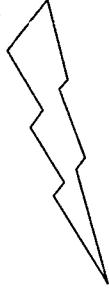
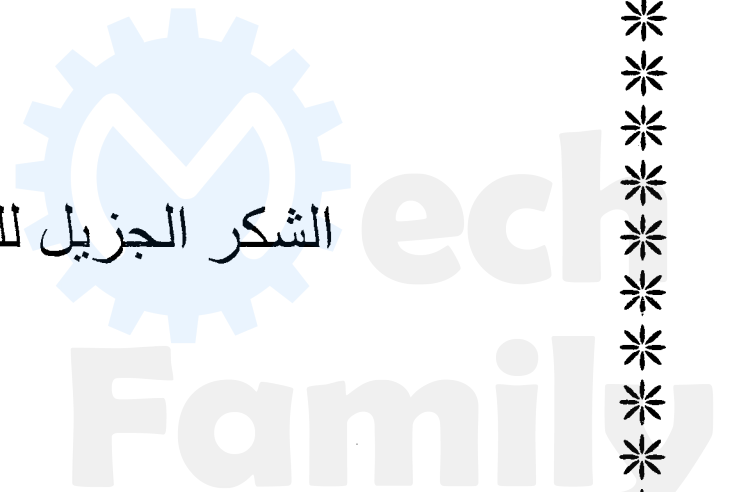


Notes For:



Electric Machines

الشكر الجزيل للطالب : احمد شاويز

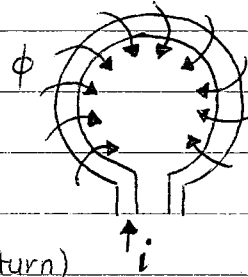
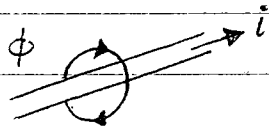


Electric Machines

DATE 7/Feb/2012 Tuesday.

* Chapter 1 : Magnetic Circuits

R.H. Rule



i = Current

ϕ = flux

Magnet-motive force (mmf) $\equiv \mathcal{F}$

$$\boxed{\mathcal{F} = Ni} \quad \text{unit} = (\text{Ampere.turn}) = \text{A.t}$$

Ohm's Law $I = \frac{V}{R}$

$$R = \rho \frac{l}{A}$$

; Electrical Circuits

$$\boxed{\phi = \frac{\mathcal{F}}{\mathcal{R}}}$$

$$\boxed{\mathcal{R} = \frac{l}{\mu A}}$$

; Magnetic Circuits

such that ϕ = Flux (Weber = Wb)

$\mathcal{R}$ = Reluctance (A.t/Wb)

μ = Permeability (Wb/A.t.m)

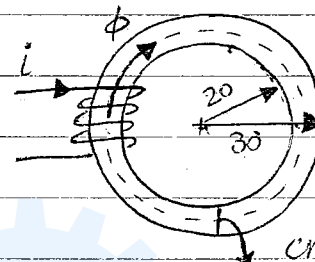
μ_0 = Permeability of Free Space - Air = $4\pi \times 10^{-7}$

μ_r = Relative Permeability

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

Ex. $i = 2\text{A}$, $N = 500$ turns, $\mu_r = 2000$

find ϕ ?



cross-section

$$\boxed{10 \times 10 \text{ cm}^2}$$

$$\text{Mean Radius} = r = \frac{20+30}{2} = 25 \text{ cm} = 0.25 \text{ m}$$

$$l = 2\pi r = 0.5\pi \text{ m}$$

$$A = 10 \times 10 \text{ cm}^2 = 0.01 \text{ m}^2$$

$$\mu = \mu_r \mu_0 = 2000 \times 4\pi \times 10^{-7} = 8000\pi \times 10^{-7}$$

$$\mathcal{R} = \frac{l}{\mu A} = \frac{0.5\pi}{8000\pi \times 10^{-7} \times 0.01} = 62500 \text{ A.t/Wb}$$

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}} = \frac{500 \times 2}{62500} = 0.016 \text{ Wb} = 16 \text{ mWb}$$

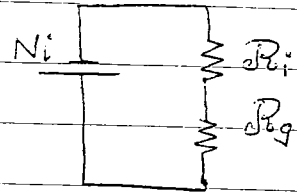
continue →

→ continue ... With Air Gap of 1mm.

$$R_g = \frac{l}{\mu_0 \mu_r A} = \frac{0.001}{4\pi \times 10^{-7} \times 0.01} = 79578$$

$$R_{\text{Total}} = R_i + R_g = 142078 \text{ A.t/Wb.}$$

$$\phi = \frac{F}{R_{\text{tot}}} = \frac{500 \times 2}{142078} = 7 \text{ mWb.}$$



* Comparison Between Electrical & Magnetic Circuits ...

• Electric

Current i (A)

Voltage emf (V)

Resistance R (Ω)

Ohm's Law $i = V/R$

$$R = \rho l / A$$

for junctions $\sum i = 0$

$$\sum \text{emf} = \sum iR$$

• Magnetic

Flux ϕ (Wb)

mmf F (A.t)

Reluctance R (A.t/Wb)

$$\phi = F/R$$

$$R = l / (\mu A)$$

for loops $\sum \phi = 0$

$$\sum F = \sum \phi R$$

Example. Find total flux?

$$i = 1 \text{ A}, N = 1000, \mu_r = 1000$$

$$l_1 = \frac{5}{2} + \frac{5}{2} + 20 = 25 \text{ cm} = 0.25 \text{ m}$$

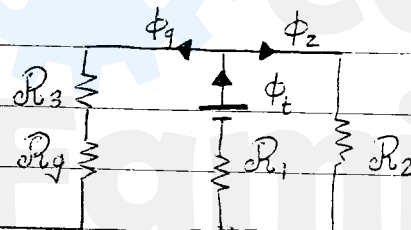
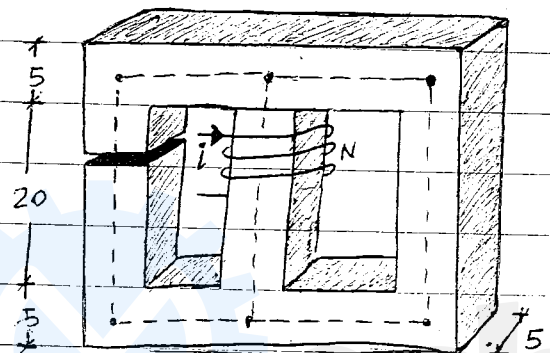
$$A_1 = 5 \times 10 \times 10^{-4} = 50 \times 10^{-4} \text{ m}^2$$

$$l_2 = 2\left(\frac{10}{2} + 30 + \frac{5}{2}\right) + l_1 = 1 \text{ m}$$

$$A_2 = 5 \times 5 \times 10^{-4} = 25 \times 10^{-4} \text{ m}^2$$

$$l_3 = 2\left(\frac{10}{2} + 20 + \frac{5}{2}\right) + l_1 = 0.8 \text{ m}$$

$$A_3 = 5 \times 5 \times 10^{-4} = 25 \times 10^{-4} \text{ m}^2$$



DATE 9/Feb/2012 Thursday.

$$R_1 = \frac{l_1}{\mu A_1} = \frac{0.25}{1000 \mu_0 \times 50 \times 10^{-4}} = \frac{0.05}{\mu_0}$$

$$R_2 = \frac{l_2}{\mu A_2} = \frac{1}{1000 \mu_0 \times 25 \times 10^{-4}} = \frac{0.4}{\mu_0}$$

$$R_3 = \frac{l_3}{\mu A_3} = \frac{0.8}{1000 \mu_0 \times 25 \times 10^{-4}} = \frac{0.32}{\mu_0}$$

$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{0.001}{\mu_0 \times 25 \times 10^{-4}} = \frac{0.4}{\mu_0}$$

$$R_{Tot} = [(R_3 + R_g) \parallel R_2] + R_1 = \left[\left(\frac{0.72}{\mu_0} \right) \parallel \left(\frac{0.4}{\mu_0} \right) \right] + \left(\frac{0.05}{\mu_0} \right)$$

$$R_{tot} = \frac{0.257}{\mu_0} + \frac{0.05}{\mu_0} = \frac{0.307}{\mu_0}$$

$$\phi_{tot} = \frac{F}{R_{tot}} = \frac{1000(1)(4\pi \times 10^{-7})}{0.307} = 4.1 \text{ mWb.}$$

$$\phi_g = \phi_{tot} \left(\frac{R_2}{R_2 + R_3 + R_g} \right) = 4.1 \times \left(\frac{0.4}{1.12} \right) = 1.46 \text{ mWb.}$$

$$\phi_2 = \phi_{tot} - \phi_g = 4.1 - 1.46 = 2.64 \text{ mWb. OR } \phi_2 = \left(\frac{R_3 + R_g}{R_2 + R_3 + R_g} \right) \phi_{tot}$$

* Flux Density $\boxed{B = \frac{\phi}{A}}$ (Wb/m²) OR (Tesla = T)

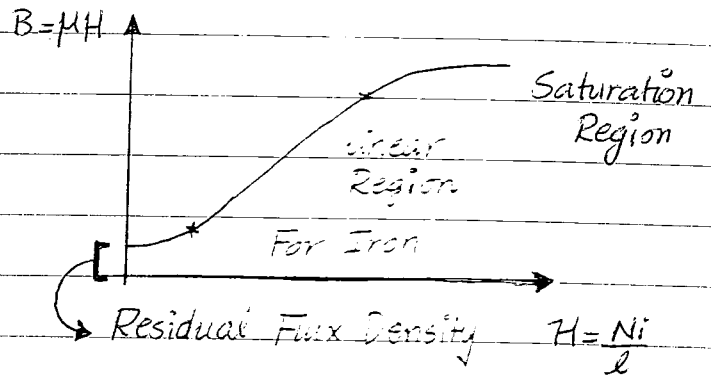
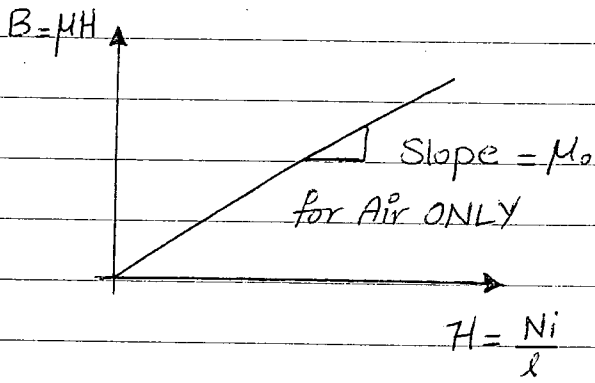
continue example $\rightarrow B_g = \frac{\phi_g}{A_g} = \frac{1.46 \times 10^{-3} \text{ Wb}}{25 \times 10^{-4} \text{ m}^2} = 0.6 \text{ T}$

But $F = \phi R \Rightarrow Ni = \phi \frac{l}{\mu A} = \frac{B l}{\mu}$

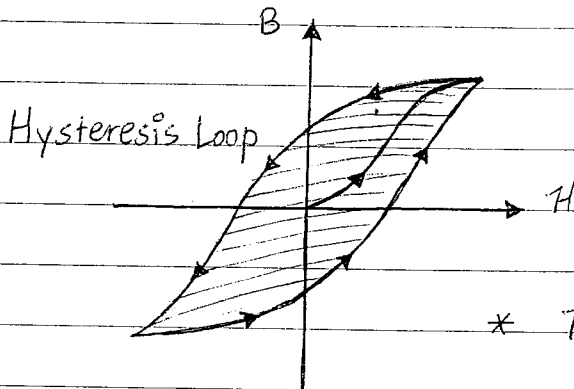
But $\frac{B}{\mu}$ is called Magnetic Intensity (H)

$$\boxed{H = \frac{B}{\mu}}$$

$\Rightarrow F = Ni = H l \leftarrow \boxed{\sum \phi R = Ni = H l}$ unit = (A.t/m)

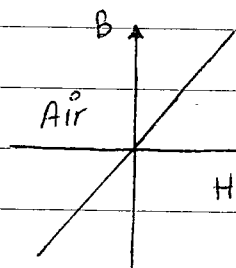


* Hysteresis



Area = Power Loss = Hysteresis Loss
كلما كانت المساحة أقل، كان (Loss) أقل
تكون الـ (Losses) على شكل حرارة.

* Hysteresis Loss



$$P_h = K_h B_{max}^n f V$$

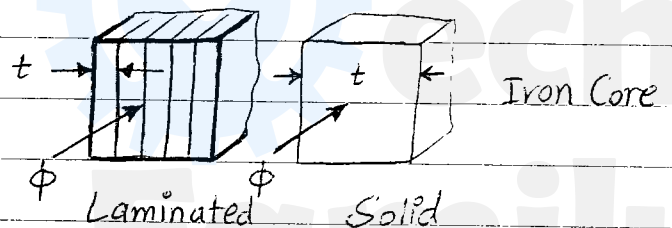
f = frequency, V = volume, $n = 1.5 \rightarrow 2.5$

* لا حاجة لـ (Air gap) في ((Loss))

* Eddy Current Loss

$$P_e = K_e B_{max}^2 f^2 V \frac{t^2}{\rho}$$

ρ = Resistivity, t = thickness



عندما تكون thickness أقل، فإننا نقل
من (Pe) !

DATE 12/Feb/2012 Sunday.

* Core Loss

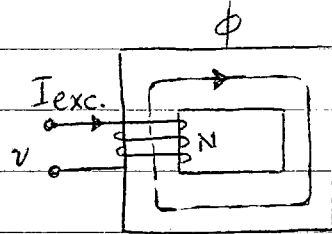
$$P_{\text{core}} = P_h + P_e$$

* Sinusoidal Excitation

e.g. $\phi(t) = \phi_{\text{max}} \sin \omega t$

$$v = e = N \frac{d\phi}{dt}$$

$$\phi = \frac{1}{N} \int v dt$$



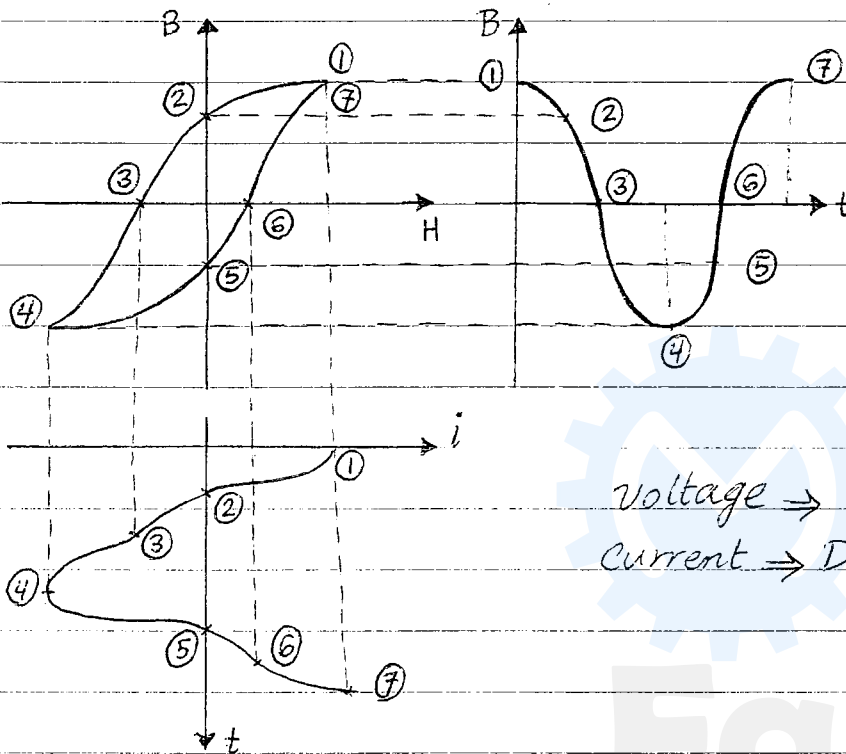
$$\Rightarrow v = N(\phi_m \omega \cos \omega t) = V_{\text{max}} \cos \omega t$$

$$B = \phi A = B_{\text{max}} \sin \omega t$$

e.g. $v = V_{\text{max}} \sin \omega t$

$$\phi = \frac{1}{N} \int v dt = -\frac{1}{N} \frac{V_{\text{max}} \cos \omega t}{\omega}$$

$$B = \phi A = B_{\text{max}} \cos \omega t$$



voltage $\Rightarrow$ sine-wave
current $\Rightarrow$ Distorted sine-wave

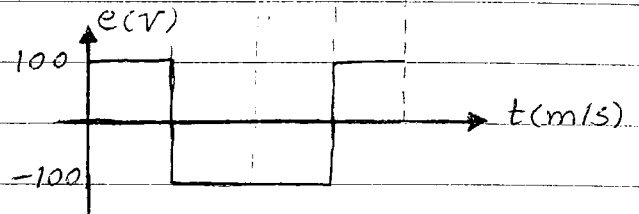
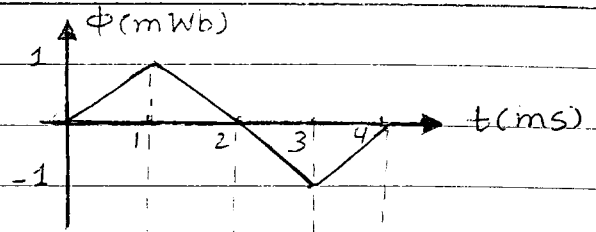
Example 3 Find v ? $N=100$

$$v = N \frac{d\phi}{dt} \quad \text{slope} = \frac{\Delta \phi}{\Delta t}$$

$$e_1 = v_1 = 100 \times \left(\frac{1 \text{ mWb}}{1 \text{ ms}} \right) = 100 \text{ V}$$

$$e_2 = v_2 = 100 \times \left(\frac{-1-1}{3-1} \right) = -100 \text{ V}$$

$$e_3 = v_3 = 100 \times \left(\frac{0+1}{4-3} \right) = 100 \text{ V}$$



Example 3- Find ϕ ? $N=100$

$$\phi = \frac{1}{N} \int e dt \quad \text{Area} = \text{المساحة}$$

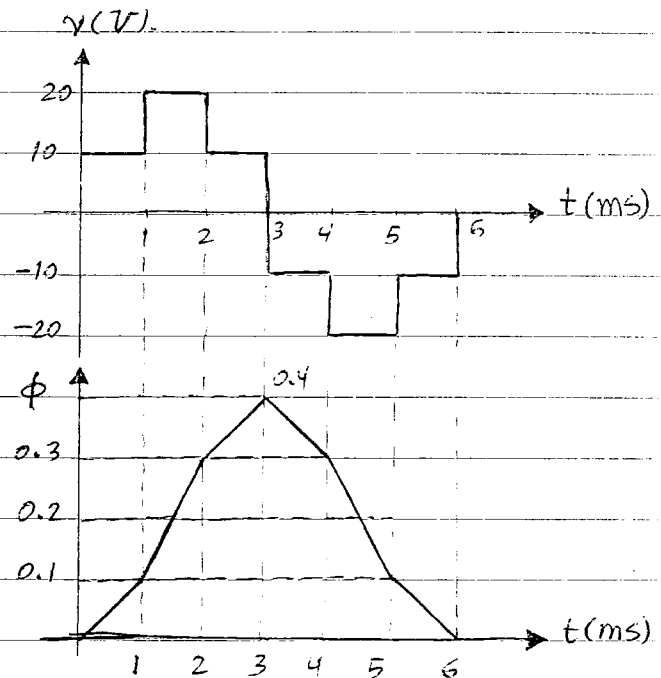
$$\phi_1 = \frac{1}{100} (10 \times 1) = 0.1 \text{ mWb}$$

$$\phi_2 = \frac{1}{100} (20 \times 1) + \phi_1 = 0.3 \text{ mWb}$$

$$\phi_3 = \left(\frac{1}{100} (10 \times 1) \right) + \phi_2 = 0.4 \text{ mWb}$$

$$\phi_4 = \frac{-1}{100} (20 \times 1) + \phi_3 = 0.2 \text{ mWb}$$

$$\phi_5 = \frac{-1}{100} (20 \times 1) + \phi_4 = 0.1 \text{ mWb}$$



Example 3

A certain Magnetic Core has 100W loss @ 50 Hz & 200W @ 80 Hz. Find the Loss @ 40 Hz?

$$P_{\text{core}} = K_1 f + K_2 f^2$$

$$100 = K_1 (50) + K_2 (2500)$$

$$200 = K_1 (80) + K_2 (6400)$$

⇒ Solving Equations: $K_1 = 1.17$

$$K_2 = 0.017$$

$$P(40) = 1.17(40) + 0.017(1600) = 74 \text{ W}$$

* Problem (1.5) _{oo}

$$i = 0.5 A, \quad N_1 = 700, \quad N_2 = 200$$

Neglect leakage Flux, Reluctance of Iron (Permeability $\approx \infty$)Find ϕ and B @ Air gap?

$$F = 700 \times 0.5 - 200 \times 0.5 = 250 \text{ A.t.}$$

$$R_{g1} = \frac{0.05 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.5 \times 2.5 \times 10^{-4}} = 636620$$

$$R_{g2} = \frac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.5 \times 2.5 \times 10^{-4}} = 1273240$$

$$\phi_{g1} = \frac{250}{636620} = 0.4 \text{ mWb}$$

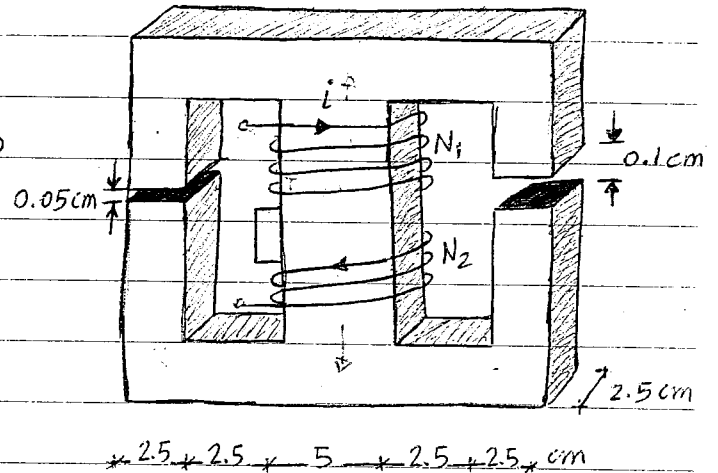
$$\phi_{g1} = \frac{0.4 \times 10^{-3}}{2.5 \times 2.5 \times 10^{-4}} = 0.63 \text{ T}$$

$$\phi_{g2} = \frac{250}{1273240} = 0.2 \text{ mWb}$$

$$\phi_{g2} = \frac{0.2 \times 10^{-3}}{2.5 \times 2.5 \times 10^{-4}} = 0.32 \text{ T}$$

$$\phi_{\text{Tot}} = \phi_{g1} + \phi_{g2} = 0.4 + 0.2 = 0.6 \text{ mWb}$$

$$B_{\text{Tot}} = \frac{0.6 \times 10^{-3}}{2.5 \times 2.5 \times 10^{-4}} = 0.96 \text{ T}$$

 $B_{\text{Tot}} \neq B_{g1} + B_{g2}$ (بسبب اختلاف Areas)* Problem (1.22) _{oo}

N=500, Find max. value of flux.

& sketch ϕ as a function of t .

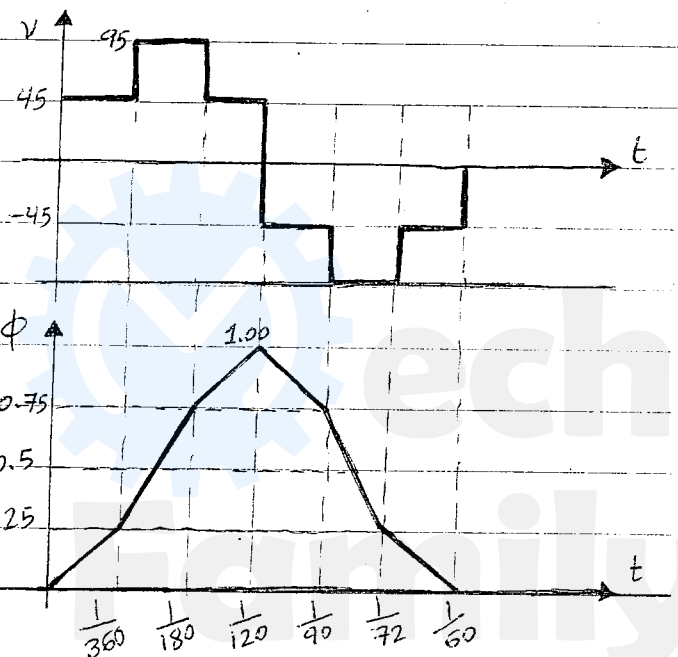
$$\phi_1 = \frac{1}{500} \left(45 \times \frac{1}{360} \right) = 0.25 \text{ mWb}$$

$$\phi_2 = \frac{1}{500} \left(90 \times \frac{1}{360} \right) + \phi_1 = 0.75 \text{ mWb}$$

$$\phi_3 = \frac{1}{500} \left(45 \times \frac{1}{360} \right) + \phi_2 = 1.00 \text{ mWb}$$

$$\phi_4 = \frac{-1}{500} \left(45 \times \frac{1}{360} \right) + \phi_3 = 0.75 \text{ mWb}$$

$$\phi_5 = \frac{-1}{500} \left(90 \times \frac{1}{360} \right) + \phi_4 = 0.25 \text{ mWb}$$



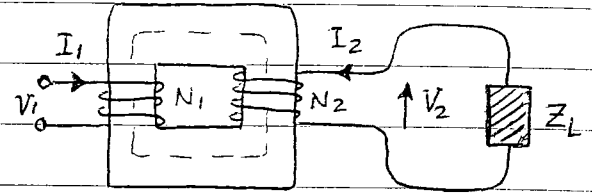
* Chapter 2 : Transformers

$$v_1 = N_1 \frac{d\phi}{dt} \quad \text{--- ①} \quad v_2 = N_2 \frac{d\phi}{dt} \quad \text{--- ②}$$

$$\frac{\text{①}}{\text{②}} \Rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2}$$

Step Up
Step Down

• Turns Ratio $a = \frac{N_1}{N_2} = \frac{v_1}{v_2}$



Iron Core

"Ideal Transformer"

for ideal transformer

$$v_1 I_1 = v_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{v_2}{v_1}$$

استعمال (Transformer)

∴ Turns Ratio

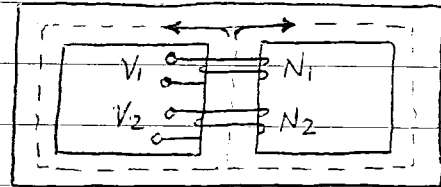
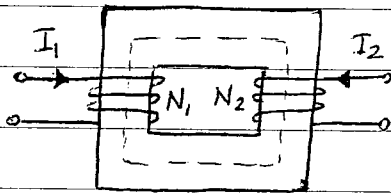
$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{I_2}{I_1}$$

كثرة نقل الطاقة
من مكان لآخر

* Types of Transformers

1 - Core Type.

2 - Shell Type.



Now, Load $Z_L = \frac{v_2}{I_2}$, But $I_2 = a I_1$ & $v_2 = \frac{v_1}{a}$

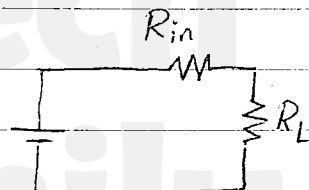
$$\therefore Z_L = \frac{(v_1/a)}{(a I_1)} = \frac{v_1}{I_1 a^2} \Rightarrow \boxed{a^2 Z_L = \frac{v_1}{I_1}} \quad \boxed{a^2 Z_L = Z_1}$$

* Impedance Matching

Note: Max. Power Transfer occurs when

$$R_{in} = R_L \quad \text{, such that } R_{in} = \text{مقاومة داخلية}$$

$$R_L = \text{مقاومة خارجية}$$



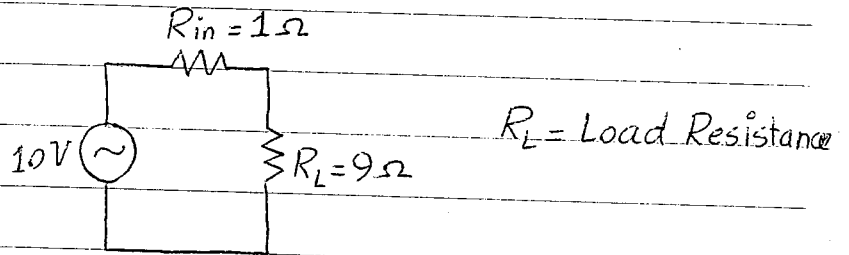
Thursday

* Note : If $a > 1 \rightarrow$ Step Down.
If $a < 1 \rightarrow$ Step Up.

Example :

$$I = \frac{V}{R} = \frac{10}{1+9} = 1A$$

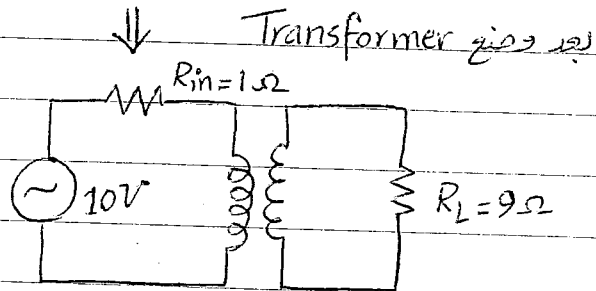
$$P_L = I^2 R_L = 1^2 \times 9 = 9W$$



نموذج وضع ال (Transformer)

$$Z_1 = a^2 Z_2$$

$$1 = a^2 \times 9 \Rightarrow \boxed{a = \frac{1}{3}}$$



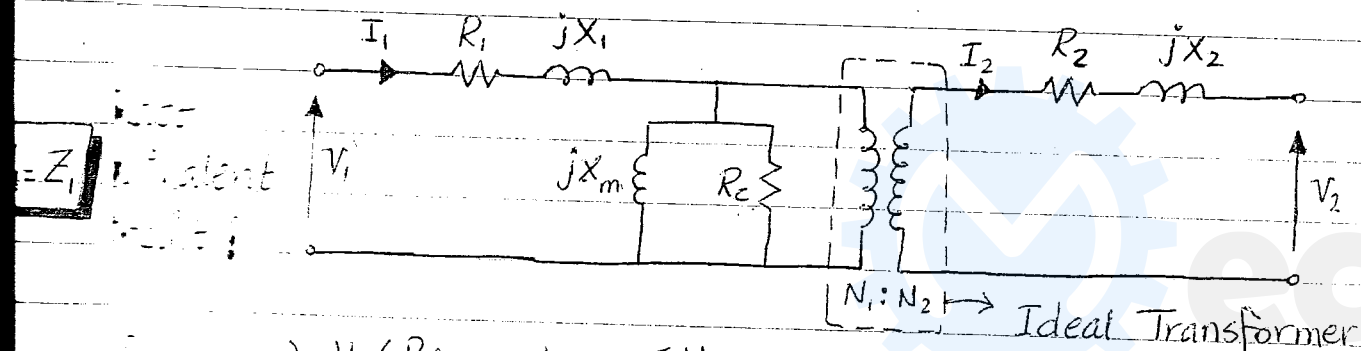
$$I_1 = \frac{10}{1+1} = 5A, \quad P_1 = I_1^2 R_1 = 25W$$

$$V_1 = I_1 R_1 = 1 \times 5 = 5V \Rightarrow a = \frac{V_1}{V_2} \Rightarrow V_2 = \frac{V_1}{a} = 5 \times 3 = 15V$$

$$I_2 = \frac{V_2}{R_2} = \frac{15}{9} = \frac{5}{3}A \Rightarrow P_2 = I_2^2 R_2 = \left(\frac{5}{3}\right)^2 \times 9 = 25W$$

تسمى هذه العملية Impedance Matching وتستخدم للحصول على أقصى قدرة.

Practical Transformers



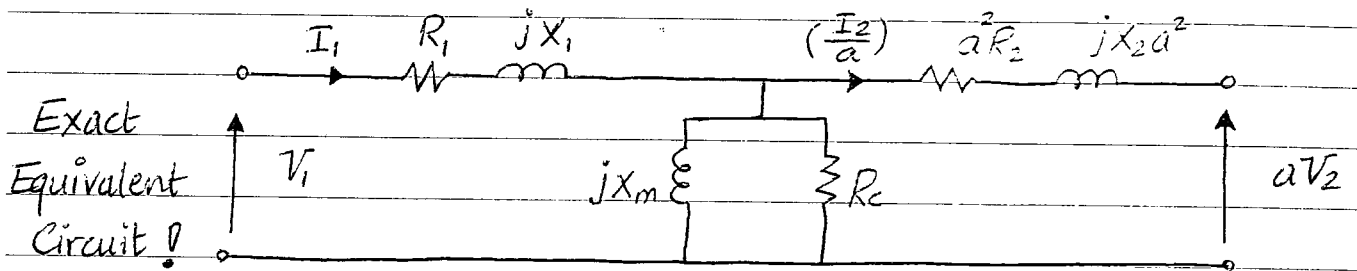
(Primary) من (Secondary) : (R, jX) للتحويل

نقسم على a^2 والعكس صحيح.

continue

DATE 19/ Feb/2012 Sunday

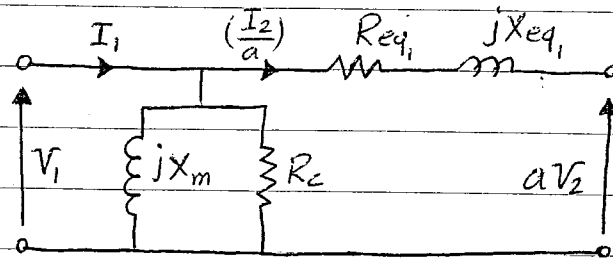
← إذا أردنا أن نكتب (Transformer) نحن نختار بأن نحول من (Primary) إلى (Secondary) أو العكس. ويجب الانتباه إلى عملية التحويل.



How :- $a = \frac{V_1}{V_2} = \frac{I_2}{I_1} \Rightarrow \boxed{I_1 = \frac{I_2}{a}}, \boxed{V_1 = aV_2}, \boxed{R_c}, \boxed{X_m}$

$a = \frac{V_1 - I_1 R_1}{V_2 - I_2 R_2} \Rightarrow a = \frac{1}{a} \times \frac{R_1}{R_2} \Rightarrow \boxed{R_1 = a^2 R_2}, \boxed{X_1 = a^2 X_2}$

! كل هذه التحويلات هي من Secondary إلى Primary

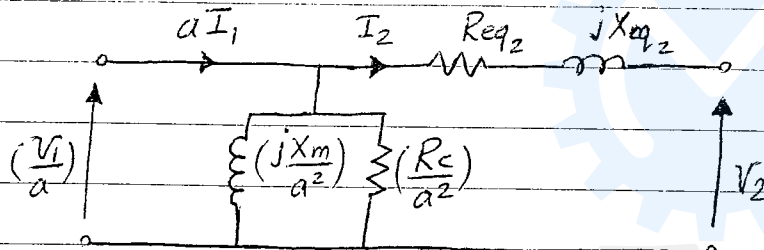


$R_{eq1} = R_1 + a^2 R_2$
 $X_{eq1} = X_1 + a^2 X_2$

Approximate Equivalent Circuit Referred to Primary !

OR : Primary → Secondary

$\boxed{V_2 = \frac{V_1}{a}}, \boxed{I_2 = aI_1}, \boxed{R_2 = \frac{R_1}{a^2}}, \boxed{X_2 = \frac{X_1}{a^2}}, \boxed{\frac{R_c}{a^2}}, \boxed{\frac{X_m}{a^2}}$



$R_{eq2} = \frac{R_1}{a^2} + R_2$
 $X_{eq2} = \frac{X_1}{a^2} + X_2$

Approximate Equivalent Circuit Referred to Secondary !

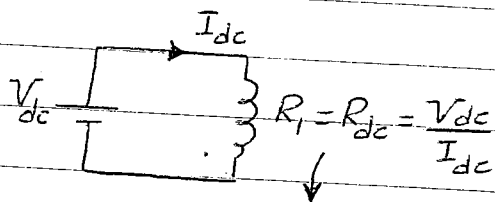
Note :- In Transformers ; there are 2 sides ...

High Voltage Side & Low Voltage Side

so ; if : step up $\rightarrow$ Primary is (Low) & Secondary is (High)
step Down $\rightarrow$ Primary is (High) & Secondary is (Low).

Testing Of Performance ...

1 - DC Test



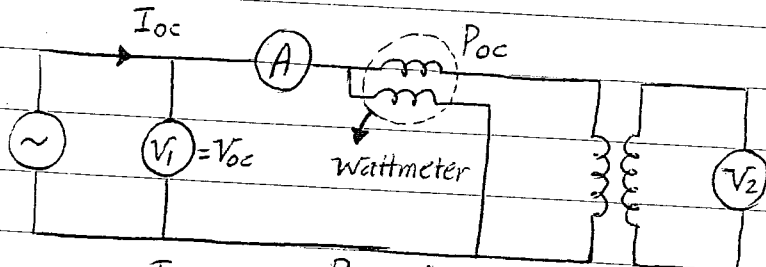
• Note

$$R_{ac} > R_{dc} ; R_{ac} \approx 1.2 R_{dc} \text{ @ } 50\text{Hz.}$$

measurement of dc resistance.

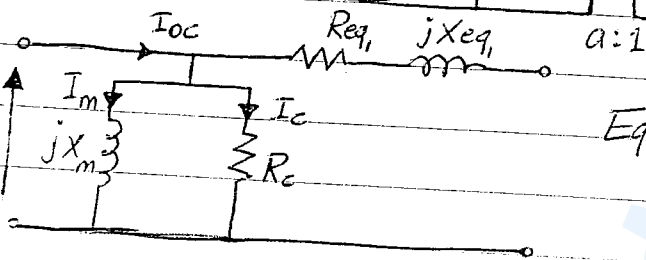
2 - Open Circuit Test (OCT) ...

OR (No Load Test) ...



$$V_1 = V_{oc}$$

$$\frac{V_1}{V_2} = a$$



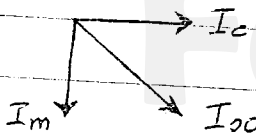
Equivalent circuit under Open circuit

$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

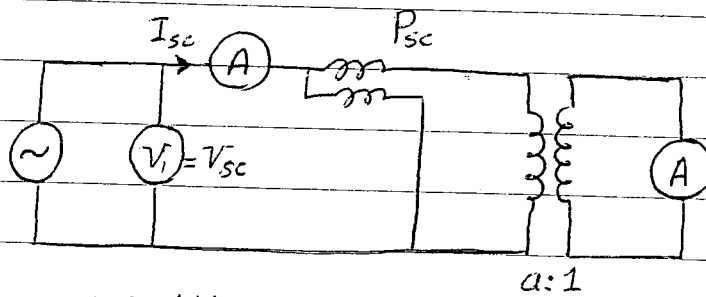
$$I_c = \frac{V_{oc}}{R_{oc}}$$

$$X_m = \frac{V_{oc}}{I_m}$$

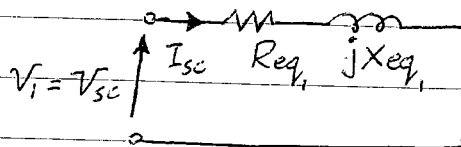
$$I_{oc} = \sqrt{I_c^2 + I_m^2}$$



3 Short Circuit Test (SCT) ∞ ∞ ∞



$$V_1 = V_{sc}$$



Equivalent Circuit @ short Circuit

$$P_{sc} = I_{sc}^2 R_{eq1} \Rightarrow \boxed{R_{eq1} = \frac{P_{sc}}{I_{sc}^2}}, \quad \boxed{Z_{eq1} = \frac{V_{sc}}{I_{sc}}}, \quad \boxed{Z_{eq1} = \sqrt{X_{eq1}^2 + R_{eq1}^2}}$$

Ar * Notes = OCT $\Rightarrow$ Rated is Voltage (V_{oc}).
 SCT $\Rightarrow$ Rated is Current (I_{sc}).

Ar * Transformer Rating ∞ ∞ ∞

A typical transformer carry the following info: 10kVA, 1100/110V.
 $\rightarrow$ It means $\therefore$ Voltage Rating indicates that the transformer has 2 winding, one rated for 1100V & the other for 110V.

$$\text{also } a = \text{turns ratio} = \frac{V_1}{V_2} = \frac{1100}{110} = 10$$

The 10kVA means that each winding is designed for 10kVA.

$$\text{then ; } I_{H_{\text{Rated}}} = \frac{10000}{1100} = 9.09 \text{ A. (Rated Current @ High Voltage Side).}$$

$$I_{L_{\text{Rated}}} = \frac{10000}{110} = 90.9 \text{ A. (Rated Current @ Low Voltage Side).}$$

Family

Tuesday

DATE

Example 8 1ϕ , 10kVA , $2200/220\text{V}$, 60Hz OCT: 220V , 2.5A , 100W . $\rightarrow$ LowSCT: 150V , 4.55A , 215W . $\rightarrow$ HighSolution 3 $V_{H\text{Rated}} = 2200\text{V}$ $V_{L\text{Rated}} = 220\text{V}$

$$I_{H\text{Rated}} = \frac{10000}{2200} = 4.55\text{A}$$

$$I_{L\text{Rated}} = \frac{10000}{220} = 45.5\text{A}$$

Ques OCT: Rated is Voltage = 220V .SCT: Rated is Current = 4.55A .Because $V_{OC} = 220\text{V}$ $\therefore$ The test was taken on Low Voltage Side, keeping High Voltage side open.Because $I_{SC} = 4.55\text{A}$ which rated current on high voltage side $\therefore$ The test was taken (done) on high voltage side, keeping Low voltage side short circuit.

$$I_{OC} = \frac{V_{OC}}{R_{CL}} \Rightarrow R_{CL} = \frac{V_{OC}^2}{P_{OC}} = \frac{(220)^2}{100} = 484\Omega$$

$$\Rightarrow I_{CL} = \frac{V_{OC}}{R_{CL}} = \frac{220}{484} = 0.45\text{A}$$

$$I_{SC} = \sqrt{I_{CL}^2 + I_m^2} \Rightarrow I_{mL} = \sqrt{I_{OC}^2 - I_{CL}^2} = \sqrt{(2.5)^2 - (0.45)^2} = 2.46\text{A}$$

$$\Rightarrow X_{mL} = \frac{V_{OC}}{I_{mL}} = \frac{220}{2.46} = 89.46\Omega$$

$$\text{Turns Ratio } (a) = \frac{2200}{220} = 10$$

$$\Rightarrow R_{CH} = a^2 R_{CL} = 100(484) = 48400\Omega$$

$$\Rightarrow X_{mH} = a^2 X_{mL} = 100(89.46) = 8946\Omega$$

continue $\rightarrow$

Date = 23/Feb/2012 Thursday

$$\Rightarrow R_{eq_H} = \frac{P_{sc}}{I_{sc}^2} = \frac{215}{(4.55)^2} = 10.4 \Omega$$

$$\Rightarrow Z_{eq_H} = \frac{V_{sc}}{I_{sc}} = \frac{150V}{4.55} = 32.97 \Omega$$

$$Z_{eq_H} = \sqrt{X_{eq_H}^2 + R_{eq_H}^2} \Rightarrow X_{eq_H} = \sqrt{Z_{eq_H}^2 - R_{eq_H}^2} = \sqrt{(32.97)^2 - (10.4)^2} = 31.3 \Omega$$

$$\Rightarrow R_{eq_L} = \frac{R_{eq_H}}{a^2} = \frac{10.4}{100} = 0.104 \Omega$$

$$\Rightarrow X_{eq_L} = \frac{X_{eq_H}}{a^2} = \frac{31.3}{100} = 0.313 \Omega$$

PF * Power Factor

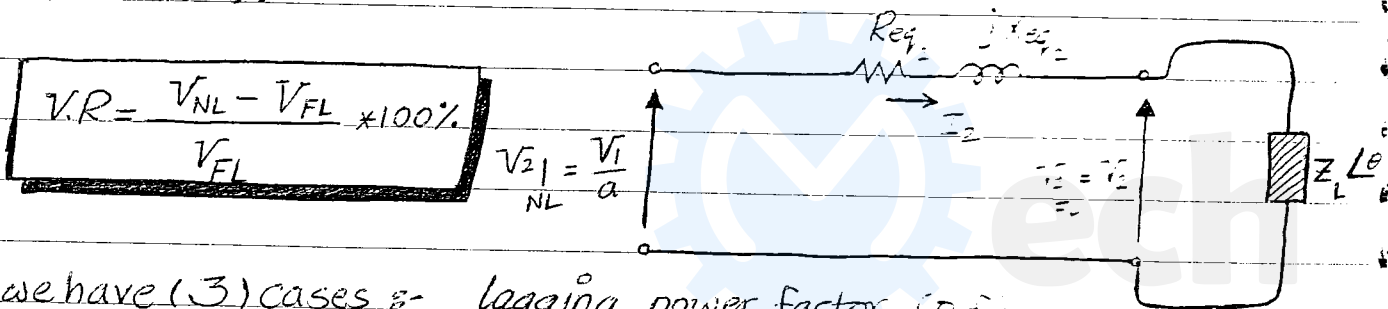
$$PF = \frac{P}{VI}$$

$$PF @ \text{No load} = \frac{P_{oc}}{V_{oc} I_{oc}} = \frac{100}{220 \times 2.5} = 0.182$$

$$PF @ \text{Short Circuit} = \frac{P_{sc}}{V_{sc} I_{sc}} = \frac{215}{150 \times 4.55} = 0.315$$

* Voltage Regulation

it's the change in magnitude of the secondary voltage as the load current changes from No Loaded to Loaded conditions.



we have (3) cases :-
 lagging power factor (p.f)
 unity power factor
 leading power factor

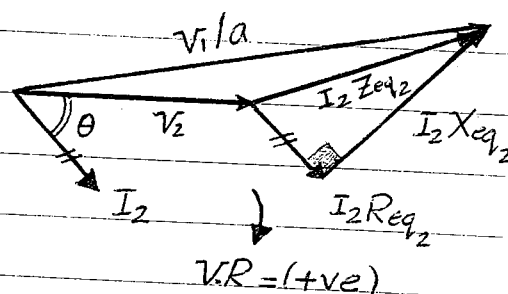
Thursday

DATE

1. Lagging power factor.

- Angle of (I_2) is negative (-ve) $\Rightarrow$ phase shift (-ve).
- Angle of (Z) is positive (+ve).

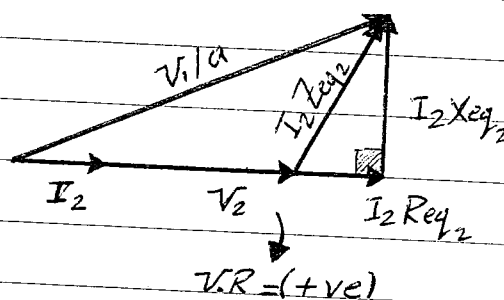
$$\Sigma (V_2 + I_2 R_{eq_2} + I_2 X_{eq_2}) = V_1/a = V_{NL}$$



2. Unity power factor.

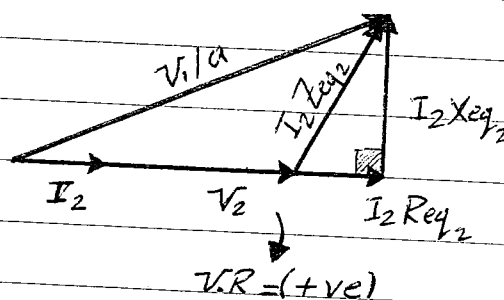
- Angle of (I_2) is zero
- Voltage & current are in-phase.

$$p.f = 1$$



3. Leading power factor.

- Angle of (I_2) is positive
- Angle of (Z) is negative.



4. (No Load Voltage) is bigger than (Full Load Voltage).

$$V_1/a = V_2 + I_2 (R_{eq_2} + jX_{eq_2})$$

$$= V_2 + I_2 (\cos \theta - j \sin \theta) (R_{eq_2} + jX_{eq_2})$$

$$V_{NL} = V_2 + I_2 (R_{eq_2} \cos \theta + X_{eq_2} \sin \theta) + j (X_{eq_2} \cos \theta - R_{eq_2} \sin \theta)$$

$$V_1/a = V_2 + I_2 (R_{eq_2} \cos \theta + X_{eq_2} \sin \theta)$$

Neglected $\Rightarrow$ 0

NL voltage $\Rightarrow$ current is Low.

VF voltage $\Rightarrow$ current is high.

* Note: $I_2 = I_{FL}$
 $V_2 = V_{FL}$

$$\therefore VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{V_1/a - V_2}{V_2} \times 100\%$$

$$= \frac{I_2 (R_{eq2} \cos \theta + X_{eq2} \sin \theta) + V_2 - V_2}{V_2}$$

$$VR = \frac{I_2 (R_{eq2} \cos \theta + X_{eq2} \sin \theta) \times 100\%}{V_2}$$

Approximate
Voltage Regulation

Example:- 10kVA, 2200/220 V. 60 Hz

OCT: 220V, 2.5A, 100W.

SCT: 150V, 4.55A, 215W.

(j*j = -1): لا تنسى *

- find V.R @
- ① 0.8 p.f lagging
 - ② unity p.f.
 - ③ 0.8 p.f Leading.

Solution :- $I_{FL} = I_2 = \frac{10000}{220} = 45.5A$, $V_2 = V_{FL} = 220V$.

$R_{eq2} = R_{eq1} = 0.104 \Omega$, $X_{eq2} = X_{eq1} = 0.313 \Omega$.

① 0.8 p.f lagging

$$\cos(\theta) = 0.8 \rightarrow \theta = -37^\circ \rightarrow \sin(-37^\circ) = -0.6$$

$$V_{NL} = \frac{V_1}{a} = 220 + 45.5 (0.8 - j0.6) (0.104 + j0.313)$$

$$= 220 + 45.5 (0.8 \times 0.104 + 0.6 \times 0.313)$$

$$+ j(45.5)(0.313 \times 0.8 - 0.104 \times 0.6) = 232.3 + j8.55$$

$$V_{NL} = \sqrt{(232.3)^2 + (j8.55)^2} = 232.1 \approx 232 \text{ V}$$

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{232 - 220}{220} \times 100\% = 5.5\%$$

DATE 26/Feb/2012 Sunday.

③ unity p.f = 1

$$\cos \theta = 1 \rightarrow \theta = 0 \rightarrow \sin(0) = 0.$$

$$V_{NL} = 220 + 45.5(1-0)(0.104-0) = 224.7 \text{ V.}$$

$$V.R = \frac{224.7 - 220}{220} = 2.15\%$$

③ 0.8 p.f leading

$$\cos \theta = 0.8 \rightarrow \theta = 37 \rightarrow \sin(37) = 0.6$$

$$\begin{aligned} V_{NL} &= 220 + 45.5(0.8 + j0.6)(0.104 + j0.313) \\ &= 215.24 + j14.23 \\ &= 215.7 \text{ V.} \end{aligned}$$

$$V.R = \frac{215.7 - 220}{220} \times 100\% = -2\%$$

--- Solution Using Approximate V.R ---

$$V.R = \frac{I_2}{V_2} \times (R_{eq2} \cos \theta + X_{eq2} \sin \theta) \times 100\%$$

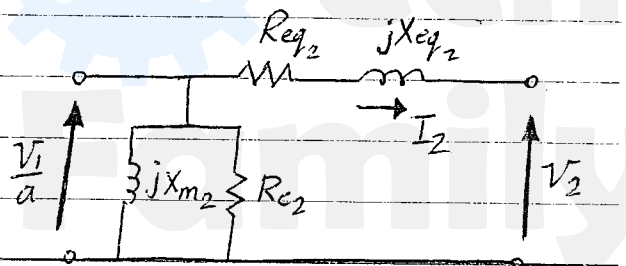
$$= \frac{45.5}{220} \times (0.104 \times 0.8 + 0.313 \times 0.6) \times 100\% = 5.6\%$$

$$V.R = \frac{45.5}{220} \times (0.104 \times 1 + 0.313 \times 0) \times 100\% = 2.15\%$$

$$V.R = \frac{45.5}{220} \times (0.104 \times 0.8 - 0.313 \times 0.6) \times 100\% = -2.16\%$$

--- Efficiency ---

$$\eta = \frac{\text{Output Power (P}_{out})}{\text{Input Power (P}_{in})} \times 100\%$$



$$P_{in} = P_{out} + P_{loss}$$

$$\therefore \eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \left(1 - \frac{P_{loss}}{P_{in}}\right) \times 100\%$$

* ملاحظة: يوجد مرتين في المصطلح Losses

هنا R_{eq2} و R_{eq2} $R_{eq2} \rightarrow$ iron loss (i) $R_{eq2} \rightarrow$ copper loss (cu)

$R_{eq2} \rightarrow$ copper loss (cu)

WHERE $\Rightarrow P_{out} = V_2 I_2 \cos \theta$

$$\Rightarrow P_{loss} = P_{iron} + P_{copper} = P_i + I_2^2 R_{eq2} \quad ; \quad P_i = \frac{V_1^2}{R_{e1}} \approx \frac{V_2^2}{R_{e2}}$$

$$\therefore \eta = \frac{V_2 I_2 \cos \theta}{V_2 I_2 \cos \theta + P_i + I_2^2 R_{eq2}} \times 100\% \quad \text{Full load}$$

NOTE: @ $I_2 = 0$ OR $I_2 = \infty \rightarrow \eta = \text{zero}$

to find maximum efficiency

take derivate w.r.t I_2 & $\theta \rightarrow \frac{\partial \eta}{\partial I_2} = 0, \frac{\partial \eta}{\partial \theta} = 0$ الاشتقاق

$$\frac{\partial \eta}{\partial I_2} = \frac{(V_2 I_2 \cos \theta + P_i + I_2^2 R_{eq2})(V_2 \cos \theta) - (V_2 I_2 \cos \theta)(V_2 \cos \theta + 2 I_2 R_{eq2})}{(V_2 I_2 \cos \theta + P_i + I_2^2 R_{eq2})^2}$$

$$\Rightarrow (V_2 I_2 \cos \theta + P_i + I_2^2 R_{eq2})(V_2 \cos \theta) - (V_2 I_2 \cos \theta)(V_2 \cos \theta + 2 I_2 R_{eq2}) = 0$$

$$V_2 I_2 \cos \theta + P_i + I_2^2 R_{eq2} - V_2 I_2 \cos \theta - 2 I_2^2 R_{eq2} = 0$$

$$\Rightarrow P_i = I_2^2 R_{eq2}$$

for Max. Efficiency, we have (2) conditions:

① $\theta = 0$ (I_2 & V_2 are in-phase).

② $P_i = I_2^2 R_{eq2}$ (Iron Loss = Copper Loss).

$$\Rightarrow \eta_{max} = \frac{V_2 I_2 \cos \theta}{V_2 I_2 \cos \theta + 2 I_2^2 R_{eq2}} \times 100\% \quad \text{Full load}$$

$$I_{2max} = I_{2fl} \times k$$

$$\therefore I_2 \Big|_{\max \eta} = \sqrt{\frac{P_i}{R_{eq2}}} \quad \text{--- (A)}$$

DATE

$$\text{In OCT :- } P_{oc} = P_i \quad \text{--- (I)}$$

$$\text{SCT :- } P_{sc} = I_{2,FL}^2 R_{eq2} \quad \text{--- (II)}$$

$$\Rightarrow \frac{P_{oc}}{P_{sc}} = \frac{P_i}{P_{oc}} = \frac{I_{2,max\eta}^2 R_{eq2}}{I_{2,FL}^2 R_{eq2}} = \frac{I_{2,max\eta}^2}{I_{2,FL}^2}$$

$$\Rightarrow \frac{I_{2,max\eta}}{I_{2,FL}} = \sqrt{\frac{P_{oc}}{P_{sc}}} = k \quad \begin{matrix} \text{(per unit loading)} \\ \text{(per unit Rating)} \end{matrix} ; k < 1$$

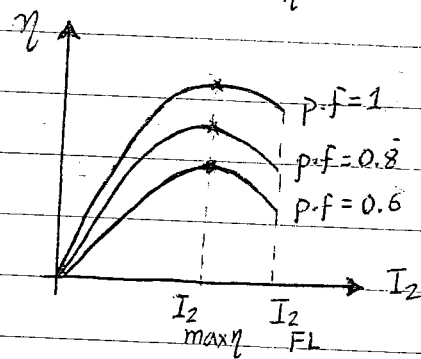
$$\therefore I_{2,max\eta} = k I_{2,FL} \quad \text{--- (B)}$$

A & B : $\frac{P_{oc}}{P_{sc}}$ * $I_{2,max\eta}$ $I_{2,max\eta}$ is always positive

because the p.f is lagging or

leading because $\cos(-\theta) = \cos\theta$

$$\eta = \frac{V_2 (k I_{2,FL} \cos\theta)}{V_2 (k I_{2,FL} \cos\theta) + 2 I_{2,FL}^2 R_{eq2}} \times 100\%$$



$$10 \text{ KVA}, 2200/220 \text{ V}, I_{FL} = I_2 = 45.5 \text{ A}$$

$$R_{eq2} = 0.104 \Omega, P_{oc} = 100 \text{ W}, P_{sc} = 215 \text{ W}$$

Full load efficiency @ 0.8 p.f lagging.

Maximum efficiency @ 0.8 p.f leading.

$$\text{① } \eta_{FL} = \frac{10000 \times 0.8}{10000 \times 0.8 + 100 + 215} \times 100\% = 96.2\%$$

$$P_i = P_{oc} = 100 \text{ W} \text{ \& } P_{copper} = P_{sc} = I_{2,FL}^2 R_{eq2} = (45.5)^2 (0.104) = 215 \text{ W}$$

$$k = \sqrt{P_{oc}/P_{sc}} = \sqrt{100/215} = 0.682 \Rightarrow I_{2,max\eta} = 31.03 \text{ A}$$

$$\text{② } \eta_{max} = \frac{10000 \times 0.8 \times 0.682}{10000 \times 0.8 \times 0.682 + 2(100)} \times 100\% = 96.5\%$$

$$P_i \text{ of } \text{Max}(\eta) \text{ is equal to } = (I_{2,max})^2 R_{eq2} = (31.03)^2 \times 0.104 = 100 \text{ W}$$

* All-DAY Efficiency (η_{AD})₀₀₀

$$\eta_{all\ day} = \frac{\text{Output Energy over 24 hours}}{\text{Input Energy over 24 hours}} \times 100\%$$

OR

$$\eta_{all\ day} = \frac{24\ \text{output energy}}{24\ \text{output energy} + 24\ P_i + 24\ \text{copper loss}}$$

Where; 24 hrs. output energy = $\sum_n V_2 I_2 \cos\theta \cdot t_n = \sum P_{out} \cdot t$
 24 hrs. copper loss = $\sum_n I_2^2 R_{eq_2} \cdot t_n = \sum P_{cu} \cdot t$

* Example: 10 kVA, 2200/220 V, $P_i = P_{oc} = 100\text{ W}$, $P_{sc} = 215\text{ W}$.

- full load @ 0.8 p.f lagging for 5 hrs.
- 0.75 full load @ 0.9 p.f lagging for 7 hrs.
- 0.5 full load @ 0.85 p.f lagging for 6 hrs.
- 0.25 full load @ unity p.f for 6 hrs.

→ Find All-day Efficiency?

Solution:

$$\eta_{all\ day} = \frac{(10000) [0.8 \times 5 + 0.75 \times 0.9 \times 7 + 0.5 \times 0.85 \times 6 + 0.25 \times 1 \times 6]}{(127750) + (24 \times 100) + [215 \times 5 + 0.75^2 \times 215 \times 7 + 0.5^2 \times 215 \times 6 + 0.25^2 \times 215 \times 6]}$$

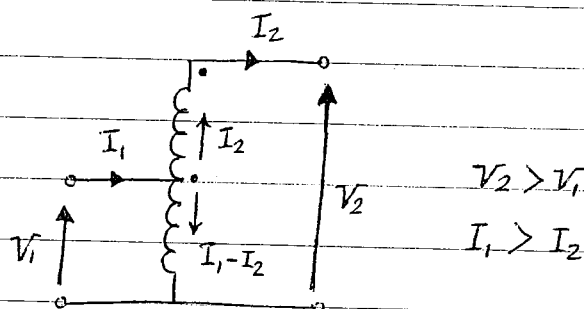
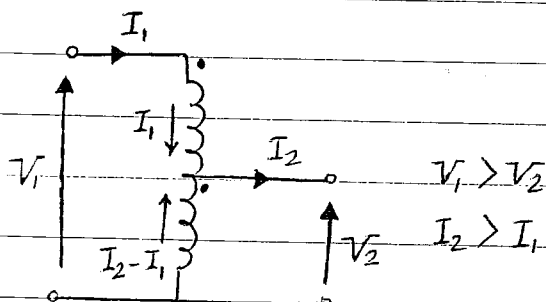
* ملاحظة: بخصوص ال (P_{cu}) عندنا يكون (Full load) فإن $P_{cu} = I_{FL}^2 R_{eq_2}$
 ولكن عندنا يكون مثلاً (0.5 Full load) فإن $P_{cu} = (0.5 I_{FL})^2 R_{eq_2}$
 أو $P_{new} = (0.5)^2 P_{cu}$

$$\therefore \eta_{all\ day} = \left(\frac{127750}{127750 + 2400 + 2324.7} \right) \times 100\% = 96.4\%$$

Tuesday

DATE

* Auto-Transformers



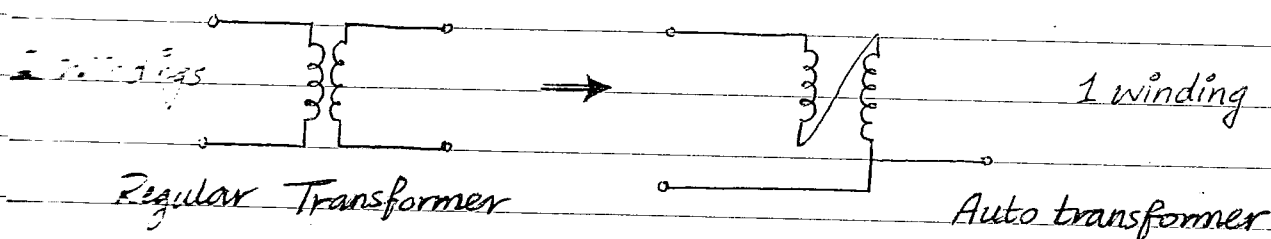
* Step Down Auto transformer

* Step Up Auto transformer

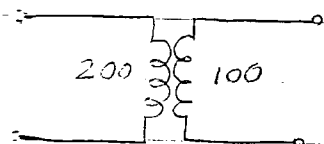
∴ Auto-Transformer has 1 winding & there's No Isolation at the middle.

∴ Coupling means → The flux (ϕ) in the first side generates an (emf) in the other side.

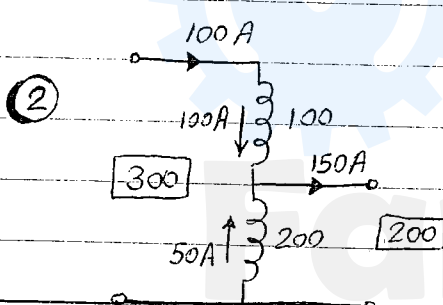
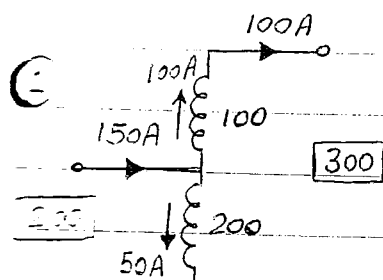
∴ can change A Regular Transformer (2 Windings) to an Autotransformer (1 winding) by the change in connections!



Ex: A 2 winding transformers, find all possible connections when converting it to Autotransformer.



⇒ There's 4 different connections.

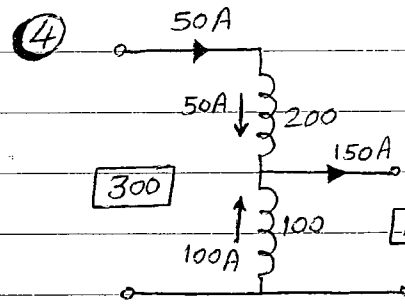
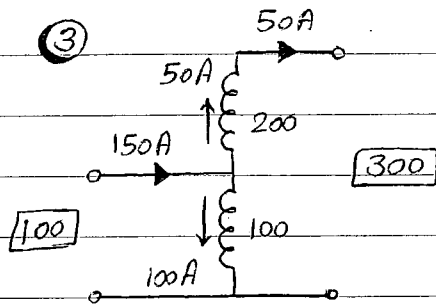


$$VA = 100 \times 300 = 30kVA$$

$$VA = 200 \times 150 = 30kVA$$

$$I_H = \frac{10000}{200} = 50A$$

$$I_L = \frac{10000}{100} = 100A$$



$$I_H = \frac{10000}{200} = 50 \text{ A}$$

$$I_L = \frac{10000}{100} = 100 \text{ A}$$

$$VA = 150 \times 100 = 15 \text{ kVA}$$

$$VA = 50 \times 300 = 15 \text{ kVA}$$

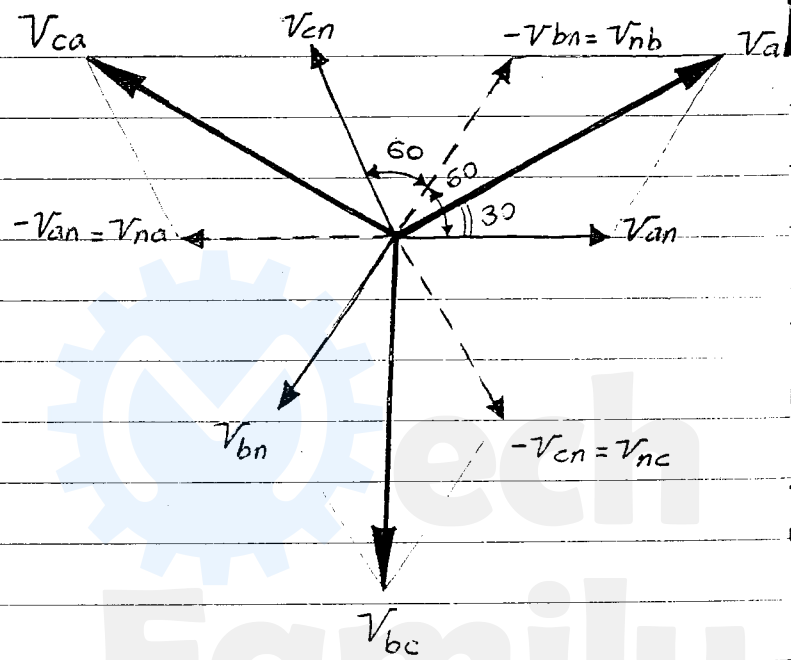
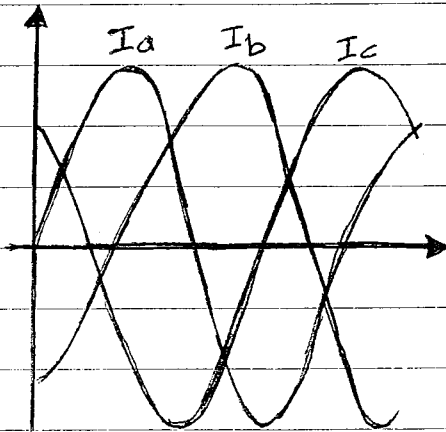
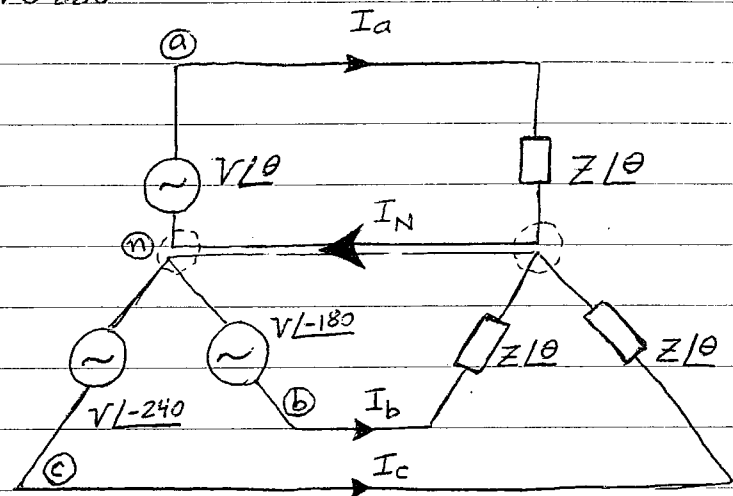
* Three-Phase Transformers

$$I_a = \frac{V/\theta}{Z/\theta} = \frac{|V|}{|Z|} \angle -\theta$$

$$I_b = \frac{V/\theta - 180}{Z/\theta} = \frac{|V|}{|Z|} \angle -\theta - 180$$

$$I_c = \frac{V/\theta - 240}{Z/\theta} = \frac{|V|}{|Z|} \angle -\theta - 240$$

$$I_a + I_b + I_c = I_N = 0$$



$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$V_{bc} = \sqrt{3} V_{bn} \angle -90^\circ$$

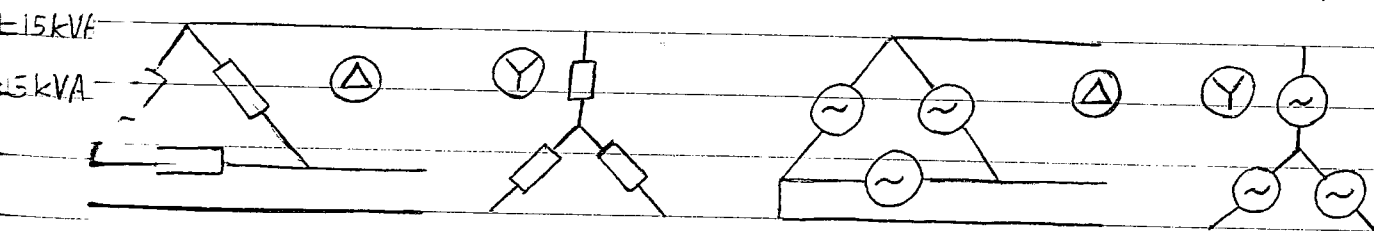
$$V_{ca} = \sqrt{3} V_{cn} \angle 150^\circ$$

nday

DATE

... in general :
= Single phase $\Rightarrow$ 3 phase

* Generators in general :
 $\Rightarrow$ Single phase $\Rightarrow$ 3 phase

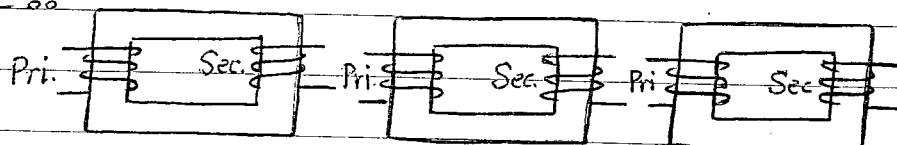


Power = $3 I_p V_p \cos \theta$
 $= \sqrt{3} I_L V_L \cos \theta$ } valid for (Y) star connection
 and for (Δ) delta connection

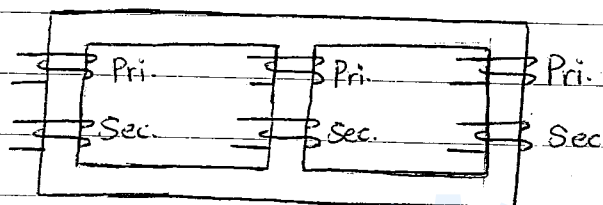
NOTE : in (Y) connection $\Rightarrow I_L = I_p$ and $V_L = \sqrt{3} V_p$
 in (Δ) connection $\Rightarrow I_L = \sqrt{3} I_p$ and $V_L = V_p$

Types of 3-phase Transformers

Core Type

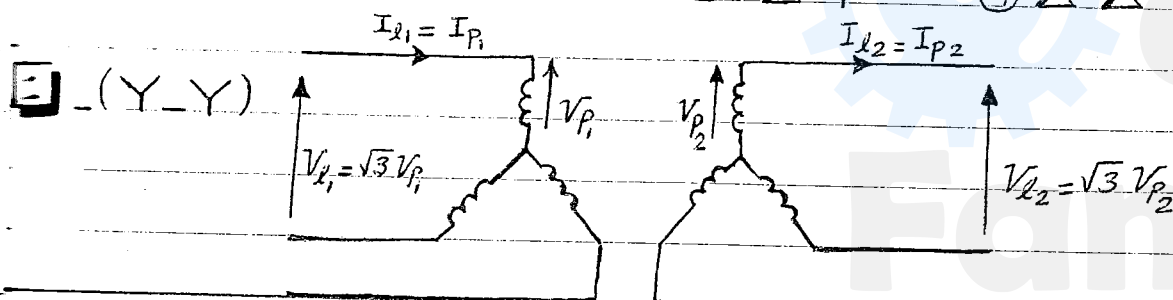


Shell Type



There are 4 possible connections of 3-phase Transformers

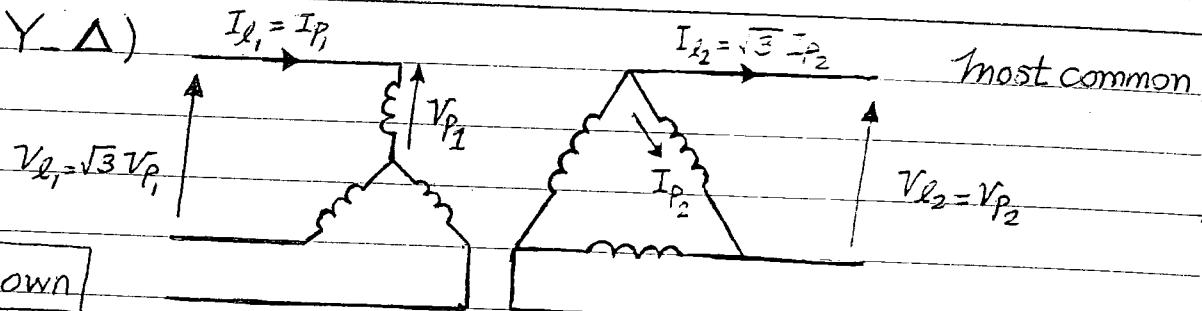
① Y-Y ② Y- Δ ③ Δ -Y ④ Δ - Δ



$$\frac{V_{P1}}{V_{P2}} = a = \frac{V_{L1}/\sqrt{3}}{V_{L2}/\sqrt{3}} = \frac{V_{L1}}{V_{L2}} = \frac{I_{L2}}{I_{L1}} = \frac{I_{P2}}{I_{P1}}$$

6/11/2012 Tuesday

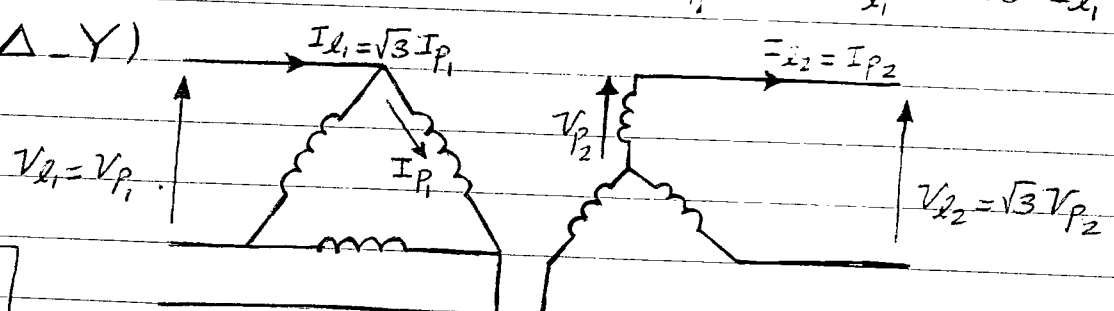
2 (Y-Δ)



Step Down

$$\frac{V_{P1}}{V_{P2}} = a = \frac{V_{L1}/\sqrt{3}}{V_{L2}} = \frac{1}{\sqrt{3}} \frac{V_{L1}}{V_{L2}} = \frac{I_{P2}}{I_{P1}} = \frac{I_{L2}/\sqrt{3}}{I_{L1}} = \frac{1}{\sqrt{3}} \frac{I_{L2}}{I_{L1}}$$

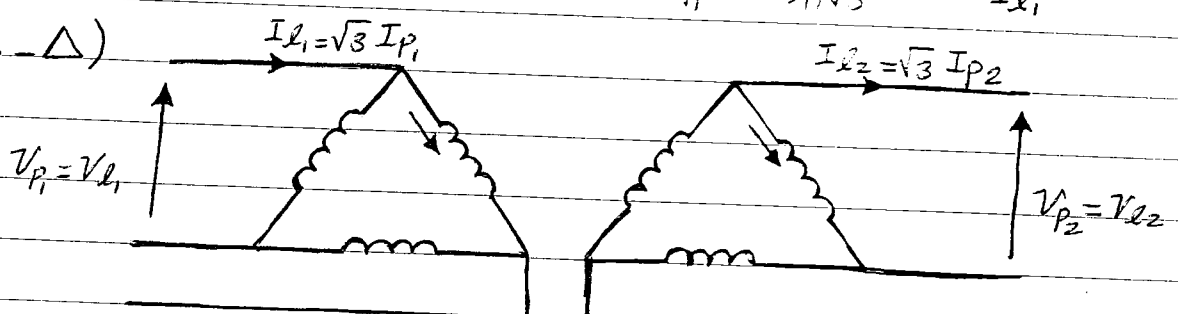
3 (Δ-Y)



Step Up

$$\frac{V_{P1}}{V_{P2}} = a = \frac{V_{L1}}{V_{L2}/\sqrt{3}} = \sqrt{3} \frac{V_{L1}}{V_{L2}} = \frac{I_{P2}}{I_{P1}} = \frac{I_{L2}}{I_{L1}/\sqrt{3}} = \sqrt{3} \frac{I_{L2}}{I_{L1}}$$

4 (Δ-Δ)



$$\frac{V_{P1}}{V_{P2}} = a = \frac{V_{L1}}{V_{L2}} = \frac{I_{P2}}{I_{P1}} = \frac{I_{L2}/\sqrt{3}}{I_{L1}/\sqrt{3}} = \frac{I_{L2}}{I_{L1}}$$

* Per-Unit (p.u) System ...

$$\text{Quantity in p.u} = \frac{\text{Actual Value}}{\text{Base Value of the quantity}}$$

* to make p.u system, we select Base values for any of:
Power (P), Voltage (V), Current (I) & Impedance (Z).

sdau

DATE

- ① Power OR VA. $\Rightarrow P_{Base} = \text{Rated VA}$
- ② Voltage (V) $\Rightarrow V_{Base1}, V_{Base2}$
- ③ Current (I) $\Rightarrow I_{Base1}, I_{Base2}$
- ④ Impedance (Z) $\Rightarrow Z_{Base1}, Z_{Base2}$

$$\begin{aligned} & \rightarrow \boxed{I_{B1} = \frac{P_B}{V_{B1}}} \text{ (Rated current of primary.)} \rightarrow \boxed{Z_{B1} = \frac{V_B}{I_{B1}}} \\ & \rightarrow \boxed{I_{B2} = \frac{P_B}{V_{B2}}} \text{ (Rated current of Secondary.)} \rightarrow \boxed{Z_{B2} = \frac{V_B}{I_{B2}}} \end{aligned}$$

Turns Ratio $a = \frac{V_{B1}}{V_{B2}} = \frac{I_{B2}}{I_{B1}}$

Impedance

$$\begin{aligned} Z_{eq1, p.u.} &= \frac{Z_{eq1}}{Z_{Base1}} ; \text{ But } Z_{B1} = \frac{V_{B1}}{I_{B1}} = \frac{a V_{B2}}{I_{B2}/a} = a^2 \frac{V_{B2}}{I_{B2}} = a^2 Z_{B2} \\ \Rightarrow Z_{eq1, p.u.} &= \frac{a^2 Z_{eq2}}{a^2 Z_{B2}} = Z_{eq2, p.u.} \Rightarrow \text{constant for primary \& Secondary} = Z_{p.u.} \end{aligned}$$

Note 3 we have the similar thing for $R_{p.u.}, X_{p.u.}, Y_{p.u.}$

Voltage Regulation (p.u.)

$$V.R. = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{I_2}{V_2} [R_{eq2} \cos \theta + X_{eq2} \sin \theta] \times 100\%$$

$$V.R. = \frac{1}{Z_{B2}} [R_{eq2} \cos \theta + X_{eq2} \sin \theta] \times 100\%$$

$$V.R. = \left[\frac{R_{eq2}}{Z_{B2}} \cos \theta + \frac{X_{eq2}}{Z_{B2}} \sin \theta \right] \times 100\%$$

$$\Rightarrow \boxed{V.R. = [R_{eq, p.u.} \cos \theta + X_{eq, p.u.} \sin \theta] \times 100\%}$$

* Efficiency (p.u) η

$$\eta = \frac{P_{out}}{P_{out} + P_i + P_{cu}} \times 100\% = \frac{\left(\frac{P_{out}}{P_B}\right)}{\left(\frac{P_{out}}{P_B}\right) + \left(\frac{P_i}{P_B}\right) + \left(\frac{P_{cu}}{P_B}\right)} \times 100\%$$

Let's see ① iron loss η

$$\Rightarrow \left(\frac{P_i}{P_B}\right) = \frac{V^2/R_c}{P_B} = \left(\frac{1}{R_{c.p.u}}\right)$$

② copper loss η

$$\Rightarrow \left(\frac{P_{cu}}{P_B}\right) = \frac{I_2^2 R_{eq2}}{I_2^2 Z_{B2}} = (R_{eq.p.u})$$

③ Input Energy

$$\Rightarrow \left(\frac{P_{in}}{P_B}\right) = \frac{V_2 I_2 \cos \theta}{V_{B2} I_{B2}}$$

$$= \frac{V_{2.p.u} I_{2.p.u} \cos \theta}{(1 \cos \theta)}$$

$$\eta = \frac{V_{2.p.u} I_{2.p.u} \cos \theta}{V_{2.p.u} I_{2.p.u} \cos \theta + (1/R_{c.p.u}) + (R_{eq.p.u})} \times 100\%$$

* Example: 10 kVA, 2200/220, $R_{eqH} = 10.4 \Omega$, $X_{eqH} = 31.3 \Omega$
 $R_{cH} = 48400 \Omega$, $X_{mH} = 8946 \Omega$.

$$\Rightarrow V_{Base1} = 2200 \text{ V}$$

$$\Rightarrow V_{Base2} = 220 \text{ V}$$

$$\Rightarrow I_{Base2} = \frac{10000}{220} = 45.5 \text{ A}$$

$$\Rightarrow I_{Base1} = \frac{10000}{2200} = 4.5 \text{ A}$$

$$\Rightarrow Z_{Base1} = \frac{V_{Base1}}{I_{Base1}} = \frac{2200}{4.55} = 484 \Omega$$

$$\Rightarrow Z_{Base2} = \frac{V_{Base2}}{I_{Base2}} = \frac{220}{45.5} = 4.84 \Omega$$

$$\Rightarrow R_{eq.p.u} = \frac{R_{eqH}}{Z_{Base1}} = \frac{10.4}{484} = 0.0215 \Rightarrow X_{eq.p.u} = \frac{X_{eqH}}{Z_{Base1}} = \frac{31.3}{484} = 0.065$$

$$\Rightarrow R_{c.p.u} = \frac{R_{cH}}{Z_{B1}} = \frac{48400}{484} = 100$$

$$\Rightarrow X_{m.p.u} = \frac{X_{mH}}{Z_{B1}} = \frac{8900}{484} = 18.4$$

DATE 8/Mar/2012 Thursday

2.5) 1 ϕ , 25 kVA, 220/440 V Transformer,

OCT : (440 V side open) 220 V, 9.5 A, 650 W $\rightarrow$ Low

SCT : (220 V side short) 37.5 V, 55 A, 950 W $\rightarrow$ High

- Derive the approximate equivalent circuit in p.u. values.
- Determine V.R @ full load, 0.8 p.f lagging.
- Draw the phasor diagram for condition (b).

Solution :- $I_{H \text{ Rated}} = \frac{25000}{440} = 56.8 \text{ A}$, $I_{L \text{ Rated}} = \frac{25000}{220} = 113.6 \text{ A}$

$a = \frac{220}{440} = \frac{1}{2} = 0.5 \Rightarrow \text{OCT (Low)} \Rightarrow \text{SCT (High)}$

$R_{CL} = \frac{V_{oc}^2}{P_{oc}} = \frac{220^2}{650} = 74.46 \Omega \Rightarrow R_{CH} = a^2 R_{CL} = (0.5)^2 (74.46) = 18.62 \Omega$

$I_{CL} = \frac{V_{oc}}{R_{CL}} = \frac{220}{74.46} = 2.95 \text{ A} \rightarrow I_{mL} = \sqrt{I_{oc}^2 - I_{cL}^2} = \sqrt{9.5^2 - 2.95^2} = 9.03 \text{ A}$

$X_{mL} = \frac{V_{oc}}{I_{mL}} = \frac{220}{9.03} = 24.40 \Omega \Rightarrow X_{mH} = a^2 X_{mL} = (0.5)^2 (24.4) = 6.09 \Omega$

$R_{eqH} = \frac{P_{sc}}{I_{sc}^2} = \frac{950}{55^2} = 0.314 \Omega \Rightarrow R_{eqL} = \frac{R_{eqH}}{a^2} = \frac{0.314}{(0.5)^2} = 1.26 \Omega$

$Z_{eqH} = \frac{V_{sc}}{I_{sc}} = \frac{37.5}{55} = 0.682 \Omega$

$X_{eqH} = \sqrt{Z_{eqH}^2 - R_{eqH}^2} = \sqrt{(0.682)^2 - (0.314)^2} = 0.61 \Omega \Rightarrow X_{eqL} = \frac{X_{eqH}}{a^2} = \frac{0.61}{(0.5)^2} = 2.42 \Omega$

Values :- $P_B = 25000 \text{ VA}$

$V_{BL} = 220 \text{ V}$, $V_{BH} = 440 \text{ V}$

$I_{BL} = 113.6 \text{ A}$, $I_{BH} = 56.8 \text{ A}$

$Z_{BL} = \frac{V_{BL}}{I_{BL}} = \frac{220}{113.6} = 1.94 \Omega$, $Z_{BH} = \frac{V_{BH}}{I_{BH}} = \frac{440}{56.8} = 7.75 \Omega$

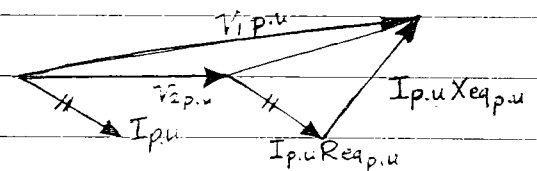
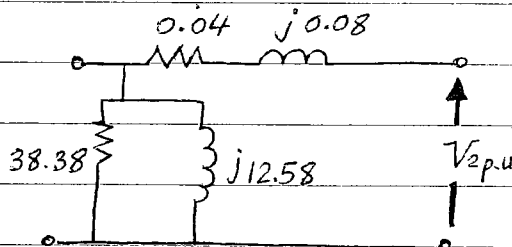
$$\Rightarrow R_{cp.u} = \frac{R_{CL}}{Z_{BL}} = \frac{74.46}{1.94} = 38.38 \text{ p.u.}$$

$$\Rightarrow X_{mp.u} = \frac{X_{mL}}{Z_{BL}} = \frac{24.4}{1.94} = 12.58 \text{ p.u.}$$

$$\Rightarrow R_{eq.p.u} = \frac{R_{eqH}}{Z_{BH}} = \frac{0.314}{7.75} = 0.04 \text{ p.u.}$$

$$\Rightarrow Z_{eq.p.u} = \frac{Z_{eqH}}{Z_{BH}} = \frac{0.682}{7.75} = 0.088 \text{ p.u.} \Rightarrow X_{eq.p.u} = \frac{X_{eqH}}{Z_{BH}} = \frac{0.61}{7.75} = 0.08 \text{ p.u.}$$

$$\bullet V.R. = (R_{eq.p.u} \cos \theta + X_{eq.p.u} \sin \theta) \times 100\% = 0.04 \times 0.8 + 0.08 \times 0.6 = 7.9\% \approx 8\%$$



P(2.8) 1 ϕ , 440 V, 80 kW having 0.8 p.f lagging is supplied through a feeder of impedance $0.6 + j1.6 \Omega$. & 1 ϕ , 100kVA 220/440 V, the Z_{eq} of transformer referred to High voltage side is $1.15 + j4.5 \Omega$.

- Draw the schematic diagram.
- Determine the voltage @ High-voltage terminal of Transformer.
- Determine the voltage @ the sending end of the feeder.

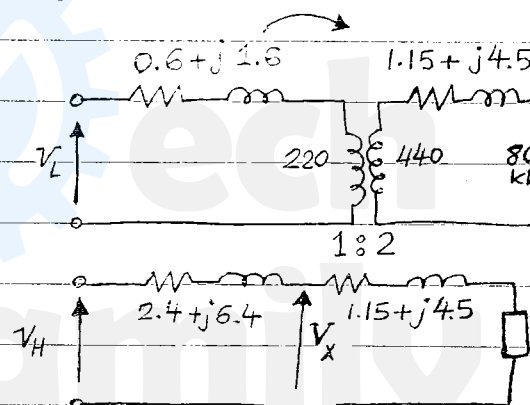
$$\text{Solution: } a = \frac{220}{440} = 0.5$$

$$Z_{eq1} = a^2 Z_{eq2}$$

$$Z_{eq2} = (0.6 + j1.6) / (0.5)^2 = 2.4 + j6.4$$

$$P_{Load} = V_L I_L \cos \theta$$

$$I_{Load} = \frac{80000}{440 \times 0.8} = 227.3 \text{ A}$$



$$V_H = 440 + 227.3(0.8 - j0.6)(2.4 + 6.4j + 1.15 + j4.5)$$

$$= 2572 + j1497.9$$

$$= 2976.4 \text{ V}$$

$$V_x = 440 + 227.3(0.8 - j0.6)(1.15 + j4.5)$$

$$= 1262.8 + j661.4$$

$$= 1425.5 \text{ V}$$

2.11) 1 ϕ , 250 kVA, 11 kV/2.2 kV transformer.

$$R_{HV} = 1.3 \Omega \quad X_{HV} = 4.5 \Omega$$

$$R_{LV} = 0.05 \Omega \quad X_{LV} = 0.16 \Omega$$

$$R_{CL} = 2.4 \text{ k}\Omega \quad X_{mL} = 0.8 \text{ k}\Omega$$

Draw the approximate equivalent circuit referred to High Voltage side.

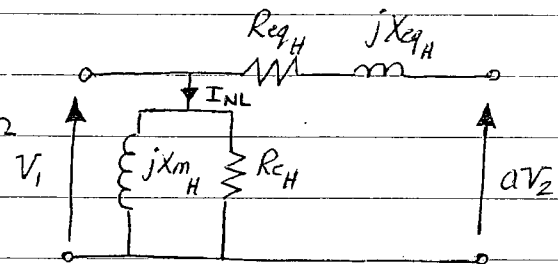
$$a = \frac{11000}{2200} = 5 \rightarrow a^2 = 25$$

$$\Rightarrow R_{eqH} = R_{HV} + a^2 R_{LV} = 1.3 + (5)^2 \times 0.05 = 2.55 \Omega$$

$$X_{eqH} = X_{HV} + a^2 X_{LV} = 4.5 + (5)^2 (0.16) = 8.5 \Omega$$

$$R_{CH} = a^2 R_{CL} = (5)^2 (2400) = 60 \text{ k}\Omega$$

$$X_{mH} = a^2 X_{mL} = (5)^2 (800) = 20 \text{ k}\Omega$$



Determine the no-load Current (High Voltage Side) & in p.u.

$$I_{NL} = \sqrt{I_c^2 + I_m^2}, \quad I_c = \frac{11000}{60000} = 0.183 \text{ A}, \quad I_m = \frac{11000}{22000} = 0.55 \text{ A}$$

$$\Rightarrow I_{NL} = \sqrt{0.183^2 + 0.55^2} = 0.5796 \text{ A}$$

$$V_{BaseH} = 11000 \text{ V}, \quad P_{Base} = 250 \text{ kW} \Rightarrow I_{BH} = \frac{250000}{11000} = 22.73 \text{ A}$$

$$\Rightarrow I_{NL \text{ p.u.}} = \frac{I_{NL}}{I_{BH}} = \frac{0.5796}{22.73} = 0.0255 \text{ p.u.}$$

If LV winding terminal shorted, find supply voltage & losses in transformer

$$Z_{eqH} = 2.55 + j8.5 = 8.874 \Omega \rightarrow V_H = Z_{eqH} I_{BH} = (8.874)(22.73) = 201.7 \text{ V}$$

$$P_c = V_H^2 / R_{CH} = 201.7^2 / 60000 = 0.678 \text{ W} \rightarrow \text{also}$$

$$P_{cu} = I_H^2 R_{eqH} = 22.73^2 \times 2.55 = 1317.4 \text{ W}$$

DATE = 11/Mar/2012 Sunday

d) HV winding is connected to 11 kV supply & a load $Z_L = 15 \angle -90^\circ$ is connected to LV winding. Determine: Load voltage & V.R.

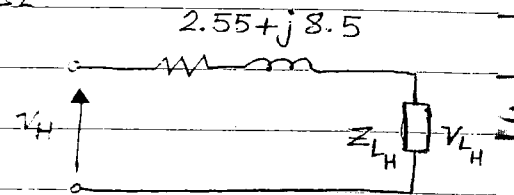
$$\Rightarrow Z_{L_H} = a^2 Z_L = 25 \times 15 \angle -90^\circ = -375j = -j375 \Omega$$

$$V_L = 11000 \text{ V.}$$

$$\Rightarrow V_{L_H} = \frac{11000}{2.55 + j8.5 - j375} * (-j375)$$

$$= 11254.6 - j76.8 = 11255 \text{ V.}$$

$$\Rightarrow \text{V.R.} = \frac{11000 - 11255}{11255} * 100\% = -2.27\%$$



P(2.15) - 1 ϕ , 10 kVA, 2400/240 V. $P_c @ FL = 100 = P_i$

$$P_{cu} @ \frac{1}{2} FL = 60 \text{ W.}$$

a) Determine (η) when it delivers full load @ 0.8 p.f lagging.

b) Determine p.u Rating @ which (η) is max. Determine (η)_{max} @ 0.9 p.f.

c) No Load for 6 hrs.

70% Load for 10 hrs @ 0.8 p.f

90% Load for 8 hrs @ 0.9 p.f $\rightarrow$ determine $\eta_{\text{all day}}$

Solution: $P_i = P_o = P_c = 100 \text{ W.}$

$$P_{cu}|_{FL} = \frac{60 \text{ W}}{(0.5)^2} = 240 \text{ W.}$$

$$\text{a) } \eta = \frac{10000 * 0.8}{10000 * 0.8 + 100 + 240} * 100\% = 95.92\%$$

$$\text{b) } k = \text{per unit Rating} = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{100}{240}} = 0.645$$

$$\eta_{\text{max}} = \frac{10000 * 0.9 * 0.645}{10000 * 0.9 * 0.645 + 2 * 100} * 100\% = 96.67\%$$

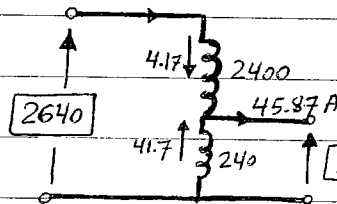
$$\begin{aligned} \text{c) } \eta_{\text{all day}} &= \frac{10000(0 * 6 + 0.7 * 10 * 0.8 + 0.9 * 8 * 0.9)}{120800 + 24(100) + (240 * 0.7^2 * 10 + 240 * 0.9^2 * 8)} * 100\% \\ &= 95.93\% \end{aligned}$$

- P(2.16) - The transformer in P(2.15) is to be used as an Auto-transformer. a) Show connections will result Max. kVA.
 b) Determine Voltage Rating of two sides.
 c) Determine kVA Rating of two sides.

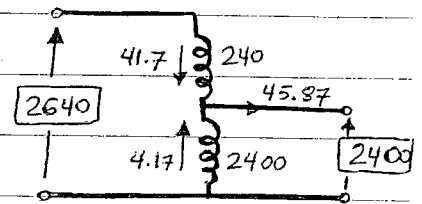
Solution :-

$$I_H = \frac{10000}{2400} = 4.17 \text{ A}$$

$$I_L = \frac{10000}{240} = 41.7 \text{ A}$$



$$\Rightarrow VA = 2640 \times 4.17 = 11009 \text{ VA}$$



$$\Rightarrow VA = 2640 \times 41.7 = 110090 \text{ VA} = \text{Max. kVA}$$

Example :- 3 ϕ Transformer, 20 kVA, 2300/230 V.

(Y-Y) connections, $Z_{eq_H} = 1 + j2 \Omega$. Low Voltage Feeder is equal to $= 0.2 + j0.4 \Omega$. Find the Supply Voltage for full load voltage of 230 V @ 0.8 p.f lagging?

$$I_2 = \frac{20000}{230} = 87 \text{ A}, a = 10$$

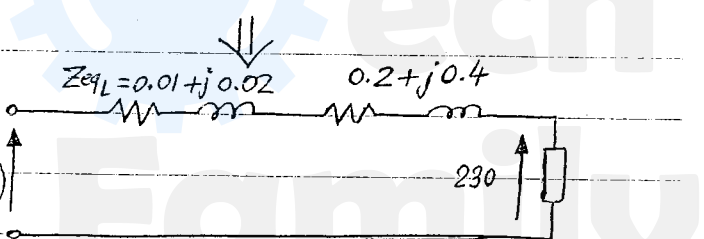
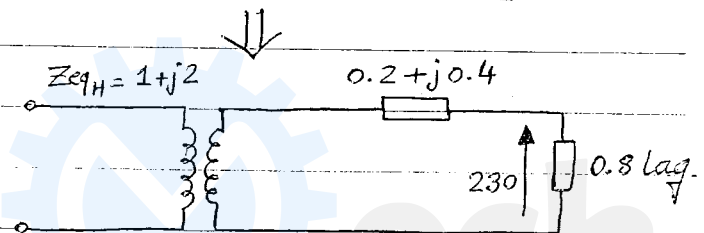
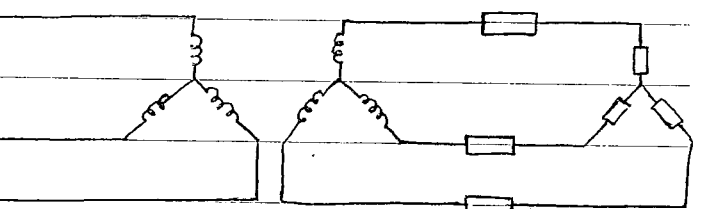
$$\begin{aligned} \frac{V_1}{a} &= V_2 + I_2 (\cos\theta - j\sin\theta) (R_{eq_2} + jX_{eq_2}) \\ &= 230 + 87 * (0.8 - j0.6) (R_{eq_2} + jX_{eq_2}) \end{aligned}$$

$$\begin{aligned} \text{But } Z_{eq_H} &= a^2 Z_{eq_L} \Rightarrow Z_{eq_L} = \frac{Z_{eq_H}}{a^2} \\ Z_{eq_L} &= 0.01 + j0.02 \end{aligned}$$

$$\begin{aligned} \therefore \frac{V_1}{a} &= \frac{V_1}{10} = 230 + 87(0.8 - j0.6) \\ &\quad * (0.01 + j0.02 + 0.2 + j0.4) \end{aligned}$$

$$\therefore \frac{V_1}{10} = 267 \Rightarrow V_1 = 10 \times 267 = 2670$$

$$\therefore V_2 = \sqrt{3} V_p = \sqrt{3} \times 2670 = 4625$$



P(2.24) - 1 ϕ Transformer, $X_{p.u} = 0.04$, $P_{cu.p.u} = 0.015$
 $P_{i.p.u} = 0.01$, 0.85 p.f lagging.

a) Determine the efficiency b) Determine the V.R.

Solution :-

$$a) \eta = \frac{1 \times 0.85}{0.85 + 0.01 + 0.015} \times 100\% = 97.14\%$$

$$b) V.R = [R_{eq.p.u} \cos \theta + X_{eq.p.u} \sin \theta] \times 100\%$$

$$\Rightarrow \text{But } P_{cu.p.u} = 0.015 = \frac{I_2^2 R_{eq2}}{I_2^2 Z_{B2}} = \frac{R_{eq2}}{Z_{B2}} = R_{eq.p.u}$$

$$\therefore V.R = (0.015 \times 0.85 + 0.04 \times 0.53) \times 100\% ; \cos \theta = 0.85$$

$$= 3.4\% \quad \sin \theta = 0.53$$

* Chapter 4 : DC Machines

1] DC Generator

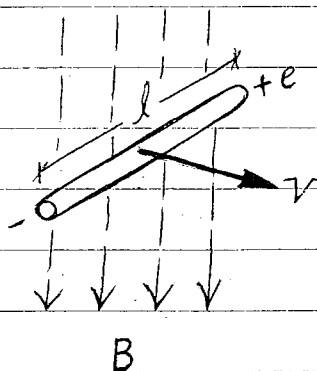
2] DC Motor

Electrical emf, i	Machine	Mechanical T, F
----------------------	---------	--------------------

Motor

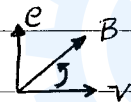
Generator

* DC Generator



Motional voltage, e

$$e = Blv$$



Right Hand Rule for (e)

such that ; B = Magnetic Field.

l = Conductor Length

v = Speed of Conductor.

• ملاحظة : إذا دُورنا الملف الواقع تحت تدفق مغناطيسي

سيكون (emf) ويكون DC Generator

DATE

$$e_t = 2Blv$$

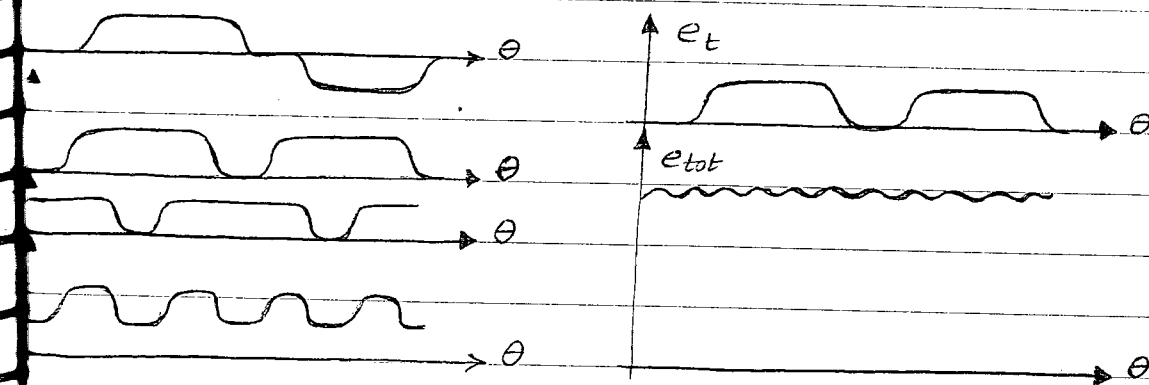
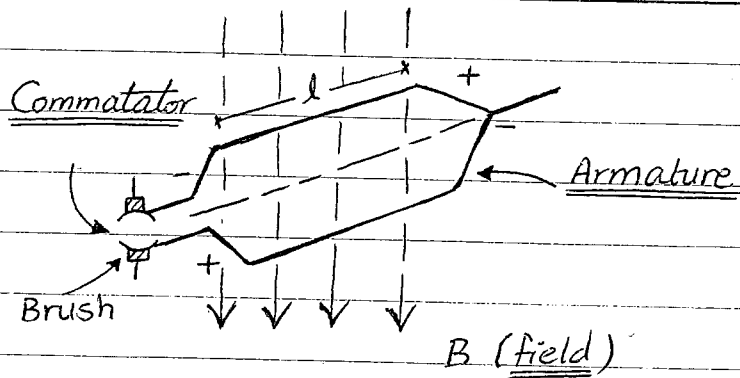
في DC Machines أجزاء *

① - Magnetic Field.

② - Armature.

③ - Commutator.

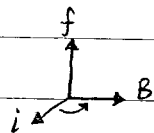
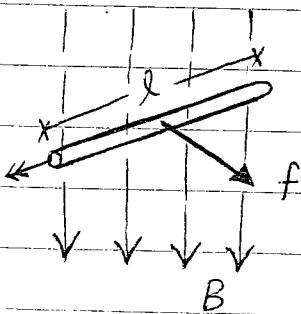
↳ converts from AC → DC.



DC Motors

Electromagnetic Force, $f = Bli$

Right Hand Rule for (f).



• ملاحظة : إذا مررنا إصبعنا في الملف الواقع تحت رادف

مغناطيسي، سيؤثر الملف حول نفسه ويكون DC Motor

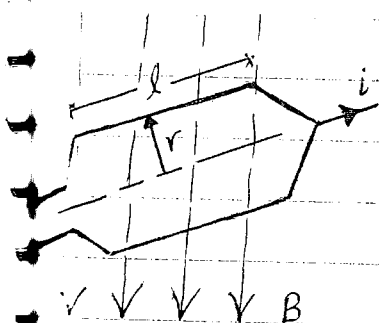
Torque $T = 2f r = 2Blir$

such that; $B \equiv$ Magnetic Field.

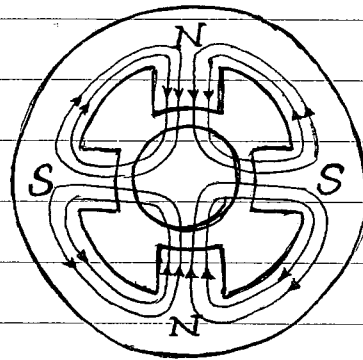
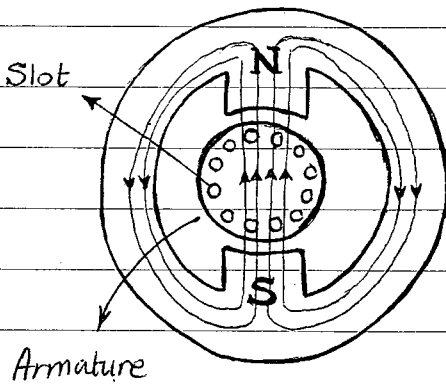
$l \equiv$ Conductor Length.

$i \equiv$ Current.

$r \equiv$ Armature Radius.



* Armature Windings ...



$p = \text{No. of poles}$

• ملاحظة: كل طراد عدد

القطبان، تتضاعف الزاوية

الكهربائية بمقدار $(\frac{P}{2})$ عن

الزاوية الميكانيكية.

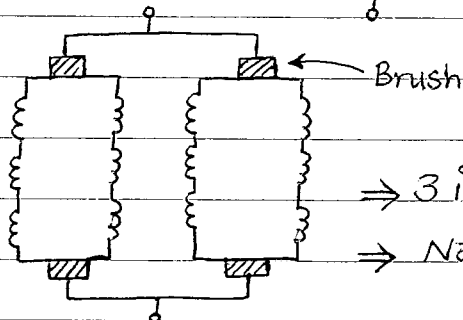
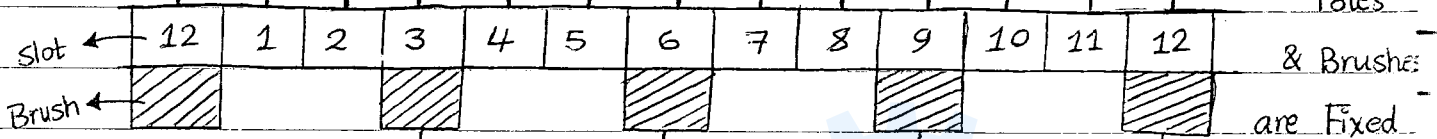
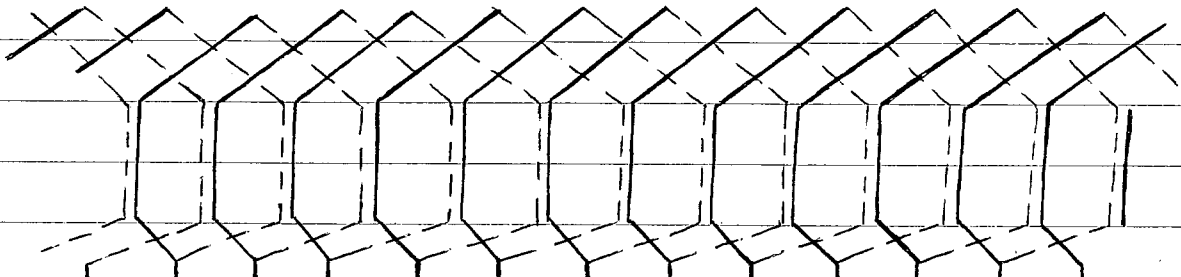
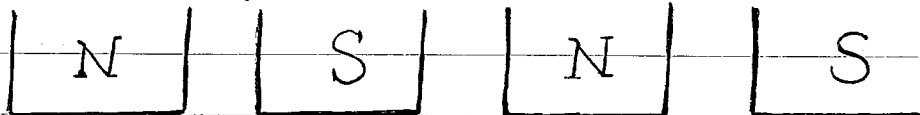
$$\Rightarrow \theta_{elec} = (\frac{P}{2}) \theta_{mech}$$

① Lab Winding ...

$a = \text{No. of parallel paths}$

$$a = p$$

$= \text{No. of Brushes}$

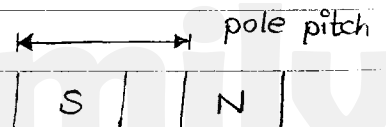
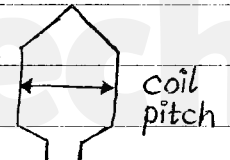


← ناول أن يكون (coil pitch) مساوي

!! 3 = (pole pitch) لا

$\Rightarrow 3 \text{ in Series, } 4 \text{ in parallel}$

$\Rightarrow \text{No. of parallel path} = 4 = a$

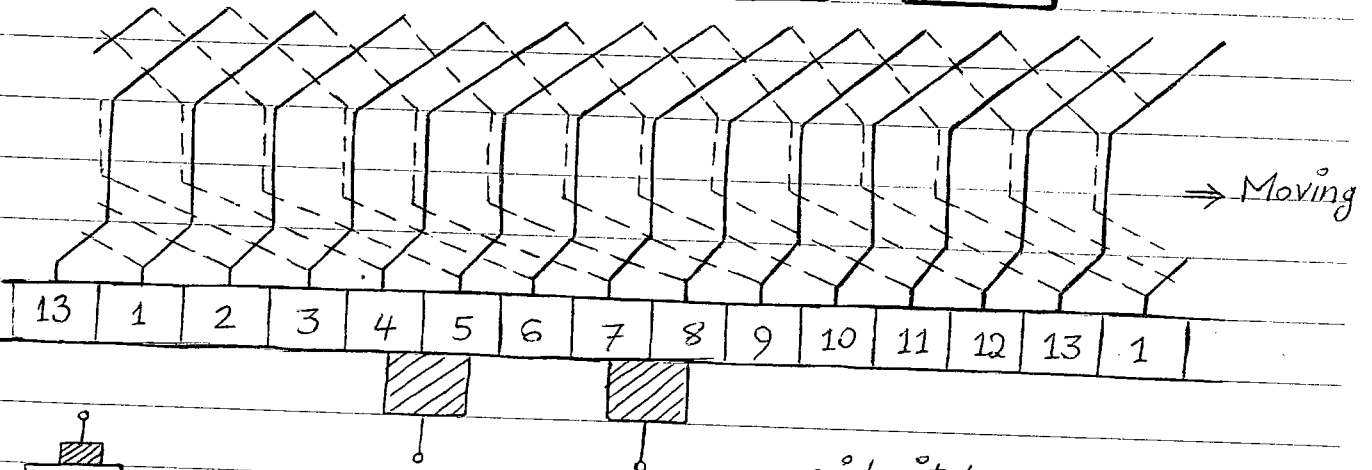
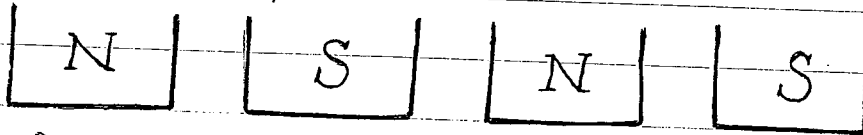


2 Wave Winding

a = No. of parallel paths

= No. of Brushes.

$a=2$



coil pitch = 3

pole pitch = $\frac{13}{4} = 3.25$

⇒ 2 in parallel = a .

⇒ 1 path has 6 in series, & 1 has 7 in series.

* Notes : Lab Winding

$a = p$

coil pitch = pole pitch

12 slots

Low Voltage

High Current

Wave Winding

$a = 2$ (always)

coil pitch $\neq$ pole pitch

13 slots

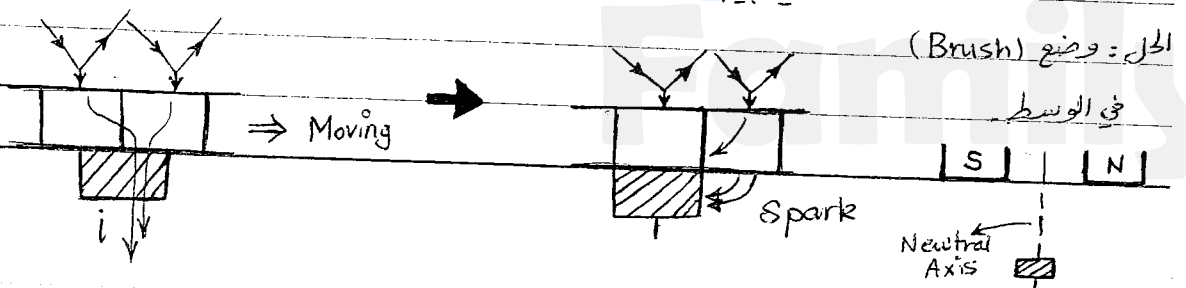
High Voltage

Low Current

Note :

المشكلة :- عندما نقطع الـ (Commutator) عن الـ (Brushes)، ينقطع التيار

الذي يمر بينها، فيحدث (Spark) !!

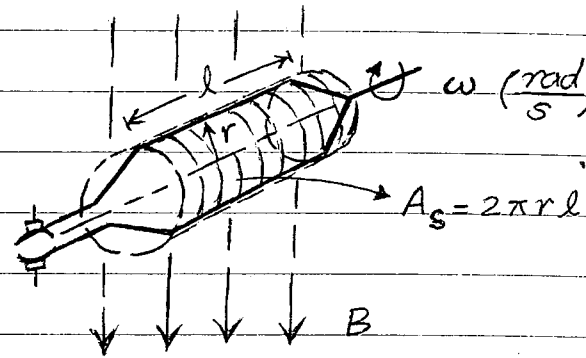


* Armature Voltage ∞∞

$$e_t = 2Blv \quad ; \quad v = \omega r$$

Let ϕ = flux per pole.

p = No. of poles.



$$\therefore B = \frac{\phi p}{A_s} = \frac{\phi p}{2\pi r l}$$

$$\Rightarrow e_t = 2Blv$$

$$= 2\left(\frac{\phi p}{2\pi r l}\right) l (\omega r) \Rightarrow \boxed{e_t = \frac{\phi p \omega}{\pi}} \quad \text{induced voltage in the turn.}$$

Let N = total No. of turns in the armature

a = No. of parallel path = p (Lab Winding).

= 2 (Wave Winding).

then, The voltage induced in all the turns connected in series into the generator is ∞∞ $E_a = \frac{N}{a} e_t = \frac{N}{a} \cdot \frac{\phi p \omega}{\pi}$

$$\boxed{E_a = \left(\frac{NP}{\pi a}\right) \omega \phi} \quad ; \quad K_a = \frac{NP}{\pi a} \quad (\text{Armature Constant}).$$

* Developed (Electromagnetic) Torque ∞∞

$$f_c = Bli \quad (\text{force per coil}).$$

$$T_t = 2f_c r \quad (\text{Torque per turn}). \quad ; \quad I_a = \text{Armature Current}$$

$$I_c = \frac{I_a}{a} \quad ; \quad \text{for } (N) \text{ turns}$$

$$\therefore T_t = 2Bli r = 2\left(\frac{\phi p}{2\pi r l}\right) l \left(\frac{I_a}{a}\right) r$$

$$\text{The total torque } T = (N T_t)$$

DATE 20/Mar/2012 Tuesday.

$$T = \left(\frac{NP}{\pi a} \right) \phi I_a ; K_a = \frac{NP}{\pi a}$$

But Power $P = E_a I_a$

* ملاحظة: للتحويل من « Mech. Torque » إلى « Elec. Power » يكون « Generator »

والعكس يكون « Motor » !

$$= (K_a \omega \phi) I_a = (K_a \phi I_a) \omega$$

$$= T \omega$$

$$P = E_a I_a = T \omega \quad \text{Mechanical Power}$$

Example :- 4 poles DC Machine, $r = 12.5 \text{ cm} = 0.125 \text{ m}$, effective length $l = 25 \text{ cm} = 0.25 \text{ m}$, the poles cover 75% of the Armature periphery, 33 coils, each coil has 7 turns 33 slots. $B = 0.75 \text{ T}$, $I_a = 400 \text{ A}$, $\omega = 1000 \text{ rpm}$.

3) Lab Winding

$$a = p = 4 \quad N = 33 \times 7 = 231 \text{ total turns.}$$

$$\Rightarrow K_a = \frac{NP}{\pi a} = \frac{231 \times 4}{\pi \times 4} = 73.53 \text{ (dimensionless)}$$

$$\Rightarrow \omega = 1000 \times \left(\frac{2\pi}{60} \right) = 104.72 \text{ rad/sec.}$$

$$\Rightarrow A_p = 75\% \left(\frac{2\pi r l}{p} \right) = \frac{2 \times \pi \times 0.125 \times 0.25 \times 0.75}{4} = 0.0368 \text{ m}^2/\text{pole.}$$

$$\Rightarrow \phi = B A_p = 0.75 \times 0.0368 = 0.0276 \text{ Wb.}$$

$$\Rightarrow E_a = K_a \omega \phi = 73.53 \times 104.72 \times 0.0276 = 212.5 \text{ V.}$$

$$\Rightarrow IP = E_a I_a ; \text{ But } I_a = 400 \text{ \& } I_c = \frac{I_a}{a} = \frac{400}{4} = 100 \text{ A per coil.}$$

$$\Rightarrow T = K_a \phi I_a = 73.53 \times 0.0276 \times 400 = 811.8 \text{ N.m.}$$

$$\Rightarrow P = E_a I_a = 212.5 \times 400 = 85000 \text{ W.}$$

b) Wave Winding ∞∞

$$a = 2 \Rightarrow K_a = \frac{NP}{\pi a} = \frac{231 \times 4}{\pi \times 2} = 147.06$$

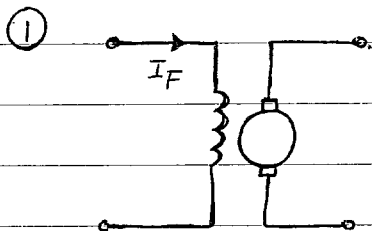
$$\Rightarrow E_a = K_a \omega \phi = 147.06 \times 104.72 \times 0.0276 = 425 \text{ V}$$

$$\Rightarrow T = K_a \phi I_a ; \text{ But } I_a = 200 \text{ A}$$

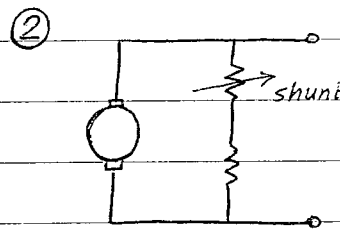
$$= 147.06 \times 0.0276 \times 200 = 811.8 \text{ N.m}$$

$$\Rightarrow P = E_a I_a = 425 \times 200 = 85000 \text{ W}$$

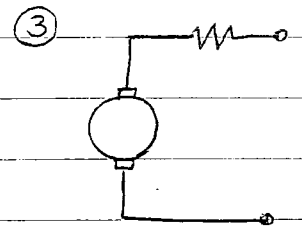
* DC Generators ∞∞



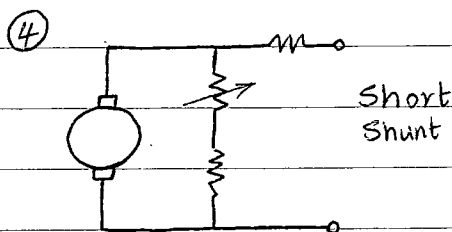
Separately Excited
DC Generator



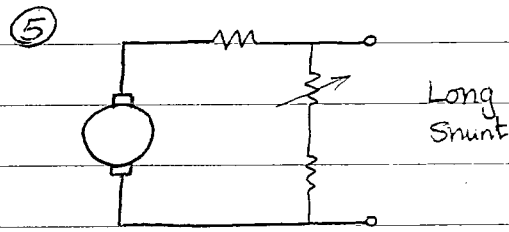
Self Excited (Shunt)
DC Generator



Series
DC Machine

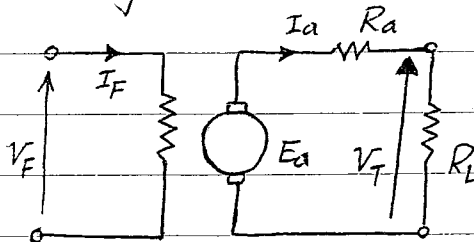


Short Compound
DC Machine (Generator)



Long Compound
DC Generator

* Separately Excited DC Generator ∞∞



$$\Rightarrow V_T = E_a - I_a R_a$$

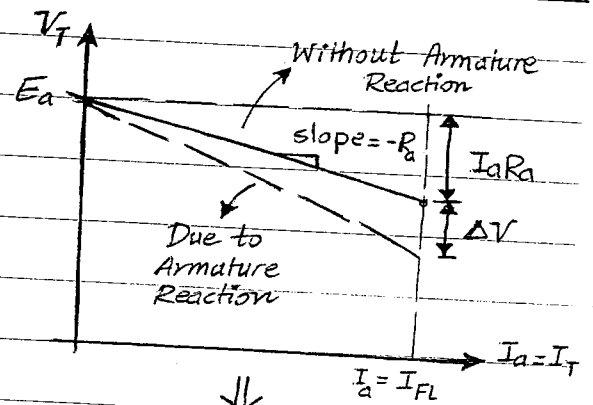
$$I_a = I_T \text{ \& } V_T = I_T R_L$$

$$V_F = I_F R_F \text{ \& } E_a = K_a \phi \omega$$

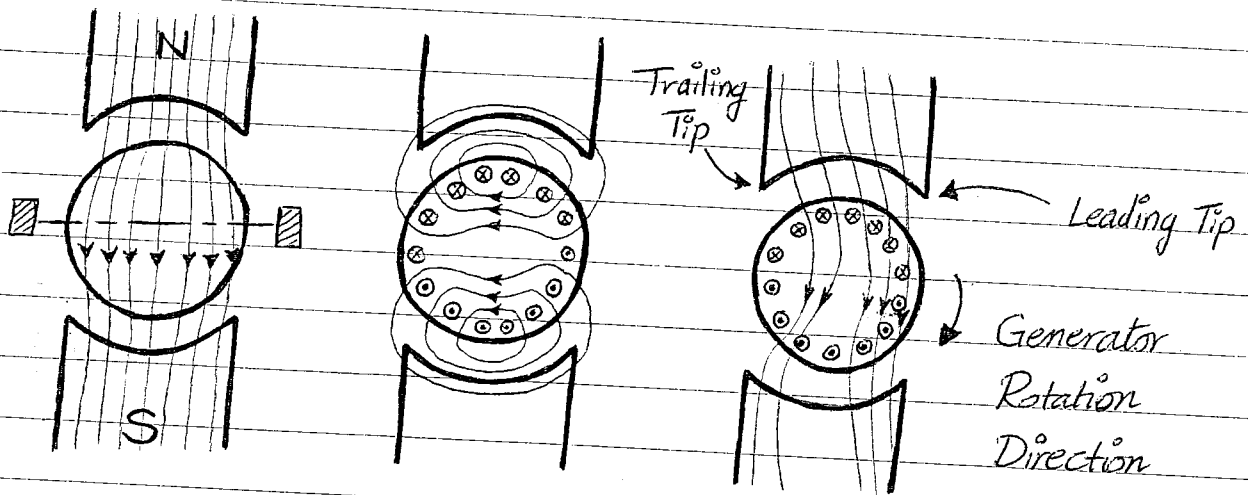
DATE 22/Mar/2012 Thursday.

* Armature Reaction

- OR (Demagnetization) → @ high values of the Armature current,
- a further voltage drop (ΔV) occurs in the terminal voltage.



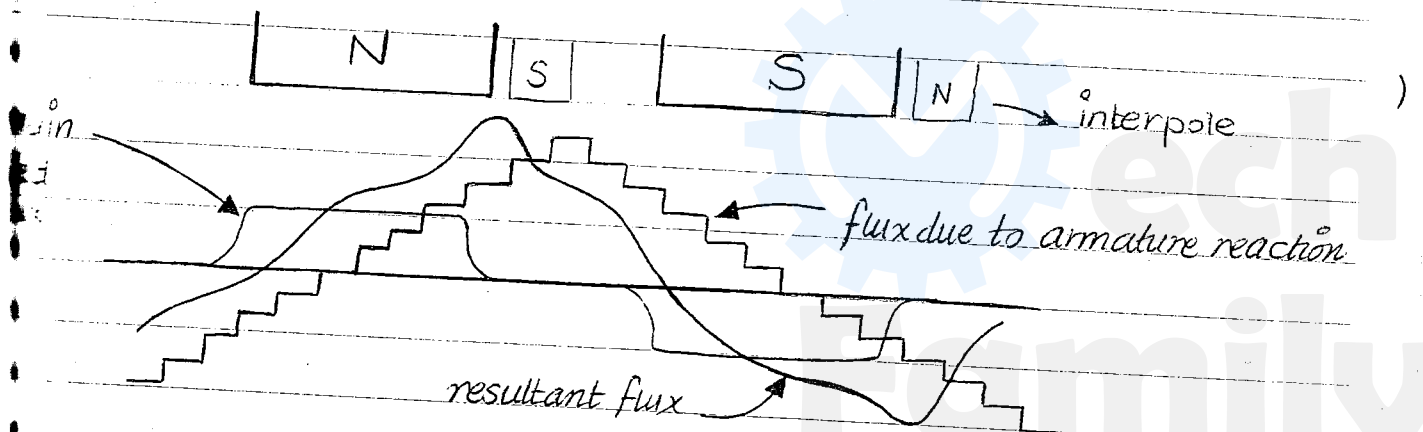
Note: This effect can be neglected for Armature currents below the Rated Current.



Main field flux

Flux due to armature reaction

resultant flux



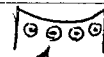
* Armature Reaction Effects ...

- 1 - Saturation of pole tips.
- 2 - Shift of Neutral Axis.
- 3 - Distortion of overall flux.
- 4 - Reduction of induced emf.

* Remedy of Armature Reaction ...

- 1 - Shift of Brushes $\Rightarrow$ Remedy for No. (2).
- 2 - Adding interpoles $\Rightarrow$ Remedy for No. (1) & (2).
- 3 - Compensating Winding. 

Note: No. (3) is the best solution.

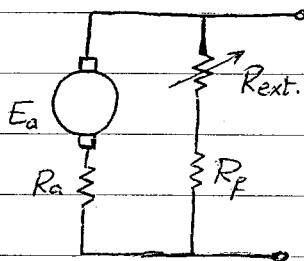


ملفلة التيار المار في اللفاف

يساوي (Armature Current)

current = Armature Current OR proportional

* Self Excited (Shunt) DC Generator ...



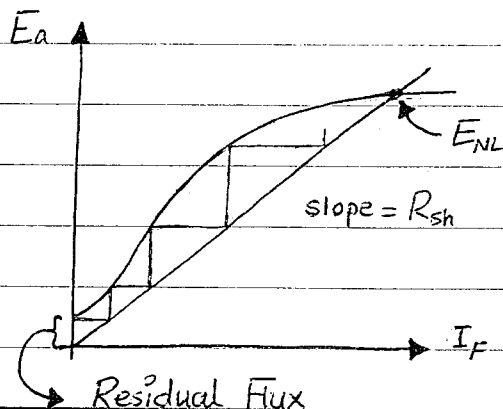
R_{ext} = External Resistance.

R_f = Field Resistance

R_{sh} = Shunt Resistance

such that ...

$$R_{sh} = R_f + R_{ext}$$



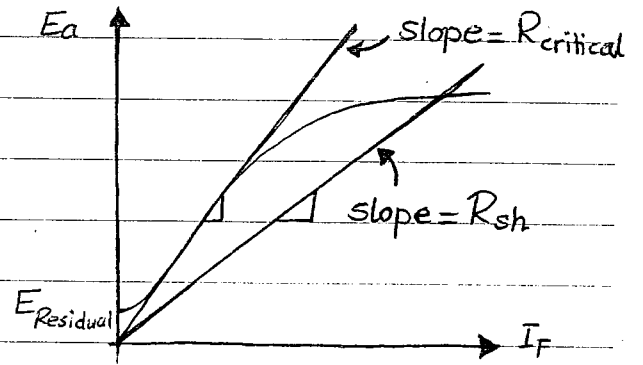
* Conditions for induced Emf ...

- 1 - Existence of Residual Flux.
- 2 - Field Flux in the same direction of Residual Flux.
- 3 - $R_{sh} < R_{critical}$

DATE 25/Mar/2012 Sunday

* $R_{critical}$ = The shunt resistance above which there will be No induced emf.

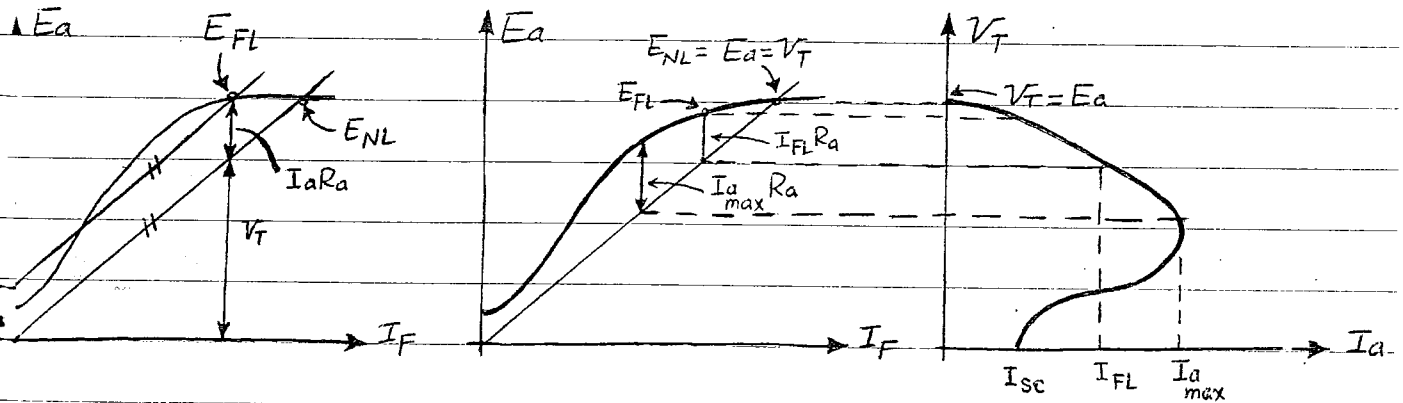
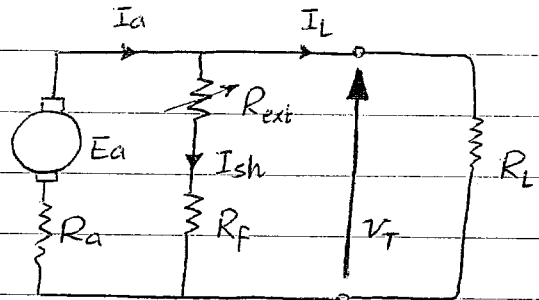
that means: if $R_{sh} > R_{critical}$, there is no induced emf.



* Voltage-Current Characteristics

$$E_a = V_T + I_a R_a$$

$$V_T = I_L R_L ; I_a = I_L + I_{sh} \\ \& I_{sh} = I_F$$



* ملاحظة : لا يمكن حل مسائل ((Shunt DC Gen.)) إلا عن طريق (Graphical Solution)
 (Shunt DC Gen) : هذا النوع الوحيد من Generator التي إذا عملنا عليها ((Short)) لا يشغل أي دائرة
 لأنه (Shunt Current) أو (field Current) يساوي صفراً. $I_F = I_{sh} = 0$

* ملاحظة : عندما تكون $I_F = 0$, فإن (induced emf) لا يساوي صفراً. بل يساوي

$$I_{sc} = \frac{E_{Residual}}{R_a} \leftarrow \text{يساوي (Short Current) ويكون}$$

Example 8: 12 kW, 100V, 1000 rpm DC Shunt Generator. $R_a = 0.1 \Omega$

$$R_F = 80 \Omega, I_F = 1 A = I_{sh}$$

- Determine the terminal voltage @ Full Load.
- Determine the Max. armature Current & the Critical Resistance.
- Determine Short Circuit Current.

Solution: $E_{NL} = V_T = 100 V$

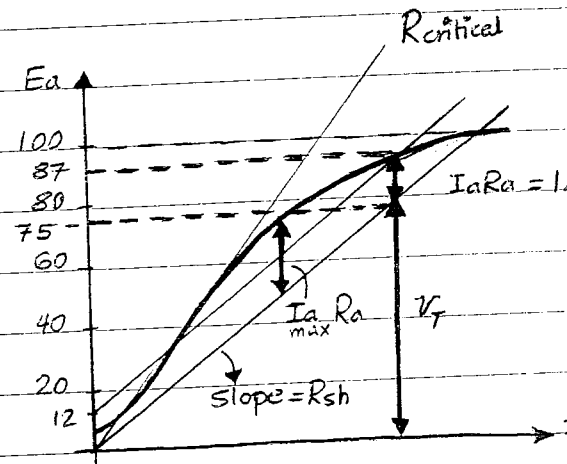
$$a) - I_{a_{FL}} = \frac{12000 \text{ kW}}{100 V} = 120 A$$

$$R_{sh} = \frac{V_T}{I_{sh}} = \frac{100 V}{1 A} = 100 \Omega$$

$$R_p + R_{ext} = R_{sh}$$

$$\Rightarrow R_{ext} = R_{sh} - R_F$$

$$R_{ext} = 100 - 80 = 20 \Omega$$



$$I_a R_a = 120 \times 0.1 = 12 V \rightarrow \text{Sketch the value on the graph.}$$

then take the line that's parallel to R_{sh} line!

the New Line crosses the characteristic Curve @ certain point.

This point is (E_{FL}) which equals to 87V.

using graph $\Rightarrow E_a = 87 V$.

$$\therefore V_T = E_a - I_a R_a = 87 - 12 = 75 V \quad \#$$

b) $I_{a_{max}} = ?? \rightarrow$ use the graph

Find the maximum distance between (R_{sh}) line and (characteristic curve), it equals to 20V.

$$\therefore I_{a_{max}} R_a = 20$$

$$I_{a_{max}} = \frac{20}{0.1} = 200 A \quad \#$$

DATE 27/Mar/2012 Tuesday

To find $R_{critical}$; Draw tangent line to Linear Region of Characteristic curve, & take the Slope.

$$\therefore \text{Slope} = R_{critical} = 150 \Omega$$

2) $I_{sc} = ?? \rightarrow$ from graph, Find $E_{Residual}$

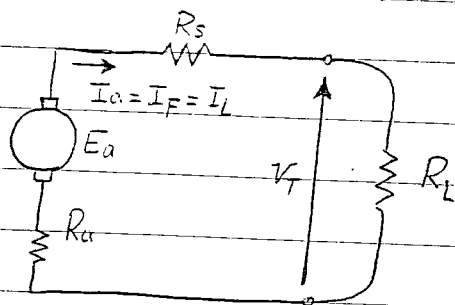
$$\Rightarrow E_{Residual} = 5V.$$

$$\therefore I_{sc} = \frac{5}{0.1} = 50A$$

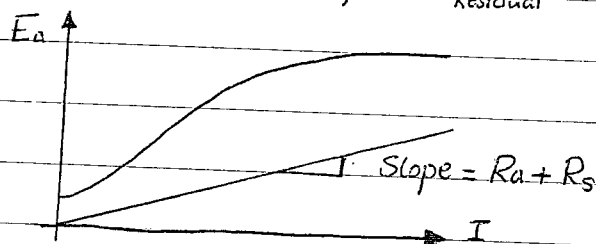
• Note that $R_{sh} < R_{critical}$

$100 < 150 \Rightarrow$ there is an induced emf.

Series DC-Generator



when $I = 0 \rightarrow \text{emf} = E_{Residual}$



1) DC Machine 1 : 120V, 1500 rpm, 4p.

DC Machine 2 : 240V, 1500 rpm, 4p.

coils are available which are rated @ 4 volts & 5A. for the same no. of coils to be used for both machines, find:-

2) Type of armature winding for each machine.

120V Machine $\rightarrow$ Lap ($a=p=4$)

240V Machine $\rightarrow$ Wave ($a=2$)

No. of coils required for each Machine.

120V M1 $120/4 = 30$ coil in series $\rightarrow 30 \times a = 120$ coils.

240V M2 $240/4 = 60$ coil in series $\rightarrow 60 \times a = 120$ coils.

Power Rating. M1 $I_a = a I_c = 4 \times 5 = 20A \rightarrow P = 120 \times 20 = 2400W$

M2 $I_a = a I_c = 2 \times 5 = 10A \rightarrow P = 240 \times 10 = 2400W.$

P(4.10) 10 kW, 250 V, 1000 rpm, $R_a = 0.2 \Omega$, $R_f = 133 \Omega$, Self-Excited

$I_f (A)$	0	0.1	0.2	0.3	0.4	0.5	0.75	1	1.5	2
$E_a (V)$	10	40	80	120	150	170	250	220	245	263

a) Determine the generated Voltage with no field current.

$$\Rightarrow E_a(\text{Residual}) = 10V.$$

b) Determine the critical field circuit resistance.

$$R_{crit} = \text{slope of linear Region} = \frac{120 - 40}{0.3 - 0.1} = 400 \Omega.$$

c) Determine the value of R_{ext} if the No-load voltage is 250V.

$$E_{NL} = 250V. \rightarrow I_f = 1.6A$$

$$R_{sh} = \frac{250}{1.6} = 156.25 \Rightarrow R_{ext} = R_{sh} - R_f = 156.25 - 133 = 23.25 \Omega$$

d) Determine the value of No load generated voltage if $\omega = 800 \text{ rpm}$ & $R_{ext} = 0$

$$R_{sh} = R_{ext} + R_f = 133 \Omega.$$

Scale down the figure by multiplying each value by $(\frac{800 \text{ rpm}}{1000 \text{ rpm}} = 0.8)$.

$$\Rightarrow E_{NL} = 195V, \text{ \& } I_f = 1.48A$$

e) Determine the speed, if $E_{NL} = 200V$, $R_{ext} = 0$

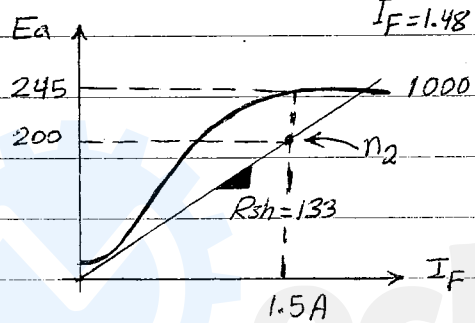
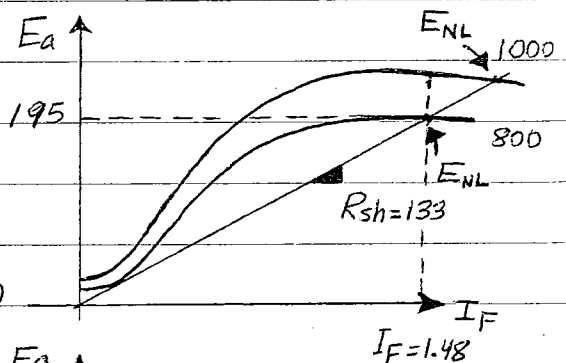
$$R_{sh} = 133 \Omega$$

Intersect the 200V with R_{sh} line, then go up to read the value on Curve.

$$\Rightarrow E_1 = 245V \text{ @ } 1000 \text{ rpm.}$$

$$\therefore n_2 = 1000 \times \frac{E_2}{E_1} = 1000 \times \frac{200}{245}$$

$$\Rightarrow n_2 = 816.3 \text{ rpm.}$$



P(4.11) - Same machine in P(4.10), the $P_{rotational} = 500W$.

a) Determine the generated voltage.

b) Determine the developed torque.

c) Determine the current in field circuit.

d) Determine the Efficiency.

DATE

Solution:- $I_a = \frac{10000 \text{ W}}{250 \text{ V}} = 40 \text{ A}$

$$V_T = E_a - I_a R_a \Rightarrow E_a = V_T + I_a R_a = 250 + 40(0.2) = 258 \text{ V}$$

$$P = E_a I_a = 258 \times 40 = 10320 \text{ W}$$

$$T = \frac{P}{\omega} = \frac{10320}{1000 \times \left(\frac{2\pi}{60}\right)} = 98.5 \text{ N.m}$$

from graph @ $E = 258 \text{ V} \rightarrow I_f \approx 1.85 \text{ A}$

$$P_{out} = V_T I_T$$

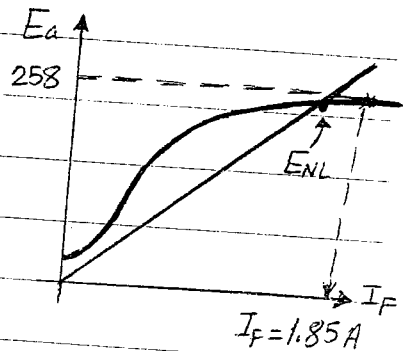
$$\text{But } I_T = I_a - I_f \text{ OR } I_T = I_a - I_{sh} \\ = 40 - 1.85 = 38.15 \text{ A}$$

$$\therefore P_{out} = V_T I_T = 250 \times 38.15 = 9538 \text{ W}$$

$$P_{loss} = P_{rot} + P_{arm} + P_{field} = P_{rot} + I_a^2 R_a + I_f^2 R_f$$

$$\therefore P_{loss} = 500 + (40)^2 \times 0.2 + (1.85)^2 \times 133 = 1275.2 \text{ W}$$

$$\Rightarrow \eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{9538}{9538 + 1275.2} \times 100\% = 88.2\%$$



4.12) 24 kW, 240 V, 1200 rpm, $R_a = 0.12 \Omega$, $N_f = 600$ turn/pole.

Separately excited dc generator, $I_f = 1.8 \text{ A}$. The No load terminal voltage 240 V, & When Full load current, the terminal voltage is 225 V.

1) Determine the generated voltage & developed torque when the generator @ full load.

$$V_T = E_a - I_a R_a \quad \therefore I_a = \frac{24000}{240} = 100 \text{ A}$$

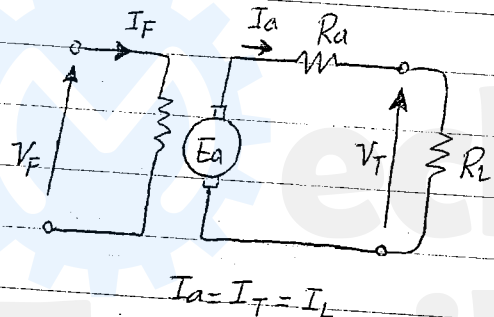
$$\text{OAD } V_T = E_{aNL} = 240 \text{ V}$$

$$\text{OAD } E_a = V_T + I_a R_a = 225 + 100(0.12)$$

$$E_{aFL} = 237 \text{ V}$$

$$P = E_a I_a = 237(100) = 23700 \text{ W}$$

$$T = \frac{P}{\omega} = \frac{23700}{1000 \times \left(\frac{2\pi}{60}\right)} = 226.3 \text{ N.m}$$



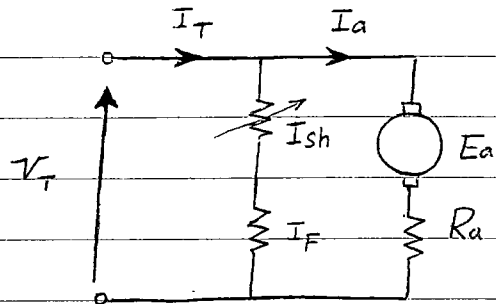
* DC - Motors ...

Shunt DC-Motor

$$E_a = V_T - I_a R_a$$

$$= K_a \phi \omega$$

$$T = K_a \phi I_a \Rightarrow I_a = \frac{T}{K_a \phi}$$

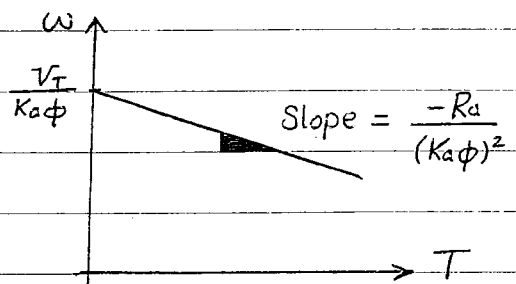


Substitute in E_a :-

$$E_a = V_T - \left(\frac{T}{K_a \phi} \right) R_a = K_a \phi \omega$$

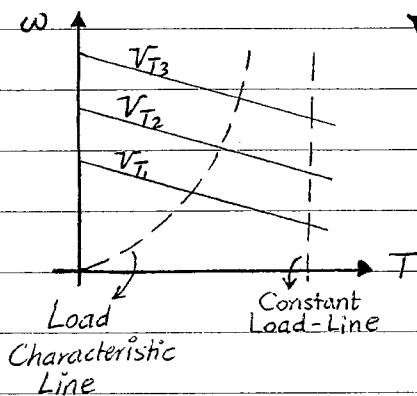
$$\Rightarrow \omega = \frac{V_T}{K_a \phi} - \frac{R_a T}{(K_a \phi)^2}$$

Linear Equation $y = b - mx$

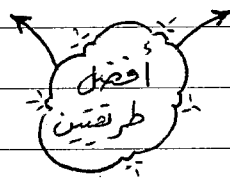
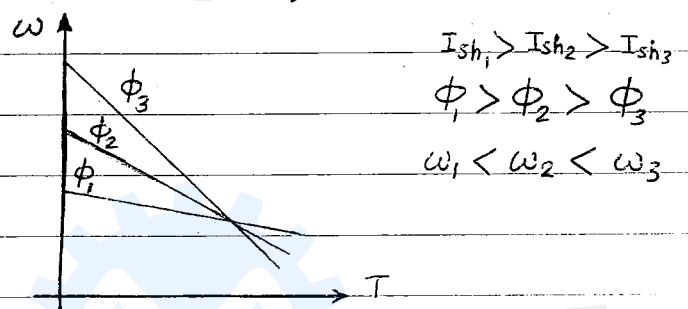


* Control of Speed of DC Shunt Motor ...

1 - By changing (V_T)



2 - By changing field current (I_F)
OR flux (ϕ)



* العلاقة بين كل من ω و ϕ : كلما زادت (ϕ) ، قلت السرعة (ω)

* العلاقة بين ω و I_{sh} : لو كانت (I_{sh}) تساوى صفراً ، تكون ($\omega \rightarrow \infty$)

يعني سرعة كبيرة جداً ، يستتبع المحرك الى أن ينكسر

$$V_{T3} > V_{T2} > V_{T1}$$

$$\omega_3 > \omega_2 > \omega_1$$

Note : To Switch off the Motor , The Armature should be disconnected before the Field!

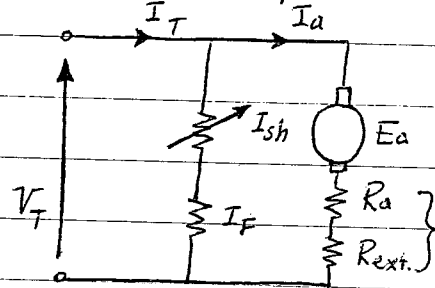
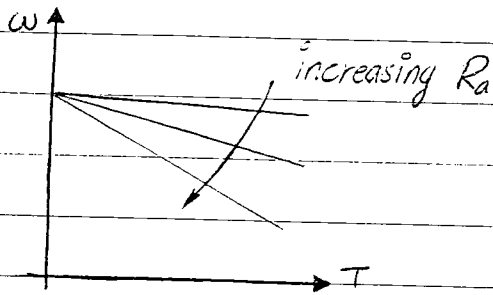
DATE

3 By changing (R_a)

* ملاحظة : فعليا نحن لا نغير (R_a) بل نضيف

مقاومة خارجية (R_{ext}) معها.

* ملاحظة : نستخدم فقط في المحركات الصغيرة.



* Power Flow & Efficiency in DC-Motors

$$P_{in} = P_{elect.} = V_T I_T$$

$$P_{out} = P_{mech} = T \cdot \omega$$

$$P_{losses} \Rightarrow P_{Armature} = I_a^2 R_a + P_{Rotational} + P_{Field} = I_F^2 R_F$$

$$Efficiency = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

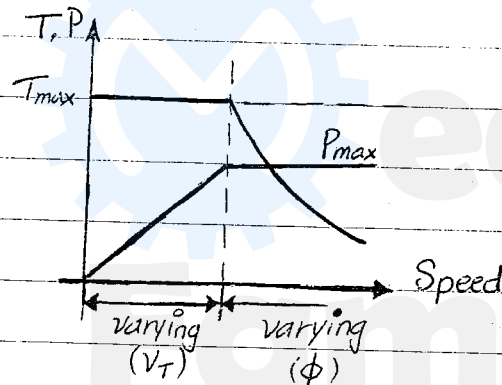
* ملاحظة : (Power-Flow) في حالة (DC Generator) يكون نفس الشيء، واعداء أنه يكون ($P_{in} = P_{mech}$) وتكون ($P_{out} = P_{elec.}$) بينما (P_{losses}) تبقى كما هي!!

$$* P = T \cdot \omega$$

$$\Rightarrow T = \frac{P}{\omega} \text{ (N.m)}$$

The shown figure yields the (Power) & (Torque) response with (speed):

- Changing Terminal voltage.
- Changing flux



Example 8:- 10 kW, 200V, $R_a = 0.2 \Omega$, $R_f = 150 \Omega$, $R_{ext} = 50 \Omega$

$$n_{FL} = 1900 \text{ rpm.}$$

$$P_e = V_T I_T \Rightarrow I_{FL} = \frac{10000}{200} = 50 \text{ A} = I_{a_{FL}} \rightarrow \text{اعتبرنا مع أنه لازم (Ia=50)}$$

(Ia=50-1=49) تكون تساوي

$$R_{sh} = R_f + R_{ext} = 150 + 50 = 200 \Omega. \quad (I_a = I_T - I_{sh})$$

$$I_{sh} = \frac{200 \text{ V}}{200 \Omega} = 1 \text{ A.}$$

① Find T. ② $R_{ext} = 100 \Omega$ find π (Speed).

$$\textcircled{1} E_a = V_T - I_a R_a$$

$$= 200 - 50(0.2) = 190 \text{ V.}$$

$$P = E_a I_a = 190 \times 50 = 9500 \text{ W.}$$

$$T = \frac{P}{\omega} = \frac{9500}{1900 \times \left(\frac{2\pi}{60}\right)} = 47.75 \text{ N.m.}$$

② Keeping Load Torque & changing $R_{ext} = 100 \Omega$.

$$I_{F_2} = \frac{200 \text{ V}}{150 + 100} = 0.8 \text{ A.}$$

← العلاقة بين (I_F) و (ϕ) علاقة Linear

$$T = \text{const.} \times \phi \times I_a$$

* العلاقة مع ثبيت الـ (Torque) وانخفاض

$$= \text{const.} \times I_F \times I_a$$

قيمة (I_F) ، سينتج أنه (I_a) ستنزله

$$\Rightarrow I_{F_1} I_{a_1} = I_{F_2} I_{a_2} \Rightarrow I_{a_2} = \frac{1 \times 50}{0.8} = 62.5 \text{ A.}$$

$$E_{a_2} = V_T - I_{a_2} R_a$$

$$= 200 - 62.5(0.2) = 187.5 \text{ V.}$$

عندنا زدنا (R_{ext}) قلت (E_a)

وزادت (I_a)

$$E_{a_2} = K_a \phi \omega = K_2 I_F n_2 \Rightarrow \frac{E_{a_2}}{E_{a_1}} = \frac{I_{F_2} n_2}{I_{F_1} n_1}$$

$$\Rightarrow n_2 = \frac{187.5}{190} \times \frac{1}{0.8} \times 1900 = 2343.75 \text{ rpm.}$$

عندنا زدنا (R_{ext}) قلت (I_F)

وزادت السرعة

③ Torque is reduced to half & No External Resistance ...

$$I_{F3} = \frac{200}{150} = 1.333 \text{ A.}$$

$$\frac{T_3}{T_1} = 0.5 = \frac{I_{F3} I_{a3}}{I_{F1} I_{a1}} \Rightarrow I_{a3} = 0.5 \frac{I_{F1} I_{a1}}{I_{F3}} = 0.5 * \frac{1 * 50}{1.333} = 18.75 \text{ A}$$

$$E_{a3} = V_T - I_{a3} R_a$$

$$= 200 - 18.75 (0.2) = 196.25 \text{ V}$$

عندما قلَّت (R_{ext}) قلَّت (I_a) وزادت (E_a)

$$\frac{E_{a3}}{E_{a1}} = \frac{I_{F3} \cdot n_3}{I_{F1} \cdot n_1} \Rightarrow n_3 = \frac{196.25}{190} * \frac{1}{1.333} * 1900 = 1471.8 \text{ rpm}$$

عندما قلَّت (R_{ext}) زادت (I_F) وقلَّت السرعة.

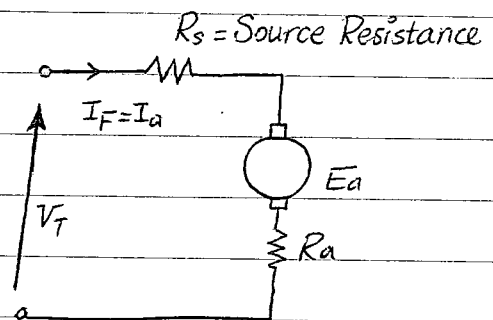
Series DC-Motor ...

$$T = \text{const} * I_a * I_F ; I_a = I_F$$

$$\therefore T = \text{const} \cdot I_a^2 = K_1 I_a^2$$

* العلاقة : في (Series Motor) كمية ال (Torque) تتناسب مع (I_a²) وليس (I_a) كما في (Shunt Motor)

$$E_a = V_T - I_a (R_a + R_s)$$



4.23) - 240V, 1200 rpm, E_a = 230V, I_a = 40A.

Is the machine functioning

as a generator or as a motor?

Note : if $V_T > E_a \Rightarrow$ Motor.

$E_a > V_T \Rightarrow$ Generator.

$V_T > E_a$, (I_a) into the machine $\rightarrow$ motor.

Determine the resistance of Armature Circuit?

$$E_a = V_T - I_a R_a$$

$$230 = 240 - 40 (R_a) \Rightarrow R_a = 0.25 \Omega$$

c). Determine power loss in Armature and Electromagnetic power?

$$P_{Arm} = I_a^2 R_a = (40)^2 \times (0.25) = 400 \text{ W.}$$

$$P_{elec.} = E_a I_a = (230) \times (40) = 9200 \text{ W}$$

d). Determine the Electromagnetic Torque (N.m)?

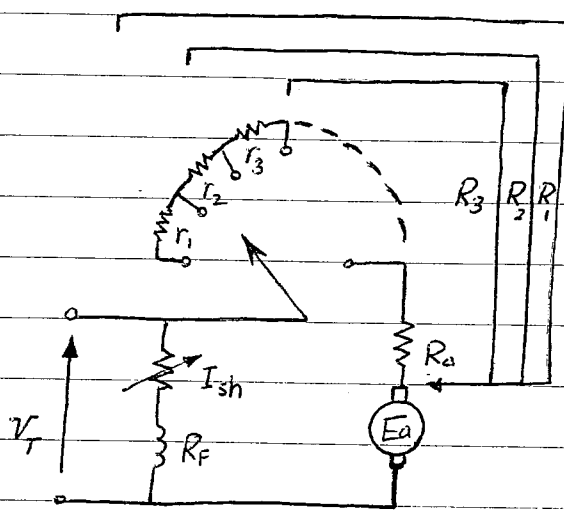
$$T = \frac{P_{elec.}}{\omega} = \frac{9200}{1200 \times \left(\frac{2\pi}{60}\right)} = 73.2 \text{ N.m.}$$

e). Assuming the load is thrown off, what will be the generated voltage & the rpm, if No Armature Reaction.

$$E_{a2} = V_T - I_a R_a$$

$$E_{a2} = 240 \text{ V.} \Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{n_2}{n_1} \Rightarrow n_2 = \left(\frac{240}{230}\right) \times 1200 = 1252 \text{ rpm.}$$

* Shunt Motor Starting



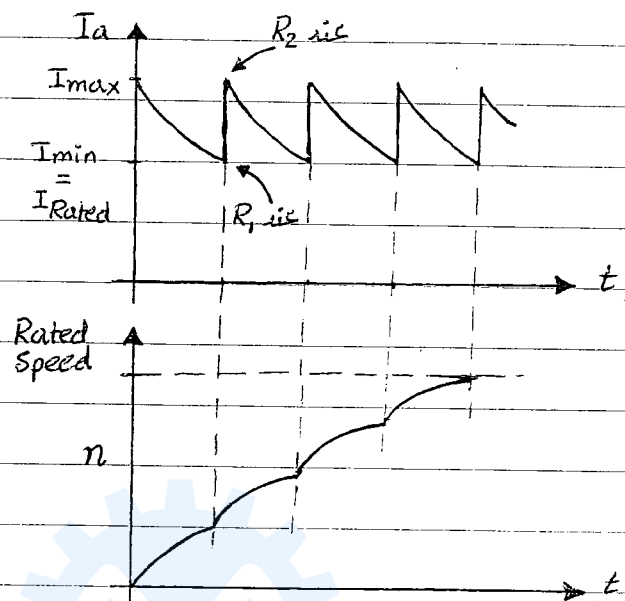
$$E_a = V_T - I_a R_a$$

$$= K_a \phi \omega$$

$$I_{max} = \frac{V_T}{R_1} \Rightarrow \boxed{R_1 = \frac{V_T}{I_{max}}}$$

$$E_{a1} = V_T - I_{min} R_1$$

$$\text{also; } E_{a1} = V_T - I_{max} R_2$$



$$\therefore V_T - I_{min} R_1 = V_T - I_{max} R_2$$

$$I_{min} R_1 = I_{max} R_2$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{I_{max}}{I_{min}} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \frac{R_4}{R_5} = \dots = \frac{R_n}{R_a}$$

* ملاحظة : أول مقاومة (R_1) وآخر مقاومة (R_a) .

* ملاحظة : مستحيل أن تكون (R_n) أقل من قيمة (R_a) .

$$\frac{R_1}{R_2} = \left(\frac{I_{\max}}{I_{\min}} \right)^n$$

and

$$n = \frac{\log(R_1/R_2)}{\log(I_{\max}/I_{\min})}$$

where: $n \equiv$ No. of Resistances (Not Speed)

Example 8- $V_T = 250 \text{ V}$, $R_a = 0.2 \Omega$, $\frac{I_{\max}}{I_{\min}} = 1.8$, $I_{\text{rated}} = 100 \text{ A}$.

$n_{\text{max}} = 2300 \text{ rpm}$.

Find all resistances, & their own emf & speed.

Solution: $E_a = V_T - I_a R_a = 250 - 100(0.2) = 230 \text{ V}$.

Because $E_a \propto n(\text{Speed})$

$$n = \text{constant} \times E_a \Rightarrow \text{constant} = \frac{2300}{230} = 10$$

$$I_{\min} = I_{\text{rated}} = 100 \text{ A}$$

$$I_{\max} = 1.8 \times 100 = 180 \text{ A}$$

$$R_1 = \frac{V_T}{I_{\max}} = \frac{250}{180} = 1.388 \Omega \rightarrow E_{a1} = 250 - 100(1.388) = 111.1 \text{ V}$$

$$\underline{OR} = 250 - 180(0.772) = 111.1 \text{ V}$$

$$R_2 = \frac{R_1}{\left(\frac{I_{\max}}{I_{\min}}\right)} = \frac{1.388}{1.8} = 0.772 \Omega \rightarrow E_{a2} = 250 - 100(0.772) = 172.8 \text{ V}$$

$$\underline{OR} = 250 - 180(0.429) = 172.8 \text{ V}$$

$$R_3 = \frac{R_2}{1.8} = \frac{0.772}{1.8} = 0.429 \Omega \rightarrow E_{a3} = 250 - 100(0.429) = 207.1 \text{ V}$$

$$\underline{OR} = 250 - 180(0.238) = 207.1 \text{ V}$$

$$R_4 = \frac{R_3}{1.8} = \frac{0.429}{1.8} = 0.238 \Omega \rightarrow E_{a4} = 250 - 100(0.238) = 226.2 \text{ V}$$

$$R_5 = \frac{R_4}{1.8} = \frac{0.238}{1.8} = 0.132 \Omega \text{ (Cancelled)} \rightarrow E_{a5} = 250 - 100(0.2) = 230 \text{ V}$$

$$< R_a = 0.2$$

$$E_{a1} = 111.1 \text{ V} \rightarrow n_1 = 10(111.1) = 1111 \text{ rpm}$$

$$E_{a2} = 172.8 \text{ V} \rightarrow n_2 = 10(172.8) = 1728 \text{ rpm}$$

$$E_{a3} = 207.1 \text{ V} \rightarrow n_3 = 10(207.1) = 2071 \text{ rpm}$$

$$E_{a4} = 226.2 \text{ V} \rightarrow n_4 = 10(226.2) = 2262 \text{ rpm}$$

$$E_{a5} = 230 \text{ V} \rightarrow n_5 = 10(230) = 2300 \text{ rpm}$$

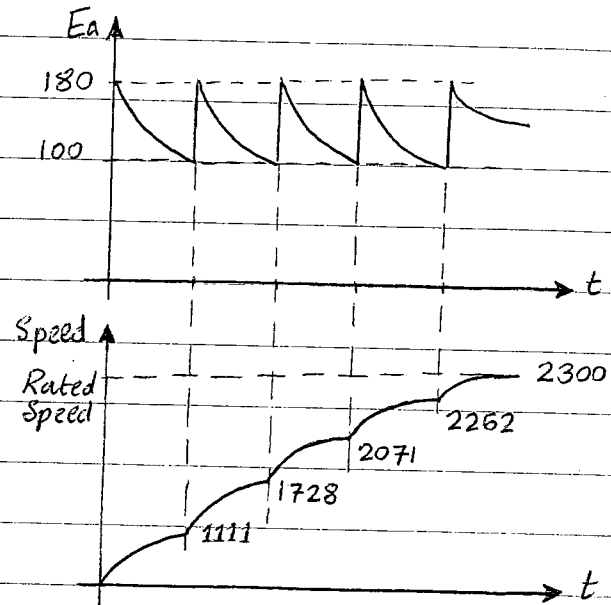
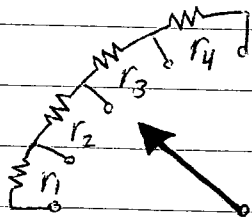
continue $\rightarrow$

$$r_1 = R_1 - R_2 = 1.388 - 0.772 = 0.616 \, \Omega$$

$$r_2 = R_2 - R_3 = 0.772 - 0.429 = 0.343 \, \Omega$$

$$r_3 = R_3 - R_4 = 0.429 - 0.238 = 0.191 \, \Omega$$

$$r_4 = R_4 - R_a = 0.238 - 0.2 = 0.038 \, \Omega$$



P(4.33) Constant Torque, $V_T = 120 \text{ V}$, $I_a = 8 \text{ A}$, 1800 rpm , $R_a = 0.08 \, \Omega$
flux (ϕ) dropped to 5%.

Solution: $E_{a1} = 120 - 8(0.08)$
 $= 119.36 \text{ V}$

$$E_{a1} = K_a \phi_1 \omega_1 \quad ; \text{ Because } \phi \downarrow 5\%$$

$$\therefore E_{a2} = 5\% E_{a1}$$

$$= 0.05 \times 119.36 = 5.968 \text{ V}$$

$$E_{a2} = V_T - I_{a2} R_a \Rightarrow I_{a2} = \frac{V_T - E_{a2}}{R_a} = \frac{120 - 5.968}{0.08} = 1425.4 \text{ A}$$

$$E_{a2} = E_{a1}$$

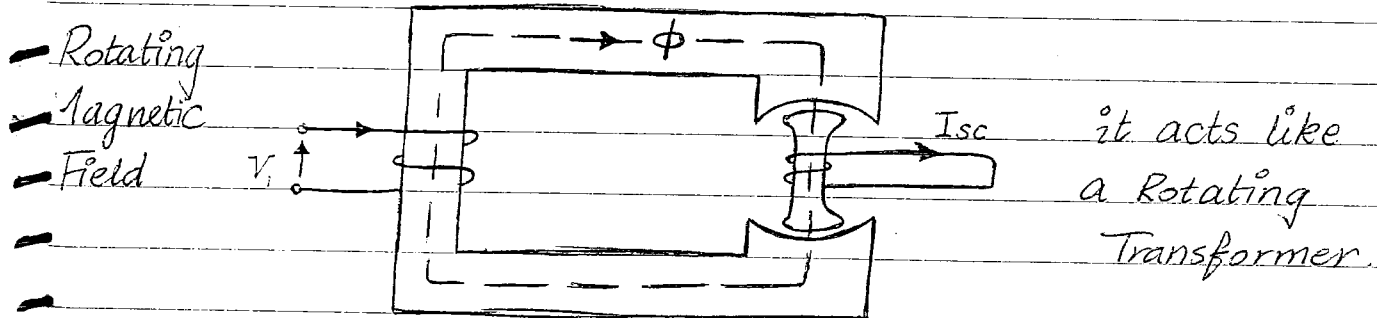
$$K_a \phi_2 n_2 = K_a \phi_1 n_1$$

$$5\% \phi_1 n_2 = \phi_1 n_1 \Rightarrow n_2 = \frac{n_1}{5\%} = \frac{1800}{0.05} = 36000 \text{ rpm}$$

DATE 5/Apr/2012 Thursday

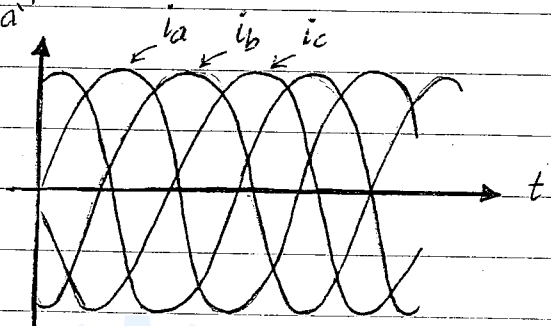
* Chapter 5 : Induction Machines ...

- The induction machine has a stator & rotor mounted on bearings & separated from the stator by an air gap.
- It can operate both as a motor & as a generator.



Single Phase	(N)	Three Phase	(a) (b) (c)	$i_a = I_m \cos \omega t$
pulsating		$\Rightarrow$ Sinusoidally distributed		$i_b = I_m \cos(\omega t - 120^\circ)$
standing				$i_c = I_m \cos(\omega t - 240^\circ)$
mmf	(S)	mmf	(a) (b) (c)	

$$\begin{aligned}
 \text{mmf} &= F(\theta) = F_a + F_b + F_c \\
 F(\theta) &= N i_a \cos \theta + N i_b \cos(\theta - 120^\circ) + N i_c \cos(\theta - 240^\circ) \\
 &= N I_m [\cos \omega t \cos \theta + \cos(\omega t - 120^\circ) \cos(\theta - 120^\circ) + \cos(\omega t - 240^\circ) \cos(\theta - 240^\circ)]
 \end{aligned}$$



$$F(\theta) = \frac{3}{2} N I_m \cos(\omega t - \theta) \quad \text{Resultant mmf wave in Air gap}$$

Rotors Winding Types :

- 1. Squirrel Cage Rotor.
- 2. Wound Rotor.

* Induced Voltage ...

The flux density distribution in the air-gap can be expressed as

$$B(\theta) = B_{\max} \cos \theta$$

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) l r d\theta = 2B_{\max} l r$$

; ϕ_p = Flux per pole.

l = axial length of stator.

r = Radius of stator @ air-gap

the induced voltage is ...

$$v = e = N \frac{d\phi}{dt} = \omega N \phi \sin \omega t$$

$$= E_{\max} \sin \omega t$$

The rms value of the induced voltage is ...

$$E_{\text{rms}} = \frac{\omega N \phi_p}{\sqrt{2}} = \frac{2\pi f N \phi_p}{\sqrt{2}} = 4.44 f N \phi_p$$

OR

$$E_{\text{rms}} = 4.44 f N \phi_p K_w$$

; K_w = Winding Factor (0.85-0.95)

K_p = Pitch Factor.

K_s = Skew Factor.

K_d = Distribution Factor

$$K_w = K_p K_s K_d$$

Note: this equation is valid for stator and for rotor.

$$\therefore \frac{E_1}{E_2} = \frac{f_1 N_1 K_{w1}}{f_2 N_2 K_{w2}}$$

① for Stator, ② for Rotor.

* Synchronous Speed (Speed of Rotating mmf) $\equiv n_s$

$$n_s = \frac{120f}{p}$$

* Mechanical Speed of the Rotor $\equiv n_m$

* Relative Speed (Slip Speed) $= (n_s - n_m)$.

* Slip
$$S = \frac{n_s - n_m}{n_s}$$

$\therefore$ Slip Speed $= S n_s$

Note: if $n_s = n_m \Rightarrow$ ~~Relative~~ Speed (Slip Speed) is equal to zero, & Induced voltage is zero.

$\Rightarrow$ No Torque Exists!

e.g.	n_s	n_m	s
	3000 rpm	2900 rpm	0.033
	1500 rpm	1450 rpm	0.033
	1000 rpm	970 rpm	0.03

• if $n_s = \frac{120 f_1}{P} \Rightarrow f_1 = \frac{P}{120} n_s$; 1 - Stator
2 - Rotor.

$$\therefore f_2 = \frac{P}{120} (n_s - n_m) = \left(\frac{P}{120} n_s \right) s$$

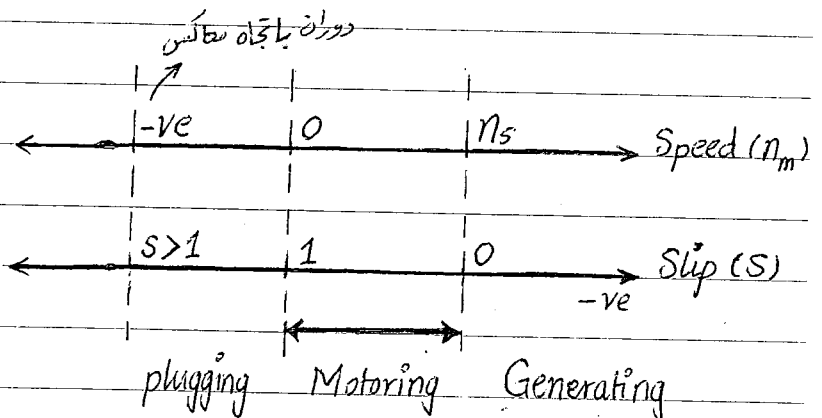
$\Rightarrow \boxed{f_2 = s f_1}$ Frequency of Rotor = Slip * frequency of Stator

Modes of Operations :-

1 - Generating.

2 - Motoring.

3 - Plugging (Retardation).



Example :- 3ϕ , 460 V, 100 hp, 4p, $s = 0.05$, $f_1 = 60$ Hz

a) - find Synchronous Speed & Motor Speed?

$$n_s = \frac{120 f}{P} = \frac{120}{4} \times 60 = 1800 \text{ rpm}$$

$$s = 0.05 = \frac{n_s - n_m}{n_s} \Rightarrow n_m = n_s - 0.05 n_s = 1710 \text{ rpm}$$

b) - find speed of Rotating air-gap field?

= 1800 rpm (Same of Synchronous Speed).

c) frequency of Rotor ?

$$f_2 = S f_1 = 0.05 \times 60 = 3 \text{ Hz}$$

d) Slip Speed ?

$$\text{Slip Speed} = \text{Slip rpm} = n_s - n_m = 1800 - 1710 = 90 \text{ rpm}$$

e) Speed of Rotor field relative to Rotor Structure = 90 rpm = Slip rpm

Speed of Rotor field relative to Stator Structure = 1800 rpm = n_s

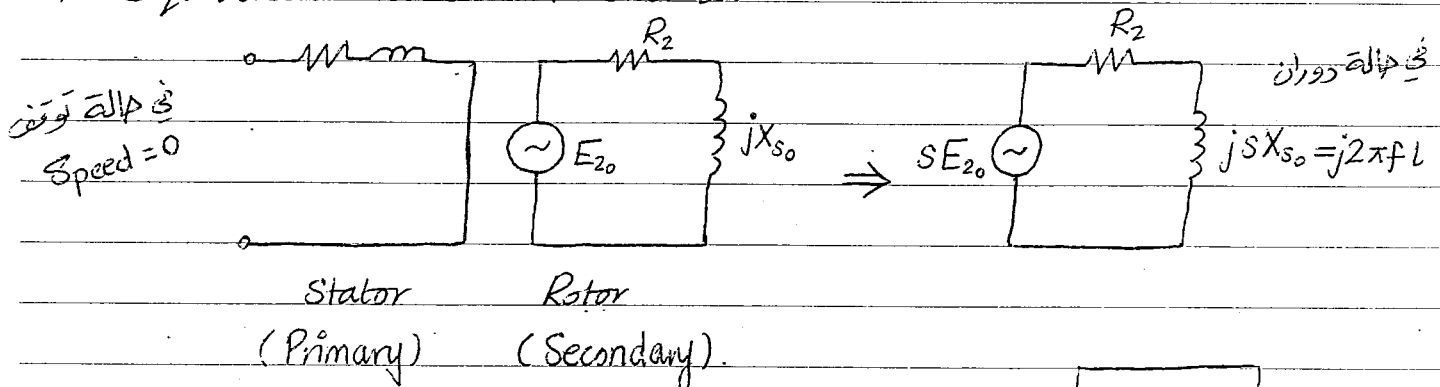
Speed of Rotor field relative to Stator Rotating Field = 0

f) Rotor induced voltage ? if stator to rotor turns ratio = 1 : 0.5

$$\frac{E_1}{E_2} = \frac{f_1 N_1 K_{w1}}{f_2 N_2 K_{w2}} \Rightarrow E_2 = \frac{f_2 N_2}{f_1 N_1} E_1$$

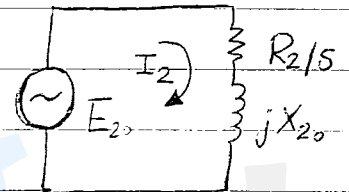
$$E_2 = \left(\frac{3}{60}\right)(0.5)\left(\frac{460}{\sqrt{3}}\right) = 6.64 \text{ V/phase}$$

* Equivalent Circuit Model

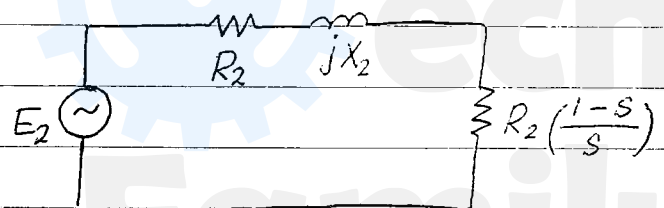


$$I_2 = \frac{SE_{20}}{R_2 + jSX_{20}}$$

$$I_2 = \frac{E_{20}}{R_2/S + jX_{20}}$$

Resistance ($\frac{R_2}{S}$)

$$\begin{aligned} & \rightarrow (R_2) \\ & \rightarrow \left(\frac{R_2(1-S)}{S}\right) \end{aligned}$$



it's like a Transformer.

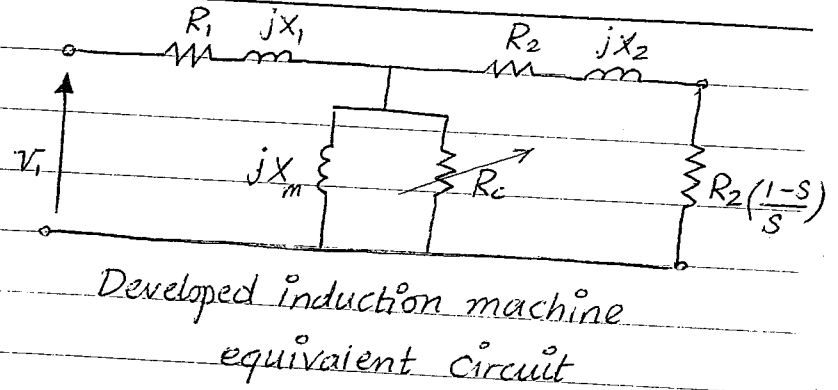
DATE

* Losses

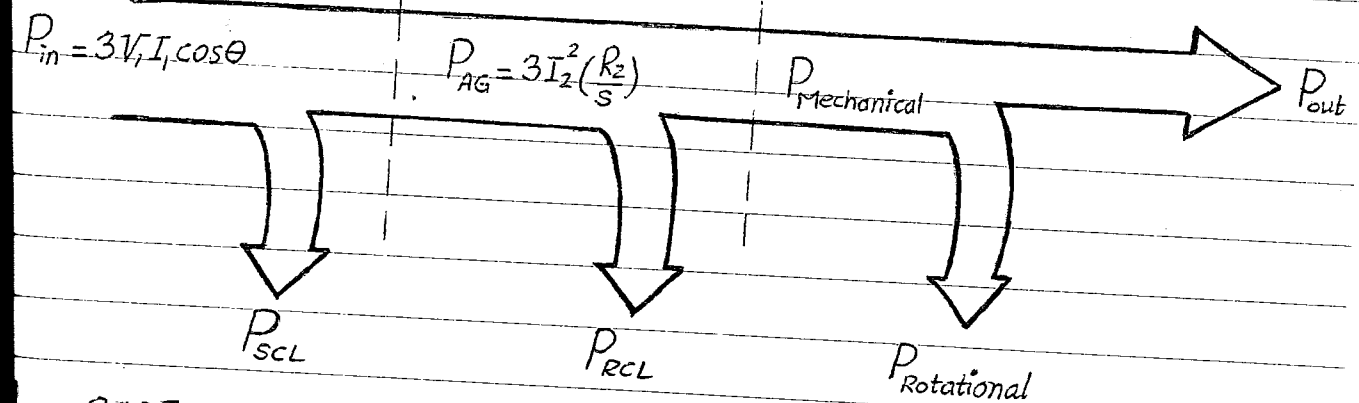
حالت محل (Loss) الموجودة في (Rc)

Rotational Loss (P_{Rot})

P_{Eddy} $P_{Hyst.}$ $P_{Friction}$ $P_{Windage}$



Power Flow



$$P_{in} = 3V_1 I_1 \cos \theta$$

$$P_{SCL} = 3I_1^2 R_1 \quad (\text{Stator Copper Loss})$$

$$P_{AG} = 3I_2^2 \left(\frac{R_2}{s} \right) \quad (\text{Air-Gap Power}) ; \quad P_{in} - P_{SCL} = P_{AG}$$

$$P_{RCL} = 3I_2^2 R_2 \quad (\text{Rotor Copper Loss})$$

$$P_{AG} = 3I_2^2 R_2$$

$$P_{mech} = 3I_2^2 R_2 \left(\frac{1-s}{s} \right) \quad (\text{Mechanical Power})$$

$$P_{AG} = (1-s) P_{AG}$$

$$P_{Rot} = s P_{AG} = 3I_2^2 R_2 \quad (\text{Rotational Loss})$$

$$P_{mech} - P_{Rot}$$

$$P_{mech} + P_{RCL} = P_{AG}$$

$$s = \frac{n_s - n_m}{n_s} \Rightarrow n_m = (1-s)n_s \Rightarrow \omega_m = (1-s)\omega_s \quad \text{rad/sec.}$$

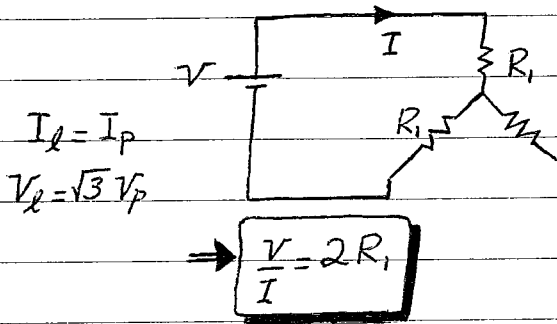
$$\frac{P}{\omega} = \frac{P_{mech}}{\omega_m} = \frac{3I_2^2 R_2 \left(\frac{1-s}{s} \right)}{(1-s)\omega_s} = \frac{P_{AG}}{\omega_s} \Rightarrow T_{dev} = \frac{P_{mech}}{\omega_m} = \frac{P_{AG}}{\omega_s}$$

* ملاحظة : في حالة Starting تكون $(\omega_m = 0)$ و $(P_m = 0)$ فلا يمكن استغلال القاطن $(T = \frac{P_m}{\omega_m})$ وتكون $(\eta_m = 0) \leftarrow (s = 1)$ وتكون $(P_{AG} = P_{RCL})$

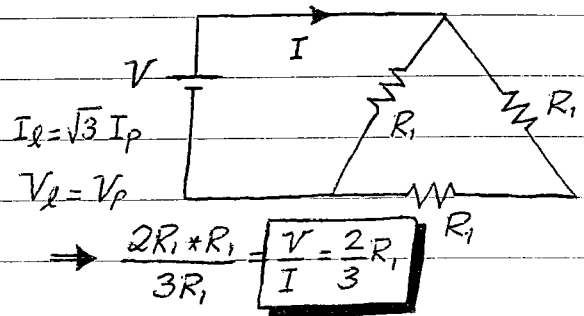
* Induction Motors Tests ...

1 DC Test ...

Y Connection of Stator



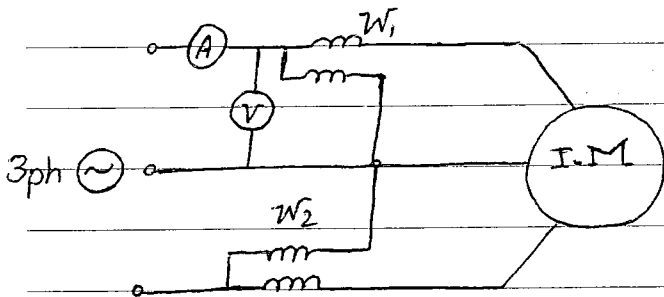
Δ Connection of Stator



2 No Load Test ...

$\Rightarrow P_{NL}, V_{NL}, I_{NL}, f_{rated}$

ملاحظة: عند هذا التحليل لا يمكن حساب القدرة الميكانيكية (No Load) ! Mechanical Power



2 Wattmeters Method

$$P_{tot} = W_1 + W_2$$

$$P_{NL} = 3I_{NL}^2 R_1 + P_{rot} = 3I_{NL}^2 R_{NL}$$

$$Z_{NL} = \frac{V_{NL}/\sqrt{3}}{I_{NL}}$$

$$R_{NL} = \frac{P_{NL}}{3I_{NL}^2}$$

$$X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2}$$

$$X_{NL} = X_1 + X_m$$

ملاحظة: عند هذه الخطوة نتوقف، لأننا لا نستطيع معرفة قيمة X_m و X_1 .

وعندها نلجأ إلى (Test) آخر طابعية الخلل.

3 Locked (Blocked) Rotor Test ...

$\Rightarrow P_{BR}, V_{BR}, I_{BR}, f_{BR}$

$$P_{BR} = 3I_{BR}^2 R_{BR} = 3I_{BR}^2 (R_1 + R_2)$$

$$Z_{BR} = \frac{V_{BR}/\sqrt{3}}{I_{BR}} = \sqrt{R_{BR}^2 + X_{BR}^2}$$

$$X_1 + X_2 = \frac{V_{BR}/\sqrt{3}}{I_{BR}} \quad (f_{BR} \text{ Tested})$$

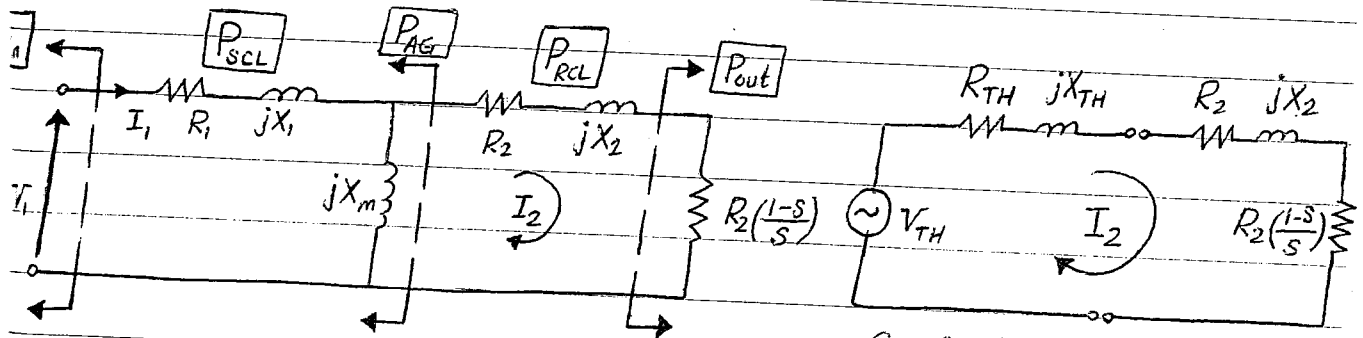
$f_{BR} < f_{rated}$

$$OR \quad (X_1 - X_2) / 2 = X_1 = X_2$$

هذا القانون يستخدم في حالة لم يذكر كم قيمة (f_{BR}) .

DATE 12/Apr/2012 Thursday

Thevenin Equivalent Circuit



Equivalent Circuit

V_1, R_1, X_1 & X_m can be replaced by Thevenin Equivalent Circuit values (V_{TH}, R_{TH} & X_{TH}).

$$V_{oc} = V_{TH} = \frac{V_1}{R_1 + j(X_1 + X_m)} \times jX_m$$

$$V_{TH} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} V_1$$

If $R_1^2 \ll (X_1 + X_m)^2$ as usually, then $V_{TH} \cong \frac{X_m}{X_1 + X_m} V_1 = K_{TH} V_1$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

If $R_1^2 \ll (X_1 + X_m)^2$ as usually, then $R_{TH} = \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 = K_{TH}^2 R_1$
and since $X_1 \ll X_m \Rightarrow X_{TH} \cong X_1$

3 ϕ , 100hp, 460V, 8p, Y-connection Squirrel Cage

NLT: 460V, 60Hz, 40A, 4.2kW

BRT: 100V, 60Hz, 140A, 8kW

(R_{DC}) between two stators terminals = 0.152 Ω

Determine the parameters of the equivalent Circuit

$$R_1 = \frac{R_{DC}}{2} = \frac{0.152}{2} = 0.076 \Omega$$

$$\Rightarrow Z_{NL} = \frac{V_{NL}/\sqrt{3}}{I_{NL}} = \frac{460/\sqrt{3}}{40} = 6.64 \Omega$$

$$P_{NL} = 3I_1^2 R_1^2 + P_{rot} \quad \& \quad P_{NL} = 3I_1^2 R_{NL}$$

$$\Rightarrow P_{rot} = P_{NL} - 3I_1^2 R_1 \quad \Rightarrow R_{NL} = \frac{4200}{3(40)^2} = 0.875 \Omega$$

$$= 4200 - 3(40)^2(0.076)$$

$$= 3835.2 \text{ W.}$$

$$\Rightarrow X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2} = \sqrt{6.64^2 - 0.875^2} = 6.58 \Omega$$

$$\Rightarrow \boxed{X_1 + X_m = 6.58}$$

• ملاحظة: إذا كان السؤال

BRT: $P_{BR} = 3I_{BR}^2 R_{BR}$

Δ -Connection

$$\Rightarrow R_{BR} = \frac{P_{BR}}{3I_{BR}^2} = \frac{8000}{3(140)^2} = 0.136 \Omega \quad (I_L = \sqrt{3}I_p) \quad (V_L = V_p) \text{ كوكب}$$

$$R_{BR} = R_1 + R_2 \Rightarrow R_2 = R_{BR} - R_1 = 0.136 - 0.076 = 0.06 \Omega$$

$$\Rightarrow Z_{BR} = \frac{V_{BR}/\sqrt{3}}{I_{BR}} = \frac{100/\sqrt{3}}{140} = 0.412 \Omega$$

$$\Rightarrow X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2} = \sqrt{0.412^2 - 0.136^2} = 0.389 \Omega$$

$$X_{BR} = X_1 + X_2 \Rightarrow X_1 = X_2 = \frac{X_{BR}}{2} = 0.194 \Omega$$

But $X_1 + X_m = 6.58$

$$\therefore X_m = 6.58 - 0.194 = 6.39 \Omega$$

$$\Rightarrow V_{TH} = \frac{V_1}{R_1 + j(X_1 + X_m)} \times jX_m = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} V_1 = \frac{6.39 \times 460/\sqrt{3}}{\sqrt{(0.076)^2 + (6.58)^2}} = 257.9 \text{ V.}$$

OR $K_{TH} = \frac{X_m}{X_1 + X_m} = \frac{6.39}{6.58} = 0.971$

$$\Rightarrow V_{TH} = K_{TH} V_1 = 0.971 \times 460/\sqrt{3} = 257.9 \text{ V.}$$

$$\Rightarrow R_{TH} = K_{TH}^2 R_1 = (0.971)^2 \times 0.076 = 0.0717 \Omega$$

$$\Rightarrow Z_{TH} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = \frac{j6.39 \times (0.076 + j0.194)}{0.076 + j(6.58)} = 0.202 \Omega$$

$$\Rightarrow X_{TH} = \sqrt{Z_{TH}^2 - R_{TH}^2} = \sqrt{0.202^2 - 0.0717^2} = 0.19 \Omega$$

$$\approx X_1$$

DATE 15/Apr/2012 Sunday

Example 2: 3ϕ , 15 hp, 460 V, 4p, 60 Hz, 1728 rpm, $P_{rot} = 750$ W.

a) Determine Mechanical power developed.

b) Air gap power.

c) Rotor Copper Loss.

Solution 2:- $P_{out} = 15 \text{ hp} \times 746 = 11190 \text{ W}$.

$$P_{out} = P_{mech} - P_{rot} \Rightarrow P_{mech} = P_{out} + P_{rot}$$

$$P_{mech} = 11190 + 750 = 11940 \text{ W}$$

$$P_{mech} = (1-s) P_{AG} ; s = \frac{n_s - n_m}{n_s} ; n_s = \frac{120 \times f}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$\therefore s = \frac{1800 - 1728}{1800} = 0.04 \Rightarrow P_{AG} = \frac{P_{mech}}{(1-s)}$$

$$\Rightarrow P_{AG} = \frac{11940}{(1-0.04)} = 12437.5 \text{ W}$$

$$\Rightarrow P_{rot} = s P_{AG} = 0.04 (12437.5) = 497.5 \text{ W}$$

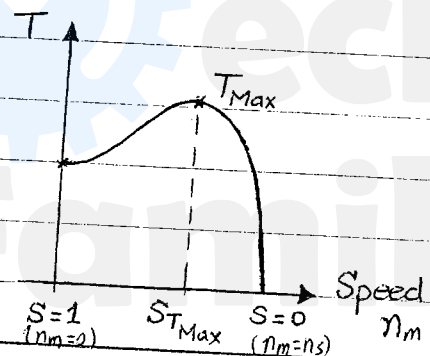
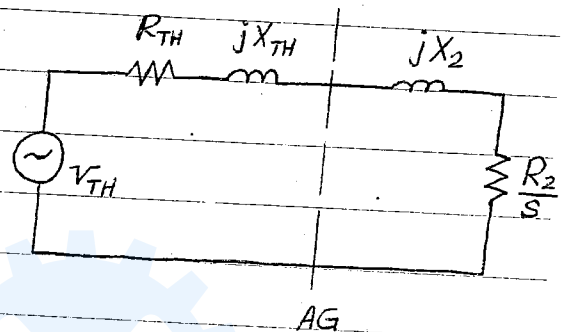
Performance (Torque-Speed) Characteristics

$$I_2 = \frac{V_{TH}}{(R_{TH} + \frac{R_2}{s}) + j(X_{TH} + X_2)}$$

$$|I_2| = \frac{V_{TH}}{\sqrt{(R_{TH} + \frac{R_2}{s})^2 + (X_{TH} + X_2)^2}}$$

$$P_{AG} = 3 I_2^2 \frac{R_2}{s} ; T_{dev} = \frac{P_{AG}}{\omega_s} = \frac{P_m}{\omega_m}$$

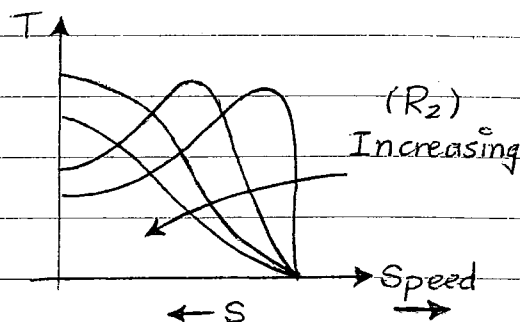
$$T_{dev} = \frac{3 V_{TH}^2 (\frac{R_2}{s})}{\omega_s [(R_{TH} + \frac{R_2}{s})^2 + (X_{TH} + X_2)^2]}$$



$$\frac{dT_{dev}}{ds} = 0 \rightarrow s = s_{T_{Max}}$$

$$T_{max} = \frac{3V_{TH}^2}{2\omega_s (R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2})}$$

$$S_{Tmax} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$



Example 8: 3ϕ , 460 V , 1740 rpm , 60 Hz , 4 p

$$R_1 = 0.25\ \Omega, R_2 = 0.2\ \Omega, X_1 = X_2 = 0.5\ \Omega, X_m = 30\ \Omega$$

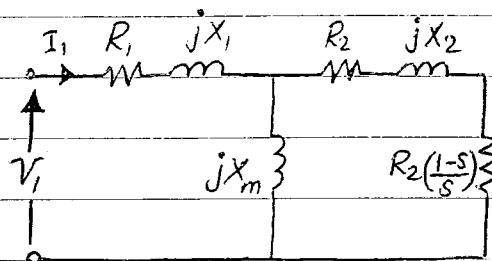
$P_{rot} = 1700\text{ W}$. Determine s-

a) Starting current when started direct on full voltage, & starting torque.

@ starting $\Rightarrow s=1$

$$Z_1 = jX_m \frac{(R_2 + jX_2)}{(R_2 + j(X_2 + X_m))} + (R_1 + jX_1)$$

$$Z_1 = j30 \frac{(0.2 + j0.5)}{(0.2 + j30.5)} + (0.25 + j0.5)$$



$$\Rightarrow Z_1 = 0.443 + j0.993 = 1.08 \angle 66^\circ\ \Omega$$

$$\Rightarrow I_{st} = \frac{V_1 / \sqrt{3}}{Z_1} = \frac{460 / \sqrt{3}}{1.08 \angle 66^\circ} = 245.9 \angle -66^\circ\text{ A}$$

$$n_s = \frac{120}{p} f = \frac{120}{4} \times 60 = 1800\text{ rpm} \Rightarrow \omega_s = 1800 \left(\frac{2\pi}{60} \right) = 188.5 \frac{\text{rad}}{\text{s}}$$

$$K_{TH} = \frac{X_m}{X_1 + X_m} = \frac{30}{0.5 + 30} = 0.98$$

$$\Rightarrow V_{TH} = K_{TH} V_1 = 0.98 (460 / \sqrt{3}) = 261.3\text{ V}$$

$$\Rightarrow R_{TH} = K_{TH}^2 R_1 = (0.98^2) (0.25) = 0.24\ \Omega$$

$$\Rightarrow Z_{TH} = \frac{(0.25 + j0.5)}{(0.25 + j30.5)} j(30) = 0.55 \angle 64^\circ\ \Omega$$

$$\Rightarrow X_{TH} = \sqrt{0.55^2 - 0.24^2} = 0.49\ \Omega \approx X_1$$

$$\therefore T_{st} = \frac{3V_{TH}^2 (R_2/s)}{\omega_s ((R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2)} = \frac{3 \times (261.3)^2 \times 0.2}{188.5 ((0.24 + 0.2)^2 + (0.49 + 0.5)^2)} = 185\text{ N.m}$$

OR $P_{in} = 3V_1 I_1 \cos \theta = 3 (460 / \sqrt{3}) (245.9) \cos(66) = 79687.5\text{ W}$

$$P_{scl} = 3I_1^2 R_1 = 3 (245.9)^2 (0.25) = 45350.1\text{ W}$$

$$P_{AG} = P_{in} - P_{scl} = 34337.4\text{ W} \Rightarrow T = \frac{P_{AG}}{\omega_s} = \frac{34337.4}{188.5} = 182\text{ N.m}$$

DATE 17/Apr/2012 Tuesday

b) Full load slip, Full Load Current, Ratio (I_{st}/I_{FL}), Full Load p.f.
Full Load Torque, Motor Efficiency @ Full load.

$$\Rightarrow S = \frac{n_s - n_m}{n_s} = \frac{1800 - 1740}{1800} = 0.033, \quad \frac{R_2}{S} = \frac{0.2}{0.033} = 6 \Omega$$

$$Z_1 = (R_1 + jX_1) + jX_m \frac{(R_2/S + jX_2)}{R_2/S + j(X_2 + X_m)} = (0.25 + j0.5) + j30 \frac{(6 + j0.5)}{(6 + j30.5)}$$

$$Z_1 = 5.839 + j2.091 = 6.2 / 19.7^\circ \Omega$$

$$\Rightarrow I_{FL} = \frac{460/\sqrt{3}}{6.2 / 19.7^\circ} = 42.8 / -19.7^\circ \text{ A.}$$

$$\Rightarrow \text{Ratio} = \frac{I_{st}}{I_{FL}} = \frac{245.9}{42.8} = 5.745$$

$$\Rightarrow \text{p.f.} = \cos(19.7^\circ) = 0.94 \text{ (lag)}$$

$$\Rightarrow T_{FL} = \frac{3V_{TH}^2 (R_2/S)}{\omega_s ((R_{TH} + \frac{R_2}{S})^2 + (X_{TH} + X_2)^2)} = \frac{3 \times 261.3^2 \times 6}{188.5 ((0.24 + 6)^2 + (0.49 + 0.5)^2)} = 163.3 \text{ N.m.}$$

OR $P_{in} = 3V_L I_L \cos \theta = 3 \times (460/\sqrt{3}) \times (42.8) \cos(-19.7^\circ) = 32105 \text{ W.}$

$$P_{scl} = 3I_L^2 R_1 = 3(42.8)^2 (0.25) = 1374 \text{ W.}$$

$$P_{AG} = P_{in} - P_{scl} = 30731 \text{ W}$$

$$\Rightarrow T_{FL} = \frac{P_{AG}}{\omega_s} = \frac{30731}{188.5} = 163 \text{ N.m.}$$

$$P_{rel} = S P_{AG} = 0.033(30731) = 1014 \text{ W.}$$

$$P_{mech} = (1 - S) P_{AG} = (1 - 0.033)(30731) = 29717 \text{ W.}$$

$$P_{out} = P_{mech} - P_{rot} = 29717 \text{ W} - 1700 \text{ W} = 28017 \text{ W.}$$

$$\Rightarrow \eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{28017}{32105} \times 100\% = 87.3\%$$

Slip @ T_{Max} , Maximum Torque developed, Speed @ S_{Tmax} .

$$\Rightarrow S_{Tmax} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = \frac{0.2}{\sqrt{(0.24)^2 + (0.49 + 0.5)^2}} = 0.1963$$

$$\Rightarrow T_{Max} = \frac{3V_{TH}^2}{2\omega_s (R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2})} = \frac{3 \times 261.3^2}{2 \times 188.5 (0.24 + \sqrt{0.24^2 + (0.49 + 0.5)^2})} = 431.6 \text{ N.m.}$$

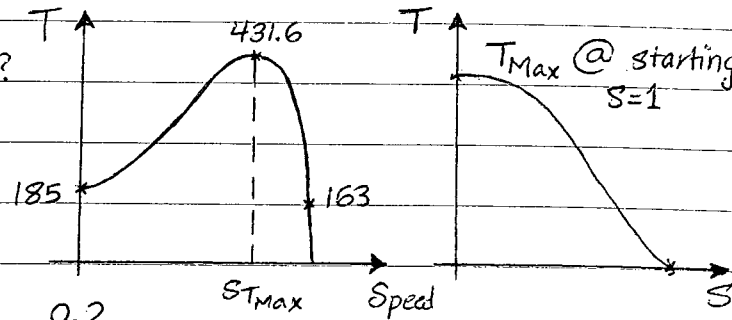
$$\Rightarrow n_m = n_s(1 - S) = 1800(1 - 0.1963) = 1447 \text{ rpm.}$$

d) How much R_{ext} should be connected to get T_{max} @ starting?

$$S_{T_{max}} = \frac{R_2 + R_{ext}}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = 1$$

$$\Rightarrow R_{ext} = \sqrt{(0.24)^2 + (0.49 + 0.5)^2} - 0.2$$

$$= 0.819 \Omega$$



ملاحظة • Note : if given (slip) & (hp) $\Rightarrow$ use slip & don't use hp!
otherwise use hp!

P(5.15) - 3 ϕ , 460 V, 60 Hz, 100 hp, 1746 rpm, $P_{rot} = 3500$ W, $P_{scl} = 3000$ W
find η (Efficiency) ?

Solution :-

$$n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s = \frac{1800 - 1746}{1800} = 0.03$$

$$\Rightarrow P_{out} = 100 \text{ hp} \times 746 = 74600 \text{ W}$$

$$P_{mech} = P_{out} + P_{rot} = 74600 + 3500 = 78100 \text{ W}$$

$$P_{mech} = (1-s) P_{AG}$$

$$P_{AG} = \frac{78100}{(1-0.03)} = 80515.5 \text{ W}$$

$$\Rightarrow P_{in} - P_{scl} = P_{AG}$$

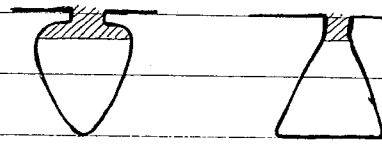
$$P_{in} = P_{AG} + P_{scl} = 80515.5 + 3000 = 83515.5 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{74600}{83515.5} \times 100\% = 89.32\%$$

* Effect of Rotor Resistance

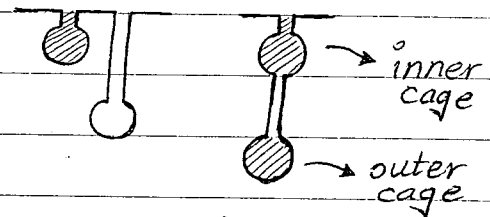
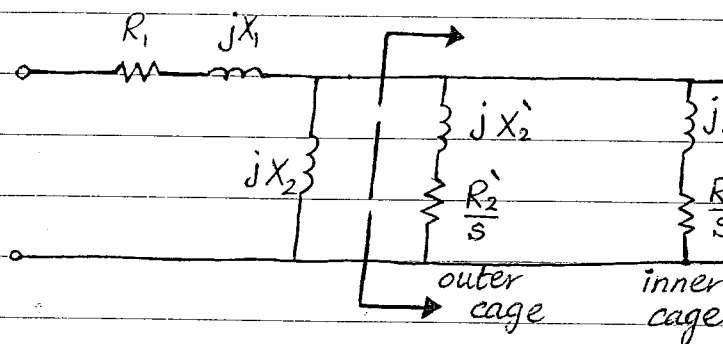
Resistance (R_2) @ start $>$ (R_2)

@ full load ; due to skin effect.
so it can be solved by choosing
the proper shape of slot.



Deep Bar Rotor

* Double cage Rotor



$$Z_o = \frac{R_2'}{s} + jX_2'$$

$$Z_i = \frac{R_2''}{s} + jX_2''$$

Ex. $Z_o = 5 + j1$, $Z_i = 1 + j5$

@ starting (s=1)

$$I_o = \frac{V}{Z_o} = \frac{V}{5 + j1} \Rightarrow |I_o| = V/\sqrt{26} \Rightarrow T_o = \frac{P_{AG}}{\omega_s} = \frac{3I_o^2 R_2'}{\omega_s}$$

$$I_i = \frac{V}{Z_i} = \frac{V}{1 + j5} \Rightarrow |I_i| = V/\sqrt{26} \Rightarrow T_i = \frac{P_{AG}}{\omega_s} = \frac{3I_i^2 R_2''}{\omega_s}$$

$$\text{Ratio } \frac{T_o}{T_i} = \frac{3(V/\sqrt{26})^2 \times 5/\omega_s}{3(V/\sqrt{26})^2 \times 1/\omega_s} = 5$$

* في حالة (Starting) : ال (outer cage) أنتج (Torque) يساوي خمس أضعاف (inner cage)

@ running (s=0.05)

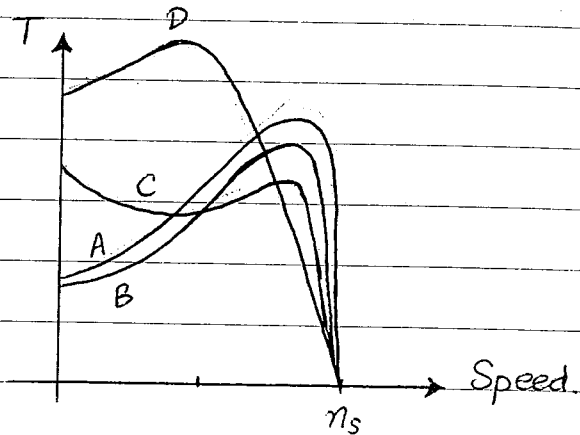
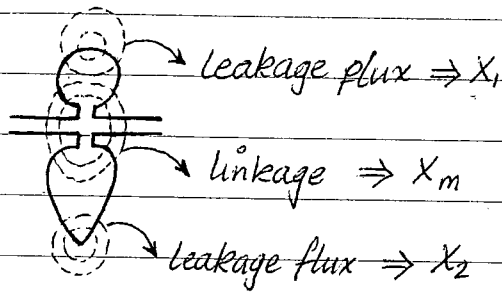
$$Z_o = \frac{5}{0.05} + j1 = 100 + j1 \Rightarrow I_o = \frac{V}{\sqrt{10001}} \Rightarrow T_o = \frac{3I_o^2 R_2'/s}{\omega_s}$$

$$Z_i = \frac{1}{0.05} + j5 = 20 + j5 \Rightarrow I_i = \frac{V}{\sqrt{425}} \Rightarrow T_i = \frac{3I_i^2 R_2''/s}{\omega_s}$$

$$\text{Ratio } \frac{T_o}{T_i} = \frac{3(V/\sqrt{10001})^2 \times 100}{3(V/\sqrt{425})^2 \times 20} = 0.212$$

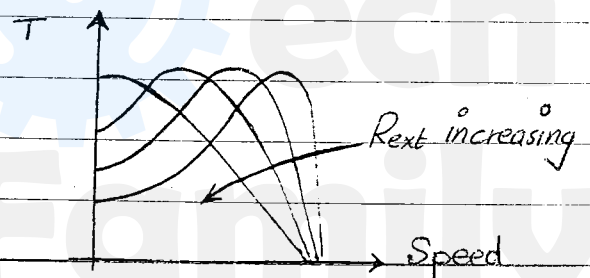
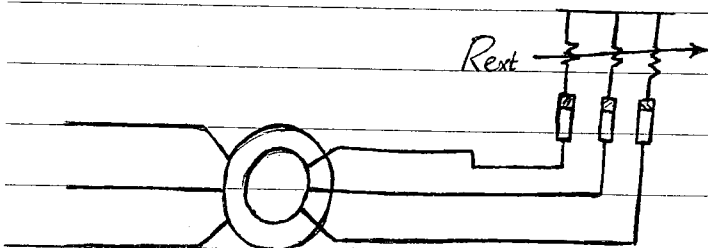
* في حالة (Running) : ال (inner cage) أنتج (Torque) تقريباً يساوي خمس أضعاف (outer)

* Classes of Squirrel-Cage Motors



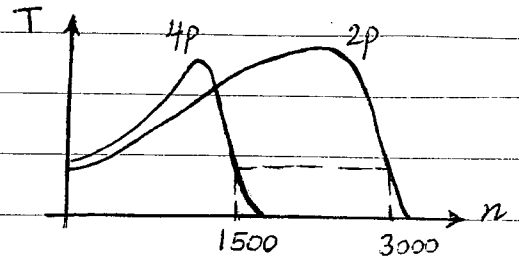
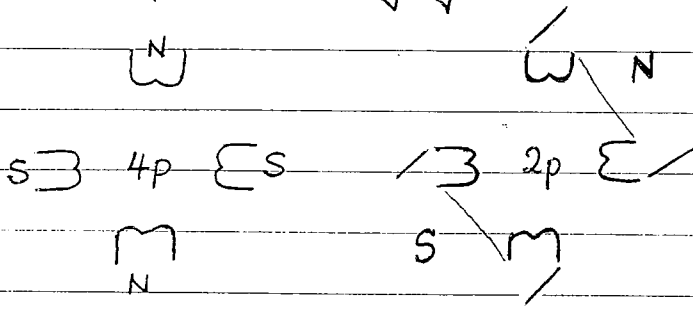
	Class "A"	Class "B"	Class "C"	Class "D"
Rotor Cage				High Resistive Material
Starting Current ($I_{st.}$)	High	Normal	Normal	Normal
Starting Torque ($T_{st.}$)	Low	Low	High	High
Slip (s)	1-5%	1-5%	1-8%	5-15%
Usage	Fan, Pump	Fan, Pump Blowers	High Torque Heavy Load Belt, Conveyors, Lift	High Acceleration Punch, Packing

* Speed Control of induction motors



DATE 22/Apr/2012 Sunday

1 - By Pole Changing



$$n_s = \frac{120f}{P}$$

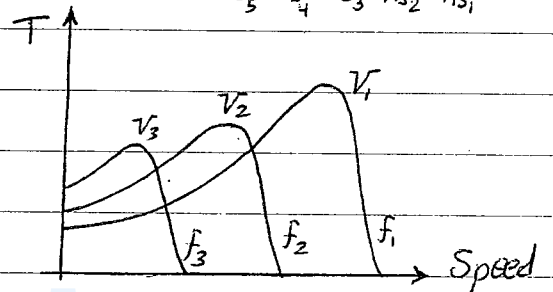
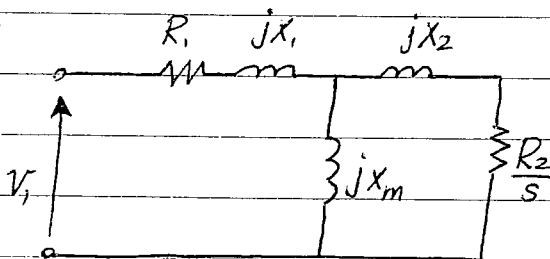
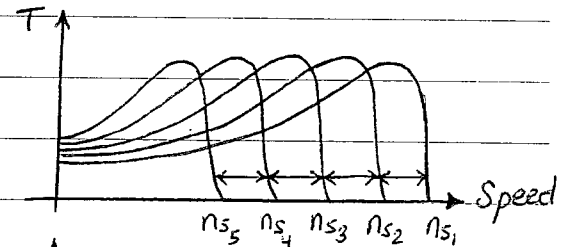
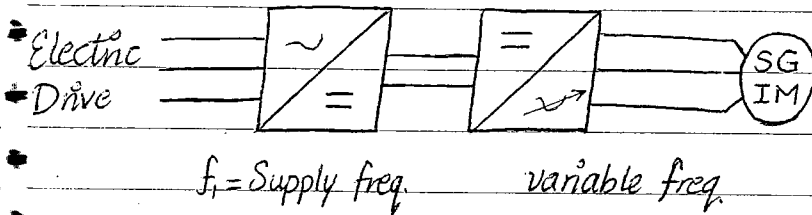
$$; f = 50 \text{ Hz}$$

$$n_{s1} = 120(50)/4 = 1500 \text{ rpm}$$

$$n_{s2} = 120(50)/2 = 3000 \text{ rpm.}$$

* أفضل طريقة هي
Changing Frequency
(Electric Drive)

2 - By Frequency Changing (Electric Drive)



$$V \propto f$$

$$T \propto V^2$$

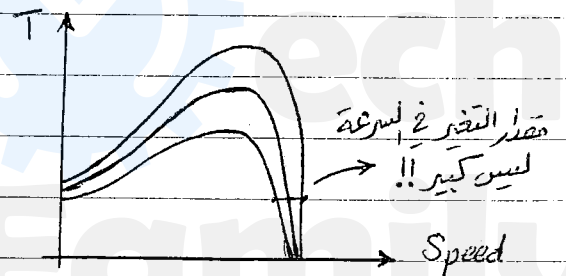
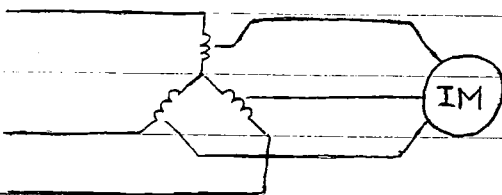
عندما تقل (f) تقل (V) ←

وال (f) بنفس النسبة

عندما تقل (f) تقل معها ←

(V) و (Tmax)

3 - By Voltage Changing



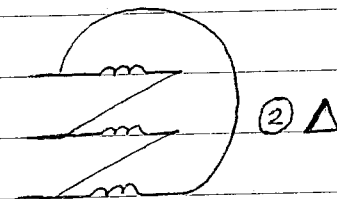
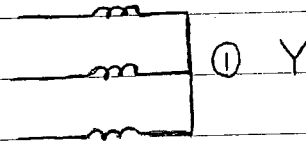
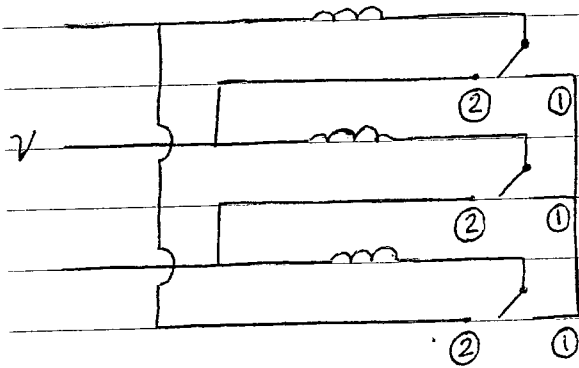
* Starting of Induction Motors

all methods of speed control can be used for starting

(Y- Δ) starting

Squirrel cage

كل طرف لتشغيل مرتبطة
بطرفي تخير السرعة.



$$(T \propto V^2)$$

$$(I \propto V)$$

only if the Motor
Normal Connection
is (Δ).

P(5.26) 3 ϕ , 60 Hz, 1710 rpm double cage rotor induction machine.

outer cage $Z_o = 4 + j1.5 \Omega$

inner cage $Z_i = 0.5 + j4.5 \Omega$

if stator impedance is neglected, determine :-

a) The Ratio of currents in the outer & inner cages

$$n_m = \frac{120(60)}{P} = 1710 \Rightarrow p = 4.21 \rightarrow 4p \Rightarrow n_s = \frac{120(60)}{4} = 1800$$

$$\text{slip } P = S = \frac{1800 - 1710}{1800} = 0.05$$

@ starting ($S=1$)

$$I_o = \frac{V}{Z_o} = \frac{V}{4 + j1.5} = \frac{V}{4.272}$$

$$I_i = \frac{V}{Z_i} = \frac{V}{0.5 + j4.5} = \frac{V}{4.528} \Rightarrow \frac{I_o}{I_i} = 1.06 \text{ @ starting.}$$

@ running ($S=0.05$)

$$I_o = \frac{V}{(\frac{4}{0.05}) + j1.5} = \frac{V}{80.014}$$

$$I_i = \frac{V}{(\frac{0.5}{0.05} + j4.5)} = \frac{V}{10.966}$$

$$\Rightarrow \frac{I_o}{I_i} = 0.137 \text{ @ Full load}$$

b) The starting torque of the motor as percent of Full-load torque.

$$T_{st} = T_{o_{st}} + T_{i_{st}} \quad , \quad T_{FL} = T_{o_{FL}} + T_{i_{FL}}$$

@ starting ($s=1$)

$$T_o = \frac{3 I_o^2 (R_2/s)}{\omega_s} = \frac{3 * \left(\frac{V}{4.272}\right)^2 * (4)}{\omega_s} = \left(\frac{3V^2}{\omega_s}\right) * \frac{4}{18.25}$$

$$T_i = \frac{3 I_i^2 (R_2/s)}{\omega_s} = \frac{3 * \left(\frac{V}{4.528}\right)^2 * (0.5)}{\omega_s} = \left(\frac{3V^2}{\omega_s}\right) * \frac{0.5}{20.5}$$

@ running ($s=0.05$)

$$T_o = \frac{3 I_o^2 (R_2/0.05)}{\omega_s} = \frac{3 * \left(\frac{V}{80.014}\right)^2 * (4/0.05)}{\omega_s} = \left(\frac{3V^2}{\omega_s}\right) * \frac{80}{6402.25}$$

$$T_i = \frac{3 I_i^2 (R_2/0.05)}{\omega_s} = \frac{3 * \left(\frac{V}{10.966}\right)^2 * (0.5/0.05)}{\omega_s} = \left(\frac{3V^2}{\omega_s}\right) * \frac{10}{120.25}$$

$$\Rightarrow T_{st} = \left(\frac{4}{18.25} + \frac{0.5}{20.5}\right) \left(\frac{3V^2}{\omega_s}\right) = 0.24357 \left(\frac{3V^2}{\omega_s}\right)$$

$$\Rightarrow T_{FL} = \left(\frac{80}{6402.25} + \frac{10}{120.25}\right) \left(\frac{3V^2}{\omega_s}\right) = 0.09566 \left(\frac{3V^2}{\omega_s}\right)$$

$$\therefore \frac{T_{st}}{T_{FL}} = \frac{0.24357}{0.09566} = 2.546 = 255\%$$

c) The Ratio of Torques in the outer & inner cages.

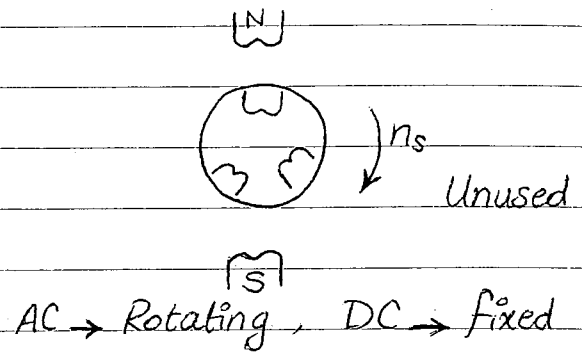
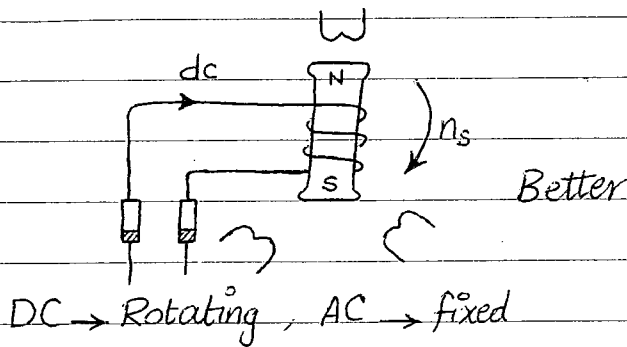
@ starting ($s=1$)

$$\frac{T_o}{T_i} = \frac{(4/18.25)}{(0.5/20.5)} = 8.486$$

@ running ($s=0.05$)

$$\frac{T_o}{T_i} = \frac{(80/6402.25)}{(10/120.25)} = 0.15$$

* Chapter 6 : Synchronous Machines ...



$$n_s = \frac{120f}{p} \Rightarrow \boxed{f = \frac{n_s p}{120}} \Rightarrow \text{constant.}$$

p	n_s	f
2	3000	50 Hz
4	1500	50 Hz
6	1000	50 Hz
12	500	50 Hz

* Infinite Busbar

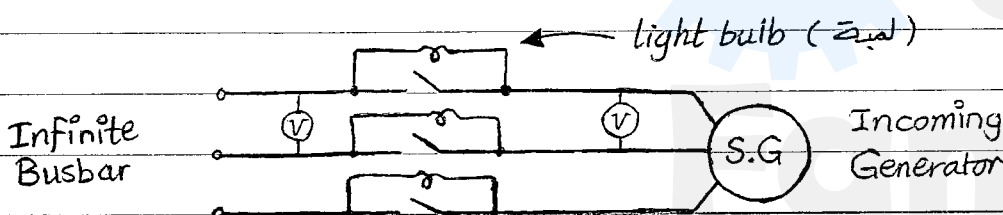
→ شبكة كبيرة من ال (Generators) مبرطة مع بعضها
يكون ال (frequency) وال (Voltage) ثابتة.
فهي تؤثر في غيرها من (generator) ولكنها لا تتأثر من غيرها

* Incoming Generator →

نريد ربط ال (Incoming Generator) مع الشبكة
(Infinite Busbar) على التوالي.

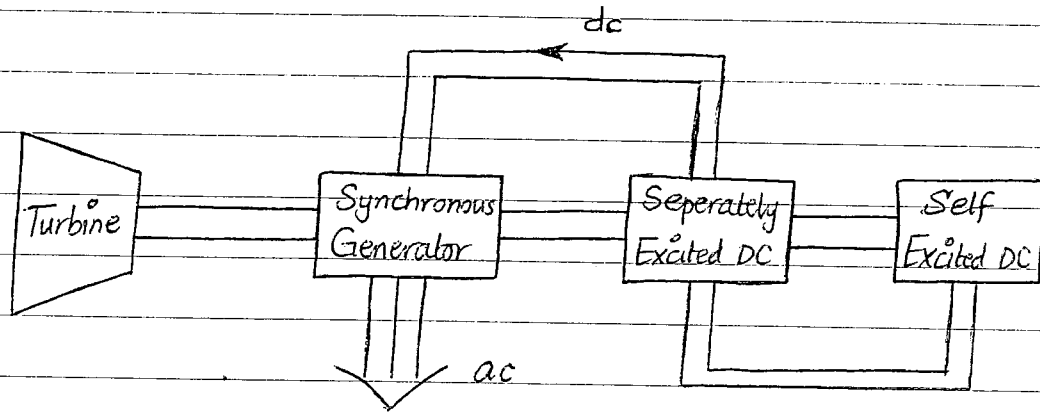
* Conditions of Synchronization of Synchronous Generators ...
(Synchronization = Connection of Generators in parallel.)

- 1 - Same phase Sequence → هذا الشرط يكفي أن نعلم مرة واحدة
- 2 - Same voltage.
- 3 - Same frequency.
- 4 - Same phase (No phase shift between the 2 generators).



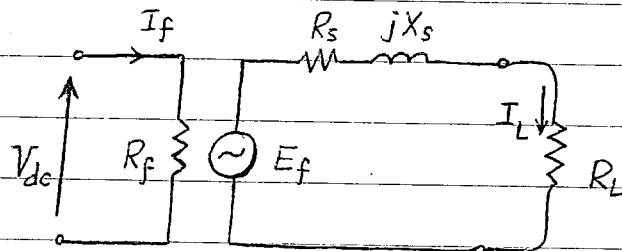
DATE 26/Apr/2012 Thursday.

* ملاحظة: أسهل طريقة لربط (Incoming Generator) مع شبكة (Infinite Busbar) هي عن طريق توصيل طابقتيهم. فإذا اشتغلوا وأطفأوا في نفس الوقت، يكون لديهم نفس ال (f, V, phase Sequence). والعكس صحيح.



Equivalent Circuit Model ...

$$X_s = X_l + X_{AR}$$



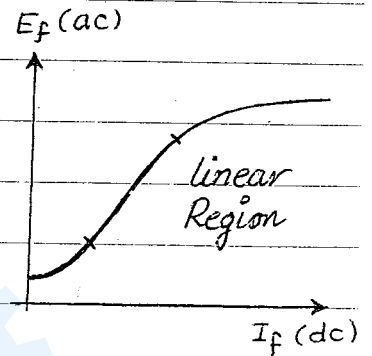
such that: $X_s \equiv$ Synchronous Reactance.

$X_l \equiv 2\pi fL \equiv$ Leakage Reactance.

$X_{AR} \equiv$ Reactance of Armature Reaction.

$Z_s = R_s + jX_s$ Synchronous Impedance.

But $X_s \gg R_s$; (R_s can be neglected)



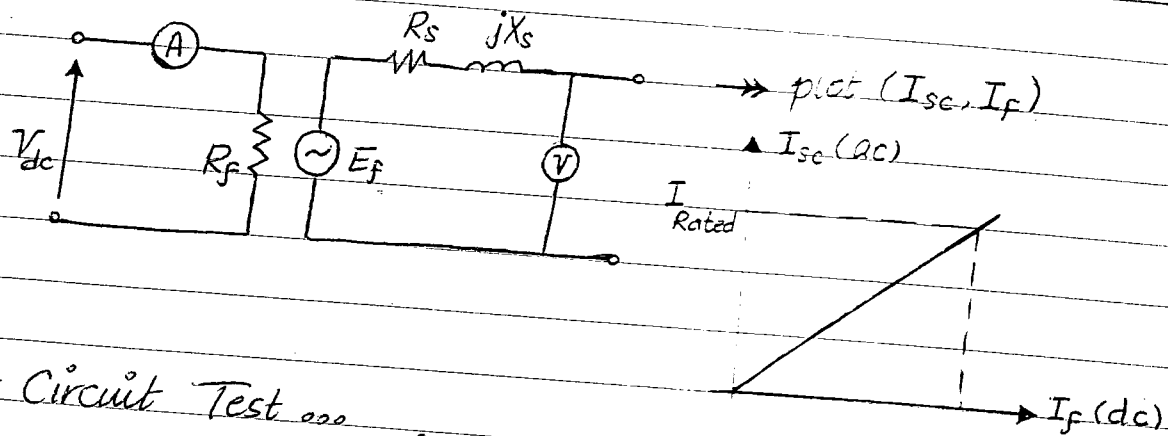
Determination of Synchronous Reactance (X_s) by Tests ...

1 - DC Test (same test of induction machine)

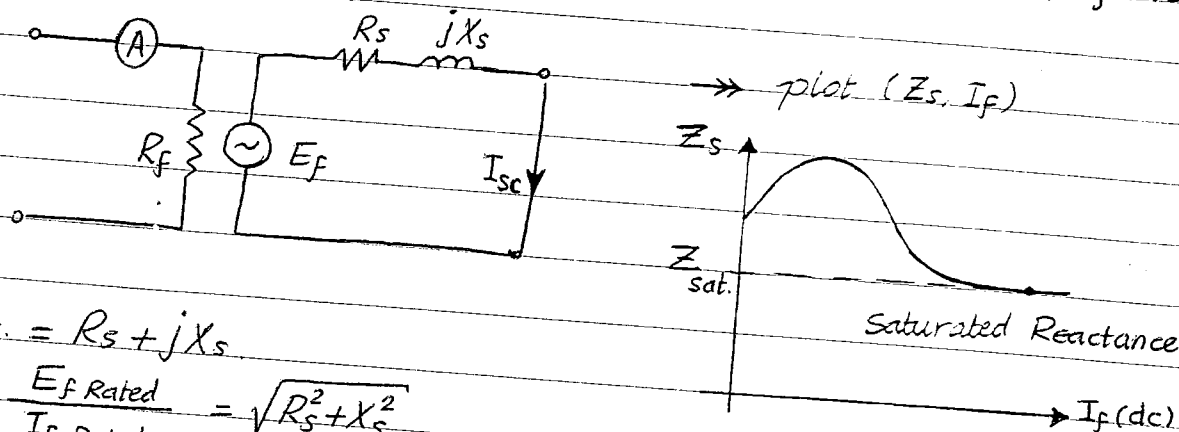
→ Find R ; $R = 2R_1$ OR $R = \frac{2}{3}R_1$

→ plot (E_f, I_f).

2 Open Circuit Test



3 Short Circuit Test



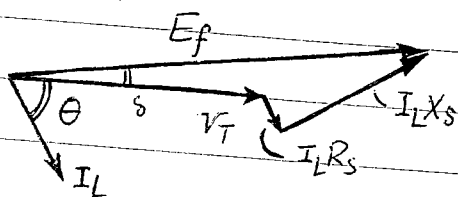
$$Z_{sat} = R_s + jX_s$$

$$\rightarrow Z_s = \frac{E_{f \text{ Rated}}}{I_{f \text{ Rated}}} = \sqrt{R_s^2 + X_s^2}$$

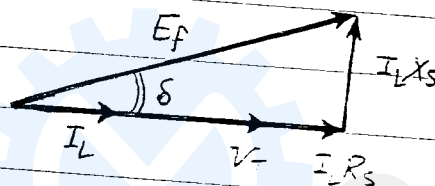
$$\rightarrow X_s = \sqrt{Z_s^2 - R_s^2} \quad ; \quad X_s = X_l + X_{AR}$$

* Phasor Diagram

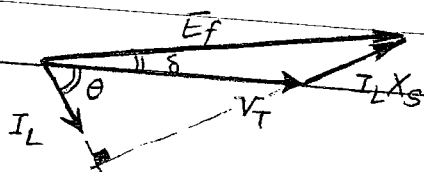
1 Lagging p.f



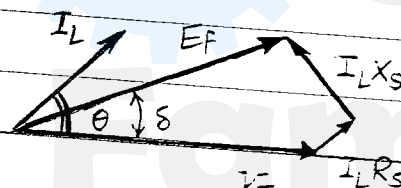
2 Unity p.f



OR with neglecting R_s

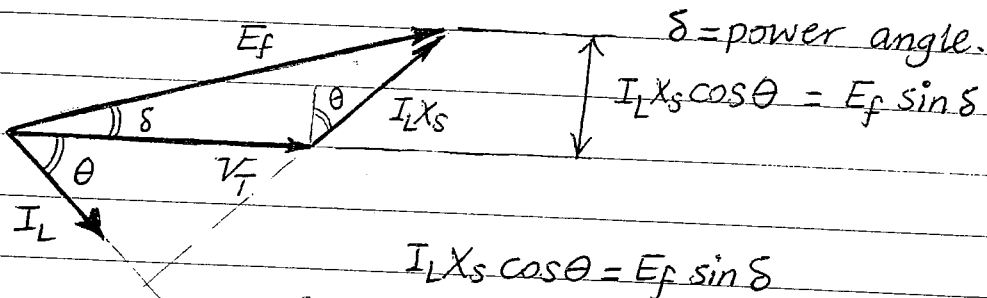


3 Leading p.f



DATE 31 May / 2012 Thursday

→ lagging p.f with neglecting (R_s) ...



$$I_L X_s \cos \theta = E_f \sin \delta$$

multiply by $\left(\frac{3V_T}{X_s}\right) \Rightarrow 3V_T I_L \cos \theta = \frac{3V_T E_f \sin \delta}{X_s}$

Power

$$P = 3V_T I_L \cos \theta = \frac{3V_T E_f \sin \delta}{X_s}$$

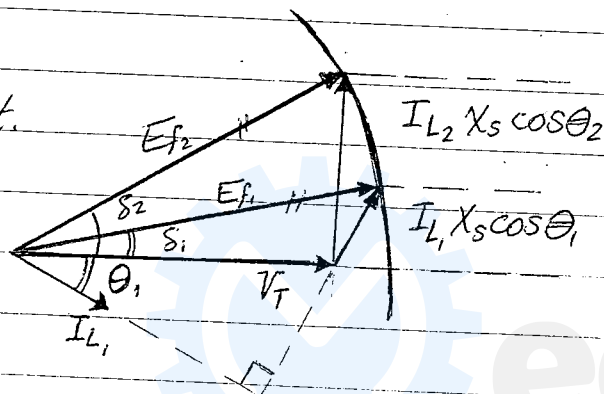
Notes : (V_T) & (X_s) can be considered constant, so Power $\equiv f(E_f, \delta)$!
 (R_s) is neglected $\Rightarrow \eta = 100\% \Rightarrow$ This power is in or out!
 Torque (T) is proportional with (P) as $\omega = \text{constant}$.
 $\Rightarrow (T \propto P)$ as $\omega = \text{constant}$!

Control of power keeping field current constant ...

By magnitude ... $E_{f1} = E_{f2}$
 frequency (f) & (V_T) $\equiv$ constant.

$$E_f = V_T + I_L Z_s$$

$$\Rightarrow I_L = \frac{E_f - V_T}{jX_s}$$



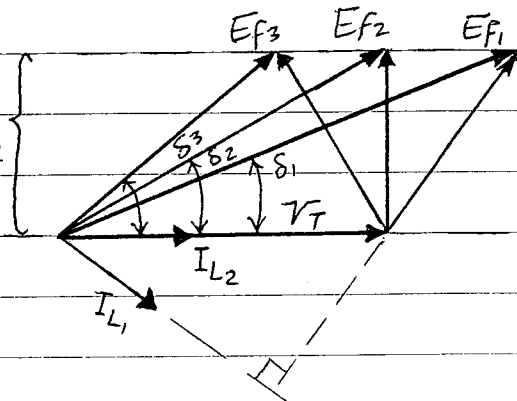
To give more power, give extra current.
 & ($T \propto P$) as $\omega = \text{constant}$.

* Control of Field Current keeping power constant ...

Keeping $(E_f \sin \delta)$ constant.

$$\therefore E_{f1} \sin \delta_1 = E_{f2} \sin \delta_2 = E_{f3} \sin \delta_3$$

power
constant



also $I_L = \frac{E_f - V_T}{jX_s}$ as a magnitude & angle ...

$$I_{L1} \cos \theta_1 = I_{L2} \cos \theta_2 + I_{L3} \cos \theta_3$$

lag.

unity

lead.

* Control to keep power factor (p.f) constant ...

Note: it increase power, also change (I_f) .

(I_{L1}) & (I_{L2}) are in the same direction

$$\Rightarrow I_{L2} > I_{L1}$$

$$P_1 = \frac{3V_T E_{f1} \sin \delta_1}{X_s}$$

$$P_2 = \frac{3V_T E_{f2} \sin \delta_2}{X_s}$$

