

Machines

Dr. Dallabeeh

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MACHINES

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• Magnetic Field:

* what?

This is the field produced by permanent magnet or electromagnetic.

"gauss weber"

* why?

Because the operation of transformers and electrical machines depend on the existence of magnetic field.

* How?

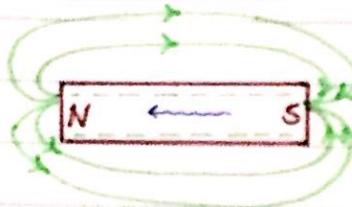
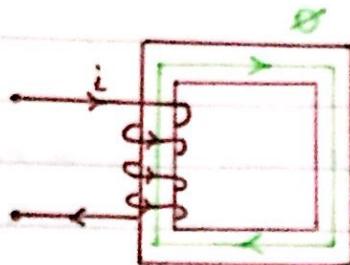
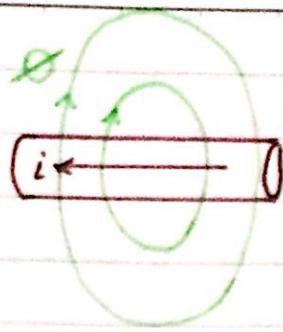
* principles:

- i. Any current (i) generates a magnetic field (Φ); where ($\Phi \propto i$).

| <u>i</u> | <u>Φ</u> |
|-----------------------|--------------------------|
| Dc | Dc |
| Ac | Ac |

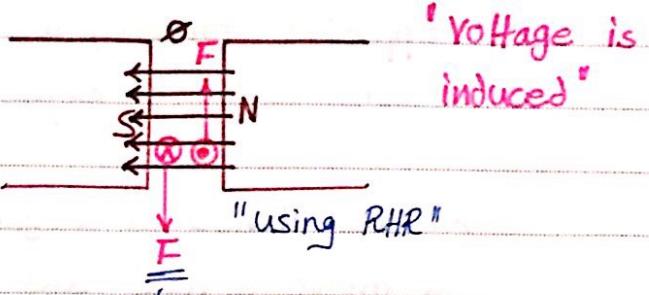
' Φ ' is represented by closed loops whose direction can be found by using "RHR" \Rightarrow Right Hand Rule.

e.g. :

* Inside " $S \rightarrow N$ "* Outside " $N \rightarrow S$ "

ii. A conductor cutting a magnetic field will have a voltage induced in it; whose polarity can be found by using "RHR" \Rightarrow right hand Rule.

* This is the generator action.



'Voltage is induced'

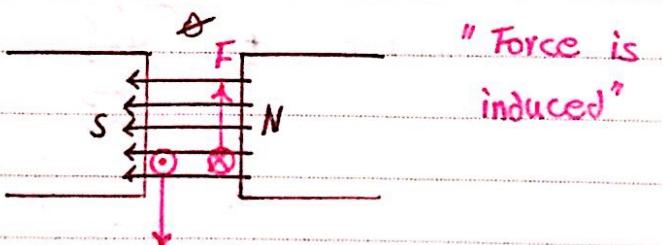
"using RHR"

Direction of applied force.
(i.e. Motion).

\odot \otimes \Rightarrow Direction of induced voltage
out of the Board. \rightarrow Into the Board.

iii. A current carrying conductor located in a magnetic field will have a force (F) induced in it; where the direction of (F) can be found by using LHR "Left Hand Rule".

* This is the motor action.



"Force is induced"

\odot = Direction of applied current.

\otimes

F = Direction of generated or developed force.



- * Motor: gets electrical but gives mechanical.
- * Generator: gets mechanical but gives electrical.

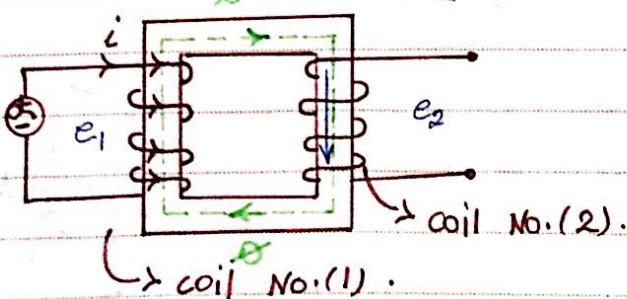
\Rightarrow Note: Because of the existence of two opposite forces; a torque will be generated. "consequently a torque will be developed".

\Rightarrow Note: Hence in a generator action; there is a motor action and in a motor action; there is a generator action.

* when we connect a voltage source with a load; a current will flow then a force will be developed which opposes the applied force.

iv. A time varying flux (Φ) linking (i.e. cutting) a conductor or a coil will induce a voltage (e) in the conductor or coil.

* This is the transformer action.



* If we connected it with a DC-voltage source, a DC flux will be developed. But, there is no voltage will be induced in the second coil depending on "Faraday's Law".

[If the flux is constant; there is no voltage induced.]

* Ex.:

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = 10$$

$e_1 = 20$ volt ; $e_2 = ?$ if connected to a DC-source.

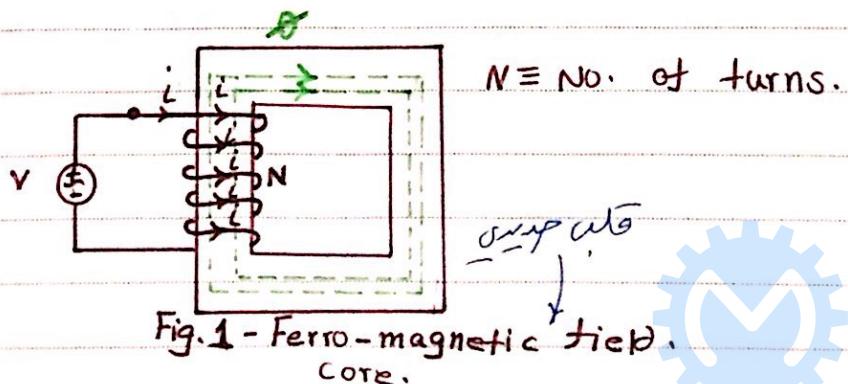
$$\Rightarrow e_2 = 0$$
 volt.

• Basic concepts of magnetic circuits.

* Amper's law $\Rightarrow \oint H \cdot dL = i$ --- (1) ; where {
↳ Line integral.

$H \equiv$ Magnetic field strength which produced by the current (i).

$L \equiv$ The length of the flux line which encloses i and illustrated as follows:-



* For (Fig. 1);

$$H L_c = \frac{N I}{L_c} \quad \dots \dots (1)$$

(Amp.-turns)

$\therefore L_c$: mean length of flux path.

\Rightarrow unit of (H) is (A. -turns)/m.

$$B = \sigma / A \quad \dots \dots (2)$$

B : Flux density. ($\text{wb.}/\text{m}^2$) or Tesla (T)

A : Cross-sectional area of the core on which (σ) is perpendicular \perp .

(cross-sectional) σ relative (flux) \perp to Area.

$$B = \mu H ; \mu = \frac{\mu_0 \mu_r}{1} \equiv \text{permability.}$$

permability of free \leftarrow permability "relative" of the space ($4\pi \times 10^{-7}$) core.

$$\bullet Ni = HL = \frac{B}{\mu} L = \frac{\sigma}{A} \cdot \frac{L}{\mu}$$

$$\therefore Ni = \frac{L}{A\mu} \sigma$$

(mmf) \leftarrow This is called "Reluctance" \mathcal{R}

$$\underbrace{P}_{\text{[Eta; sensa]}} \text{ (mmf)} = \mathcal{R} \sigma ; \text{ where } \mathcal{R} = \frac{L}{A\mu}$$

$\mu = \mu_0 \mu_r ; \mu_r = \frac{\mu}{\mu_0}$

$$\sigma = \frac{L}{\mathcal{R}} ; \text{ As } \mu \uparrow \Rightarrow \text{ Then } \mathcal{R} \downarrow \Rightarrow \text{ Then } \sigma \uparrow$$

* That is why in transformers and machines;
Ferromagnetic materials (i.e. high μ_r) are used.

* Analogy between electrical and magnetic ccts. :

| Electrical ccts. | Magnetic ccts. |
|--|---------------------------------|
| emf ; e or V i | mmf ; \mathcal{P} σ |
| \mathcal{R} or χ or \mathcal{Z} \leftarrow Resistance. | \mathcal{R} (Reluctance) |

* This is the table that can be used to draw the equivalent magnetic circuits for any magnetic structure.

+ consequently one can apply cat. laws to magnetic cat's.

- Ohm's law $\Rightarrow P = I \mathcal{E}$.

- KVL $\Rightarrow \sum I = 0$ around a loop.

- KCL $\Rightarrow I_{in} = I_{out}$ at a given junction.

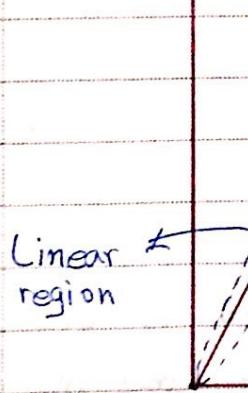
★ Don't forget parallel and series connection. :-

The same characteristics can be applied to series and parallel connected elements.

$$B = \mu H$$

+ Graphical relationship between B and H.

$$\Delta \text{ or } B = \frac{\mathcal{E}}{A} \quad (\mathcal{E} \text{ is current} \times B \times \text{length})$$



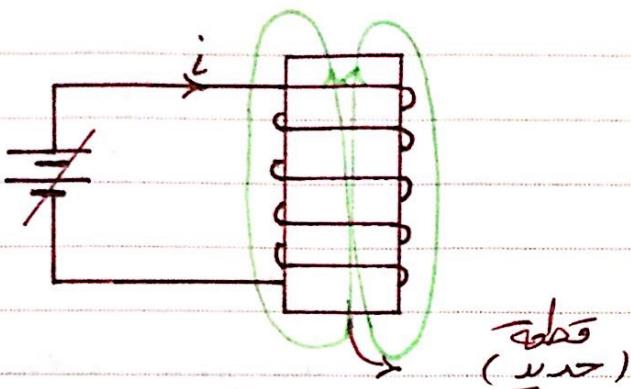
+ This curve is called (B-H) curve or Magnetization Curve.

$$N_i = HL$$

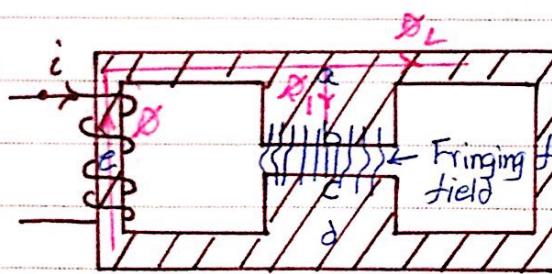
$$H = \frac{N_i}{L}$$

* Hence curve is used in the solution of magnetic cct's.

* Each material has it's own (B-H) curve.

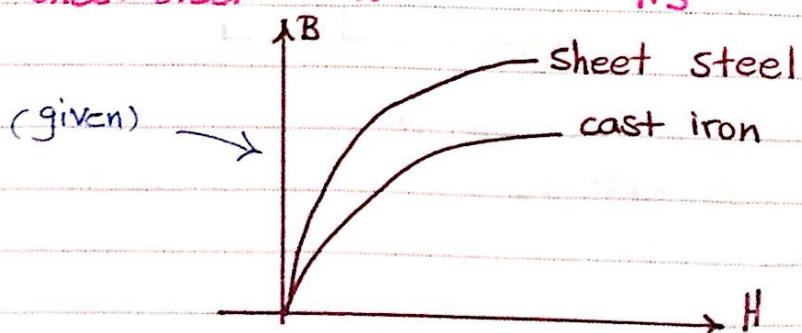


* Ex. 1



* In this case $\mu_g > \mu_c$; in this e.g. (fringing is neglected).

| Path | Material | Length (cm) | Area (cm^2) | Flux (kL) ^{kilo Line or vibers} |
|------|-------------|-------------|---|--|
| ab | cast iron | 10 | 10 | 100 |
| bc | Air-gap | ? | use the obtained results to find $l_{bc} = ?$ | 100 |
| cd | cast iron | 10 | 10 | 100 |
| dea | sheet steel | 60 | 17.5 | 225 |
| afd | sheet steel | 60 | 7.5 | ? |



B (KL/cm²) H (AT 1cm) HL P

10 120 1200

10 120 1200

18 150 9000

16.7 90 5400

$$B = \propto / A$$

\Rightarrow To find (H) use the given curve

(P) By KVL:

$$P = P_{\text{rea}} + P_{\text{atd}}$$

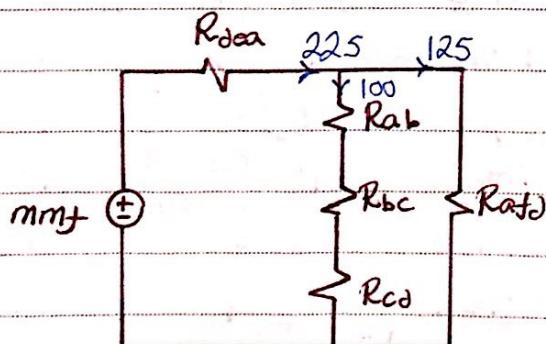
$$= 9000 + 5400$$

$$= 14400$$

* mmf (air gap)

mmf (bc)

$$R_{(ab, bc, cd)} = R_{\text{atd}}$$



* $l_{bc} = 0.376 \text{ cm}$, check it?

Flux (atd) = 125 KL

• Losses in magnetic materials:

1. Eddy current losses; P_e .

* $\mathcal{E}(t) \Rightarrow$ induces a voltage (e) in the core; since the core is made of conducting material, then a current (i) "This is called eddy current". Due to (e) will circulate within the core in such away to oppose $\mathcal{E}(t)$.

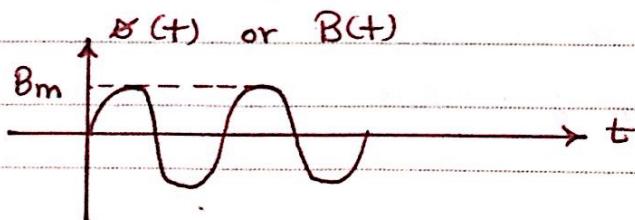
* This is in going to cause losses and heat the core.

* These losses are called P_e .

\Rightarrow It can be found that $P_e = k_e B_m^2 f^2$

k_e \equiv constant depends on the type of ferromagnetic material.

$B_m \equiv$ Peak or Maximum value of flux density.

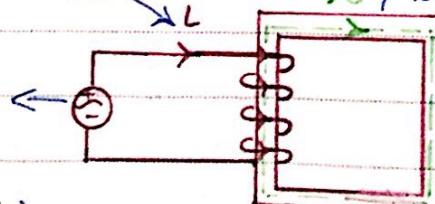


$f \equiv$ Frequency of applied voltage or current.

Applied Current i \rightarrow $\mathcal{E}; \mathcal{E} = \mathcal{E}_m \sin(\omega t)$

sinusoidal voltage source

$$V = V_m \sin(\omega t)$$



* There is another current inside the core called (eddy current).



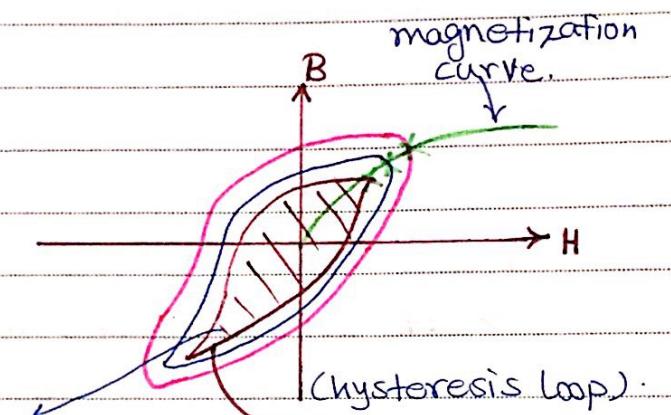
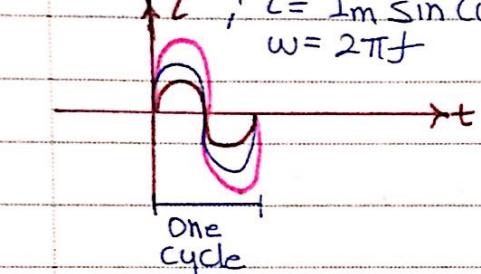
* P_e can be reduced by making the core of laminations between which there is an insulating material.

• Wile Tablem Eddy currents (Laminated core) \rightarrow \downarrow \uparrow
 (Eddy-current Losses) \rightarrow losses

2. Hysteresis Losses:

$$i; i = I_m \sin(\omega t)$$

$$\omega = 2\pi f$$



"Area of the hysteresis loop represents energy density per cycle; it represents the energy supplied or consumed in the processes of magnetization of (hysteresis loop) magnetic material."

\Rightarrow As the period of each cycle gets longer; the area of each loop becomes larger.

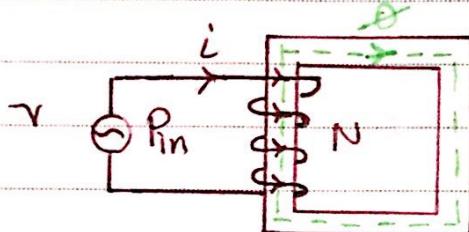
* This is called hysteresis losses (P_h).

* If we connect the loops tips together; we will get (Magnetization curve)
 \Rightarrow It can be found that $P_h = k_h B_m^n f$

- k_h = constant "Depends on the type of material"
- n = constant depends on the type of material
($1.5 \leq n \leq 2.5$)

↙ By definition

* Core losses; $P_c \triangleq P_e + P_h$



P_h losses:

1. To generate flux
2. core losses (Inside the core)
3. Electrical losses (Inside the coil).

* Fill in the blanks:

$$\Rightarrow \oint H dL = Ni$$

\Rightarrow i needs an applied voltage

$\Rightarrow e$ = Induced voltage due to $\dot{\phi}$

* By Faraday's Law $\Rightarrow e \triangleq \frac{\omega L}{\Delta t}$
 \hookrightarrow to specify the polarity.

[If the flux is DC; there is no induced (e) \Rightarrow
 only in AC]

If $\theta = \theta_m \sin(\omega t)$

Derive it

$$e = N\theta_m \omega \cos(\omega t)$$

$$e = E_m \cos(\omega t) \text{, where } E_m = N\theta_m \omega \equiv \text{Peak Value.}$$

$$E_{rms} = \frac{E_m}{\sqrt{2}}$$

$$E_m = N\theta_m \omega = N\theta_m (2\pi f)$$

$$\therefore E_{rms} = 4.44 N\theta_m f.$$

• **Comments :**

1. There is a rated value for (B_m) in order not to exceed the rated value of (P_e) and (P_h).

2. It was found that $E = 4.44 Nf \Phi_m$ [rms].

$$\therefore \Phi_m = \frac{E}{4.44 Nf} \quad \therefore \text{if } (f) \downarrow \Rightarrow \text{Then } (\Phi_m) \uparrow \Rightarrow$$

In order not to exceed rated value of Φ_m ; then (E) should be decreased. \therefore (Derating of the device concerned. Freq. In Jordan 50 Hz U.S.A 60 Hz

★ **Transformers** ★

what ?

It is a device which is used to convert a voltage or a current from one value to another. That is step-up or step-down.

why?

1. It can be used in process of electrical energy transfer, In this case it's called (power Transformer). *

2. It can be used in process of measuring voltage and current, In this case it's called (Instrument Transformer).

3. It can be used in process of changing the magnitude of Impedance, In this case it's called (Impedance matching Transformer).*

How?

Construction and analysis of single phase and three phase transformers (1-ph., 3-ph.).

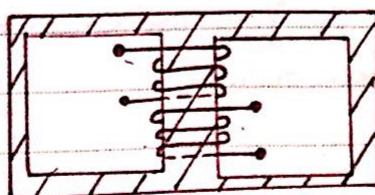
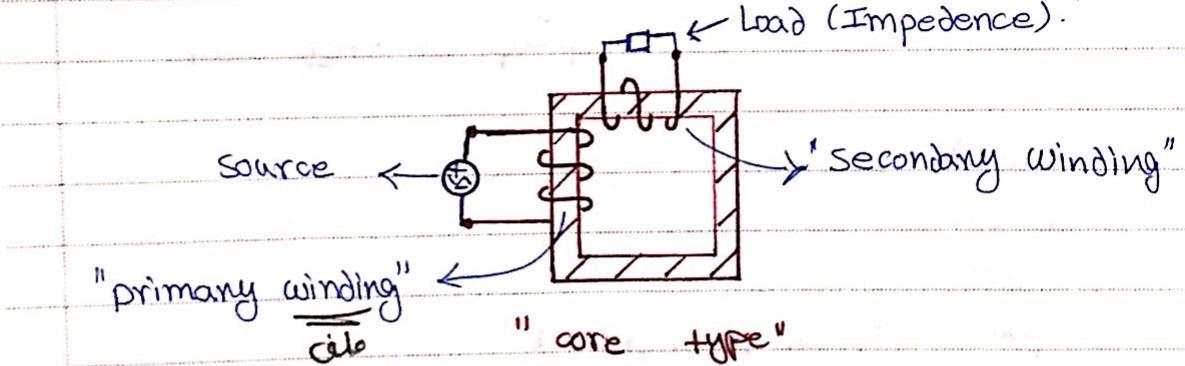
* 1-ph. Transformer:

=> construction:

It consists of:

1. A magnetic core.
2. Two or more coils.

core could be one of two basic types:



"shell type"

- Primary winding: (P.w.) It is the winding connected to a source.

- Secondary winding: (S.w.) It is the winding connected to a load.

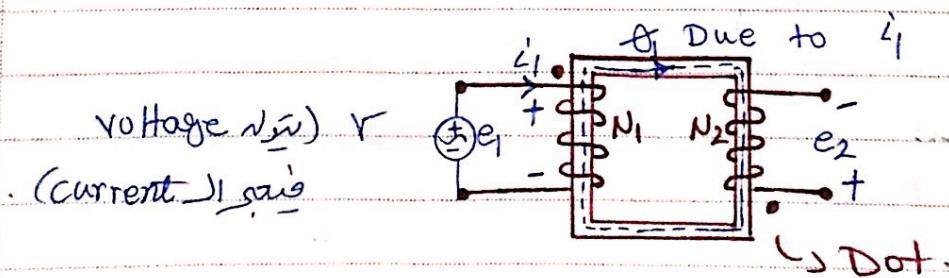
- Notes:

- If the transformer is step-up; then the primary is Low voltage (L.v.); secondary is High voltage (H.v.).

- If the transformer is step-down; then the primary is High voltage (H.v.); secondary is Low voltage (L.v.).

* Ideal 1-Ph. Transformer.:

1. A lossless transformer (No core or electrical losses.)
2. All the flux is within the core.
3. $\mu_r = \infty$



where;

V \equiv Applied voltage.

I \equiv Applied current.

$N_1, N_2 \equiv$ no. of turns.

$e_1, e_2 \equiv$ Induced voltages due to θ_1 .

where (as a magnitude):

$$\text{mag. law of coil 1: } e_1 = N_1 \frac{\partial \theta_1}{\partial t} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \therefore \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

$$\text{mag. law of coil 2: } e_2 = N_2 \frac{\partial \theta_1}{\partial t}$$



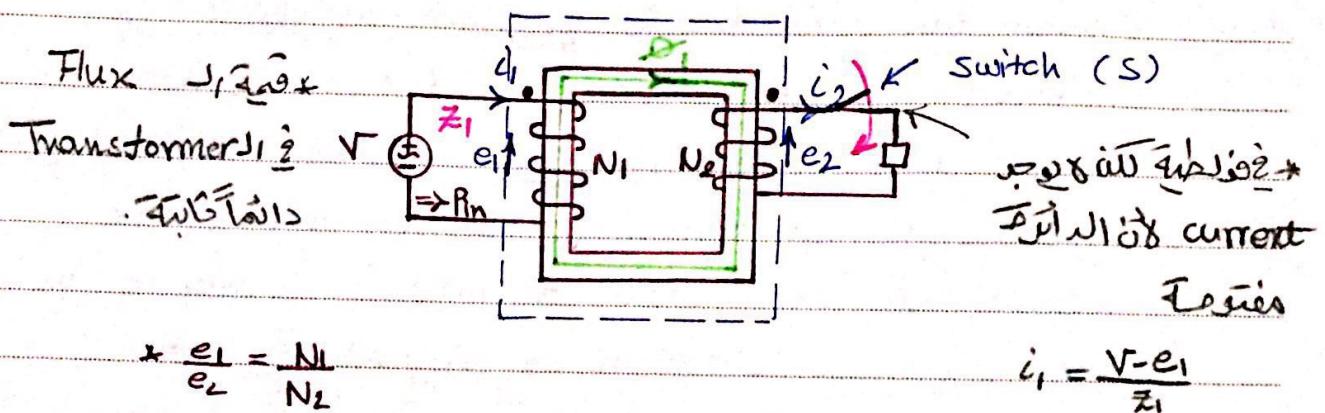
The voltage direction can be found by knowing the current flow direction (+ve - sign at the current entrance).

*polarity of e_1 and e_2 ?

This is defined by Dot convention (•) which is defined as:

- If a current enters a dotted terminal, it will induce a voltage in the second coil where (+ve) is at its dotted terminal.

- If a current enters an un-dotted terminal, it will induce a voltage in the second coil where (+ve) is at its undotted terminal.



- Dot concept is based on Flux Aiding (i.e. if currents enter dotted terminals, they will generate flux in the same direction).

• Note! The previous analysis was based on No-Load condition.

* Loading conditions:

close \leq "switch" $\Rightarrow i_2$ is going to flow $\Rightarrow i_2$ generates \vec{B}_2 $\Rightarrow \vec{B}_2$ opposes $\vec{B}_1 \Rightarrow$ Resultant flux in the core \downarrow .

$\therefore e_1 \downarrow \Rightarrow i_1$ is going to increase by an amount say i_1' ; in order to oppose mmf of i_2 . In order to bring the flux in the core to its original value.

i_1' is called "Load component of primary current", when

$$i_1' N_1 = i_2 N_2 \quad \dots \quad (1)$$

Summary

when the switch is open, a current i_1 is induced then θ_1 developed in the two coils.

Then the switch is closed. So, θ_2 is developed opposes θ_1 . So, $\Phi_{tot.}$ is reduced and e_1 is reduced as well. Then an mmf is developed in the opposite direction of secondary's coil mmf to let $\Phi_{tot.}$ returns to it's original value.

∴ In general primary i_1 consists of two components:

$$i_1 = \underline{i_{1,NL}} + \underline{i_1'} \quad \text{--- (2)}$$

no - load

Since

$$\underline{i_1'} \gg \underline{i_{1,NL}} \Rightarrow \therefore \underline{i_1'} \approx i_1 \quad \text{--- (3)}$$

Sub. (3) into (1) !

$$i_{1,NL} = i_{2,NL}$$

$$\therefore \frac{i_1}{i_2} = \frac{N_L}{N_1}$$

NO.

Fill in the blanks:

what is the function of i_1 (to generate flux Φ_1)

i_1 (?)

\Rightarrow Small Letters (Time Domain)

Capital Letters (Phasor Domain)

$$\therefore \frac{e_1}{e_2} = \frac{i_2}{i_1} = \frac{N_1}{N_2} = a$$

a is called "Turns ratio".

* Since a is a real number then e_1 & e_2 are in phase, also i_1 & i_2 are in phase.

$$\therefore \frac{E_1}{E_2} = \frac{I_2}{I_1} = a ; E, I \equiv \text{phasors.}$$

For ideal transformers $U = E_1$ and $U_L = E_2$

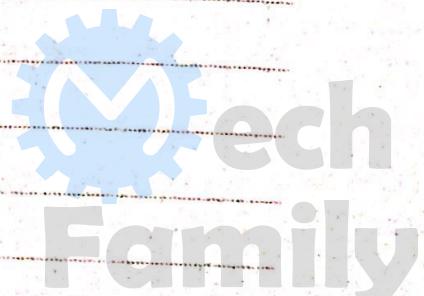
\hookrightarrow No resistance or reactance.

$$U = E + I_2 U_2$$

$$\therefore \frac{U}{U_L} = \frac{I_2}{I_1}$$

$\therefore U I_1 = U_L I_2 \Rightarrow$ apparent power.

$$|U| |I_1| = |U_L| |I_2|$$



- Apparent power of primary and secondary at HV (or) LV sides are equal.

$$P_{in} = U_1 I_1 \cos \theta_1 \quad | \quad \text{since } \theta_1 = \theta_2$$

$$P_{out} = U_2 I_2 \cos \theta_2 \quad | \quad \therefore P_{in} = P_{out}$$

$$P_{in} = U_1 I_1 \cos \theta_1 \quad | \quad \text{since } \theta_1 = \theta_2$$

$$P_{out} = U_2 I_2 \cos \theta_2 \quad | \quad \therefore P_{in} = P_{out}$$

$$\therefore \eta \text{ (i.e. efficiency)} = 100\% \Rightarrow \text{IDEAL} (P_{in} = P_{out})$$

* Impedance Relationship

$$Z = \frac{\text{phasor voltage}}{\text{phasor current}}$$

$$Z \triangleq \frac{U}{I} \Rightarrow \text{capital (I)}$$

$$Z_1 = \frac{U}{I_1} \quad (1) ; \quad Z_1 = \text{Impedance of transformer seen from primary side.}$$

$$\Rightarrow \frac{U_2}{I_2} = \frac{I_2}{I_1} = a \quad (2)$$

Sub. (2) into (1) ;

$$Z_1 = \frac{a U_2}{I_2/a} = a^2 \frac{U_2}{I_2} = a^2 Z_2 \Rightarrow Z_1 = a^2 Z_2$$

$$* \frac{U_1}{U_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$$

$$* Z_1 = \frac{U_1}{I_1} = \frac{aU_2}{I_2/a} = a^2 \frac{U_2}{I_2} = a^2 Z_2$$

$$Z_1 = a^2 Z_2 \quad \text{OR} \quad Z_2 = \frac{1}{a^2} Z_1$$

\therefore To reflect Z_2 (secondary impedance) to (primary side) multiply it by a^2 ; OR to reflect Z_1 (primary impedance) to (secondary side) divide it by a^2 .

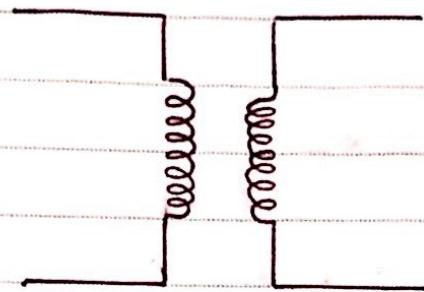
$$U_1 = aU_2 \quad \text{OR} \quad U_2 = \frac{1}{a} U_1$$

\therefore To reflect U_2 (secondary voltage) to (primary side) multiply it by a ; OR to reflect U_1 (primary voltage) to (secondary side) divide it by a .

$$I_1 = \frac{1}{a} I_2 \quad \text{OR} \quad I_2 = a I_1$$

\therefore To reflect I_2 (secondary current) to (primary side) divide it by a ; OR to reflect I_1 (primary current) to (secondary side) multiply it by a .

* This concept of reflection can be used to find a single equivalent circuit of Transformer by reflecting primary to secondary or the other way round. OR by reflecting "High voltage" to "Low voltage" or the other way round.



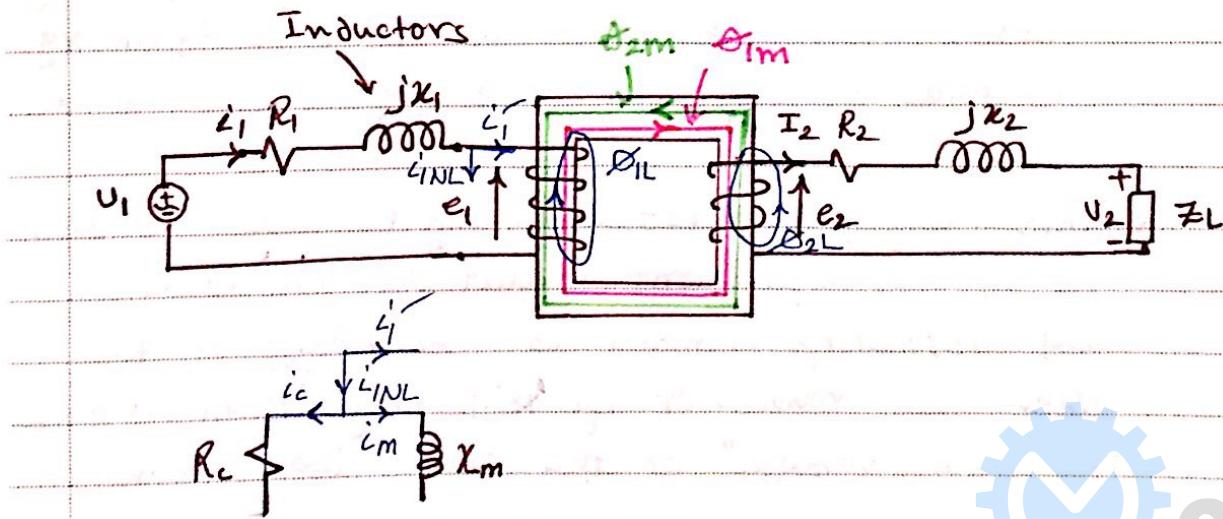
N_1, N_2

$$1 : \frac{N_2}{N_1} = a \quad \text{OR} \quad a = \frac{N_1}{N_2} 1 : 1$$

• In general:

1. To reflect from the "1" side to "a", then multiply "Z" by a^2 , multiply "V" by a and divide "I" by a . (primary to secondary).
2. To reflect from the "a" side to "1" side, then divide "Z" by a^2 , divide "V" by a , and multiply "I" by a .

* Practical or Real Transformers:



1. Each coil has resistance, R_1 and R_2 .
2. δ_{1L}, δ_{2L} "Leakage flux in the coils", they are represented by reactances (jx_1) and (jx_2)
- primary current \equiv two components (load + no load).

v_1 \equiv Primary applied voltage.

i_1 \equiv Primary applied current $= i_{1NL} + i_1'$

i_{1NL} \equiv is called excitation current
No Load

$i_{1NL} = i_m$ (magnetizing current) + i_c (core loss component).

R_c \equiv Resistance to represent current losses.

χ_m \equiv To represent mutual flux being generated.

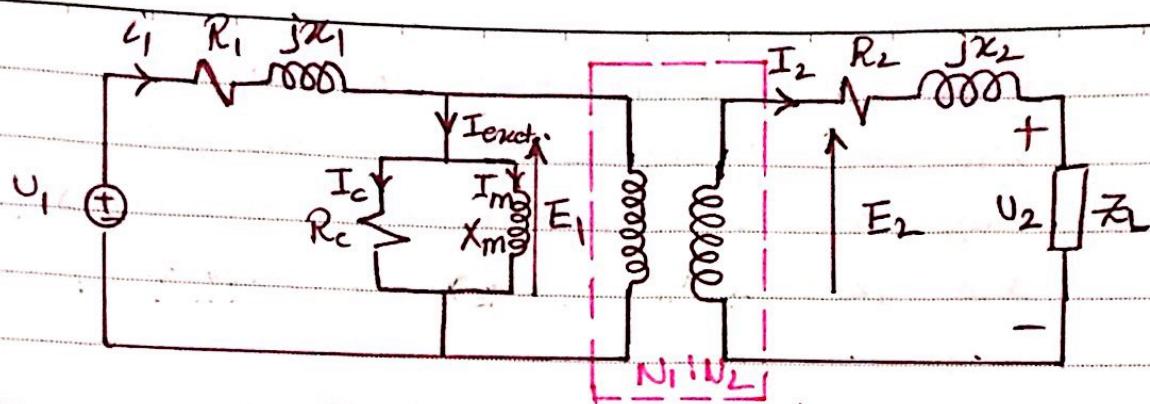
i_1' \equiv Load component of i_1 .

e_1, e_2 \equiv Induced voltage.

I_2 \equiv Secondary or Load current.

U_2 \equiv Secondary "terminal" voltage.

\therefore A practical transformer can be represented by an ideal transformer in series with passive elements on both sides as follows:-



Practical Transformer.

* Equivalent circuit by using impedance, voltage, and current reflection:

Let $a = \frac{N_1}{N_2}$; let us reflect to the 'a' side

• "a" side "a-side" \rightarrow Left side

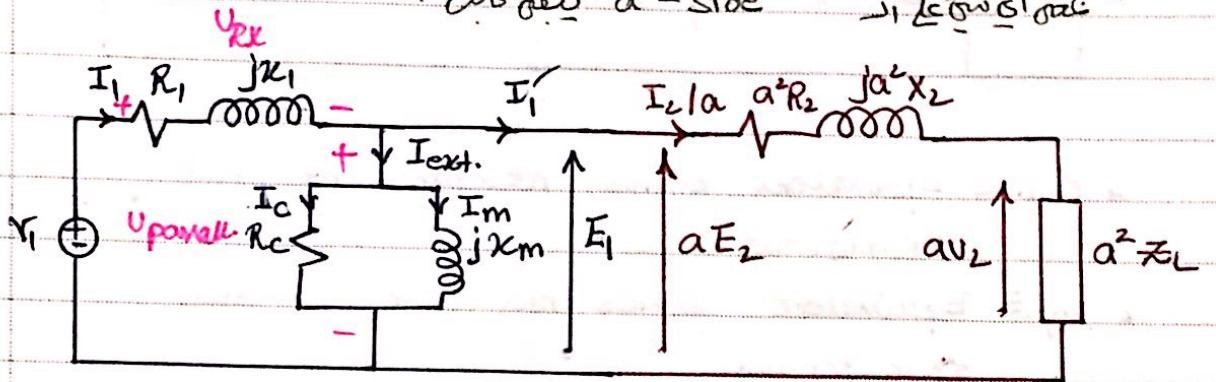


Fig. 1

* Fig. 1 is the exact equivalent cct. of the transformer.

* It can be observed that:

$$1. E_1 = a E_2$$

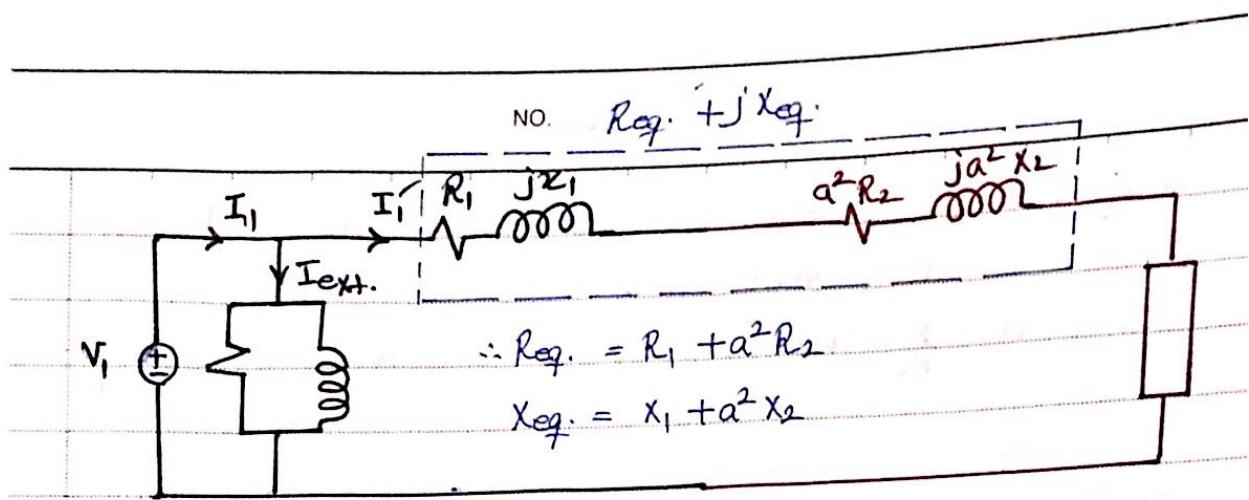
$$\therefore \frac{E_1}{E_2} = a = \frac{N_1}{N_2}$$

$$2. I_1' = \frac{I_2}{a}$$

$$\therefore \frac{I_1'}{I_2} = \frac{1}{a} = \frac{N_2}{N_1}$$

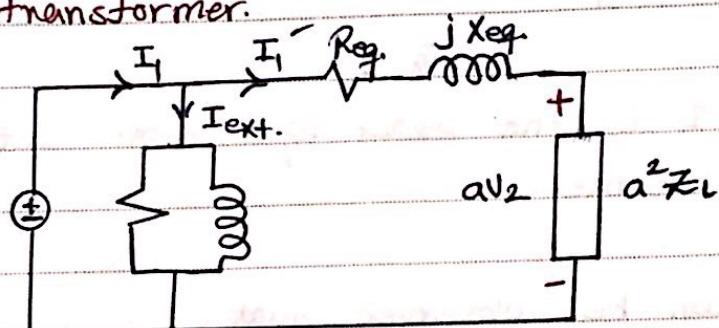
1. Since voltage drop in R_1 and X_1 (V_{RX}) is very small, then one may say that V_1 appears across branch of $R_C \parallel X_m$ (i.e. $V_1 \approx V_{\text{parallel}}$).

\therefore This parallel branch can be shifted to the input.

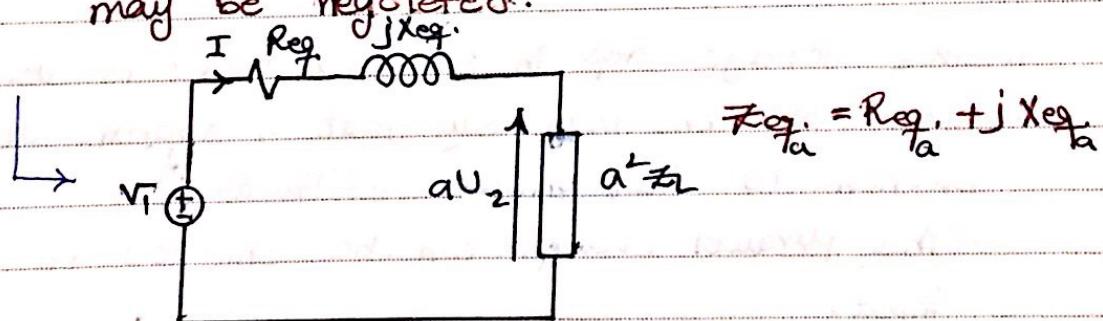


* $\text{Req.} \equiv$ Equivalent series resistance of the transformer.

* $X_{\text{eq.}} \equiv$ Equivalent series reactance of the transformer.



2. Since ($I' \gg I_{\text{ext.}}$), Then parallel branches may be neglected.



* Note: If the reflection was to "1" side.

$$\therefore R_{eq.1} = R_2 + \frac{R_1}{a^2}$$

$$\therefore R_{eq.2} \neq R_{eq.1}$$

$$X_{eq.1} = X_2 + \frac{X_1}{a^2}$$

$$X_{eq.2} \neq X_{eq.1}$$

* Exam:

Find $Z_{eq.}$ referred to low or high voltage.

$$\text{Ansatz: } \frac{N_2}{N_1} = \frac{1}{a} \quad \text{or} \quad a = \frac{N_2}{N_1} \quad \text{is ok} \rightarrow$$

\Rightarrow Try at home $a = \frac{N_2}{N_1}$; reflect once at a and once on 1.

* Tests on the transformers:

* what?

In order to determine its parameters $R_{eq.}$, $X_{eq.}$, R_c and X_m . So, that it's:

Voltages, currents, Power

Voltage regulation and efficiency can be determined.

* How?

By performing open-circuit (O/C) and short circuit (S/C) tests.

* (O/C) test.

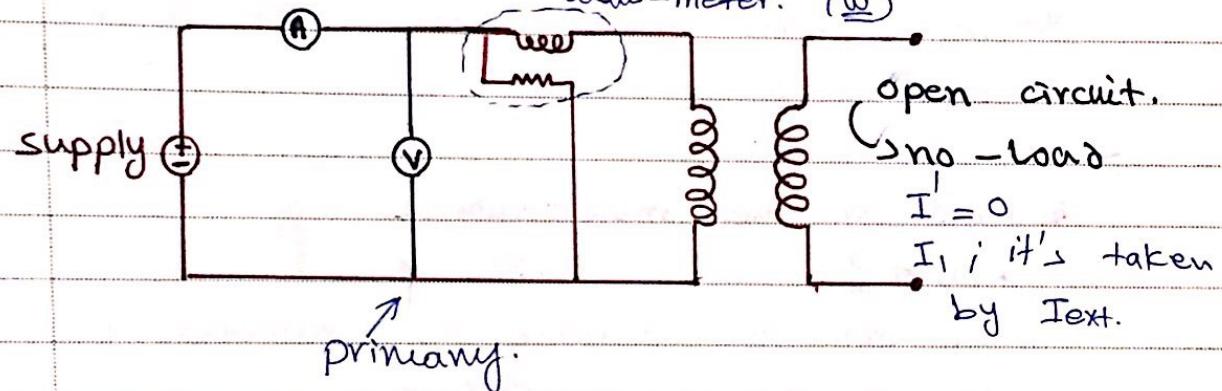
1. Open circuit one of its sides (i.e. LV or HV);

Since it's easier to obtain LV in the Lab,

then in conventional test HV is made open and noted voltage is applied to LV side.

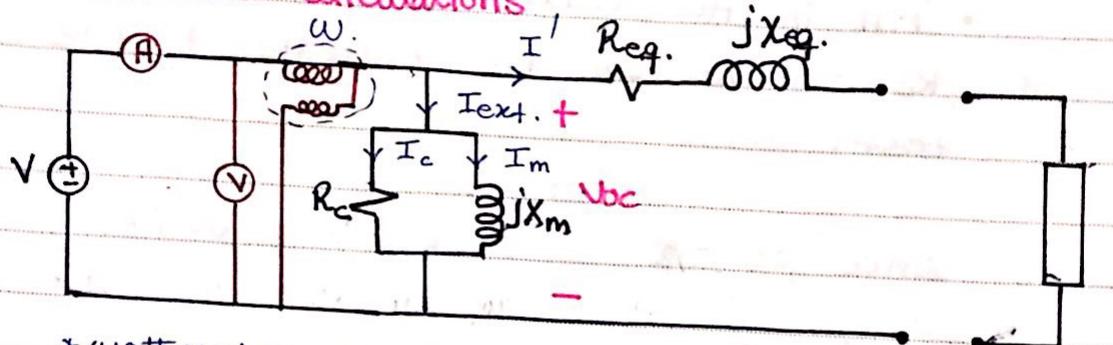
→ it's up to you to choose whether the HV or LV is open unless it was required to use the "conventional test".

Watt-meter. (W)



2. Take the reading of input current (A), voltage (V) and power (W).

• (O/c) test calculations



* wattmeter consists of two coils "current coil in series + potential coil".

$\Rightarrow I_{ext.} \equiv$ No load current.

- In red \equiv Measuring instruments

- Ammeter

- Voltmeter

- watt meter

$$\nexists I' \models 0$$

- Ammeter Reading, $I_{ac} = I_{ext}$. (A)

- Voltmeter Reading, $V_{OC} \equiv V$ across $R_C \parallel X_m$ (u)

- wattmeter Reading, $P_{ac} \equiv$ Power consumed in the cct (w).

Conventional Test \Rightarrow low voltage \rightarrow I_L \rightarrow \bar{I}_L \rightarrow I_L \rightarrow I_L

∴ In the (O/C) test, the parameters which can be evaluated are P_C and X_m as follows:

100, 110, 120 amper losses \rightarrow Electrical losses \rightarrow opt. 1

جار (jXm) لیزی \bar{N}_0 (no-load I) لیزی \bar{N}_1 لیزی \bar{N}_2

• P_C J1/2 core losses

- Fill in the blanks! 'Exam'

R_c and X_m can be evaluated by open circuit test.

$$\text{Since } P \triangleq U^2/R$$

$$\therefore P_{oc} = \frac{U_{oc}^2}{R_c}$$

$$\therefore R_c = U_{oc}^2 / P_{oc}$$

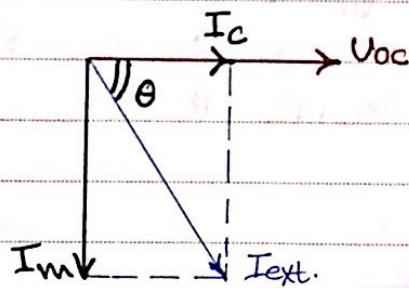
$$R_c = \frac{(\text{voltmeter reading})^2}{\text{wattmeter reading}}$$

$$|I_c| = \frac{U_{oc}}{R_c}$$

$|I_{ext.}| \equiv \text{Ammeter reading.}$

$$I_{ext.} = I_m + I_c \text{ (phasor summation)}$$

I_a and U_{oc} are in phase.



+ which lags and leads:

1. R 2. L 3. C

$$I_c = I_{ext.} \cos \theta$$

$$I_m = I_{ext.} \sin \theta$$

$$\therefore |I_{ext.}| = \sqrt{(I_c)^2 + (I_m)^2}$$

$$\therefore |I_m| = \sqrt{|I_{ext.}|^2 - |I_c|^2}$$

$$\therefore X_m = \frac{U_{oc}}{|I_m|}$$

• Resistance \Rightarrow (I and U are in phase).

• Inductor \Rightarrow (I lags U)

• capacitor \Rightarrow (I leads U).



comments:

calculated R_c and X_m are referred to the side at which the readings are taken.

- Referred to low voltage
- Referred to high voltage

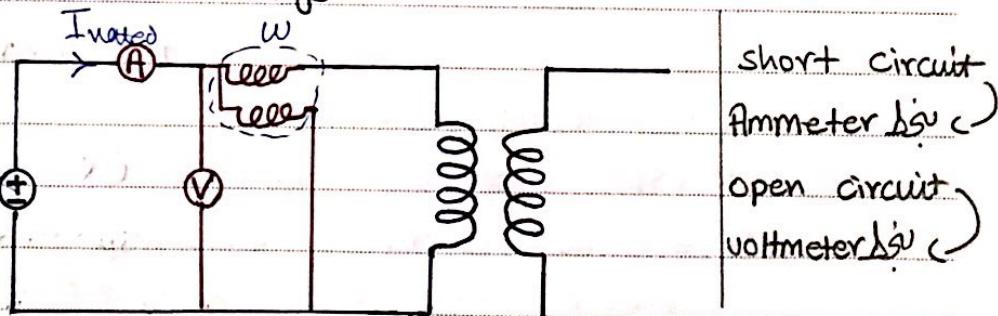
E_{ip}^2 (Referred to what) \rightarrow E_{ip} (rating voltage)

• High voltage \Rightarrow low current

Low voltage \Rightarrow high current.

• Short circuit test (S/C) :

1. short circuit (S/C) the terminals at one of the windings.



2. Apply the rated current to the other winding.

3. Take the reading of the input current, voltage and power.

Note: Since it's easier to measure low current than high current, [Then in conventional SIC test] the LV voltage and supply is applied to HV side.

4. calculate the parameters.

• Rated current = Rated apparent power, [VA]
Rated voltage

Ex.:

Transformer 20 kVA, (240 / 110) volt.

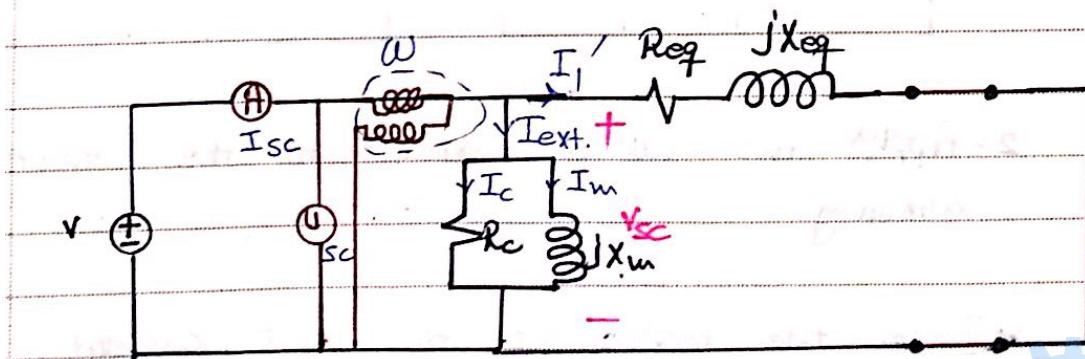
Rated current:

$$I_{HV} = \frac{20 \times 10^3}{240} \quad I_{LV} = \frac{240 \times 10^3}{110}$$

+ short circuit $\underline{Z_L} = 0$

in (SIC) test we evaluate (R_{eq} / X_{eq} .)

P_{sc} = losses (core + Electrical).

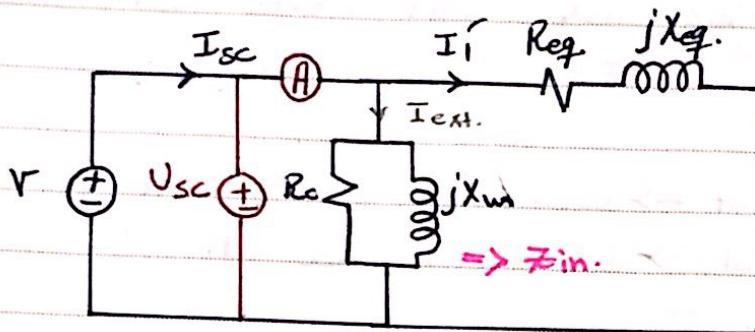


$$P_{sc} = \omega - \text{Reading}$$

$$I_{sc} = A - \text{Reading}$$

$$V_{sc} = V - \text{Reading.}$$

- Short circuit test:



Since $I_{ext.} \ll I'$

$\therefore I'_L = I_{sc} \equiv$ Amm. Reading.

$\therefore P_{sc} \equiv$ wattmeter Reading \equiv Losses in $Req.$

$$\therefore P_{sc} = I_{sc}^2 Req. \quad \therefore Req. = \frac{P_{sc}}{I_{sc}^2}$$

$$\therefore V_{sc} = |Z_{in}| \rightarrow \text{magnitude}$$

$$I_{sc} = \sqrt{Req^2 + X_{eq}^2} \quad \text{since } Z_{eq.} = Req + jX_{eq.}$$

$$\therefore X_{eq.} = \sqrt{|Z_{in}|^2 - Req^2}$$

- The short circuit test, one evaluate $X_{eq.}$ and $Req.$.

$\Rightarrow (O/c)$ Test \sim we apply rated voltage.

$\Rightarrow (s/c)$ Test \sim we apply rated current.

* Ex: A 1380VA, (230/115) volt, 50Hz, 1-Ph transformer has been tested with the following results.

- * (O/c) test \Rightarrow $V_{OC} = 230\text{V}$, $I_{OC} = 0.45\text{A}$, $P_{OC} = 30\text{W}$
- * (S/c) test \Rightarrow $V_{SC} = 13.2\text{V}$, $I_{SC} = 6\text{A}$, $P_{SC} = 20.1\text{W}$

Find the equivalent circuit of the transformer referred to the "LV-side".

\Rightarrow (O/c) test \rightsquigarrow primary was "High voltage".

• Since in (O/c) test, one apply rated voltage, then in this example the readings were taken on the "HV-side".

\therefore The calculated R_o and X_m will be referred to HV side.

\therefore Since in (S/c) test (one apply rated current).

$$I_{LV, \text{Rated}} = \frac{1380}{230} = 12\text{A}$$

$$I_{HV, \text{Rated}} = \frac{115}{230} = \underline{\underline{6\text{A}}} \quad \therefore \text{In this example}$$

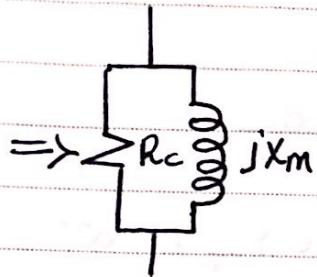
the readings were taken on the HV side.

\therefore The calculated R_{eq} and X_{eq} will be referred to the HV side.

• (O/C)

$$P_{oc} = \frac{V_{oc}^2}{R_c}$$

$$\therefore R_c = \frac{V_{oc}^2}{P_{oc}} = \frac{230^2}{80} = 1763.3 \Omega = R_{ch} \quad \dots \dots 1$$



$$\star Y = Y_1 + Y_2 = \frac{1}{R_c} + \frac{1}{jX_m} = \frac{1}{R_c} - \frac{j}{X_m}$$

$$\therefore |Y| = \sqrt{\left(\frac{1}{R_c}\right)^2 + \left(\frac{1}{X_m}\right)^2} \quad \dots \dots 3$$

$$\text{but } |Y| = \frac{I_{oc}}{V_{oc}} = \frac{0.45}{230} = 0.001957 \text{ S or } \text{V} \quad \dots \dots 2$$

Sub. (1) and (2) into (3) to find X_m .

$$\therefore X_m = 533.9 \Omega = X_{ch}$$

• (S/C)

$$P_{sc} = I_{sc}^2 R_{eq}$$

$$\therefore R_{eq} = \frac{P_{sc}}{I_{sc}^2} = \frac{20.1}{(6)^2} = 0.558 \Omega = R_{ch} \quad \dots \dots (4)$$

$$|Z_{sc}| = \frac{V_{sc}}{I_{sc}} = \frac{13.2}{6} = 2.2 \Omega \quad \dots \dots (5)$$

$$X_{eq} = \sqrt{|Z_{sc}|^2 - R_{eq}^2} \quad \dots \dots (6)$$

Sub. (4) and (5) into (6) .

$$\therefore X_{eq.} \doteq 2.128 \Omega = X_{eq.H}$$

\Rightarrow To refer to "LV-side"; find \underline{a} .

(Reflect to \underline{a} side).

$$\text{Let } a = \frac{N_L}{N_H} = \frac{115}{230} = \frac{1}{2} = 0.5$$

$$\therefore R_{eq.L} = R_{eq.H} \cdot a^2, \quad X_{eq.L} = X_{eq.H} \cdot a^2$$

$$\therefore R_{c.L} = R_{c.H} \cdot a^2, \quad X_{m.L} = X_{m.H} \cdot a^2$$

$$R_{eq.L} = 0.139 \Omega$$

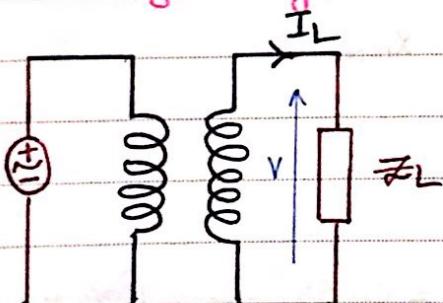
$$R_{c.L} = 440.8 \Omega$$

$$X_{eq.L} = 0.532 \Omega$$

$$X_{m.L} = 133.3 \Omega$$



* Voltage Regulation: VR



$$VR \triangleq \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|} * 100\%$$

\Rightarrow Magnitudes

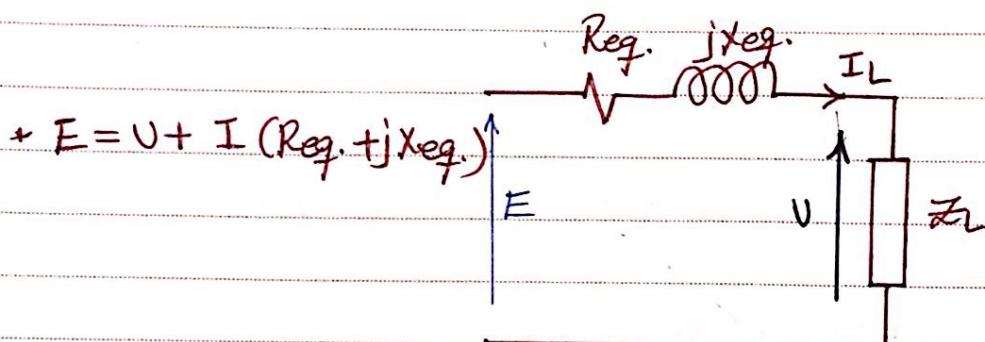
* V_{NL} = No Load voltage.

V_{FL} = Full Load voltage.

I_L = Load current.

To find V_R use the equivalent circuit.

→ referred to the load side (secondary side).

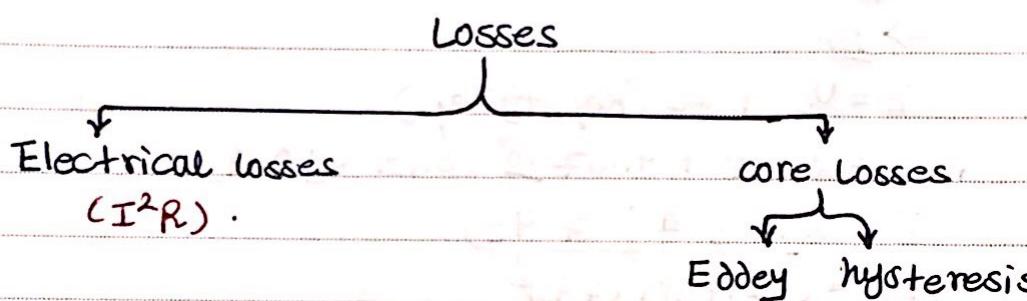


• Voltage Regulation:

$$VR = \frac{|V_{NL}| - |V_{FL}|}{|V_{FL}|} = \frac{|E| - |V_{FL}|}{|V_{FL}|}$$

• Efficiency:

$$\eta = \frac{P_{out}}{P_{out} + \text{losses}} = \frac{P_{out}}{P_{input}}$$

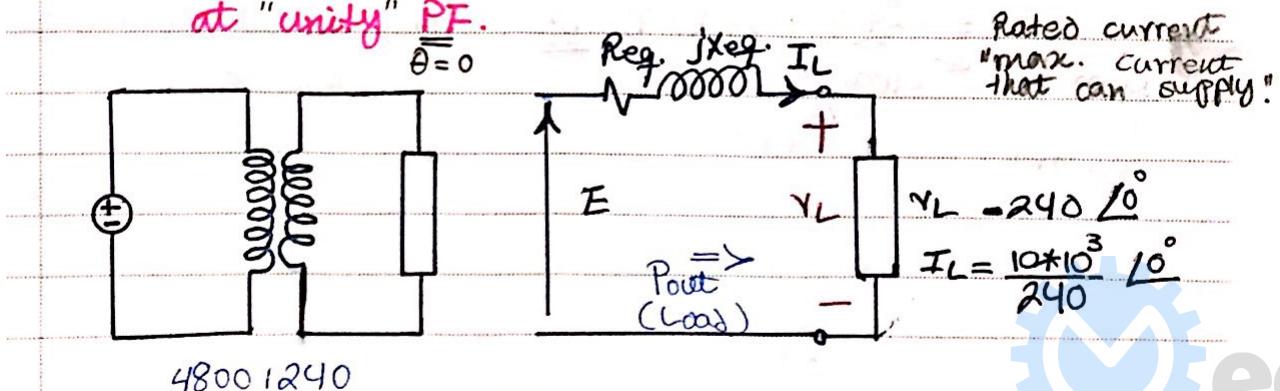


* Example: A 10kVA, (4800 / 240) v, 50 Hz Transformer
Apparent power (rated).
 was tested with the following results:

* 0/c \Rightarrow 240V, 1.5A, 60W

* 50/c \Rightarrow 180V, 2.083A, 180W

i) Find the voltage regulation if this transformer is used as a step-down to supply a full load at "unity" PF.



+ $R_{eq,L}$ and $X_{eq,L}$ can be evaluated from the given (0/c) and (s/c) results. which it can be found as—

$$R_{eq,L} = 0.104 \Omega, X_{eq,L} = 0.19 \Omega$$

$$+VR = \frac{|E| - |V_L|}{|V_L|}$$

فیصلہ ملکہ نہیں بلکہ
ایکی قدرتی

$$\Rightarrow |V_L|$$

$$E = V_L + I_L (R_{eq.} + jX_{eq.})$$

$$= 240 \angle 0^\circ + 41.67 \angle 0^\circ (0.104 + j0.19)$$

$$E = 244.3 + j(7.93)$$

$$|E| = \sqrt{(244.3)^2 + (7.93)^2}$$

$$= 244.4$$

$$UR = \frac{244.4 - 240}{240} \times 100\% = 1.83\%$$

II) Find η at full load with a $PF = 0.9$ lagging.

$$\eta = \frac{P_{out}}{P_{out} + \text{Electrical + core}} \quad \text{losses}$$

+ Full load \Rightarrow multiply it by (1). (current)

$$P_{out} = V_L \times I_L \times \cos(\theta)$$

+ Half load $\Rightarrow \times (0.5)$

$$= (240)(41.67)(0.9) \quad \text{OR}$$

$\times 0.25$ of load $\Rightarrow \times (0.25)$

$$9000.72 \quad \leftarrow (10)(10^3)(0.9)$$

$\times PF = \cos(\theta)$

"Apparent power!"

* Electrical Losses = $I_L^2 \text{Req. } L$

$$= 41.67^2 \times 0.104 = 180.5 \text{ W OR}$$

P_{sc} at full load only

(rated current) $\text{no load (S/C) } 180.5 \text{ W}$

because I_L is rated

$$P_{sc} = 180 \text{ W.}$$

* core losses:

$$\text{Core. Losses} = \frac{V_{oc}^2}{R_c} = \frac{V_{inH}^2}{R_{ch}H} = \frac{V_{inL}^2}{R_{cl}L} \text{ OR regulation } \text{ at high or low.}$$

$$= P_{oc} = 60 \text{ watt}$$

"the same answer"

at full load or no load, it

doesn't matter since I'm applying
rated voltage.

$$\eta = 97.4\%$$



FIVE APPLE

ech
Family

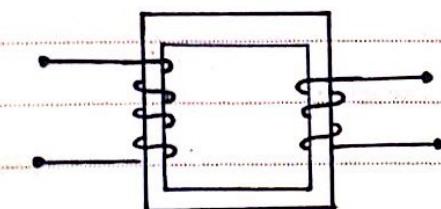
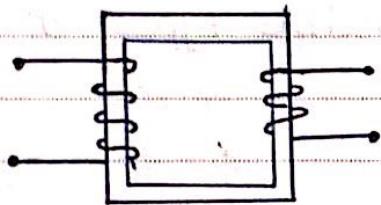
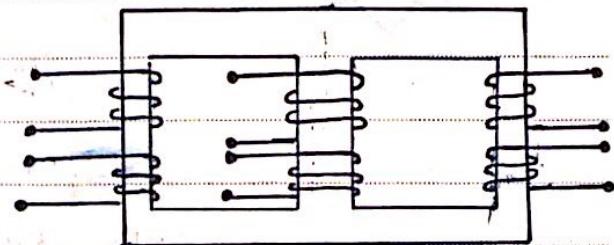
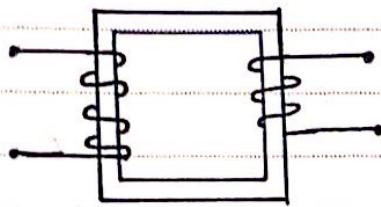
* 3-ph. Transformer:

- Construction:

It could be one of two structures:

1. Using three 1-ph. transformer, OR

2. A single core with three pairs of windings.



This type is more lighter and more economical.

1. It needs less space.
2. It is cheaper.

"Bank of 3-ph. transformer"

→ 3 - single phase transformer.

1. more reliable

2. if one 1-ph. transformer fail, then a

3-ph. power can still

be supplied but with a reduced input (58%).

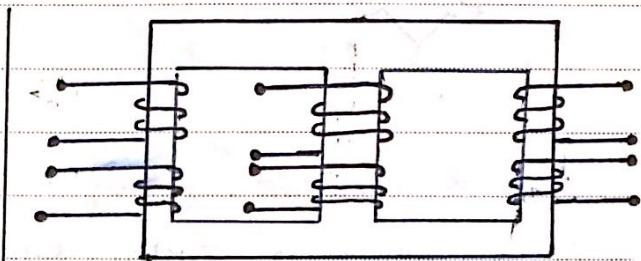
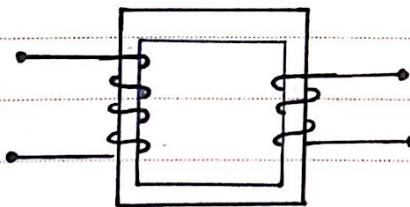
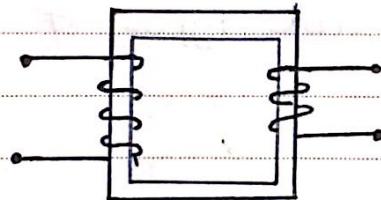
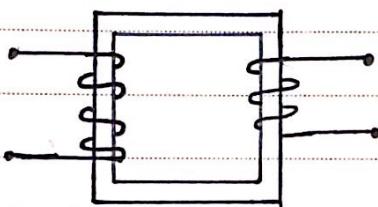
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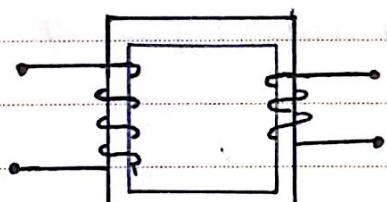
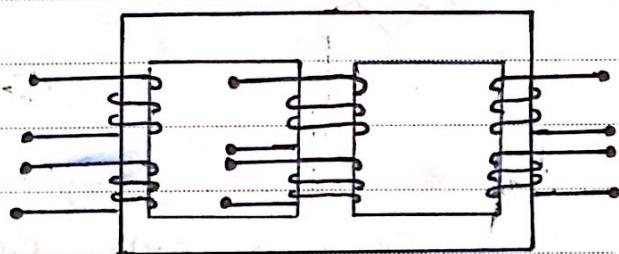
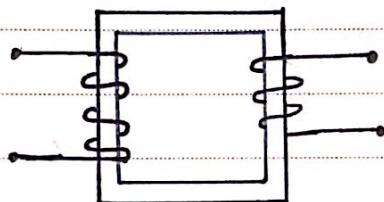
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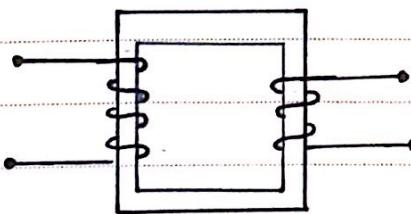
2. A single core with three pairs of windings.



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"Bank of 3-ph. transformer"

→ 3 - single phase transformer.

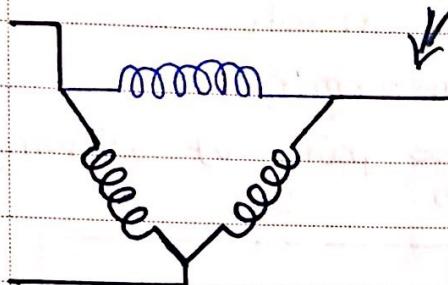
1. more reliable

2. if one 1-ph. transformer fail, then a

3-ph. power can still

be supplied but with a reduced input (58%).

* using the called ∇ -connection.



* Connection types:

Since a 3-Ph. winding can be connected as Y or Δ then there are 4-possible types of transformers:

1. $Y \rightarrow \Delta \Rightarrow$ Step Down
2. $\Delta \rightarrow Y \Rightarrow$ Step up
3. $\Delta \rightarrow \Delta \Rightarrow \nabla$ -connection.
4. $Y \rightarrow Y \Rightarrow$ Rarely being used.

* Markings:

1. usually HV terminals are represented by capital letters (A, B, C)
2. usually LV terminals are represented by small letters (a, b, c).

* Voltage - current relationship:

consider a Y-Δ connection

A.

B.

C.

N

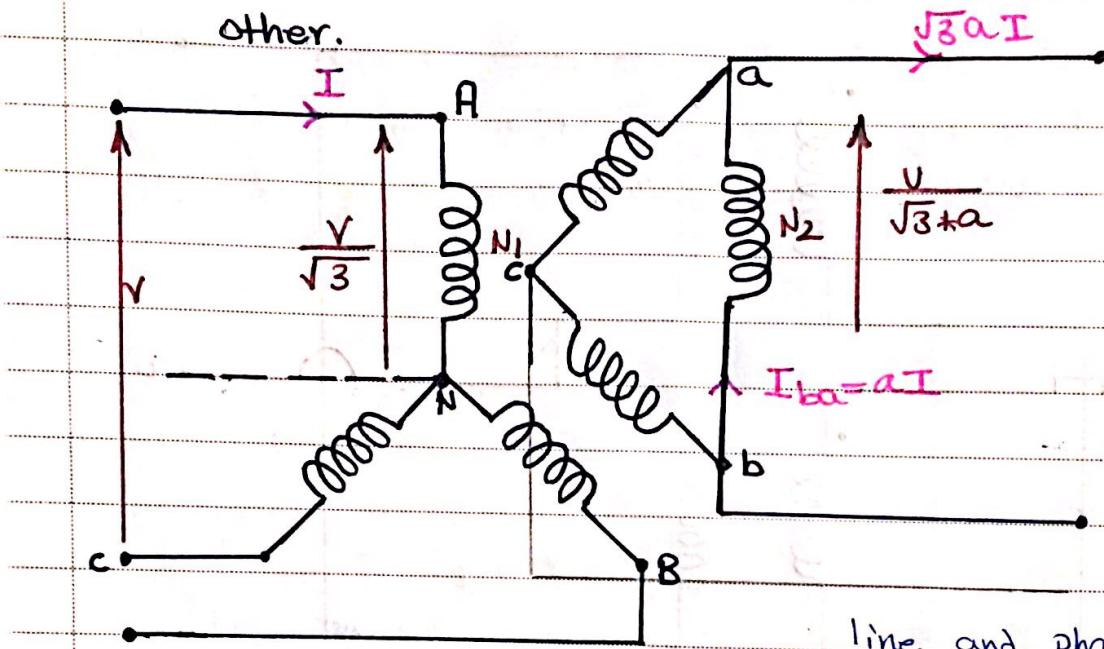
"Neutral"

"connection or wiring

Diagram."

\parallel \Rightarrow means (parallel)

* **Schematic Diagram:** Here the winding of each phase are drawn parallel to each other.



line and phase current are not the same.

line and phase current are the same.

* Fill in the blanks.

VAN is the Phase voltage.

N ≡ The common point between the three phases.

$N_1, N_2 \equiv$ No. of turns of HV and LV windings.

$$\text{Let } a = \frac{N_1}{N_2}$$

U ≡ Line voltage.

$$\star \frac{|V_{AN}|}{|V_{ab}|} = \frac{N_1}{N_2} = a$$

$$\therefore |V_{ab}| = \frac{|V_{AN}|}{a} = \frac{U}{\sqrt{3} + a} \quad \text{magnitude.}$$

$$\star \frac{I}{I_{ab}} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$\therefore I_{ab} = aI$$

- The same procedure can be repeated to the other connections.
- However the Y - Δ or Δ - Y introduce a phase shift of 30° , as follows!

Assume +ve phase sequence.

$$\text{let } V_{AN} = V_m \angle 0^\circ$$

$$\therefore V_{AB} = \sqrt{2} \angle 30^\circ$$

Since ($V_{ab} \parallel V_{AN}$)

$\therefore |V_{ab}| = |V_{AN}| = 0^\circ$; Parallel branches have voltages in the same phase.

• **Comments:**

In the case of $Y - \Delta$ or $\Delta - Y$ transformer

- The terminal markings are made in such away that HV leads LV by 30° in the (C+ve) ph. sequence.

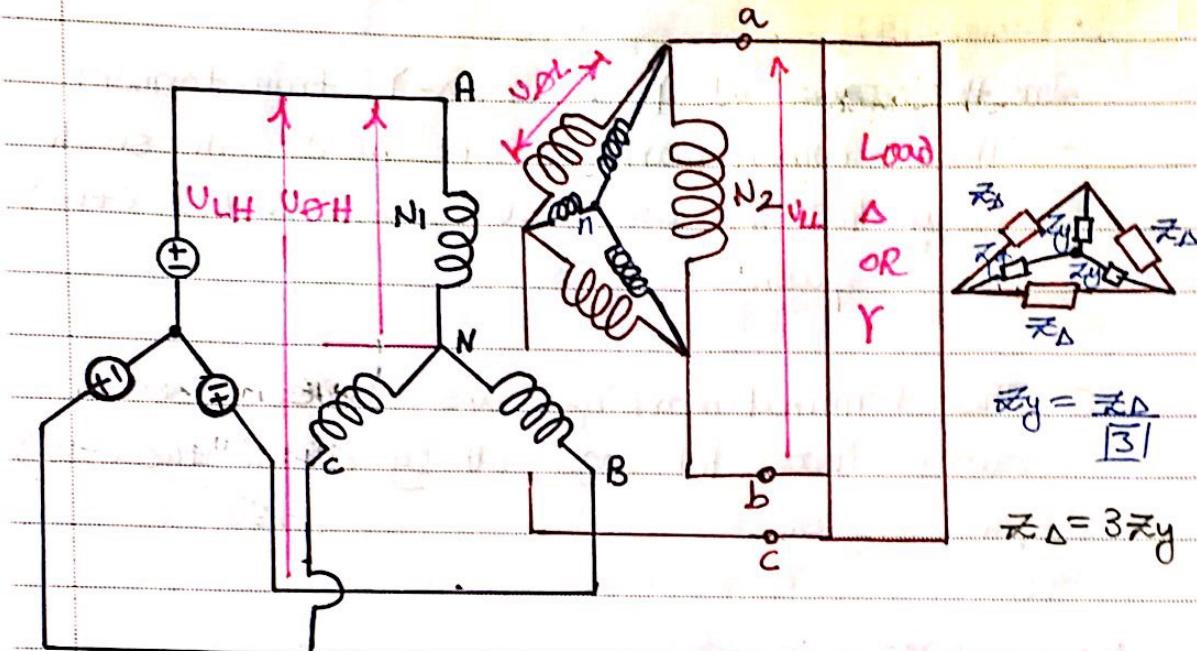
- The terminal markings are made in such away that HV lags LV by 30° in the (C-ve) ph. sequence.

* **per-phase concept:**

In balanced systems which contain 3-ph. transformers, then the per-phase or single-phase act. can be used to solve the problem, as follows:

Phase _____
Neutral _____

* Note in the case of Δ -connection convert it to it's equivalent Y .
consider the following system.

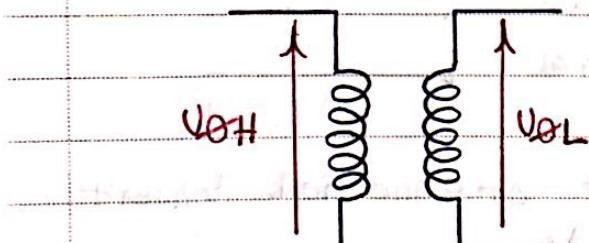


U_{LH} ≡ Line voltage of HV side.

V_{PH} ≡ phase voltage of HV side.

U_{LL} ≡ Line voltage of LV side.

V_{PL} ≡ phase voltage of LV side.



$$N_1 / N_2 / \sqrt{3} \Rightarrow N_1 / N_2 / \sqrt{3} \text{ II}$$

act. II

$$\rightarrow \frac{V_{LH}}{V_{LL}} = \sqrt{3} \frac{V_{PH}}{V_{LL}} = \sqrt{3} \frac{V_{PH}}{V_{LL}} = \sqrt{3} \frac{N_1}{N_2} = \sqrt{3} a = \frac{N_1}{N_2} = a_{eff.}$$

↑ magnitudes.

$$a \equiv \text{Turns ratio} = \frac{N_1}{N_2}$$

$$\rightarrow \text{Also, } \frac{V_{PH}}{V_{LL}} = \frac{V_{PH}}{V_{LL}/\sqrt{3}} = \frac{N_1}{N_2/\sqrt{3}} = a_{eff.} \equiv \text{The Effective turns ratio.}$$

(→ Ratio between the line voltages $(\frac{V_{PH}}{V_{LL}})$).

* In the 4-cases \Rightarrow the effective turns ratio \equiv line voltage ratio.

• Exam question:

Derive the turns ratio of the phase voltages for $\Delta-Y \Rightarrow \left(\frac{N_1/\sqrt{3}}{N_2}\right)$.

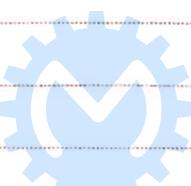
• Conclusion:

It can be shown for any type of connection ($Y-Y$, $\Delta-\Delta$, $\Delta-Y$, $Y-\Delta$), that $a_{eff.} \triangleq \text{Ratio of line voltages.}$

(Make sure of them ?!

$$\left[\begin{array}{l} \text{* } Y-Y \\ \Delta-\Delta \end{array} \right] \frac{N_1/\sqrt{3}}{N_2/\sqrt{3}} = \frac{N_1}{N_2} = a_{eff.}$$

$$\left[\begin{array}{l} \text{* } \Delta-Y \\ \text{* } Y-\Delta \end{array} \right] \frac{\frac{N_1/\sqrt{3}}{N_2}}{\frac{N_1}{N_2/\sqrt{3}}}$$



NO.
→ Always line voltages.

* Ex. 1 A (13800/480) v, Y-Δ transformer Bank

consists of three identical single phase

100 kVA, $\frac{HV}{7967}$ $\frac{LV}{480}$ v transformers:

It is supplied with power directly along
constant supply.

+ In the (S/Ic) test on the HV side of one
of the 1-ph. transformer, the following
results were obtained:

$\Rightarrow V_{sc} = 500$ v, $I_{sc} = 12.6$ A, $P_{sc} = 3300$ watt.

If this Bank is used as "step-down" to
supply rated load at 0.88 PF lagging at
rated voltage.

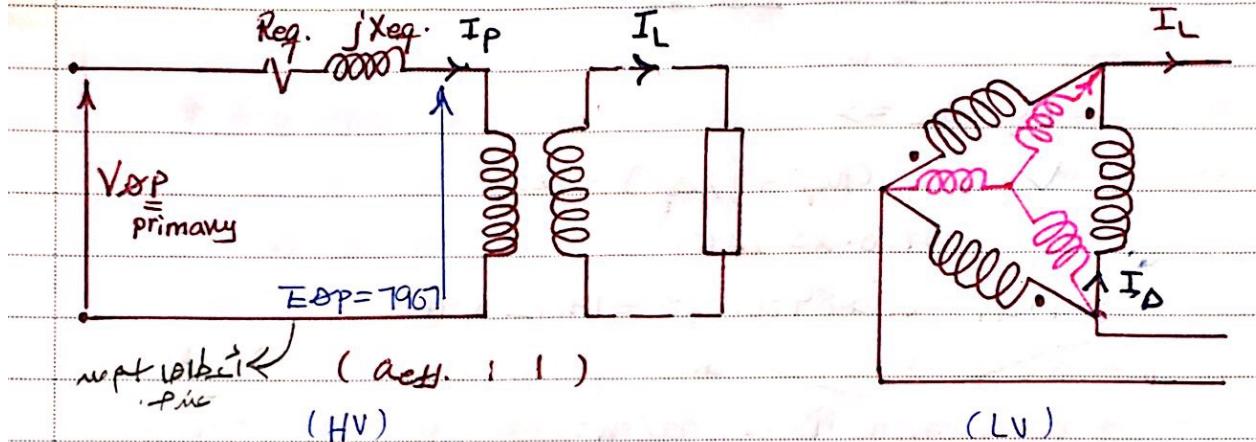
+ Find line voltage on the primary side.

• comment:

It can be deduced that HV side of 1-ph.
transformers are connected in Y and LV side
of 1-ph. connected in Δ to give the required
line voltages (13800/480)

- Objective: To find the equivalent circuit reflected to HV.

Opere in circuito



$$a_{eff.} = \frac{13800}{480} = 28.75$$

From the given (s/c) test, it can be found that:

$$Reg. = 20.79 \Omega$$

$$\chi_{eq} = 39.2 \Omega$$

$$I_{\Delta} = \frac{100 \times 10^3}{480} = 208.3 A$$

$$\rightarrow I_L = \sqrt{3} + I_{\Delta} = \sqrt{3} + 208.3 = 360.79 A$$

$$\therefore \frac{I_p}{I_L} = \frac{1}{a_{eff.}}$$

$$\therefore I_p = \frac{I_L}{a_{eff.}} = \frac{\sqrt{3} (208.3)}{28.75} = 12.55 A$$

$$\Rightarrow \text{Let } E_{op} = 7967 \angle 0^\circ$$

$$\therefore I_p = 12.55 \angle -\cos^{-1}(0.88)$$

$$= 12.55 \angle -28.36^\circ$$

\therefore By EUL \Rightarrow

$$V_{op} = I_p (\text{Req.} + j\chi_{\text{eq.}}) + E_{op}$$

$$= 8436.25 \angle 2.1^\circ$$

$$\therefore |V_{op}| = \sqrt{3} \times 8436.25 = 14612 \text{ volt.}$$

* Connection Designation:

- These are letters used to represent type of 3-ph. connection.

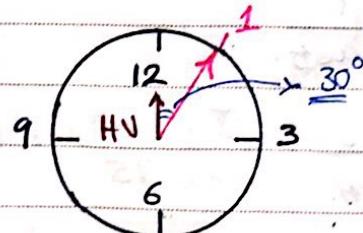
* capital letters for HV \equiv D or Y
Delta wye

* small letters for LV \equiv d or y
delta wye

* If "neutral line" exist, then use N or n.

- Phase shift between HV and LV represented by the clock arms.

J1 HV J1' LV
reference



→ Hour arm pointing at 12:00 used to represent HV.

* Minutes arm represent LV.

• Designation \boxed{HV} \boxed{LV} $\boxed{\text{ph. shift}}$

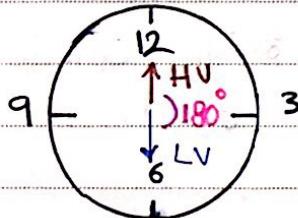
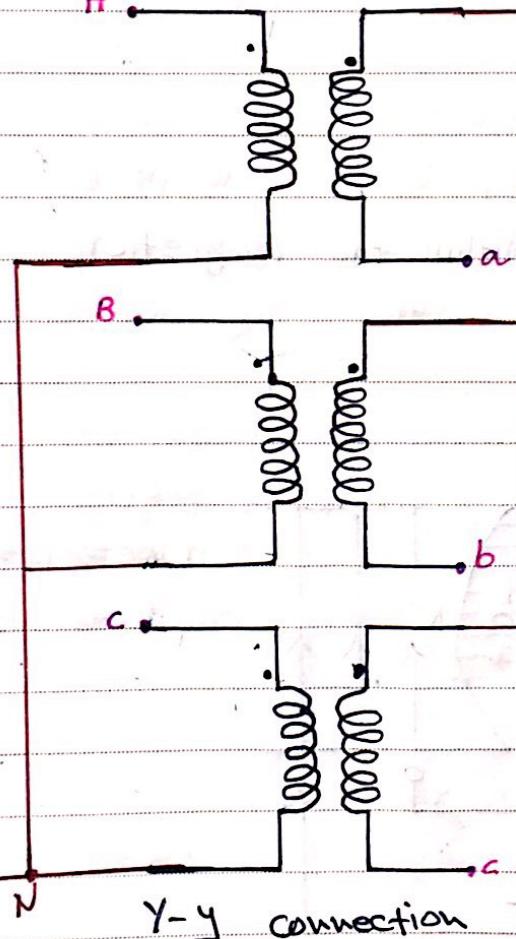
* Illustration:

For example $\Delta-Y \equiv$ step up, find connection

designation for $\Delta - Y$ connection assuming
+ve ph. sequence. $\begin{matrix} \uparrow & \uparrow \\ LV & HV \end{matrix}$

Ans. $\equiv YNd_1$

$\begin{matrix} \nearrow & \searrow \\ \text{Lead} & \text{ref.} \end{matrix}$ $\begin{matrix} \nearrow & \searrow \\ \text{lag} & \text{+ve seq.} \Rightarrow \end{matrix}$
LV Lags HV
by 30° .



→ The phase shift between HV and LV is 180°

Ans. $\equiv YNyn_6$



Electrical Machines (motor or generator) (AC or DC)

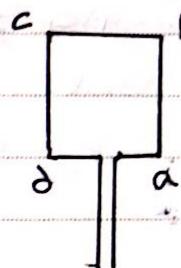
- Basic Fundamentals!

- Induced Voltage

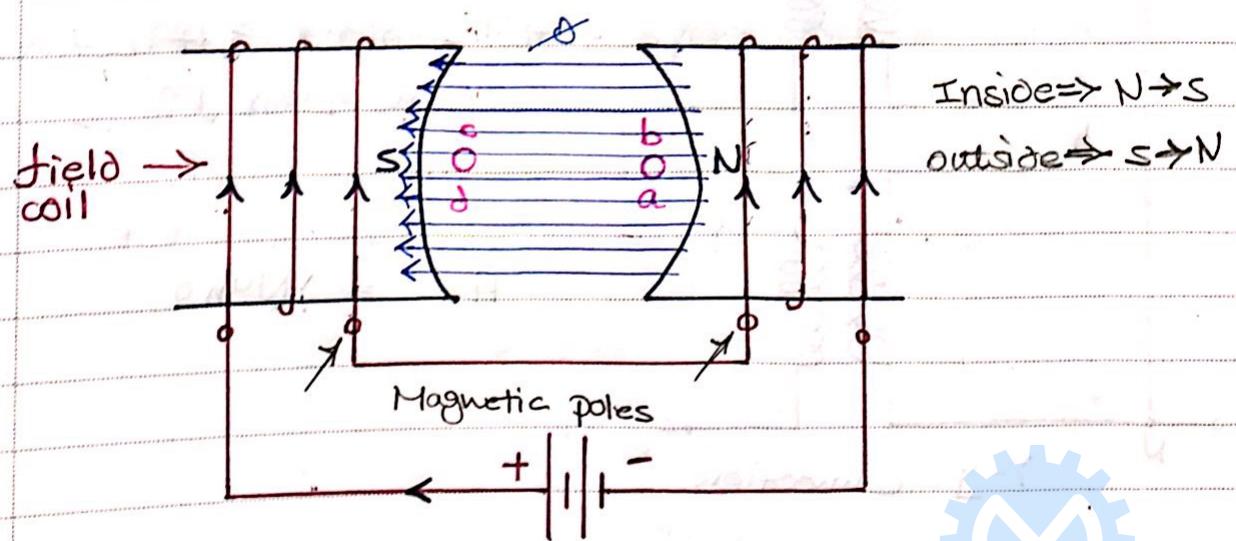
- Structure :

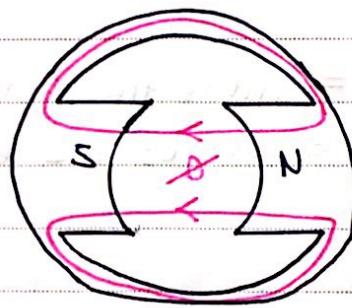
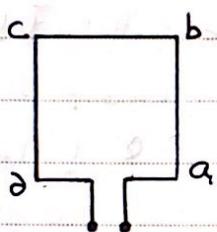
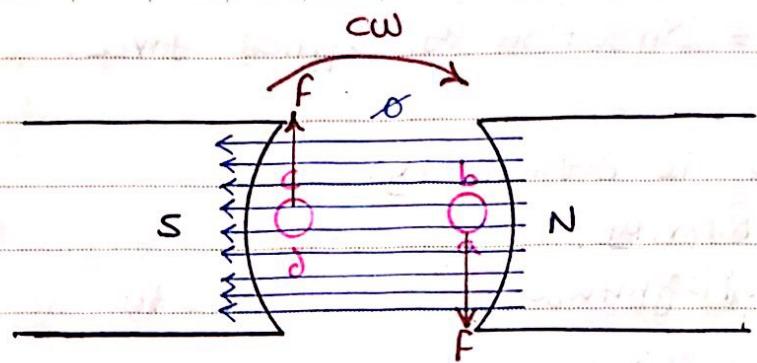
1. Magnetic field "stationary" called stator produced by electromagnet to produce &

2. conductors by means of coil



3. The coil is rotated within the magnetic field.





* Field coils \equiv stationary called stator.

* ab, cd \equiv called coil sides.

• Let the coil "abcd" to rotate cw or ccw;
it's called Rotor.

- Armature windings: These are the windings in which voltage is induced in the case of "generator" OR windings to which current is applied in the case of "motor".

- Let the Armature to rotate cw. since it's rotating then it will cut $\theta \Rightarrow$ Hence a voltage (e) will be induced in it.

$\rightarrow f \equiv$ Direction of applied force.

- e is defined by:

1. Polarity; by using RHR

2. Magnitude; $\otimes \odot \equiv$ gives direction of induced voltage.

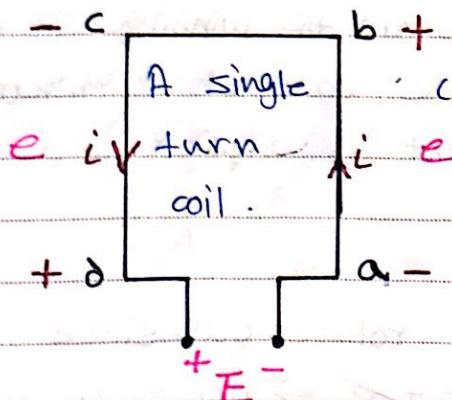
$\otimes \equiv$ Into the board. $\odot \equiv$ Out of the board.

$\odot \equiv$ out of the board. $\otimes \odot, \odot \odot, \odot \odot$

Dot concept: applied for two mutual coupled coils only.

* Convention:

If coil abcd is connected to a load, then a current will flow in the same direction of induced voltage.



- Since in a source, current flows from (-) to (+)

∴ polarity of induced voltage in each coil side can be determined as shown in fig. 1.

- If (e = voltage induced per coil side)

∴ E = voltage generated by the coil

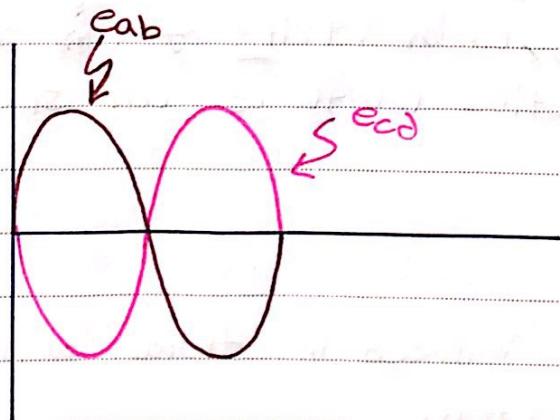
(i.e. voltage at coils terminals) is

by KVL:

$$\therefore E = 2e$$

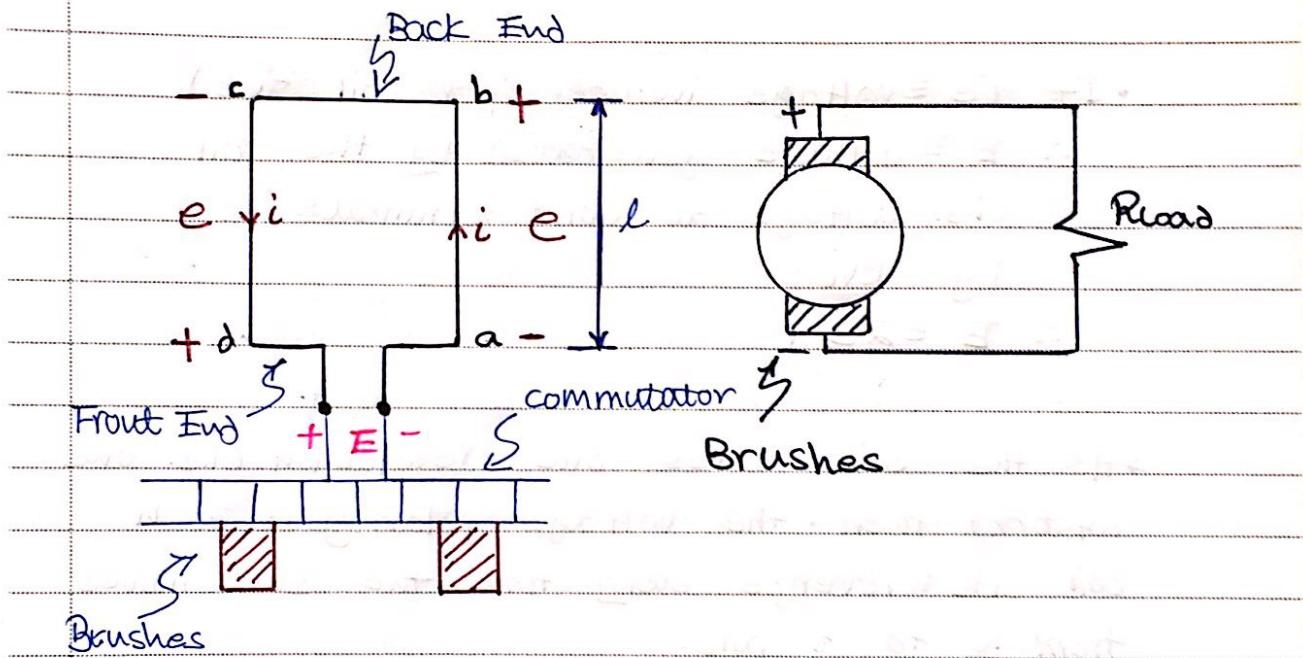
* As the coil rotates one revolution (i.e. one rotation) then the voltage polarity in each coil side reverse every time the side moves from N to S pole.

∴ Internal induced voltage is AC.



But in DC machines this interval AC is converted to DC voltage by using mechanical rectifier which consists of:

1. commutator which is rotating with A.W.
2. Brushes which are stationary but making continuous contacts with commutator.



• Magnitude:

$$e = \frac{B l v}{T} \rightarrow \text{Linear velocity of coil side.}$$

Flux Density \downarrow Active length of coil side.

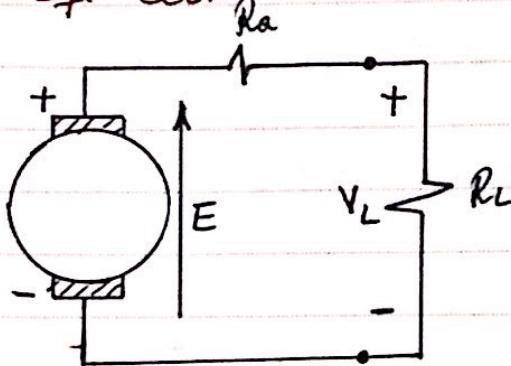
(Tesla or $\frac{wb}{m^2}$).

+ No voltage induced in front and back end connection.

* E 1 turn $= 2BLV$ (for single turn).

* $E = 2NB\ell V \Rightarrow$ for N turns.

\Rightarrow Eq. cct.



$R_a \equiv$ Internal resistance of armature winding

$R_L \equiv$ Load resistance

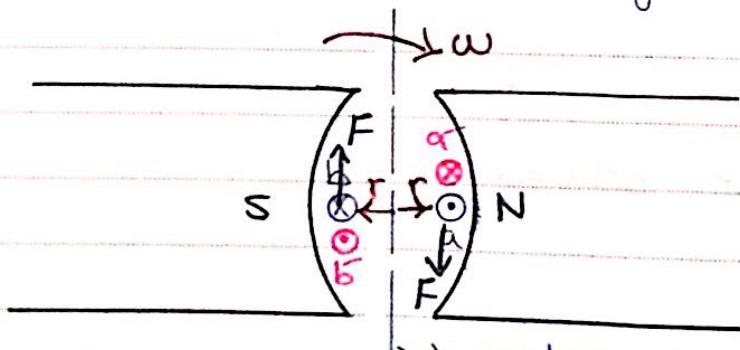
$V_L \equiv$ Terminal or Load's voltage.

• Generated or Induced Force and Torque:

If a current is applied to armature windings

then a force will be produced on each coil

side. It's direction by LHR.



* LHR \equiv using 3 fingers

$\odot, \otimes \equiv$ Direction of applied current (motor Action).

$F \equiv$ Induced or produced force.

$F \triangleq BIL$

$$T \triangleq BIL$$

$B \equiv$ Flux Density

$I \equiv$ Applied Current

$L \equiv$ Active length of conductor.

$\omega \equiv$ Direction of produced motion with a radian speed $= \underline{\omega}$.

$$\rightarrow \underline{\omega}_1 = BILr$$

$$\rightarrow \underline{\omega}_2 = BILr$$

\therefore Tongue produced, ($\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2$)

$$\therefore \underline{\omega} = 2BILr.$$

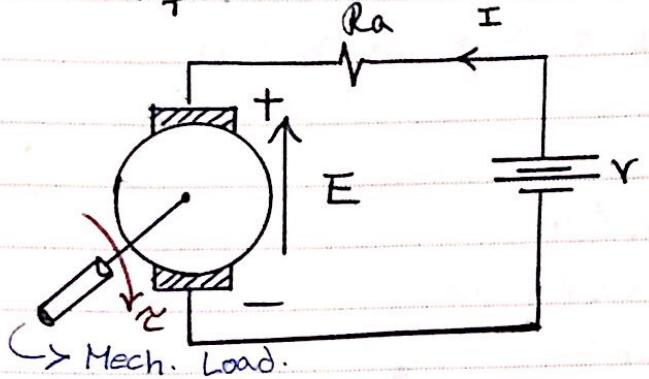
+ Consequently due to tongue ($\underline{\omega}$) \rightarrow motor is going to rotate and cutting magnetic field \Rightarrow Hence a voltage will be induced in these conductors whose direction can be found by RHR as follows!

① $\Phi \equiv$ Direction of generated voltage

(i.e. generator action) This is called Back emf.

\therefore Hence in a motor there is a generator action.

= Equivalent cct. :



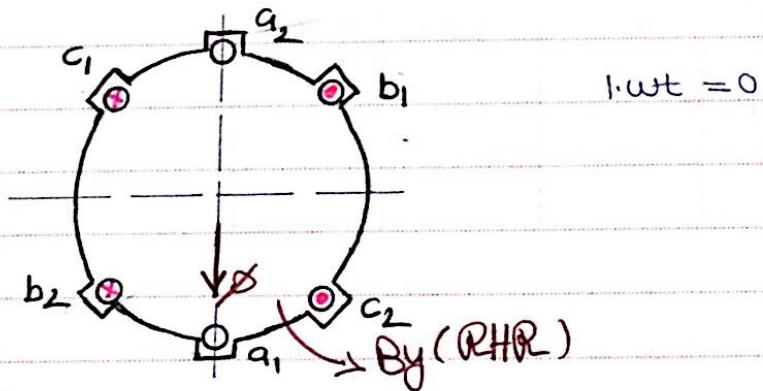
V = Applied voltage

I = Applied current

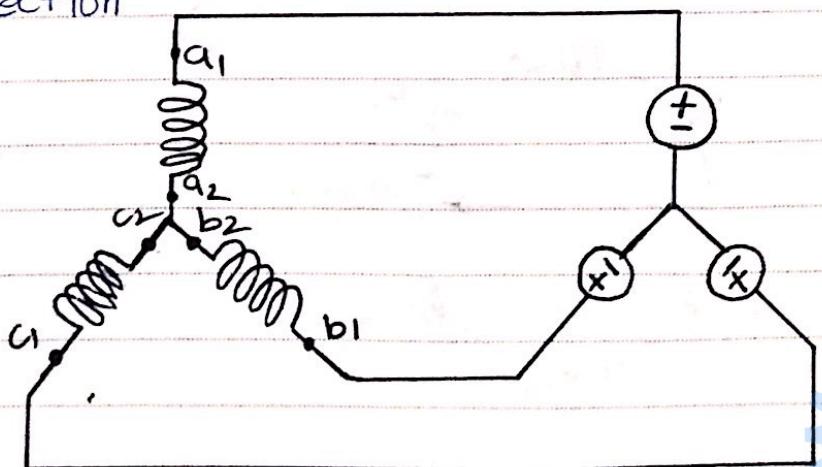
E = Back emf

• Rotating magnetic field:

If a balanced 3-ph. current is applied to a 3-ph. winding, then a rotating magnetic field is produced within the winding as follows:



Y-connection



Exam question
→ show how its rotating graphically?

Ans.1

1. let $i_{a_1, a_2} = I_m \sin(\omega t)$

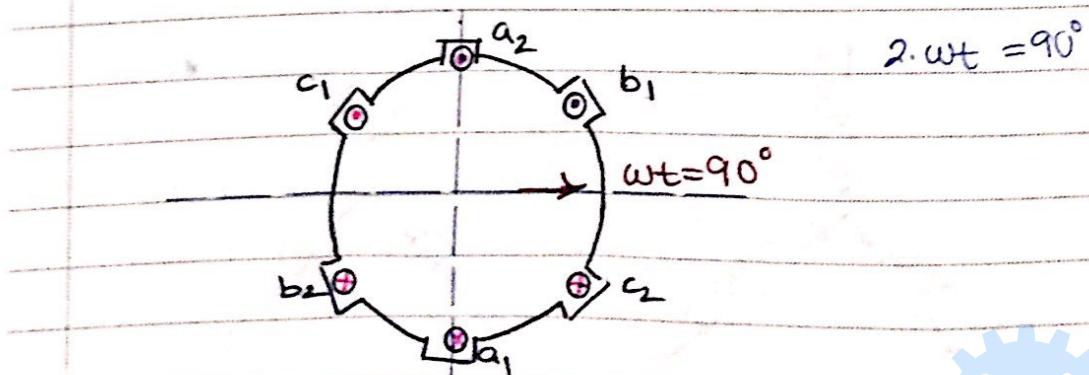
$i_{b_1, b_2} = I_m \sin(\omega t - 120^\circ)$

$i_{c_1, c_2} = I_m \sin(\omega t - 240^\circ)$

let the current changes over one cycle.

| wt | i_{a_1, a_2} | i_{b_1, b_2} | i_{c_1, c_2} | LR |
|-------------|----------------|----------------|----------------|-------------|
| 0 | 0 | -ve | +ve | -90 |
| 90° | +ve | -ve | -ve | 0 |
| 180° | 0 | +ve | -ve | |
| 270° | | | | |
| 360° | | | | |

let $(+ve) \Rightarrow \otimes$
 $(-ve) \Rightarrow \circlearrowleft$



* Another way of imagination:

[N]

[S]

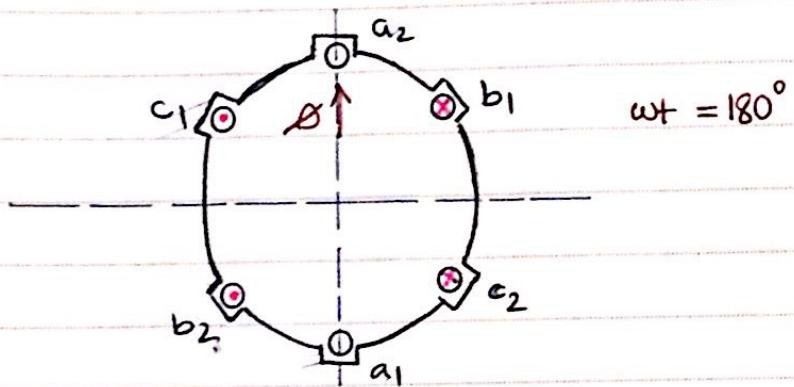
$wt = 0$

[N] [S]

$wt = 90^\circ$

[S]

[N]
 $wt = 180^\circ$



∴ In one cycle of current, it makes one mech. rotation.

• Comments:

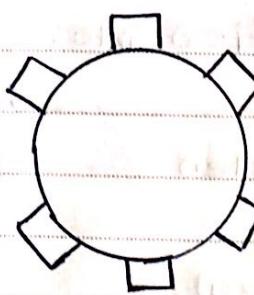
I) The 3-ph. windings represent 2 rotating poles.
 II) For one cycle of applied current there were one mechanical rotation at Δ .

$\therefore F_e = F_m \rightarrow$ Mechanical frequency (r.p.s) "revolution per second".
 \hookrightarrow Electrical frequency (Hz)

III) The three phase winding can be made to have a larger no. of poles.

• General relationships:

$F_e = P F_m$, $P \equiv$ No. of pole pairs "class"
 \hookrightarrow frequency of generated voltage
 \downarrow frequency of rotation of the motor.



Rotor types

- 2 poles \rightarrow 1 pair
- 4 poles \rightarrow 2 pairs



*DC - Machines:

⇒ construction:

It consists of:

I) Stator: carrying field windings, which are excited by field current, I_F . Explained before.

II) Rotor: carry Armature windings.

*Fill in the blanks:

Armature windings are: They are windings in which voltage is induced in case of "generator" and in which current is applied in case of "motor" to produce torque.

*Classification:

According to the source of I_F , DC Machines are classified into:

1. Separately Excited

DC - Machines:

Here an external DC - source is used to supply I_F .

2. Self Excited

DC - Machines:

This depends on the existence of residual magnetism in the poles.

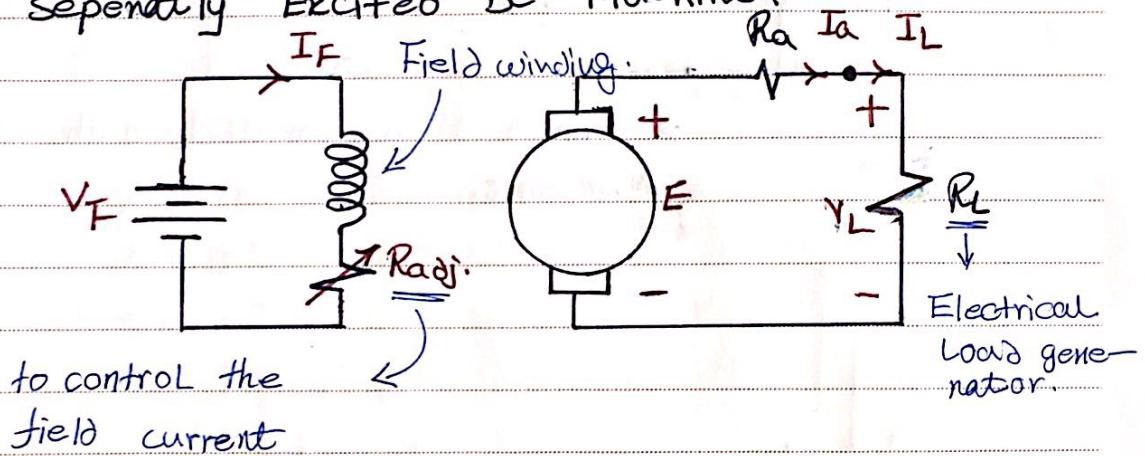
Series, shunt



2. self Excited DC -
Machines:

shunt DC Series DC compound
machine. machine. DC machine

I) Separately Excited DC - Machine:



$E \equiv$ Generated or induced voltage
 \rightarrow (Generator)

* Mech. load
(Motor).

$E \equiv$ Back emf
 \rightarrow (Motor)

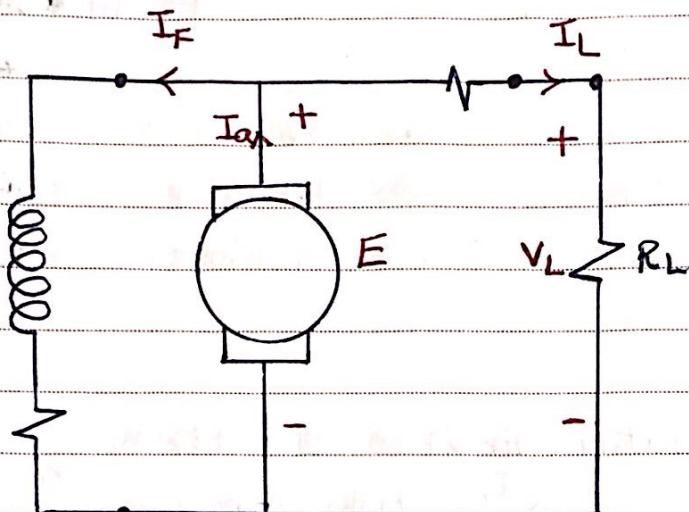
$$I_F = \frac{V_F}{R_F} ; R_F \equiv \text{Resistance of the field cct.}$$

$$R_F = R_{FW} + R_{adj.}$$

$$E = V_L + I_a R_a$$

$$\rightarrow E = R N B L V$$

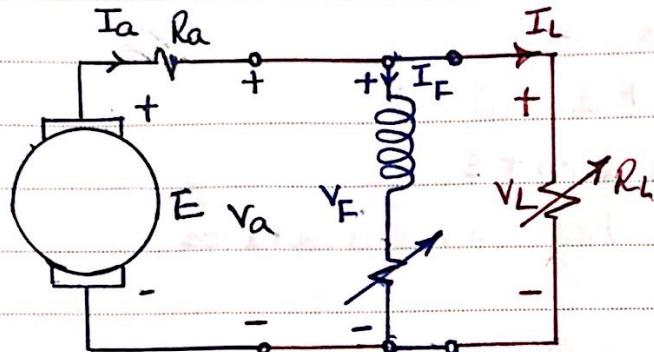
II) shunt Dc-Machine (self excited):



↳ connected in parallel with the brushes.

• Self Excited:

- shunt machines:



+ If it is connected

with a DC power supply "Motor", with a Load resistance "generator".

Fig. 1

Fig. 1 ≡ DC shunt generator.

$$\Rightarrow V_L = V_F = V_a$$

$$I_a = I_F + I_L$$

$$E = I_a R_a + V_a$$

$$V_L = I_L R_L$$

$$I_F = V_F / R_F$$

R_F ≡ Field - cct. resistance.

= Field winding (F.W.) resistance + R_{adj.}

- Series machines:

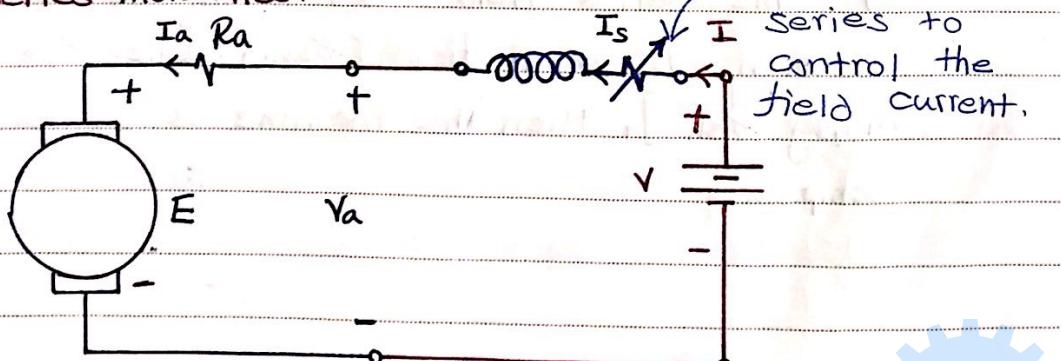


Fig. 2

In parallel or series to control the field current.



Fig. 2 \equiv DC - series motor.

$V, I \equiv$ Applied voltage and current.

$$I = I_s = I_a$$

$$\therefore V_a = I_a R_a + E$$

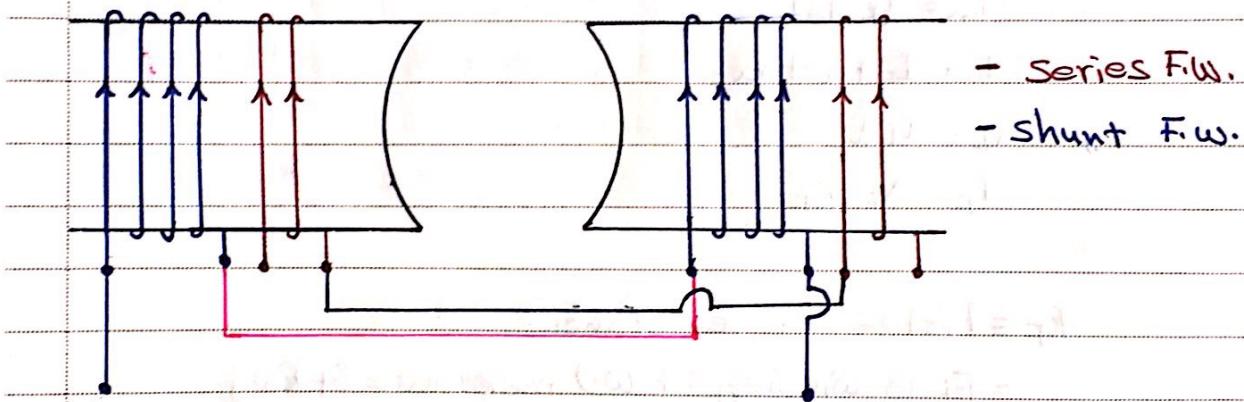
$$V = I_s R_s + I_a R_a + E$$

R_s = Series field cct. resistance.

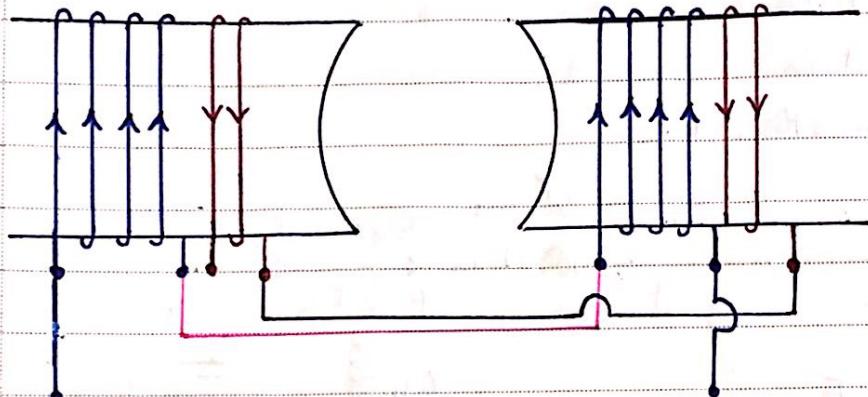
- compound machines:

Here there are 2 field windings:

Shunt and series fields.



If the series field (\mathcal{B}_{se}) is in the same direction of shunt field ($\mathcal{B}_{sh.}$). [i.e. \mathcal{B}_{se} is aiding $\mathcal{B}_{sh.}$], then the machine is cumulative compound.



If \mathcal{E}_s opposes [i.e. not aiding] \mathcal{B}_{sh} , then the machine is called (Differential compound).

*According to the method of connecting shunt and series fields, there are two types:

1. Long shunt compound machine.

2. Short shunt compound machine. $\leftarrow (\mathcal{E}_s\right)$

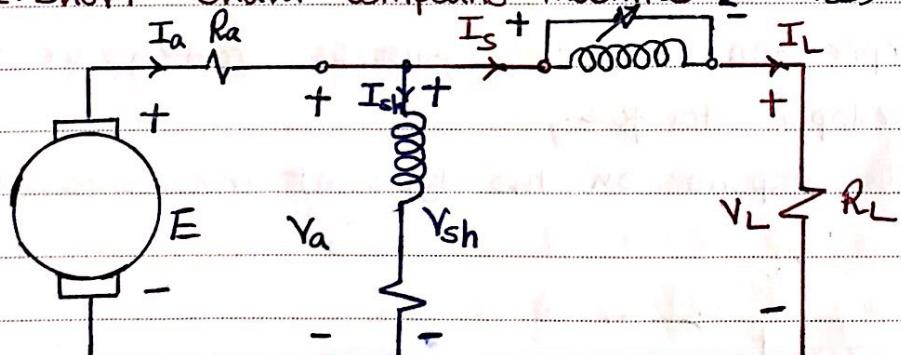


Fig. 3

Fig. 3 = Short shunt compound machine or generator.

$$I_s = I_L$$

$$I_a = I_{se.} + I_s$$

$$E = I_a R_a + V_a$$

$$V_a = V_{sh.} = I_{se.} R_{se.} + V_L$$

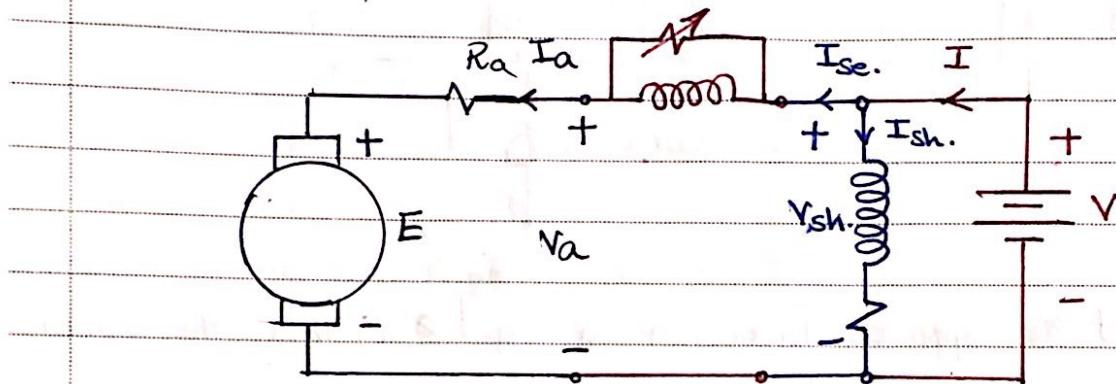


Fig. 4

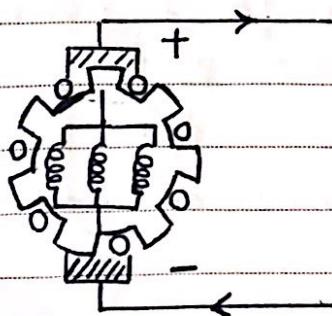
Fig. 4 \equiv Long shunt compound DC-motor.

$$V = V_{sh.} = I_{se.} R_{se.} + I_a R_a + E$$

$$I = I_{se.} + I_{sh.}$$

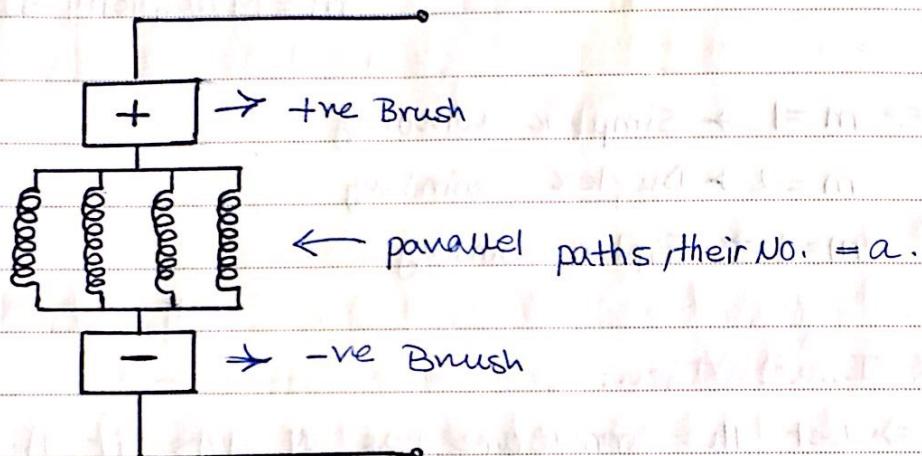
- Expression for the generated voltage or developed torque.

This depends on the type of armature winding.

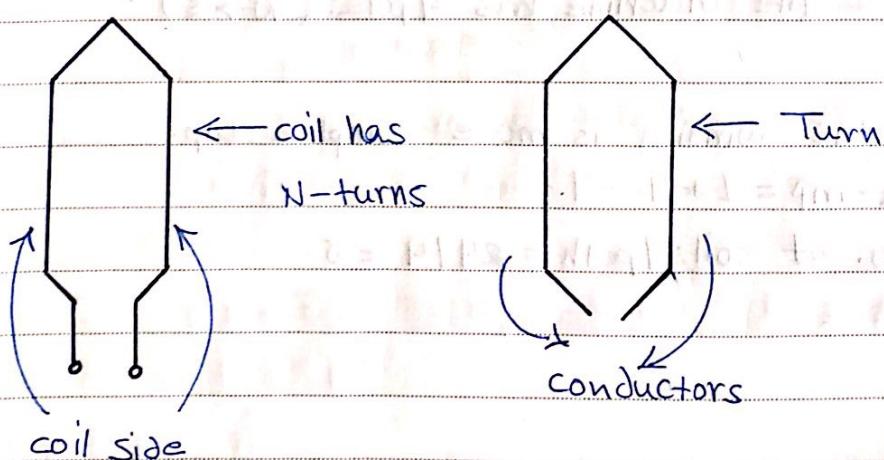


• Armature windings (A.w.):

A.w. are wound in such away to create a number of parallel paths (a) between the (+ve) and (-ve) brushes.



A.w. consists of a set of coils, their No. = c. Each coil consist of N turns.



\therefore Total No. of turns = CN

\therefore Total No. of conductors = $\mathcal{Z} = 2CN$

• There are types of A.W.:

1. Lap winding, $a = mp$

; where

2. wave winding, $a = 2m$

$P \equiv$ No. of poles

$m \equiv$ Multiplicity of winding

$\Rightarrow m = 1 \rightarrow$ Simplex winding

$m = 2 \rightarrow$ Duplex winding

$m = 3 \rightarrow$ Triplex winding

* Illustration:

\Rightarrow Let the armature has 24 slots, if A.W. consists of 24 coils.

$$\therefore c = 24$$

\Rightarrow Let the rated voltage of each coil = e

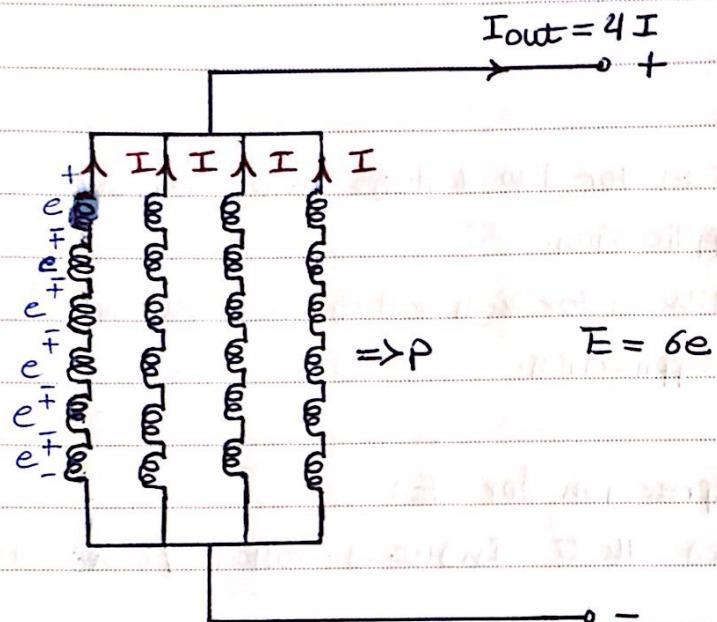
the rated current of each coil = I

the machine has 4 poles ($N \rightarrow S$) .

(If) the winding is made simplex Lap.

$$\therefore a = mp = 1 \times 4 = 4$$

$$\therefore \text{No. of coils / path} = 24 / 4 = 6$$

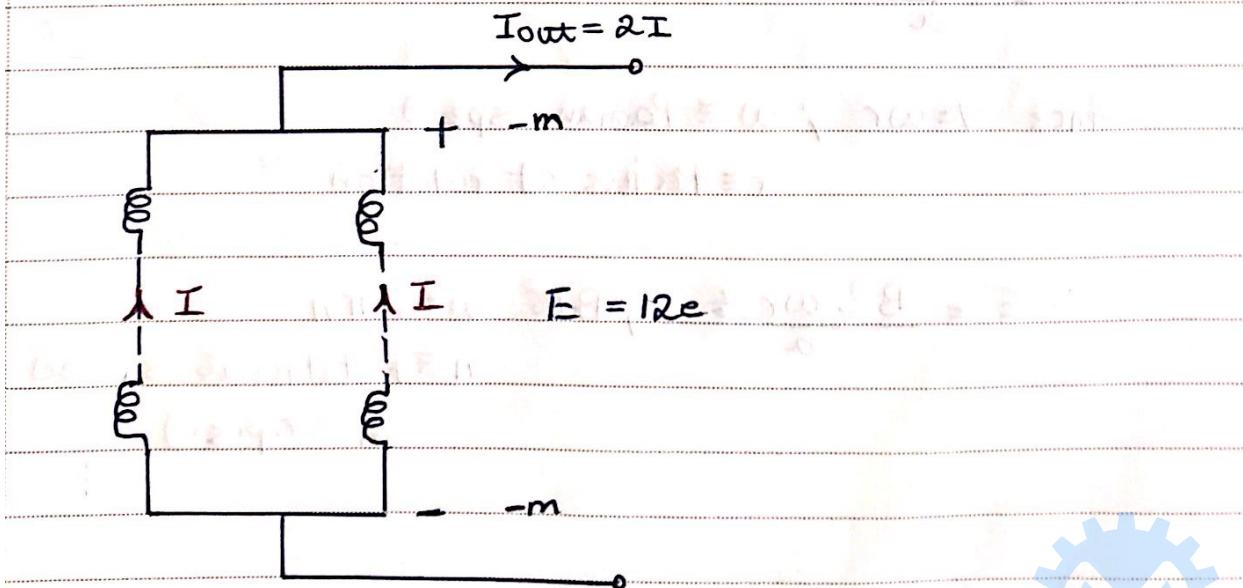


$$* P = E * I_{out} = 6e * 4I = 24 Ie.$$

(If) A.w. is simplex wave.

$$\therefore a = 2m = 2+1 = 2$$

$$\therefore \text{No. of coil/path.} = 12$$



$$\therefore P = I_{\text{out}} * E = 12e * 2I = 24EI$$

* In general:

1-Lap winding: used for low voltage high current application.

2-wave winding: used for high voltage low current application.

* General Expression for E :

It was found that Induced voltage/conductor

$$E_c \stackrel{\Delta}{=} BLV$$

\Rightarrow If total No. of conductors = Z

If No. of parallel paths = a

∴ $E \equiv$ voltage per path.

$$= E_c + \frac{Z}{a}$$

$$= \frac{BLVZ}{a}$$

Since $V = wr$; $w \equiv$ Radian speed

$r \equiv$ Radius of rotation

$$\therefore E = \frac{BLwrZ}{a}; \text{ But } w = 2\pi n$$

$n \equiv$ Rotational speed
in (r.p.s.)

$$E = \frac{BL 2\pi n r \phi}{a} \quad \text{--- (1)}$$

If the no. of poles = P

Flux / Pole = ϕ

$$\therefore B = \frac{P\phi}{2\pi r L} \quad \text{--- (2)}$$

Sub. (2) into (1)

$$E = \frac{P\phi}{2\pi L r} * \frac{2\pi L r n \phi}{a}$$

$$E = \frac{P\phi^2 n}{a}$$

$$E = \frac{P \theta z n}{a}$$

* For a given machine (P, z, a) constants:

$$\therefore E = k \theta n, k = \frac{P z}{a}$$

* General Expression for the torque, τ :

It was found that Torque produced by a conductor,

$$\tau_{\text{cond.}} \triangleq B I_c l r$$

I_c = conductor's current.

$$\therefore \text{Torque, } \tau = z \tau_{\text{cond.}}$$

$$= z B I_c l r$$

$$\tau = z * \frac{P \theta}{2 \pi L r} + \frac{I}{a} * l r, \text{ since } I_c = \frac{I}{a} \Rightarrow I \equiv \text{Applied current to the brushes.}$$

$$\tau = \frac{z P \theta I}{2 \pi a} \quad \begin{bmatrix} \text{Retarding torque} \Rightarrow \text{generator.} \\ \text{Generated torque} \Rightarrow \text{motor.} \end{bmatrix}$$

* For a given machine (P, z , and a) are constants:

$$\therefore \tau = k_1 \theta I, k_1 = \frac{z P}{2 \pi a}$$

$\therefore \tau$ can be controlled by means of (θ) and (I).

• Example: Two DC-Machines are noted as follows:

Mch. 1 : 120 V, 1500 rpm, 4 poles.

Mch. 2 : 240 V, 1500 rpm, 4 poles.

The available armature coils are noted at 4V, 5A. For the same number of coils to be used for both machines, Determine:

+ Rating \equiv Max. that the machine could handle.

NO.

I) Type of A.w. II) Number of coils required
III) KW rating of each machine.

→ For HV use wave winding.

\Rightarrow For Lv use Lap winding.

*Mch. 2.

I) For machine.2, use simplex wave winding (S.W.W.)

$$\text{II) No. of coils / path} = \frac{240}{4} = 60 \text{ coils}$$

$$a = 2m = 2*1 = 2$$

$$\text{No. of coils required} = (2)(60) = 120$$

$$\text{III) } \text{kW Rating} = \frac{E * I}{1000} = \frac{240 * (5)(2)}{1000} = 2.4 \text{ kW.}$$

(Total).

* Mch. 1.

I) For machine 1, use simplex lap winding.

$$\text{II) No. of coils / path} = \frac{120}{4} = 30$$

$$a = mb = 1 + 4 = 4$$

$$\text{Total No. of coils required} = 4 \times 80 = 120$$

$$\text{III) KW Rating} = \frac{E + I}{1000} = \frac{120 + (5)(4)}{1000} = \frac{120 + 20}{1000}$$

$$= \frac{2400}{1900} = 2.4 \text{ kW.}$$

و يضمن، لضمان عدم نفعها، أن Windows قد لا تلبي المعايير المطلوبة.

.(No. of coils)

Exam Question:

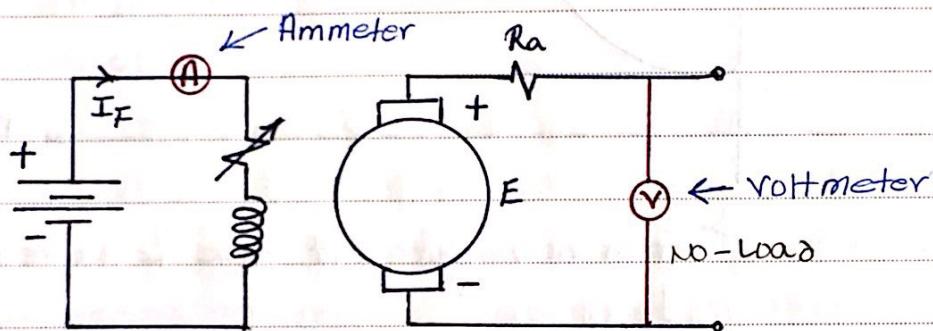
→ Design a DC - Machine with the following rating.

*Load characteristics of DC - Machines:

DC-generators.

No-Load characteristics!

I. Separately Excited Generator:



• Procedure:

I) Run the generator at constant speed (i.e. $n = \text{const.}$)

II) vary I_F

III) Take readings of E .

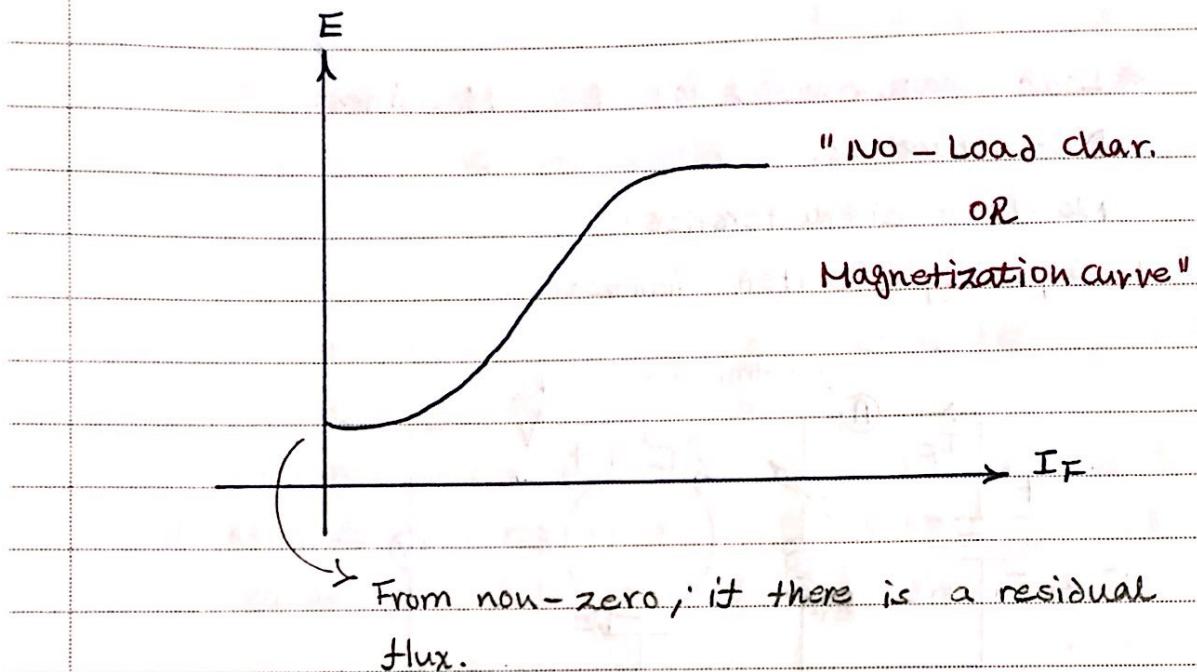
| Input $\rightarrow I_F$ | $E \rightarrow$ output |
|-------------------------|------------------------|
| I_1 | E_1 |
| I_2 | E_2 |
| ⋮ | ⋮ |
| I_n | E_n |

IV) Plot E vs. I_F .

NO. _____

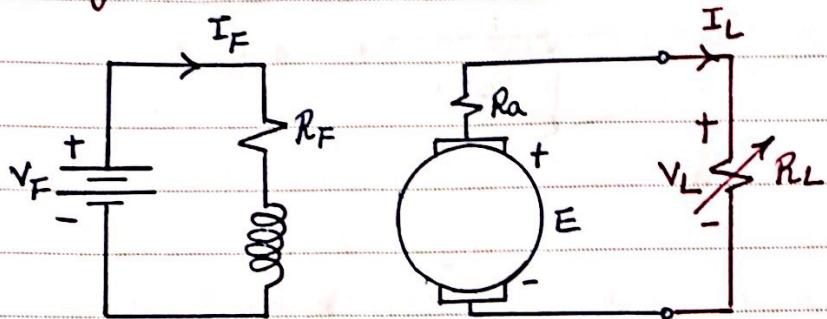
$$+ E = k\sigma n = k'\sigma ; k' = kn. \quad \sigma \uparrow \Rightarrow E \uparrow$$

$$\sigma \propto I_F$$



From zero; if there is no residual flux.

- Load characteristics of s.ext. gen. "Separately Excited generator."



- Objectives

Find V_L versus I_L .

- procedure

I) Run generator at constant speed.

II) For given I_F ,

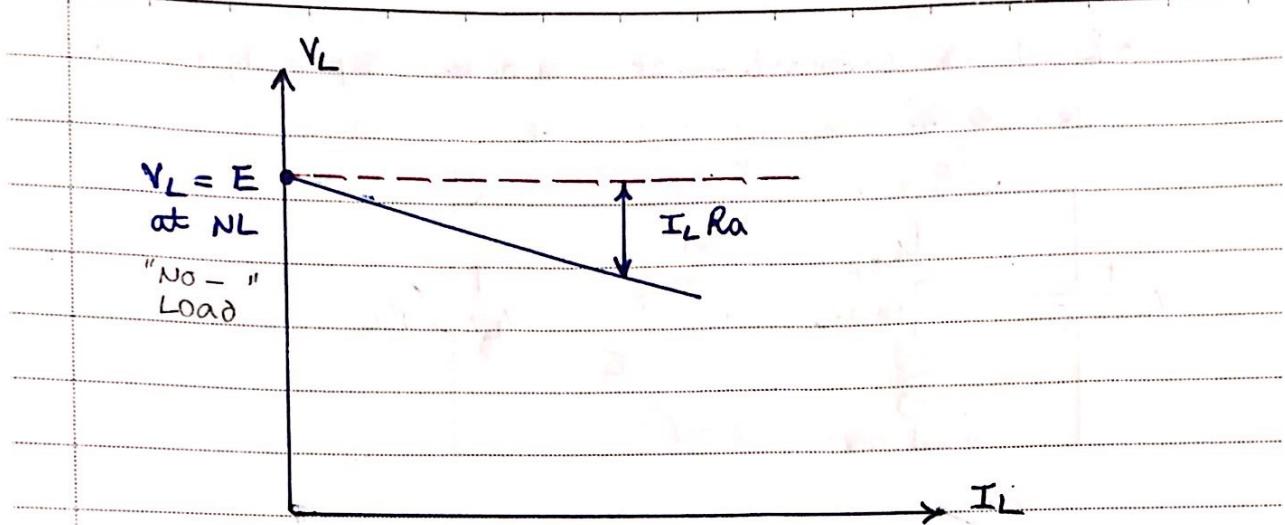
Since $E = k_a n$; E is constant.

II) vary R_L and take the readings of I_L and R_L .

| I_L | V_L |
|----------|----------|
| I_1 | V_1 |
| I_2 | V_2 |
| \vdots | \vdots |
| I_n | V_n |

II) plot V_L with I_L .

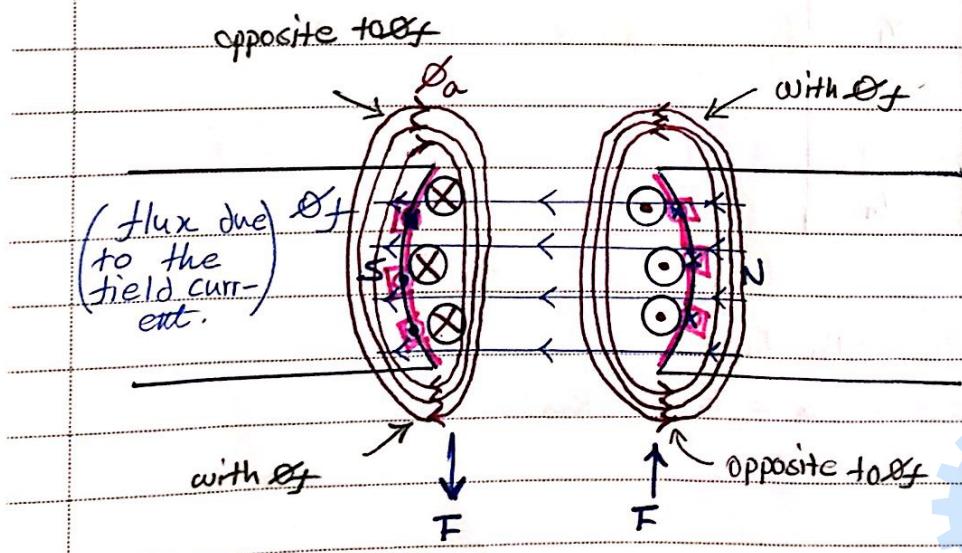
$$V_L = E - I_a R_a = E - I_L R_a$$



* Since R_a is usually very small, then up to the rated value of I_a , the drop ($I_L R_a$) is small.

∴ This gen. can be considered as a constant DC - voltage source.

- comment: In addition to the drop of $(I_L R_a)$, there is a drop due to armature reaction explained as follows:



Φ_F ≡ Flux due to I_F

Φ_a ≡ Armature flux due to I_a

○ ⊗ ≡ Direction of induced voltages and direction of I_a .

$$\textcircled{Q} = \Phi_a \text{ (RHR)}$$

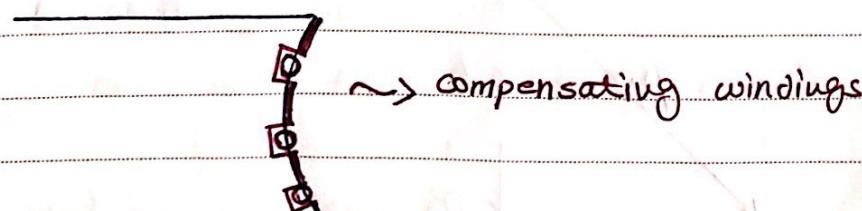
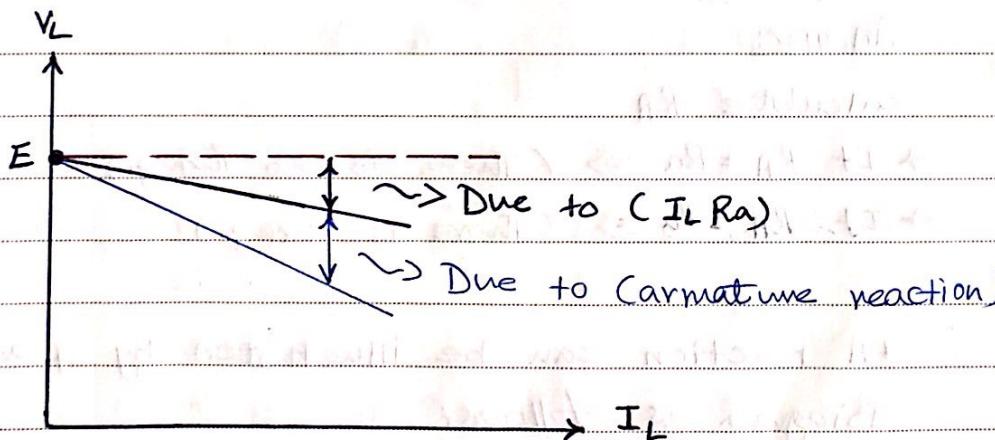
Φ_a = opposes Φ_F at one pole tip and aid Φ_F at the other pole tip.

* Due to saturation, the resultant effect of Φ_a is a reduction in Φ_F .

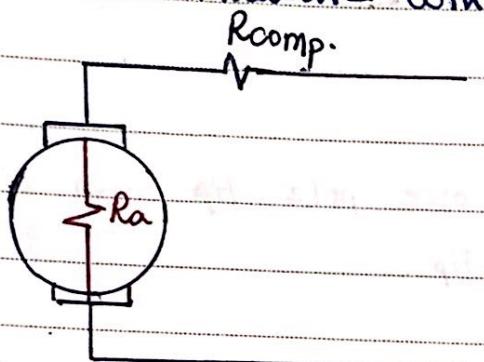
• consequently as resultant flux:

$$\Phi_R = \Phi_F + \Phi_a \downarrow \text{ (Decreases)}$$

then $E \downarrow \Rightarrow V_L \downarrow$



- Armature Reaction can be reduced or eliminated by means of compensating windings which are conductors located at the pole faces and connected in series with the armature windings.



($R_{comp.}$ and R_a are in series.)

- Armature cct. resistance, $R_a = R_a + R_{comp.}$

Armature winding resistance ↑

* Exam Question:

Separately Excited gen., Does it have $R_{comp.}$?

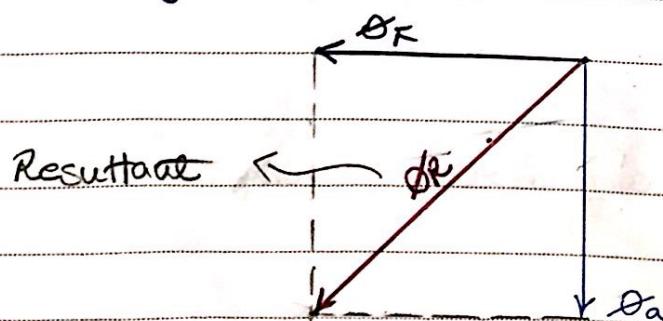
Answer:

calculate R_a

→ If $R_a = R_a \Rightarrow$ (There is no $R_{comp.}$)

→ If $R_a > R_a \Rightarrow$ (There is $R_{comp.}$)

- * A reaction can be illustrated by phasors Diagram as follows:

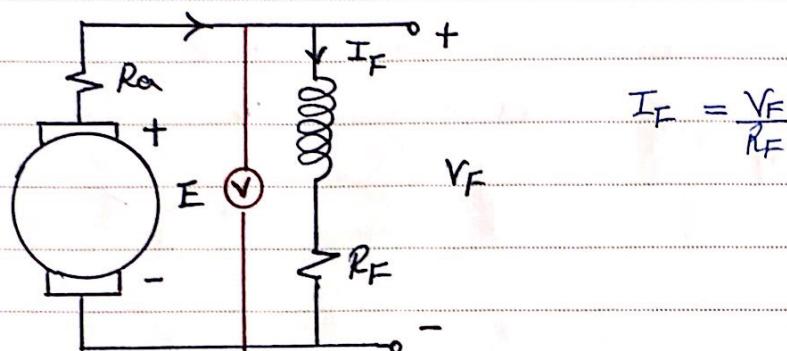


→ If No-Load

$$I_a = 0 \Rightarrow \delta_a = 0$$

* This is based on the existence of residual magnetism.

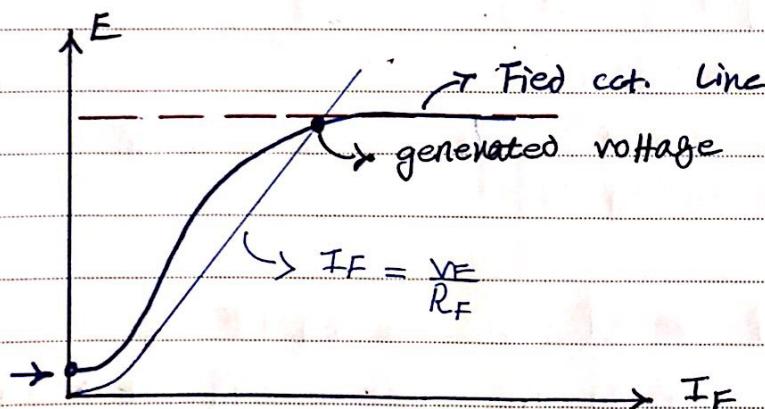
Build up voltage: saturation of field, V_F



$$I_F = \frac{V_F}{R_F}$$

- I) Run the gen. at constant speed.
- II) observe the generated voltage $E = k\omega n$ by using the voltmeter V .

$$I_F \uparrow \quad E_F \uparrow \quad E \uparrow$$

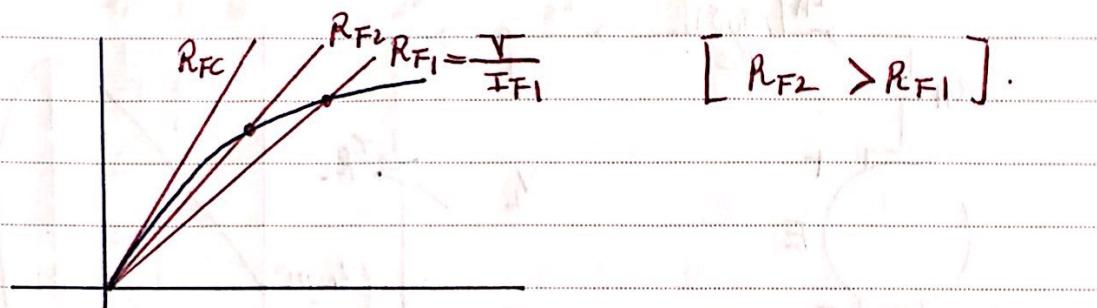


"Build up of self exciting generator"

* proof \Rightarrow The Brushes - - - ?

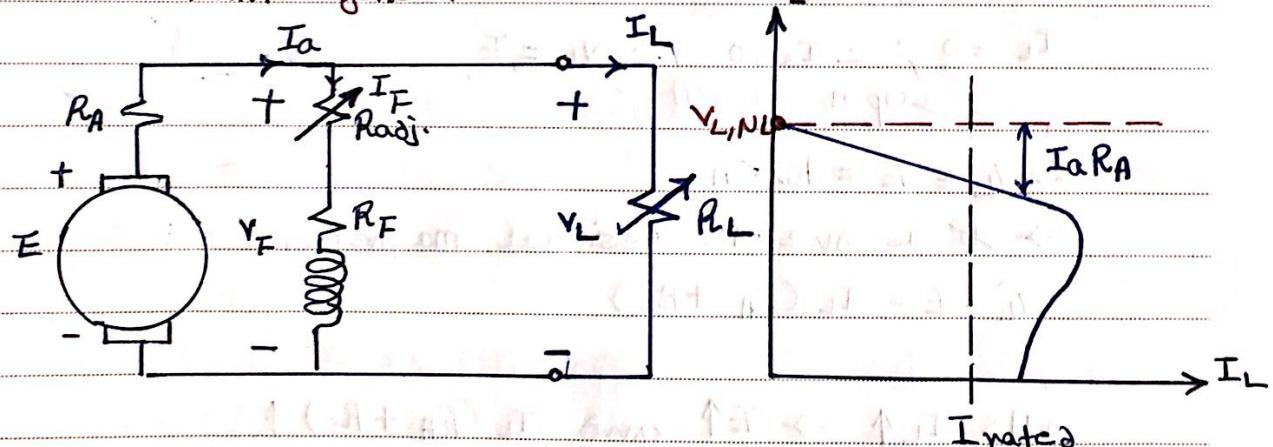
* Causes of Failure to build up:

- I) Lack of residual magnetism.
- II) Armature windings are connected in reverse connection w.r.t field windings.
- III) O/C in armature windings.
- IV) R_F of the field windings is greater ($>$) than the critical field resistance.



* Load characteristics of:

- Shunt generator:



$$V_L = E - I_a R_A$$

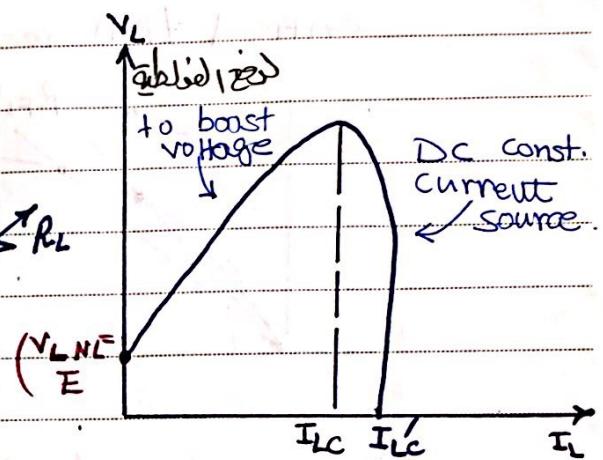
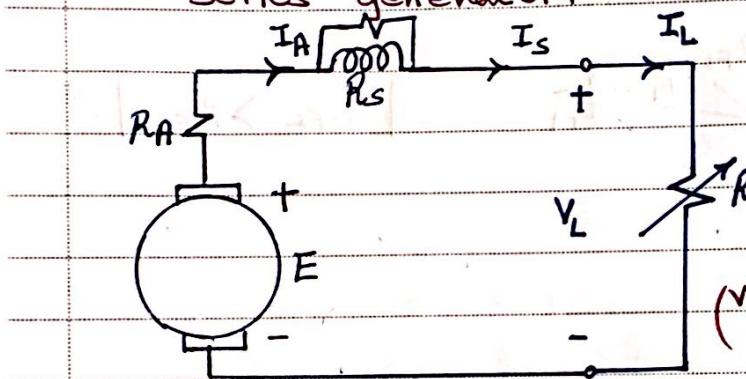
$$I_a = I_F + I_L$$

\therefore As $I_L \uparrow \rightarrow I_a \uparrow \rightarrow I_a R_A \uparrow \rightarrow V_L \downarrow$

* Since $V_F = V_L$, then as $V_L \downarrow \rightarrow V_F \downarrow \rightarrow I_F = \frac{V_F}{R_F} \downarrow \rightarrow E \propto I_F \therefore E \downarrow$

$\therefore I_L$ is increased after a certain point, then V_L is going to collapse.

- Series generators



$$I_L = I_S = I_A$$

$$I_L = 0; \therefore I_S = 0 \therefore V_L = E$$

$\left. \begin{matrix} \text{Open circuit.} \end{matrix} \right\}$

$$\therefore V_L = E = k \theta n$$

$\Rightarrow \theta$ is due to residual magnetism.

$$V_L = E - I_L (R_A + R_S)$$

* As $I_L \uparrow \rightarrow E \uparrow$ and $I_L (R_A + R_S) \uparrow$.

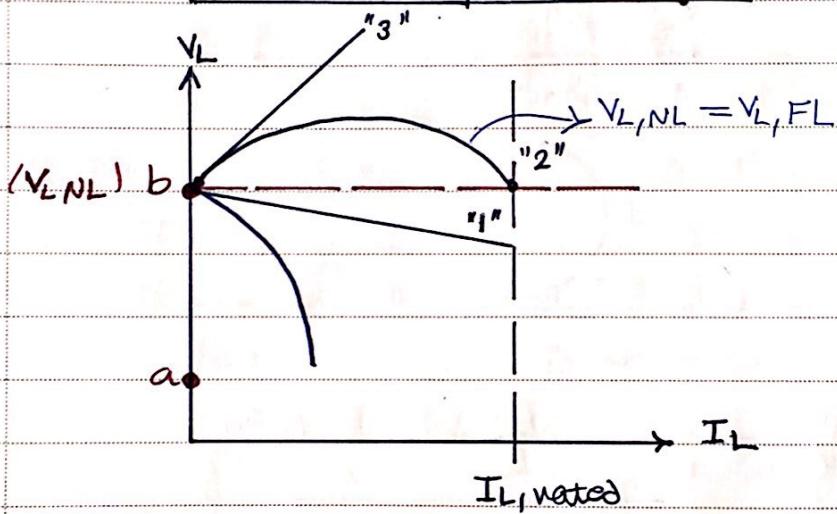
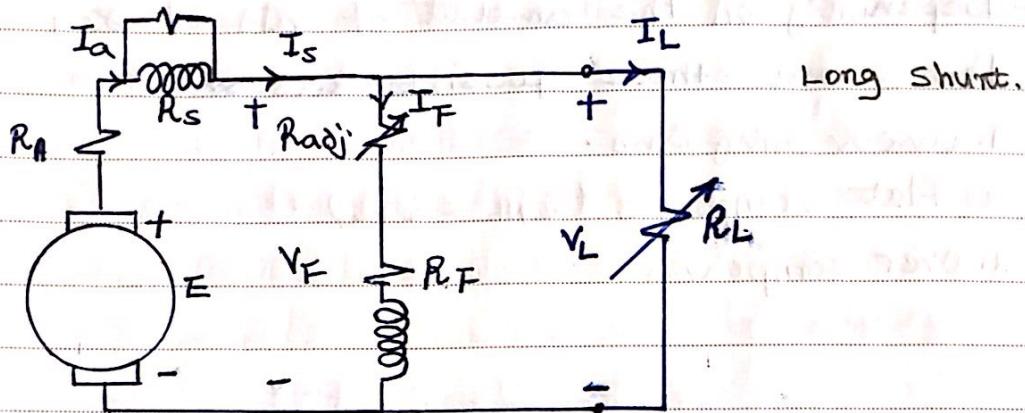
However initially the increase in ($E >$ voltage

drop of $I_L (R_A + R_S)$ up to the current of I_{Lc} .

* After I_{Lc} , the situation will be reversed.

\Rightarrow consequently (V_L) will drop.

- compound generator!



$$V_L = E - I_a (R_A + R_s)$$

$$I_a = I_F + I_L$$

$$V_L = V_F$$

$I_L \uparrow \rightarrow E = k (\theta_{sh} + \theta_{se}) n \downarrow$ in differential,

also $I_a (R_A + R_s) \uparrow \rightarrow$ continuous drop in V_L .

θ_{se} opposes θ_{sh} .

$E = k(\alpha_{sh} + \alpha_{se})n \uparrow$ in case of cumulative.

→ Depending on the amount of aid of α_s ,
there are three possible cases:

1. Under compound.
2. Flat compound ($V_{NL} = V_{FL}$).
3. Over compound.

* Example: A separately excited DC generator with compensating winding is rated at 172 kW, 480 V, 400A and 1800 rpm.

It has the magnetization curve shown in Fig. 1.

The machine has the following data:

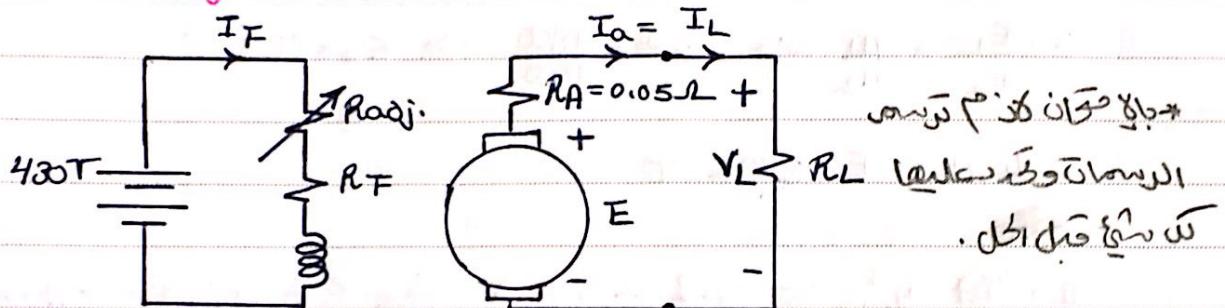
$$R_a = 0.05 \Omega$$

$$V_F = 480 \text{ V}$$

$$R_F = 20 \Omega$$

$$N_F = 1000 \text{ turns/pole.}$$

$$\text{Radius} = 0 \rightarrow 300 \Omega$$



Field cct.

Armature cct.

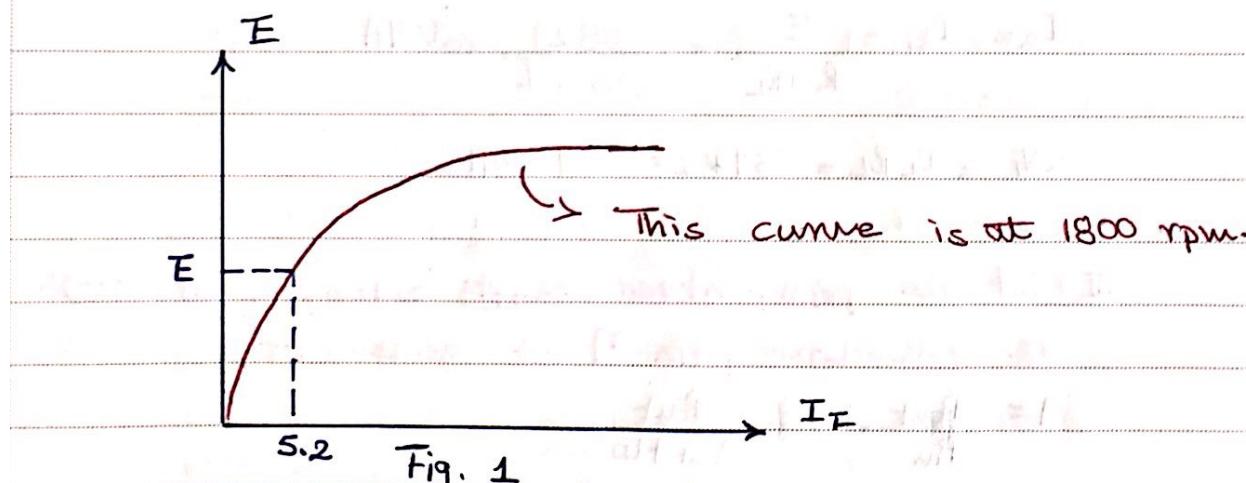


Fig. 1

I) If $R_{adj.}$ is set at 63Ω and the prime mover driving the generator speed is 1600 rpm. Find No-terminal voltage ($V_L - N_L = ?$)



FIVE APPLE

Tech
Family

* No load = open circ.

$$\therefore V_{L, NL} = E$$

$$I_F = \frac{V_F}{R_{adj} + R_F} = \frac{430}{63 + 20} = 5.2 \text{ A}$$

∴ From Fig. 1, for $I_F = 5.2 \text{ A}$. It can be found that $E = 430 \text{ V}$ at 1800 rpm.

$$(E = k \theta n)$$

∴ For same I_F (i.e. θ is constant)

$$\therefore \frac{E_1}{E_2} = \frac{n_1}{n_2} \Rightarrow \frac{430}{E_2} = \frac{1800}{1600} \Rightarrow E_2 = 382 \text{ V}$$

$$\therefore V_{L, NL} = E = 382 \text{ V}$$

II) If the load = 1.2, find the terminal voltage

$$(R_L = 1.2)$$

$$I_a = I_L = \frac{E}{R_{adj} + R_L} = \frac{382}{0.05 + 1} = 364 \text{ A}$$

$$\therefore V_L = I_L R_L = 364 \times 1 = 364 \text{ volt.}$$

III) If the prime mover supply a torque of 1000 N.m

at full-load, find η at generator.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{losses}}$$

$$P_{out} = 172 \text{ kW (given)}$$

$$P_{in} = \omega T = \frac{(2\pi)(1800)}{60} \times 1000 = 60\pi \times 10^3$$

$$\eta = \frac{172}{60\pi} \approx 91 \%$$

- Note: Read a study about the power flow diagram of DC-machines.

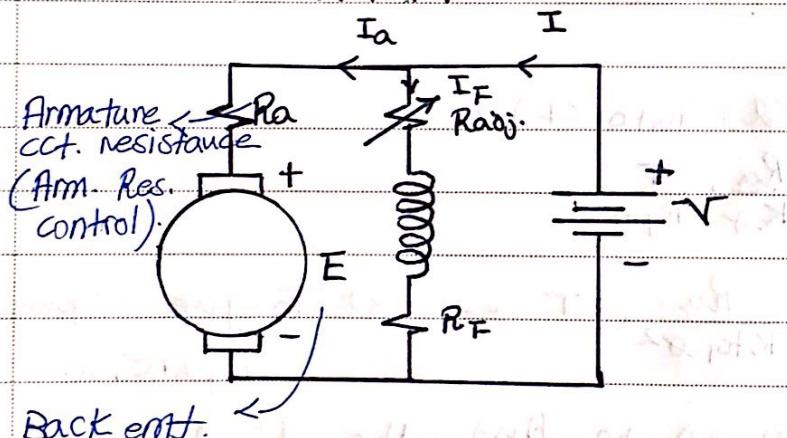
⇒ Generator gets a mechanical input to move the rotor then produce electrical output.

• DC - Motors:

- objectives:

1. Voltage - Current relationship. ⇒ Explained Before.
2. Speed equation.
3. Speed - Torque relationship.

- Shunt Motor:



* Exam Question

Derive speed egn.?

1. Draw the cct.
2. currents directions.
3. write the eqn's.

$$E = V - I_a R_a$$

$$K \theta \omega = V - I_a R_a$$

$$\therefore \omega = \frac{V - I_a R_a}{K \theta} \quad \text{--- (1) Speed equation of DC-motors.}$$

+ Speed equation shows that there are 3 methods to control the speed of the motor:

1. vary V , Armature voltage control.

2. vary R_a , Armature resistance control.

3. vary Φ , (Field control), i.e. varying I_F .

• Comment: If the field cct. is opened suddenly.

$I_F = 0 \rightarrow \Phi \downarrow \rightarrow [\omega]$ is going to reach a very dangerous speed (never ever open the field cct. suddenly).

+ since $T = k_1 \Phi I_a$

$$\therefore I_a = \frac{T}{k_1 \Phi} \quad \text{--- (2)}$$

\Rightarrow substitute (2) into (1) :

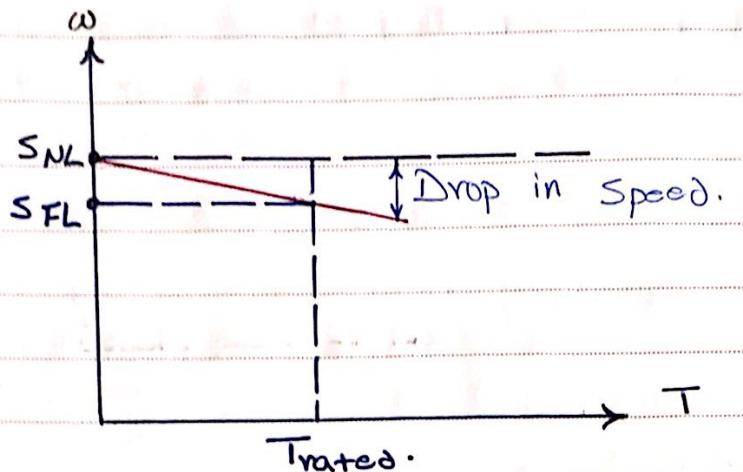
$$\omega = \frac{V}{k_2 \Phi} - \frac{R_a}{k_2 \Phi} \cdot \frac{T}{k_1 \Phi}$$

$$\omega = \frac{V}{k_2 \Phi} - \frac{R_a}{k_1 k_2 \Phi^2} \cdot T \quad \text{--- (3) Torque - speed equation.}$$

\therefore (3) can be used to find the load characteristics (i.e. ω vs. T)

\therefore For a given applied voltage and given R_{adj} .

$\therefore V$ and Φ are constants.



$$\therefore \text{Speed Regulation} = \frac{S_{NL} - S_{FL}}{S_{FL}}$$

\therefore If the drop in voltage is very small, then the shunt motor is a "constant speed motor!"

*Load characteristics:

$$E = V - I_a R_a$$

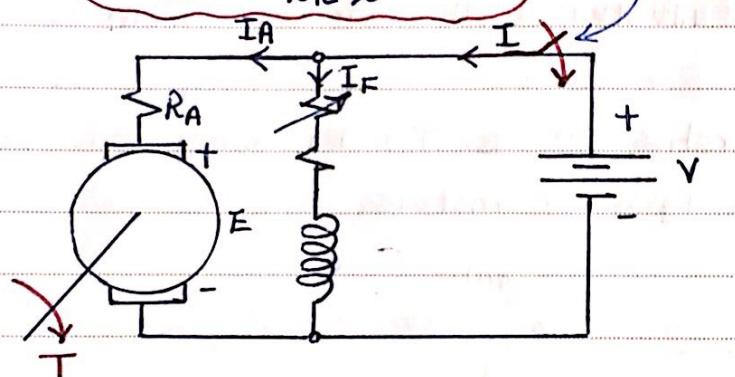
$$K \cdot \omega = V - I_a R_a \quad \text{"The diagram should be given!"}$$

$$\omega = \frac{V - I_a R_a}{K \cdot \delta} \quad \dots \dots (1)$$

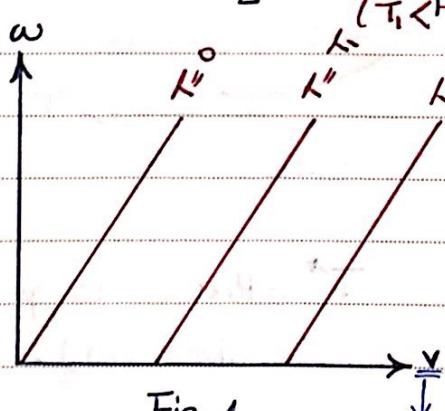
$$T = K' \cdot \delta \cdot I_a$$

$$\therefore \omega = \frac{V}{K \cdot \delta} - \frac{R_a}{K \cdot \delta} \cdot \frac{T}{K' \cdot \delta}$$

$$\omega = \frac{V}{K \cdot \delta} - \frac{R_a}{K \cdot K' \cdot \delta^2} \cdot T \quad \dots \dots (2)$$



• For a given torque: by using eqn. (2)



$[R_a \& \delta \equiv \text{const.}]$ Armature voltage control

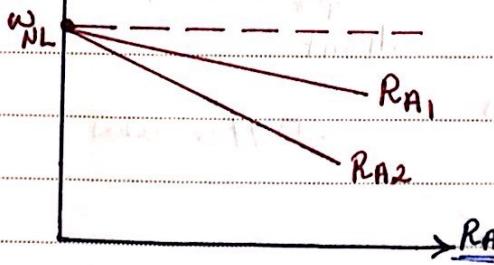


Fig. 2

Armature
Resistance
control.



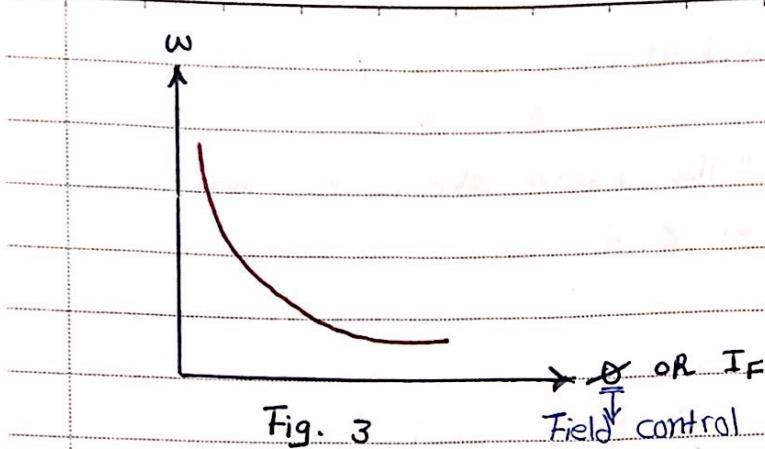
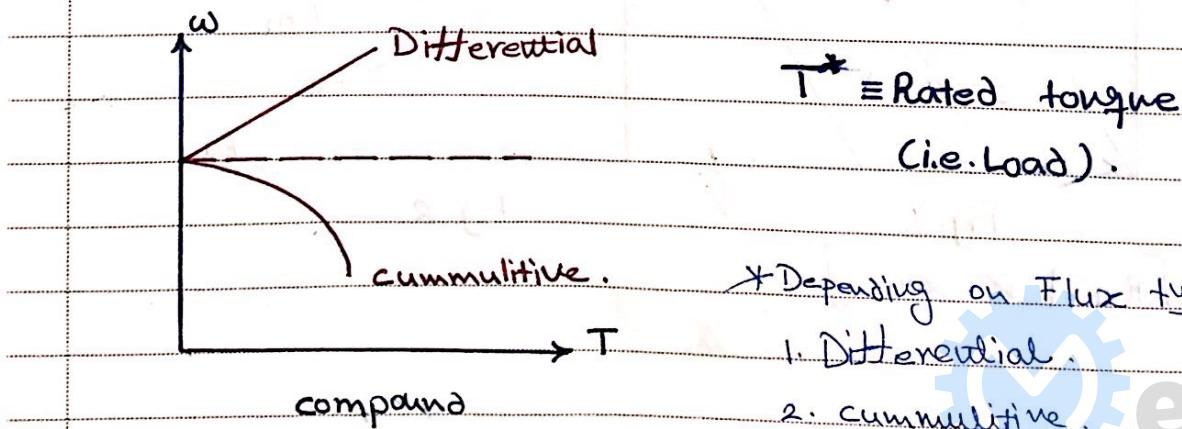
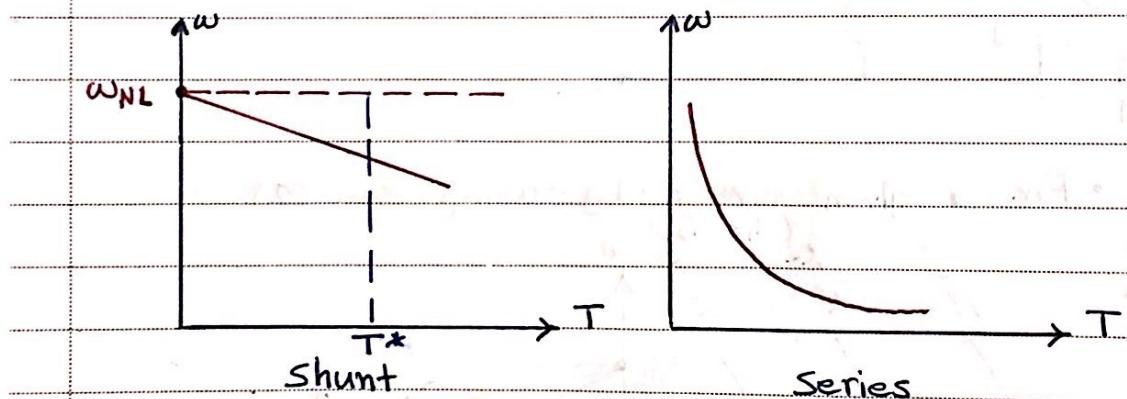


Fig. 3

ω or I_F
Field control

• Comment:

- Fig. 1, 2 and 3 illustrates the various methods of speed control.
- Load characteristics (ω vs. T) by using eqn. (2).
- For the various types of motors.



T^* = Rated torque
(i.e. Load).

- * Depending on Flux type:
 1. Differential.
 2. cumulative.



* when $T \uparrow \rightarrow E \downarrow$

$$E = k \varphi w$$

$$T = k \varphi I_A$$

$$I_A = \frac{V - E}{R_a} \uparrow$$

$$I_A = I_s \text{ (series)} \uparrow \rightarrow \delta \uparrow$$

• Starting of DC-motors:

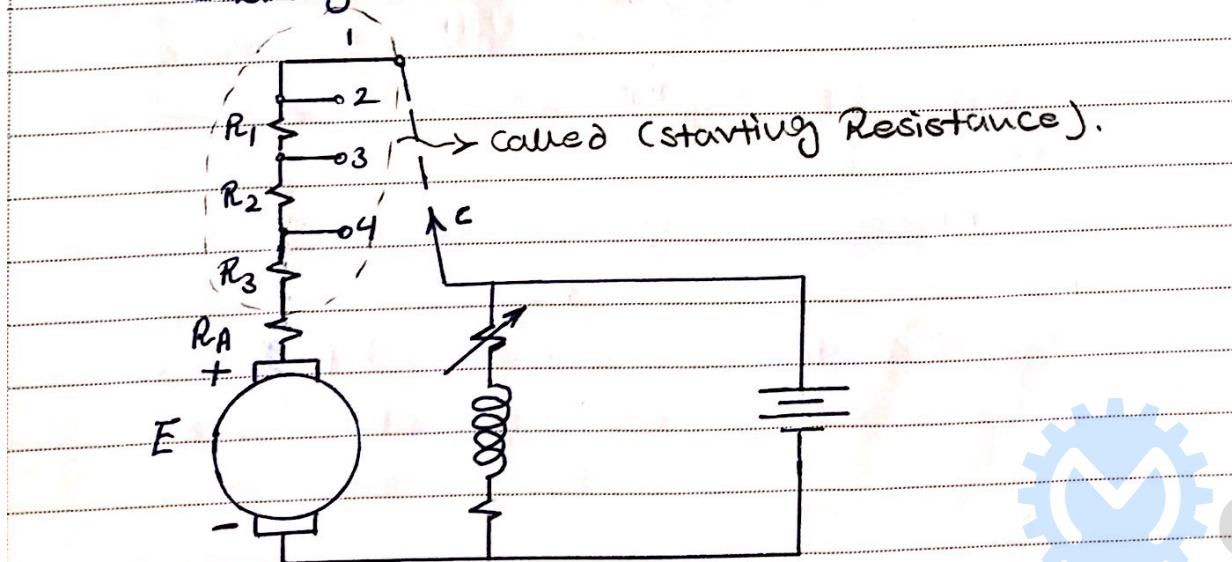
Initially when a voltage (V) is applied to a DC motor, it doesn't rotate due to its inertia.

$$\therefore E = k \varphi n = 0; n = 0$$

$$\therefore I_A = \frac{V - E}{R_a} = \frac{V}{R_a}$$

$$I_{A \text{ starting}} \gg I_{A \text{ rated}}$$

\therefore Initially I_A should be limited by using external starting resistance.



* Initially c is at position *1 \Rightarrow when motor rotated $I_A \downarrow \rightarrow *2 \dots$ until position *4 is reached.

→ See the example in the book.

Power \Rightarrow convert (Electrical \Leftrightarrow Mechanical)

\Rightarrow consider (Shunt Motor)!

$$(E \triangleq V - I_A R_A) + I_A$$

$$E I_A = V I_A - I_A^2 R_A$$

$$= V(I - I_F) - I_A^2 R_A$$

$$= V I - (V I_F + I_A^2 R_A) = R_{shunt} - \text{Electrical Losses} = P_{out}$$

Input Field Armature
Losses Losses

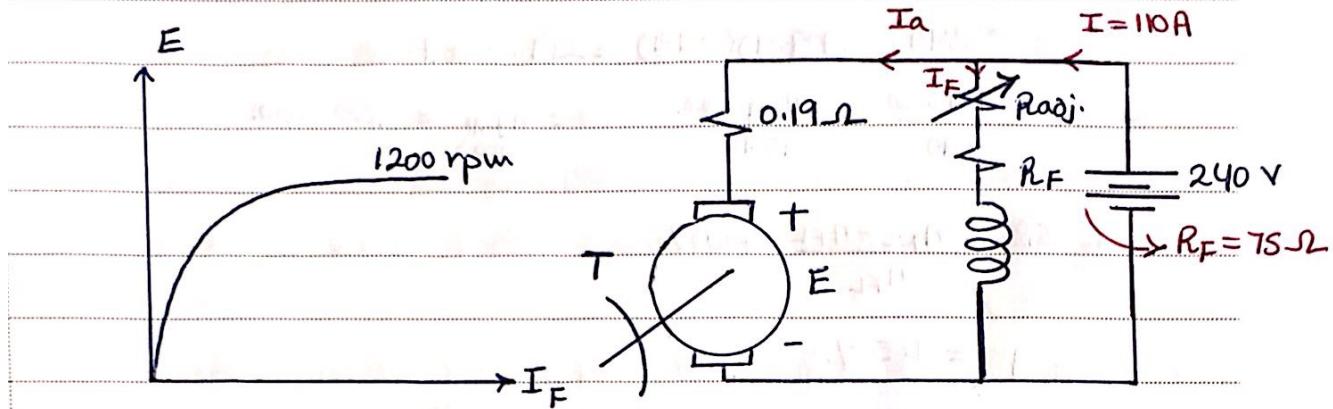
Electrical Losses

\Rightarrow For shunt.

* solve it for series and compound.

• Note: The same expression at power can be found for all types of (motors).

* Ex. 1 For the given system and magnetization curve, what is the speed at no-load? given $R_{adj.} = 175 \Omega$.



I.

$$\text{No-load} \Rightarrow T=0$$

At no-load (I_a) is very small, assumed to be zero.

$$\therefore E = V = 240 \text{ volt}$$

$$I_F = \frac{240}{75 + 175} = 0.96 \text{ A}$$

By using the magnetization curve.

$$I_F = 0.96 \Rightarrow E = 277 \text{ volt at } 1200 \text{ rpm.}$$

$$\Rightarrow \text{since } E = k \cdot \theta \cdot n$$

$$\therefore \frac{E_1}{E_2} = \frac{n_1}{n_2} \Rightarrow \frac{277}{240} = \frac{1200}{n_2}$$

$$\therefore n_2 = 1040 \text{ rpm. (Field control, } I_F \text{ (line) is } 0)$$

II. Assuming no-armature reaction, what is the speed at full-load, given $I_{full-load} = 110 \text{ A}$ for the same $R_{adj.}$!

$$I_A = I - I_F$$

$$= 110 - 0.96 = 109.4 \text{ A}$$

$$\therefore E = V - I_A R_A$$

$$= 240 - (109.4)(0.19) = 219.3 \text{ volt.}$$

$$\therefore \frac{E_{\text{full load}}}{240} = \frac{n_{\text{full load}}}{1040} \quad \therefore n_{\text{full load}} = 950 \text{ rpm}$$

$$\text{SR} = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100\%$$

$$= 9.5\%$$

III. Find I_A start?

$$I_A \text{ start} = \frac{V - 0}{R_A} = \frac{240}{0.19} = 1263 \text{ A}$$

If I_A start is to be limited [$2 I_A$ rated],
find R_A ext.?

$$I_A \text{ rated} = 109.04 \text{ A}$$

$$\therefore 2 I_A \text{ rated} = 109.04 + 2 = \frac{240}{0.19 + R_A \text{ ext.}}$$

$$\therefore R_A \text{ ext.} = 0.906 \Omega$$

• AC - Machines:

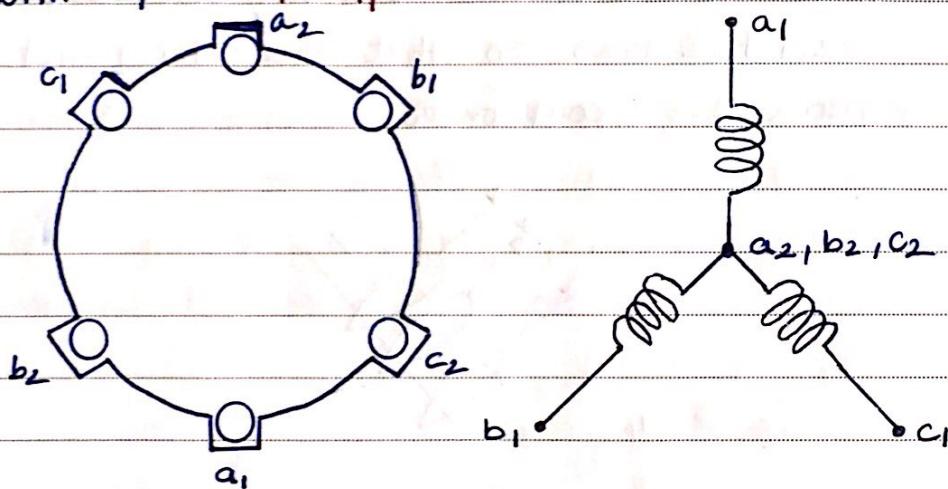
3-ph. induction motor.

- construction:

It consists of:

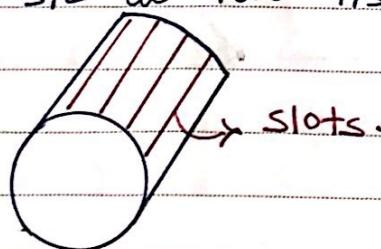
1) Stator {which consists of 3-ph. windings into which a balanced 3-ph. voltage is applied. As shown before this produce a rotating magnetic field.}

with speed $f = np$



2) Rotor { There are two types:

a) Squirrel-cage: (चिरु, ग्रीड), Rotor consists of a cylinder at its surface there are slots at which conductors are located. These conductors are s/c at two its ends.

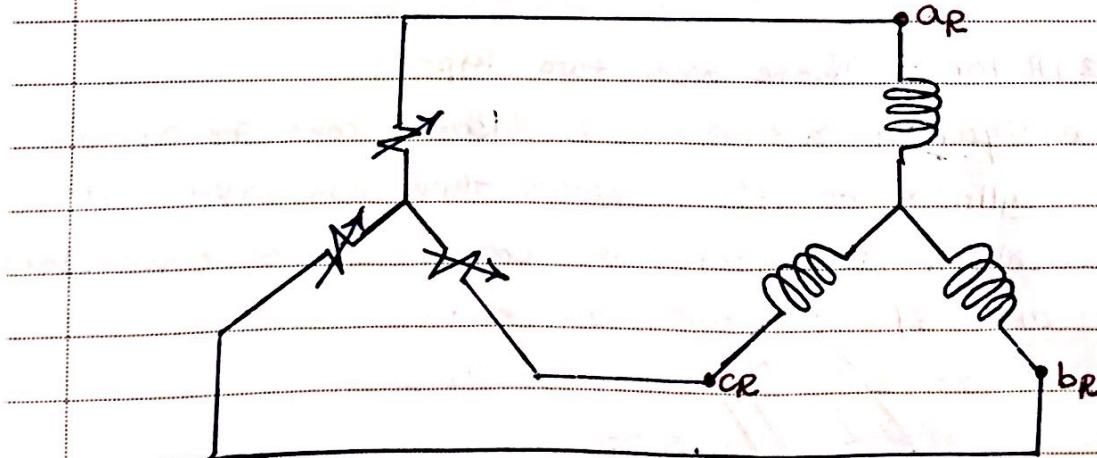
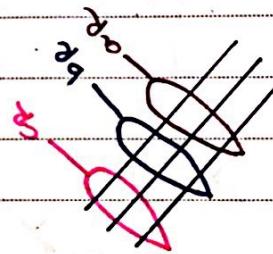


* Hence for this type the motor is called SCIM (squirrel cage Inductor Motor).

b) Wound-Type: Rotor consists of 3-ph. windings, whose terminals are connected to 3 slip rings.

These rings make contact with 3 stationary brushes.

The brushes are connected to 3-ph. resistive load. so that the rotor resistance can be controlled.



* operation of Induction Motor:

a 3-ph voltage V_s is applied to stator winding (s.w.)



current I_s will flow in (s.w.)



I_s is going to generate Rotating Magnetic Field, B_s .

The speed of $B_s = \frac{f}{p}$; $n_s = \frac{f}{p}$

$\Rightarrow f_s \equiv$ Frequency of V_s \rightarrow synchronous speed.

$P \equiv$ No. of pairs (pole pairs) of s.w.



B_s is going to cut Rotor conductors at windings (R.w.).



A voltage E_R will be induced in R.w. --



A current I_R due to E_R will flow in the R.w.



I_R will produce a flux B_R .



Hence, due to I_R and B_s , a torque "T" will be produced or one can say due to the interaction between B_s and B_R a torque will be produced.

$$\Rightarrow T \triangleq K (B_R + B_s)$$



∴ Due to (T), the rotor is going to rotate with a speed $\frac{n_{motor}}{rps}$ or $\frac{\omega_m}{rad/s}$ \Rightarrow Mechanical speed

* n_s is called synchronous speed $\Rightarrow B_s$.

where ($n_m < n_s$), In order to have relative motion between B_s and R.W.

. Induction \rightarrow ~~force~~ \rightarrow I_R

• **Comments:**

1. Since E_R and I_R are generated by induction effect.

Hence the motor is called Induction motor.

2. Hence the I_M can be considered as a rotating transformer where stator represent primary and rotor represent secondary.

3. Slip speed $\triangleq n_s - n_m$

$$\text{Slip} \triangleq \frac{\text{Slip speed}}{n_s}$$

$$S = \frac{n_s - n_m}{n_s}$$

* If the Rotor is blocked (i.e. $n_m = 0$)

$$\therefore S = 1$$

$$S = 1 \text{ i.e. } n_m = 0 \text{ i.e. } 1 - 1 = 0$$

∴ In this case frequency of the induced voltage in R.W (f_r) is the same as that of voltage applied to S.W (i.e. f_s)
 $\hookrightarrow "f_r = f_s"$

* If the Rotor is rotating at speed $= n_s$.

$$\therefore s = 0 \quad "n_m = n_s"$$

$$\underline{0} = \underline{n_m} \text{ (slip)} \quad \text{لأن } n_m = n_s \quad \text{لأن } \underline{0} = \underline{n_s}$$

\therefore In this case $f_r = 0 = f_s$

$$\therefore 0 \leq s < \frac{1}{2}$$

\therefore In general $f_r = s f_s$. Rotor \rightarrow slip \rightarrow stator \rightarrow slip

* Example: A 220 V, 3-ph., 6 pole, 50 Hz, IM is running at a slip of 3.5%.

a. Find the speed of B_s (n_s):

$$f = np$$

$$\therefore n = \frac{f}{p} = \frac{50}{3} \text{ nps}$$

$$\text{pairs of poles} \leftarrow = \frac{50}{3} \times 60 = 1000 \text{ rpm.}$$

b. n_m

$$s = \frac{n_s - n_m}{n_s} \Rightarrow \frac{3.5}{100} = \frac{1000 - n_m}{1000}$$

$$\therefore n_m = 965 \text{ rpm.}$$

c. slip speed

$$= n_s - n_m$$

$$= 1000 - 965 = 35 \text{ rpm.}$$



d. Rotor frequency.

$$f_r = s f_s$$

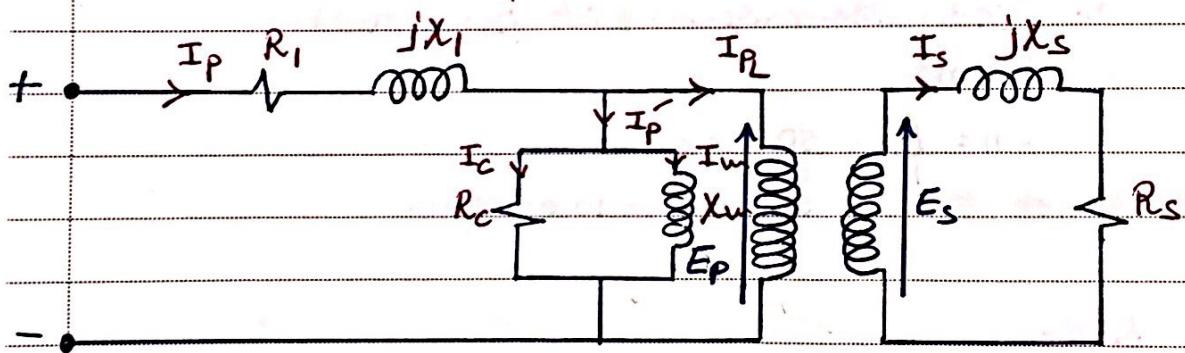
$$= \frac{3.5}{100} \times 50 = 1.75 \text{ Hz}$$

* Equivalent cct of I_H .

This cct. can be used to evaluate the performance of I_H .

since I_H can be considered as a transformer, then the equivalent cct. of transformer will be used, where: Stator = primary.

Rotor = secondary.



P ≡ Primary (Stator)

S ≡ Secondary (Rotor)

* V_p ≡ phase voltage applied to stator.

R_1, X_1 ≡ Resistance and Reactance of s.w.

I_p ≡ stator current

I_p' ≡ No-load component of I_p

I_c, I_m ≡ to represent core losses and component of I_p to generate B_s respectively.

I_{p_L} ≡ Load component of I_p

E_p \equiv Induced voltage in stator winding.
(primary)

E_s \equiv Induced voltage in secondary winding.
(secondary)

R_s, jX_s \equiv Resistance and Reactance of Rotor.

• Equivalent cct. of Rotor:

Let at locked (blocked) condition (i.e. $n_m = 0$) where

$s = 1$, where $f_r = f_e$

+ f_e = Electrical frequency of voltage applied to stator.

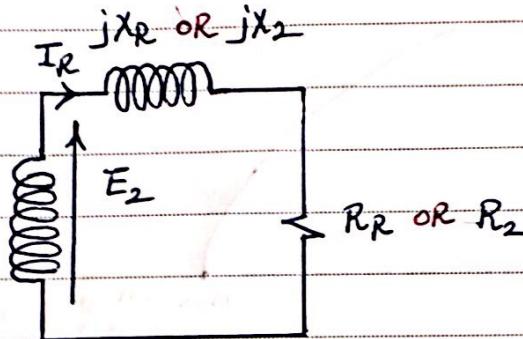


Fig. 1

$\Rightarrow E_2 = E_0$, since in general $f_r = s f_e$

\therefore In general $E_2 = s E_0$

$$\begin{aligned}
 \Rightarrow X_R &= \omega L_R & + L_R &= \text{Inductance of} \\
 &= 2\pi f_r L_R & & \text{Rotor} \\
 &= 2\pi (s f_e) L_R & & \\
 &= s (2\pi f_e L_R) = s X_0
 \end{aligned}$$

\therefore Fig. 1 can be redrawn as follows:

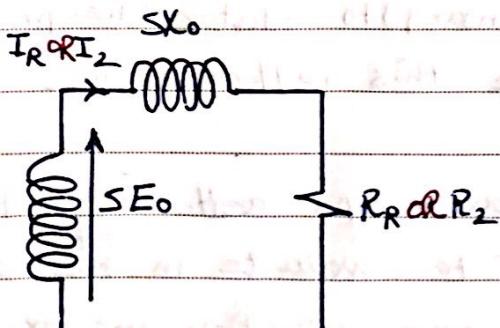
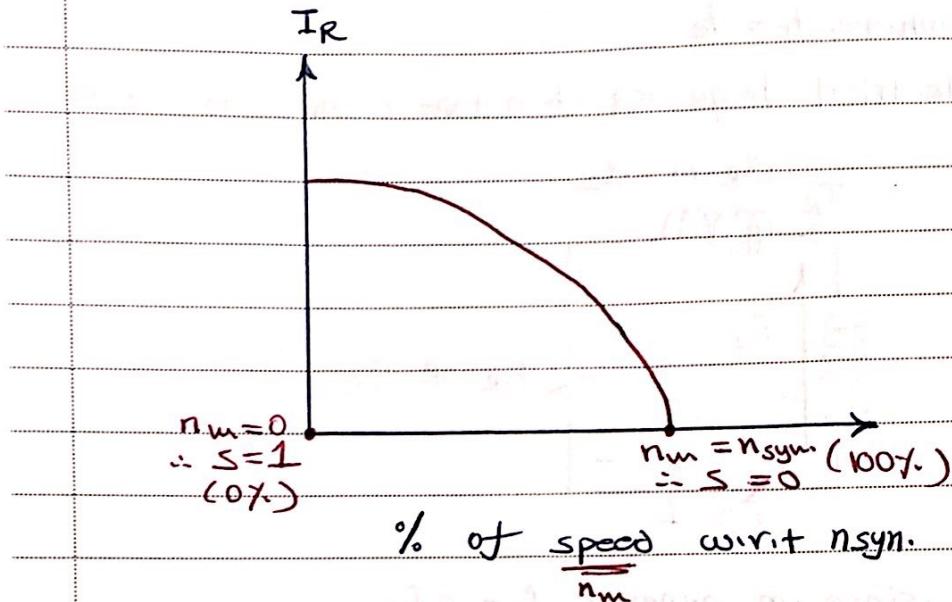


Fig. 2

$$I_R = \frac{SE_0}{R_2 + jSx_0} = \frac{E_0}{\frac{R_2}{S} + jx_0}$$



• **Comment:**

It can be observed that I_R depends on R_2 (i.e. Rotor Resistance). This is where the advantage of wound type IM which can be used to control the value of R_2 .

• **Final Equivalent circ. of Induction Motor:**

This can be obtained by reflecting Rotor (i.e. secondary) to stator (i.e. primary).

• To make this reflection, one should know a.e.f.

(λ for wound type a.e.f. = $\frac{N_s}{N_p}$). However it is difficult to evaluate in the case of squirrel cage induction motor.

* Reflect ($R \rightarrow s$)

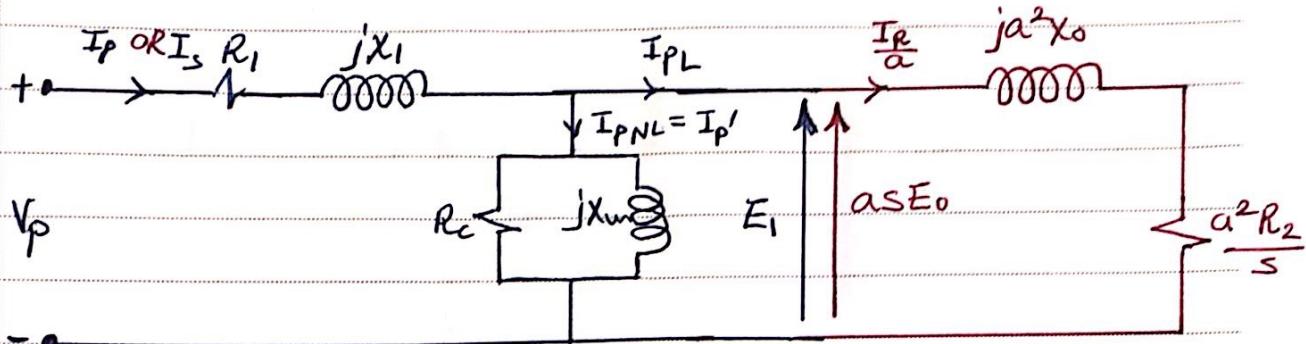


Fig. 3

"per-phase cct."

$j\alpha^2 X_o \neq jX_1$

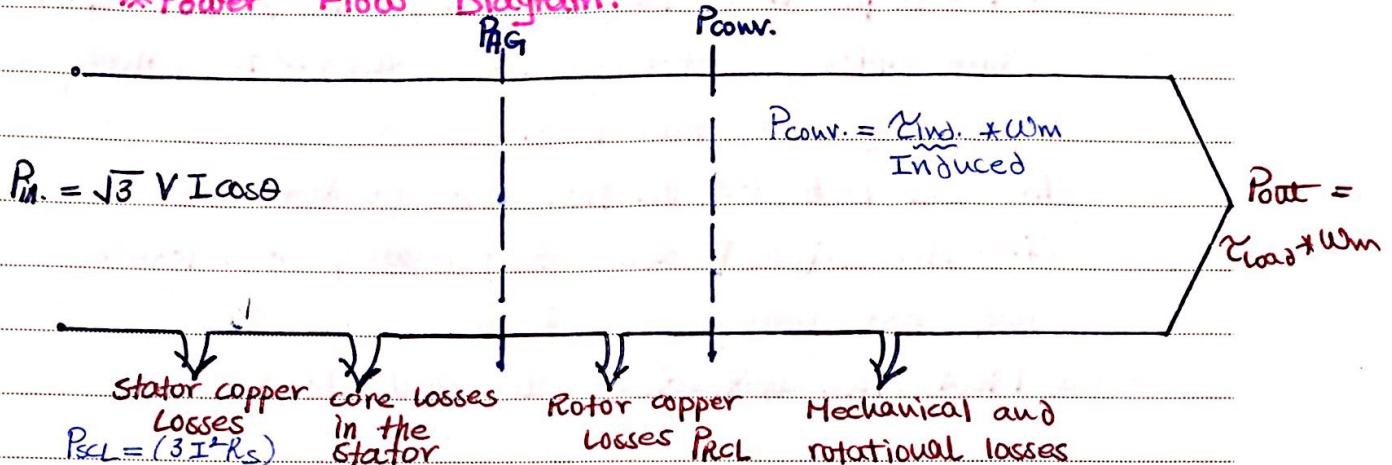
(\rightarrow they have the same frequency.

* Reflection when $f_{E_o} = f_{E_1}$

.. reflection due to frequency \rightarrow load is

\therefore Fig. 3 will be used to find voltage, current, power, and torque.

*Power Flow Diagram:



$V, I \equiv$ Applied line voltage & current.

$\theta \equiv$ PF angle

$P_{AG} \equiv$ Power transferred through Air Gap to motor.

$P_{scl.} \equiv$ Power stator copper losses.

$P_{conv.} \equiv$ Amount of power converted to mechanical power.

∴ Power flow diagram + equ. cct. can be used to find:

1. Relationship between P and T "torque".

2. Relationship between T and w .

\Rightarrow Rotor copper losses $= 3I^2R$ (in case of wound type only not squirrel cage).

• Note! If core losses is given as a number (i.e. not as R_c) then it is usually combined with the mechanical losses.

* Example: A 50 kW, 440 volt, 50 Hz, 2 pole I.M. (Induction motor) has a slip = 6%, when operating at full load.

At full load, the friction and windage losses (i.e. Mechanical) are 520 watt, core losses are 500 watt.

Find the followings at full load.

I) Power in watts.

$$P_{\text{full}} = 50 \text{ kW.}$$

II) Load torque.

$$P = \gamma \omega_m \quad \dots \quad (1)$$

$$\omega_m = \omega_{\text{syn.}} (1 - s) \quad \dots \quad (2)$$

$$\omega_{\text{syn.}} = 2\pi n_{\text{syn.}} = 2\pi \frac{f}{P} = \frac{2\pi (50)}{2} \quad \dots \quad (3)$$

(\rightarrow No. of pole pairs.

by substitute (2) and (3) into (1)

$$\rightarrow \gamma_{\text{load}} = 169.4 \text{ N.m}$$

III) Find $\gamma_{\text{max.}}$.

$$P_{\text{core}} = \gamma_{\text{max.}} * \omega_m \quad \dots \quad (4)$$

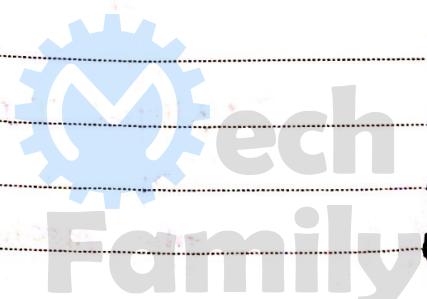
$$P_{\text{core}} = P_{\text{full}} + P_{\text{mech.}} + P_{\text{core}}$$

$$= (50 + 0.52 + 0.5) \text{ kW}$$

$$= 51.02 \quad \dots \quad (5)$$

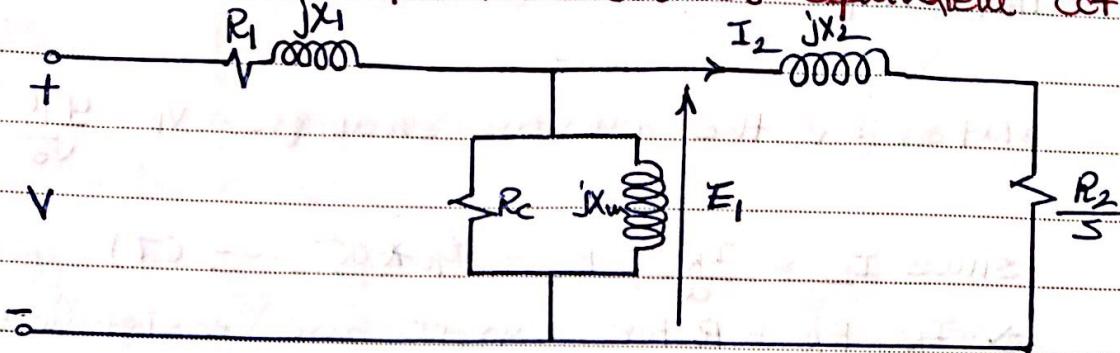
Substitute (4) into (5)

$$\gamma_{\text{max.}} = 172.9 \text{ N.m.}$$



• Power and torque relationship

Consider the previous deduced equivalent circuit.



per phase circuit

I_2 = Secondary (i.e. Rotor) current reflected to primary.

X_2, R_2 = Secondary Resistance and Reactance reflected to primary.

I_1 = Primary current.

$\frac{R_2}{s}$ = Resistance which consumes P_{ag} .

$$\therefore Z_1 = \frac{V_1}{I_1} = R_1 + jX_1 + \frac{1}{\frac{1}{R_c} + \frac{1}{jX_m} + \frac{1}{\frac{R_2}{s} + jX_2}}$$

$$\therefore I_1 = \frac{V_1}{Z_1} \quad \text{--- (1)}$$

* Note: $\theta = \angle Z$, V_1 = Applied phase voltage.

$$Z_1 = Z_{eq.} \quad \text{--- (2)}$$

$$P_{sL} = \frac{3 I_1^2 R_1}{s} \quad \text{--- (3)}$$

stator core losses.

$$P_{core} = (3) \frac{E_1^2}{R_c} \quad \text{--- (4)}$$

per phase circuit.

$$P_{AG} = 3 I_2^2 \frac{R_2}{s} \quad \dots \dots (5)$$

$$P_{AG} = P_{Inp.} - P_{RCL} - P_{Conv.} \quad \dots \dots (6)$$

*Note! In the previous example ($V_1 = \frac{440}{\sqrt{3}}$).

$$\text{since } I_2 = \frac{I_R}{a}; R_2 = R_R * a^2 \quad \dots \dots (7)$$

$\Rightarrow I_R, R_R =$ Rotor current and resistance.

$a \equiv$ Effective turns ratio.

Substitute (7) into (5).

$$P_{AG} = 3 \left(\frac{I_R^2}{a^2} \right) + R_R a^2 \frac{1}{s}$$

$$P_{AG} = 3 I_R^2 R_R + \frac{1}{s} \quad \dots \dots (8)$$

From these equations, it can be found that

$$P_{RCL} = s P_{AG} \quad \dots \dots (9)$$

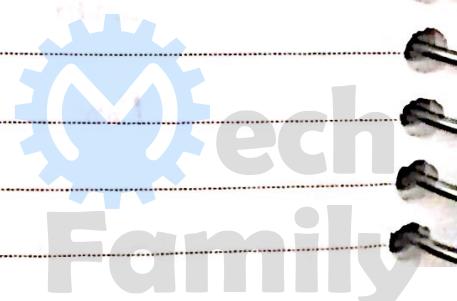
$$\text{since } P_{Conv.} = P_{AG} - P_{RCL} \quad \dots \dots (10)$$

Substitute (9) into (10).

$$\begin{aligned} P_{Conv.} &= P_{AG} - s P_{AG} \\ &= (1-s) P_{AG} \end{aligned}$$

$$\text{since } P_{Conv.} = \gamma_{ind.} \cdot \omega_m$$

$$\therefore \gamma_{ind.} = \frac{P_{Conv.}}{\omega_m} = \frac{(1-s) P_{AG}}{\omega_m}$$



- Conclusion:

$$P_{\text{conv.}} = \gamma_{\text{ind.}} + \omega_m$$

$$\gamma_{\text{ind.}} = \frac{P_{\text{conv.}}}{\omega_m}$$

$$P_{\text{conv.}} = P_{\text{AG}} - P_{\text{CRL}} = P_{\text{AG}} - s P_{\text{AG}}$$

$$= P_{\text{AG}} (1 - s)$$

$$\text{Since } \omega_m = \omega_{\text{syn.}} (1 - s)$$

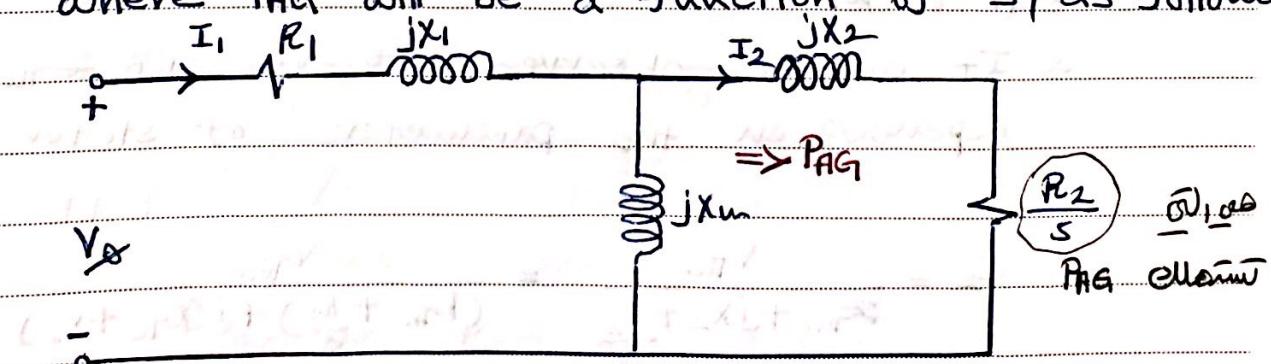
$$\therefore \gamma_{\text{ind.}} = \frac{P_{\text{AG}} (1 - s)}{\omega_{\text{syn.}} (1 - s)} = \frac{P_{\text{AG}}}{\omega_{\text{syn.}}} \quad \text{--- (1)}$$

∴ Advantage of (1) is that for a given f and No. of poles of stator winding then, $\omega_{\text{syn.}}$ is const.

$$\therefore \gamma_{\text{ind.}} \propto P_{\text{AG}}$$

* Torque vs. arm or η_m :

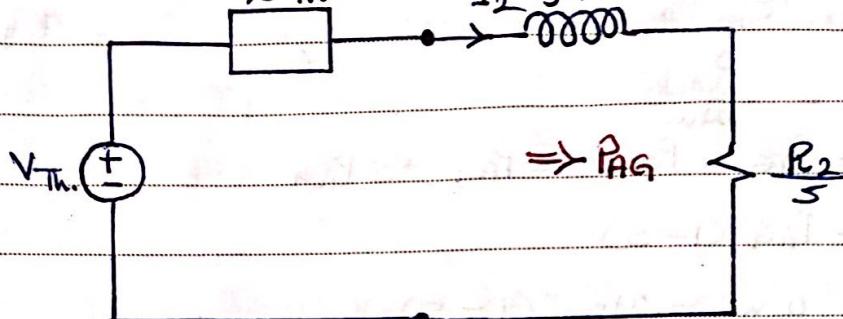
* Since $\gamma_{\text{ind.}} = \frac{P_{\text{AG}}}{\omega_{\text{syn.}}}$, Then P_{AG} is going to be derived by using a simplified equivalent circuit, where P_{AG} will be a function of s , as follows!



$$\gamma_{\text{ind.}} = \frac{P_{\text{AG}}}{\omega_{\text{syn.}}} \quad \text{--- (1)}$$

$$P_{\text{AG}} = \frac{3|I_2|^2 R_2}{s} \quad \text{--- (2)}$$

I_2 can be found by using Théor. eqn. cct.



$$\therefore V_{Th.} = V_2 \cdot \frac{jX_m}{R_1 + jX_1 + jX_m} \quad \therefore |V_{Th.}| = \frac{V_2 X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}$$

$$Z_{Th.} = jX_m \parallel (R_1 + jX_1)$$

$$\therefore Z_{Th.} = R_{Th.} + jX_{Th.}$$

by using the approximation where $X_m \gg X_1$ and $X_m \gg R_1$. It will be found that:

$$R_{Th.} \approx R_1 \left(\frac{X_m}{X_1 + X_m} \right)^2$$

$$X_{Th.} \approx X_1$$

∴ It can be observed that $V_{Th.}$ and $Z_{Th.}$ depending on the parameters of stator cct.

$$\therefore I_2 = \frac{V_{Th.}}{Z_{Th.} + jX_2 + \frac{R_2}{s}} = \frac{V_{Th.}}{(R_{Th.} + \frac{R_2}{s}) + j(X_{Th.} + X_2)}$$

$$\therefore |I_2| = \frac{|V_{Th.}|}{\sqrt{(R_{Th.} + \frac{R_2}{s})^2 + (X_{Th.} + X_2)^2}} \quad \text{--- (3)}$$

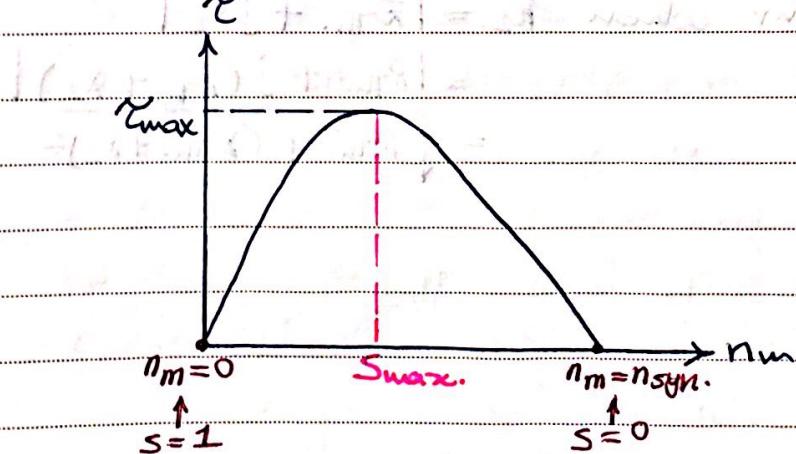
* Sub. (3) into (2) to give PAG as a function of $r_{th.}$, $R_{th.}$ and $x_{th.}$. Let this be equation (4).

* Then sub. (4) into (1) to give the following general expression!

$$\therefore \chi_{ind.} = 3 \frac{|r_{th.}|^2 + \frac{R_2}{s}}{\omega_{syn.} \left[(R_{th.} + \frac{R_2}{s})^2 + (x_{th.} + x_2)^2 \right]} \quad \dots \dots (5)$$

* Comments:

I) equation (5) gives the relationship between $\chi_{ind.}$ vs. n_m or ω_m , since $s = \frac{n_{syn.} - n_m}{n_{syn.}}$.



$\Rightarrow \chi_{max.} = \text{Max. value of tongue and it's called (Pull-out tongue).}$

II) One can find variation of tongue (χ) as (s) vary.

III) For a given slip (s), it can be found that $\gamma_{ind.}$ can be controlled by varying $V_{th.}$ (i.e. Applied voltage) or Reactance at the motor.

$\Rightarrow s_{max.} \equiv$ value of (s) when ($\gamma = \gamma_{max.}$)

IV). Evaluation of $s_{max.}$

$$\text{Since } \gamma_{ind.} = \frac{P_{AG}}{\omega_{syn.}}$$

$\therefore \gamma_{max.}$ is when P_{AG} is max.

According to the max. power theorem.

This occur when $R_2 = |\bar{Z}_{th.} + jX_2|$

$$\begin{aligned} &= |R_{th.} + j(X_{th.} + X_2)| \\ &= \sqrt{R_{th.}^2 + (X_{th.} + X_2)^2} \end{aligned}$$

* $\gamma_{max.} \Rightarrow P_{avg. max.} \Rightarrow$ when power supplied to R_2 is max. For max. power $\frac{R_2}{s} = \sqrt{R_{Th.}^2 + (X_{Th.} + X_2)^2}$

$$\therefore S_{max.} = \frac{R_2}{\sqrt{R_{Th.}^2 + (X_{Th.} + X_2)^2}} \quad \text{--- (1)}$$

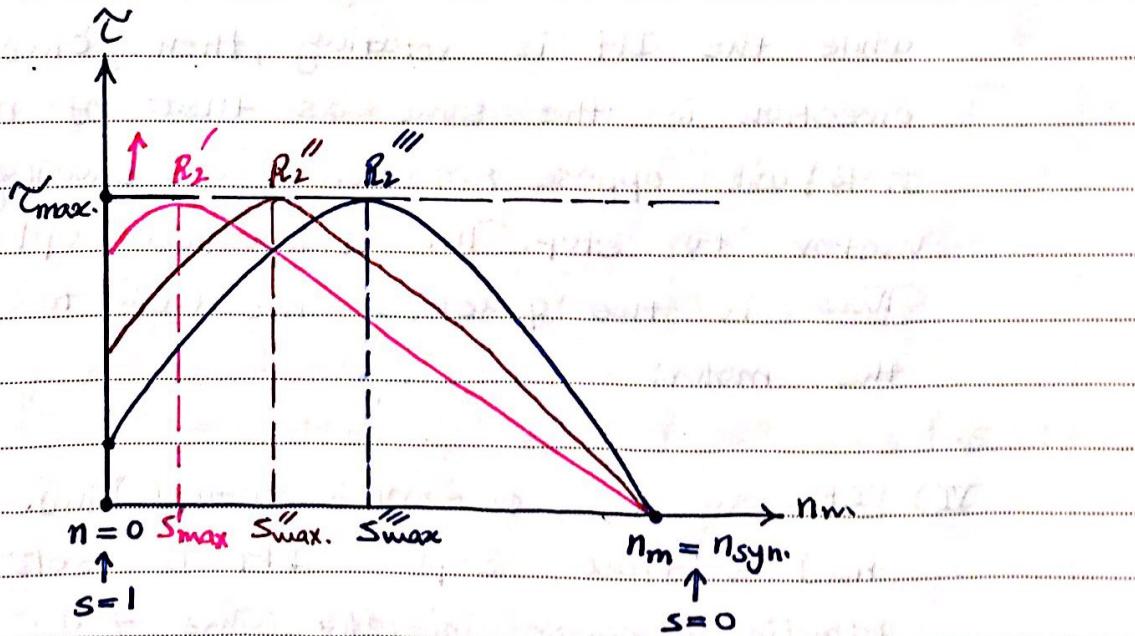
If $S_{max.}$ is substituted in the expression of $\gamma_{ind.}$, it can be found:

$$\gamma_{max.} = \frac{3V_{Th.}^2}{2\omega_{syn.} [\sqrt{R_{Th.}^2 + (X_{Th.} + X_2)^2} + R_{Th.}]} \quad \text{--- (2)}$$

• **Comments:**

I) $S_{max.}$ depends on R_2 but torque $\gamma_{max.}$ doesn't depend on R_2 .

∴ $S_{max.}$ can determine the location of $\gamma_{max.}$



$$\therefore R_2' > R_2'' > R_2'''$$

• n_{max} must not be >= 1/2 n_{max} in order for it to be

II) :- For a given input voltage and stator parameters, γ_{max} is constant.

III) By varying R_L , one can change the location of γ_{max} .

IV) This is the advantage of wound type IM. Is that γ_{max} can occur at low speed. Therefore the characteristics can be used to derive heavy loads. As the motor rotates then (R_L) is decreased, until n_m reaches near n_{syn} .

V) If the rotation of magnetic field is reversed while the IM is rotating, then γ_{ind} (whose direction is the same as that of magnetic field) will oppose rotation and causing the motor to stop. This is called (plugging). This is the quickest method to stop the motor.

VI) It can be observed from (γ_{ind} vs. n_m) is that 3-ph. IM is self starting. Because initially $\gamma_{ind} \neq 0$.

- squirrel cage, no brushes or slip rings, so, maintenance cost is very low.
- wound type, it can be used to operate at heavy loads.

Line Voltage

*Ex.: A 208 V, 4 poles, 60 Hz, Y connected rotor IM.

Rated at 15 hp. Its equivalent circuit has:

$$R_1 = 0.22 \Omega$$

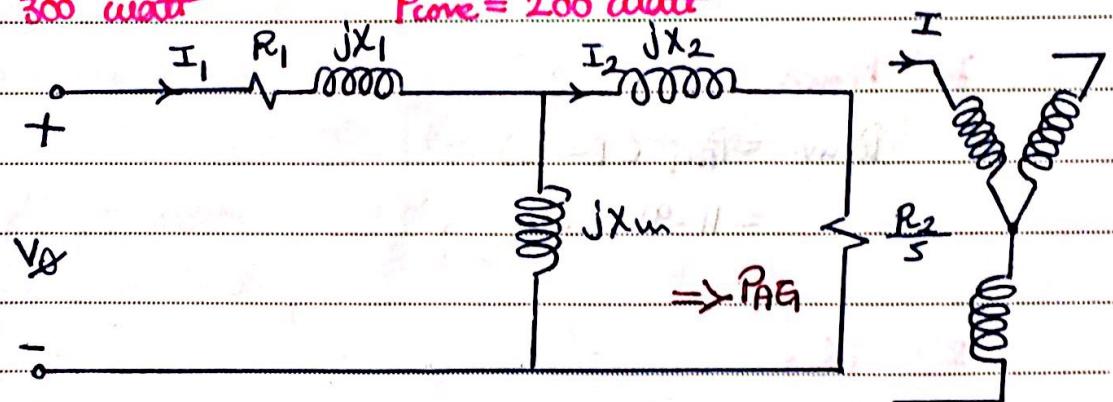
$$R_2 = 0.127 \Omega, X_m = 15 \Omega$$

$$X_1 = 0.43 \Omega$$

$$X_2 = 0.43 \Omega$$

$$P_{\text{mech.}} = 300 \text{ watt}$$

$$P_{\text{cone}} = 200 \text{ watt}$$



per phase acc.

For a slip $s = 0.05$, find the following:

I) The line current. (stator)

Find $I_1 = ?$

$$I_1 = V_s / Z_{\text{eq.}} ; V_s = 208 \angle 0^\circ \text{ (Ref. to stator)}$$

$$Z_{\text{eq.}} = R_1 + jX_1 + (jX_m \parallel (R_2 \frac{\sqrt{3}}{s} + jX_2))$$

$$= 2.84 \angle 25.7^\circ \Omega$$

$$I_1 = 42.28 \angle -25.7^\circ \text{ A}$$

II) P_{SCL}

$$P_{SCL} = 3 |I_1|^2 R_1 = 1179.8 \text{ watt.}$$

III) P_{AG}

$$P_{AG} = 3 |I_2|^2 \frac{R_2}{s}$$

$$I_2 = I_1 * \frac{jX_m}{jX_m + jX_2 + \frac{R_2}{s}}$$

$$I_2 = 40.65 / -16.4 \text{ A}$$

$$P_{AG} = 12.59 \text{ kW.}$$

IV) $P_{conv.}$

$$P_{conv.} = P_{AG} (1 - s)$$

$$= 11.91 \text{ kW.}$$

V) $\gamma_{ind.}$

$$\gamma_{ind.} = \frac{P_{conv.}}{\omega_m}; \quad \omega_m = \omega_{syn.} (1 - s)$$

$$\omega_{syn.} = 2\pi n_s = 2\pi \frac{f}{P} = 2\pi \frac{(60)}{2} = 60\pi$$

$$\gamma_{ind.} = 66.5 \text{ N.m.}$$

VI) γ_{load}

$$\gamma_{load} = \frac{P_{out}}{\omega_m}; \quad P_{out} = P_m - \text{losses.}$$

$$\gamma = \frac{P_{out}}{P_{in}}$$

$$P_m = 3 |Y_0| |I_1| \cos \angle I_1 \rightarrow (I_1, \gamma_{load})$$

OR

$$P_{out} = P_{conv.} - P_{mech.} - P_{core}$$

$$= 11.41 \text{ kW}; \quad \therefore \gamma_{load} = 63.75 \text{ N.m}$$

III) Find slip at pull-out torque and the value of this torque?

$$S_{\max.} = \frac{R_2}{\sqrt{R_{\text{Th.}}^2 + (X_{\text{Th.}} + X_2)^2}} ; R_{\text{Th.}} = R_1 \left(\frac{X_m}{X_1 + X_m} \right)^2$$

$$X_{\text{Th.}} = X_1$$

* by substitution:

$$R_{\text{Th.}} = 0.2 \Omega$$

$$X_{\text{Th.}} = 0.43 \Omega$$

$$\Rightarrow S_{\max.} = 0.144$$

$$\gamma_{\text{pull out}} = \gamma_{\max.} = \frac{3V_{\text{Th.}}^2}{2w_{\text{syn.}} [R_{\text{Th.}} + \sqrt{R_{\text{Th.}}^2 + (X_{\text{Th.}} + X_2)^2}]} \quad \text{Industry} \rightarrow 3\text{-ph is supplied.}$$

* by substitution:

$$\Rightarrow \gamma_{\max.} = 100 \text{ N.m.}$$

* Single-phase Induction Motor: Industry \rightarrow 3-ph is supplied.

Since 1-ph. voltage is distributed to residential section, then most domestic appliances use 1-ph. I.M.

- construction!

As any machine, it consists of:

I) stator

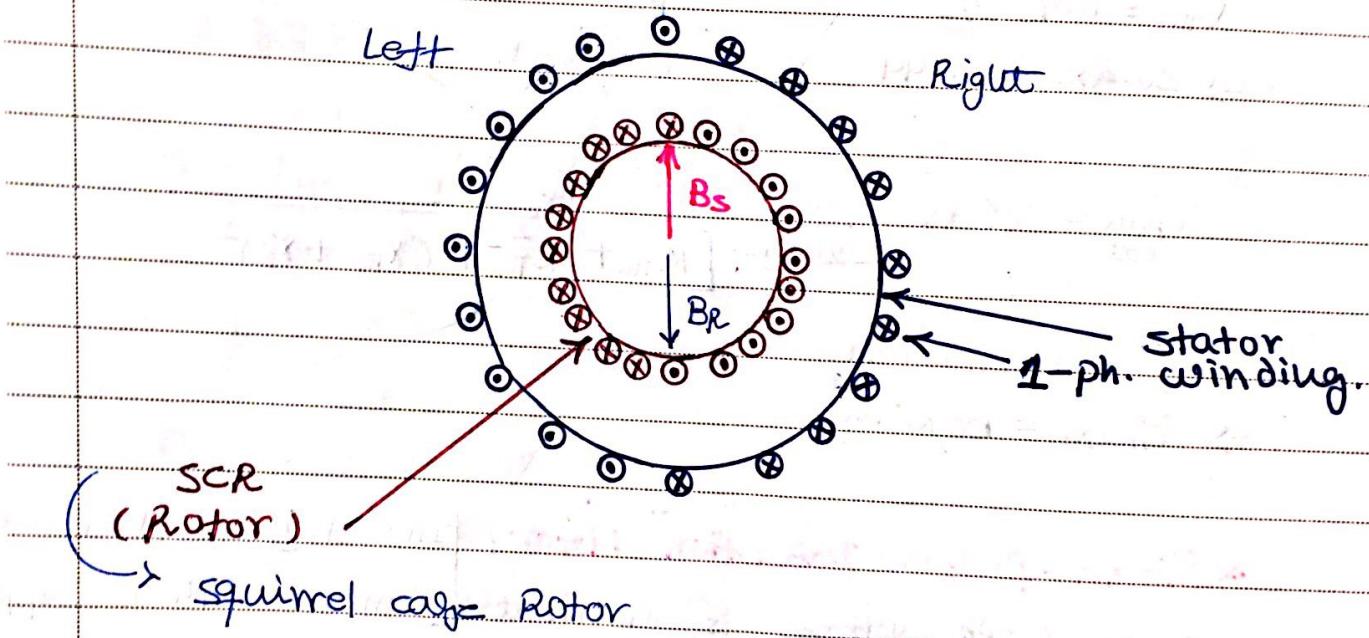
II) Rotor.

I) Stator:

It consists of a distributed 1-ph. winding into which 1-ph. voltage is applied, and consequently a 1-ph. current flow.

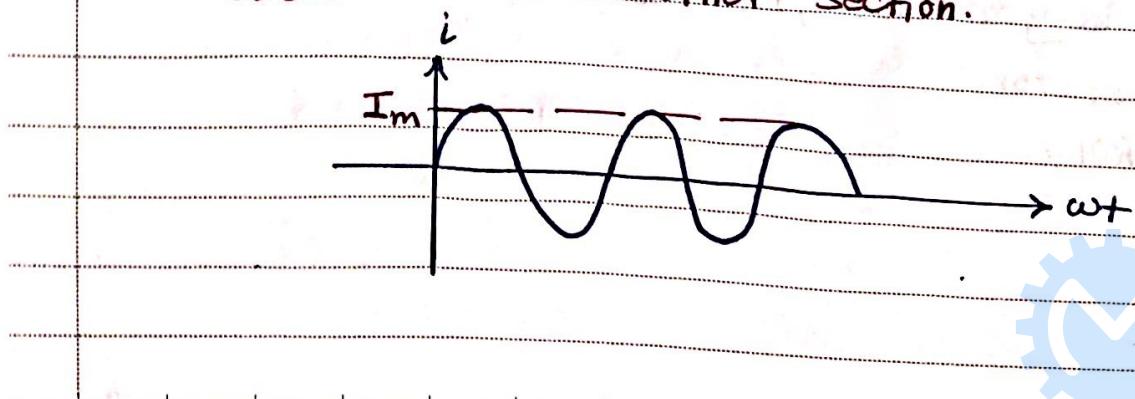
II) Rotor:

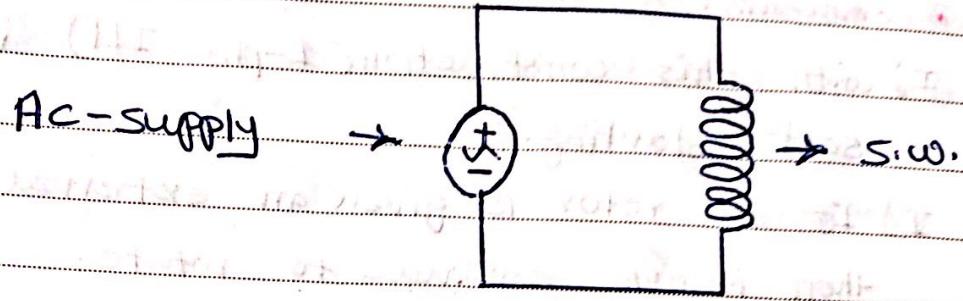
It is a squirrel cage rotor.



• Operation:

when a current is applied to s.w., then it will enter from a certain section and leave from the other section.





→ i_s is going to generate a flux Φ_s or B_s , by RHR direction of B_s will be along the vertical axis. It's magnitude vary in sinusoidal manner, since $(B_s \propto i_s)$. $\therefore B_s$ is pulsating.



* Since Φ_s is pulsating (i.e. $\frac{d\Phi_s}{dt} \neq 0$) then a voltage is going to be induced in the rotor, causing i_R to flow. i_R will generate Φ_R .

By (Lenz's law) $\Rightarrow \Phi_R$ opposes Φ_s .

$$\begin{aligned}
 \text{E}_{\text{developed}} &\triangleq k (B_R \cdot B_s) \quad \xrightarrow{\text{Dot product}} \\
 \text{induced} &= k |B_R||B_s| \sin(\angle B_R - \angle B_s) \quad \xrightarrow{\text{180}^\circ} \\
 &\quad \sin(180) = 0 \quad \xrightarrow{\Phi_R \text{ or } B_R}
 \end{aligned}$$

$$= 0.$$

• **Comments:**

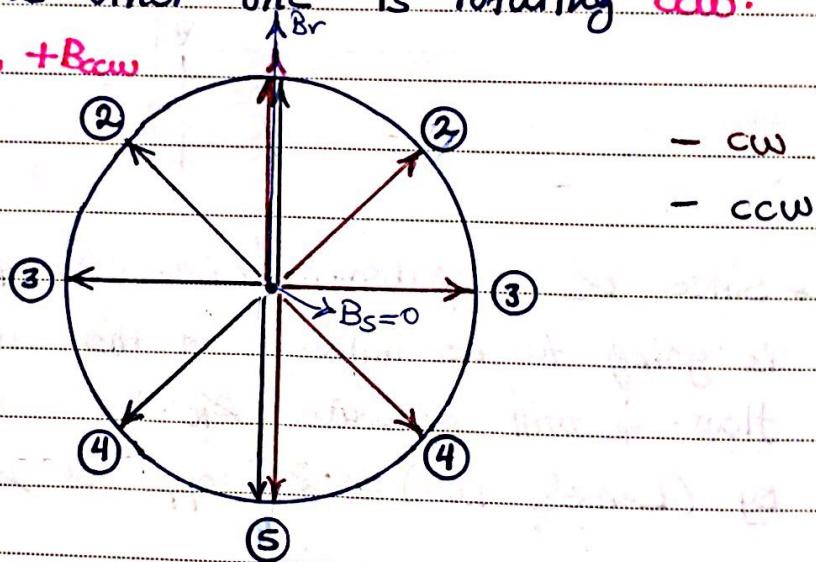
I) with this construction (1-ph. IM) is not a self starting.

II) If the rotor is given an external torque, then it will continue to rotate.

[This will be explained by Two-Revolving field theory.]

III) Pulsating (B_s) can be represented by 2-equal rotating magnetic fields; one is rotating cw and the other one is rotating ccw.

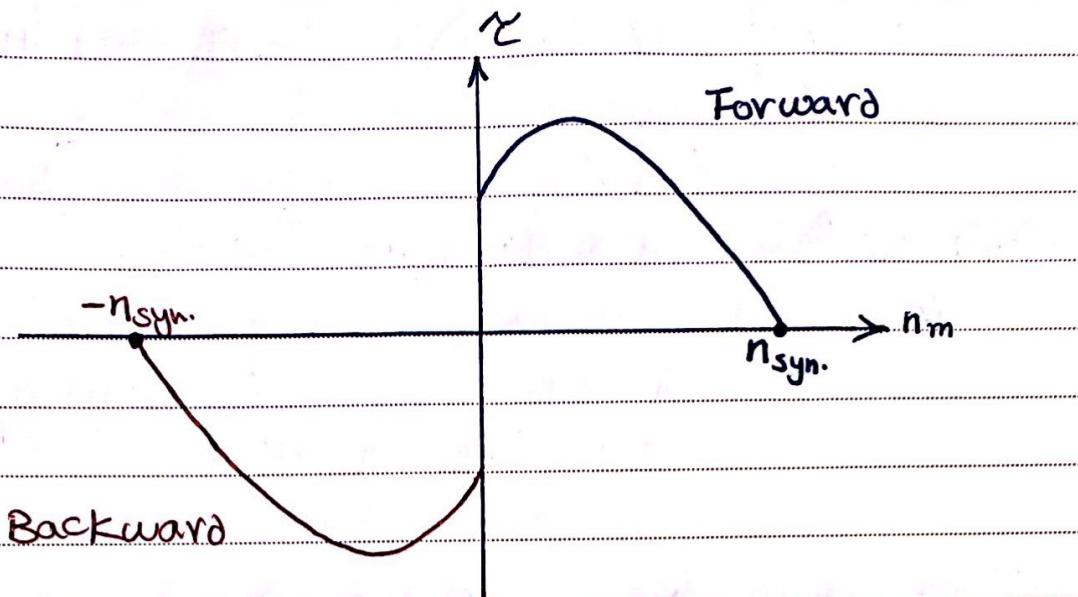
$$B_s = B_{cw} + B_{ccw}$$



* Hence, when each rotating magnetic field complete one rotation, then B_s complete one cycle (+ve max. $\rightarrow 0 \rightarrow -ve$ max $\rightarrow 0$).

cw is called forward]
ccw is called Backward]

* Each rotating field produce its torque the same as the case of the 3-ph. I.M., as follows:



$$\Sigma_{1\text{-ph. IM}} = \Sigma_B + \Sigma_F$$

* Motor: Device converts from electrical to mechanical.

* Rotor: The rotating part.

• Comment:

→ To find I_{start} for 3-ph. I.M., then let $s=1$, find Z_{eq} . then calculate I_1 .

→ For the previous example, it can be found that $|Z_{\text{eq}}| = 0.9\Omega$

$$\therefore |I_{\text{start}}| = \frac{|V_{\phi}|}{|Z_{\text{eq}}|} = \frac{208}{0.9}$$

\Rightarrow It can be found in general, $[I_{\text{start}} = (6-8) \text{ times } I_{\text{steady state}}]$.

* Starting methods of 1-ph. I.M.:

I) Mechanical method: A rope is used to pull the shaft.

This method is usually used to test if there is a defect in the other methods.

II) Electrical method: There are various methods, among which is the so called (split-phase method), explained as follows:

* Rotating field by two-phase system:

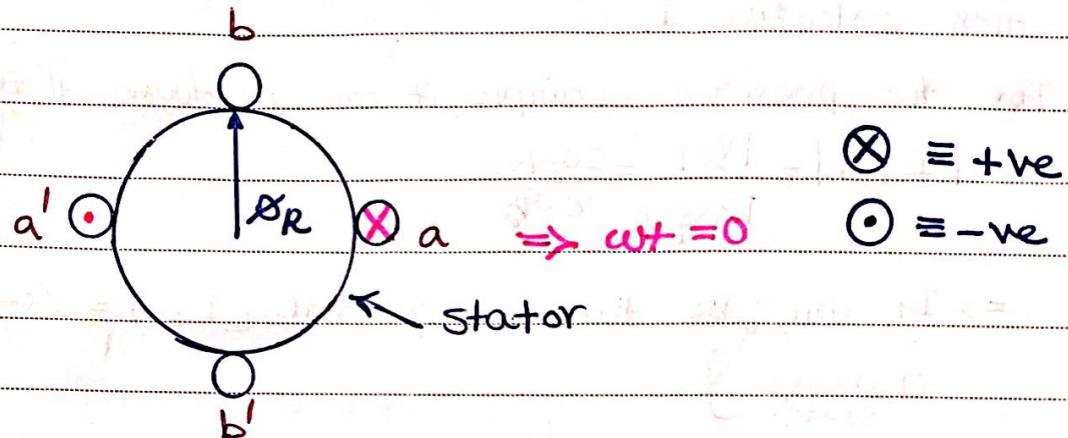
Two coils (a, a') and (b, b') are located on the stator with 90° between them.

They are supplied by currents with 90° phase shift.

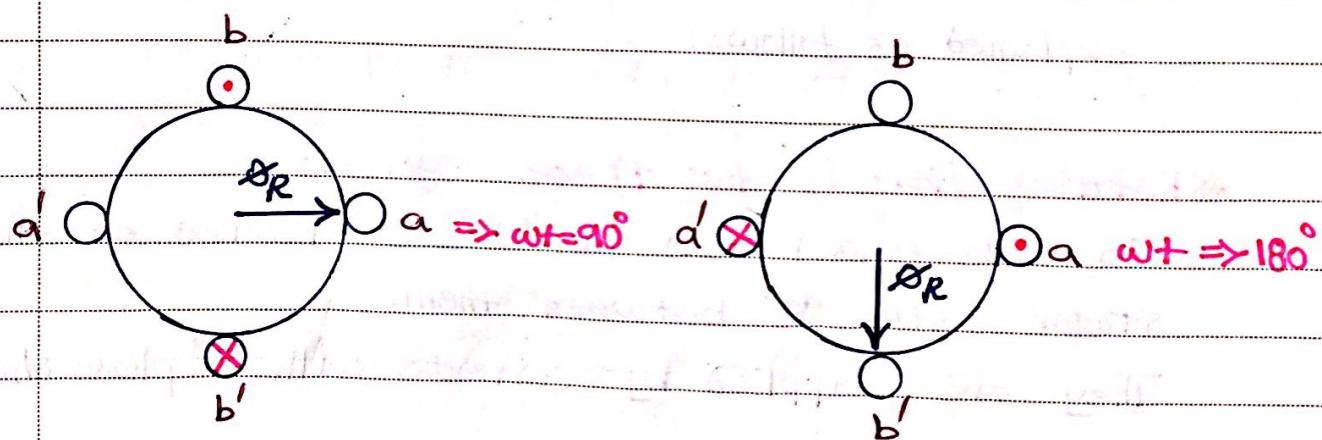
$$\text{If } i_{aa'} = I_m \cos(\omega t) \Rightarrow I_{aa'} = I_m \angle 0^\circ$$

$$\therefore i_{bb'} = I_m \cos(\omega t + 90^\circ) \Rightarrow I_{bb'} = I_m \angle 90^\circ$$

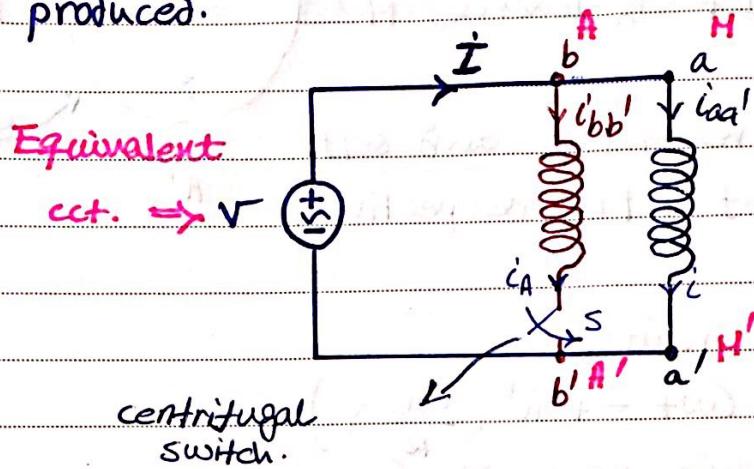
* Since ($\theta \propto i$), let us see what will happen when (ωt) change over one cycle.



| ωt | $i_{aa'}$ | $i_{bb'}$ | $\theta_R = \theta_{aa'} + \theta_{bb'}$ |
|-------------|-----------|-----------|--|
| 0 | +ve | 0 | ↑ |
| 90° | 0 | -ve | → |
| 180° | -ve | 0 | ↓ |
| 270° | 0 | | ← |
| 360° | | | ↑ |



1. Hence, with this two phase system, a Rotating magnetic field with constant magnitude is produced.



2. If the currents are not equal in magnitude and/or phase shift $\neq 90^\circ$.

Still a rotating magnetic field is produced but with varying magnitude.

3. with this construction and assumption ω_R rotate cw.

If one of the coils connection is reversed, then the rotation of ω_R will be reversed.

*Split-phase method:

Stator 1 consists of 2 coils! -

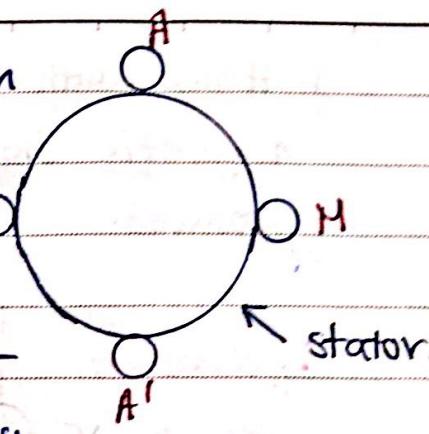
1) one is called Main winding.

2) other is called Auxilliary winding.

A is connected in series with a centrifugal switch (s).

* The coils are selected in such away that $\frac{R_A}{L_A} > \frac{R_M}{L_M}$ in order to let i_A leads i_M where;

$(R \& L)$ are resistance and self inductance of $A \& M$ respectively.

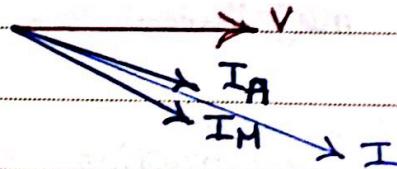


\Rightarrow For RL circuit:

$$i = I_M \cos(\omega t - \tan^{-1} \left(\frac{\omega L}{R} \right))$$

* After certain speed \Rightarrow (s) is open and only (M) winding is kept in the cct.

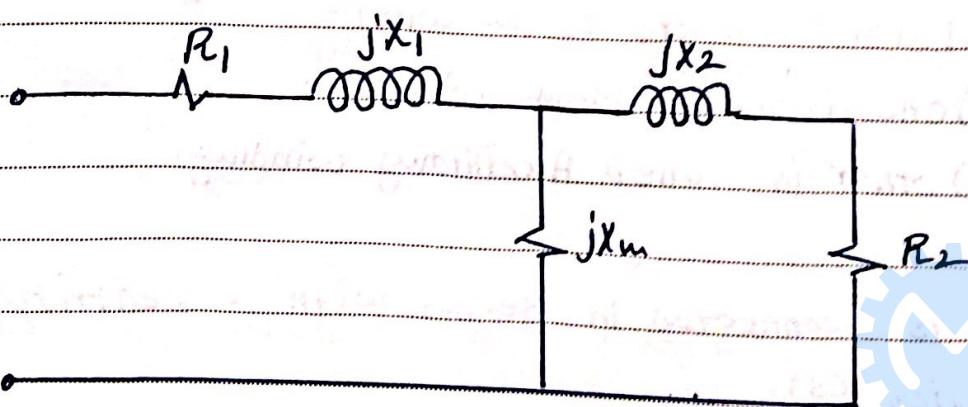
• ~~at~~ \Rightarrow ~~by~~ \Rightarrow ~~is not~~ \Rightarrow ~~Switch~~



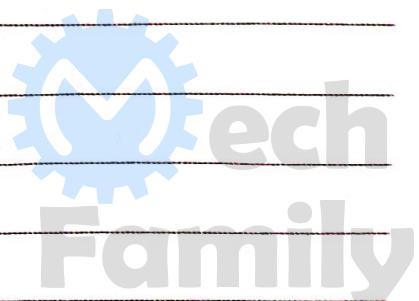
* Equivalent cct. of 1-ph. induction motor:

let initially the rotor is stalled (i.e. blocked, not rotating).

\therefore The motor is \equiv 1-ph. transformer.



* when a voltage is applied to (M) main winding, then a pulsating field is produced. This field is represented by two oppositely rotating fields.



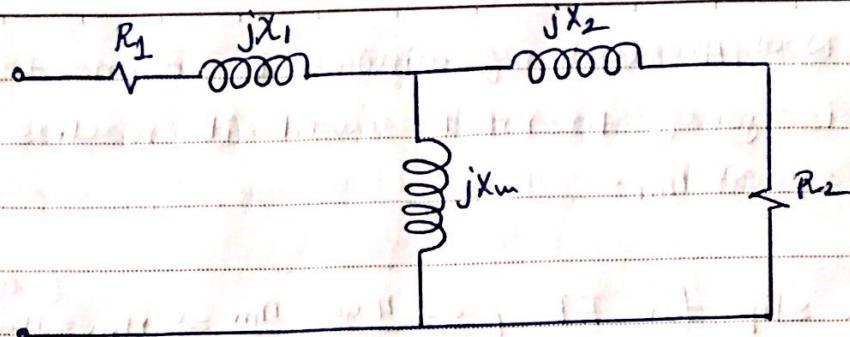


Fig. 1

* Fig. 1 = without Aw, ψ_b is applied to stator \rightarrow Pulsating field (PF) is produced.

* PF is represented by 2 revolving magnetic fields (i.e. FF \Rightarrow Forward field and BF \Rightarrow Backward field).

* Each FF and BF induces equal voltage in the Rotor cct.

* Hence Fig. 1 is modified as follows as shown in Fig. 2:

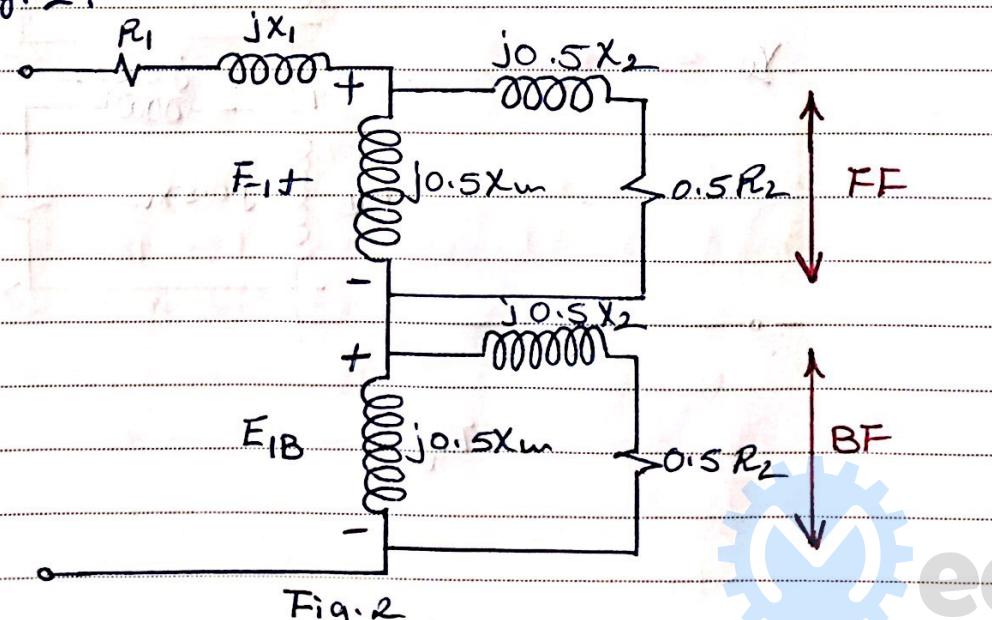


Fig. 2

+ Now, Auxiliary winding is connected \Rightarrow Motor is going to rotate until it reaches certain speed n_m .

$$\therefore \text{slip for FF, } s = \frac{n_{\text{syn.}} - n_m}{n_{\text{syn.}}} \Rightarrow n_m = n_{\text{syn.}}(1-s) \quad \text{---(1)}$$

$$\therefore \text{slip for BF, } s_1 = \frac{-n_{\text{syn.}} - n_m}{-n_{\text{syn.}}} \Rightarrow -s_1(-n_{\text{syn.}}) = -n_{\text{syn.}} - n_m \quad \text{---(2)}$$

$$n_m = n_{\text{syn.}}(s_1 - 1)$$

* By equating (1) and (2),

$$n_{\text{syn.}}(1-s) = n_{\text{syn.}}(s_1 - 1)$$

$$\therefore s_1 = 2-s \quad \text{---(3)}$$

\therefore Fig. 2 to be modified by introducing slip.

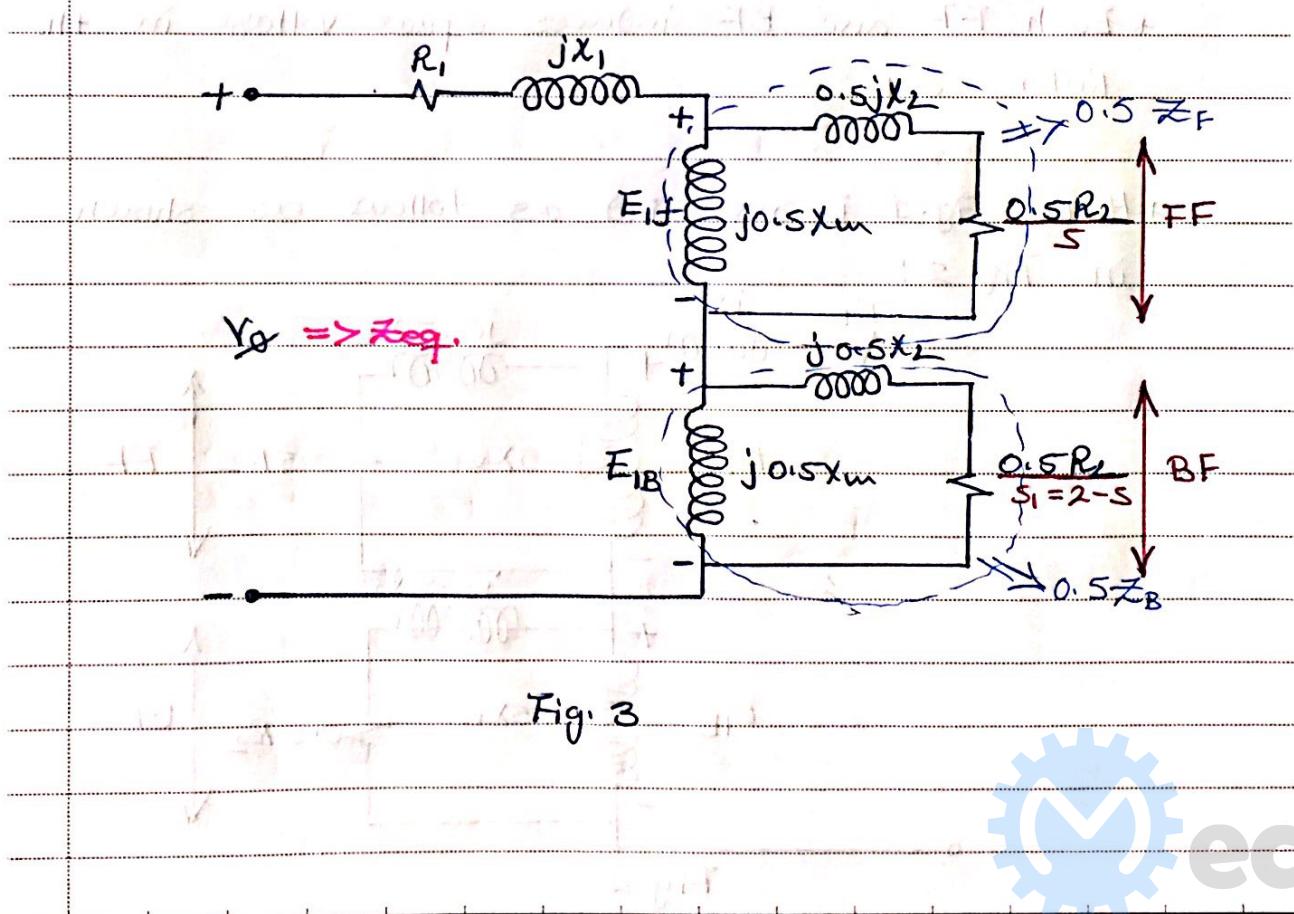


Fig. 3

∴ Fig. 3 equivalent circ. of single phase IM.

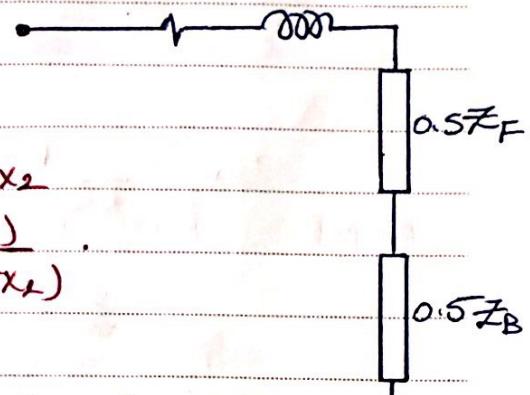
This circ is used in the analysis of this motor as follows:

$$Z_{eq.} = \frac{V_o}{I}$$

$$Z_{eq.} = R_1 + jX_1 + 0.5Z_F + 0.5Z_B$$

$$0.5Z_F = j0.5X_m // \frac{0.5R_2}{s} + j0.5X_2$$

$$= \frac{j0.5X_m \left(\frac{0.5R_2}{s} + j0.5X_2 \right)}{j0.5X_m + \left(\frac{0.5R_2}{s} + j0.5X_2 \right)}$$



* similarly Z_B can be found.

∴ where one can write $Z_F = R_F + jX_F$

$$Z_B = R_B + jX_B$$

* Since, each rotating field (i.e. FF and BF) has its own power flow diagram.

$$\therefore P_{AG} = \underbrace{P_{AG, FF}}_{\frac{P_{supplied}}{s}} - \underbrace{P_{AG, BF}}_{\frac{P_{supplied}}{2-s}}$$

$$\left(\frac{0.5R_2}{s} \right) \quad \left(\frac{0.5R_2}{2-s} \right)$$

∴ The rest of variables can be found as in 3-ph. I.M i-

$$P_{out} = P_{AG} (1-s)$$

$$P_{out} = \gamma_{ind.} * \omega_m$$

NO. 21. 4. 2018

* Ex. 1 A 120 V, $\frac{1}{3}$ hp, 60 Hz, 4 pole, split phase IM has the following parameters:

$$R_1 = 2 \Omega \quad X_1 = 2.56 \Omega \quad X_m = 60.5 \Omega$$

$$R_2 = 2.8 \Omega \quad X_2 = 2.56 \Omega$$

At $slip = 0.05$, Rotational mech. losses = 51 watt.

Find the following:

I. Input current:

$$I_1 = \frac{V_A}{R_1 + jX_1 + 0.5Z_f + 0.5Z_B}$$

by substitution it can be found that

$$Z_f = 28.81 + j28.05 \Omega$$

$$Z_B = 1.32 + j2.49 \Omega$$

$$\therefore I_1 = 4.86 / -46.26 A$$

II. Input power:

$$|V_A| |I_1| \cos(46.26) = 403.22 \text{ watt.}$$

III. PAG:

$$P_{AG} = P_{AG,F} - P_{AG,B}$$

$$= I_1^2 (0.5R_f) - I_1^2 (0.5R_B)$$

$$= 324.65 \text{ watt.}$$

IV. Power:

$$P_{com.} = (1 - s) P_{AG} = 308.42 \text{ watt}$$

\uparrow
0.05

V. P_{out} :

$$\begin{aligned}
 P_{out} &= P_{cav.} - P_{mech.} \\
 &= 308.42 - 51 \\
 &= 257.42 \text{ watt}
 \end{aligned}$$

VI. $\gamma_{ind.}$:

$$\gamma_{ind.} = \frac{P_{cav.}}{\omega_m = \omega_{syn.}(1-s)} = 1.72 \text{ N.m} \quad n = \omega \cdot s$$

$\omega = \text{rad/s}$
 $\equiv \text{Revolution}$
 per sec.

$$\frac{2\pi(60)(1-0.05)}{2}$$

VII. η :

$$\eta = \frac{P_{out}}{P_{in}} = 63.8\%$$

VIII. PF of the load:

$$\begin{aligned}
 \cos(\delta - \phi) &= \cos(0 + 46.26) \quad ; \text{Standard PF} \\
 &= 0.69 \angle 0.8 \quad (0.8 - 0.85) \\
 &\quad \text{v}
 \end{aligned}$$

*** Synchronous Machines:****• construction:**

I. stator: It consists of 3-ph. windings;

— In the case of generator: 3-ph. voltage is induced in them.

— In the case of motor: 3-ph. current is applied to them.

II. Rotor: It consists of two or more poles

"Electromagnet poles" into which DC current is applied to its field windings.

Through brushes and slip rings

- Introduction to 3-ph. syn. generator:

This is the type of generator which is used in conventional power stations (i.e. steam, hydro, gas -- etc.).

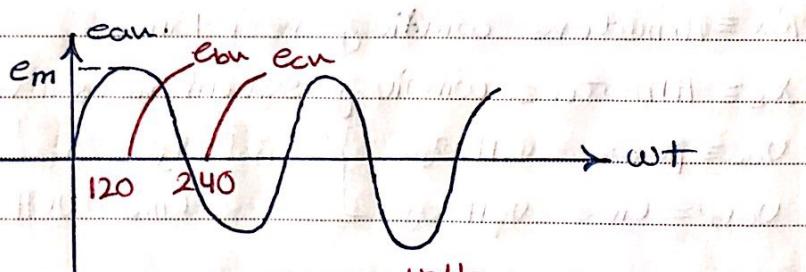
- Operation:

Rotor field windings produce sinusoidal flux $\Phi_f \Rightarrow \Phi_f$ cuts stator windings and induce voltage in them \Rightarrow given by $e_{au} = E_m \sin(\omega t)$

$$e_{bu} = E_m \sin(\omega t - 120)$$

$$e_{cu} = E_m \sin(\omega t - 240)$$

$E_m \equiv$ Peak of induced voltage.

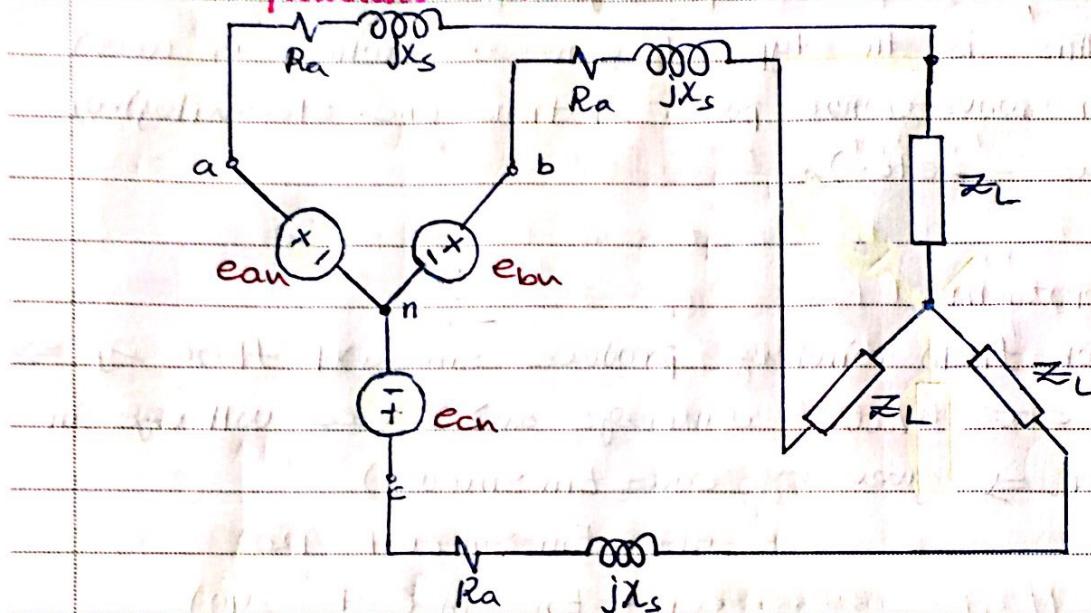


Since $E_{rms.} = \frac{E_m}{\sqrt{2}} \equiv V_{rms.}$

* For this type of generators, it can be found that $E_{rms} = 4.44 f N \Phi k_w$ ← winding factor.

frequency ↑
No. of turns ↑
Flux per pole ↑

* Equivalent circuit:



R_a ≡ Armature winding resistance.

X_s ≡ Armature winding synchronous reactance.

V_{an} ≡ phase voltage

V_{ab} ≡ line voltage

Line voltage

Line voltage

* Commercial generators generate voltage which could be in the range (11 - 25) kV

Line voltage

* 3-ph. synch. motor:

• Construction:

1. A balanced 3-ph. voltage is applied to the stator and generating a rotating magnetic field, B_s
2. A DC-current is applied to the rotor windings, producing magnetic field, B_r .

• Operation:

Due to B_r and B_s , a torque is developed or induced:

$$\tau_{ind.} \triangleq K (B_r * B_s) \\ = K |B_r| |B_s| \sin (\angle B_r - \angle B_s).$$

Direction of $\tau_{ind.}$ can be found by using RHR as follows:

Thumb \equiv Direction of B_r .

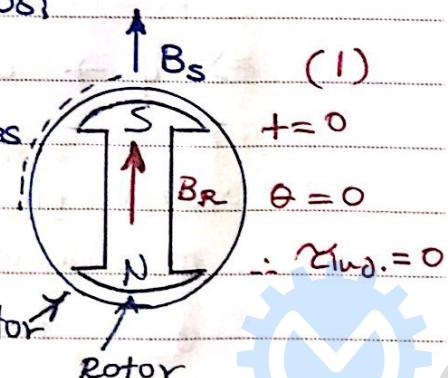
Fore \equiv Direction of B_s .

Middle \equiv Direction of $\tau_{ind.}$

* It can be shown that this motor is not self starting as follows:

\Rightarrow If the stator has two poles, windings:

\Rightarrow If the applied voltage has $f = 50 \text{ Hz}$.



$\therefore B_s$ is going to rotate at speed $n = \frac{f}{p} = 50 \text{ rps}$
 $= 2000 \text{ rpm.}$

(2)



$$t = \frac{1}{4} T \quad \rightarrow \text{periodic time.}$$

$$\theta = 90^\circ \text{ (or } -90^\circ)$$

$$\therefore \gamma_{\text{ind.}} \equiv \text{ccw}$$

(3)

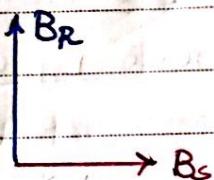


$$t = \frac{1}{2} T$$

$$\theta = 180^\circ$$

$$\therefore \gamma_{\text{ind.}} = 0$$

(4)



$$t = \frac{3}{4} T$$

$$\theta = -90^\circ$$

$$\therefore \gamma_{\text{ind.}} \equiv \text{cw}$$

(5) $t = T$

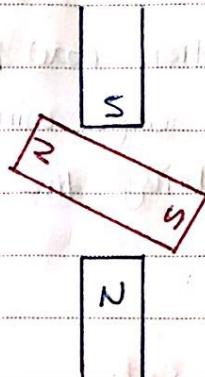
$$\theta = 0^\circ$$

$$\therefore \gamma_{\text{ind.}} = 0$$

$$\therefore 0 \rightarrow \text{ccw} \rightarrow 0 \rightarrow \text{cw} \rightarrow 0 \text{ (i.e. one cycle)}$$

\therefore over one cycle of rotation at B_s , $\gamma_{\text{ind.}}$ once ccw and then cw, hence Rotor is going to vibrate.

∴ Hence a starting mechanism is needed to start the motor.



* Methods of starting:

1. use a frequency converter that is applying stator voltage with low frequency, in order to give the rotor the chance to chase B_s .

* when the motor starts to rotate, increase the frequency.

2. Use an external prime mover to rotate rotor, when the rotor reaches near n_{syn} , then switch on DC supply at rotor.

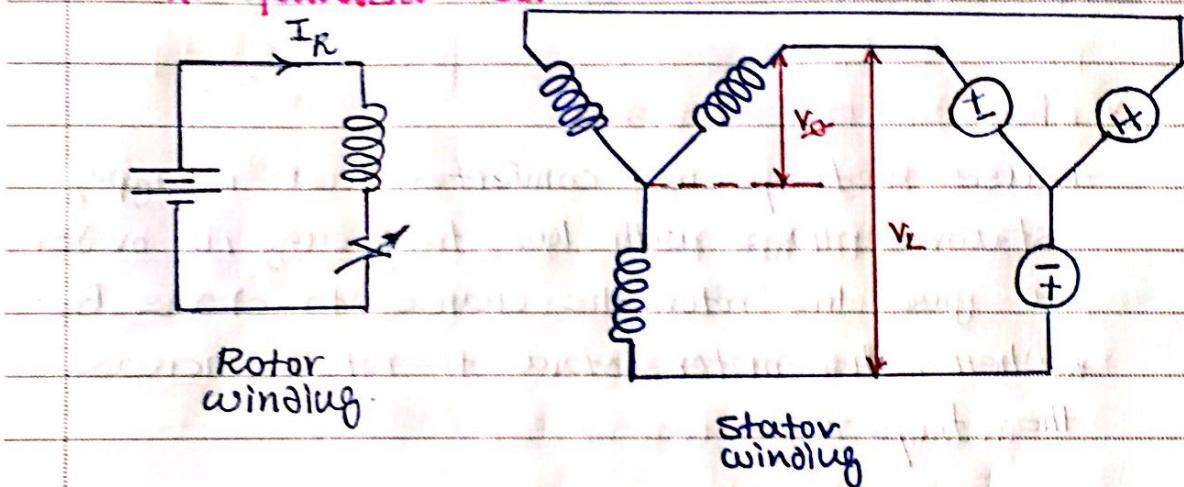
3. Start the motor as IM, that is by adding a 3rd winding (i.e. squirrel cage winding) on the shaft of the motor.

Hence the motor is going to start as IM and the rotor is going to follow B_s , until it reaches the speed at n_{syn} .

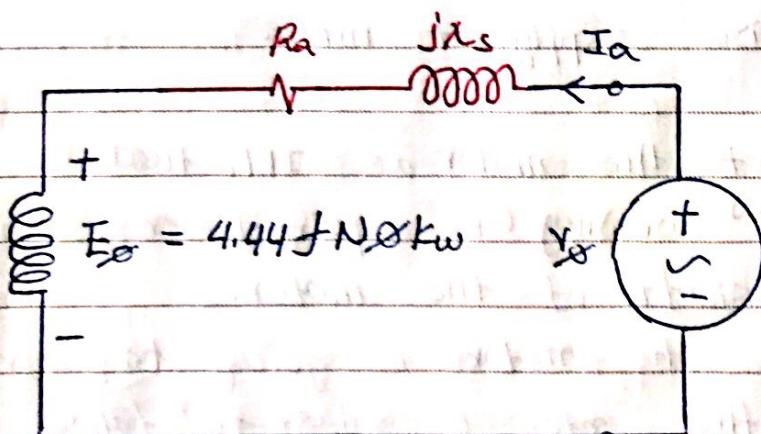
∴ Rotor always run at constant speed irrespective of the load.

* For e.g. when load $\uparrow \Rightarrow$ $n_{rot} \downarrow$ - current induces in squirrel cage winding and producing additional torque to bring the speed at rotor back to $n_{syn.}$

* Equivalent cat.:



* Since the system is balanced, then one may use per-phase cat.



Per-phase. cat. "Fig. 1"

R_a \equiv Armature winding resistance.

χ_s \equiv Synchronous reactance

E_a \equiv Induced voltage in stator winding.

Note: The equ. cct. for the generator is reversed.

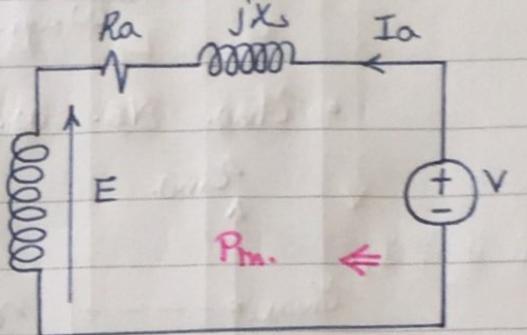
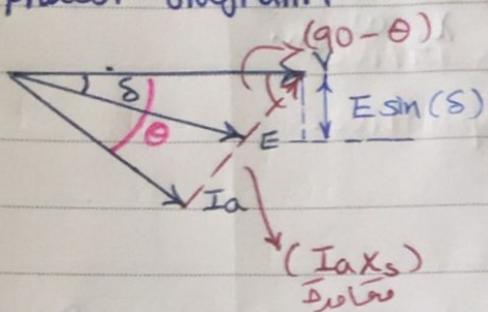
• Load \rightarrow Yedurige Talukar, M&B

* One may use Fig. 1 to produce the phasor diagram.

NO. 26.4.2018

* Analysis of 3-ph. Synch. motor:

This can be presented by means of the following phasor diagram:



$$V = E + Ia(Ra + jXs)$$

θ = PF angle.

δ = Tongue angle.

* If R_a is neglected

$$P_m = 3V I_a \cos(\theta) \quad \dots (1)$$

* It can be found that:

$$\therefore E \sin(\delta) = I_a X_s \sin(90^\circ - \theta)$$

$$E \sin(\delta) = I_a X_s \cos(\theta) \quad \dots (2)$$

$$\therefore I_a \cos(\theta) = \frac{E \sin(\delta)}{X_s} \quad \dots (3)$$

* Sub. (3) into (1):

$$P_m = \frac{3V E \sin(\delta)}{X_s}$$

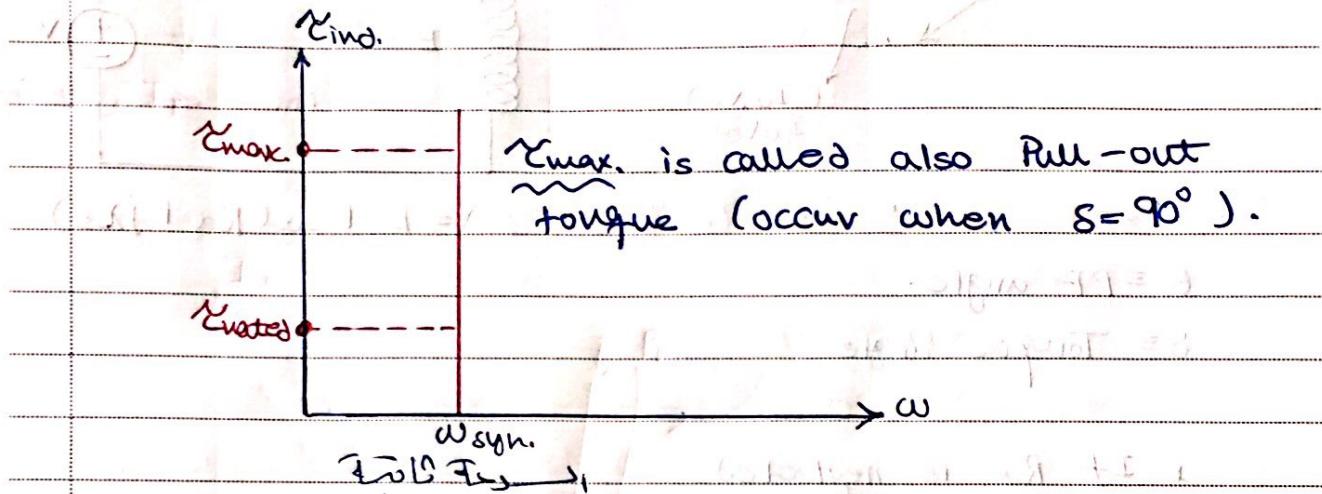
$$P_{\text{curr.}} = P_m - \underbrace{P_{\text{elect. losses}}}_{(3I_a R_a = 0)}$$

$$\therefore P_{\text{conv.}} = P_{\text{in.}}$$

$$\text{Since } P_{\text{conv.}} = \mathcal{E}_{\text{ind.}} \times \omega_{\text{syn.}}$$

$$\therefore \mathcal{E}_{\text{ind.}} = \frac{P_{\text{conv.}}}{\omega_{\text{syn.}}} = \frac{3VE \sin(\delta)}{X_s \omega_{\text{syn.}}}$$

$$\bullet \mathcal{E}_{\text{ind.}} \text{ vs. } \omega :$$



\therefore 3-ph. synch. motor is constant speed drive.

$$\therefore \mathcal{E}_{\text{max.}} = \frac{3VE(1)}{X_s \omega_{\text{syn.}}} ; \delta = 90^\circ \quad (\sin(90^\circ) = 1)$$

* Effect of field current I_f (i.e. I_R) on the performance of 3-ph. synch. motor

• problem:

For a given load, what will happen when the field current is increased?

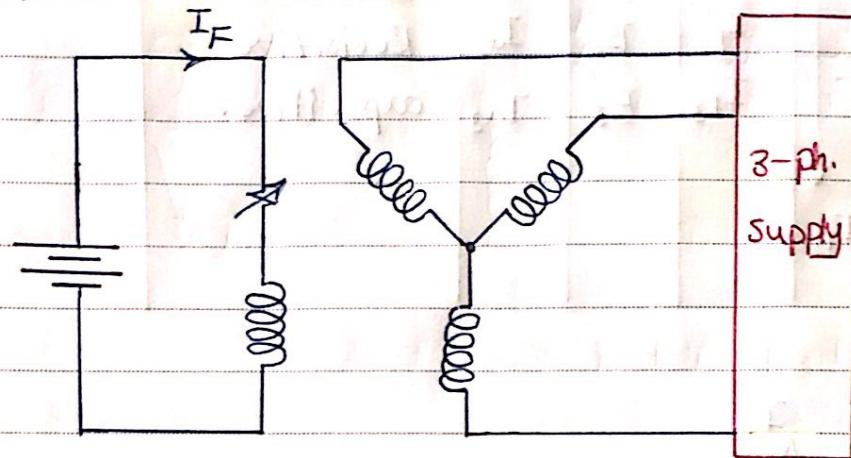
* Since the load doesn't change, the P constant.

* Since $P \propto E \sin(\delta) \propto I_a \cos(\theta)$

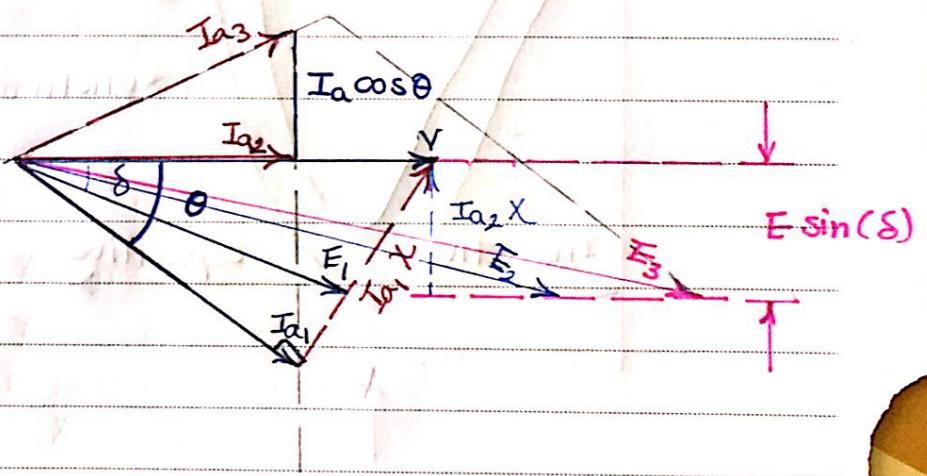
$\therefore E \sin(\delta)$ and $I_a \cos(\theta)$ should remain constant.

* since $\{ E = k \omega \}$

\therefore As $I_F \uparrow \Rightarrow \delta \uparrow \Rightarrow \therefore E \uparrow$



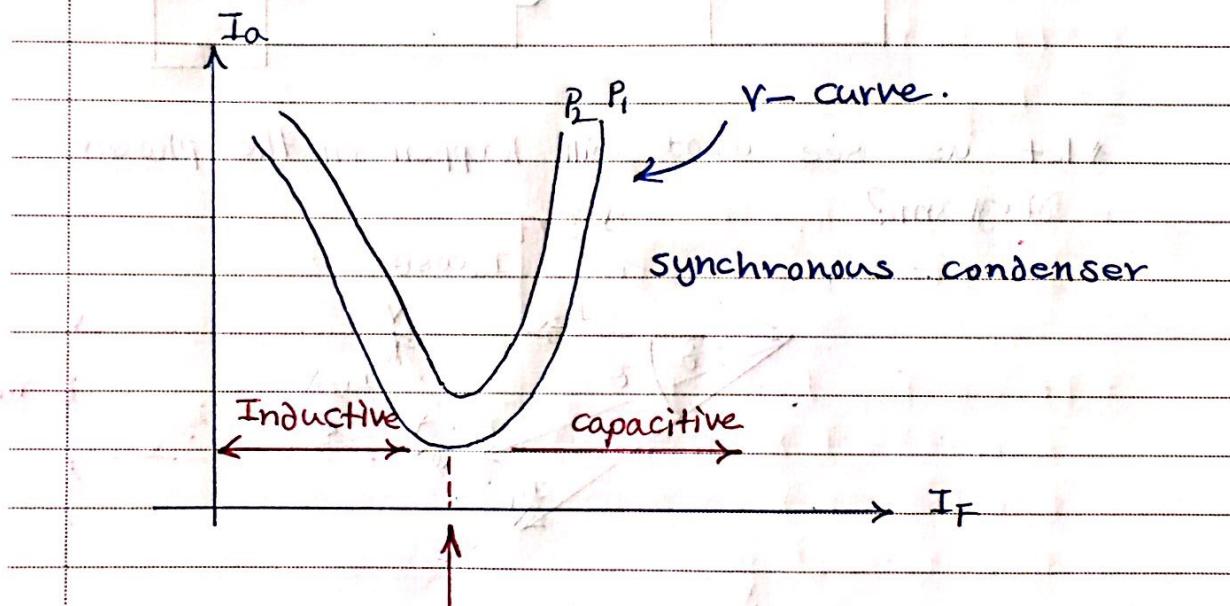
* let us see what will happen on the phasor diagram?



\therefore As $I_F \uparrow$, Then E should increase by sliding along the constant line of $(E \sin \delta)$.

| I_F | E | I_a | Type of load |
|----------|-------|----------|--------------|
| I_{F1} | E_1 | I_{a1} | Inductive |
| I_{F2} | E_2 | I_{a2} | Resistive |
| I_{F3} | E_3 | I_{a3} | capacitive. |

جذب I_F



NO. 29. 4. 2018

- Effect of load changes on motor performance!

As the load $\uparrow \Rightarrow$ initially its speed $\downarrow \Rightarrow$ $\chi_{ind.} \uparrow$ in order to maintain the speed on $n_{syn.}$

* Since $E = k\Phi\omega$

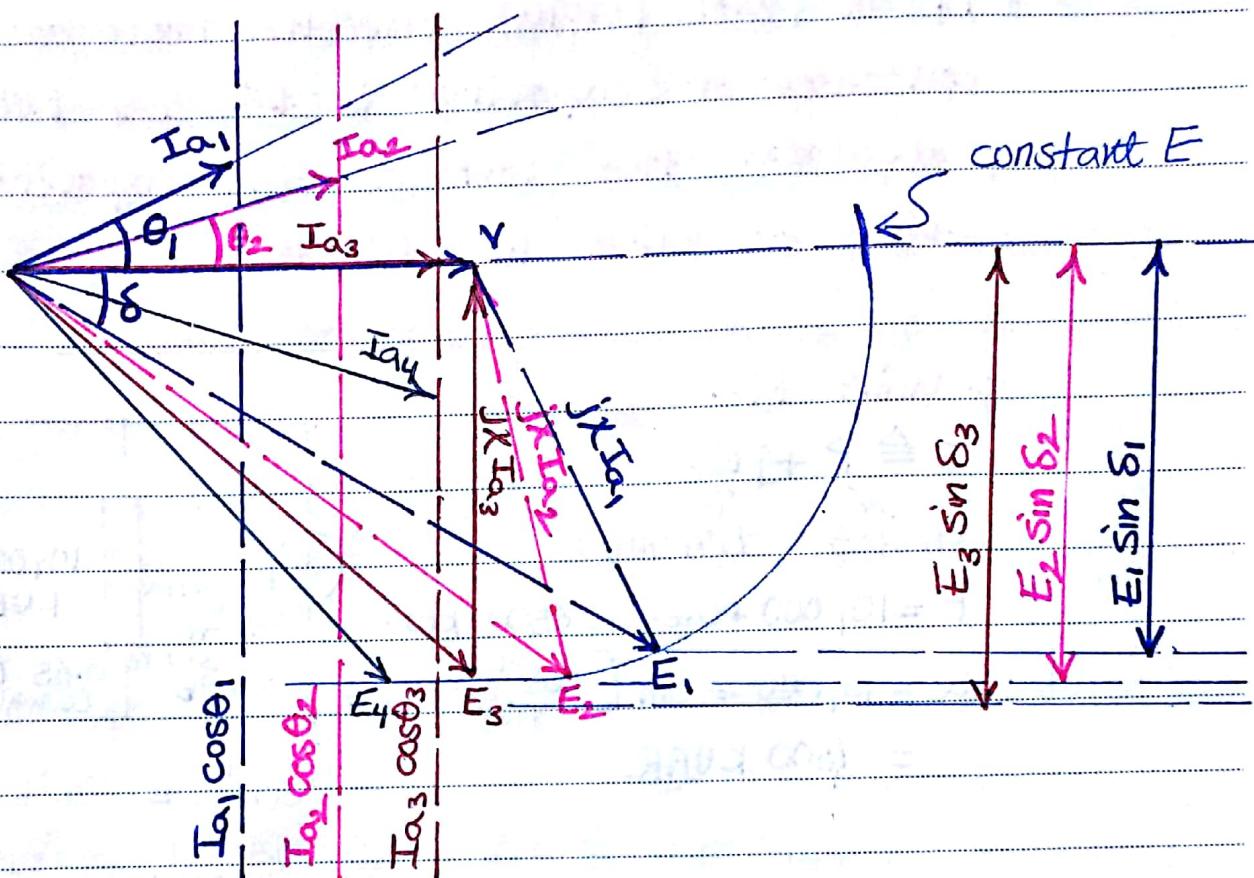
if I_F (i.e. Φ) is kept constant, the E will remain constant.

* Hence, performance can be explained by means of the following phasor diagram.

Knowing that:

$$V = E + j\chi I_a$$

* Assume initially that I_a is leading.



* As the load changes (i.e. $I_a \cos \theta$, and $E \sin \theta$), then E is going to move along a circle.

∴ As the load $\uparrow \Rightarrow$ the PF is going to change from leading \Rightarrow unity \Rightarrow lagging.

* Exam Question: Show the performance of 3-ph. syn. motor using phasor diagram?

↳ Ans: Draw the complete figure "Leading \Rightarrow unity \Rightarrow lagging".

* Example: A 10,000 kVA system is operating at PF of 0.65 lagging and the cost of synch. condenser or capacitor is \$60 per kVA.

Calculate the cost of the capacitor in order to raise the PF to 1.

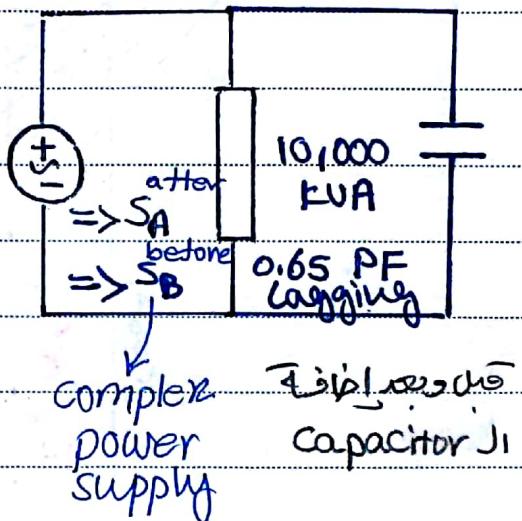
a. Unity PF.

$$S \triangleq P + jQ$$

$$\text{KVA cost} \quad \text{KVA} \sin \theta$$

$$P = 10,000 \times 0.65 = 6500 \text{ kW.}$$

$$Q = 10,000 \times \sin [\cos^{-1} 0.65] \\ = 7600 \text{ kVAR}$$



∴ For unity PF, $S_A = 6500 + j0$

∴ The capacitor should absorb $= -7600 \text{ kVAR}$

or the capacitor should supply $= 7600 \text{ kVAR}$

$$\text{cost} = 7600 \times 60 = 456000 \text{ $}$$

$$\text{PBP (Payback period)} = \frac{456000}{5000}$$

b. 0.85 PF.

$$\theta = \cos^{-1} (0.85) = 31.8$$

$$P_B = 6500 \text{ kW} = P_A$$

$$\therefore Q_A = P_A \tan(31.8) = 4030 \text{ kVAR}$$

\therefore capacitor should supply $= 7600 - 4030 = 3570 \text{ kVAR}$.

$$\therefore \text{cost} = 3570 \times 60 = 214200 \text{ \$}$$

* Example: 480 V, 60 Hz, 6 pole Synch. motor takes 80 A from the supply and unity PF and full load. Assume no losses.

I. Find τ_{out} .

$$P_{\text{out}} = \tau_{\text{out}} \times \omega_{\text{syn.}}$$

$$P_{\text{out}} = P_m = \sqrt{3} V I \cos \theta = \sqrt{3} (480)(80)(1) = 66.51 \text{ kW.}$$

$$\omega_{\text{syn.}} = 2\pi n_{\text{syn.}} = 2\pi \frac{f}{p} = 2\pi \left(\frac{60}{3}\right)$$

by substitution:

$$\tau_{\text{out}} = 529.5 \text{ N.m}$$

II. what will be the current magnitude if PF is adjusted to 0.8 leading.

$$P_{\text{out}} = \sqrt{3} V_L I_L \cos \theta$$

$$66.51 = 480 \sqrt{3} (I_L) 0.8$$

$$\hookrightarrow I_L = 99.98 \text{ A.}$$

• Special Motors:

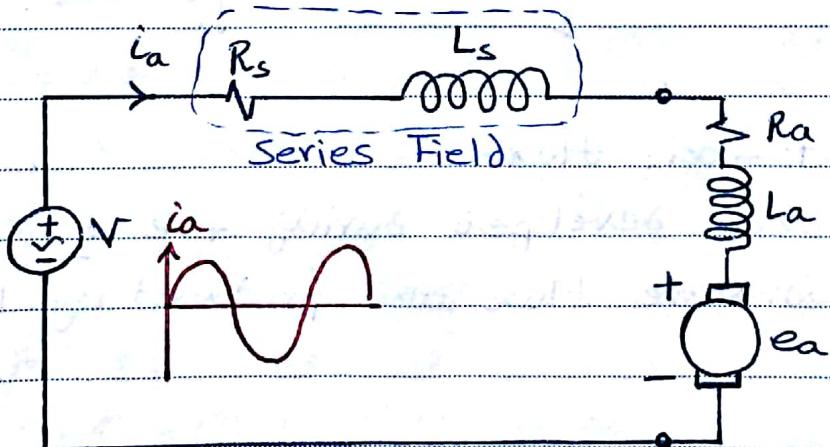
These are fractional hp motors, used mainly in control systems in order to control rotation on linear motion.

⇒ Among these types:

1. Series motor.
2. Stepper motor.
3. Brushless motor.
4. Reluctance.

1* Series or universal motors:

It is the same structure as previous DC-Series motors:



R_s, L_s = Resistance and self inductance of series field.

R_a, L_a = Resistance and self inductance of armature cct.

V, e_a = Applied voltage and back emf.

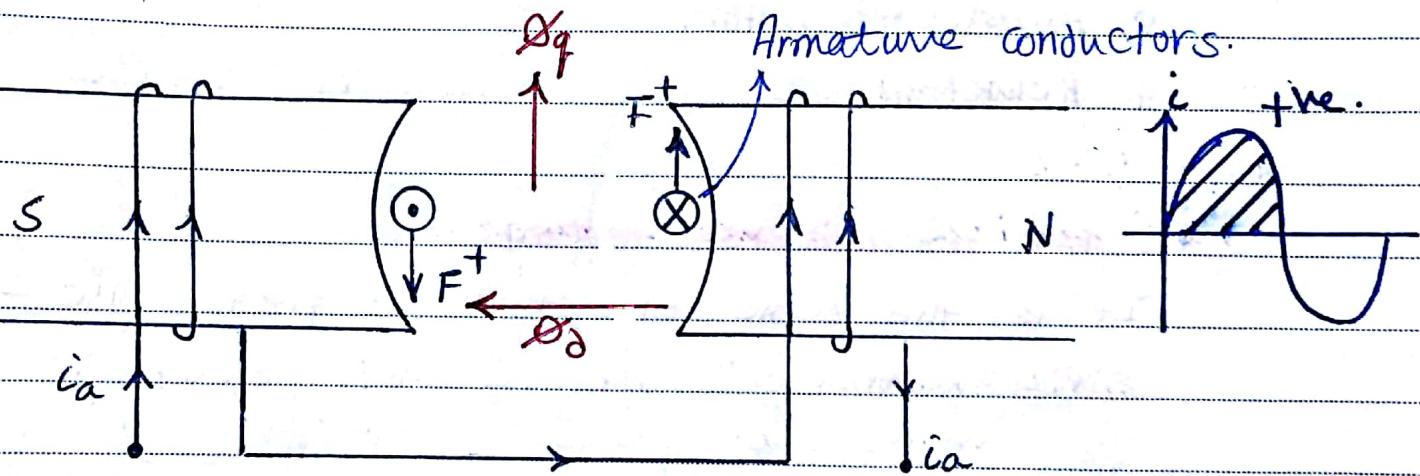
By KVL:

$$V = R_s i_a + l_s \frac{di_a}{dt} + R_a i_a + L_a \frac{di_a}{dt} + e_a$$

* The performance of this motor, can be explained as follows!

=> Qualitative!

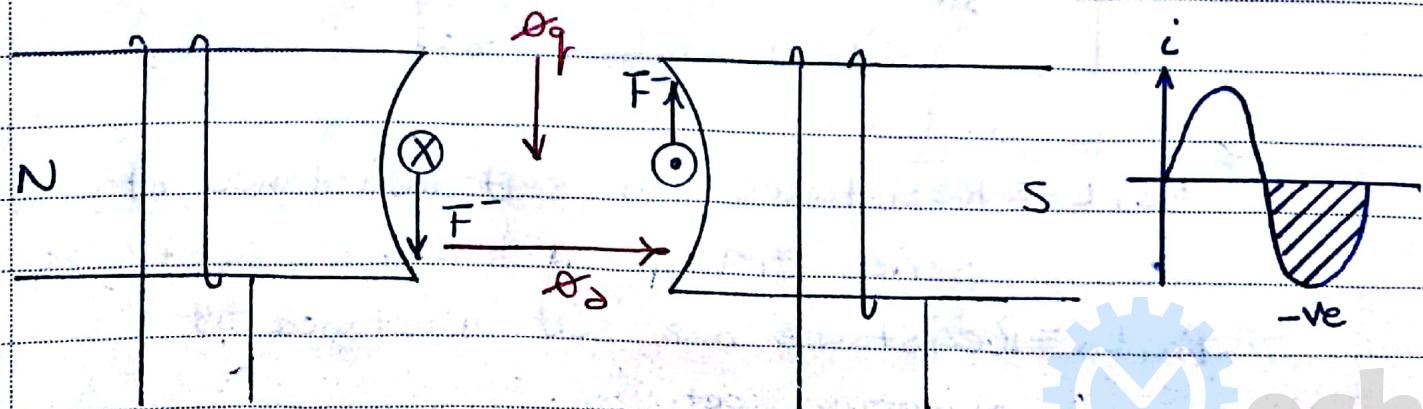
Consider the following schematic diagram:



$\Phi_d \equiv$ Direct-axis flux.

$F^+ \equiv$ Force developed during +ve $\frac{1}{2}$ cycle.

$\Phi_q \equiv$ Quadrature flux and produced by A.W.



$F^- \equiv$ Force developed during -ve $\frac{1}{2}$ cycle.

$F^+ = F^-$; Although current reverses then induced torque always in the same direction.

• Note 1 In the case of AC series motor, stator and rotor are made of laminated cores.

* Since it is used for DC and AC voltage, then series motor is called "universal".

⇒ Quantitative!

$$\text{let } i_a = I_m \cos \omega t.$$

$$\text{if } \theta_d \propto i_a.$$

$$\therefore \theta_d = \theta_{dm} \cos \omega t, \quad \omega = 2\pi f.$$

$$e_a = k \theta_d n = k \theta_{dm} n \cos \omega t.$$

$\therefore i_a, \theta_d$ and e_a are in phase.

But instantaneous torque:

$$\tau = k \theta_d i_a$$

$$\begin{aligned} \tau &= k \theta_{dm} \cos \omega t \cdot I_m \cos \omega t \\ &= k \theta_{dm} I_m \cos^2 \omega t. \end{aligned}$$

$$\tau = k \theta_{dm} I_m \cdot \frac{1}{2} (1 + \cos 2\omega t).$$

\therefore Average value of $\tau = \tau_{avg.} = \frac{1}{2} k \theta_{dm} I_m$.

$\therefore \tau_{avg.}$ shows always a torque in the same direction is produced.

2* Stepper motor

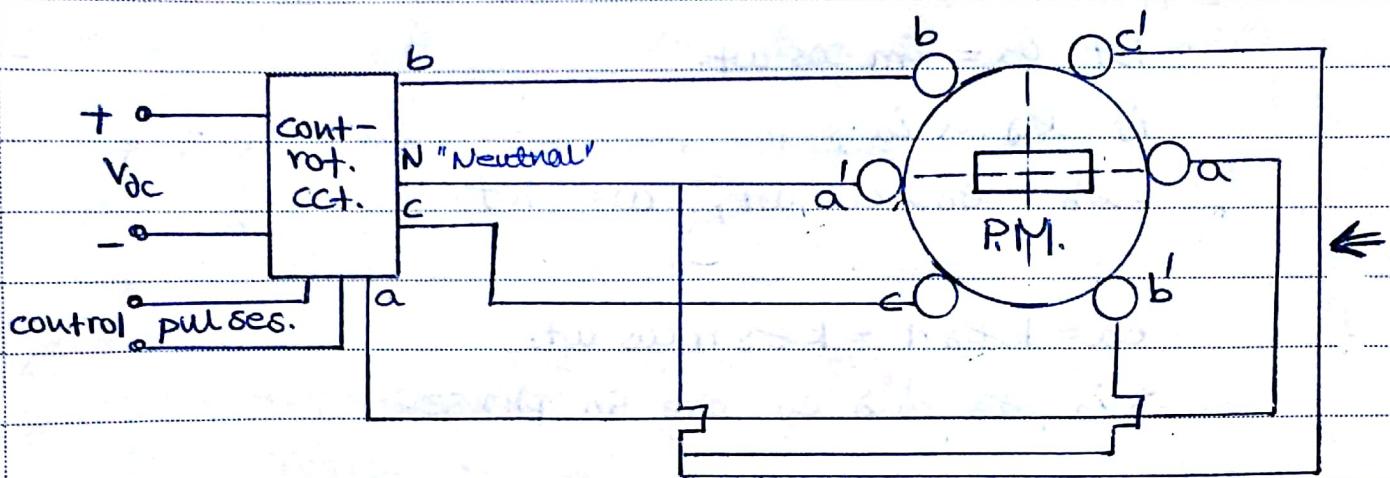
This is a special type of 3-ph. synch. motor.

Its stator consists of 3-ph. winding.

Its rotor is usually a permanent magnet.

→ Rotor moves in step every time one phase is excited by DC voltage.

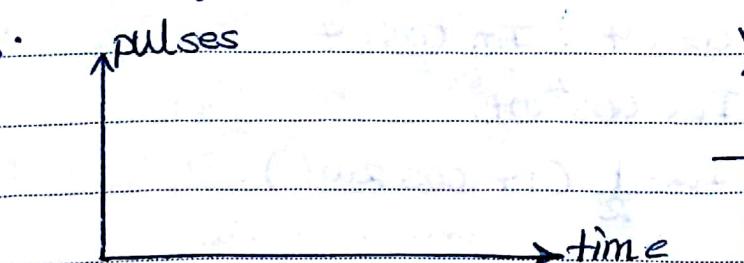
This voltage is controlled by pulses as follows:



P.M. = Permanent magnet.

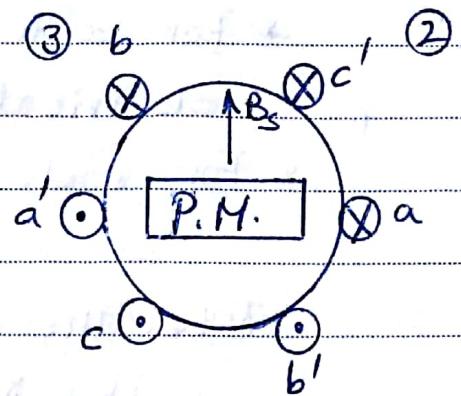
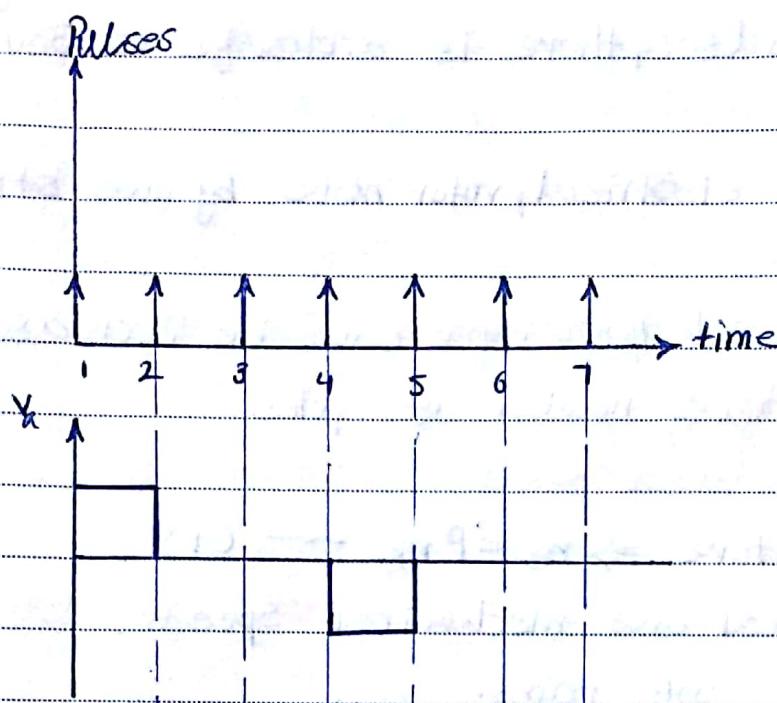
* P.M. always aligns itself with stator field, 2 pole stator winding.

Bs: pulses



V_a

N_t



2 pole stator winding.

- P.M. (i.e. Rotor) going to be aligned with B_s .

- For each pulse one DC voltage is applied to one phase causing deflection at rotor.

| Pulse | V_a | V_b | V_c | Position at rotor |
|-------|-------|-------|-------|-------------------|
| 1 | V | 0 | 0 | 0° |
| 2 | 0 | 0 | $-V$ | 60° |
| 3 | 0 | V | 0 | 120° |
| 4 | $-V$ | 0 | 0 | 180° |
| 5 | 0 | 0 | V | 240° |
| 6 | 0 | $-V$ | 0 | 300° |

* For each 6 pulses, there is a change of 360° electrical.

* For each 60° electrical, rotor move by one step.

• Note! The step of the rotor can be decreased by increasing number of poles.

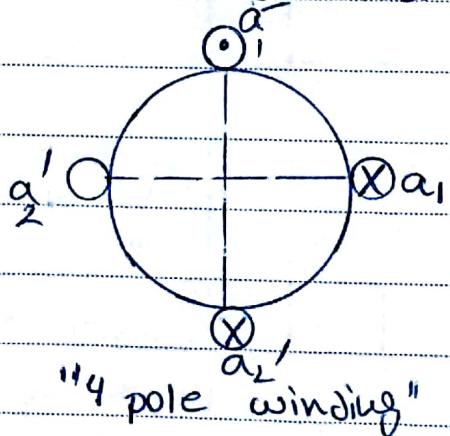
* As found before $\Rightarrow n_e = P n_m$ --- (1)

n_e, n_m = Electrical and mechanical speeds.

P = Number of pole pairs.

$\therefore \theta_e = P \theta_m$ --- (2)

θ_e, θ_m = Electrical and mechanical displacement.



* Since there are 6 pulses per electrical cycle.

No. of pulses per second, $n_{pulse} = 6 n_e$

$\therefore n_e = n_{pulse} / 6$ --- (3)

Sub. (3) into (1)

$$\frac{n_{pulse}}{6} = P n_m$$

$$\therefore n_{pulse} = 6 P n_m --- (4)$$

one electrical cycle \Rightarrow 6-pulses
 2 elect. cyc. \Leftarrow 12-pulses

* Ex. : A 3-ph. P.M. stepper motor required for one particular application must be capable controlling the position of a shaft in steps of 7.5° , and must be capable of running at speed of 300 rpm.

a. How many poles are required for this motor?

Since 60° electrical \Rightarrow one mechanical step.

$$\theta_e = 60^\circ, \theta_m = 7.5^\circ$$

$$\text{from (2)} \Rightarrow P = \frac{\theta_e}{\theta_m} = \frac{60}{7.5} = 8$$

$$\therefore \text{Number of poles} = 2 \times 8 = 16.$$

b. At what rate must control pulses be received in the motor's control unit if it is to be running at 300 rpm?

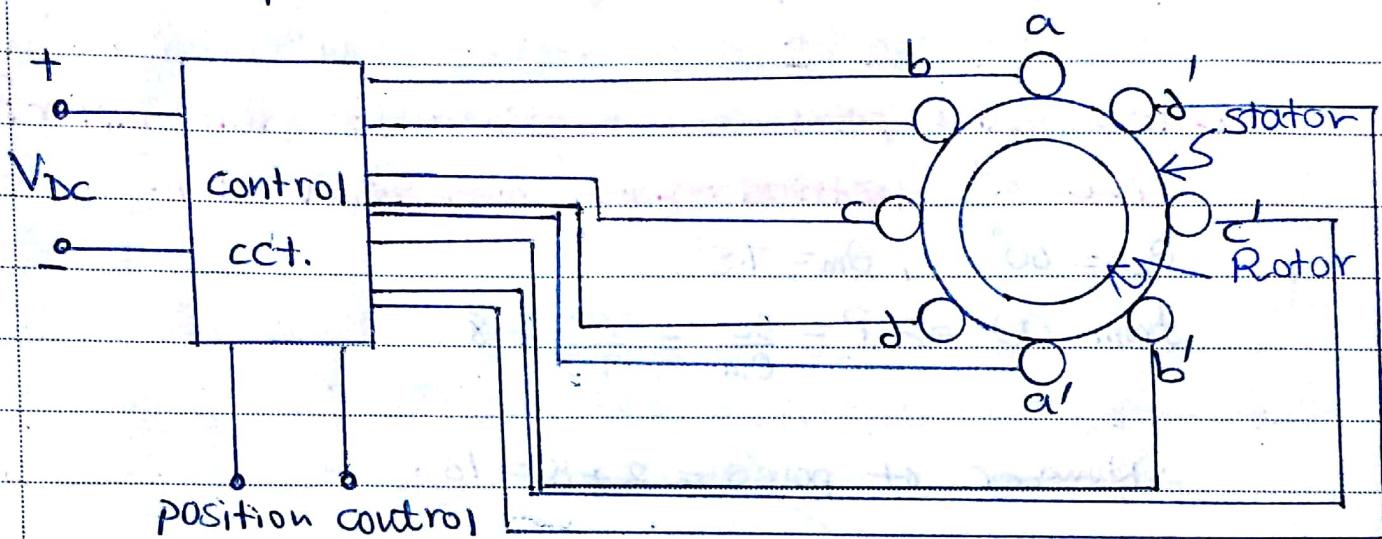
$$N_{\text{pulse}} = 6 \times 8 \times \frac{300}{60} = 240 \text{ pulses/sec.}$$

$$\therefore \text{No. of electrical cycles} = \frac{240}{60} = 40 \text{ cycle.}$$

3* Brushless DC-Motor!

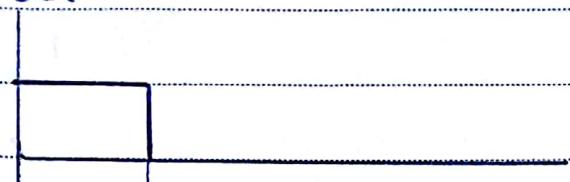
Construction:

1. It consists of stator with 3 or more phases.
2. A cylindrical P.M. rotor.
3. DC - supply.
4. position control unit.

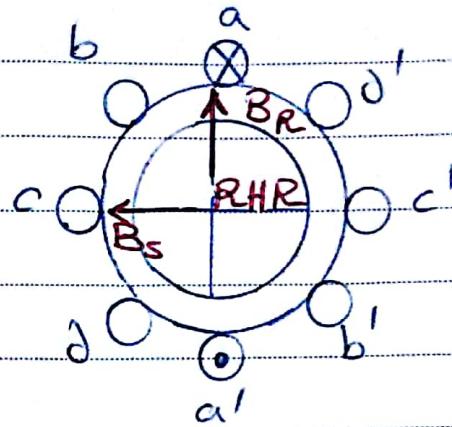


"4-phases stator"

$V_{aa'}$



$V_{bb'}$



$$* \mathcal{E} = k (B_R * B_S)$$

LHR \Rightarrow Direction of \mathcal{E}

$V_{cc'}$



* when B_R is aligned with B_S , then position control changes the voltage from a to b.

$V_{dd'}$



4* Reluctance Motor:

The same of stepper motor, but (RM. \rightarrow Electrical magnet.) P.M. \Rightarrow E.M.