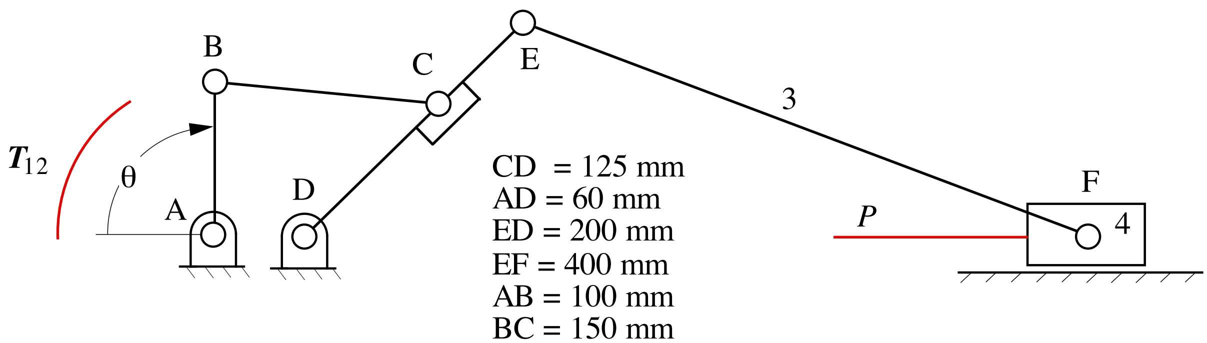
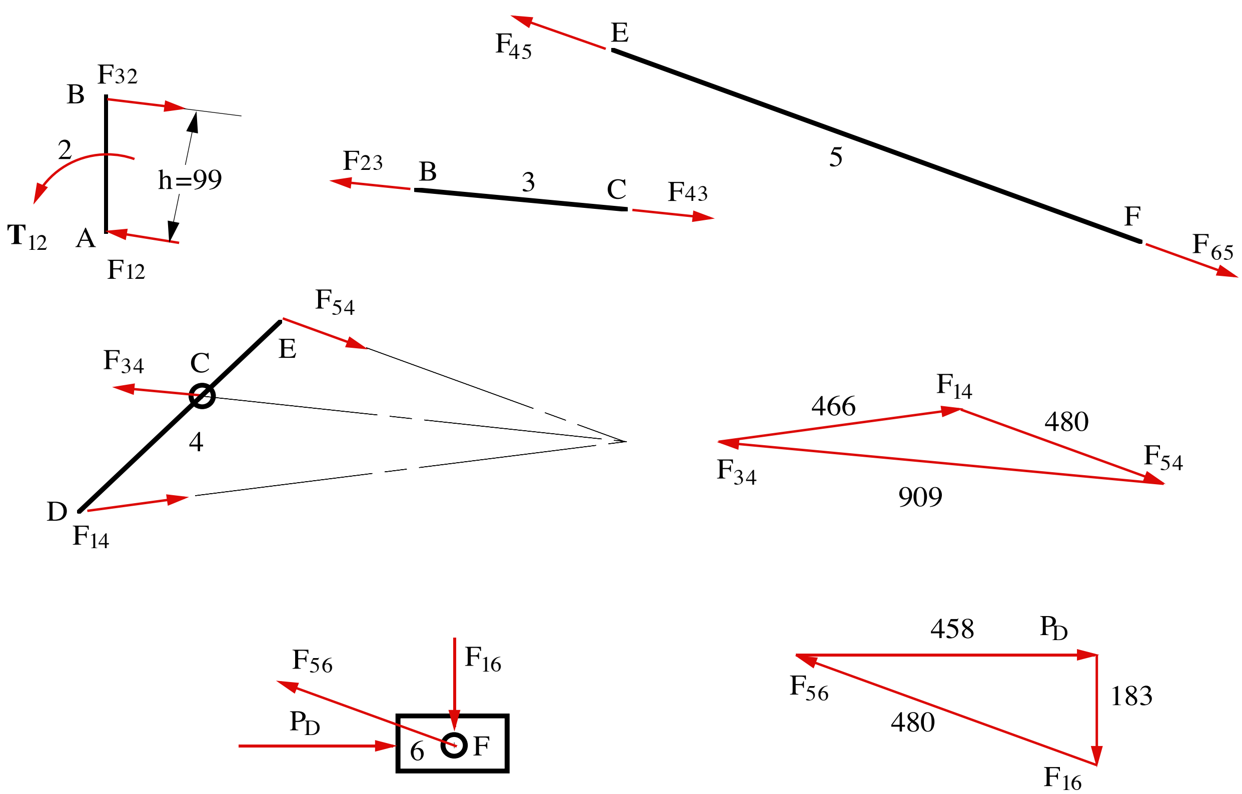
Problem 13.1

In the mechanism shown, sketch a free-body diagram of each link, and determine the force P that is necessary for equilibrium if T_{12}=90 N-m and $\theta = 90^{\circ}$.



Solution:

The freebody diagram of each link is shown below.



First sum moments about point A of the free-body diagram for link 2. From this, we get

$$T_{12} = hF_{32} \Rightarrow F_{32} = \frac{T_{12}}{h} = \frac{90}{0.099} = 909N$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:

$$F_{32} = -F_{23} = F_{43} = -F_{34}$$

and

$$F_{54} = -F_{45} = F_{65} = -F_{56}$$

and

$$F_{12} = -F_{32}$$

By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum F = 0 = F_{14} + F_{34} + F_{54}$$

By summing forces vectorially on link 6, the magnitudes of all the forces can be determined. The force summation equation is

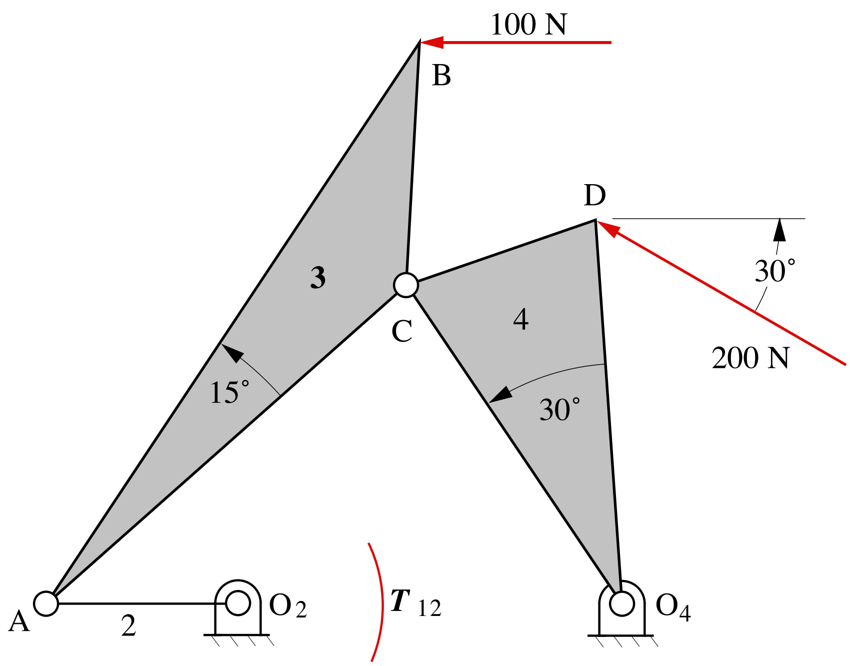
$$\sum F = 0 = F_{16} + P_D + F_{56}$$

Then we get

P_D=458 N, to the right.

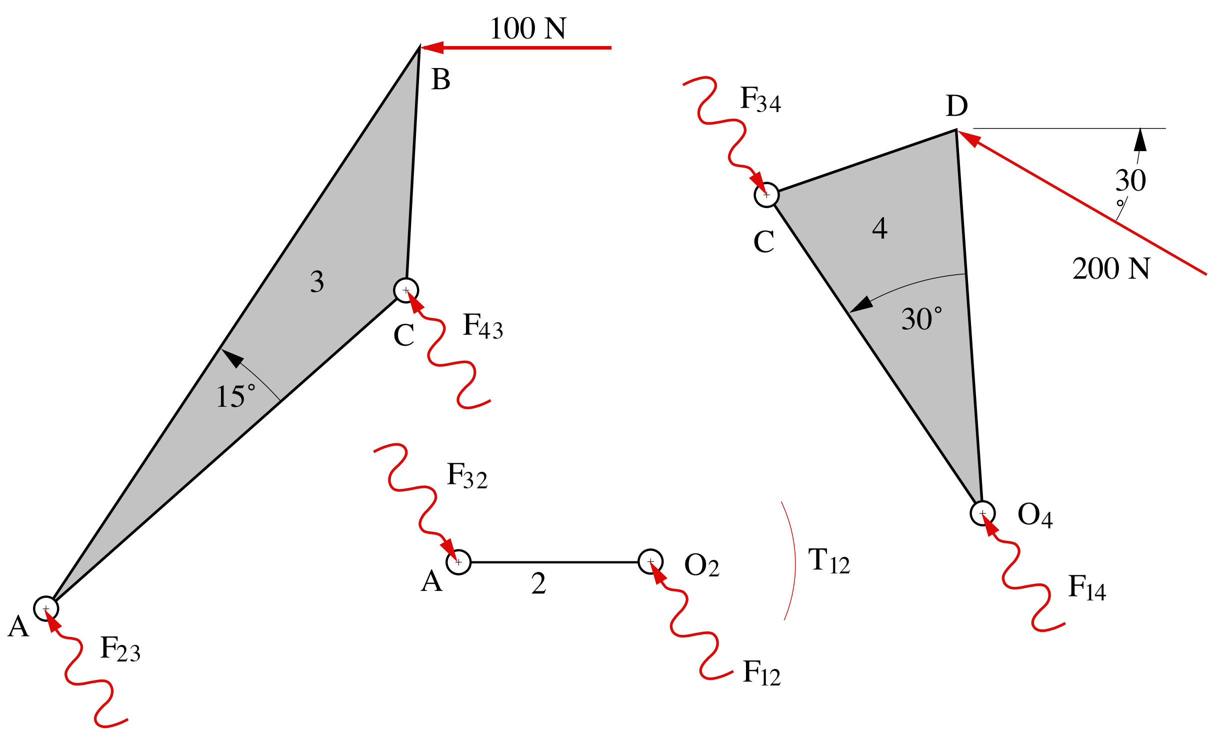
Problem 13.2

Draw a free-body diagram for each of the members of the mechanism shown, and find the magnitude and direction of all the forces and moments. Compute the torque applied to link 2 to maintain static equilibrium. Link 2 is horizontal.



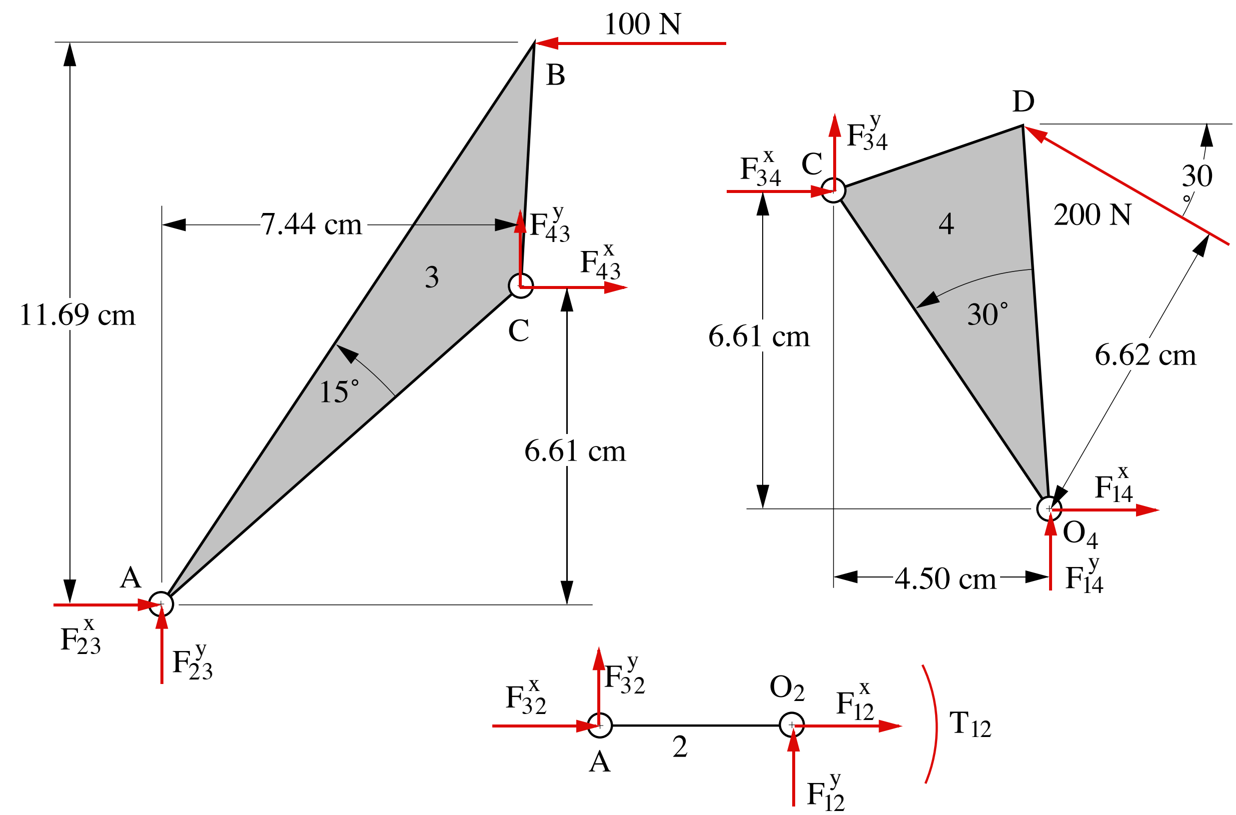
AO_2 = 4.0 cm
O_4O_2 = 8.0 cm
CO_4 = 8.0 cm
DO_4 = 8.0 cm
AB = 14.0 cm
AC = 10.0 cm

Solution:



The freebody diagram of each link is shown above. From the freebody diagrams, it is clear that no single free body can be analyzed separately because in each case, four unknowns result. Therefore, we must write the equilibrium equations for each freebody, and solve the equations as a set.

To proceed, resolve each force into X and Y components and graphically determine the distances as shown below .



For the freebody diagram for link 4, assume initially that all of the unknown forces are in the positive x and y directions. Then a negative result will indicate that the forces are in the negative direction. Summing forces in the X and Y directions gives

$$\sum \mathbf{F}_x = 0 \Rightarrow F_{14}^x + F_{34}^x - 200\cos 30 = 0 \Rightarrow F_{14}^x + F_{34}^x = 173.2$$
$$\sum \mathbf{F}_y = 0 \Rightarrow F_{14}^y + F_{34}^y + 200\sin 30 = 0 \Rightarrow F_{14}^y + F_{34}^y = -100 \qquad (1)$$

and summing moments (CCW positive) about point O_4 gives

$$\sum M_{O4} = 0 \Rightarrow -F_{34}^x(6.61) - F_{34}^y(4.50) + 200(6.62) = 0 \Rightarrow F_{34}^x(6.61) + F_{34}^y(4.50) = 1324 \qquad (2)$$

Between links 3 and 4,

$$F_{43}^x = -F_{34}^x$$
$$F_{43}^y = -F_{34}^y \qquad (3)$$

Now move to the free body diagram for link 3. Summing forces in the X and Y directions gives:

$$\sum \mathbf{F}_x = 0 \Rightarrow F_{23}^x + F_{43}^x - 100 = 0 \Rightarrow F_{23}^x + F_{43}^x = 100$$
$$\sum \mathbf{F}_y = 0 \Rightarrow F_{23}^y + F_{43}^y = 0 \tag{4}$$

and summing moments about point A gives

$$\sum M_{A_2} = 0 \Rightarrow -F_{43}^x(6.61) + F_{43}^y(7.44) + 100(11.69) = 0 \Rightarrow F_{43}^x(6.61) - F_{43}^y(7.44) = 1169 \tag{5}$$

Now using Eqs. (3),

$$\begin{aligned} &F_{23}^x - F_{34}^x = 100 \\ &F_{23}^y - F_{34}^y = 0 \\ &-F_{34}^x(6.61) + F_{34}^y(7.44) = 1169 \end{aligned} \tag{6}$$

Between links 2 and 3,

$$\begin{aligned} &F_{32}^x = -F_{23}^x \\ &F_{32}^y = -F_{23}^y \end{aligned} \tag{7}$$

For link 2, the equilibrium equations are

$$\begin{aligned} &\sum \mathbf{F}_x = 0 \Rightarrow F_{32}^x + F_{12}^x = 0 \\ &\sum \mathbf{F}_y = 0 \Rightarrow F_{32}^y + F_{12}^y = 0 \end{aligned} \tag{8}$$

and summing moments about point O_2 gives

$$\sum M_{O_2} = 0 \Rightarrow -F_{32}^y(4) + T_{12} = 0 \tag{9}$$

Now using Eqs. (7),

$$\begin{aligned} &\sum \mathbf{F}_x = 0 \Rightarrow -F_{23}^x + F_{12}^x = 0 \\ &\sum \mathbf{F}_y = 0 \Rightarrow -F_{23}^y + F_{12}^y = 0 \\ &\sum M_{O_2} = 0 \Rightarrow F_{23}^y(4) + T_{12} = 0 \end{aligned} \tag{10}$$

Equations (1), (2), (6), and (10) can be written in matrix form for solution. This gives,

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.61 & 4.50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -6.61 & 7.44 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_{14}^x \\ F_{14}^y \\ F_{34}^x \\ F_{34}^y \\ F_{23}^x \\ F_{23}^y \\ F_{12}^x \\ F_{12}^y \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 173.2 \\ -100 \\ 1324 \\ 100 \\ 0 \\ 1169 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (11)$$

Equation (11) can be easily solved using MATLAB. The results are

$F_{14}^x = 115.0\text{ N}$
$F_{14}^y = -308.8\text{ N}$

$F_{34}^x = -F_{43}^x = 58.16\text{ N}$
$F_{34}^y = -F_{43}^y = 208.8\text{ N}$

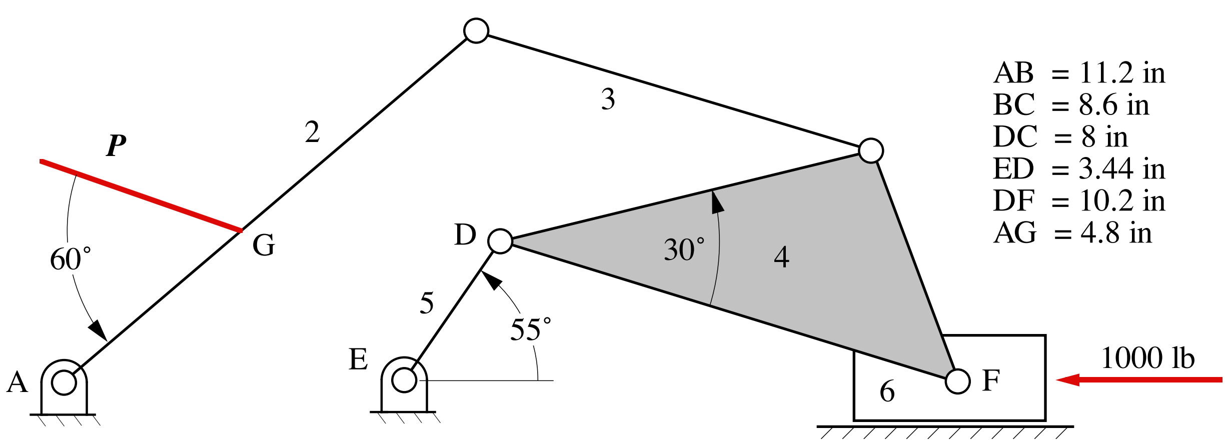
$F_{23}^x = -F_{32}^x = 158.2\text{ N}$
$F_{23}^y = -F_{32}^y = 208.8\text{ N}$

$F_{12}^x = 158.2\text{ N}$
$F_{12}^y = 208.8\text{ N}$

$T_{12} = -835.2\text{ N}-\text{cm}$

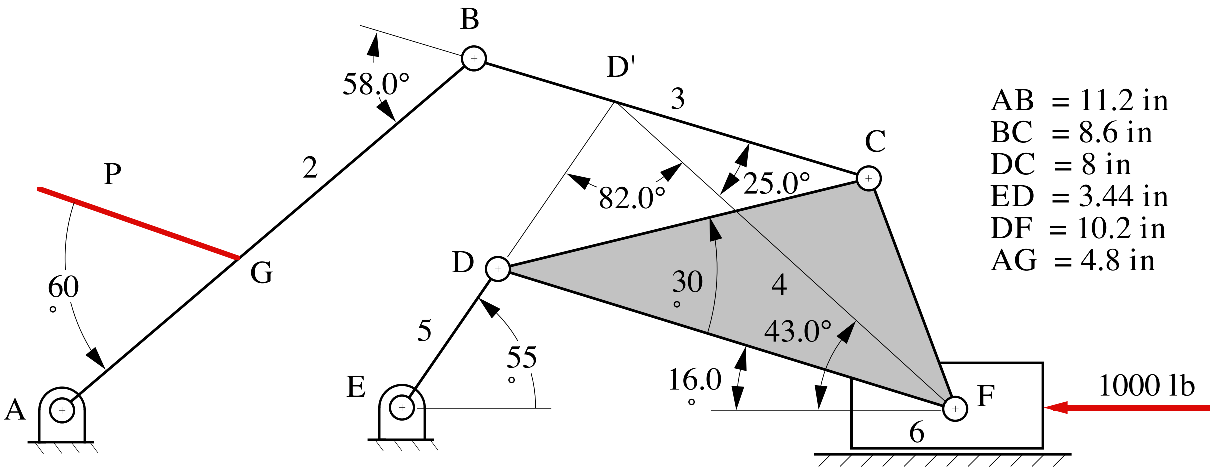
Problem 13.3

If a force of 1000 lb is applied to the slider as shown, determine the force ***P*** required for static equilibrium.



Solution Using FBDs:

Using the procedures discussed in Chapter 4, we can determine the position information shown in the picture below,



The known information is:

$r_{B/A}$=11.2; $r_{C/B}$=8.6; $r_{C/D}$=8;
$r_{D/E}$=3.44; $r_{F/D}$=10.2; $r_{G/A}$=4.8;

After the position analysis, we get

$\angle$DFE=16°; $\angle$EFD=43°; $\angle$EDF=82°;
$\angle$CDF=25°; $\angle$ABC=122°;

The freebody diagram of each member is shown as follows. Then,

$$F_{46} = F / \cos\angle EFD' = 1000 / \cos 43° = 1367.3$$

and

$$F_{64} = F_{46}$$

and in the force triangular, we use the sine law,

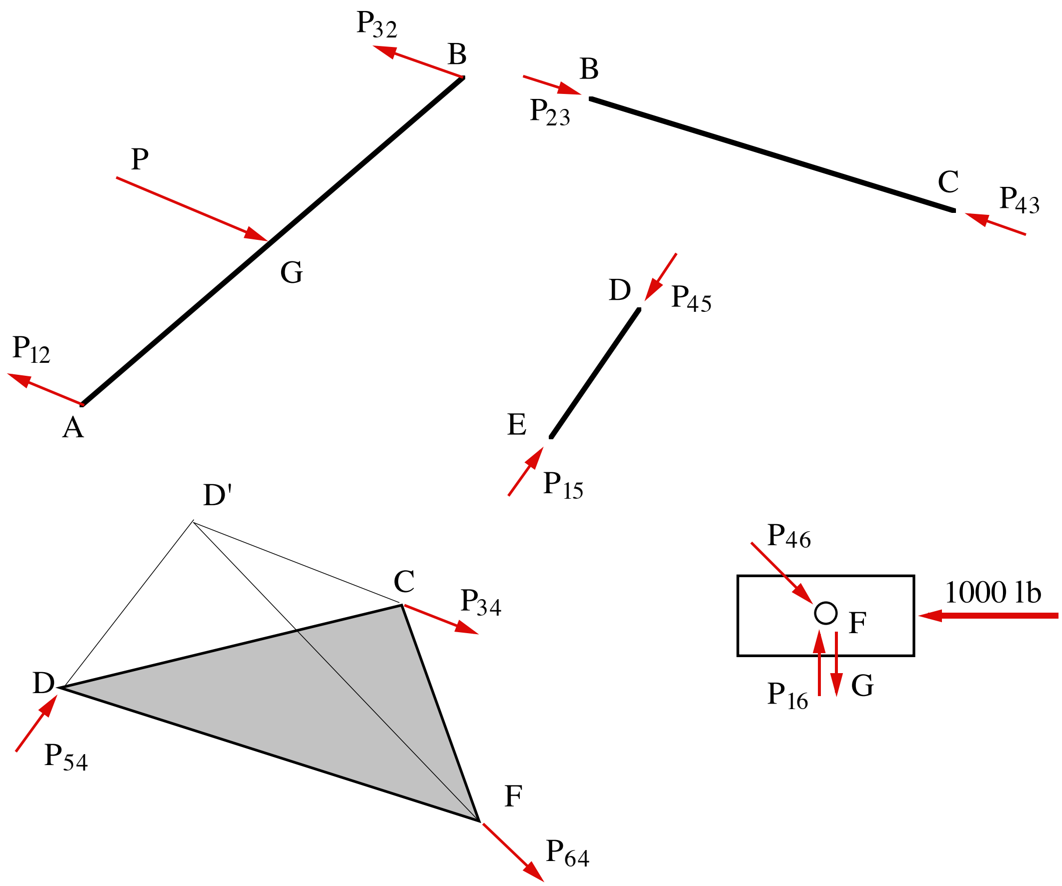
$$\frac{F_{64}}{\sin(180° - \angle ED'F - \angle CD'F)} = \frac{F_{34}}{\sin(\angle ED'F)}$$

Then,

$$F_{34} = \frac{F_{64}\sin(\angle ED'F)}{\sin(\angle ED'F + \angle CD'F)} = \frac{1367.3\sin(82°)}{\sin(82° + 25°)} = 1415.9$$

and

$$F_{43} = F_{34}$$



From the equilibrium of link 3, we know that

$$F_{23} = F_{43}$$

Summing moments about point A, we get

$$\sum M_A = 0 \Rightarrow P \cdot r_{G/A} \sin 60° = F_{32} \cdot r_{B/A} \sin \angle ABC$$

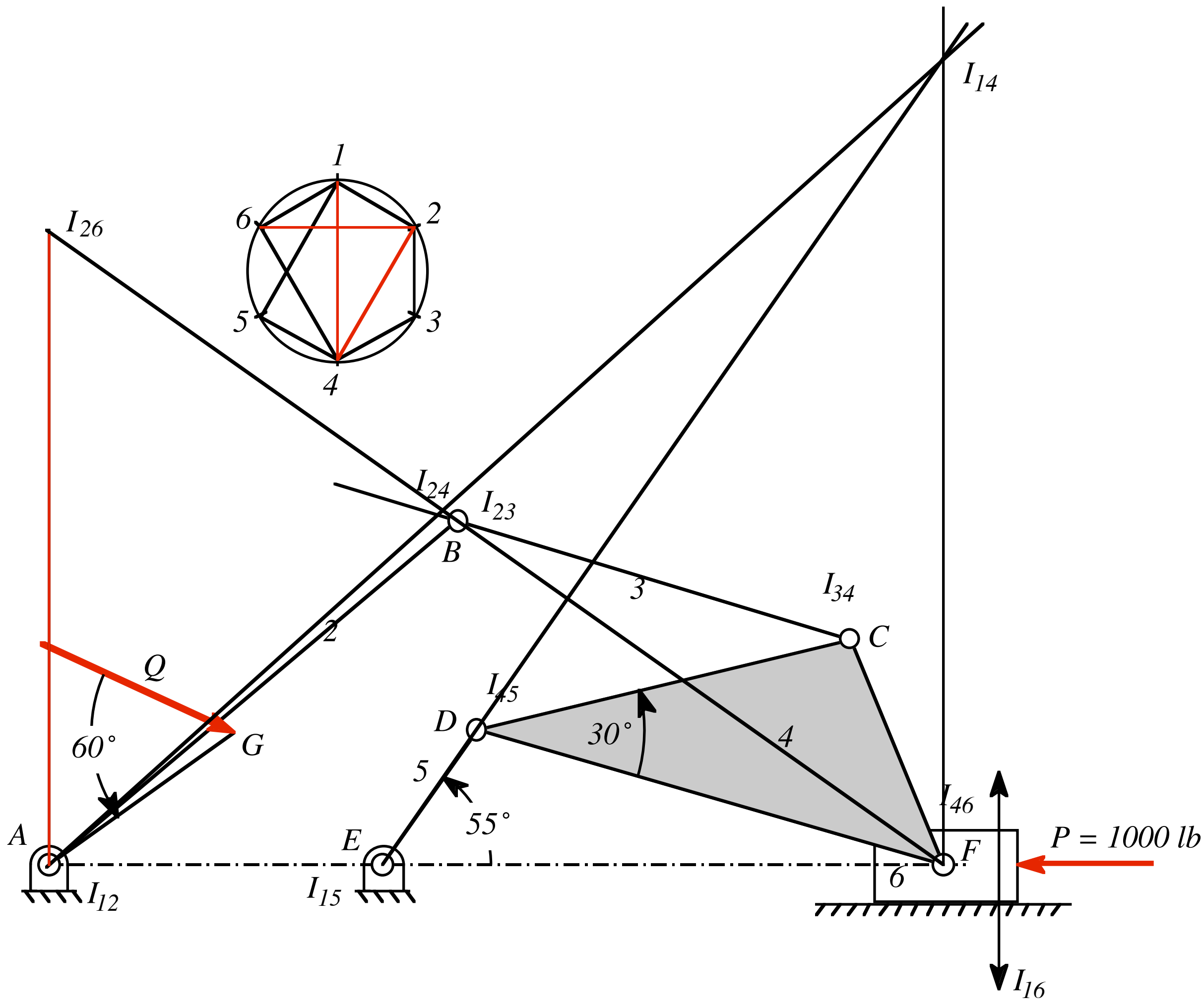
Therefore,

$$P = \frac{F_{32} \cdot r_{B/A} \sin \angle ABC}{r_{G/A} \sin 60°} = \frac{1415.9 \cdot 11.2 \sin 122°}{4.8 \sin 60°} = 3240\, lbs$$

and P=3240 lbs in the direction shown.

Solution Using Conservation of Power:

For conservation of power, we need to find the relationships among the velocities, and this can be done most easily using instant centers of velocity. Power is involved at links 2 and 6; therefore, we need to find I_{12}, I_{16}, and I_{26}. Using the procedures discussed in Chapter 4, the instant centers are shown in the figure below.



From conservation of power,

$$\mathbf{P}\cdot\mathbf{v}_{F_6}+\mathbf{Q}\cdot\mathbf{v}_{G_2}=0 \qquad (1)$$

Also,

$$\mathbf{v}_{F_6}=\mathbf{v}_{I_{26}}=\mathbf{v}_{I_{26}/I_{12}}={}^1\omega_2\times\mathbf{r}_{I_{26}/I_{12}}$$

Assume that ${}^1\omega_2$ is clockwise. Then power is put into the system at link 2 and taken out at link 6. Point G on link 2 moves toward the right so that Q also moves to the right. Since the direction of Q is then known, we need only determine the magnitude. From Eq. (1), the magnitudes are related by

$$Q\left|\mathbf{v}_{F_6}\right|=P\left|\sin 60°\right|\left|\mathbf{v}_{G_2}\right|$$

or

$$P = Q\frac{|\mathbf{v}_{F_6}|}{|\sin 60°||\mathbf{v}_{G_2}|} = Q\frac{|\mathbf{v}_{I_{26}}|}{|\sin 60°||{}^1\omega_2||\mathbf{r}_{G/I_{12}}|} = Q\frac{|\mathbf{v}_{I_{26}}|}{|\sin 60°||\mathbf{v}_{I_{26}}|/|\mathbf{r}_{I_{26}/I_{12}}||\mathbf{r}_{G/I_{12}}|}$$

$$= Q\frac{|\mathbf{r}_{I_{26}/I_{12}}|}{|\sin 60°||\mathbf{r}_{G/I_{12}}|} = 1000\frac{|13.35|}{|\sin 60°||4.8|} = 3210\,lbs$$

Therefore,

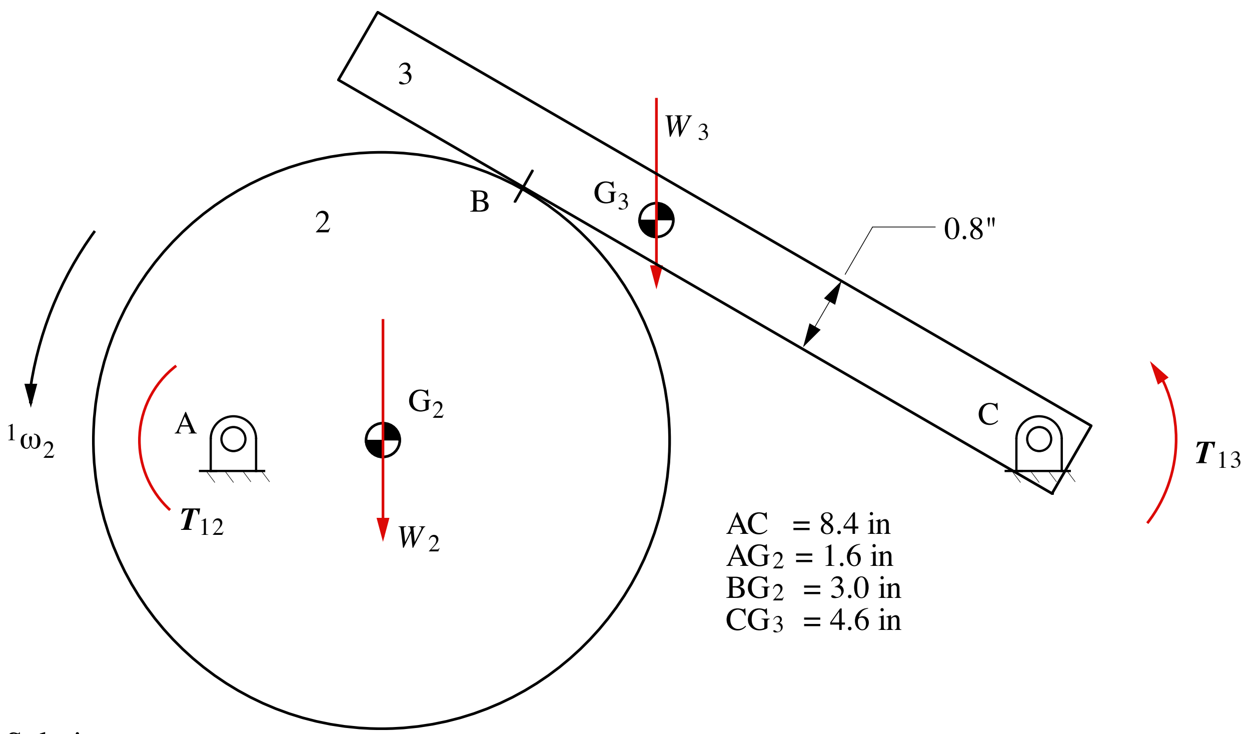
$P = 3210\,lbs$ generally to the right as shown

Problem 13.4

For the mechanism and data given, determine the cam torque, $\boldsymbol{T}_{12}$, and the forces on the frame at points A and C ($\boldsymbol{F}_{21}$ and $\boldsymbol{F}_{31}$). Assume that there is friction between the cam and follower only.

$\boldsymbol{T}_{14}$ = 50 in-lb
μ = 0.13

W_2 = 16.1 lb
W_3 = 32.2 lb



Solution

After a position analysis, we get the position data shown in the figure below. This is followed by the freebody diagrams for links 2 and 3.

Summing moments about point C on link 3 gives,

$$\sum M_C = 0 \Rightarrow T_{13} + W_4(4) - N_{23}(5.89) - F_{f23}(0.4) = 0$$

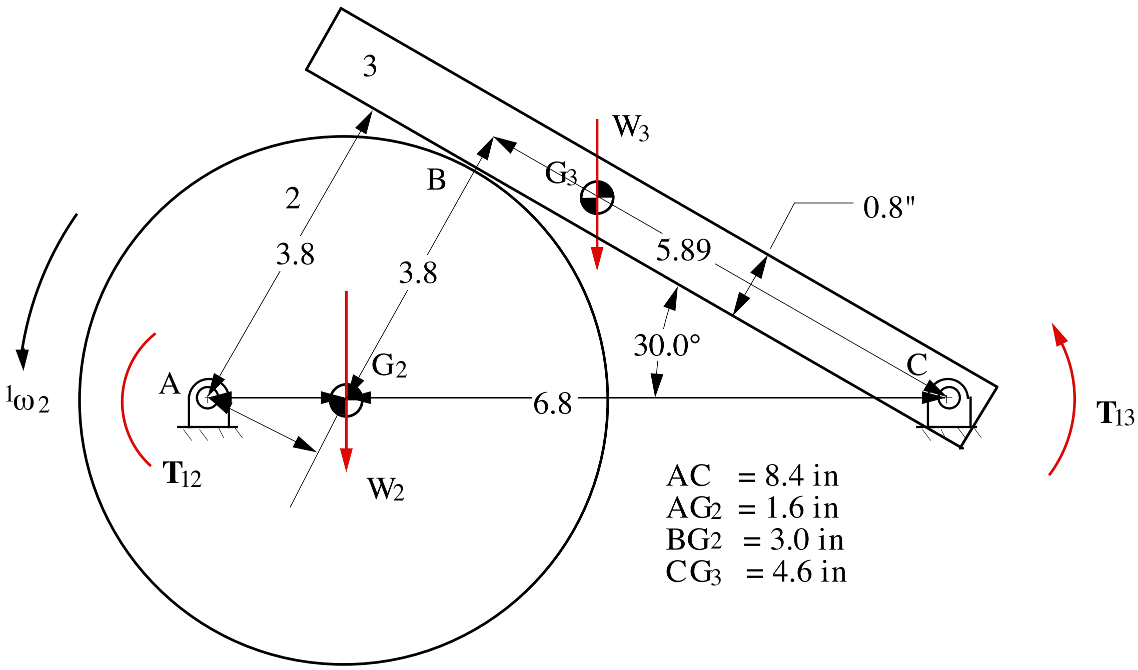
The friction and normal forces are related through

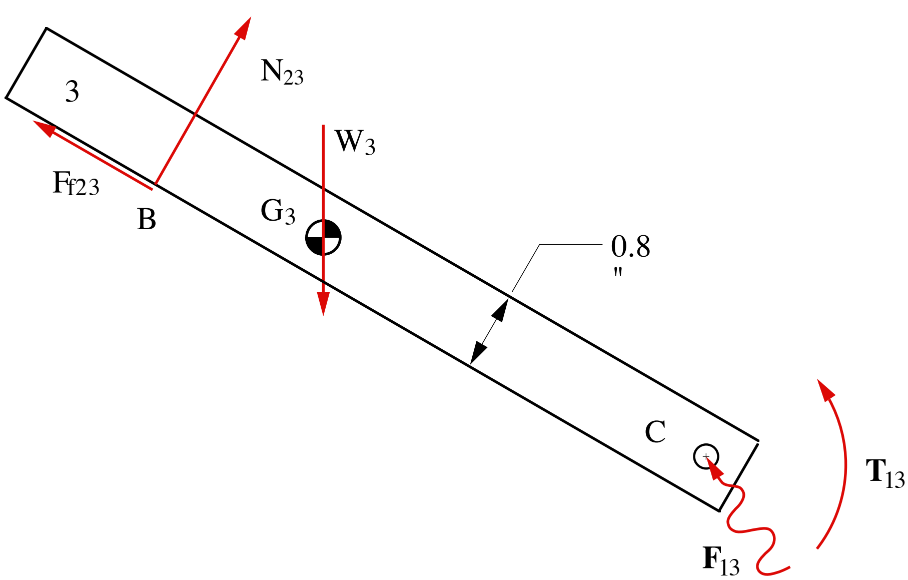
$$F_{f23} = \mu N_{23} = 0.13N_{23}$$

From the two equations above, we get

$$N_{23} = \frac{T_{13} + W_3(4)}{5.89 + 0.4 \cdot 0.13} = \frac{50 + 32.2(4)}{5.89 + 0.4 \cdot 0.13} = 30$$

$$F_{f23} = 0.13N_{23} = 3.9$$





The free body diagram of cam 2 is shown below. From equilibrium,

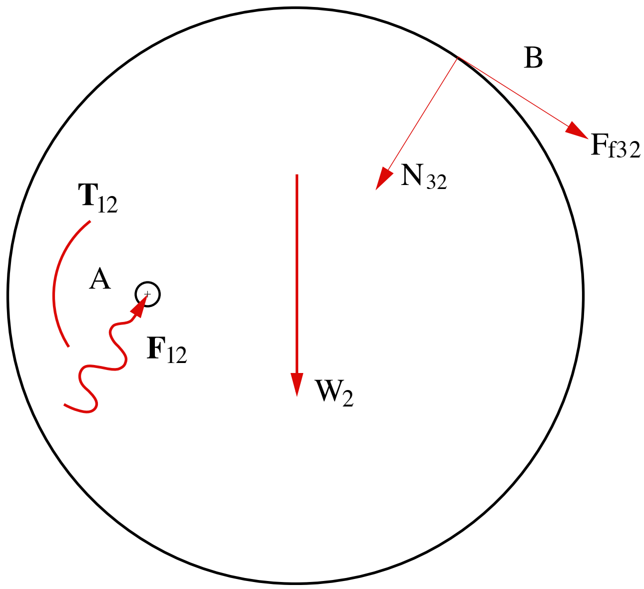
$$\mathbf{N}_{32} = -\mathbf{N}_{23}$$

and

$$\mathbf{F}_{f32} = -\mathbf{F}_{f23}$$

Assume that T_{12} is CCW. Then sum moments about point A,

$$\sum M_A = 0 \Rightarrow T_{12} - W_2(1.6) - N_{32}(1.39) - F_{f32}(3.8) = 0$$



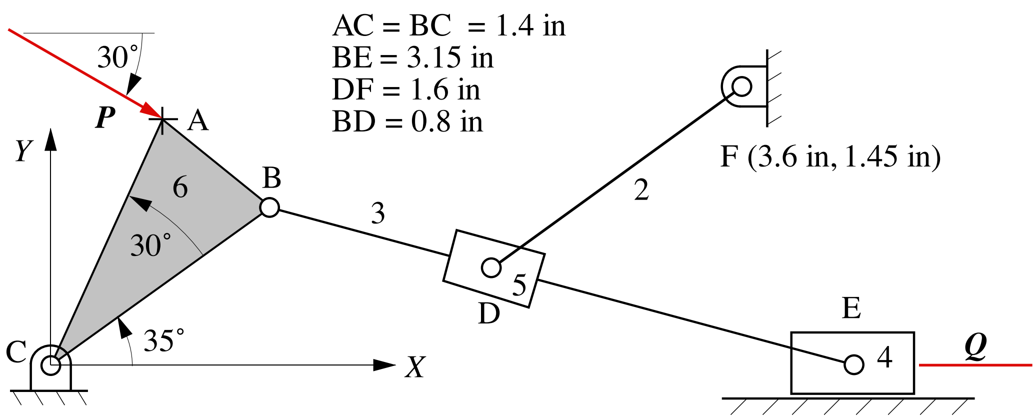
Then,

$$T_{12} = W_2(1.6) + N_{32}(1.39) + F_{f32}(3.8) = (16.1)(1.6) + (30)(1.39) + (3.9)(3.8) = 82.3$$

Therefore $\mathbf{T}_{12}$=82.3 in-lb CCW

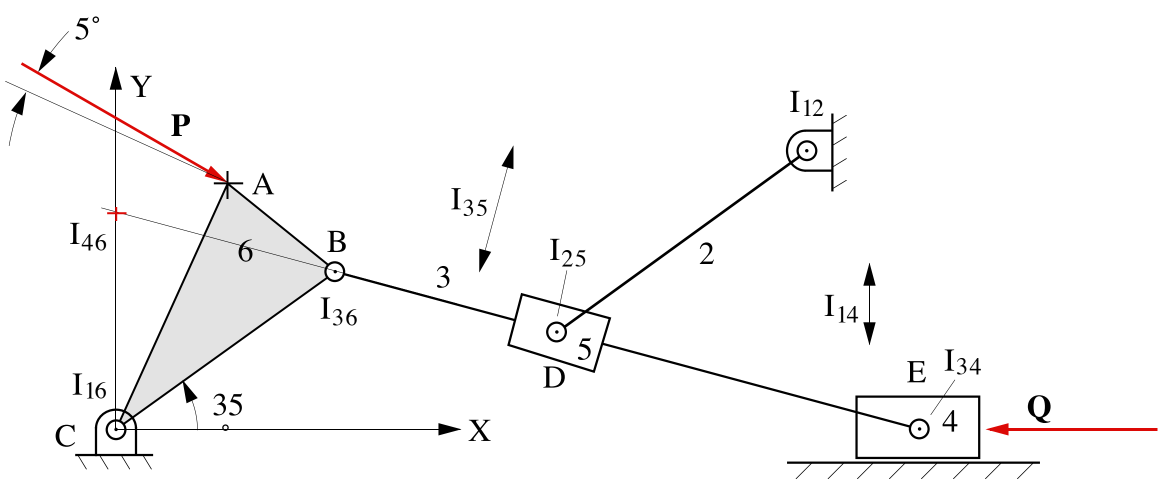
Problem 13.5

In the mechanism shown, $\boldsymbol{P}$ = 100 lb. Find the value of the force $\boldsymbol{Q}$ on the block for equilibrium. Use energy methods.



Solution:

Because only the force **Q** is of interest, this problem can be solved easily using energy methods. For this we need the instant centers I_{16}, I_{46}, and I_{14}. We can get I_{16} and I_{14} by inspection. To find I_{46}, redraw the mechanism to scale and use I_{16} and I_{14}, and I_{36} and I_{43}. The results are given below.



From conservation of power,

$$\mathbf{P} \cdot \mathbf{v}_{A6} + \mathbf{Q} \cdot \mathbf{v}_{E4} = 0 \qquad (1)$$

Also,

$$\mathbf{v}_{E4} = \mathbf{v}_{I46} = \mathbf{v}_{I46/I16} = {}^{1}\omega_6 \times \mathbf{r}_{I46/I16}$$

and

$$\mathbf{v}_{A6} = \mathbf{v}_{A6/I16} = {}^{1}\omega_6 \times \mathbf{r}_{A6/I16}$$

Assume that $^1\omega_6$ is clockwise. Then power is put into the system at link 6 and taken out at link 4. Link 4 moves toward the right so that Q moves to the left. Since the direction of Q is then known, we need only determine the magnitude. From Eq. (1), the magnitudes are related by

$$P\cos 5^\circ \left|{}^1\omega_6\right|\left|\mathbf{r}_{A6/I16}\right| = Q\left|{}^1\omega_6\right|\left|\mathbf{r}_{I46/I16}\right|$$

or

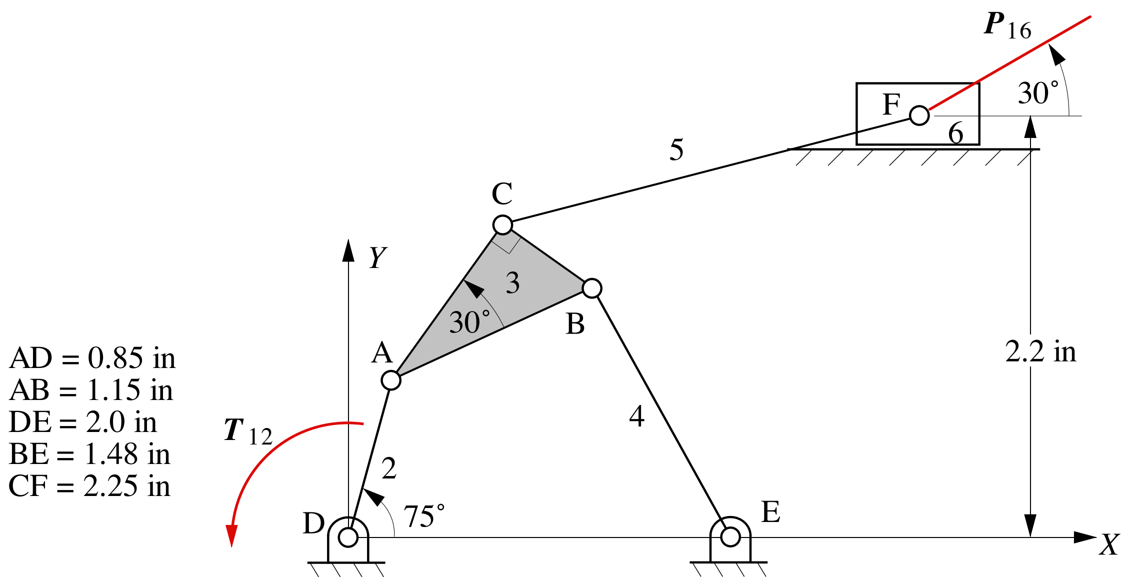
$$Q = P\frac{\cos 5^\circ \left|{}^1\omega_6\right|\left|\mathbf{r}_{A6/I16}\right|}{\left|{}^1\omega_6\right|\left|\mathbf{r}_{I46/I16}\right|} = P\frac{\cos 5^\circ \left|\mathbf{r}_{A6/I16}\right|}{\left|\mathbf{r}_{I46/I16}\right|} = P\frac{\cos 5^\circ 1.4}{1.128} = 1.24P$$

Therefore,

$$\mathbf{Q} = 1.24P = 1.24(100) = 124 \text{ lbs} \text{ to the left}$$

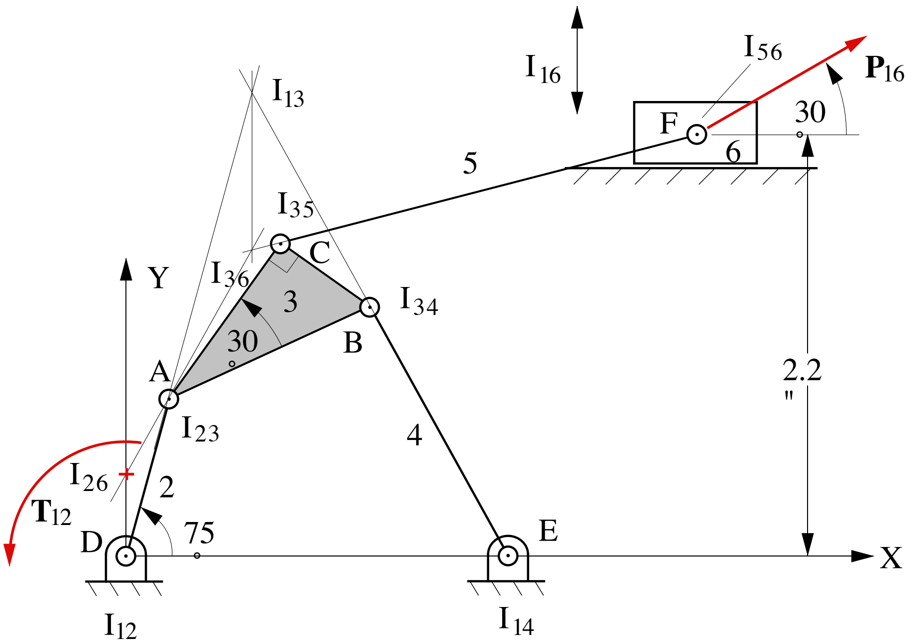
Problem 13.6

If T_{12} is 1 in-lb, find $\boldsymbol{P}_{16}$ using energy methods.



Solution:

Draw the mechanism to scale. For conservation of power, we need to find the relationships among the velocities, and this can be done most easily using instant centers of velocity. Power is involved at links 2 and 6; therefore, we need to find I_{12}, I_{16}, and I_{26}. Using the procedures discussed in Chapter 4, the instant centers are shown in the figure below.



From the conservation of power,

$$\mathbf{T}_{12} \cdot {}^1\omega_2 + \mathbf{P}_{16} \cdot \mathbf{v}_{F_6} = 0 \tag{1}$$

From the instant centers,

$$\mathbf{v}_{F6} = \mathbf{v}_{I26} = \mathbf{v}_{I26/I12} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{I26/I12}$$

Arbitrarily assume that ${}^1\omega_2$ is CCW. Then, power is put into the linkage at link 2 and taken out at link 6. Point F_6 is moving to the left so the component of P16 in the direction $\mathbf{v}_{F6}$ is to the right. This direction is indicated in the figure. Given the direction of P16, we need only determine the magnitude. From Eq. (1),

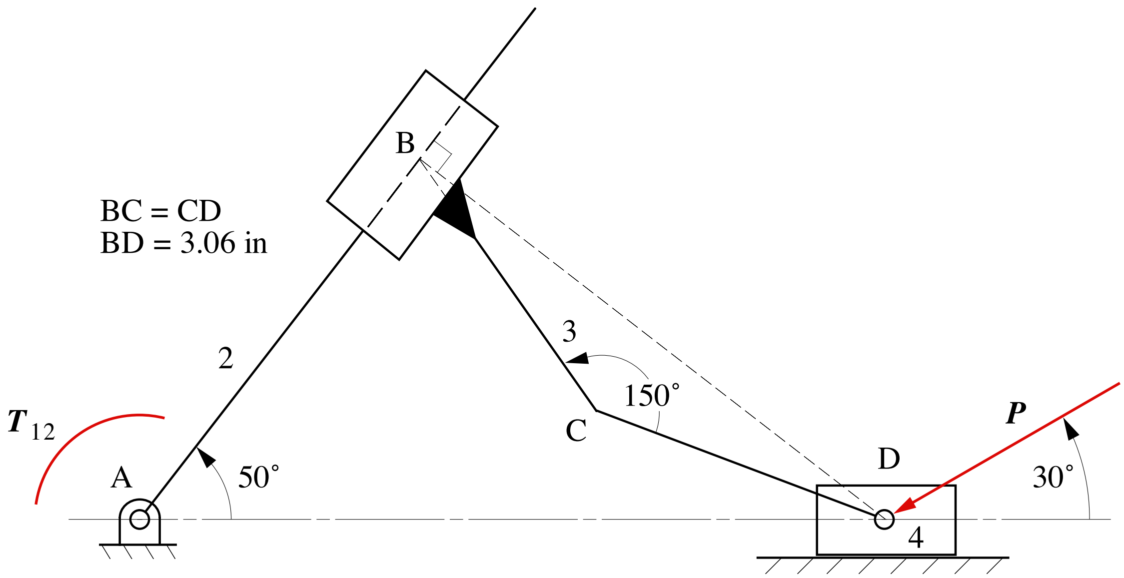
$$T_{12}\left|{}^1\boldsymbol{\omega}_2\right| = P_{16}\cos 30^\circ \left|{}^1\boldsymbol{\omega}_2\right|\left|\mathbf{r}_{I26/I12}\right|$$

or

$$P_{16} = \frac{T_{12}}{\cos 30^\circ \left|\mathbf{r}_{I26/I12}\right|} = \frac{1}{0.427\cos 30^\circ} = 2.7\,\text{lbs} \text{ in the direction shown.}$$

Problem 13.7

Assume that the force ***P*** is 10 lb. Use energy methods to find the torque T_{12} required for equilibrium.

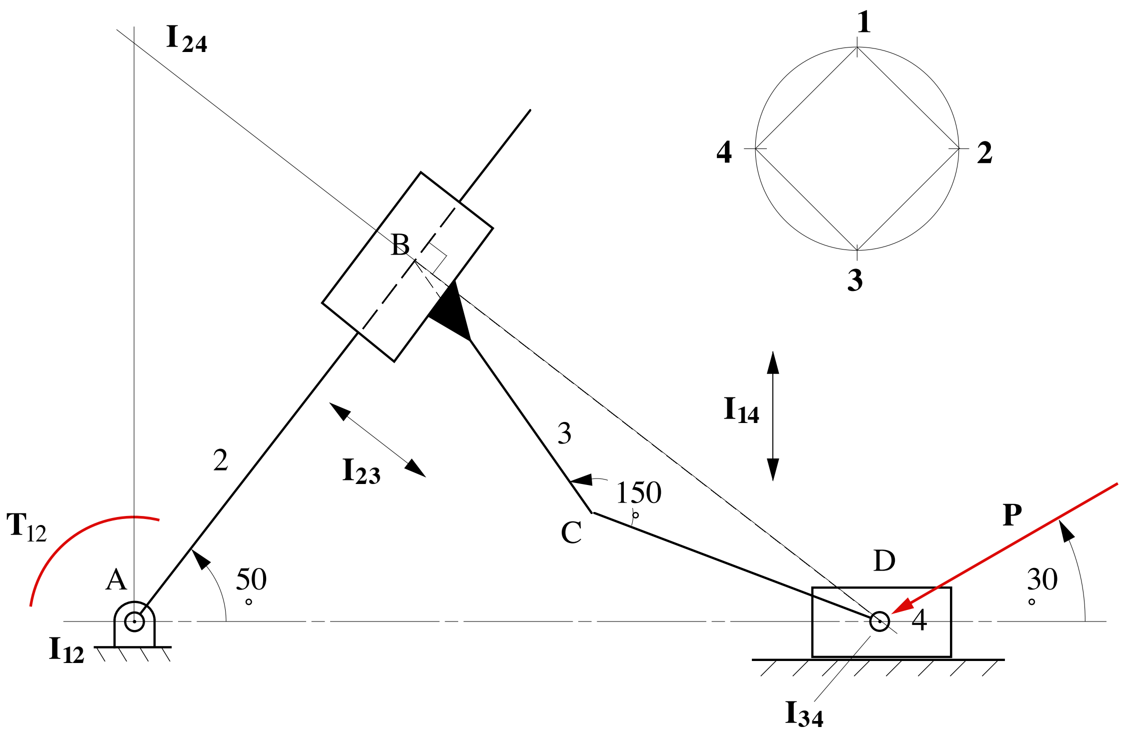


Solution

To solve the problem using energy methods, we need $I_{1,2}$, $I_{1,4}$, and $I_{2,4}$. Only $I_{2,4}$ needs to be determined. $I_{1,2}$ and $I_{1,4}$ are found by inspection.

From conservation of power,

$$\mathbf{P} \cdot {}^1\mathbf{v}_{E4} + \mathbf{T}_{12} \cdot {}^1\omega_2 = 0$$



To determine the sign of T_{12}, assume that the angular velocity of link 2 is counter-clockwise. Then the velocity of D_4 will be generally in the same direction as the force P and power will be taken out of the system at link 4. Therefore, power will be taken out of the system at link 2, and the torque T_{12} must be clockwise.

To compute the magnitude of the torque, use:

$$|P\cos 30°||{}^1v_{D4}| = |T_{12}||{}^1\omega_2|$$

Now,

$$|{}^1v_{D4}| = |{}^1v_{I24}| = |{}^1\omega_2||r_{24/A}|$$

So, substituting for ${}^1v_{D4}$

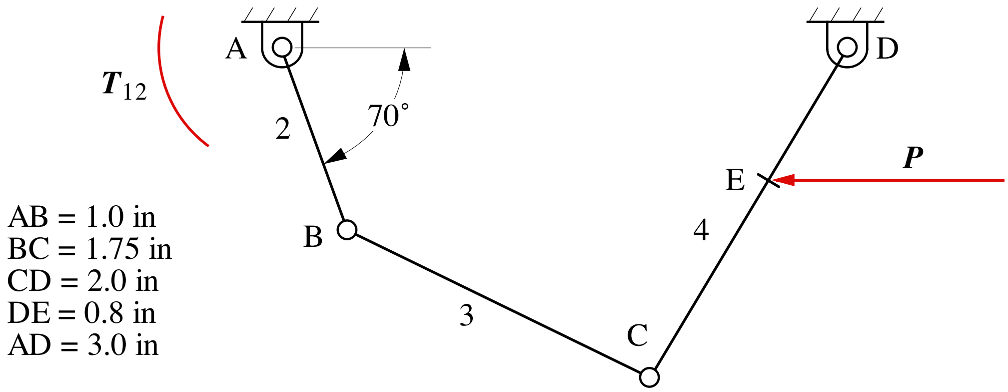
$$|P\cos 30°||{}^1\omega_2||r_{24/A}| = |T_{12}||{}^1\omega_2|$$

or

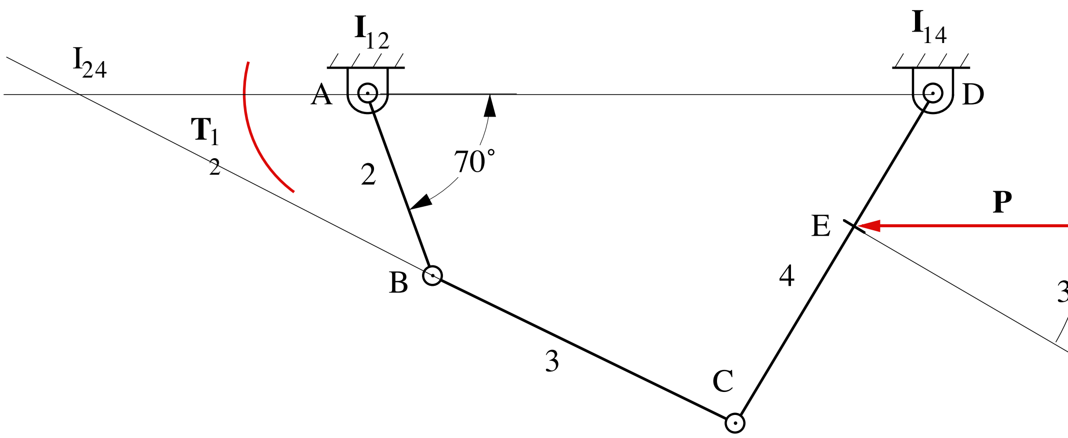
$$|T_{12}| = |P\cos 30°||r_{24/A}| = 10(\cos 30°)3.104 = 26.88 \text{ in} - \text{lbs CW}$$

Problem 13.8

In the four-bar linkage shown, the force P is 100 lb. Use energy methods to find the torque $\boldsymbol{T}_{12}$ required for equilibrium.



Solution



To solve the problem using energy methods, we need $I_{1,2}$, $I_{1,4}$, and $I_{2,4}$.

From conservation of power,

$$\mathbf{P} \cdot {}^1\mathbf{v}_{E_4} + \mathbf{T}_{12} \cdot {}^1\boldsymbol{\omega}_2 = 0$$

To determine the sign of T_{12}, assume that the angular velocity of link 4 is counter-clockwise. Then the velocity of E_4 will be generally opposite to the force P and power will be taken out of the system at link 4. Therefore, power will be put into the system at link 2. Because the instant center I_{24} is outside of the instant centers I_{12} and I_{14}, the angular velocity of link 2 is in the same direction (CCW) as that of link 4. Therefore, the torque T_{12} must also be counter-clockwise.

To compute the magnitude of the torque, use:

$$|P\cos 30°||{}^1v_{E_4}| = |T_{12}||{}^1\omega_2|$$

Now,

Also,

$$\left|{}^1\omega_4\right|\left|r_{(I_{2,4})/(I_{1,4})}\right| = \left|{}^1\omega_2\right|\left|r_{(I_{2,4})/(I_{1,2})}\right|$$

Since,

$$\left|{}^1\omega_4\right| = \left|{}^1\omega_2\right|\frac{\left|r_{(I_{2,4})/(I_{1,2})}\right|}{\left|r_{(I_{2,4})/(I_{1,4})}\right|}$$

Then

$$\left|{}^1v_{E_4}\right| = \left|{}^1v_{E_4/D_4}\right| = \left|{}^1\omega_4\right|\left|r_{E/D}\right| = \left|{}^1\omega_2\right|\frac{\left|r_{(I_{2,4})/(I_{1,2})}\right|}{\left|r_{(I_{2,4})/(I_{1,4})}\right|}\left|r_{E/D}\right|$$

So, substituting for ${}^1v_{E4}$

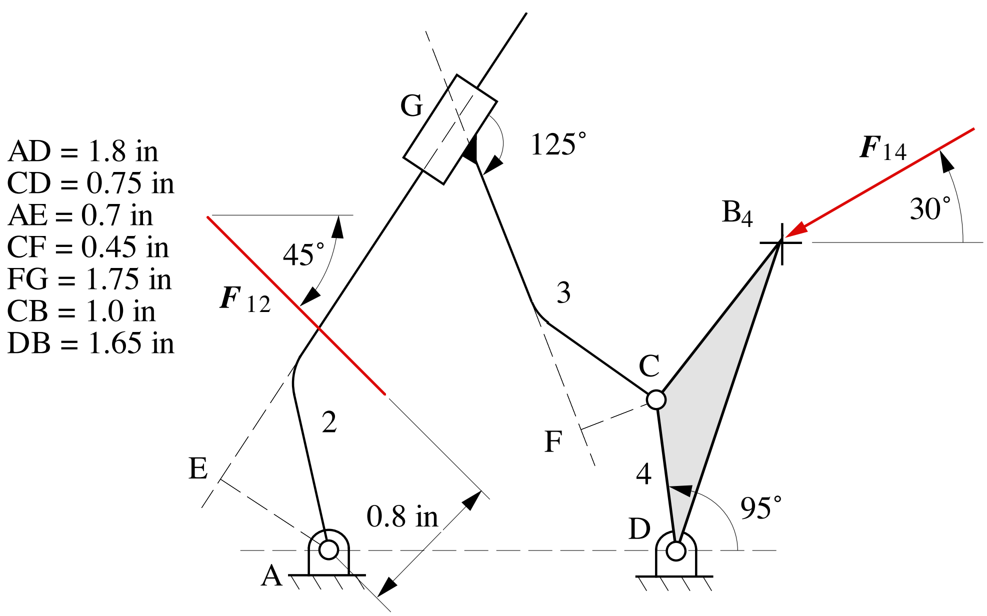
$$\left|P\cos 30^\circ\right|\left|{}^1\omega_2\right|\frac{\left|r_{(I_{2,4})/(I_{1,2})}\right|}{\left|r_{(I_{2,4})/(I_{1,4})}\right|}\left|r_{E/D}\right| = \left|T_{12}\right|\left|{}^1\omega_2\right|$$

or

$$\left|T_{12}\right| = \left|P\cos 30^\circ\right|\frac{\left|r_{(I_{2,4})/(I_{1,2})}\right|}{\left|r_{(I_{2,4})/(I_{1,4})}\right|}\left|r_{E/D}\right| = 100(\cos 30^\circ)\frac{1.5139}{4.4583}1.009 = 29.672 \text{ in} - \text{lbs CCW}$$

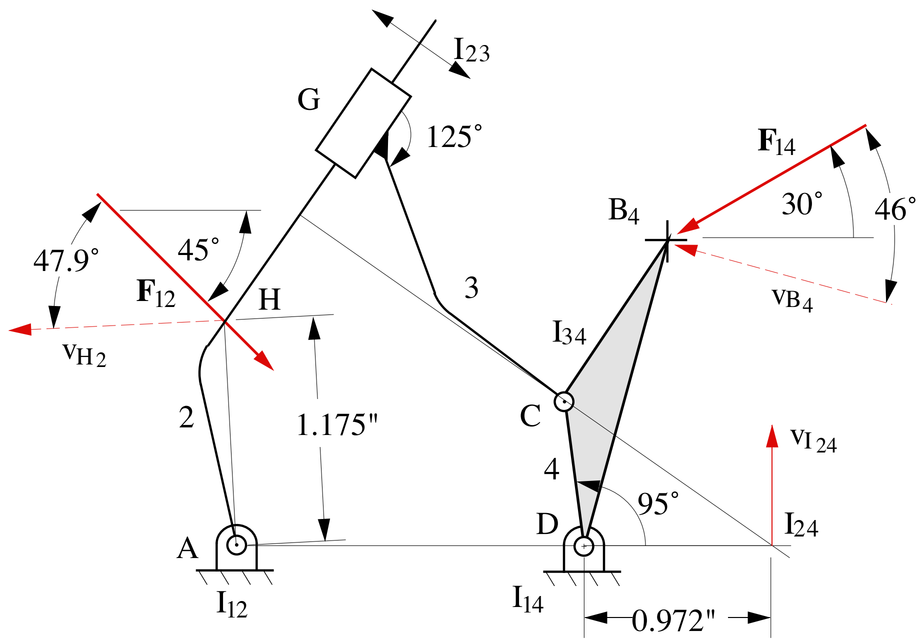
Problem 13.9

If $\boldsymbol{F}_{14}$ is 100 lb, find the force $\boldsymbol{F}_{12}$ required for static equilibrium.



Solution:

Because only the force $\mathbf{F}_{12}$ is of interest, this problem can be solved easily using energy methods and instant centers. For this we need the instant centers I_{12}, I_{24}, and I_{14}. We can get I_{12} and I_{14} by inspection. To find I_{24}, redraw the mechanism to scale and use I_{12} and I_{14}, and I_{32} and I_{34}. The results are given below.



Assume that link 4 rotates CCW. Then $\mathbf{v}_{I_{24}}$, $\mathbf{v}_{B_4}$, and $\mathbf{v}_{H_2}$ and will be in the directions shown above. From conservation of power,

$$\mathbf{F}_{12}\cdot\mathbf{v}_{H_2}+\mathbf{F}_{14}\cdot\mathbf{v}_{B_4}=0 \tag{1}$$

Also,

$$\mathbf{v}_{B_4}={}^1\boldsymbol{\omega}_4\times\mathbf{r}_{B_4/I_{14}}\,, \tag{2}$$

$$\mathbf{v}_{H_2}={}^1\boldsymbol{\omega}_2\times\mathbf{r}_{H_2/I_{12}}\,, \tag{3}$$

$$\mathbf{v}_{I_{24}}={}^1\boldsymbol{\omega}_2\times\mathbf{r}_{I_{24}/I_{12}}={}^1\boldsymbol{\omega}_4\times\mathbf{r}_{I_{24}/I_{14}} \tag{4}$$

and

$$\frac{|{}^1\boldsymbol{\omega}_2|}{|{}^1\boldsymbol{\omega}_4|}=\frac{|\mathbf{r}_{I_{24}/I_{14}}|}{|\mathbf{r}_{I_{24}/I_{12}}|}$$

If ${}^1\boldsymbol{\omega}_4$ is clockwise, then power is put into the system at link 4 and taken out at link 2. Therefore, the force $\mathbf{F}_{12}$ is in the direction shown. Since the direction of $\mathbf{F}_{12}$ is then known, we need only determine the magnitude. From Eq. (1), the magnitudes are related by

$$|\mathbf{F}_{12}|\cos 47.9^\circ|{}^1\boldsymbol{\omega}_2||\mathbf{r}_{H_2/I_{12}}|=|\mathbf{F}_{14}|\cos 46^\circ\,|{}^1\boldsymbol{\omega}_4|\,|\mathbf{r}_{B_4/I_{14}}|$$

Then

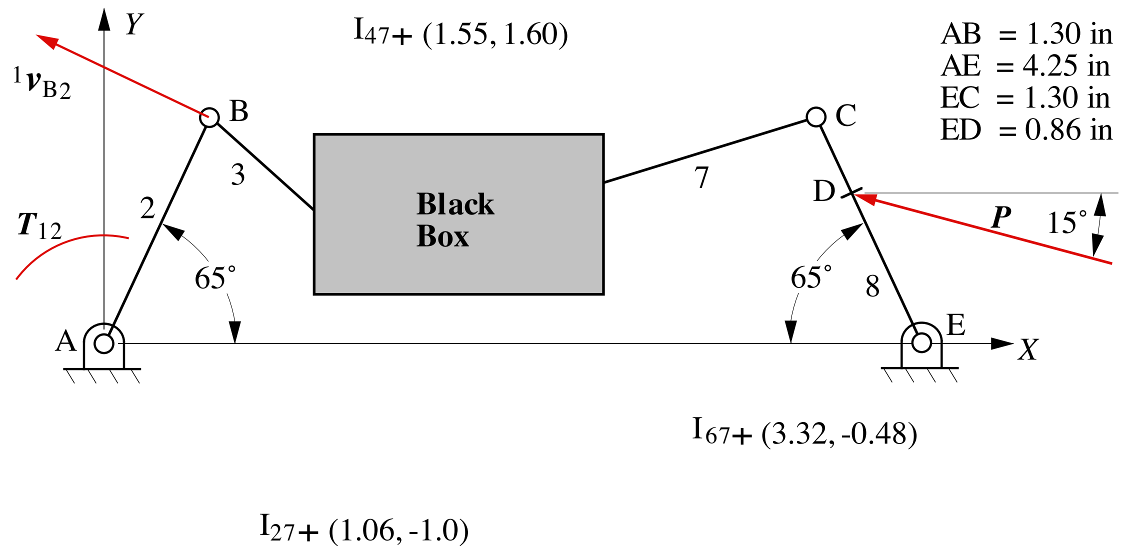
$$|\mathbf{F}_{12}|=\frac{|\mathbf{F}_{14}|\cos 46^\circ\,|{}^1\boldsymbol{\omega}_4|\,|\mathbf{r}_{B_4/I_{14}}|}{\cos 47.9^\circ|{}^1\boldsymbol{\omega}_2||\mathbf{r}_{H_2/I_{12}}|}=\frac{|\mathbf{F}_{14}|\cos 46^\circ\;|\mathbf{r}_{B_4/I_{14}}|}{\cos 47.9^\circ|\mathbf{r}_{H_2/I_{12}}|}\frac{|\mathbf{r}_{I_{24}/I_{12}}|}{|\mathbf{r}_{I_{24}/I_{14}}|}$$

Then

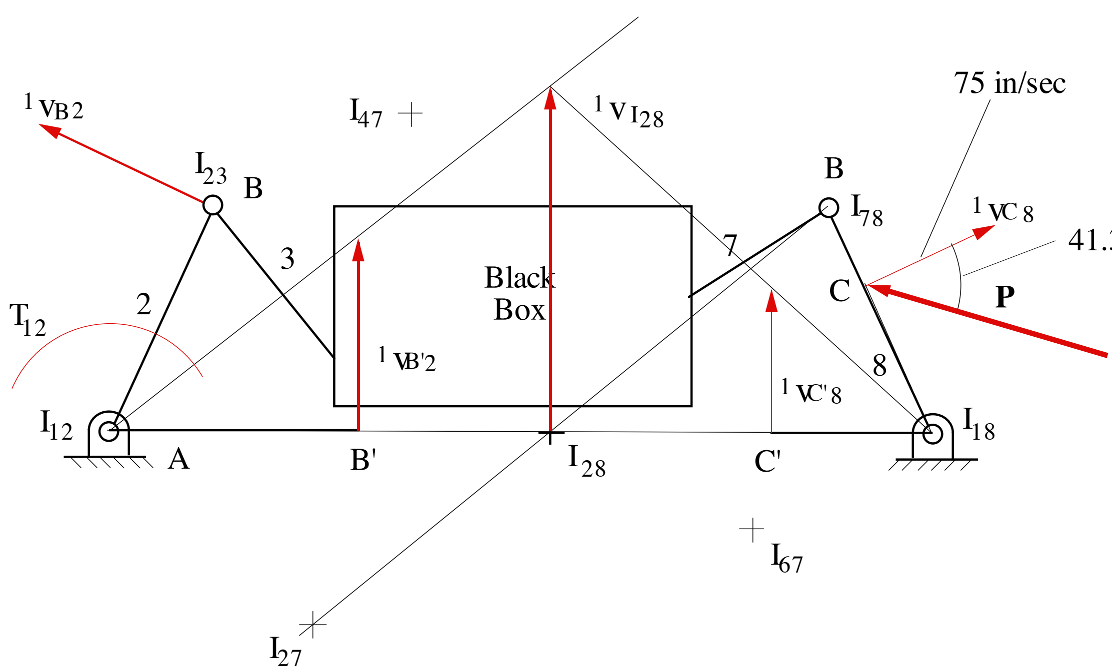
$$\mathbf{F}_{12} = \frac{100\cos 46^\circ(1.65)}{\cos 47.9^\circ(1.1753)}\frac{2.7861}{0.9861} = 410\text{ lbs in the direction shown}$$

Problem 13.10

In the eight-link mechanism, most of the linkage is contained in the black box and some of the instant centers are located as shown. The force P is 100 lb and is applied to point D on link 8. If ${}^1\mathbf{v}_{B_2} = 100$ in/s in the direction shown, compute the velocity of point C_8 and determine the torque T_{12} necessary for equilibrium.



Solution



Find instant centers I_{12}, I_{18}, and I_{28}. Then use the rotating radius method to find the velocity of C8. The results are shown on the figure. From the figure,

$$^1r_{B/I_{12}} = 1.2926"$$
$$^1r_{C/I_{18}} = 0.8454"$$
$$^1v_{C_8/I_{18}} = 75\ in/sec$$

Then,

$$^1\omega_2 = \frac{\left|^1v_{B_2/I_{12}}\right|}{\left|r_{B_2/I_{12}}\right|} = \frac{100}{1.2926} = 77.36 rad/s\ \ CCW$$

and

$$T_{12} \bullet {}^1\omega_2 - P\cos 41.32^\circ v_{C_8/I_{18}} = 0$$

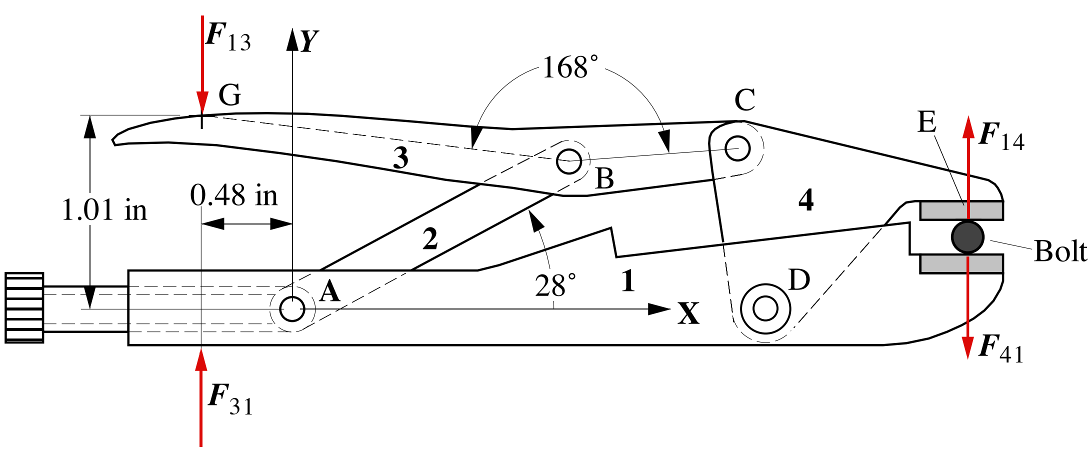
or

$$T_{12} = \frac{P\cos 41.32^\circ v_{C_8/I_{18}}}{^1\omega_2} = \frac{100(\cos 41.32)75}{77.36} = 72.8\text{ in} - \text{lbs}$$

Problem 13.11

The mechanism shown is called a vice grip because a very high force at E can be generated with a relatively small force at G when points A, B, and C are collinear (toggle position). In the position shown, determine the ratio $\boldsymbol{F}_{14}/\boldsymbol{F}_{13}$.

AB = 1.65 in
BC = 0.88 in
CD = 0.85 in
AD = 2.46 in
CE = 1.26 in
DE = 1.16 in
BF = 1.94 in



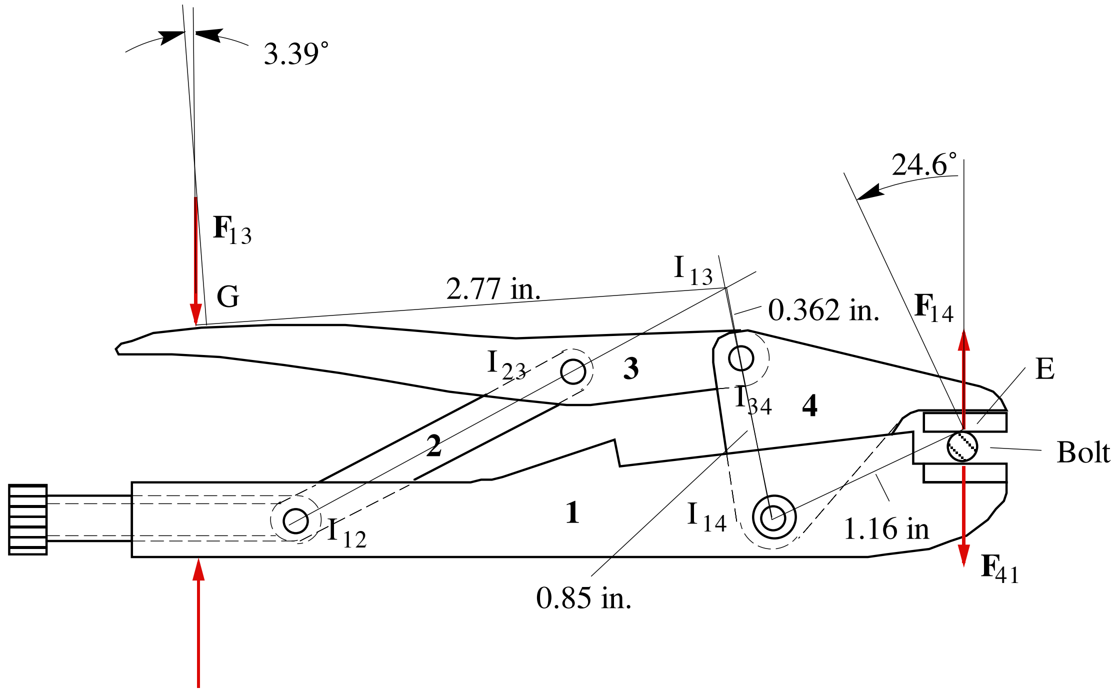
Solution

From conservation of power,

$$\mathbf{F}_{12} \bullet {}^1\mathbf{v}_{G3} + \mathbf{F}_{14} \bullet {}^1\mathbf{v}_{E4} = 0$$

and

$^1v_{G_3} = {}^1\omega_3 \times r_{G/I_{13}}$



$^1v_{B_4} = {}^1\omega_4 \times r_{E/I_{14}}$

Because I_{34} is an instant center,

$$^1\omega_4 \times r_{I_{34}/I_{14}} = {}^1\omega_3 \times r_{I_{34}/I_{13}}$$

Assume that we are putting power in at A. Then V_{A3} is down. Because of the location of instant centers I_{13}, I_{14}, and I_{34}, the angular velocity $^1\omega_3$ must be CCW which means that $^1\omega_4$ must be CW. Then, this makes the velocity of B_4 (v_{B4}) generally down and the force F_{14} generally up as shown. Knowing the directions of the force and velocity terms, we can deal with magnitudes. Then,

$$F_{12}\cos 3.39({}^1\omega_3)(r_{G/I_{13}}) = F_{14}\cos 24.6({}^1\omega_4)(r_{E/I_{14}})$$

or

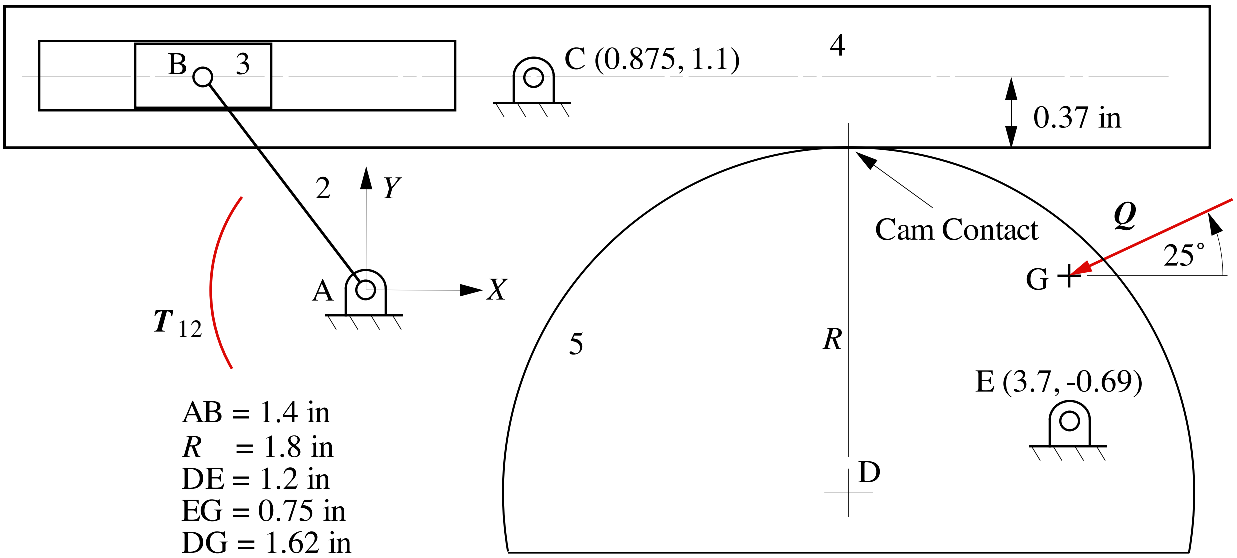
$$F_{12}\cos 3.39({}^1\omega_3)(r_{G/I_{13}}) = F_{14}\cos 24.6({}^1\omega_3)\frac{r_{I_{34}/I_{13}}}{r_{I_{34}/I_{14}}}(r_{E/I_{14}})$$

and finally,

$$F_{14} = F_{12}\frac{\cos 3.39}{\cos 24.6}\frac{r_{G/I_{13}}}{r_{E/I_{14}}}\frac{r_{I_{34}/I_{14}}}{r_{I_{34}/I_{13}}} = 20\frac{0.998}{0.909}\frac{2.77}{1.16}\frac{0.85}{0.362} = 123.12 \text{ lbs in the direction shown}$$

Problem 13.12

If $\boldsymbol{Q}$ is 100 in-lb the direction shown, use energy methods to find the torque $\boldsymbol{T}_{12}$ required for equilibrium.



Solution

We will use instant centers to determine the relationships among the velocities. For this, we need I_{15}, I_{12}, I_{25}. These are shown in the figure below. From the drawing, the distances which will be needed are:

$r_{I_{25}/I_{15}} = 2.170$ in
$r_{I_{25}/I_{12}} = 1.562$ in
$r_{G/I_{15}} = 0.75$ in

Assume that link 2 rotates CCW. Then, because of the locations of I_{15}, I_{12}, and I_{25}, link 5 will rotate CW. The velocity of point G will then be in the direction shown.

From conservation of power,

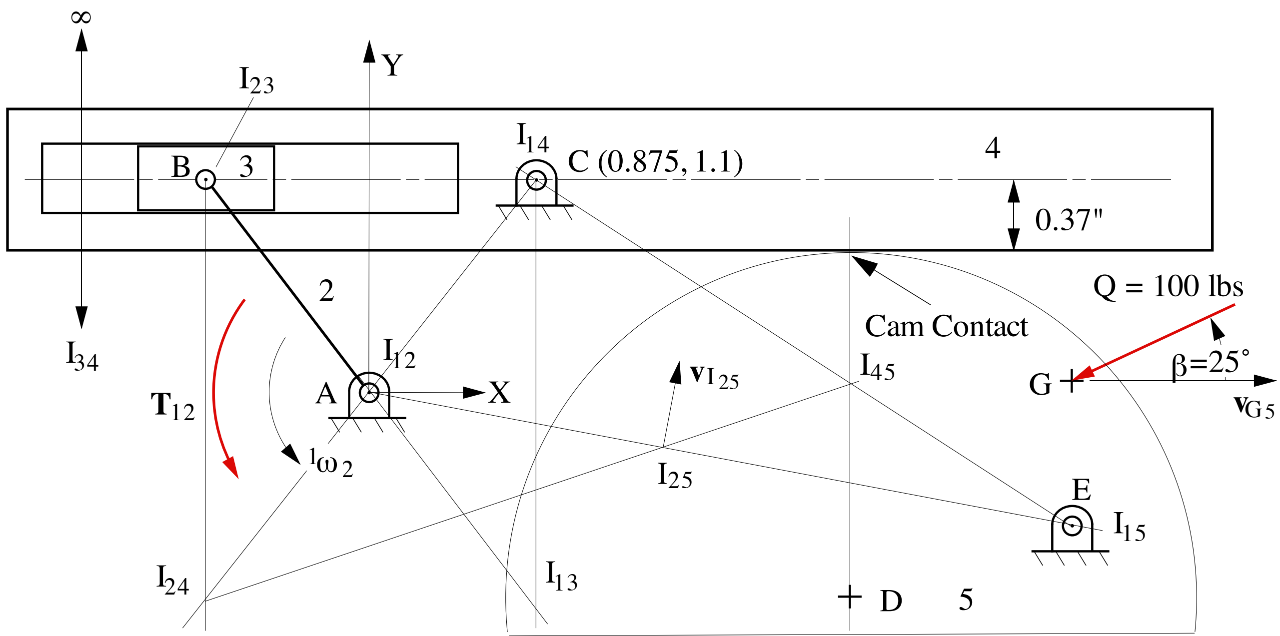
$$\mathbf{Q}\cdot\mathbf{v}_{G_5/I_{15}} + \mathbf{T}_{12}\cdot{}^1\boldsymbol{\omega}_2 = 0 \tag{1}$$

Also,

$$\mathbf{v}_{G_5/I_{15}} = {}^1\boldsymbol{\omega}_5 \times \mathbf{r}_{G_5/I_{15}}, \tag{2}$$

$$\mathbf{v}_{I_{25}} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{I_{25}/I_{12}} = {}^1\boldsymbol{\omega}_5 \times \mathbf{r}_{I_{25}/I_{15}} \tag{3}$$

and



$$\frac{|{}^1\omega_5|}{|{}^1\omega_2|} = \frac{|\mathbf{r}_{I25/I12}|}{|\mathbf{r}_{I25/I15}|} \tag{4}$$

If ${}^1\boldsymbol{\omega}_2$ is clockwise, then the component of $\mathbf{Q}$ in the direction of $\mathbf{v}_{G5/I15}$ will be negative so that power is taken out of the system at link 5 and put into the system at link 2. Therefore, the torque $\mathbf{T}_{12}$ is in the direction shown. Since the direction of $\mathbf{T}_{12}$ is then known, we need only determine the magnitude. From Eqs. (1) and (2), the magnitudes are related by

$$|\mathbf{Q}|\cos\beta\,|{}^1\omega_5|\,|\mathbf{r}_{G5/I15}| = |T_{12}|\,|{}^1\omega_2|$$

Then

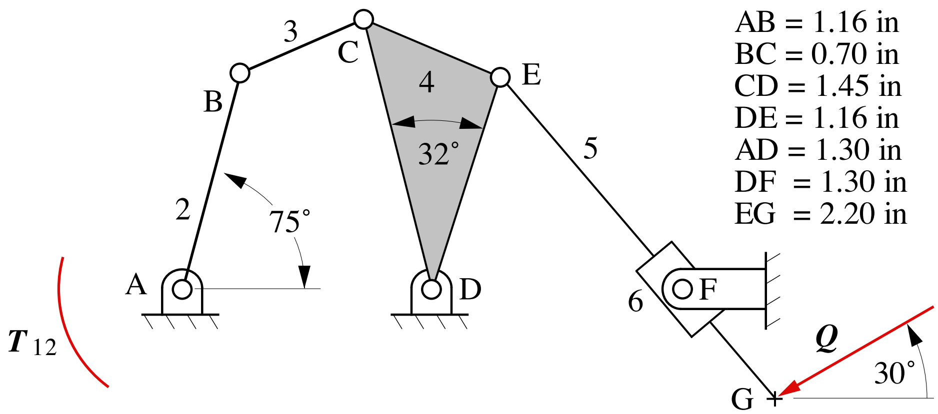
$$|T_{12}| = \frac{|\mathbf{Q}|\cos\beta\,|{}^1\omega_5|\,|\mathbf{r}_{G5/I15}|}{|{}^1\omega_2|}$$

From Eq. (4),

$$|T_{12}| = \frac{|\mathbf{Q}|\cos\beta\,|\mathbf{r}_{I25/I12}|\,|\mathbf{r}_{G5/I15}|}{|\mathbf{r}_{I25/I15}|} = \frac{100\cos 25^\circ(1.562)(0.75)}{2.170} = 48.9 \text{ in-lbs, CCW}$$

Problem 13.13

If $\boldsymbol{Q}$ is 100 in-lb the direction shown, use energy methods to find $\boldsymbol{T}_{12}$.



Solution

We will use instant centers to determine the relationships among the velocities. For this, we need I_{15}, I_{12}, I_{25}. These are shown in the figure below. From the drawing, the distances which will be needed are:

$$r_{I_{25}/I_{15}} = 2.373 \text{ in}$$
$$r_{I_{25}/I_{12}} = 0.634 \text{ in}$$
$$r_{G/I_{15}} = 2.477 \text{ in}$$

Assume that link 2 rotates CCW. Then, because of the locations of I_{15}, I_{12}, and I_{25}, link 5 will also rotate CCW. The velocity of point G will then be in the direction shown.

From conservation of power,

$$\mathbf{Q}\cdot\mathbf{v}_{G_5/I_{15}} + \mathbf{T}_{12}\cdot{}^1\boldsymbol{\omega}_2 = 0 \tag{1}$$

Also,

$$\mathbf{v}_{G_5/I_{15}} = {}^1\boldsymbol{\omega}_5 \times \mathbf{r}_{G_5/I_{15}}, \tag{2}$$

$$\mathbf{v}_{I_{25}} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{I_{25}/I_{12}} = {}^1\boldsymbol{\omega}_5 \times \mathbf{r}_{I_{25}/I_{15}} \tag{3}$$

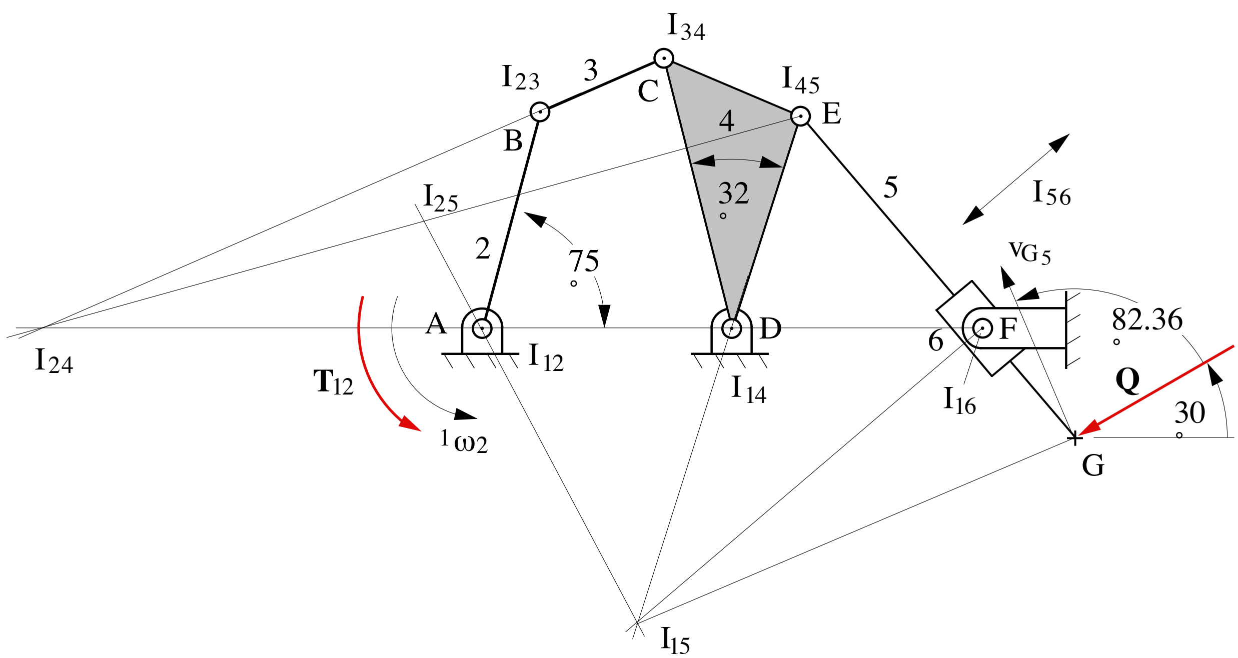
and

$$\frac{|{}^1\boldsymbol{\omega}_5|}{|{}^1\boldsymbol{\omega}_2|} = \frac{|\mathbf{r}_{I_{25}/I_{12}}|}{|\mathbf{r}_{I_{25}/I_{15}}|} \quad (4)$$

If ${}^1\omega_2$ is clockwise, then the component of $\mathbf{Q}$ in the direction of $\mathbf{v}_{G_5/I_{15}}$ will be negative so that power is taken out of the system at link 5 and put into the system at link 2. Therefore, the torque $\mathbf{T}_{12}$ is in the direction shown. Since the direction of $\mathbf{T}_{12}$ is then known, we need only determine the magnitude. From Eqs. (1) and (2), the magnitudes are related by

$$|\mathbf{Q}|\cos 82.36^\circ |{}^1\boldsymbol{\omega}_5| |\mathbf{r}_{G_5/I_{15}}| = |\mathbf{T}_{12}||{}^1\boldsymbol{\omega}_2|$$

Then



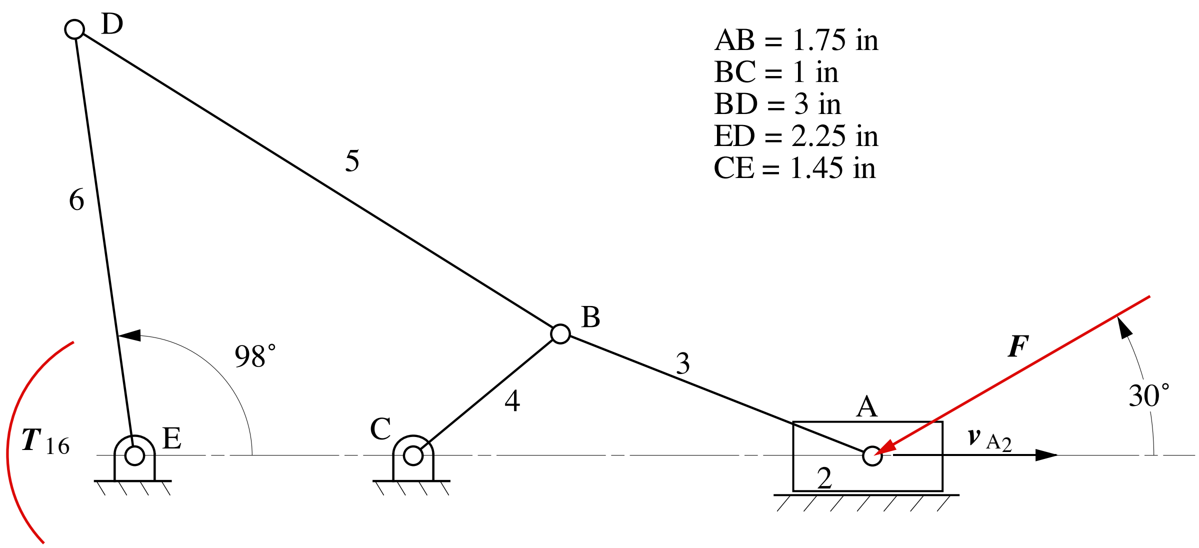
$$|T_{12}| = \frac{|\mathbf{Q}|\cos 82.36^\circ |{}^1\omega_5| |\mathbf{r}_{G5/I15}|}{|{}^1\omega_2|}$$

From Eq. (4),

$$|T_{12}| = \frac{|\mathbf{Q}|\cos 82.36^\circ |\mathbf{r}_{I25/I12}| |\mathbf{r}_{G5/I15}|}{|\mathbf{r}_{I25/I15}|} = \frac{100\cos 82.36^\circ (0.634)(2.477)}{2.373} = 8.80 \text{ in-lbs, CCW}$$

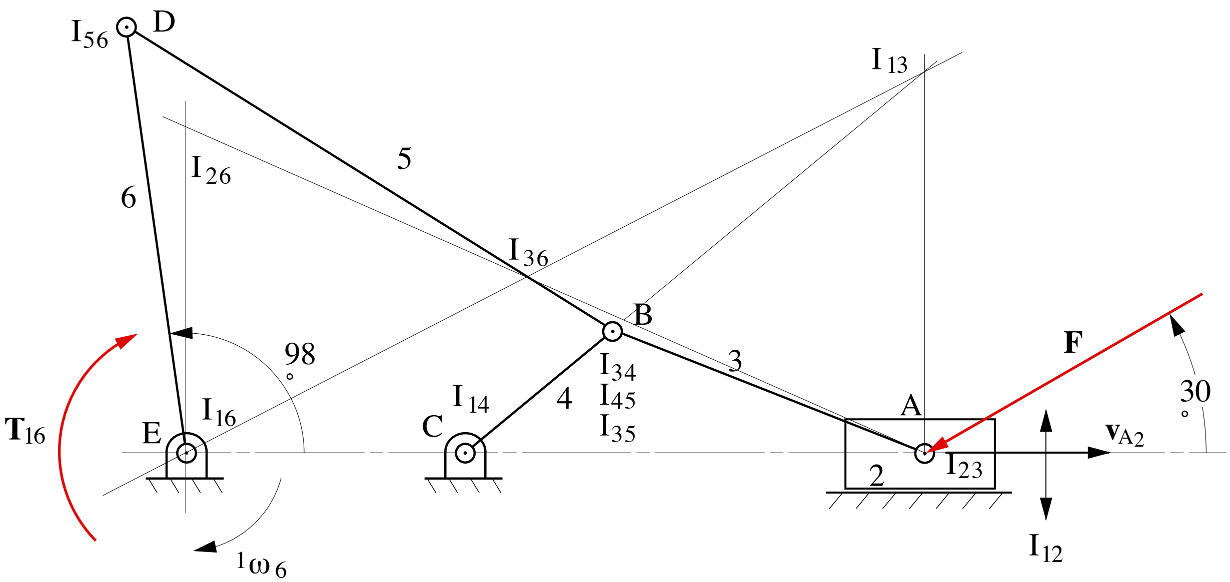
Problem 13.14

If the velocity of link 2 is 10 in/s, and the force on link 2 is 100 in-lb the direction shown, find the torque on link 6 required to maintain equilibrium in the mechanism.



Solution:

Because only the force $\mathbf{T}_{12}$ is of interest, this problem can be solved easily using energy methods and instant centers. For this we need the instant centers I_{12}, I_{26}, and I_{16}. We can get I_{12} and I_{16} by inspection. To find I_{26}, redraw the mechanism to scale and locate the instant centers as shown in the figures below.



If link 2 moves to the right, then $^1\omega_6$ will be in the direction shown above (CW). From conservation of power,

$$\mathbf{T}_{16} \cdot {}^1\boldsymbol{\omega}_6 + \mathbf{F} \cdot \mathbf{v}_{A_2} = 0 \tag{1}$$

Also,

$$\mathbf{v}_{I_{26}} = \mathbf{v}_{A_2} = {}^1\boldsymbol{\omega}_6 \times \mathbf{r}_{I_{26}/I_{12}}, \tag{2}$$

From the figure above, the force F is generally opposite to the velocity $\mathbf{v}_{A_2}$. Therefore, power is taken out of the system at link 2 and put into the system at link 6. Therefore, $\mathbf{T}_{16}$ and $^1\boldsymbol{\omega}_6$ are in the same direction (CW). Since the direction of $\mathbf{T}_{16}$ is then known, we need only determine the magnitude. From Eq. (1), the magnitudes are related by

$$|\mathbf{T}_{16}||{}^1\omega_6| = |\mathbf{F}|\cos 30^\circ|\mathbf{v}_{A_2}| = |\mathbf{F}|\cos 30^\circ|{}^1\omega_6||\mathbf{r}_{I_{26}/I_{12}}|$$

Then

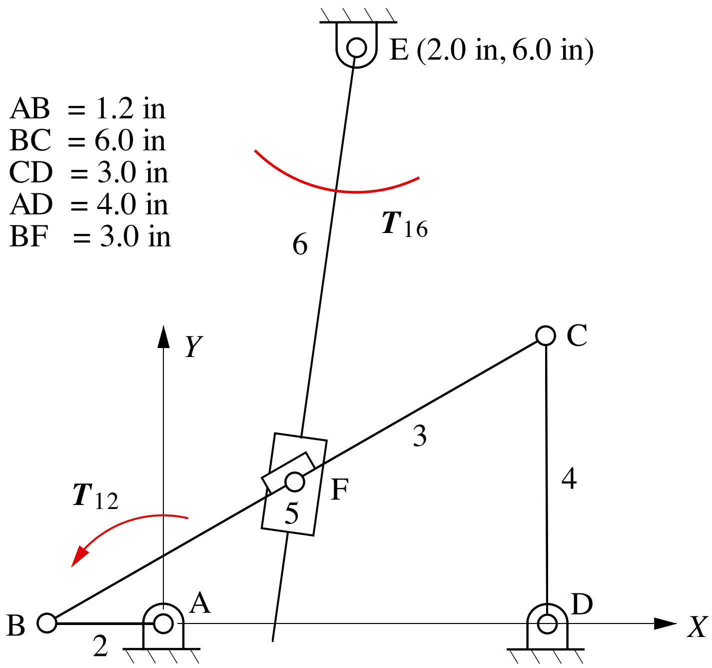
$$|\mathbf{T}_{16}| = \frac{|\mathbf{F}|\cos 30^\circ|{}^1\omega_6||\mathbf{r}_{I_{26}/I_{12}}|}{|{}^1\omega_6|} = |\mathbf{F}|\cos 30^\circ|\mathbf{r}_{I_{26}/I_{12}}|$$

From the drawing, $\mathbf{r}_{I_{26}/I_{12}}$ = 1.6945 in. Therefore,

CW.

Problem 13.15

In the mechanism shown, point F is a swivel at the midpoint of Link 3 that carries link 5. The motion of the four-bar linkage causes arm 6 to oscillate. If link 2 rotates counterclockwise at 12 rad/s and is driven by a torque of 20 ft-lb, determine the resisting torque on link 6 required for equilibrium.



Solution:

Because only the torque $\mathbf{T}_{16}$ is of interest, this problem can be solved easily using energy methods and instant centers. For this we need the instant centers I_{12}, I_{26}, and I_{16}. We can get I_{12} and I_{16} by inspection. To find I_{26}, redraw the mechanism to scale and as indicated in the figure below. Distances which will be needed in the analysis are

$$r_{I_{26}/I_{16}} = 5.991 \text{ in}$$

and

$$r_{I_{26}/I_{12}} = 0.324 \text{ in}$$

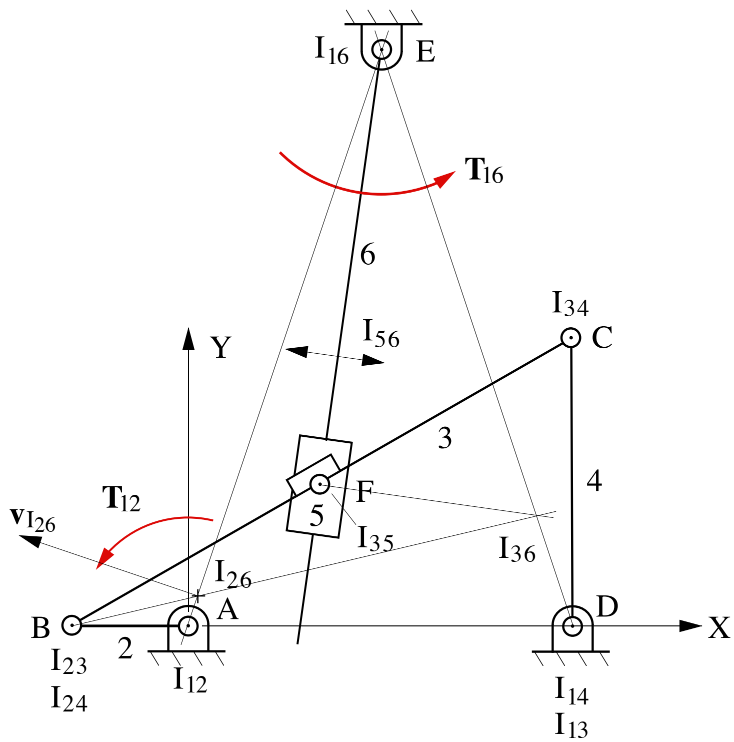
Link 2 rotates CCW and power is put into the linkage at link 2. Therefore, $\mathbf{T}_{12}$ is also CCW. From conservation of power,

$$\mathbf{T}_{12} \cdot {}^{1}\boldsymbol{\omega}_{2} + \mathbf{T}_{16} \cdot {}^{1}\boldsymbol{\omega}_{6} = 0 \qquad (1)$$

Also,

$$\mathbf{v}_{I_{26}} = {}^{1}\boldsymbol{\omega}_{2} \times \mathbf{r}_{I_{26}/I_{12}} = {}^{1}\boldsymbol{\omega}_{6} \times \mathbf{r}_{I_{26}/I_{16}} \qquad (2)$$

Power is taken out at link 6, and therefore, $\mathbf{T}_{16}$ and ${}^1\omega_6$ are in opposite directions. Because of the location of the instant centers I_{12}, I_{26}, and I_{16}, ${}^1\omega_6$ is CW. Therefore, $\mathbf{T}_{16}$ must be CCW. Because we know the direction of $\mathbf{T}_{16}$, we need only determine the magnitude. From Eq. (1),



$$|\mathbf{T}_{16}| = \frac{|\mathbf{T}_{12}||{}^1\omega_2|}{|{}^1\omega_6|}$$

and from Eq. (2),

$$\frac{|{}^1\omega_2|}{|{}^1\omega_6|} = \frac{|\mathbf{r}_{I26/I16}|}{|\mathbf{r}_{I26/I12}|}$$

Therefore,

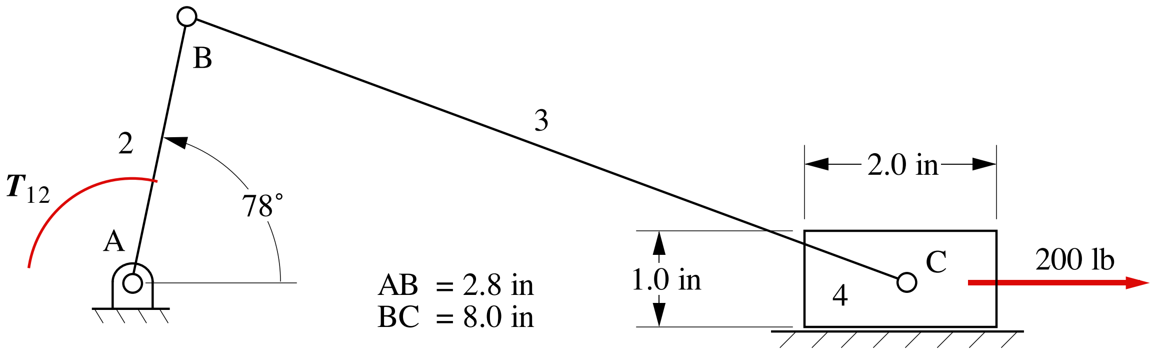
$$|\mathbf{T}_{16}| = \frac{|\mathbf{T}_{12}||\mathbf{r}_{I26/I16}|}{|\mathbf{r}_{I26/I12}|}$$

or

$$\mathbf{T}_{16} = \frac{|\mathbf{T}_{12}||\mathbf{r}_{I26/I16}|}{|\mathbf{r}_{I26/I12}|} = \frac{20(5.991)}{0.324} = 369.8 \text{ ft} - \text{lbs CCW}$$

Problem 13.16

Find the torque $\boldsymbol{T}_{12}$ for a coefficient of friction μ of 0.0 and 0.2. Assume that the radius of each pin is 1 in, and consider both pin and slider friction. Link 2 rotates CW.



Solution:

When μ = 0.0, the free-body diagram of the zero friction mechanism is shown below.

By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:

$$\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34}$$

and

$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$

This gives us the general direction of all forces at the joints. To determine the torque $\mathbf{T}_{12}$ for equilibrium in the non-friction case, sum moments about point A of the free-body diagram for link 2. From this, we get

$$T_{12} = hF_{32} = (2.76)(213) = 588 \text{ in-lbs}$$

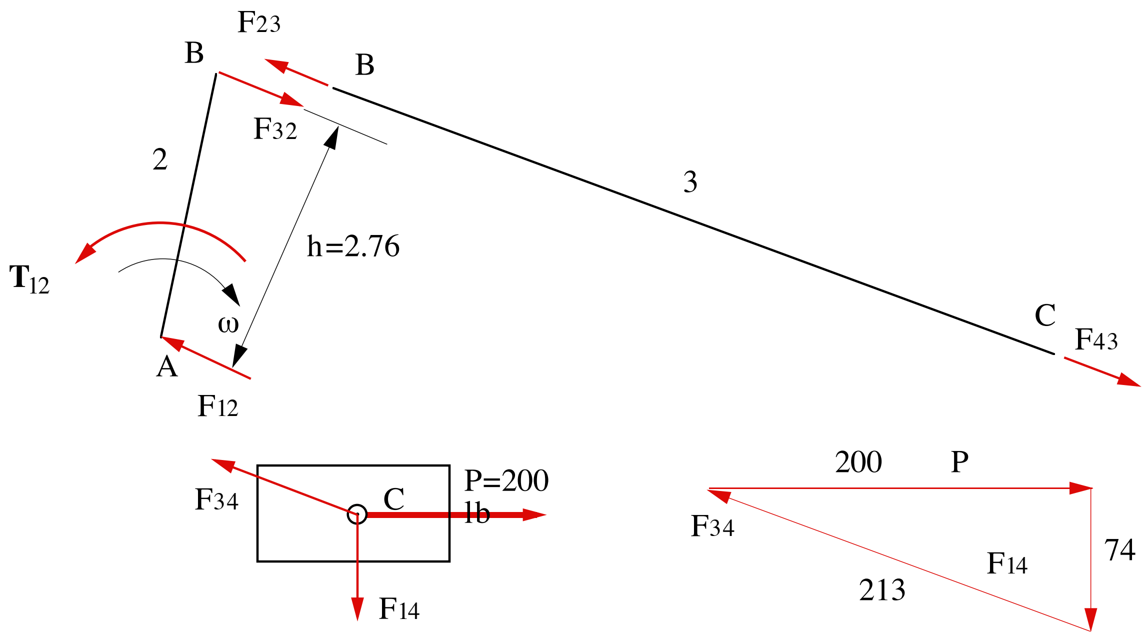
By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=588 in-lbs, CCW (no friction case)

To analyze the system with friction, we need to compute the friction angle and friction circle radius for each joint. The friction angle is

$$\phi = \tan^{-1}(\nu) = \tan^{-1}(0.2) = 11.31°$$

and the friction circle radius at each joint is



$$R_f = R\sin\phi = 1\sin(11.31°) = 0.12\text{in}$$

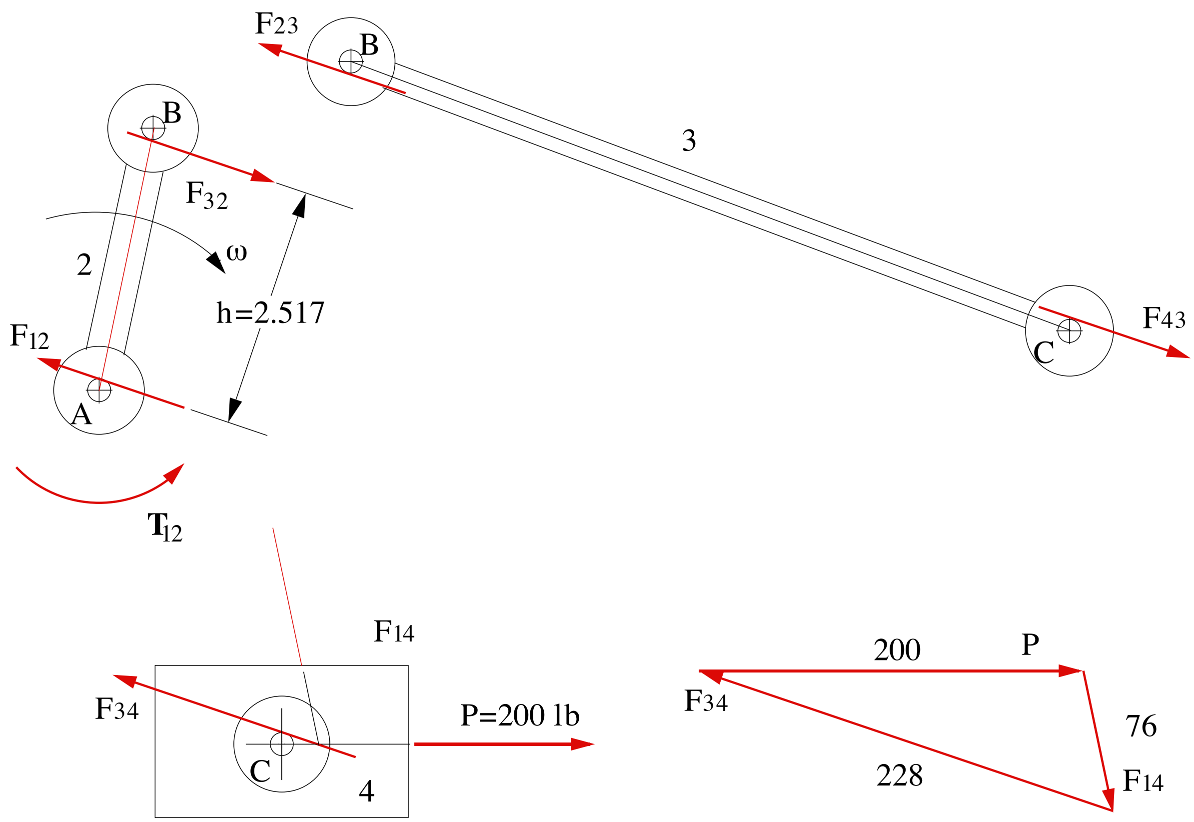
After we determine the relative motion at each joint and the force directions for the non-friction mechanism, we can draw the free-body diagram of the mechanism with friction, as shown below.

We can sum the forces on link 4 again, and solve for the unknowns as shown in the force triangle. The magnitude of $\mathbf{T}_{12}$ is then given by

$$T_{12} = hF_{32} = (2.517)(228) = 573 \text{ in-lbs}$$

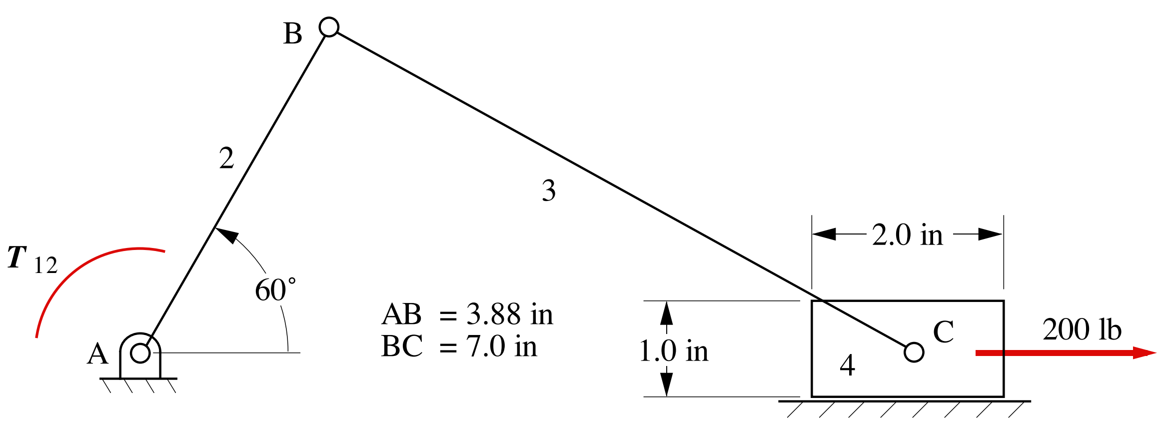
By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=573 in-lbs, CCW (friction case)



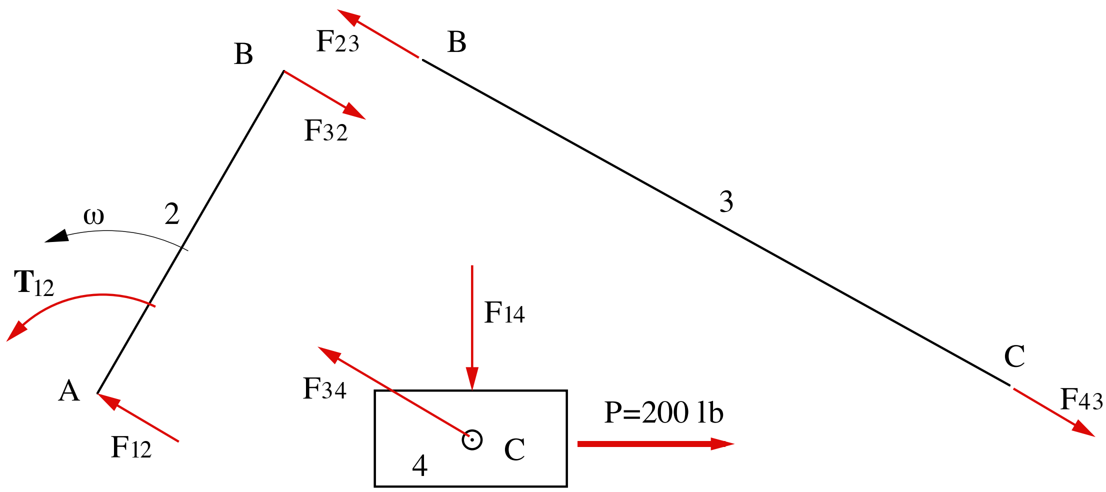
Problem 13.17

For the position given for the slider crank mechanism, find the torque T_{12} required for equilibrium. The radius of each pin is 1 in, and the friction coefficient at the pin between links 3 and 4 and between the block and the frame is 0.3. Elsewhere, the coefficient of friction is 0. Link 2 rotates CCW.



Solution:

The freebody diagram of the zero friction mechanism is shown below.



To analyze the system with friction, we need to compute the friction angle and friction circle radius for each joint. The friction angle is

$$\phi = \tan^{-1}(\nu) = \tan^{-1}(0.3) = 16.7°$$

and the friction circle radius at thejoint at point C (the only pin joint with friction) is

$$R_f = R\sin\phi = 1\sin(16.7°) = 0.29\text{in}$$

After we determine the relative motion at each joint, and the force direction of the non-friction mechanism, we can draw the free-body diagram of the mechanism with friction, as shown below. Note that friction is involved only at the joints of the slider. The force $\mathbf{F}_{34}$ is located on the bottom of the friction circle because the force must create a moment about point C which opposes the relative motion of link 4 relative to link 3.

By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. In particular, $F_{34} = 269\text{ lbs}$.

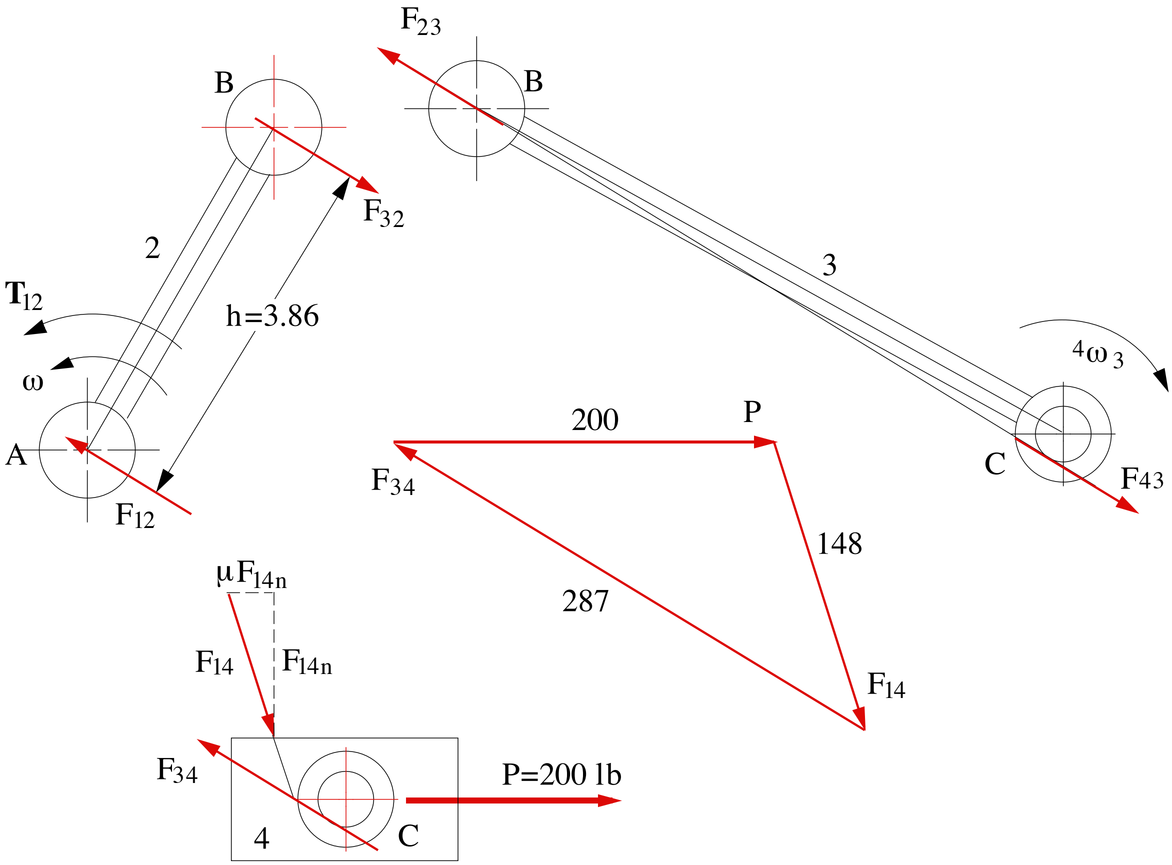
From equilibrium considerations at each joint, we know:

$$\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34}$$

and

$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$

This gives us the general direction of all forces at the joints. To determine the torque $\mathbf{T}_{12}$ for equilibrium in the friction case, sum moments about point A of the free-body diagram for link 2. From this we get



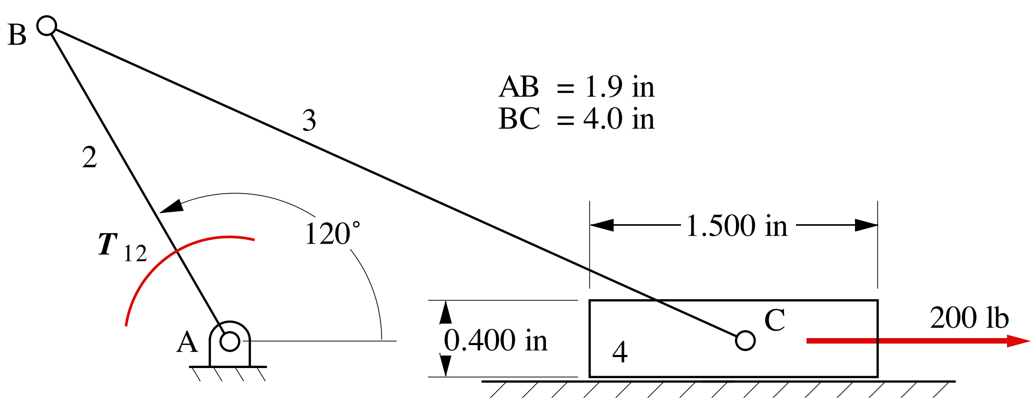
$$T_{12} = hF_{32} = (3.86)(287) = 1108 \text{ in-lbs}$$

By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=1108 in-lbs, CCW (friction case)

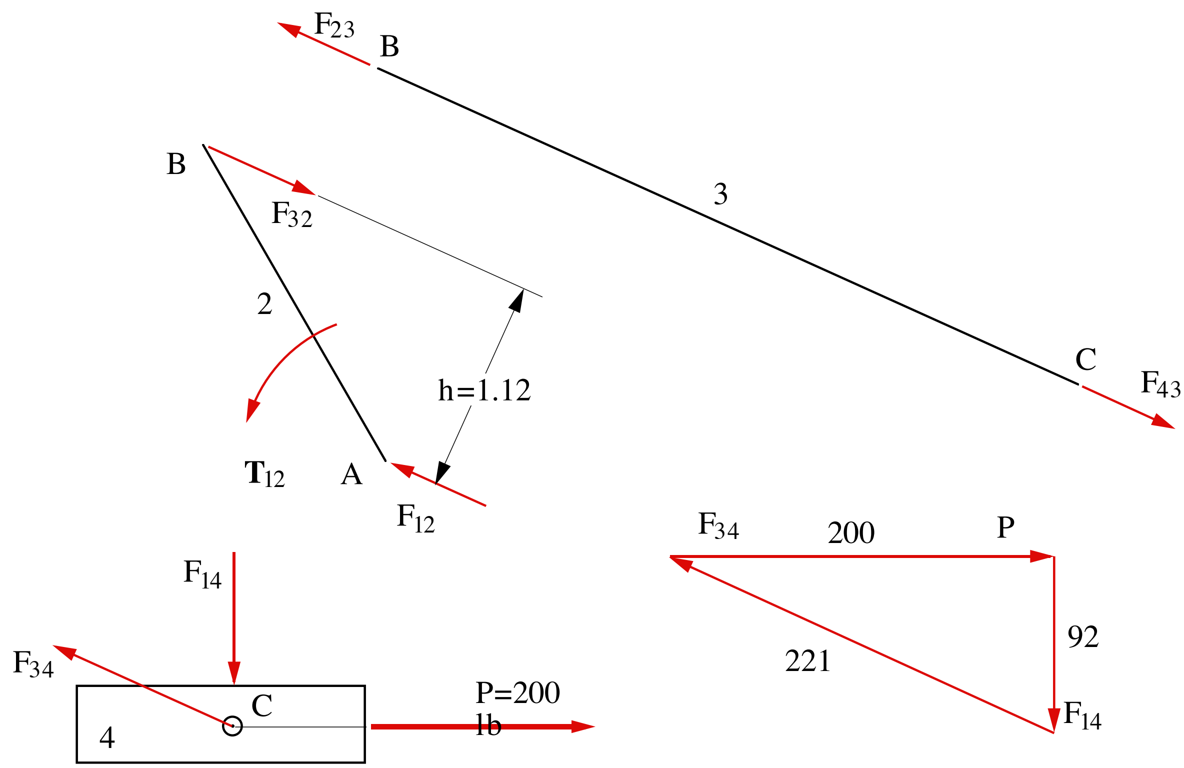
Problem 13.18

If the radius of each pin is 0.9 in and the coefficient of friction at all joints is 0.15, find the torque $\boldsymbol{T}_{12}$ required for equilibrium in the position shown. Link 2 rotates CCW.



Solution:

First perform a force analysis of the mechanism without friction to determine the general direction of the forces. A free-body diagram of the friction free mechanism is shown below.



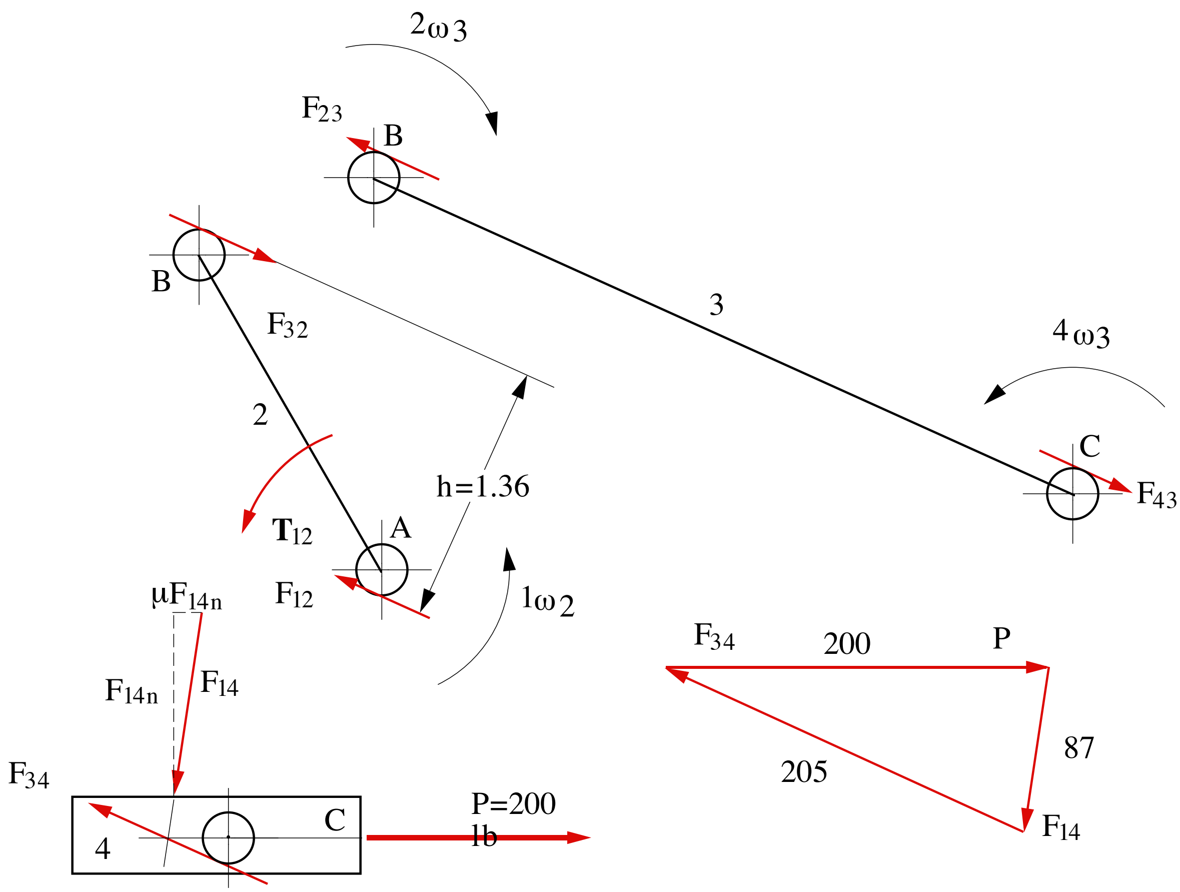
For the friction case, the friction angle is

$$\phi = tan^{-1}(\mu) = tan^{-1}(0.15) = 8.53°$$

The friction circle radius at each joint is

$$R_f = R\sin\phi = 0.9\sin(8.53) = 0.13 \text{ in}$$

To determine the location of the forces relative to the friction circles, we must determine the direction of the relative motion at each joint. The forces will then be located to create a torque which opposes the relative motion. Free-body diagrams of each of the links are shown in the following.



By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:

$$\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 205 \text{ lbs}$$

and

$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$

This gives us the general direction of all forces at the joints. To determine the torque $\mathbf{T}_{12}$ for equilibrium, sum moments about point A of the free-body diagram for link 2. From this we get

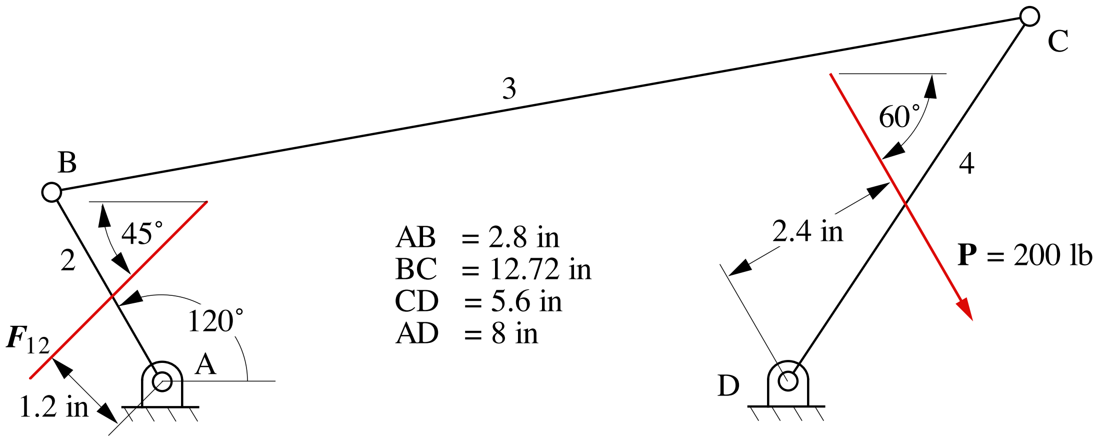
$$T_{12} = hF_{32} = (1.36)(205) = 279 \text{ in-lbs}$$

By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=279 in-lbs, CCW

Problem 13.19

If the coefficient of friction is 0.4 at each pin and each pin radius is 1 in, determine the force ***F*** required for equilibrium. Link 2 rotates CCW.



Solution:

The free-body diagrams and force polygons for the zero friction mechanism are shown below. This gives the approximate directions for each of the forces.

To analyze the system with friction, we need to compute the friction angle and friction circle radius for each joint. The friction angle is

$$\phi = \tan^{-1}(\mu) = \tan^{-1}(0.4) = 21.8°$$

and the friction circle radius at each joint is

$$R_f = R\sin\phi = 1\sin(21.8°) = 0.37\text{in}$$

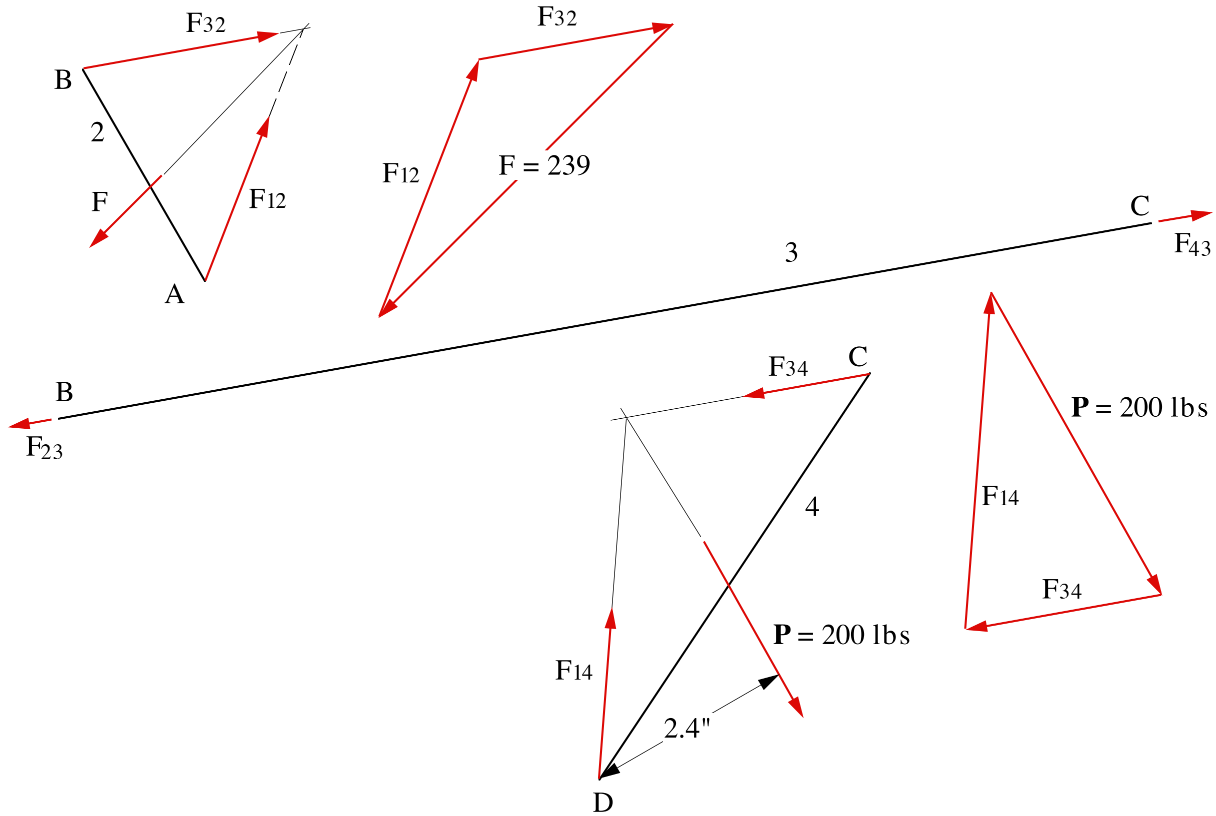
After we determine the relative motion at each joint, and the force direction of the non-friction mechanism, we can draw the free-body diagram of the mechanism with friction.

The relative motion at each joint can be determined by inspection by visualizing how the two links at each joint move relative to each other when link 2 is rotating CCW. If it is not possible to determine the relative velocities by inspection, we could do a velocity analysis. It is not necessary to determine the magnitudes of the angular velocities; we need only determine the directions.

By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:



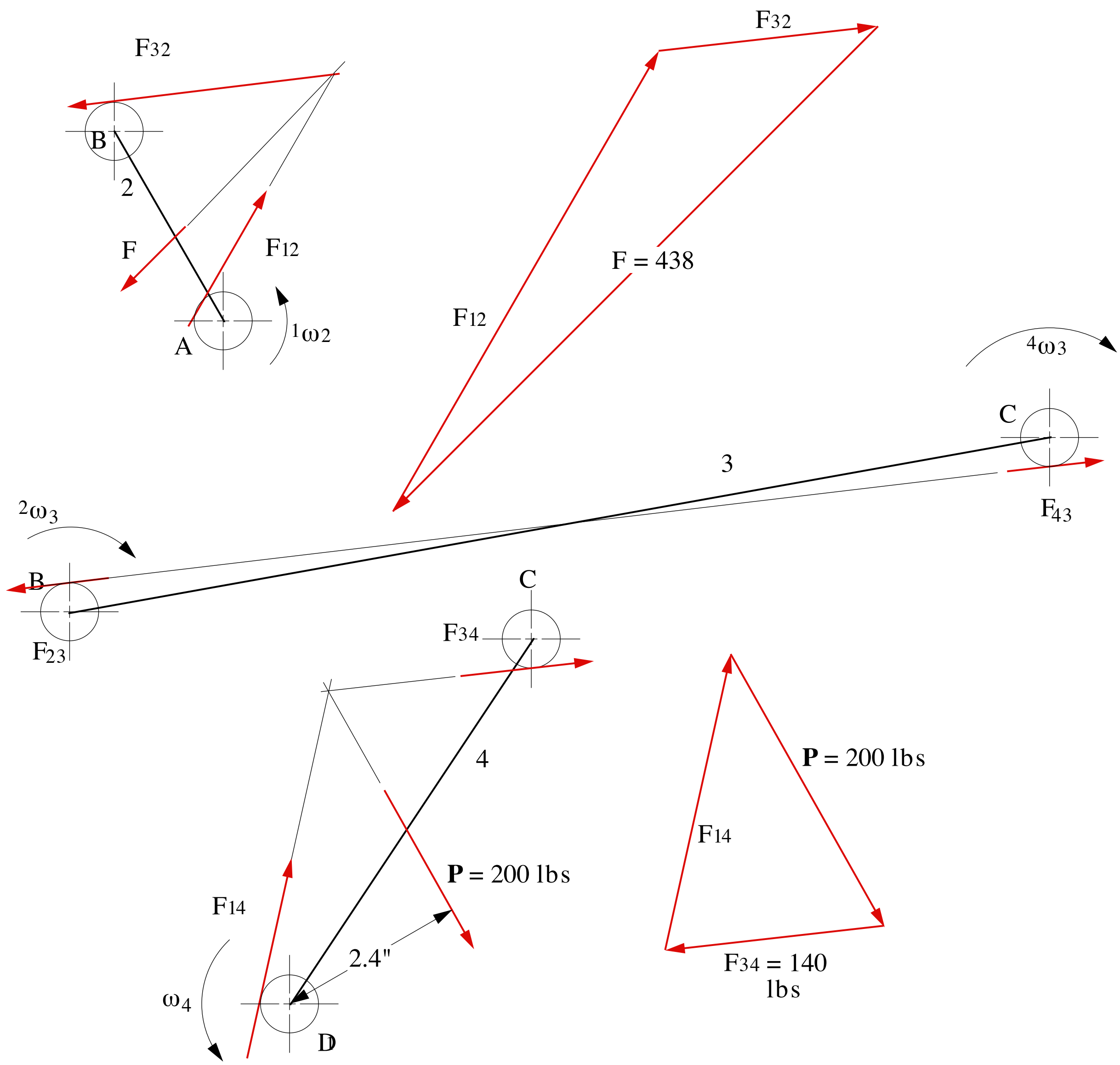
$$\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 140 \text{ lbs}$$

This gives us the general direction of all forces at the joints. To determine **F** for equilibrium in the friction case, sum forces vectorially on link 2. From this we get

$$\sum \mathbf{F} = 0 = \mathbf{F}_{12} + \mathbf{F} + \mathbf{F}_{32}$$

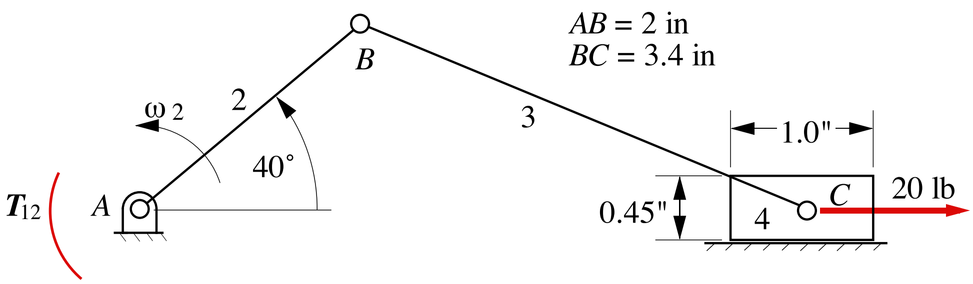
By inspection, the force **F** must be down and to the left. Therefore,

F=438 lbs in the direction shown.



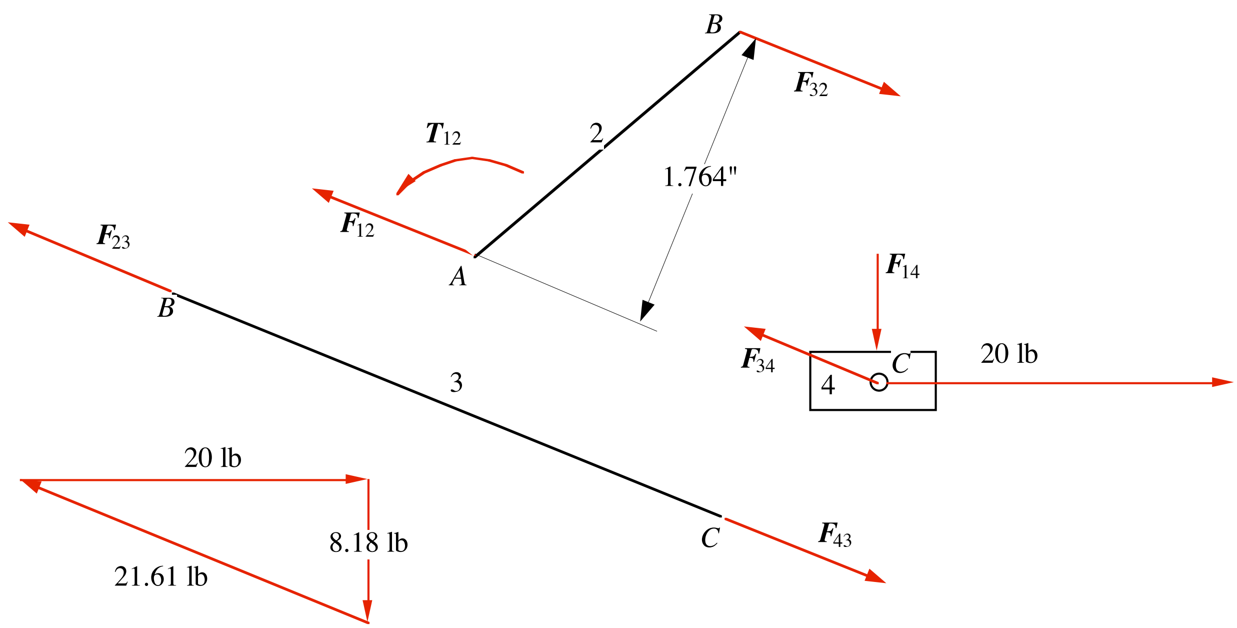
Problem 13.20

Find the torque T_{12} for a coefficients of friction μ of 0.0 and 0.2. Consider friction at the slider only and neglect the masses of the links.



Solution:

First perform a force analysis of the mechanism without friction to determine the general direction of the forces. A free-body diagram of the friction free mechanism is shown below.



By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:

$$\boldsymbol{F}_{32} = -\boldsymbol{F}_{23} = \boldsymbol{F}_{43} = -F_{34} = 21.61\, lbs$$

and

$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$

To determine the torque $\boldsymbol{T}_{12}$ for equilibrium, sum moments about point A of the free-body diagram for link 2. From this we get

$$T_{12} = hF_{32} = (1.764)(21.61) = 38.12 \text{ in-lbs.}$$

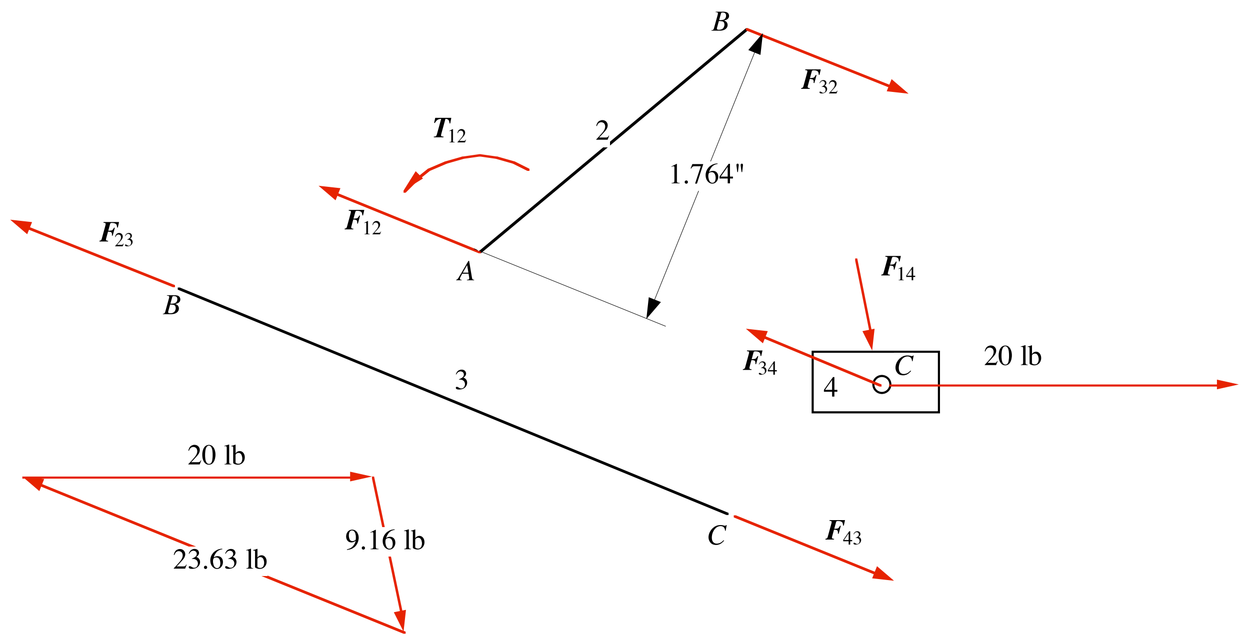
By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=38.12 in-lbs, CCW

For the friction case, the friction angle is

$$\phi = tan^{-1}(\mu) = tan^{-1}(0.2) = 11.31°$$

There is friction between the slider and the frame only. The slider is moving to the left. The new free body diagrams are given in the following.



By summing forces vectorially on link 4, the magnitudes of all the forces can be determined. The force summation equation is

$$\sum \mathbf{F} = 0 = \mathbf{F}_{14} + \mathbf{P} + \mathbf{F}_{34}$$

The force polygon gives the magnitude and direction for each of the vectors. From equilibrium considerations at each joint, we know:

$$\boldsymbol{F}_{32} = -\boldsymbol{F}_{23} = \boldsymbol{F}_{43} = -F_{34} = 23.63\,lbs$$

and

$$\mathbf{F}_{12} = -\mathbf{F}_{32}$$

This gives us the general direction of all forces at the joints. To determine the torque $\mathbf{T}_{12}$ for equilibrium, sum moments about point A of the free-body diagram for link 2. From this we get

$$T_{12} = hF_{32} = (1.764)(23.63) = 41.68 \text{ in-lbs}$$

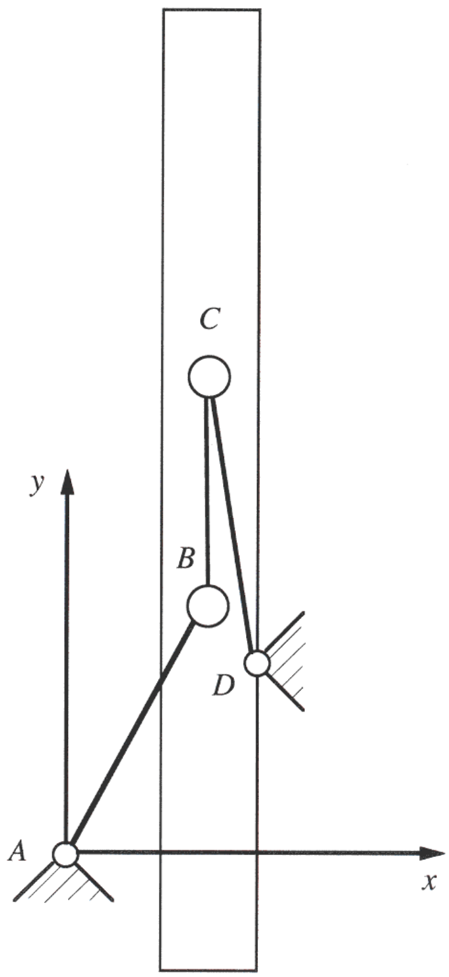
By inspection, the torque must be CCW. Therefore,

$\mathbf{T}_{12}$=41.68 in-lbs, CCW

Problem 13.21

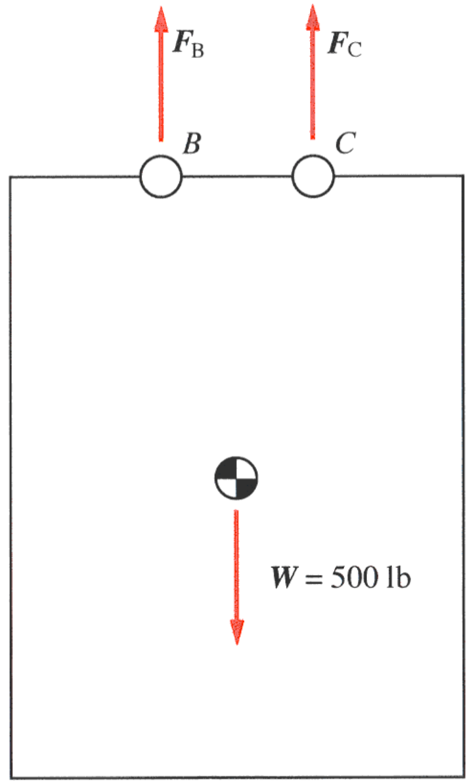
The linkage shown is a type I double-rocker four-bar linkage in which the two coupler pivots, *B* and *C* have been replaced by spherical joints. It moves in the *horizontal* plane and is used to guide and support a heavy door that is hung from the coupler. The vertical crank shafts are each supported on a tapered roller bearing that supports both radial and thrust loads. The centers of these bearings are 1.5 in below the horizontal plane in which the spherical joint centers lie. The crank shafts are also supported by ball bearings that support only radial loads. The central plane of the ball bearings is 2 in above the plane of the spherical joint centers. The door weighs 500 lb, including the weight of the coupler, and its center of mass is directly below the mid-point of the coupler. Each of the cranks *AB* and *CD* weighs 30 lb and can be modeled as a uniform rod. Find the radial loads carried by the ball bearings and the radial and thrust loads carried by the tapered roller bearings when the linkage is in the position shown.

AB = *CD* = 18.0 in, *BC* = 14.5 in, *AD* = 17.0 in. The line *AD* is at 45‡ to the *x* axis.

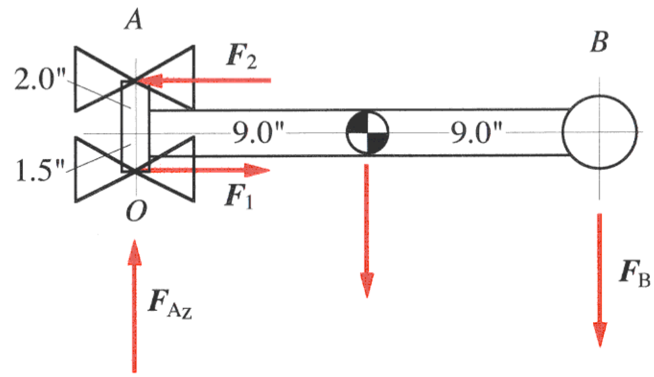


Solution:

Free body diagrams



Door



Arm

For the door:

$$\sum F_z = 0: \quad F_B + F_C$$
$$\sum M_G = 0: \quad 7.25F_C - 7.25F_B = 0$$

Hence $F_B = F_C = 250$ lb

For either arm:

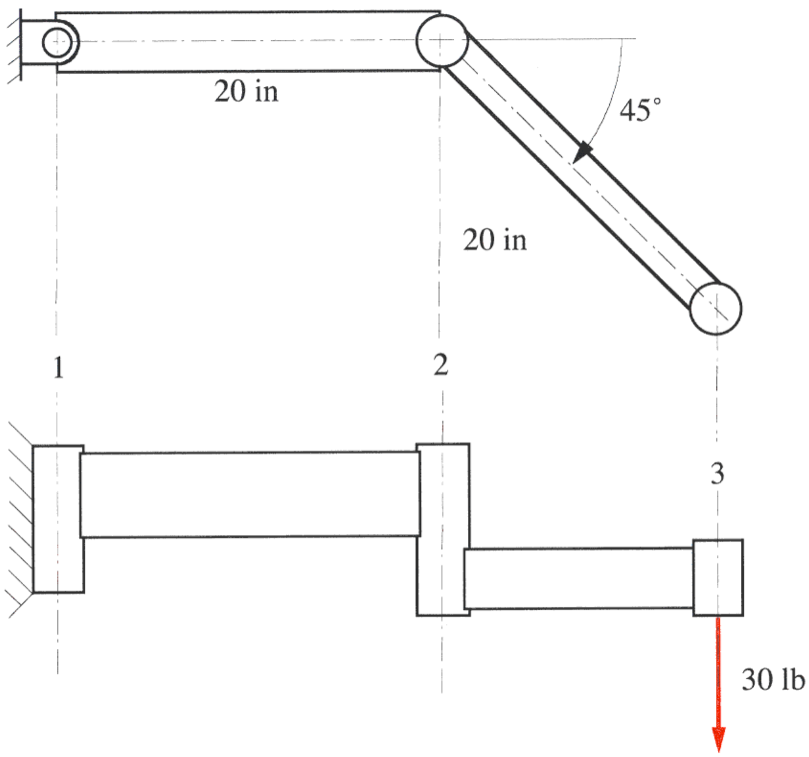
$$\sum F_z = 0: \quad F_z = 250 + 30 = 280 \text{ lb}$$
$$\sum F_y = 0: \quad F_1 = F_2$$
$$\sum M_O = 0: \quad 3.5F_2 = 9.0 \times 30 + 18.0 \times 250$$
$$F_1 = F_2 = 1363 \text{ lb}$$

Problem 13.22

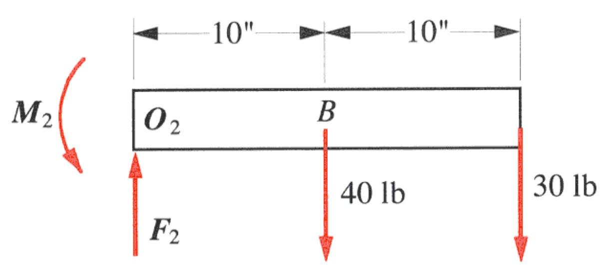
The robot of Example 13.8 is positioned so that the angle between members *A* and *B* is 45◊, and the payload is 30 lb located on the axis of joint 3. Characterize the loads on members *A* and *B* in this position.

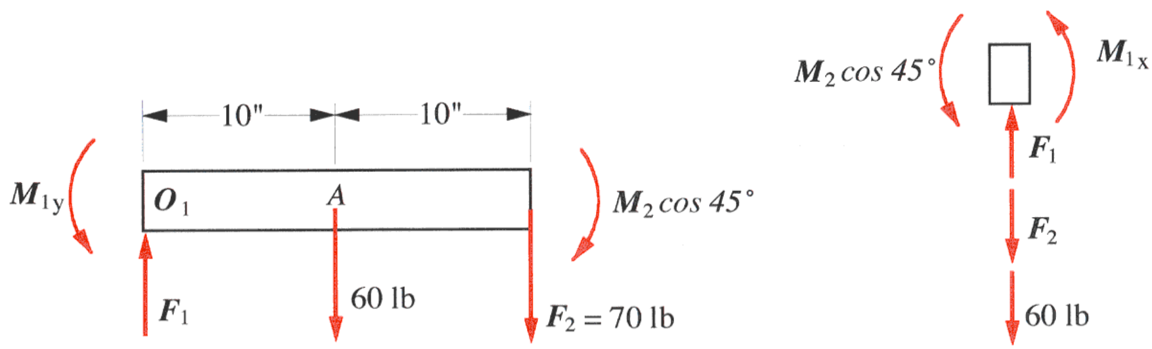
Solution:

Configuration:



Free body diagrams:



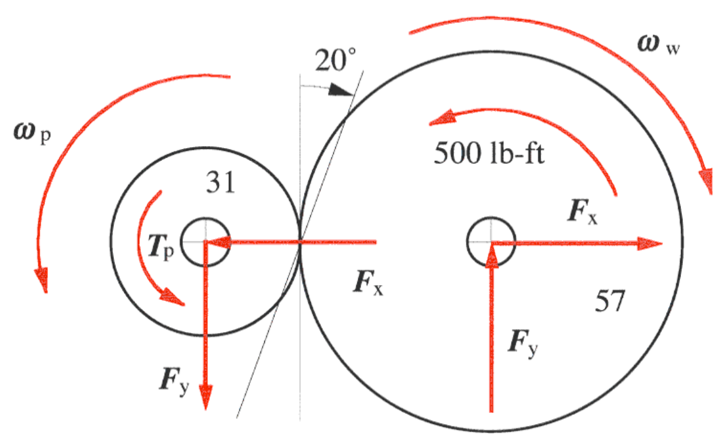


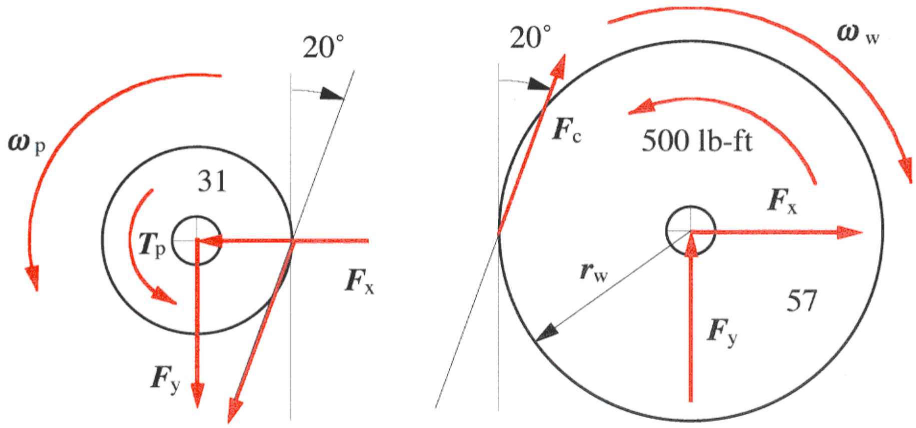
Problem 13.23

A pair of spur gears with 31 and 57 teeth is cut to standard 20° pressure angle dimensions with diametral pitch 16. The wheel carries a load torque of 500 lb ft. Find the reaction loads carried by the bearings on which the gears turn.

Solution:

Free body diagrams





For wheel

$$d_p = \frac{N}{P_d} = \frac{57}{16}$$

so

$$r_w = \frac{57}{32} = 1.7813 \text{ in}$$

Hence

$$F_x + F_c \sin 20° = 0$$
$$F_y + F_c \cos 20° = 0$$
$$r_w F_c \cos 20° = 500 \text{ lb - ft}$$

so

$$F_c = 298.7 \text{ lb}$$

$$F_x = -298.7 \sin 20° = -102.2 \text{ lb}$$
$$F_y = -298.7 \cos 20° = -280.7 \text{ lb}$$

For pinion

$$M_p = \frac{31}{57} 500 = 271.9 \text{ lb - ft}$$

Problem 13.24

A helical gear is cut with a standard 25◊ pressure angle, diametral pitch 8 spur gear cutter at a helix angle of 30◊. The gear has 50 teeth and transmits 150 hp at a speed of 1500 rpm. Calculate the radial and thrust loads that must be carried by the mounting bearings. Also, compute the couple due to the axial component of the tooth loads. If the shaft bearings are to be 5 in apart, compute the additional radial bearing load from this source.

Solution:

$$r_p = \frac{N}{2P} = \frac{50}{2\times 8} = 3.125 \text{ in}$$

$$W_t = \frac{(150\times 550)12\times 60}{2\pi 1500\times 3.125} = 2017 \text{ lb}$$

$$W_r = W_t \tan\psi_t = W_t \frac{\tan\psi_n}{\cos\alpha} = 2017\frac{\tan 25^\circ}{\cos 30^\circ} = 1086 \text{ lb}$$

$$W_a = 2017\tan 30^\circ = 1165 \text{ lb}$$

Component in the plane of the wheel

$$W = \sqrt{W_r^2 + W_t^2} = \sqrt{1086^2 + 2017^2} = 2291 \text{ lb}$$

$$T_a = 2017\times 3.125\tan 30^\circ = 3639 \text{ in} - \text{lb}$$

$$F = \frac{3639}{5} = 728 \text{ lb}$$