Design of Machine Element

Design of Shafts

Fatigue Strength

- Bending, torsion, and axial stresses may be present in both midrange and alternating components.
- For analysis, it is simple enough to combine the different types of stresses into alternating and midrange von Mises stresses.
- It is sometimes convenient to customize the equations specifically for shaft applications.
- Axial loads are usually comparatively very small at critical locations where bending and torsion dominate, so they will be left out of the derived equations.

Fatigue Strength

- The fluctuating stresses due to bending and torsion are given by:

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I}$$

$$\tau_a = K_{fs} \frac{T_a c}{J} \qquad \tau_m = K_{fs} \frac{T_m c}{J}$$

- where M_m and M_a are the midrange and alternating bending moments, T_m and T_a are the midrange and alternating torques, and K_f and K_{fs} are the fatigue stress concentration factors for bending and torsion, respectively.

Fatigue Strength

Assuming a solid shaft with round cross section, appropriate geometry terms can be introduced for c, I, and J resulting in

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3}$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}$$

Fatigue Strength

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Fatigue Strength

Combining these stresses in accordance with the distortion energy failure theory, the von Mises stresses for rotating round, solid shafts, neglecting axial loads, are given by

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma'_a = \left[\sigma_{xa}^2 + 3\tau_{xya}^2\right]^{1/2} = 16/\pi d^3\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} = 16A/\pi d^3$$

$$\sigma'_m = \left[\sigma_{xm}^2 + 3\tau_{xym}^2\right]^{1/2} = 16/\pi d^3\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2} = 16B/\pi d^3$$

Fatigue Strength

$$\sigma'_a = \left[\sigma_{xa}^2 + 3\tau_{xya}^2\right]^{1/2} = 16/\pi d^3\left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2} = 16A/\pi d^3$$

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where A and B are defined as

$$A = \left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2}$$

$$B = \left[4(K_f M_m)^2 + 3(K_{fs}T_m)^2\right]^{1/2}$$

The Gerber fatigue failure criterion

The Gerber fatigue failure criterion is:

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = \frac{n\sigma'_a}{S_e} + \left(\frac{n\sigma'_m}{S_{ut}}\right)^2 = \frac{16nA}{\pi d^3 S_e} + \left(\frac{16nB}{\pi d^3 S_{ut}}\right)^2 = 1$$

The critical shaft diameter is given by

$$d = \frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}^{1/3}$$

The Gerber fatigue failure criterion

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = \frac{n\sigma'_a}{S_e} + \left(\frac{n\sigma'_m}{S_{ut}}\right)^2 = \frac{16nA}{\pi d^3 S_e} + \left(\frac{16nB}{\pi d^3 S_{ut}}\right)^2 = 1$$

or, solving for 1/*n*, the factor of safety is given by

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

The Gerber fatigue failure criterion

$$d = \frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

$$r = \frac{\sigma'_a}{\sigma'_m} = \frac{A}{B}$$

The Gerber fatigue failure criterion

For a rotating shaft with constant bending and torsion, the bending stress is completely reversed and the torsion is steady.

Previous Equations can be simplified by setting *M_m = 0 and T_a = 0, which simply drops out some of the terms.*

$$\begin{cases} A = 2K_f M_a \\ B = \sqrt{3}\, K_{fs} T_m \end{cases}$$

The Gerber fatigue failure criterion

Critical Shaft Diameter

$$d = \frac{16nK_f M_a}{\pi S_e}\left\{1+\left[1+3\left(\frac{K_{fs}T_m S_e}{K_f M_a S_{ut}}\right)^2\right]^{1/2}\right\}^{1/3}$$

Safety Factor

$$\frac{1}{n} = \frac{16K_f M_a}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{K_{fs}T_m S_e}{K_f M_a S_{ut}}\right)^2\right]^{1/2}\right\}$$

$$r = \frac{\sigma'_a}{\sigma'_m} = \frac{2K_f M_a}{\sqrt{3}K_{fs}T_m}$$

The DE-Elliptic Criterion

The Elliptic fatigue-failure criterion is defined by

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = \left(\frac{n\sigma'_a}{S_e}\right)^2 + \left(\frac{n\sigma'_m}{S_y}\right)^2 = \left(\frac{16nA}{\pi d^3 S_e}\right)^2 + \left(\frac{16nB}{\pi d^3 S_y}\right)^2 = 1$$

Remember:

$$\sigma'_a = \left[\sigma_{xa}^2 + 3\tau_{xya}^2\right]^{1/2} = 16/\pi d^3 \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} = 16A/\pi d^3$$

$$\sigma'_m = \left[\sigma_{xm}^2 + 3\tau_{xym}^2\right]^{1/2} = 16/\pi d^3 \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2} = 16B/\pi d^3$$

$$A = \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2}$$

$$B = \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}$$

The DE-Elliptic Criterion

Substituting for A and B gives expressions for *d*, *1/n* and *r*:

$$d = \left\{ \frac{16n}{\pi} \left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2 \right]^{1/2} \right\}^{1/3}$$

$$\frac{1}{n} = \frac{16}{\pi d^3} \left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2 \right]^{1/2}$$

The DE-Elliptic Criterion

Substituting for A and B gives expressions for *d*, *1/n* and *r*:

$$r = \frac{\sigma'_a}{\sigma'_m} = \frac{A}{B} = \sqrt{\frac{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2}{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}}$$

The DE-Elliptic Criterion

For a rotating shaft with constant bending and torsion, the bending stress is completely reversed and the torsion is steady.

Previous Equations can be simplified by setting *M_m = 0 and T_a = 0, which simply drops out some of the terms.*

$$\begin{cases} A = 2K_f M_a \\ B = \sqrt{3}\, K_{fs} T_m \end{cases}$$

The DE-Elliptic Criterion

$$d = \left\{ \frac{16n}{\pi} \left[4\left(\frac{K_f M_a}{S_e} \right)^2 + 3\left(\frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2} \right\}^{1/3}$$

$$\frac{1}{n} = \frac{16}{\pi d^3} \left[4\left(\frac{K_f M_a}{S_e} \right)^2 + 3\left(\frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2}$$

$$r = \frac{\sigma'_a}{\sigma'_m} = \frac{A}{B} = \sqrt{\frac{4\left(K_f M_a\right)^2}{3\left(K_{fs} T_m\right)^2}} = \frac{2K_f M_a}{\sqrt{3}K_{fs} T_m}$$

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Assignment

Write a general Matlab code (with a GUI) to carry out all calculations relevant to the Static and Dynamic analysis of Shafts based on the material provided in the class notes.

Due. Sunday, 11.11.2012 [group work, same as previous groups. A mind map MUST accompany the work!]

Assessment will be based on:

1. **Thoroughness of code**
2. **Friendliness of code**
3. **Ease of use of the code**

Submit your final code as an executable file (.exe)