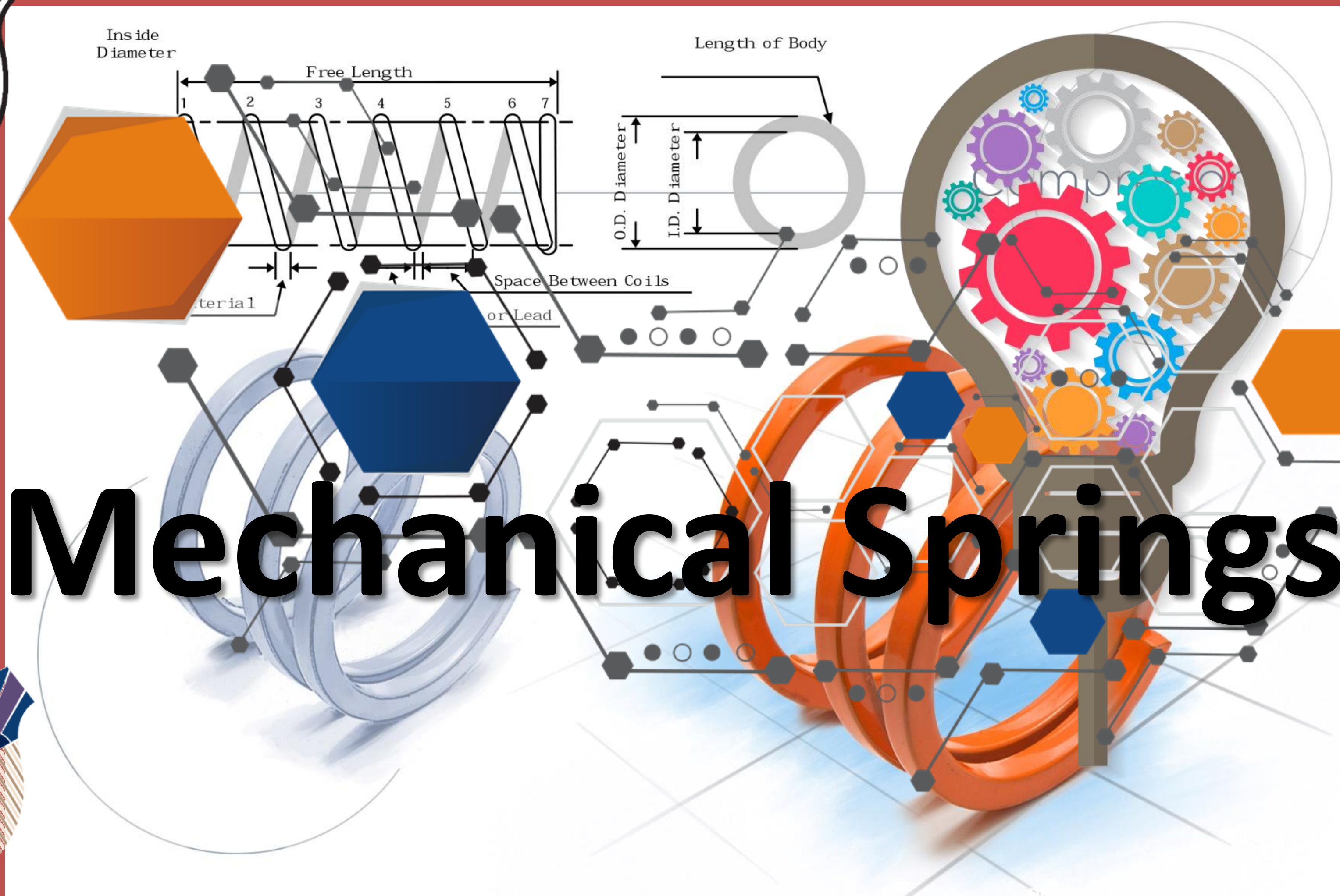
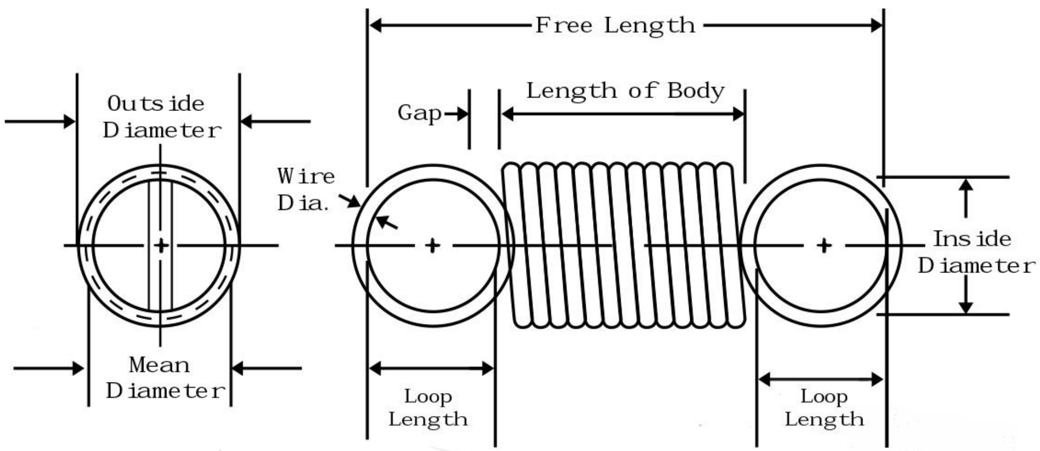
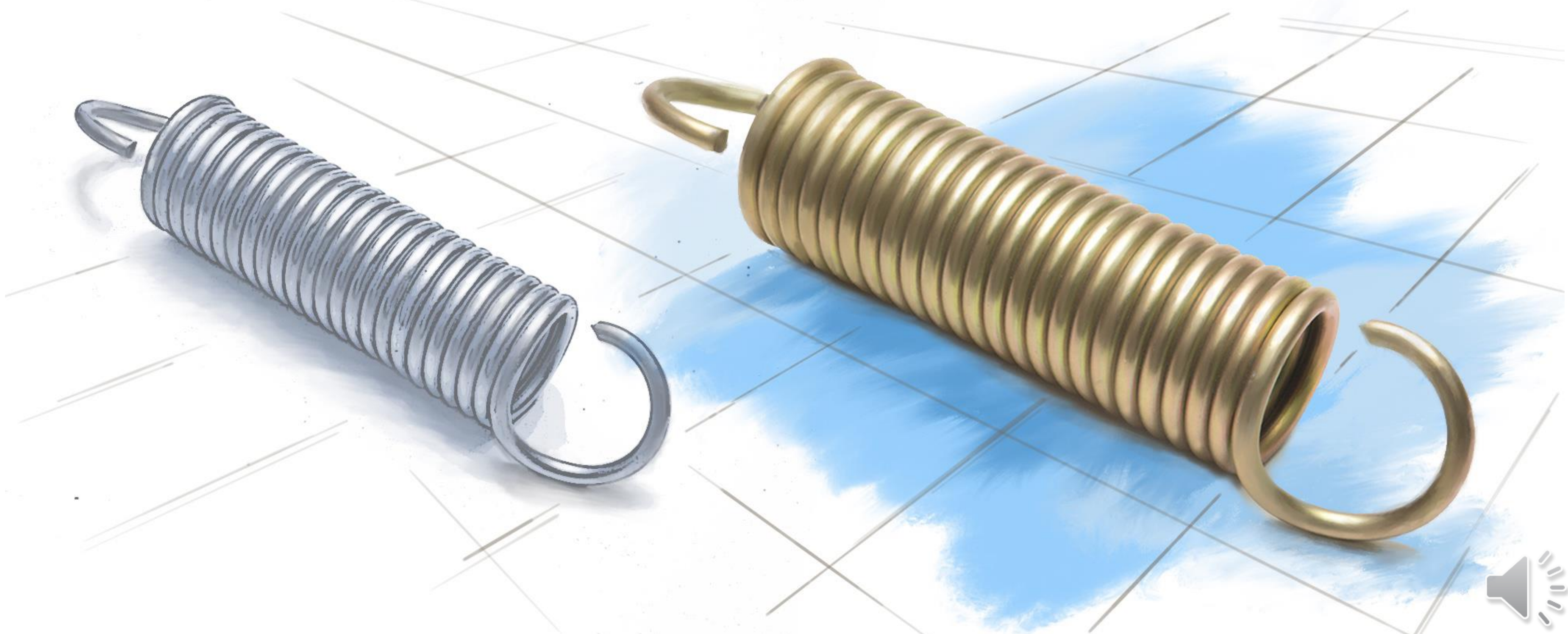
University of Jordan – School of Engineering – Mechanical Engineering Department

Mechanical Springs



Mechanical Springs





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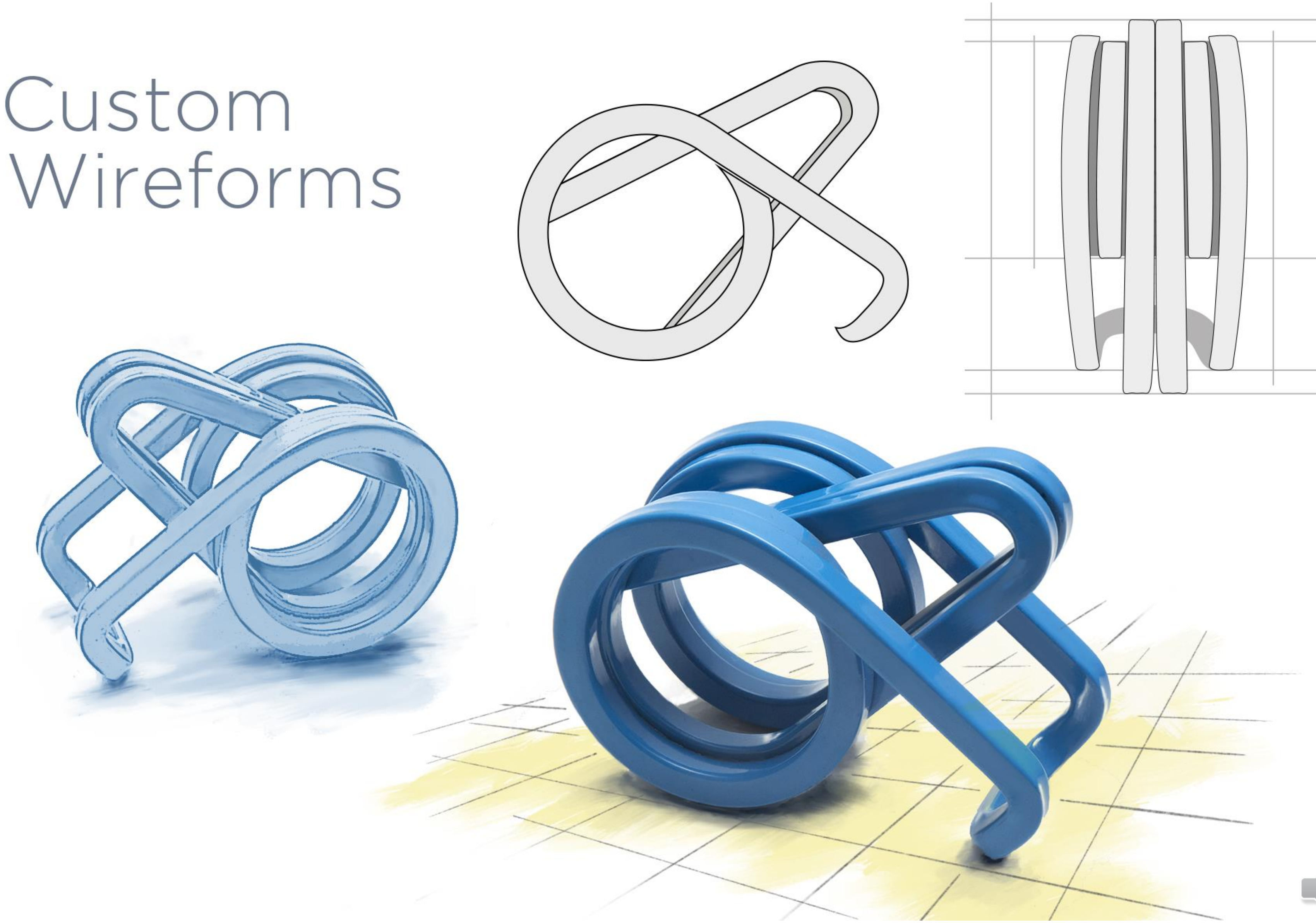

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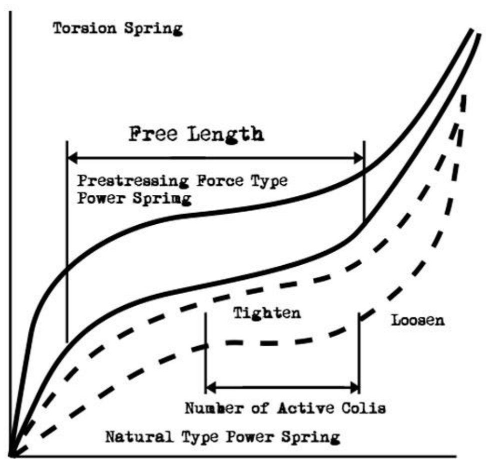
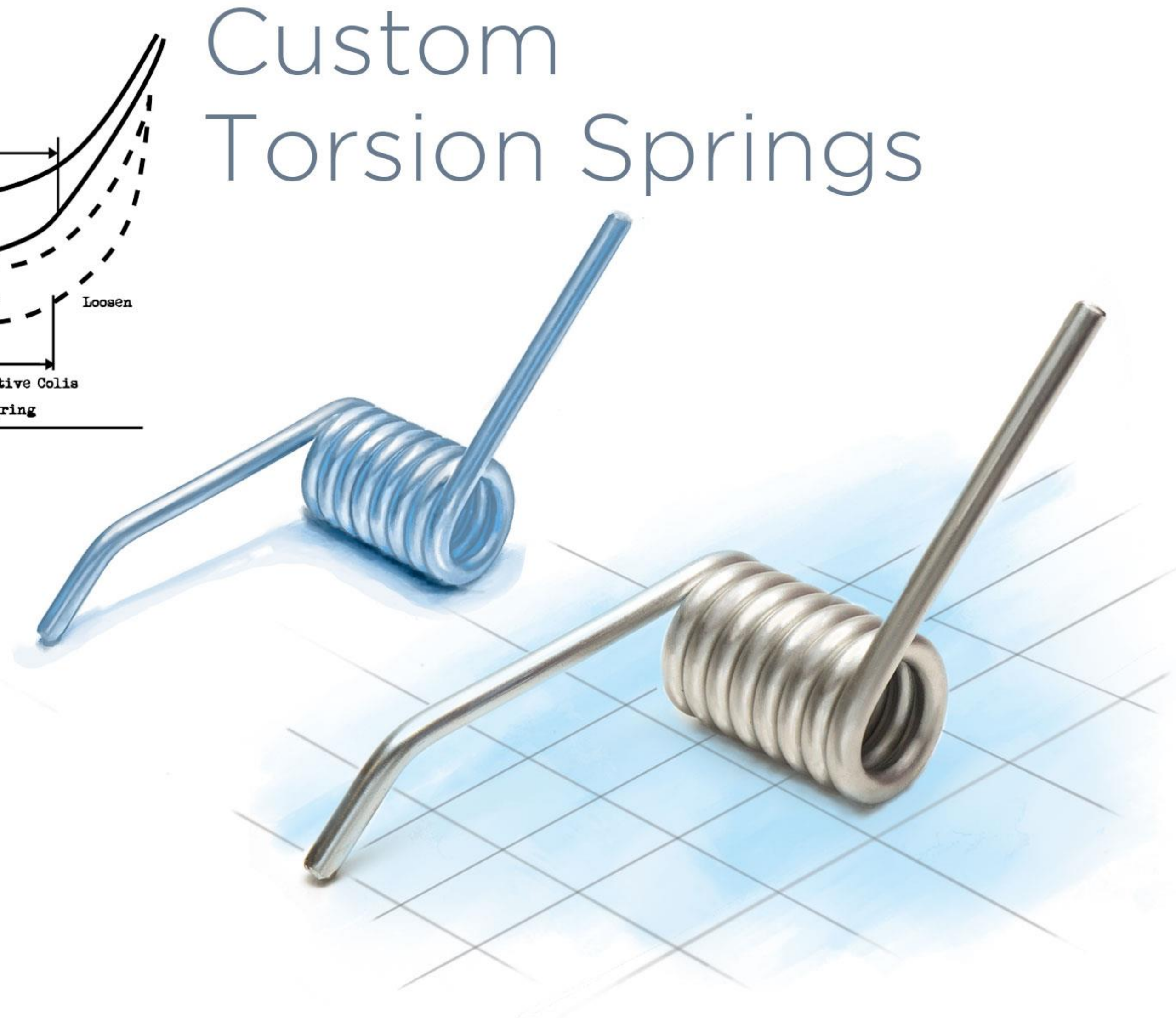
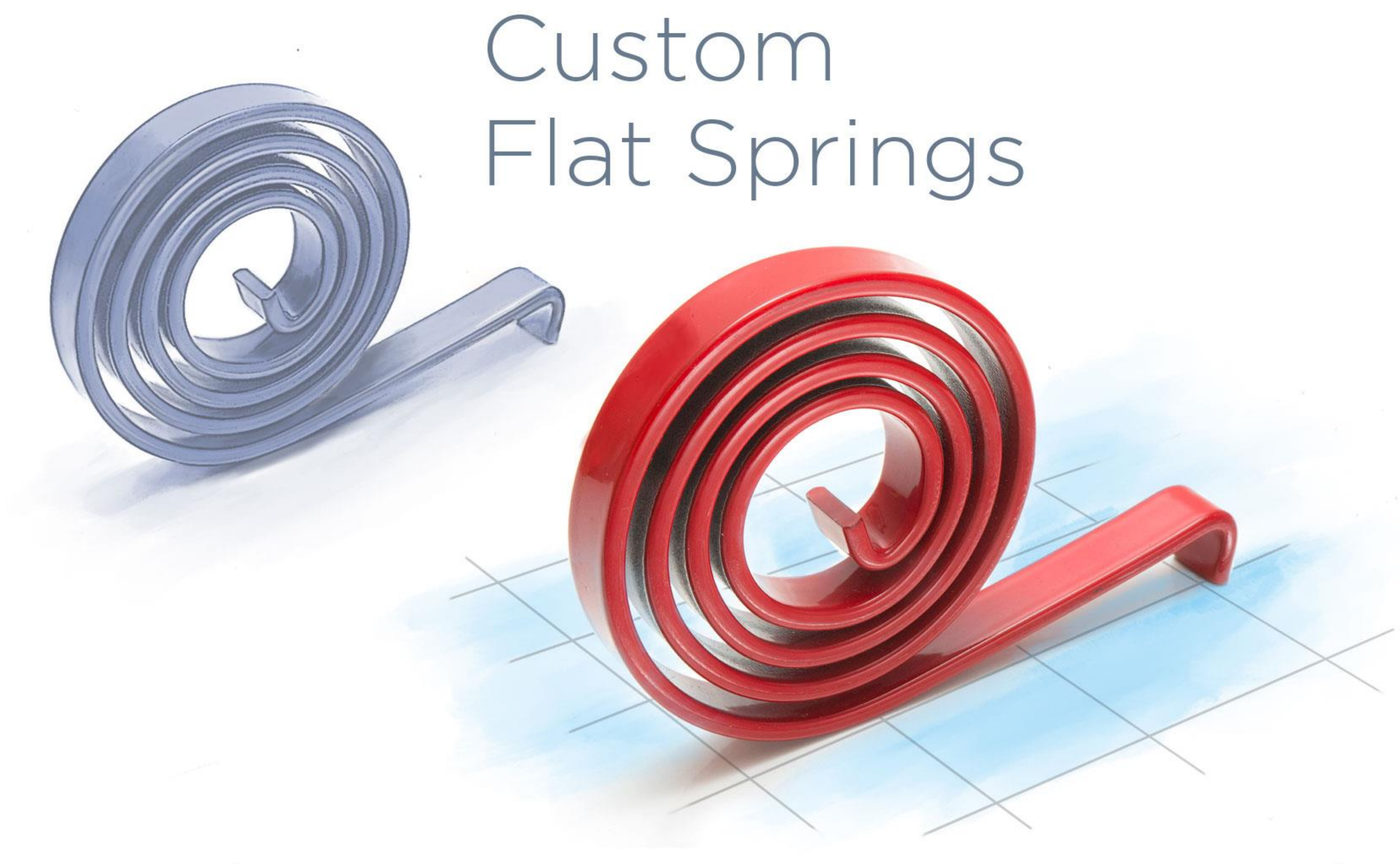
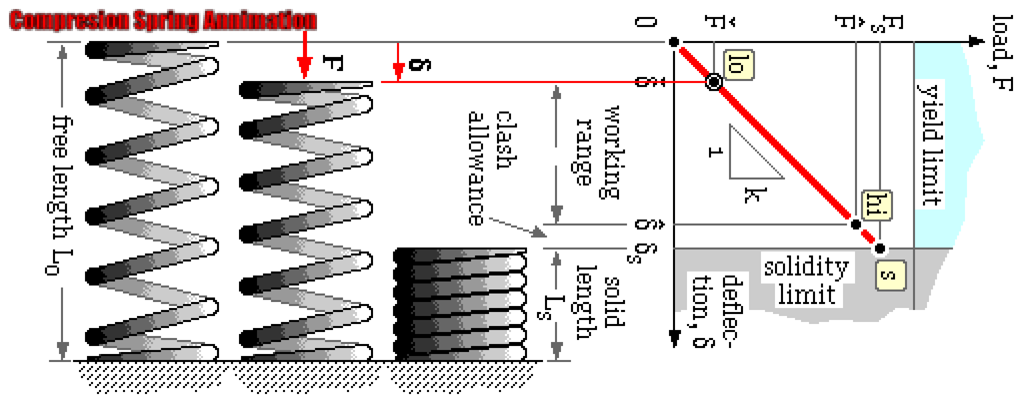




Mechanical Springs



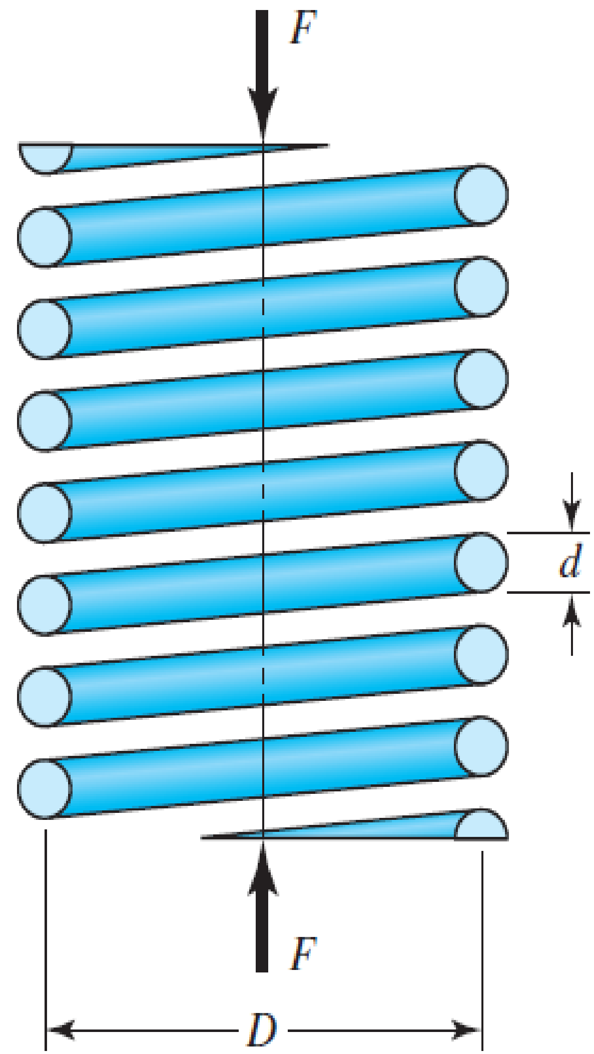
Why do we need Springs?



Stresses in Helical Springs

The figure shows a round-wire helical compression spring loaded by the axial force *F*.

Let *D* be the *mean coil diameter* and *d* as the *wire diameter*. Isolate a section in the spring.



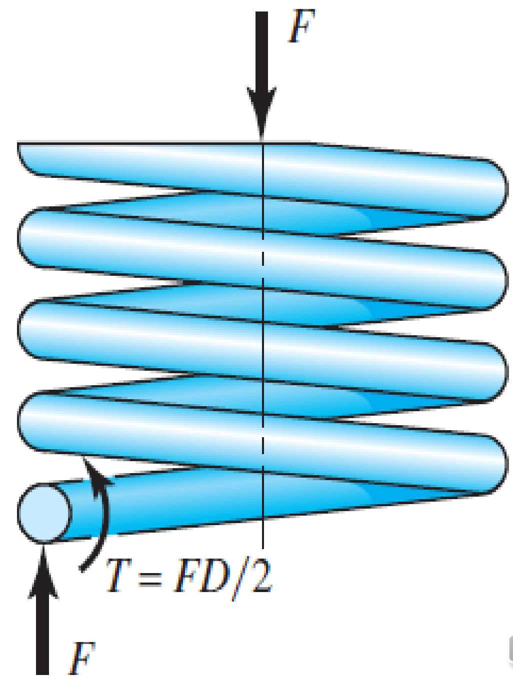
Stresses in Helical Springs

For equilibrium, the isolated section contains a direct shear force *F* and a torsional moment *T* = *FD*/2.

The maximum shear stress in the wire may be computed by superposition of the direct shear stress (*V* =*F*) and the torsional shear stress:

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A}$$

$$= \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$



Stresses in Helical Springs

Now we define the *spring index C,* which is a measure of coil curvature. The preferred value of *C* ranges from 4 to 12.

$$C = \frac{D}{d} \qquad \text{thus,} \qquad \tau = K_s \frac{8FD}{\pi d^3}$$

where K_s is a *shear stress-correction factor* and is defined by the equation:

$$K_s = \frac{2C + 1}{2C}$$

The Curvature Effect

The curvature of the wire causes a localized increase in stress on the inner surface of the coil, which can be accounted for with a curvature factor.

This factor can be applied in the same way as a stress concentration factor.

For static loading, the curvature factor is normally neglected because any localized yielding leads to localized strain strengthening.

For fatigue applications, the curvature factor should be included.

The Curvature Effect

Suppose K_s is replaced by another K factor, which corrects for both curvature and direct shear.

Then this factor is given by either of the equations:

$$K_W = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

$$K_B = \frac{4C + 2}{4C - 3}$$

The first of these is called the *Wahl factor,* and the second, the *Bergsträsser factor.*

The Curvature Effect

Since the results of these two equations differ by the order of 1% percent, K_B is used

The curvature correction factor can now be obtained by canceling out the effect of the direct shear.

Thus, the curvature correction factor is found to be:

$$K_c = \frac{K_B}{K_s} = \frac{2C(4C + 2)}{(4C - 3)(2C + 1)}$$

Now, K_s, K_B or K_W, and K_c are simply stress-correction factors applied multiplicatively to *Tr/J* at the critical location to estimate a particular stress.

There is *no* stress concentration factor.

The Curvature Effect

To predict the largest shear stress: $$\tau = K_B \frac{8FD}{\pi d^3}$$

Deflection of Helical Springs

The deflection-force relations are quite easily obtained by using Castigliano's theorem.

The total strain energy for a helical spring is composed of a torsional component and a shear component:

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG} \qquad \text{or:} \qquad U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

Where $l = \pi DN$ and $N = N_a$ (number of active coils)

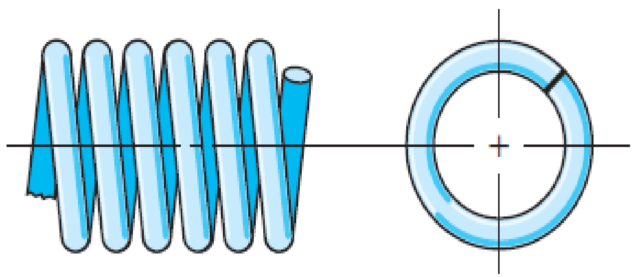
to find total deflection: $$y = \frac{\partial U}{\partial F} = \frac{8FD^3N}{d^4G} + \frac{4FDN}{d^2G}$$

$$y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3N}{d^4G} \qquad k \approx \frac{d^4G}{8D^3N}$$

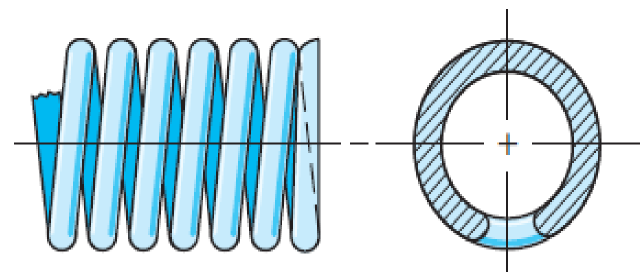
Compression Springs

The four types of ends generally used for compression springs are:

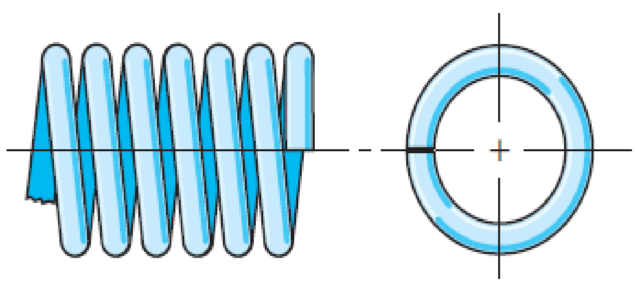
The terminal end of each spring is only shown on the right-end of the spring.



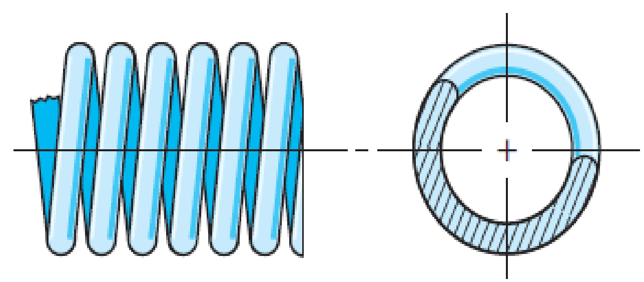
(*a*) Plain end, right hand



(*c*) Squared and ground end, left hand



(*b*) Squared or closed end, right hand



(*d*) Plain end, ground, left hand

Compression Springs

A spring with *plain ends* has a noninterrupted helicoid; the ends are the same as if a long spring had been cut into sections.

A spring with plain ends that are *squared* or *closed* is obtained by deforming the ends to a zero-degree helix angle.

Springs should always be both squared and ground for important applications, because a better transfer of the load is obtained.

Table 10–1 shows how the type of end used affects the number of coils and the spring length. Note that the digits 0, 1, 2, and 3 appearing in Table 10–1 are often used without question. *Some of these need closer scrutiny as they may not be integers.*

Compression Springs

Table 10–1

Formulas for the Dimensional Characteristics of Compression-Springs. (N_a = Number of Active Coils)

Source: From *Design Handbook,* 1987, p. 32. Courtesy of Associated Spring.

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Stability of Springs

compression coil springs may buckle when the deflection becomes too large.

The critical deflection is given by the equation:

$$y_{cr} = L_0 C'_1 \left[1 - \left(1 - \frac{C'_2}{\lambda_{eff}^2} \right)^{1/2} \right]$$

where y_{cr} is the deflection corresponding to the onset of instability.

λ_{eff} is the *effective slenderness ratio:* $\lambda_{eff} = \frac{\alpha L_0}{D}$

C'_1 and C'_2 are dimensionless elastic constants defined by the equations:

$$C'_1 = \frac{E}{2(E - G)} \qquad C'_2 = \frac{2\pi^2(E - G)}{2G + E}$$

Stability of Springs

α is the *end-condition constant*. It depends upon how the ends of the spring are supported. Table 10–2 gives values of α for usual end conditions.

Table 10–2

End-Condition Constants α for Helical Compression Springs*

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

Stability of Springs

Absolute stability occurs when the term $C'_2/\lambda^2_{\text{eff}} > 1$

This means that the condition for absolute stability is that:

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

For steels, this turns out to be

$$L_0 < 2.63\frac{D}{\alpha}$$

For squared and ground ends supported between flat parallel surfaces, $\alpha = 0.5$ and $L_o < 5.26D$.

Spring Material

Springs are manufactured either by hot- or cold-working processes, depending upon the size of the material, the spring index, and the properties desired.

In general, prehardened wire should not be used if $D/d < 4$ or if $d > 0.25$ in.

Winding of the spring induces residual stresses through bending, but these are normal to the direction of the torsional working stresses in a coil spring.

Quite frequently in spring manufacture, they are relieved, after winding, by a mild thermal treatment.

Spring Material

A great variety of spring materials are available to the designer, including plain carbon steels, alloy steels, and corrosion-resisting steels, as well as nonferrous materials such as phosphor bronze, spring brass, beryllium copper, and various nickel alloys.

Descriptions of the most commonly used steels will be found in Table 10–3.

The UNS steels listed in Appendix A should be used in designing hot-worked, heavy-coil springs, as well as flat springs, leaf springs, and torsion bars.

Spring Material

Table 10–3

High-Carbon and Alloy Spring Steels

Source: From Harold C. R. Carlson, "Selection and Application of Spring Materials," *Mechanical Engineering*, vol. 78, 1956, pp. 331–334.

Name of Material	Similar Specifications	Description
Music wire, 0.80–0.95*C*	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures.

Spring Material

Spring materials may be compared by an examination of their tensile strengths; these vary so much with wire size that they cannot be specified until the wire size is known.

The material and its processing also, of course, have an effect on tensile strength.

It turns out that the graph of tensile strength versus wire diameter is almost a straight line for some materials when plotted on log-log paper.

Writing the equation of this line as $S_{ut} = \frac{A}{d^m}$ furnishes a good means of estimating minimum tensile strengths when the intercept *A* and the slope *m* of the line are known.

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Spring Material

Values of these constants have been worked out from recent data and are given for strengths in units of kpsi and MPa in Table 10–4.

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook*, 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

*Surface is smooth, free of defects, and has a bright, lustrous finish.

†Has a slight heat-treating scale which must be removed before plating.

‡Surface is smooth and bright with no visible marks.

§Aircraft-quality tempered wire, can also be obtained annealed.

‖Tempered to Rockwell C49, but may be obtained untempered.

#Type 302 stainless steel.

**Temper CA510.

Spring Material

Although the torsional yield strength is needed to design the spring and to analyze the performance, spring materials customarily are tested only for tensile strength— perhaps because it is such an easy and economical test to make.

A very rough estimate of the torsional yield strength can be obtained by assuming that the tensile yield strength is between 60 and 90 percent of the tensile strength.

Then the distortion energy theory can be employed to obtain the torsional yield strength ($S_{sy} = 0.577 S_y$).

For steels, this approach results in the range

$$0.35 S_{ut} \leq S_{sy} \leq 0.52 S_{ut}$$

Spring Material

For wires listed in Table 10–5, the maximum allowable shear stress in a spring can be seen in column 3.

Music wire and hard-drawn steel spring wire have a low end of range $S_{sy} = 0.45S_{ut}$.

Table 10–5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Elastic Limit, Percent of S_{ut} Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2

Spring Material

For specific materials for which you have torsional yield information use Table 10-6 as a guide.

Table 10–6

Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Source: Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design*, 3rd ed., McGraw-Hill, New York, 2004.

Material	Maximum Percent of Tensile Strength: Before Set Removed (includes K_W or K_B)	Maximum Percent of Tensile Strength: After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

Joerres provides set-removal information, that $S_{sy} \geq 0.65 S_{ut}$ increases strength through cold work, but at the cost of an additional operation by the springmaker.

Spring Material

Sometimes the additional operation can be done by the manufacturer during assembly.

Some correlations with carbon steel springs show that the tensile yield strength of spring wire in torsion can be estimated from $0.75S_{ut}$.

The corresponding estimate of the yield strength in shear based on distortion energy theory is $S_{sy} = 0.577(0.75)S_{ut} = 0.433S_{ut} \approx 0.45S_{ut}$.

Samónov discusses the problem of allowable stress and shows that $S_{sy} = \tau\ t_{all} = 0.56S_{ut}$ for high-tensile spring steels, which is close to the value given by Joerres for hardened alloy steels.

Design for Static Service

- The preferred range of the spring index is $4 \le C \le 12$, with the lower indexes being more difficult to form (because of the danger of surface cracking) and springs with higher indexes tending to tangle often enough to require individual packing.

- This can be the first item of the design assessment.

- The recommended range of active turns is $3 \le Na \le 15$.

- To maintain linearity when a spring is about to close, it is necessary to avoid the gradual touching of coils (due to nonperfect pitch).

Design for Static Service

A helical coil spring force-deflection characteristic is ideally linear.

Practically, it is nearly so, but not at each end of the force-deflection curve.

The spring force is not reproducible for very small deflections, and near closure, nonlinear behavior begins as the number of active turns diminishes as coils begin to touch.

The designer confines the spring's operating point to the central 75% of the curve between no load, $F = 0$, and closure, $F = F_s$.

Design for Static Service

- The maximum operating force should be limited to $F_{max} \leq \frac{7}{8} F_s$.

- Defining the fractional overrun to closure as ξ , where:

$$F_s = (1 + \xi)F_{max} = (1 + \xi)\left(\frac{7}{8}\right)F_s$$

- it is recommended that $\xi \geq 0.15$

In addition to the relationships and material properties for springs, we now have some recommended design conditions to follow, namely:

$$3 \leq Na \leq 15$$

$$4 \leq C \leq 12$$

$$\xi \geq 0.15$$

$n_s \geq 1.2$ (factor of safety at closure (solid height))

Design for Static Service

When considering designing a spring for high volume production, the *figure of merit* can be the cost of the wire from which the spring is wound.

The **fom** would be proportional to the relative material cost, weight density, and volume:

$$\text{fom} = -(\text{relative material cost})\frac{\gamma\pi^2 d^2 N_t D}{4}$$

Spring design is an open-ended process.

There are many decisions to be made, and many possible solution paths as well as solutions.

Design for Static Service

In the past, charts, nomographs, and “spring design slide rules” were used by many to simplify the spring design problem.

Today, the computer enables the designer to create programs in many different formats—direct programming, spreadsheet, MATLAB, etc.

Design Strategy

STATIC SPRING DESIGN

Choose d

Over-a-rod

As-wound or set

$$D = d_{rod} + d + \text{allow}$$

Free

As-wound

$$S_{sy} = \text{const}(A)/d^{m\,\dagger}$$

$$C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

$$\alpha = \frac{S_{sy}}{n_s} \qquad \beta = \frac{8(1+\xi)F_{max}}{\pi d^2}$$

$$D = Cd$$

Set removed

$$S_{sy} = 0.65A/d^m$$

$$D = \frac{S_{sy}\pi d^3}{8n_s(1+\xi)F_{max}}$$

In-a-hole

As-wound or set

$$D = d_{hole} - d - \text{allow}$$

† const is found from Table 10–6.

Design Strategy

$$C = D/d$$

$$K_B = (4C + 2)/(4C - 3)$$

$$\tau_s = 8K_B(1 + \xi)F_{max}D/(\pi d^3)$$

$$n_s = S_{sy}/\tau_s$$

$$OD = D + d$$

$$ID = D - d$$

$$N_a = Gd^4 y_{max}/(8D^3 F_{max})$$

N_t: Table 10–1

L_s: Table 10–1

L_O: Table 10–1

$$(L_O)_{cr} = 2.63D/\alpha$$

$$\text{fom} = -(\text{rel. cost})\,\gamma\pi^2 d^2 N_t D/4$$

Print or display: d, D, C, OD, ID, N_a, N_t, L_s, L_O, $(L_O)_{cr}$, n_s, fom

Build a table, conduct design assessment by inspection

Eliminate infeasible designs by showing active constraints

Choose among satisfactory designs using the figure of merit

Design Strategy

Make the a priori decisions, with hard-drawn steel wire the first choice (relative material cost is 1.0).

Choose a wire size *d*. With all decisions made, generate a column of parameters: d, D, C, OD or ID, N_a, L_s, L_o, $(L_o)_{cr}$, n_s, and fom.

By incrementing wire sizes available, we can scan the table of parameters and apply the design recommendations by inspection.

After wire sizes are eliminated, choose the spring design with the highest figure of merit.

This will give the optimal design despite the presence of a discrete design variable *d* and aggregation of equality and inequality constraints.

Design Strategy

It is general enough to accommodate to the situations of as-wound and set-removed springs, operating over a rod, or in a hole free of rod or hole.

In as-wound springs the controlling equation must be solved for the spring index as follows:

$$\frac{S_{sy}}{n_s} = K_B \frac{8F_s D}{\pi d^3} = \frac{4C+2}{4C-3}\left[\frac{8(1+\xi)F_{\max}C}{\pi d^2}\right]$$

Let $\alpha = \dfrac{S_{sy}}{n_s}$ and $\beta = \dfrac{8(1+\xi)F_{\max}}{\pi d^2}$ then:

$$C = \frac{2\alpha-\beta}{4\beta} + \sqrt{\left(\frac{2\alpha-\beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

EXAMPLES

Textbook: Example 10-1 to 10-3