

Lecturer's Solutions Manual

Gas Turbine Theory

Sixth edition

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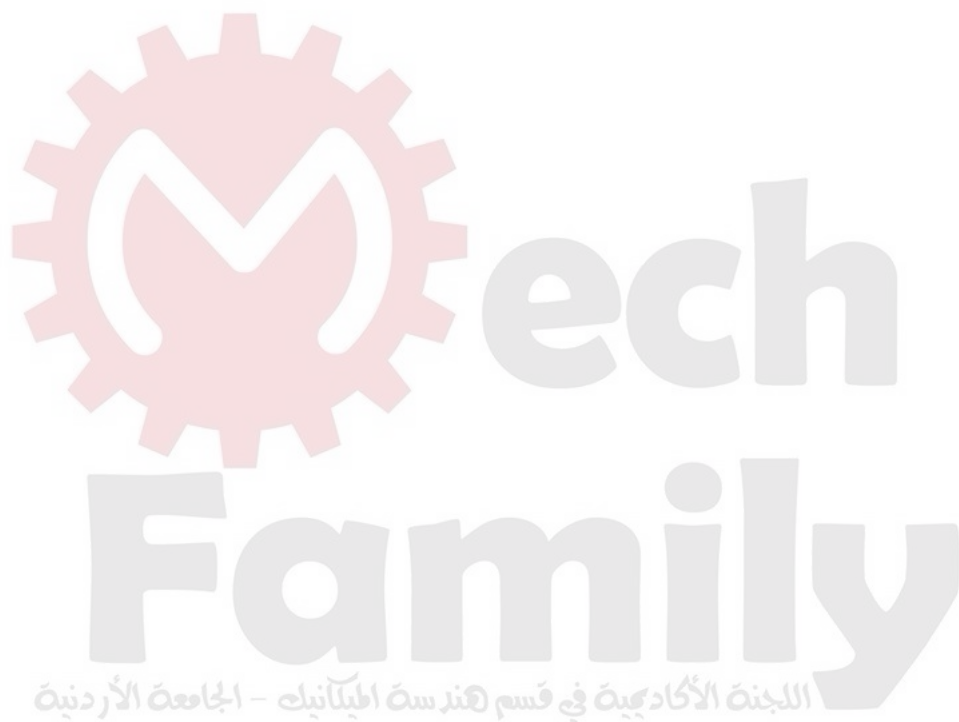
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Preface

Since the introduction of the Second Edition in 1972 many requests for solutions have been received. The advent of modern word processing systems has now made it convenient to prepare these in an electronic format and I am glad to do so.

All significant gas turbine calculations carried out in industry are universally done by digital computer and the purpose of these problems is to provide an understanding of the engineering principles involved. It is perhaps of historical significance that all of these calculations were originally done on a slide rule and many were previous examination questions.

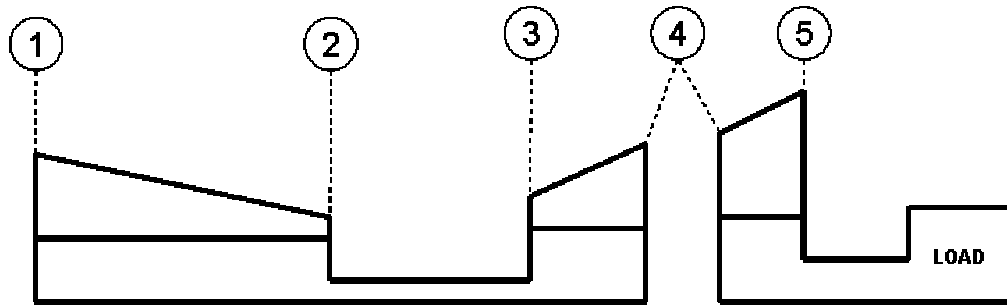
This manual will be available to instructors adopting the main text, who are then permitted to photocopy the material, but it is hoped that students will tackle the problems before looking at the manual. I will be very glad to hear of any corrections needed.

My sincere thanks to Mr. Hariharan Hanumanthan, a doctoral student at Cranfield University, for his invaluable assistance in the preparation of this manual.

H.I.H. Saravanamuttoo
Ottawa, June 2008



Problem 2.2



$$T_{02} - T_a = \frac{T_a}{\eta_c} \left[\left(\frac{P_{02}}{P_a} \right)^{\left(\frac{\gamma-1}{\gamma} \right)} - 1 \right]$$

$$= \frac{288}{0.82} \left[\left(11 \right)^{\left(\frac{1}{3.5} \right)} - 1 \right] = 345.598 \text{ K}$$

Compressor and turbine work required per unit mass flow is:

$$W_{tc} = \frac{C_{pa}(T_{02} - T_a)}{\eta_m} = C_{pg}(T_{03} - T_{04})$$

$$T_{03} - T_{04} = \frac{1.005 \times 345.598}{0.98 \times 1.147} = 308.992 \text{ K}$$

$$T_{04} = 1150 - 309 = 841 \text{ K}$$

$$T_{03} - T_{04} = \eta_t T_{03} \left[1 - \left(\frac{1}{P_{03}/P_{04}} \right)^{\left(\frac{\gamma-1}{\gamma} \right)} \right]$$

$$308.992 = 0.87 \times 1150 \left[1 - \left(\frac{1}{P_{03}/P_{04}} \right)^{\left(\frac{1}{4} \right)} \right]$$

$$\frac{P_{03}}{P_{04}} = 4.382$$

$$P_{03} = 11.0 - 0.4 = 10.6 \text{ bar}$$

$$P_{04} = 2.418 \text{ bar}, P_{05} = P_a$$

$$T_{04} - T_{05} = 0.89 \times 841 \left[1 - \left(\frac{1}{2.418} \right)^{\left(\frac{1}{4} \right)} \right] = 148.254 \text{ K}$$

Specific power output:

$$W_N = 1.147 \times 0.98 \times 148.254 = 166.64 \text{ kWs/kg}$$

$$\text{Hence mass flow required} = \frac{20 \times 10^3}{166.64} = 120.019 \text{ kg/sec}$$

$$T_{02} = 288 + 345.6 = 633.6 \text{ K}$$

$$T_{03} - T_{02} = 1150 - 633.6 = 516.4 \text{ K}$$

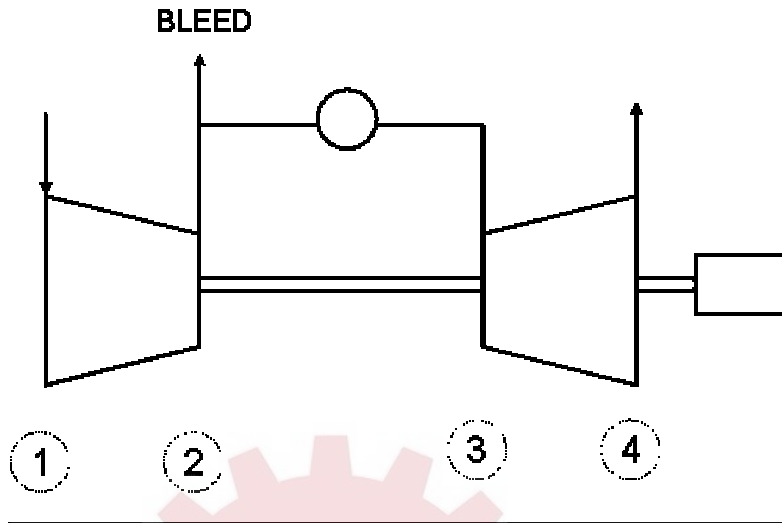
Theoretical $f = 0.01415$ (from Fig. 2.15)

$$\text{Actual } f = 0.01415 / 0.99 = 0.01429$$

$$S.F.C = \frac{3600f}{W_N} = \frac{3600 \times 0.01429}{166.64} = 0.308 \text{ kg/kW - hr}$$



Problem 2.3



$$T_{02} - T_a = \frac{288}{0.85} [3.8^{\frac{1}{3.5}} - 1] = 157.3 \text{ K}$$

$$P_{03} = 3.8 - 0.12 = 3.68 \text{ bar}$$

$$\frac{P_{03}}{P_a} = 3.68$$

$$T_{03} - T_{04} = 1050 \times 0.88 [1 - (\frac{1}{3.68})^{\frac{1}{4}}] = 256.87 \text{ K}$$

Net work output

$$W = \eta_{m(\text{load})} [(m - m_c) C_{pg} (T_{03} - T_{04}) - \frac{m C_{pa} (T_{02} - T_a)}{\eta_{m(\text{comp. rotor})}}]$$

$$200 = 0.98 [(m - 1.5) \times 1.147 \times 256.87 - \frac{m \times 1.005 \times 157.3}{0.99}]$$

$$200 = 288.73m - 433.1 - 156.5m$$

$$(a) \quad m = 4.788 \text{ kg/sec}$$

With no bleed flow:

$$\begin{aligned} \text{Net work output} &= 0.98 \times 4.788 [1.147 \times 256.87 - \frac{1.005 \times 157.3}{0.99}] \\ &= 633.11 \text{ kW} \end{aligned}$$

$$(b) \quad \text{The power output with no bleed} = 633.11 \text{ kW}$$

Problem 2.4

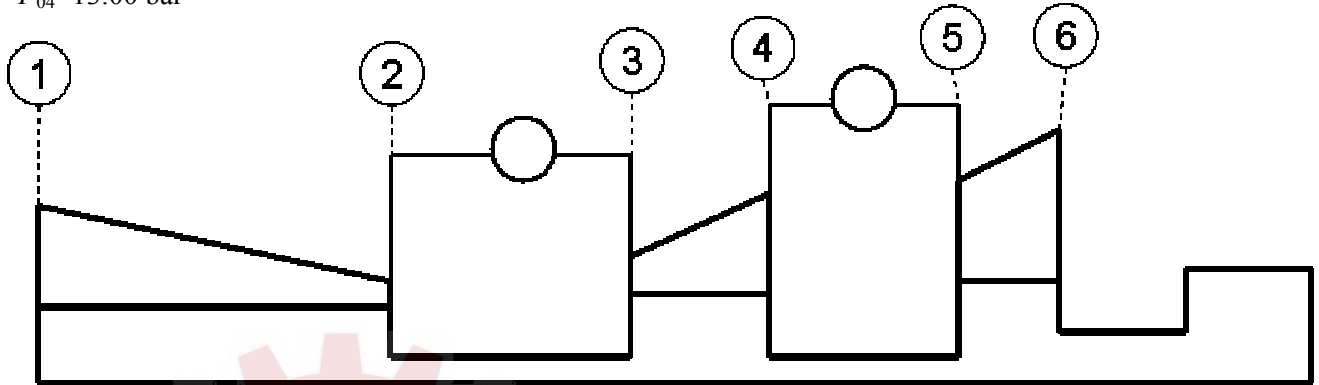
	A	B	C
$\left(\frac{n-1}{n}\right)_c$	0.3287	0.3250	0.3213
$\left(\frac{n-1}{n}\right)_t$	0.2225	0.2205	0.2205
T_{02}/T_{01}	2.059	2.242	2.437
ΔT_{012} (K)	305	357.8	413.9
T_{02} (K)	593	645.8	701.9
P_{03}/P_{04}	8.55	11.40	15.2
$T_{03}/T_{04} = \left(\frac{P_{03}}{P_{04}}\right)^{\frac{n-1}{n}}$	1.612	1.708	1.820
T_{03}	1150	1400	1600
T_{04}	713.4	819.6	879.2
ΔT_{034}	436.6	580.4	720.7
$W_c = (1.005 \times \Delta T_{12}) / 0.99$	309.6	363.2	420.2
$W_t = 1.148(1 - B)\Delta T_{034}$	501.2	649.6	786.0
$W_{net} = W_t - W_c$	191.6	286.4	365.8
Power	14,370	22,912	31,093
	Base	+59.4%	+116%
ΔT_{cc}	557	754	898
f/a	0.0162	0.0219	0.0268
$m_f = m_a \times f \times (1 - B)$	4374	6150	7791
SFC (kg/kwhr)	0.304	0.268	0.251
	base	11.8%	17.4%
EGT ($^{\circ}C$)	440	547	606

Problem 2.5

$$T_{03}=1525 \text{ K}$$

$$P_{03}=29.69 \text{ bar}$$

$$P_{04}=13.00 \text{ bar}$$



$$\therefore \frac{P_{03}}{P_{04}} = 2.284 \quad \therefore \frac{T_{03}}{T_{04}} = (2.284)^{0.2223} = 1.202$$

$$T_{04}=1268.7 \text{ K}, \Delta T_{hp} = 256.3$$

$$\frac{P_{05}}{P_{06}} = \frac{13.00 \times 0.96}{1.02} = 12.24 \quad \therefore \frac{T_{05}}{T_{06}} = (12.24)^{0.223} = 1.745$$

$$T_{03} - T_{04} = 256.3$$

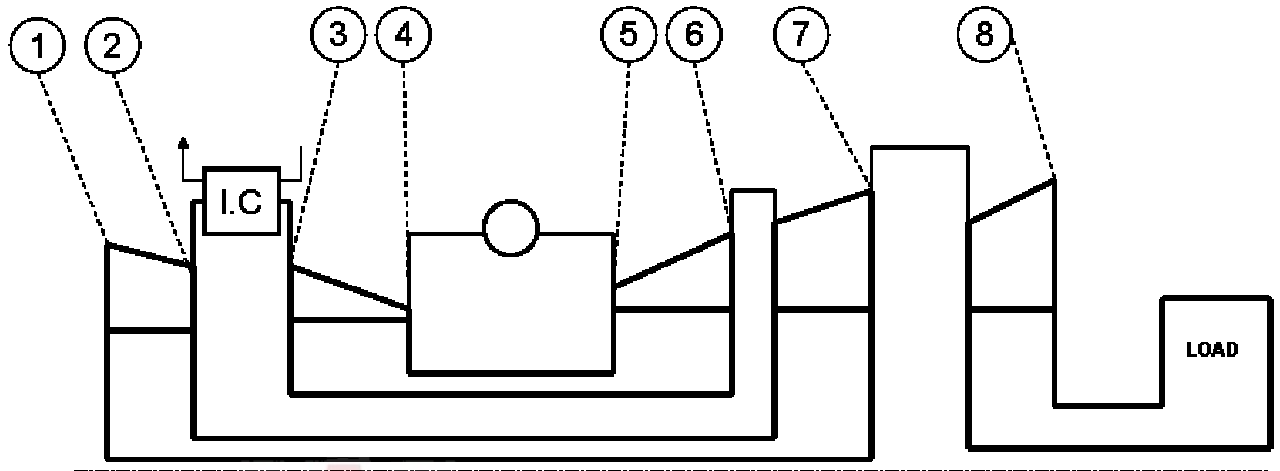
T_{05}	1525	1425	1325
T_{06}	874	816.6	759.3
$T_{05}-T_{06}$	651.0	608.4	565.7
$T_{03}-T_{04}$	256.3	256.3	256.3
ΔT_{total}	907.3	864.7	822.0
$\Delta T_{\text{total}} \times 1.148 \times 0.99$	1031.1	982.7	934.2
$-\Delta T_c \times 1.004$	573.0	573.0	573.0
	458.1	409.7	361.2

$$\therefore m \text{ for } 240 \text{ MW} = \frac{240000}{458.1} = 523.9 \text{ kg/s}$$

MW	240.0	214.6	189.2
$(f/a)_1$	0.0197	0.0197	0.0197
$(f/a)_2$	0.0085	0.0050	0.0030
	0.0282	0.0247	0.0227
η_{th}	458.1	409.7	361.2
	0.0282×43100	0.0247×43100	0.0227×43100
	37.7	38.5	36.9

So η_{th} remains high as power reduced. May be difficult to control low fuel: air ratio in 2nd combustor.

Problem 2.6



$$\Delta T_{12} = \frac{288}{0.875} \left[(5.5)^{0.286} - 1 \right] = 206.8 \quad T_2 = 494.8 \text{ K}$$

$$\Delta T_{34} = \frac{300}{0.870} \left[(7.5)^{0.286} - 1 \right] = 268.7 \quad T_4 = 568.7 \text{ K}$$

$$\Delta T_{cc} = 1550 - 569 = 981^\circ \text{ C}$$

$$\Delta T_{56} = \frac{268.7 \times 1.005}{1.148 \times 0.95 \times 0.99} = 250.1 \quad T_6 = 1550 - 250 = 1300 \text{ K}$$

$$\Delta T_{67} = \frac{206.8 \times 1.005}{1.148 \times 0.99} = 182.9 \quad T_7 = 1300 - 182.9 = 1117.1 \text{ K}$$

$$HPT, 250.1 = 0.88 \times 1550 \left[1 - \frac{1}{R^{0.25}} \right] \quad R_{hp} = 2.248$$

$$CDP = 1.00 \times 5.5 \times 7.5 \times 0.95 = 39.19 \text{ bar} \quad P_6 = \frac{39.19}{2.248} = 17.43 \text{ bar}$$

$$LPT, 182.9 = 0.89 \times 1300 \left[1 - \frac{1}{R^{0.25}} \right] \quad RLP = 1.990 \quad P_7 = 8.76 \text{ bar}$$

$$P_8 = P_1 = 1.00$$

$$\therefore \Delta T_{78} = 0.89 \times 1117.1 \left[1 - \frac{1}{8.76^{0.25}} \right] = 416.3 \text{ K}$$

$$\therefore m \times 1.148 \times 0.99 \times 416.3 = 100,000 \quad \therefore m = 211.3 \text{ kg/s}$$

$$f/a = \frac{0.028}{0.99} = 0.0283$$

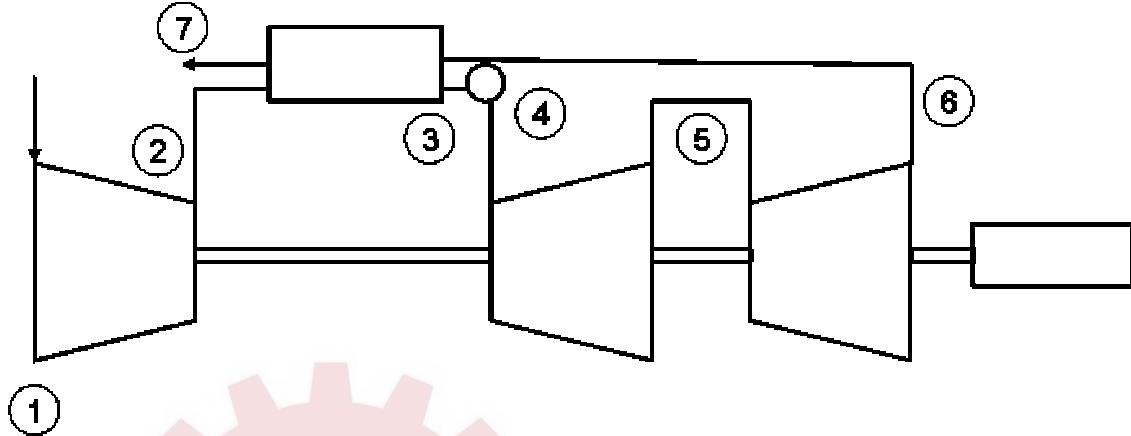
$$\text{Specific output} = 1.148 \times 0.99 \times 416.3 = 473.1$$

$$\therefore \eta_{th} = \frac{473.1}{0.283 \times 43100} = 38.8\%$$

$$EGT = 1117.7 - 416.3 = 701.4 \text{ K} = 428.4^\circ \text{ C}$$

Problem 2.8

$$P_a = 1.013 \text{ bar}, T_a = 15^\circ \text{C}$$



$$\Delta T_{12} = \frac{288}{0.87} [8.5^{0.286} - 1] = 279.5 \text{ K}$$

$$T_{02} = 567.5 \text{ K}$$

$$P_{04} = 1.013 \times 8.5 \times [1 - 0.015] \times [1 - 0.042] = 8.125 \text{ bar}$$

$$\Delta T_{45} = \frac{279.5 \times 1.004}{1.147 \times 0.99} = 247.1 \text{ K}, T_5 = 1037.8 \text{ K}$$

$$247.1 = 0.87 \times 1285 \left[1 - \frac{1}{(P_4/P_5)^{0.25}} \right] \Rightarrow \frac{P_4}{P_5} = 2.716, P_5 = 2.991 \text{ bar}$$

$$P_6 = 1.013 \times 1.02 = 1.0333 \Rightarrow \frac{P_5}{P_6} = 2.895$$

$$\therefore \Delta T_{56} = 0.88 \times 1037.8 \left[1 - \frac{1}{2.895^{0.25}} \right] = 213.1 \therefore T_6 = 824.7 \text{ K}$$

$$\therefore \text{Power} = 112.0 \times 1.147 \times 0.99 \times 213.1 = 27,106 \text{ kW}$$

$$T_3 - T_2 = 0.90(824.7 - 567.5) = 231.5$$

$$T_3 = 231.5 + 567.5 = 799 \text{ K}$$

$$\Delta T_{cc} = 1285 - 799 = 486^\circ \text{C}$$

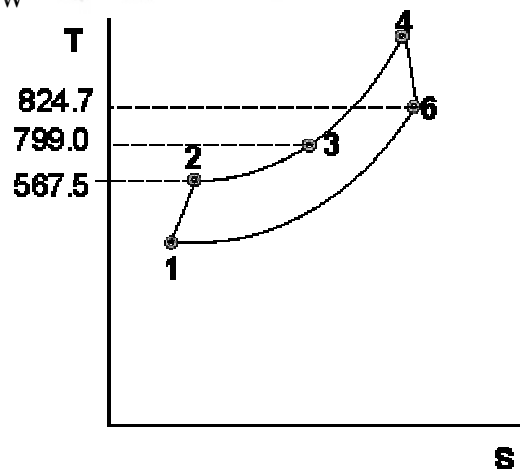
$$T_{in} = 799 - 273 = 526^\circ \text{C}$$

$$(f/a)_{id} = 0.0138 \quad f/a = \frac{0.0138}{0.99} = 0.0139$$

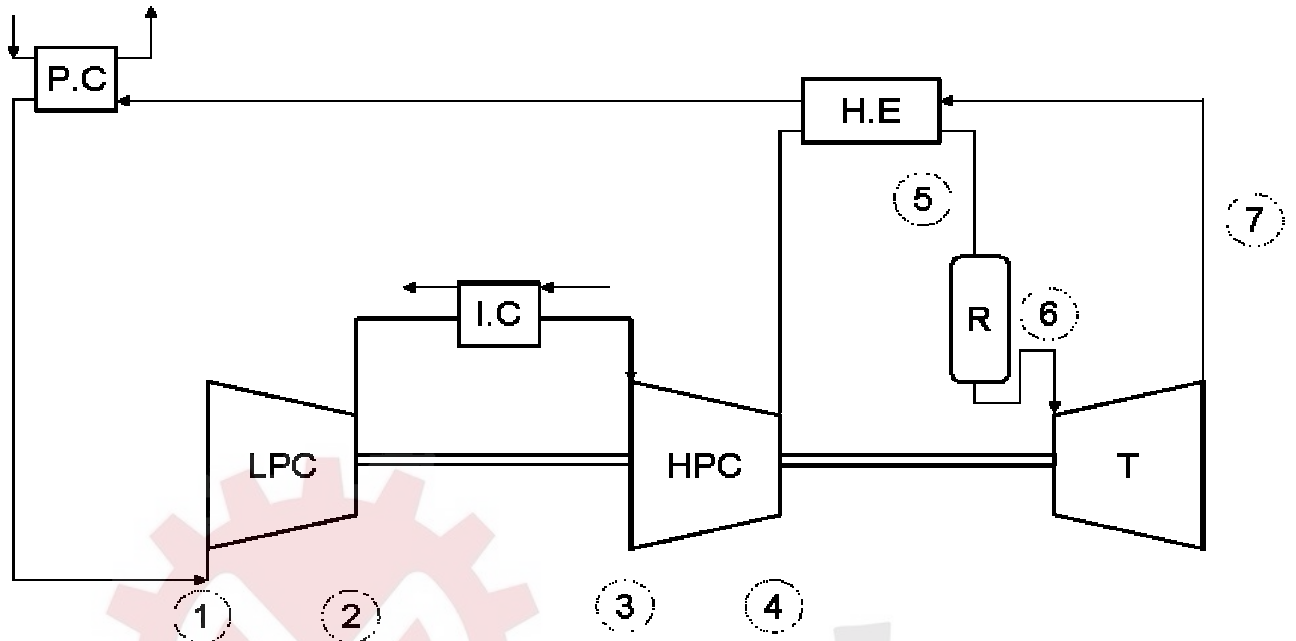
$$m_f = 0.0139 \times 112.0 = 1.56 \text{ kg/s}$$

$$\text{Heat input} = 1.56 \times 43,100 \frac{\text{kJ}}{\text{kg}} \frac{\text{kg}}{\text{s}} = 67,288 \text{ kW}$$

$$\therefore \text{Efficiency} = 40.3 \%$$



Problem 2.9



$$\text{For compression: } \frac{n-1}{n} = \frac{1}{\eta_{\infty c}} \left(\frac{\gamma-1}{\gamma} \right) = \frac{0.66}{0.88 \times 1.66} = 0.4518$$

$$\text{For expansion: } \frac{n-1}{n} = \eta_{\infty t} \left(\frac{\gamma-1}{\gamma} \right) = \frac{0.88 \times 0.66}{1.66} = 0.3498$$

$$T_{02} - T_{01} = T_{01} \left[\left(\frac{P_{02}}{P_{01}} \right)^{\frac{n-1}{n}} - 1 \right] = 310 [2^{0.4518} - 1] = 114 \text{ K}$$

$$T_{04} - T_{03} = 300 [2^{0.4518} - 1] = 110.3 \text{ K}$$

$$T_{04} = 410.3 \text{ K} \quad \text{and} \quad T_{05} = 700 \text{ K} \quad (\text{given})$$

$$T_{06} - T_{05} = \frac{Q}{m C_p} = \frac{500 \times 10^3}{180 \times 5.19} = 535.2 \text{ K}$$

$$\therefore T_{06} = 1235.2 \text{ K}$$

$$P_{03} = 2 \times 14.0 - 0.34 = 27.66 \text{ bar}$$

$$P_{04} = 2 \times 27.66 = 55.32 \text{ bar}$$

$$P_{06} = 55.32 - (0.27 + 1.03) = 54.02 \text{ bar}$$

$$P_{07} = 14.0 + 0.34 + 0.27 = 14.61 \text{ bar}$$

$$\therefore \frac{P_{06}}{P_{07}} = 3.697$$

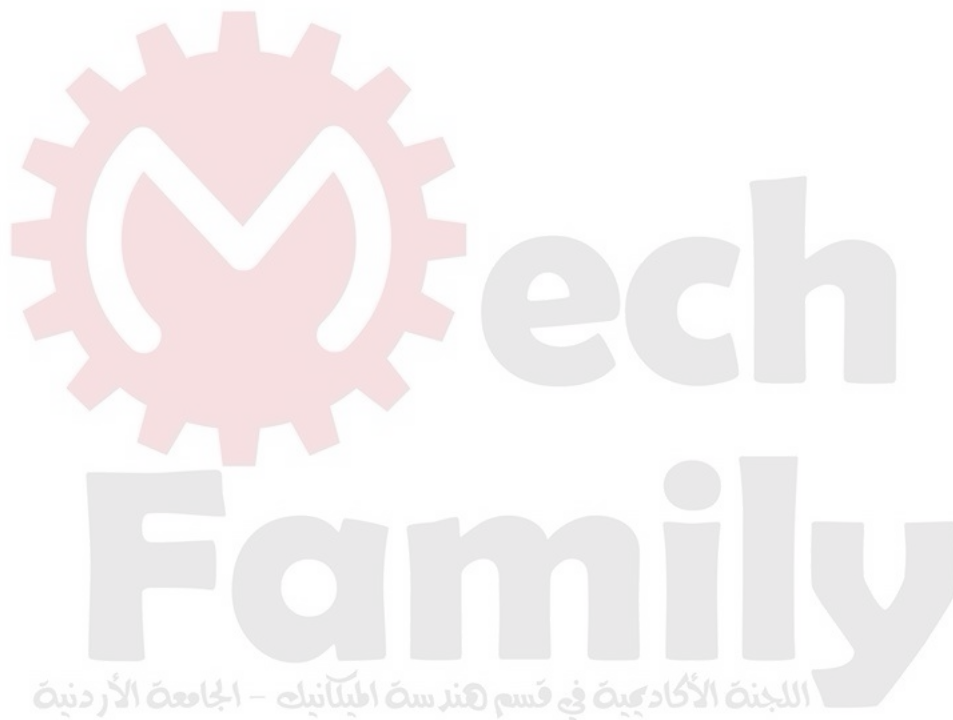
$$T_{06} - T_{07} = 1235.2 \left[1 - \left(\frac{1}{3.697} \right)^{0.3498} \right] = 453.3 \text{ K}$$

$$T_{07} = 781.82 \text{ K}$$

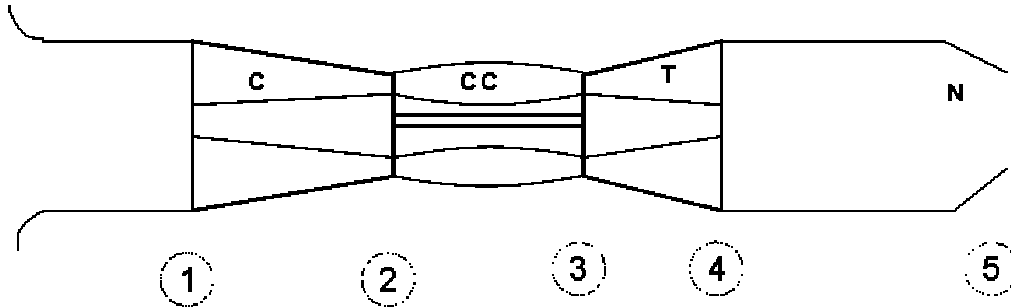
$$\begin{aligned}\text{Power output} &= mC_p[\Delta T_{067} - \Delta T_{034} - \Delta T_{012}] = 180 \times 5.19 \times 229.0 \\ &= 213996 \text{ kW or } 213.996 \text{ MW}\end{aligned}$$

$$\text{Thermal efficiency} = \frac{213.996}{500} = 0.4279 \quad \text{or} \quad 42.8\%$$

$$\text{H.E. effectiveness} = \frac{T_{05} - T_{04}}{T_{07} - T_{04}} = \frac{700 - 410.3}{781.8 - 410.3} = 0.7798 \quad \text{or } 78\%$$



Problem 3.1



For compression: $\frac{n-1}{n} = \frac{1}{0.87 \times 3.5} = 0.3284$

For expansion: $\frac{n-1}{n} = \frac{0.87}{4} = 0.2175$ and $\frac{n}{n-1} = 4.5977$

At 7000 m $P_a = 0.411 \text{ bar}$; $T_a = 242.7 \text{ K}$

$$\frac{C_a^2}{2c_p} = \frac{260^2}{2 \times 1.005 \times 1000} = 33.63 \text{ K}$$

$$T_{01} = 242.7 + 33.63 = 276.33 \text{ K}$$

$$P_{01} = P_a \left[1 + \eta_i \frac{C_a^2}{2c_p T_a} \right]^{\frac{\gamma}{\gamma-1}}$$

$$P_{01} = 0.411 \left[1 + \frac{0.95 \times 33.63}{242.7} \right]^{3.5} = 0.633 \text{ bar}$$

$$P_{02} = 8 \times 0.633 = 5.068 \text{ bar}$$

$$T_{02} - T_{01} = 276.33 [8.0^{0.3284} - 1] = 270.7 \text{ K}$$

$$T_{02} = 547.02 \text{ K}$$

$$T_{03} - T_{04} = \frac{c_{pa}(T_{02} - T_{01})}{c_{pg}\eta_m} = \frac{1.005 \times 270.7}{1.147 \times 0.99} = 239.6 \text{ K}$$

$$T_{04} = 1200 - 239.6 \quad T_{04} = 960.4 \text{ K} \quad P_{03} = 5.068(1 - 0.06) = 4.763 \text{ bar}$$

$$\frac{P_{03}}{P_{04}} = \left(\frac{T_{03}}{T_{04}} \right)^{\frac{n}{n-1}} = \left(\frac{1200}{960.4} \right)^{4.5977} = 2.784$$

$$P_{04} = \left(\frac{4.763}{2.784} \right) = 1.710 \text{ bar}$$

Nozzle pressure ratio $\frac{P_{04}}{P_a} = \frac{1.710}{0.411} = 4.16$

(Since the critical pressure ratio is of the order of 1.9 the nozzle is clearly choked)

$$\frac{P_{04}}{P_c} = \frac{1}{\left[1 - \frac{1}{0.95} \left(\frac{0.333}{2.333}\right)\right]^4} = 1.918$$

$$P_5 = P_c = \frac{1.710}{1.918} = 0.891 \text{ bar}$$

$$T_5 = T_c = \frac{2}{\gamma + 1} T_{04} = \frac{2 \times 960.4}{2.333} = 823.3 \text{ K}$$

$$\rho_5 = \frac{P_c}{RT_c} = \frac{100 \times 0.891}{0.287 \times 823.3} = 0.377 \text{ kg/m}^3$$

$$C_5 = (\gamma RT_c)^{\frac{1}{2}} = (1.333 \times 0.287 \times 823.3 \times 1000)^{\frac{1}{2}} = 561.22 \text{ m/sec}$$

$$A_5 = \frac{m}{\rho_5 C_5} = \frac{15}{0.377 \times 561.22} = 0.0708 \text{ m}^2$$

$$F = m(C_j - C_a) + A_j(P_j - P_a)$$

$$F = 15(561.22 - 260) + 0.0708(0.891 - 0.411)10^5$$

$$F = 4518.3 + 3398.4 = 7916.7 \text{ N}$$

$$T_{02} = 547.02 \text{ K}$$

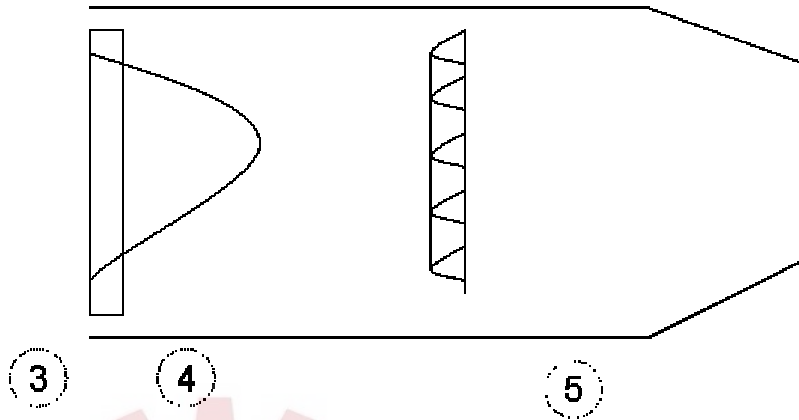
$$T_{03} - T_{02} = 1200 - 547.02 = 652.8 \text{ K}$$

Therefore theoretical $f = 0.01785$ (Fig. 2.15)

$$\text{Actual } f = \frac{0.01785}{0.97} = 0.0184$$

$$S.F.C = \frac{0.0184 \times 3600 \times 15}{7916.7} = 0.01255 \text{ kg/kN}$$

Problem 3.2



From problem 3.1 $P_{04}=1.710$ bar

$$P_{05} = 1.17(1 - 0.03) = 1.658 \text{ bar}$$

$$\frac{P_{05}}{P_c} = 1.918 \quad \text{as before}$$

$$\frac{P_{05}}{P_c} = \frac{1.658}{1.918} = 0.8644 \text{ bar}$$

Since

$$T_{05} = 2000 \text{ K}, \quad T_6 = T_c = \frac{2}{2.333} \times 2000 = 1714.5 \text{ K}$$

$$\rho_5 = \frac{100 \times 0.8644}{0.287 \times 1714.5} = 0.1756 \text{ kg/m}^3$$

$$C_5 = (1.333 \times 0.287 \times 1714.5 \times 1000)^{\frac{1}{2}} = 809.88 \text{ m/s}$$

$$A_5 = \frac{15}{0.1756 \times 809.88} = 0.1054 \text{ m}^2$$

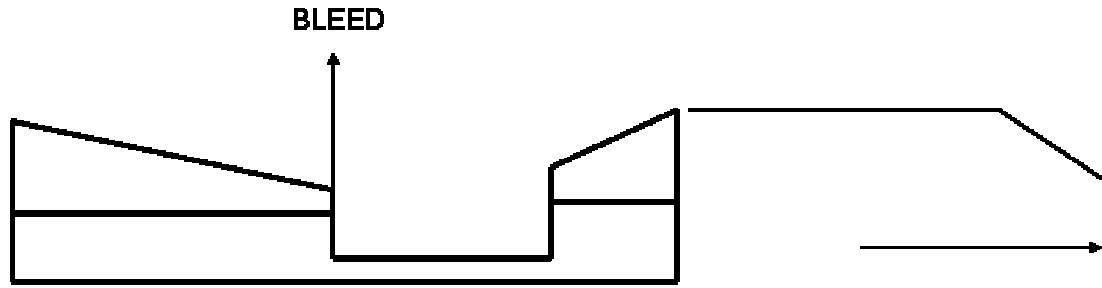
Percentage increase in area = $0.1054 - 0.0708 = 48.97\%$

$$F = 15(809.88 - 260) + 0.1054(0.8644 - 0.411) \times 10^5$$

$$F = 8248.2 + 4778.83 = 13027.03 \text{ N}$$

$$\text{Percentage increase in thrust} = \frac{13027.03 - 7916.7}{7916.7} = 64.56\%$$

Problem 3.3



Since $P_{01} = P_a = 1 \text{ bar}$

$$T_{01} = T_a = 288 \text{ K}$$

$$T_{02} - T_{01} = \frac{288}{0.82} \left[9^{\frac{1}{3.5}} - 1 \right] = 306.77 \text{ K}$$

$$0.85 m c_{pg} (T_{03} - T_{04}) = \frac{m C_{pa}}{\eta_m} (T_{02} - T_{01})$$

$$T_{03} - T_{04} = \frac{1.005 \times 306.77}{0.85 \times 1.147 \times 0.98} = 322.678 \text{ K}$$

$$T_{03} - T_{04} = \eta_j T_{03} \left[1 - \left(\frac{1}{P_{03}/P_{04}} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$322.678 = 0.87 \times 1275 \left[1 - \left(\frac{1}{P_{03}/P_{04}} \right)^{\frac{1}{4}} \right] \Rightarrow \frac{P_{03}}{P_{04}} = 3.955$$

$$P_{03} = 9.0 - 0.45 = 8.55 \text{ bar}$$

$$P_{04} = 2.161 \text{ bar}$$

$$\frac{P_{04}}{P_a} = 2.161 \quad \text{and} \quad \frac{P_{04}}{P_a} = \frac{1}{\left[1 - \frac{1}{0.95} \left(\frac{0.333}{2.333} \right) \right]^4}$$

So nozzle is choking even while landing

$$P_5 = P_c = \frac{2.161}{1.918} = 1.126 \text{ bar}$$

$$T_{04} = 1275 - 322.678 = 952.3 \text{ K}$$

$$T_5 = T_c = \left(\frac{2}{\gamma + 1} \right) \times T_{04} = \frac{2 \times 952.322}{2.333} = 816.4 \text{ K}$$

$$\rho_5 = \frac{P_c}{RT_c} = \frac{100 \times 1.126}{0.287 \times 816.4} = 0.480 \quad \text{kg/m}^3$$

$$C_5 = (\gamma RT_c)^{\frac{1}{2}} = (1.333 \times 0.287 \times 816.4 \times 1000)^{\frac{1}{2}} = 555.86 \text{ m/s}$$

$$(m - m_b) = \rho_5 A_5 C_5 = 0.480 \times 0.13 \times 555.86 = 34.87 \text{ kg/sec}$$

$$m_{\text{intake}} = \frac{34.87}{0.85} = 41.02$$

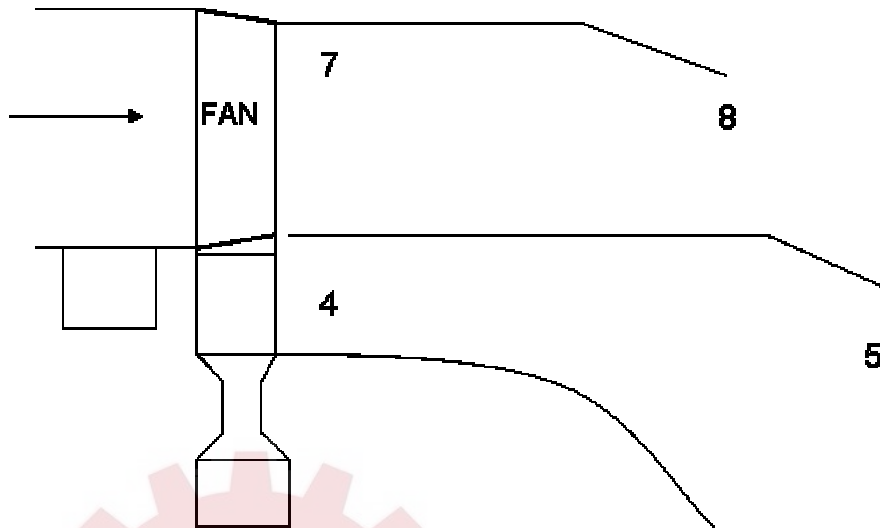
(N.B. – m_b , bleed at 90° produces no thrust. Also, ram drag based on *intake flow*)

$$F = (34.87 \times 555.86 - 41.02 \times 55) + 0.13(1.126 - 1) \times 10^5$$

$$F = (19487.44 - 2256.1) + 1638$$

$$F = 18869.34 \text{ N} \quad \text{or} \quad F = 18.87 \text{ kN}$$

Problem 3.4



Isentropic expansion in nozzle:

$$\frac{P_{04}}{P_a} = \frac{2.4}{1.01} = 2.376$$

$$\frac{P_{04}}{P_c} = \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2.333}{2} \right)^4 = 1.851$$

$$\text{Choking; So } P_5 = P_c = \frac{2.4}{1.851} = 1.296 \text{ bar}$$

$$\frac{T_{04}}{T_c} = \frac{\gamma+1}{2} = \frac{2.333}{2} \quad \text{and} \quad T_5 = T_c = \frac{1000 \times 2}{2.333} = 857.26 \text{ K}$$

$$\rho_5 = \frac{P_c}{RT_c} = \frac{100 \times 1.296}{0.287 \times 857.26} = 0.526 \text{ kg/m}^3$$

$$C_5 = (\gamma RT_c)^{\frac{1}{2}} = (1.333 \times 0.287 \times 857.26 \times 1000)^{\frac{1}{2}} = 572.68 \text{ m/sec}$$

$$A_5 = \frac{m}{\rho_5 C_5} = \frac{23}{0.526 \times 572.68} = 0.0763 \text{ m}^2$$

$$F = 23 \times 572.68 + 0.0763(1.296 - 1.01)10^5$$

$$F = 13171.64 + 2182.18 = 15353.82 \text{ N}$$

$$\text{or } F = 15.35 \text{ kN}$$

$$\frac{P_{07}}{P_a} = 1.75 \quad \text{and} \quad P_{07} = 1.01 \times 1.75 = 1.768 \text{ bar}$$

$$T_{07} - T_a = \frac{T_a}{\eta_c} \left[\left(\frac{P_{07}}{P_a} \right)^{\frac{1}{3.5}} - 1 \right] = \frac{288}{0.88} \left[(1.75)^{\frac{1}{3.5}} - 1 \right] = 56.74 \text{ K}$$

$$m_h C_{pg} (T_{04} - T_{05}) = m_c C_{pa} (T_{07} - T_a)$$

$$\text{But} \quad \frac{m_c}{m_h} = 2.0$$

$$T_{04} - T_{05} = \frac{2 \times 1.005 \times 56.74}{1.147} = 99.43 \text{ K}$$

$$T_{05} = 1000 - 99.43 = 900.57 \text{ K}$$

$$99.43 = 0.90 \times 1000 \left[1 - \left(\frac{1}{P_{04}/P_{05}} \right)^{\frac{1}{4}} \right] \Rightarrow \frac{P_{04}}{P_{05}} = 1.597$$

$$P_{05} = \frac{2.4}{1.597} = 1.503 \text{ bar}$$

$$\frac{P_{05}}{P_a} = \frac{1.503}{1.01} = 1.488$$

Hot nozzle is now unchoked, so $P_6 = P_a = 1.01 \text{ bar}$

$$\frac{T_{05}}{T_6} = \left(\frac{P_{05}}{P_6} \right)^{\frac{\gamma-1}{\gamma}} = 1.488^{\frac{1}{4}} = 1.1045$$

$$T_6 = \frac{900.57}{1.1045} = 815.36 \text{ K}$$

$$T_{05} - T_6 = 85.2 \text{ K}$$

$$C_6 = \left[2 C_p (T_{05} - T_{06}) \right]^{\frac{1}{2}} = (2 \times 1.147 \times 85.21 \times 1000)^{\frac{1}{2}}$$

$$C_6 = 442.12 \text{ m/s}$$

Hot nozzle thrust:

$$F_h = 23 \times 442.12 = 10,168.8 \text{ N}$$

The cold nozzle pressure ratio = 1.75, while the critical pressure ratio is :

$$\frac{P_{07}}{P_c} = \left(\frac{1.4 + 1}{2} \right)^{3.5} = 1.893$$

So this nozzle is also unchoked. Since the expansion is isentropic:

$$\frac{T_{07}}{T_8} = 1.75^{\frac{1}{3.5}} = 1.1733$$

$$T_8 = \frac{288 + 56.74}{1.1733} = 293.8 \text{ K} \quad \text{and} \quad T_{07} = 344.74$$

$$T_{07} - T_8 = 50.94 \text{ K}$$

$$C_8 = \left[2C_p (T_{07} - T_8) \right]^{\frac{1}{2}} = (2 \times 1.005 \times 50.94 \times 1000)^{\frac{1}{2}} = 320 \text{ m/s}$$

Cold thrust

$$F_c = 46 \times 320 = 14720 \text{ N}$$

$$\text{Total thrust: } = 14720 + 10168.8 = 24888.8 \text{ N}$$

$$\text{or } F = 24.89 \text{ kN}$$



Problem 3.5

From solution of turbofan example in Chapter 3:

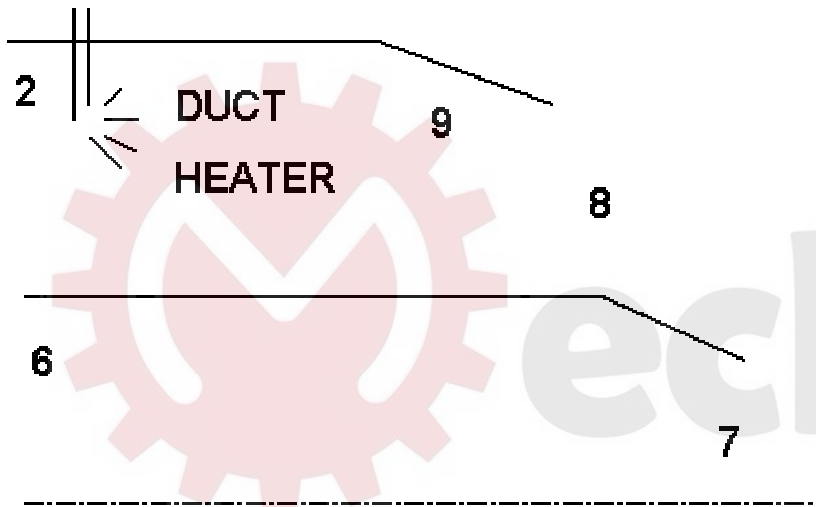
$$T_{03}=800 \text{ K}$$

$$T_{04}-T_{03}=1550 - 800=750 \text{ K}$$

$$\text{Actual } f = \frac{0.0221}{0.99} = 0.0223$$

$$m = 35.83 \text{ kg/s and } F_s=71,463 \text{ N}$$

$$S.F.C = \frac{0.0223 \times 35.83 \times 3600}{71463} = 0.0403 \text{ kg/Nh}$$



$$P_{09}=P_{02} - 0.05=1.65-0.05=1.60 \text{ bar}$$

$$\text{So nozzle is unchoked and } \frac{P_{09}}{P_a} = 1.60 \left(< \frac{P_{05}}{P_c} \right)$$

$$\begin{aligned} T_{09} - T_8 &= \eta_j T_{09} \left[1 - \left(\frac{1}{P_{09}/P_a} \right)^{3.5} \right] \\ &= 0.95 \times 1000 \left[1 - \left(\frac{1}{1.6} \right)^{3.5} \right] = 119.4 \text{ K} \end{aligned}$$

$$C_8 = (2 \times 1.005 \times 119.4 \times 1000)^{\frac{1}{2}} = 490 \text{ m/s}$$

$$m_c = 179.2 \text{ kg/s}$$

$$\text{New thrust: } F_c = 179.2 \times 490 = 87,808 \text{ N}$$

$$\begin{aligned} \text{Total thrust} &= \text{Unchanged } F_h + \text{new } F_c \\ &= 18931 + 87808 = 106,739 \text{ N} \end{aligned}$$

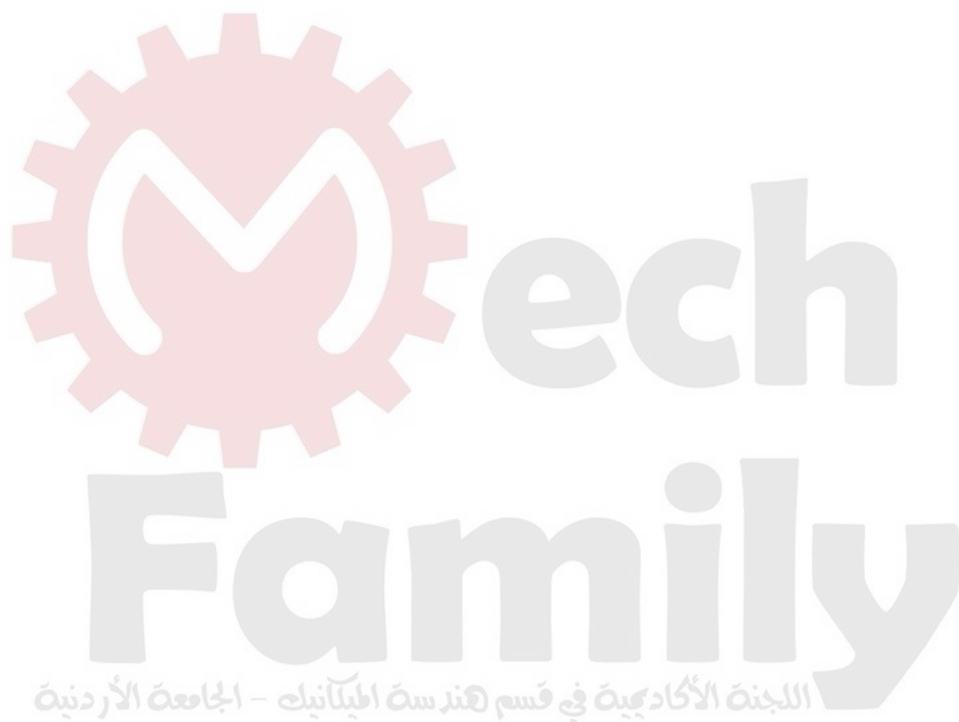
$$T_{02}=337.7 \text{ and } T_{09} - T_{02}=1000-337.7=662.3 \text{ K}$$

From Fig. 2.15, theoretical $f = 0.01715$

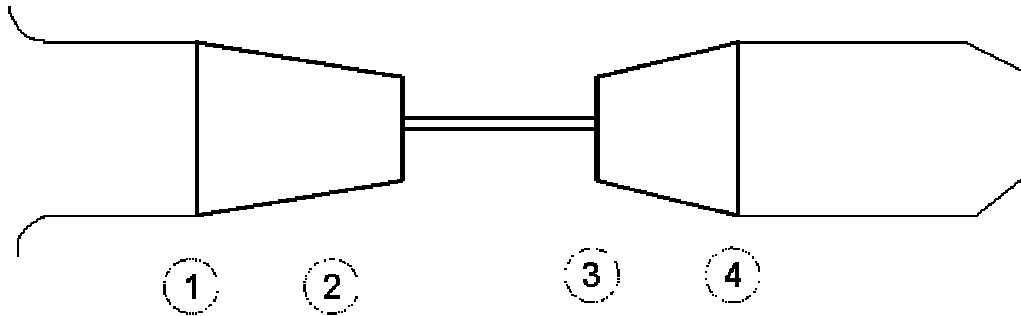
$$\text{Actual } f = \frac{0.01715}{0.97} = 0.01768 \text{ for by-pass chamber}$$

$$S.F.C = \frac{\text{Total Fuel}}{\text{Total thrust}} = \frac{((0.01768 \times 179.2 + 0.0223 \times 35.83) \times 3600}{106,739}$$

$$S.F.C = 0.1338 \text{ kg/kN}$$



Problem 3.6



$$P_a = 0.899 \text{ bar}$$

$$T_a = 281.7 \text{ K}$$

$$C_a = 336.4 \text{ m/s} \quad M = 0.70$$

$$\left(\frac{n-1}{n}\right)_c = \frac{1}{0.89} \left(\frac{0.4}{1.4}\right) = 0.3210 \quad \left(\frac{n-1}{n}\right)_t = 0.90 \left(\frac{0.333}{1.333}\right) = 0.2248$$

$$T_{01} = T_a \left(1 + \frac{\gamma-1}{2} M^2\right) = 281.7(1 + 0.2 \times 0.7^2) = 309.3 \text{ K} \quad \Delta T_r = 27.61^\circ \text{C}$$

$$\frac{P_{01}}{P_a} = \left(1 + \frac{0.95 \times 27.61}{281.7}\right)^{\frac{1.4}{0.4}} = 1.366 \quad \therefore P_{01} = 1.228 \text{ bar}$$

$$\frac{T_{02}}{T_{01}} = (5.0)^{0.3210} = 1.676 \quad \therefore T_{02} = 472.2 \quad \Delta T_{12} = 162.9^\circ \text{C}$$

$$\Delta T_{34} = \frac{162.9 \times 1.005}{1.148 \times 0.99} = 144.1 \quad T_{04} = 1250 - 144.1 = 1105.9 \text{ K}$$

$$\frac{T_{03}}{T_{04}} = \left(\frac{P_{03}}{P_{04}}\right)^{0.2248} \quad \therefore \frac{P_{03}}{P_{04}} = \left(\frac{1250}{1105.9}\right)^{\frac{1}{0.2248}} = 1.724$$

$$P_{03} = 1.366 \times 5.0 \times 0.945 = 6.454 \text{ bar} \quad \therefore P_{04} = \frac{6.454}{1.724} = 3.744 \text{ bar}$$

$$\text{Critical } P.R. = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1.333}{2}\right)^{4.003} = 1.852$$

$$\frac{P_{04}}{P_a} = \frac{3.744}{0.899} = 4.164 \quad \therefore \text{choked}$$

$$P_4 = \frac{3.744}{1.852} = 2.022 \text{ bar} \quad T_4 = 1105.9 \times \frac{2}{2.333} = 948.0 \text{ K}$$

$$\rho = \frac{100 \times 2.022}{0.287 \times 948.0} = 0.7432 \text{ kg/m}^3$$

$$C = \sqrt{\gamma RT} = \sqrt{1.333 \times 0.287 \times 1000 \times 948.0} = 602.2 \text{ m/s}$$

$$A = \frac{\pi}{4} (0.15)^2 = 0.01767$$

$$m = \rho AC = 0.7432 \times 0.01767 \times 602.2 = 7.91 \text{ kg/s}$$

$$C_a = 0.70 \times 336.4 = 235.5 \text{ m/s}$$

$$\begin{aligned} F &= m(C_j - C_a) + (P_n - P_a)A \times 10^5 \\ &= 7.91(602.2 - 235.5) + (2.022 - 0.899) \times 0.01767 \times 10^5 \\ &= 2900 \text{ N} + 1984 \text{ N} = 4884 \text{ N} \end{aligned}$$

$$\Delta T_{cc} = 1250 - 472 = 778, \quad T_{in} = 472 \text{ K} \rightarrow (f/a)_{id} = 0.0212 \approx f/a = \frac{0.0212}{0.99} = 0.02141$$

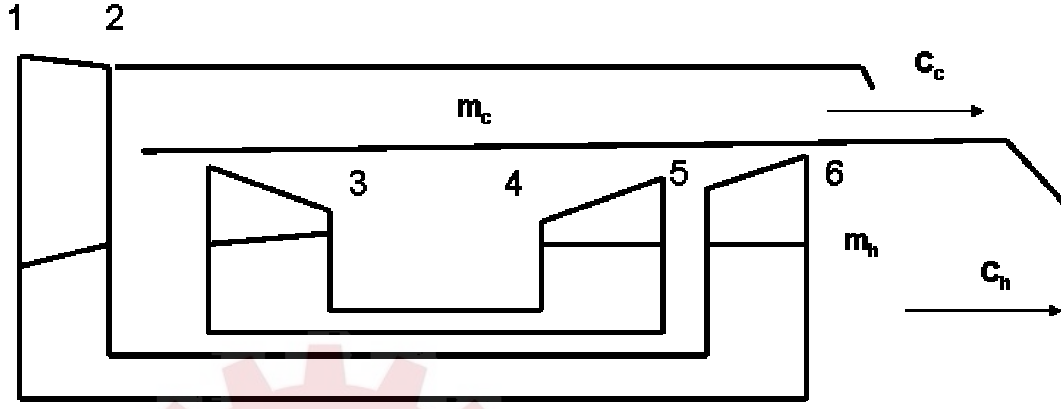
$$\therefore m_f = 7.91 \times 0.02141 \times 3600 = 609.8 \text{ kg/hr}$$

$$\therefore SFC = \frac{609.8}{4884} = 0.1249 \text{ kg/Nh}$$

Problem 3.7

$$M=0.75; \text{BPR}=3.8; \text{FPR}=1.7$$

$$C_a=0.75 \times 295.1 = 221.3 \text{ m/s}$$



$$T_{01} = T_a \left(1 + \frac{\gamma-1}{2} M^2 \right) = 216.7(1 + 0.2 \times 0.75^2) = 241.08 \text{ K}$$

$$\Delta T_r = 24.38^\circ \text{C}$$

$$\frac{P_{01}}{P_a} = \left(1 + \frac{0.95 \times 24.38}{216.7} \right)^{3.5} = 1.427 \therefore P_{01} = 0.2558 \text{ bar}$$

Fan

$$\frac{T_{02}}{T_{01}} = (1.7)^{\frac{n-1}{n}} = 1.1837 \quad \frac{n-1}{n} = \frac{0.286}{0.90} = 0.3178$$

$$\therefore T_{02} = 285.36 \text{ K} \quad \Delta T_f = 44.28 \text{ K} \quad P_{02} = 0.4400$$

HPC

$$\left(\frac{T_{03}}{T_{02}} \right) = (7.9)^{0.3178} = 1.9287 \quad T_{03} = 550.38 \text{ K} \quad P_{03} = 3.4757$$

$$\Delta T_{023} = 265.02$$

$$\text{Combustor } P_{04} = 3.4757(1 - 0.005) = 3.3019$$

$$\text{HP turbine } \Delta T_{45} = \frac{1.004 \times 265.02}{1.147 \times 0.99} = 234.32 \text{ K} \quad T_{05} = 985.7 \text{ K}$$

$$\text{LP turbine } \Delta T_{56} = \frac{1.004 \times 44.28(3.8 + 1)}{1.147 \times 0.99} = 187.92 \text{ K} \quad T_{06} = 797.8 \text{ K}$$

$$\frac{P_{04}}{P_{05}} = \left(\frac{T_{04}}{T_{05}} \right)^{\frac{n}{n-1}} \quad \frac{n-1}{n} = 0.286 \times 0.9 = 0.2232$$

$$\frac{n}{n-1} = 4.4803$$

$$\frac{P_{04}}{P_{05}} = \left(\frac{1220}{985.7} \right)^{4.4803} = 2.60$$

$$\frac{P_{05}}{P_{06}} = \left(\frac{985.7}{797.8} \right)^{4.4803} = 2.579$$

$$\therefore P_{06} = \frac{3.3019}{2.60 \times 2.579} = 0.4924 \text{ bar}$$

Cold Stream (both choked)

$$P_{02} = 0.4400 \text{ } T_{02} = 285.4$$

$$P_2 = \frac{0.4400}{1.893} = 0.2324$$

$$T_2 = \frac{285.4}{1.2} = 237.8$$

$$\rho = \frac{100 \times 0.2324}{0.287 \times 237.8} = 0.3405$$

$$m_c = \frac{9.6 \times 3.8}{4.8} = 7.60$$

$$C_c = \sqrt{1.4 \times 287 \times 237.8} = 309.1 \text{ m/s}$$

$$A = \frac{7.6}{0.3405 \times 309.1} = 0.0722 \text{ m}^2$$

$$Fc = 7.6(309.1 - 221.3) + 0.0722(0.2324 - 0.1793) \times 10^5$$

$$= 667.3 + 383.4 = 1050.7$$

$$Fh = 2.0(511.5 - 221.3) + 0.02886(0.2659 - 0.1793) \times 10^5$$

$$= 580 + 250.0 = 830$$

$$F = 1880.7 \text{ N}$$

$$T_{04} = 1220, T_{03} = 550 \rightarrow f = 0.0186$$

$$m_f = 2.00 \times 0.0186 \times 3600 = 133.63 \text{ kg/hr}$$

$$SFC = \frac{133.63}{1880.7} = 0.0711 \text{ kg/Nh}$$

Hot Stream

$$P_{06} = 0.4924 \text{ bar } T_{06} = 797.8 \text{ K}$$

$$P_6 = \frac{0.4924}{1.852} = 0.2659$$

$$T_6 = 797.8 \left(\frac{2}{2.333} \right) = 683.9$$

$$\rho = \frac{100 \times 0.2659}{0.287 \times 683.9} = 0.1355$$

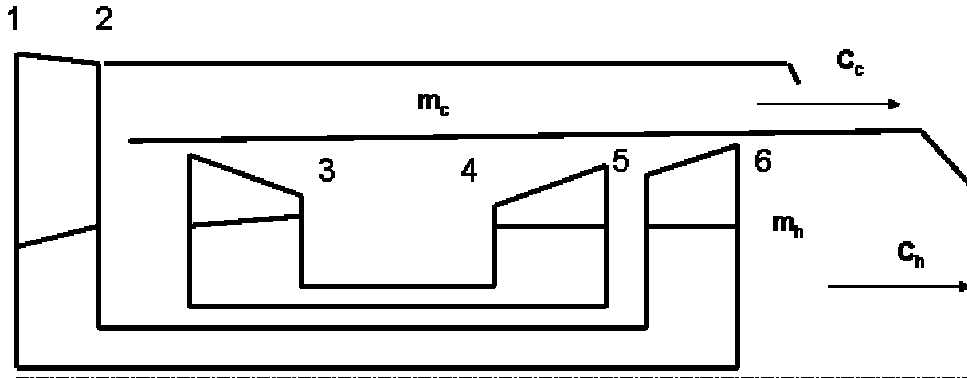
$$m_h = \frac{9.6 \times 1}{4.8} = 2.00$$

$$C_h = \sqrt{1.4 \times 287 \times 683.9} = 511.5 \text{ m/s}$$

$$A = \frac{2.0}{0.1355 \times 511.5} = 0.02886 \text{ m}^2$$

Problem 3.8

$$M=0.85; BPR=5.7; FPR=1.77$$



$$\left(\frac{n-1}{n}\right)_c = \frac{0.286}{0.90} = 0.3178$$

$$\left(\frac{n-1}{n}\right)_t = \frac{0.333 \times 0.9}{1.333} = 0.2248$$

$$T_{01} = T_a \left(1 + \frac{\gamma-1}{2} M^2\right) = 216.8(1 + 0.2 \times 0.85^2) = 248.13 \text{ K}$$

$$\Delta T_r = 31.33 \text{ C}$$

$$\frac{P_{01}}{P_a} = \left(1 + \frac{0.95 \times 31.33}{216.8}\right)^{3.5} = 1.569 \therefore P_{01} = 0.227 \times 1.569 = 0.3561 \text{ bar } P_{02} = 0.6303$$

$$\frac{T_{02}}{T_{01}} = (1.77)^{0.31718} = 1.1989 \therefore \Delta T_{12} = 49.33 \text{ K}$$

$$\therefore T_{02} = 297.5$$

$$\left(\frac{P_{03}}{P_{02}}\right) = \frac{34}{1.77} = 19.21$$

$$\left(\frac{T_{03}}{T_{02}}\right) = (19.21)^{0.3178} = 2.558, \Delta T_{23} = 463.4 \quad T_{03} = 761 \text{ K}$$

$$\Delta T_{023} = 265.02$$

$$P_{04} = 0.3561 \times 1.77 \times 19.21 \times 0.97 = 11.74 \text{ bar}$$

$$\Delta T_{45} = \frac{1.004 \times 463.4}{1.148 \times 0.99} = 409.4 \quad T_{05} = 940.6 \text{ K}$$

$$\frac{P_{04}}{P_{05}} = \left(\frac{1350}{940.6}\right)^{\frac{1}{0.2248}} = 4.99$$

$$\Delta T_{56} = \frac{1.004 \times 49.33(5.7 + 1)}{1.148 \times 0.99} = 292.0 \quad T_{06} = 648.6 \text{ K}$$

$$T_6 = 556.0 \text{ K}$$

$$\frac{P_{05}}{P_{06}} = \left(\frac{940.6}{648.6} \right)^{\frac{1}{0.2248}} = 5.22$$

$$\therefore P_{06} = \frac{11.74}{4.99 \times 5.22} = 0.4505 \text{ bar} \quad \frac{P_{06}}{P_a} = 1.984 \quad \therefore \text{choked}$$

$$C_a = 0.85 \times 295.2$$

Cold Stream (both choked)

Hot Stream

$$m_c = \frac{220 \times 5.7}{6.7} = 187.2$$

$$m_h = \frac{220}{6.7} = 32.8$$

$$P_2 = \frac{P_{02}}{P_{cr}} = \frac{0.6303}{1.893} = 0.3330$$

$$P_6 = \frac{0.4505}{1.852} = 0.2433$$

$$T_2 = \frac{297.5}{1.2} = 247.9$$

$$\rho = \frac{100 \times 0.3330}{0.287 \times 247.9} = 0.468$$

$$\rho = \frac{100 \times 0.2433}{0.287 \times 556} = 0.1525$$

$$C_c = \sqrt{1.4 \times 287 \times 247.9} = 315.6 \text{ m/s}$$

$$C_h = \sqrt{1.333 \times 287 \times 556} = 461.2 \text{ m/s}$$

$$A = \frac{187.2}{0.468 \times 315.6} = 1.267 \text{ m}^2$$

$$A = \frac{32.8}{0.1525 \times 461.2} = 0.466 \text{ m}^2$$

$$F_c = 187.2(315.6 - 250.9) + 1.267(0.333 - 0.227) \times 10^5 \\ = 12.108 + 13430 = 25,538$$

$$F_h = 32.8(461.2 - 250.9) + 0.466(0.2433 - 0.227) \times 10^5 \\ = 6898 + 760 = 7658$$

$$F = 33,196 \text{ N} \quad (F_s = 150.9)$$

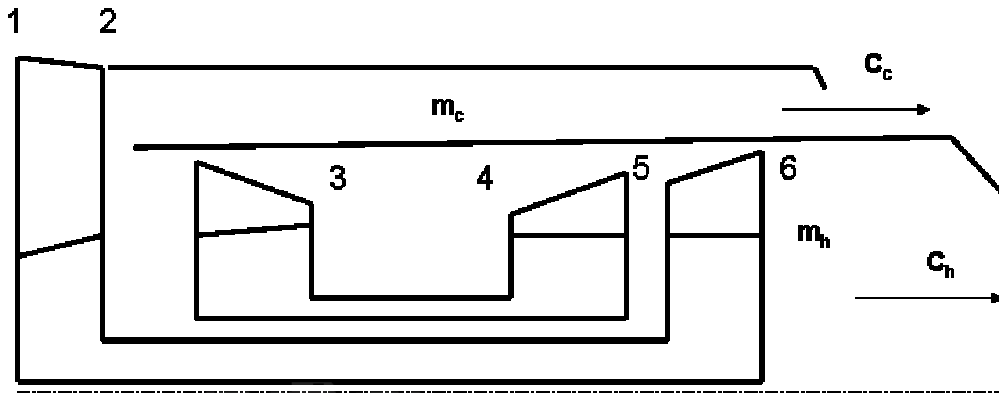
$$f = \frac{0.017}{0.99} = 0.0172$$

$$m_f = 0.0172 \times 32.8 \times 3600 = 2028 \text{ kg/hr}$$

$$SFC = \frac{2028}{33196} = 0.0611 \text{ kg/Nh}$$

Problem 3.9

M 1.4 Turbofan ($BPR=2.0$, $FPR=2.2$)



Assume full expansion, i.e. no pressure thrust

$$\left(\frac{n-1}{n}\right)_c = \frac{0.4}{0.90 \times 1.4} = 0.3175$$

$$\left(\frac{n-1}{n}\right)_t = \frac{0.333 \times 0.9}{1.333} = 0.2248$$

$$T_{01} = T_a \left(1 + \frac{\gamma-1}{2} M^2\right) = 216.7(1 + 0.2 \times 1.4^2) = 301.65 K$$

$$\Delta T_r = 84.95 K$$

$$\frac{P_{01}}{P_a} = \left(1 + \frac{0.93 \times 84.95}{216.7}\right)^{3.5} = 2.968 \therefore P_{01} = 0.3594 \text{ bar}$$

$$\frac{T_{02}}{T_{01}} = (2.2)^{0.3175} = 1.2845 \therefore T_{02} = 387.46 K \therefore \Delta T_{12} = 85.8 K$$

$$\frac{T_{03}}{T_{02}} = (9.0)^{0.3175} = 2.009 \therefore T_{03} = 778.39 K \therefore \Delta T_{23} = 390.9 K$$

Assuming no cooling, $1.004 \times 390.9 = 1.147 \times 0.99 \times \Delta T_{45}$ $\Delta T_{45} = 345.6$ $T_5 = 1154.3 K$

$$\frac{P_{04}}{P_{05}} = \left(\frac{1500}{1154.3}\right)^{\frac{1}{0.2248}} = 3.206$$

$$LP \text{ system } (B+1) \times 1.004 \times 85.8 = 1.147 \times 0.99 \times \Delta T_{56} \quad \Delta T_{56} = 227.6 \quad T_6 = 926.7 K$$

$$\frac{P_{05}}{P_{06}} = \left(\frac{1154.3}{926.7}\right)^{\frac{1}{0.2248}} = 2.656$$

$$P_{04} = 0.3594 \times 2.2 \times 9.0 \times 0.93 = 6.618 \text{ bar}$$

$$P_{02} = 0.3594 \times 2.2 = 0.7907 \text{ bar}$$

$$P_{06} = \frac{6.618}{3.206 \times 2.656} = 0.7772 \text{ bar} \quad \frac{P_{06}}{P_a} = 6.418 \quad \frac{P_{02}}{P_a} = 6.529$$

With full expansion to P_a

$$\frac{T_{02}}{T_2} = (6.529)^{0.286} = 1.710 \quad T_2 = 226.6K ; C_c^2 = 2 \times 1.004 \times 10^3 [387.4 - 226.6] = 568 \text{ m/s}$$

$$\frac{T_{06}}{T_6} = (6.418)^{0.25} = 1.582 \quad T_6 = 582.2K ;$$

$$C_h^2 = 2 \times 1.147 \times 10^3 [926.7 - 582.2] = 888.9 \text{ m/s}$$

$$m_c = 70 \times \frac{2}{2+1} = 46.67 ; m_c C_c = 46.67 \times 568 = 26508 \text{ N}$$

$$m_h = 70 \times \frac{1}{2+1} = 23.33 ; m_h C_h = 23.33 \times 888.9 = 20738 \text{ N}$$

$$47246 \text{ N (gross)}$$

$$C_a = 1.4 \times 295.1 = 413.1 \text{ ram drag} = 70 \times 413.1 = 28920 \text{ N}$$

$$[Specific Thrust = 261.8 \text{ N / kg / hr}] \quad 18326 \text{ N (nett)}$$

$$T_{04} - T_{03} = 1500 - 778 - 722 \quad \therefore f_{ideal} = 0.023 \quad f = \frac{0.023}{0.99} = 0.0232$$

$$\therefore m_f = 23.33 \times 0.0232 \times 3600 = 1951 \text{ kg/hr}$$

$$SFC = \frac{1951}{18326} = 0.1065 \text{ kg/Nh}$$

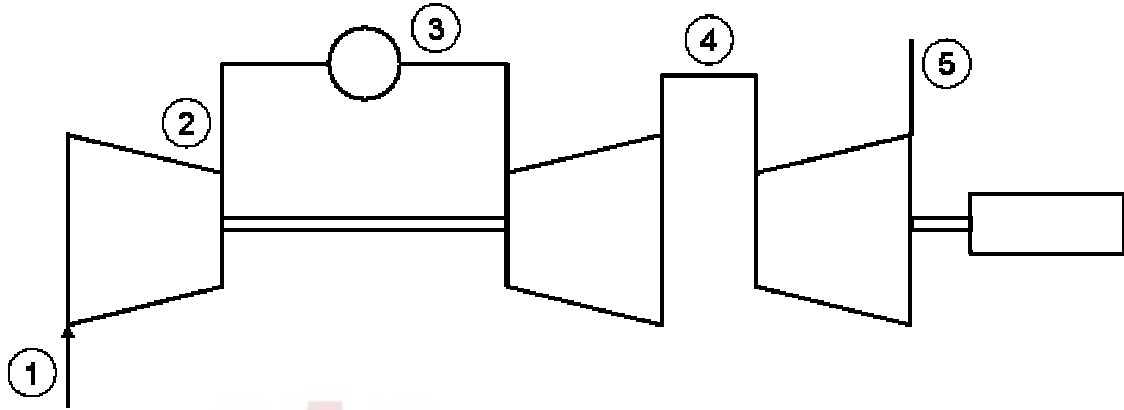
$$C_a = 413 \text{ m/s} = (413/1000) \times 3600 \text{ km/h} = 1486 \text{ km/h}$$

$$\therefore \text{Flight time} = \frac{5600}{1486.8} = 3.766 \text{ hrs}$$

$$\therefore \text{Fuel flow} = 3.766 \times 2 \times 1951 = 14,697 \text{ kg}$$

Problem 3.10

$$P_a = 1.013 \text{ bar} ; T_a = 303 \text{ K}$$



$$\Delta T_{12} = \frac{303}{0.81} [15^{0.286} - 1] = 437.5$$

$$\therefore T_2 = 740.5 \text{ K} \quad \Delta T_{cc} = 1600 - 740.5 = 859.5^\circ \text{C}$$

$$\Delta T_{34} = \frac{437.5 \times 1.005}{1.148 \times 0.99 \times 0.97} = 398.8 \quad \therefore 398.8 = 0.87 \times 1600 \left[1 - \frac{1}{R^{0.25}} \right]$$

$$\therefore R_{hp} = 3.86$$

$$P_3 = 1.013 \times 15.0 \times 0.95 = 14.43 \text{ bar} \quad P_4 = \frac{14.43}{3.86} = 3.74 \text{ bar}$$

$$\frac{P_4}{P_5} = \frac{3.74}{1.013} = 3.692$$

$$T_4 = 1600 - 398.8 = 1201.2 \text{ K}$$

$$\therefore \Delta T_{45} = 1201.2 \times 0.89 \left[1 - \frac{1}{3.692^{0.25}} \right] = 297.8 \text{ K}$$

$$\therefore \text{Specific output} = 1.148 \times 297.8 \times 0.99 \times 0.985 = 33.4 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta T_{cc} = 860, T_{in} = 467^\circ \text{C} \quad (f/a)_{ideal} = 0.0254$$

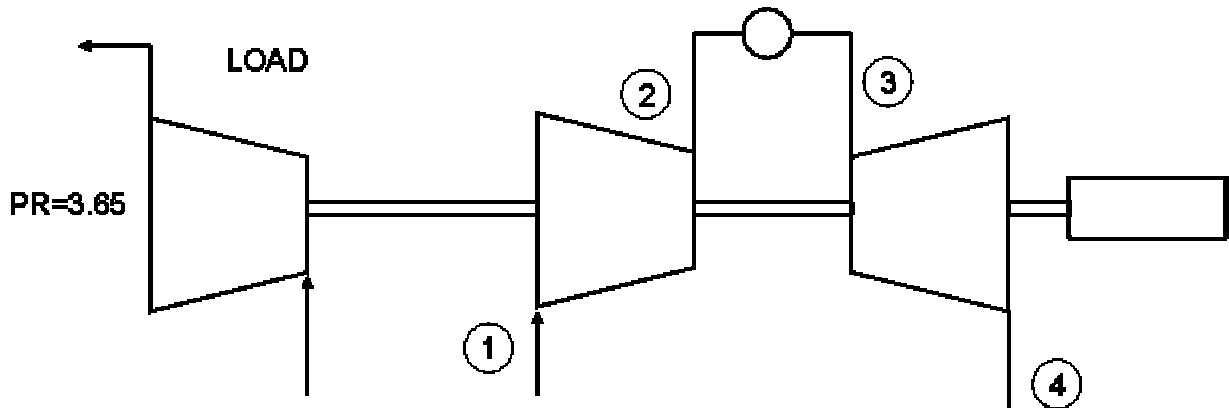
$$\therefore fa = \frac{0.0254}{0.99} = 0.0257$$

$$\text{Airflow} = 11.0 \text{ kg/s} \quad \therefore \text{Power} = 333.4 \times 11.0 = 3667 \text{ kW}$$

$$m_f = 0.0257 \times 11.0 \times 3600 = 1018.9 \text{ kg/hr}$$

$$\therefore SFC = 0.278 \text{ kg/kWh}$$

Problem 3.11



$$\text{Load compressor } \Delta T = \frac{308}{0.88} [3.65^{0.286} - 1] = 156.9 \text{ K}$$

$$\text{Power} = 3.5 \times 1.004 \times 156.9 = 551.2 \text{ kW}$$

$$\text{For gas turbine } \Delta T_{12} = \frac{308}{0.84} [12^{0.286} - 1] = 379.6 \quad T_{02} = 688 \text{ K}$$

$$\frac{P_{03}}{P_{04}} = (12 \times 1.01 \times 0.95) / 1.04 = 11.07$$

$$\therefore \Delta T_{34} = 1390 \times 0.87 \left[1 - \frac{1}{11.07^{0.25}} \right] = 546$$

$$m \times 1.148 \times 546.0 \times 0.99 = m \times 1.004 \times 379.6 + 200 + 551.2$$

$$m[620.5 - 381.1] = 755.8 \quad \therefore m = \frac{755.8}{239.4} = 3.16 \text{ kg/s}$$

$$f/a(690, \Delta T = 700) = \frac{0.02}{0.99}$$

$$\therefore m_f = \frac{0.02}{0.99} \times 3.16 \times 3600 = 229.6 \text{ kg/hr}$$

$$SFC = \frac{229.6}{755.8} = 0.304 \text{ kg/kwhr}$$

Problem 4.1

$$\text{Flow area} = \pi(15^2 - 6.5^2) = 574 \text{ cm}^2 = 0.0574 \text{ m}^2$$

$$C = \frac{8.0}{\rho \times 0.0574} = \frac{139.37}{\rho} \text{ m/s}$$

$$T_s = 288 - \frac{C^2}{2010}, \quad \frac{P_0}{P_s} = \left(\frac{T_0}{T_s} \right)^{3.5}, \quad \theta_c = \frac{C^2}{2c_p}$$

$$\rho = \frac{P_s}{RT_s} = \frac{100 \times P_s}{0.287 \times T_s} = 348.4 \frac{P_s}{T_s}$$

We must find the axial velocity by iteration to satisfy continuity:

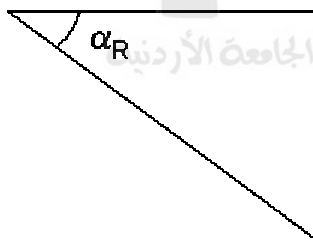
Let $C_{\max} = 150 \text{ m/s}$

C	θ	T_s	T_0/T_s	P_0/P_s	P_s	ρ	C_{calc}
150	11.12	276.8	1.0405	1.149	0.870	1.095	127.3
127.3	8.05	279.94	1.0288	1.104	0.905	1.127	123.7
123.7	7.61	280.39	1.0271	1.098	0.910	1.131	123.1
123.1	7.54	280.46	1.0269	1.097	0.911	1.132	123.1

$$\therefore C_a = 123.1 \text{ m/s}, \quad T_s = 280.46 \text{ K}$$

$$\text{At root, } U = \pi \times (0.065 \times 2) \times 270 = 110.3 \text{ m/s}$$

110.3



123.1

$$\tan \alpha_r = \frac{123.1}{110.3} = 1.116$$

$$\alpha_r = 48^\circ 10'$$

At tip, $U = \pi \times (0.15 \times 2) \times 270 = 254.4 \text{ m/s}$

$$\tan \alpha_t = \frac{123.1}{254.4} = 0.4839$$

$$\alpha_t = 25^\circ 48'$$

At tip $V^2 = 254.4^2 + 123.1^2 = 79720.42 \Rightarrow V = 282.3 \text{ m/s}$



$$a = \sqrt{1.4 \times 0.287 \times 280.46 \times 10^3} = 335.7 \text{ m/s}$$

$$\therefore M = \frac{282.3}{335.7} = 0.841$$

Problem 4.2

$$T_1 = T_{01} - \frac{C_1^2}{2C_p}; T_{01} = 217 + \frac{230^2}{2 \times 1.005 \times 10^3} = 243.3 \text{ K}$$

$$P_{01} = 0.23 \left(\frac{243.3}{217} \right)^{3.5} = 0.343 \text{ bar}$$

$$A = \frac{\pi}{4} (0.33^2 - 0.18^2) = 0.060 \text{ m}^2$$

$$m = \rho A C_a = \rho A C \cos 25 \Rightarrow \rho C = \frac{3.6}{0.060 \times 0.906}$$

$$\rho C = 66.22$$

But based on stagnation conditions, for a first estimate

$$\rho = \frac{100}{0.287} \times \frac{P_0}{T_0} = \frac{100 \times 0.343}{0.287 \times 243.3} = 0.491 \text{ kg/m}^3$$

$$\therefore C = \frac{66.22}{0.491} = 134.87 \text{ m/s}$$

First trial: $C_{\max} = 140 \text{ m/s}$

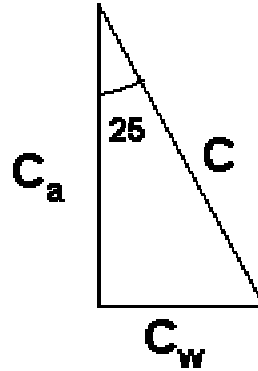
C	θ	T_s	T_0/T_s	P_0/P_s	P_s	ρ	ρC
140	9.7	233.7	1.042	1.155	0.297	0.443	62
150	11.2	232.2	1.049	1.182	0.290	0.435	65.2
160	12.7	230.7	1.056	1.21	0.287	0.434	69.2
152.4	11.5	231.9	1.051	1.19	0.288	0.433	66.0

$$\text{Take } C = 152.4 \times \frac{66.22}{66.0} = 152.90 \text{ m/s}$$

$$C_{1w} = 152.9 \times \sin 25 = 152.9 \times 0.4226 = 64.61 \text{ m/s}$$

$$\frac{P_{02}}{P_{01}} = \left\{ 1 + \frac{\psi \eta_c}{C_p T_{01}} \left[\sigma U^2 - C_w (r \omega)_{\text{mean}} \right] \right\}^{3.5}$$

$$U_e = (r \omega)_{\text{mean}} = \left(\frac{18 + 33}{2 \times 100} \right) \times \frac{1}{2} \times 270 \times 2\pi = 216.3 \text{ m/s}$$

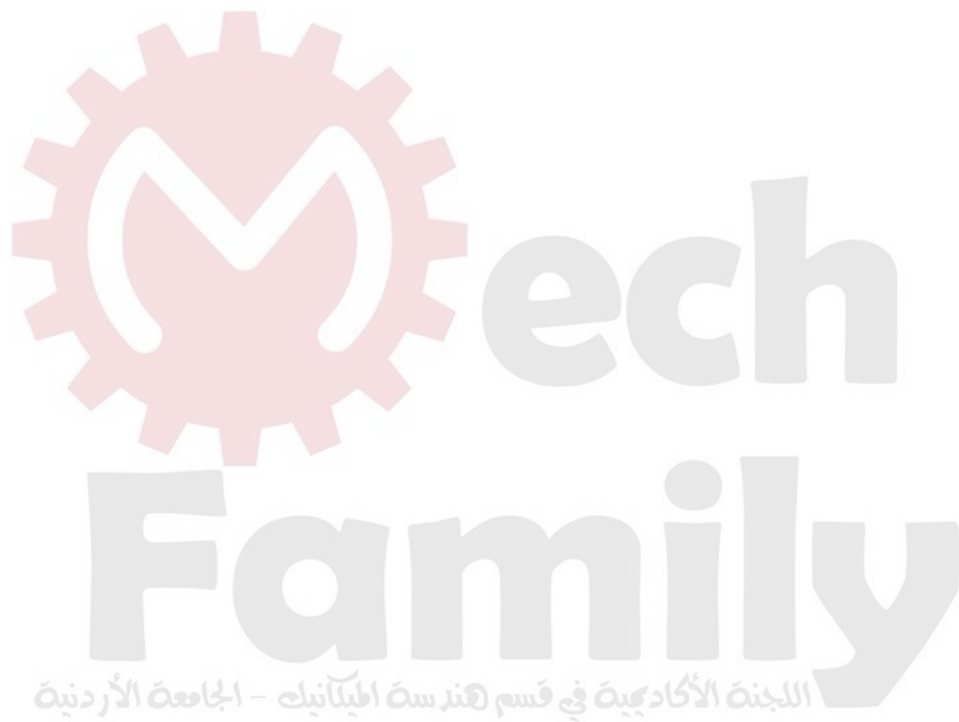


$$U = \pi \times 270 \times 0.54 = 458 \text{ m/s}$$

$$\frac{P_{02}}{P_{01}} = \left\{ 1 + \frac{1.04 \times 0.8}{1.005 \times 243.3 \times 10^3} [0.9 \times 458^2 - 64.61 \times 216.3] \right\}^{3.5}$$

$$\frac{P_{02}}{P_{01}} = (1.595)^{3.5} = 5.12$$

$$\therefore P_{02} = 5.12 \times 0.343 = 1.756 \text{ bar}$$



Problem 4.3

$$P_{01} = P_a = 0.99 \text{ bar and } T_{01} = T_a = 288 \text{ K}$$

$$T_{02} - T_{01} = 429 - 288 = \frac{T_{01}}{\eta} \left[\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\therefore \eta_c = \frac{288}{141} \left[\left(\frac{2.97}{0.99} \right)^{\frac{1}{3.5}} - 1 \right] = 0.753$$

$$\sigma = 1 - \frac{0.63\pi}{17} = 0.8835$$

$$U_2 = \pi \times \frac{16.5}{100} \times 765 = 397.1 \text{ m/s}$$

$$\text{Hence: } C_{2w} = \sigma U_2 = 0.8835 \times 397.1 = 351 \text{ m/s}$$

$$T_2 = T_{02} - \frac{C_{2w}^2}{2C_p} = 429 - \left(\frac{351^2}{2 \times 1.005 \times 10^3} \right) - \frac{C_{r2}^2}{2C_p}$$

$$\text{Therefore } T_2 = 367.7 - \theta_r$$

$$\rho_2 = \frac{100 \times 1.92}{0.287 \times T_2} = \frac{669}{T_2}$$

$$A_2 = \frac{\pi \times 16.5 \times 1.0}{10^4} = 51.83 \times 10^{-4} \text{ m}^2$$

$$C_{r2} = \frac{m}{\rho_2 A_2} = \frac{0.60}{51.83 \times 10^{-4}} \times \frac{1}{\rho_2} = \frac{115.76}{\rho_2}$$

$$\theta_r \text{ is small - For the first trial let } T_2 = 366 \text{ K}$$

$$\therefore \rho_2 = \frac{669}{366} = 1.83 \quad \text{Hence } C_{r2} = 63.26 \text{ m/s}$$

Final trial:

C_{r2}	θ	T_2	ρ	C_{r2}
64.0	2.03	366	1.827	63.5

$$\therefore T_2 = 366 \text{ K}$$

$$\frac{P_{2t}}{P_2} = \left(\frac{T_{2t}}{T_2} \right)^{3.5} = \left(\frac{429}{366} \right)^{3.5} = 1.743$$

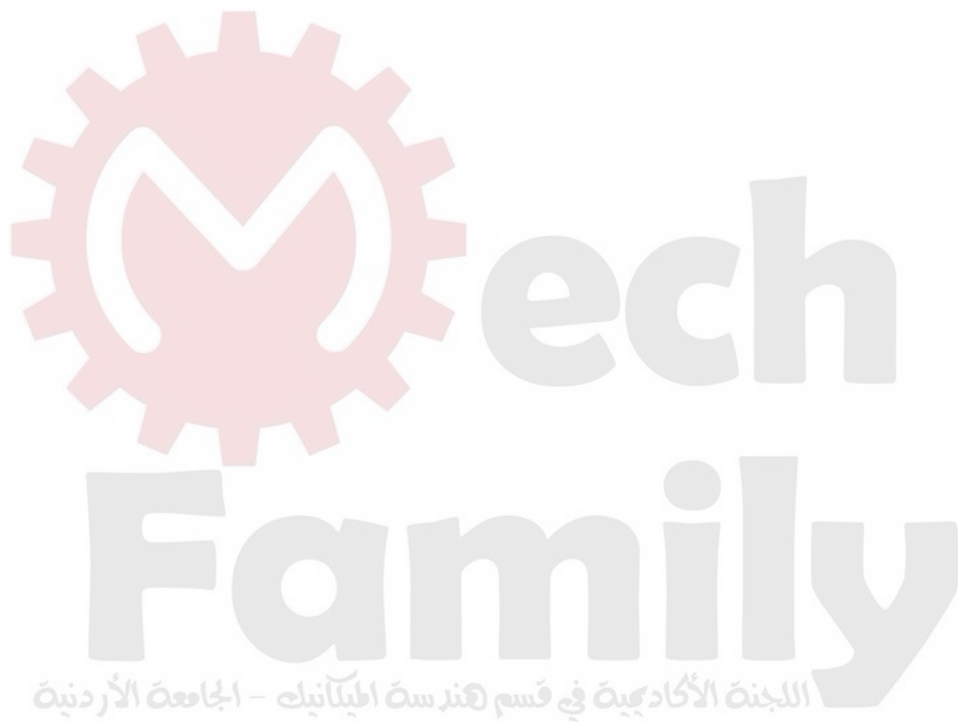
$$P_{2t} = 1.92 \times 1.743 = 3.347 \text{ bar}$$

$$\frac{P_{02}}{P_{01}} = \left[1 + \eta_{imp} \times \frac{141}{288} \right]^{3.5}$$

$$\frac{3.347}{0.99} = \left[1 + \eta_{imp} \times \frac{141}{288} \right]^{3.5}$$

$$\therefore \eta_{imp} = 0.850$$

$$\text{Fraction of loss in impeller} = \frac{1 - 0.87}{1 - 0.753} = 0.607$$



Problem 4.4

(a) $U_2 = \pi \times 0.50 \times 270 = 424.12 \text{ m/s}$

$$\Delta T = \frac{1.04 \times 0.9 \times 424.12^2}{1.005 \times 10^3} = 167.5 \text{ K}$$

$$T_{02} = 455.5 \text{ K}$$

$$\frac{P_{02}}{P_{01}} = \left[1 + \frac{0.90 \times 167.5}{288} \right]^{3.5} = (1.523)^{3.5} = 4.36$$

$$P_{02} = 4.36 \times 1.01 = 4.40 \text{ bar}$$

(b) Flow area at tip: $= \frac{\pi \times 50 \times 5}{10^4} = 0.0785 \text{ m}^2$

$$C_{2w} = 0.9 \times 424.12 = 381.7 \text{ m/s}$$

If $C_{2r} = 96$ and $\theta_c = \left(\frac{381.7}{44.9} \right)^2 + \left(\frac{96}{44.9} \right)^2 = 76.84$

(noting $\sqrt{2 \times 1.005 \times 10^3} = 44.9$)

$$T_2 = 455.5 - 76.84 = 378.66 \text{ K}$$

$$\frac{P_{02}}{P_2} = \left(\frac{455.5}{378.66} \right)^{3.5} = 1.909 \quad P_2 = \frac{4.40}{1.909} = 2.30 \text{ bar}$$

$$\rho_2 = \frac{100 \times 2.30}{0.287 \times 378.66} = 2.12 \quad \rho_2 A_2 C_{r2} = 2.12 \times 0.0785 \times 96 = 15.97$$

The agreement is close since the required flow is 16.0 kg/sec.

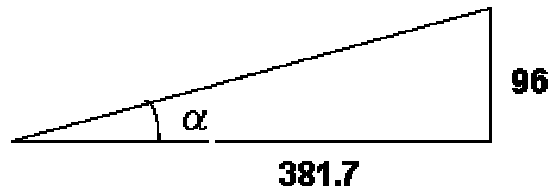
$\therefore C_{r2} = 96 \text{ m/s}$

$$a = \sqrt{0.287 \times 1.4 \times 378.66 \times 10^3} = 390 \text{ m/s}$$

$$C_2^2 = 381.7^2 + 96^2 \quad \therefore C_2 = 393.6 \text{ m/s}$$

$$M = \frac{C_2}{a} = \frac{393.6}{390} = 1.01$$

$$\tan \alpha = \frac{96}{381.7} = 0.251 \quad \therefore \alpha = 14^\circ 5'$$



(c) $P_{3t}=P_{2t}=4.40$ bar

$$A = \frac{\pi \times 58 \times 5}{10^4} = 0.0911 \text{ m}^2$$

$$\rho_3 C_{3r} = \frac{16.0}{0.0911} = 175.6 \quad C_{3w} = \frac{381.7 \times 25}{29} = 329 \text{ m/s}$$

We must find C_{3r} by trial and error; as a first guess assume:

$$C_{3r} = 96 \times \frac{25}{29} = 82.75 \text{ m/s}$$

$$T_{03}=T_{02}=455.5 \text{ K}$$

C_{3w}	θ_w	C_{3r}	θ_r	θ	T_3	T_0/T_s	P_0/P_s	P_s	ρ	ρC_r
329	53.6	82.75	3.4	57.0	398.4	1.143	1.596	2.755	2.41	199.5
		73	2.64	56.2	399.2	1.140	1.581	2.78	2.43	177.5
		72.4	2.60	56.2	399.1	1.140	1.581	2.78	2.43	176

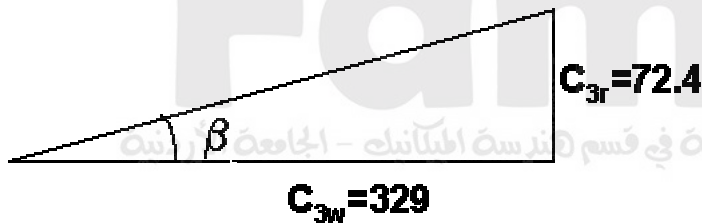
$$\therefore C_{3r} = 72.4 \text{ m/s}, \quad T_3 = 399.1 \text{ K}$$

$$C_3^2 = 329^2 + 72.4^2 \Rightarrow C_3 = 336.87 \text{ m/s}$$

$$a = \sqrt{1.4 \times 0.287 \times 399.1 \times 10^3} = 400.44 \text{ m/s}$$

$$M = \frac{336.87}{400.44} = 0.841$$

$$\tan \beta = \frac{72.4}{329} = 0.22 \quad \beta = 12^\circ 24'$$



$$\therefore P_{02} = 1 \times 4.607 = 4.607 \text{ bar}$$

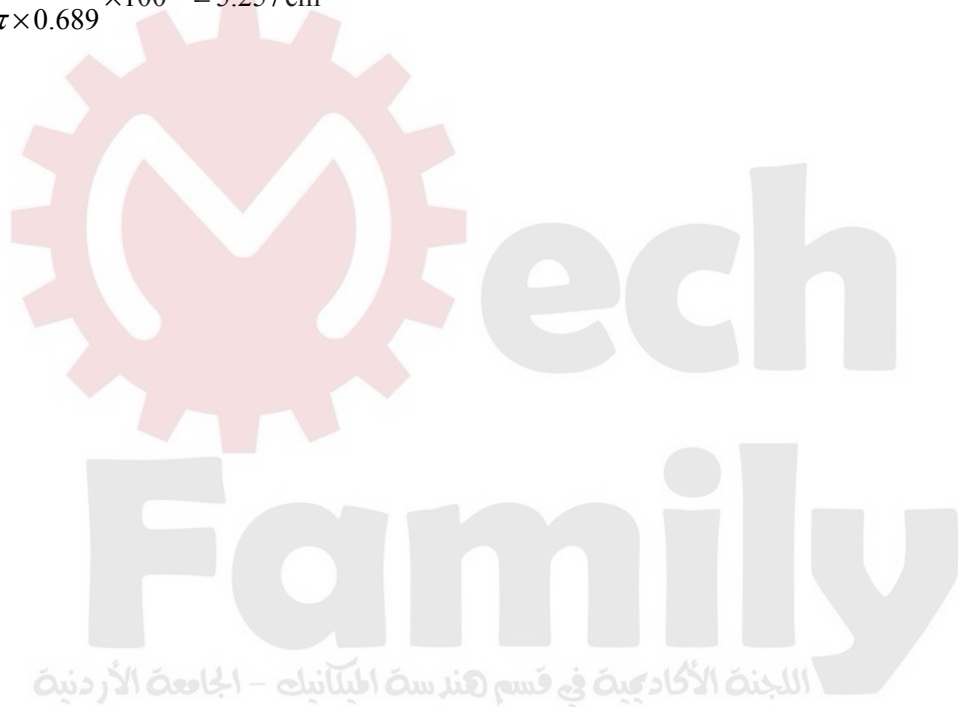
$$\frac{P_{02}}{P_2} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}} = 1.2^{3.5} = 1.893$$

$$P_2 = \frac{4.607}{1.893} = 2.434 \text{ bar}$$

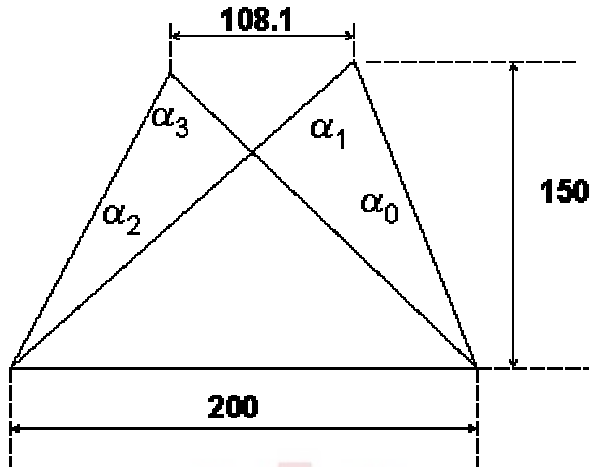
$$\rho_2 = \frac{100 \times 2.434}{0.287 \times 385.9} = 2.198 \text{ kg/m}^3$$

$$m = \rho A C_{2r} \quad \therefore A = \frac{14.0}{2.198 \times 55.97} = 0.1138 \text{ m}^2$$

$$\therefore h = \frac{0.1138}{\pi \times 0.689} \times 100 = 5.257 \text{ cm}$$



Problem 5.1



At mean radius, $m\Omega\Delta C_w = mC_p\Delta T$

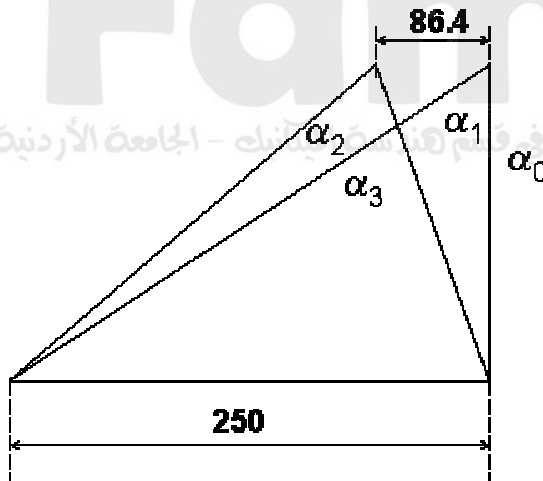
$$\therefore \Delta C_w = \frac{1.005 \times 20 \times 10^3}{0.93 \times 200} = 108.1 \text{ m/s}$$

$$C_{0w} = \frac{200 - 108.1}{22} = 45.95 \text{ m/s}$$

$$\tan \alpha_0 = \frac{45.95}{150} = 0.307 \quad \alpha_0 = 17^\circ 4' (= \alpha_2)$$

$$\tan \alpha_1 = \frac{200 - 45.95}{150} = 1.027 \quad \alpha_1 = 45^\circ 44' (= \alpha_3)$$

With free vortex, $C_w r = \text{constant}$ i.e. $C_w U = \text{constant}$



Tip

$$C_{0w} \times 250 = 46.0 \times 200$$

$$\therefore C_{0w} = 36.8 \text{ m/s}$$

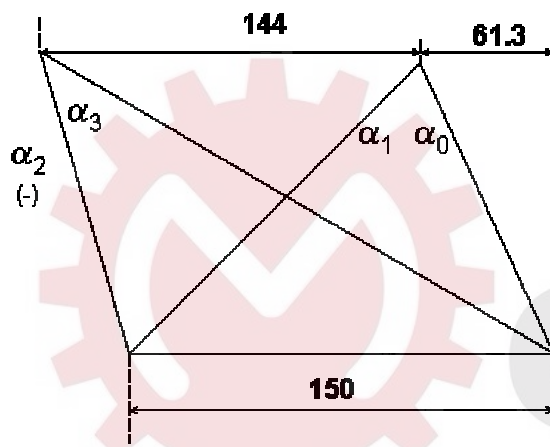
$$\Delta C_{wu} = \text{constant} \therefore \Delta C_w = \frac{108.1 \times 200}{250} = 86.4 \text{ m/s}$$

$$\tan \alpha_0 = \frac{36.8}{150} = 0.245, \alpha_0 = 13^\circ 46'$$

$$\tan \alpha_1 = \frac{250 - 36.8}{150} = 1.421, \alpha_1 = 54^\circ 53'$$

$$\tan \alpha_2 = \frac{36.8 + 86.4}{150} = 0.845, \alpha_2 = 40^\circ 14'$$

$$\tan \alpha_3 = \frac{36.8 + 86.4}{150} = 0.821, \alpha_3 = 39^\circ 26'$$



Root

$$C_{0w} \times 150 = 46 \times 200 \therefore C_{0w} = 61.3 \text{ m/s}$$

$$\Delta C_w = \frac{108 \times 200}{150} = 144 \text{ m/s}$$

$$\tan \alpha_0 = \frac{61.3}{150} = 0.409, \alpha_0 = 22^\circ 15'$$

$$\tan \alpha_1 = \frac{150 - 61.3}{150} = 0.591, \alpha_1 = 30^\circ 36'$$

$$\tan \alpha_2 = \frac{150 - (144 + 61.3)}{150} = -0.369, \alpha_2 = -20^\circ 15'$$

$$\tan \alpha_3 = \frac{144 + 61.3}{150} = 1.369, \alpha_3 = 53^\circ 51'$$

$$\Lambda = \frac{\text{static enthalpy rise in rotor}}{\text{Stagnation enthalpy rise in stage}} = \frac{T_1 - T_0}{20}$$

$$\text{Now } (T_0 + \frac{C_0^2}{2C_p} + 20) = T_1 + \frac{C_1^2}{2C_p} \therefore T_1 - T_0 = 20 + \left(\frac{C_0^2}{2C_p} - \frac{C_1^2}{2C_p} \right) \frac{1}{10^3}$$

Tip

$$C_0 = \frac{150}{\cos 13^\circ 46'} = \frac{150}{0.9712} = 154.5 \quad T_1 - T_0 = 20 + 11.19 - 18.8 = 13.1$$

$$C_1 = \frac{150}{\cos 39^\circ 26'} = \frac{150}{0.7724} = 194.1 \quad \therefore \Lambda = \frac{13.1}{20} = 65.5\%$$

Root

$$C_0 = \frac{150}{\cos 22^\circ 15'} = \frac{150}{0.9255} = 162.1$$

$$T_1 - T_0 = 20 + \frac{1}{10^3} \left(\frac{162.1^2}{2 \times 1.005} - \frac{254.2^2}{2 \times 1.005} \right)$$

$$C_1 = \frac{150}{\cos 53^\circ 51'} = \frac{150}{0.5899} = 254.2 \quad T_1 - T_0 = 20 + 13.07 - 32.15$$

$$\therefore T_1 - T_0 = 0.92, \quad \Lambda = \frac{0.92}{20} = 4.6\%$$

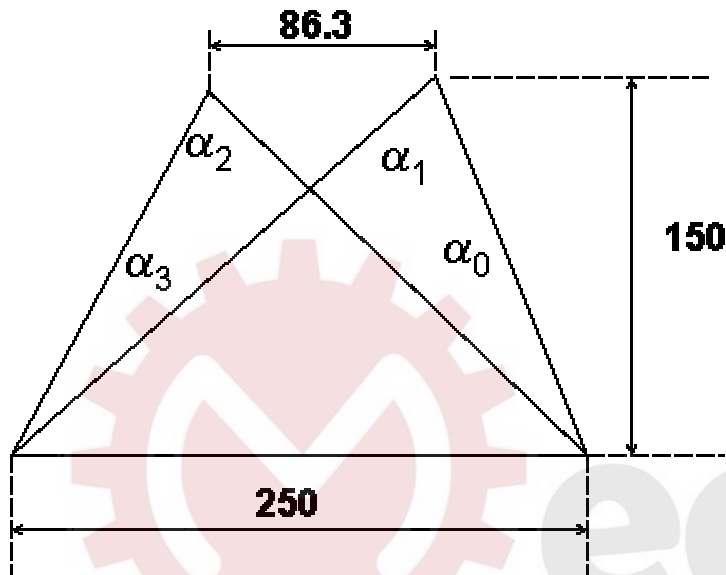
(N.B.: Λ is very sensitive to small errors in C_0 and C_1)

Problem 5.2

Velocity diagram at mid span is as in previous question.

ΔC_w : mean = 108, tip = 86.3, root = 144 m/s

Equal work at all radius



Tip

$$C_{0w} = \frac{250 - 86.3}{2} = 81.85 \text{ m/s}$$

$$\tan \alpha_0 = \frac{81.85}{150} = 0.546, \quad \alpha_0 = 28^\circ 36' (= \alpha_2)$$

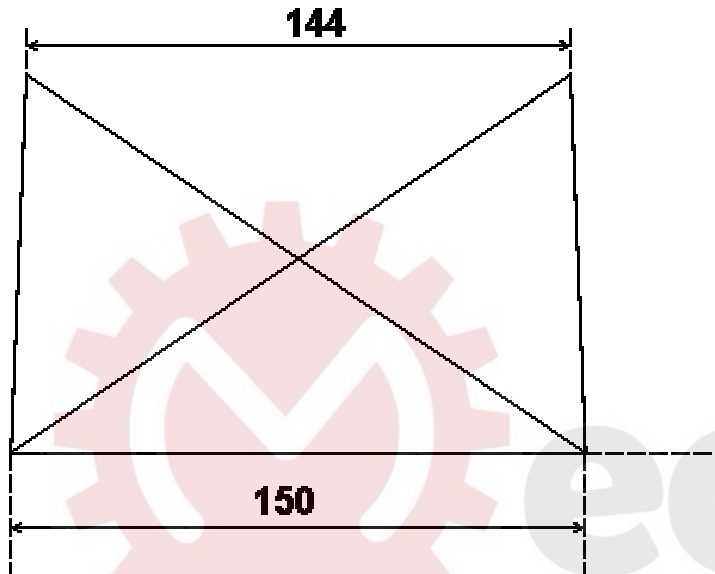
$$\tan \alpha_1 = \frac{250 - 81.85}{150} = 1.121, \quad \alpha_1 = 48^\circ 16' (= \alpha_3)$$

Root

$$C_{0w} = \frac{150 - 144}{2} = 3 \text{ m/s}$$

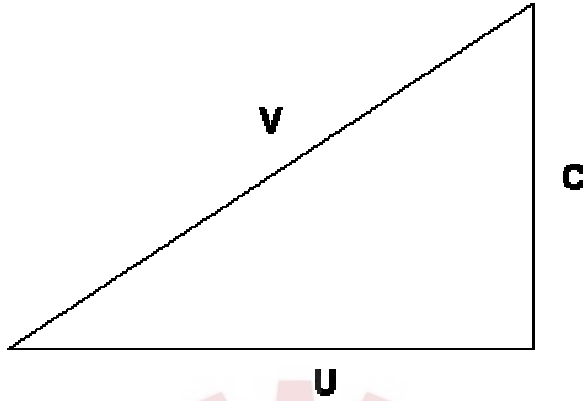
$$\tan \alpha_0 = \frac{3}{150} = 0.0020, \quad \alpha_0 = \alpha_2 = 0^\circ 7'$$

$$\tan \alpha_1 = \frac{150 - 3}{150} = 0.980, \quad \alpha_1 = \alpha_3 = 44^\circ 25'$$



Problem 5.3

(a) With no inlet guide vanes the inlet velocity is *axial*.



$$T = 288 - \frac{140 \times 140}{2 \times 1.005} \times \frac{1}{10^3} = 278.25 \text{ K}$$

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 0.287 \times 278.25 \times 10^3} = 334.4 \text{ m/s}$$

$$\therefore V = 0.95 \times 334.4 = 317.7 \text{ m/s}$$

$$V^2 = U^2 + C^2 \therefore U^2 = 317.7^2 - 140^2 = 81333.3$$

$$\therefore U = 285.2 \text{ m/s}$$

$$\pi \times D \times N = 285.2 \therefore D = \frac{285.2}{\pi \times 100} = 0.908 \text{ m}$$

$$\therefore \text{Tip radius} = 0.454 \text{ m or } 45.40 \text{ cm}$$

$$(b) \frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{3.5} = \left(\frac{288}{278.25} \right)^{3.5} = 1.128$$

$$P = \frac{1.01}{1.128} = 0.895 \text{ bar}$$

$$\rho = \frac{0.895 \times 100}{0.287 \times 278.25} = 1.121 \text{ kg/m}^3$$

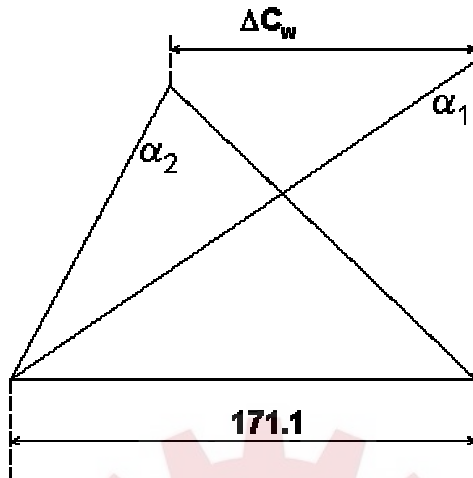
$$D_{\text{hub}} = 0.60 \times 0.908 = 0.545 \text{ m}$$

$$A = \frac{\pi}{4} (0.908^2 - 0.545^2) = 0.4143 \text{ m}^2$$

$$\therefore m = 1.121 \times 0.4143 \times 140 = 65.02 \text{ or } m = 65 \text{ kg/s}$$

$$(c) \frac{P_{02}}{P_{01}} = \left[1 + \frac{0.89 \times 20}{288} \right]^{3.5} = 1.2335$$

- (d) At the root we have axial inlet velocity. Free vortex gives constant work at all radii and constant axial velocity.



$$U_h = 0.60 \times 285.2 = 171.1 \text{ m/s}$$

$$m U \Delta C_w \Omega \frac{1}{10^3} = m C_p \Delta T$$

$$\Delta C_w = \frac{C_p \Delta T \times 10^3}{u \Omega} = \frac{1.005 \times 20 \times 10^3}{171.1 \times 0.93} = 126.3 \text{ m/s}$$

$$\tan \alpha_1 = \frac{171.1}{140} = 1.2221, \alpha_1 = 50^\circ 44'$$

$$\tan \alpha_2 = \frac{171.1 - 126.3}{140} = 0.320, \alpha_2 = 17^\circ 46'$$

At tip

$$\Delta C_w = 126.3 \times 0.60 = 75.78 \text{ m/s}$$

$$\tan \alpha_1 = \frac{285.2}{140} = 2.0371, \alpha_1 = 63^\circ 50'$$

$$\tan \alpha_2 = \frac{285.2 - 75.78}{140} = 1.4958, \alpha_2 = 56^\circ 14'$$

$$\text{Power input} = 65 \times 1.005 \times 20 = 1306.5 \text{ kW}$$

Problem 5.4

$$\frac{n-1}{n} = \frac{0.286}{0.88} = 0.325$$

$$\frac{T_{02}}{T_{01}} = (4.0)^{0.325} = 1.569$$

$$\Delta T_{012} = 288[1.569 - 1] = 163.9 \text{ K}$$

$$\therefore \text{Number of stages} = \frac{163.9}{25} = 6.556 \text{ i.e. 7 stages}$$

$$\frac{\Delta T}{\# \text{ of stages}} = \frac{163.9}{7} = 23.4 \text{ K/stage}$$

$$\text{For first stage: } \frac{T_{out}}{T_{in}} = \frac{288 + 23.4}{288} = 1.0812$$

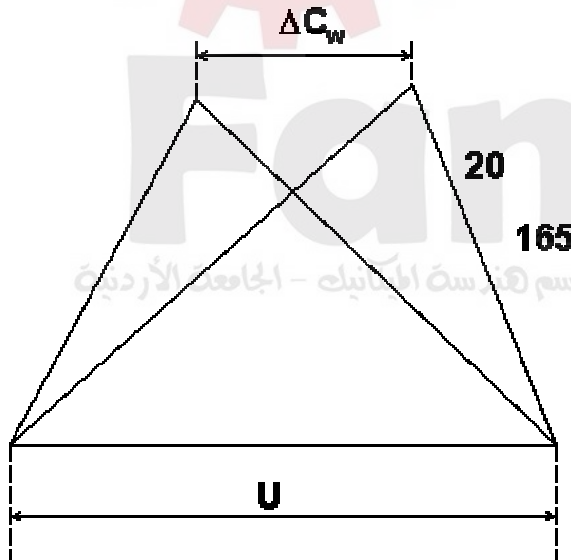
$$1.0812 = (R)^{0.325} \therefore R_{\text{first}} = 1.271$$

For last stage:

$$T_{out} = 288 + 163.9 = 451.9 \text{ K}$$

$$T_{in} = 451.9 - 23.4 = 428.5 \text{ K}; \therefore \frac{T_{out}}{T_{in}} = 1.0546$$

$$1.0546 = (R)^{0.325} \therefore R_{\text{last}} = 1.178$$



$$mU\Delta C_w \Omega \frac{1}{10^3} = mC_p \Delta T ; (\text{note that } C_a = 165 \cos 20)$$

$$\Delta C_w = U - 2C \sin 20 = U - 330 \sin 20$$

$$\Omega U (U - 112.9) \frac{1}{10^3} = C_p \Delta T$$

$$U^2 - 112.9U = \frac{1.005 \times 23.4 \times 10^3}{0.83}$$

$$U^2 - 112.9U - 28.3 \times 10^3 = 0$$

$$\therefore U = \frac{112.9 \pm \sqrt{112.9^2 + 4 \times 28.3 \times 10^3}}{2} = \frac{112.9 \pm 354.9}{2}$$

$$\therefore U = 233.9 \text{ m/s}$$

$$\frac{\pi DN}{60} = 233.9 \quad \therefore N = \frac{60 \times 233.9}{\pi \times 0.18} = 24817.5 \text{ rpm}$$

At the entry to last rotor:

$$T_0 = 428.5; P_0 = \frac{4 \times 1.01}{1.178} = 3.43 \text{ bar}$$

$$C = 165 \text{ m/s} \quad \therefore T = 428.5 - \frac{165^2}{2.01 \times 10^3} = 414.96 \text{ K}$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{3.5} = \left(\frac{428.5}{414.96} \right)^{3.5} = 1.1189$$

$$P = \frac{3.43}{1.1189} = 3.07 \text{ bar}$$

$$\therefore \rho = \frac{100 \times 3.07}{0.287 \times 414.96} = 2.58 \text{ kg/m}^3$$

$$(\pi Dh) \rho \times C_a = m$$

$$\pi \times 0.18 \times h \times 2.58 \times 165 \cos 20 = 3.0$$

$$\therefore h = \frac{3.0}{\pi \times 0.18 \times 2.58 \times 165 \times 0.9397} = 0.01326 \text{ m}$$

$$h = 1.326 \text{ cm}$$

Problem 5.5

$$\text{Axial: } \frac{n-1}{n} = \frac{0.286}{0.92} = 0.311 \quad \text{Centrifugal: } \frac{n-1}{n} = \frac{0.286}{0.83} = 0.3446$$

$$\text{For axial } \Delta T = 4 \times 30 = 120 \text{ K} \quad \therefore T_2 = 288 + 120 = 408 \text{ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{408}{288} = 1.417 = \left(\frac{P_{02}}{P_{01}} \right)^{0.311} \quad \therefore \frac{P_{02}}{P_{01}} = 3.07$$

$$\therefore \frac{P_{03}}{P_{02}} = \frac{10.0}{3.07} = 3.257$$

Axial :

$$mU\Delta C_w \Omega \frac{1}{10^3} = mC_p \Delta T$$

$$\Delta C_w = U - 2C_a \tan 20 = U - 300 \times 0.364 = U - 109.2$$

$$U(U - 109.2) = \frac{10^3 \times 1.005 \times 30}{0.86} = 35.05 \times 10^3$$

$$U^2 - 109.2U - 35.05 \times 10^3 = 0$$

$$\therefore U = \frac{109.2 \pm \sqrt{109.2^2 + 4 \times 35.05 \times 10^3}}{2} = \frac{109.2 \pm 390}{2}$$

$$U = 249.6 \text{ m/s}$$

$$249.6 = \frac{\pi DN}{60} \quad \therefore N = \frac{60 \times 249.6}{\pi \times 0.25} = 19068 \text{ rpm}$$

Centrifugal:

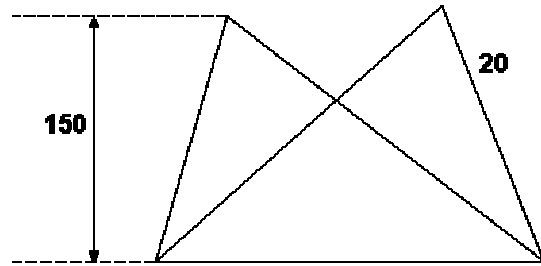
$$\frac{T_{03}}{T_{02}} = \left(\frac{P_{03}}{P_{02}} \right)^{0.3446} = (3.257)^{0.3446} = 1.502$$

$$\Delta T_{023} = 0.502 \times 408 = 204.82 \text{ K}$$

$$\Delta T = \frac{\sigma \psi U_2^2}{C_p} = \frac{0.90 \times 1.04 \times U_2^2}{1.005 \times 10^3} \quad \therefore U_2^2 = \frac{1.005 \times 10^3 \times 204.82}{0.90 \times 1.04}$$

$$U_2 = 468.95 \text{ m/s}$$

$$\therefore N(\text{Centrifugal}) = \frac{60 \times 468.95}{\pi \times 0.33} = 27140 \text{ rpm}$$



Problem 6.1

- (a) From Fig. 2.15, for $T_1=298\text{ K}$ and $\Delta T = 583\text{ K}$, theoretical $f=0.0147$

$$\eta_b = \frac{0.0147}{0.0150} = 0.980$$

- (b) From Fig. 2.15, for $f=0.0150$ and $T_1=298\text{ K}$, theoretical $\Delta T = 593\text{ K}$

$$\eta_b = \frac{583}{593} = 0.983$$

- (c) mass of CO per kg of fuel:

$$m = \frac{28}{12} \times 0.04 \times 0.8608 = 0.0803\text{ kg}$$

$$\text{Actual released energy} = 43100 - (0.0803 \times 10110)$$

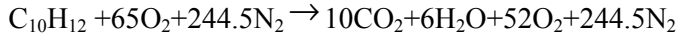
Efficiency based on released energy:

$$\eta = \frac{43100 - (0.0803 \times 10110)}{43100} = 1 - 0.0188$$

$$\eta = 0.9812$$

Problem 6.2

The combustion equation:



132kg 2080 6846 440 108 1664 6846

$$\text{Fuel/ air ratio} = \frac{132}{(2080 + 6846)} = 0.0148$$

Since the H_2O will be in vapour phase, we require ΔH_{vap} , given by:

$$\Delta H_{\text{vap}} = -42500 + \frac{108}{132} \times 2442 = -40500 \text{ KJ/kg of } \text{C}_{10}\text{H}_{12}$$

Energy equation is:

$$(H_{p2} - H_{p0}) + \Delta H_0 + (H_{R_0} - H_{R_1}) = 0$$

Where the first stage is 325 K for $\text{C}_{10}\text{H}_{12}$ and 450 K for air

$$\text{Mean temperature of reactants } \text{O}_2 \text{ and } \text{N}_2 \text{ is } \frac{450 + 298}{2} = 374 \text{ K}$$

From fig. 2.15 for $f=0.0148$ and for an initial temperature of 450 K

For air, temperature rise $\Delta T = 565 \text{ K}$.

Hence approximate final temperature $T_2 = 565 + 450 = 1015 \text{ K}$

First guess at mean temperature of products = $(1015 + 298)/2 = 656.5 \text{ K}$. Mean values of C_p from tables are

: R- reactants, P- products. اللجنة الأكاديمية في قسم هندسة الميكانيك -

	R's	P's	1 st Guess	2 nd Guess	mC_p
O_2	0.934	CO_2	1.105	1.047	460.7
N_2	1.042	H_2O	2.051	2.041	220.4
		O_2	1.019	1.014	1687.2
		N_2	1.088	1.084	7421.1
					9789.4

$$(T_2 - 298) \{ (440 \times 1.105) + (108 \times 2.051) + (1664 \times 1.019) + (6846 \times 1.088) \}$$

$$- (132 \times 40500) - (450 - 298) [(2080 \times 0.934) + (6846 \times 1.042)]$$

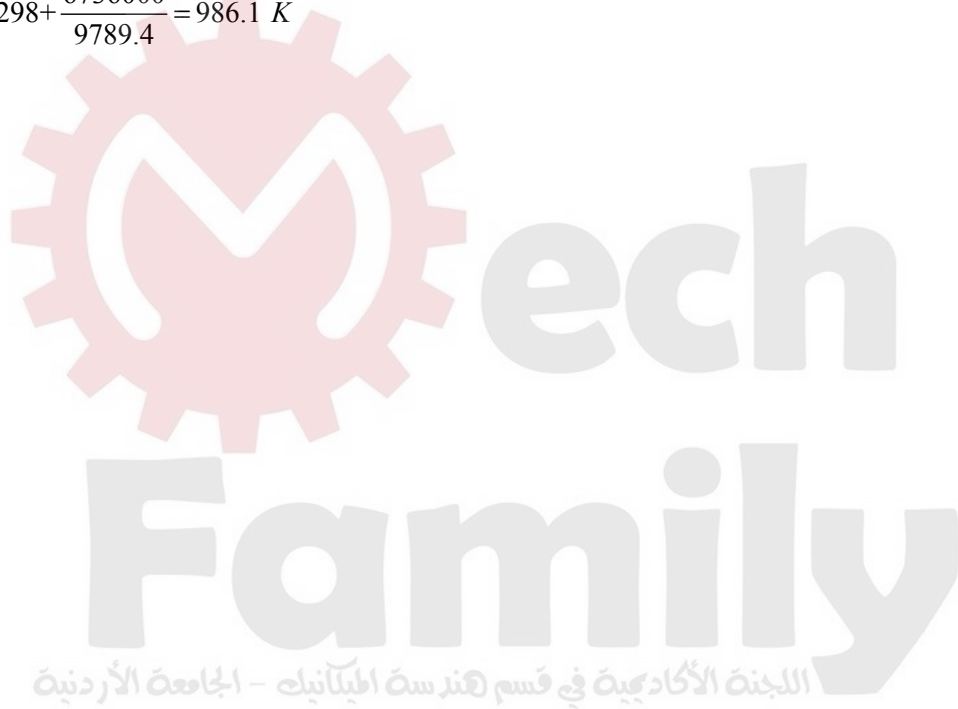
$$- (325 - 198) [132 \times 1.945]$$

$$= (T_2 - 298) \times 985.5 - 5,35,000 - 152 \times 9073 - 27 \times 257 = 0$$

$$\therefore T_2 = 298 + \frac{6736000}{9855} = 982 \text{ K}$$

Second guess at mean temperature of products = $1280/2 = 640 \text{ K}$

$$\therefore T_2 = 298 + \frac{6736000}{9789.4} = 986.1 \text{ K}$$



Problem 6.3

Under design conditions:

$$T_{01} = T_1 + \frac{C_1^2}{2C_p} = \frac{P_1}{R\rho_1} + \frac{m^2}{2\rho_1^2 A_1^2 C_p}$$

$$475 = \frac{4.47 \times 100}{0.287 \rho_1} + \frac{9.0^2}{2\rho_1^2 \times 0.0389^2 \times 1.005 \times 10^3}$$

$$475 = \frac{1557}{\rho_1} + \frac{26.70}{\rho_1^2}$$

$$475 \rho_1^2 - 1557 \rho_1 - 26.70 = 0$$

$$\text{From which : } \rho_1 = \frac{1557 \pm \sqrt{1557^2 - 4 \times 475 \times (-26.76)}}{2 \times 475}$$

$$\rho_1 = 3.294 \text{ kg/m}^3$$

$$\text{Hence: } \frac{0.27 \times 10^5}{9.0^2 / 2 \times 3.294 \times 0.0975^2} = 19.0 + K_2 \left(\frac{1023}{475} - 1 \right)$$

$$20.88 = 19.0 + 1.153 K_2$$

$$\therefore K_2 = 1.630$$

Part-load conditions:

$$439 = \frac{3.52 \times 100}{0.287 \rho_1} + \frac{7.4^2}{2\rho_1^2 \times 0.0389^2 \times 1.005 \times 10^3} = \frac{1226.5}{\rho_1} + \frac{18.004}{\rho_1^2}$$

$$439 \rho_1^2 - 1226.5 \rho_1 - 18 = 0$$

$$\therefore \rho_1 = 2.808 \text{ kg/m}^3$$

$$10^5 \Delta P_0 = \frac{7.4^2}{2 \times 2.808 \times 0.0975^2} \left[19.0 + 1.630 \left(\frac{900}{439} - 1 \right) \right]$$

$$\Delta P_0 = 0.213 \text{ bar}$$

$$\text{At design : } C_1 = \frac{m}{\rho_1 A_1} = \frac{9.0}{3.294 \times 0.0389} = 70.24 \text{ m/s}$$

$$\text{At part load: } C_1 = \frac{7.4}{2.808 \times 0.0389} = 67.75 \text{ m/s}$$

$$\text{At design: } T_1 = \frac{P_1}{R\rho_1} = \frac{4.47 \times 100}{0.287 \times 3.294} = 472.8 \text{ K}$$

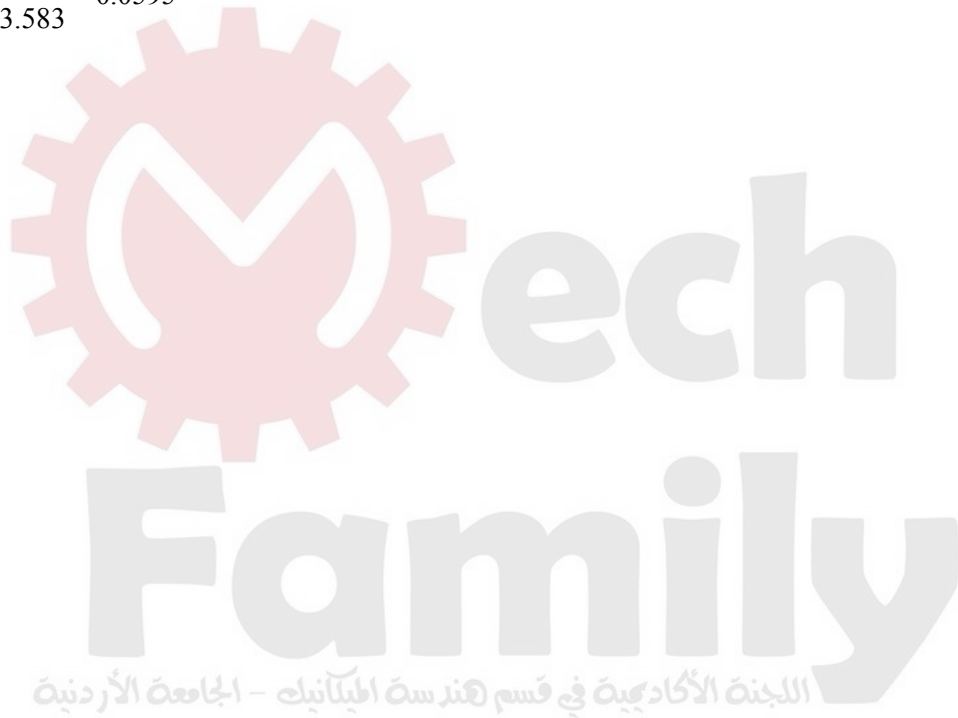
$$P_{01}=P_1 \left(\frac{T_{01}}{T_1} \right)^{\gamma/\gamma-1} = 4.47 \left(\frac{475}{472.8} \right)^{3.5} = 4.543 \text{ bar}$$

$$\frac{\Delta P_0}{P_{01}} = \frac{0.27}{4.543} = 0.0594$$

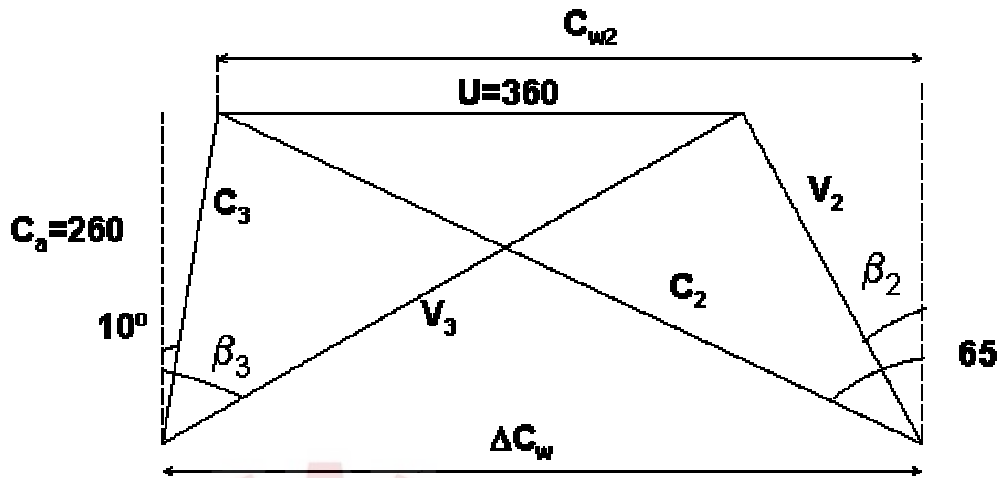
$$\text{At part load: } T_1 = \frac{3.52 \times 100}{0.287 \times 2.808} = 436.78 \text{ K}$$

$$P_{01} = 3.52 \left(\frac{439}{436.78} \right)^{3.5} = 3.583 \text{ bar}$$

$$\frac{\Delta P_0}{P_{01}} = \frac{0.213}{3.583} = 0.0595$$



Problem 7.1



$$C_2 = \frac{C_a}{\cos 65} = \frac{260}{0.4226} = 615.2 \text{ m/s}$$

$$C_{w2} = \sqrt{615.2^2 - 260^2} = 557.5 \text{ m/s}$$

$$\tan \beta_2 = \frac{557.5 - 360}{260} = 0.7596 \therefore \beta_2 = 37^\circ 13'$$

$$C_{w3} = 260 \tan 10 = 260 \times 0.1763 = 45.84 \text{ m/s}$$

$$\tan \beta_3 = \frac{360 + 45.84}{260} = 1.5609 \therefore \beta_3 = 57^\circ 22'$$

Since $C_1 = C_3$ and C_a is constant:

$$\Lambda = \frac{C_a}{2U} (\tan \beta_3 - \tan \beta_2) = \frac{260}{2 \times 360} (1.5609 - 0.7596)$$

$$\Lambda = 0.289$$

Temperature drop coefficient:

$$\psi = \frac{2C_p \Delta T_{0s}}{U^2} = \frac{2U \Delta C_w}{U^2} = \frac{2(557.5 + 45.84)}{360} = 3.35$$

$$\text{Power output} = mU \Delta C_w = 20 \times 360 \times (557.5 + 45.84)$$

$$P = 4343760 \text{ W or } 4344 \text{ KW}$$

$$T_2 = T_{01} - \frac{C_2^2}{2C_p} = 1000 - \frac{615.2^2}{2294} = 835 \text{ K}$$

$$T_2 - T_2' = \lambda_N \frac{C_2^2}{2C_p} = 0.05 \times \frac{615.2^2}{2294} = 8.25 \text{ K}$$

$$T_2' = 835 - 8.25 = 826.75 \text{ K}$$

$$\frac{P_{01}}{P_2} = \left(\frac{T_{01}}{T_2'} \right)^{\gamma/\gamma-1} = \left(\frac{1000}{826.75} \right)^4 = 2.14$$

$$\therefore P_2 = \frac{4}{2.140} = 1.869 \text{ bar}$$

For isentropic flow, critical pressure ratio $P_c/P_a = 1.853$.

$$\therefore \frac{P_{01}}{P_2} > 1.853. \therefore \text{The nozzle is choking}$$

The throat conditions are:

$$P_c = \frac{4.0}{1.853} = 2.158 \text{ bar}, T_c = \left(\frac{2}{\gamma+1} \right) \times T_{01} = \frac{2}{2.333} \times 1000 = 857 \text{ K}$$

$$\rho_c = \frac{P_c}{RT_c} = \frac{2.158 \times 100}{0.287 \times 857} = 0.877 \text{ kg/m}^3$$

$$C_c = \sqrt{1.333 \times 0.287 \times 857 \times 10^3} = 572.6 \text{ m/s}$$

$$A_{\text{throat}} = \frac{m}{\rho_c C_c} = \frac{20}{0.877 \times 572.6} = 0.0398 \text{ m}^2$$

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Problem 7.2

$$\Delta T_{013} = \eta_t T_{01} \left[1 - \left(\frac{1}{P_{01}/P_{03}} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$= 0.88 \times 1050 \left[1 - \left(\frac{1}{2} \right)^{\frac{1}{4}} \right] = 147 \text{ K}$$

$$T_{03} = 1050 - 147 = 903 \text{ K}$$

With zero outlet swirl:

$$W_s = UC_{w2} = C_p \Delta T_{013}$$

And with free vortex,
 $C_w r = \text{constant}$

Hence at root

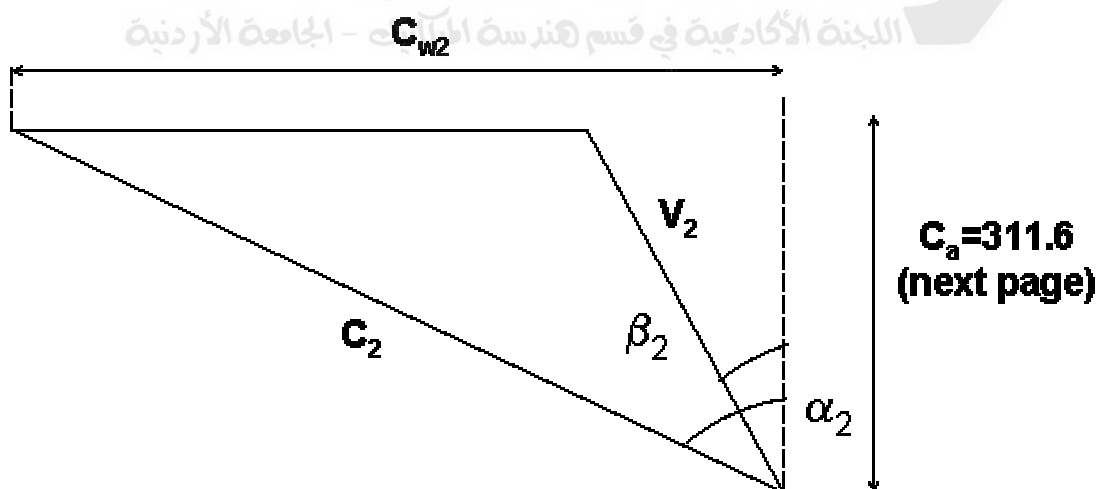
$$(C_{w2})_r = \frac{C_p \Delta T_{013}}{U} = \frac{1.147 \times 147 \times 10^3}{300} = 562 \text{ m/s}$$

Now at root, $\Lambda = 0 \therefore \frac{T_2 - T_3}{T_1 - T_3} = 0$ hence $T_2 = T_3$

$$\therefore T_{02} - \frac{C_{2r}^2}{2C_p} = T_{03} - \frac{C_3^2}{2C_p}; T_{02} = T_{01} \text{ and } C_3 = C_{a3} = 275 \text{ m/s}$$

$$\text{Hence } \frac{C_{2r}^2}{2C_p} = \frac{275^2}{2294} + 147 = 180 \text{ K}$$

$$\therefore C_{2r} = 642.6 \text{ m/s}$$



$$\alpha_{2r} = \sin^{-1} \frac{562}{642.6} = \sin^{-1} 0.8746 = 61^\circ$$

$$(C_{a2})_r = \sqrt{642.6^2 - 562^2} = 311.6 \text{ m/s}$$

$$\tan \beta_{2r} = \frac{(C_{w2})_r - U_r}{(C_{a2})_r} = \frac{562 - 300}{311.6} = 0.8406$$

$$\beta_{2r} = 40^\circ 4'$$

$$(C_{a2})_t = (C_{a2})_r = 311.6 \text{ m/s and } \frac{r_t}{r_r} = 1.4$$

$$(C_{w2})_t = \frac{562}{1.4} = 401.4 \text{ m/s}$$

$$\alpha_{2t} = \tan^{-1} \left(\frac{401.4}{311.6} \right) = \tan^{-1} (1.2882)$$

$$\therefore \alpha_{2t} = 52^\circ 10'$$

$$C_{2t} = \sqrt{401.4^2 + 311.6^2} = 508.15 \text{ m/s}$$

$$\text{With no exit swirl } T_{3t} = T_{3r} = 903 - \frac{275^2}{2294} = 870 \text{ K}$$

$$T_{2t} = 1050 - \frac{508.15^2}{2294} = 937.5 \text{ K}$$

$$(T_1 - T_3) = \Delta T_{013} \text{ because } C_1 = C_3 \text{ with no inlet or exit swirl}$$

$$\Lambda_{tip} = \frac{T_{2t} - T_{3t}}{T_1 - T_3} = \frac{937.5 - 870}{147} = 0.46$$

$$\text{At the root } T_{2r} - T_{2r} = \lambda_n \frac{C_{2r}^2}{2Cp} = 0.05 \times \frac{642.6^2}{2294}$$

$$T_{2r} - T_{2r} = 9 \text{ K}$$

$$T_{2r} = (1050 - 180) - 9.0 = 861 \text{ K}$$

$$\frac{P_{01}}{P_2} = \left(\frac{T_{01}}{T_2} \right)^{\gamma/\gamma-1} = \left(\frac{1050}{861} \right)^4 = 2.21 \therefore P_2 = \frac{3.8}{2.21} = 1.719 \text{ bar}$$

$$\frac{P_{03}}{P_3} = \left(\frac{T_{03}}{T_3} \right)^{\gamma/\gamma-1} = \left(\frac{903}{870} \right)^4 = 1.16 \therefore P_3 = \frac{3.8/2}{1.16} = 1.638 \text{ bar}$$

Problem 7.3

$$T_3 = T_{03} - \frac{C_2^2}{2C_p} = (1200 - 150) - \frac{400^2}{2294} = 980.2 \text{ K}$$

$$U_m = 2\pi r_m N \text{ or } r_m = \frac{320}{2\pi \times 250} = 0.2037 \text{ m}$$

$$\Delta T_{013} = \eta_t T_{01} \left[1 - \left(\frac{1}{P_{01}/P_{03}} \right)^{\frac{1}{4}} \right]$$

$$\left(\frac{1}{P_{01}/P_{03}} \right)^{\frac{1}{4}} = 1 - \frac{\Delta T_{013}}{\eta_t T_{01}} = 1 - \frac{150}{0.90 \times 1200} = 0.8611$$

$$\frac{P_{01}}{P_{03}} = 1.818 \therefore P_{03} = \frac{8}{1.818} = 4.4 \text{ bar}$$

$$\frac{P_{03}}{P_3} = \left(\frac{T_{03}}{T_3} \right)^{\gamma/\gamma-1} = \left(\frac{1050}{980.25} \right)^4 = 1.316 \therefore P_3 = \frac{4.4}{1.316} = 3.343 \text{ bar}$$

$$\rho_3 = \frac{P_3}{RT_3} = \frac{3.343 \times 100}{0.287 \times 980.25} = 1.19 \text{ kg/m}^3$$

$$A_3 = \frac{m}{\rho_3 C_3} = \frac{36}{1.19 \times 400} = 0.0756 \text{ m}^2 \text{ (N.B. } C_3 = C_{a3})$$

$$h = \frac{A}{2\pi r_m} = \frac{0.0756}{2\pi \times 0.2037} = 0.0591 \text{ m}$$

$$\frac{r_t}{r_r} = \frac{0.2037 + (0.0591/2)}{0.2037 - (0.0591/2)} = \frac{0.233}{0.1741} = 1.34$$

$$A_2 = A_3$$

$$C_{a2} \text{ must satisfy } C_{a2} = \frac{m}{\rho_2 A_2} \text{ and } C_{a2} = \sqrt{C^2 - C_{w2}^2}$$

$$W_s = C_{w2} U = C_p \Delta T_{013} \text{ (because } \alpha_3 = 0)$$

$$C_{w2} = \frac{1.147 \times 150 \times 10^3}{320} = 537.6 \text{ m/s}$$

$$\text{If } C_{a2} = 346 \text{ m/s } C_2^2 = 346^2 + 537.6^2 \Rightarrow C_2 = 639 \text{ m/s}$$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = T_{01} - \frac{C_2^2}{2C_p} = 1200 - \frac{639^2}{2294} = 1022 \text{ K}$$

$$T_2 = T_2 - \lambda_N \frac{C_2^2}{2C_p} = 1022 - 0.07 \times 178 = 1009.5 \text{ K}$$

$$P_2 = \frac{P_{01}}{(T_{01}/T_2)^4} = \frac{8.0}{(1200/1009.5)^4} = \frac{8.0}{1.995} = 4.008 \text{ bar}$$

$$\rho_2 = \frac{4.008 \times 100}{0.287 \times 1022} = 1.366 \text{ kg/m}^3$$

$$C_{a2} = \frac{m}{C_2 A_3} = \frac{36}{1.366 \times 0.0756} = 348 \text{ m/s (agrees with given 346 m/s)}$$

At the root, for free vortex design, we have:

$$\frac{r_m}{r_i} = \frac{0.2037}{0.1741} = 1.17$$

$$(C_{w2})_r = (C_{w2})_m \times \frac{r_m}{r_r} = 537.6 \times 1.17 = 629 \text{ m/s}$$

C_{a2} is constant at 346 m/s. Hence:

$$C_{2r} = \sqrt{629^2 + 346^2} = 717.9 \text{ m/s}$$

$$T_{2r} = 1200 - \frac{718^2}{2294} = 975.3 \text{ K}$$

$$U_R = \frac{320}{1.17} = 273.5 \text{ m/s}$$

$$V_{2r} = \sqrt{346^2 + (629 - 273.5)^2} = 496.1 \text{ m/s}$$

$$a_{2r} = \sqrt{\gamma R T_{2r}} = \sqrt{1.333 \times 0.287 \times 975.3 \times 10^3} = 610.86 \text{ m/s}$$

$$(M_{v2})_r = \frac{496.1}{610.86} = 0.812$$

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	$\left(\frac{r_m}{r}\right)_2$	$\left(\frac{r_m}{r}\right)_2^2$	$\tan \beta_2$	β_2
Root	1.164	1.357	0.709	35 ° 20'
Tip	0.877	0.769	0	0

We therefore have untwisted nozzles.

$C_{w2} r^x = \text{constant}$ where $x = \sin^2 \alpha_2 = \sin^2 58^\circ 23' = 0.726$ With

$\alpha_2 = \text{constant}$, we also have $C_{a2} r^x = \text{constant}$.

Hence, $C_{a2} = (C_{a2})_m \left(\frac{r_m}{r}\right)_2^x$

This with

$U = U_m \left(\frac{r}{r_m}\right)_2$ yields

$\tan \beta_2 = \tan \alpha_2 - \left(\frac{r}{r_m}\right)_2^{x+1} \left(\frac{M}{C_{a2}}\right)_m$

	$\left(\frac{r_m}{r}\right)_2^{1.726}$	$\tan \beta_2$	β_2
Root	1.164	0.662	35 ° 20'
Tip	0.797	0.056	3 ° 12'

Problem 7.5

$$m = \rho_3 A C_3 = \frac{P_3}{RT_3} A \sqrt{2C_p} \times \sqrt{T_{03} - T_3}$$

$$= \frac{P_3}{RT_3} A \sqrt{2C_p} \times \sqrt{T_{01} - \Delta T_w - T_3}$$

Assuming isentropic expansion,

$$P_3 = P_{01} \left(\frac{T_3}{T_{01}} \right)^{(\gamma/\gamma-1)}$$

$$m = \frac{A \sqrt{2C_p}}{R} \times \frac{P_{01}}{T_{01}^{(\gamma/\gamma-1)}} \sqrt{T_3^{\frac{2}{\gamma-1}} (T_{01} - \Delta T_w - T_3)}$$

$$m = \frac{A \sqrt{2C_p}}{R} \times \frac{P_{01}}{T_{01}^{(\gamma/\gamma-1)}} \sqrt{T_3^{\frac{2}{\gamma-1}} (T_{01} - \Delta T_w) - T_3^{\frac{\gamma+1}{\gamma-1}}}$$

For the given inlet conditions m is a maximum when $\frac{dm}{dT_3} = 0$ i.e when:

$$\frac{2}{\gamma-1} (T_{01} - \Delta T_w) T_3^{(3-\gamma)/(\gamma-1)} - \frac{\gamma+1}{\gamma-1} T_3^{\frac{2}{\gamma-1}} = 0$$

$$\therefore T_3 = \frac{2}{\gamma+1} (T_{01} - \Delta T_w)$$

Hence, writing $K = \frac{A \sqrt{2C_p}}{R} \times \frac{P_{01}}{T_{01}^{(\gamma/\gamma-1)}}$

$$m_{\max} = K \sqrt{(T_{01} - \Delta T_w)^{\frac{\gamma+1}{\gamma-1}} \left[\left(\frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \right]}$$

$$m_{\max} = K \sqrt{(T_{01} - \Delta T_w)^{\frac{\gamma+1}{\gamma-1}} \left[\left(\frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} \left(\frac{2}{\gamma+1} \right) \right]}$$

$$m_{\max} = K \sqrt{(T_{01} - \Delta T_w)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} \left(\frac{\lambda-1}{\gamma+1} \right)}$$

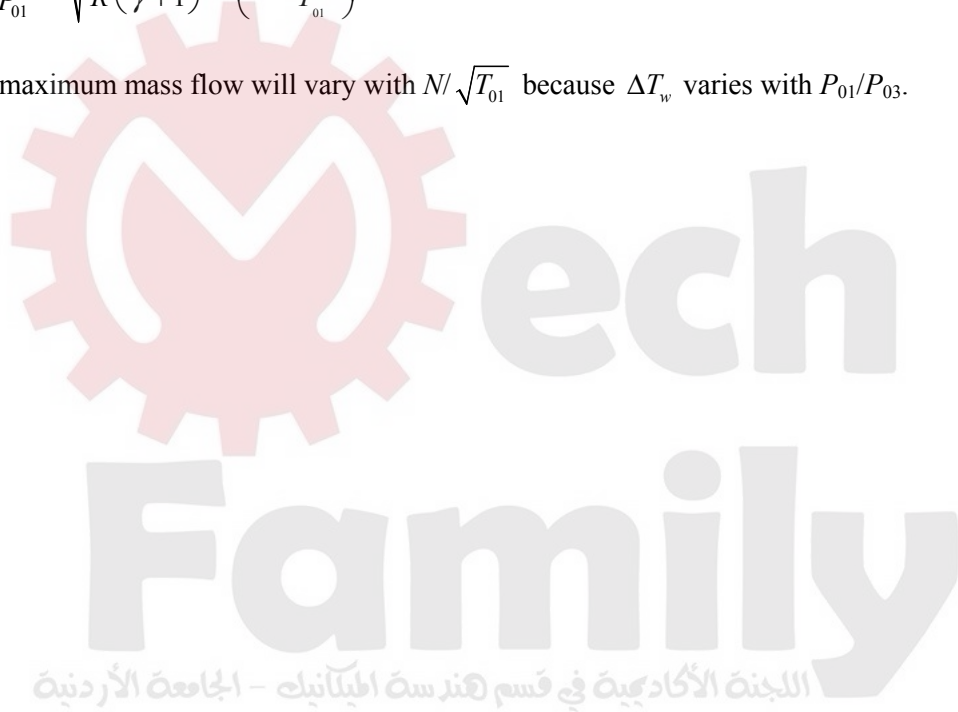
But $\frac{C_p}{R} = \frac{\gamma}{\gamma-1}$

$$m_{\max} = \frac{AP_{01}}{T_{01}^{(\gamma/\gamma-1)}} \sqrt{(T_{01} - \Delta T_w)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{2}{R} \frac{\gamma}{\gamma-1} \right)^{\frac{2}{\gamma-1}} \frac{2^{\frac{\gamma+1}{\gamma-1}} (\gamma-1)}{(\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}}$$

$$m_{\max} \frac{\sqrt{T_{01}}}{AP_{01}} = \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \frac{\Delta T_w}{T_{01}} \right)^{\frac{\gamma+1}{\gamma-1}} \frac{T_{01}^{\frac{\gamma+1}{\gamma-1}}}{T_{01}^{\frac{\gamma+1}{\gamma-1}}}}$$

$$m_{\max} \frac{\sqrt{T_{01}}}{AP_{01}} = \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \frac{\Delta T_w}{T_{01}} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

Hence maximum mass flow will vary with $N/\sqrt{T_{01}}$ because ΔT_w varies with P_{01}/P_{03} .



Problem 8.1

$N=250 \text{ rev/s} \Rightarrow \text{Angular velocity } \omega = 2\pi N = 1570.5 \text{ rad/s}$

$R_r=0.1131\text{m}$

$R_t=0.2262 \text{ m}$

$\rho = 4500 \text{ kg/m}^3$ (Ti-6Al-4V) ref table page 407

Centrifugal stress at the root

$$\sigma_r = \frac{\rho \omega^2}{2} (R_t^2 - R_r^2) K$$

$$= \frac{4500(1570.5)^2}{2} (0.2262^2 - 0.1131^2)(0.6)$$

$\cong 127.8 \text{ MPa}$

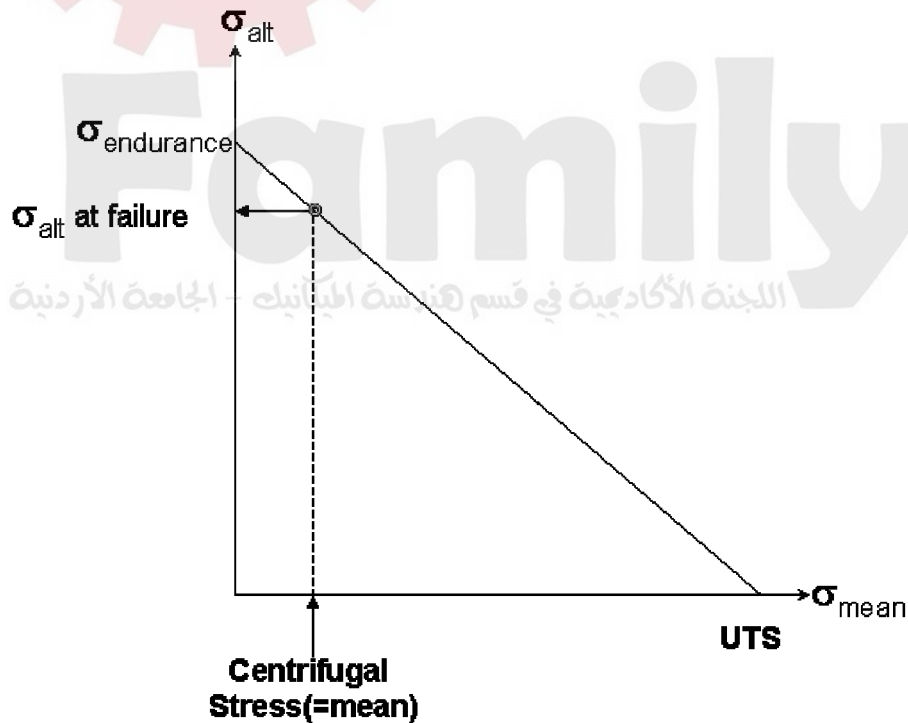
Material $UTS=880 \text{ MPa}$

Endurance limit $=350 \text{ MPa}$

Assess the vibratory stress at failure – Goodman diagram

-vibratory stress at failure $\approx 300 \text{ MPa}$ -high

-Allowable approximately $\frac{1}{3} \cong 100 \text{ MPa}$



Problem 8.2

$$N=250 \text{ rev/s}$$

$$R_0=0.113\text{m}; R_i=0.105\text{m}$$

$$\text{Width}=0.025\text{m}$$

$$\text{Blade } m \cong 0.020 \text{ kg}$$

$$r_{cg} \cong 0.163 \text{ m}$$

$$\text{Material } \rho = 4500 \text{ kg/m}^3, \quad \text{UTS} = 880 \text{ MPa}$$

For a thin ring rotor (eq 8.29)

$$\sigma_t = \rho \omega^2 R_{ring}^2 + \frac{F_{rim}}{2\pi A_{ring}}$$

$$R_{ring} = \frac{0.113 + 0.105}{2} = 0.109\text{m}$$

$$A_{ring} = 0.025(0.113 - 0.105)$$

$$= 2 \times 10^{-4} \text{ m}^2$$

$$F_{rim} = 43(mr\omega^2)$$

$$= 43(0.02 \times 0.163 \times 1570.5^2) \cong 345,750 \text{ N}$$

$$\sigma_t = 4500(1570.5)^2 (0.109)^2 + \frac{345,750}{2\pi(2 \times 10^{-4})}$$

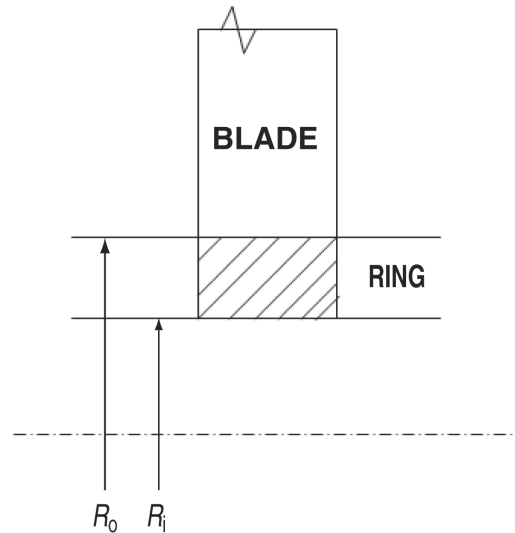
$$= 131.87 \times 10^6 + 275 \times 10^6 \cong 407.06 \times 10^6 \text{ Pa}$$

$$\sigma_t = 407 \text{ MPa}$$

Considering burst, for UTS = 880MPa and burst margin of 1.2, the maximum allowable σ_t

$$(\sigma_t)_{allowable} = \frac{880}{(1.2)^2} = 611 \text{ MPa} \gg 407 \text{ MPa}$$

The design is conservative; but acceptable for a conceptual design.

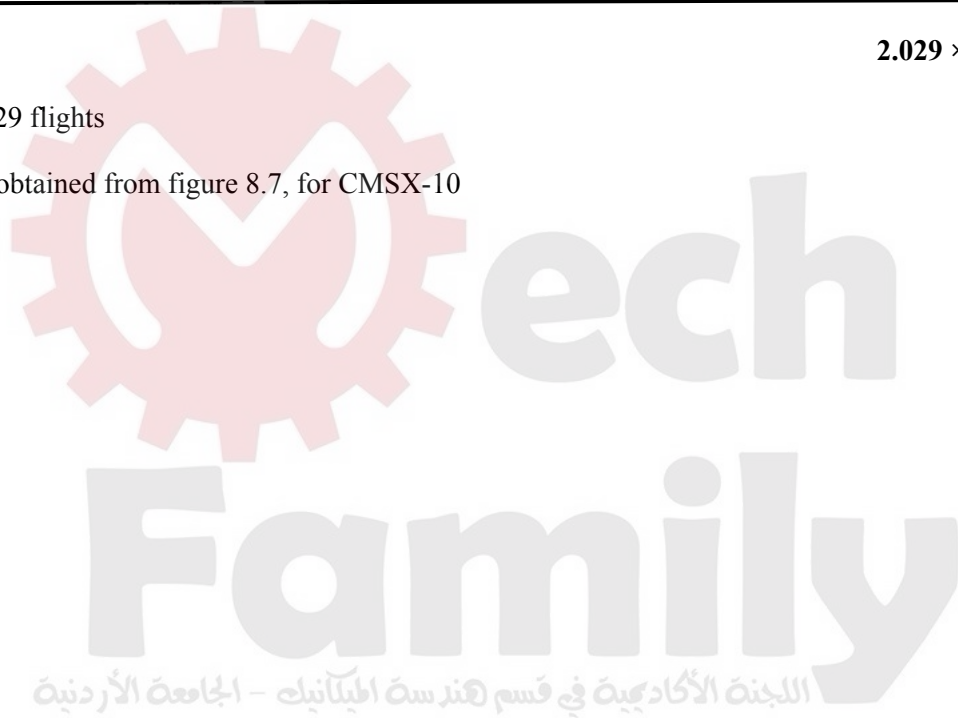


Problem 8.3

	$t[\text{hrs}]$	$\sigma[\text{MPa}]$	P_{LM}	$T[\text{K}]$	$t_{1\%}[\text{hrs}]$	$\frac{t}{t_{1\%}}$
TO+Climb	0.33	300	26.8	1150	2,015	1.489×10^{-4}
Cruise	5	220	28	1000	100,000	5×10^{-5}
Descent+L	0.5	150	29.5	1050	124,520	4×10^{-6}
						2.029×10^{-4}

$\therefore$ 4,929 flights

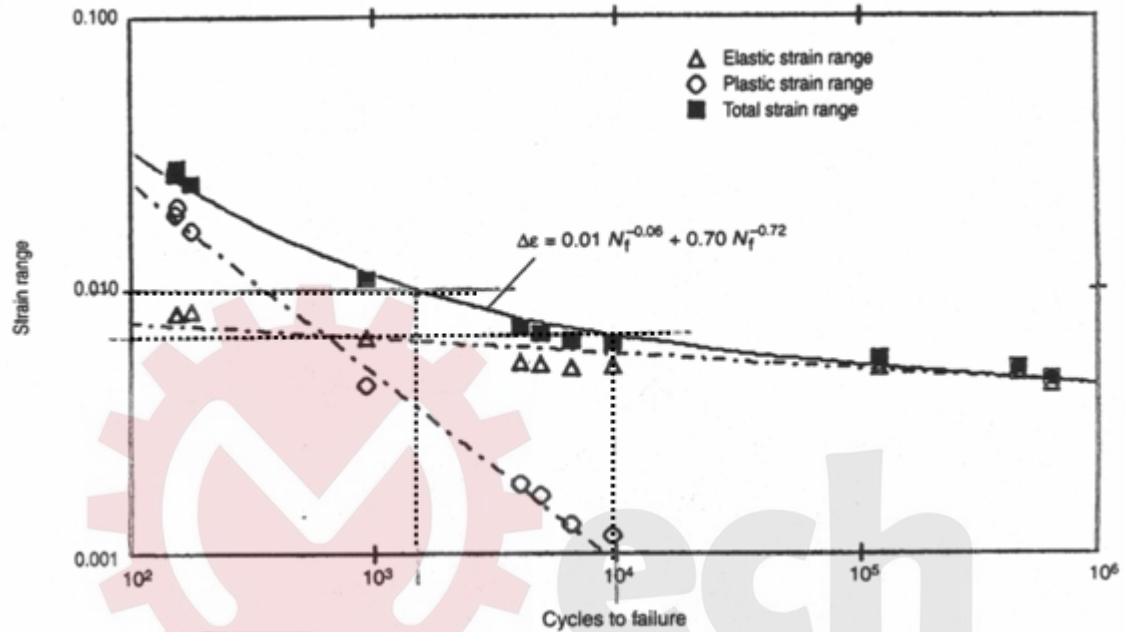
P_{LM} is obtained from figure 8.7, for CMSX-10



Problem 8.4

Haynes 188 at 1033K

$$\Delta \epsilon = 0.01 N_f^{-0.06} + 0.7 N_f^{-0.72}$$



Source: Manson, S. S. and Halford, G. R., *Fatigue and durability of structural materials* (ASM International, 2006), where it is Figure 6.15(c). Reprinted with permission of ASM International ©. All rights reserved. www.asminternational.org

$$\Delta \epsilon_1 = 0.009 \rightarrow N_{f1} = 2 \times 10^3 \text{ cycles}$$

$$\Delta \epsilon_2 = 0.007 \rightarrow N_{f2} = 1 \times 10^4 \text{ cycles}$$

Using Miner's rule (linear damage accumulation)

$$\frac{n}{N_{f1}} + \frac{n}{N_{f2}} = 1 = \frac{n}{2 \times 10^3} + \frac{n}{1 \times 10^4}$$

$$1 = \frac{1 \times 10^4 n + 2 \times 10^3 n}{2 \times 10^7 n} = n \left(\frac{12 \times 10^3}{2 \times 10^7} \right)$$

$\therefore n \approx 1,670$ cycles of each loading.

Problem 8.5

Rigid bearings : K_r = rotor stiffness

$$\omega = \sqrt{\frac{K_r}{m}} = \left[\frac{N}{mkg} \right]^{\frac{1}{2}} = \left[\frac{kgm}{s^2 mkg} \right]^{\frac{1}{2}} = \frac{1}{s}$$

$$m = 20 \text{ kg} ; \omega = \frac{\pi N}{30} = 5,235 \text{ rad/s}$$

$$5,235 = \left(\frac{K_r}{20} \right)^{\frac{1}{2}} \Rightarrow K_r = 548 \times 10^6 \text{ N/m}$$

Soft bearings: total stiffness

$$\frac{1}{K_t} = \frac{K_r + K_s}{K_r K_s} = \frac{548 \times 10^6 + 1 \times 10^7}{548 \times 10^6 \times 1 \times 10^7}$$
$$= 1.018 \times 10^{-7}$$

$$K_t = 9.821 \times 10^6 \text{ N/m}$$

First critical speed

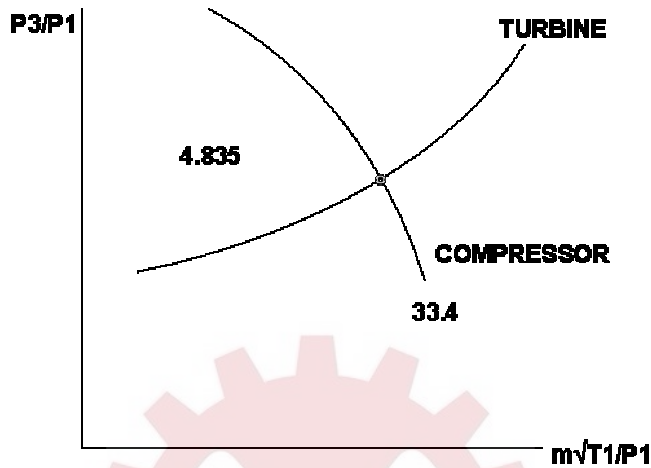
$$\omega = \sqrt{\frac{9.821 \times 10^6}{20}} = 700.7 \text{ rad/s} = 6,693 \approx 6,700 \text{ rpm}$$

Plot the critical speed as a function of support stiffness!

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Problem 9.1

Note: In the problems for chapter 9, we are dealing with stagnation conditions at all times and the suffix '0' has dropped throughout.



Flow compatibility is expressed by:

$$\frac{m\sqrt{T_1}}{P_1} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_1} \times \sqrt{\frac{T_1}{T_3}}$$

And

$$\frac{P_3}{P_1} = \frac{P_2}{P_1} - \frac{P_2 - P_3}{P_1} = \frac{P_2}{P_1} - 0.05 \frac{P_2}{P_1} = 0.95 \frac{P_2}{P_1}$$

$$\therefore \frac{m\sqrt{T_1}}{P_1} = 14.2 \times 0.95 \frac{P_2}{P_1} \times \sqrt{\frac{288}{1100}} = 6.903 \frac{P_2}{P_1}$$

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$\frac{P_2}{P_1}$	$6.903 \frac{P_2}{P_1}$	$\frac{m\sqrt{T_1}}{P_1}$
5.0	34.515	32.9
4.7	32.444	33.8
4.5	31.064	34.3

From curves, equilibrium point is at:

$$\frac{P_2}{P_1} = 4.835, \quad \frac{m\sqrt{T_1}}{P_1} = 33.4, \quad \eta_c = 0.795$$

$$\frac{P_3}{P_1} = 0.95 \times 4.835 = 4.59$$

And from turbine characteristic graph: $\eta_t = 0.8497$

$$m = \frac{33.4 \times 1.01}{\sqrt{288}} = 1.988 \text{ kg/s}$$

$$\text{Net power output} = m C_{pg} \Delta T_{34} - \frac{1}{\eta_m} m C_{pa} \Delta T_{12}$$

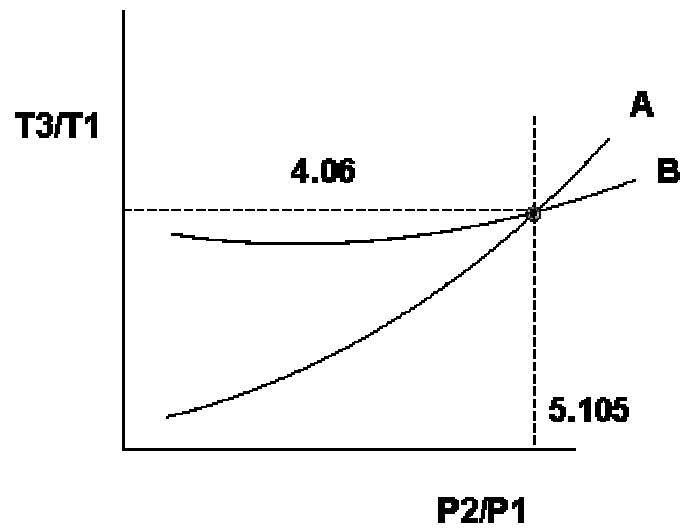
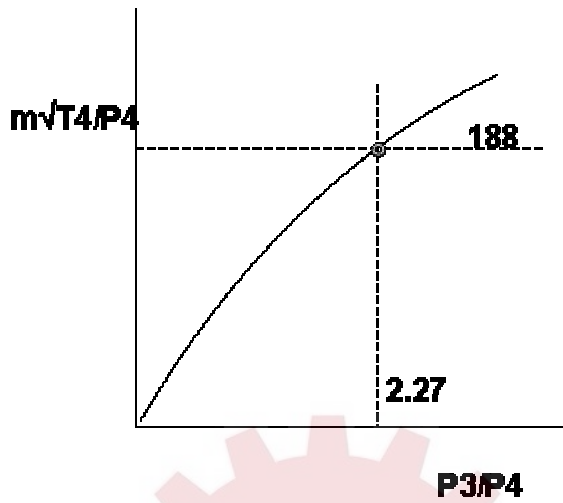
$$P_{\text{output}} = 1.988 \times 1.147 \times 0.8497 \times 1100 \left[1 - \left(\frac{1}{4.59} \right)^{\frac{1}{4}} \right]$$

$$- 1.988 \times 1.005 \times \frac{288}{0.795} \left[(4.835)^{\frac{1}{3.5}} - 1 \right]$$

$$P_{\text{output}} = 675.189 - 411.620 = 263.989 \text{ KW}$$

$$\text{Net power output} = 264 \text{ KW}$$

Problem 9.2



$$\frac{m\sqrt{T_4}}{P_4} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_4} \times \sqrt{\frac{T_4}{T_3}}$$

$$\frac{T_4}{T_3} = 1 - \frac{\Delta T_{34}}{T_3}$$

$$\frac{\Delta T_{34}}{T_3} = \eta_t \left[1 - \frac{1}{r^{\frac{1}{\gamma}} \right]; \quad r = \frac{P_3}{P_4}; \text{ and } \eta_t = 0.85$$

$\frac{P_3}{P_4}$	$\left(\frac{P_3}{P_4}\right)^{\frac{1}{4}}$	$\frac{\Delta T_{34}}{T_3}$	$\frac{T_4}{T_3}$	$\sqrt{\frac{T_4}{T_3}}$	$\frac{m\sqrt{T_3}}{P_3}$	$\frac{m\sqrt{T_4}}{P_4}$
2.00	1.189	0.135	0.865	0.930	88.2	164.05
2.25	1.2247	0.156	0.844	0.918	90.2	186.31
2.50	1.257	0.174	0.826	0.909	90.2	205.0

$$\therefore \frac{m\sqrt{T_4}}{P_4} = 188 \quad \text{when } \frac{P_3}{P_4} = 2.270 \text{ (graphical solution)}$$

$$\therefore \frac{m\sqrt{T_3}}{P_3} = 90.2$$

$$\frac{\Delta T_{34}}{T_3} = 0.85 \left[1 - \left(\frac{1}{2.27} \right)^{\frac{1}{4}} \right] = 0.1575$$

From compatibility of flow:

$$\frac{m\sqrt{T_1}}{P_1} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_1} \times \sqrt{\frac{T_1}{T_3}} \text{ and } P_3 = P_2$$

$$\therefore \sqrt{\frac{T_3}{T_1}} = \frac{(m\sqrt{T_3}/P_3)(P_2/P_1)}{(m\sqrt{T_1}/P_1)}$$

$$\sqrt{\frac{T_3}{T_1}} = \frac{90.2(P_2/P_1)}{(m\sqrt{T_1}/P_1)} \text{-----(A)}$$

From compatibility of work:

$$\frac{\Delta T}{T_1} = \frac{\Delta T}{T_3} \frac{T_3}{T_1} \frac{C_{pg}\eta_m}{C_{pa}} = \frac{0.1575 \times 1.147 \times 0.98}{1.005} \frac{T_3}{T_1} = 0.1762 \frac{T_3}{T_1} \text{-----(B)}$$

$$\frac{\Delta T_{12}}{T_1} = \frac{1}{\eta_c} \left[\left(\frac{P_2}{P_1} \right)^{0.286} - 1 \right]$$

$\frac{P_2}{P_1}$	$\frac{m\sqrt{T_1}}{P_1}$	$\sqrt{\frac{T_3}{T_1}}$	$\frac{T_3}{T_1}$	η_c	$\left(\frac{P_2}{P_1} \right)^{0.286}$	$\frac{\Delta T_{12}}{T_1}$	$\frac{T_3}{T_1} \left(= \left(\frac{1}{0.1762} \right) \left(\frac{\Delta T}{T_1} \right) \right)$
	(A)						(B)
5.2	220	2.13	4.54	0.82	1.602	0.734	4.17
5.0	236	1.91	3.65	0.83	1.584	0.704	3.99
4.8	244	1.77	3.15	0.82	1.566	0.690	3.92

$$\therefore \frac{P_2}{P_1} = 5.105 \text{ and } \frac{T_3}{T_1} = 4.06, \text{ from graphical solution}$$

Hence $T_3 = 1170 \text{ K}$

Problem 9.3

At outlet of gas generator turbine

$$\frac{m\sqrt{T_4}}{P_4} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_4} \times \sqrt{\frac{T_4}{T_3}}$$

$$\text{and } \frac{T_4}{T_3} = 1 - \eta_t \left[1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{1}{4}} \right];$$

$$\therefore \frac{m\sqrt{T_4}}{P_4} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_4} \times \sqrt{1 - \eta_t \left[1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{1}{4}} \right]};$$

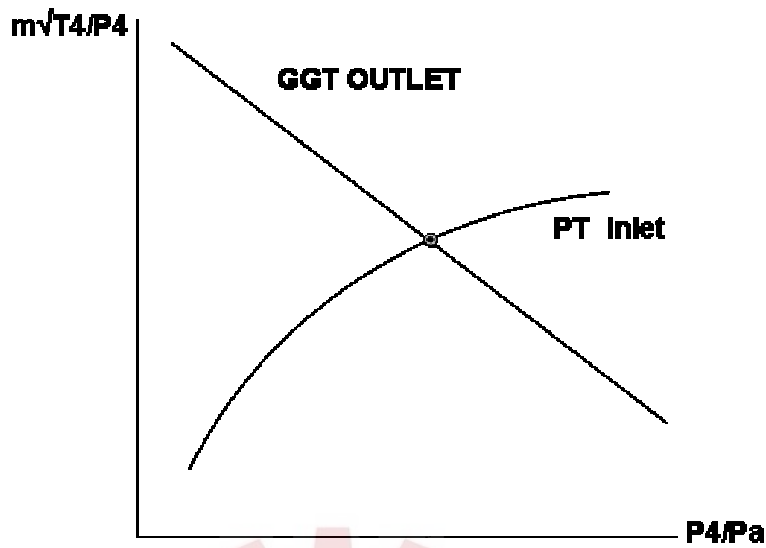
$$\eta_t = 0.85$$

$\frac{P_3}{P_4}$	$\left(\frac{P_3}{P_4} \right)^{\frac{1}{4}}$	$1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{1}{4}}$	$\left\{ 1 - \eta_t \left[1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{1}{4}} \right] \right\}^{\frac{1}{2}}$	$\frac{m\sqrt{T_3}}{P_3}$	$\frac{m\sqrt{T_4}}{P_4}$	$\frac{P_4}{P_a}$
1.3	1.0678	0.064	0.972	20.0	25.27	1.92
1.5	1.1067	0.097	0.958	44.0	63.22	1.664
1.8	1.1583	0.137	0.940	62.0	104.90	1.387

The value of P_4/P_a in the table is found from

$$\frac{P_4}{P_a} = \frac{P_4}{P_3} \frac{P_3}{P_2} \frac{P_2}{P_a} = \frac{P_4}{P_3} \times 0.96 \times 2.60 = 2.496 \frac{P_4}{P_3}$$

Hence curves – from which power turbine pressure ratio is $r = 1.55$



And gas generator turbine pressure ratio $r=1.61$

Use work compatibility equation to find T_3

$$\frac{\Delta T_{12}}{T_1} = \frac{\Delta T_{34}}{T_3} \times \frac{T_3}{T_1} \left[\frac{C_{pg} \eta_m}{C_{pa}} \right]$$

$$\therefore C_{pa} \frac{T_1}{\eta_c} \left[\left(\frac{P_2}{P_1} \right)^{\frac{1}{3.5}} - 1 \right] = \eta_m C_{pg} T_3 \left[1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{1}{4}} \right]$$

T_1 must be given. Compressor efficiency from compressor characteristic once operating point

found from $\frac{m\sqrt{T_1}}{P_1} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_1} \times \sqrt{\frac{T_1}{T_3}}$ where T_3 unknown, so trial and error method required.

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Problem 9.4

For gas generator turbine

$$\frac{m\sqrt{T_4}}{P_4} = \frac{m\sqrt{T_3}}{P_3} \times \frac{P_3}{P_4} \times \sqrt{\frac{T_4}{T_3}}$$

$$\text{where } \frac{T_4}{T_3} = 1 - \frac{\Delta T_{34}}{T_3} = 1 - \eta_t \left[1 - \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} \right];$$

and η_t is constant.

$$\text{Thus: } \frac{m\sqrt{T_4}}{P_3} = f \left(\frac{P_3}{P_4} \right) \text{ from gas generator turbine characteristics.}$$

Since power turbine is choking at all conditions considered, the gas generator turbine is operating at a fixed pressure ratio and hence fixed value of $\frac{\Delta T_{34}}{T_3}$.

(a) At 95% of speed, work compatibility yields

$$\eta_m C_{pg} \Delta T_{34} = C_{pa} \frac{T_1}{\eta_c} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\Delta T_{34} = C_{pa} \frac{T_1}{\eta_m C_{pg} 0.863} \left[(4.0)^{\frac{1}{3.5}} - 1 \right]$$

$$= \frac{C_{pa} T_1 0.486}{\eta_m C_{pg} 0.863}$$

At 100% speed we have similarly:

$$\Delta T_{34} = C_{pa} \frac{T_1}{\eta_m C_{pg} 0.859} \left[(4.6)^{\frac{1}{3.5}} - 1 \right] = \frac{C_{pa} T_1 0.5465}{\eta_m C_{pg} 0.859}$$

$$\text{But: } \left(\frac{\Delta T_{34}}{1075} \right)_{95\% \text{ speed}} = \left(\frac{\Delta T_{34}}{T_3} \right)_{100\% \text{ speed}}$$

Therefore;

$$T_3 = 1075 \times \frac{0.5465}{0.859} \times \frac{0.863}{0.486} = 1214.5 \text{ K}$$

(b) 95% mechanical speed at 273 K.

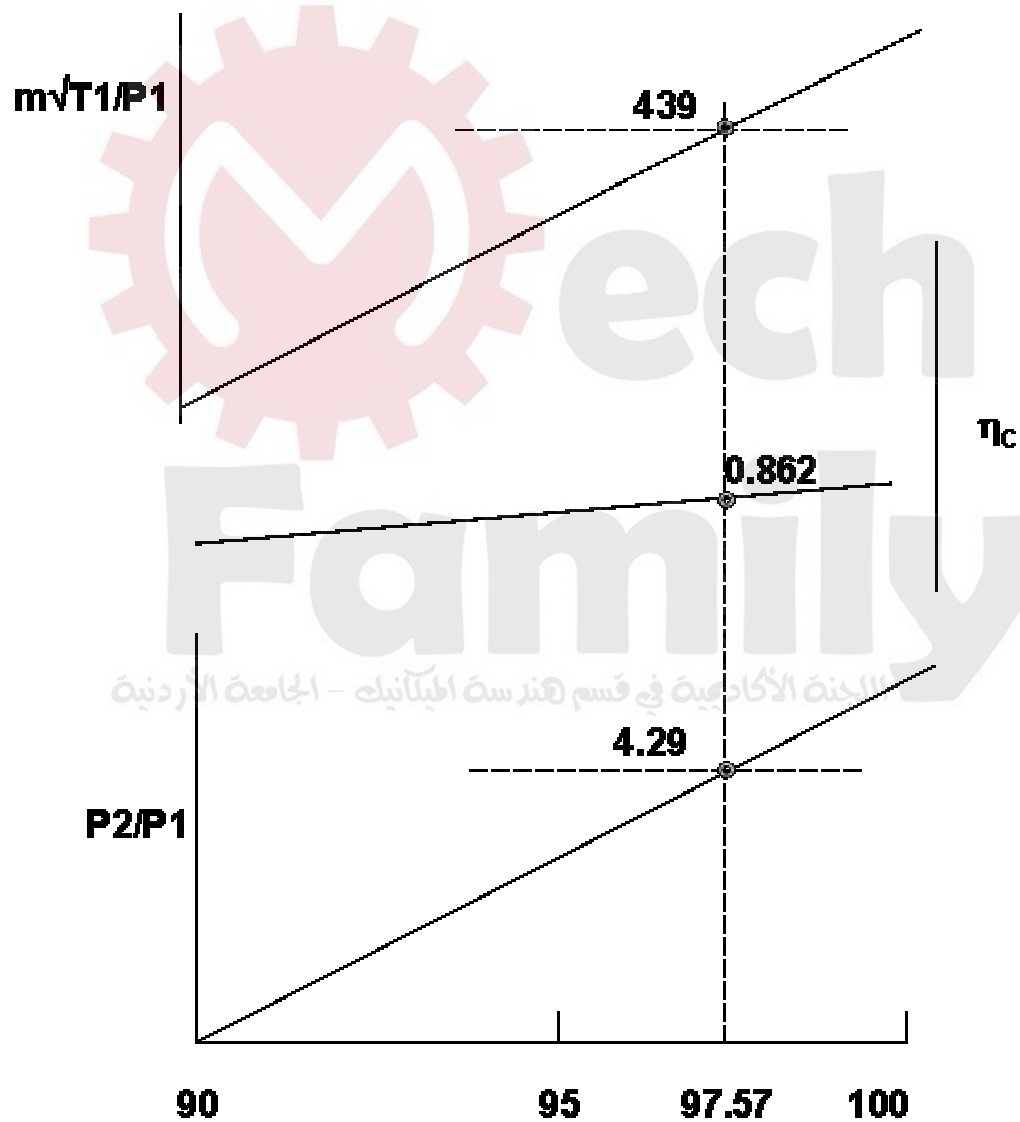
$$\frac{N}{\sqrt{T_1}} (\%design) = 95 \sqrt{\frac{288}{273}} = 97.57 \%$$

From the operating line $\frac{m\sqrt{T_1}}{P_1} = 439$, $\frac{P_2}{P_1} = 4.29$ and $\eta_c = 0.862$

$$\Delta T = \frac{273}{0.862} [(4.29)^{0.286} - 1] = 163.6_K$$

$$m = \frac{439 \times 0.76}{\sqrt{273}} = 20.19 \text{ kg/s}$$

$$\text{Power} = 20.19 \times 1.005 \times 163.5 = 3318 \text{ kw.}$$



Problem 9.5

Work compatibility yields:

$$m\eta_t T_3 \left[1 - \left(\frac{1}{P_3/P_4} \right)^{\frac{\gamma-1}{\gamma}} \right] = m_c \frac{T_1}{\eta_c} \left[\left(\frac{P_2}{P_1} \right)^{\frac{1}{3.5}} - 1 \right]$$

$$\text{Let: } \frac{P_3}{P_4} = \frac{P_2}{P_1} = r$$

At design point:

$$\frac{m_c - m_b}{m_c} = \frac{1}{\eta_t \eta_c} \frac{T_1 \left[r^{\frac{1}{3.5}} - 1 \right]}{T_3 \left[1 - \left(\frac{1}{r} \right)^{\frac{1}{3.5}} \right]} = \frac{1}{0.8 \times 0.85} \times \frac{1}{3.3} \times \frac{\left[4^{\frac{1}{3.5}} - 1 \right]}{\left[1 - \left(\frac{1}{4} \right)^{\frac{1}{3.5}} \right]}$$

$$\frac{m}{m_c} = \frac{m_c - m_b}{m_c} = 0.6622$$

$$\frac{m_b}{m_c} = 1 - 0.6622 = 0.3378 = \frac{m_b}{m_c} \frac{\sqrt{T_1/P_1}}{\sqrt{T_1/P_1}}$$

Therefore at design point:

$$\frac{m_b \sqrt{T_1}}{P_1} = 0.3378 \times 22.8 = 7.70$$

$$\left[\frac{(m_c - m_b) \sqrt{T_3}}{P_3 \sqrt{1 - \frac{1}{r^2}}} \right]_d = \frac{(m_c - m_b) \sqrt{T_3}}{P_3 \sqrt{1 - \frac{1}{r^2}}}$$

And $\sqrt{T_3}$ is constant.

$$\frac{m_c - m_b}{(m_c - m_b)_d} = \frac{P_3}{(P_3)_d} \frac{\sqrt{1 - \frac{1}{r^2}}}{\sqrt{1 - \frac{1}{r_d^2}}} = \frac{r \sqrt{1 - \frac{1}{r^2}}}{r_d \sqrt{1 - \frac{1}{r_d^2}}} \quad \text{Thus : } P_3 = P_2$$

$$\frac{m_c \sqrt{T_1/P_1} - m_b \sqrt{T_1/P_1}}{22.8 - 7.7} = \frac{r \sqrt{1 - 1/r^2}}{4 \sqrt{1 - 1/16}} = \frac{\sqrt{r^2 - 1}}{\sqrt{15}}$$

$$m_c \sqrt{T_1/P_1} - m_b \sqrt{T_1/P_1} = 3.899 \sqrt{r^2 - 1} \quad \text{-----(1)}$$

Also from work compatibility:

$$\frac{m_c \sqrt{T_1} / P_1 - m_b \sqrt{T_1} / P_1}{m_c \sqrt{T_1} / P_1} = \frac{0.4457(r^{1/3.5} - 1)}{\left[1 - \left(\frac{1}{r}\right)^{\frac{1}{3.5}}\right]} \quad \text{-----(2)}$$

Now: $m_b \sqrt{T_1} / P_1 = \frac{3}{4} \times 7.7 = 5.775$

From (1): $m_c \sqrt{T_1} / P_1 = 5.775 + 3.899\sqrt{r^2 - 1}$

(2) becomes:

$$1 - \frac{5.775}{5.775 + 3.899\sqrt{r^2 - 1}} = \frac{0.4457(r^{1/3.5} - 1)}{\left[1 - \left(\frac{1}{r}\right)^{\frac{1}{3.5}}\right]}, \text{ solve by trial and error}$$

r	r^2	$\sqrt{r^2 - 1}$	$3.899\sqrt{r^2 - 1}$	$5.775 + 3.899\sqrt{r^2 - 1}$	left-hand-side
3.5	12.25	3.35	13.08	18.85	0.694
3.0	9.0	2.83	11.05	16.82	0.657
2.5	6.25	2.29	8.94	14.71	0.607
2.25	5.06	2.015	7.865	13.63	0.576

r	$r^{\frac{1}{3.5}}$	$0.4457[r^{1/3.5} - 1]$	$\left(1 - \frac{1}{r^{1/3.5}}\right)$	right-hand-side
3.5	1.430	0.192	0.301	0.638
3.0	1.369	0.164	0.270	0.607
2.5	1.299	0.133	0.230	0.578
2.25	1.261	0.116	0.207	0.560

From curves, $r = 2.08$

Hence;

$$m_c \sqrt{T_1} / P_1 = 5.775 + 3.899\sqrt{2.08^2 - 1} = 12.886$$