \delta_{RP3})$ $\qquad R_{P3x} = 0.000\ in$

$R_{P3y} := R_{P3} \cdot sin(\theta_3 + \delta_{RP3})$ $\qquad R_{P3y} = 0.000\ in$

$R_{P4x} := R_{P4} \cdot cos(\theta_4 + \delta_{RP4})$ $\qquad R_{P4x} = -1.293\ in$

$R_{P4y} := R_{P4} \cdot sin(\theta_4 + \delta_{RP4})$ $\qquad R_{P4y} = 7.895\ in$

2. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$a_{G2x} := a_{G2} \cdot cos(\theta_{AG2})$ $\qquad a_{G2x} = -593.948\ in \cdot sec^{-2}$

$a_{G2y} := a_{G2} \cdot sin(\theta_{AG2})$ $\qquad a_{G2y} = -537.427\ in \cdot sec^{-2}$

$a_{G3x} := a_{G3} \cdot cos(\theta_{AG3})$ $\qquad a_{G3x} = -1.490 \times 10^3\ in \cdot sec^{-2}$

$a_{G3y} := a_{G3} \cdot sin(\theta_{AG3})$ $\qquad a_{G3y} = -800.355\ in \cdot sec^{-2}$

$a_{G4x} := a_{G4} \cdot cos(\theta_{AG4})$ $\qquad a_{G4x} = -724.459\ in \cdot sec^{-2}$

$a_{G4y} := a_{G4} \cdot sin(\theta_{AG4})$ $\qquad a_{G4y} = -658.513\ in \cdot sec^{-2}$

3. Calculate the $x$ and $y$ components of the external force on links 3 and 4 in the CGS.

$F_{P3x} := F_{P3} \cdot cos(\delta_{FP3})$ $\qquad F_{P3x} = 0.000\ lbf$

$F_{P3y} := F_{P3} \cdot sin(\delta_{FP3})$ $\qquad F_{P3y} = 0.000\ lbf$

$F_{P4x} := F_{P4} \cdot cos(\delta_{FP4})$ $\qquad F_{P4x} = 34.641\ lbf$

$F_{P4y} := F_{P4} \cdot sin(\delta_{FP4})$ $\qquad F_{P4y} = -20.000\ lbf$

4. Substitute these given and calculated values into the matrix equation 11.9, modified for additional terms. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ (m_3 \cdot a_{G3x} - F_{P3x}) \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{P3y}) \cdot lbf^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{P3x} \cdot F_{P3y} + R_{P3y} \cdot F_{P3x} - T_3) \cdot lbf^{-1} \cdot in^{-1} \\ (m_4 \cdot a_{G4x} - F_{P4x}) \cdot lbf^{-1} \\ (m_4 \cdot a_{G4y} - F_{P4y}) \cdot lbf^{-1} \\ (I_{G4} \cdot \alpha_4 - R_{P4x} \cdot F_{P4y} + R_{P4y} \cdot F_{P4x} - T_4) \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot lbf$ $\quad F_{12x} = -124.0\, lbf$ $\quad F_{12y} := R_2 \cdot lbf$ $\quad F_{12y} = -62.3\, lbf$

$F_{32x} := R_3 \cdot lbf$ $\quad F_{32x} = 122.8\, lbf$ $\quad F_{32y} := R_4 \cdot lbf$ $\quad F_{32y} = 61.2\, lbf$

$F_{43x} := R_5 \cdot lbf$ $\quad F_{43x} = 93.0\, lbf$ $\quad F_{43y} := R_6 \cdot lbf$ $\quad F_{43y} = 45.2\, lbf$

$F_{14x} := R_7 \cdot lbf$ $\quad F_{14x} = -14.10\, lbf$ $\quad F_{14y} := R_8 \cdot lbf$ $\quad F_{14y} = -0.676\, lbf$

$T_{12} := R_9 \cdot lbf \cdot in$ $\quad T_{12} = 176.4\, lbf \cdot in$

5. Calculate the shaking force and shaking torque using equations 11.15.

$\mathbf{F_{21}} := -F_{12x} - j \cdot F_{12y}$ $\qquad \mathbf{F_{41}} := F_{14x} + j \cdot F_{14y}$

$\mathbf{F_s} := \mathbf{F_{21}} + \mathbf{F_{41}}$ $\mathbf{F_s} = 109.868 + 61.581j\ lbf$

Magnitude: $F_s := |\mathbf{F_s}|$ $F_s = 125.949\ lbf$

Angle: $\theta_{Fs} := arg(\mathbf{F_s})$ $\theta_{Fs} = 29.271\ deg$

$\mathbf{T_s} := -T_{12}$ $\mathbf{T_s} = -176.4\ lbf \cdot in$

## PROBLEM 11-6a

**Statement:** Tables P11-3 and P11-4 show kinematic and geometric data for several pin-jointed fourbar linkages of the type and orientation shown in Figure P11-2. All have $\theta_1 = 0$. The point locations are defined as described in the text. For row *a* in the table, solve for input torque on link 2, using the method of virtual work, at the position shown. Work in any units system you prefer.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$

**Given:** Link lengths:

Link 2 ($O_2$ to $A$) $a := 4.00 \cdot in$ Link 3 ($A$ to $B$) $b := 12.00 \cdot in$

Link 4 ($B$ to $O_4$) $c := 8.00 \cdot in$ Link 3 ($O_2$ to $O_4$) $d := 15.00 \cdot in$

Link angles: $\theta_2 := 45 \cdot deg$ $\theta_3 := 24.97 \cdot deg$ $\theta_4 := 99.30 \cdot deg$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Rocker point: $R_{P4} := 8.0 \cdot in$ $\delta_{RP4} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.100 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$ $I_{G4} := 0.50 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

$R_{CG4} := 4.00 \cdot in$ $\delta_4 := 30 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $F_{P4} := 40 \cdot lbf$ $\delta_{FP4} := -30 \cdot deg$

$T_3 := -15 \cdot lbf \cdot in$ $T_4 := 25 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 801.00 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 222.14 \cdot deg$

$\alpha_3 := 75.29 \cdot rad \cdot sec^{-2}$ $a_{G3} := 1691.49 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 208.24 \cdot deg$

$\alpha_4 := 244.43 \cdot rad \cdot sec^{-2}$ $a_{G4} := 979.02 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 222.27 \cdot deg$

Velocities: $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $v_{G2} := 40.0 \cdot in \cdot sec^{-1}$ $\theta_{VG2} := 135.0 \cdot deg$

$\omega_3 := -5.62 \cdot rad \cdot sec^{-1}$ $v_{G3} := 54.44 \cdot in \cdot sec^{-1}$ $\theta_{VG3} := 145.19 \cdot deg$

$\omega_4 := 3.56 \cdot rad \cdot sec^{-1}$ $v_{G4} := 14.23 \cdot in \cdot sec^{-1}$ $\theta_{VG4} := 219.30 \cdot deg$

$v_{P3} := 80.00 \cdot in \cdot sec^{-1}$ $\theta_{VP3} := 135.00 \cdot deg$

$v_{P4} := 28.45 \cdot in \cdot sec^{-1}$ $\theta_{VP4} := 189.30 \cdot deg$

**Solution:** See Figure P11-2, Table P11-3, Table P11-4, and Mathcad file P1106a.

1. Calculate the *x* and *y* components of the velocity vectors.

$v_{G2x} := v_{G2} \cdot cos(\theta_{VG2})$ $v_{G2x} = -28.284 \, in \cdot sec^{-1}$

$v_{G2y} := v_{G2} \cdot sin(\theta_{VG2})$ $v_{G2y} = 28.284 \, in \cdot sec^{-1}$

$v_{G3x} := v_{G3} \cdot \cos(\theta_{VG3})$ $\quad v_{G3x} = -44.698\ in \cdot sec^{-1}$

$v_{G3y} := v_{G3} \cdot \sin(\theta_{VG3})$ $\quad v_{G3y} = 31.077\ in \cdot sec^{-1}$

$v_{G4x} := v_{G4} \cdot \cos(\theta_{VG4})$ $\quad v_{G4x} = -11.012\ in \cdot sec^{-1}$

$v_{G4y} := v_{G4} \cdot \sin(\theta_{VG4})$ $\quad v_{G4y} = -9.013\ in \cdot sec^{-1}$

$v_{P3x} := v_{P3} \cdot \cos(\theta_{VP3})$ $\quad v_{P3x} = -56.569\ in \cdot sec^{-1}$

$v_{P3y} := v_{P3} \cdot \sin(\theta_{VP3})$ $\quad v_{P3y} = 56.569\ in \cdot sec^{-1}$

$v_{P4x} := v_{P4} \cdot \cos(\theta_{VP4})$ $\quad v_{P4x} = -28.076\ in \cdot sec^{-1}$

$v_{P4y} := v_{P4} \cdot \sin(\theta_{VP4})$ $\quad v_{P4y} = -4.598\ in \cdot sec^{-1}$

2. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2})$ $\quad a_{G2x} = -593.948\ in \cdot sec^{-2}$

$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2})$ $\quad a_{G2y} = -537.427\ in \cdot sec^{-2}$

$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3})$ $\quad a_{G3x} = -1.490 \times 10^{3}\ in \cdot sec^{-2}$

$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3})$ $\quad a_{G3y} = -800.355\ in \cdot sec^{-2}$

$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4})$ $\quad a_{G4x} = -724.459\ in \cdot sec^{-2}$

$a_{G4y} := a_{G4} \cdot \sin(\theta_{AG4})$ $\quad a_{G4y} = -658.513\ in \cdot sec^{-2}$

3. Calculate the $x$ and $y$ components of the external force on links 3 and 4 in the CGS.

$F_{P3x} := F_{P3} \cdot \cos(\delta_{FP3})$ $\quad F_{P3x} = 0.000\ lbf$

$F_{P3y} := F_{P3} \cdot \sin(\delta_{FP3})$ $\quad F_{P3y} = 0.000\ lbf$

$F_{P4x} := F_{P4} \cdot \cos(\delta_{FP4})$ $\quad F_{P4x} = 34.641\ lbf$

$F_{P4y} := F_{P4} \cdot \sin(\delta_{FP4})$ $\quad F_{P4y} = -20.000\ lbf$

4. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \begin{bmatrix} m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \ldots \\ + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \ldots \\ + -(F_{P3x} \cdot v_{P3x} + F_{P3y} \cdot v_{P3y}) - (F_{P4x} \cdot v_{P4x} + F_{P4y} \cdot v_{P4y}) \ldots \\ + -T_3 \cdot \omega_3 - T_4 \cdot \omega_4 \end{bmatrix}$$

$T_{12} = 166.3\ lbf \cdot in$

## PROBLEM 11-7a

***Statement:*** For row *a* in Table P11-3, input the associated disk file to program FOURBAR, calculate the linkage parameters for crank angles from zero to 360 deg by 5 deg increments, and design a steel disk flywheel to smooth the input torque using a coefficient of fluctuation of 0.05. Minimize the flywheel weight.

***Units:*** $blob := lbf \cdot sec^2 \cdot in^{-1}$

***Given:*** Link lengths:

| | | | |
|---|---|---|---|
| Link 2 ($O_2$ to $A$) | $a := 4.00 \cdot in$ | Link 3 ($A$ to $B$) | $b := 12.00 \cdot in$ |
| Link 4 ($B$ to $O_4$) | $c := 8.00 \cdot in$ | Link 3 ($O_2$ to $O_4$) | $d := 15.00 \cdot in$ |

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\quad \omega_2 := 20 \cdot rad \cdot sec^{-1}$

Other link angles: $\theta_3 := 24.97 \cdot deg$ $\quad \omega_3 := -5.62 \cdot rad \cdot sec^{-1}$ $\quad \theta_4 := 99.30 \cdot deg$ $\quad \omega_4 := 3.56 \cdot rad \cdot sec^{-}$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\quad \delta_{RP3} := 0.0 \cdot deg$

Rocker point: $R_{P4} := 8.0 \cdot in$ $\quad \delta_{RP4} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $\quad m_3 := 0.020 \cdot blob$ $\quad m_4 := 0.100 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $\quad I_{G3} := 0.20 \cdot blob \cdot in^2$ $\quad I_{G4} := 0.50 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\quad \delta_2 := 0 \cdot deg$ $\quad R_{CG3} := 5.00 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

$R_{CG4} := 4.00 \cdot in$ $\quad \delta_4 := 30 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\quad \delta_{FP3} := 0 \cdot deg$ $\quad F_{P4} := 40 \cdot lbf$ $\quad \delta_{FP4} := -30 \cdot deg$

$T_3 := -15 \cdot lbf \cdot in$ $\quad T_4 := 25 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $\quad a_{G2} := 801.00 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG2} := 222.14 \cdot deg$

$\alpha_3 := 75.29 \cdot rad \cdot sec^{-2}$ $\quad a_{G3} := 1691.49 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG3} := 208.24 \cdot deg$

$\alpha_4 := 244.43 \cdot rad \cdot sec^{-2}$ $\quad a_{G4} := 979.02 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG4} := 222.27 \cdot deg$

Coefficient of fluctuation: $k := 0.05$

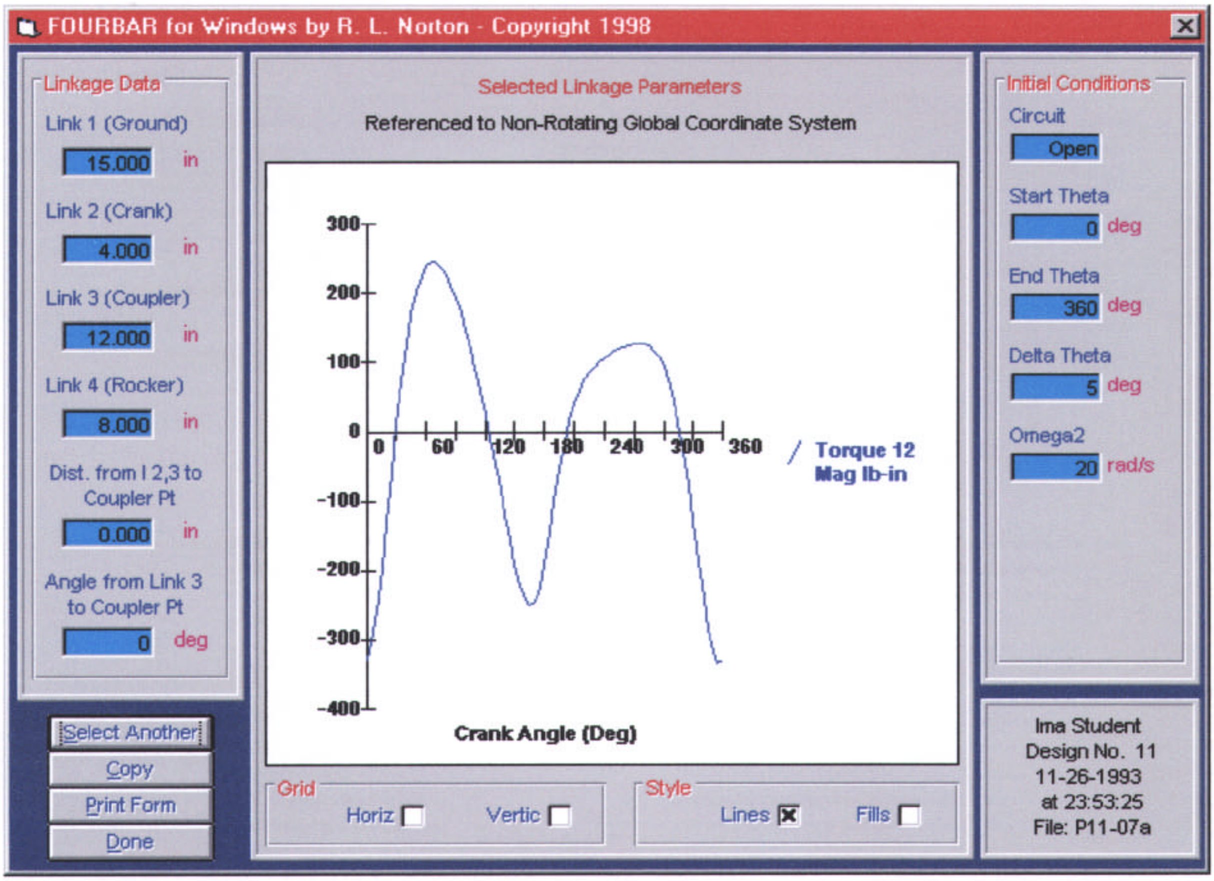
Desired average speed: $\omega_{avg} := \omega_2$

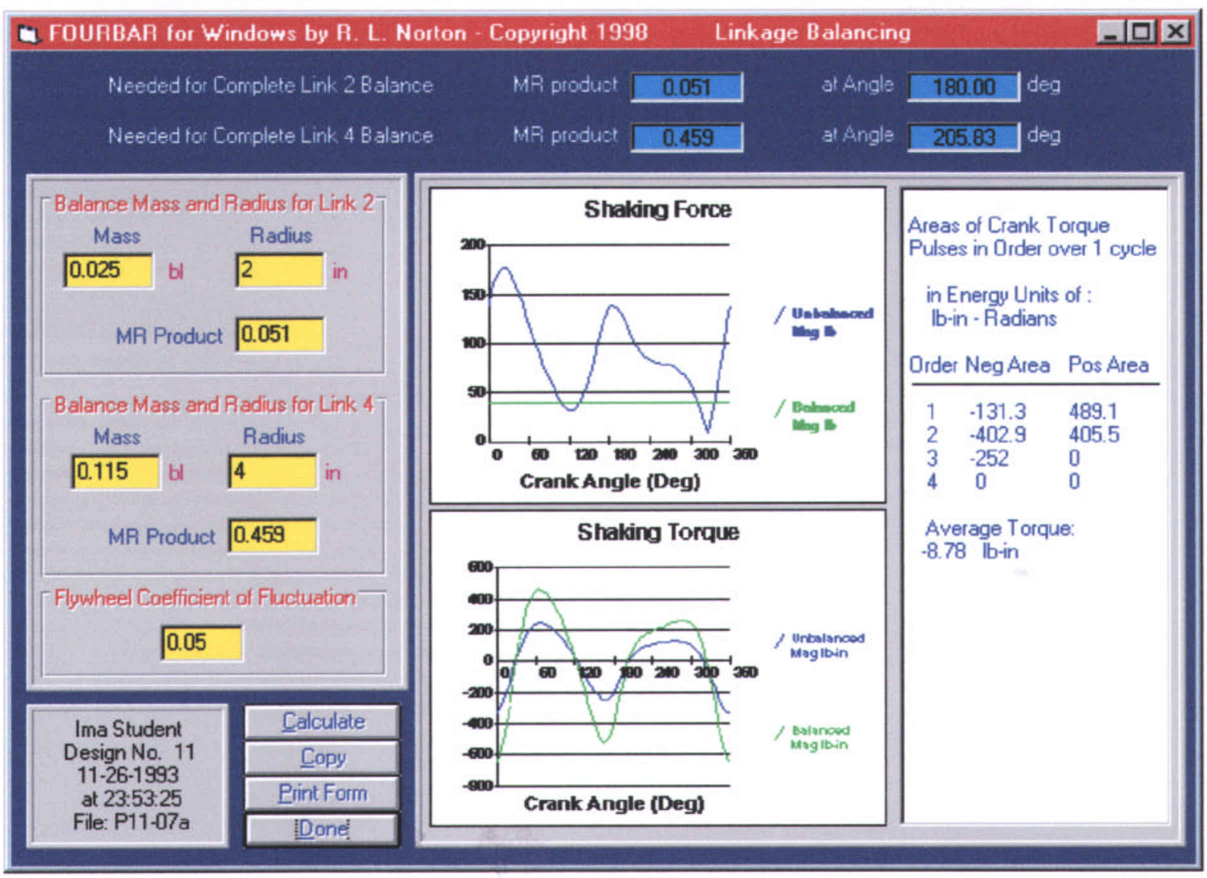
***Solution:*** See Figure P11-2, Table P11-3 and Mathcad file P1107a.

1. Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed. The input torque vs crank angle graph is shown on the next page. The linkage balance screen is shown just below the input torque graph. A table of torque pulse areas is shown on the balancing screen. Use the data from it and equation 11.22 to determine the required system mass moment of inertia.

$$E := 402.9 \cdot lbf \cdot in \qquad I_s := \frac{E}{k \cdot \omega_{avg}^2} \qquad I_s = 20.1\ blob \cdot in^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.


FOURBAR for Windows by R. L. Norton - Copyright 1998
Linkage Data
Link 1 (Ground) 15.000 in
Link 2 (Crank) 4.000 in
Link 3 (Coupler) 12.000 in
Link 4 (Rocker) 8.000 in
Dist. from I 2,3 to Coupler Pt 0.000 in
Angle from Link 3 to Coupler Pt 0 deg
Select Another
Copy
Print Form
Done
Selected Linkage Parameters
Referenced to Non-Rotating Global Coordinate System
Torque 12 Mag lb-in
Crank Angle (Deg)
Grid Horiz Vertic
Style Lines Fills
Initial Conditions
Circuit Open
Start Theta 0 deg
End Theta 360 deg
Delta Theta 5 deg
Omega2 20 rad/s
Ima Student
Design No. 11
11-26-1993
at 23:53:25
File: P11-07a


FOURBAR for Windows by R. L. Norton - Copyright 1998 Linkage Balancing
Needed for Complete Link 2 Balance MR product 0.051 at Angle 180.00 deg
Needed for Complete Link 4 Balance MR product 0.459 at Angle 205.83 deg
Balance Mass and Radius for Link 2
Mass 0.025 bl Radius 2 in
MR Product 0.051
Balance Mass and Radius for Link 4
Mass 0.115 bl Radius 4 in
MR Product 0.459
Flywheel Coefficient of Fluctuation 0.05
Shaking Force
Unbalanced Mag lb
Balanced Mag lb
Crank Angle (Deg)
Shaking Torque
Unbalanced Mag lb-in
Balanced Mag lb-in
Crank Angle (Deg)
Areas of Crank Torque Pulses in Order over 1 cycle
in Energy Units of : lb-in - Radians
Order Neg Area Pos Area
1 -131.3 489.1
2 -402.9 405.5
3 -252 0
4 0 0
Average Torque: -8.78 lb-in
Ima Student
Design No. 11
11-26-1993
at 23:53:25
File: P11-07a
Calculate
Copy
Print Form
Done

## PROBLEM 11-8

***Statement:*** Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank $O_2A$ has $\omega$ = 40 rad/sec and $\alpha$ = -20 rad/sec$^2$. There is a horizontal force at $P$ of $F$ = 50 lb. Find all pin forces and the torque needed to drive the crank at this instant.

***Units:*** $blob := lbf \cdot sec^2 \cdot in^{-1}$

***Given:*** Link lengths:

Link 2 ($O_2$ to $A$) $a := 5.00 \cdot in$ Link 3 ($A$ to $B$) $b := 4.40 \cdot in$

Link 4 ($B$ to $O_4$) $c := 5.00 \cdot in$ Link 1 ($O_2$ to $O_4$) $d := 9.50 \cdot in$

Coupler point: $R_{pa} := 8.90 \cdot in$ $\delta_3 := 56 \cdot deg$ $F := 50 \cdot lbf$ $T_4 := 0 \cdot lbf \cdot in$

Crank angle and motion: $\theta_2 := 50 \cdot deg$ $\omega_2 := 40 \cdot rad \cdot sec^{-1}$ $\alpha_2 := -20 \cdot rad \cdot sec^{-2}$

Link cross-section dims:

$w_2 := 1.00 \cdot in$ $t_2 := 0.50 \cdot in$ $t_3 := 0.75 \cdot in$ $w_4 := 1.00 \cdot in$ $t_4 := 0.50 \cdot in$

Material specific weight: steel $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$ aluminum $\gamma_a := 0.1 \cdot lbf \cdot in^{-3}$

***Solution:*** See Figure P11-3 and Mathcad file P1108.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 10.105 \cdot deg$ $\omega_3 := -41.552 \cdot rad \cdot sec^{-1}$ $\alpha_3 := -335.762 \cdot rad \cdot sec^{-2}$

$\theta_4 := 113.008 \cdot deg$ $\omega_4 := 26.320 \cdot rad \cdot sec^{-1}$ $\alpha_4 := 2963.667 \cdot rad \cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 2.500\, in$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 2.500\, in$

Link 3: $$R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \qquad R_{CG3x'} = 3.126\, in$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \qquad R_{CG3y'} = 2.459\, in$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \qquad R_{CG3} = 3.977\, in$$

At an angle with respect to the local $x'$ axis of

$$\delta_{33} := atan2(R_{CG3x'}, R_{CG3y'}) \qquad \delta_{33} = 38.199\, deg$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \qquad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

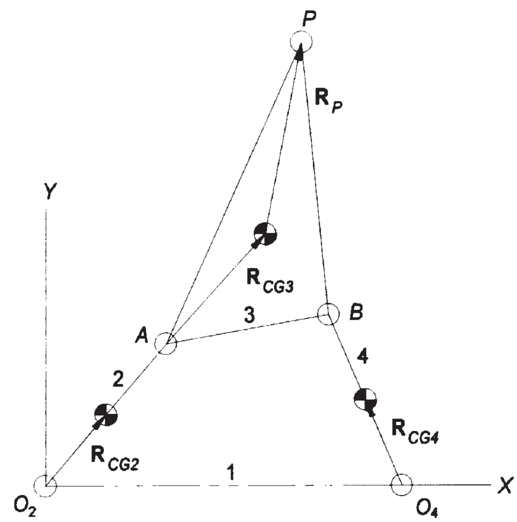
$$m_2 = 1.943 \times 10^{-3}\, blob \qquad m_3 = 3.153 \times 10^{-3}\, blob \qquad m_4 = 1.943 \times 10^{-3}\, blob$$

$$I_{G2} := \frac{m_2}{12}\cdot\left(w_2^{\,2} + a^2\right) \qquad I_{G2} = 4.209\times10^{-3}\,blob\cdot in^2$$

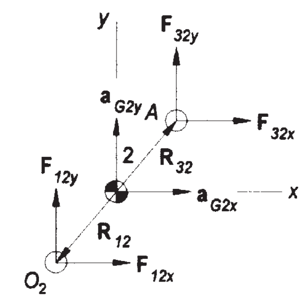
$$I_{G3} := \frac{m_3}{6}\cdot\left[b^2 + \left(R_{pa}\cdot sin(\delta_3)\right)^2\right] \qquad I_{G3} = 0.039\,blob\cdot in^2$$

$$I_{G4} := \frac{m_4}{12}\cdot\left(w_4^{\,2} + c^2\right) \qquad I_{G4} = 4.209\times10^{-3}\,blob\cdot in^2$$

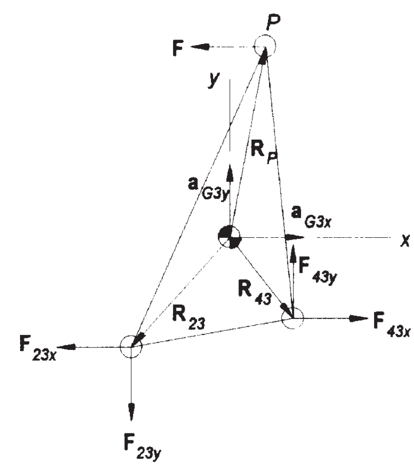
4. Set up an LNCS *xy* coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



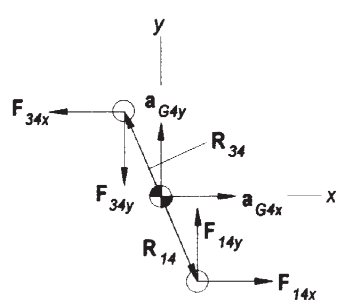

(a) The complete linkage with GCS




(b) FBD of Link 2




(c) FBD of Link 3




(d) FBD of Link 4

5. Calculate the *x* and *y* components of the position vectors.

$$R_{12x} := R_{CG2}\cdot cos(\theta_2 + 180\cdot deg) \qquad R_{12x} = -1.607\,in$$

$$R_{12y} := R_{CG2}\cdot sin(\theta_2 + 180\cdot deg) \qquad R_{12y} = -1.915\,in$$

$$R_{32x} := R_{CG2}\cdot cos(\theta_2) \qquad R_{32x} = 1.607\,in$$

$$R_{32y} := R_{CG2}\cdot sin(\theta_2) \qquad R_{32y} = 1.915\,in$$

$$R_{23x} := R_{CG3}\cdot cos(\delta_{33} + \theta_3 + 180\cdot deg) \qquad R_{23x} = -2.646\,in$$

$R_{23y} := R_{CG3} \cdot sin(\delta_{33} + \theta_3 + 180 \cdot deg)$ $\quad R_{23y} = -2.970\ in$

$R_{43x} := b \cdot cos(\theta_3) - R_{CG3} \cdot cos(\theta_3 + \delta_{33})$ $\quad R_{43x} = 1.686\ in$

$R_{43y} := -(R_{CG3} \cdot sin(\theta_3 + \delta_{33}) - b \cdot sin(\theta_3))$ $\quad R_{43y} = -2.198\ in$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $\quad R_{34x} = -0.977\ in$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $\quad R_{34y} = 2.301\ in$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $\quad R_{14x} = 0.977\ in$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $\quad R_{14y} = -2.301\ in$

$R_{Px} := R_{pa} \cdot cos(\theta_3 + \delta_3) - |R_{23x}|$ $\quad R_{Px} = 0.959\ in$

$R_{Py} := R_{pa} \cdot sin(\theta_3 + \delta_3) - |R_{23y}|$ $\quad R_{Py} = 5.167\ in$

6. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $\quad a_{G2x} = -5.104 \times 10^3 \frac{in}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $\quad a_{G2y} = -6.160 \times 10^3 \frac{in}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3 + \delta_{33}) + j \cdot cos(\theta_3 + \delta_{33})) \ldots + -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3 + \delta_{33}) + j \cdot sin(\theta_3 + \delta_{33}))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $\quad a_{G3x} := Re(\mathbf{a_{G3}})$ $\quad a_{G3x} = -8.636 \times 10^3 \frac{in}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $\quad a_{G3y} = -1.221 \times 10^4 \frac{in}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $\quad a_{G4x} = -5.466 \times 10^3 \frac{in}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $\quad a_{G4y} = -6.084 \times 10^3 \frac{in}{sec^2}$

7. Calculate the $x$ and $y$ components of the external force at $P$ in the CGS.

$F_{Px} := -F$ $\quad F_{Py} := 0 \cdot lbf$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ \left(m_3 \cdot a_{G3x} - F_{Px}\right) \cdot lbf^{-1} \\ \left(m_3 \cdot a_{G3y} - F_{Py}\right) \cdot lbf^{-1} \\ \left(I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}\right) \cdot lbf^{-1} \cdot in^{-1} \\ m_4 \cdot a_{G4x} \cdot lbf^{-1} \\ m_4 \cdot a_{G4y} \cdot lbf^{-1} \\ \left(I_{G4} \cdot \alpha_4 - T_4\right) \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

| | | | |
|---|---|---|---|
| $F_{12x} := R_1 \cdot lbf$ | $F_{12x} = -37.1\ lbf$ | $F_{12y} := R_2 \cdot lbf$ | $F_{12y} = 42.4\ lbf$ |
| $F_{32x} := R_3 \cdot lbf$ | $F_{32x} = 27.2\ lbf$ | $F_{32y} := R_4 \cdot lbf$ | $F_{32y} = -54.4\ lbf$ |
| $F_{43x} := R_5 \cdot lbf$ | $F_{43x} = 50.0\ lbf$ | $F_{43y} := R_6 \cdot lbf$ | $F_{43y} = -92.9\ lbf$ |
| $F_{14x} := R_7 \cdot lbf$ | $F_{14x} = 39.3\ lbf$ | $F_{14y} := R_8 \cdot lbf$ | $F_{14y} = -104.7\ lbf$ |
| $T_{12} := R_9 \cdot lbf \cdot in$ | $T_{12} = 279\ lbf \cdot in$ | | |

## PROBLEM 11-9

***Statement:*** Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank $O_2A$ has $\omega$ = 10 rad/sec and $\alpha$ = 5 rad/sec$^2$. There is a vertical force at $P$ of $F$ = 100 N. Find all pin forces and the torque needed to drive the crank at this instant.

***Given:*** Link lengths:

Link 2 ($O_2$ to $A$) $a := 1.00 \cdot m$ Link 3 ($A$ to $B$) $b := 2.06 \cdot m$

Link 4 ($B$ to $O_4$) $c := 2.33 \cdot m$ Link 1 ($O_2$ to $O_4$) $d := 2.22 \cdot m$

Coupler point: $R_{pa} := 3.06 \cdot m$ $\delta_3 := -31 \cdot deg$ $F := 100 \cdot N$ $T_4 := 0 \cdot N \cdot m$

Crank angle and motion: $\theta_2 := 60 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 5 \cdot rad \cdot sec^{-2}$

Link cross-section dims:

$w_2 := 50 \cdot mm$ $t_2 := 25 \cdot mm$ $t_3 := 25 \cdot mm$ $w_4 := 50 \cdot mm$ $t_4 := 25 \cdot mm$

Material specific weight: steel $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$ aluminum $\gamma_a := 0.1 \cdot lbf \cdot in^{-3}$

***Solution:*** See Figure P11-4a and Mathcad file P1109.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 44.732 \cdot deg$ $\omega_3 := -3.669 \cdot rad \cdot sec^{-1}$ $\alpha_3 := 55.752 \cdot rad \cdot sec^{-2}$

$\theta_4 := 96.322 \cdot deg$ $\omega_4 := 1.442 \cdot rad \cdot sec^{-1}$ $\alpha_4 := 67.103 \cdot rad \cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 0.500\, m$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 1.165\, m$

Link 3:

$$R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \qquad R_{CG3x'} = 1.561\, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \qquad R_{CG3y'} = -0.525\, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^{\,2} + R_{CG3y'}^{\,2}} \qquad R_{CG3} = 1.647\, m$$

At an angle with respect to the local $x'$ axis of

$$\delta_{33} := atan2(R_{CG3x'}, R_{CG3y'}) \qquad \delta_{33} = -18.600\, deg$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{1}{2} \cdot b \cdot \left| R_{pa} \cdot \sin(\delta_3) \right| \cdot t_3 \cdot \frac{\gamma_a}{g} \qquad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

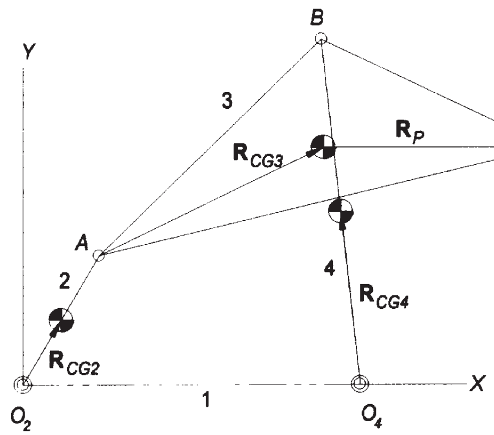
$m_2 = 10.380\, kg$ $m_3 = 112.332\, kg$ $m_4 = 24.185\, kg$

$$I_{G2} := \frac{m_2}{12} \cdot \left( w_2^{\,2} + a^2 \right) \qquad I_{G2} = 0.867\ kg \cdot m^2$$

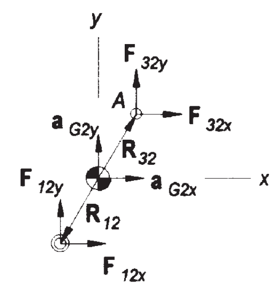
$$I_{G3} := \frac{m_3}{6} \cdot \left[ b^2 + \left( R_{pa} \cdot sin(\delta_3) \right)^2 \right] \qquad I_{G3} = 125.951\ kg \cdot m^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left( w_4^{\,2} + c^2 \right) \qquad I_{G4} = 10.947\ kg \cdot m^2$$

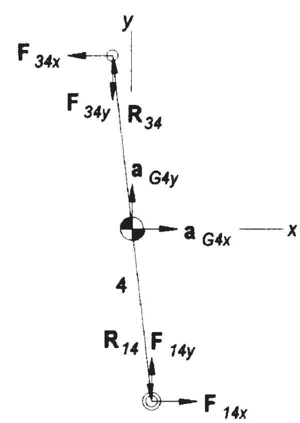
4. Set up an LNCS *xy* coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



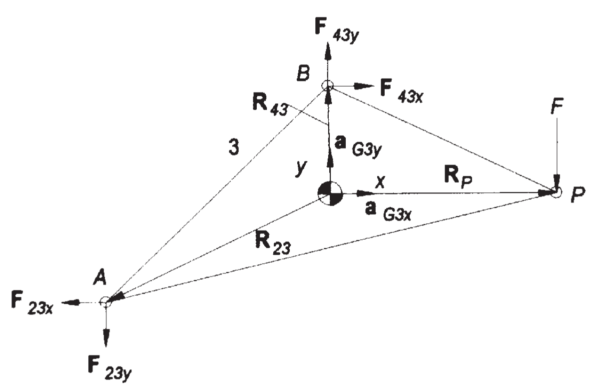

(a) The complete linkage with GCS




(b) FBD of Link 2




(d) FBD of Link 4




(c) FBD of Link 3

5. Calculate the *x* and *y* components of the position vectors.

$$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg) \qquad R_{12x} = -0.250\ m$$

$$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg) \qquad R_{12y} = -0.433\ m$$

$$R_{32x} := R_{CG2} \cdot cos(\theta_2) \qquad R_{32x} = 0.250\ m$$

$$R_{32y} := R_{CG2} \cdot sin(\theta_2) \qquad R_{32y} = 0.433\ m$$

$$R_{23x} := R_{CG3} \cdot cos(\delta_{33} + \theta_3 + 180 \cdot deg) \qquad R_{23x} = -1.479\ m$$

$R_{23y} := R_{CG3} \cdot sin(\delta_{33} + \theta_3 + 180 \cdot deg)$ $R_{23y} = -0.725\, m$

$R_{43x} := b \cdot cos(\theta_3) - R_{CG3} \cdot cos(\theta_3 + \delta_{33})$ $R_{43x} = -0.015\, m$

$R_{43y} := -(R_{CG3} \cdot sin(\theta_3 + \delta_{33}) - b \cdot sin(\theta_3))$ $R_{43y} = 0.724\, m$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $R_{34x} = -0.128\, m$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $R_{34y} = 1.158\, m$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $R_{14x} = 0.128\, m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $R_{14y} = -1.158\, m$

$R_{Px} := R_{pa} \cdot cos(\theta_3 + \delta_3) - |R_{23x}|$ $R_{Px} = 1.494\, m$

$R_{Py} := R_{pa} \cdot sin(\theta_3 + \delta_3) - |R_{23y}|$ $R_{Py} = 9.865 \times 10^{-4}\, m$

6. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $a_{G2x} = -52.165 \dfrac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $a_{G2y} = -85.353 \dfrac{m}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3 + \delta_{33}) + j \cdot cos(\theta_3 + \delta_{33})) \ldots$$
$$+ -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3 + \delta_{33}) + j \cdot sin(\theta_3 + \delta_{33}))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $a_{G3x} := Re(\mathbf{a_{G3}})$ $a_{G3x} = -114.678 \dfrac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $a_{G3y} = -11.429 \dfrac{m}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $a_{G4x} = -77.166 \dfrac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $a_{G4y} = -13.424 \dfrac{m}{sec^2}$

7. Calculate the $x$ and $y$ components of the external force at $P$ in the CGS.

$F_{Px} := 0 \cdot N$ $F_{Py} := -F$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot N \qquad F_{12x} = -13559\,N \qquad F_{12y} := R_2 \cdot N \qquad F_{12y} = -12294\,N$

$F_{32x} := R_3 \cdot N \qquad F_{32x} = 13018\,N \qquad F_{32y} := R_4 \cdot N \qquad F_{32y} = 11408\,N$

$F_{43x} := R_5 \cdot N \qquad F_{43x} = 136\,N \qquad F_{43y} := R_6 \cdot N \qquad F_{43y} = 10224\,N$

$F_{14x} := R_7 \cdot N \qquad F_{14x} = -1731\,N \qquad F_{14y} := R_8 \cdot N \qquad F_{14y} = 9899\,N$

$T_{12} := R_9 \cdot N \cdot m \qquad T_{12} = 5587\,N \cdot m$

## PROBLEM 11-10

**Statement:** Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank $O_2A$ has $\omega = 15$ rad/sec and $\alpha = -10$ rad/sec$^2$. There is a horizontal force at $P$ of $F = 200$ N. Find all pin forces and the torque needed to drive the crank at this instant.

**Given:** Link lengths:

Link 2 ($O_2$ to $A$) $a := 0.72 \cdot m$ Link 3 ($A$ to $B$) $b := 0.68 \cdot m$

Link 4 ($B$ to $O_4$) $c := 0.85 \cdot m$ Link 1 ($O_2$ to $O_4$) $d := 1.82 \cdot m$

Coupler point: $R_{pa} := 0.97 \cdot m$ $\delta_3 := 54 \cdot deg$ $F := 200 \cdot N$ $T_4 := 0 \cdot N \cdot m$

Crank angle and motion: $\theta_2 := 30 \cdot deg$ $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := -10 \cdot rad \cdot sec^{-2}$

Link cross-section dims:

$w_2 := 50 \cdot mm$ $t_2 := 25 \cdot mm$ $t_3 := 25 \cdot mm$ $w_4 := 50 \cdot mm$ $t_4 := 25 \cdot mm$

Material specific weight: steel $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$ aluminum $\gamma_a := 0.1 \cdot lbf \cdot in^{-3}$

**Solution:** See Figure P11-4b and Mathcad file P1110.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 23.290 \cdot deg$ $\omega_3 := -16.412 \cdot rad \cdot sec^{-1}$ $\alpha_3 := -138.628 \cdot rad \cdot sec^{-2}$

$\theta_4 := 132.283 \cdot deg$ $\omega_4 := 1.570 \cdot rad \cdot sec^{-1}$ $\alpha_4 := 427.881 \cdot rad \cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 0.360\, m$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 0.425\, m$

Link 3:

$$R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \qquad R_{CG3x'} = 0.417\, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \qquad R_{CG3y'} = 0.262\, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \qquad R_{CG3} = 0.492\, m$$

At an angle with respect to the local $x'$ axis of

$$\delta_{33} := atan2(R_{CG3x'}, R_{CG3y'}) \qquad \delta_{33} = 32.117\, deg$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{1}{2} \cdot b \cdot \left| R_{pa} \cdot \sin(\delta_3) \right| \cdot t_3 \cdot \frac{\gamma_a}{g} \qquad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

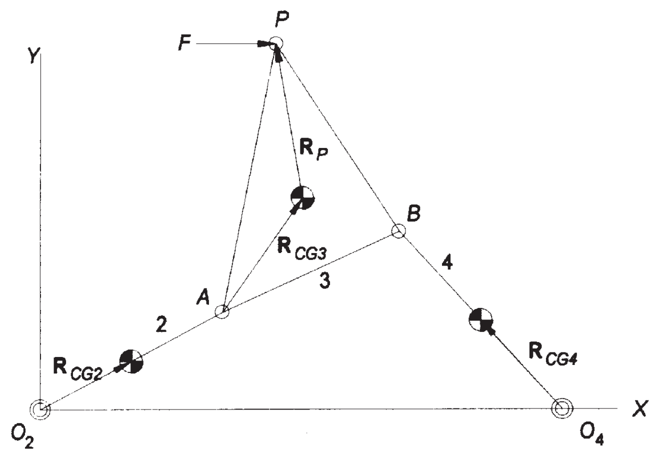
$m_2 = 7.474\, kg$ $m_3 = 18.463\, kg$ $m_4 = 8.823\, kg$

$$I_{G2} := \frac{m_2}{12} \cdot \left( w_2^{\,2} + a^2 \right) \qquad I_{G2} = 0.324\, kg \cdot m^2$$

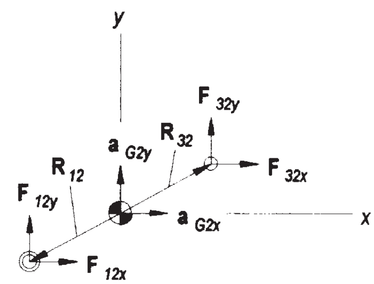
$$I_{G3} := \frac{m_3}{6} \cdot \left[ b^2 + \left( R_{pa} \cdot sin(\delta_3) \right)^2 \right] \qquad I_{G3} = 3.318\, kg \cdot m^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left( w_4^{\,2} + c^2 \right) \qquad I_{G4} = 0.533\, kg \cdot m^2$$

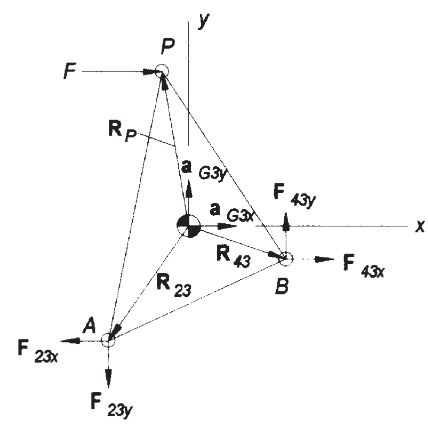
4. Set up an LNCS *xy* coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



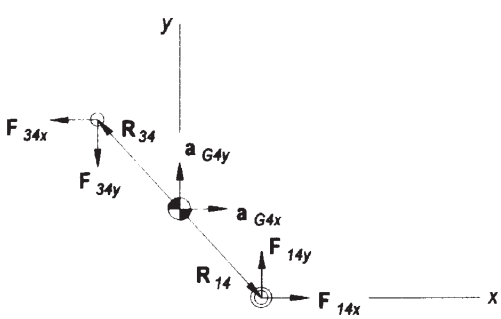

(a) The complete linkage with GCS




(b) FBD of Link 2




(c) FBD of Link 3




(d) FBD of Link 4

5. Calculate the *x* and *y* components of the position vectors.

$$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg) \qquad R_{12x} = -0.312\, m$$

$$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg) \qquad R_{12y} = -0.180\, m$$

$$R_{32x} := R_{CG2} \cdot cos(\theta_2) \qquad R_{32x} = 0.312\, m$$

$$R_{32y} := R_{CG2} \cdot sin(\theta_2) \qquad R_{32y} = 0.180\, m$$

$$R_{23x} := R_{CG3} \cdot cos(\delta_{33} + \theta_3 + 180 \cdot deg) \qquad R_{23x} = -0.279\, m$$

$R_{23y} := R_{CG3} \cdot sin(\delta_{33} + \theta_3 + 180 \cdot deg)$ $\quad R_{23y} = -0.405\,m$

$R_{43x} := b \cdot cos(\theta_3) - R_{CG3} \cdot cos(\theta_3 + \delta_{33})$ $\quad R_{43x} = 0.345\,m$

$R_{43y} := -(R_{CG3} \cdot sin(\theta_3 + \delta_{33}) - b \cdot sin(\theta_3))$ $\quad R_{43y} = -0.136\,m$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $\quad R_{34x} = -0.286\,m$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $\quad R_{34y} = 0.314\,m$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $\quad R_{14x} = 0.286\,m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $\quad R_{14y} = -0.314\,m$

$R_{Px} := R_{pa} \cdot cos(\theta_3 + \delta_3) - |R_{23x}|$ $\quad R_{Px} = -0.066\,m$

$R_{Py} := R_{pa} \cdot sin(\theta_3 + \delta_3) - |R_{23y}|$ $\quad R_{Py} = 0.541\,m$

6. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $\quad a_{G2x} = -138.496 \dfrac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $\quad a_{G2y} = -84.118 \dfrac{m}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3 + \delta_{33}) + j \cdot cos(\theta_3 + \delta_{33})) \ldots$$
$$+ -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3 + \delta_{33}) + j \cdot sin(\theta_3 + \delta_{33}))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $\quad a_{G3x} := Re(\mathbf{a_{G3}})$ $\quad a_{G3x} = -155.788 \dfrac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $\quad a_{G3y} = -235.056 \dfrac{m}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $\quad a_{G4x} = -133.128 \dfrac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $\quad a_{G4y} = -123.897 \dfrac{m}{sec^2}$

7. Calculate the $x$ and $y$ components of the external force at $P$ in the CGS.

$F_{Px} := F$ $\quad F_{Py} := 0 \cdot N$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot N$ $\quad F_{12x} = -5484\,N$ $\qquad F_{12y} := R_2 \cdot N$ $\quad F_{12y} = -5050\,N$

$F_{32x} := R_3 \cdot N$ $\quad F_{32x} = 4449\,N$ $\qquad F_{32y} := R_4 \cdot N$ $\quad F_{32y} = 4422\,N$

$F_{43x} := R_5 \cdot N$ $\quad F_{43x} = 1373\,N$ $\qquad F_{43y} := R_6 \cdot N$ $\quad F_{43y} = 81.8\,N$

$F_{14x} := R_7 \cdot N$ $\quad F_{14x} = 198\,N$ $\qquad F_{14y} := R_8 \cdot N$ $\quad F_{14y} = -1011\,N$

$T_{12} := R_9 \cdot N \cdot m$ $\quad T_{12} = -1168\,N \cdot m$

## PROBLEM 11-11

***Statement:*** Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank $O_2A$ has $\omega$ = 15 rad/sec and $\alpha$ = -10 rad/sec². There is a vertical force at $P$ of $F$ = 500 N. Find all pin forces and the torque needed to drive the crank at this instant.

***Given:*** Link lengths:

Link 2 ($O_2$ to $A$) $a := 0.785\cdot m$ Link 3 ($A$ to $B$) $b := 0.356\cdot m$

Link 4 ($B$ to $O_4$) $c := 0.950\cdot m$ Link 1 ($O_2$ to $O_4$) $d := 0.544\cdot m$

Coupler point: $R_{pa} := 1.09\cdot m$ $\delta_3 := 0\cdot deg$ $F := 500\cdot N$ $T_4 := 0\cdot N\cdot m$

Crank angle and motion: $\theta_2 := 96\cdot deg$ $\omega_2 := 15\cdot rad\cdot sec^{-1}$ $\alpha_2 := -10\cdot rad\cdot sec^{-2}$

Link cross-section dims:

$w_2 := 50\cdot mm$ $t_2 := 25\cdot mm$ $w_3 := 50\cdot mm$ $t_3 := 25\cdot mm$ $w_4 := 50\cdot mm$ $t_4 := 25\cdot mm$

Material specific weight: $\gamma_s := 0.3\cdot lbf\cdot in^{-3}$

***Solution:*** See Figure P11-5a and Mathcad file P1111.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 20.261\cdot deg$ $\omega_3 := -6.830\cdot rad\cdot sec^{-1}$ $\alpha_3 := 106.282\cdot rad\cdot sec^{-2}$

$\theta_4 := 107.906\cdot deg$ $\omega_4 := 12.023\cdot rad\cdot sec^{-1}$ $\alpha_4 := 49.372\cdot rad\cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5\cdot a$ $R_{CG2} = 0.393\,m$ $R_{CG4} := 0.5\cdot c$ $R_{CG4} = 0.475\,m$

Link 3: $R_{CG3} := 0.5\cdot R_{pa}$ $R_{CG3} = 0.545\,m$

3. Determine the mass and moment of inertia of each link.

$m_2 := w_2\cdot t_2\cdot a\cdot \dfrac{\gamma_s}{g}$ $m_3 := w_3\cdot t_3\cdot R_{pa}\cdot \dfrac{\gamma_s}{g}$ $m_4 := w_4\cdot t_4\cdot c\cdot \dfrac{\gamma_s}{g}$

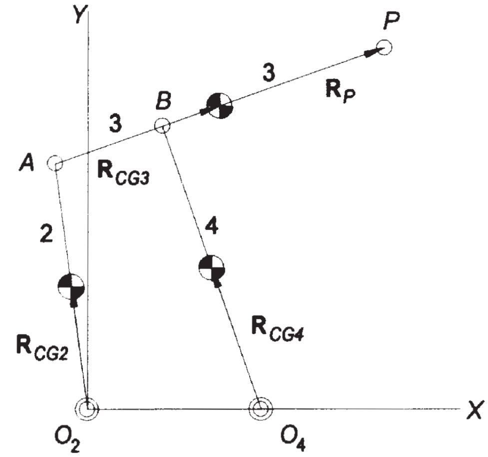
$m_2 = 8.148\,kg$ $m_3 = 11.314\,kg$ $m_4 = 9.861\,kg$

$$I_{G2} := \frac{m_2}{12}\cdot\left(w_2^{\,2} + a^2\right) \qquad I_{G2} = 0.420\,kg\cdot m^2$$

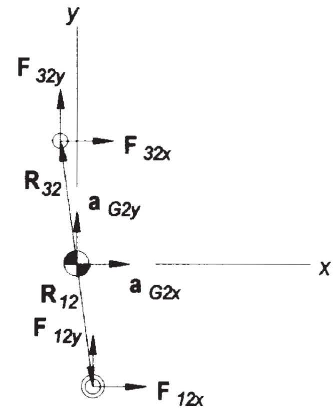
$$I_{G3} := \frac{m_3}{12}\cdot\left(w_3^{\,2} + R_{pa}^{\,2}\right) \qquad I_{G3} = 1.123\,kg\cdot m^2$$

$$I_{G4} := \frac{m_4}{12}\cdot\left(w_4^{\,2} + c^2\right) \qquad I_{G4} = 0.744\,kg\cdot m^2$$

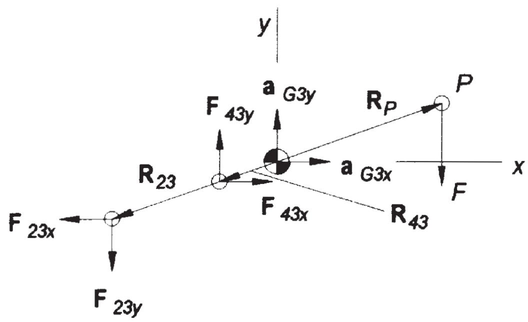
4. Set up an LNCS $xy$ coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



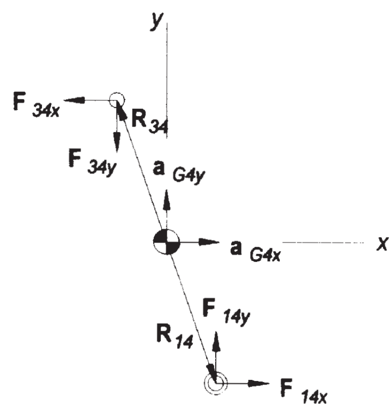

(a) The complete linkage with GCS




(b) FBD of Link 2




(c) FBD of Link 3




(d) FBD of Link 4

5. Calculate the *x* and *y* components of the position vectors.

| | |
|---|---|
| $R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg)$ | $R_{12x} = 0.041\, m$ |
| $R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg)$ | $R_{12y} = -0.390\, m$ |
| $R_{32x} := R_{CG2} \cdot cos(\theta_2)$ | $R_{32x} = -0.041\, m$ |
| $R_{32y} := R_{CG2} \cdot sin(\theta_2)$ | $R_{32y} = 0.390\, m$ |
| $R_{23x} := R_{CG3} \cdot cos(\theta_3 + 180 \cdot deg)$ | $R_{23x} = -0.511\, m$ |
| $R_{23y} := R_{CG3} \cdot sin(\theta_3 + 180 \cdot deg)$ | $R_{23y} = -0.189\, m$ |
| $R_{43x} := (R_{CG3} - b) \cdot cos(\theta_3 + 180 \cdot deg)$ | $R_{43x} = -0.177\, m$ |
| $R_{43y} := (R_{CG3} - b) \cdot sin(\theta_3 + 180 \cdot deg)$ | $R_{43y} = -0.065\, m$ |
| $R_{34x} := R_{CG4} \cdot cos(\theta_4)$ | $R_{34x} = -0.146\, m$ |
| $R_{34y} := R_{CG4} \cdot sin(\theta_4)$ | $R_{34y} = 0.452\, m$ |

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $\qquad R_{14x} = 0.146\,m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $\qquad R_{14y} = -0.452\,m$

$R_{Px} := (R_{pa} - R_{CG3}) \cdot cos(\theta_3)$ $\qquad R_{Px} = 0.511\,m$

$R_{Py} := (R_{pa} - R_{CG3}) \cdot sin(\theta_3)$ $\qquad R_{Py} = 0.189\,m$

6. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $\qquad a_{G2x} = 22.366 \frac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $\qquad a_{G2y} = -175.247 \frac{m}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3) + j \cdot cos(\theta_3)) \ldots$$
$$+ -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3) + j \cdot sin(\theta_3))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $\qquad a_{G3x} := Re(\mathbf{a_{G3}})$ $\qquad a_{G3x} = -17.640 \frac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $\qquad a_{G3y} = -129.301 \frac{m}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $\qquad a_{G4x} = 19.906 \frac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $\qquad a_{G4y} = -137.884 \frac{m}{sec^2}$

7. Calculate the *x* and *y* components of the external force at *P* in the CGS.

$F_{Px} := 0 \cdot N$ $\qquad F_{Py} := -F$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ \left(m_3 \cdot a_{G3x} - F_{Px}\right) \cdot N^{-1} \\ \left(m_3 \cdot a_{G3y} - F_{Py}\right) \cdot N^{-1} \\ \left(I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}\right) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ \left(I_{G4} \cdot \alpha_4 - T_4\right) \cdot N^{-1} \cdot m^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot N \qquad F_{12x} = -231\,N \qquad F_{12y} := R_2 \cdot N \qquad F_{12y} = -2231\,N$

$F_{32x} := R_3 \cdot N \qquad F_{32x} = 413\,N \qquad F_{32y} := R_4 \cdot N \qquad F_{32y} = 803\,N$

$F_{43x} := R_5 \cdot N \qquad F_{43x} = 214\,N \qquad F_{43y} := R_6 \cdot N \qquad F_{43y} = -160\,N$

$F_{14x} := R_7 \cdot N \qquad F_{14x} = 410\,N \qquad F_{14y} := R_8 \cdot N \qquad F_{14y} = -1519\,N$

$T_{12} := R_9 \cdot N \cdot m \qquad T_{12} = 372\,N \cdot m$

## PROBLEM 11-12

***Statement:*** Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank $O_2A$ has $\omega$ = -10 rad/sec and $\alpha$ = 10 rad/sec$^2$. There is a horizontal force at $P$ of $F$ = 300 N. Find all pin forces and the torque needed to drive the crank at this instant.

***Given:*** Link lengths:

Link 2 ($O_2$ to $A$) $a := 0.86 \cdot m$ Link 3 ($A$ to $B$) $b := 1.85 \cdot m$

Link 4 ($B$ to $O_4$) $c := 0.86 \cdot m$ Link 1 ($O_2$ to $O_4$) $d := 2.22 \cdot m$

Coupler point: $R_{pa} := 1.33 \cdot m$ $\delta_3 := 0 \cdot deg$ $F := 300 \cdot N$ $T_4 := 0 \cdot N \cdot m$

Crank angle and motion: $\theta_2 := -36 \cdot deg$ $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Link cross-section dims:

$d_{link} := 50 \cdot mm$

Material specific weight: $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$

***Solution:*** See Figure P11-5b and Mathcad file P1112.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$\theta_3 := 46.028 \cdot deg$ $\omega_3 := 3.285 \cdot rad \cdot sec^{-1}$ $\alpha_3 := -109.287 \cdot rad \cdot sec^{-2}$

$\theta_4 := 106.189 \cdot deg$ $\omega_4 := 11.417 \cdot rad \cdot sec^{-1}$ $\alpha_4 := -43.426 \cdot rad \cdot sec^{-2}$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 0.430\, m$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 0.430\, m$

Link 3: $R_{CG3} := 0.5 \cdot b$ $R_{CG3} = 0.925\, m$

3. Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{link}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{\pi \cdot d_{link}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \qquad m_4 := \frac{\pi \cdot d_{link}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

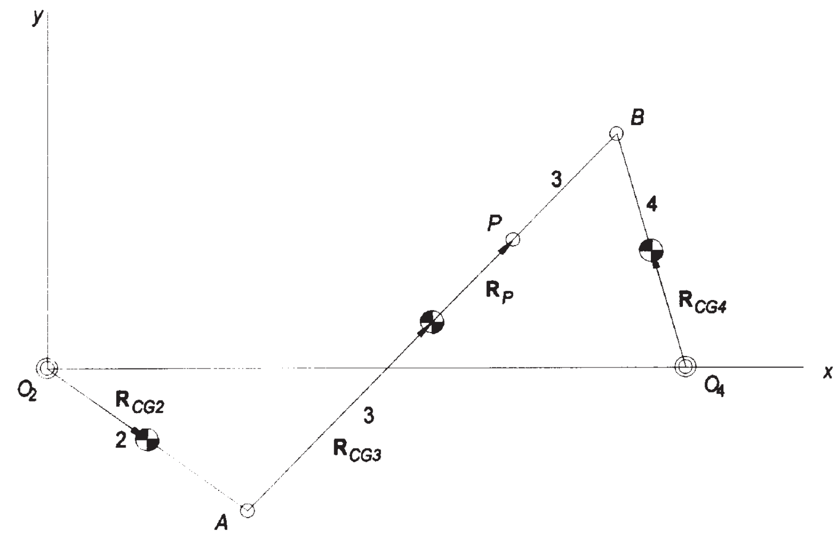
$m_2 = 14.022\, kg$ $m_3 = 30.164\, kg$ $m_4 = 14.022\, kg$

$$I_{G2} := \frac{m_2}{12} \cdot \left( \frac{3}{4} \cdot d_{link}^2 + a^2 \right) \qquad I_{G2} = 0.866\, kg \cdot m^2$$

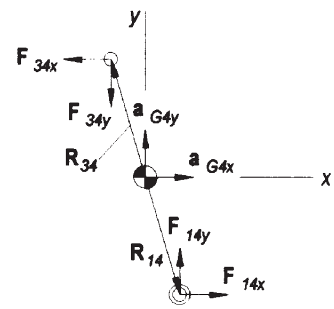
$$I_{G3} := \frac{m_3}{12} \cdot \left( \frac{3}{4} \cdot d_{link}^2 + b^2 \right) \qquad I_{G3} = 8.608\, kg \cdot m^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left( \frac{3}{4} \cdot d_{link}^2 + c^2 \right) \qquad I_{G4} = 0.866\, kg \cdot m^2$$

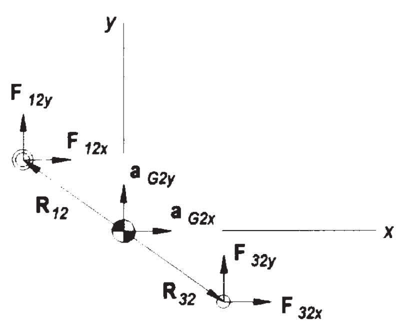
4. Set up an LNCS *xy* coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



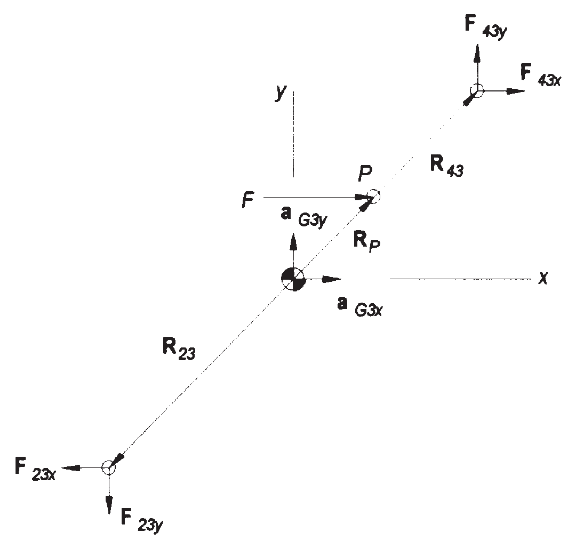

(a) The complete linkage with GCS




(d) FBD of Link 4




(b) FBD of Link 2




(c) FBD of Link 3

5. Calculate the *x* and *y* components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot deg) \qquad R_{12x} = -0.348\, m$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot deg) \qquad R_{12y} = 0.253\, m$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2) \qquad R_{32x} = 0.348\, m$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2) \qquad R_{32y} = -0.253\, m$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + 180 \cdot deg) \qquad R_{23x} = -0.642\, m$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + 180 \cdot deg) \qquad R_{23y} = -0.666\, m$$

$$R_{43x} := (R_{CG3} - b) \cdot \cos(\theta_3 + 180 \cdot deg) \qquad R_{43x} = 0.642\, m$$

$$R_{43y} := (R_{CG3} - b) \cdot \sin(\theta_3 + 180 \cdot deg) \qquad R_{43y} = 0.666\, m$$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $\quad R_{34x} = -0.120\, m$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $\quad R_{34y} = 0.413\, m$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $\quad R_{14x} = 0.120\, m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $\quad R_{14y} = -0.413\, m$

$R_{Px} := (R_{pa} - R_{CG3}) \cdot cos(\theta_3)$ $\quad R_{Px} = 0.281\, m$

$R_{Py} := (R_{pa} - R_{CG3}) \cdot sin(\theta_3)$ $\quad R_{Py} = 0.291\, m$

6. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $\quad a_{G2x} = -67.048 \frac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $\quad a_{G2y} = 54.028 \frac{m}{sec^2}$

$$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3) + j \cdot cos(\theta_3)) \ldots$$
$$+ -R_{CG3} \cdot \omega_3^{\,2} \cdot (cos(\theta_3) + j \cdot sin(\theta_3))$$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $\quad a_{G3x} := Re(\mathbf{a_{G3}})$ $\quad a_{G3x} = 1.302 \frac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $\quad a_{G3y} = -19.864 \frac{m}{sec^2}$

$$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^{\,2} \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $\quad a_{G4x} = 49.187 \frac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $\quad a_{G4y} = -102.448 \frac{m}{sec^2}$

7. Calculate the *x* and *y* components of the external force at *P* in the CGS.

$F_{Px} := F$ $\quad F_{Py} := 0 \cdot N$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ \left(m_3 \cdot a_{G3x} - F_{Px}\right) \cdot N^{-1} \\ \left(m_3 \cdot a_{G3y} - F_{Py}\right) \cdot N^{-1} \\ \left(I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}\right) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ \left(I_{G4} \cdot \alpha_4 - T_4\right) \cdot N^{-1} \cdot m^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot N$ $\qquad F_{12x} = -1246\,N$ $\qquad F_{12y} := R_2 \cdot N$ $\qquad F_{12y} = 940\,N$

$F_{32x} := R_3 \cdot N$ $\qquad F_{32x} = 306\,N$ $\qquad F_{32y} := R_4 \cdot N$ $\qquad F_{32y} = -183\,N$

$F_{43x} := R_5 \cdot N$ $\qquad F_{43x} = 45.1\,N$ $\qquad F_{43y} := R_6 \cdot N$ $\qquad F_{43y} = -782\,N$

$F_{14x} := R_7 \cdot N$ $\qquad F_{14x} = 735\,N$ $\qquad F_{14y} := R_8 \cdot N$ $\qquad F_{14y} = -2219\,N$

$T_{12} := R_9 \cdot N \cdot m$ $\qquad T_{12} = 7.14\,N \cdot m$

## PROBLEM 11-13

***Statement:*** Figure P11-6 shows a water jet loom laybar drive mechanism driven by a pair of Grashof crank rocker fourbar linkages. The crank rotates at 500 rpm. The laybar is carried between the coupler-rocker joints of the two linkages at their respective instant centers $I_{3,4}$. The combined weight of the reed and laybar is 29 lb. A 540-lb beat-up force from the cloth is applied to the reed as shown. The steel links have a 2 x 1 in uniform cross-section. Find the forces on the pins for one revolution of the crank. Find the torque-time function required to drive the system.

***Units:*** $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

***Given:*** Link lengths:

Link 2 ($A$ to $B$) $\quad a := 2.00 \cdot in$ $\quad$ Link 3 ($B$ to $C$) $\quad b := 8.375 \cdot in$

Link 4 ($C$ to $D$) $\quad c := 7.187 \cdot in$ $\quad$ Link 1 ($A$ to $D$) $\quad d := 9.625 \cdot in$

Coupler point: $\quad R_{pa} := 0.0 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Crank angle and motion: $\quad \omega_2 := 500 \cdot rpm$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Link cross-section dims: $\quad w := 2.00 \cdot in$ $\quad t := 1.00 \cdot in$

Material specific weight: $\quad$ steel $\quad \gamma := 0.3 \cdot lbf \cdot in^{-3}$

***Solution:*** See Figure P11-6 and Mathcad file P1113.

1. Determine the distance to the CG in the LRCS on each of the three moving links. All three are located on the $x'$ axis in the LRCS and their angle is zero deg.

$R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 1.000\ in \qquad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 4.188\ in$

$R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 3.594\ in$

2. Determine the mass and moment of inertia of each link.

$m_2 := w \cdot t \cdot a \cdot \frac{\gamma}{g} \qquad m_3 := w \cdot t \cdot b \cdot \frac{\gamma}{g} \qquad m_4 := w \cdot t \cdot c \cdot \frac{\gamma}{g}$

$m_2 = 3.108 \times 10^{-3}\ blob \qquad m_3 = 0.013\ blob \qquad m_4 = 0.011\ blob$

$$I_{G2} := \frac{m_2}{12} \cdot \left(w^2 + a^2\right) \qquad I_{G2} = 0.00207\ blob \cdot in^2$$

$$I_{G3} := \frac{m_2}{12} \cdot \left(w^2 + b^2\right) \qquad I_{G3} = 0.01920\ blob \cdot in^2$$

Include one half of the mass of the laybar as a lumped mass at the end of link 4.

$$I_{G4} := \frac{m_4}{12} \cdot \left(w^2 + c^2\right) + R_{CG4}{}^2 \cdot \frac{14.5 \cdot lbf}{g} \qquad I_{G4} = 0.53677\ blob \cdot in^2$$

3. Define any external forces, their locations and directions.

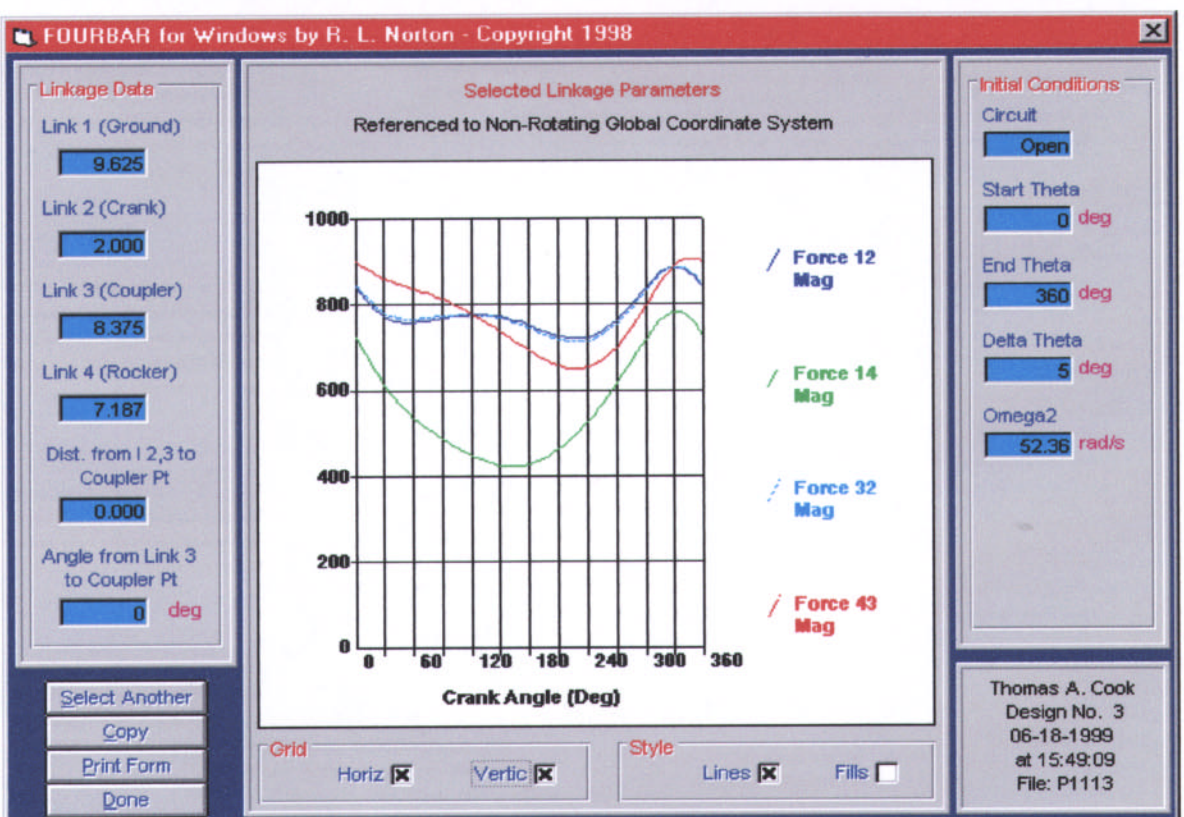
Beat-up force $\quad F := 590 \cdot lbf$ $\quad$ acting on link 4 at a distance $\quad R := c + 3.75 \cdot in$

$R = 10.937\ in$

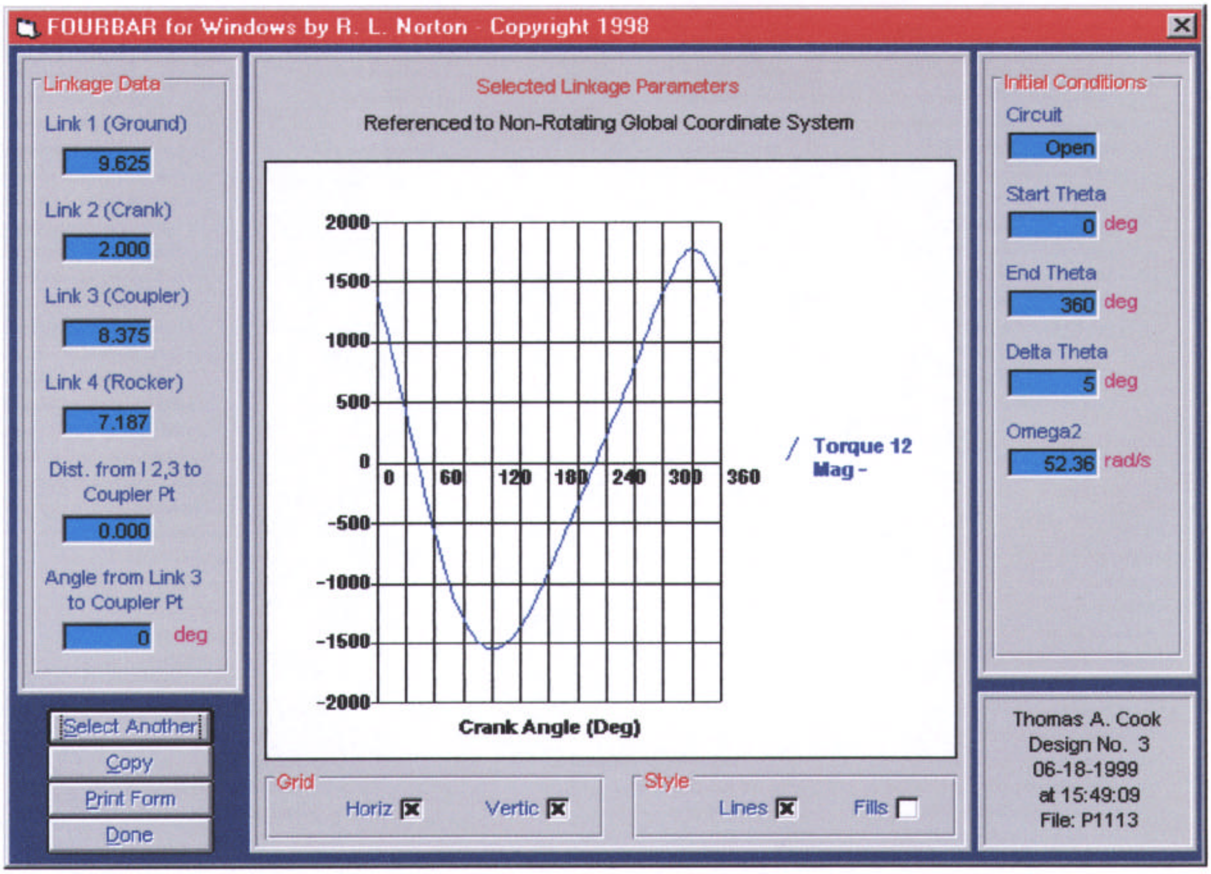
The angle in the CGS is 180 deg.

4. Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..


FOURBAR for Windows by R. L. Norton - Copyright 1998 Dynamics Screen
Link 2 3 4 Units
Mass Properties
Mass of Link 0.00311 0.013 0.011
Mass Moment of Inertia about CG 0.00207 0.0192 0.537
CG Locations
Distance to CG from Pivot 1.000 4.188 3.594
Angle to CG in LRCS* 0.00 0.0 0.00 degrees
External Forces
Force Magnitude 0 590
Angle of Force in GCS* 0 180 degrees
External Force Locations
Distance to Force Point from CG 0 7.343
Angle to Force Point in LRCS* 0 0 degrees
External Torques
Torque on Link 0 0
Friction
Friction Force on Link 0
* LCRS = Local Rotating Coordinate System. GCS = Global Coordinate System
Solve
Show Matrix
Copy
Print Form
Done
Cancel


FOURBAR for Windows by R. L. Norton - Copyright 1998
Linkage Data
Link 1 (Ground) 9.625
Link 2 (Crank) 2.000
Link 3 (Coupler) 8.375
Link 4 (Rocker) 7.187
Dist. from I 2,3 to Coupler Pt 0.000
Angle from Link 3 to Coupler Pt 0 deg
Selected Linkage Parameters
Referenced to Non-Rotating Global Coordinate System
1000
800
600
400
200
0
0 60 120 180 240 300 360
Crank Angle (Deg)
Force 12 Mag
Force 14 Mag
Force 32 Mag
Force 43 Mag
Initial Conditions
Circuit Open
Start Theta 0 deg
End Theta 360 deg
Delta Theta 5 deg
Omega2 52.36 rad/s
Select Another
Copy
Print Form
Done
Grid Horiz Vertic
Style Lines Fills
Thomas A. Cook
Design No. 3
06-18-1999
at 15:49:09
File: P1113

5. The input torque is plotted below.



## PROBLEM 11-14

***Statement:*** Figure P11-7 shows a crimping tool. Find the force $F_{hand}$ needed to generate a 2000 lb $F_{crimp}$. Find the pin forces. What is the linkage's joint force transmission index (JFI) in this position?

***Given:*** Link lengths:

Link 2 (*A* to *B*) $a := 0.80 \cdot in$ Link 3 (*B* to *C*) $b := 1.23 \cdot in$

Link 2 (*C* to *D*) $c := 1.55 \cdot in$ Link 2 (*A* to *D*) $d := 2.40 \cdot in$

Link 2 angle: $\theta_2 := 49 \cdot deg$

Distance to crimp force from pivot *D*: $R_{P4} := 1.00 \cdot in$ $\delta_4 := 0 \cdot deg$

Crimp force: $F_{P4} := 2000 \cdot lbf$ (perpendicular to link 4)

Distance to hand force from pivot *A*: $R_{P2} := 4.26 \cdot in$ $\delta_2 := 0 \cdot deg$

***Solution:*** See Figure P11-7 and Mathcad file P1114.

1. Enter the above data into program FOURBAR to determine link 3 and 4 angles and calculate the angle that the crimping force makes with respect to the fourbar coordinate frame..

$\theta_3 := 34.039 \cdot deg$ $\theta_4 := 123.518 \cdot deg$

$\theta_{FP4} := \theta_4 + 90 \cdot deg$ $\theta_{FP4} = 213.518\, deg$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $R_{CG2} = 0.400\, in$ $R_{CG4} := 0.5 \cdot c$ $R_{CG4} = 0.775\, in$

Link 3: $R_{CG3} := 0.5 \cdot b$ $R_{CG3} = 0.615\, in$

3. Calculate the *x* and *y* components of the position vectors.

$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg)$ $R_{12x} = -0.262\, in$

$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg)$ $R_{12y} = -0.302\, in$

$R_{32x} := R_{CG2} \cdot cos(\theta_2)$ $R_{32x} = 0.262\, in$

$R_{32y} := R_{CG2} \cdot sin(\theta_2)$ $R_{32y} = 0.302\, in$

$R_{23x} := R_{CG3} \cdot cos(\theta_3 + 180 \cdot deg)$ $R_{23x} = -0.510\, in$

$R_{23y} := R_{CG3} \cdot sin(\theta_3 + 180 \cdot deg)$ $R_{23y} = -0.344\, in$

$R_{43x} := (b - R_{CG3}) \cdot cos(\theta_3)$ $R_{43x} = 0.510\, in$

$R_{43y} := (b - R_{CG3}) \cdot sin(\theta_3)$ $R_{43y} = 0.344\, in$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $R_{34x} = -0.428\, in$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $R_{34y} = 0.646\, in$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $R_{14x} = 0.428\, in$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $R_{14y} = -0.646\, in$

$R_{P2y} := (R_{P2} + R_{CG2}) \cdot sin(\theta_2 + 180 \cdot deg)$ $R_{P2y} = -3.517\, in$

$R_{P2x} := (R_{P2} + R_{CG2})\cdot cos(\theta_2 + 180\cdot deg)$ $\qquad R_{P2x} = -3.057\, in$

$R_{P4x} := (R_{P4} - R_{CG4})\cdot cos(\theta_4)$ $\qquad R_{P4x} = -0.124\, in$

$R_{P4y} := (R_{P4} - R_{CG4})\cdot sin(\theta_4)$ $\qquad R_{P4y} = 0.188\, in$

4. Calculate the *x* and *y* components of the external crimp force at *P on link 4* in the CGS.

$F_{P4x} := F_{P4}\cdot cos(\theta_{FP4})$ $\qquad F_{P4x} = -1667.4\, lbf$

$F_{P4y} := F_{P4}\cdot sin(\theta_{FP4})$ $\qquad F_{P4y} = -1104.4\, lbf$

5. Substitute these given and calculated values into the matrix equation 11.9 modified to omit all mass and inertia terms. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & \frac{R_{P2x}}{in} & \frac{-R_{P2y}}{in} \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & tan\left(\theta_2 + \frac{\pi}{2}\right) & -1 \end{pmatrix}$$

$$F := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -F_{P4x}\cdot lbf^{-1} \\ -F_{P4y}\cdot lbf^{-1} \\ (-R_{P4x}\cdot F_{P4y} + R_{P4y}\cdot F_{P4x})\cdot lbf^{-1}\cdot in^{-1} \\ 0 \end{bmatrix} \qquad R := C^{-1}\cdot F$$

$F_{12x} := R_1\cdot lbf$ $\quad F_{12x} = 1029\, lbf$ $\qquad F_{12y} := R_2\cdot lbf$ $\quad F_{12y} = 757\, lbf$

$F_{32x} := R_3\cdot lbf$ $\quad F_{32x} = -1069\, lbf$ $\qquad F_{32y} := R_4\cdot lbf$ $\quad F_{32y} = -722\, lbf$

$F_{43x} := R_5 \cdot lbf$ $F_{43x} = -1069\ lbf$ $F_{43y} := R_6 \cdot lbf$ $F_{43y} = -722\ lbf$

$F_{14x} := R_7 \cdot lbf$ $F_{14x} = 598\ lbf$ $F_{14y} := R_8 \cdot lbf$ $F_{14y} = 382\ lbf$

$F_{handx} := R_9 \cdot lbf$ $F_{handx} = 40.1\ lbf$ $F_{handy} := R_{10} \cdot lbf$ $F_{handy} = -34.9\ lbf$

6. Calculate the pin forces.

Pin at *A*: $F_{12} := \sqrt{F_{12x}^2 + F_{12y}^2}$ $F_{12} = 1278\ lbf$

Pin at *B*: $F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$ $F_{32} = 1290\ lbf$

Pin at *C*: $F_{43} := \sqrt{F_{43x}^2 + F_{43y}^2}$ $F_{43} = 1290\ lbf$

Pin at *D*: $F_{14} := \sqrt{F_{14x}^2 + F_{14y}^2}$ $F_{14} = 710\ lbf$

7. Calculate the hand force.

$F_{hand} := \sqrt{F_{handx}^2 + F_{handy}^2}$ $F_{hand} = 53.1\ lbf$

8. Use equation 11.23a to calculate the joint force index.

$JFI := \frac{F_{32}}{F_{P4}}$ $JFI = 0.645$

## PROBLEM 11-15

***Statement:*** Figure P11-8 shows a walking beam conveyor mechanism that operates a slow speed (25 rpm). The boxes being pushed each weigh 50 lb. Determine the pin forces in the linkage and the torque to drive the mechanism through one revolution. Neglect the masses of the links.

***Solution:*** No solution is given for this problem, which is suited to solution using the *Working Model* program.

## PROBLEM 11-16

***Statement:*** Figure P11-9 shows a surface grinder table drive that operates at 120 rpm. The crank radius is 22 mm, the coupler is 157 mm, and its offset is 40 mm. The mass of the table and workpiece combined is 50 kg. Find the pin forces, slider side loads, and driving torque over one revolution.

***Solution:*** No solution is given for this problem, which is suited to solution using the *Working Model* program.

## PROBLEM 11-17

***Statement:*** Figure P11-10 shows a power hacksaw that operates at 50 rpm. The crank is 75 mm, the coupler is 170 mm, and its offset is 45 mm. Find the pin forces, slider side loads, and driving torque over one revolution for a cutting force of 250 N in the forward direction and 50 N during the return stroke.

***Solution:*** No solution is given for this problem, which is suited to solution using the *Working Model* program.

## PROBLEM 11-18

***Statement:*** Figure P11-11 shows a paper roll off-loading station. The paper rolls have a 0.9-m OD, 0.22-m ID, are 3.23 m long, and have a density of 984 kg/m$^3$. The forks that support the roll are 1.2 m long. The motion is slow so inertial loading can be neglected. Find the force required of the air cylinder to rotate the roll through 90 deg.

***Solution:*** No solution is given for this problem, which is suited to solution using the *Working Model* program.

## PROBLEM 11-19

***Statement:*** Derive an expression for the relationship between flywheel mass and the dimensionless parameter radius/thickness (*r/t*) for a solid disk flywheel of moment of inertia *I*. Plot this function for an arbitrary value of *I* and determine the optimum *r/t* ratio to minimize flywheel weight for that *I*.

***Solution:*** No solution is provided to this algebraic exercise.

## PROBLEM 11-20

**Statement:** Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob-in$^2$; both including the counterweight. Its CG is on the link cenerline, 13.2 in from $O_2$. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from $A$. It has a mass moment of inertia of 150 blob-in$^2$. Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob-in$^2$: both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. At the instant shown in the figure the crank angle is at 45 deg with respect to the global coordinate system. Find all pin forces and the torque needed to drive the crank for the position shown. Include gravity forces.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

**Given:** Link lengths:

Link 2 ($O_2$ to $A$): $a := 14.0 \cdot in$ Link 3 ($A$ to $B$): $b := 80.0 \cdot in$

Link 4 ($B$ to $O_4$): $c := 51.3 \cdot in$ Link 1 ($O_2$ to $O_4$): $d := 79.7 \cdot in$

Link 1 offsets: $d_X := -47.5 \cdot in$ $\quad d_Y := 64 \cdot in$

External load data: $F := 2300 \cdot lbf$ $\quad T_4 := 0 \cdot lbf \cdot in$

Crank angle and motion: $\theta_{2xy} := -81.582 \cdot deg$ $\quad \omega_2 := 4 \cdot rpm$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Link CG positions: $R_{CG2} := 13.2 \cdot in$ $\quad R_{CG3} := 40.0 \cdot in$ $\quad R_{CG4} := 79.22 \cdot in$

Link weights: $W_2 := 598.3 \cdot lbf$ $\quad W_3 := 108 \cdot lbf$ $\quad W_4 := 2706 \cdot lbf$

Moments of inertia: $I_{G2} := 11.8 \cdot blob \cdot in^2$ $\quad I_{G3} := 150 \cdot blob \cdot in^2$ $\quad I_{G4} := 10700 \cdot blob \cdot in^2$

Angle between $O_4B$ and $CG_4B$: $\alpha := 143.11 \cdot deg$ $\quad R_{34} := 32.00 \cdot in$

Angle between $O_4B$ and $CG_4O_4$: $\beta := -14.03 \cdot deg$ $\quad R_{14} := 79.22 \cdot in$

Angle between $O_4B$ and $CG_4P$: $\delta := 156.62 \cdot deg$ $\quad R_p := 124.44 \cdot in$

**Solution:** See Figure P11-12 and Mathcad file P1120.

1. Use Problems 6.84c and 7.70b with $\omega_2 = 4$ rpm to determine the position, velocity, and acceleration of links 3 and 4. The angles are calculated in the $xy$ coordinate system and then rotated into the $XY$ coordinate system after calculating accelerations.

   Coordinate rotation angle: $\gamma := atan2(d_X, d_Y)$ $\quad \gamma = 126.582\,deg$

   $\theta_{3xy} := 332.475 \cdot deg$ $\quad \omega_3 := -0.0214 \cdot rad \cdot sec^{-1}$ $\quad \alpha_3 := -0.0250 \cdot rad \cdot sec^{-2}$

   $\theta_{4xy} := 262.482 \cdot deg$ $\quad \omega_4 := 0.0986 \cdot rad \cdot sec^{-1}$ $\quad \alpha_4 := -0.0272 \cdot rad \cdot sec^{-2}$

2. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot \left(-\sin(\theta_{2xy}) + j \cdot \cos(\theta_{2xy})\right) - R_{CG2} \cdot \omega_2^2 \cdot \left(\cos(\theta_{2xy}) + j \cdot \sin(\theta_{2xy})\right)$$

$a_{G2} := |\mathbf{a_{G2}}|$ $\quad a_{G2} = 2.316\,in \cdot sec^{-2}$ $\quad \theta_{aG2} := arg(\mathbf{a_{G2}}) + \gamma$ $\quad \theta_{aG2} = 225.000\,deg$

$a_{G2x} := a_{G2} \cdot \cos(\theta_{aG2})$ $\quad a_{G2y} := a_{G2} \cdot \sin(\theta_{aG2})$

$a_{G2x} = -1.638\,in \cdot sec^{-2}$ $\quad a_{G2y} = -1.638\,in \cdot sec^{-2}$

$$\mathbf{a_A} := a\cdot\alpha_2\cdot\left(-\sin(\theta_{2xy}) + j\cdot\cos(\theta_{2xy})\right) - a\cdot\omega_2^{\,2}\cdot\left(\cos(\theta_{2xy}) + j\cdot\sin(\theta_{2xy})\right)$$

$$\mathbf{a_{CG3A}} := R_{CG3}\cdot\alpha_3\cdot\left(-\sin(\theta_{3xy}) + \mathrm{j}\cdot\cos(\theta_{3xy})\right) - R_{CG3}\cdot\omega_3^{\,2}\cdot\left(\cos(\theta_{3xy}) + \mathrm{j}\cdot\sin(\theta_{3xy})\right)$$

$$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$$

$a_{G3} := \left|\mathbf{a_{G3}}\right|$ $\quad a_{G3} = 1.763\ in\cdot sec^{-2}$ $\quad \theta_{aG3} := arg(\mathbf{a_{G3}}) + \gamma$ $\quad \theta_{aG3} = 244.955\ deg$

$a_{G3x} := a_{G3}\cdot\cos(\theta_{aG3})$ $\quad a_{G3y} := a_{G3}\cdot\sin(\theta_{aG3})$

$a_{G3x} = -0.747\ in\cdot sec^{-2}$ $\quad a_{G3y} = -1.598\ in\cdot sec^{-2}$

$$\mathbf{a_{G4}} := R_{CG4}\cdot\alpha_4\cdot\left(-\sin(\theta_{4xy} + \beta) + j\cdot\cos(\theta_{4xy} + \beta)\right) \ldots$$
$$+ -R_{CG4}\cdot\omega_4^{\,2}\cdot\left(\cos(\theta_{4xy} + \beta) + j\cdot\sin(\theta_{4xy} + \beta)\right)$$

$a_{G4} := \left|\mathbf{a_{G4}}\right|$ $\quad a_{G4} = 2.288\ in\cdot sec^{-2}$ $\quad \theta_{aG4} := arg(\mathbf{a_{G4}}) + \gamma$ $\quad \theta_{aG4} = 265.366\ deg$

$a_{G4x} := a_{G4}\cdot\cos(\theta_{aG4})$ $\quad a_{G4y} := a_{G4}\cdot\sin(\theta_{aG4})$

$a_{G4x} = -0.185\ in\cdot sec^{-2}$ $\quad a_{G4y} = -2.281\ in\cdot sec^{-2}$

Transform the link angles to the global *XY* system:

$\theta_2 := \theta_{2xy} + \gamma$ $\quad \theta_3 := \theta_{3xy} + \gamma - 360\cdot deg$ $\quad \theta_4 := \theta_{4xy} + \gamma - 360\cdot deg$

$\theta_2 = 45.000\ deg$ $\quad \theta_3 = 99.057\ deg$ $\quad \theta_4 = 29.064\ deg$

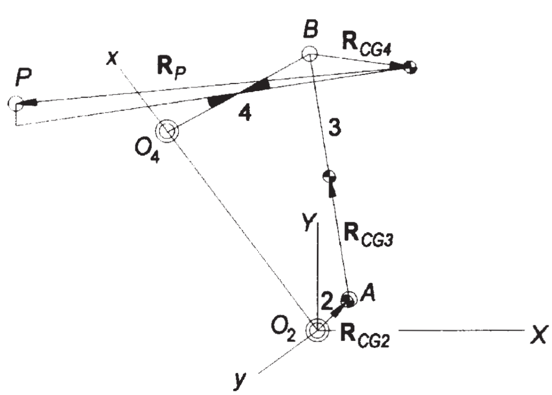
3. Set up an LNCS *xy* coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.

4. Calculate the *x* and *y* components of the position vectors.

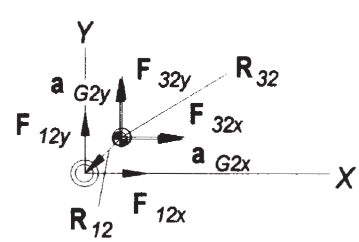
$R_{12x} := R_{CG2}\cdot\cos(\theta_2 + 180\cdot deg)$ $\quad R_{12x} = -9.334\ in$

$R_{12y} := R_{CG2}\cdot\sin(\theta_2 + 180\cdot deg)$ $\quad R_{12y} = -9.334\ in$

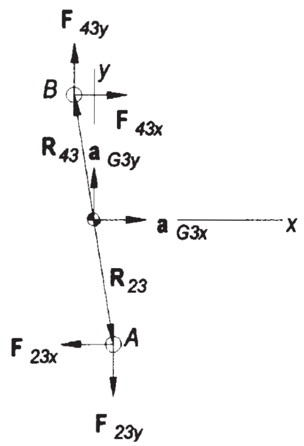
$R_{32x} := (a - R_{CG2})\cdot\cos(\theta_2)$ $\quad R_{32x} = 0.566\ in$



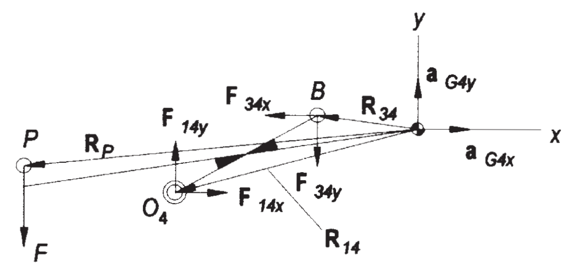
(a) The complete linkage with GCS



(b) FBD of Link 2




(c) FBD of Link 3




(d) FBD of Link 4

$R_{32y} := (a - R_{CG2})\cdot sin(\theta_2)$ $\quad R_{32y} = 0.566\ in$

$R_{23x} := R_{CG3}\cdot cos(\theta_3 + 180\cdot deg)$ $\quad R_{23x} = 6.297\ in$

$R_{23y} := R_{CG3}\cdot sin(\theta_3 + 180\cdot deg)$ $\quad R_{23y} = -39.501\ in$

$R_{43x} := (R_{CG3} - b)\cdot cos(\theta_3 + 180\cdot deg)$ $\quad R_{43x} = -6.297\ in$

$R_{43y} := (R_{CG3} - b)\cdot sin(\theta_3 + 180\cdot deg)$ $\quad R_{43y} = 39.501\ in$

$R_{34x} := R_{34}\cdot cos(\theta_4 + \alpha)$ $\quad R_{34x} = -31.702\ in$

$R_{34y} := R_{34}\cdot sin(\theta_4 + \alpha)$ $\quad R_{34y} = 4.357\ in$

$R_{14x} := R_{14}\cdot cos(\theta_4 + \beta + 180\cdot deg)$ $\quad R_{14x} = -76.508\ in$

$R_{14y} := R_{14}\cdot sin(\theta_4 + \beta + 180\cdot deg)$ $\quad R_{14y} = -20.550\ in$

$R_{Px} := (R_p)\cdot cos(\theta_4 + \delta)$ $\quad R_{Px} = -123.828\ in$

$R_{Py} := (R_p)\cdot sin(\theta_4 + \delta)$ $\quad R_{Py} = -12.326\ in$

7. Calculate the *x* and *y* components of the external force at *P and the weight forces at the CGs.* in the CGS.

$F_{Px} := 0\cdot lbf$ $\quad F_{Py} := -F$

$F_{G2y} := -W_2$ $\quad F_{G3y} := -W_3$ $\quad F_{G4y} := -W_4$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$m_2 := \frac{W_2}{g}$ $\quad m_2 = 1.55\ blob$ $\quad m_3 := \frac{W_3}{g}$ $\quad m_3 = 0.28\ blob$

$m_4 := \frac{W_4}{g}$ $\quad m_4 = 7.01\ blob$

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ \left(m_2 \cdot a_{G2y} - F_{G2y}\right) \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ m_3 \cdot a_{G3x} \cdot lbf^{-1} \\ \left(m_3 \cdot a_{G3y} - F_{G3y}\right) \cdot lbf^{-1} \\ I_{G3} \cdot \alpha_3 \cdot lbf^{-1} \cdot in^{-1} \\ \left(m_4 \cdot a_{G4x} - F_{Px}\right) \cdot lbf^{-1} \\ \left(m_4 \cdot a_{G4y} - F_{Py} - F_{G4y}\right) \cdot lbf^{-1} \\ \left[I_{G4} \cdot \alpha_4 - \left(R_{Px} \cdot F_{Py} - R_{Py} \cdot F_{Px}\right)\right] \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix} \qquad R := C^{-1} \cdot F$$

$F_{12x} := R_1 \cdot lbf$ $\quad F_{12x} = -327\ lbf$ $\qquad F_{12y} := R_2 \cdot lbf$ $\quad F_{12y} = 2682\ lbf$

$F_{32x} := R_3 \cdot lbf$ $\quad F_{32x} = 324\ lbf$ $\qquad F_{32y} := R_4 \cdot lbf$ $\quad F_{32y} = -2086\ lbf$

$F_{43x} := R_5 \cdot lbf$ $\quad F_{43x} = 324\ lbf$ $\qquad F_{43y} := R_6 \cdot lbf$ $\quad F_{43y} = -1978\ lbf$

$F_{14x} := R_7 \cdot lbf$ $\quad F_{14x} = 323\ lbf$ $\qquad F_{14y} := R_8 \cdot lbf$ $\quad F_{14y} = 3012\ lbf$

$T_{12} := R_9 \cdot lbf \cdot in$ $\quad T_{12} = 29442\ lbf \cdot in$

## PROBLEM 11-21

***Statement:*** Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob-in$^2$; both including the counterweight. Its CG is on the link cenerline, 13.2 in from $O_2$. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from $A$. It has a mass moment of inertia of 150 blob-in$^2$. Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob-in$^2$: both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. Find and plot all pin forces and the torque needed to drive the crank for one revolution of the crank. Include gravity forces.

***Units:*** $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

***Given:*** Link lengths:

| | | | |
|---|---|---|---|
| Link 2 ($O_2$ to $A$): | $a := 14.0 \cdot in$ | Link 3 ($A$ to $B$): | $b := 80.0 \cdot in$ |
| Link 4 ($B$ to $O_4$): | $c := 51.3 \cdot in$ | Link 1 ($O_2$ to $O_4$): | $d := 79.7 \cdot in$ |
| Link 1 offsets: | $d_X := -47.5 \cdot in$ | $d_Y := 64 \cdot in$ | |

| | | | |
|---|---|---|---|
| External load data: | $F := 2300 \cdot lbf$ | $T_4 := 0 \cdot lbf \cdot in$ | |
| Crank angle and motion: | $\theta_2 := 45 \cdot deg$ | $\omega_2 := 4 \cdot rpm$ | $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$ |
| Link CG positions: | $R_{CG2} := 13.2 \cdot in$ | $R_{CG3} := 40.0 \cdot in$ | $R_{CG4} := 32.0 \cdot in$ |
| Link weights: | $W_2 := 598.3 \cdot lbf$ | $W_3 := 108 \cdot lbf$ | $W_4 := 2706 \cdot lbf$ |
| Moments of inertia: | $I_{G2} := 11.8 \cdot blob \cdot in^2$ | $I_{G3} := 150 \cdot blob \cdot in^2$ | $I_{G4} := 10700 \cdot blob \cdot in^2$ |

| | | |
|---|---|---|
| Angle between $O_4B$ and $CG_4B$: | $\alpha := 143.11 \cdot deg$ | $R_{34} := 32.00 \cdot in$ |
| Angle between $O_4B$ and $CG_4O_4$: | $\beta := -14.03 \cdot deg$ | $R_{14} := 79.22 \cdot in$ |
| Angle between $O_4B$ and $CG_4P$: | $\delta := 156.62 \cdot deg$ | $R_p := 124.44 \cdot in$ |

***Solution:*** See Figure P11-12 and Mathcad file P1121.

1. No solution is given for this problem, which is suited to solution using the *Working Model* program.

## ✍ PROBLEM 11-22

**Statement:** Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob-in$^2$; both including the counterweight. Its CG is on the link cenerline, 13.2 in from $O_2$. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from $A$. It has a mass moment of inertia of 150 blob-in$^2$. Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob-in$^2$: both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. At the instant shown in the figure the crank angle is at 45 deg with respect to the global coordinate system. Find the torque needed to drive the crank for the position shown using the method of virtual work. Include gravity forces.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

**Given:** Link lengths:

| | | | |
|---|---|---|---|
| Link 2 ($O_2$ to $A$): | $a := 14.0 \cdot in$ | Link 3 ($A$ to $B$): | $b := 80.0 \cdot in$ |
| Link 4 ($B$ to $O_4$): | $c := 51.3 \cdot in$ | Link 1 ($O_2$ to $O_4$): | $d := 79.7 \cdot in$ |
| Link 1 offsets: | $d_X := -47.5 \cdot in$ | $d_Y := 64 \cdot in$ | |

External load data: $F_{P4y} := -2300 \cdot lbf$

Crank angle and motion: $\theta_2 := 45 \cdot deg$ $\quad \omega_2 := 4 \cdot rpm$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Link CG positions: $R_{CG2} := 13.2 \cdot in$ $\quad R_{CG3} := 40.0 \cdot in$ $\quad R_{CG4} := 32.0 \cdot in$

Link weights: $W_2 := 598.3 \cdot lbf$ $\quad W_3 := 108 \cdot lbf$ $\quad W_4 := 2706 \cdot lbf$

Moments of inertia: $I_{G2} := 11.8 \cdot blob \cdot in^2$ $\quad I_{G3} := 150 \cdot blob \cdot in^2$ $\quad I_{G4} := 10700 \cdot blob \cdot in^2$

Angle between $O_4B$ and $CG_4B$: $\alpha := 143.11 \cdot deg$ $\quad R_{34} := 32.00 \cdot in$

Angle between $O_4B$ and $CG_4O_4$: $\beta := -14.03 \cdot deg$ $\quad R_{14} := 79.22 \cdot in$

Angle between $O_4B$ and $CG_4P$: $\delta := 156.62 \cdot deg$ $\quad R_p := 124.44 \cdot in$

**Solution:** See Figure P11-12 and Mathcad file P1122.

1. Use Problems 6.84c and 7.70b with $\omega_2 = 4$ rpm to determine the position, velocity, and acceleration of links 3 and 4. The angles are calculated in the $xy$ coordinate system and then rotated into the $XY$ coordinate system, which is used in the remainder of the problem.

Coordinate rotation angle: $\gamma := atan2(d_X, d_Y)$ $\quad \gamma = 126.582\, deg$

$\theta_3 := 99.057 \cdot deg$ $\quad \omega_3 := -0.0214 \cdot rad \cdot sec^{-1}$ $\quad \alpha_3 := -0.0250 \cdot rad \cdot sec^{-2}$

$\theta_4 := 29.064 \cdot deg$ $\quad \omega_4 := 0.0986 \cdot rad \cdot sec^{-1}$ $\quad \alpha_4 := -0.0272 \cdot rad \cdot sec^{-2}$

Accelerations: $a_{G2} := 2.316 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG2} := 225.00 \cdot deg$

$a_{G3} := 1.764 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG3} := 244.915 \cdot deg$

$a_{G4} := 2.288 \cdot in \cdot sec^{-2}$ $\quad \theta_{AG4} := 265.366 \cdot deg$

$a_{P4} := 1.391 \cdot in \cdot sec^{-2}$ $\quad \theta_{AP4} := 60.394 \cdot deg$

Velocities: $v_{G2} := 5.529 \cdot in \cdot sec^{-1}$ $\quad \theta_{VG2} := 135.0 \cdot deg$

$v_{G3} := 5.406 \cdot in \cdot sec^{-1}$ $\theta_{VG3} := 127.628 \cdot deg$

$v_{G4} := 7.809 \cdot in \cdot sec^{-1}$ $\theta_{VG4} := 105.034 \cdot deg$

$v_{P4} := 4.753 \cdot in \cdot sec^{-1}$ $\theta_{VP4} := 245.646 \cdot deg$

2. Calculate the *x* and *y* components of the velocity vectors.

$v_{G2x} := v_{G2} \cdot cos(\theta_{VG2})$ $v_{G2x} = -3.910\, in \cdot sec^{-1}$

$v_{G2y} := v_{G2} \cdot sin(\theta_{VG2})$ $v_{G2y} = 3.910\, in \cdot sec^{-1}$

$v_{G3x} := v_{G3} \cdot cos(\theta_{VG3})$ $v_{G3x} = -3.301\, in \cdot sec^{-1}$

$v_{G3y} := v_{G3} \cdot sin(\theta_{VG3})$ $v_{G3y} = 4.282\, in \cdot sec^{-1}$

$v_{G4x} := v_{G4} \cdot cos(\theta_{VG4})$ $v_{G4x} = -2.026\, in \cdot sec^{-1}$

$v_{G4y} := v_{G4} \cdot sin(\theta_{VG4})$ $v_{G4y} = 7.542\, in \cdot sec^{-1}$

$v_{P4x} := v_{P4} \cdot cos(\theta_{VP4})$ $v_{P4x} = -1.960\, in \cdot sec^{-1}$

$v_{P4y} := v_{P4} \cdot sin(\theta_{VP4})$ $v_{P4y} = -4.330\, in \cdot sec^{-1}$

3. Calculate the *x* and *y* components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$a_{G2x} := a_{G2} \cdot cos(\theta_{AG2})$ $a_{G2x} = -1.638\, in \cdot sec^{-2}$

$a_{G2y} := a_{G2} \cdot sin(\theta_{AG2})$ $a_{G2y} = -1.638\, in \cdot sec^{-2}$

$a_{G3x} := a_{G3} \cdot cos(\theta_{AG3})$ $a_{G3x} = -0.748\, in \cdot sec^{-2}$

$a_{G3y} := a_{G3} \cdot sin(\theta_{AG3})$ $a_{G3y} = -1.598\, in \cdot sec^{-2}$

$a_{G4x} := a_{G4} \cdot cos(\theta_{AG4})$ $a_{G4x} = -0.185\, in \cdot sec^{-2}$

$a_{G4y} := a_{G4} \cdot sin(\theta_{AG4})$ $a_{G4y} = -2.281\, in \cdot sec^{-2}$

4. Calculate the mass of each link.

$m_2 := \frac{W_2}{g}$ $m_3 := \frac{W_3}{g}$ $m_4 := \frac{W_4}{g}$

5. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \begin{bmatrix} m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \ldots \\ + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \ldots \\ + (W_2 \cdot v_{G2y} + W_3 \cdot v_{G3y} + W_4 \cdot v_{G3y}) - (F_{P4y} \cdot v_{P4y}) \end{bmatrix}$$

$T_{12} = 10219\, lbf \cdot in$

## PROBLEM 11-23

***Statement:*** Use the information in problem 11-20 to find and plot the torque needed to drive the crank for one revolution of the crank using the method of virtual work.

***Solution:*** See Figure P11-12 and Mathcad file P1123.

No solution for this problem is provided. This problem is more suitable for a longer-term project than for a short-term homework problem.

## PROBLEM 11-24

**Statement:** In Figure P11-13, links 2 and 4 each weigh 2 lb and there are 2 of each (another set behind). Their CGs are at their midpoints. Link 3 weighs 10 lb. The moments of inertia of links 2, 3, and 4 are 0.071, 0.430, and 0.077 blob-in$^2$, respectively. Find the torque needed to begin a slow CCW rotation of link 2 from the position shown using the method of virtual work. Include gravity forces.

**Units:** $blob := lbf \cdot sec^2 \cdot in^{-1}$

**Given:** Link lengths:

Link 2 ($O_2$ to $A$) $a := 9.174 \cdot in$ Link 3 ($A$ to $B$) $b := 12.971 \cdot in$

Link 4 ($B$ to $O_4$) $c := 9.573 \cdot in$ Link 3 ($O_2$ to $O_4$) $d := 7.487 \cdot in$

Link angles (LCS): $\theta_2 := -26.0 \cdot deg$ $\theta_3 := 72.239 \cdot deg$ $\theta_4 := 60.491 \cdot deg$

Weight: $W_2 := 4 \cdot lbf$ $W_3 := 10 \cdot lbf$ $W_4 := 4 \cdot lbf$

Mass: $m_2 := \dfrac{W_2}{g}$ $m_3 := \dfrac{W_3}{g}$ $m_4 := \dfrac{W_4}{g}$

$m_2 = 0.010\, blob$ $m_3 = 0.026\, blob$ $m_4 = 0.010\, blob$

Moment of inertia: $I_{G2} := 0.071 \cdot blob \cdot in^2$ $I_{G3} := 0.430 \cdot blob \cdot in^2$ $I_{G4} := 0.077 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 4.587 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 7.086 \cdot in$ $\delta_3 := -23.758 \cdot deg$

$R_{CG4} := 4.786 \cdot in$ $\delta_4 := 0 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $F_{P4} := 0 \cdot lbf$ $\delta_{FP4} := 0 \cdot deg$

$T_3 := 0 \cdot lbf \cdot in$ $T_4 := 0 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$ $a_{G2} := 0.0 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 85.879 \cdot deg$

$\alpha_3 := 0.00206 \cdot rad \cdot sec^{-2}$ $a_{G3} := 0.0174 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 100.159 \cdot deg$

$\alpha_4 := 0.0025 \cdot rad \cdot sec^{-2}$ $a_{G4} := 0.0159 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 123.308 \cdot deg$

Velocities: $\omega_2 := 0.010 \cdot rad \cdot sec^{-1}$ $v_{G2} := 0.0459 \cdot in \cdot sec^{-1}$ $\theta_{VG2} := -41.21 \cdot deg$

$\omega_3 := 0.0347 \cdot rad \cdot sec^{-1}$ $v_{G3} := 0.284 \cdot in \cdot sec^{-1}$ $\theta_{VG3} := 52.251 \cdot deg$

$\omega_4 := 0.0466 \cdot rad \cdot sec^{-1}$ $v_{G4} := 0.0223 \cdot in \cdot sec^{-1}$ $\theta_{VG4} := 82.367 \cdot deg$

**Solution:** See Figure P11-13 and Mathcad file P1124.

1. Calculate the $x$ and $y$ components of the velocity vectors (global coordinate system).

$v_{G2x} := v_{G2} \cdot cos(\theta_{VG2})$ $v_{G2x} = 0.035\, in \cdot sec^{-1}$

$v_{G2y} := v_{G2} \cdot sin(\theta_{VG2})$ $v_{G2y} = -0.030\, in \cdot sec^{-1}$

$v_{G3x} := v_{G3} \cdot cos(\theta_{VG3})$ $v_{G3x} = 0.174\, in \cdot sec^{-1}$

$v_{G3y} := v_{G3} \cdot sin(\theta_{VG3})$ $v_{G3y} = 0.225\, in \cdot sec^{-1}$

$v_{G4x} := v_{G4} \cdot cos(\theta_{VG4})$ $\qquad v_{G4x} = 2.962 \times 10^{-3}\ in \cdot sec^{-1}$

$v_{G4y} := v_{G4} \cdot sin(\theta_{VG4})$ $\qquad v_{G4y} = 0.022\ in \cdot sec^{-1}$

2. Calculate the $x$ and $y$ components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$a_{G2x} := a_{G2} \cdot cos(\theta_{AG2})$ $\qquad a_{G2x} = 0.000\ in \cdot sec^{-2}$

$a_{G2y} := a_{G2} \cdot sin(\theta_{AG2})$ $\qquad a_{G2y} = 0.000\ in \cdot sec^{-2}$

$a_{G3x} := a_{G3} \cdot cos(\theta_{AG3})$ $\qquad a_{G3x} = -3.069 \times 10^{-3}\ in \cdot sec^{-2}$

$a_{G3y} := a_{G3} \cdot sin(\theta_{AG3})$ $\qquad a_{G3y} = 0.017\ in \cdot sec^{-2}$

$a_{G4x} := a_{G4} \cdot cos(\theta_{AG4})$ $\qquad a_{G4x} = -8.731 \times 10^{-3}\ in \cdot sec^{-2}$

$a_{G4y} := a_{G4} \cdot sin(\theta_{AG4})$ $\qquad a_{G4y} = 0.013\ in \cdot sec^{-2}$

3. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \begin{bmatrix} m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \ldots \\ + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \ldots \\ + -T_3 \cdot \omega_3 - T_4 \cdot \omega_4 + (W_2 \cdot v_{G2y} + W_3 \cdot v_{G3y} + W_4 \cdot v_{G4y}) \end{bmatrix}$$

$T_{12} = 221.32\ lbf \cdot in$

**PROBLEMS 11-3 AND 11-4**

| Row | $T_{12}$ |
|---|---|
| a | 99.69 |
| b | -56.59 |
| c | -84.01 |
| d | -133.69 |
| e | 13.13 |
| f | -2612.7 |
| g | -574.90 |

**PROBLEMS 11-5 AND 11-6**

| Row | $T_{12}$ |
|---|---|
| a | 176.37 |
| b | -1266.0 |
| c | 1245.2 |
| d | 266.02 |
| e | 58.56 |
| f | -21137 |
| g | 34.23 |