Machines

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MechFamily

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Notebooks

⇒ In this course, the following topics will be covered:

[1] Transformers

Electrical energy (V_1, f_1) ⟶ [Transformer] ⟶ Electrical energy (V_2, f_1)

- Transformers are used to step up/step down the Voltage V.
- They can only operate if they are supplied with AC power [They can't operate if they are supplied with DC power].
- They cannot change the AC power supply frequency f

[2] Motors

Electrical energy "Source" ⟶ [Motor] ⟶ Mechanical energy "Load"

$V, i \Rightarrow P$ "input power"

$\left.\begin{matrix} T, \omega \\ F, v \end{matrix}\right\}$ P "output power"

- Electrical motors are used to convert electrical energy into mechanical energy.
- They can be supplied with either DC power or AC power.

[3] Generators

Mechanical energy "Source" ⟶ [Generator] ⟶ Electrical energy "Load"

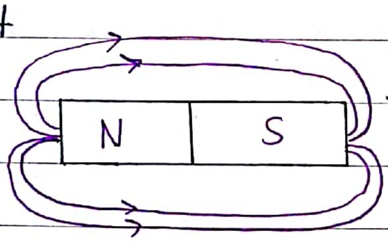
- Generators are used to convert mechanical energy into electrical energy.
- They can produce either DC power or AC power.

⇒ Notes:

- Transformers are stationary devices [∵ They don't contain moving parts]. However, motors and generators contain rotating parts.
- All electrical machines (transformers, motors, generators) can't operate without the existence of magnetic field. Thus, we shall first talk about magnetic fields.

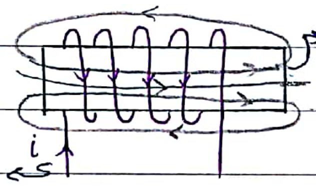
⊛ Sources of magnetic field

[1] Permanent magnet



→ Magnetic field lines always:
[1] Form closed loops
[2] Leave the North pole and enter the South pole

[2] Electromagnet



→ Core (Usually made of ferromagnetic material)

Wire

magnetic field is produced by an electric current.

Ferromagnetic materials: are materials that can be magnetized [become a magnet]. E.g: iron.

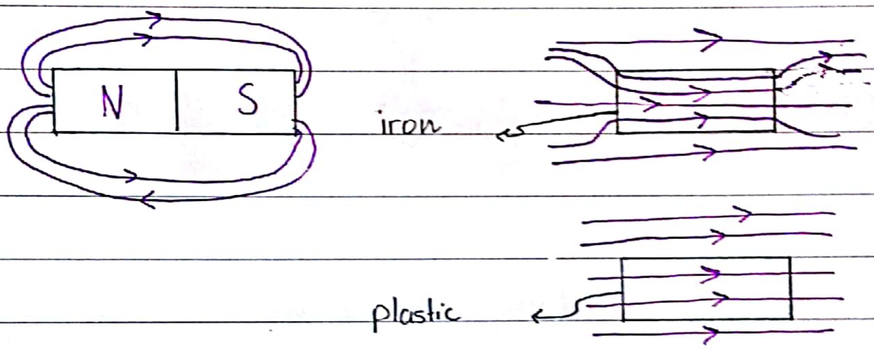
- When a wire is wrapped around an iron rod, and an electric current flows in the wire, the iron rod becomes a magnet.
- Right hand rule can be used to find the direction of the magnetic field. Wrap your fingers along the coil in the direction of the current, then your thumb will point in the direction of the magnetic field
- The main difference between the permanent magnet and the electromagnet:
  - For permanent magnets, the strength of the magnetic field is fixed
  - For electromagnets, the strength of the magnetic field can be increased by: Increasing the current / Increasing the number of turns / Using a ferromagnetic material core instead of air core

⊛ Electrical and magnetic properties of materials:

[1] Conductivity σ : the degree to which a material conducts electricity

[2] Permittivity ϵ : a property that measures the level of resistance which is experienced whenever developing an electrical field inside a medium [This property is related to capacitors]

[3] Permeability μ : a property that measures the ability of a material to support the formation of a magnetic field within it self. Or it is a measure of the ability of the material to be magnetized

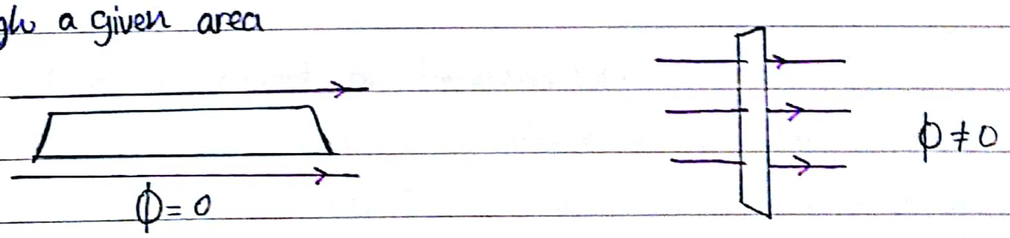


→ If you put a piece of iron or plastic inside the magnetic field of a magnet, the magnetic field lines will look like as shown in the adjacent figure

$\mu_{iron} > \mu_{plastic}$

⊛ Magnetic field can be described by the following parameters

[1] Flux ϕ [Wb] = weber. It can be thought as the number of magnetic field lines passing through a given area



[2] Flux density B [Wb/m^2 = T] = Tesla. It is the magnetic flux per unit area

[3] Flux intensity H [A/m] : It represents the strength of the magnetic flux

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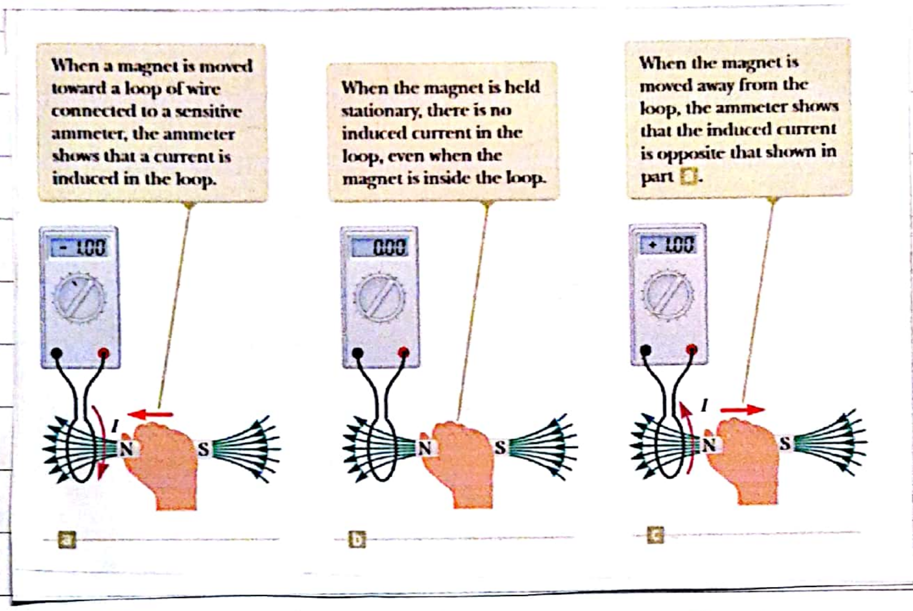
(*) Principles of magnetic field

[1] Faraday's law (Transformer action) مبدأ

• Faraday's law can be explained by the following experiment



• Faraday's law states that an emf (electromotive force) is induced in a loop when the magnetic flux through the loop changes with time.

Note: emf means voltage source.

• Mathematically

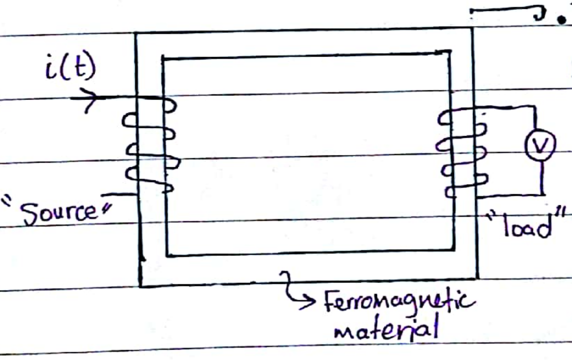
$$emf = \underbrace{N}_{\text{number of turns}} \frac{d\phi}{dt}$$

note: • if $\phi = \phi(t)$ is given, we can calculate emf

• If $emf = emf(t)$ is given, we can calculate ϕ

$$\phi = \frac{1}{N} \int emf \, dt$$

• Transformer action is based on Faraday's law.



• This is a transformer. An electric current is flowing through the coils of the source, and as a result, a magnetic field is generated. Since the coils of the source are supplied with an alternating current i(t), the generated magnetic field (or magnetic flux) will vary with time, and hence, an electromotive force will be produced in the coils of the load. ⇒

• If the coils of the source are supplied with direct current I, the generated magnetic flux will not vary with time and hence no electromotive force will be produced in the coils of the ~~load~~ load.

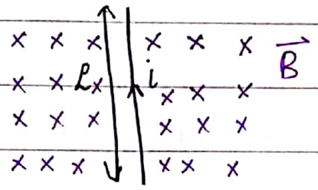
⇓

This is why transformers can only operate if they are supplied with AC power.

[2] Motor action

• The magnetic force acting on a wire carrying an electric current is given by

$$\vec{F} = i\,\vec{\ell} \times \vec{B}$$



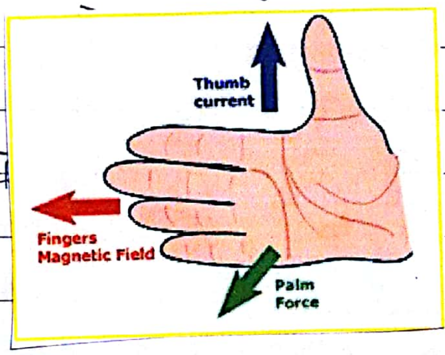
where

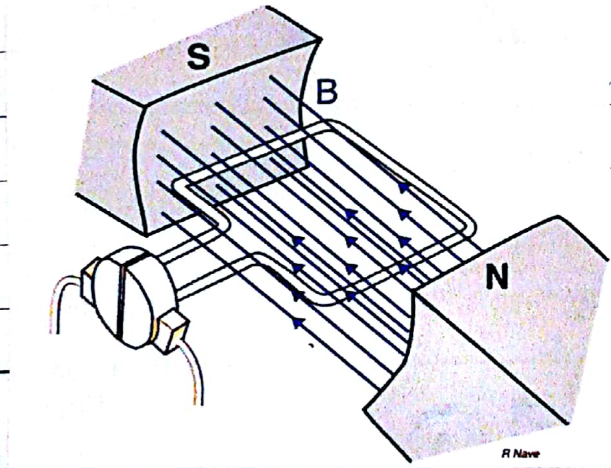
i = current flowing through the wire

$\vec{\ell}$ = a vector that points in the direction of the current i and has a magnitude equal to the length of the wire

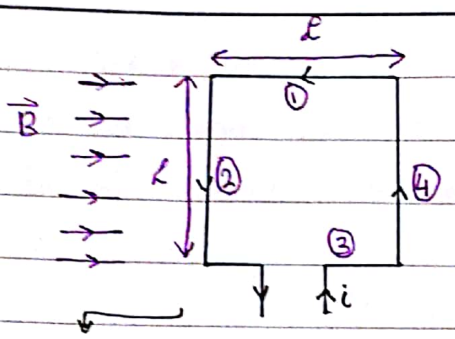
$\vec{B}$ = flux density

• To determine the direction of the magnetic force: Your thumb must point in the direction of $\vec{\ell}$ and the extended fingers in the direction of $\vec{B}$. The force $\vec{F}$ extends outward from the palm of your hand. [use your right hand]





→ In every motor, you can find a rectangular loop carrying a current (i) in the presence of magnetic field B



- No magnetic forces act on sides ① and ③ because these sides are parallel to $\vec{B} \Rightarrow i \vec{\ell} \times \vec{B} = 0$ (because $\theta = 0, 180$)
- Side ② is subjected to magnetic force (out of the page •)
- Side ④ is subjected to magnetic force (into the page x)
- Notice that the two forces point in opposite directions but are not directed along the same line of action. These two forces produce a torque

Input: i, B
Output: Force F

[3] Generator action

- Recall from physics:

The magnetic force acting on a charged particle is given by

$\vec{F} = q\vec{v} \times \vec{B}$ $\quad |\vec{F}| = q v B \sin\theta$

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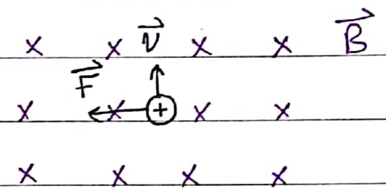
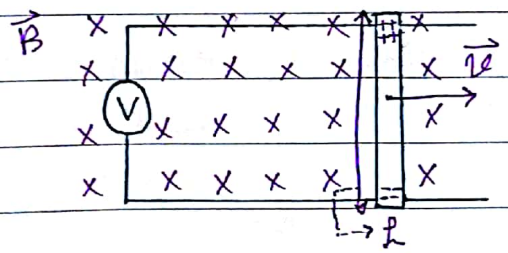


To determine the direction of the magnetic force: your thumb must point in the direction of the velocity of the particle $\vec{v}$, while the other fingers in the direction of the magnetic field $\vec{B}$. The force $\vec{F}$ extends "outward" from the palm of your hand if q is positive and extends "inward" towards the palm if q is −negative.

- Consider a wire of length ℓ is moving to the right through a magnetic field directed into the page.



- From the magnetic force acting on moving charged particles, the electrons in the wire experience a magnetic force that is directed downward, and as a result, electrons move to the lower end of the wire, leaving a net positive charge at the upper end. The voltage difference across the ends of the wire is given by

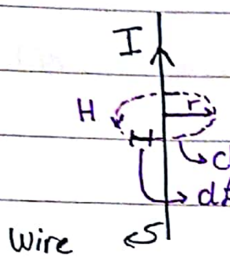
$\Delta V = emf = (v \times B)\,\ell$

[4] Ampere's law:

The line integral of $H \cdot d\ell$ around any closed path equals NI, wher NI represents the total current passing through the surface bounded by the closed path

$$\oint H \cdot d\ell = NI$$

• Example:



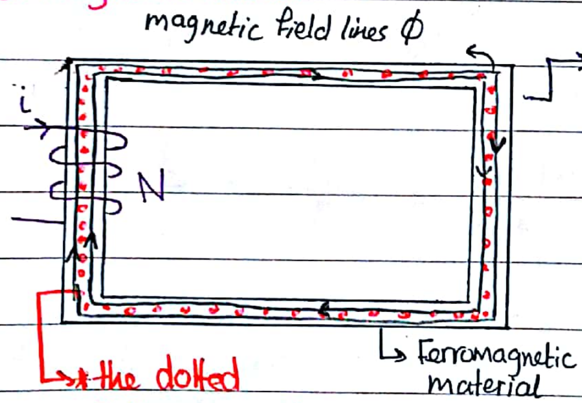
• H is uniform

$$H \oint d\ell = NI \Rightarrow H * 2\pi r = I \Rightarrow H = \frac{I}{2\pi r}$$

• Note: The following diagram represent the relation between I, H, B, ϕ

$$I \xrightarrow[\text{Ampere's law}]{} H \xrightarrow[\mu]{} B \xrightarrow[\text{Area}(A)]{} \phi$$

(*) Magnetic circuit



* the dotted path represents the closed path and it has a length L.

* L is the mean path length

• This is a magnetic circuit. It consists of a core that is made of a ferromagnetic material + a wire that is wrapped around one of the sides of the core.

• Applying Ampere's law to the closed dotted path

$$HL = Ni$$, where N = number of turns of the coil

$$H = \frac{Ni}{L}$$

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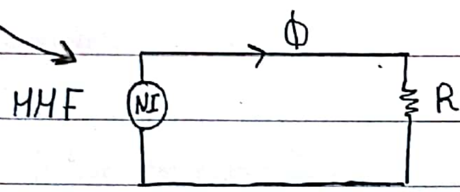
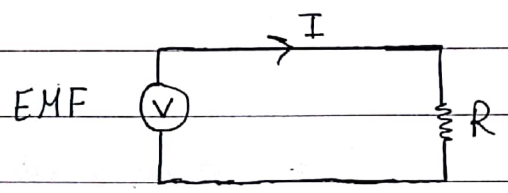
- For the previous magnetic circuit

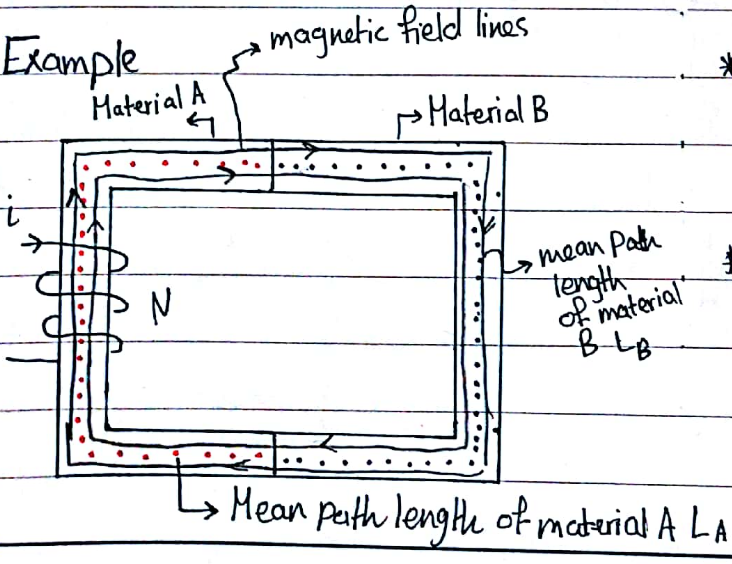
$$\Phi = B \cdot A = (\mu H) \cdot A = \mu \left(\frac{NI}{L}\right) A = NI \left(\frac{\mu A}{L}\right) = \frac{NI}{\frac{L}{\mu A}}$$

$$\Phi = \frac{NI}{\frac{L}{\mu A}} \rightarrow \text{Similar to Ohm's law } I = \frac{V}{R}$$

Hence, the previous magnetic circuit can be represented as

Magnetic circuit	Electrical circuit
	
MMF = magnetic motive force = NI [Ampere.turn]	EMF = electro-motive force = V [Volts]
Φ [Weber]	i [Ampere]
R = Relcutance = $\frac{L}{\mu A}$ $\left[\frac{\text{Ampere.turn}}{\text{Weber}}\right]$	R = Resistance = $\frac{L}{\sigma A}$ [Ohm Ω]

Example

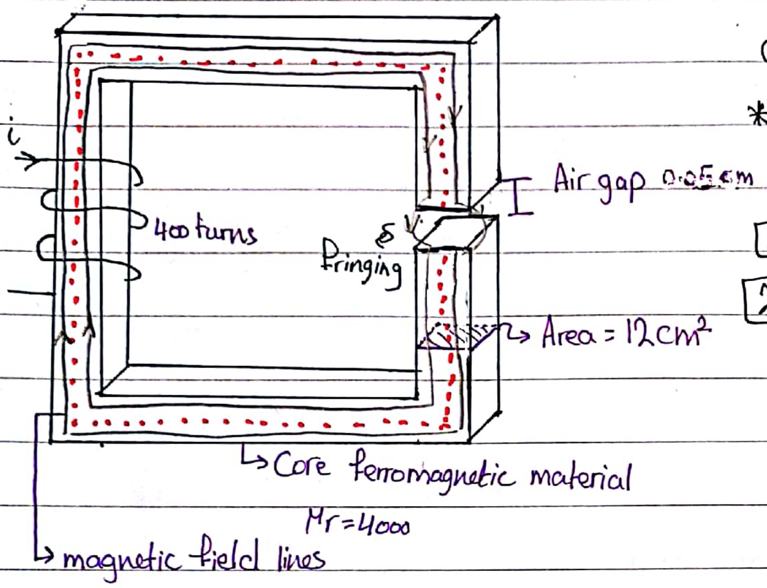


* Applying Ampere's law

$$H_A L_A + H_B L_B = N i$$

* Notice that $\Phi_A = \Phi_B$ [the same number of magnetic field lines are passing through A and B]

Example



* If the mean path length of the core (dotted path) = 40 cm
* Fringing increase air gap area by 5%

Find

1. Total reluctance (Core + air gap)
2. Current required to produce flux density = 0.5T in the air gap
   ↳ tesla

Notes:

- μ_r = relative permeability $= \frac{\mu}{\mu_0}$, μ = Permeability of the specified material
  μ_0 = permeability of free space (vacuum)
  $= 4\pi * 10^{-7}$ Henry/m
- $\Phi_{gap} = \Phi_{core}$ [The same number of magnetic field lines passes through the gap and the core]

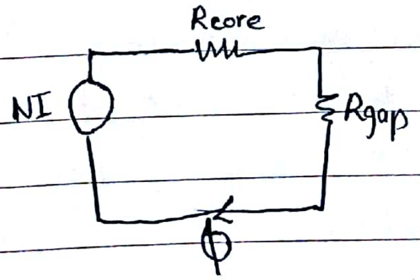
due to fringing $A_{gap} > A_{core}$, Recall $B = \frac{\Phi}{A}$ $\therefore$ $A_{gap} = 1.05 A_{core}$

$\therefore B_g < B_c$

- Apply Ampere's law to the magnetic circuit $H_c L_c + H_g L_g = NI$

Solution:

* Draw the equivalent magnetic circuit



$\boxed{1}$ $R = \frac{L}{\mu A}$, $R_c = \frac{40 * 10^{-2}}{4000 * 4\pi * 10^{-7} (12 \times 10^{-4})}$

$= 66314$ A·turn/weber

≈ 6600 A·t/Wb

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$$R_g = \frac{L}{\mu A} = \frac{0.05 * 10^{-2}}{4\pi * 10^{-7}\left(1.05 * 12 * 10^{-4}\right)} = 315783 \approx 316000 \ A.t/Wb$$

$\boxed{2}$ $B_g = 0.5$ Tesla

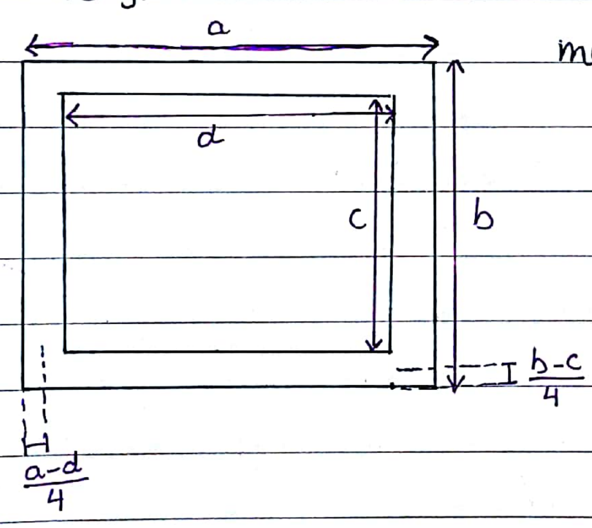
$$\phi_g = B_g * A = 0.5 * \left(1.05 * 12 * 10^{-4}\right) = 6.3 \times 10^{-4} \ Wb$$

$$\phi_g = \phi_{core} = \phi$$

$$\phi = \frac{NI}{R_{tot}} \Rightarrow 6.3 * 10^{-4} = \frac{I * 400}{316000 + 66000} \Rightarrow I = 0.6 \ A$$

Notes:

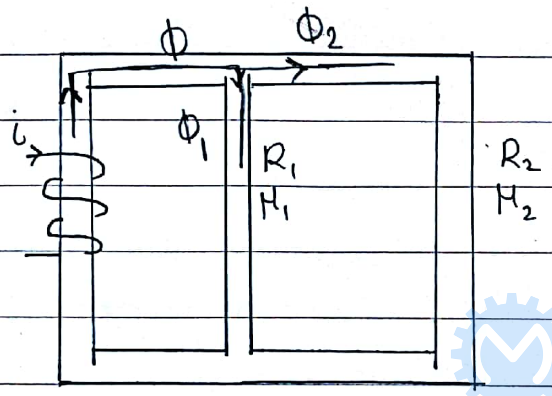
* Sometimes, the mean path length is not given. Instead, the dimensions of the circuit are given:



$$\text{mean path length} = 2\left(a - \frac{(a-d)}{2}\right) + 2\left(b - \frac{(b-c)}{2}\right)$$

* When $\mu \to \infty \Rightarrow R \to 0$

– In the adjacent figure, the total magnetic flux = the sum of the fluxes through each path ($\phi = \phi_1 + \phi_2$)



→ Continue

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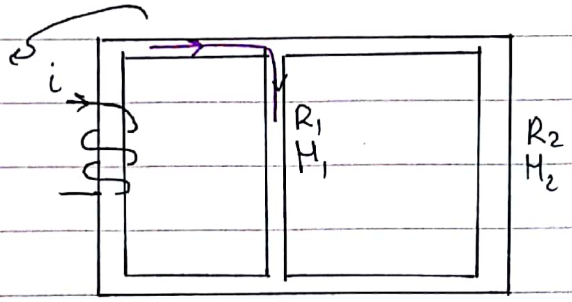
- If $\mu_1 \to \infty$, $R_1 \to 0$ $\Rightarrow$ Result: Short circuit

the total magnetic flux will flow in this direction



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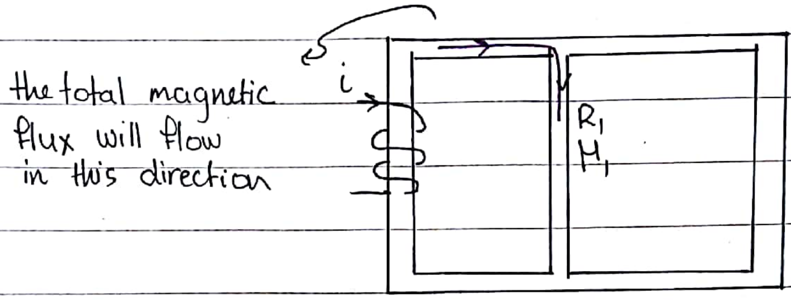
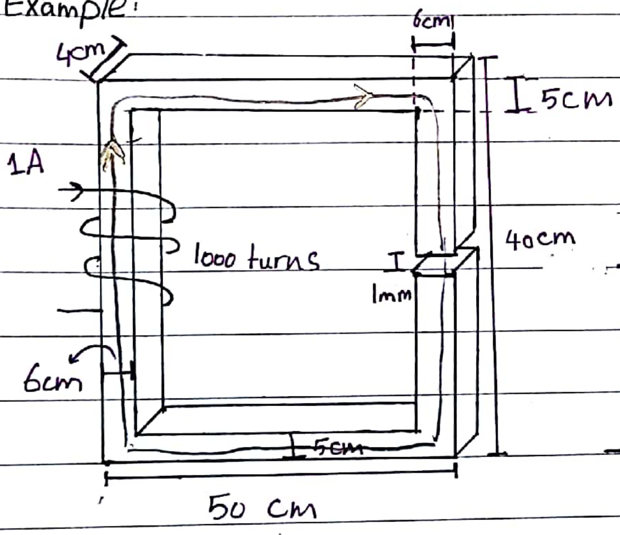


Example:



* If μ_r of the core = 2000, find ϕ

→ Notice that the mean path length + Area are not given

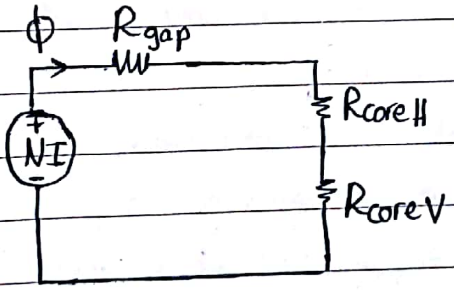
→ A (Area) is changing along the core

A in the vertical direction = $A_V = 6 \times 4 = 24\ cm^2$

A in the horizontal direction $A_H = 5 \times 4 = 20\ cm^2$

→ $\mu_0 = 4\pi * 10^{-7}$ Henry/m [Memorize this number]

→ Draw the equivalent magnetic circuit



* $R_{coreV} = \dfrac{L_V}{\mu A_V}$, $A_V = 24 * 10^{-4}\ m^2$

$\mu = 2000 * 4\pi * 10^{-7}$ Henry/m

$L_V = (40cm - 2.5cm - 2.5cm) * 2 - 1mm$

$= 70cm - 1mm = 69.9\ cm$

$= 69.9 \times 10^{-2} m$

$\therefore R_{coreV} = 115884.69 \approx 116000$ At/Wb

* $R_{coreH} = \dfrac{L_H}{\mu A_H} = 175070 \approx 175000$ At/Wb , $A_H = 20 * 10^{-4}\ m^2$

$\mu = 2000 * 4\pi * 10^{-7}$ Henry/m

$L_H = (50 - 3 - 3) * 2 = 88cm = 88 \times 10^{-2} m$

* $R_{gap} = \frac{L}{\mu A} = 331572 \approx 332000$ At/Wb , $L = 1 \times 10^{-3}$ m

$\mu = 4\pi \times 10^{-7}$ Henry/m

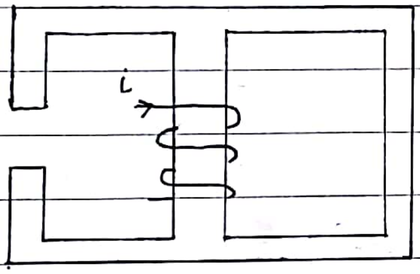
$A = 24 \times 10^{-4}$ m² [notice that $R_{gap} > R_{core}$]

← مقاومة الهوا للتدفق المغناطيسي أكبر

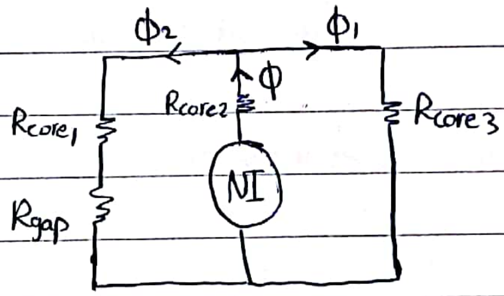
$$\phi = \frac{NI}{\Sigma R_{tot}} = \frac{1000 \times 1}{116000 + 175000 + 331572}$$

$$= 1.6 \times 10^{-3} \text{ Wb} = 1.6 \text{ mWb}$$

Example



*Draw the equivalent magnetic circuit, assume that the cross sectional area is not varying



→ In the previous analysis, several approximations or assumptions were made, Such as:

[1] Using the mean path length in calculations [accurate solution involves taking all possible paths into consideration].

[2] Assuming no leakage in the magnetic flux [i.e it is assumed that the magnetic flux flows in the core only, it doesn't leak out from the core to the surrounding air.

[3] Assuming a linear relationship between B and H ($B = \mu H$)

- H = flux intensity, which represents the ability of an electric current to produce a magnetic field [$\oint H \cdot dl = NI$ ⇒ Notice I produces H, if $I = 0 \Rightarrow H = 0$)
- B = flux density, which measures how much the material is magnetized
- Recall $\phi = \frac{NI}{R}$ was derived from Amperés law + "$B = \mu H$" → this assumption may, sometimes, lead to wrong answers, why? ⇒

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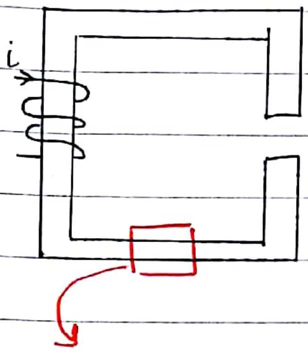
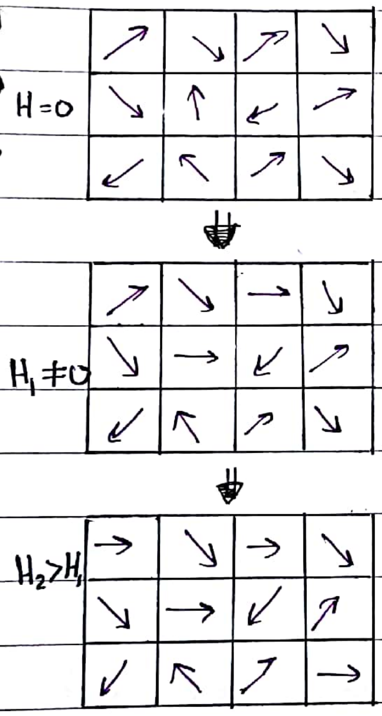


- Every magnetic material contains magnetic domains that have random orientations [You can think of magnetic domains as mini magnets within the material].
- When $H = \text{zero}$ (i.e $i = \text{zero}$) $\Rightarrow$ Ideally, $B = \text{zero}$

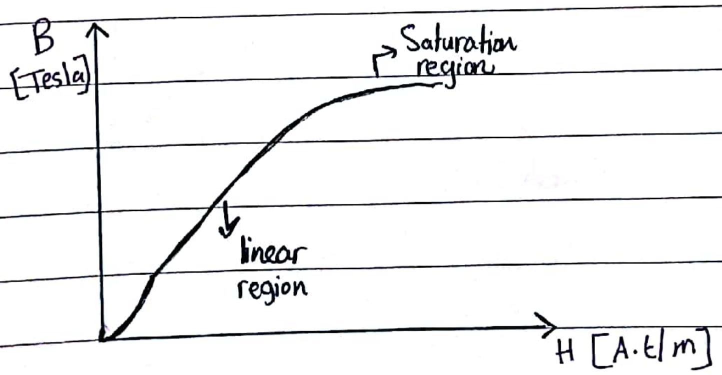
In reality, a little number of magnetic domains are aligned causing B to be nonzero.

بس بتكون قيمة B صغيرة كثير



- When $H_1 \neq 0$ (i.e $i_1 \neq 0$), Some magnetic domains become aligned $\therefore B_1 \neq 0$
- When $H_2 > H_1$ (i.e $i_2 > i_1$), more magnetic domains become aligned (i.e $B_2 > B_1$)
- As $H \uparrow \Rightarrow B \uparrow$
- We can continue increasing H by increasing i [We can control the value of H].
- However, we can't control the value of B, since it depends on the type of the material (Recall B: measures how much the 'material' is magnitized). The value of B can't continue increasing, since every material has a certain capacity.

- Every magnetic material has a B-H curve



- From the curve, we can see that the relation between B and H is not always linear, and hence assuming that the relation is linear (as in the previous analysis) is not always valid.

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• Notice:

$\mu = \frac{\Delta B}{\Delta H}$ = Slope in the B-H curve → In the linear region
$\mu = const \Rightarrow R = \frac{\ell}{\mu A} = const$

→ In the saturation region
$\mu \neq const$, $\mu = f(H) \Rightarrow R = \frac{\ell}{\mu A} \neq const$

• In the Saturation region
when $\Delta B \to 0 \Rightarrow \mu \to 0 \Rightarrow R \to \infty$ (High resistance to magnetic flux

• When we solve problems, we must first determine whether we are in the linear region or saturation region

If $\mu = const$ → We are in the linear region → We can use $\phi = \frac{NI}{R}$

If $\mu \neq const$ → We are in the saturation region → We use Ampere's law ($NI = H\ell$)

This formula is applicable for both linear and saturation regions.

• In the saturation region, all magnetic domains become aligned

• If a material has the following B-H curve

$\mu = \frac{\Delta B}{\Delta H} \neq const \quad \therefore R$ is not const

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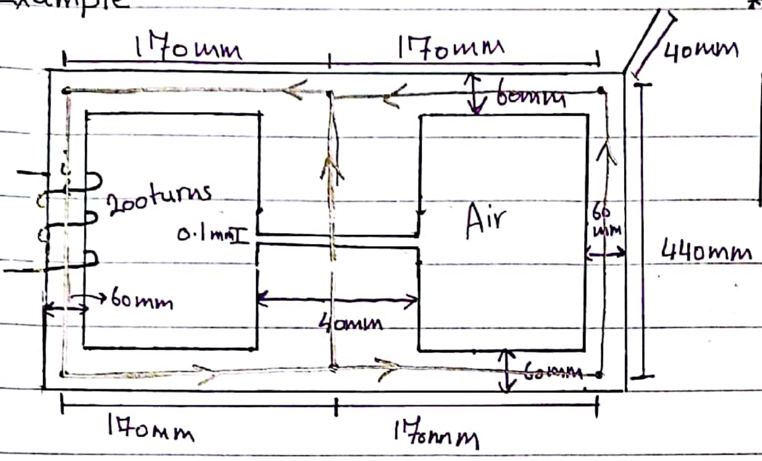
Example

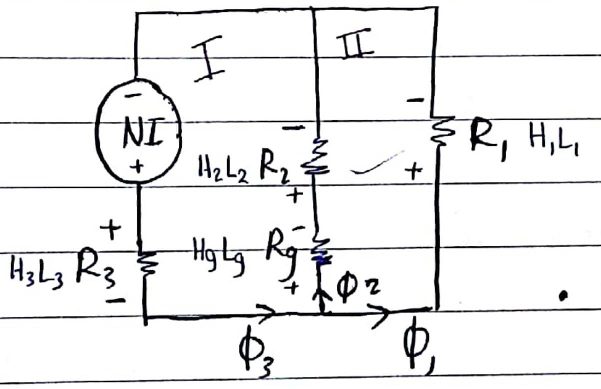


* Some values of B and H for the magnetic material

B	0.39	0.8	0.925
H	363.67	500	562.5

440mm * Find I so that $\Phi_g = 1.28$ mWb

- For the magnetic material $\mu = \frac{\Delta B}{\Delta H} \neq$ const ∴ Use Ampere's law
- Draw the equivalent magnetic circuit



- $\Phi_3 = \Phi_2 + \Phi_1$
- Apply Ampere's law on loop I

$$-NI + H_3L_3 + H_gL_g + H_2L_2 = 0 \quad eq(1)$$

- $\Phi_2 = \Phi_g = 1.28$ mWb

$$B_2 = \frac{\Phi_2}{A_2} = \frac{1.28 * 10^{-3}}{40*40*10^{-6}} \doteq 0.8\,T$$

$$B_g \doteq B_2 = 0.8T \quad (\text{No fringing})$$

- $H_2 = 500$ A.t/m (from the above table)
- $H_g = \frac{B_g}{\mu_0} = \frac{0.8}{4\pi * 10^{-7}} = 63.66*10^4$ (μ_0 is always const)

- Apply Ampere's law on loop II

$$-H_2L_2 - H_gL_g + H_1L_1 = 0 \Rightarrow L_2 = 440 - 0.1 = 439.9 \text{ mm}$$

$$L_g = 0.1 \text{ mm}$$

$$L_1 = 170 + 440 + 170 = 780 \text{mm}$$

⇒

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$$-500*439.9-63.66*10^4*0.1+H_1*780=0 \Rightarrow H_1=363.60\ At/m$$

$B_1=0.39\ T$ (check the table)

$$\phi_1=B_1*A_1=0.39*60*40*10^{-6}=9.36\times10^{-4}\ Wb=0.936\ mWb$$

$$\phi_3=\phi_1+\phi_2=0.936+1.28=2.216\ mWb$$

$$\downarrow$$

$$B_3=\frac{\phi_3}{A_3}=\frac{2.216*10^{-3}}{60*40*10^{-6}}=0.923\ T \Rightarrow H_3=562.5\ A.t/m$$

Substitute in eq (1)

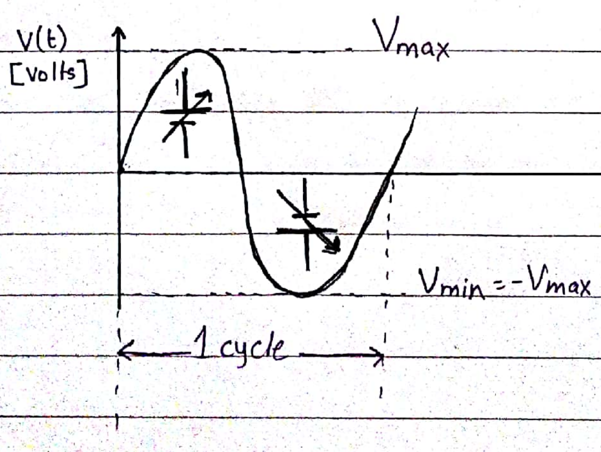
$$-NI+H_2L_2+H_gL_g+H_3L_3=0 \Rightarrow L_3=170+170+440=780\ mm$$

$$-200*I+500*439.9*10^{-3}+63.66*10^4*0.1*10^{-3}+562.5*780*10^{-3}=0$$

$$I=3.61\ A$$

⊛ AC circuits

- Previously, we were dealing with DC magnetic circuits [i.e the magnitude of current flowing through the coils is constant]. Now, we will consider AC magnetic circuits.
- Recall:



→ In AC circuits, voltage can be described by a sinusoidal wave signal.

→ Frequency f: represents the number of cycles per unit time [1 cycle is shown in the adjacent figure]

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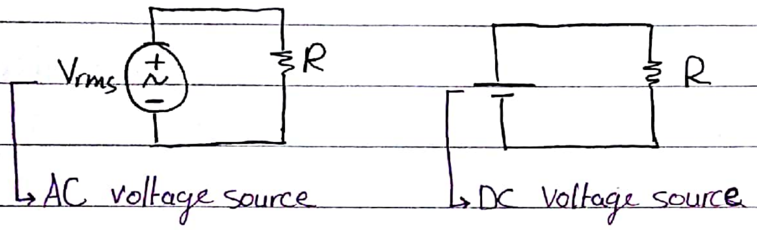
→ In Jordan, household circuits have a frequency of 50 Hz [i.e the cycle will be repeated 50 times in 1 second].

→ Period = $T = \frac{1}{f} = \frac{1}{50} = 0.02$ sec = $\underbrace{20 \text{ ms}}$

20 ms is required to complete 1 cycle

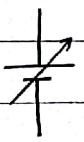
→ Root mean square voltage = $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

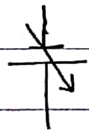
V_{rms} represents the equivalent DC voltage which gives the same effect.



For example, a resistance connected to a 5 volts rms AC supply will dissipate the same power when connected to a steady 5 volts DC supply.

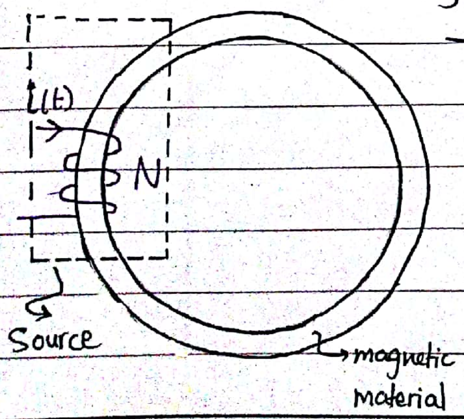
→ In the previous page, you can see 2 symbols in the figure:


* The diagonal arrow represents variable voltage.

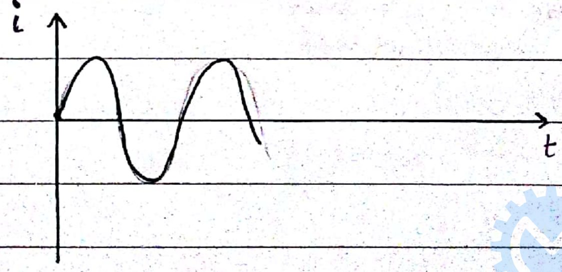

* This symbol is used to show that the polarity of the voltage source is reversed.

⊛ No load losses

→ Consider the following AC magnetic circuit:



→ In this circuit, i can be described by a sinusoidal signal wave.



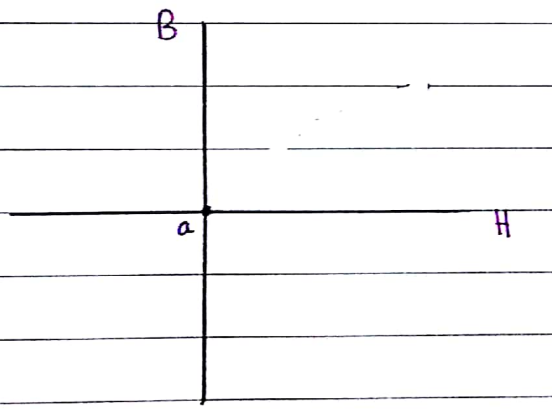
→ In every AC magnetic circuit, some power will be lost, even if there is no load. This type of losses is called "No load losses".

* شاحن الموبايل بيحتوي على magnetic circuit . لو حطيت الشاحن بالكهرباء، من غير ما أشبك فيه الموبايل ك الشاحن لو تركته رح يحمى ويصير ساخن، وهاد بيدل على إنه في losses مع إنه ما في load (اللي هو الموبايل).

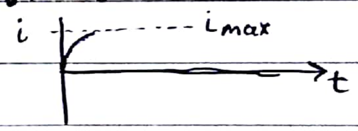
→ No load losses → losses measured under no load conditions
→ Consist of 2 types of losses → Hysteresis losses
→ Eddy current losses

[1] Hysteresis losses

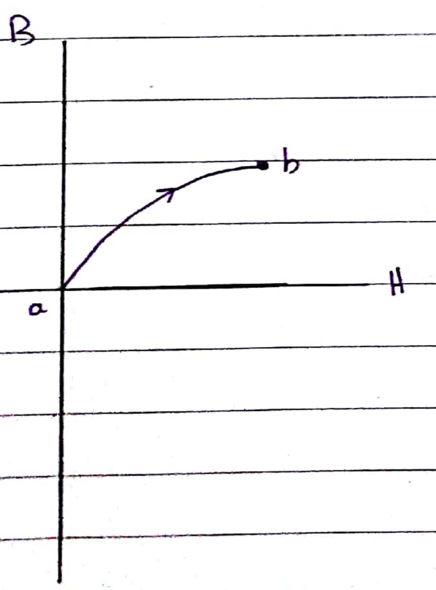
The B-H curve for the AC magnetic circuit is generated as follows:



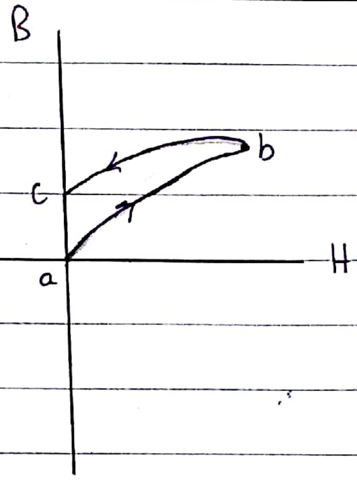
- Initially $i=0 \Rightarrow H=0 \Rightarrow B=0$ (The magnetic material is unmagnetized) غير ممغنط [Point a]
- As $i \uparrow$



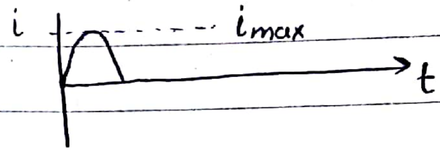
$H \uparrow \Rightarrow$ Some magnetic domains start to align themselves (i.e $B \uparrow$)



- When i reaches its maximum value $\Rightarrow H$ is maximum $\Rightarrow$ Almost all of the magnetic domains are aligned (B is maximum). [Point b]. The magnetic material may or may not reach Saturation. This depends on → The type of the magnetic material
→ The value of i_{max} (Sometimes the value of i_{max} is not sufficient to allow the material to reach Saturation)

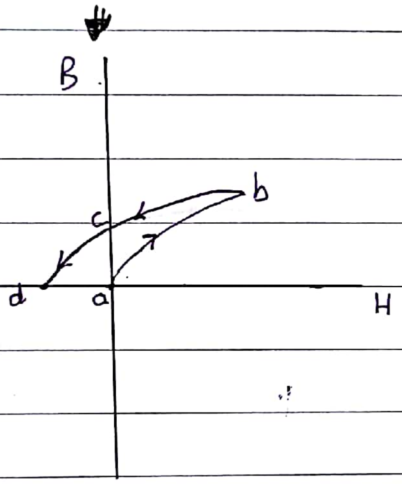


- Now, H will be reduced to zero (since i is reduced to zero)

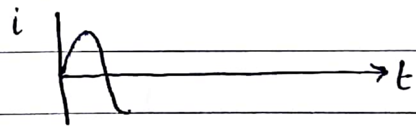


The curve doesn't follow its original path from point b to point a. Instead, it follows the path bc. Why? The magnetic domains within the material try to maintain their alignment [The magnetic material resists losing its magnetization]. So, at point C, some of the magnetic domains remain aligned but some have lost their alignment [i.e $B\downarrow$, but it is still non-zero].

At point C $\Rightarrow$ H = zero, B = non zero

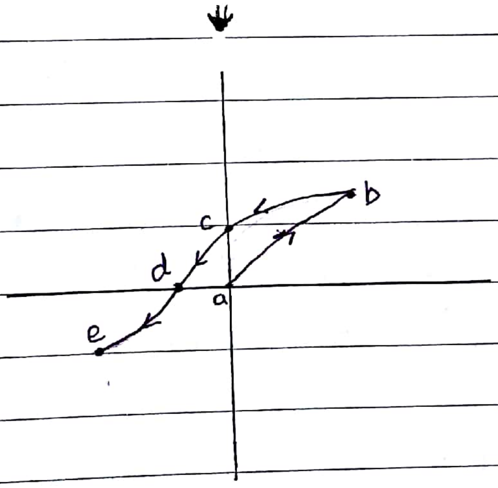


- When i is reversed



H becomes negative, the curve moves to point d. At this point, all magnetic domains have lost their alignment (B=0)

↳ The magnetic material returns unmagnetized.



- As $i\uparrow$ in the negative direction, H also $\uparrow$ in the negative direction $\Rightarrow$ magnetic domains start to align themselves in the opposite direction
- When $i = -i_{max} \Rightarrow$ H is max. $\Rightarrow$ B is max. [Point e]

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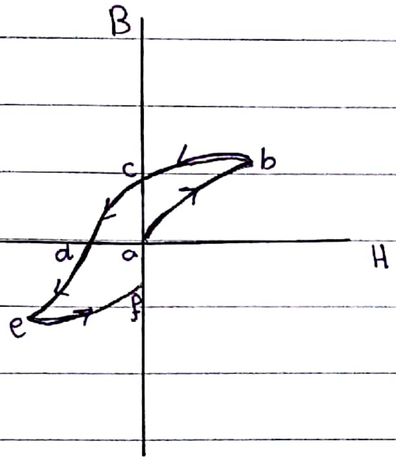
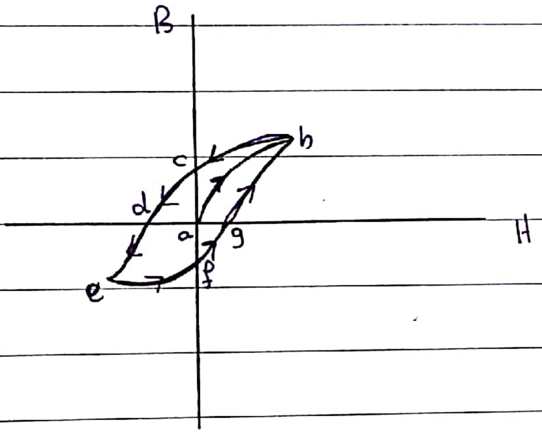


- H will be reduced to zero. The magnetic material resists to lose its magnetization. At point f ⇒ H = zero, B = nonzero
- Increasing H back in the positive direction will return B to zero (Point g). As H↑ ⇒ B↑ until it returns to point b.
- The loop bcdefgb is called hysteresis loop.
- Assuming that this magnetic circuit has a frequency of 50 Hz, this loop will be repeated every 20 mS
- Points C and f can be designated by B_r and $-B_r$ respectively. B_r ≡ residual flux density = the magnetic flux density that remains in a material when the magnetic flux intensity H = zero



- points g and d can be designated by H_c and $-H_c$ respectively.
  H_c ≡ Coercive force. The amount of magnetic field intensity that must be applied to the magnetic material to make it unmagnetized (B=0)
- Hysteresis losses represent the work done to align the magnetic domains in a certain direction and then re-align them in the opposite direction.
- The area enclosed by this loop has units of

$$[B] * [H]$$

$$\text{Tesla} * \frac{A.\text{turns}}{m}$$

$$\frac{N}{A.m} * \frac{A.\cancel{\text{turn}}}{m} \quad \rightarrow \text{assume } N = 1 \text{ turn}$$

$$\frac{N}{m^2} = \frac{J}{m^3} = \text{Energy density}$$

Recall: • $F = BIL$

$$B = \frac{F}{IL}, [B] = \frac{N}{A.m} = \text{Tesla}$$

• Work = F*d

$$F = \frac{\text{Work}}{d}, [F] = \frac{J}{m} = N$$

→ Energy density = Energy per unit volume of the core)

→ Total energy per 1 loop = Area enclosed by the loop * Volume of the core

→ Power (hysteresis losses) = Area enclosed by the loop * Volume of the core

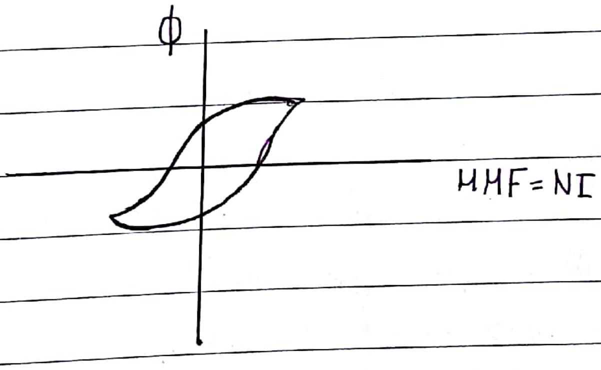
$* \frac{1}{T}$ or f

→ T = period = time required to complete 1 loop

→ It is difficult to find the area enclosed by the loop. Thus, we will use the following formula to estimate hysteresis losses.

P_h = hysteresis losses = $K_h B_{max}^n f$, where
- K_h = constant
- n = exponent, $1.5 < n < 2.5$
- B_{max} = B at point b in the B-H curve
- f = frequency.

→ The same loop is obtained if ϕ is plotted against MMF



→ The area enclosed by this loop has units of

$[\phi] \cdot [MMF]$

Wb $\cdot$ turns * A

$\frac{V \cdot S}{\cancel{turns}} \cdot \cancel{turns} * A$

$A \cdot V \cdot S$

Watts $\cdot$ s = Joule = Energy

Recall: Faraday's law:

$emf = N \frac{d\phi}{dt}$

$[\phi] = \frac{Volts \cdot sec}{turns} = \frac{V \cdot S}{turns}$

$= Wb$

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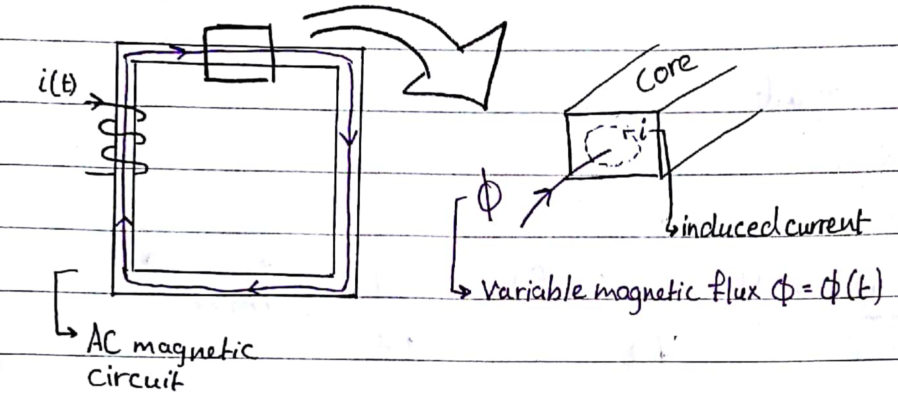
[2] Eddy Current losses



- Eddy currents are loops of electrical current induced within the core due to Changes in magnetic flux flowing through the core. This These eddy currents create magnetic flux that opposes the change in magnetic flux that created it.

- Eddy current losses is given by

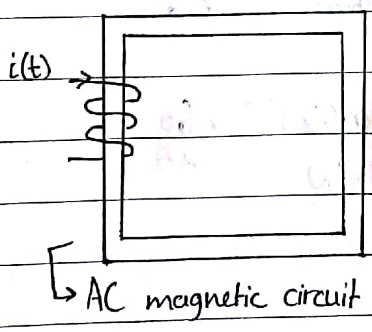
$$P_e = K_e B_{max}^2 f^2 \quad , \text{Where } K_e = \text{const}$$

B_{max} = B at point b in the B-H curve

f = Frequency

- No load losses = Core losses = $P_{core} = P_e + P_h$

Example: if $V_{rms} = 220$ volts, $f = 50\ Hz$, find $\phi(t)$



→ $V = V_{max} \sin(\omega t)$, $V_{max} = \sqrt{2}\ V_{rms}$

$V(t) = \sqrt{2} * 220 \sin(2\pi * 50 t)$, $\omega = 2\pi f$

→ Using Faraday's law

$V = emf = N \frac{d\phi}{dt}$ → by integration, we can find $\phi(t)$

when $V(t)$ is sinusoidal (as in AC magnetic circuits), $\phi(t)$ is also sinusoidal

كدا الشي متغير، نستنتجه من Faraday's law

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$\rightarrow$ let $\phi = \phi_{max} \sin \omega t$

$$emf = N \frac{d\phi}{dt} = \underbrace{N \phi_{max} \omega}_{V_{max}} \cos\omega t$$

$$V_{rms} = \frac{N \phi_{max} \omega}{\sqrt{2}} = \frac{N \phi_{max} (2\pi f)}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}} N \phi_{max} f$$

$\boxed{V_{rms} = 4.44 N \phi_{max} f}$ (Applicable only in AC circuits)

Example

$P_{h_1} = 846$ W, $P_{e_1} = 642$ W @ 240 V, 25 Hz, Find P_{h_2}, P_{e_2} @ 60 Hz, flux density is 62% of its value at 25 Hz, $n = 1.4$

and Find V_2

Solution:

- $P_h = K_h B_{max}^n f \Rightarrow P_{h_1} = K_h B_{max_1}^{1.4} * 25$

$$P_{h_2} = K_h B_{max_2}^{1.4} * 60 = K_h * (0.62 B_{max_1})^{1.4} * 60$$

$$\frac{P_{h_2}}{P_{h_1}} = (0.62)^{1.4} * \frac{60}{25} \Rightarrow P_{h_2} = 846 * (0.62)^{1.4} * \frac{60}{25} = 1039.75 \text{ W}$$

- $P_e = K_e B_{max}^2 f^2 \Rightarrow P_{e_1} = K_e B_{max_1}^2 * (25)^2$

$$P_{e_2} = K_e (0.62 * B_{max_1})^2 * 60^2 \Rightarrow$$

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$$\frac{P_{e_2}}{P_{e_1}} = 0.62^2 * \left(\frac{60}{25}\right)^2 \Rightarrow P_{e_2} = 642 * 0.62^2 * \left(\frac{60}{25}\right)^2 = 1421.48\ W$$

Find V_2? $V_1 = 240$ Volt

"In AC circuits, any given value for the voltage is an rms value"

$$V_{rms} = 4.44 N \phi_{max} f = 4.44 N A B_{max} f$$

$$V_1 = 4.44 NA\ B_{max_1} * 25$$
$$V_2 = 4.44 NA * 0.62 B_{max_2} * 60$$

$$\frac{V_2}{V_1} = 0.62 * \frac{60}{25} \Rightarrow V_2 = 240 * 0.62 * \frac{60}{25} = 357.12\ \text{Volts}$$

→ let $V_2 = 4.44 NA f_2 B_{max_2}$

$V_1 = 4.44 NA f_1 B_{max_1}$

$$\frac{V_2}{V_1} = \frac{f_2}{f_1} \cdot \frac{B_{max_2}}{B_{max_1}} \Rightarrow \boxed{\frac{V_2 / f_2}{V_1 / f_1} = \frac{B_{max_2}}{B_{max_1}}}$$

Example

$P_{core_1} = 500\ W$ @ $240\ V, 25\ Hz$

Find $P_{h_1}, P_{e_1}, P_{h_2}, P_{e_2}$

$P_{core_2} = 1400\ W$ @ $480\ V, 50\ Hz$

Solution

There are 4 unkowns $\Rightarrow$ 4 equations are required to find them

- $P_{core_1} = 500 = P_{h_1} + P_{e_1}$ ---- (1)

  $P_{core_2} = 1400 = P_{h_2} + P_{e_2}$ ---- (2)

- $P_h = K_h B_{max}^n f \longrightarrow \dfrac{P_{h_2}}{P_{h_1}} = \left(\dfrac{B_{max_2}}{B_{max_1}}\right)^n * \dfrac{f_2}{f_1}$

This is the previous derived equation $\leftarrow$ $\dfrac{V_2/f_2}{V_1/f_1} = \dfrac{B_{max_2}}{B_{max_1}} = \dfrac{480/50}{240/25} = 1$

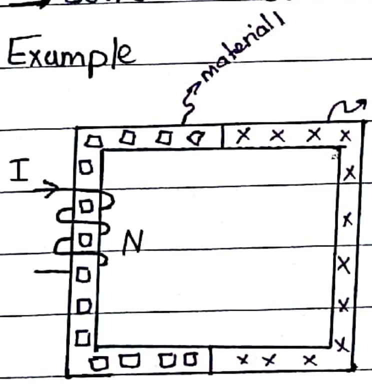
$$\therefore \frac{P_{h_2}}{P_{h_1}} = (1)^n * \frac{50}{25} = 2 \Rightarrow P_{h_2} = 2P_{h_1} \quad \text{--- (3)}$$

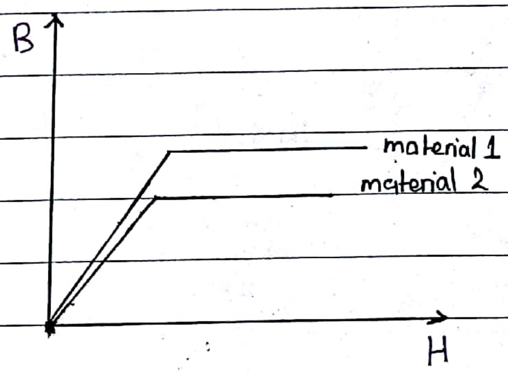
- $P_e = K_e B_{max}^2 f^2 \longrightarrow \dfrac{P_{e_2}}{P_{e_1}} = \underbrace{\left(\dfrac{B_{max_2}}{B_{max_1}}\right)^2}_{1} * \underbrace{\left(\dfrac{f_2}{f_1}\right)^2}_{2}$

$$P_{e_2} = 4 P_{e_1} \quad \text{--- (4)}$$

$\rightarrow$ Solve the above equations.

Example





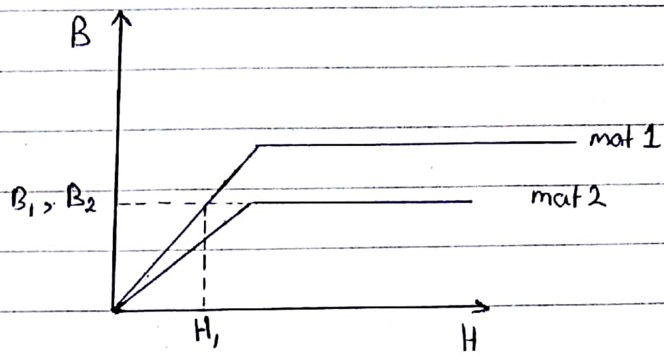
$\rightarrow$ Assuming that:

* Material 2 is Saturated.
* The cross sectional area of the core is const $(A_1 = A_2 = A)$

Find $\phi_1, \phi_2, B_1, B_2, H_1, H_2$

Solution

- Material 2 is saturated
  $\Downarrow$
  We can find B_2 from the B-H curve of material 2

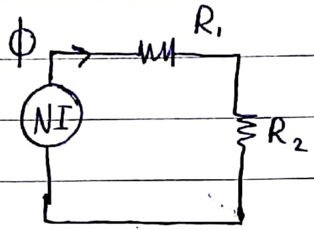


- Find ϕ_2:

$$\phi_2 = B_2 * A$$

- Since the reluctances of the 2 materials are in series

$$\therefore \phi_1 = \phi_2 = \phi$$



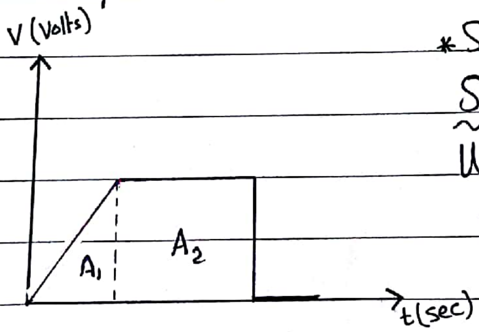
- $A_1 = A_2 = A$

$$\therefore B_1 = \frac{\phi_1}{A_1} = \frac{\phi_2}{A_2} = B_2 \Rightarrow$$ From the B-H curve of material 1, we can find H_1

- Using Ampere's law:

$$H_1 L_1 + H_2 L_2 = NI \Rightarrow \text{Find } H_2$$

Example



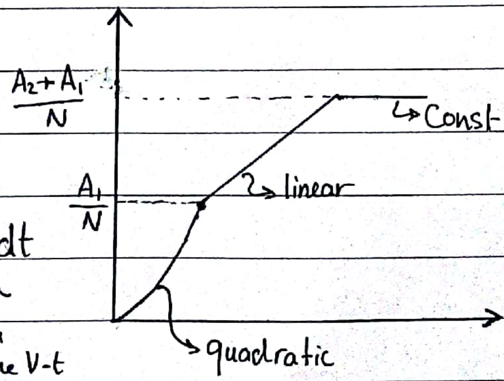
* Sketch $\phi(t)$

Solution:

Using Faraday's law

$$V(t) = N\frac{d\phi}{dt}$$

$$\therefore \phi(t) = \frac{1}{N}\underbrace{\int V(t)\,dt}_{\text{The area under the V-t curve}}$$



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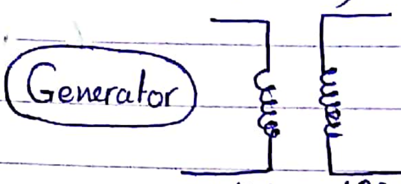
(*) Transformers

→ Transformers are used to step up / step down the voltage V.

E.g:

(Step up)

*



22 KV 132 KV

العقبة عمان

- Transformers are used to increase the voltage, when we want to transmit electrical power over long distances. (Why?) because less power will be lost if high voltages are used.

كيف هاد الاشي بصير؟

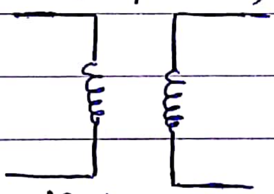
- Assume the power to be transmitted is P, & and the resistance of the transmission wire is R, if the power is transmitted with voltage V, then the current flow through the transmission wire is $I = P/V$.

The power loss $P_{loss} = I^2 R = \left(\frac{P}{V}\right)^2 * R$

less power will be lost if high voltages are used.

(Step down)

*

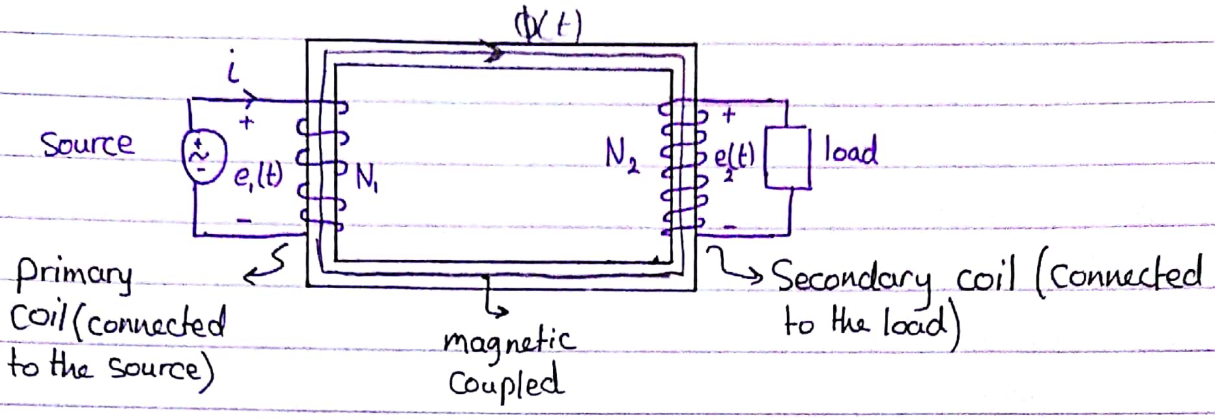


132 KV 33 KV

- Transformers are used to step down the voltage when we want to distribute the transmitted electrical power

لما توصل الكهربا لعمان في محطة
التوزيع، منستخدم محولات
عشان نخفض الفولتية
· 132 KV → 33 KV
وقبل ما نوزع الكهربا عالبيوت
منستخدم كمان محولات
كمان عشان نخفض الفولتية
33 KV → 230 V

(*) Ideal Transformers (Lossless transformers)



→ A Transformer is basically a "pair" of coils wrapped separately around a ferromagnetic core.

→ The basic idea is:

- AC current comes in through the primary coil
- This will create varying magnetic flux in the core ($\phi = \phi(t)$)
- As a result, an AC current in the secondary coil will be created.

→ According to Faraday's law:

- Primary coil : $e_1(t) = N_1 \dfrac{d\phi}{dt} \rightarrow \dfrac{d\phi}{dt} = \dfrac{e_1(t)}{N_1} \rightarrow \phi(t) = \dfrac{1}{N_1} \int e_1(t)\, dt$

- Secondary coil $e_2(t) = N_2 \dfrac{d\phi}{dt}$

* Ideally, The same magnetic flux is passing through the primary coil and the secondary coil

$$\therefore \frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} = \frac{d\phi}{dt}$$

$$\therefore e_2(t) = N_2 * \left(\frac{e_1(t)}{N_1}\right) \rightarrow \frac{e_2(t)}{e_1(t)} = \frac{N_2}{N_1}$$

⇓ In reality $\phi_1 \neq \phi_2$ because; leakage in magnetic flux occurs

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→ Notice that if DC source is used

↓

The generated magnetic flux will be constant (i.e $\phi = const$)

↓

$\frac{d\phi}{dt} = zero$ ⇒ According to Faraday's law $e_2(t) = N_2 * \frac{d\phi}{dt} = 0$

↓

No electromotive force will be induced in the secondary coil, and hence no current will flow through the secondary coils (i.e $i_2 = 0$)

Conclusion: Transformers can only work for AC (not DC)

→ $\frac{e_2(t)}{e_1(t)} = \frac{N_2}{N_1}$ ⇒ $e_2(t) = \frac{N_2}{N_1} e_1(t)$

if $e_1(t) = e_{max} \sin(\omega t) = e_{max} \sin(2\pi f t)$, then:

$e_2(t) = \frac{N_2}{N_1} * e_{max} \sin(2\pi f t)$

- Both voltages $e_1(t)$, $e_2(t)$ have the same frequency

→

Recall: In AC circuits $E_{rms} = 4.44 f \phi N$

- For the primary coil $E_{1\,rms} = 4.44 f_1 \phi_1 N_1$
- For the secondary coil $E_{2\,rms} = 4.44 f_2 \phi_2 N_2$

$f_1 = f_2 = f$

$\phi_1 = \phi_2 = \phi$

$$\frac{E_{1\,rms}}{E_{2\,rms}} = \frac{N_1}{N_2}$$

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Example: If $E_{1rms} = 33KV$, $E_{2rms} = 11\ KV$, Find $\frac{N_1}{N_2}$

step down

$$\frac{E_{1rms}}{E_{2rms}} = \frac{N_1}{N_2} \Rightarrow \frac{N_1}{N_2} = \frac{33}{11} = \frac{3}{1}$$ ⇒ Secondary coil كل لفة بال
Primary coil بيقابلها 3 لفات بال

[*] Current relations:

→ Since we are dealing with losseless transformers, then:

$P_1 = P_2$

$Q_1 = Q_2$

$S_1 = S_2$

→ Recall:

P = average power. It has units of Watts (W)

Q = reactive power. It has units of Volt-amperes reactive (VAR)

S = Complex power. It has units of Volt-amperes (VA)

→

$$S = V * I^* \quad , \quad |S| = |V| * |I|$$

$$|S_1| = |S_2|$$

$$|I_1||V_1| = |I_2||V_2| \Rightarrow \frac{|V_1|}{|V_2|} = \frac{|I_2|}{|I_1|} = \frac{N_1}{N_2}$$

* Note: these are rms values

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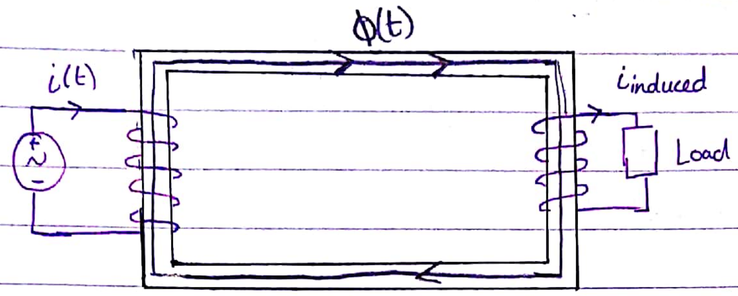
[*] How to determine the direction of induced current in the Secondary coil?

[1]



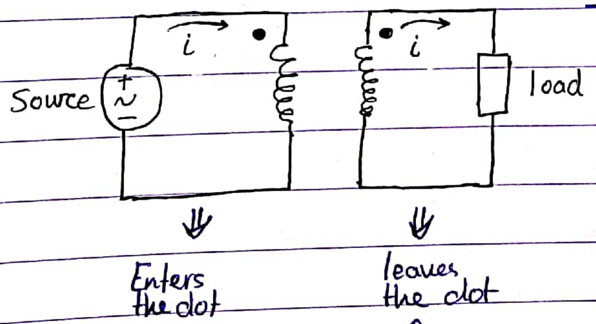
→ The induced current in the secondary coil will have a direction, such that it will produce a magnetic flux that opposes the change in the original magnetic flux.

→ E.g: Assume that the magnetic flux produced by the primary coil has the direction shown in the above figure + This magnetic flux is increasing

⇓

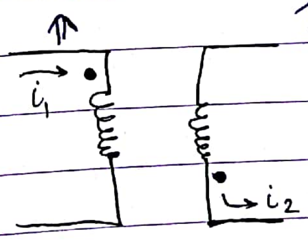
The induced current in the secondary coil will have a direction, Such that it will produce a magnetic flux that opposes the "increase" in the original magnetic flux [i.e it will have the direction shown in the above figure]

[2] Dot Convention



→ Transformers use the dot convention to tell the direction of current on the secondary side of the transformer. The relationship is as follows:

"If the primary current flows into the dot, the secondary current will flow out of the dot"



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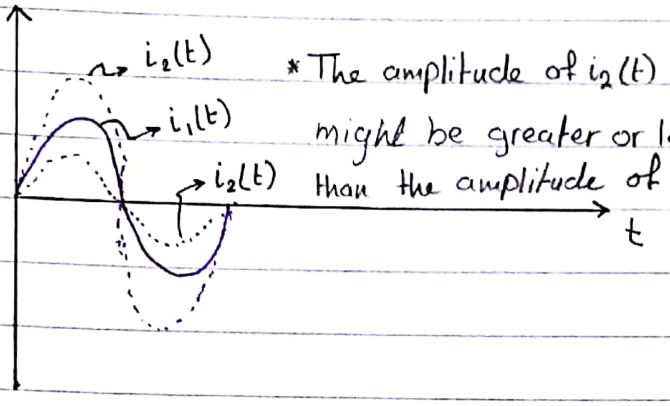
→ In single phase ideal transformers, $i_1(t)$ and $i_2(t)$ are in phase



* The amplitude of $i_2(t)$ might be greater or less than the amplitude of $i_1(t)$

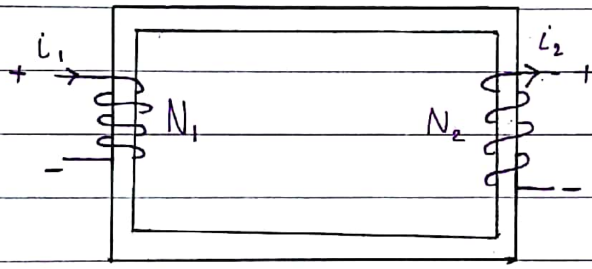
$$\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1}$$

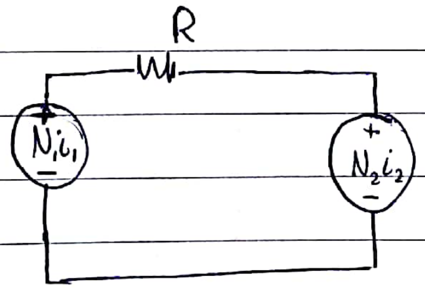
if $i_1(t) = I\sin(\omega t + \Theta)$

then $i_2(t) = \frac{N_1}{N_2} * \left(I\sin(\omega t + \Theta)\right)$

↳ both $i_1(t)$, $i_2(t)$ have the same Phase angle Θ

Question: Show that $\frac{i_2}{i_1} = \frac{N_1}{N_2}$ using magnetic circuit method





→ Since this is an ideal transformer

$\mu \to \infty \Rightarrow R \to 0$

$$N_1 i_1 - N_2 i_2 = 0 \Rightarrow \frac{N_1}{N_2} = \frac{i_2}{i_1}$$

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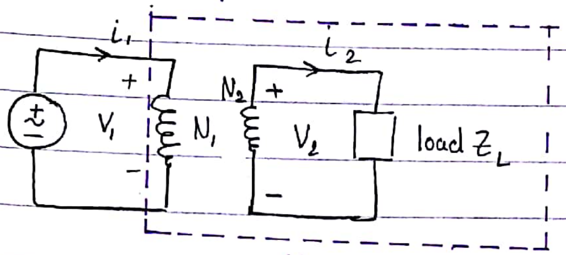
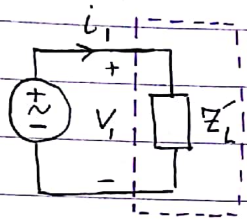
[*] Reflected impedence



⇓ Replaced with

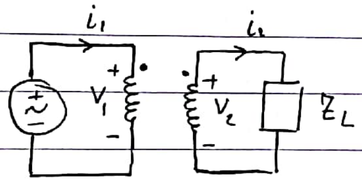


Reflected impedence →

$$Z_L' = \frac{V_1}{I_1} = \frac{V_2\left(\frac{N_1}{N_2}\right)}{I_2\left(\frac{N_2}{N_1}\right)}$$

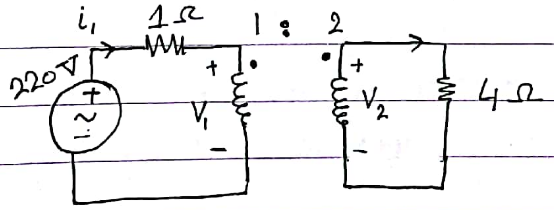
$$= \frac{V_2}{I_2}\left(\frac{N_1}{N_2}\right)^2 = Z_L\left(\frac{N_1}{N_2}\right)^2$$

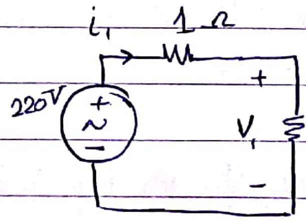
*Summary



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \quad \frac{i_1}{i_2} = \frac{N_2}{N_1}, \quad Z_L' = Z_L\left(\frac{N_1}{N_2}\right)^2$$

Example: Find i_1, i_2, V_1, V_2





$$4 * \left(\frac{1}{2}\right)^2 = 1\ \Omega$$

$$i_1 = \frac{220}{1+1} = 110\ A, \quad V_1 = 1 * 110 = 110\ V$$

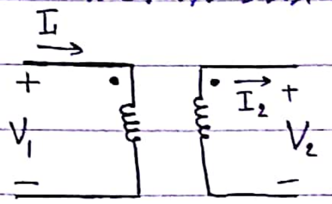
step up

Increase in Voltage at the expense of current ←

$$i_2 = 110 * \left(\frac{1}{2}\right) = 55\ A, \quad V_2 = 110 * \left(\frac{2}{1}\right) = 220\ V$$

notice that $i_1 * V_1 = i_2 * V_2$ (lossless transformer)

→ When you deal with any transformer, you must know the following specifications
120KVA , 220V/110V



$\boxed{1}$ 120KVA ≡ Apparent power rating (or complex power rating) $|S|$.
- The word "rating" means that this is the highest power allowed to be transformed in this transformer
- $|S| = \sqrt{P^2+Q^2}$; P is the average power , Q is the reactive power

P = the amount of power that is actually consumed in the circuit. (useful power)
Q = the amount of power that flows back and forth between the inductor (or capacitor) and the source (unuseful power)

* If an AC circuit contains an inductor, this inductor will store a certain amount of power, then it will discharge it back to the source. This amount of unused power is called reactive power. It can't be used or consumed

$\boxed{2}$ 220 V (This is an rms value) ≡ Rating of the HV (High Voltage) side
- The word "rating" means that this is the highest voltage allowed on the HV side. (can't be exceeded)

$\boxed{3}$ 110 V (This is an rms value) ≡ Rating of the LV (Low Voltage) side.

$$|S| = |V||I| \Rightarrow I_{rating,HV} = \frac{120*10^3}{220} = 545.4\ A$$

$$I_{rating,LV} = \frac{120*10^3}{110} = 1090.9A$$

These values can't be exceeded, otherwise, the transformer becomes damaged.

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(*) Practical Transformers

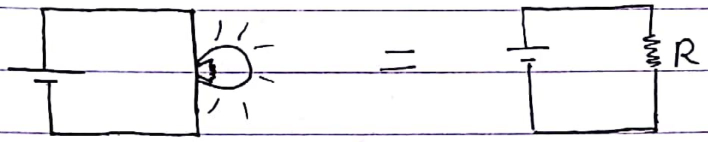
→ Practical transformer = ideal transformer + losses

→ We will look for an "electrical model" to represent the practical transformer

- What does "electrical model" mean?

To represent an electrical device by electrical components such as resistors, inductors or capacitors.

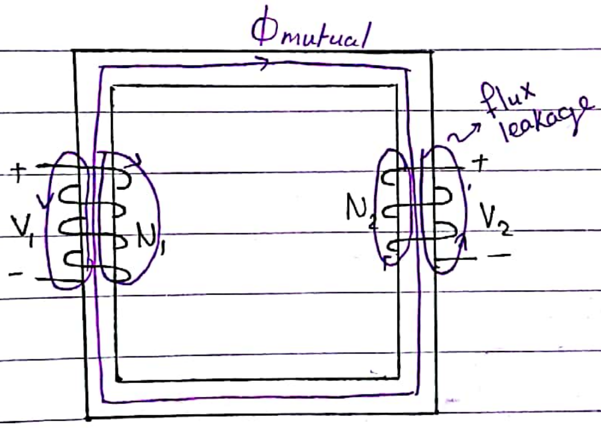
E.g



- A resistor is a good electrical model for a light bulb.

→ This electrical model is needed in order to analyse practical transformers

→ In order to build up the electrical model, we first need to determine the losses that exist in practical transformers



[1] Copper losses: the 2 coils are made of copper. This material has a resistance and it dissipates power. This type of losses is called Copper losses. Resistors are good electrical models for these losses.

[2] Flux leakage: in ideal transformers, it is assumed that μ of the core $\to \infty$

$\therefore R \to 0$ $\therefore$ No flux leakage

However, in practical transformers, μ of the core = finite

$\therefore R \neq 0 \Rightarrow$ There will be flux leakage.

Inductors are good electrical models to represent flux leakage

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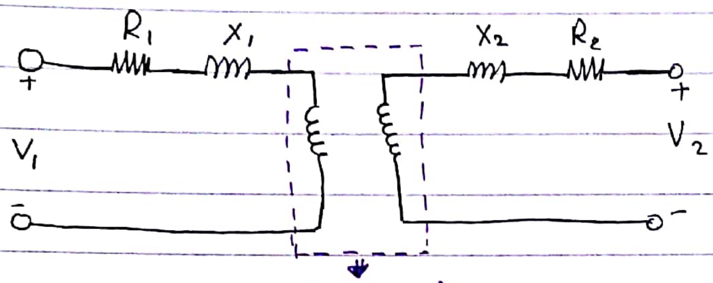
[3] Core losses = Hysteresis losses + Eddy current losses. Core losses appear in the form of heat (The core gets hot due to core losses). Resistors are good electrical models to represent ~~ ~~ Core losses.

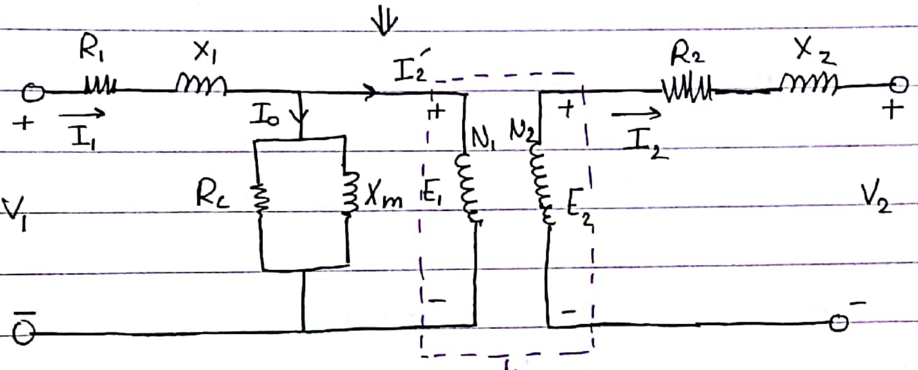


ideal transformer

- $R_1, R_2 \Rightarrow$ Electrical models for Copper losses
- $X_1, X_2 \Rightarrow$ Electrical models for flux leakage

if $\phi_{leakage} = 0$, $X_1 = X_2 = 0$

$\Downarrow$



ideal transformer

- $R_c \Rightarrow$ Electrical model for Core losses
- $X_m \Rightarrow$ Electrical model to represent mutual flux

Practical transformer

→ Notice that practical transformer = ideal transformer + losses

→ $\frac{N_1}{N_2} \neq \frac{V_1}{V_2}$ but $\frac{N_1}{N_2} = \frac{E_1}{E_2}$

→ $\frac{N_1}{N_2} \neq \frac{I_2}{I_1}$ but $\frac{N_1}{N_2} = \frac{I_2}{I_2'}$

→ I_o is called excitation current

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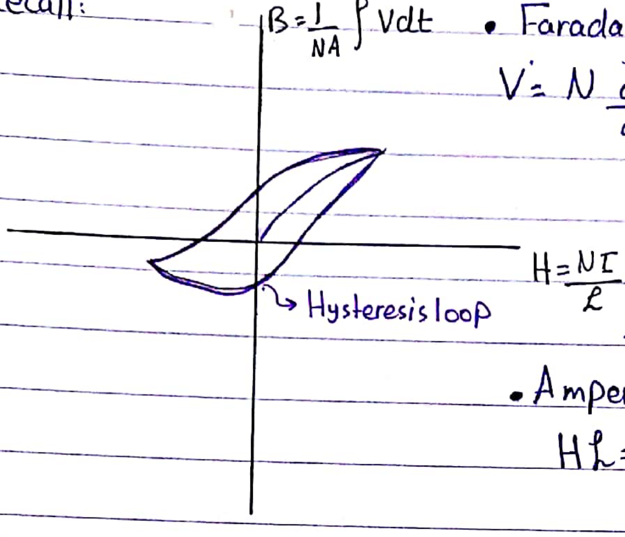
→ Why are R_c, X_m in parallel (not in series)?

Recall:



- Faraday's law

$$V = N\frac{d\phi}{dt} \Rightarrow \phi(t) = \frac{1}{N}\int V dt$$
$$\Downarrow$$
$$B = \frac{\phi}{A} = \frac{1}{NA}\int V dt$$

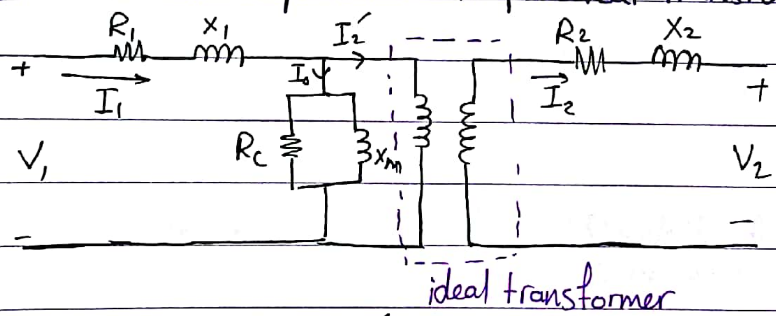
$\therefore$ B is related to $\int V$

- Ampere's law

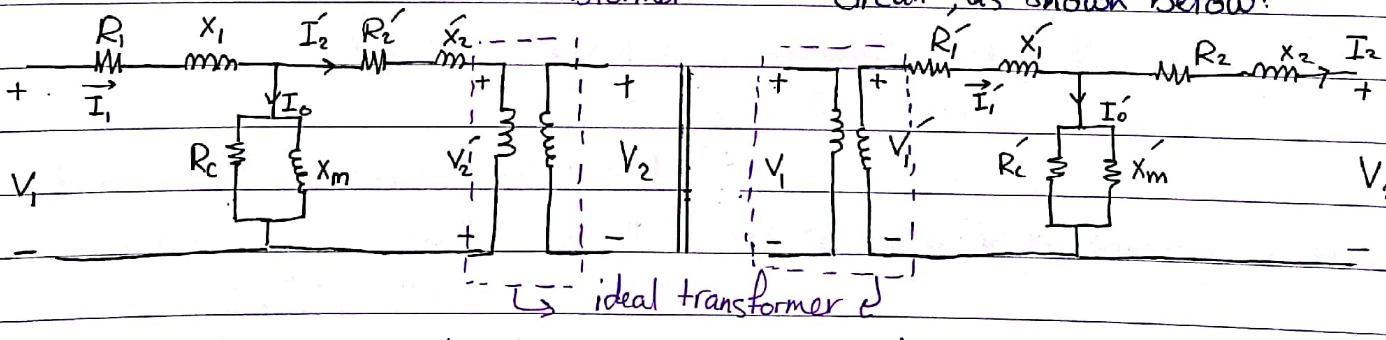
$H\ell = Ni \quad \therefore$ H is related to i

- R_c, X_m are in parallel to ensure that the relation between the current flowing through R_c "i" and the integration of the voltage across R_c "$\int V$" will give the hysteresis loop if B is plotted against H

[*] Reflected impedance in practical transformers



⇒ In the practical transformer equivalent circuit, the ideal transformer can be moved out to the right or to the left of the equivalent circuit, as shown below:



→ Notice that all parameters (R_1, X_1, R_2, X_2, R_c, X_m) in the above 2 circuits are either on the HV side or LV side

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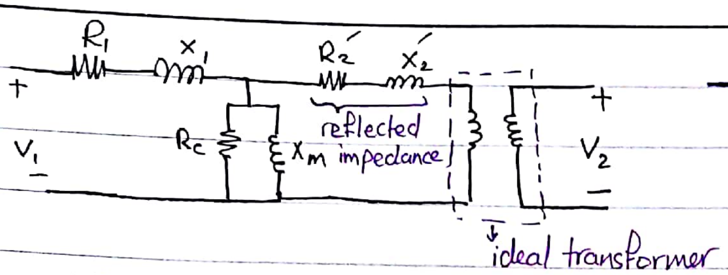
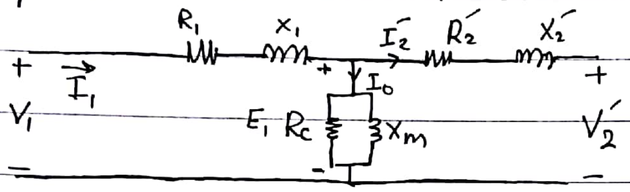


→ In this circuit, the ideal transformer is moved out to the right

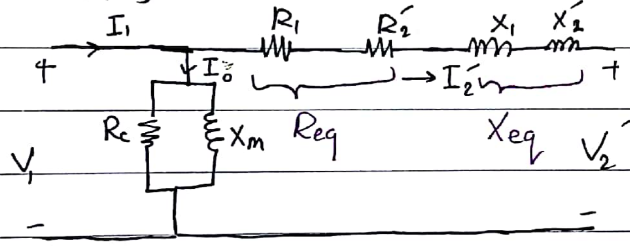
→ For convenience, the ideal transformer is usually not shown and the equivalent circuit is drawn as shown below:



⇒ "Exact equivalent circuit"

→ We can do approximations in the above circuit

* The voltage drops I_1R_1 and I_1X_1 are normally small and hence $E_1 \approx V_1$.
As a result, the branch (composed of R_c and X_m) can be moved to the supply terminal as shown below

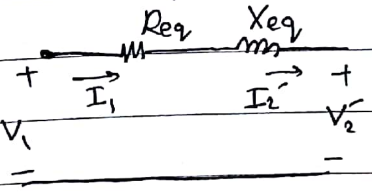


⇒ "Approximate equivalent circuit"

* Since $R_c \gg R_1$, $R_c \gg R_2$ (Recall R_1, R_2 represents wire resistances which are very small)
and
$X_m \gg X_1$, $X_m \gg X_2$ (Recall X_1, X_2 represents leakage flux. In a good design $\Phi_{mutual} \gg \Phi_{leakage}$)

then I_0 is very small and $I_1 \approx I_2'$

∴ A further approximation of the circuit can be made by removing the branch



⇒ "Approximate equivalent circuit"

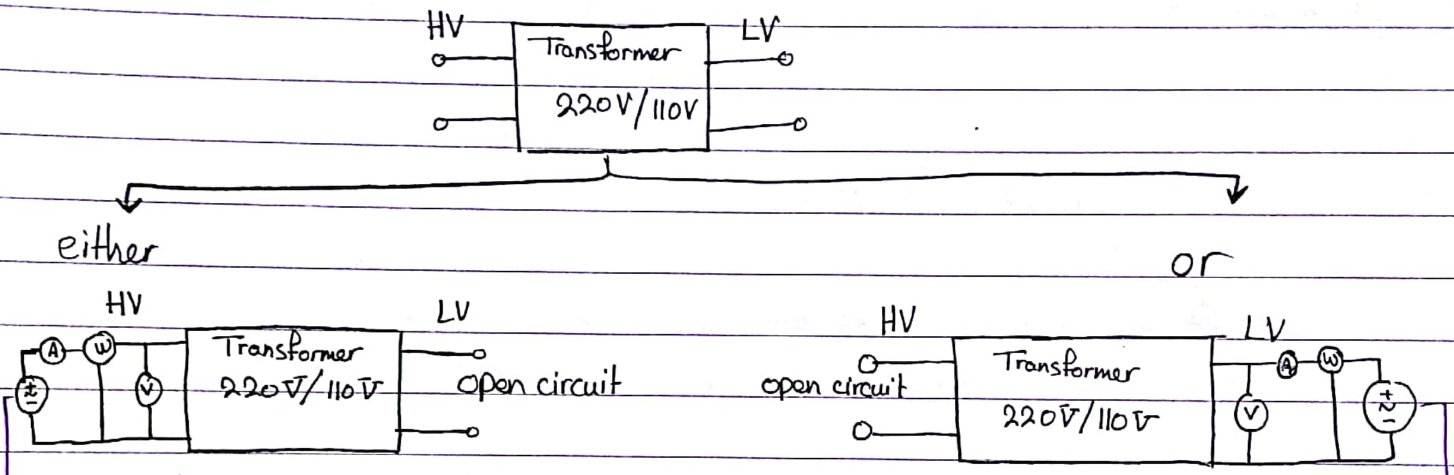
→ Two tests can be performed to a practical transformer to determine the Values of X_m, R_c, R_{eq}, X_{eq}:

[1] Open Circuit Test (OCT) ⇒ used to find X_m, R_c

[2] Short circuit Test (SCT) ⇒ used to find R_{eq}, X_{eq}

⊛ Open circuit Test (OCT)

→ The OCT can be performed in 2 ways:



↳ AC source ($V = 220$ volt = Rated voltage)

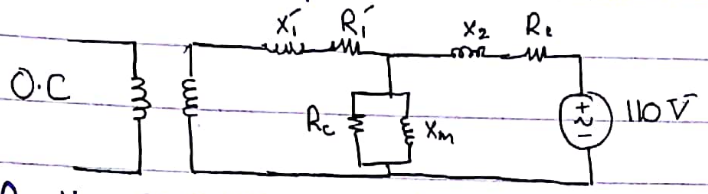
AC source ($V = 110$ V = Rated Voltage)

* An AC source is connected to the HV side, while the LV side is left with no load (open circuit)
* V_{rms} of the AC source must be equal to the rated voltage on the HV side
* A = ammeter: measures i
* V ≡ Voltmeter: measures V
* W = Wattmeter: measures the average power (or active power) P. It doesn't measure the complex power |S|.

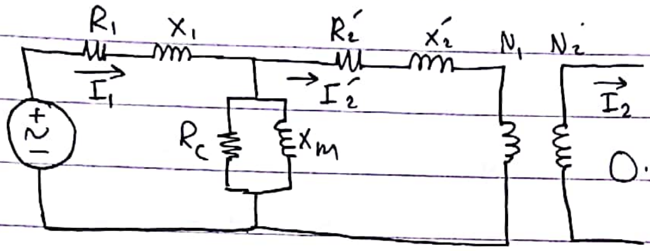
$$|S| = \sqrt{P^2 + Q^2} = |I||V|$$

* An AC source is connected to the LV side, while the HV side is left with no load (open circuit)
* V_{rms} of the AC source must be equal to the rated Voltage on the LV side.
* It is preferred to connect the AC source to the LV side, since for LV side, rated voltage required will be less compared to High voltage side.

→ If the O.C.T is performed at the LV side, the equivalent circuit is shown below:



→ If the O.C.T is performed at the HV side, the equivalent circuit is shown below

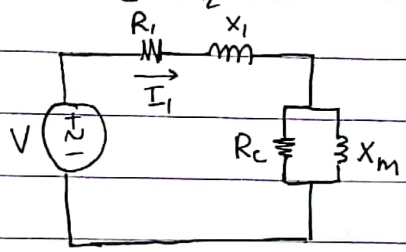


$\Rightarrow I_2 = 0$ (open circuit)

$\dfrac{I_2'}{I_2} = \dfrac{N_2}{N_1} \Rightarrow I_2' = 0$ (since $I_2 = 0$)

⇓

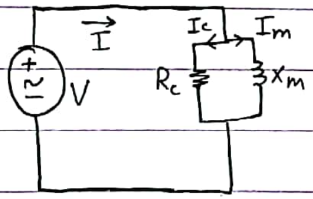
since $I_2 = I_2' = 0 \Rightarrow$ the circuit is simplified as follows:



$\Rightarrow R_1, X_1$ are small compared to R_c, X_m

$\therefore R_1, X_1$ are neglected

⇓



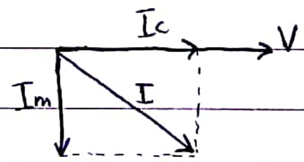
$\Rightarrow V, I, P$ are measured by the voltmeter, ammeter and the wattmeter

$\Rightarrow$ Recall: the OCT is used to find R_c, X_m

$P = \dfrac{V^2}{R_c}$? $\Rightarrow R_c = \dfrac{V^2}{P}$

$I_c = \dfrac{V}{R_c} \Rightarrow I_m = \sqrt{I^2 - I_c^2} \Rightarrow X_m = \dfrac{V}{I_m}$

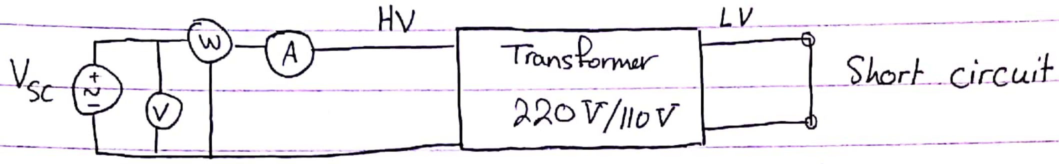
Note: $I \neq I_c + I_m$ (since I_c, I_m are not in phase)



$\Rightarrow$ Phasor diagram showing I_c, I_m, I and V

⊛ Short circuit Test (SCT)

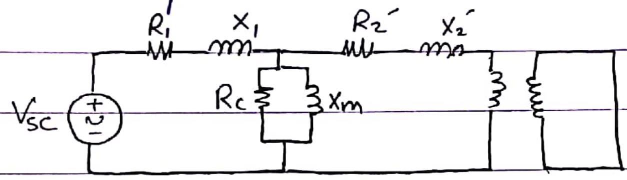
→ SCT is either performed at the HV side or LV side.



→ In the above transformer, the SCT is performed at the HV side. An AC source is connected to the HV side, while the LV side is short circuited.

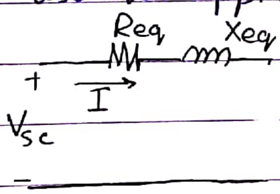
→ V_{rms} of the AC source (called V_{sc} ≡ short circuit voltage) is increased gradually until the ammeter's reading reaches $I_{rated,HV}$ ($V_{sc} \ll 220$ V (Rated Voltage at the HV side))

→ The equivalent circuit for the above transformer



⇓

Use the approximate equivalent circuit model



⇒ I, V_{sc}, P are measured by the ammeter, Voltmeter and the Wattmeter

⇒ Recall: SCT is used to find R_{eq} X_{eq}

$$P = I^2 R_{eq} \Rightarrow R_{eq} = \frac{P}{I^2}$$

$$Z = \frac{V_{sc}}{I} \Rightarrow X_{eq} = \sqrt{Z^2 - R_{eq}^2}$$

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Example: A Transformer with the following specifications: 20KVA, 8000V/240V, 60 Hz. OCT and SCT are performed on this transformer: Find $R_{c,HV}$, $X_{m,HV}$, $R_{eq,HV}$, $X_{eq,HV}$

	OCT	SCT
V (V)	8000	489
I (A)	0.214	2.5
P (W)	400	240

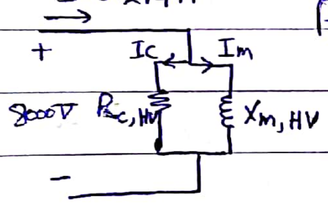
Solution:

* OCT is performed at the HV side since $V = 8000\,V$ = Rated voltage at the HV side
* SCT: $I_{rated,HV} = \dfrac{|S|}{|V|} = \dfrac{20*10^3}{8000} = 2.5\,A$

$I_{rated,LV} = \dfrac{|S|}{|V|} = \dfrac{20*10^3}{240} = 83.33\,A$

$\therefore$ SCT is performed at the HV side, since $I = I_{rated,HV} = 2.5A$

* OCT



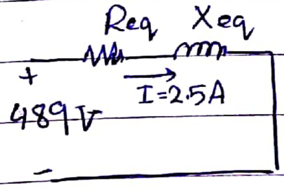
$P = 400\,W$ (consumed by R_c "core losses")

$P = \dfrac{V^2}{R_{c,HV}} \Rightarrow R_{c,HV} = \dfrac{8000^2}{400} = 160.\ K\Omega$

$I_c = \dfrac{V}{R_{c,HV}} = \dfrac{8000}{160*10^3} = 0.05A \Rightarrow I_m = \sqrt{0.214^2 - 0.05^2} = 0.208\,A$

$X_{m,HV} = \dfrac{V}{I_m} = \dfrac{8000}{0.208} = 38.46\,K\Omega$

* SCT



$P = 240\,W$ (Consumed by $R_{eq} = R_1 + R_2'$ "Copper losses")

Notice that $V = 489 < 8000\,V$ (Rated voltage at the HV side)

$P = R_{eq} I^2 \Rightarrow R_{eq} = \dfrac{240}{2.5^2} = 38.4\,\Omega$

$|Z| = \dfrac{|V|}{|I|} = \dfrac{489}{2.5} = 195.6\,\Omega \Rightarrow X_{eq} = \sqrt{Z^2 - R_{eq}^2}$

$= \sqrt{195.6^2 - 38.4^2}$

$= 19\ .59\,\Omega$

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$R_{c,HV} = 160\,k\Omega$

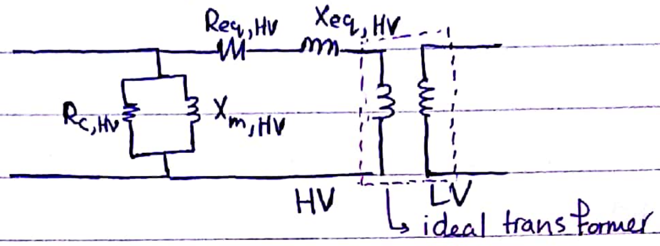
$X_{m,HV} = 38.46\,k\Omega$

$R_{eq,HV} = 38.4\,\Omega$

$X_{eq,HV} = 191.69\,\Omega$

$\Rightarrow$ Notice that $R_c \gg R_{eq}$, $X_m \gg X_{eq}$ (متل ما حكينا قبل)

$\Rightarrow$ The approximate equivalent circuit ~~can~~ for this transformer can be represented as follows:



To find $R_{c,LV}$, $X_{m,LV}$, $R_{eq,LV}$, $X_{eq,LV}$:

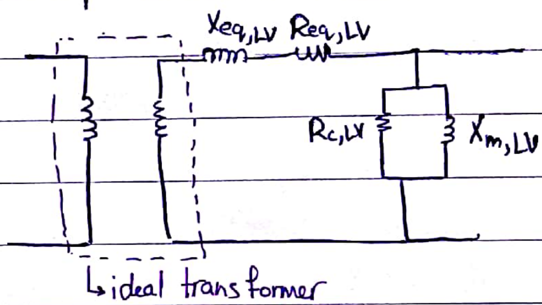
$$R_{c,LV} = R_{c,HV} * \left(\frac{240}{8000}\right)^2 = 144\,\Omega$$

$$X_{m,LV} = X_{m,HV} * \left(\frac{240}{8000}\right)^2 = 34.614\,\Omega$$

$$R_{eq,LV} = R_{eq,HV} * \left(\frac{240}{8000}\right)^2 = 0.3456\,\Omega$$

$$X_{eq,LV} = X_{eq,HV} * \left(\frac{240}{8000}\right)^2 = 0.1725\,\Omega$$

$\Rightarrow$ Another representation of this transformer



Note:

$$\frac{R_{c,LV}}{R_{c,HV}} = \frac{X_{m,LV}}{X_{m,HV}} = \frac{R_{eq,LV}}{R_{eq,HV}} = \frac{X_{eq,LV}}{X_{eq,HV}} = \left(\frac{N_{LV}}{N_{HV}}\right)^2 = \left(\frac{\text{Rated voltage @ LV side}}{\text{Rated voltage @ HV side}}\right)^2$$

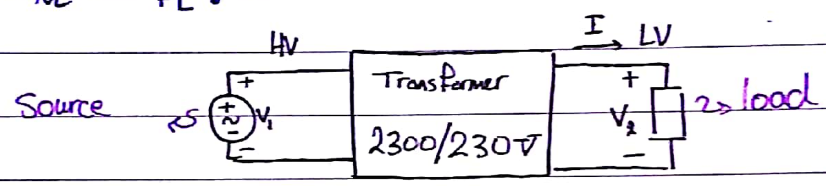
(*) Voltage regulation

→ Voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage.

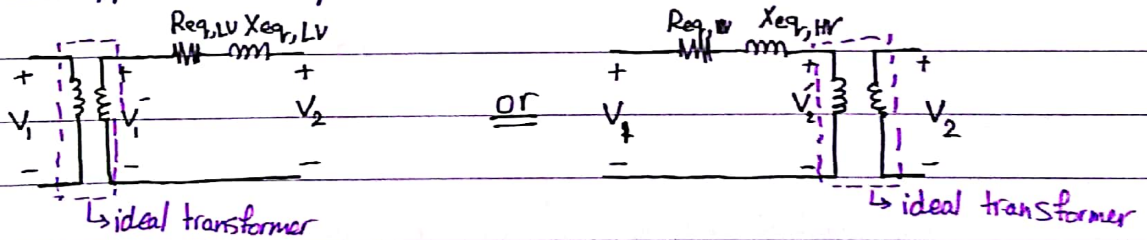
Mathematically speaking:

$$\text{Voltage Regulation: } V.R\% = \frac{V_{NL} - V_{FL}}{V_{FL}} * 100\% \quad ; \quad NL = \text{No Load},\ FL = \text{Full Load}$$

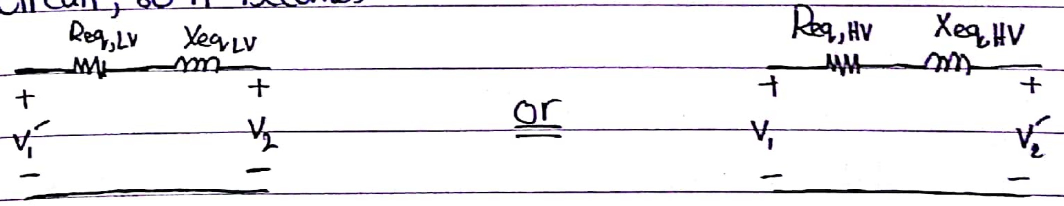
→ What are V_{NL}, V_{FL}?



* The load in the above figure is considered as "full load" when:
  - The voltage across the load terminals V_2 is equal to the rated voltage @ LV side (i.e 230 V)
  - The current passing through the load I is equal to the rated current @ the LV side
* The approximate equivalent circuit of the above transformer is as follows:



- For convience, we will not draw the ideal transformer in the approximate equivalent circuit, so it becomes



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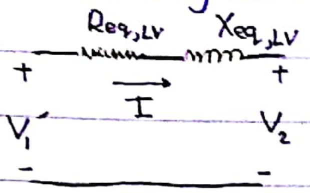
→ Consider again the following equivalent circuit



[1] In the case of "full load"

$V_{FL} = V_2 = V_{rated,LV}$, $I = I_{rated}$

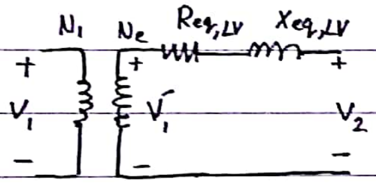
$$V_1' = I_{rated}\left(R_{eq,LV} + X_{eq,LV}\hat{i}\right) + \underbrace{V_2}_{V_{FL} = V_{rated}}$$

[2] In the case of "no load"

$I = 0$ $\therefore$ No voltage drop will occur due to $R_{eq,LV}$, $X_{eq,LV}$

$$\therefore V_2 = \underbrace{V_1'}_{\text{This value hasn't Changed, why?}} = I_{rated}\left(R_{eq,LV} + X_{eq,LV}\hat{i}\right) + V_2$$

This value hasn't Changed, why?



V_1 = the voltage of the source. In the case of no load, we only remove the load, without changing the source (i.e V_1 = fixed)

$V_1' = \frac{N_2}{N_1} V_1$ (V_1, N_2, N_1 are all fixed $\therefore$ V_1' is also a fixed value)

→ Finally:

$$V.R\% = \frac{V_1' - V_{2FL}}{V_{2FL}} * 100\%$$

Example: A transformer having the following specifications 15 KVA , 2300 V/ 230 V

$R_{eq,LV} = 0.0445\ \Omega$, $X_{eq,LV} = 0.0645\ \Omega$

Find V.R% as the load increases from no load to full load @

[1] 0.8 PF lag

[2] 0.8 PF lead

[3] Unity PF

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Solution

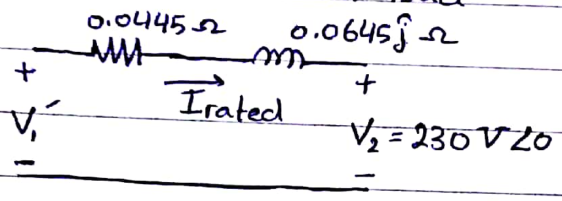
- Recall :

- $PF = \frac{P}{|S|} = \cos(\angle V - \angle I)$

	if $\angle V = 0$:	
• lag means I lags V	$\angle I = -\cos^{-1}(PF)$	∴ The load consumes Q
• lead means I leads V	$\angle I = +\cos^{-1}(PF)$	The load injects Q
• Unity means I inphase with V	$\angle I = 0$	$Q = 0$

[1] The load has PF=0.8 lag

In the case of full load



$|I_{rated}| = \frac{|S|}{|V_{rated,LV}|} = \frac{15 * 10^3}{230} = 65.2\ A$

Since PF of the load is 0.8, then I_{rated} lags V_2

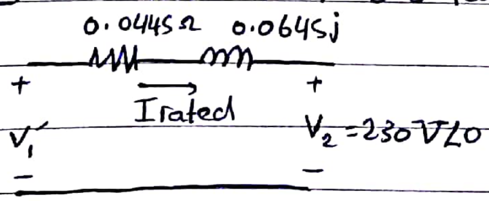
$\angle I_{rated} = -\cos^{-1}(0.8) = -37°$

$\therefore I_{rated} = 65.2 \angle -37°$

$V_1' = 65.2 \angle -37° (0.0445 + 0.0645\hat{j}) + 230 \angle 0 = 235 \angle 0.4\ V$

$\therefore VR\% = \frac{V_{NL} - V_{FL}}{V_{FL}} * 100 = \frac{235 - 230}{230} * 100\% = 2.17\%$

[2] The load has PF=0.8 lead



$I_{rated} = 65.2 \angle + \cos^{-1}(0.8) = 65.2 \angle +37°$

$V_1' = 65.2 \angle 37 (0.0445 + 0.0645\hat{j}) + 230 \angle 0$

$= 229.84\ V \angle 1.2$

$\therefore VR\% = \frac{229.84 - 230}{230} * 100 = -0.065\%$

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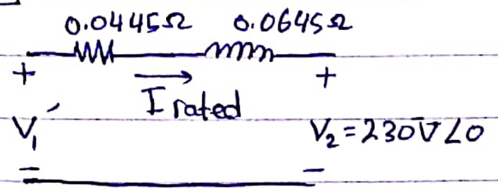
$\boxed{3}$ The load has PF = 1



$$I_{rated} = 65.2 \angle \cos^{-1}(1) = 65.2\angle 0$$

$$V_1' = 65.2\angle 0\,(0.0445 + 0.0645 j) + 230\angle 0$$
$$= 233 \angle 1.04\ V$$

$$VR\% = \frac{233 - 230}{230} * 100 = 1.3\%$$

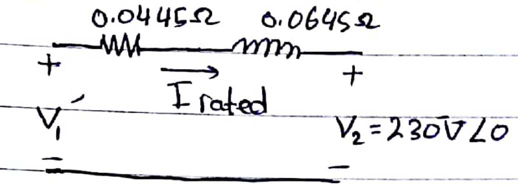
- Note:

* When $VR\% = \frac{V_{NL} - V_{FL}}{V_{Fl}} = \frac{1.3}{100} \Rightarrow V_{NL} - V_{FL} = 0.013\, V_{FL} \Rightarrow V_{NL} = 1.013\, V_{FL}$

This means V@ No load is 1.013 * V@ full load

* Voltage regulation can be defined as: the measure of how a transformer can maintain constant secondary voltage V_2 given a constant primary voltage V_1 and variance in load (from no load to full load). The lower the V.R% (closer to zero), the more stable the secondary voltage (i.e if $V.R\% \approx 0 \therefore V_{NL} \approx V_{FL}$)

[3] The load has PF = 1



$I_{rated} = 65.2 \angle \cos^{-1}(1) = 65.2 \angle 0$

$V_1' = 65.2\angle 0 \, (0.0445 + 0.0645j) + 230\angle 0$
$= 233 \angle 1.04 \ V$

$$VR\% = \frac{233-230}{230} * 100 = 1.3\%$$

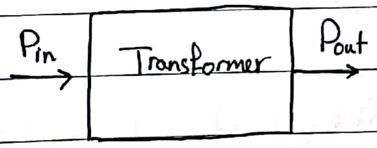
- Note:

* When $VR\% = \dfrac{V_{NL} - V_{FL}}{V_{FL}} = \dfrac{1.3}{100} \Rightarrow V_{NL} - V_{FL} = 0.013 V_{FL} \Rightarrow V_{NL} = 1.013 V_{FL}$

This means V@ No load is 1.013 * V@ full load

* Voltage regulation can be defined as: the measure of how a transformer can maintain constant secondary voltage V_2 given a constant primary voltage V_1 and variance in load (from no load to full load). The lower the V.R% (closer to zero), the more stable the secondary voltage (i.e if $V.R\% \approx 0 \therefore V_{NL} \approx V_{FL}$)

⊛ Efficiency



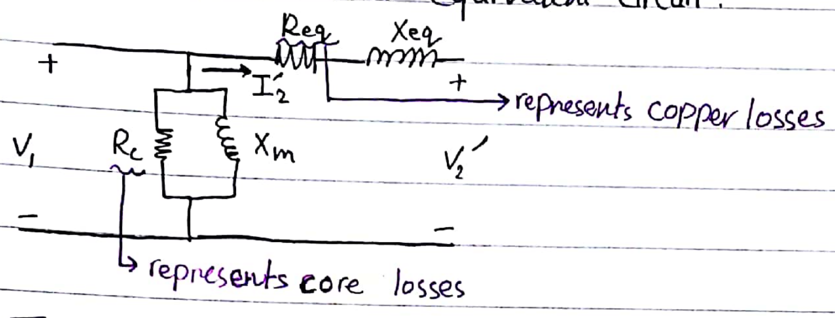
⇒ $P_{out} < P_{in}$ due to losses

⇒ There are 2 types of losses
- → Copper losses P_{cu}
- → Core losses P_{core}

$$\Rightarrow \text{Efficiency } \eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{losses}} = \frac{P_o}{P_o + P_{cu} + P_{core}}$$

notice that η is given in terms of the average power "P" (not in terms of the complex power S or the reactive power Q, because P represents the useful power that can be consumed)

⇒ Recall: the transformer equivalent circuit:



⇒ From the above circuit:

- P_{Cu} (copper losses) $= (I_2')^2 * R_{eq}$ ⇝ In the case of no load $I_2' = 0 \Rightarrow P_{Cu} = 0$. As we increase the load $\Rightarrow I_2' \uparrow \Rightarrow P_{Cu} \uparrow$
  $\therefore P_{Cu}$ depends on the load
- $P_{Cu,rated}$ (maximum value of copper losses) $= (I_{2,rated}')^2 * R_{eq}$

⇓

- P_{Cu} can be related to $P_{Cu,rated}$ as follows:
  → Define a constant $K \equiv$ loading percentage ; $0 < K < 1$ (0 → No load, 1 → Full load)
  ⇝ $I_2' = K \, I_{2,rated}'$

$$P_{Cu} = (K \, I_{2,rated}')^2 * R_{eq} = K^2 * I_{2,rated}^2 * R_{eq}$$

$$P_{Cu} = K^2 \, P_{Cu,rated} \;;\; \text{since } 0 < K < 1 \Rightarrow 0 < P_{Cu} < P_{Cu,rated}$$

- $P_{Cu,rated} = I_{2,rated}'^{2} * R_{eq}$ ⇒ Can be calculated from the short circuit test
  لأنه من الـ S.C.T منقدر نطلع R_{eq}
  + في الـ S.C.T بيكون قراءة الأميتر $= I_{2,rated}$

- P_{core} (Core losses) $= P_{Noload} = \dfrac{V_1^2}{R_c}$ ⇝ This type of losses is called "No load" losses since it always exists, even if there is no load.

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$\Rightarrow$ Continue, from the circuit in the previous page:

- $P_{out} = V_2' I_2' \, PF \sim$ $|S_{out}| = (\text{complex power}) = |V_2'||I_2'|$

$$P_{out} = |S_{out}| * PF$$

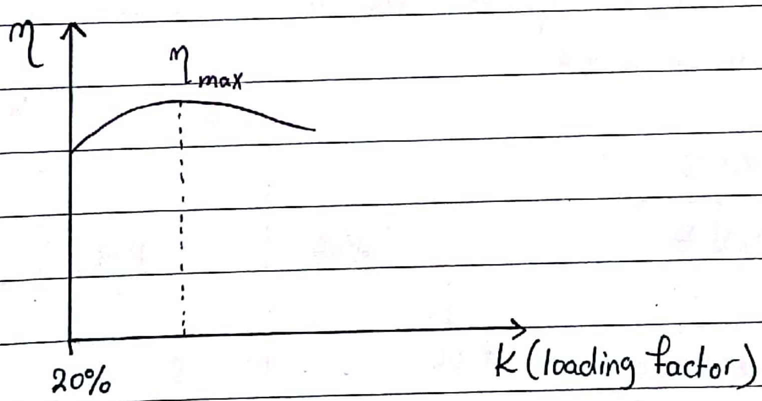
$$\therefore \eta = \frac{P_{out}}{P_{out} + P_{core} + P_{cu}} = \frac{V_2' I_2' \, PF}{V_2' \underbrace{I_2'}_{K * I'_{2,rated}} PF + \frac{V_1^2}{R_c} + K^2 P_{cu,rated}} * 100\%$$

$$\Downarrow$$

$$\eta = \frac{K * S_{rated} * PF}{K * \underbrace{S_{rated}}_{V_2' I_{2,rated}} * PF + \frac{V_1^2}{R_c} + K^2 * P_{cu,rated}} * 100\%$$

$\Rightarrow$ notice that η depends on the load (or K = loading factor)

$\Rightarrow$ If η is plotted against K, we will get the following curve:



$\Rightarrow$ The maximum efficiency occurs when $\frac{d\eta}{dI_2'} =$ zero. If this derivative is applied to the above equation, you will find that:

The maximum efficiency occurs when $P_{core} = P_{Cu} \Rightarrow P_{core} = K^2 P_{Cu,rated} \Rightarrow K = \sqrt{\frac{P_{core}}{P_{cu,rated}}}$

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→ The maximum efficiency occurs @ $K = \sqrt{\dfrac{P_{core}}{P_{cu\,rated}}}$ P_{core} → Calculated from the O.C.T, $P_{cu\,rated}$ → Calculated from the S.C.T

(*) All day efficiency (Energy efficiency)

→ From the previous discussion, we found that Core losses of a transformer is always constant, but copper losses depend on the load. As a result, copper loss varies as the load is varying through the day

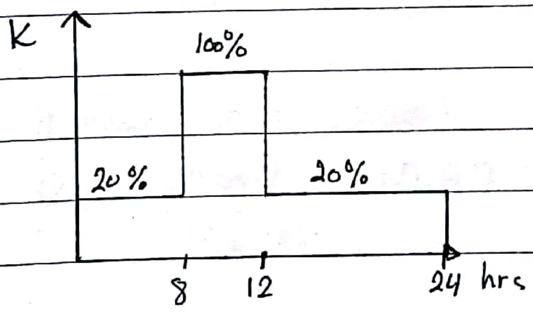
→ A new term has been defined, called "all day efficiency", in order to take the variation of the load into consideration. η_{AD} is defined as:

$$\eta_{AD} = \frac{\sum V_2' I_{2,t}' * pf * \underbrace{\Delta t}_{\text{in hours}}}{\sum V_2' I_{2,t}' * pf * \underbrace{\Delta t}_{\text{in hours}} + 24 * P_{core} + \sum (K_t)^2 P_{cu,rated} * \underbrace{\Delta t}_{\text{in hours}}}$$

$\boxed{\text{Power} * \text{time} = \text{Energy}}$

Example

A Transformer 100 KVA, 2300/230 V, $P_{core} = 500$ Watt, $P_{cu,rated} = 400$ Watt, $pf = 1$.
The loading factor varies through the day as follows
* Find the all day efficiency.



$$\eta_{AD} = \frac{\sum K * S_{rated} * pf * \Delta t}{\sum K * S_{rated} * pf * \Delta t + 24 * P_{core} + \sum K^2 * P_{cu,rated} * \Delta t}$$

$$= \frac{0.2 * 100k * 1 * 8 + 100k * 1 * 4 + 0.2 * 100k * 12}{(\text{نفس البسط}) + 24 * 0.5k + [0.2^2 * 0.4k * 8 + 0.4k * 4 + 0.2^2 * 0.4k * 12]}$$

$= 98.28\%$

Example: A 10 KVA, 2300/230 V transformer with

$R_1 = 5.8\,\Omega$, $X_1 = X_2' = 12\,\Omega$

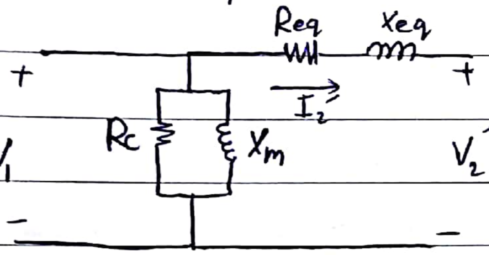
$R_2' = 6.05\,\Omega$, $R_c = 75.6\,K\Omega$, $X_m = 69.4\,K\Omega$

Find η @ full load, 0.8 Pf lag

Solution:

→ Full load means $K = 1$, $I_2' = I'_{2,rated} = \dfrac{10\,KVA}{2300\,V} = 4.35A \angle -\cos^{-1}(0.8) \approx 37°$

→ Transformer equivalent circuit:



$R_{eq} = R_1 + R_2' = 11.85\,\Omega$

$X_{eq} = X_1 + X_2' = 24\,\Omega$

$P_o = K S_{rated} * PF = 1 * 100 * 10^3 * 0.8 = 80 * 10^3\,W = 80KW$

$P_{cu} = (I_2')^2 * R_{eq} = I'^2_{2,rated} * (R_1 + R_2') = (4.35)^2 * (5.8 + 6.05) = 224.23\,W$

$P_{core} = \dfrac{V_1^2}{R_c}$, $V_1 = 4.35 \angle -37° (11.85 + 24j) + 2300\angle 0°$ (using KVL)

$= 2404 \angle 1.2°\,V$

However, use $V_1 \approx 2300\,V$ (This is an approximation)

$P_{core} = \dfrac{(2300)^2}{75.6 * 10^3} = 69.97\,W.$

$\eta = \dfrac{P_o}{P_o + P_{core} + P_{cu}} = \dfrac{80 * 10^3}{80*10^3 + 69.97 + 224.3} * 100\% = 99.63\,\%$

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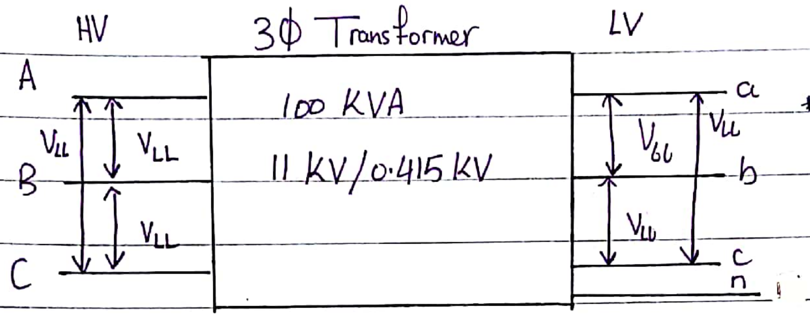
(*) 3 phase transformers (3ϕ transformers)

→ 3ϕ means 3 phase, 1ϕ means single phase



*LL, (*) LL means line to line

→ The above figure shows a 3ϕ Transformer (Step down transformer)

→ A 3ϕ transformer consists of 3 single phase transformers

→ The above transformer has the following properties:

- $|S_{3\phi}| = 100\,KVA$ = the rated or maximum complex power that can be delivered by the transformer. $|S_{3\phi}|$ represents the sum of the complex powers of the 3 single phase transformers that form the 3ϕ transformer
- $V_{LL,p}|_{rated} = 11\,KV$ = the rated or maximum line to line voltage on the primary side
- $V_{LL,s}|_{rated} = 0.415\,KV$ = the rated or maximum line to line voltage on the secondary side

Question • If $V_{LL,s} = 400\,V$, Find $V_{LL,p}$?

$$\frac{V_{LL,p}}{V_{LL,s}} = \frac{11}{0.415} = \text{transformation ratio}, \quad V_{LL,p} = \frac{11}{0.415} * 0.4\,KV = 10.6\,KV$$

• If $V_{LL,p} = 10\,KV$, find $V_{LL,s}$?

$$V_{LL,s} = \frac{0.415}{11} * 10\,KV = 0.377\,KV$$

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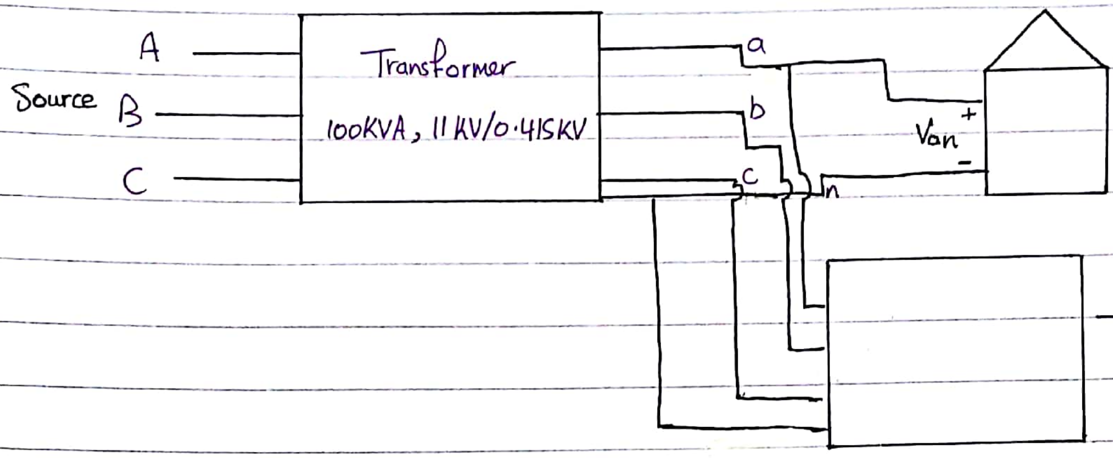
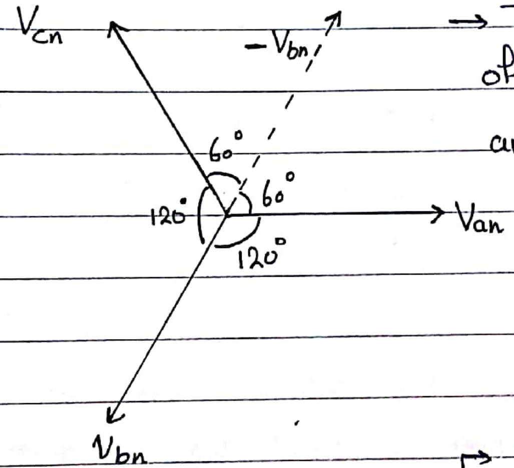


→ Houses require 1ϕ Power

→ Large buildings and factories require 3ϕ power

* What is the magnitude of V_{an} (the voltage that is provided to our homes)?



→ The adjacent figure shows a phasor diagram of the voltages V_{an}, V_{bn}, V_{cn}. These voltages are sometimes designated by V_{Ln} ≡ line to neutral voltage

→ V_{ab}, V_{bc}, V_{ca} are line to line Voltages V_{LL}

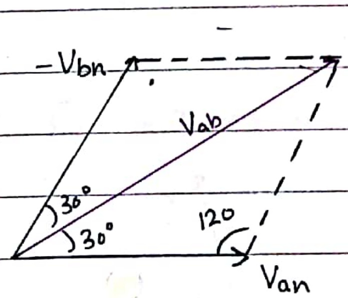
$$V_{ab} = V_{an} - V_{bn} = V_{an} + (-V_{bn})$$

$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

From the law of cosines

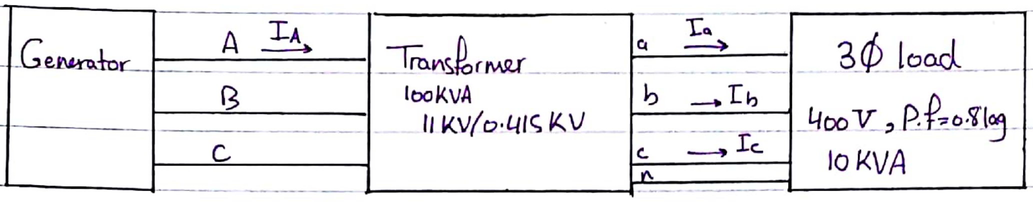
$$V_{ab}^2 = V_{an}^2 + V_{bn}^2 - 2V_{an}V_{bn}\cos 120$$

$$= 3V_{an}^2 = 3V_{bn}^2$$



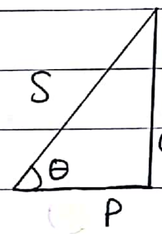
$V_{ab} = \sqrt{3}\,V_{an} \Rightarrow$ Conclusion $V_{LL} = \sqrt{3}\,V_{Ln}$

→ Assuming that $V_{LL,p} = 10\,kV$, $V_{LL,s} = 377\,V \Rightarrow V_{an} = V_{Ln} = \dfrac{377}{\sqrt{3}} = 217\,V$



→ From the above figure, find I_a, I_b, I_c, I_A?

- $S_{3\phi} = 3S_{1\phi} = 3V_pI_p$ ⇝ note V_p = V phase voltage = V_{Ln}
  $= \sqrt{3}\, V_L I_L$ $\qquad I_p$ = phase current
- $P_{3\phi} = \sqrt{3}\, V_L I_L * P.f$ $\qquad V_L$ = line to line voltage
- $\qquad I_L$ = line to line current
- $Q_{3\phi} = \sqrt{3}\, V_L I_L * \sin(\cos^{-1}(p.f))$
  - → Q is negative if I leads V
  - → Q is positive if I lags V



⇝ $Pf = \frac{|P|}{|S|} = \cos\theta$

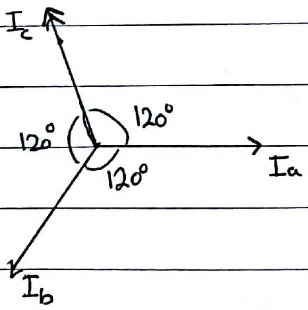
$|P| = |S| * \cos\theta = |S| * Pf$

$|Q| = |S| * \sin\theta = |S| * \sin(\cos^{-1}(Pf))$

- For the above 3ϕ load, 10KVA represents the power consumed by the load. 400V represents line to line voltage V_L

$S_{3\phi} = \sqrt{3}\, V_L I_L$

$10KVA = \sqrt{3} * 0.400KV * I_L \Rightarrow |I_L| = 14.43\ A \Rightarrow I_a = 14.43 \angle -\cos^{-1}(Pf)$



$I_b = 14.43 \angle -\cos^{-1}(Pf) - 120°$

$I_c = 14.43 \angle -\cos^{-1}(Pf) + 120°$

or $= 14.43 \angle -\cos^{-1}(P.f) - 240°$

→ Continue

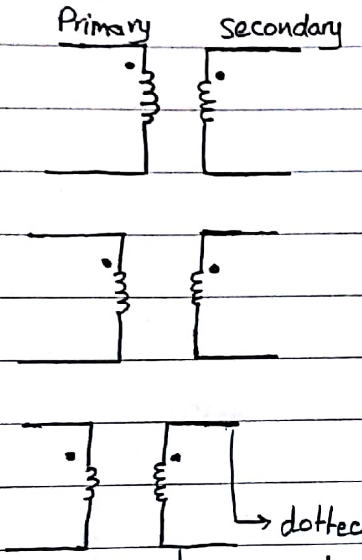
- To find I_A, use the transformation ratio:

$$|I_A| = |I_a| * \left[\frac{0.415}{11}\right] \qquad \frac{I_{L,HV}}{I_{L,LV}} = \frac{V_{L,LV}}{V_{L,HV}}$$

$$= 14.43 * \left[\frac{0.415}{11}\right] = 0.54\ A$$

$$I_A = 0.54\ A \angle -\cos^{-1}(Pf)$$

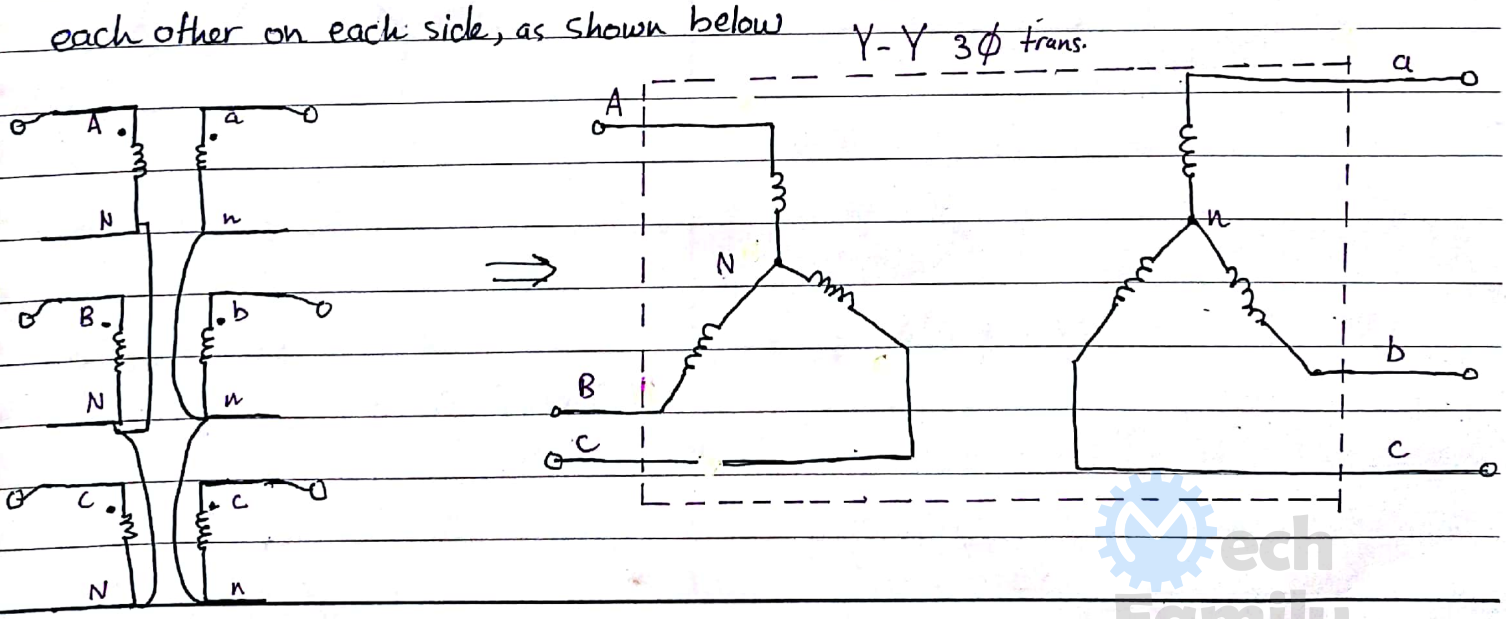
→ A 3 phase transformer consists of 3 single phase trans formers:



→ We can connect these 3 single phase transformers in order to form:

1. Y-Y 3ϕ transformer
2. Y-Δ 3ϕ transformer
3. Δ-Y 3ϕ transformer
4. Δ-Δ 3ϕ transformer

(*) Y-Y 3ϕ transformer is formed by connecting the undotted terminals to each other on each side, as shown below



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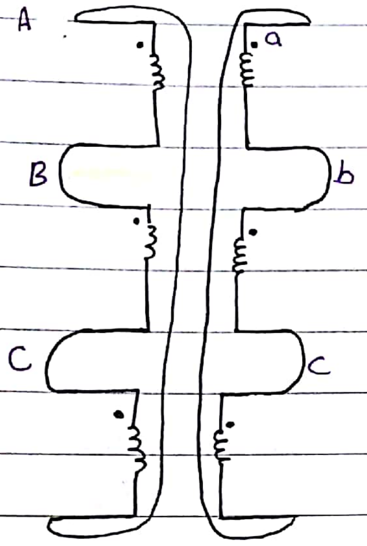
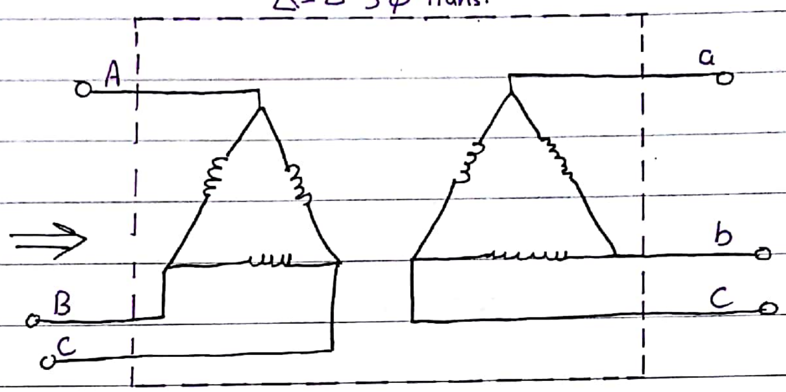
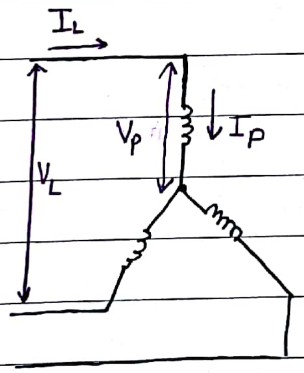
(*) Δ-Δ 3ϕ transformer is formed by connecting the undotted terminal with the dotted terminal on each side, as shown below:





→ Voltage, current, Power relations in Y and Δ connections:

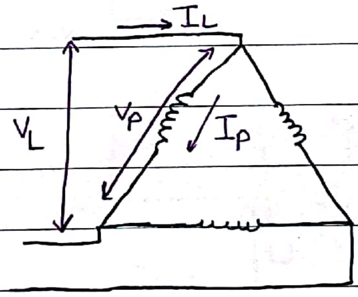
Y connection



L: line

P: phase

Δ Connection



→ $I_L = I_p$

→ $V_L = \sqrt{3}\, V_p$

→ $S_{3\phi} = 3 S_{1\phi} = 3 I_p V_p$

$= 3 I_L \left(\frac{V_L}{\sqrt{3}}\right) = \sqrt{3}\, I_L V_L$

→ $I_L = \sqrt{3}\, I_p$

→ $V_p = V_L$

→ $S_{3\phi} = 3 S_{1\phi} = 3 I_p V_p = 3\left(\frac{I_L}{\sqrt{3}}\right) V_L$

$= \sqrt{3}\, I_L V_L$

DATE

→ Transformation ratio $= \dfrac{V_{LL,P}}{V_{LL,S}}$, Turns ratio $= \dfrac{V_{phase,P}}{V_{phase,S}}$

Example: 3 single phase transformers each 100kVA, 2300V/230V are connected to form 3ϕ transformer. What are the ratings of the:

[1] Y-Y 3ϕ transformer $(3\times100\,kVA, \sqrt{3}*2300V/\sqrt{3}*230V)$

[2] Y-Δ 3ϕ transformer $(3\times100\,kVA, \sqrt{3}*2300V/230V)$

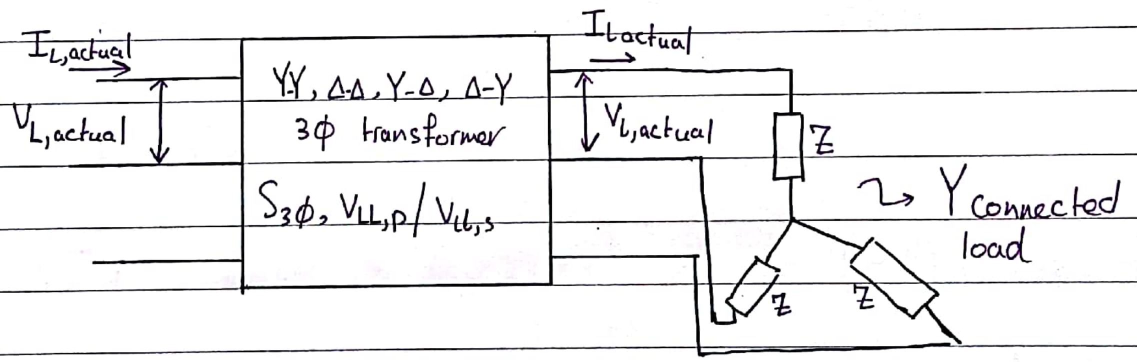
[3] Δ-Y 3ϕ transformer $(3*100\,kVA, 2300V/\sqrt{3}*230V)$

[4] Δ-Δ 3ϕ transformer $(3*100\,kVA, 2300V/230V)$

* Remember that the ratings of a 3ϕ transformer are $(S_{3\phi}, V_{LL,P}/V_{LL,S})$

$S_{3\phi} = 3*S_{1\phi}$

$V_{LL} = \sqrt{3}*V_{phase}$ in Y connection , $V_{LL} = V_{phase}$ in Δ connection



$$\rightarrow V'_{L,actual} = V_{L,actual} * \left(\frac{V_{LL,P}}{V_{LL,S}}\right)$$

$$\rightarrow I'_{L,actual} = I_{L,actual}\left(\frac{V_{LL,S}}{V_{LL,P}}\right)$$

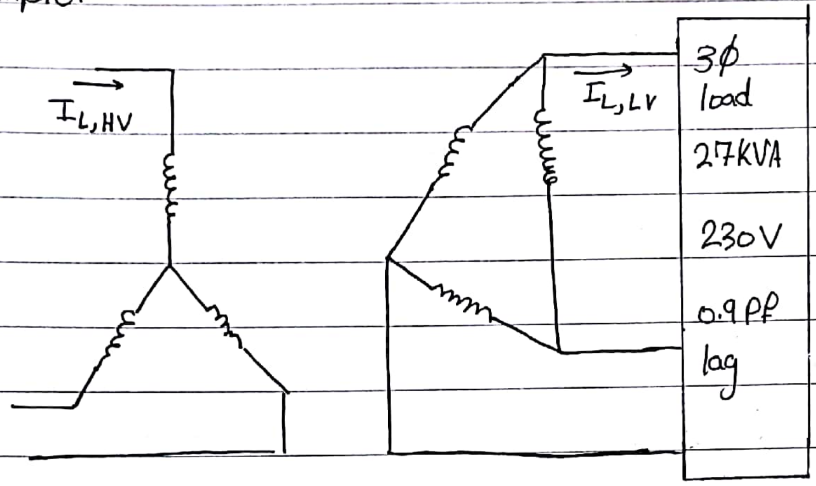
These 2 relations are applicable to all types of 3ϕ transformer (Y-Y, Δ-Δ, Y-Δ, Δ-Y)

$$\rightarrow Z'_{reflected} = Z * \left(\frac{V_{LL,P}}{V_{LL,S}}\right)^2$$

⇝ This relation is applicable to Y connected load. If the load is Δ connected, then you have to convert it into Y connected load ..... Continue ⇒

→ You have to convert it into a Y-connected load, in order to be able to use this relation $\left(Z_{y\,connection} = \frac{Z_{\Delta\,connection}}{3}\right)$

Example:



* 3, 1ϕ transformers ($10KVA$, $1330V/230V$) are connected to form Y-Δ 3ϕ transformer

[1] Find the ratings of the 3ϕ transformer

$S_{3\phi} = 3 * S_{1\phi} = 3 * 10 = 30KVA$, $V_{LL,P} = \sqrt{3} * 1330V$, $V_{LL,S} = 230V$

[2] Find $I_{L,LV}$

$$S_{3\phi} = \sqrt{3} * I_L * V_L$$

$$27\,KVA = \sqrt{3} * I_L * 230V \Rightarrow I_{L,LV} = \frac{27 * 10^3}{\sqrt{3} * 230} \angle -\cos^{-1}(P.f) = 67.78 \angle -25.8\,A$$

[3] Find $I_{L,HV}$, $V_{L,HV}$

$$I_{L,HV} = I_{L,LV} * \left(\frac{V_{LL,S}}{V_{LL,P}}\right) = \frac{27 * 10^3}{\sqrt{3} * 230} * \left(\frac{230}{\sqrt{3} * 1330}\right) = 6.77\,A$$

$$V_{L,HV} = V_{L,LV} * \left(\frac{V_{LL,P}}{V_{LL,S}}\right) = 230 * \left(\frac{\sqrt{3} * 1330}{230}\right) = 2303.62\,V$$

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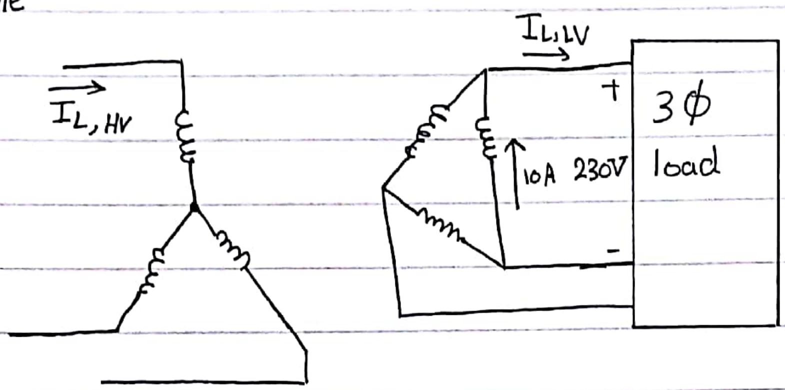
[4] Find $I_{Phase,HV}$, $I_{Phase,LV}$

$I_{phase,HV} = I_{line} = 6.77\ A$, $I_{phase,LV} = \sqrt{3} * I_{L,LV} = \sqrt{3} * 67.78 = 117.39\ A$

Example



* A 3 1ϕ transformer (10KVA, 1330 V/230 V) are connected to form Y-Δ 3ϕ transformer

[1] Find $I_{L,LV}$, $I_{L,HV}$

$I_{L,LV} = \sqrt{3} * I_{phase} = \sqrt{3} * 10 = 17.32\ A$

$$I_{L,HV} = I_{L,LV} * \left(\frac{V_{LL,S}}{V_{LL,P}}\right) = 17.32 * \left(\frac{230}{\sqrt{3} * 1330}\right) = 1.73\ A$$

[2] Find $I_{P,HV}$

$$I_{P,HV} = I_{L,HV} = \sqrt{3} * 10 * \left(\frac{230}{\sqrt{3} * 1330}\right) = I_{P,LV} * \underbrace{\left(\frac{230}{1330}\right)}_{\text{turns ratio}}$$

Conclusion

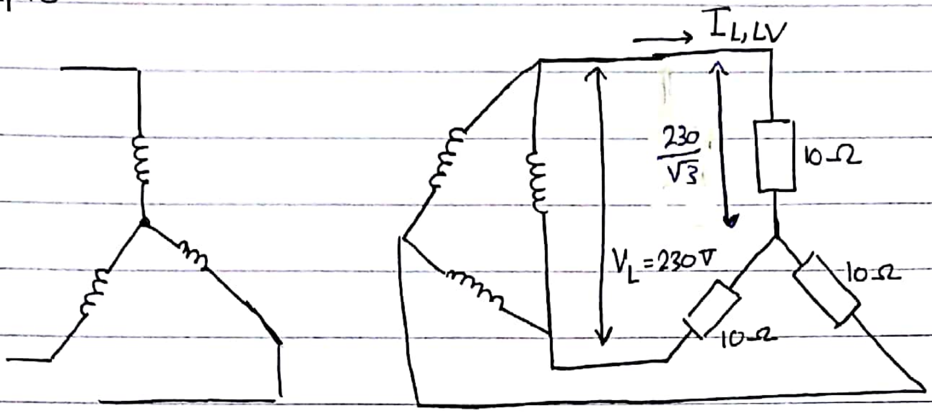
$I_{P,HV} = I_{P,LV} *$ turns ratio

[3] Find S_{load}

$S_{load} = \sqrt{3}\ I_L V_L = \sqrt{3} * \sqrt{3} * 10 * 230 = 3 * 10 * 230 = 6900\ VA$

$= 3 * I_p * V_p$

Example



* A 3 1ϕ transformer (10kVA, 1330 V/230 V) are connected to form Y-Δ 3ϕ transformer

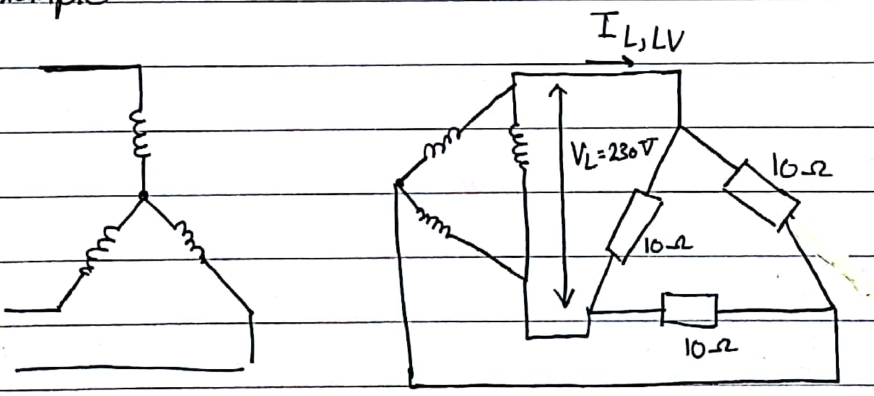
[1] Find $I_{L,LV}$

$$I_{L,LV} = \frac{V_{\text{phase of the load}}}{R} = \frac{230/\sqrt{3}}{10} = 13.27\,A$$

[2] Find Z as seen from the primary side

$$Z' = Z * \left(\frac{V_{LL,P}}{V_{LL,S}}\right)^2 = 10 * \left(\frac{\sqrt{3} * 1330}{230}\right)^2 = 1003.16\,\Omega$$

Example



* A 3,1ϕ transformer (10 kVA, 1330 V/230 V) are connected to form Y-Δ 3ϕ transformer

[1] Find $I_{L,LV}$

$$I_{L,LV} = \sqrt{3} * I_p = \sqrt{3} * \left(\frac{230}{10}\right) = 39.84\,A$$

[2] Find Y equivalent Z' seen from the primary side

$$Z_y = \frac{Z_\Delta}{3} = \frac{10}{3}\,\Omega \Rightarrow Z' = Z * \left(\frac{V_{LL,P}}{V_{LL,S}}\right)^2 = \frac{10}{3} * \left(\frac{\sqrt{3} * 1330}{230}\right)^2 = 334.38\,\Omega$$

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→ So far, we were dealing with "ideal" 3ϕ transformers. Now, we will consider "Practical" 3ϕ transformers.

Example: A 3ϕ transformer (11000V/660V, 600 KVA). The following results were obtained after performing the O.C.T and S.C.T.

	V_{LL}	I_{LL}	$P_{3\phi}$
O.C.T	660 V	16 A	4.8 KW
S.C.T	500 V	30A	8.2 KW

* Assuming Star/delta 3ϕ transformer, Find the equivalent parameters @ HV side

Solution:

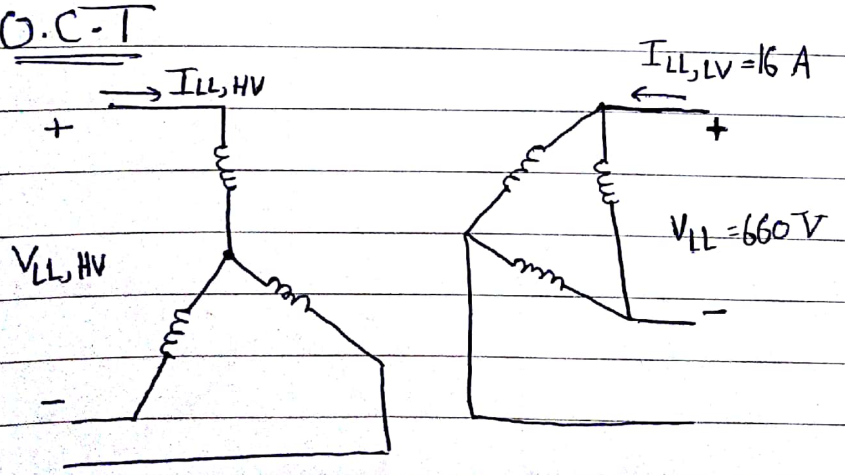
→ In O.C.T, $V_{LL} = 660\,V$ ∴ This test was performed @ the LV side

→ $S = \sqrt{3}\, I_L V_L \rightsquigarrow I_{L,LV}\Big|_{rated} = \dfrac{S_{rated}}{\sqrt{3}\, V_{L,LV}\big|_{rated}} = \dfrac{600*10^3}{\sqrt{3}*660} = 524.8\,A$

$I_{L,HV}\Big|_{rated} = \dfrac{600*10^3}{\sqrt{3}*11000} = 31.49\,A$ ⇝ This value is close to $I_{LL} = 30\,A$ (obtained from S.C.T)

∴ The S.C.T was performed @ the HV side

O.C.T



$(P_{3\phi} = 4.8\,KW)$

→ Since we have to find the equivalent parameters @ the HV side, $I_{LL,HV}$ and $V_{LL,HV}$ must be calculated

$V_{LL,HV} = V_{LL,LV} * \left(\dfrac{V_{LL,P}}{V_{LL,S}}\right)$

$= 660 * \left(\dfrac{11000}{660}\right) = 11000\,V$

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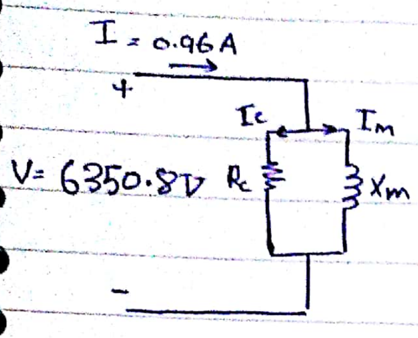
$\rightsquigarrow I_{LL,HV} = I_{LL,LV} * \left(\frac{V_{LL,S}}{V_{LL,P}}\right) = 16 * \frac{660}{11000} = 0.96\,A$

$\rightsquigarrow$ Draw the equivalent circuit (Per phase)



$I_{Phase,HV} = I_{L,HV} = 0.96\,A$, $V_{Phase,HV} = \frac{V_{line,HV}}{\sqrt{3}} = \frac{11000}{\sqrt{3}}$

$= 6350.85\,V$

$P_{3\phi} = 4.8\,kW \Rightarrow P_{1\phi} = \frac{4.8}{3} = 1.6\,KW$

$P_{1\phi} = \frac{V^2}{R_c} \Rightarrow R_c = \frac{(6350.8)^2}{1.6 * 10^3} = 25.2\,k\Omega$ per phase

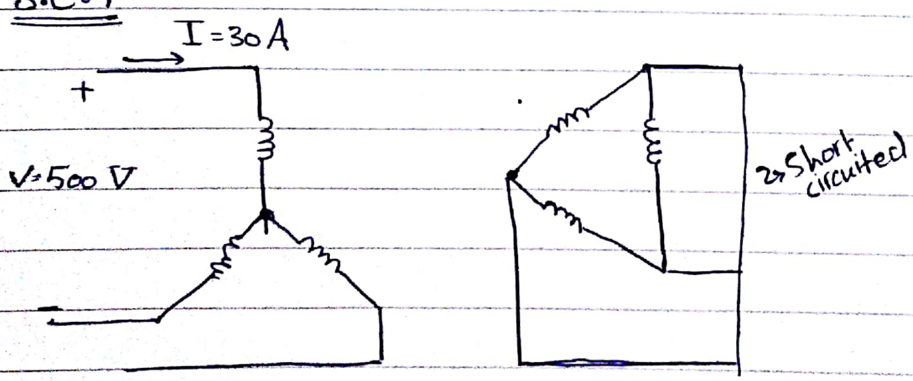
$= 25.2\,k\Omega/\text{phase}$

$I_c = \frac{V}{R_c} = \frac{6350.8}{25.2 * 10^3} = 0.25\,A$, $I_m = \sqrt{I^2 - I_c^2}$

$= \sqrt{0.96^2 - 0.25^2} = 0.93\,A$

$X_m = \frac{V}{I_m} = \frac{6350.8}{0.93} = 6.9\,K\Omega/\text{phase}$

S.C.T



$(P_{3\phi} = 8.2\,KW)$

$\rightarrow$ The measurements in S.C.T were obtained from the H.V side, and hence we can find X_{eq}, R_{eq} directly.

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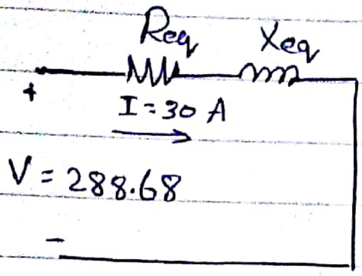
-> Draw the equivalent circuit (per phase)



$I_{phase} = I_{line} = 30A$, $V_{phase} = \frac{V_{line}}{\sqrt{3}} = \frac{500}{\sqrt{3}} = 288.68\ V$

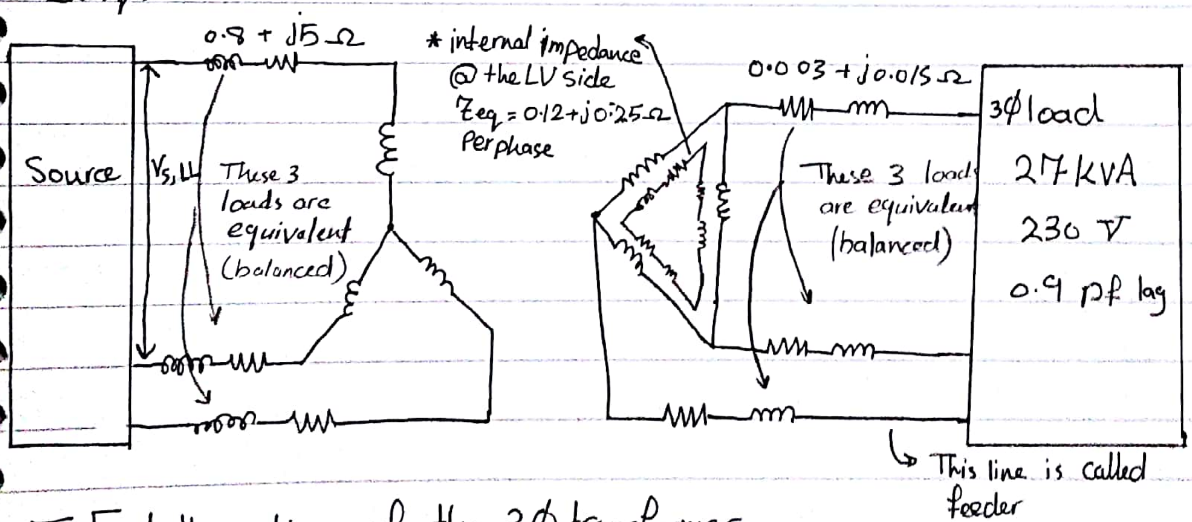
$P_{3\phi} = 8.2\ KW \Rightarrow P_{1\phi} = \frac{8.2}{3} = 2.73\ KW$

$P_{1\phi} = I^2 * Req \Rightarrow R_{eq} = \frac{2.73*10^3}{30^2} = 3.03\ \Omega/Phase$

$|Z| = \frac{V}{I} = \frac{288.68}{30} = 9.62\ \Omega$

$X_{eq} = \sqrt{Z^2 - R_{eq}^2} = \sqrt{9.62^2 - 3.03^2} = 9.13\ \Omega/phase$

Example:



* 3, 1φ transformer (10kVA, 1330V/ 230V) are connected to form Y-Δ 3φ transformer

[1] Find the ratings of the 3φ transformer

$S_{3\phi} = 3 * S_{1\phi} = 3*10 = 30KVA$

$V_{LL,P} = \sqrt{3} * 1330 = 2303.63\ V$

$V_{LL,S} = 230\ V$

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[2] Find $I_{L,LV}$, $I_{L,HV}$

$S = \sqrt{3}\, I_L V_L \Rightarrow 27 * 10^3 = \sqrt{3}\, I_{L,LV} * 230 \Rightarrow I_{L,LV} = 67.78\ A$

$I_{L,HV} = I_{L,LV} * \left(\frac{V_{LL,s}}{V_{LL,P}}\right) = 67.78 * \left(\frac{230}{\sqrt{3} * 1330}\right) = 6.77\ A \angle \underbrace{-\cos^{-1}(P.f)}_{25.8^\circ}$

[3] Z seen by the source (Reflect all the loads to the primary side)

$Z = 0.8 + j5 + \underbrace{(0.12 + j0.25)}_{\text{Phase impedance}} * \left(\frac{1330}{230}\right)^2 + \underbrace{(0.003 + j0.015)}_{\text{line impedance}} \left(\frac{1330\sqrt{3}}{230}\right)^2$

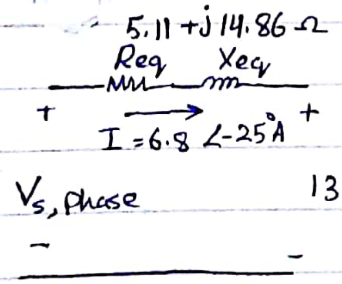
Phase impedance
You multiply it by the turns ratio to reflect it to the primary side

line impedance
you multiply it by the transformation ratio to reflect it to the primary side

$= 5.11 + j14.86\ \Omega/\text{phase}$

[3] Find the sending Voltage $V_{s,LL}$ (Check the previous figure to recognize the sending Voltage)

→ Draw the transformer equivalent circuit (per phase)



$V_{s\,phase} = 6.8\angle{-25}\,(5.11 + 14.86\hat{j}) + 1330\angle 0$

$= 1407 \angle 3^\circ$

$V_{s,LL} = \sqrt{3} * V_{s\,phase} = \sqrt{3} * 1407 = 2437\ V$

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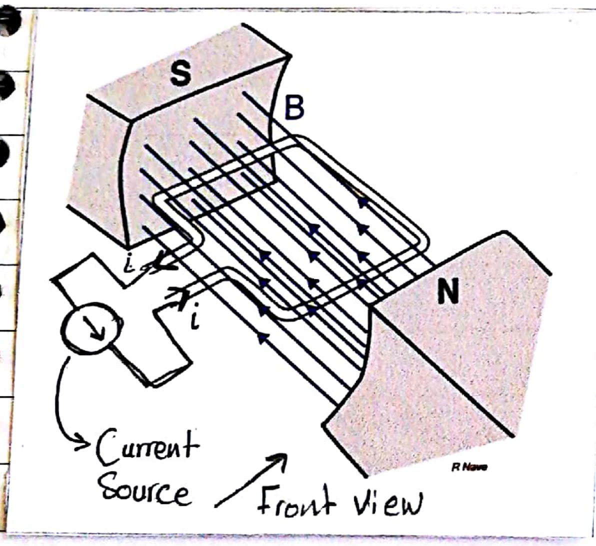
⊛ DC machines:

→ DC machines include:

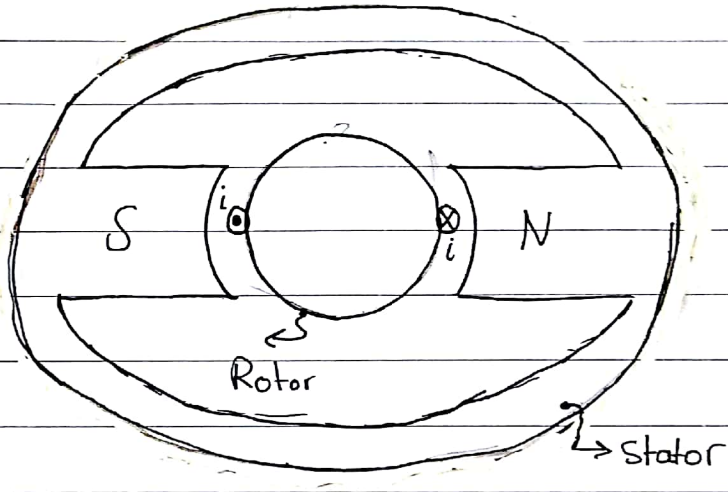
1. DC motors: Convert electrical energy into mechanical energy.
2. DC generators: Convert mechanical energy into electrical energy.

⊛ DC motor:

→ The following figures show a simple DC motor:



"3D view"



"2D view (front view)"

→ The main components of a motor:

1. Stator (Stationary component)
2. Rotor (Rotating component)

→ The motor must be provided with

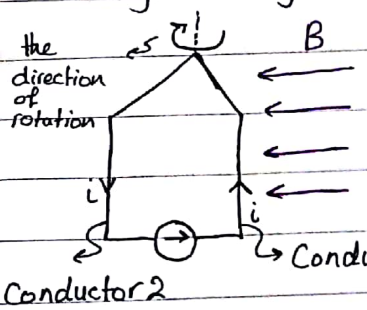
1. Current (provided by the current source) ⟹ "Electrical energy"
2. Magnetic flux (provided by a magnet "Stator")

→ Current flowing inside the conductor + flux = Force acting on the conductor

$(F = (i \times B) \cdot L)$ ⟹ "Mechanical energy"

→ How to determine the direction of the force?

Using the right hand rule



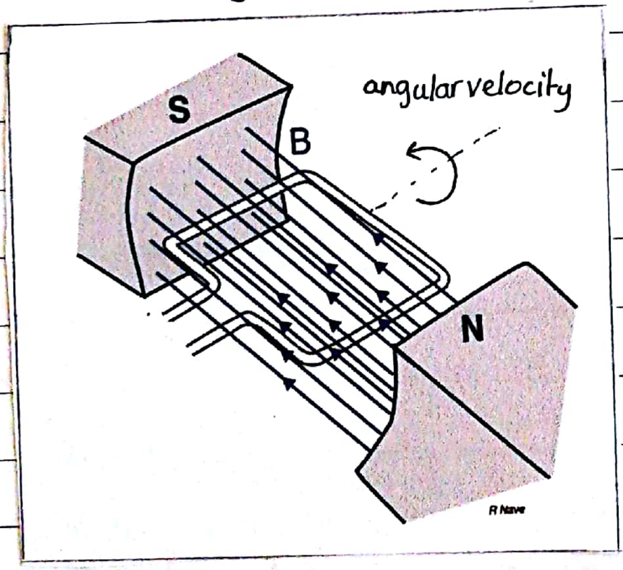
⇝ Direct your thumb in the direction of the current, while the other fingers in the direction of the magnetic field. The force F extends "outward" from the palm of your hand.

- The force acting on conductor 1 is directed out of the page ⊙
- The force acting on conductor 2 is directed into the page ⊗

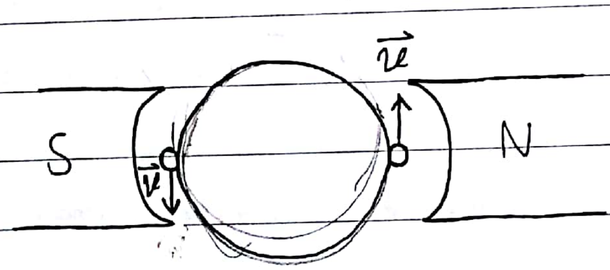
→ This pair of forces produces torque, and hence, the loop will rotate in the direction shown in the adjacent figure

⊛ DC generator

→ The following figures show a DC generator



"3D view"

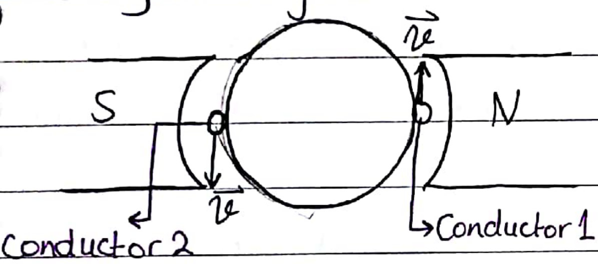


"2D view (front view)"

→ The generator must be provided with
[1] External force or torque to rotate the loop ⇒ "Mechanical energy"
[2] Magnetic flux (provided by a magnet)

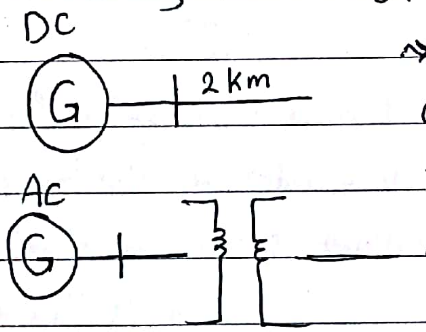
→ Motion (rotation) + flux = emf (electromotive force) emf $= (\vec{v} \times \vec{B}) \cdot L$
(A current will flow in the wires, if the wires form a closed loop) ⇒ "Electrical energy"

→ How to determine the direction of the current?
Using the right hand rule



↝ Direct your thumb in the direction of $\vec{v}$, while the other fingers in the direction of the magnetic flux. The direction of the current i extends "outward" from the palm of your hand.
• The current flowing in conductor 1 is directed out of the page ⊙
• The current flowing in conductor 2 is directed into the page ⊗

→ Nowadays, DC machines are rarely used compared to AC machines which are commonly used, why?



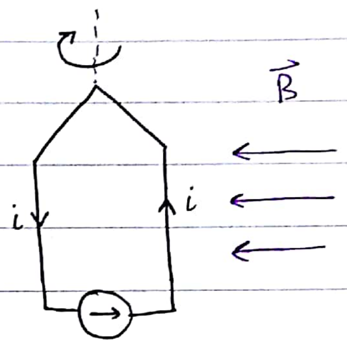
↝ The voltage generated by a DC generator is limited $V \leq 1000\,V$ due to certain reasons + Power can't be transferred for more than 2 Km, since power will be lost.
↝ The voltage generated by an AC generator may reach 20kV + transformers may be used to step up the generated voltage, and hence power may be transferred for long distances.

→ Summary:

- In motors: current + flux = motion (or force)
- In generators motion + flux = current (or emf)
- A generator action is developed in every motor. How?

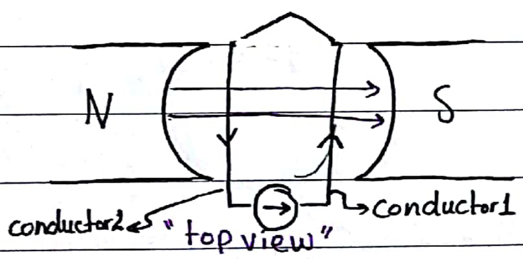
In the presence of a magnetic field in a DC motor, when the current flows in the loop in the direction determined by the DC current source, the loop will rotate (current + flux = motion).

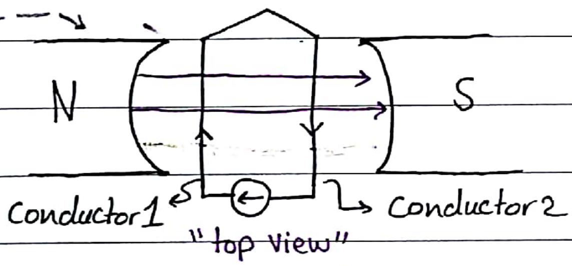
Since the loop is rotating + flux exists, current will be produced (motion + flux = current "generator action"). The direction of the produced current is opposite to the direction of the original current

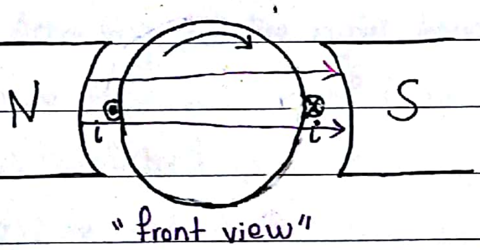


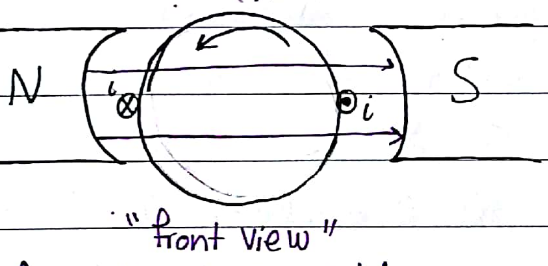
- Also, in the same manner, a motor action is developed in every generator.

→ let's take a look at a DC motor







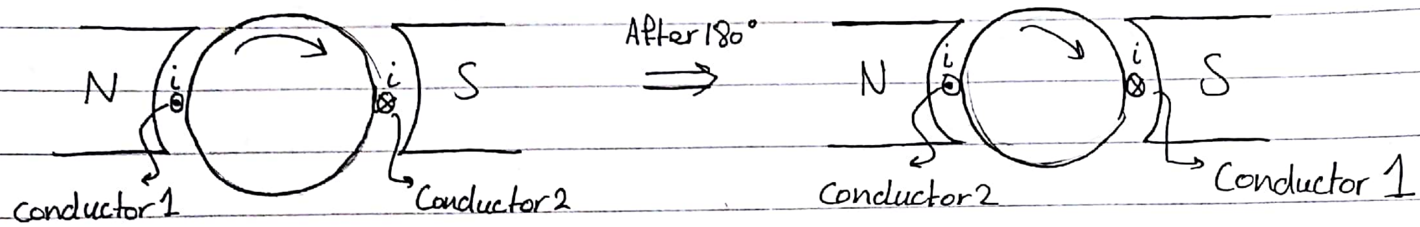


* According to the right hand rule, the rotor will rotate in the cw direction

* After 180°

* According to the right hand rule, the rotor will rotate in the ccw direction!

→ Notice that we are facing a problem. The direction of rotation of the rotor is reversed every 180°. In order to maintain the direction of rotation of the rotor, the direction of the current in both conductors 1 and 2 must be reversed every 180°. يعني المفروض هاد السيناريو يصير

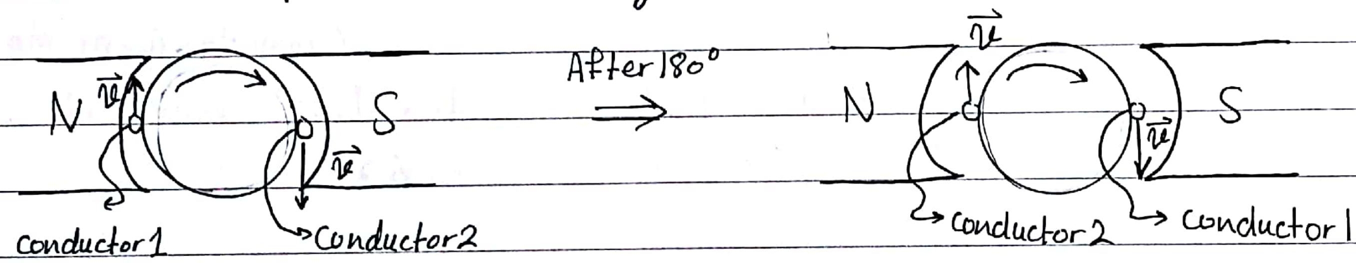


Initially, the direction of i in conductor 2 is ⊗ $\xrightarrow{\text{after } 180^\circ}$ ⊙

the direction of i in conductor 1 is ⊙ → ⊗

كأنه محتاج AC current بدل DC current عشان نحل المشكلة

→ A similar problem occurs in generators:



* According to the right hand rule, the direction of i in
  1. conductor 1 ⊗
  2. conductor 2 ⊙

* According to the right hand rule, the direction of i in
  1. conductor 1 ⊙
  2. Conductor 2 ⊗

* It seems that the DC generator is providing us with AC current instead of DC current. (مشكلة)

→ To solve these problems, 2 "electrical" devices were invented

* For motors:

DC → Inverter → AC

• Inverters: are electronic devices that change direct current (DC) to alternating current (AC)

* For generators

AC → Rectifier → DC

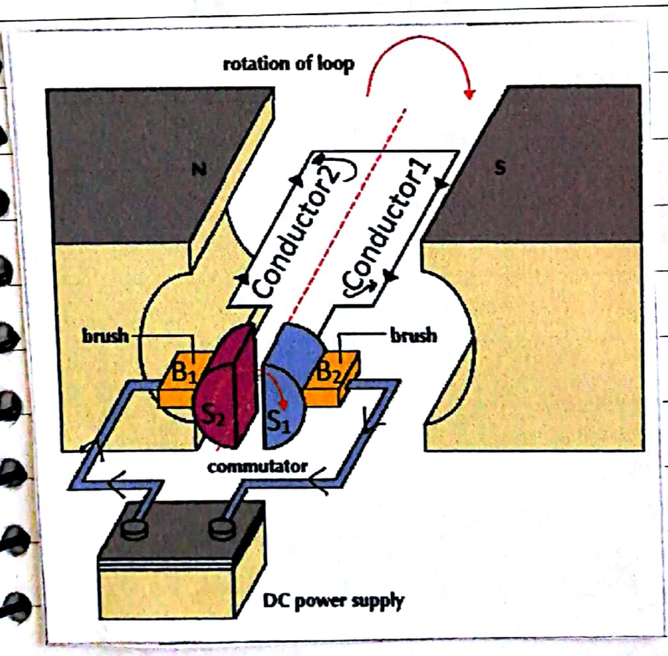
• Rectifiers: are electronic devices that change alternating current (AC) to direct current (DC)

(*) Mechanical commutators

→ Commutators are mechanical inverters (i.e they are mechanical devices, not electronic devices)

→ They were used before inventing inverters

ال Commutators اشي كثير قديم



→ B_1, B_2 are called brushes. They are stationary components and they conduct electricity

→ S_1, S_2 are commutator segments

S_1 is connected to conductor 1

S_2 is connected to conductor 2

These commutators rotate with the conductors. In the adjacent figure B_1 is connected to S_2, while B_2 is connected to S_1

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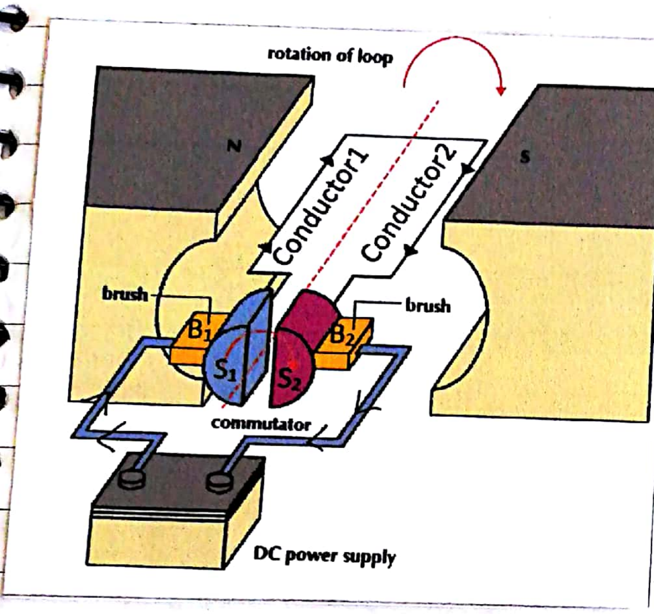
→ After 180°:



~ Notice that B_1 and B_2 are stationary, while S_1 and S_2 rotate with the conductors. Now, B_1 is connected to S_1, B_2 is connected to S_2.

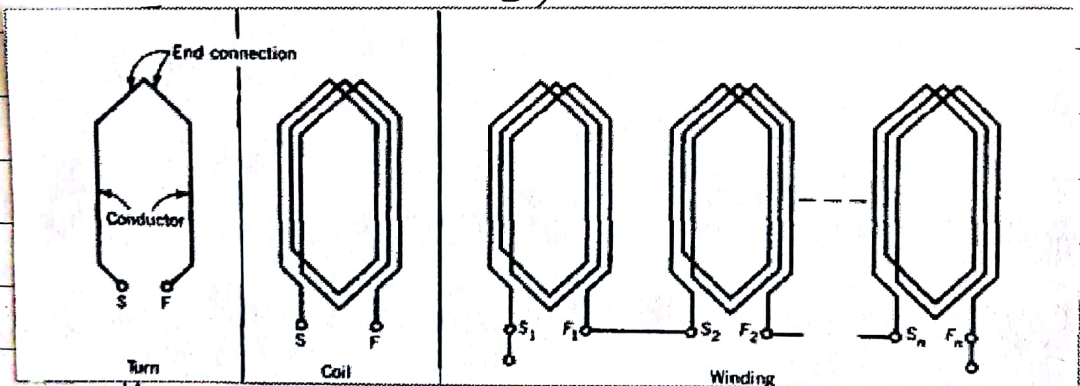
~ Initially, the direction of current in

1. Conductor 1 ↙ after 180° ↗
2. Conductor 2 ↗ after 180° ↙

} The results the direction of rotation hasn't changed.

* Note:

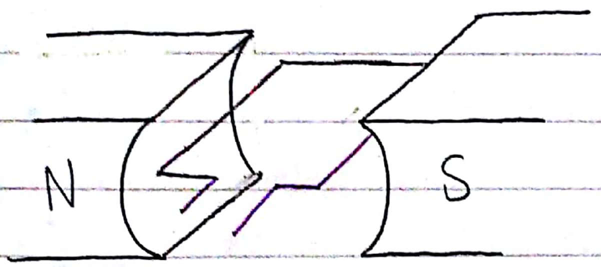
The following figures show the difference between the following terms: (conductor / turn / coil / winding)



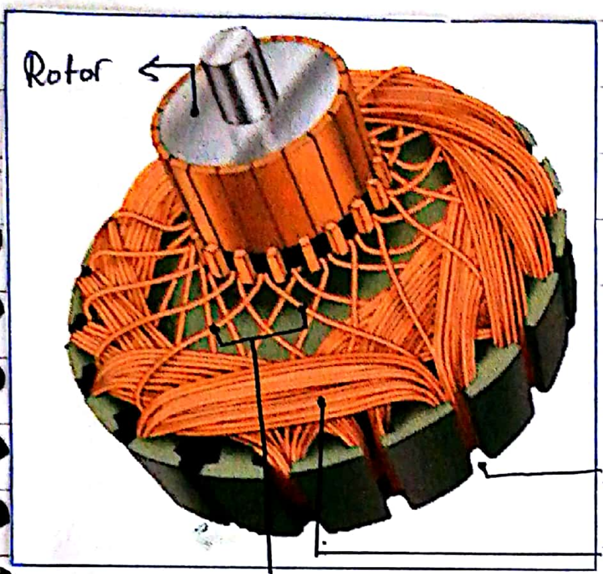
A "turn" consists of 2 conductors that are connected to each other by end connection

A "coil" is formed by connecting several turns

A "winding" is formed by connecting several coils in series



⇒ These schematics are "idealized", in order to explain the principles of motors and generators easily. They are "idealized" because the rotor contains only 1 turn of wire. Real motor uses the same principles, but the rotor contains large number of coils, as shown here



⇒ This large number of coils is called the "armature winding"

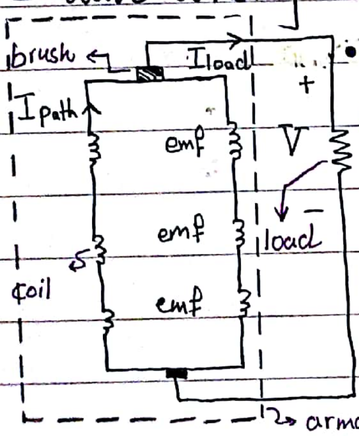
→ These openings are called slots

→ 1 coil

→ the ends of the coil

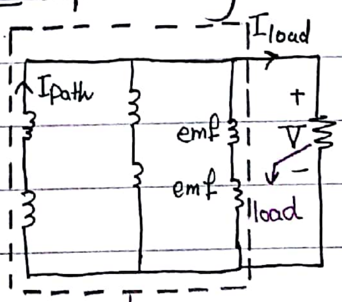
⇒ The design of the armature winding depends on the requirements of the load that would be connected to the motor. The design of the armature winding can be of 2 types: [1] Wave winding [2] Lap winding

[1] Wave winding



- # of parallel paths = 2
- $V = 3\,emf$
- $I_{path} = \dfrac{I_{load}}{\#\ \text{Parallel Paths}}$

[2] Lap winding



- # of parallel paths = 3
- $V = 2\,emf$
- $I_{path} = \dfrac{I_{load}}{\#\ \text{Parallel Paths}}$

* The above figures show "schematic diagrams" of the wave and lap armature windings.

DATE

→ Notice that:

- The total emf = V in the wave winding design is greater than the total emf in the lap winding design.
- I_{path} in the lap winding design is less than I_{path} in the wave winding design.

⇓

Assume that $I_{load} = 600A$, $I_{rated\ per\ path} = 210A$.

$I_{path}(\text{wave winding}) = \frac{600}{2} = 300A > I_{rated\ per\ path}$ → the wire burns

$I_{path}(\text{lap winding}) = \frac{600}{3} = 200A < I_{rated\ per\ path}$

لو ال load بيحتاج تيار عالي ← بستخدم ال lap winding

Conclusions:

- Lap winding design is preferable for high current, low voltage load requirement
- Wave winding design is preferable for low current, high voltage load requirement

* حسب احتياجات ال load بختار ال lap winding أو wave winding

→ In the following example, we will learn how to draw a schematic diagram for an armature winding

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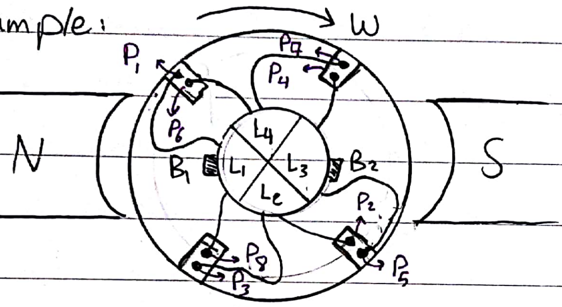
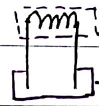
Example:



* B_1 and B_2 → 2 brushes (fixed)
* L_1, L_2, L_3, L_4 → 4 commutator segments
* $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ → represent the ends of 4 coils



* The back view of the rotor shows that ← من ورا مشوف هاي الشبكة

P_1 is connected to P_2 — P_1 ~~ P_2

P_3 is connected to P_4 — P_3 ~~ P_4

P_5 is connected to P_6 — P_5 ~~ P_6

P_7 is connected to P_8 — P_7 ~~ P_8

* There are 4 slots, and 2 poles (or 1 pair of poles)
* At the moment shown, B_1 is connected to L_1, while B_2 is connected to L_3
* Draw a schematic diagram for the above armature winding.

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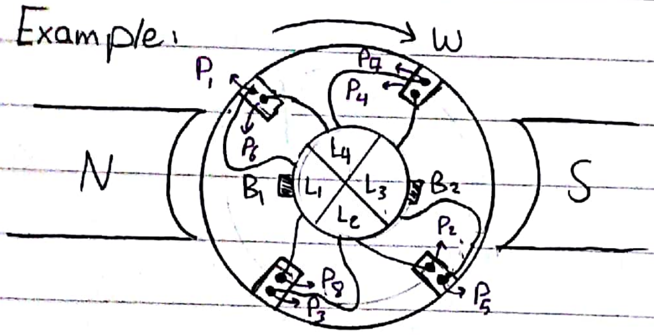
Example:



* B_1 and B_2 → 2 brushes (fixed)
* L_1, L_2, L_3, L_4 → 4 commutator segments
* P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 → represent the ends of 4 coils

* The back view of the rotor shows that ← من ورا منشوف هاي الشبكة

P_1 is connected to P_2 — P_1 mm P_2

P_3 is connected to P_4 — P_3 mm P_4

P_5 is connected to P_6 — P_5 mm P_6

P_7 is connected to P_8 — P_7 mm P_8

* There are 4 slots, and 2 poles (or 1 pair of poles)
* At the moment shown, B_1 is connected to L_1, while B_2 is connected to L_3
* Draw a schematic diagram for the above armature winding.

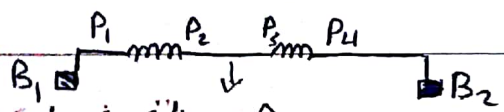
→ The schematic diagram for the armature winding shows the parallel paths that form the armature winding

← زي ما شفنا قبل ، الـSchematic diagram بورجينا الـ Parallel paths الي بتكوّن الـarmature winding ← يعني ، لو لقينا الـ parallel paths متكونة ، صفّينا الـ diagram

→ To construct the first parallel path, you have to start at B_1 and stop at B_2: (look at the above figure and read the following:)

B_1 is connected to L_1 → L_1 is connected to P_1 → P_1 is connected to P_2 → P_2 is connected to L_2 → L_2 is connected to P_3 → P_3 is connected to P_4 → P_4 is connected to L_3 → L_3 is connected to B_2 (STOP).

∴ The first parallel path looks like this

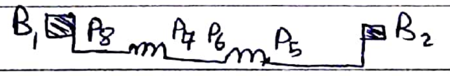


هو مربوط في L_2 ، بس احنا ما منرسمها

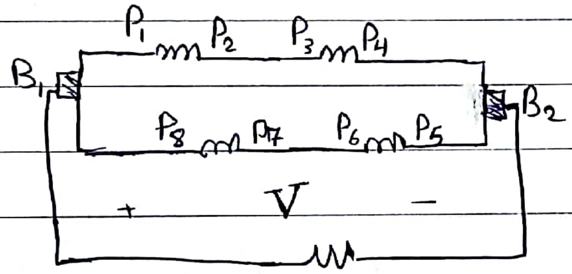
→ To construct the second parallel path, Start at B_1 and Stop at B_2:
B_1 is connected to L_1 → L_1 is connected to P_8 → P_8 is connected to P_7 → P_7 is connected to L_4 → L_4 is connected to P_6 → P_6 is connected to P_5 → P_5 is connected to L_3 → L_3 is connected to B_2 (STOP)

بمجرد ما وصلنا L_3 منكون وصلنا B_2 فمنوقف

∴ The second parallel path looks like this

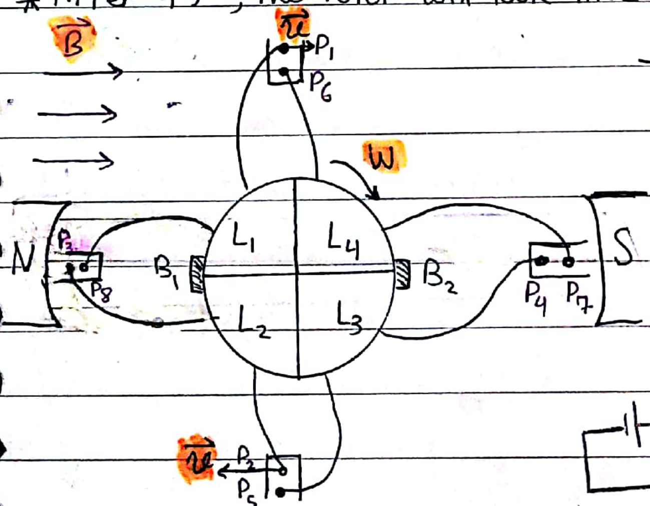


∴ The schematic diagram for the armature winding:

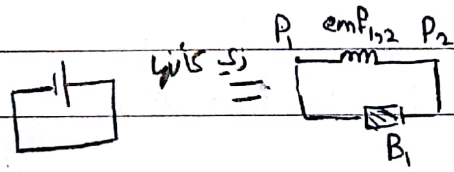


$$V = emf_{1,2} + emf_{3,4} \text{ or } emf_{8,7} + emf_{6,5}$$

* After 45°, the rotor will look like this



→ If you try to construct the first parallel path:
B_1 is connected to L_1 → L_1 is connected to P_1
P_1 is connected to P_2 → P_2 is connected to L_2
L_2 is connected to B_1!

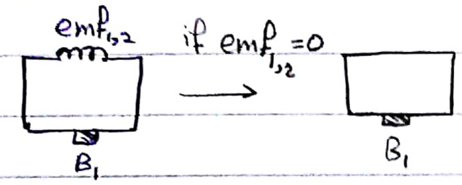


⇒ Short circuit

ل يكون عنا تيار عالي

To avoid the formation of the short circuit, $emf_{1,2}$ must be equal to zero at this instant

⇒



⇒ No current will flow through this loop
∴ the formation of the short circuit is avoided

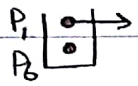
Recall: $emf = (\vec{v} \times \vec{B}) \cdot L$ → $emf = 0$ when θ between $\vec{v}$ and $\vec{B} = 0, 180$

∴ If we let the angle θ between $\vec{v}$ and $\vec{B} = 0; 180$; at this instant → $emf_{1,2} = 0$

→ The formation of the short circuit is avoided.

* بالصفحة الي قبل ، مثلاً:

($\vec{B}$ اتجاه) $\vec{B}$



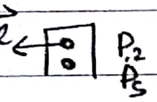
$\vec{v}$ (the direction of the velocity of the conductor P_1)

$$\vec{v} \times \vec{B} = 0$$

∴ at this instant $emf_{1,2} = 0$

وهو المطلوب

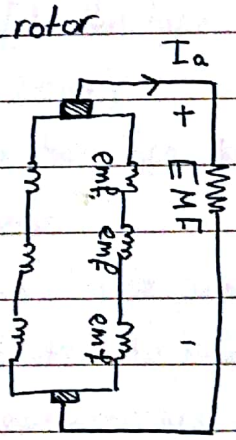
(the direction of $\vec{v}$ of P_2) $\vec{v}$



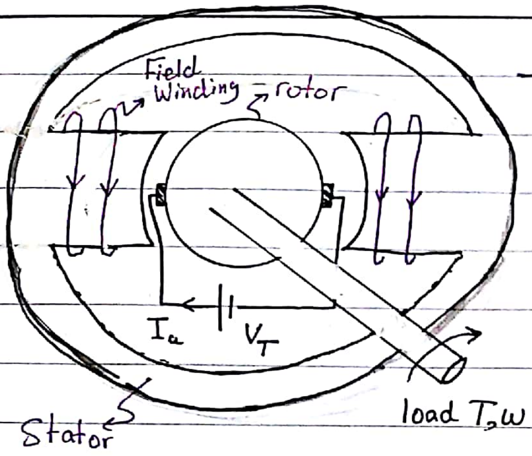
Useful relations in DC machines:

1- The torque produced by a DC motor is given by $T = K \phi I_a$, where k is a constant, ϕ is the magnetic flux produced by the magnet, I_a is the armature current.

2- The total emf produced by a generator is given by $EMF = K \phi \omega_m$, where k is a constant, ϕ is the magnetic flux produced by the magnet, ω_m is the angular speed of the rotor



⇒ The total emf $= \sum emf_i = EMF = K \phi \omega_m$



→ This figure shows a DC motor, since electrical power is supplied to the rotor by the voltage source V_T [Recall: DC motors convert electrical energy into mechanical energy]

→ A shaft is connected to the rotor. This shaft carrys a load (الـ load ممكن يكون مروحة)

- T = represents the torque acting on the shaft by the load (i.e $T = T_{load}$)
- w = represents the angular speed of the load (or the shaft) المروحة

→ The stator of the motor provides the magnetic flux, and hence, the stator might be an electromagnet (as shown above) or permanent magnet: N S

→ 2 types of windings may exist in a DC machine:

[1] Armature winding (which is attached to the rotor)

[2] Field winding (which is attached to the stator) ⇒ "Check the above figure"

→ DC motors are classified into:

[1] Separately excited DC motor [In this type, the armature winding circuit and the field winding circuit are not connected to each other]

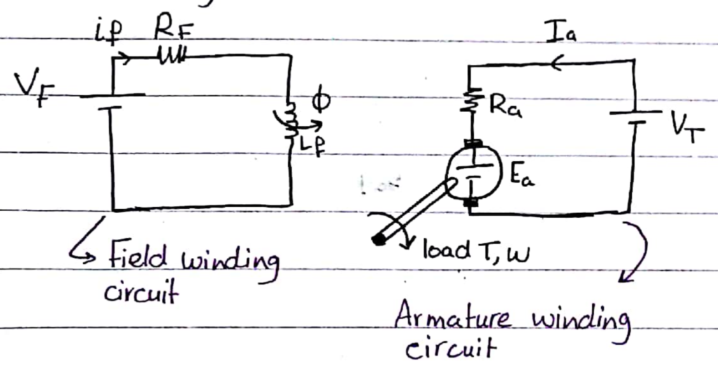
[2] Series DC motor [In this type, the field winding circuit is connected in series to the armature winding circuit)

[3] Shunt DC motor [In this type, the field winding circuit is connected in parallel to the armature winding circuit]

[4] Compound DC motor [it is a combination of series + shunt DC motors]

→ We will learn [1] how to draw the equivalent circuit + [2] the characteristics of each type

[*] Separately excited DC motor

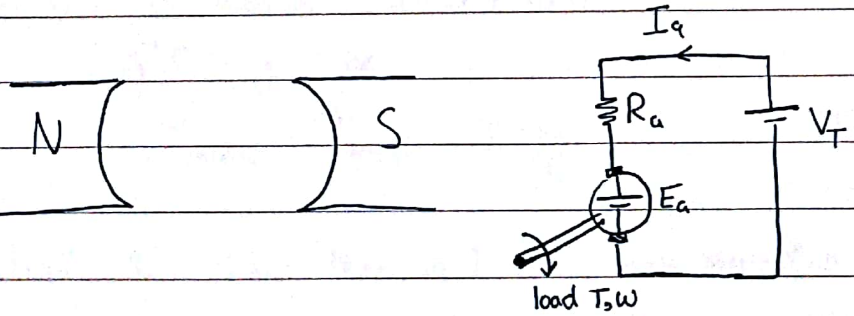


→ This figure shows the equivalent circuit of a separately excited DC motor.
→ Notice that the field winding circuit is separated from the armature winding circuit.

→ In the field winding circuit:
V_F = field voltage source , R_F = the resistance of the wires that form the coils of the field wind
i_F = field current , L_F = the inductor that produces the magnetic flux ϕ
↳ This inductor is an electrical representation of the flux produced by the coils of the electromagnet

→ In the armature winding circuit:
V_T = represents the terminal voltage source
I_a = armature current
R_a = the resistance of the wires that form the coils of the armature winding
E_a = armature emf [Recall: every motor has a generator action. E_a represents the emf produced by the generator action]. Notice that the polarity of E_a opposes the polarity of V_T. $E_a = k\phi\omega_m$

→ If the stator provides magnetic flux ϕ by a permanent magnet (instead of electromagnet), the equivalent circuit will look like this:

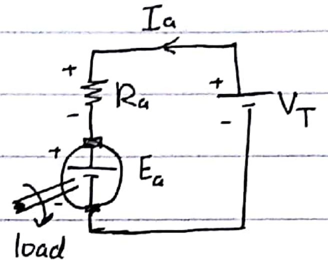


→ This type is considered as separately excited DC motor

→ @ motor starting $\omega_m = zero$ (the rotor is not rotating) $\Rightarrow E_a = K\phi\,\omega_m = zero$.
While running $\omega_m \neq zero \Rightarrow E_a = K\phi\,\omega_m \neq zero$

$\omega_m \uparrow \Rightarrow E_a \uparrow$

→ Apply KVL to the armature winding circuit



$$V_T - R_a I_a - E_a = 0$$

$$\Downarrow$$

$$I_a = \frac{V_T - E_a}{R_a} \longrightarrow @\text{ Starting } E_a = 0$$

$$\therefore I_a = \frac{V_T}{R_a}\ (\text{Max}) \Rightarrow T_{motor} = K\phi I_a\ (\text{Max, since } I_a \text{ is maximum})$$

* We can see that both the armature current and the torque produced by motor are very high at starting (maximum values).
The increased torque results in a sudden mechanical stress on the machine which leads to a reduced service life. + The high current stresses the power supply, which may lead to voltage dip.
هبوط في الفولتية

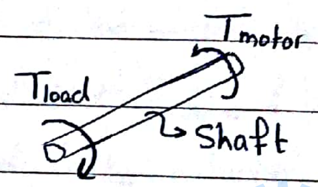
← الهبوط في الفولتية ـ أشي مش منيح ، لأنه الموتور حتى يشتغل لازم نزوده على الأقل بفولتية معينة
V_{min} حتى يشتغل (يعني لو زودنا بفولتية أقل من V_{min} ما رح يشتغل) ، و الهبوط في الفولتية ممكن
يخلي $V_{supplied\ to\ the\ motor} < V_{min}$ ⇐ ∴ ما رح يشتغل الموتور

* Later, we will discuss how to solve this problem.

→ According to Newton's second law

$$\Sigma T = J\,\alpha$$

$$T_{motor} - T_{load} = J\,\frac{d\omega_m}{dt}$$



Initially $\omega_m = zero$ then it becomes non zero (i.e the motor is accelerating)
(In order to accelerate or to start the motor T_{motor} must be > than T_{load})

$$\rightarrow \quad T_{motor} - T_{load} = J \frac{d\omega_m}{dt}$$

at starting $E_a = 0$ since $\omega_m = 0$, while $T_{motor} = K\phi I_a$ is maximum since I_a is max, $I_a = \frac{V_T}{R_a}$
then,
E_a will start to increase as $\omega_m \uparrow$ $(E_a = K\phi\,\omega_m)$

$\Downarrow$

$$I_a = \frac{V_T - E_a \uparrow}{R_a} \Rightarrow I_a \text{ will decrease} \Rightarrow T_{motor} = K\phi I_a \downarrow \text{ will decrease}$$

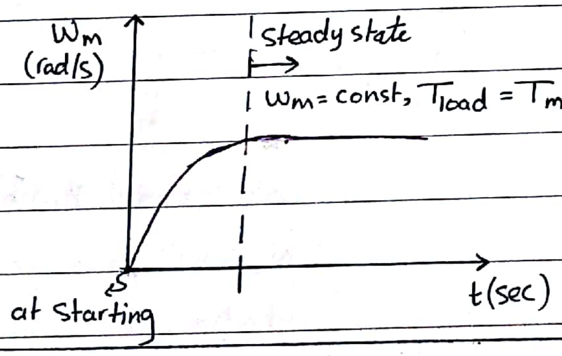
$\Downarrow$

$T_{motor} \downarrow$ until $T_{motor} = T_{load}$ (steady state)

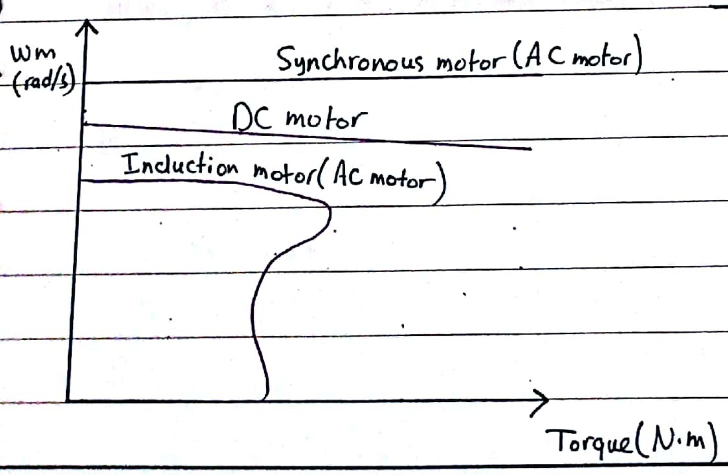
$$\Sigma T = T_{motor} - T_{load} = 0 \Rightarrow \text{i.e } \frac{d\omega_m}{dt} = 0 \Rightarrow \omega_m = \text{constant}$$

(No acceleration)

$\rightarrow$ ω_m can be plotted against t



[*] Charcteristic curves of motors

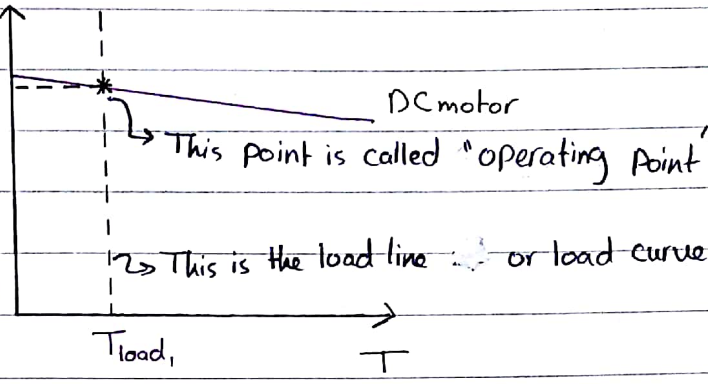


$\rightarrow$ These curves represent the charcterist curves of different types of motors. ω_m is the angular speed of the motor, while the torque represents T_{motor} (Note: at steady state $T_{motor} = T_{load}$).

$\Rightarrow$ Continue

→ From these curves, we can find the angular speed of the motor at steady state for a certain T_{load}, e.g if $T_{load} = T_{load_1}$
at steady state $T_{motor} = T_{load_1} \rightarrow \omega_m = \omega_{m_1}$ from the curve

→ For DC motors, we can see from its curve:

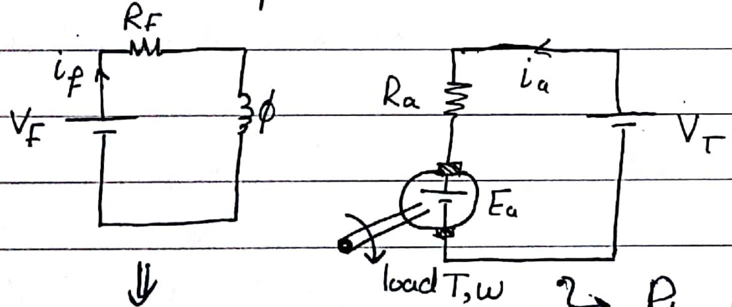


at steady state as $T_{load} \uparrow$, $\omega_m \downarrow$ $\left[\begin{array}{l} T_{load} = T_{motor} \\ @\ \text{steady state} \end{array}\right]$

منطقياً بس أزيد ال load الي مركبه
عالموتور، سرعة الموتور رح تقل

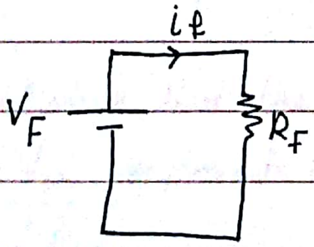
→ In the next lectures, we will learn how to construct these charcteristic curves.

→ Recall: The equivalent circuit of a separately excited DC motor:



$P_{load} = T_{load} * \omega_m = E_a * i_a$
This load might be a fan, elevator ... etc

Recall: from electrical engineering course, in DC circuits, inductors were replaced with wires, so we can draw the field winding circuit as:



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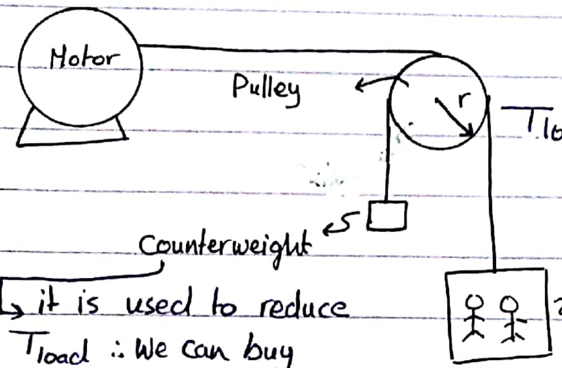
→ Example:



$T_{load} = mgr$

→ Elevator load $= mg$

↳ it is used to reduce T_{load} ∴ We can buy a motor with a less T_{rated}, motor

The largest torque that can be supplied by the motor

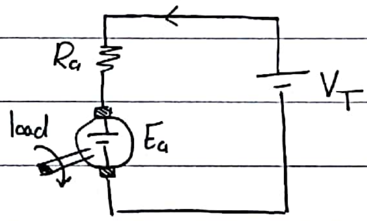
@ Starting T_{motor} must be greater than T_{load}, in order to accelerate the motor

@ Steady State

$T_{motor} = T_{load}$

[*] Charcteristic curve of a separately excited DC motor

→ Apply KVL on the armature winding circuit



$$V_T - I_a R_a - E_a = 0 \quad , E_a = K\phi\, w$$

$$\Downarrow$$

$$V_T - I_a R_a = K\phi\, w_m$$

$$\Downarrow$$

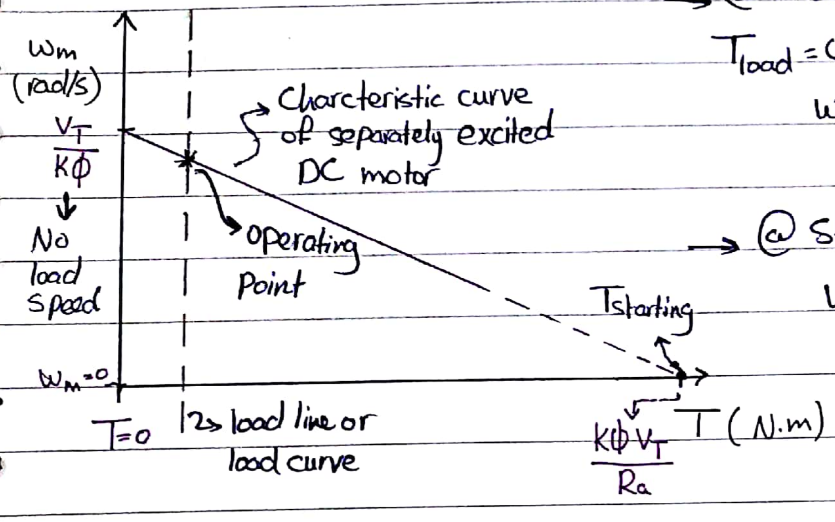
$$w_m = \frac{V_T}{K\phi} - \frac{R_a}{K\phi} I_a \quad , T_{motor} = K\phi I_a \Rightarrow I_a = \frac{T_m}{K\phi}$$

$$w_m = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_m$$

↳ T_{motor} (@ steady state $T_{motor} = T_{load}$)

→ Notice that the relation is linear between w_m and T_m

→ $\frac{V_T}{K\phi}$ represents the y-intercept, $\frac{-R_a}{(K\phi)^2}$ represents the Slope of the line



→ @ Steady state, in the case of no load (i.e $T_{load} = 0$) $\Rightarrow T_{motor} = T_{load} = 0$

$$w_m @_{T=0} = \frac{V_T}{K\phi} \text{ (No load speed)}$$

→ @ Starting $w_m = \text{zero}$

$$w_m = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_m$$

$$\Downarrow$$

$$0 = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_m \rightarrow T_m = \frac{K\phi V_T}{R_a}$$

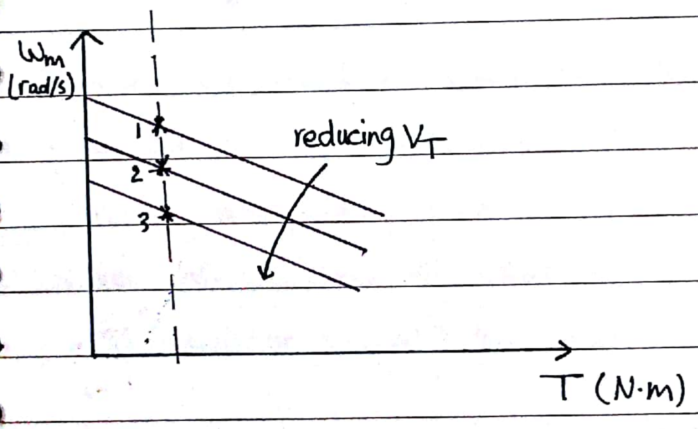
$$\therefore T_{starting} = \frac{K\phi V_T}{R_a}$$

→ @ Steady state

$$w_m = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_m = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T_{load}$$

- For the same T_{load}, How can we reduce the speed of the motor w_m?

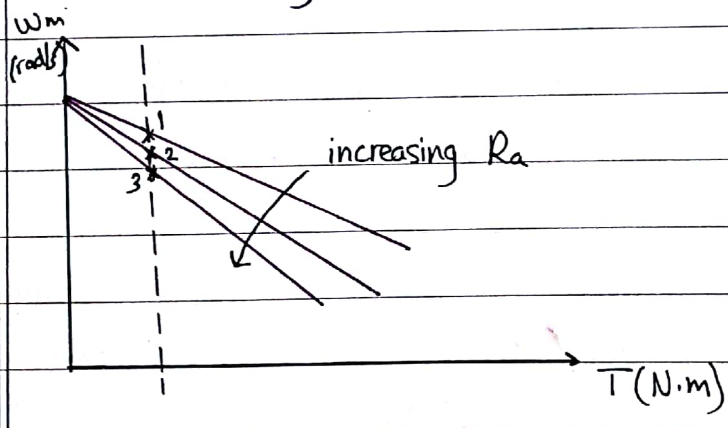
[1] By reducing V_T



→ Notice that reducing V_T will reduce the y intercept, but it will not affect the slope $\left(\frac{-R_a}{(K\phi)^2}\right)$

→ $w_{m_3} < w_{m_2} < w_{m_1}$

[2] By increasing R_a

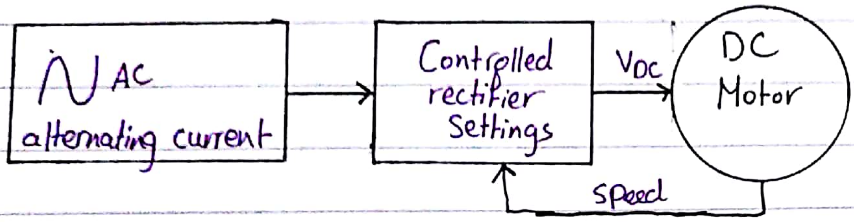


→ Notice that increasing R_a will increase the slope $\left(\frac{-R_a}{(K\phi)^2}\right)$, but it will not affect the y-intercept $= \frac{V_T}{K\phi}$

→ $w_{m_3} < w_{m_2} < w_{m_1}$

→ How can we reduce V_T (or control V_T)?

- Note: again, we control V_T to control the speed ω_m
- We control V_T using rectifier



→ The rectifier has to do 2 tasks

$\boxed{1}$ Converting the alternating current into direct current

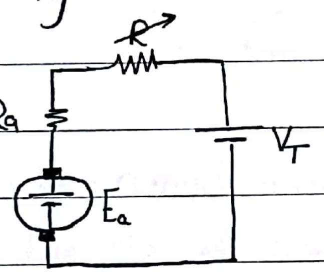
لأنه الكهربا اللي بتوصل من شركة
الكهربا بتكون AC مش DC

$\boxed{2}$ Changing V_T, in order to change the speed ω_m

← المروحة مثلا ، لما أغير سرعتها ، ال rectifier بيغير V_T فبتتغير سرعة المروحة

→ How can we increase R_a?

by adding a variable resistance



$R^{\nearrow}$ = a symbol for the variable resistance

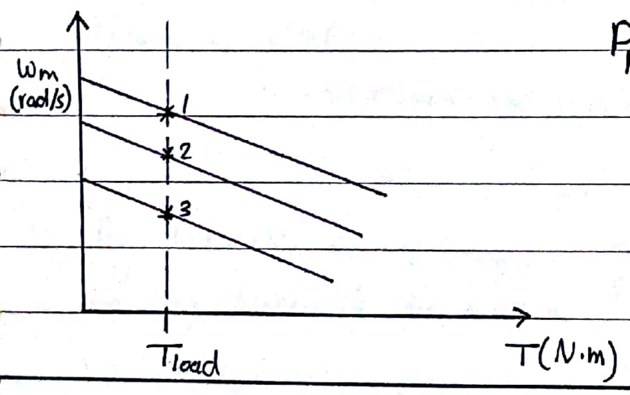
مقاومة متغيرة ممكن نتحكم بقيمتها

→ When we want to reduce ω_m, we often reduce V_T instead of increasing R_a, why?

- because increasing R_a would dissipate more power, while reducing V_T doesn't involve dissipating more power.

→ Why do we need to reduce (or to control) ω_m?

- To reduce (or control) the power of the load

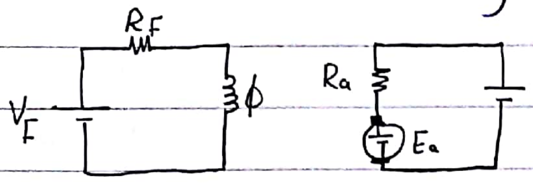


$P_{load} = T\omega$, $P_{load_3} < P_{load_2} < P_{load_1}$

$T_{load_1} = T_{load_2} = T_{load_3}$, but

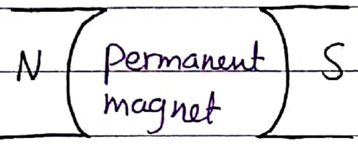
$\omega_{m_3} < \omega_{m_2} < \omega_{m_1}$

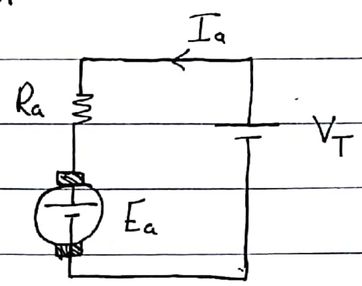
• Note: In separately excited DC motor, Changing V_T will not affect the magnetic flux ϕ, since the field winding circuit is separated from the armature winding circuit.



→ لو غيرنا V_T مش رح يأثر ع ϕ
لانه الدائرتين الكهربائيتين
مش متصلات ببعض

Recall: @ Starting $E_a = zero$, since $\omega_m = zero$





$$I_{starting} = \frac{V_T}{R_a} \ (max) \quad , \ T_{starting} = K\phi I_a = K\phi\left(\frac{V_T}{R_a}\right) \ (max)$$

• High starting current and high starting torque will cause electrical and mechanical damage. How can we solve the problem?

ملاحظة: أنا بس بحتاج انه $T_{starting} > T_{load}$ حتى يشتغل الموتور ، بس ما بحتاج
$T_{starting}$ تكون عالية كثير

We can solve the problem by reducing the starting current temporarily. How?

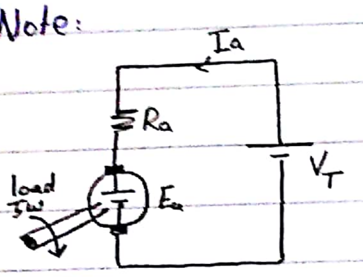
[1] By reducing V_T @ starting [We can supply the motor with $0.2\,V_T$ instead of V_T @ starting $\rightarrow I_{starting} = \frac{V_T}{R_a} \downarrow \rightarrow T_{starting} \downarrow \rightarrow$ Once the motor starts to rotate

↳ We can increase V_T again

* لما انزّل V_T ، لازم أكون ضامنة في البداية انه $V_{starting} > V_{load}$

[2] By increasing $R \rightarrow I_{starting} = \frac{V_T}{R_a} \downarrow \rightarrow T_{starting} \downarrow \rightarrow$ Once the motor starts to rotate we can disconnect the resistance.

* Note:



→ $P_{load} = T\omega = E_a I_a = V_T I_a - I_a^2 R_a$

• Apply KVL $V_T - E_a - I_a R_a = 0$

$\Downarrow$

$E_a = V_T - I_a R_a$ (multiply both sides by I_a)

$E_a I_a = V_T I_a - I_a^2 R_a$

→ You may be asked to find I_a, given P_{load}, V_T, R_a

$P_{load} = V_T I_a - I_a^2 R_a$ ⇒ You have to solve a quadratic equation to solve for I_a

→ If the direction of I_a is reversed, the motor will run as a generator

(الموتور رح يتحول لمولد)

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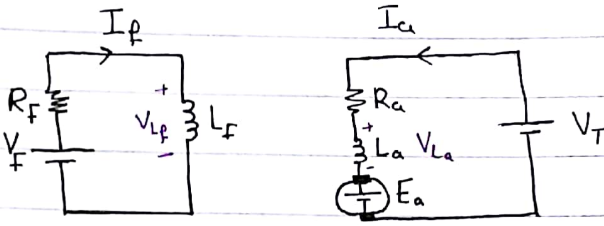
[*] Summary

→ The equivalent circuit of a separately excited DC motor is shown as follows:



"Field winding" circuit — "Armature winding" circuit

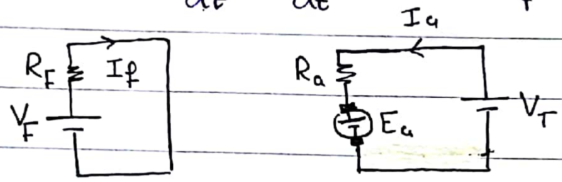
→ L_f : This element represents the field winding which produces the magnetic flux ϕ

→ L_a : represents the armature winding.

→ The voltage across L_f and L_a is given by:

$V_{Lf} = L_f \frac{di_f}{dt}$, $V_{La} = L_a \frac{di_a}{dt}$ ⟹ In DC motors $\frac{di_f}{dt} = \frac{di_a}{dt} = 0$ ∴ $V_{Lf} = V_{La} = 0$

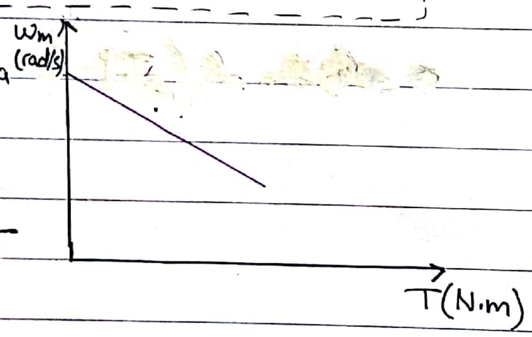
thus, these elements can be replaced by wires, as shown in the adjacent figure



→ Applying KVL on the armature winding circuit $V_T - I_a R_a - E_a = 0 \Rightarrow V_T - I_a R_a = k\phi\omega_m$

$$V_T - \left(\frac{T}{k\phi}\right) R_a = k\phi\omega_m \Rightarrow \omega_m = \frac{V_T}{k\phi} - \frac{T}{(k\phi)^2} R_a$$

The relation is linear between ω_m and T



→ We control the speed of the motor ω_m by controlling the voltage V_T.

In separately excited DC motors, change V_T will not affect I_f

∴ The magnetic flux ϕ will not be affected by changing V_T

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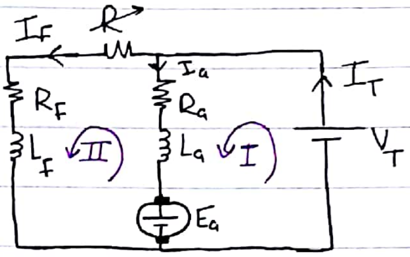
[✳] Shunt DC motors

→ The equivalent circuit of shunt DC motors is shown as follows:



→ I_T = terminal current
I_a = armature current
I_F = field current

→ Shunt DC motors are cheaper than separately excited DC motors, because shunt DC motors contain 1 voltage source V_T, while separately excited DC motors contain 2 voltage sources V_T, V_F.

→ We control the speed of the motor ω_m by controlling V_T. Changing V_T will lead to a change in I_F ∴ ϕ will change when V_T changes

⇓

To solve this problem, a variable resistance is added R to keep I_F unchanged

← لو زدنا دة V_T تؤدي إلى زيادة I_F ⇐ بالتالي بزيد قيمة R عشان أحافظ على قيمة I_F الأصلية

← لو قللنا V_T ، I_F رح تقل ⇐ بقلل قيمة R عشان أحافظ على قيمة I_F

→ Apply KVL on loop I

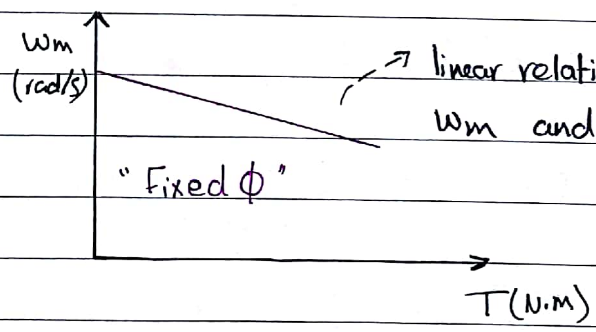
$$V_T - R_a I_a - E_a = 0 \Rightarrow V_T - R_a I_a = k\phi\omega_m \Rightarrow V_T - R_a\left(\frac{T}{k\phi}\right) = k\phi\omega_m$$

↓

$$\omega_m = \frac{V_T}{k\phi} - \frac{R_a}{(k\phi)^2}T$$

↓

نفس الـ equivalent DC motor



linear relation between ω_m and T

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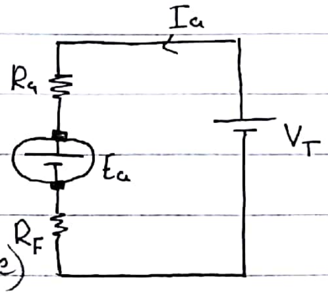
[*] Series DC motors

→ The equivalent circuit of Series DC motor is shown as follows:



→ $R_{F,series} \ll R_{F,shunt}$, why? ($R_F$ = field resistance)

@ Steady state, $T_{load} = T_{motor} = K\phi I_a$, I_a must be kept high enough in order to deliver the required torque needed by the load.

Hence, to keep I_a high $R_{F,series}$ must be very low, while $R_{F,shunt}$ must be high [In shunt DC motor $I_T = I_F + I_a$, when $R_{F,shunt}$ is high, we ensure that I_F is small while I_a is high]

In addition, since $R_{F,series}$ is small, it will cause a small voltage drop. For $R_{F,shunt}$, the voltage across $R_{F,shunt}$ will always be V_T. However, since $R_{F,shunt}$ is high, it will dissipate a little amount of power $\Rightarrow P = \dfrac{V_T^2 \to \text{fixed}}{R_{F,shunt} \to \text{high}}$ low ⇓

* Recall: $R = \dfrac{\rho L}{A}$ ⇒ Resistance can be reduced by increasing the cross sectional area.

$\therefore A_{R_{F,series}} > A_{R_{F,shunt}}$

→ Notice: in series DC motors $I_a = I_F$

→ Apply KVL on the equivalent circuit: $V_T - I_a(R_F + R_a) - E_a = 0$

$$V_T - I_a(R_a + R_F) = E_a = K\phi\omega_m \Rightarrow V_T - \left(\frac{T}{K\phi}\right)(R_a + R_F) = K\phi\omega_m$$

$$\Downarrow$$

$$\omega_m = \frac{V_T}{K\phi} - \frac{T}{(K\phi)^2}(R_a + R_F)$$

→ We know that the field current I_F produces the required magnetic flux ϕ.

(As $I_F \uparrow \Rightarrow \phi \uparrow$)

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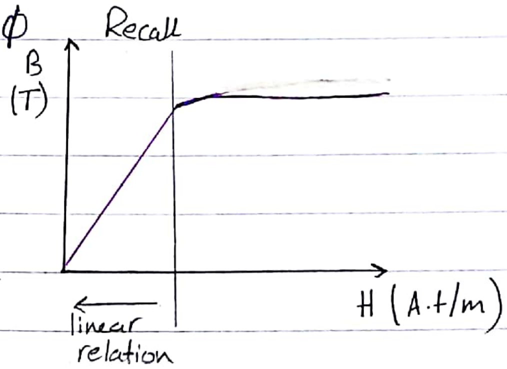
→ We can assume that the relation between I_F and ϕ is linear, since there is a linear relation between B (related to ϕ) and H (related to I_F) in the B-H curve



$\therefore \phi = C I_F \Rightarrow$ since $I_F = I_a$ in Series DC motors $\Rightarrow \phi = C I_a$

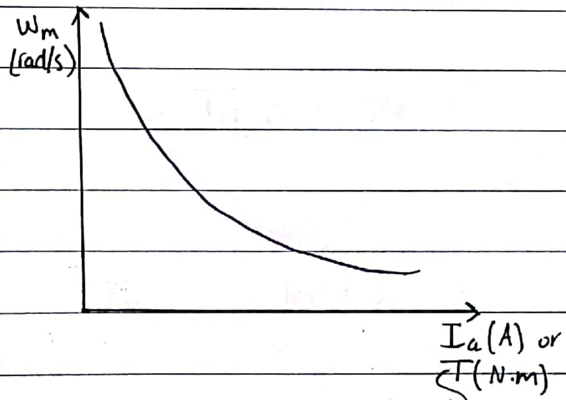
→ Thus, we can write:

• $T = K\phi I_a = KCI_a^2$

• $\omega_m = \dfrac{V_T}{K\phi} - \dfrac{(R_a+R_F)}{(K\phi)^2} T \Rightarrow \omega_m = \dfrac{V_T}{KCI_a} - \dfrac{(R_a+R_F)}{K^2C^2\cancel{I_a^2}} * \cancel{KC}\cancel{I_a^2}$

$$\omega_m = \underbrace{\frac{V_T}{KC}}_{\text{Const}}\frac{1}{I_a} - \underbrace{\frac{(R_a+R_F)}{KC}}_{\text{const}}$$

* الرسمة اللي بتبين العلاقة بين ω_m و I_a زي رسمة $y=\frac{1}{x}$



→ 2 conclusions can be drawn from this plot:

$\boxed{1}$ @ No load (i.e $I_a \to 0$) $\Rightarrow \omega \to \infty$

$\therefore$ Do not operate series DC motors @ no load

$\boxed{2}$ @ Starting (i.e $\omega \to 0$) $\Rightarrow T_{starting} \to \infty$

⇓

This type of DC motors is useful in applications where starting torque required is high such as tractors and cranes

$T = KCI_a^2 \rightarrow I_a = \sqrt{\dfrac{T}{KC}}$

$$\therefore \omega_m = \underbrace{\frac{V_T(\sqrt{KC})}{KC}}_{\text{const}}\frac{1}{(\sqrt{T})} - \underbrace{\frac{(R_a+R_F)}{KC}}_{\text{const}}$$

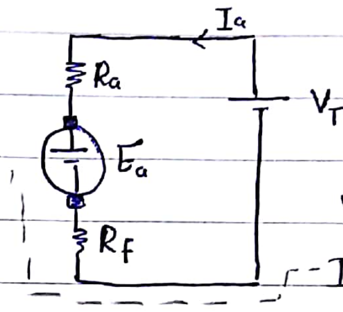
* الرسمة اللي بتمثل العلاقة بين ω_m و T زي رسمة $y=\frac{1}{\sqrt{x}}$ ، رسمة $y=\frac{1}{\sqrt{x}}$ بتشبه رسمة $y=\frac{1}{x}$

→ prove that $T_{starting,series} > T_{starting,shunt}$.

- Series: $T_{starting} = K\phi I_{a,starting} = KC I_{f,starting} I_{a,starting}$, $I_a = I_f$

$\therefore T_{st,series} = KC I_{a,st}^2$

$$T_{st,series} = KC\left(\frac{V_T}{R_a+R_F}\right)^2$$



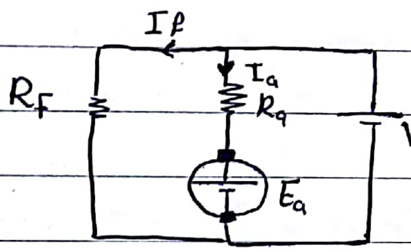
@ Starting $E_a = 0$

Apply KVL:

$V_T - I_a(R_a + R_F) = 0$

$I_a = \dfrac{V_T}{R_a+R_F}$

- Shunt: $T_{st} = K\phi I_{a,st} = KC I_{f,st} I_{a,st}$, $I_a \neq I_f$



@ starting, $E_a = 0$

$\Rightarrow I_{f,st} = \dfrac{V_T}{R_F}$, $I_{a,st} = \dfrac{V_T}{R_a}$

$$\therefore T_{st,shunt} = KC \frac{V_T}{R_F}\frac{V_T}{R_a} = KC\frac{V_T^2}{R_F R_a}$$

$T_{st,series} = KC\left(\dfrac{V_T}{\underbrace{R_a}_{\text{Small} \approx 0.1\,\Omega}+\underbrace{R_F}_{\text{small}}}\right)^2$, $T_{st,shunt} = KC\dfrac{V_T^2}{\underbrace{R_F}_{\text{large}}\underbrace{R_a}_{\text{Small} \approx 0.1\,\Omega}}$, Recall: $R_{F_{series}} \ll R_{F_{shunt}}$

$\therefore T_{st,series} > T_{st,shunt}$ → If the load requires high starting torque, then use series DC motors, otherwise, don't use them, since they will cause mechanical damage.

[*] Speed control of DC motors

→ The speed of a DC motor is given by

$$\omega = \frac{V_T}{K\phi} - \frac{R_a}{(K\phi)^2} T \quad \text{(general)}$$

↳ general equation means it is applicable for all types of DC motors

→ From the above equation, we can see that changing the speed can be attained by:

[1] Changing the voltage V_T [@ constant flux] which is achieved using controlled rectifiers

[2] Using variable resistance R

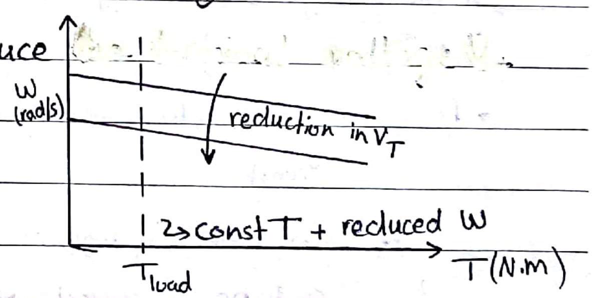
[3] Changing the flux ϕ, which is achieved by changing I_{Field}.

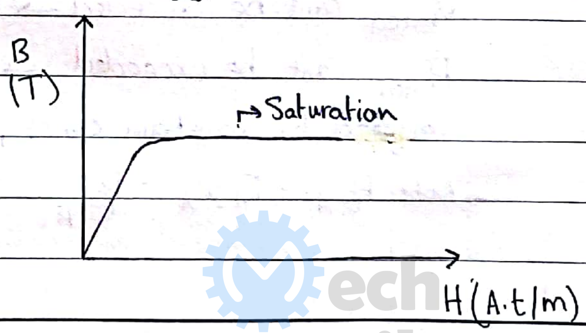
→ Also, from the above equation, we can see that:

- Increasing V_T increase ω
- Increasing R reduce ω
- Increasing ϕ reduce ω ⇒ In the electrical machines lab, when you deal with DC motors, the instructors will ask you to open the "armature" circuit **first**, then you can open the "field" circuit. why? If you open the field circuit first, ϕ becomes zero, from the above equation $\omega = \frac{Const}{0} \Rightarrow \omega \to \infty$ (which is dangerous). Thus, you have to open the armature circuit before opening the field circuit

من هون منستنتج انه الانقصان في ϕ يؤدي إلى زيادة ω والعكس صحيح ← يعني زيادة ϕ تؤدي لنقصان ω

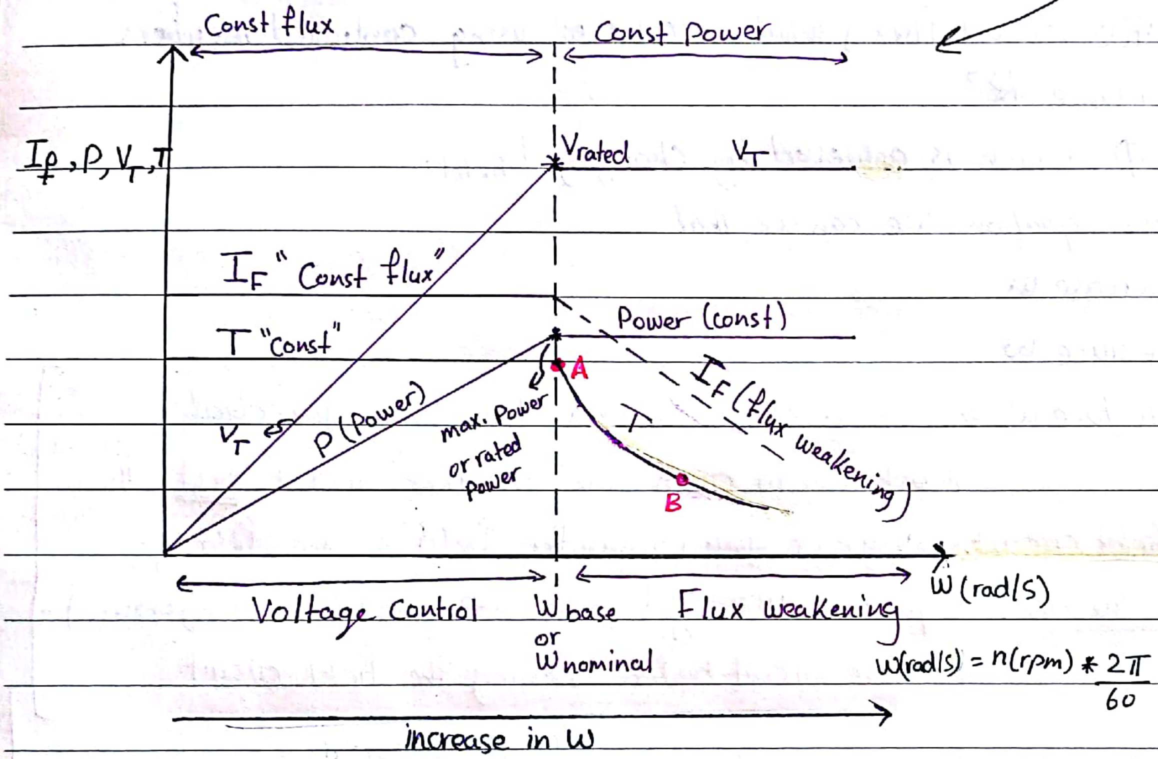
→ When we want to reduce ω, we tend to reduce the terminal voltage V_T. We rarely reduce ω by increasing ϕ, why? Increasing ϕ is achieved by increasing I_f. If we increase I_f significantly, we will reach the saturation region (i.e further increase in I_f will not change ϕ). If ϕ remains unchanged, ω_m will not change (لا تتغير)





→ When we want to increase ω significantly, we tend to weaken the magnetic flux instead of increasing V_T, why?
Every machine has a rated voltage V_{rated} which can't be exceeded.
Hence, we can increase ω by increasing V_T until we reach $V_{T,rated}$. After that, further increase in ω can be attained by flux weakening.

وهاد الرسم موضح هوم



→ In the voltage control region: we increase V_T until we reach V_{rated} @ const flux [i.e I_f is const]

- $P_{load} = \underbrace{T_{load}}_{const} \omega$, ω is increasing $\therefore$ P will increase until P_{rated} is reached

ما بقدر التجاوزه

→ In the flux weakening region: I_f is reduced to reduce ϕ
V_{rated} can't be exceeded → V_T stays fixed
P_{rated} can't be exceeded → $P_{load} = T_{load}\,\omega$, ω is increasing $\therefore$ T must be reduced to maintain const power

→ Note $P_{@A} = T_A \omega_A = P_{@B} = T_B \omega_B$ (الدكتور حكى ممكن يجيب سؤال ع هاد الرسم)

Example: 220 V DC shunt motor, $R_a = 0.2\ \Omega$ $R_F = 110\ \Omega$

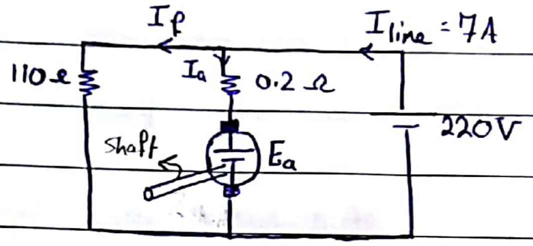
@ no load $\Rightarrow$ $n_{NL} = 1000$ rpm , I_{line} (or $I_{terminal}$) = 7A

@ full load $\Rightarrow$ $P_{in} = 11$ kW and air gap flux is fixed

Find: Speed, Speed regulation, $T_{developed}$ @ Full load

Solution

@ no load



$$I_f = \frac{V_T}{R_F} = \frac{220}{110} = 2A$$

$$\therefore I_a = 7 - 2 = 5A$$

note: We know that @ no load, $T_{load} = 0$. We also know that at steady state $T_{load} = T_m = K\phi I_a$. Since $T_{load} = 0 \Rightarrow I_a$ must be = zero! However, in our example $I_a \neq 0$. This indicates that there are some mechanical losses due to friction [i.e there is no actual load that is attached to the shaft, but since there is friction between the shaft and other mechanical components $\rightarrow T_{load} \neq 0 \Rightarrow I_a \neq 0$]

such as bearings

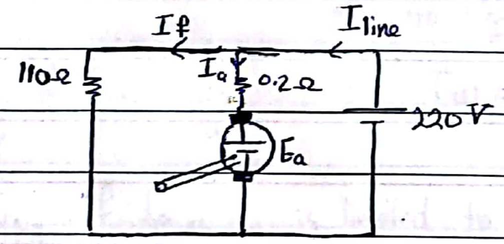
مش مركب اشي

عال Shaft (يعني مش مركب مروحة مثلا)

$$E_a\Big|_{@ \text{no load}} = 220 - 5 * 0.2 = 219\ V$$

@ Full load

$I_f = 2A$ (air gap flux is fixed)



$$P_{in} = 11 * 10^3 = V_T * I_{line} = 220 * I_{line}$$

$$I_{line, \text{Full load}} = 50\ A$$

$$I_{a,FL} = 50 - 2 = 48A$$

$$E_a\Big|_{@ \text{full load}} = 220 - 0.2(48) = 210.4\ V$$

* To find n_{FL} :

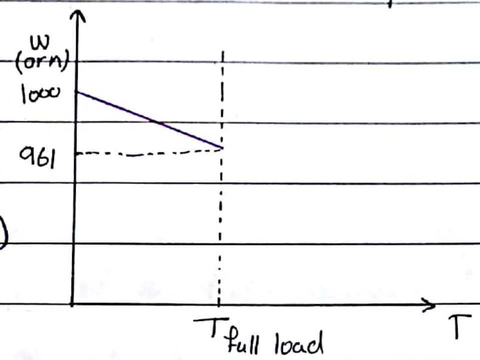
$$E_a = K\phi\, w \Rightarrow \frac{E_{a,FL}}{E_{a,NL}} = \frac{K\phi\, w_{FL}}{K\phi\, w_{NL}} = \frac{\cancel{K\phi}\, \frac{n_{FL} * \cancel{2\pi}}{\cancel{60}}}{\cancel{K\phi}\, \frac{n_{NL} * \cancel{2\pi}}{\cancel{60}}}$$

$$\frac{210.4}{219} = \frac{n_{FL}}{\underbrace{1000}_{\text{given}}} \Rightarrow n_{FL} = 961 \text{ rpm}$$

* الدكتور حكى انه دايماً نكتب
السرعة بوحدة rpm

* notice that $n_{@\text{full load}} < n_{@\text{no load}}$

(For shunt DC motors, the increase in the load, will reduce the speed of the motor)



* speed regulation $= \frac{n_{NL} - n_{FL}}{n_{FL}} * 100 = \frac{1000 - 961}{961} * 100 = 4.09\%$

This value indicates that

$n_{NL} = n_{FL} + 0.0409\, n_{FL} = 1.0409\, n_{FL}$

* T developed @ Full load

$$P_{load} = E_a I_a = T w \Rightarrow T = \frac{210.4 * 48}{961 * \frac{2\pi}{60}} = 100.3 \text{ N.m}$$

$P_{elect} = P_{mech}$

or

$$E_a = K\phi\, w \Rightarrow K\phi = \frac{E_a}{w} = \frac{210.4}{961 * \frac{2\pi}{60}} = 2.09$$

$$T = K\phi\, I_a = \frac{E_a}{w} I_a \Rightarrow T w = I_a E_a \text{ (The same result)}$$

* Find $T_{starting}$, if $I_{a,starting}$ is limited to 150% of $I_{a@FL}$

→ If $I_{a,starting}$ is not limited.

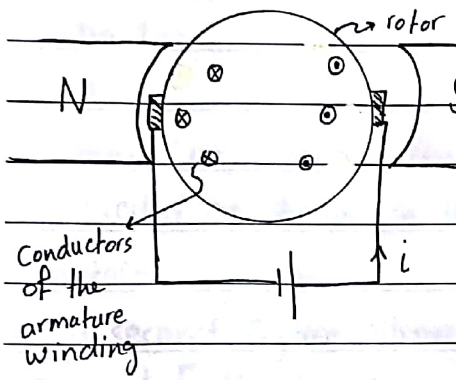
@starting $E_a = 0$, $I_a = \frac{V_T}{R_a} = \frac{220}{0.2} = 1100$ A → Recall: high $I_{starting}$ and $T_{a,starting}$ may cause mechanical and electrical damage. This effect may be reduced by reducing V_T or increasing R temporarily

→ However, $I_{a,starting}$ is limited. $I_{a,starting} = 1.5 * I_{a@FL} = 1.5*48 = 72\ A$

$$T_{starting} = K\phi\, I_{a\,starting} = 2.09 * 72 = 150.48\ N.m$$

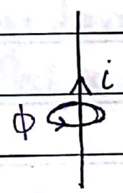
Notice that $T_{starting} > T_{@\text{Full load}}$ [$T_{starting}$ must be high enough to accelerate the motor]

[*] Armature reaction



→ So far, we said that:
In motors, when current flows through the conductors of the armature winding + in the presence of the field magnetic flux ϕ_f which is formed by the permanent magnet or the electromagnet, Torque will be produced and hence, the rotor will rotate.
→ There is an approximation in this model.

→ We know that any wire that carries current will produce magnetic flux as shown below:

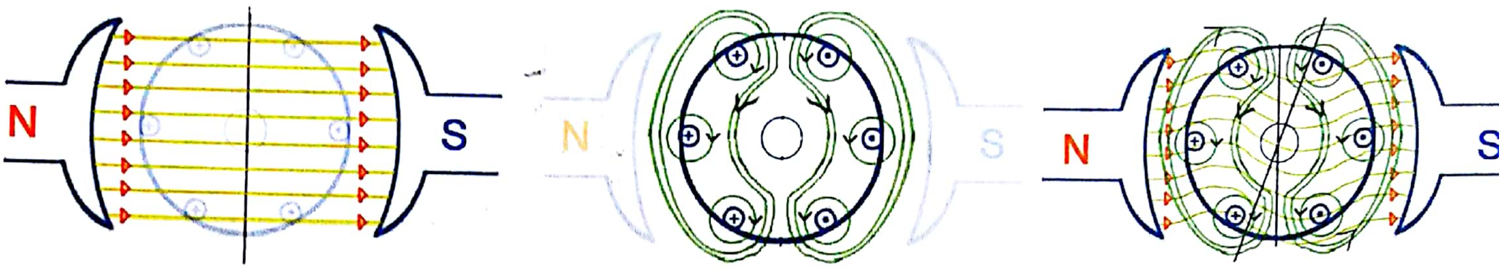


⇒ Use your right hand to determine the direction of ϕ. Point your thumb in the direction of i, curl your fingers around the wire. Your fingers indicate the direction of ϕ

→ Since there is a current I_a flowing through the conductors of the armature winding, we can say that 2 kinds of flux are present in motors:
[1] Main field flux (resulting from the field winding)
[2] Armature flux (resulting from the armature winding)
i.e $\phi_{net} = \phi_{Field} + \phi_{armature}$

→ There was an approximation in the previous model, because we were neglecting the armature flux.

→ The effect of the armature flux on the main field flux is called armature reaction.



Main field flux
Fig. 1

Armature Flux
Fig. 2

Distortion of main field flux due to armature flux - Armature reacti
Fig. 3

* Consider no current is flowing in the armature conductors and only the field winding is excited as shown in the 1st figure. In this case, magnetic flux lines are uniform

* The second figure shows only the armature flux lines due to the armature current [The 1st figure shows ϕ_{field} only while the 2nd figure shows $\phi_{armature}$ only]

* Now, when a DC motor is running, both ϕ_{field} and $\phi_{armature}$ will be present at a time. The armature flux superimposes with the main field flux and hence, distorts the main field flux (as shown in the 3rd figure). This effect is called armature reaction

→ Notice that $\phi_{armature}$ depends on I_a

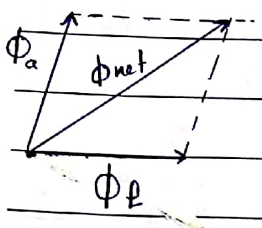
@ No load $I_a = 0 \Rightarrow \phi_{armature} = 0$ (No armature reaction)

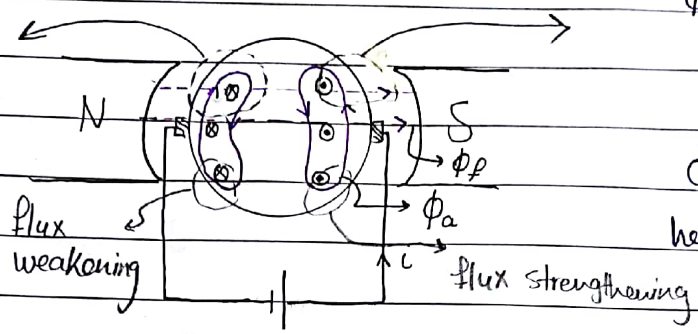
@ Full load I_a is max $\Rightarrow \phi_{armature}$ is max (Armature reaction effect is max)

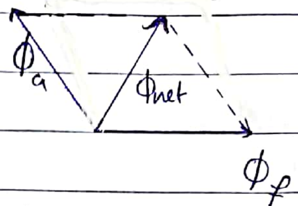
Conclusion: As we increase the load, the effect of the armature reaction will ↑.

→ Armature reaction has 2 effects:

[1] It weakens the main flux



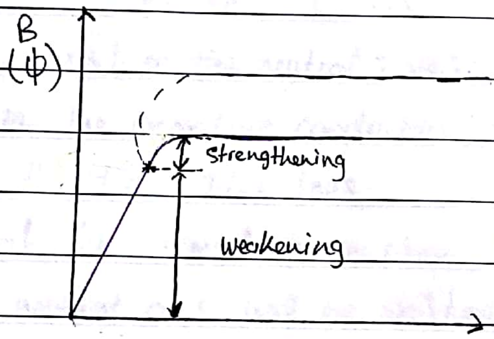




$\phi_{net} > \phi_f$
here, the flux is strengthened

$\phi_{net} < \phi_f$
here the flux is weakened

* بس لاحظة ال armature reaction بيضعف ال main flux ككل ؟



→ Assume that this is the operating point of the motor @ no load (i.e $\phi_{net} = \phi_{field}$).
When I_a starts to flow, $\phi_{armature}$ will be developed and it will
[1] Strengthen the main flux @ the upper left and lower right regions (As shown in the above figure)

↳ As we increase I_a (i.e increase ϕ_a), these regions will reach saturation [i.e further increase in I_a will not affect the magnetic flux in these regions]

[2] Weaken the main flux @ the upper right and lower left regions (As shown in the above figure). Further increase in I_a will reduce the magnetic flux in those regions more and more.

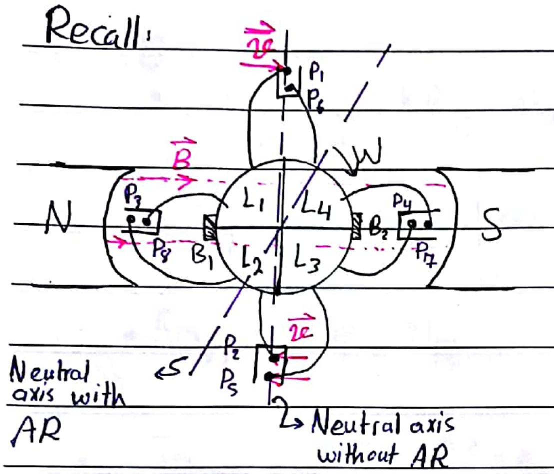
* Overall, the armature reaction weakens the main flux, since as I_a starts to flow
  → The strengthened regions reach saturation immediately and hence further increase in I_a will not affect the flux in these regions
  ↳ The magnetic flux, however, will be reduced more and more in the weakened regions

* In motors, weakening ϕ will increase the speed of the motor

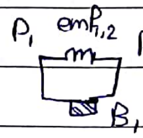
→ Armature reaction has 2 effects

$\boxed{2}$ It distorts the main flux and hence the position of neutral axis gets shifted.

Recall:



→ When we tried to construct the schematic diagram of the armature winding, we found that



a short circuit is formed, because B_1 is in contact with L_1 and L_2

→ However, we avoid it by letting the conductors move $\vec{v}$ parallel to the field lines $\vec{B}$ and hence $emf_{1,2} = (\vec{v} \times \vec{B}) \cdot \ell = 0$

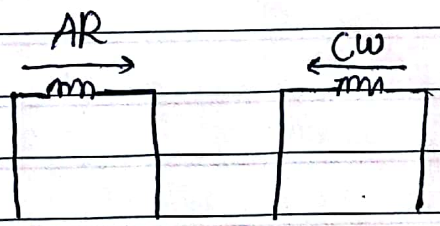
→ The neutral axis (or the neutral zone) is the axis along which no emf is generated in the armature conductors, since these conductors move parallel to the field flux lines

→ Because of the armature reaction, the main flux gets distorted and hence the neutral axis will be shifted (as shown in the above figure)

• Note AR = armature reaction

→ Since the neutral axis is shifted, the formation of the short circuit will not be avoided! This will lead to sparking at the surface of the brushes

→ To solve this problem, a compensating winding (CW) is added to produce a magnetic flux that opposes the flux of the armature winding, without affecting the main flux or field flux



*Reconsider the previous example and Find the speed, speed regulation, T developed at full load, if the armature reaction reduces the magnetic flux by 50% at full load

Solution:

- $E_{a}|_{@\text{no load}} = K\phi_{NL}\, W_{NL}$ at no load, armature reaction doesn't affect the main flux
- $\phi_{FL} = 0.5 * \phi_{NL}$ (due to armature reaction)
- $E_{@FL} = K\phi_{FL}\, W_{FL}$
- $\frac{E_{@NL}}{E_{@FL}} = \frac{K\cancel{\phi_{NL}} * \cancel{\frac{2\pi}{60}} * n_{NL}}{\cancel{K} * 0.5 * \cancel{\phi_{NL}} * \cancel{\frac{2\pi}{60}}\, n_{FL}} = \frac{n_{NL}}{0.5 * n_{FL}} \Rightarrow \frac{219}{210.4} = \frac{1000}{0.5 * n_{FL}}$

$n_{FL} = \left(\frac{1000 * 210.4}{219}\right) * 2 = 961 * 2$
$= 1922\ \text{rpm}$

- Speed regulation $= \frac{n_{NL} - n_{FL}}{n_{FL}} * 100\% = \frac{1000 - 1922}{1922} = -47.97\%$ السرعة زادت

- T developed

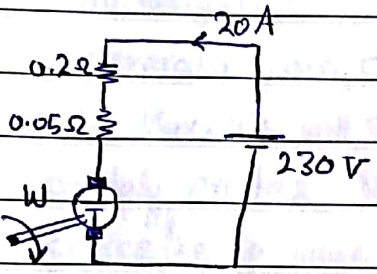
$P_{mech} = P_{elect} \Rightarrow T * w = E_a I_a$

$T * 1922 * \frac{2\pi}{60} = 210.4 * 48 \Rightarrow T = 50.17\ N.m$

نفس القيمة لم تتأثر مع انه ال AR

* Example: 230 V DC series motor with $R_a = 0.2\,\Omega$, $R_F = 0.05\,\Omega$ ↳ Notice how small is this value
$I_a = 20A$, $W = 1500$ rpm, $P_{rot\ losses} = 400\,W$. Find η

$$\eta = \frac{P_{out}}{P_{in}} * 100\% = \frac{P_{in} - P_{loss}}{P_{in}} * 100\%$$



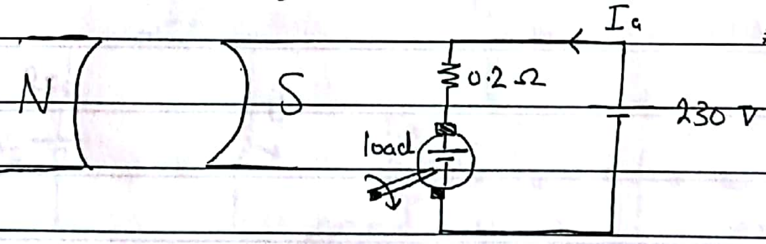
$$P_{in} = V_T * I_T = 230 * 20 = 4600\,W = 4.6\,KW$$

$$P_{loss} = P_{cu,losses} + P_{rotational\ losses}$$

$$= (20)^2 * 0.2 + (20)^2 * 0.05 + 400 = 500 \text{ Watts}$$

$$\therefore \eta = \frac{4600 - 500}{4600} * 100 = 89.13\%$$

Example: Separately excited DC motor



* Assume that P_{load} is given, Find I_a

Solution:

$$E_a I_a = T\omega$$

$$P_{in} = P_{cu,loss} + P_{load} \Rightarrow V_T I_a = I_a^2 * R_a + P_{load}$$

$$230 I_a = I_a^2 (0.2) + P_{load} \text{ given}$$

Solve for I_a, you will get 2 answers I_{a_1}, I_{a_2}

To choose the right answer

known $P_{load} = I_a E_a$

↳ Substitute I_{a_1}, I_{a_2}

Find E_{a_1}, E_{a_2}

Select I_a that will give you a value for E_a which is closer to $V_T = 230$ V

Since R_a is small, the voltage drop across R_a is also small
$\therefore E_a \approx V_T$

[*] DC generators:

→ Generators are electrical machines, used to convert mechanical energy into electrical energy.

→ Any generator must be provided with external source or torque to rotate the rotor (Mechanical energy) and magnetic flux (provided by a magnet) to induce emf (Electrical energy).

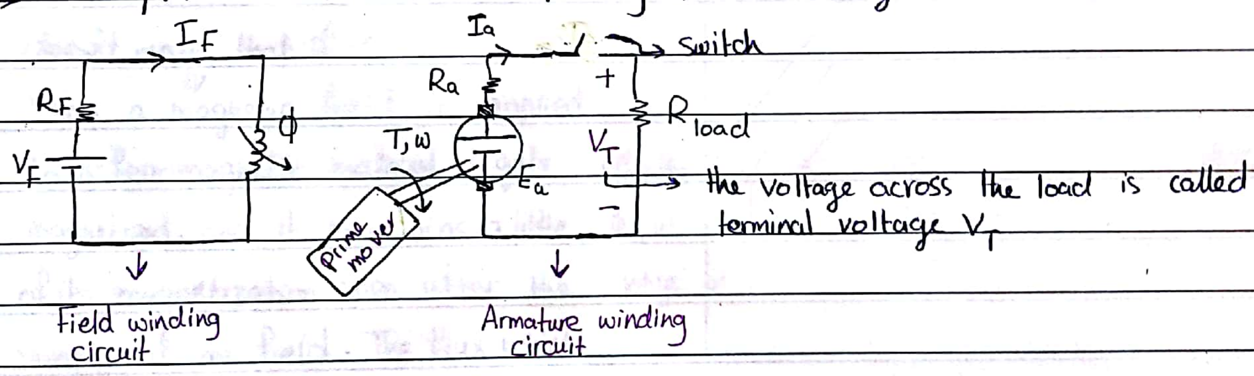
→ DC generators are classified according to the way of excitation of their magnetic flux. We will study 2 types:

[1] Separately excited DC generator: Field winding is excited by external (separate) DC source (i.e I_f, which produces ϕ_f, will be generated by external DC source)

[2] Self excited DC generator: Field winding is excited by the generator itself. (i.e I_f, which produces ϕ_f, will be generated by the generator itself)

[1] Separately excited DC generator:

→ The equivalent circuit of a separately excited DC generator is shown below:

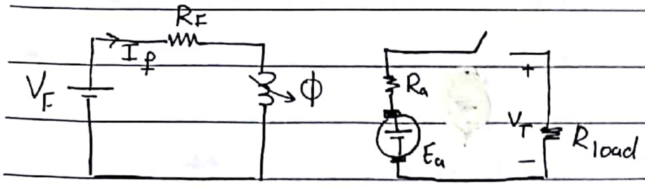


→ The field winding circuit provides the required magnetic flux.

→ A prime mover is a device that delivers motion (mechanical energy) to the generator. In electrical machines lab, you will find that the prime mover is a motor (i.e a motor is operated to supply the generator with mechanical energy). However, in real life, the prime mover might be a windmill [wind energy is used to rotate the blades of the windmill]

→ We will study the characteristics of a separately excited DC generator:

* under no load (i.e when the switch is open) ⇝ it is called open circuit charcteristic
* with load (i.e when the switch is closed) ⇝ it is called load curve charcteristic



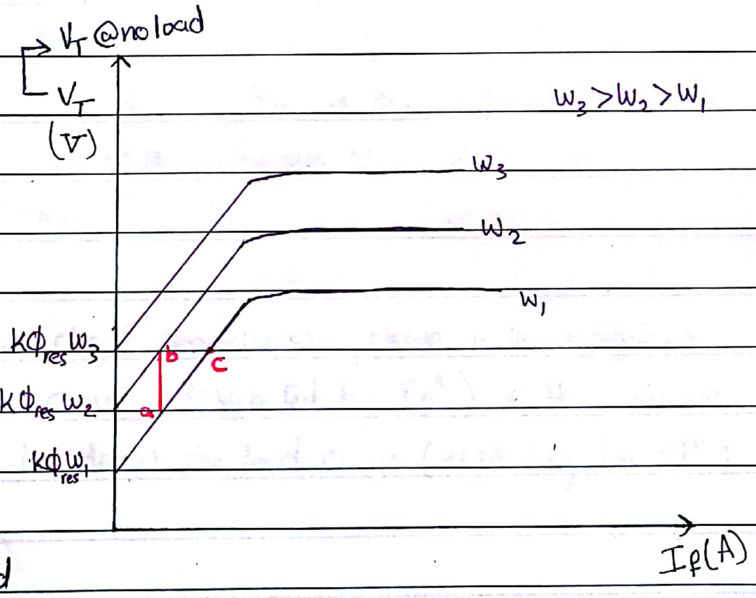
* Open circuit Characteristic (OCC) → [No load]

→ This Characteristic gives the variation of the terminal voltage V_T with field current I_f at a constant speed ω under no load.

→ The data for OCC curve is obtained by operating the generator at no load [opening the switch] and letting the generator be driven at a constant speed by the prime mover. Field current is gradually increased and the corresponding terminal voltage is recorded.

→ When the switch is open
$I_a = 0$, $V_T = E_a = K\phi\omega$

→ Initially, $I_f = 0$ [$I_f = 0$ doesn't mean that $\phi = 0$.
⇓
When a magnetic field is applied to a ferromagnetic material, it gets magnetized, and it maintains a little of its magnetization even after the removal of the field. The flux which remains after the removal of the field is called residual flux. ϕ_{res} . Hence, intially
$V_T = E_a = K\phi_{res}\omega$

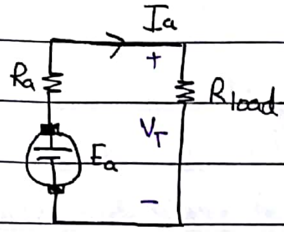


→ As we increase $I_f \Rightarrow \phi\uparrow \Rightarrow V_T\uparrow$. ϕ continues to increase with increasing I_f until saturation is reached. At this point, ϕ remains fixed even if $I_f\uparrow$, thus E_a stays fixed.

→ We can increase V_T by increasing ω (Check the above curves and notice that we can go from a to b by increasing ω). Also increasing V_T may occur by increasing ϕ or I_f (We can go from a to C by increasing I_f)

* Load curve characteristic (with load)

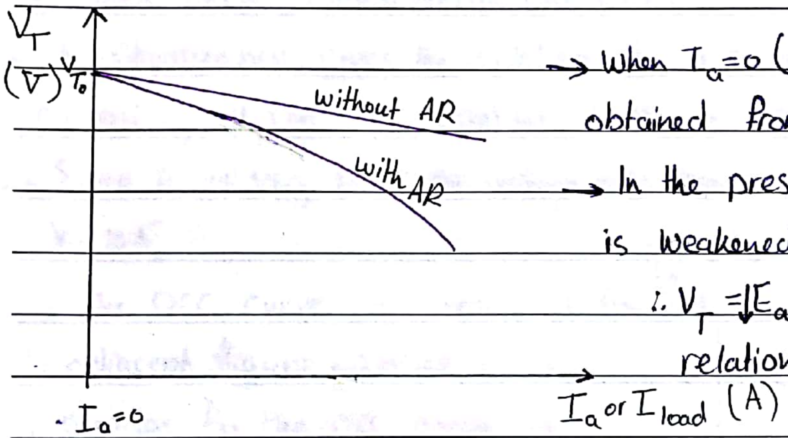
→ This characteristic gives the variation between V_T and I_a



→ Apply KVL to the armature winding circuit

$$V_T = E_a - I_a R_a = K\phi\omega - I_a R_a$$

* Notice the relation is linear between V_T and I_a

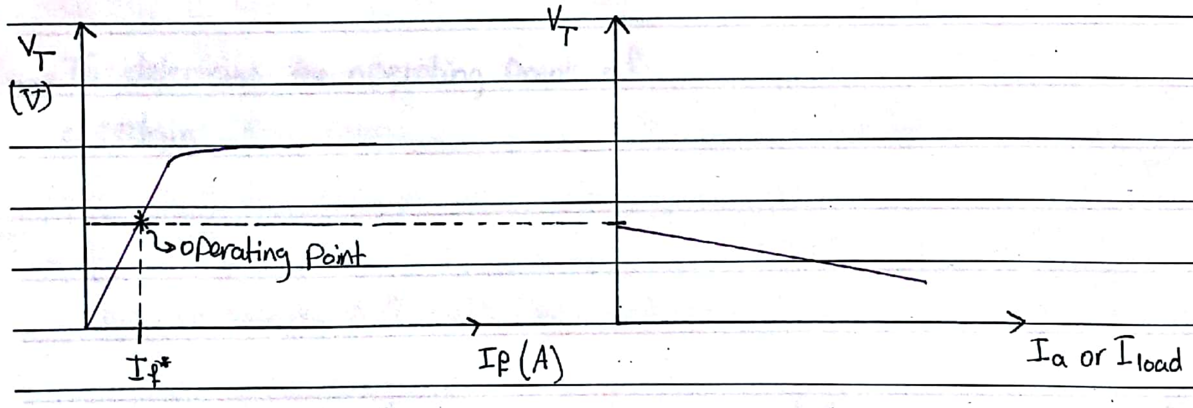


→ When $I_a = 0$ (no load) $V_T = V_{T_o}$ ⇒ This value can be obtained from the open circuit characteristic curve

→ In the presence of armature reaction, the flux is weakened, hence $E_a = K\phi\omega$ is reduced

∴ $V_T = \downarrow E_a - I_a R_a$ is also reduced and the relation between V_T and I_a becomes non-linear

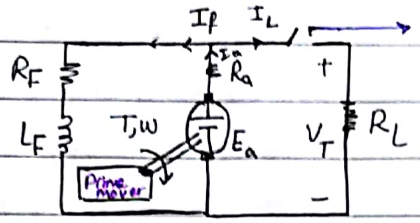
* Suppose you were given the open circuit characteristic curve + the magnitude of I_f which flows in the field winding circuit (designated by I_f^*) + the armature resistance R_a, and you were asked to draw the load curve (neglecting the AR)



→ From the OCC curve find the value of V_T @ $I_f = I_f^*$. This value represents V_{T_o} (the value of V_T at no load i.e @ $I_a = 0$). V_{T_o} is the y-intercept in the load curve. Now, you can draw a line with a slope $m = -R_a$ [because $V_T = E_a - \underbrace{R_a}_{\text{This is the slope}} I_a$]

[2] Self excited DC generator (or Shunt DC generator)

→ The equivalent circuit of a shunt DC generator is shown below



→ We will study the characteristics of shunt DC generators as we did in self excited DC generator.

* Open circuit characteristic (No load)

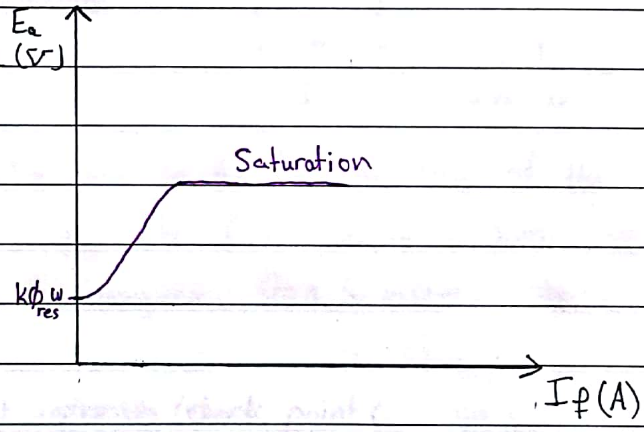
→ This characteristic gives the variation of the armature voltage E_a with the field current I_f at constant speed w under no load (i.e the switch is open).

→ Since R_a is very small, the voltage drop across it is also small. Thus, we will assume that $V_T \approx E_a$.

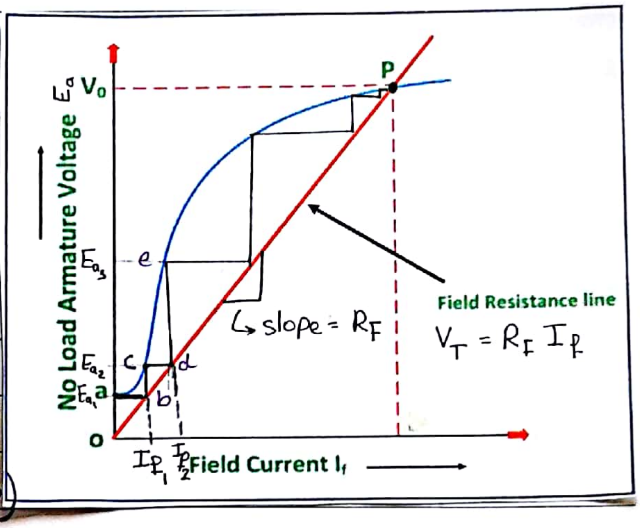
→ The OCC curve is shown in the adjacent figure. Notice, it is similar to the OCC curve of a separately excited DC generator.

Initially, when $I_f = 0 \Rightarrow E_a = K\phi_{res} w$

As $I_f \uparrow \Rightarrow \phi \uparrow \Rightarrow E_a \uparrow$ until saturation is reached

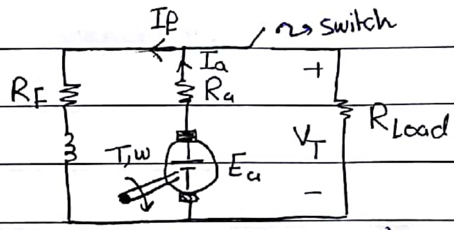


→ To determine the operating point of a certain Shunt DC generator, we will draw the field resistance line ($V_T = R_F I_f$). The point of intersection between the 2 curves represents the operating point P, since at steady state $V_T = E_a$ (neglect R_a)



→ Let's see what happens before reaching the steady state (i.e before reaching the point P)

• Assume that the generator is working at no load (the switch is open) and the prime mover drives the rotor of the generator.



$I_a = I_f$ (under no load)

• Initially, $I_f = 0$ + The generator will generate a voltage $E_a = k\phi_{res}\,w$ (Check point a in the above OCC curve).

• This voltage causes a current I_{f_1} to flow in the field winding of the generator (check point b in the OCC curve). The field current is given by $I_{f_1} = \frac{V_T \text{ or } E_a}{R_F}$. This current causes the magnetic flux to increase. As a result, the armature voltage $E_a = k\phi w$ increases (check point C in the OCC curve)

• The increased E_a increases V_T. With the increase in V_T, the field current increases further to I_{f_2} (check point d in the OCC curve)

• This, in turn, increases the flux ϕ and hence E_a is further increased (Check point e)

• This process continues until the point p is reached (Steady State, $V_T = E_a$)

• Note:

Notice that at point C (unsteady state) ⇒ $I_f = I_{f_1}$, $E_a = E_{a_2}$, $V_T = R_F * I_{f_1} = E_{a_1}$

↓

at this instant $V_T < E_a$ (since we didn't reach the steady state)

at point d ⇒ $I_f = I_{f_2}$, $E_a = E_{a_2}$, $V_T = I_{f_2} * R_F = E_{a_2}$ (at this instant $V_T = E_a$)

في ال unsteady ، E_a بتزيد وال V_T بتلحقها ، وبتصير الشغلة حتى نوصل steady state

لما نوصل ال steady state ⇐ $V_T = E_a$ و ما بتتغير مع الزمن (بتثبت)

[*] Failure to self excite shunt DC generator

↳ Produce magnetic flux

Self excitation in shunt DC generator may fail due to the following reasons:

[1] Insufficient ϕ_{res}

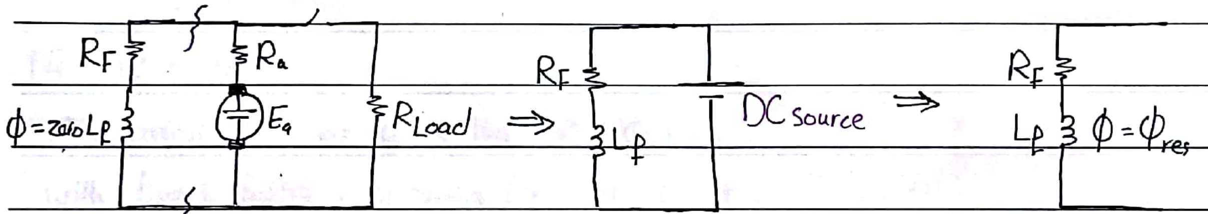
If $\phi_{res} = 0$, and even if you drive the generator at a certain speed ω

$E_a = K\phi_{res}\,\omega = 0$ (initially). Since $E_a = 0 \Rightarrow$ No current will flow in the field winding $I_f = 0 \Rightarrow$ No increase in the magnetic flux will occur $\Rightarrow E_a$ will stay zero

$\therefore$ The DC generator fails to excite itself

$\Downarrow$

To solve this problem, disconnect the field winding from the circuit and apply a DC voltage to the field winding to magnetize it. Then, you can return the field winding back to the circuit (a little amount of flux ϕ_{res} is maintained after the removal of the DC source)



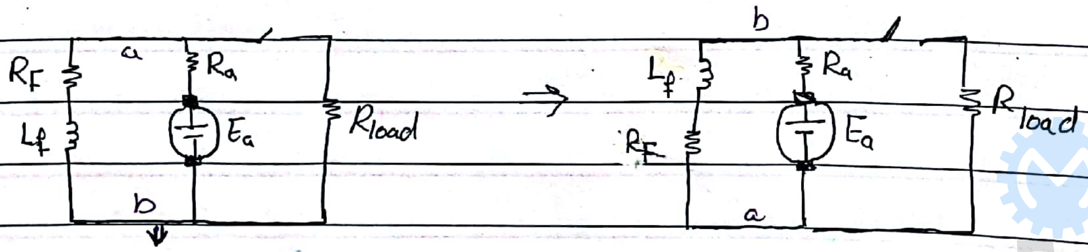
↳ disconnection "magnetization process" "After the removal of DC source, a little amount of flux Φ_{res} is maintained"

[2] Polarity of the field flux reduces the residual flux

If the direction of the field flux is opposite to the direction of the residual flux, the magnetic flux will be weakened

$\Downarrow$

To solve this problem, the field winding terminals should be connected in such a way that the field current increases the flux in the direction of residual flux

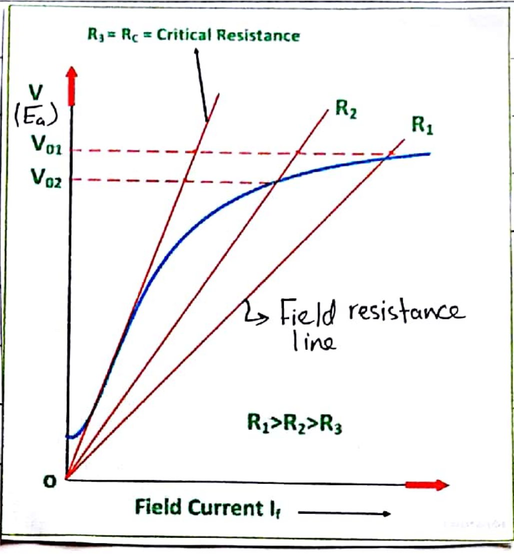


Assume that this configuration reduces the magnetic flux

Reverse the connection of the field winding to reverse the polarity

[3] $R_F > R_{F,critical}$

The OCC curve in Shunt DC generator for various field resistances R_F is shown in the adjacent figure. As we can see, an increase in the field resistance R_F increases the slope of the field resistance line, resulting in a lower voltage. If the field resistance is increased to critical resistance $R_{F,critical}$, the field resistance line becomes tangent to the initial part of the OCC curve. If the value of R_F is higher than $R_{F,critical}$, the generator fails to excite.

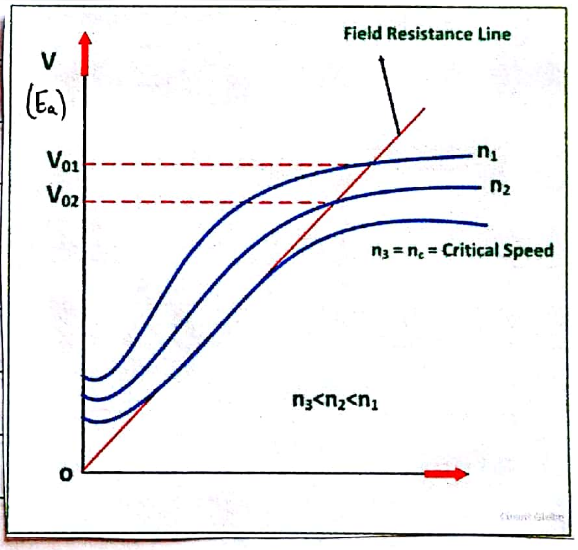


⇓

To solve this problem, always let $R_F < R_{F,critical}$

[4] $w < w_{critical}$

The adjacent figure gives the OCC curves with fixed field resistance R_F and variable speed. If R_F is kept constant and the speed is reduced, all the points on the OCC curve are lowered. At a certain speed, called critical speed, the field resistance line becomes tangent to the OCC curve. Below the critical speed, the generator fails to excite.

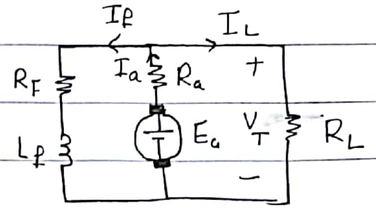


⇓

To solve this problem, always let $w > w_{critical}$

* Load curve Characteristic in Shunt DC generator

→ This curve shows the relation between V_T and I_a in the presence of the load (i.e the switch is closed)

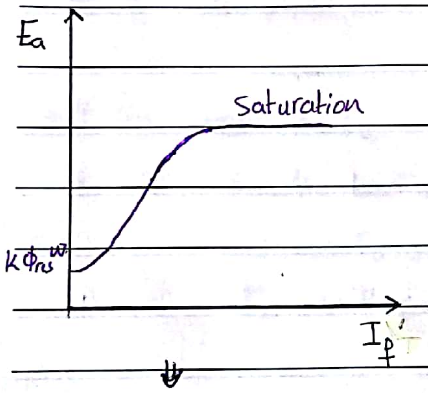


→ In the presence of the load

$I_a = I_f + I_L$ (In the case of no load $I_a = I_f$)

$V_T = E_a - R_a I_a$ (In the case of no load, we neglected R_a. Now we will take R_a into consideration)

→ In the presence of the load the relation between E_a and I_f is shown below



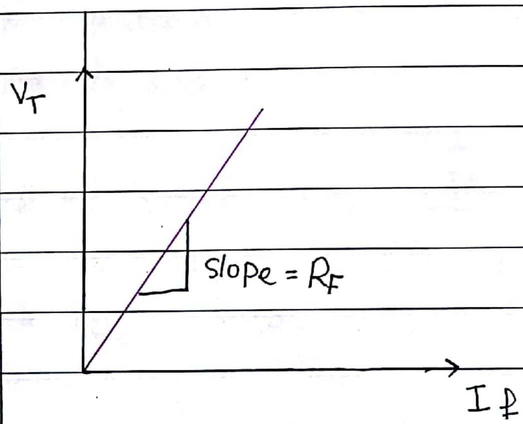
⇓

Increasing I_f will increase Φ

∴ $E_a = K\Phi w$ will increase until saturation is reached

→ In the presence of the load

$V_T = R_F * I_f$



Combining these 2 plots will give

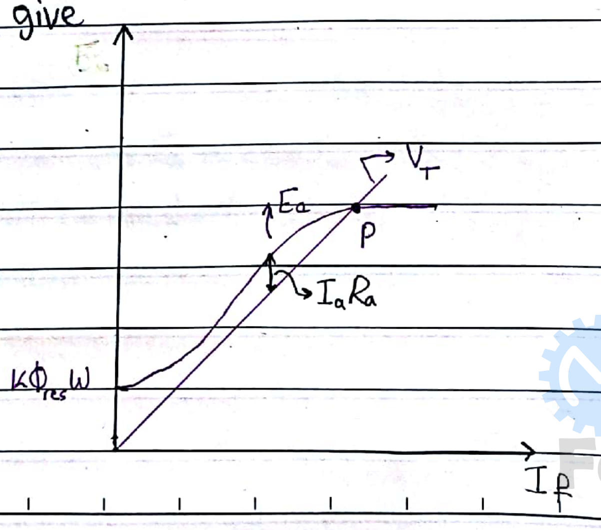
→ From the above equation

$V_T = E_a - R_a I_a$

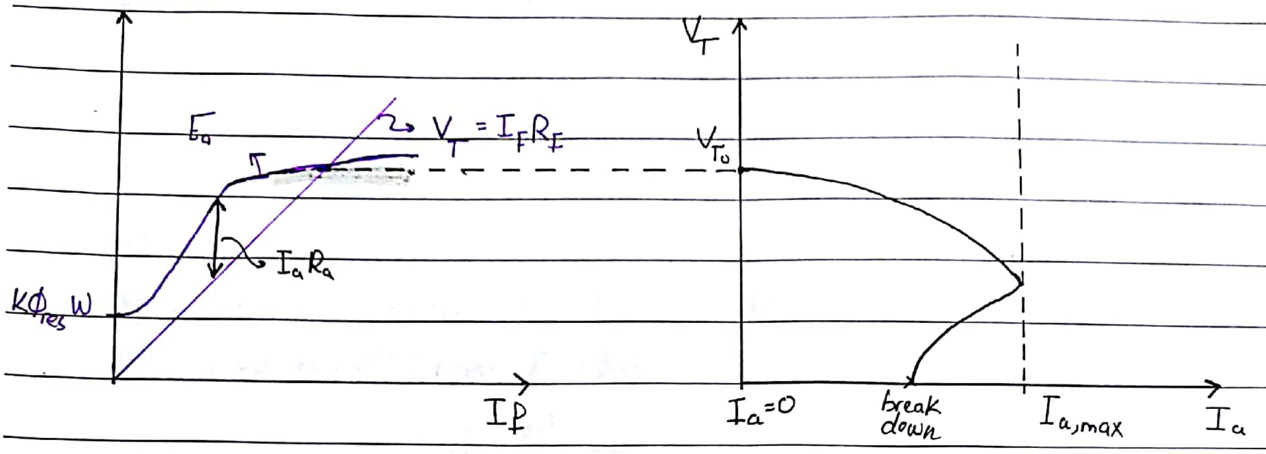
The difference between the 2 curves represents $R_a I_a$.

→ At the point of intersection

$V_T = E_a \Rightarrow \therefore I_a = 0$



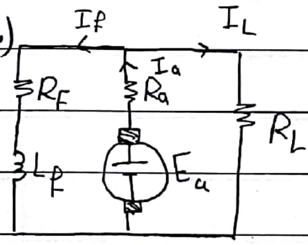
→ We will draw the load curve as shown below



"load curve"

* $V_T = E_a - I_a R_a$ ⟹ @ the point of intersection $V_T = E_a$ ⟹ ∴ $I_a = 0$ (i.e the value of V_T @ the point of intersection represents V_T @ $I_a = 0$)

* if we increase I_a ⟹ $V_T = E_a - I_a R_a$ will decrease ($V_T\downarrow$) as shown in the load curve



* We can increase I_a until we reach $I_{a,max}$ ∴ Further increase in I_a will cause V_T to drop to zero (breakdown)

⇓

Why does this dramatic drop in V_T occur if I_a is increased above $I_{a,max}$?

because increasing I_a causes V_T to decrease

if $V_T\downarrow$ ⟹ $I_f = \frac{V_T\downarrow}{R_F}$ (I_f will ↓)

if $I_f\downarrow$ ⟹ $\phi\downarrow$ ⟹ $E_a = k\phi\downarrow w$ (E_a will decrease)

$V_T = E_a - I_a R_a$ →

* في البداية مع زيادة I_a ← $V_T\downarrow$

← $I_f\downarrow$ ← $E_a\downarrow$ (يعني I_a بتزيد و E_a بتقل بنفس الوقت، ف V_T رح تقل بشكل (nonlinear)

لكن لو تجاوزنا $I_{a,max}$ بتصير E_a تقل بمعدل أسرع، وهذا الاشي بخلي V_T تنهار

Example: The following data is obtained from OCC curve for a shunt DC generator with $R_a = 0.4$, $n = 300$ rpm. Find the OCC curve @ 375 rpm

I_f (A)	0	2	3	4	5	6	7
V_T (V)	75	92	132	162	183	190	212

Solution

* @ $I_f = 0$ $n = 300$ rpm $E_a = k\phi\omega = 75$ ($\omega = \frac{2\pi * 300}{60}$)

@ $I_f = 0$ $n = 375$ rpm $E_a = k\phi\omega$ ($\omega = \frac{2\pi * 375}{60}$)

E_a ??

$$\frac{E_a}{75} = \frac{375}{300} \Rightarrow E_a(@I_f=0, n=375\text{rpm}) = 75 * \frac{375}{300} = 93.75 \text{ V}$$

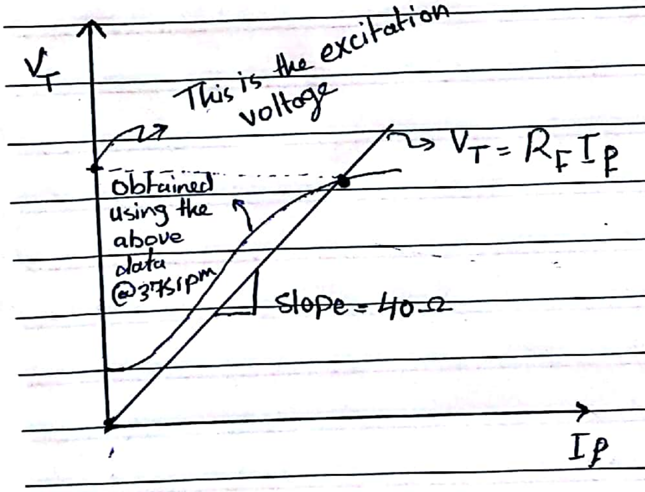
* $E_a(@I_f = 2\text{A}, n = 375) = 92 * \frac{375}{300} = 115$ V ...

* @ $n = 375$ rpm:

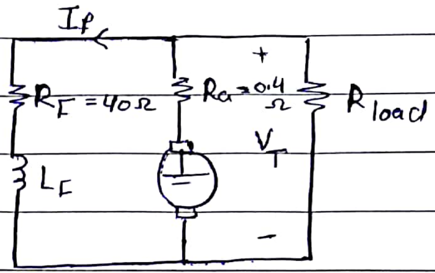
I_f (A)	0	2	3	4	5	6	7
E_a (V)	93.75	115	165	202.5	228.75	237.5	265

→ Determine the voltage @ which the machine will excite when $n = 375$ rpm given $R_F = 40\,\Omega$

↳ Using this table, we have to plot the $E_a - I_f$ curve



→ Find the load current where $V_T = 200\ V$, $R_F = 40\ \Omega$ when $n = 375\ rpm$



$I_f = \frac{V_T}{R_F} = \frac{200}{40} = 5A$

$E_a(@ I_f = 5A, n = 375\ rpm) = 229\ V$
↓
Check the previous table

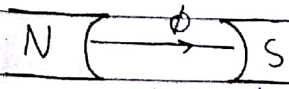
$V_T = E_a - I_a R_a \Rightarrow 200 = 229 - I_a * 0.4 \Rightarrow I_a = 72\ A$

$I_a = I_f + I_L \Rightarrow I_L = 72 - 5 = 67\ A$

[*] 3ϕ induction motor:

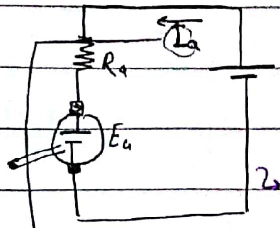
→ Recall:

- In motors, torque (mechanical energy) is produced if i (electrical energy) + ϕ exist.
- In DC motors:



⇓ ↳ stator → Permanent magnet / → electromagnet

The stator provides the motor with the required flux ϕ. The magnitude and direction of this flux don't change with time

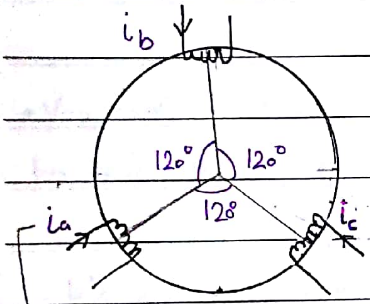


↳ armature winding circuit

→ I_a is generated by the voltage source V_T

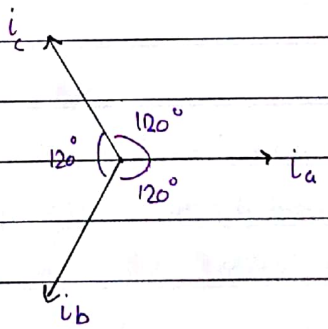
→ 3ϕ induction motor components are:

[1] Stator:



→ التيار بيجي من شركة الكهربا

- The stator is provided with 3 phase power. The phase angles of i_a, i_b, i_c are shown in the following Phasor diagram



- The stator is provided with alternating current. i_a, i_b, i_c are rms values.

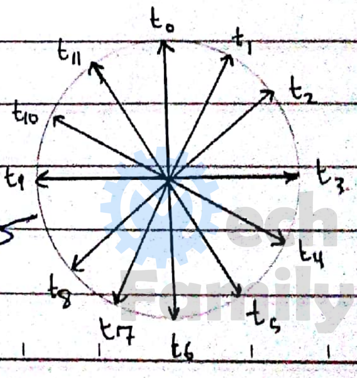
$|i_a| = |i_b| = |i_c|$

- The coils of the stator are equally distributed on the circumference of the stator (ie the angle between any 2 coils = 120°)
- These coils generate magnetic flux when current flows through them.

The total generated Flux = $\phi_{net} = \phi_A + \phi_B + \phi_C$

- The magnitude of ϕ_{net} doesn't change with time. However its direction changes with time, as shown in the adjacent figure.

it is called "rotating magnetic flux"

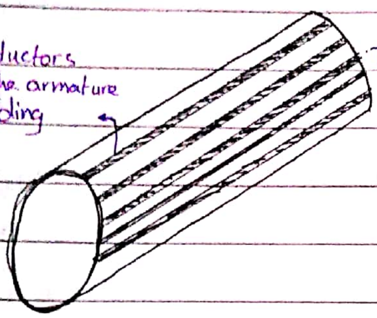


• Later, we will discuss how the coils of the stator can generate rotating magnetic flux

• Note: Rotating magnetic flux may be produced by rotating a permanent magnet! | N | S |

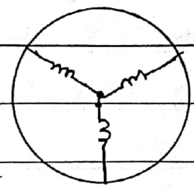
[2] Rotor

• The rotor of an induction motor has 2 types:



⇒ Rotor of induction motor has the following types:

1) Squirrel cage
2) Wound rotor



Schematic diagram of a wound rotor

• You don't have an access to the rotor (i.e You can't open it if you want to add a coil or a resistance)

→ Working principle of 3ϕ induction motor:

• The stator is provided with 3ϕ current $\Rightarrow i_a + i_b + i_c$

• The coils generate flux when current flows through them $\Rightarrow \phi_A + \phi_B + \phi_C = \phi_{net}$

• ϕ_{net} is rotating inside the air gap (i.e the direction of ϕ_{net} is changing with time) → متغير مع الزمن flux *

• The variation of the direction of ϕ_{net} induces emf in the armature winding of the rotor.

• The induced emf induces a current the flows in the armature winding

induced current + flux = Torque

→ Notes:

- In DC motor, voltage source V_T generates the armature current I_a.
- In "induction" motor, I_a is induced by induced emf.

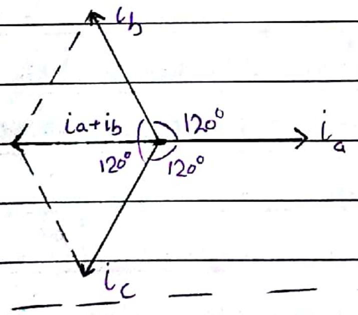
ناتجة عن التغير في ال flux

- They are called "induction" motor, since I_a is generated by "induced" emf
- Induction motors are sometimes called "rotating" transformers, since both have the same working principle.

- Alternating current is provided to the primary coil. The coil generates magnetic flux (The magnitude of the magnetic flux is changing with time). This variable flux induces emf in the secondary coil, which produces current the flows through the secondary coil.
- The only difference is: transformers are stationary devices, while induction motors are rotating devices.

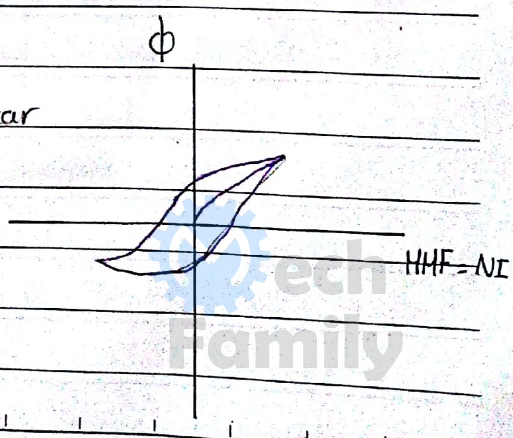
→ Rotating field flux:

- As we said before, $i_a + i_b + i_c$ generate rotating flux ϕ_{net}. The question that comes to our minds: How is this rotating flux generated?
- The following figure shows a phasor diagram for i_a, i_b, i_c



- Notice that $i_a + i_b + i_c = 0$
- Assume that the relation is linear between i and ϕ: $\phi = C i$

↳ In fact, the relation between i and ϕ is not linear (according to the hysteresis loop). However, we will "assume" that the relation is linear



• $\phi = Ci$, $i_a + i_b + i_c = 0$

$\Downarrow$

However, $\phi_a + \phi_b + \phi_c \neq 0$! why? before we answer this question, we will first plot the magnitudes of ϕ_A, ϕ_B, ϕ_C against time

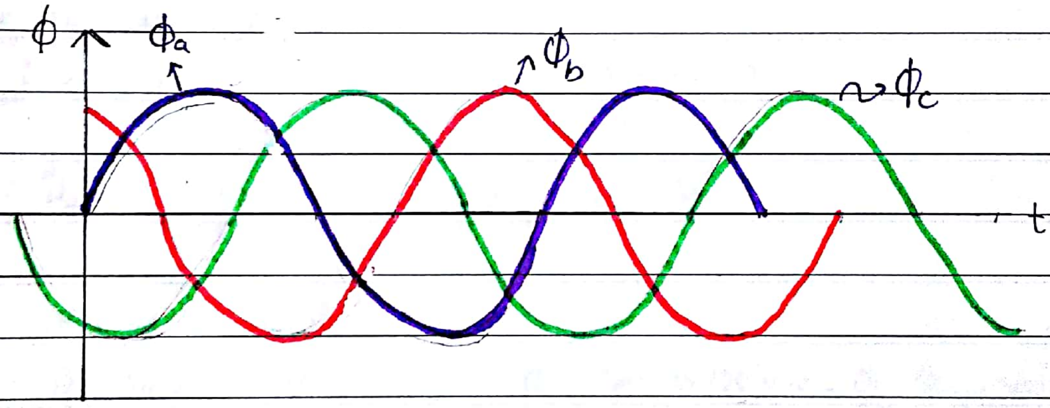
• Since i_a, i_b, i_c are alternating currents (i.e they can be described by sinusoidal functions), then ϕ_a, ϕ_b, ϕ_c will also be described by sinusoidal functions.

لأنه ناهي اللي بتولد ϕ ، فلو i كانت sinusoidal $\Leftarrow$ ϕ رح تكون sinusoidal

Hence, we can write

$$\phi_a = \phi_{max} \sin \omega t$$
$$\phi_b = \phi_{max} \sin(\omega t - 120)$$
$$\phi_c = \phi_{max} \sin(\omega t + 120)$$

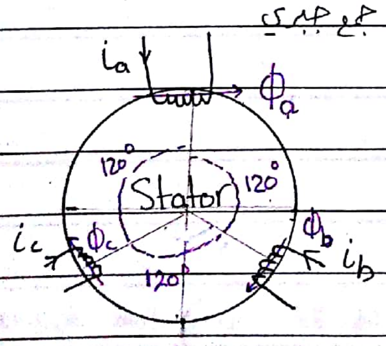
→ If we plot them, we will obtain the following:



$\Downarrow$

• These plots represent the "magnitudes" of ϕ_A, ϕ_B, ϕ_C. We also have to consider the directions of ϕ_a, ϕ_b, ϕ_c:

← هدول كميات متجهة ما بيقع نجمعهم جمع جبري



• As we can see; ϕ_a, ϕ_b, ϕ_c have different directions and we have to add them vectorially

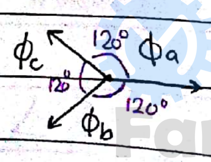
→ the angle between any 2 coils in space = 120°

⇒ Notice that the angle between

ϕ_a and $\phi_b = 120°$

ϕ_b and $\phi_c = 120°$

ϕ_c and $\phi_a = 120°$



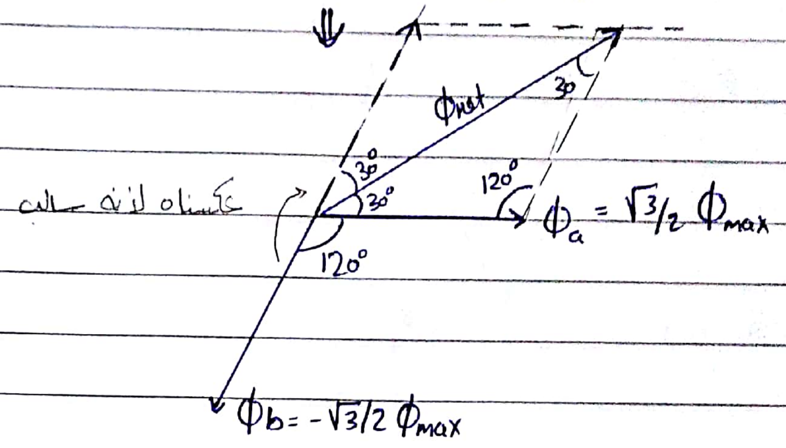
• Let's find Φ_{net} at 3 different instants

@ t_0, assume $wt = 60°$

$\Phi_a = \Phi_{max} \sin 60 = \frac{\sqrt{3}}{2}\Phi_{max}$, $\Phi_b = \Phi_{max}\sin(60-120) = -\frac{\sqrt{3}}{2}\Phi_{max}$

$\Phi_c = \Phi_{max}\sin(60+120) = 0 \Rightarrow \Phi_{net} = \Phi_a + \Phi_b + \Phi_c$

$\Downarrow$



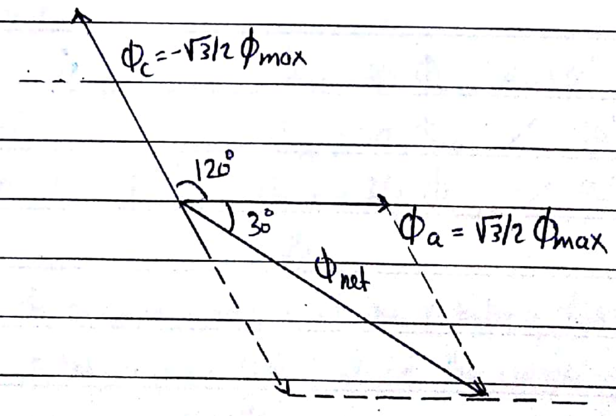
using the law of cosines

$$\Phi_{net}^2 = \Phi_a^2 + \Phi_b^2 - 2\Phi_a\Phi_b\cos 120 = \frac{3*3}{4}\Phi_{max}^2$$

$$\therefore \Phi_{net} = \frac{\sqrt{3}*\sqrt{3}}{2}\Phi_{max} = \frac{3}{2}\Phi_{max}\angle +30°$$

@ t_1, assume $wt = 120°$

$\Phi_a = \Phi_{max}\sin 120 = \sqrt{3}/2\ \Phi_{max}$, $\Phi_b = \Phi_{max}\sin(120-120) = 0$, $\Phi_c = \Phi_{max}\sin(120+120) = -\frac{\sqrt{3}}{2}\Phi_{max}$



$\Phi_{net} = \frac{3}{2}\Phi_{max}\angle -30$

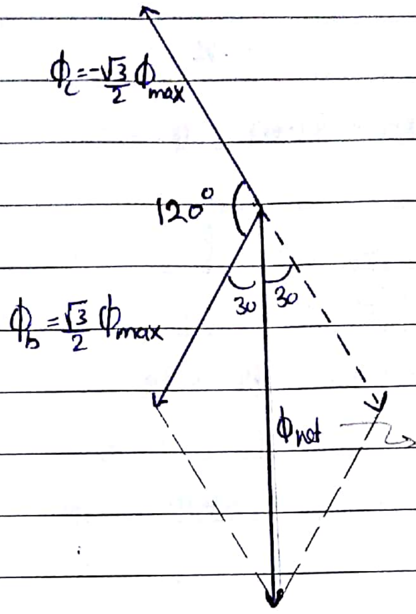
→ Notice that @ t_0 $wt = 60°$ changed by 60° $\Phi_{net} = 3/2\ \Phi_{max}\angle 30$ Changed by 60°

@ t_1 $wt = 120°$ $\Phi_{net} = 3/2\ \Phi_{max}\angle -30$

$\therefore$ if @ t_2 $wt = 180°$, we expect $\Phi_{net} = 3/2\ \Phi_{max}\angle -90°$

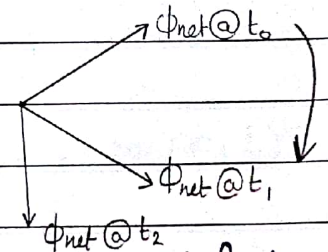
@ t_2, assume $\omega t = 180°$

$\phi_a = \phi_{max} \sin(180) = 0$, $\phi_b = \phi_{max} \sin(180-120) = \sqrt{3}/2\ \phi_{max}$, $\phi_c = \phi_{max} \sin(180+120) = -\frac{\sqrt{3}}{2}\phi_{max}$



$\phi_{net} = 3/2\ \phi_{max} \angle -90°$

Notice that ϕ_{net} is rotating



- Conclusions:

- Rotating field flux can be generated, if the following conditions are satisfied:

[1] 3ϕ currents are flowing through the coils

[2] The angle between the coils in space $= 120°$

- The magnitude of the rotating field flux doesn't change with time ($\phi_{net} = \frac{3}{2}\phi_{max}$). However, its direction changes.

- We noticed that:

@ t_0	$\omega t = 60°$	Changed by $60°$	$\phi_{net} = 3/2\ \phi_{max} \angle +30°$	Changed by $60°$
@ t_1	$\omega t = 120°$	Changed by $60°$	$\phi_{net} = 3/2\ \phi_{max} \angle -30°$	Changed by $60°$
@ t_2	$\omega t = 180$		$\phi_{net} = 3/2\ \phi_{max} \angle -90°$	

angular speed of the current source

$\therefore$ the angular speed ω of the rotating field flux ϕ_{net}

= the angular speed ω of the current source

$\Downarrow$ since both have the same ω, both will have the same frequency f

$$\omega_{\phi_{net}} = \omega_{current\ source}$$

$$\cancel{2\pi} f_{\phi_{net}} = \cancel{2\pi} f_{current\ source}$$

التردد الكهربا الي بتجي من شركة الكهربا

→ Synchronous Speed N_s

- The speed of the rotating magnetic flux is sometimes called air gap flux speed or Synchronous speed

$$N_s = \frac{1\ rev}{T}$$

T → Period: time required to complete one rev (sec) $T = \frac{1}{f}$

$$N_s = \frac{1\ rev}{1/f} = f\ rev/s = 60f\ rpm \Rightarrow \boxed{N_s = 60f\ rpm}$$

↳ This formula is only applicable when the number of poles = 2 (or the number of pole pairs = 1)

- The general equation of N_s is given by

$$N_s = \frac{60f}{PP} = \frac{120f}{P}$$

PP → # of pole pairs

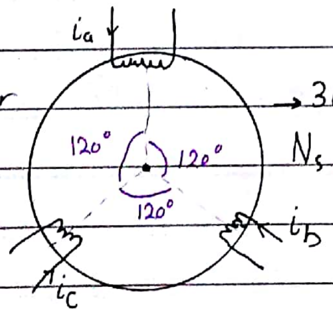
P → # of poles → عدد الاقطاب ذاتا زوجي

→ Notice that if $P = 2$ (or $PP = 1$)
$N_s = 60f$ rpm

- The following two figures show the configuration of the stator in the case of 2 poles and 4 poles

* What does 2 Poles stator mean?

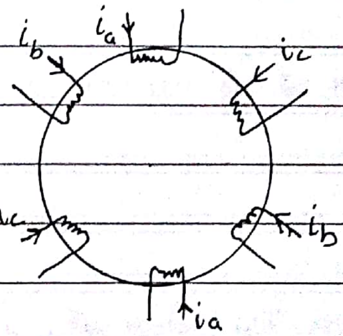
(2 Poles)



→ 3 coils

$N_s = \frac{60f}{1} = 60f$

(4 Poles)



→ 6 coils

$N_s = \frac{60f}{2} = 30f$

↳ For every 3 coils: 3φ currents flow through them ⇒ they generate rotating flux
⇒ this rotating flux may also be produces by rotating 1 magnet (This magnet has 2 poles) [i.e every 3 coils may be considered as 1 rotating magnet]. Hence, if we have 6 coils, then, they may be considered as 2 rotating magnets (4 poles)

- $N_s|_{2\ Poles} > N_s|_{4\ Poles}$

• Induction motors involve 2 types of speeds:

[1] Synchronous speed N_s = the speed of the rotating flux

[2] Mechanical speed N_m = the speed of the rotor

• Recall: the variation of the direction of ϕ_{net} induces emf in the armature winding of the rotor. This induced emf is a function of $\Delta n = N_s - N_m$ ($emf = f(\Delta n)$)

$\Downarrow$

if $\Delta n = zero \Rightarrow$ induced $emf = 0$

if $\Delta n \neq zero \Rightarrow$ induced $emf \neq 0 \Rightarrow$ current i will flow in the armature winding

• @ Starting (standstill): the rotor is stationary $N_m = zero$

$\Downarrow$

$$\Delta n = N_s - \underbrace{N_m}_{zero} \Rightarrow \Delta n = N_s \text{ (max)}$$

$\Downarrow$

$emf = f(\Delta n)$, since Δn is high $\Rightarrow$ emf is high

$\Downarrow$

emf generates i, since emf is high $\Rightarrow$ i is also high

$\Downarrow$

Torque is produced when
- → magnetic flux exists
- → current i flows through the armature winding

i.e $T_m = f(\phi, i)$

since i is high → the torque produced by the motor is high

$$T_m - T_L = J \frac{d\omega_m}{dt}$$

assuming that $T_m > T_L$, the motor will accelerate

- While operation "@steady state":

$T_m = T_L$

if $T_L \uparrow$ $\Rightarrow$ T_m must increase to maintain steady state conditions

زي ما عاوزين الحمولة في الصعد

$\Downarrow$

$T_m = f(\phi, i)$ $\Rightarrow$ i must increase in order to increase T_m

$\Downarrow$

induced emf must increase in order to increase i

$\Downarrow$

$emf = f(\Delta n)$ $\Rightarrow$ Δn must increase in order to increase emf

$\Downarrow$

$\Delta n = N_s - N_m$, $N_s = \frac{120 * f}{P}$ (the number of poles and the frequency don't change while operation $\therefore N_s$ is const)

to increase Δn, N_m must decrease

*Conclusion: if $T_L \uparrow \Rightarrow N_m \Downarrow$

→ Slip (Concept related to induction motors)

- Slip is defined as the relative difference between N_s and N_m

$$S = \frac{N_s - N_m}{N_s} * 100\%$$

, the value of S often ranges between 5-10%

Example: 2 pole, 60 Hz induction motor, if $S = 0.02$, find the rotor speed N_m

Solution

$N_s = \frac{120 * f}{P} = \frac{120 * 60}{2} = 3600$ rpm , $S = \frac{N_s - N_m}{N_s} \Rightarrow N_m = (1-S)N_s$

$N_m = (1-0.02) * 3600 = 3528$ rpm (notice that $N_m < N_s$)

دائماً N_m أقل من N_s + قيمة N_m بتكون قريبة من N_s

Example: 50 Hz induction motor, if $N_m = 2950$ rpm, find N_s

Solution:

- We know that N_m is always less than N_s + the value of N_m is close to the value of N_s ⇝ We will use this information to find N_s

- Assume that

$P = 2$ (أقل عدد الأقطاب زوجي + P=2 هو أقل عدد)

$N_s = \frac{120 * 50}{2} = 3000$ rpm → Notice that $N_m < N_s$ + the value of N_m is close to the value of N_s. This indicates that the answer is $N_s = 3000$ rpm

if you try:

$P = 4$

$N_s = \frac{120 * 50}{4} = 1500$ rpm $< N_m$ → $N_s = 1500$ rpm is wrong since $N_s < N_m$, which is impossible. The value of N_s @ $P = 4$ poles confirms that $N_s = 3000$ rpm is the correct answer

→ Equivalent circuit of induction motor

• Recall: in induction motors:

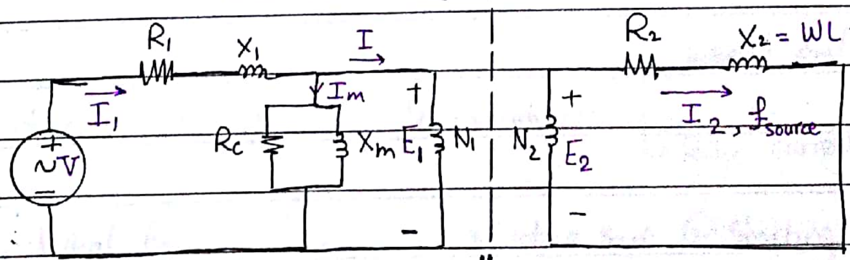
Stator	Rotor
The stator is supplied with 3ϕ voltage source	Emf is induced in the armature winding of the rotor $Emf = f(\Delta n) = f(N_s - N_m)$
↓ which generates current I	↓
↓ This current produces rotating flux ϕ	Emf generates current that flows through the armature winding

↳ The interaction between ϕ and I produces torque

• Induction motors are called rotating transformers. Why?

- When voltage is supplied to the stator → Emf is induced in the rotor
- Stator winding is equivalent to the primary coil of the transformer, while the rotor winding is equivalent to the secondary coil
- The only difference between induction motors and transforms: In induction motor, the rotor is rotating, while the secondary coil of a transformer is stationary

• The following figure shows the equivalent circuit (per phase) for induction motor @ starting (or @ stand still $N_m = 0$)



V phase voltages with frequency: f_{source}

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

* Remember $V_{line} = \sqrt{3}\, V_{phase}$

- Notice that the equivalent circuit of 1ϕ induction motor is similar to that of a transformer.
- E_2 represents the induced emf in the rotor $E_2 = f(\Delta n) = f(N_s - \cancel{N_m}) = f(N_s) = f\left(\frac{120 * f}{P}\right)$

- I_2 is the current produced by the induced emf E_2. I_2 is called rotor current. I_2 is an alternating current and it has a frequency $= f_{source}$

This statement is true only @ starting

- R_1, R_2 represent copper losses / X_1, X_2 represent leakage flux
- R_c represents core losses / X_m represents the mutual flux

• While operation:

- The induced emf becomes a function of $N_s - N_m$

@ starting the induced emf is a function of N_s only

- We will designate the induced emf
  - @ Starting by $E_{2,standstill}$ or $E_{2,s} = f(N_s)$
  - while operation by $E_{rotor} = f(N_s - N_m)$

$$\frac{E_{rotor}}{E_{2,s}} = \frac{N_s - N_m}{N_s} = \text{Slip}(S) \Rightarrow \boxed{E_{rotor} = S E_{2,s}}$$

- @ Starting, the frequency of the rotor current $= f_{source}$, but when the rotor starts to rotate (while operation), the frequency depends upon the slip:

$$\frac{f_{rotor\ current}}{f_{source}} = S \Rightarrow \boxed{f_{rotor\ current} = S\, f_{source}}$$

* Summary:

- In the above discussion, We discussed 2 parameters
  - Induced emf
  - Rotor current frequency

and we found that $E_{rotor} = S E_{2,s}$

$f_{rotor} = S\, f_{source}$

Notice that @ Starting $N_m = 0$ and

$$S = \frac{N_s - \overset{zero}{N_m}}{N_s} = 1$$

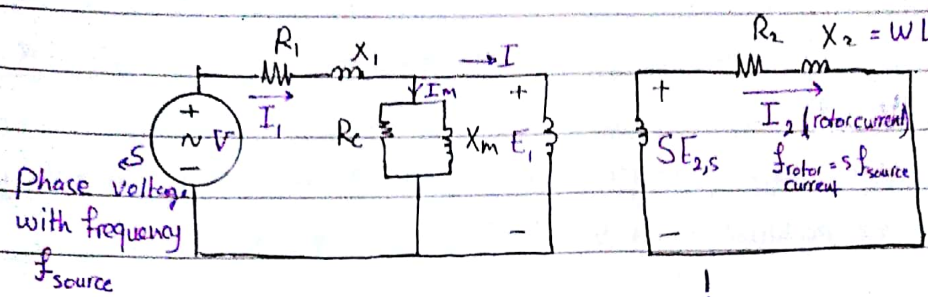
* These 2 equations relate the induced emf + rotor current frequency @ starting with the induced emf + rotor current frequency while operation

$\therefore E_{rotor} = E_{2,s}$

$f_{rotor} = f_{source}$

طلعت معنا نفس القيم اللي حكينا عنها

→ A general equivalent circuit [i.e it is applicable @ starting + while operation] for 1ϕ induction motor is shown as follows:



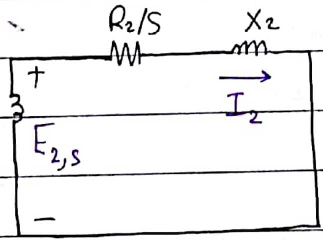
$X_2 = \omega L = 2\pi f_{rotor\ current} L$

$= 2\pi f_{source} * L * S$

$= S * X_{2,\text{standstill}} = S * X_{2,S}$

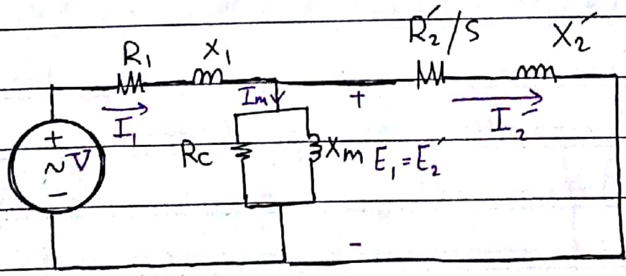
↳ In the rotor circuit, the rotor current $I_2 = \dfrac{S E_{2,S} \div S}{(R_2 + S X_{2,S})/S} = \dfrac{E_{2,S}}{\frac{R_2}{S} + X_{2,S}}$

$I_2 = \dfrac{E_{2,S}}{\frac{R_2}{S} + X_{2,S}}$ ⇒ We can redraw the rotor circuit, such that it becomes as follows:



↳ This circuit and the above rotor circuit are equivalent

→ We can do "reflection" in the induction motor circuit, as we did in transformers:



→ The resistance $\dfrac{R_2'}{S}$ may be represented as $\dfrac{R_2'}{S} = \underbrace{R_2'}_{\text{represents copper losses}} + \underbrace{R_2'\left(\dfrac{1-S}{S}\right)}_{\text{electrical representation of the mechanical load.}}$

كيف عرفنا؟

→ Continue

→ $R_2'\left(\frac{1-S}{S}\right)$ is an electrical representation of the mechanical load. How did we know that?

- We know that electrical loads may consume
  - → active power (such as resistors)
  - → reactive power (such as inductors + capacitors)

while mechanical loads consume active power only

$\therefore$ Resistors are good electrical representation of mechanical loads

- In addition, notice that $R_2'\left(\frac{1-S}{S}\right)$ is a variable resistance.

$\Downarrow$

$S = \frac{N_s - N_m}{N_s}$, N_s is const, N_m may change

$\therefore R_2'\left(\frac{1-S}{S}\right)$ Changes when N_m Changes

Recall: When $T_L \uparrow \Rightarrow N_m \downarrow$ (i.e when T_L changes, N_m changes)

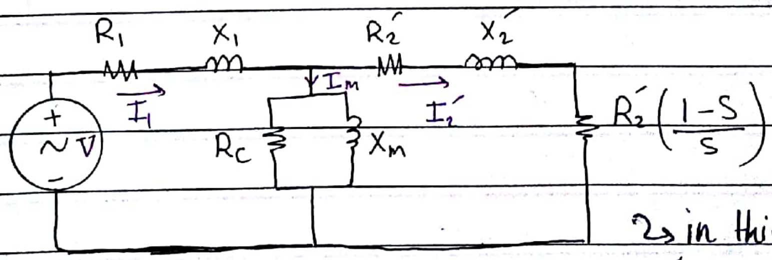
Since the value of $R_2'\left(\frac{1-S}{S}\right)$ Changes when N_m changes

دهاي أحمال بتتغير مع تغير T_L

$\Downarrow$

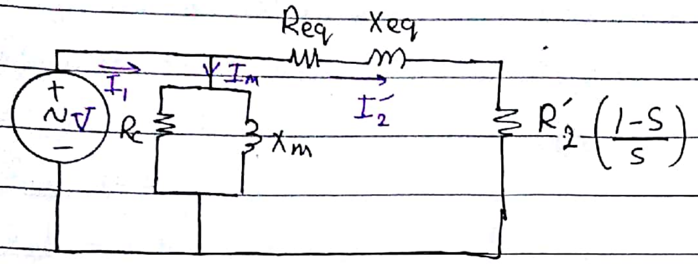
This indicates that $R_2'\left(\frac{1-S}{S}\right)$ represents the mechanical load.

→ Hence, we can redraw the reflected equivalent circuit as follows:



→ in this circuit we replaced $\frac{R_2'}{S}$ with $R_2' + R_2'\left(\frac{1-S}{S}\right)$

→ We can make an approximation in the previous circuit, by shifting the branch that contains $X_m + R_c$ to the left, such that it becomes closer to the voltage source:



→ $R_{eq} = R_1 + R_2'$. It represents copper losses

→ $X_{eq} = X_1 + X_2'$. It represents leakage flux

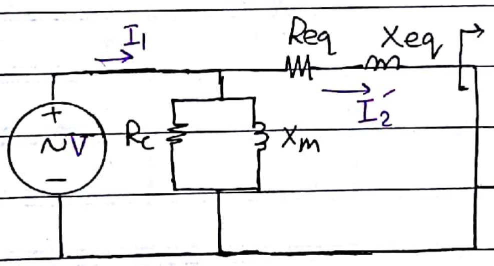
→ In transformers, we studied 2 types of tests: Open circuit test + Short circuit test. In induction motors, there are similar tests. One of these tests is called "locked rotor test"

↓

In this test, the rotation of the rotor is locked (i.e $N_m = 0$)

$\therefore S = \frac{N_s - \cancel{N_m}^{\,zero}}{N_s} = 1$, when $S = 1 \rightarrow R_2'\left(\frac{1-S}{S}\right) = \text{zero}$

The circuit becomes



* صارت زي كأنها short circuit

+ قيمة I_2' بتصير عالية

→ This condition also occurs:

@ Starting (since at starting $N_m = 0$, $S = 1$)

→ Power flows in induction motor

Power flows in Stator circuit → Input power (P_{in})

- P_{Cu} losses in Stator (P_{Cu_1})

  $P_{Cu_1} = 3 I_1^2 R_1$
- P_{core} losses

  $P_{core} = 3 \dfrac{V^2}{R_c}$

Power flows in rotor circuit → P_{airgap} (P_g) $= 3 (I_2')^2 * \dfrac{R_2'}{S}$

$\dfrac{R_2'}{S} = R_2' + R_2' \left(\dfrac{1-S}{S} \right)$

- P_{Cu} losses in rotor (P_{Cu_2})

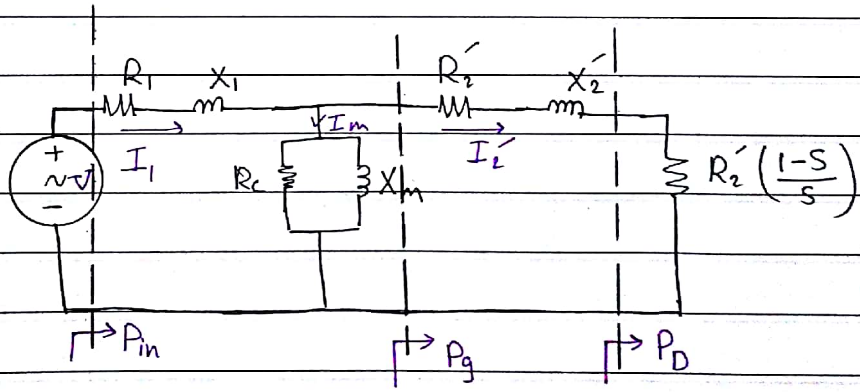
  $P_{Cu_2} = 3 I_2'^2 R_2' = S P_g$
- Developed power P_D

  $P_D = 3 I_2'^2 R_2' \left(\dfrac{1-S}{S} \right)$

  $= (1-S) P_g$
  - Rotational losses
  - Output power P_{out}

* ملاحظة : ضربنا قيم الـpower بـ 3 ، لأنه احنا متعاملين مع 3φ مش 1φ

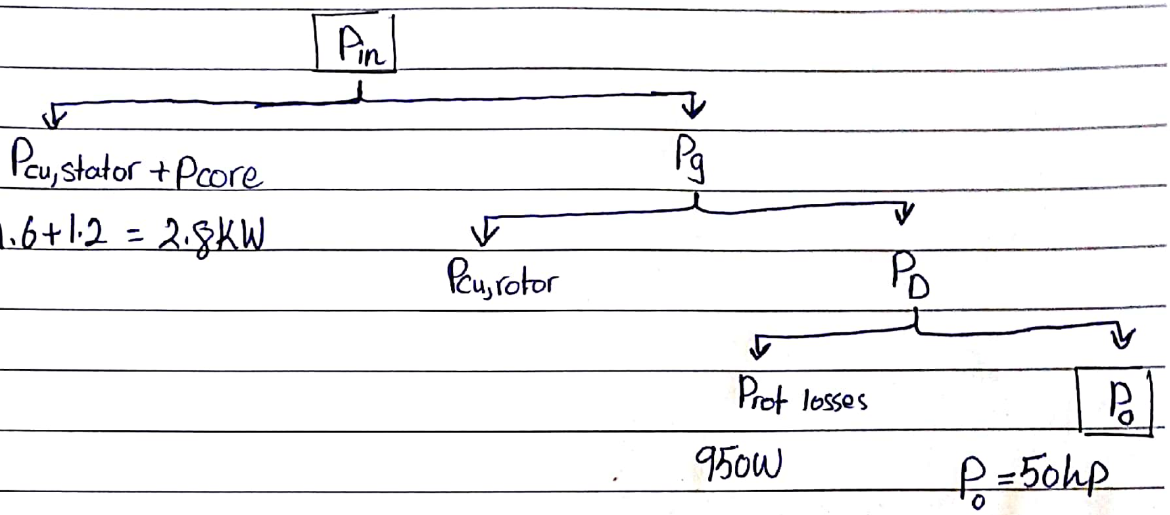
→ The following circuit shows the power flows in induction motor



Example: 50 hp, 60 Hz, Y connected induction motor, operate @ full load @ speed 1764 rpm.

$P_{rot\ losses} = 950\ W$, $P_{sator\ copper\ losses} = 1.6\ KW$, $P_{iron\ losses} = 1.2\ KW$, Find η

Draw the power flow diagram:



$* \ \eta = \frac{P_o}{P_{in}} * 100\%$

$P_o = 50\,hp = 50 * 0.746 = 37.3\ KW \quad (1hp = 0.746\ KW)$

↳ represents the power consumed by the mechanical load.

The power consumed by the mechanical load is always given in hp

$P_D = P_{rot\ losses} + P_o = 0.950 + 37.3 = 38.25\ KW$

$P_D = P_g(1-S) \Rightarrow P_g = \frac{P_D}{1-S}$, $S = \frac{N_s - N_m}{N_s}$

↳ N_m(given) = 1764 rpm, to find N_s, assume:

$P = 2 \quad N_s = \frac{120 * f}{P} = \frac{120 * 60}{2} = 3600\ rpm$ ✗

$P = 4 \quad N_s = \frac{120*60}{4} = 1800\ rpm$ ✓

$P = 6 \quad N_s = \frac{120 * 60}{6} = 1200\ rpm$ ✗

* N_s must be greater than N_m + it must be close to N_m

$\therefore N_s = 1800\ rpm$

$$\therefore S = \frac{N_s - N_m}{N_s} = \frac{1800 - 1764}{1800} = 0.02$$

$$P_g = \frac{P_D}{1-S} = \frac{38.25}{1-0.02} = 39 \text{ KW}$$

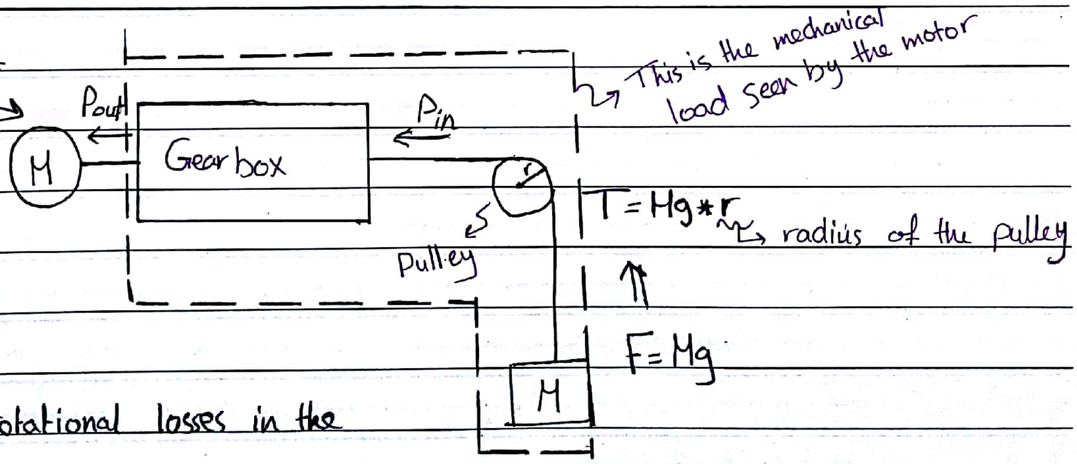
$$P_{in} = (P_{cu,stator} + P_{core}) + P_g = 2.8 + 39 = 41.8 \text{ KW}$$

$$\eta = \frac{37.3}{41.8} * 100\% = 89.2\%$$

* Note: We can find $P_{cu,rotor}$ as follows:

$$P_g = P_{cu,rotor} + P_D \Rightarrow P_{cu,rotor} = 39 - 38.25 = 0.750 \text{ KW} = 750 \text{ W}$$

* هلا حكالك الدكتور
عن ال مدحام →



* Assuming no rotational losses in the gear box

$$P_{in} = P_{out}$$

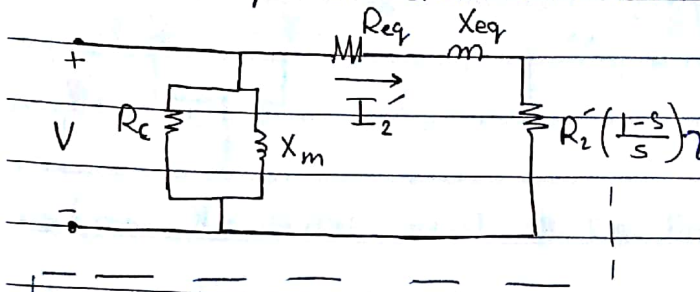
$$T_{in} W_{in} = T_{out} W_{out}$$

$$(Mg*r) * W_{in} = \underbrace{T_{out}}_{\text{represents } T_L \text{ seen be the motor}} * W_{out}:$$

* $T_m - T_L = J * \frac{dw_m}{dt}$ * @ steady state $T_m = T_L$

[*] Torque-speed characteristic of induction motor:

→ Recall: the equivalent circuit of induction motor (per phase) is shown below:



↳ represents the mechanical load

↳ The power delivered by the motor to the load: $P_D = 3 * (I_2')^2 * \underbrace{R_{load}}$

$$= 3\left(I_2'\right)^2 * R_2'\left(\frac{1-S}{S}\right)$$

↳ We multiply by 3, because we are dealing with 3ϕ induction motor

→ The power consumed by the mechanical load:

$P_{load} = T\,W_m$ = Power delivered by the motor to the load

$$= 3\left(I_2'\right)^2 * R_2'\left(\frac{1-S}{S}\right)$$

→ The torque delivered by the motor is given by

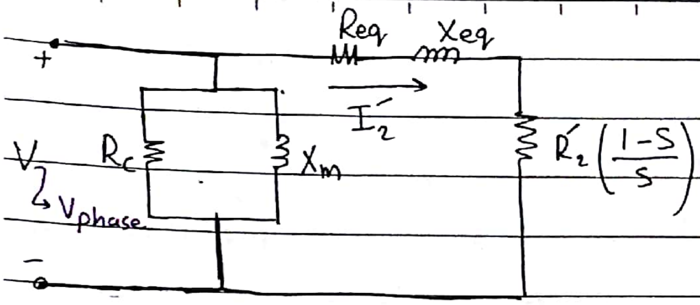
$$T = \frac{3(I_2')^2 * R_2'\left(\frac{1-S}{S}\right)}{W_m}$$

W_m ↳ represents the angular speed of the load or the rotor

$$S = Slip = \frac{N_s - N_m}{N_s} = \frac{W_s - W_m}{W_s} \Rightarrow W_m = (1-S)W_s$$

∴ T can be rewritten as follows

$$T = \frac{3(I_2')^2 * R_2'\left(\frac{\cancel{1-S}}{S}\right)}{\cancel{(1-S)} * W_s} = \frac{3(I_2')^2 * R_2'}{S\,W_s}$$



→ From the above circuit, we can find that I_2' is given by:

$$I_2' = \frac{V}{|Z|}, \text{ where } |Z| = \sqrt{\left(R_{eq} + R_2'\left(\frac{1-S}{S}\right)\right)^2 + X_{eq}^2}$$

$$= \sqrt{\left(R_1 + \underbrace{R_2' + R_2'\left(\frac{1-S}{S}\right)}_{\frac{R_2'}{S}}\right)^2 + X_{eq}^2} = \sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + X_{eq}^2}$$

$$\therefore I_2' = \frac{V}{\sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + X_{eq}^2}}$$

V → V_{phase} or $V_{line\ to\ neutral}$

* Note: In Y-connection $V_{line\ to\ neutral} = \frac{V_{line\ to\ line}}{\sqrt{3}}$

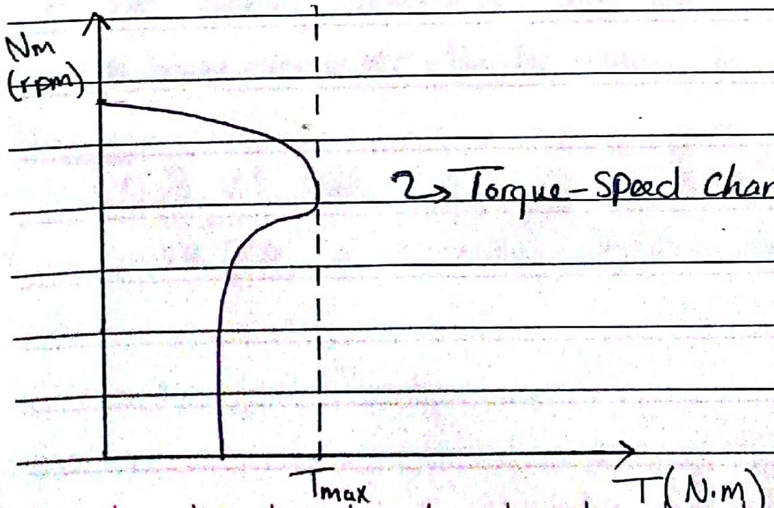
→ From the previous page, the torque developed by the motor:

substitute

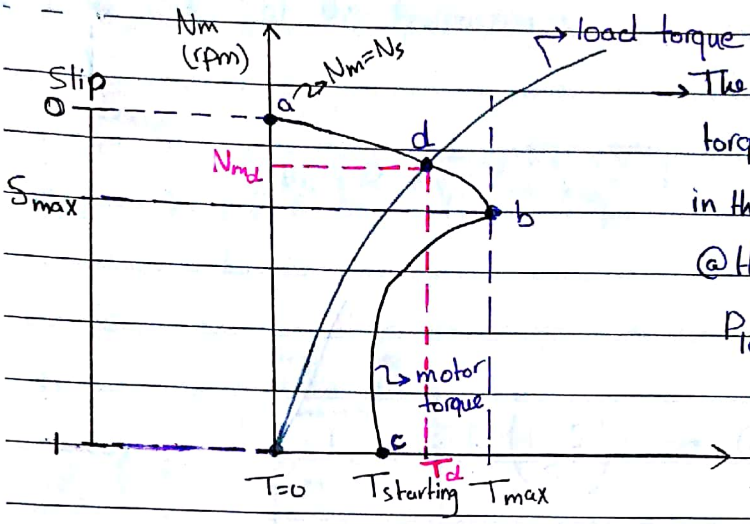
$$T = \frac{3(I_2')^2 * R_2'}{S\omega_s} = \frac{3V^2 * R_2'}{S\omega_s\left(\left(R_1 + \frac{R_2'}{S}\right)^2 + X_{eq}^2\right)}$$

$$S = \frac{N_s - N_m}{N_s}$$

→ Plotting the rotor speed "N_m" against the developed torque "T" gives:



→ Torque-Speed characteristic curve of induction motor



→ The point of intersection between the motor torque curve and the load torque curve (point d' in the adjacent figure) represents the operating point.
@ this point

$$P_{load} = \underbrace{T\omega}_{rad/s} = T_d * \underbrace{\underbrace{N_{md}}_{rpm} * \frac{2\pi}{60}}_{rad/S}$$

→ @ starting $N_m = 0$, Slip $= S = \frac{N_s - \cancel{N_m}}{N_s} = \frac{N_s}{N_s} = 1$. In the above curve

$T = T_{starting}$ when $N_m = 0$ (Check point "c" in the above curve)

→ $T \equiv$ torque developed by the motor $= 0$ when $\Delta n = N_s - N_m = 0$, because:

emf $= f(\Delta n)$

if $\Delta n = 0 \Rightarrow$ emf $= 0 \Rightarrow$ no current will flow through the armature winding of the rotor $\Rightarrow$ No torque will be produced

↳ ليش torque ال

torque (طاقة $\Leftarrow$ تيار $\Leftarrow$ طاقة $\Leftarrow \phi + i$

$\therefore T = 0$ when $\Delta n = N_s - N_m = 0$, Slip $= S = \frac{N_s - N_m}{N_s} = \frac{0}{N_s} = 0$ (Check point a in the above curve. @ this point $N_m = N_s$)

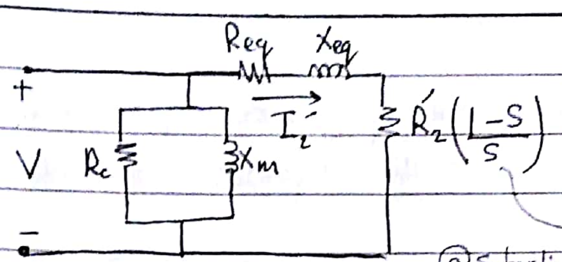
→ The torque developed by the motor:

$$T = \frac{3 V^2 * R_2'}{s\omega_s \left(\left(R_1 + \frac{R_2'}{s}\right)^2 + X_{eq}^2 \right)}$$

. To find Tmax and S_{max} (Point b in the above figure). find the derivative $\frac{dT}{ds}$ (slip) and set $\frac{dT}{ds} = 0$

→ You will get the following :

$$T_{max} = \frac{3V^2}{2W_s\left[R_1 + \sqrt{R_1^2 + X_{eq}^2}\right]} \quad , \quad S_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$

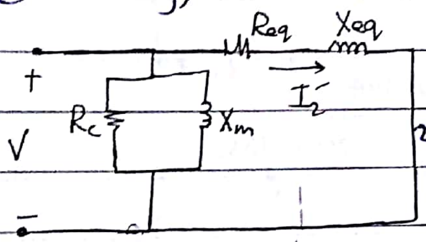


⇒ @ Starting $N_m = 0$, $S = \frac{N_s - N_m}{N_s} = \frac{N_s}{N_s} = 1$

@ starting → $R_2'\left(\frac{1-1}{1}\right) = \text{zero}$

⇓

@ Starting, the circuit becomes.



→ Short ⇒ This causes I_2' @ starting to be very high

→ I_2' @ starting is given by $I_2'\Big|_{@starting} = \frac{V}{\sqrt{R_{eq}^2 + X_{eq}^2}}$

→ How can we reduce I_2' @ starting?

• By reducing V :

$$I_2\Big|_{@starting} \downarrow = \frac{\downarrow V}{\sqrt{R_{eq}^2 + X_{eq}^2}}$$

→ V is reduced using a device called "Soft Starter"

Soft starter : a device that is used to reduce the voltage @ starting

• We can reduce I_2' @ starting by increasing R "resistance". However, we don't tend to use this method, because in squirrel cage rotor type, we don't have access to the rotor

→ The torque delivered by the motor @ starting is given by

$$T_{st} = \frac{3V^2 * R_2'}{S W_s\left(\left(R_1 + \frac{R_2'}{S}\right)^2 + X_{eq}^2\right)} = \frac{3V^2 R_2'}{W_s\left(R_{eq}^2 + X_{eq}^2\right)}$$

@ starting $N_m = 0$, $S = \frac{N_s - \overset{zero}{N_m}}{N_s} = 1$ ↳ $R_{eq} = R_1 + R_2'$

→ In the previous page, we said that we reduce I_1' @ starting by reducing V. However, reducing V will cause $T_{starting}$ to decrease

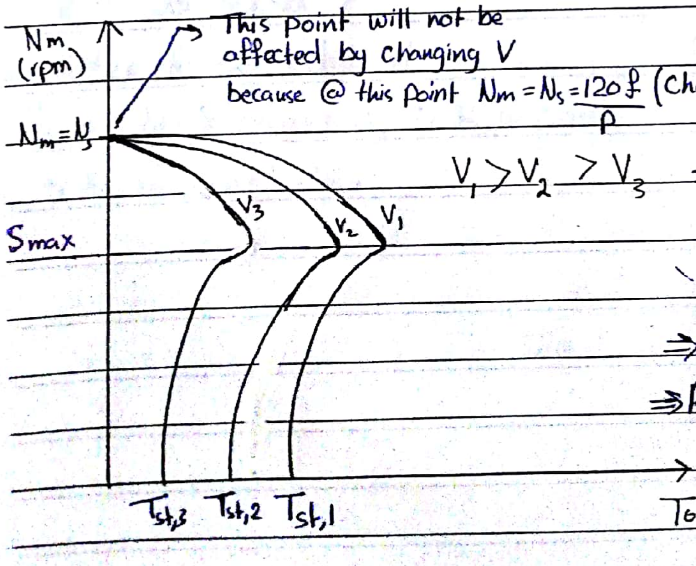
$$T_{starting}\downarrow = \frac{3V^2\downarrow * R_2'}{W_s\left(R_{eq}^2 + X_{eq}^2\right)}$$

↳ If $T_{m,starting}$ becomes less than T_{Load}, the motor will not accelerate ~ (ما راح يشتغل الموتور)

⇓ (عشان تصير $T_{motor,starting} > T_{load}$ ، بالتالي بيتسارع الموتور)

To solve this problem, we can set $T_{load} = 0$ or small load @ starting. Once the rotor starts to rotate, we can set the desired T_{load}

→ The following figure shows the effect of changing V (Voltage) on the torque-speed characteristic curve of induction motor



$V_1 > V_2 > V_3$ ⇒ Reducing V reduces $T_{starting}$

$$T_{starting}\downarrow = \frac{3V^2\downarrow R_2'}{W_s\left(R_{eq}^2 + X_{eq}^2\right)}$$

⇒ Reducing V doesn't affect $S_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$

⇒ Reducing V reduces T_{max}

$$T_{max}\downarrow = \frac{3V^2\downarrow}{2\left[R_1 + \sqrt{R_1^2 + X_{eq}^2}\right]}$$

Torque (N.m)

→ Note: if $T_{st,2}$, $T_{st,3}$ are given in the previous figure, we can find the ratio $\frac{V_2}{V_3}$:

$$T_{st} = \frac{3V^2 R_2'}{W_s(R_{eq}^2 + X_{eq}^2)} \qquad \frac{T_{st,2}}{T_{st,3}} = \frac{\frac{\cancel{3} V_2^2 \cancel{R_2'}}{\cancel{W_s(R_{eq}^2+X_{eq}^2)}}}{\frac{\cancel{3} V_3^2 \cancel{R_2'}}{\cancel{W_s(R_{eq}^2+X_{eq}^2)}}} = \frac{V_2^2}{V_3^2}$$

$$\therefore \frac{V_2}{V_3} = \sqrt{\frac{T_{st,2}}{T_{st,3}}}$$

[*] Controlling the speed of induction motor

→ We can change the speed of induction motor by:

[1] Changing the voltage V (@ const frequency)

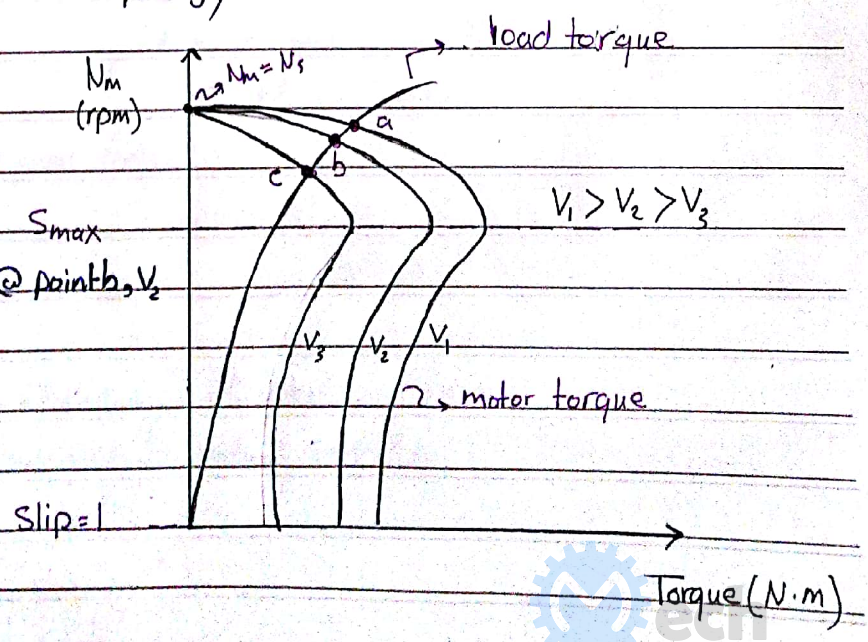
[2] Changing the frequency (@ const voltage)

[3] $\frac{V}{F}$ scalar control

[1] Changing the Voltage (@ const frequency)

→ From the previous figure, we can see that as $V\uparrow \Rightarrow$ speed $\uparrow$.

→ Notice in the adjacent figure N_m @ point a, V_1 > N_m @ point b, V_2 > N_m @ point c, V_3.



[2] Changing the frequency (@ const voltage)

→ if $f\uparrow$:

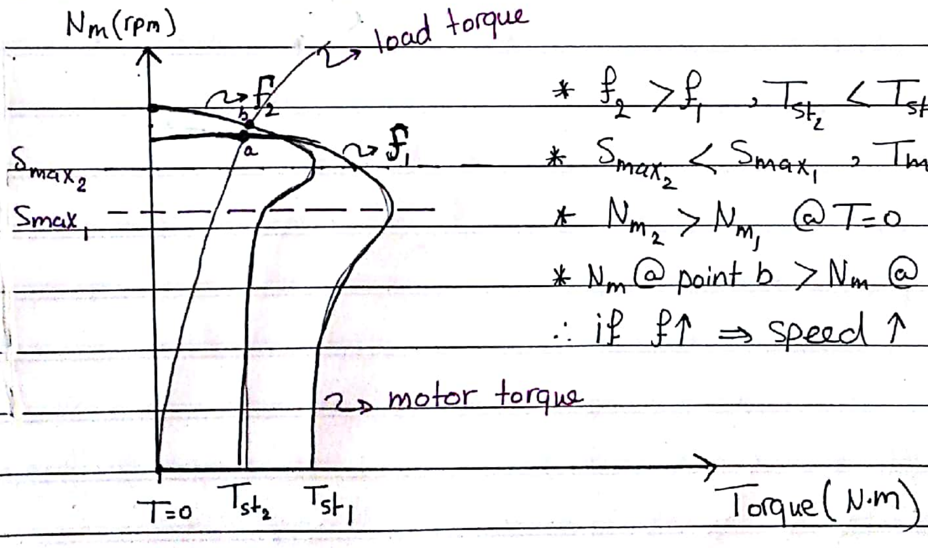
- @ $T=0$ $\quad N_m = N_s = \frac{120 f}{P} \Rightarrow N_m \uparrow$ @ $T=0$

- $S_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$, when $f\uparrow \Rightarrow X_{eq} = L_{eq} w = L_{eq} 2\pi f \Rightarrow X_{eq}\uparrow \quad \therefore S_{max}\downarrow$

- $T_{st} = \frac{3 V^2 R_2'}{W_s (R_{eq}^2 + X_{eq}^2)}$, when $f\uparrow \Rightarrow X_{eq}\uparrow$, $W_s = \frac{120 f}{P} * \frac{2\pi}{60} \Rightarrow W_s \uparrow \quad \therefore T_{st}\downarrow$

- $T_{max} = \frac{3 V^2}{2 W_s [R_1 + \sqrt{R_1^2 + X_{eq}^2}]}$, when $f\uparrow \Rightarrow W_s\uparrow$, $X_{eq}\uparrow \quad \therefore T_{max}\downarrow$

$\therefore$ The torque-speed characteristic curves @ different frequencies is shown below:



* $f_2 > f_1$, $T_{st_2} < T_{st_1}$
* $S_{max_2} < S_{max_1}$, $T_{max_2} < T_{max_1}$
* $N_{m_2} > N_{m_1}$ @ $T=0$
* N_m @ point b $> N_m$ @ point a

$\therefore$ if $f\uparrow \Rightarrow$ speed $\uparrow$

→ Recall

$V_{rms} = 4.44 f N \phi$, we change f @ const voltage

increasing f means reducing ϕ (flux weakening)

V_{rms} ↓ Voltage per Phase

[3] V/f scalar control

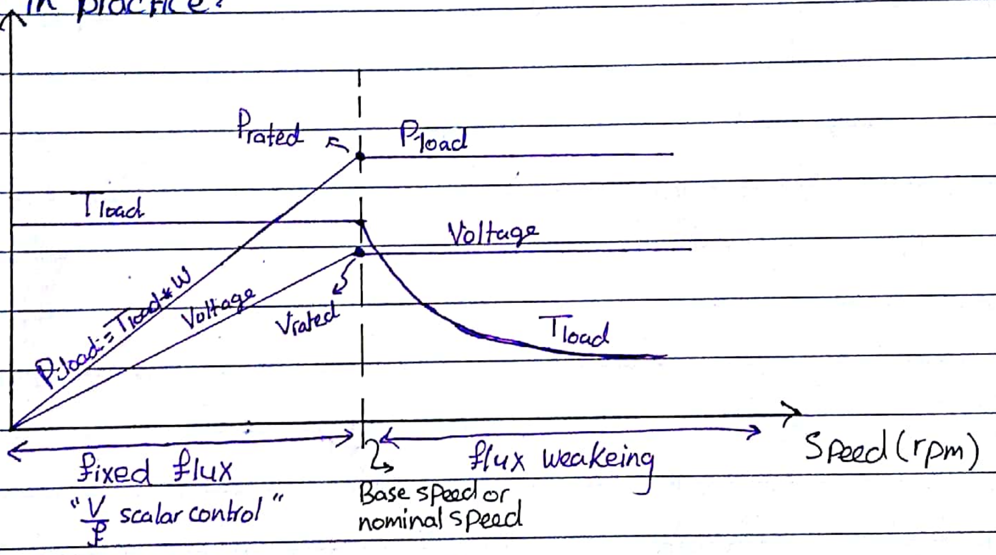
→ In this method, the speed of the motor is increased by increasing both f and V, such that the ratio $\frac{V}{f}$ is maintained constant.

$$V_{rms} = 4.44 f \phi N$$

$$\frac{V_{rms}}{f} = 4.44 \phi N \Rightarrow \text{Const } \frac{V_{rms}}{f} \text{ ratio means constant flux}$$

→ In practice, in this method, T_{load} is maintained constant while the speed is varied

The following figure shows how do we control the speed of induction motor in practice:



→ In the fixed flux region, both frequency and voltage are increased to increase the speed such that $\frac{V}{f}$ stays const. The voltage is increased until we reach the rated voltage ما بقدر أتجاوزها

- $P_{load} = T_{load} * W$, T_{load} = const, but $W \uparrow$ ∴ $P_{load} \uparrow$ until Prated is reached

→ In the flux weakening region, the frequency is increased at constant voltage V_{rated}.

- V_{rated} can't be exceeded → V stays constant
- P_{rated} can't be exceeded → $P_{load} = T_{load} * W$, w is increasing ∴ T_{load} must be reduced to maintain const power.

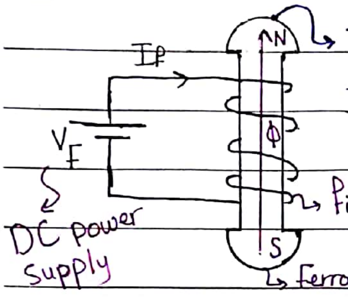
[*] Synchronous generators:

→ Recall: generators are used to convert "mechanical" power to "electrical" power.

→ Synchronous generators generate AC electrical power.

→ Basic parts of a synchronous generator are:

[1] Rotor



↳ The rotor is made of ferromagnetic material. When a current I_F flows through the coils of the field winding, a magnetic flux ϕ is produced and the rotor becomes electromagnet.

*Notes:

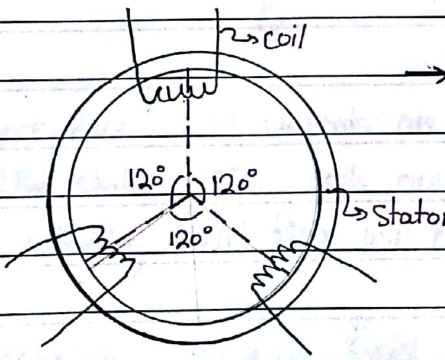
• The magnitude of the produced magnetic flux ϕ is const, because this flux results from Direct current (DC) ⇝ لإنه التيار I_F ثابت ف رح يولد ϕ ثابت

• When we operate a synchronous generator, the rotor is rotated (mechanical power) by a prime mover.

• When the rotor is rotated, the magnetic flux ϕ will also rotate. This will form a "rotating magnetic flux".

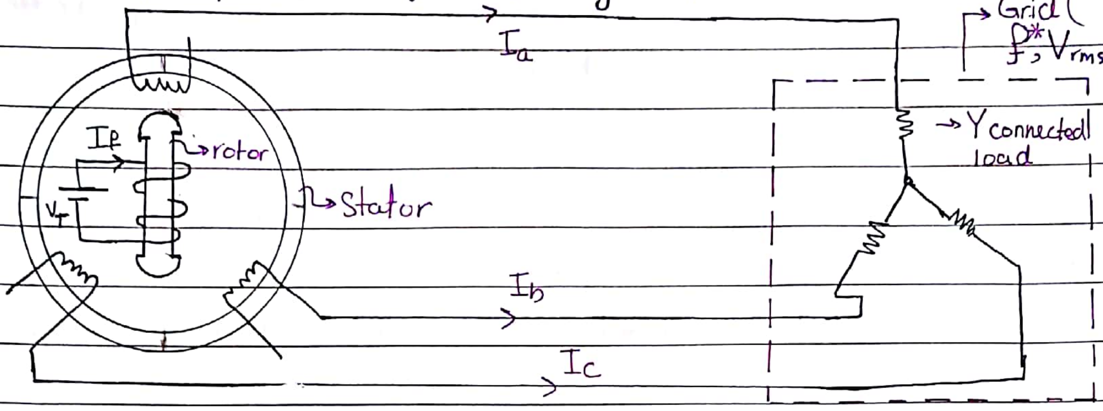
↳ The rotating flux has constant magnitude. However, its direction changes with time

[2] Stator



→ The coils are separated in space by 120°

→ Operation principle of a synchronous generator



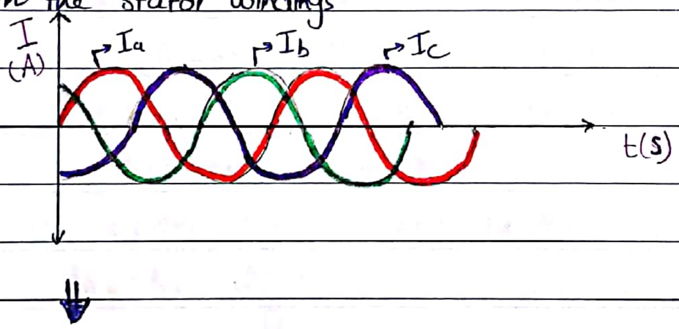
[1] A DC current is flowing in the field winding of the rotor

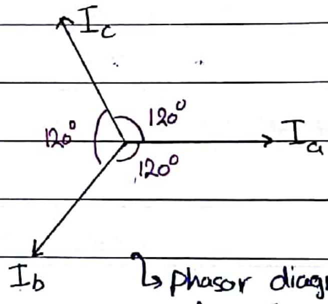
[1] The rotor of the generator is rotated by a prime mover

[2] A DC current is flowing in the field winding of the rotor, which produces a rotating magnetic field within the machine

[3] The rotating magnetic field induces a 3ϕ emf in the stator windings of the generator

[4] If the generator is connected to a load, 3ϕ currents I_a, I_b, I_c will flow in the stator windings





↳ phasor diagram of I_a, I_b, I_c

notice: the phase shift between them $= 120^\circ$

Since there 3ϕ currents are flowing through the coils of the stator + The coils are separated by 120° in space, a rotating field flux will be produced.

Conclusion: 2 rotating field fluxes exist in a synchronous generator:

[1] Rotating field flux that results from rotating the rotor

[2] Rotating field flux that results from the windings of the stator

$$\rightarrow \vec{\phi}_{net,\,air\,gap} = \underbrace{\vec{\phi}_{rotor}}_{\text{rotating field flux}} + \underbrace{\vec{\phi}_{stator}}_{\text{rotating field flux}}$$

متطابقة/متزامن

→ Synchronous generators are called "Synchronous", because: the speed of the rotor (or the rotating field of the rotor) and the speed of the rotating field of the stator are equal

$$N_m = N_s$$

→ Check the figure in the previous page and notice that the grid has a certain frequency f^* and certain voltage V_{rms}.

→ Synchronous generator must supply the grid with electrical power of frequency f^* and voltage V_{rms} (i.e the generator must provide an electrical power of frequency and voltage that are consisent with the requirements of the grid.

e.g: In Jordan, the frequency of the grid = 50 Hz. Hence, generators in Jordan must provide electrical power with frequency = 50 Hz. On the other hand, the frequency of the grid in Saudi Arabia = 60 Hz, hence, generators in Saudi Arabia must provide electrical Power with frequency = 60 Hz.)

→ How can we control the frequency and the voltage induced in the generator? [Assume that the requirements of the grid $f = 50\,Hz$, $V = 20\,kV/\text{phase}$]

• We control the frequency by controlling the speed of the rotor:

$$N_m = N_s = \frac{120 f}{P}$$

We rotate the rotor at a certain speed, such that the frequency of the electricity produced by the generator = 50 Hz

• We control the Voltage induced by the generator by controlling I_f:

$$\underbrace{V_{rms}}_{\substack{\text{induced voltage}\\ \text{by the generator}\\ \text{(per phase)}}} = 4.44\, f N \phi_m$$

→ Changing the magnitude of the rotating field flux of the rotor is achieved by changing I_f

→ Note:

مثل تشغيل المكيفات في الصيف

* If a high load is connected to the synchronous generator, high current will be drawn by the load. This high current increases the magnitude of the rotating field flux of the stator.

In the presence of the rotating field flux of the stator + current flowing through the field winding of the rotor → Torque will be produced (motor action)

هاد الـ torque بأثر عالـ rotor

This torque opposes the torque applied by the prime mover, hence N_m (the speed of the rotor) will decrease.

$\downarrow N_m = N_s = \frac{120 f}{P}$ → As a result, the frequency will decrease

واحنا لازم نحافظ على قيمة f

To solve this problem, we can increase the torque applied by the prime mover ∴ N_m will increase again and f retains its value.

الحل الثاني انه شركة الكهربا تقطع علينا الكهربا عشان نخفف الـ load

→ Note: We know that:

[1] The "field" winding produces the main field flux

[2] The generated Voltage (emf) is induced in the "armature" winding.

* Notice that in Synchronous generators, the winding of the rotor produces the main field flux (hence the rotor winding = field winding), while the generated emf is induced in the windings of the Stator (hence the Stator winding = armature winding)

→ Comparison between Synchronous machines and DC machines

Synchronous machines

	Winding
Rotor	field
Stator	armature

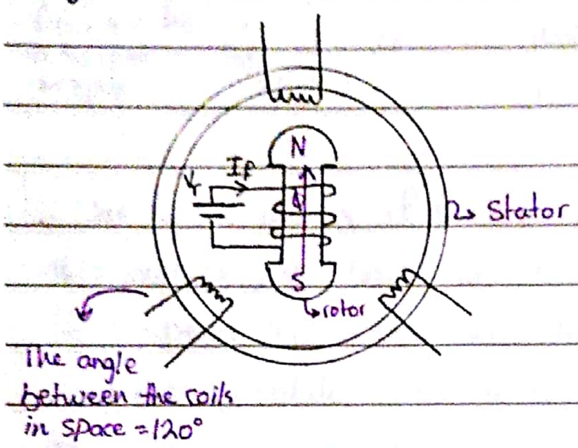
DC machines

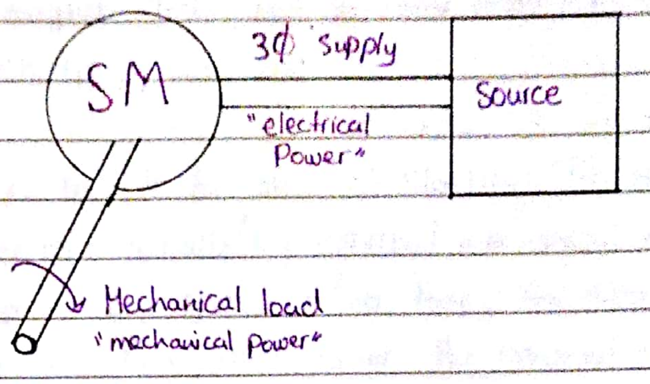
	Winding
Rotor	armature
Stator	field

[*] Synchronous motor (SM)

→ Motors are used to convert electrical power to mechanical power.

→ The components of SM are the same as the components of the Synchronous generator





→ Operation principle of a Synchronous motor:

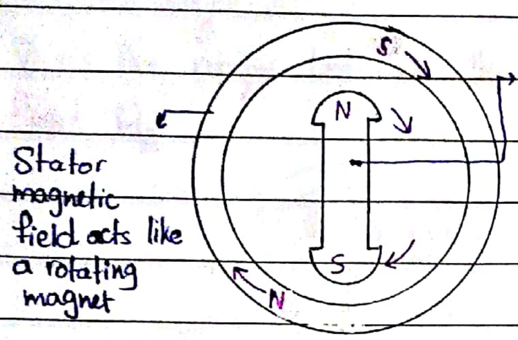
[1] A 3ϕ voltage is supplied to the windings of the stator, which produces 3ϕ currents that flow in the windings. Since the angle between the coils in space = 120° + 3ϕ currents are flowing through the coils → a rotating field flux is produced (notice the stator of SM is the same as the stator of IM)

SM: Synchronous motor; IM: induction motor

This rotating flux has constant magnitude, but its direction changes with time. The rotating flux acts like a rotating magnet (Note: the rotating flux has a speed N_s(synchronous speed) $= \frac{120f}{P}$.

[2] A DC voltage is supplied to the winding of the rotor, which produces a field current I_f. This current produces a magnetic flux ϕ, which has constant magnitude. The rotor becomes electromagnet. It has 2 poles N and S (check the above figure).

[3] While the rotating field is rotating, the rotor and stator poles might be of opposite polarities (S-N or N-S) at a certain instant, causing an attractive force acting on the rotor, and hence, the rotor will rotate (mechanical power) at a speed $N_m = N_s$.



→ Rotor magnetic field acts like a bar magnet.

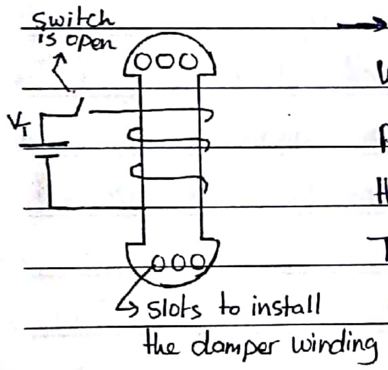
→ Since the stator magnetic field is rotating, the rotor magnetic field (and the rotor itself) will try to catch up ⇝ بيحاول يلحقه

* بس في عنا مشكلة بال SM ، شو هي؟ ------

→ Due to the inertia of the rotor, it is unable to rotate at starting. To solve this problem, we have to rotate the rotor initially (by external mechanical means) to a speed that is very close to synchronous speed. After that, the rotor continues to rotate with the rotating field flux, even after the removal of the external mechanical means.

⇓

One of the methods to rotate the rotor @ starting is to use "damper winding"



→ Additional winding is placed in the rotor poles. Initially, when the rotor is not rotating, an induced emf is produced in the damper winding, which produces a current that flows in the winding. Current + flux → Torque. This torque rotates the rotor to a speed that is very close to N_s. Then, the damper winding is disconnected. [We can say that the motor first runs as induction motor!]

→ Notes:

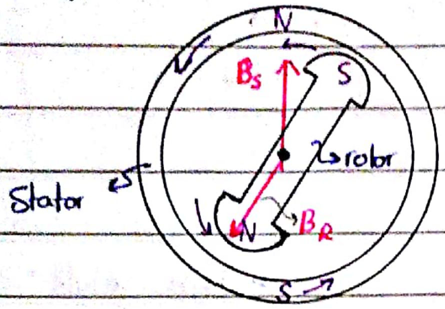
- Notice that the stators of IM and SM are identical, however, their rotors are different.
- SM is sometimes called double excited motor, because 2 electrical inputs are provided to it:

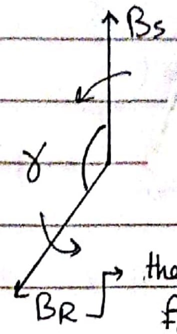
[1] DC power supply, which produces the magnetic flux in the rotor

[2] 3Ø power supply, which produces rotating field flux in the stator

→ Load angle γ:

γ is the angle between the "stator" rotating field B_s and the "rotor" rotating field B_R





→ the rotor field lags the stator field

→ Both fields are rotating @ $N_s = \frac{120 f}{P}$, however, there is a phase shift between them.

→ Recall: In induction motor as $T_{load}\uparrow$ $N_m\downarrow$

but in SM as $T_{load}\uparrow$ $N_m = const = N_s$, γ changes

as $T_{load}\uparrow$ $\gamma\uparrow$

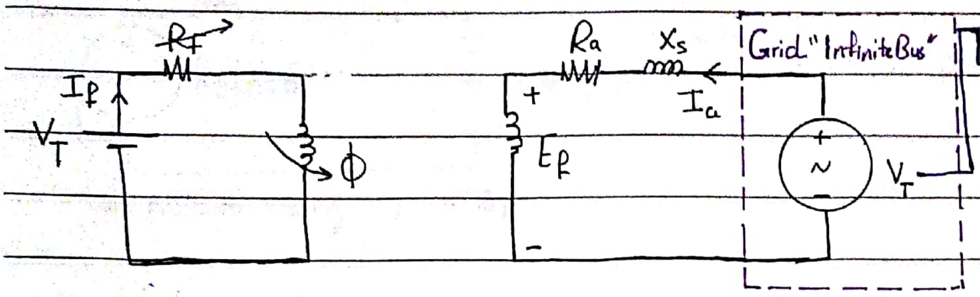
@ no load $\gamma = 0$

- T_{load} does ~~↑~~ affect the speed of the motor. It only affects the load angle γ
- Actually, synchronous motors are considered as const speed motors. To change the speed of SM ⇒ $N_s = \frac{120 f}{P}$ ⇝ you have to change f or P

لأ عشان هيك زيادة T_{load} ما بتأثر عالسرعة

→ Equivalent circuit of SM:

* ملاحظة : (Grid = شبكة)



"Rotor circuit" "Stator circuit per phase"

↳ V_T represents the phase voltage.

→ The grid provides the motor with the required V_T.

→ Infinite Bus means the grid is strong → الشبكة قوية

↳ An infinite bus is a power system, that its voltage and frequency don't change regardless of how much power is drawn or supplied to it

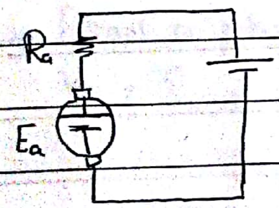
→ In the rotor circuit: Variable resistance is added R_F to control I_f

∴ We can control the flux ϕ generated by I_f.

→ In the Stator circuit:

- X_s is called synchronous reactance.
- R_a is the armature winding (stator winding) resistance. We will set $R_a = 0$ in the following analysis.
- E_f is called excitation voltage (induced emf). It is similar to E_a in DC machines:

Recall:



⇒ This is the armature winding circuit of a separately excited DC motor. E_a represents the induced emf because of the generator action. Recall:

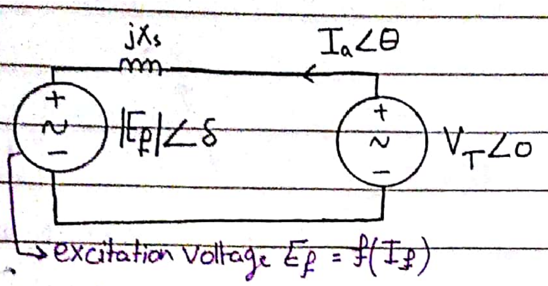
$$E_a = K\phi\omega_m \Rightarrow E_a = f(\phi, \omega_m)$$

⇓

In SM, the speed of the rotor is fixed ∴ E_f doesn't depend on ω_m, it only depends on ϕ

$$E_f = f(\phi) \quad \text{or} \quad E_f = f(I_f)$$

↳ ϕ produced by I_f



"Stator circuit per phase"

→ Synchronous motors always consume real electrical power "P", because mechanical loads require real power (not reactive power).

→ However, synchronous motors may consume or deliver reactive power Q

→ δ is called the power angle. It represents the difference between the phase angle of the induced voltage E_f and the phase angle of the terminal voltage V_T

- $|\delta| = |\gamma| =$ load angle, and hence if $T_{load}\uparrow \Rightarrow |\delta|\uparrow$
- δ is −ve in Synchronous motors

  δ is +ve in Synchronous generators

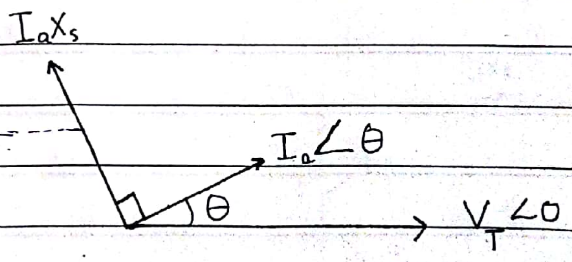
→ In the following analysis, we will investigate the real and reactive power consumed by the SM.

[*] Real power and reactive power equations in SM:

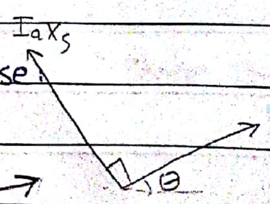
→ Apply KVL on the above circuit:

$$E_f\angle\delta = V_T\angle 0 - I_a\angle\theta\,(jX_s)$$

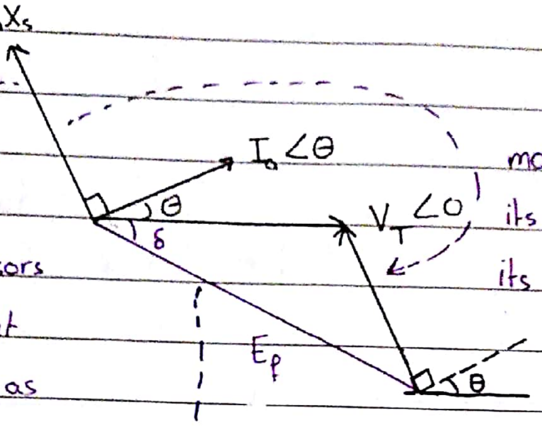
→ Draw a phasor diagram, showing the following parameters:
V_T, I_a, I_aX_s



I_a represents the current flowing through the reactance X_s, while I_aX_s represents the voltage drop across the reactance X_s. Recall from electrical course voltage leads current by 90° in inductors (or current lags voltage by 90°)



لهاد السبب العلاقة بين
I_a و I_aX_s هيك



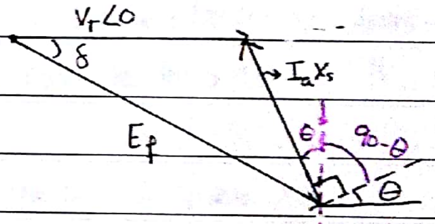
*Note: These arrows are called phasors (not vectors), but we treat them as Vectors.

move this phasor, without changing its magnitude (length of the phasor) and its orientation

→ This phasor represents E_f, since $E_f\angle\delta = V_T\angle 0 - I_a(jX_s)$

→ Notice that the power angle δ is -ve [Recall: δ represents the phase angle between V_T and E_f]

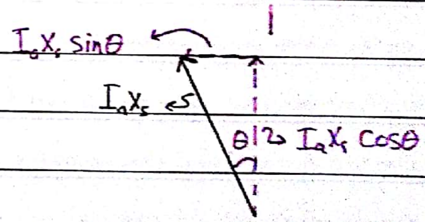
→ From the adjacent phasor diagram:



* $I_aX_s\sin\theta = E_f\cos\delta - V_T$ (Phasor components in the X-direction)

$$\downarrow$$

$$I_a\sin\theta = \frac{E_f\cos\delta - V_T}{X_s}$$

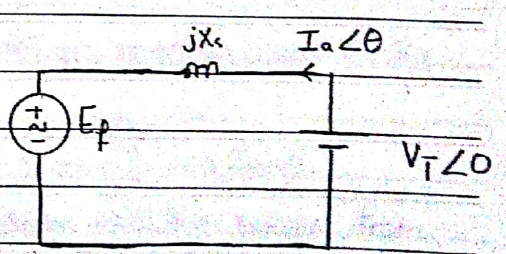


* $I_aX_s\cos\theta = E_f\sin\delta$ (Phasor components in the y-direction)

$$\downarrow$$

$$I_a\cos\theta = \frac{E_f\sin\delta}{X_s}$$

→ From the adjacent circuit:



$$P = 3V_T I_a\cos\theta = 3V_T\frac{E_f\sin\delta}{X_s}$$

$$Q = 3V_T I_a\sin\theta = 3V_T * \frac{E_f\cos\delta - V_T}{X_s}$$

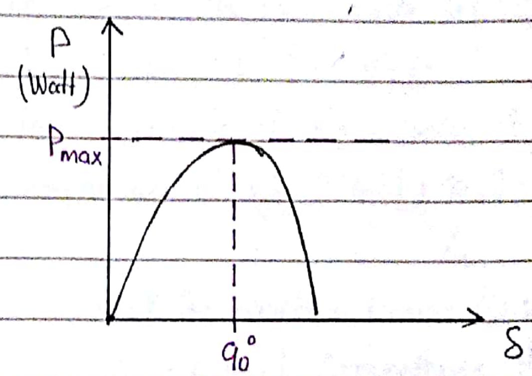
"Stator circuit per phase"

→ ضربنا بـ 3

لأنها 3ϕ

→ The real power consumed by the SM is given by

$$P = \frac{3\, V_T\, E_P \sin\delta}{X_s}$$



notice that $p = f(\delta)$

if $T_{load}\uparrow \Rightarrow \delta\uparrow \Rightarrow P\uparrow$

منطقي انه لما $T_{load}\uparrow \Rightarrow \uparrow P$ لازم

ال load بيستهلك real Power

$$T_{load} = \frac{P}{\omega_s} = \frac{3\, V_T\, E_P \sin\delta}{\omega_s X_s}$$

$\omega_s = \omega_m$

↳ This equation indicates that as $T_{load}\uparrow \Rightarrow \delta\uparrow$

This angle plays an important role for the stability of the SM. If the angle goes beyond 90° (i.e $\delta > 90^\circ$), the SM becomes unstable. Hence, we always operate the SM @ $\delta < 90$

عادةً بنشتغل $\delta \leq 30^\circ$

→ The reactive power (consumed or delivered) by the SM is given by

$$Q = \frac{3\, V_T\, (E_P \cos\delta - V_T)}{X_s}$$

- When $E_P \cos = V_T \Rightarrow Q = \text{zero}$ [The SM neither consumes nor delivers reactive power]
- When $E_P \cos\delta > V_T \Rightarrow Q$ is +ve (Positive) [The SM delivers reactive power to the grid]

  $SM \xrightarrow{Q} Grid$
- When $E_P \cos\delta < V_T \Rightarrow Q$ is −ve (negative) [The SM consumes reactive power from the grid]

  $SM \xleftarrow{Q} Grid$

→ Notes:

- The excitation voltage E_f has.
  - → magnitude: * We control the magnitude of E_f by controlling I_f (Recall: $E_f = f(I_f)$).
    * We control the magnitude of E_f, since it affects the magnitude of $Q = \frac{3V_T(E_f\cos\delta - V_T)}{X_s}$
    - → The importance of the reactive power Q: Delivering Q to the grid strengthens the grid ⤳ وهاد الاشي بوفّر مصاري لأنه ما منحتاج نرفع مصاري مرة نحسن الشبكة
  - → phase angle δ: δ is controlled by T_{load}
    if $T_{load}\uparrow \Rightarrow \delta\uparrow$

[*] V curves of synchronous motors:

→ V curves of a SM represents the relation between the field current I_f (in the rotor circuit) and the armature current I_a (in the stator circuit)

→ In V curves, I_a is plotted against I_f. However we will plot I_a against $E_f = f(I_f)$, since it is easier to predict the curves.

- V curve under no load:

→ under no load $T_{load} = 0$, $T_{load} = \frac{P}{\omega_s} \Rightarrow P = 0$

∴ under no load P is always = zero

→ However Q might be zero, +ve or -ve

under no load δ = zero

Q = zero when $E_f\cos\delta = V_T$

• V curve under no load:

→ Under no load $T_{load} = 0$, $T_{load} = \frac{P}{\omega} \Rightarrow P = 0$

$\therefore$ Under no load, P is always $P = 0$

→ However, Q might be zero, +ve or −ve

Under no load $\delta = zero$

* $Q = zero$ when $E_f \cos\delta = V_T$ ($\delta = zero$ "no load") $\Rightarrow \therefore Q = zero$ when $E_f = V_T$

$Q = 3 V_T I_a \sin\theta$

when $Q = zero \Rightarrow I_a = 0$

* Q is +ve when $E_f \cos\delta > V_T$ ($\delta = zero$ "no load") $\Rightarrow \therefore Q$ is +ve when $E_f > V_T$

$$Q = \frac{3 V_T (E_f \cos\delta - V_T)}{X_s} = \frac{3 V_T (E_f - V_T)}{X_s} \quad (\delta = zero \text{ "no load"})$$

Assuming that $E_f > V_T$: as $E_f \uparrow \Rightarrow Q \uparrow \Rightarrow Q = 3 V_T I_a \sin\theta$

when $Q \uparrow \Rightarrow I_a \uparrow$

$\therefore$ when $E_f > V_T$, if $E_f \uparrow \rightarrow Q \uparrow \rightarrow I_a \uparrow$

* Q is −ve when $E_f \cos\delta < V_T$ ($\delta = zero$ "no load") $\Rightarrow \therefore Q$ is −ve when $E_f < V_T$

$$Q = \frac{3 V_T (E_f - V_T)}{X_s}$$

Assuming that $E_f < V_T$: as $E_f \downarrow$, Q becomes more negative

يعني مثلاً Q بتكون -2 بتصير -3

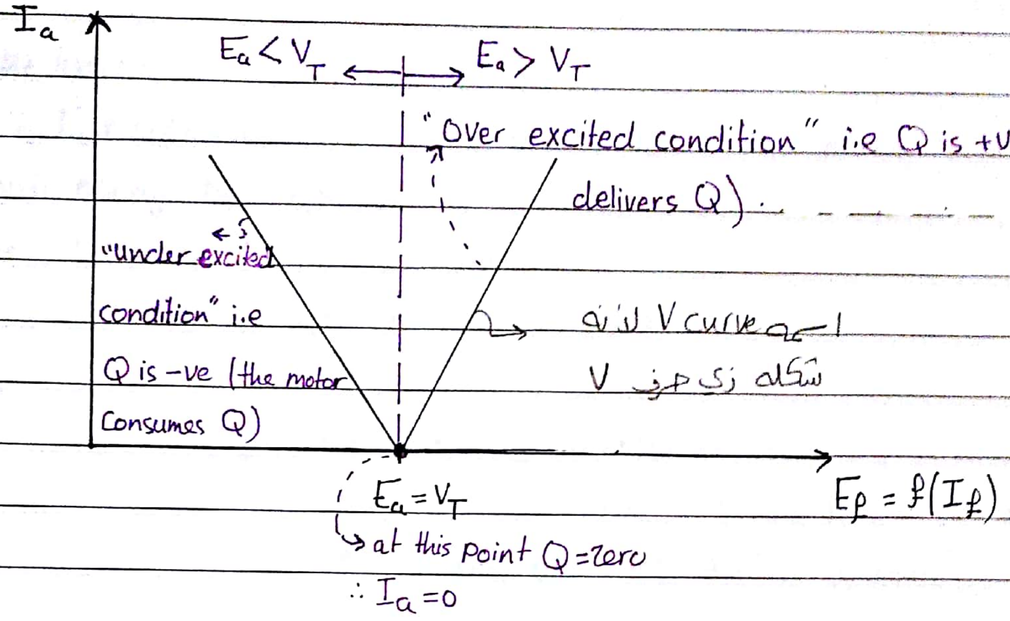
The negative sign indicates only that the motor is "consuming" reactive power. as $E_f \downarrow \Rightarrow |Q| \uparrow$ i.e the motor is consuming more reactive power.

↳ $|Q| = |3 V_T I_a \sin\theta| \Rightarrow$ as $E_f \downarrow \Rightarrow |Q| \uparrow \Rightarrow I_a \uparrow$

$\therefore$ when $E_f < V_T$ as $E_f \downarrow \Rightarrow I_a \uparrow$

→ From the above discussion, we can draw the V-curve $\Rightarrow$

- V curve under no load ($T_{load} = 0$, $P = 0$)



→ Notice that when $E_a > V_T \Rightarrow$ as $E_a \uparrow \Rightarrow I_a \uparrow$

when $E_a < V_T \Rightarrow$ as $E_a \downarrow \Rightarrow I_a \uparrow$

→ Recall from electrical course:

Capacitors consumes only reactive power (i.e P = zero, Q = −ve)

$\therefore$ SM acts like capacitors when $T_{load} = 0$ (i.e P = zero) + $E_a < V_T$

لأنو بهاي الحالة ، Q بتكون سالبة

→ From the above curve:

when $E_a < V_T \Rightarrow P = 0$ "no load", Q = −ve, I_a = non zero

$E_a = V_T \Rightarrow P = 0$ "no load", Q = zero, I_a = zero

$E_a > V_T \Rightarrow P = 0$ "no load", Q = +ve, I_a = non zero

→ When:

$E_a < V_T$, I_a = non zero, because $Q = 3 V_T I_a \sin\theta \Rightarrow Q$ = nonzero $\therefore I_a$ = nonzero

$P = 3 V_T I_a \cos\theta$ = zero

$E_a = V_T$, I_a = zero because $Q = 3 V_T I_a \sin\theta$ = zero, $P = 3 V_T I_a \cos\theta$ = zero } both are equal to zero $\therefore I_a$ = zero

$E_a > V_T$, I_a = zero because $Q = 3 V_T I_a \sin\theta \Rightarrow Q$ = non zero $\therefore I_a$ = non zero

$P = 3 V_T I_a \cos\theta = 0$

Conclusion: $I_a = 0$, only if both P and Q = zero

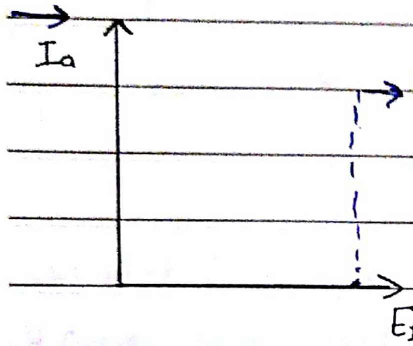
• V curves under load $\left(T_{load} \neq 0,\ P = \frac{T_{load}}{W_s} \neq 0\right)$

→ Under load:

$P = 3 V_T I_a \cos\theta$ = nonzero. Since under load P is always non zero $(P \neq 0)$, I_a will always be nonzero $(I_a \neq 0)$

لأنه حتى تكون $I_a = 0$ لازم $P = Q = 0$ في حالة وجود

$\therefore I_a \neq 0 \Leftarrow P \neq 0 \Leftarrow$ load



→ @ the x-axis $I_a = 0$. Under load I_a is always non zero
∴ Under load, V curve will not touch the x-axis
(i.e V curve will be shifted upward)

→ Under load:

$P \neq 0$, but Q might be zero, +ve or −ve:

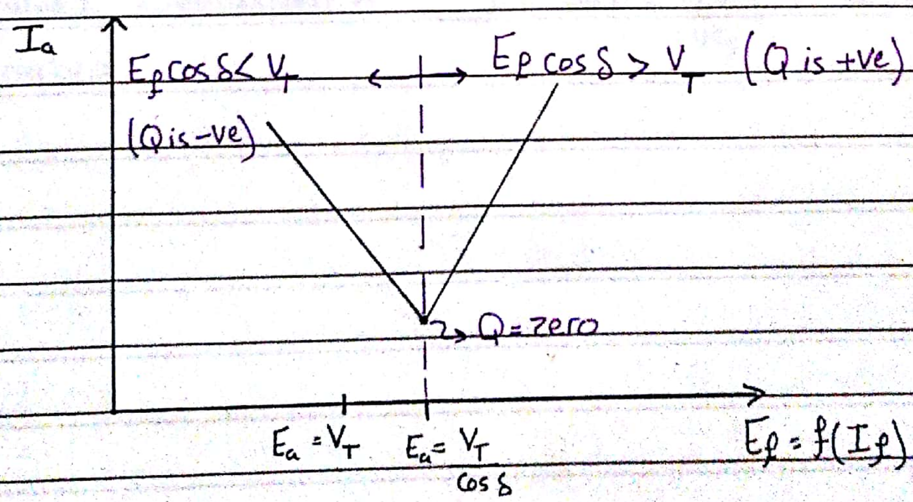
* Q = zero when $E_f \cos\delta \doteq V_T$ (under load $\delta \neq 0$)

$$E_f = \frac{V_T}{\cos\delta} \quad \therefore E_f > V_T$$

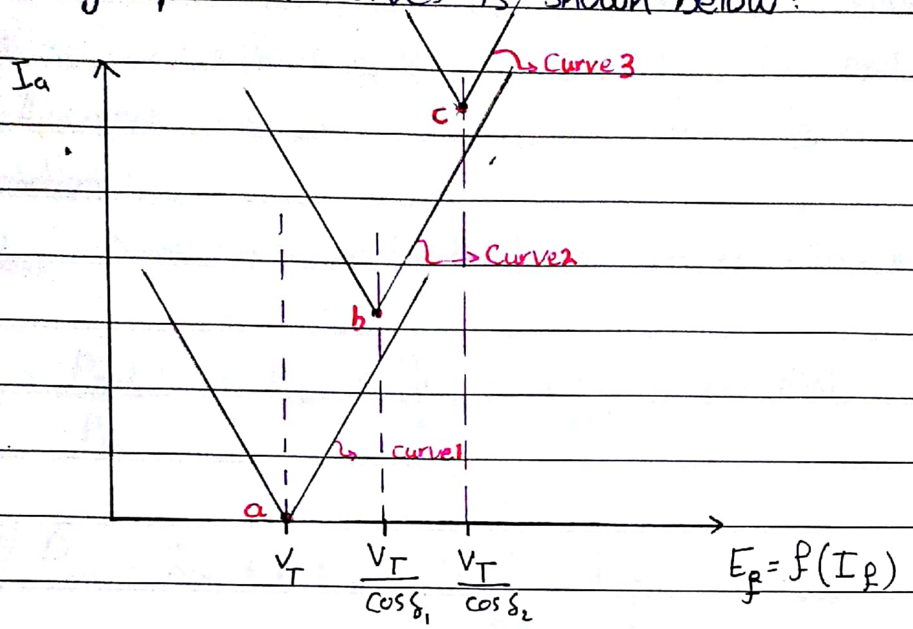
$\cos\delta$ → قيمته أقل من 1

* Q = +ve when $E_f \cos\delta > V_T$. As $E_f \uparrow \Rightarrow Q \uparrow \Rightarrow I_a \uparrow$
* Q = −ve when $E_f \cos\delta < V_T$. As $E_f \downarrow \Rightarrow Q$ becomes more negative → $I_a \uparrow$

∴ V curve under load is shown below



→ A group of V-curves is shown below:



→ Curve 1 represensts V curve under no load, while curves 2,3 represent V curves under load.

→ @ point a (curve 1): $E_f = V_T$, $Q=0$, $P=0$

→ @ point b (curve2): $E_f = \frac{V_T}{\cos\delta_1}$, $Q=0$, $P\neq 0$

→ @ point c (curve 3): $E_f = \frac{V_T}{\cos\delta_2}$, $Q=0$, $P\neq 0$

→ $\cos\delta_2 < \cos\delta_1$ $\therefore$ E_f @ Point C > E_f @ point b

→ $P = 3 I_a V_T \cos\theta$

I_a @ Point b < I_a @ point C ⇒ This indicates that P @ point b < P @ point C. Conclusion: if $T_{load}\uparrow \Rightarrow P = T_{load}\uparrow$ / ω_s ⇒ V curve will be shifted upward

Example: 3000 hp, 6600 V, 3ϕ Y connected SM operates @ full load @ leading pf of 0.8 and efficiency $\eta = 74.6\%$, $X_s = 11\,\Omega$, Find

it means SM injects or delivers Q

① Apparent power per phase ② line current ③ E_f ④ δ

Solution:

P_{out} = power consumed by the load = 3000 hp = 3000 * 0.746 = 2238 kW

$$\eta = \frac{P_{out}}{P_{in}} \Rightarrow P_{in} = \frac{P_{out}}{\eta} = \frac{2238}{0.746} = 3000\,kW$$

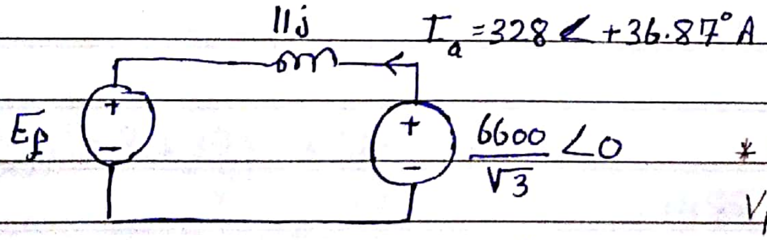
$$① \; P_{in} = S_{in} * P.f \Rightarrow S_{in}\Big|_{3\phi} = \frac{P_{in}}{Pf} = \frac{3000}{0.8} = 3750\,KVA$$

Apparent power per phase: $S_{in}\Big|_{1\phi} = \frac{S_{in}\big|_{3\phi}}{3} = \frac{3750}{3} = 1250\,KVA$

$$② \; S_{in}\Big|_{3\phi} = \sqrt{3}\, V_L\, I_L$$

$$3750*10^3 = \sqrt{3} * 6600 * I_L \Rightarrow I_L = 328 \angle \underset{\text{leading}}{+} \cos^{-1}(0.8) \; (\rightarrow pf)$$
$$= 328 \angle +36.87^\circ \; A$$

③



"Stator circuit per phase"

* In Y connection
$V_{phase} = \frac{V_{line}}{\sqrt{3}}$, $I_{Phase} = I_{line}$

$$E_f = \frac{6600}{\sqrt{3}} \angle 0 - (11j)(328 \angle +36.87^\circ) = 6635.9 \angle \overset{\delta}{-25.78} \; V/phase$$

زاوية سالبة معناها حلها صح

$$E_{f_{LL}} = \sqrt{3} * E_{f_{LN}} = \sqrt{3} * 6635.9$$

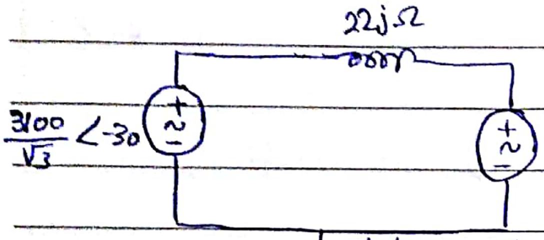
* Notice that $E_f \cos\delta > V_T \Rightarrow$ This indicates that SM delivers Q

Example: 3ϕ, 4 pole, 60 Hz, Y connected SM is connected to a 3980 V 3ϕ grid. The motor generates $E_f|_{LL}$ = emf$|_{LL}$ = 3100 V when $I_f = 25\,A$, $X_s = 22\,\Omega$, $\delta = 30$. Find I_a, pf, T_{load}

notice that $E_f \cos\delta < V_T$ $\therefore$ SM consumes Q

$\Downarrow$

we will get lagging power factor



$V_T = \dfrac{3980}{\sqrt{3}} \angle 0 \ V$

↳ stator circuit per phase

$* \ E_f = V_T - (X_s j) I_a \Rightarrow \dfrac{3100}{\sqrt{3}} \angle -30 = \dfrac{3980}{\sqrt{3}} - 22j\, I_a$

$I_a = 53 \angle -39.89 \ A$

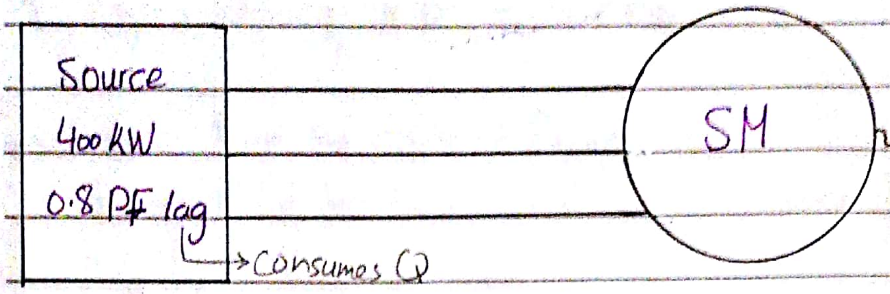
$* \ pf = \cos(\angle V - \angle I_a) = \cos(0 - (-39.89)) = 0.77$ lag

$* \ T_{load} = \dfrac{3 V_T E_f \sin\delta}{X_s W_s}$, $N_s = \dfrac{120 f}{P} = \dfrac{120 * 60}{4} = 1800\ rpm = 1800 * \dfrac{2\pi}{60}$

$= 188.49\ rad/s$

$T_{load} = \dfrac{3 * \frac{3980}{\sqrt{3}} * \frac{3100}{\sqrt{3}} * \sin 30}{22 * 188.49} = 1488\ N.m$

Example



$X_s = 2\,\Omega/\text{phase}$
6600 V, 50 Hz, 6 pole
Y connected SM.

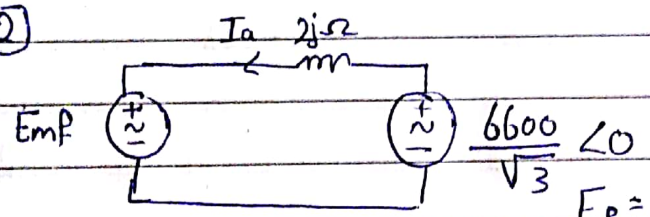
Find: ① I_a ② $Emf = E_f$ ③ T_{max} ④ If E_{mf} is increased by 25% + $P_{in} = 400\,kW$
Find δ

① $P_{in} = \sqrt{3}\, V_L I_L * pf \Rightarrow 400*10^3 = \sqrt{3} * 6600\, I_L * 0.8$

$I_L = 43.73 \angle -\cos^{-1}(0.8)$ A
$= 43.73 \angle -36.86$ A

$I_a = I_L = 43.73 \angle -36.86$ A

②



$Emf = \frac{6600}{\sqrt{3}} \angle 0 - (2j)(43.73 \angle -36.86)$

$E_f = |Emf|_{LN} = 3758.7 \angle -1^\circ$

→ -ve, which is correct

③ T_{max}

$T = \frac{3\, V_T E_f \sin\delta}{X_s w_s}$, T is maximum when $\delta = 90^\circ \Rightarrow \sin\delta = 1$

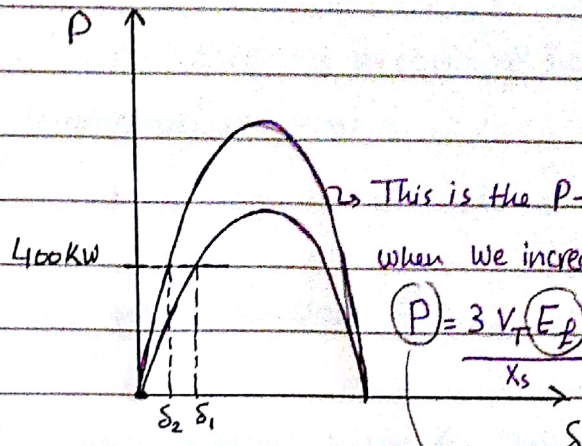
$$\therefore T_{max} = \frac{3 * \frac{6600}{\sqrt{3}} * 3758.7 * (1)}{2 * \underbrace{\frac{120*50}{6} * \frac{2\pi}{60}}} = 205155\ N.m$$

$w_s = \frac{120 * f}{P} * \frac{2\pi}{60}$

زيادة عن طريق زيادة I_f

(4) $E_{f_2} = 1.25\, E_{f_1} \Rightarrow P_1 = P_2 = 400\,KW$

→ Notice from the adjacent figure δ_2 must be less than $\delta_1 = 1^\circ$



→ This is the P-δ curve when we increase E_f

$$P = \frac{3 V_T E_f \sin\delta}{X_s}$$

→ if $E_f \uparrow \Rightarrow P \uparrow$

$$P_1 = P_2$$

$$\frac{\cancel{3V_{T_1}}\,\cancel{E_{f_1}} \sin\delta_1}{\cancel{X_s}} = \frac{\cancel{3V_{T_2}}\,(1.25\,\cancel{E_{f_1}}) \sin\delta_2}{\cancel{X_s}}$$

$$\sin\delta_1 = 1.25 \sin\delta_2 \Rightarrow \sin(1) = 1.25 \sin(\delta_2) \Rightarrow \delta_2 = 0.8^\circ < 1^\circ$$

$\delta_2 = -0.8^\circ$ لأنه اصلا متقاطعة موجود