Fundamentals of Heat and Mass Transfer

Chapter 2

Fourier's Law and the Heat Equation

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Fourier's Law

- Heat flux vector is in a direction perpendicular to the isothermal surfaces.

- An alternative form of Fourier's law is therefore:

$$\boldsymbol{q}'' = q_n'' \boldsymbol{n} = -k\frac{\partial \mathrm{T}}{\partial \mathrm{n}}\boldsymbol{n}$$

where q''_n is the heat flux in a direction n, which is normal to an isotherm, and n is the unit normal vector in that direction.

This is illustrated for the two-dimensional case in Figure 2.3. The heat transfer is sustained by a temperature gradient along n.

Fourier's Law

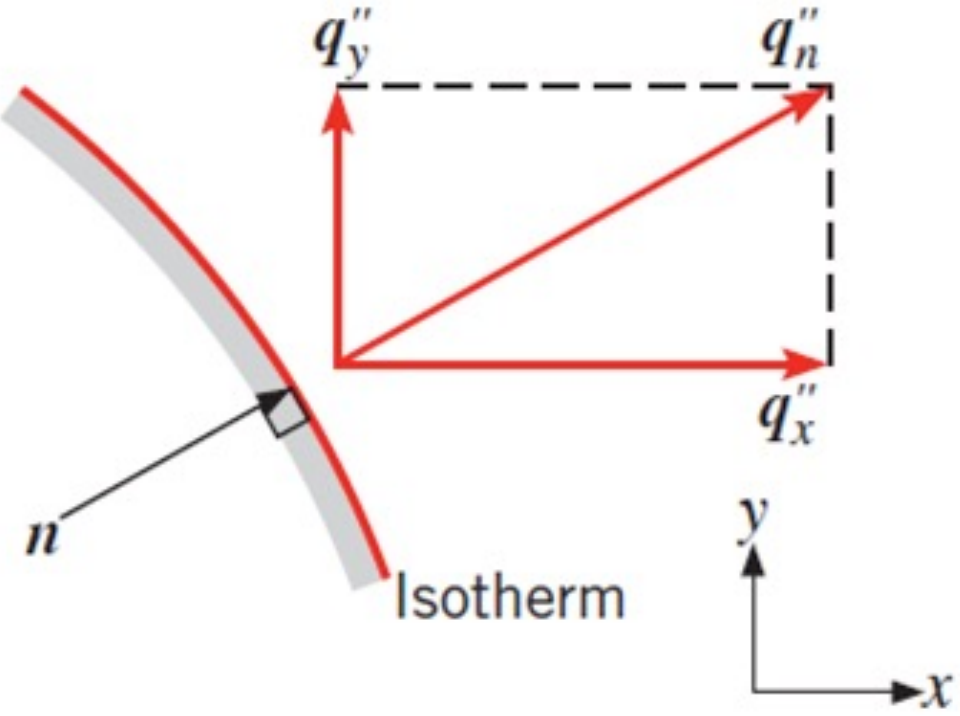


FIGURE 2.3 The heat flux vector normal to an isotherm in a two-dimensional coordinate system.

Fourier's Law

- Note also that the heat flux vector can be resolved into components such that, in Cartesian coordinates, the general expression for q″ is

$$\mathbf{q}'' = \mathbf{i}q_x'' + \mathbf{j}q_y'' + \mathbf{k}q_z''$$

$$q_x'' = -k\frac{\partial T}{\partial x} \quad q_y'' = -k\frac{\partial T}{\partial y} \quad q_z'' = -k\frac{\partial T}{\partial z}$$

Equation 2.3

Fourier's Law

- Each of those expressions relates the heat flux across a surface to the temperature gradient in a direction perpendicular to the surface.

- It is also implicit in Equation 2.3 that the medium considered here is isotropic. For such a medium, the value of the thermal conductivity is independent of the coordinate direction.

Fourier’s Law

Summary:

Fourier's law is the cornerstone of conduction heat transfer, and its key features are summarized as follows:

1- It is not an expression that may be derived from first principles; it is instead a generalization based on experimental evidence.

2- It is an expression that defines an important material property, the thermal conductivity.

3- Fourier's law is a vector expression indicating that the heat flux is normal to an isotherm and in the direction of decreasing temperature.

4- Fourier's law applies for all matter, regardless of its state (solid, liquid, or gas).

The Thermal Properties of Matter

- To use Fourier's law, the thermal conductivity of the material must be known.

- This property, which is referred to as **a transport property**, **depends on the atomic and molecular structure of matter**, and therefore, on the state of the matter.

Thermal Conductivity

- From Fourier's law, the thermal conductivity associated with conduction in the x-direction is defined as

$$k_x \equiv -\frac{q''_x}{(\partial T / \partial x)}$$

The Thermal Properties of Matter

- Similar definitions are associated with thermal conductivities in the y- and z-directions (ky, kz), but for an isotropic material the thermal conductivity is independent of the direction of transfer, $k_x = k_y = k_z \equiv k$.

- From the foregoing equation, it follows that, **for a prescribed temperature gradient, the conduction heat flux increases with increasing thermal conductivity.**

- In general, the **thermal conductivity of a solid is larger than that of a liquid, which is larger than that of a gas.**

The Thermal Properties of Matter

- As illustrated in Figure 2.4, the thermal conductivity of a solid may be more than four orders of magnitude larger than that of a gas.

- This trend is due largely to differences in intermolecular spacing for the two states.

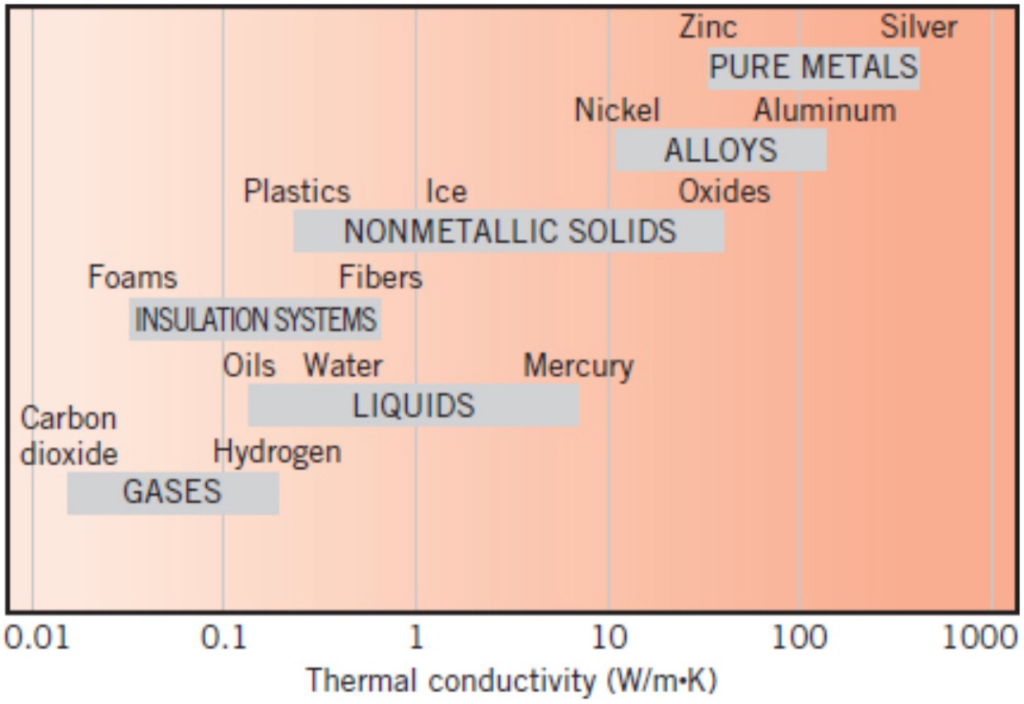


FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

The Thermal Properties of Matter

The Solid State

- In the modern view of materials**, a solid may be comprised of free electrons and atoms bound in a periodic arrangement called the lattice.**

- Accordingly, **transport of thermal energy may be due to two effects: the migration of free electrons and lattice vibrational waves.**

- **The lattice vibration quanta are termed phonons.**

The Thermal Properties of Matter

The Solid State

- In pure metals, **the electron contribution to conduction heat transfer dominates, whereas in nonconductors and semiconductors, the phonon contribution is dominant.**

- Kinetic theory yields the following expression for the thermal conductivity

$$k = \frac{1}{3} \mathrm{C}\ \bar{c} \lambda_{\mathrm{mfp}}$$

The Thermal Properties of Matter

- For conducting materials such as metals, $C \equiv C_e$ is the electron specific heat per unit volume,
- τ is the mean electron velocity,
- $\lambda_{mfp} \equiv \lambda_e$ is the electron mean free path, which is defined as **the average distance traveled by an electron before it collides with either an imperfection in the material or with a phonon.**
- In nonconducting solids, $C \equiv C_{ph}$ is the phonon specific heat,
- τ is the average speed of sound,
- $\lambda_{mfp} \equiv \lambda_{ph}$ is the phonon mean free path, which again is determined by collisions with imperfections or other phonons.

The Thermal Properties of Matter

- In all cases, **the thermal conductivity increases as the mean free path of the energy carriers (electrons or phonons) is increased.**

- When electrons and phonons carry thermal energy leading to conduction heat transfer in a solid, the thermal conductivity may be expressed as

$$k=k_e+k_{ph}$$

The Thermal Properties of Matter

- To a first approximation, k_e is inversely proportional to the electrical resistivity, ρ_e.
- **For pure metals**, ρ_e, k_e is much larger than k_{ph}.
- In contrast**, for alloys,** which are of substantially larger ρ_e, the contribution of k_{ph} to k is no longer negligible.
- For nonmetallic solids, k is determined primarily **by k_{ph}**, which increases as the frequency of interactions between the atoms and the lattice decreases.

The Thermal Properties of Matter

- **The regularity of the lattice arrangement has an important effect on k_{ph}, with crystalline (well-ordered) materials like quartz having a higher thermal conductivity than amorphous materials like glass.**

- In fact, for crystalline, nonmetallic solids such as diamond and beryllium oxide, k_{ph} can be quite large, exceeding values of k **associated with good conductors, such as aluminum.**

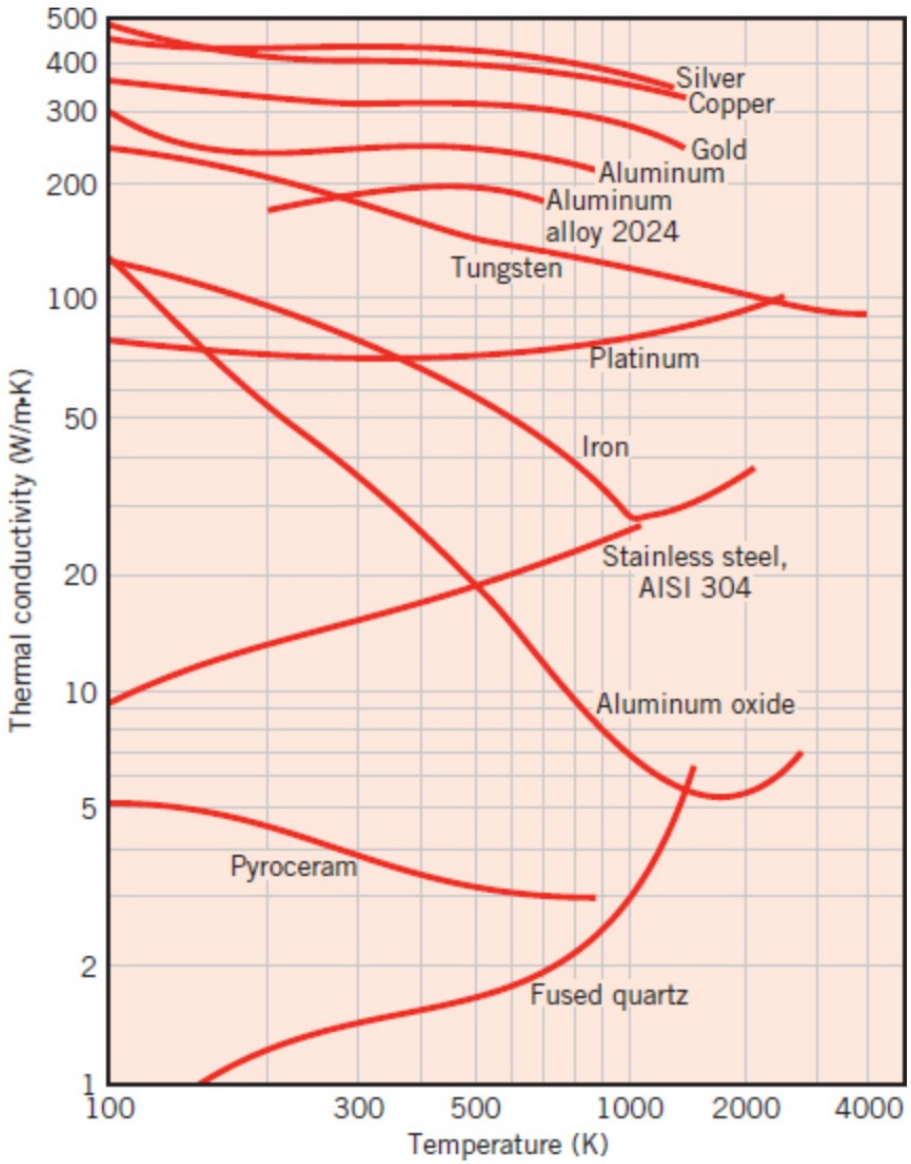
The Thermal Properties of Matter

- The temperature dependence of k is shown in Figure 2.5 for representative metallic and nonmetallic solids.

- Values for selected materials of technical importance are also provided in Table A.1 (metallic solids) and Tables A.2 and A.3 (nonmetallic solids).

- More detailed treatments of thermal conductivity are available in the literature

FIGURE 2.5 The temperature dependence of the thermal conductivity of selected solids.



Other Relevant Properties

- In our analysis of heat transfer problems, it will be necessary to use several properties of matter.

- These properties are generally referred to as thermophysical properties and include two distinct categories, transport and thermodynamic properties.

- The transport properties include the diffusion rate coefficients such as k, the thermal conductivity (for heat transfer), and ν, the kinematic viscosity (for momentum transfer).

- Thermodynamic properties, on the other hand, pertain to the equilibrium state of a system. Density (ρ) and specific heat (cp) are two such properties used extensively in thermodynamic analysis.

Other Relevant Properties

- The product ρc_p (J/m3 · K), commonly termed **the volumetric heat capacity**, measures the ability of a material to store thermal energy.

- Because substances of large density are typically characterized by small specific heats, many solids and liquids, which are very good energy storage media, have comparable heat capacities ($\rho c_p > 1$ MJ/m3 · K).

- Because of their very small densities, however, gases are poorly suited for thermal energy storage ($\rho c_p \approx 1$ kJ/m3 · K).

- Densities and specific heats are provided in the tables of Appendix A for a wide range of solids, liquids, and gases.

Other Relevant Properties

- In heat transfer analysis, the ratio of the thermal conductivity to the heat capacity is an important property termed **the thermal diffusivity α**, which has units of m2/s:

$$\alpha = \frac{k}{\rho c_p}$$

- It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy.

- Materials of large α will respond quickly to changes in their thermal environment, whereas materials of small α will respond more sluggishly, taking longer to reach a new equilibrium condition.

Other Relevant Properties

- The accuracy of engineering calculations depends on the accuracy with which the thermophysical properties are known [16–18].

- Numerous examples could be cited of flaws in equipment and process design or failure to meet performance specifications that were attributable to misinformation associated with the selection of key property values.

- Selection of reliable property data is an integral part of any careful engineering analysis. Recommended data values for many thermophysical properties can be obtained from Reference 19.

Example 2.1

The thermal diffusivity α is the controlling transport property for transient conduction. Using appropriate values of k, ρ, and cp from Appendix A, calculate α for the following materials at the prescribed temperatures: pure aluminum, 300 and 700 K; silicon carbide, 1000 K; paraffin, 300 K.

Solution

Known: Definition of the thermal diffusivity α.

Find: Numerical values of α for selected materials and temperatures.

Properties: Table A.1, pure aluminum (300 K):

TABLE A.1 Thermophysical Properties of Selected Metallic Solids[a]

						Properties at Various Temperatures (K)									
		Properties at 300 K				k (W/m · K)/c_p (J/kg · K)									
Composition	**Melting Point (K)**	**ρ (kg/m³)**	**c_p (J/kg · K)**	**k (W/m · K)**	**$\alpha \cdot 10^6$ (m²/s)**	**100**	**200**	**400**	**600**	**800**	**1000**	**1200**	**1500**	**2000**	**2500**
Aluminum															
Pure	933	2702	903	237	97.1	302	237	240	231	218					
						482	798	949	1033	1146					
Alloy 2024-T6	775	2770	875	177	73.0	65	163	186	186						
(4.5% Cu, 1.5% Mg, 0.6% Mn)						473	787	925	1042						
Alloy 195, Cast		2790	883	168	68.2			174	185						
(4.5% Cu)								—	—						
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8	78.7			
						203	1114	2191	2604	2823	3018	3227	3519		

TABLE A.2 Thermophysical Properties of Selected Nonmetallic Solids

c_p			1122			364	757	1431						
Carbon														
Amorphous	1500	1950	—	1.60	—	0.67	1.18	1.89	2.1	2.37	2.53	2.84	3.48	
						—	—	—	—	—	—	—	—	
Diamond, type IIa insulator	—	3500	509	2300	—	10,000	4000	1540						
						21	194	853						
Graphite, pyrolytic	2273	2210												
k, ∥ to layers				1950		4970	3230	1390	892	667	534	448	357	262
k, ⊥ to layers				5.70		16.8	9.23	4.09	2.6	2.01	1.60	1.34	1.08	0.81
c_p			709			136	411	992	1406	1650	1793	1890	1974	2043
Graphite fiber epoxy (25% vol) composite	450	1400												
k, heat flow ∥ to fibers				11.1		5.7	8.7	13.0						
k, heat flow ⊥ to fibers				0.87		0.46	0.68	1.1						
c_p			935			337	642	1216						
Pyroceram, Corning 9606	1623	2600	808	3.98	1.89	5.25	4.78	3.64	3.2	3.08	2.96	2.87	2.79	
						—	—	908	1038	1122	1197	1264	1498	
Silicon carbide	3100	3160	675	490	230			—	—	—	87	58	30	
								880	1050	1135	1195	1243	1310	
Silicon dioxide, crystalline (quartz)	1883	2650												

TABLE A.2 Thermophysical Properties of Selected Nonmetallic Solids

Leather (sole)	300	998	0.159	—
Paper	300	930	0.180	1340
Paraffin	300	900	0.240	2890
Rock				
Granite, Barre	300	2630	2.79	775
Limestone, Salem	300	2320	2.15	810
Marble, Halston	300	2680	2.80	830
Quartzite, Sioux	300	2640	5.38	1105
Sandstone, Berea	300	2150	2.90	745
Rubber vulcanized				
Soft	300	1100	0.13	2010
Hard	300	1190	0.16	—
Sand	300	1515	0.27	800
Soil	300	2050	0.52	1840

$$\left.\begin{aligned} \rho &= 2702\ \text{kg/m}^3 \\ c_p &= 903\ \text{J/kg}\cdot\text{K} \\ k &= 237\ \text{W/m}\cdot\text{K} \end{aligned}\right\} \alpha = \frac{k}{\rho c_p} = \frac{237\ \text{W/m}\cdot\text{K}}{2702\ \text{kg/m}^3 \times 903\ \text{J/kg}\cdot\text{K}}$$

$$= 97.1 \times 10^{-6}\ \text{m}^2/\text{s}$$

Table A.1, pure aluminum (700 K):

$$\begin{aligned} \rho &= 2702\ \text{kg/m}^3 && \text{at 300 K} \\ c_p &= 1090\ \text{J/kg}\cdot\text{K} && \text{at 700 K (by linear interpolation)} \\ k &= 225\ \text{W/m}\cdot\text{K} && \text{at 700 K (by linear interpolation)} \end{aligned}$$

$$\alpha = \frac{k}{\rho c_p} = \frac{225\ \text{W/m}\cdot\text{K}}{2702\ \text{kg/m}^3 \times 1090\ \text{J/kg}\cdot\text{K}} = 76 \times 10^{-6}\ \text{m}^2/\text{s}$$

Table A.2, silicon carbide (1000 K):

$$\left.\begin{aligned} \rho &= 3160\ \text{kg/m}^3 && \text{at 300 K} \\ c_p &= 1195\ \text{J/kg}\cdot\text{K} && \text{at 1000 K} \\ k &= 87\ \text{W/m}\cdot\text{K} && \text{at 1000 K} \end{aligned}\right\} \alpha = \frac{87\ \text{W/m}\cdot\text{K}}{3160\ \text{kg/m}^3 \times 1195\ \text{J/kg}\cdot\text{K}}$$

$$= 23 \times 10^{-6}\ \text{m}^2/\text{s}$$

Table A.3, paraffin (300 K):

$$\left.\begin{aligned} \rho &= 900\ \mathrm{kg/m^3} \\ c_p &= 2890\ \mathrm{J/kg \cdot K} \\ k &= 0.24\ \mathrm{W/m \cdot K} \end{aligned}\right\} \alpha = \frac{k}{\rho c_p} = \frac{0.24\ \mathrm{W/m \cdot K}}{900\ \mathrm{kg/m^3} \times 2890\ \mathrm{J/kg \cdot K}}$$

$$= 9.2 \times 10^{-8}\ \mathrm{m^2/s}$$

Comments:

1. Note the temperature dependence of the thermophysical properties of aluminum and silicon carbide. For example, from Table A.2 for silicon carbide, $\alpha(1000\ \text{K}) \approx 0.1 \times \alpha(300\ \text{K})$; hence properties of this material have a strong temperature dependence.

2. The physical interpretation of α is that it provides a measure of heat transport (k) relative to energy storage (ρc_p). In general, metallic solids have higher α, whereas nonmetallics (e.g., paraffin) have lower values of α.

3. Linear interpolation of property values is generally acceptable for engineering calculations.

Comments:

4. Use of the low-temperature (300 K) density at higher temperatures ignores thermal expansion effects but is also acceptable for engineering calculations.

5. The IHT software provides a library of thermophysical properties for selected solids, liquids, and gases that can be accessed from the toolbar button, Properties.