Fundamentals of Heat and Mass Transfer

Chapter 1

Dr. Osaid Matar

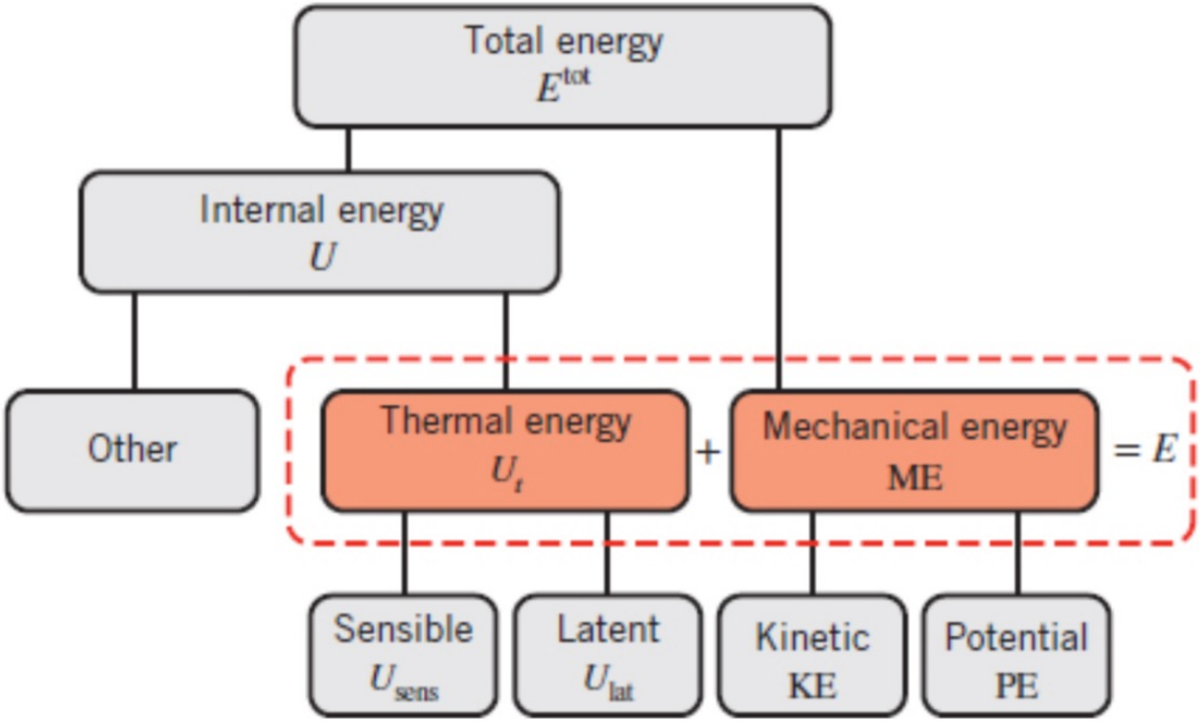
Thermal Resistance Concept

For example, in conduction, $q_x = \Delta T/R_{t,\ cond}$, where $R_{t,cond} = L/kA$ is a thermal resistance associated with conduction, having the units K/W.

The thermal resistance concept will be considered in detail later and will be seen to have great utility in solving complex heat transfer problems.

Conservation of Energy (First Law of Thermodynamics)

- The first law of thermodynamics states that the total energy of a system is conserved



Conservation of Energy (First Law of Thermodynamics)

- An important tool in heat transfer analysis, often providing the basis for determining the temperature of a system.
- Alternative Formulations

Time Basis:

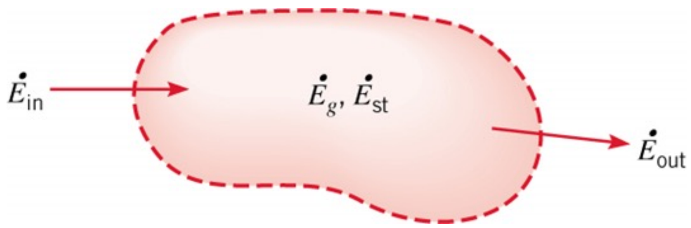
At an instant or Over a time interval

Control Volume: a region of space bounded by a control surface (fixed volume) through which mass flows in and out

Note: Mass entering and leaving the control volume carries energy with it; this process, termed energy advection,

Application to a Control Volume (1 of 2)

- At an Instant of Time:



Note representation of system by a control surface (dashed line) at the boundaries.

Surface Phenomena

$\dot{E}_{in}, \dot{E}_{out}$: rate of thermal and/or mechanical energy transfer across the control surface due to heat transfer, fluid flow and/ or work interactions.

Volumetric Phenomena

$\dot{E}_g$: rate of thermal energy generation due to conversion from another energy from (example: electrical, nuclear, or chemical); energy conversion process occurs within the system.

$\dot{E}_{st}$: rate of change of energy storage in the system.

Application to a Control Volume (2 of 2)

Conservation of Energy

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = \frac{dE_{\text{st}}}{dt} \equiv \dot{E}_{\text{st}} \qquad \textbf{(1.12c)}$$

Each term has units of J/s or W.

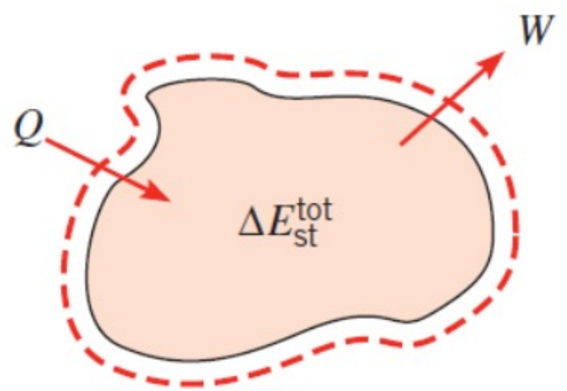
- Over a Time Interval

$$E_{\text{in}} - E_{\text{out}} + E_g = \Delta E_{\text{st}} \qquad \textbf{(1.12b)}$$

Each term has units of J.

Closed System

- Special Cases (Linkages to Thermodynamics)
  - (i) Transient Process for a Closed System of Mass (m) *Assuming Heat Transfer to the System (Inflow) and Work Done by the System (Outflow).*



Over a time interval

$$Q - W = \Delta E_{st}^{tot}$$

For negligible changes in potential or kinetic energy

$$Q - W = \underbrace{\Delta U_t}_{\text{Internal thermal energy}}$$

At an instant

$$q - \dot{W} = \frac{dU_t}{dt}$$

Thermal and Mechanical Energy, E

- Mechanical energy is the sum of kinetic energy ($KE = \frac{1}{2} mV^2$, where m and V are mass and velocity, respectively) and potential energy ($P_E = mg_z$, where g is the gravitational acceleration and z is the vertical coordinate).

- Thermal energy consists of a sensible component U_{sens}, which accounts for translational, rotational, and/or vibrational motion of the atoms/molecules comprising the matter

- and a latent component U_{lat}, which relates to intermolecular forces influencing phase change between solid, liquid, and vapor states.

- The sensible energy is the portion associated mainly with changes in temperature (although it can also depend on pressure).
- For example, if the material in the control volume increases in temperature, its sensible energy increases.
- The latent energy is the component associated with changes in phase.
- If the material in the control volume changes from solid to liquid (melting) or from liquid to vapor (vaporization, evaporation, boiling), the latent energy increases. (Endothermic)
- Conversely, if the phase change is from vapor to liquid (condensation) or from liquid to solid (solidification, freezing), the latent energy decreases. (Exothermic)

- The change in stored thermal and mechanical energy is given by $\Delta E_{st} = \Delta(KE + PE + Ut)$, where Ut = Usens + Ulat.
- In many heat transfer problems, the changes in kinetic and potential energy are small relative to changes in the thermal energy, and can be neglected.
- In addition, if there is no phase change, then the only relevant term will be the change in sensible energy, that is, $\Delta E_{st} = \Delta U_{sens}$.
- For either an ideal gas or an incompressible substance (one whose density can be treated as constant), the change in sensible energy can be expressed as $\Delta U_{sens} = mc_v \Delta T$ or in terms of rates, $dU_{sens}/dt = mc_v dT/dt$.

Energy generation, E_g,

- It is associated with conversion from some other form of internal energy (chemical, nuclear, electrical, or magnetic) to thermal or mechanical energy. It is a volumetric phenomenon.

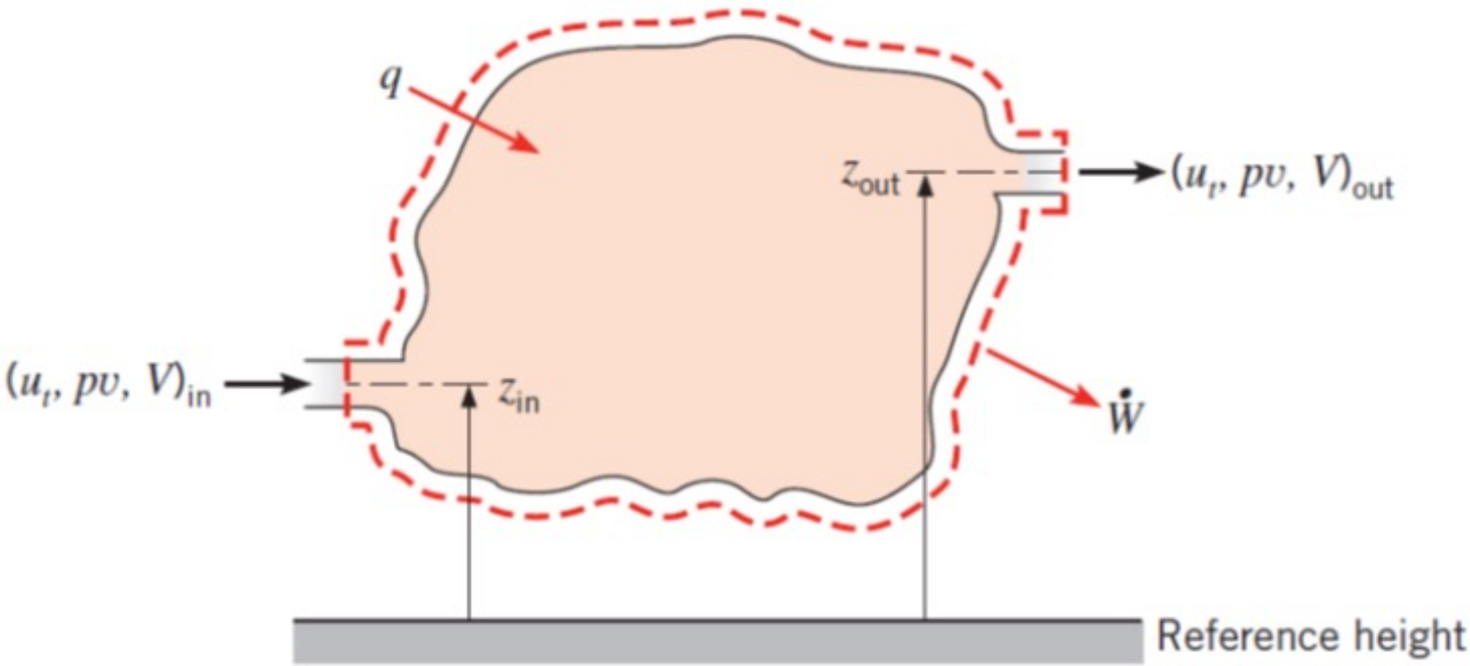
- That is, it occurs within the control volume and is generally proportional to the size of this volume.

Open System

Inflow and Outflow, E_{in} and E_{out} (surface phenomena)

ii. Steady State for Flow through an Open System without Phase Change or Generation:

At a steady state:



- When the first law is applied to a control volume with fluid crossing its boundary, it is customary to divide the work term into two contributions:

- The first contribution, termed flow work, is associated with the work done by pressure forces moving fluid through the boundary.

- For a unit mass, the amount of work is the product of the pressure and the specific volume of the fluid (pv).

- The second contribution has a symbol W. It is traditionally used for the rate at which the remaining work (not including flow work) is performed.

- If operation is under steady-state conditions ($dE_{st}/dt = 0$) and there is no thermal or mechanical energy generation

For instance, if the mass flow rate entering through the boundary is $\dot{m}$, then the rate at which thermal and mechanical energy enters with the flow is,

$$\dot{m}(u_t + \frac{1}{2}V^2 + gz),$$

where u_t is the thermal energy per unit mass. The mass flow rate may be expressed as

$$\dot{m} = \rho V A_c$$

where ρ is the fluid density and A_c is the cross-sectional area of the channel through which the fluid flows. The volumetric flow rate is simply .

$$\dot{\forall} = V A_c = \dot{m}/\rho.$$

$$\dot{m}\left(u_t + p\upsilon + \frac{V^2}{2} + gz\right)_{\text{in}} + q - \dot{m}\left(u_t + p\upsilon + \frac{V^2}{2} + gz\right)_{\text{out}} - \dot{W} = 0$$

- $(p\upsilon) \rightarrow$ flow work
- $(u_t + p\upsilon) \equiv i \rightarrow$ enthalpy

- For an ideal gas with constant specific heat:

$$i_{\text{in}} - i_{\text{out}} = c_{\text{p}}\left(T_{\text{in}} - T_{\text{out}}\right)$$

- For an incompressible liquid:

$$u_{\text{in}} - u_{\text{out}} = c\left(T_{\text{in}} - T_{\text{out}}\right)$$

$$\left(pv\right)_{\text{in}} - \left(pv\right)_{\text{out}} \approx 0$$

- For systems with significant heat transfer:

$$\left(\frac{V^2}{2}\right)_{\text{in}} - \left(\frac{V^2}{2}\right)_{\text{out}} \approx 0$$

$$\left(gz\right)_{\text{in}} - \left(gz\right)_{\text{out}} \approx 0$$

Simplified Steady-Flow Thermal Energy Equation

Having already assumed steady-state conditions and no thermal or mechanical energy generation, we also adopt the approximation $i_{in} - i_{out} = c_p(T_{in} - T_{out})$

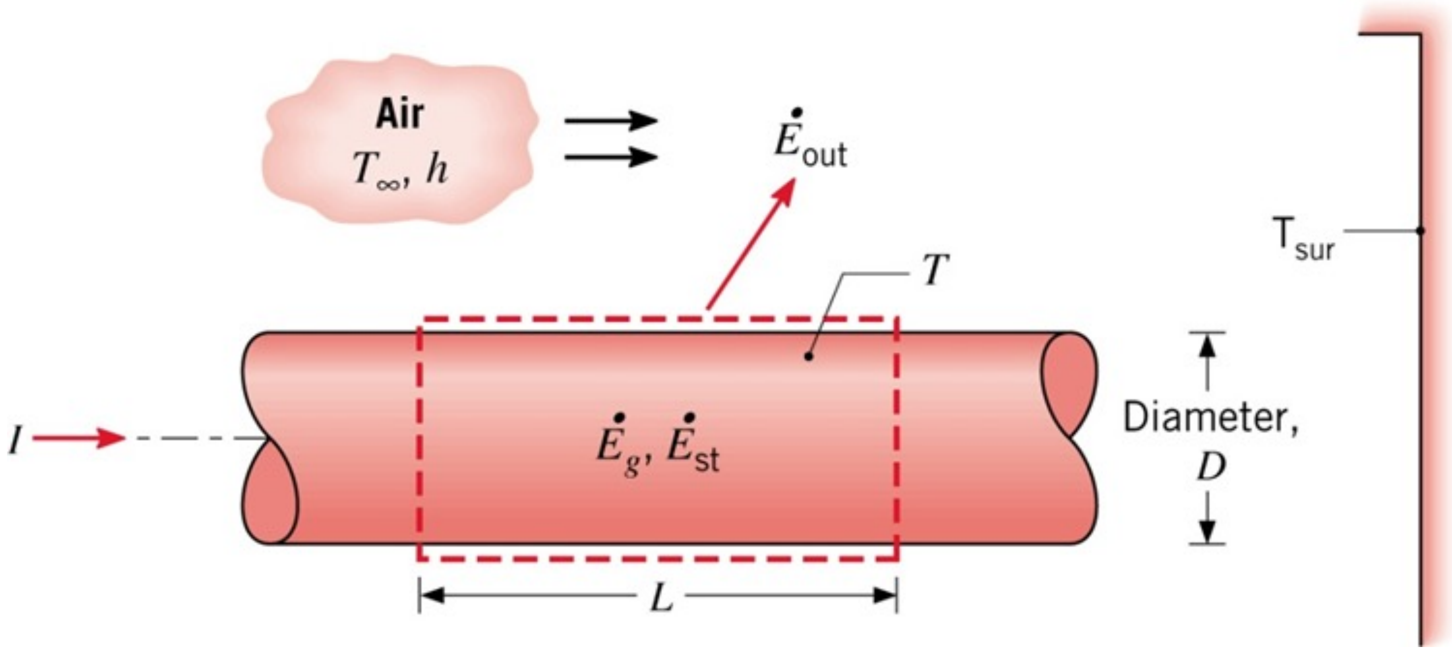
Further assuming negligible changes in kinetic and potential energy and negligible work. Then, the simplified steady-flow thermal energy equation:

$$q = \dot{m} c_p (T_{out} - T_{in})$$

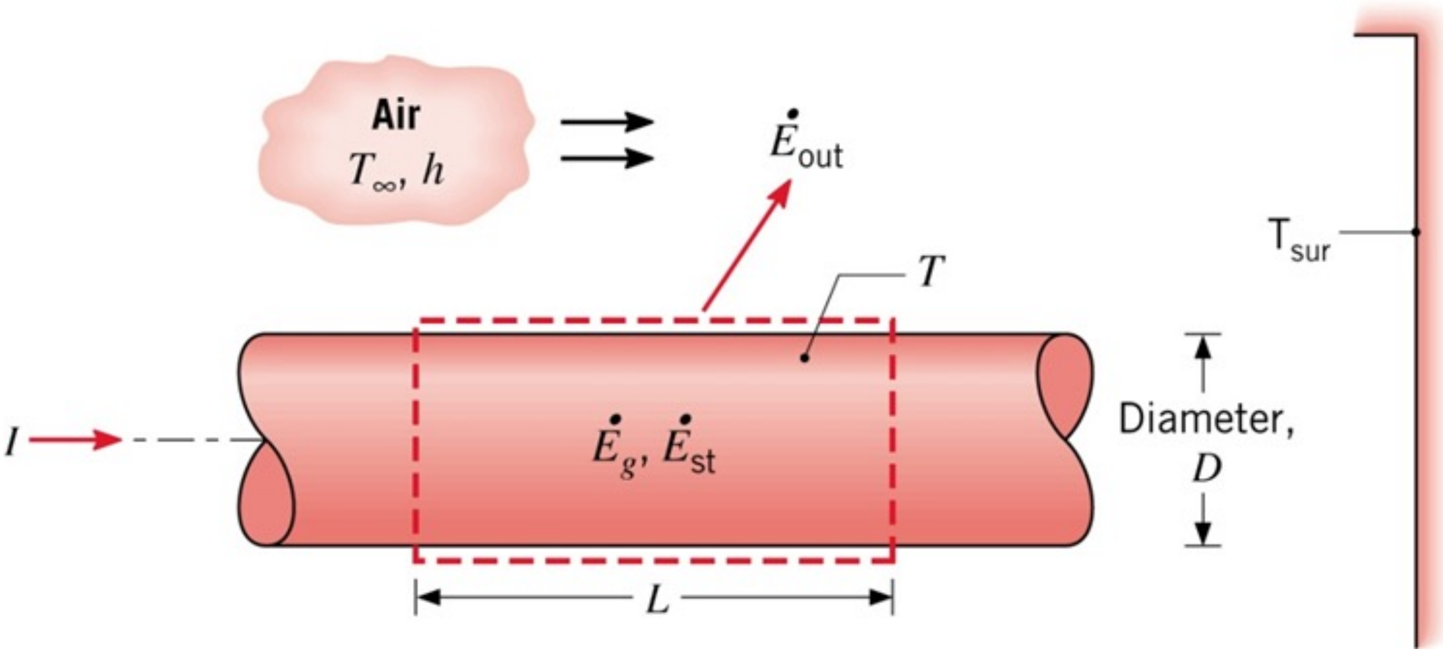
Problem: Volumetric Heating

Example 1.4: Application to thermal response of a conductor with Ohmic heating (generation)

Ohmic heating: Electrical resistance heating, the process of passing electric currents materials



A long conducting rod of diameter D and electrical resistance per unit length R'_e is initially in thermal equilibrium with the ambient air and its surroundings. This equilibrium is disturbed when an electrical current I is passed through the rod. Develop an equation that could be used to compute the variation of the rod temperature with time during the passage of the current.



SOLUTION

Known: Temperature of a rod of prescribed diameter and electrical resistance changes with time due to passage of an electrical current.

Find: Equation that governs temperature change with time for the rod.

Assumptions:

1. At any time t, the temperature of the rod is uniform.
2. Constant properties (ρ, cv, $\varepsilon = \alpha$).
3. Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and a large enclosure.

$$\dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \qquad \dot{E}_g = I^2 R_e' L$$

$$\dot{E}_{out} = h(\pi DL)(T - T_\infty) + \varepsilon\sigma(\pi DL)(T^4 - T_{sur}^4)$$

The change in energy storage is due only to sensible energy change,

$$\dot{E}_{st} = \frac{dU_{sens}}{dt} = mc_v \frac{dT}{dt} = \rho V c_v \frac{dT}{dt}$$

$$I^2 R_e' L - h(\pi DL)(T - T_\infty) - \varepsilon\sigma(\pi DL)(T^4 - T_{sur}^4) = \rho c_v \left(\frac{\pi D^2}{4}\right) L \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{I^2 R_e' - \pi D h(T - T_\infty) - \pi D \varepsilon\sigma(T^4 - T_{sur}^4)}{\rho c_v (\pi D^2 / 4)}$$

Comments:

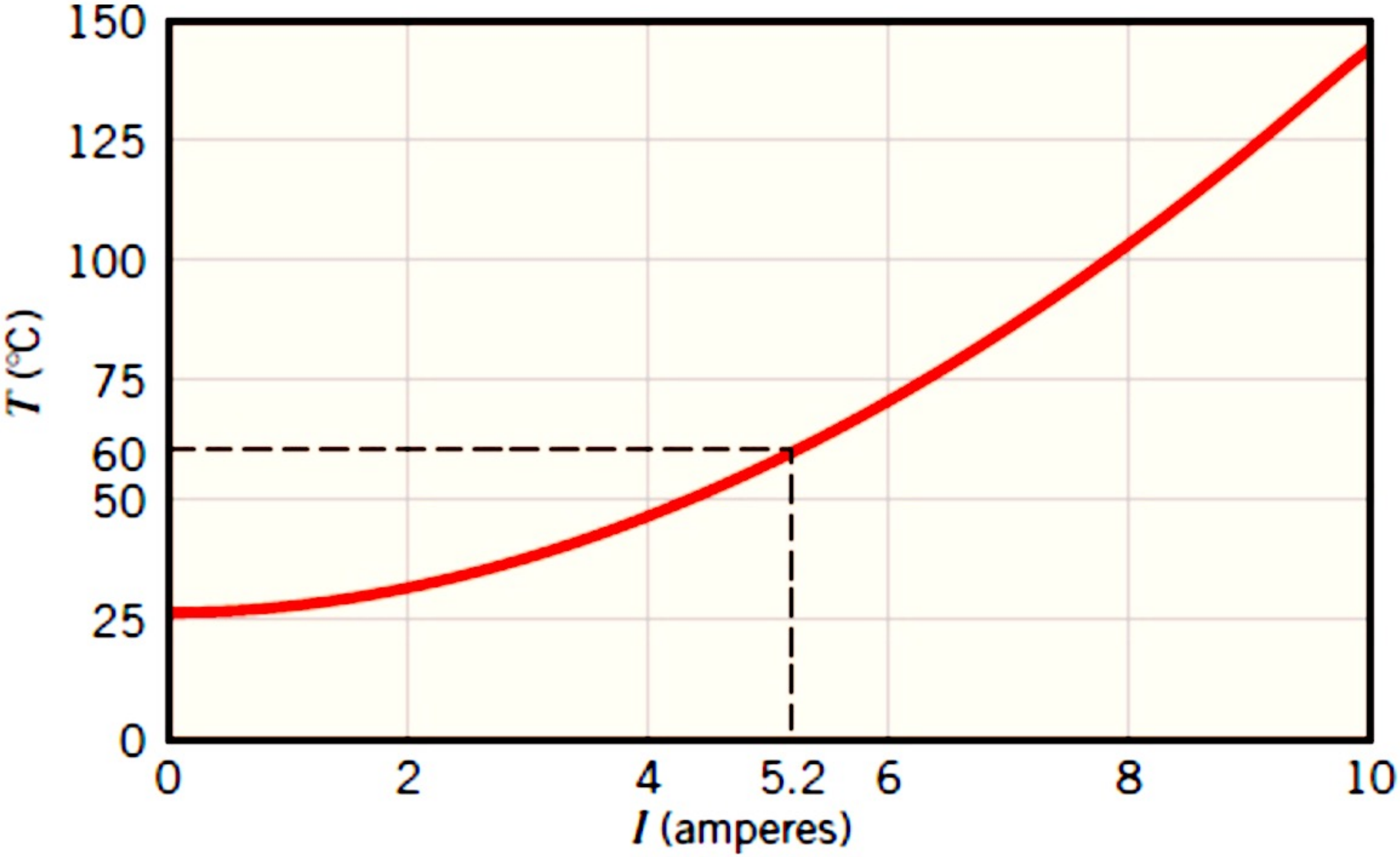
1. Solids and liquids can usually be treated as incompressible, in which case $c_v = c_p = c$. This assumption will be made in the remainder of this text.

2. The preceding equation could be solved for the time dependence of the rod temperature by integrating numerically. A steady-state condition would eventually be reached for which dT/dt = 0. The rod temperature is then determined by an algebraic equation of the form:

$$\pi D h (T - T_\infty) + \pi D \varepsilon \sigma (T^4 - T_{sur}^4) = I^2 R_e'$$

3. For fixed environmental conditions (h, T_∞, T_{sur}), as well as a rod of fixed geometry (D) and properties , the steady-state temperature depends on the rate of thermal energy generation and hence on the value of the electric current.

Consider an uninsulated copper wire (D = 1 mm, $\varepsilon = 0.8$, $R'_e = 0.4\ \Omega/m$) in a relatively large enclosure ($T_{sur} = 300$ K) through which cooling air is circulated (h = 100 W/m2 · K, T∞ = 300 K). Substituting these values into the foregoing equation, the rod temperature has been computed for operating currents in the range $0 \leq I \leq 10$ A, and the following results were obtained:


150
125
100
75
60
50
25
0
T (°C)
0
2
4
5.2
6
8
10
I (amperes)

4. If a maximum operating temperature of $T = 60°C$ is prescribed for safety reasons, the current should not exceed 5.2 A.

At this temperature, heat transfer by radiation (0.6 W/m) is much less than heat transfer by convection (10.4 W/m).

Hence, if one wished to operate at a larger current while maintaining the rod temperature within the safety limit, the convection coefficient would have to be increased by increasing the velocity of the circulating air. For $h = 250\ W/m^2 \cdot K$, the maximum allowable current could be increased to 8.1 A.