Fundamentals of Heat and Mass Transfer

Chapter 1

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Methodology of First Law Analysis

- 1. The appropriate control volume must be defined, with the control surfaces represented by a dashed line or lines.
- 2. The appropriate time basis must be identified.
- 3. The relevant energy processes must be identified, and each process should be shown on the control volume by an appropriately labeled arrow.
- 4. The conservation equation must then be written, and appropriate rate expressions must be substituted for the relevant terms in the equation.
- Solve for the unknown quantity.

Problem 1.46:

A furnace for processing semiconductor materials is formed by a silicon carbide chamber that is zone-heated on the top section and cooled on the lower section.

With the elevator in the lowest position, a robot arm inserts the silicon wafer on the mounting pins.

In a production operation, the wafer is rapidly moved toward the hot zone to achieve the temperature-time history required for the process recipe.
In this position, the top and bottom surfaces of the wafer exchange radiation with the hot and cool zones, respectively, of the chamber.

The zone temperatures are $T_h = 1500$ K and $T_c = 330$ K, and the emissivity and thickness of the wafer are $\varepsilon = 0.65$ and $d = 0.78$ mm, respectively.

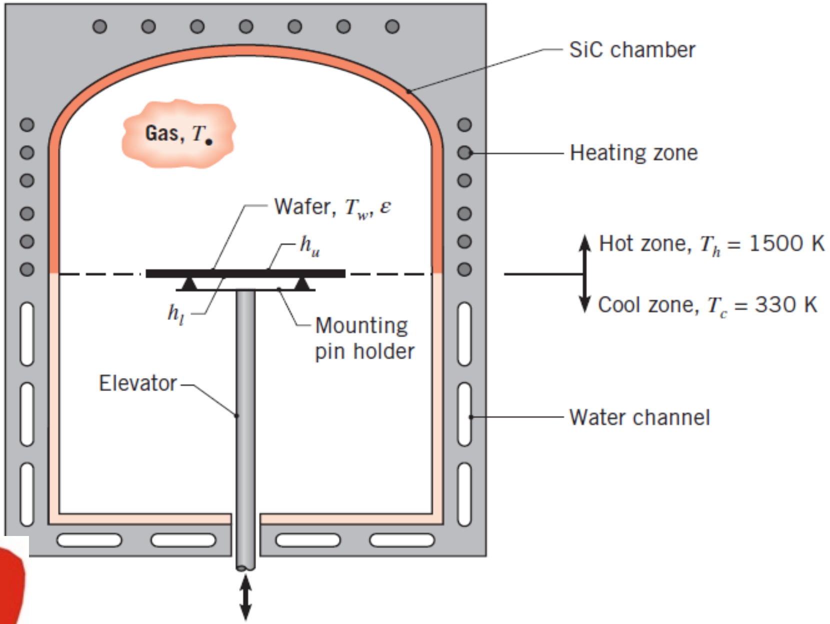
With the ambient gas at $T_\infty = 700$ K, convection coefficients at the upper and lower surfaces of the wafer are 8 and 4 W/m2 · K, respectively. The silicon wafer has a density of 2700 kg/m3 and a specific heat of 875 J/kg · K.

Problem: Silicon Wafer (1 of 3)

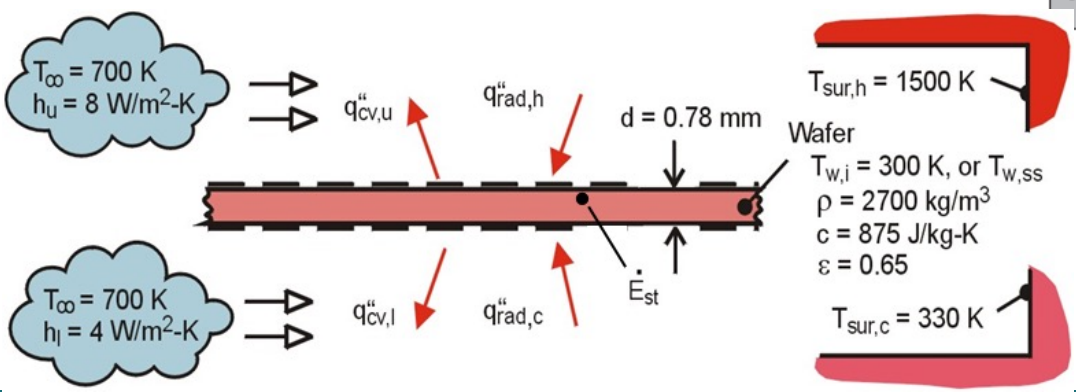
Thermal processing of silicon wafers in a two-zone furnace. Determine (a) the initial rate of change of the wafer temperature and (b) the steady-state temperature.

KNOWN: Silicon wafer positioned in furnace with upper and lower surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of wafer temperature from a value of $T_{w,i} = 300$ K, and (b) steady-state temperature. Is convection significant? Sketch variation of wafer temperature with vertical distance.



SCHEMATIC:



Problem: Silicon Wafer (2 of 3)

ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Hot and cool zones have uniform temperatures. (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone). (4) $\alpha = \varepsilon$, and (5) Negligible heat losses from Wafer to pin holder.

ANALYSIS: The energy balance on the wafer includes convection to the upper (u) and lower (1) surfaces from the ambient gas, radiation exchange with the hot- and cool-zones and an energy storage term for the transient condition. Hence, from Equation (1.12c),

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

or, per unit surface area

$$q''_{rad,h} + q''_{rad,c} - q''_{cv,u} - q''_{cv,l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon\sigma\left(T_{sur,h}^4 - T_w^4\right) + \varepsilon\sigma\left(T_{sur,c}^4 - T_w^4\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of change of the wafer temperature is determined using the foregoing energy balance with $T_w = T_{w,i} = 300\text{K}$,

$$0.65 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(1500^4 - 300^4\right)\text{K}^4 + 0.65 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(330^4 - 300^4\right)\text{K}^4$$

$$-8\ \text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} - 4\ \text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} =$$

$$2700\ \text{kg/m}^3 \times 875\ \text{J/kg} \cdot \text{K} \times 0.00078\ \text{m}\ \left(dT_w / dt\right)_i$$

$$\left(dT_w / dt\right)_i = 104\ \text{K/s} \qquad <$$

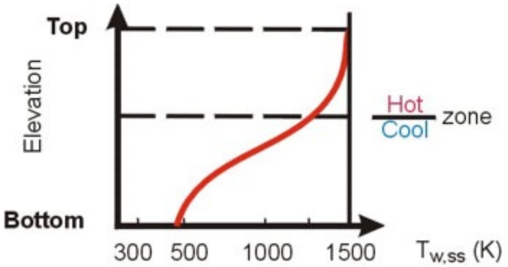
Problem: Silicon Wafer (3 of 3)

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)\text{K}^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)\text{K}^4$$
$$-8\,\text{W/m}^2\cdot\text{K}\left(T_{w,ss} - 700\right)\text{K} - 4\,\text{W/m}^2\cdot\text{K}\left(T_{w,ss} - 700\right)\text{K} = 0$$
$$T_{w,ss} = 1251\ \text{K} \qquad <$$

To assess the relative importance of convection, solve the energy balances assuming no convection. With $(dT_w / dt)_i = 101\text{K/s}$ and $T_{w,\,ss} = 1262$ K, we conclude that the radiation exchange processes control the initial rate of change and the steady-state temperature.

If the wafer elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone. Conversely, by lowering the wafer, the upper surface would experience les radiant exchange with the hot zone, and its temperature would decrease. The temperature-distance relation might appear as shown in the sketch.

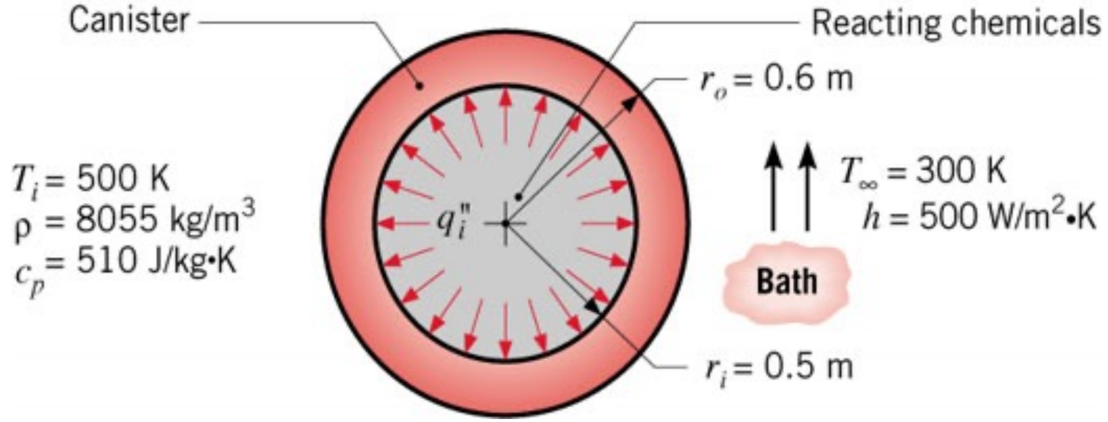


Problem: Cooling of Spherical Canister (1 of 4)

Problem 1.50: A spherical, stainless steel (AISI 302) canister is used to store reacting chemicals that provide for a uniform heat flux to its inner surface. The canister is suddenly submerged in a liquid bath of temperature $T_\infty < T_i$, where T_i is the initial temperature of the canister wall. Cooling of spherical canister used to store reacting chemicals.

Determine (a) the initial rate of change of the canister temperature,

(b) the steady-state temperature



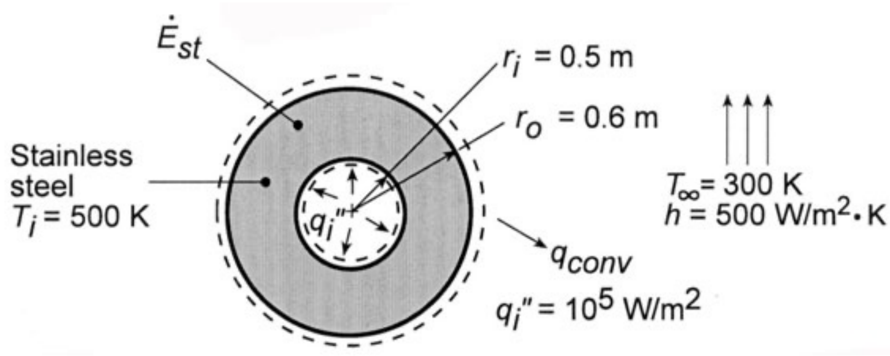
Problem: Cooling of Spherical Canister (1 of 4)

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time and the initial rate of change, (b) Steady-state wall temperature

Problem: Cooling of Spherical Canister (2 of 4)

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time independent heat flux at inner surface.

PROPERTIES: *Table A1,* AISI 302 Stainless: $\rho = 8055\,\text{kg/m}^3, c_p = 510\,\text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$.

Identifying relevant processes and solving for $\frac{dT}{dt}$,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho \frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right] \qquad <$$

Problem: Cooling of Spherical Canister (3 of 4)

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right|_i = \frac{3\left[10^5 \dfrac{\text{W}}{\text{m}^2}(0.5\text{m})^2 - 500\dfrac{\text{W}}{\text{m}^2\cdot\text{K}}(0.6\text{m})^2(500-300)\text{K}\right]}{8055\dfrac{\text{kg}}{\text{m}^3}510\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\left[(0.6)^3-(0.5)^3\right]\text{m}^3}$$

$$\left.\frac{dT}{dt}\right|_i = -0.084\text{K/s}$$ <

(b) Under steady-state conditions with $\dot{E}_{\text{st}} = 0,$ it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300\text{K} + \frac{10^5\ \text{W/m}^2}{500\text{W/m}^2\cdot\text{K}}\left(\frac{0.5\text{m}}{0.6\text{m}}\right)^2 = 439\text{K}$$ <

Problem: Cooling of Spherical Canister (4 of 4)

COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with coefficients. Its solution is

$$\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}, \text{where } \theta \equiv T - T_\infty,$$

$S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right), R = 3hr_o^2 / \rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \rightarrow \infty$ and for $S = 0$.