Fundamentals of Heat and Mass Transfer

Chapter 3

One Dimensional, Steady State Conduction

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Problem 3.23

The performance of gas turbine engines may be improved by increasing the tolerance of the turbine blades to hot gases emerging from the combustor.

One approach to achieving high operating temperatures involves application of a thermal barrier coating (TBC) to the exterior surface of a blade, while passing cooling air through the blade.

Typically, the blade is made from a high-temperature superalloy, such as Inconel ($k \approx 25$ W/m · K), while a ceramic, such as zirconia ($k \approx 1.3$ W/m · K), is used as a TBC.

Problem 3.23

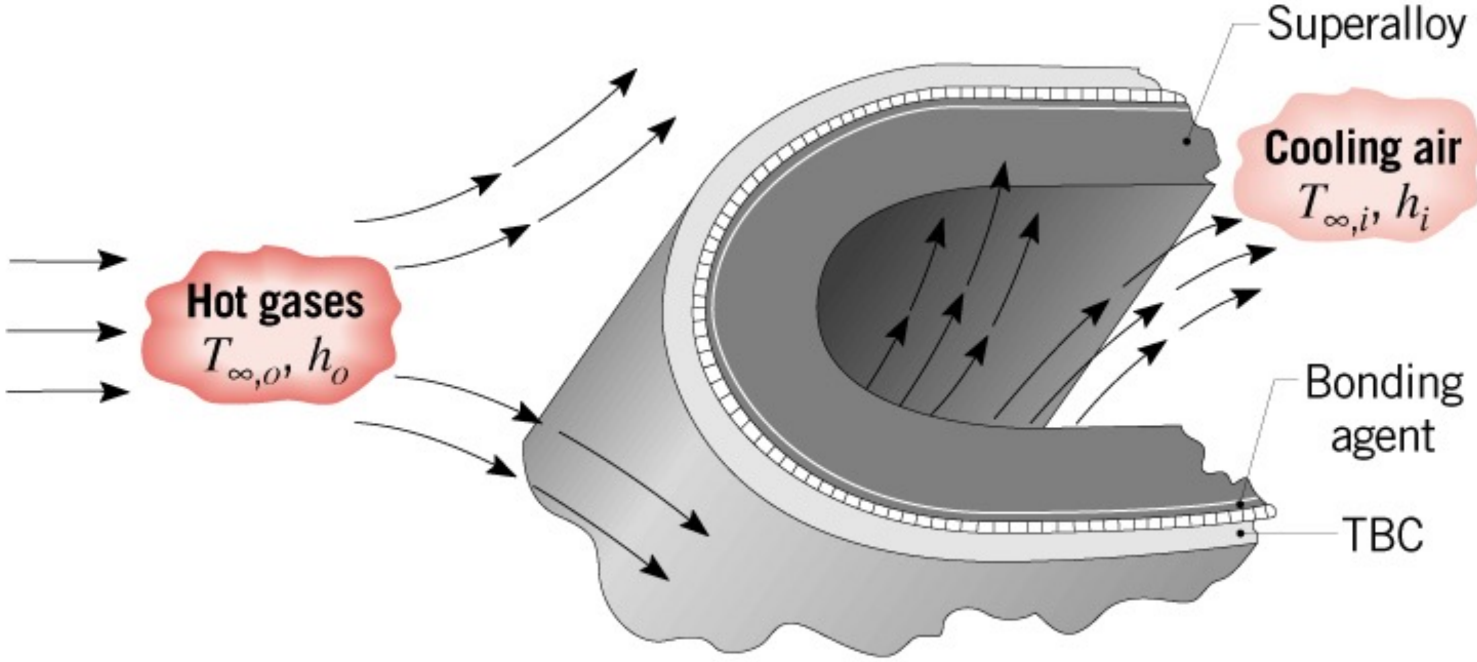
Consider conditions for which hot gases at $T_{\infty,o}$ = 1700 K and cooling air at $T_{\infty,i}$ = 400 K provide outer and inner surface convection coefficients of h_o = 1000 W/m2 · K and h_i = 500 W/m2 · K, respectively.

If a 0.5-mm-thick zirconia TBC is attached to a 5-mm-thick Inconel blade wall by means of a metallic bonding agent, which provides an interfacial thermal resistance of $R''_{t,c} = 10^{-4}$ m^2.K/W, **can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250 K?**

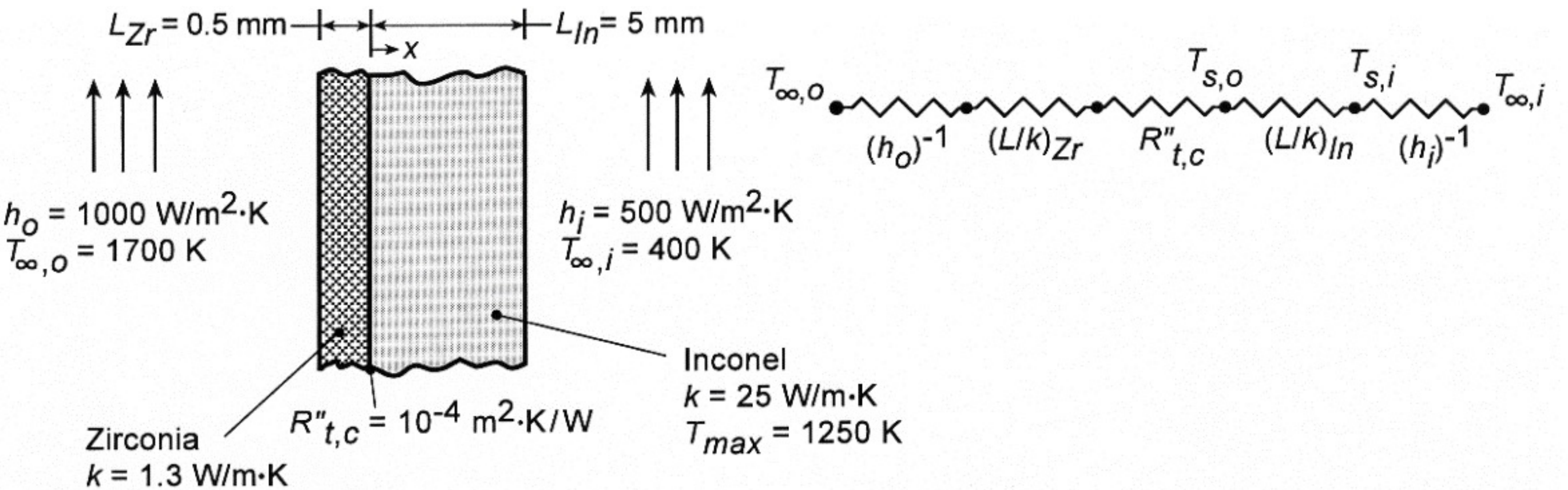
Radiation effects may be neglected, and the turbine blade may be approximated as a plane wall.

Plot the temperature distribution with and without the TBC. Are there any limits to the thickness of the TBC?

Problem 3.23


Superalloy
Cooling air
$T_{\infty,i}$, h_i
Hot gases
$T_{\infty,o}$, h_o
Bonding agent
TBC

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{\text{tot},w} = h_o^{-1} + (L/k)_{\text{Zr}} + R''_{t,c} + (L/k)_{\text{In}} + h_i^{-1}$$

$$R''_{\text{tot},w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right)\text{m}^2\cdot\text{K/W} = 3.69\times10^{-3}\ \text{m}^2\cdot\text{K/W}$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{\text{tot},w}} = \frac{1300\,\text{K}}{3.69\times10^{-3}\ \text{m}^2\cdot\text{K/W}} = 3.52\times10^5\ \text{W/m}^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(\text{w})} = T_{\infty,i} + (q''_w/h_i) = 400\,K + \left(\frac{3.52\times10^5\,W/m^2}{500\,W/m^2\cdot K}\right) = 1104\,K$$

$$T_{s,o(\text{w})} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{\text{In}}\right]q''_w$$

$$= 400\,\text{K} + \left(2\times10^{-3} + 2\times10^{-4}\right)\text{m}^2\cdot\text{K/W}\left(3.52\times10^5\ \text{W/m}^2\right) = 1174\,\text{K}$$

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Problem: Thermal Barrier Coating (3 of 3)

Without the TBC,

$$R''_{\text{tot,wo}} = h_o^{-1} + (L/k)_{\text{In}} + h_i^{-1} = 3.20 \times 10^{-3}\ \text{m}^2 \cdot \text{K/W}$$

$$q''_{\text{wo}} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{\text{tot,wo}} = 4.06 \times 10^5\ \text{W/m}^2$$

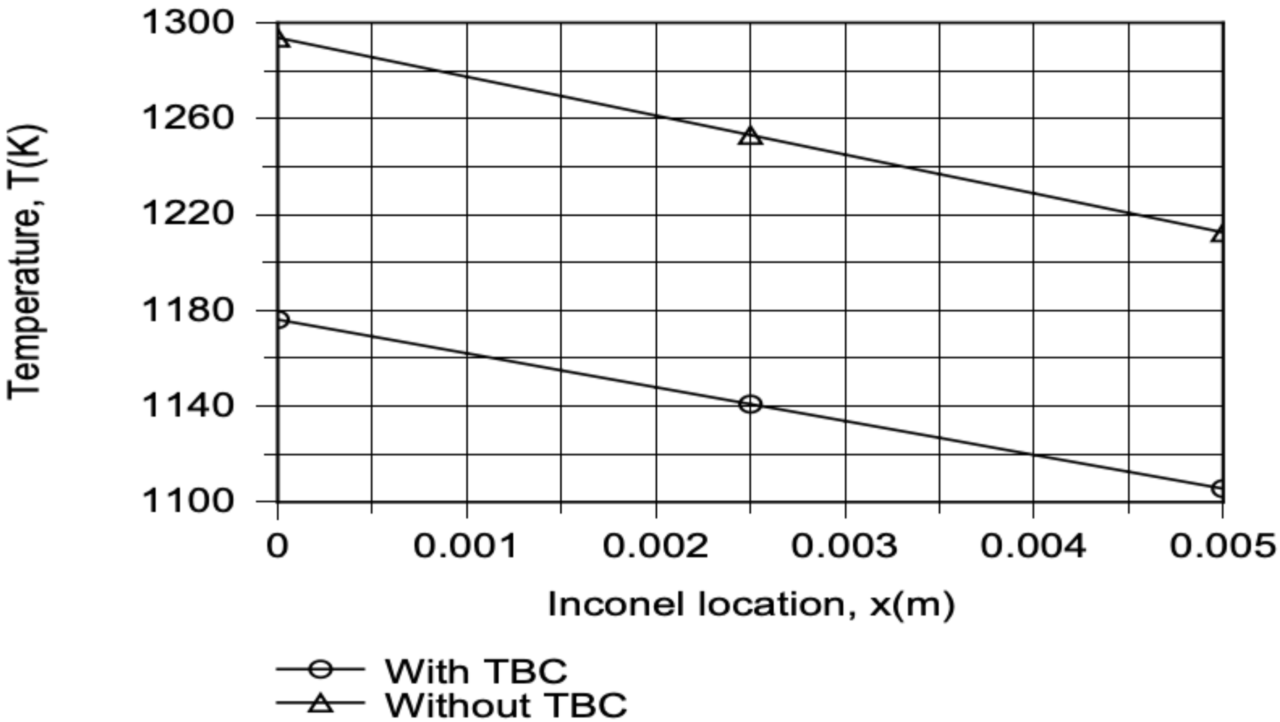
The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(\text{wo})} = T_{\infty,i} + \left(q''_{wo}/h_i\right) = 1212\ \text{K}$$

$$T_{s,o(\text{wo})} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{\text{In}}\right] q''_{wo} = 1293\ \text{K}$$

$<$

Use of the TBC facilitates operation of the Inconel below Tmax = 1250 K.



COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to its thickness are associated with reliability considerations. In this application, thermal contact resistance is desirable.

Problem 3.40

To maximize production and minimize pumping costs, crude oil is heated to reduce its viscosity during transportation from a production field.

(a) Consider a pipe-in-pipe configuration consisting of concentric steel tubes with an intervening insulating material. The inner tube is used to transport warm crude oil through cold ocean water. The inner steel pipe (ks = 35 W/m · K) has an inside diameter of $D_{i,1}$ = 150 mm and wall thickness t_i = 10 mm while the outer steel pipe has an inside diameter of $D_{i,2}$ = 250 mm and wall thickness to = t_i.

Determine the maximum allowable crude oil temperature to ensure the polyurethane foam insulation (kp = 0.075 W/m · K) between the two pipes does not exceed its maximum service temperature of $T_{p,max}$ = 70°C. The ocean water is at $T_{\infty,o}$ = −5°C and provides an external convection heat transfer coefficient of h_o = 500 W/m^2 · K. The convection coefficient associated with the flowing crude oil is h_i = 450 W/m^2 · K.

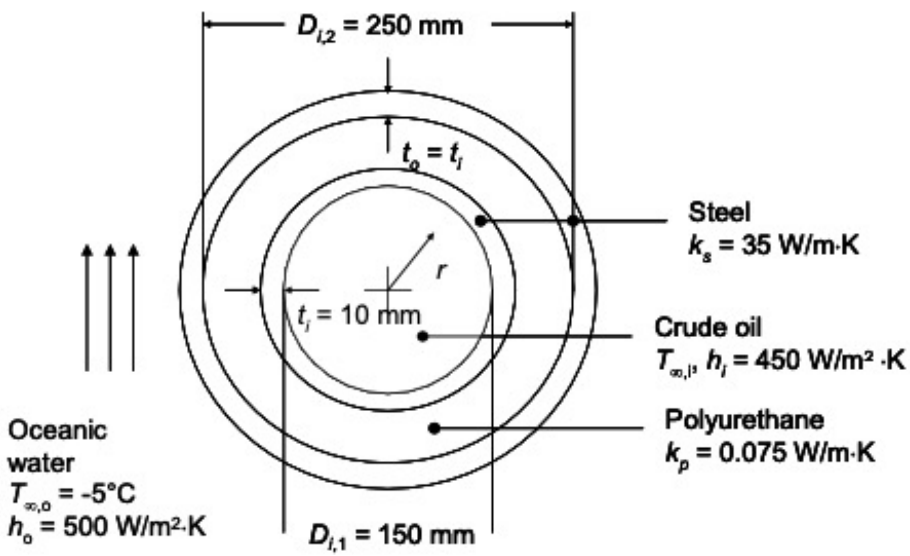
(b) It is proposed to enhance the performance of the pipe-in-pipe device by replacing a thin (t_a = 5 mm) section of polyurethane located at the outside of the inner pipe with an aerogel insulation material (ka = 0.012 W/m · K). Determine the maximum allowable crude oil temperature to ensure maximum polyurethane temperatures are below $T_{p,max}$ = 70°C.

PROBLEM 3.40

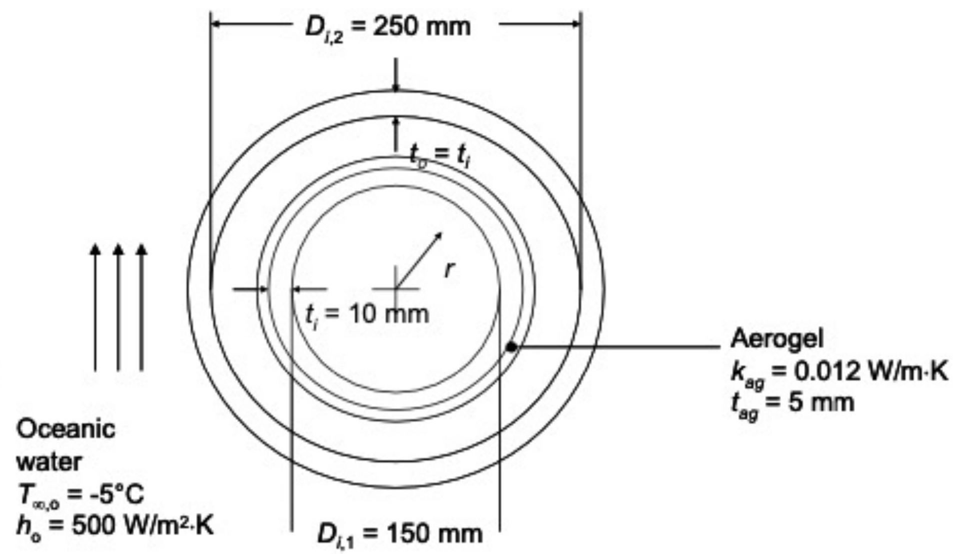
KNOWN: Dimensions of components of a pipe-in-pipe device. Thermal conductivity of materials, inner and outer heat transfer coefficients, outer fluid temperature.

FIND: (a) Maximum crude oil temperature to not exceed allowable service temperature of polyurethane. (b) Maximum crude oil temperature to not exceed allowable service temperature of polyurethane after insertion of aerogel layer.

SCHEMATIC:



Without Aerogel

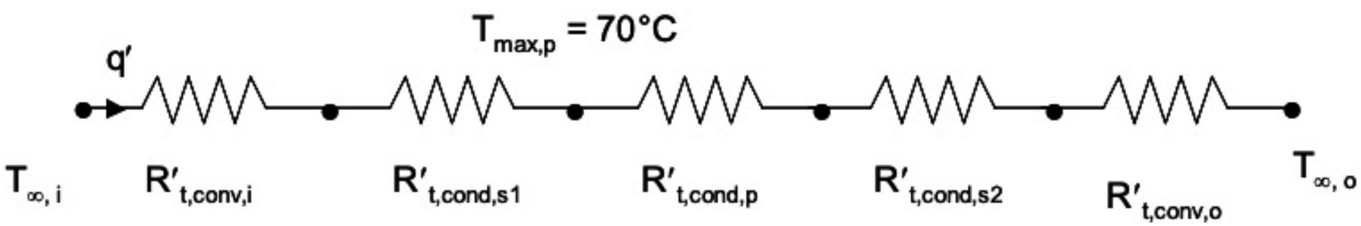


With Aerogel

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible contact resistances, (3) Constant properties.

PROPERTIES: *Given*, Steel: k = 35 W/m·K; polyurethane: k = 0.075 W/m·K; aerogel: k = 0.012 W/m·K.

ANALYSIS: (a) The thermal resistance network for the case without the aerogel is shown below. The maximum polyurethane temperature occus at its inner surface.



Equating the heat rate per unit length of tubing from the crude oil to the inner surface of the polyurethane with the heat rate from the inner surface of the polyurethane to the oceanic waters yields

$$q' = \frac{T_{\infty,i} - T_{\max,p}}{R'_{t,\text{conv},i} + R'_{t,\text{cond},s1}} = \frac{T_{\max,p} - T_{\infty,o}}{R'_{t,\text{cond},p} + R'_{t,\text{cond},s2} + R'_{t,\text{conv},o}}$$

which may be rearranged to give

$$T_{\infty,i} = \frac{(T_{\max,p} - T_{\infty,o})(R'_{t,\text{conv},i} + R'_{t,\text{cond},s1})}{(R'_{t,\text{cond},p} + R'_{t,\text{cond},s2} + R'_{t,\text{conv},o})} + T_{\max,p} \qquad (1)$$

Continued…

The various thermal resistances are evaluated as follows.

$$R'_{t,\text{conv},i} = \frac{1}{450\text{W}/\text{m}^2\cdot\text{K}\times\pi\times0.150\text{m}} = 4.716\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} \;;\; R'_{t,\text{cond},s1} = \frac{\ln\left[(150+20)/150\right]}{2\times\pi\times35\text{W/m}\cdot\text{K}} = 569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

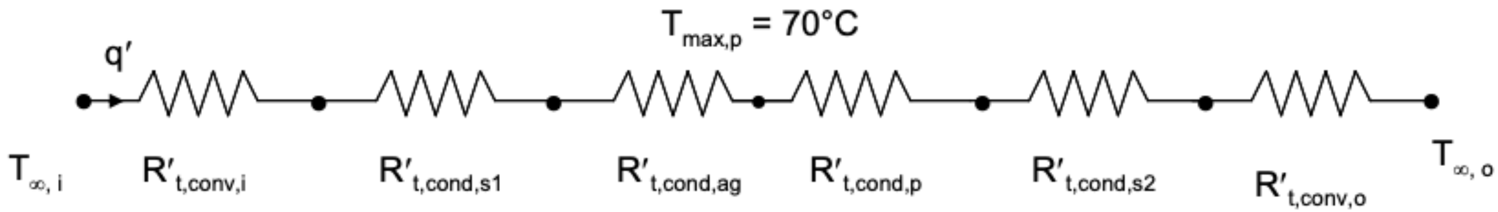
$$R'_{t,\text{cond},p} = \frac{\ln\left[250/(150+20)\right]}{2\times\pi\times0.075\text{W/m}\cdot\text{K}} = 818.4\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} \;;\; R'_{t,\text{cond},s2} = \frac{\ln\left[(250+20)/250\right]}{2\times\pi\times35\text{W/m}\cdot\text{K}} = 350.0\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

$$R'_{t,\text{conv},o} = \frac{1}{500\text{W/m}^2\cdot\text{K}\times\pi\times0.270\text{m}} = 2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}}$$

Substituting into Equation (1) yields

$$T_{\infty,i} = \frac{\left[70^\circ\text{C}-(-5^\circ\text{C})\right](4.716\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}}+569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}})}{(818.4\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}}+350\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}}+2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})} + 70^\circ\text{C} = 70.5^\circ\text{C} \qquad <$$

(b) The thermal resistance network for the case with the aerogel is shown below.



The thermal resistance values are as before, except the conduction resistance per unit length in the polyurethane is decreased, since its thickness is reduced relative to part (a). In addition, the conduction resistance for the aerogel must be evaluated. These two resistances are:

$$R'_{t,\mathrm{conv},ag} = \frac{\ln\left[(150+20+10)/(150+20)\right]}{2\times\pi\times 0.012\,\mathrm{W/m\cdot K}} = 758\times10^{-3}\,\frac{\mathrm{m\cdot K}}{\mathrm{W}} \;;\; R'_{t,\mathrm{cond},p} = \frac{\ln[250/180]}{2\times\pi\times 0.075\,\mathrm{W/m\cdot K}} = 697\times10^{-3}\,\frac{\mathrm{m\cdot K}}{\mathrm{W}}$$

Incorporating the aerogel resistance, Equation (1) becomes

$$T_{\infty,i} = \frac{(T_{\max,p} - T_{\infty,o})(R'_{t,\mathrm{conv},i} + R'_{t,\mathrm{cond},s1} + R'_{t,\mathrm{cond},ag})}{(R'_{t,\mathrm{cond},p} + R'_{t,\mathrm{cond},s2} + R'_{t,\mathrm{conv},o})} + T_{\max,p}$$

Substituting values yields

Continued…

PROBLEM 3.40 (Cont.)

$$T_{\infty,i} = \frac{[70°\text{C} - (-5°\text{C})](4.716\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 569.2\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 758\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})}{(697\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 350\times10^{-6}\,\frac{\text{m}\cdot\text{K}}{\text{W}} + 2.358\times10^{-3}\,\frac{\text{m}\cdot\text{K}}{\text{W}})} + 70°\text{C}$$

$$= 151.8°\text{C}$$ <

COMMENTS: Assuming the dynamic viscosity of crude oil is similar to that of engine oil, we may evaluate the viscosity of the oil at the two maximum operating temperatures. From Table A-5 at T = 70.5°C = 343 K, μ = 0.046 N·s/m^2. At T = 151.8°C = 425 K, μ = 0.517 N·s/m^2. The viscosity of the oil with the aerogel insulation is 0.046/0.517 = 0.09, or only 9% of the viscosity of the oil without the aerogel. Savings in pumping costs and/or increases in oil production rates could be realized with use of the aerogel pipe-in-pipe concept.