Fundamentals of Heat and Mass Transfer

Chapter 11

HEAR EXCHANGERS

Dr. Osaid Matar

INTRODUCTION

- The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications.

- The device used to implement this exchange is termed a heat exchanger, and specific applications may be found in space heating and air-conditioning, power production, waste heat recovery, and chemical processing.

- **In this chapter our objectives are to introduce performance parameters for assessing the efficacy of a heat exchanger and to develop methodologies for designing a heat exchanger or for predicting the performance of an existing exchanger operating under prescribed conditions.**

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- **Heat exchangers are typically classified according to flow arrangement and type of construction.**

- The simplest heat exchanger is one for which the hot and cold fluids move in the same or opposite directions in a concentric tube (or double-pipe) construction.

- **In the parallel-flow arrangement** of Figure 11.1a, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end.

- **In the counterflow arrangement** of Figure 11.1b, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends

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- **Alternatively, the fluids may move in cross flow (perpendicular to each other), as shown by the finned and unfinned tubular heat exchangers of Figure 11.2.**

- The two configurations are typically differentiated by an idealization that treats fluid motion over **the tubes as unmixed or mixed.**

- **In Figure 11.2a, the fluid is said to be unmixed because the fins inhibit motion in a direction (y) that is transverse to the main-flow direction (x).**

- In this case the fluid temperature varies with x and y.

- In contrast, for the unfinned tube bundle of Figure 11.2b, fluid motion, hence mixing, in the transverse direction is possible, and temperature variations are primarily in the main- flow direction.

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- Since the tube flow is unmixed, both fluids are unmixed in the finned exchanger, while one fluid is mixed and the other unmixed in the unfinned exchanger.

- The nature of the mixing condition can significantly influence heat exchanger performance.

- Another common configuration is the shell-and-tube heat exchanger [1].
- Specific forms differ according to the number of shell-and-tube passes, and the simplest form, which involves single tube and shell passes, is shown in Figure 11.3.

- Baffles are usually installed to increase the convection coefficient of the shell-side fluid by inducing turbulence and a cross-flow velocity component.

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- In addition, the baffles physically support the tubes, reducing flow-induced tube vibration.

- Baffled heat exchangers with one shell pass and two tube passes and with two shell passes and four tube passes are shown in Figures 11.4a and 11.4b, respectively.
- A special and important class of heat exchangers is used to achieve a very large ($\geq$400 m^2/m^3 for liquids and ($\geq$ 700 m^2/m^3 for gases) heat transfer surface area per unit volume.

- Termed compact heat exchangers, these devices have dense arrays of finned tubes or plates and are typically used when at least one of the fluids is a gas, and is hence characterized by a small convection coefficient.

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- The tubes may be flat or circular, as in Figures 11.5a and 11.5b, c, respectively, and the fins may be plate or circular, as in Figures 11.5a, b and 11.5c, respectively.

- Parallel-plate heat exchangers may be finned or corrugated and may be used in single-pass (Figure 11.5d) or multipass (Figure 11.5e) modes of operation.

- Flow passages asso- ciated with compact heat exchangers are typically small ($D_h \leq 5$ mm), and the flow is usually laminar.

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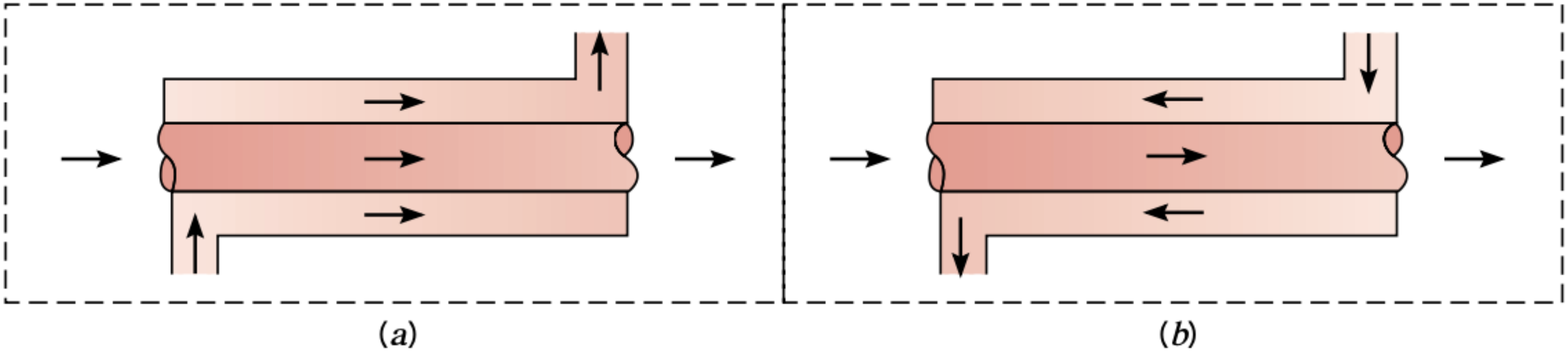

FIGURE 11.1 Concentric tube heat exchangers. (*a*) Parallel flow. (*b*) Counterflow.

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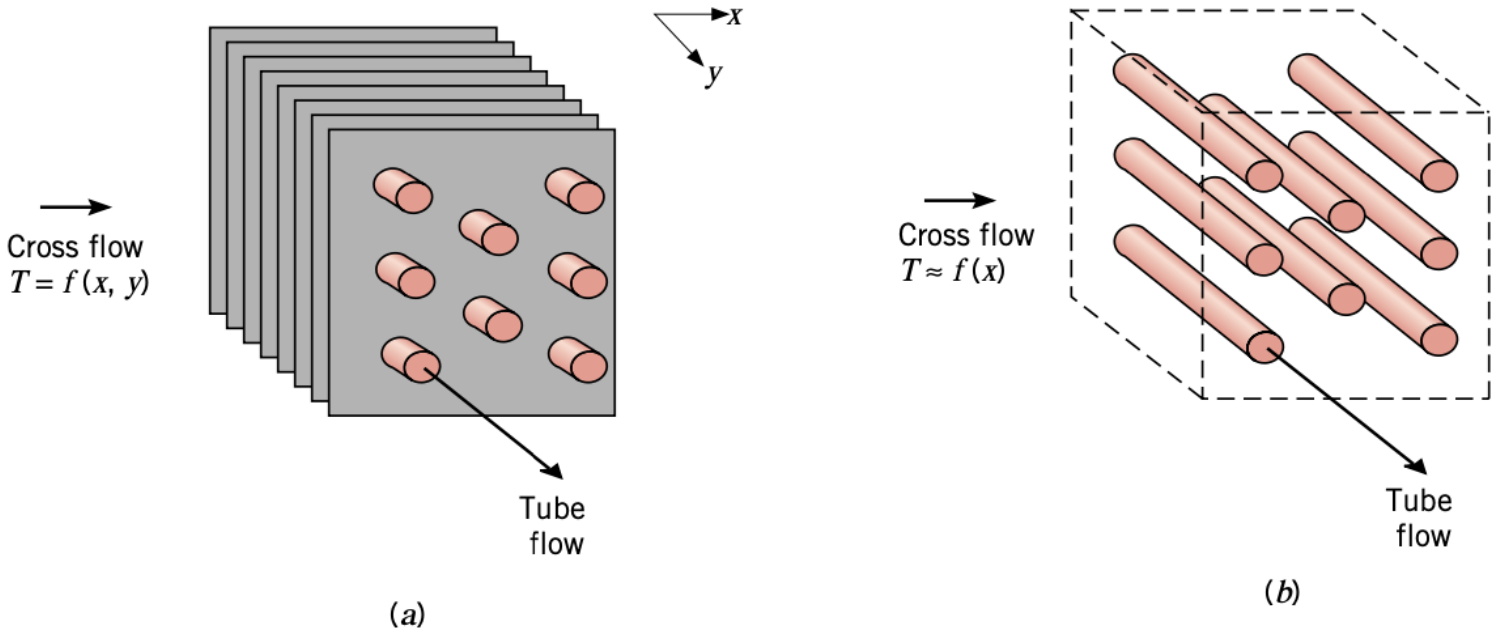

FIGURE 11.2 Cross-flow heat exchangers. (***a***) Finned with both fluids unmixed. (***b***) Unfinned with one fluid mixed and the other unmixed.

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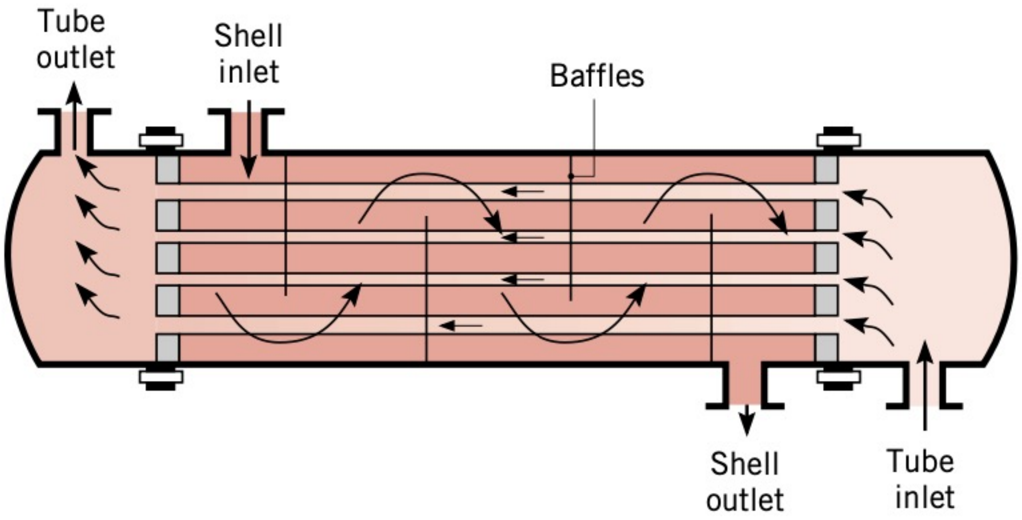


FIGURE 11.3 Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of operation).

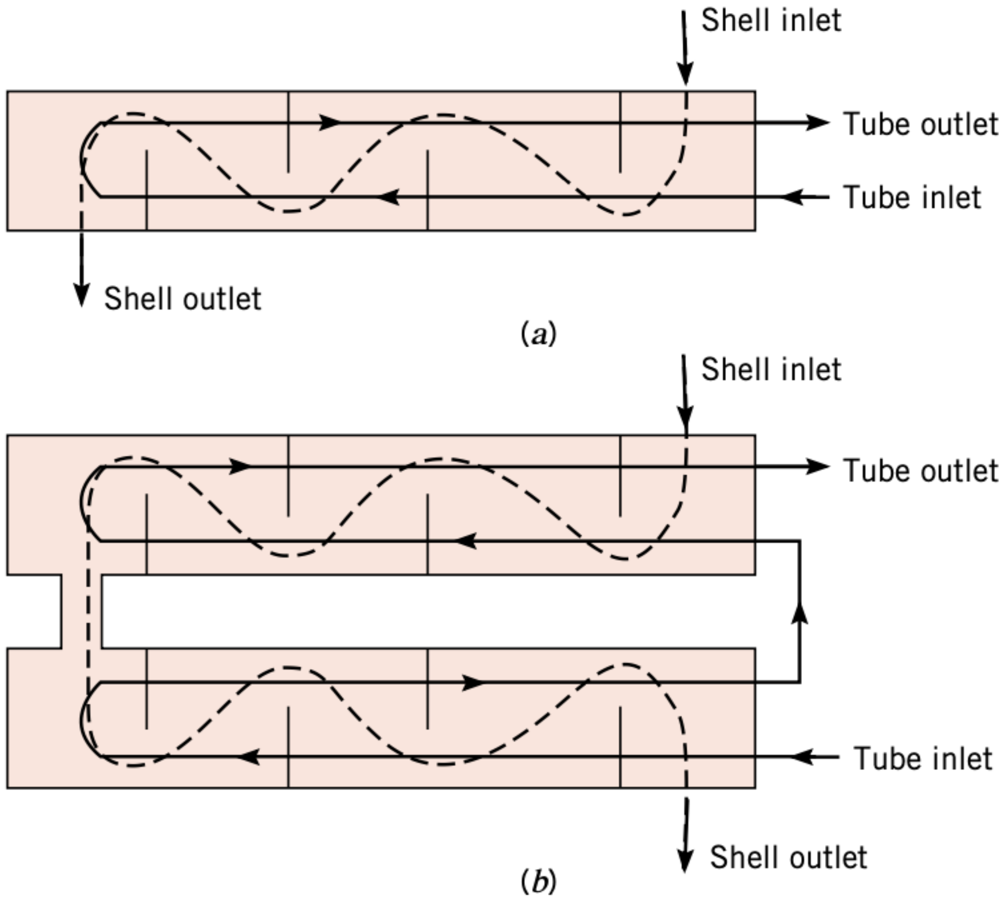


FIGURE 11.4 Shell-and-tube heat exchangers. (*a*) One shell pass and two tube passes. (*b*) Two shell passes and four tube passes.

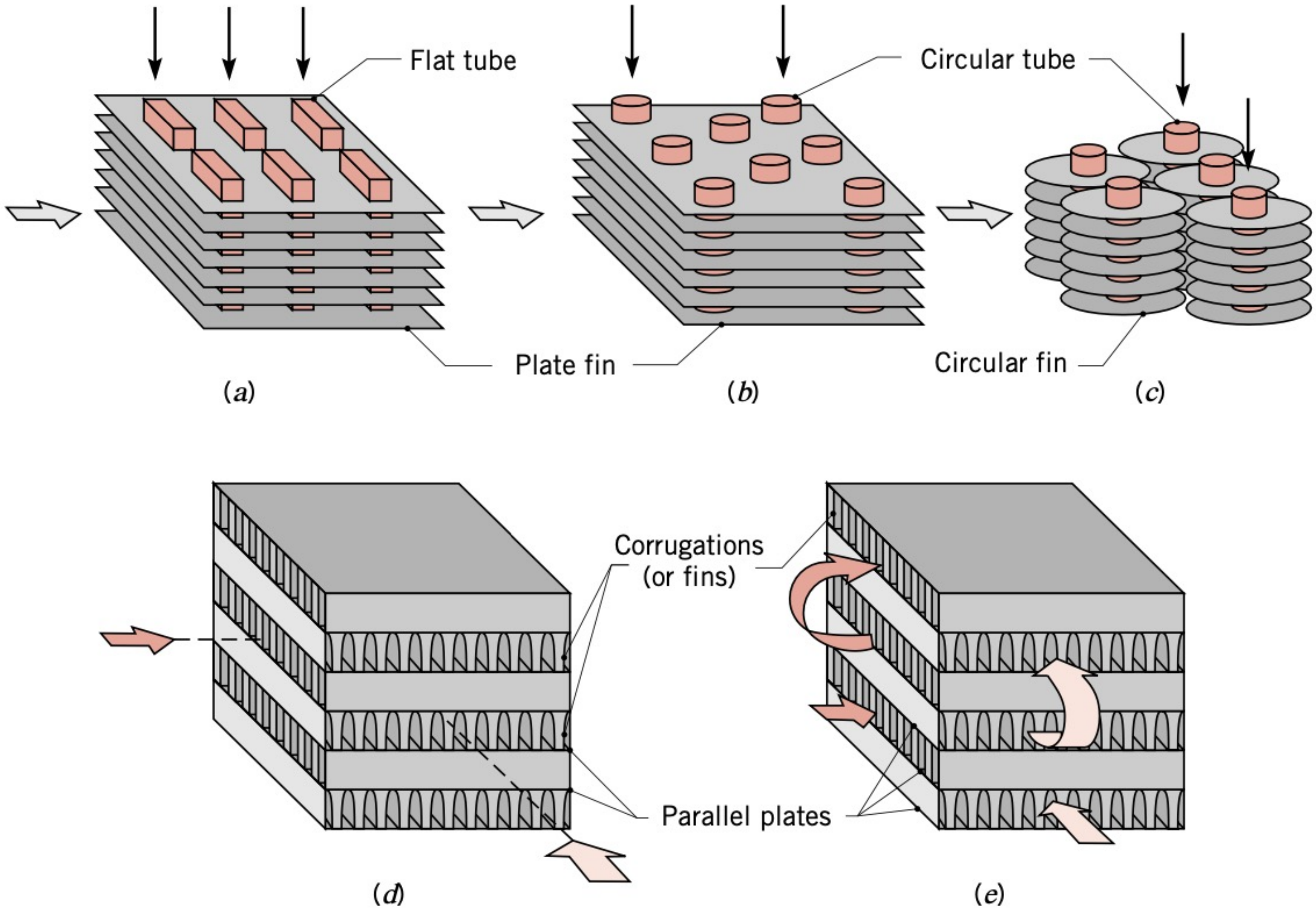


FIGURE 11.5 Compact heat exchanger cores. (*a*) Fin–tube (flat tubes, continuous plate fins). (*b*) Fin–tube (circular tubes, continuous plate fins). (*c*) Fin–tube (circular tubes, circular fins). (*d*) Plate–fin (single pass). (*e*) Plate–fin (multipass).

The Overall Heat Transfer Coefficient

$$\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{U_h A_h}$$

$$= \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h} \qquad (11.1)$$

The quantity η_0 in Equation 11.1 is termed the overall surface efficiency or temperature effectiveness of a finned surface. It is defined such that, for the hot or cold surface without fouling, the heat transfer rate is

$$q = \eta_o hA(T_b - T_\infty) \qquad (11.2)$$

where T_b is the base surface temperature (Figure 3.20) and A is the total (fin plus exposed base) surface area. The quantity was introduced in Section 3.6.5, and the following expression was derived:

$$\eta_o = 1 - \frac{A_f}{A}(1 - \eta_f) \qquad (11.3)$$

The Overall Heat Transfer Coefficient

TABLE 11.1 Representative Fouling Factors [1]

Fluid	R''_f ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

If a straight or pin fin of length L (Figure 3.16) is used and an adiabatic tip is assumed, Equations 3.76 and 3.86 yield

$$\eta_f = \frac{\tanh(mL)}{mL} \qquad (11.4)$$

The Overall Heat Transfer Coefficient

TABLE 11.2 **Representative Values of the Overall Heat Transfer Coefficient**

Fluid Combination	**U ($W/m^2 \cdot K$)**
Water to water	850–1700
Water to oil	110–350
Steam condenser (water in tubes)	1000–6000
Ammonia condenser (water in tubes)	800–1400
Alcohol condenser (water in tubes)	250–700
Finned-tube heat exchanger (water in tubes, air in cross flow)	25–50

The Overall Heat Transfer Coefficient

For the unfinned, tubular heat exchangers of Figures 11.1 through 11.4, Equation 11.1 reduces to

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$= \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o} \quad (11.5)$$

Analysis: Use of the Log Mean Temperature Difference

- **To design or to predict the performance of a heat exchanger, it is essential to relate the total heat transfer rate to quantities such as the inlet and outlet fluid temperatures, the overall heat transfer coefficient, and the total surface area for heat transfer.**

- Two such relations may readily be obtained by applying overall energy balances to the hot and cold fluids, as shown in Figure 11.6.

- In particular, if q is the total rate of heat transfer between the hot and cold fluids and there is negligible heat transfer between the exchanger and its surroundings, as well as negligible potential and kinetic energy changes, application of the steady flow energy equation, Equation 1.11d, gives

$$q = \dot{m}_h(i_{h,i} - i_{h,o}) \qquad (11.6a)$$

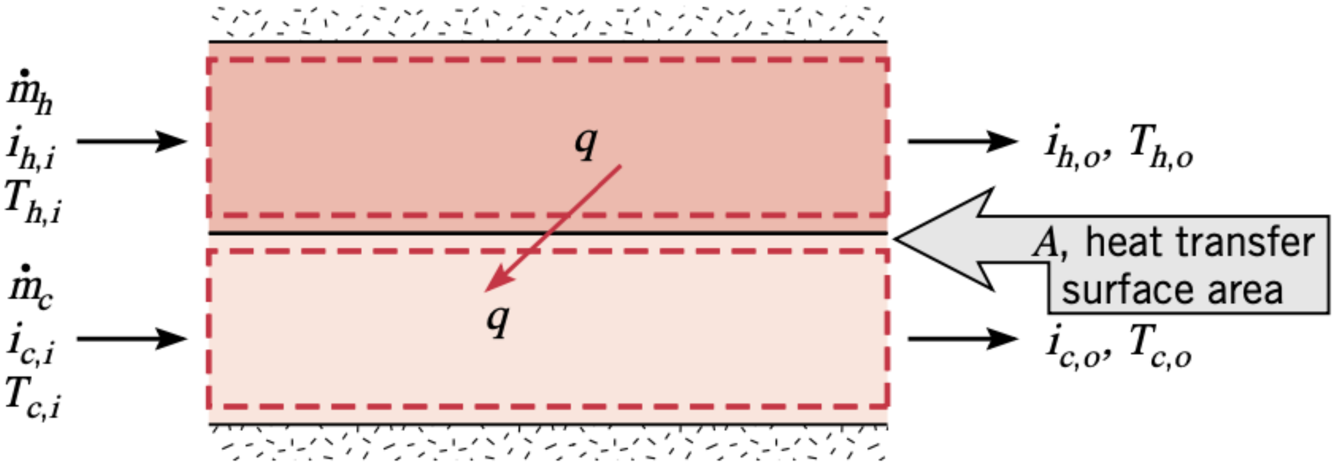


FIGURE 11.6 Overall energy balances for the hot and cold fluids of a two-fluid heat exchanger.

$$q = \dot{m}_c(i_{c,o} - i_{c,i}) \tag{11.7a}$$

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) \tag{11.6b}$$

and

$$q = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) \tag{11.7b}$$

$$\Delta T \equiv T_h - T_c \tag{11.8}$$

$$q = UA\,\Delta T_m \tag{11.9}$$

The Parallel-Flow Heat Exchanger

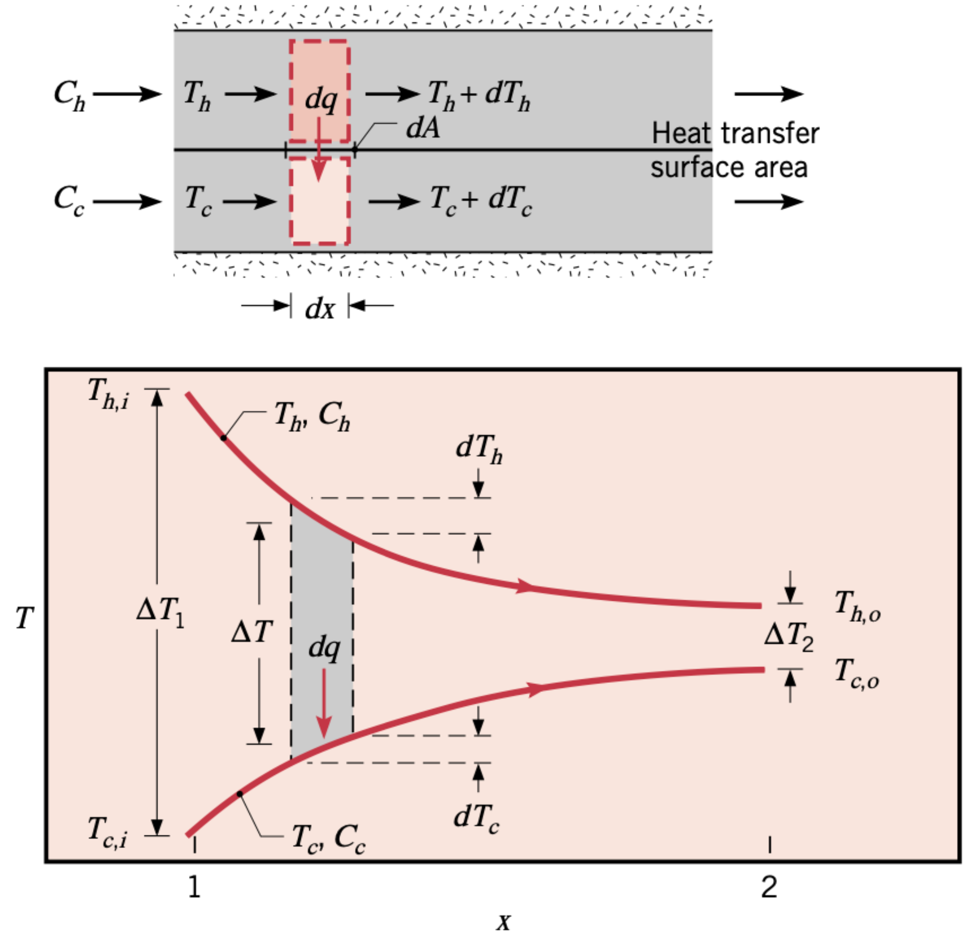


Figure 11.7 Temperature distributions for a parallel-flow heat exchanger.

The Parallel-Flow Heat Exchanger

The energy balances and the subsequent analysis are subject to the following assumptions.

1. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids.
2. Axial conduction along the tubes is negligible.
3. Potential and kinetic energy changes are negligible.
4. The fluid specific heats are constant.
5. The overall heat transfer coefficient is constant.

Applying an energy balance to each of the differential elements of Figure 11.7, it follows that

$$dq = -\dot{m}_h c_{p,h}\, dT_h \equiv -C_h\, dT_h \tag{11.10}$$

and

$$dq = \dot{m}_c c_{p,c}\, dT_c \equiv C_c\, dT_c \tag{11.11}$$

where Ch and Cc are the hot and cold fluid heat capacity rates, respectively.

The Parallel-Flow Heat Exchanger

The heat transfer across the surface area dA may also be expressed as

$$dq = U\,\Delta T\,dA \tag{11.12}$$

where $\Delta T = T_h - T_c$ is the *local* temperature difference between the hot and cold fluids.

To determine the integrated form of Equation 11.12, we begin by substituting Equations 11.10 and 11.11 into the differential form of Equation 11.8

$$d(\Delta T) = dT_h - dT_c$$

to obtain

$$d(\Delta T) = -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Substituting for dq from Equation 11.12 and integrating across the heat exchanger, we obtain

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

or

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \tag{11.13}$$

The Parallel-Flow Heat Exchanger

Substituting for C_h and C_c from Equations 11.6b and 11.7b, respectively, it follows that

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right)$$
$$= -\frac{UA}{q}[(T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})]$$

Recognizing that, for the parallel-flow heat exchanger of Figure 11.7, $\Delta T_1 = (T_{h,i} - T_{c,i})$ and $\Delta T_2 = (T_{h,o} - T_{c,o})$, we then obtain

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

Comparing the above expression with Equation 11.9, we conclude that the appropriate average temperature difference is a *log mean temperature difference*, ΔT_{lm}. Accordingly, we may write

$$q = UA\,\Delta T_{\text{lm}} \tag{11.14}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{11.15}$$

Remember that, for the *parallel-flow exchanger*,

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o} \end{bmatrix} \tag{11.16}$$

For the Counterflow Heat Exchanger

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i} \end{bmatrix}$$

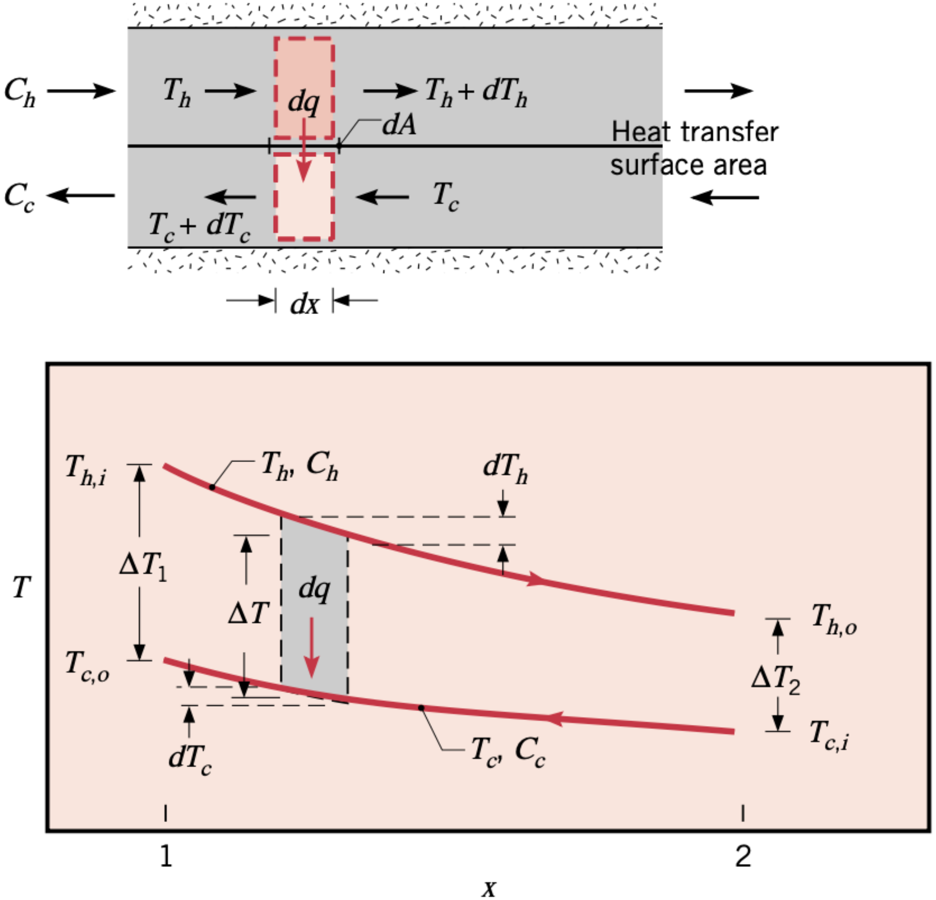


FIGURE 11.8 Temperature distributions for a counterflow heat exchanger.

Special Operating Conditions

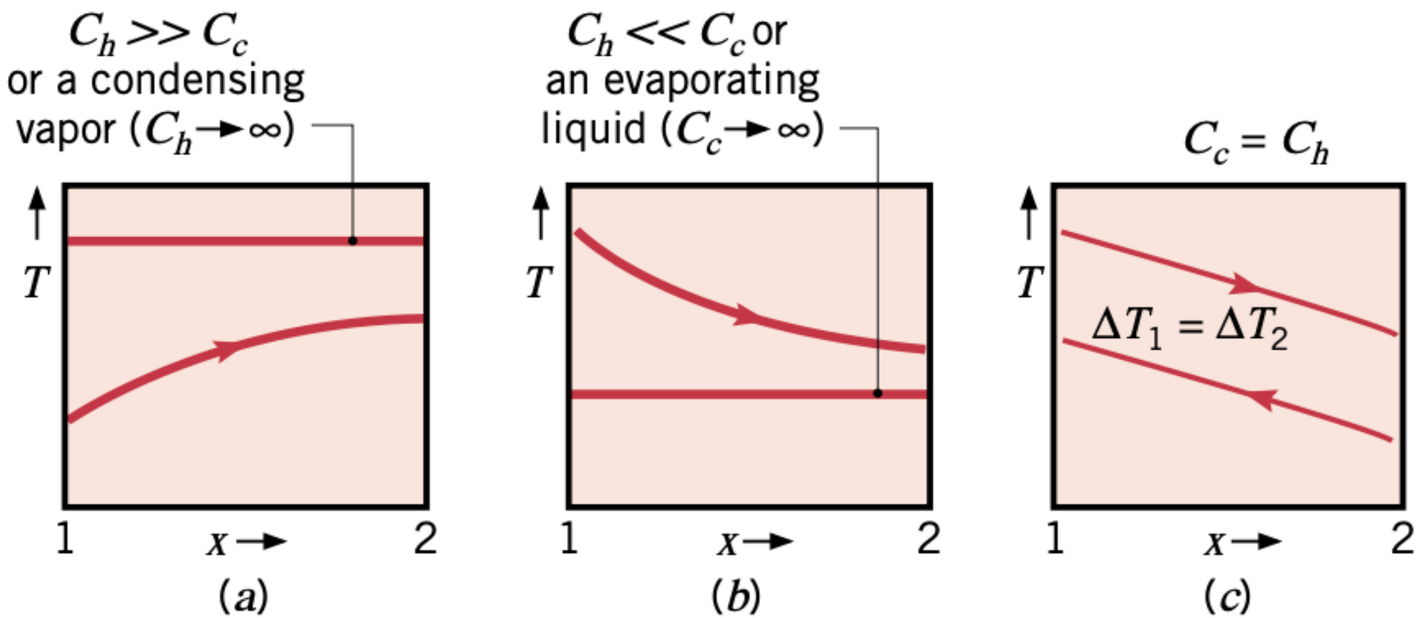


FIGURE 11.9 Special heat exchanger conditions. (*a*) $C_h \gg C_c$ or a condensing vapor. (*b*) An evaporating liquid or $C_h \ll C_c$. (*c*) A counterflow heat exchanger with equivalent fluid heat capacities ($C_h = C_c$).

Example 11.1

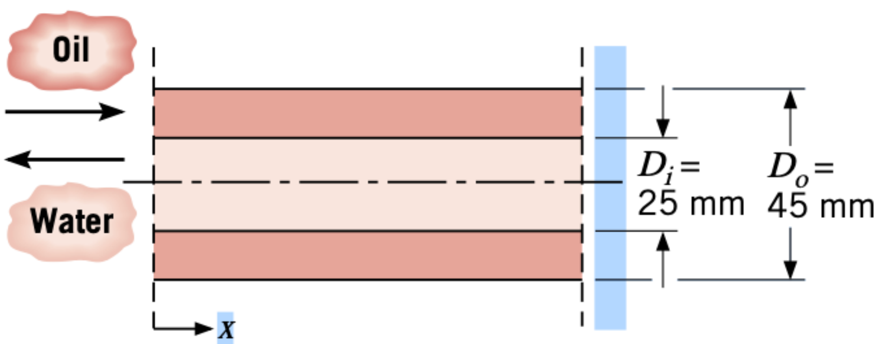
A counterflow, concentric tube heat exchanger is used to cool the lubricating oil for a large industrial gas turbine engine. The flow rate of cooling water through the inner tube ($D_i = 25$ mm) is 0.2 kg/s, while the flow rate of oil through the outer annulus ($D_o = 45$ mm) is 0.1 kg/s. The oil and water enter at temperatures of 100 and 30°C, respectively. How long must the tube be made if the outlet temperature of the oil is to be 60°C?

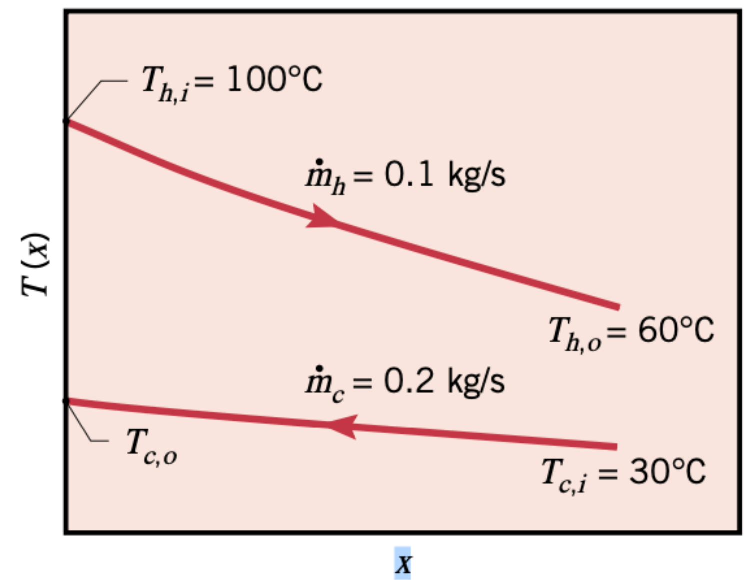
Solution

Known: Fluid flow rates and inlet temperatures for a counterflow, concentric tube heat exchanger of prescribed inner and outer diameter.

Find: Tube length to achieve a desired hot fluid outlet temperature.

Schematic:





Assumptions:

1. Negligible heat loss to the surroundings.
2. Negligible kinetic and potential energy changes.
3. Constant properties.
4. Negligible tube wall thermal resistance and fouling factors.
5. Fully developed conditions for the water and oil (U independent of x).

Analysis: The required heat transfer rate may be obtained from the overall energy balance for the hot fluid, Equation 11.6b.

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o})$$
$$q = 0.1\ \text{kg/s} \times 2131\ \text{J/kg} \cdot \text{K}\ (100 - 60)°\text{C} = 8524\ \text{W}$$

Applying Equation 11.7b, the water outlet temperature is

$$T_{c,o} = \frac{q}{\dot{m}_c c_{p,c}} + T_{c,i}$$
$$T_{c,o} = \frac{8524\ \text{W}}{0.2\ \text{kg/s} \times 4178\ \text{J/kg} \cdot \text{K}} + 30°\text{C} = 40.2°\text{C}$$

Accordingly, use of $\overline{T}_c = 35°\text{C}$ to evaluate the water properties was a good choice. The required heat exchanger length may now be obtained from Equation 11.14,

$$q = UA\,\Delta T_{\text{lm}}$$

where $A = \pi D_i L$ and from Equations 11.15 and 11.17

$$\Delta T_{\text{lm}} = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln[(T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,i})]} = \frac{59.8 - 30}{\ln(59.8/30)} = 43.2°\text{C}$$

From Equation 11.5 the overall heat transfer coefficient is

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

For water flow through the tube,

$$Re_D = \frac{4\dot{m}_c}{\pi D_i \mu} = \frac{4 \times 0.2\ \text{kg/s}}{\pi(0.025\ \text{m})725 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2} = 14{,}050$$

Accordingly, the flow is turbulent and the convection coefficient may be computed from Equation 8.60

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{0.4}$$

$$Nu_D = 0.023\, (14{,}050)^{4/5}\, (4.85)^{0.4} = 90$$

Hence

$$h_i = Nu_D \frac{k}{D_i} = \frac{90 \times 0.625\ \text{W/m} \cdot \text{K}}{0.025\ \text{m}} = 2250\ \text{W/m}^2 \cdot \text{K}$$

For the flow of oil through the annulus, the hydraulic diameter is, from Equation 8.71, $D_h = D_o - D_i = 0.02$ m, and the Reynolds number is

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\rho(D_o - D_i)}{\mu} \times \frac{\dot{m}_h}{\rho\pi(D_o^2 - D_i^2)/4}$$

$$Re_D = \frac{4\dot{m}_h}{\pi\,(D_o + D_i)\,\mu} = \frac{4 \times 0.1\ \text{kg/s}}{\pi(0.045 + 0.025)\ \text{m} \times 3.25 \times 10^{-2}\ \text{kg/s} \cdot \text{m}} = 56.0$$

For turbulent Flow inside a tube:

$$Nu_D = 0.023 Re_D^{4/5} Pr^n \quad (8.60)$$

where $n = 0.4$ for heating ($T_s > T_m$) and 0.3 for cooling ($T_s < T_m$). These equations have been confirmed experimentally for the range of conditions

$$\begin{bmatrix} 0.7 \lesssim Pr \lesssim 160 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

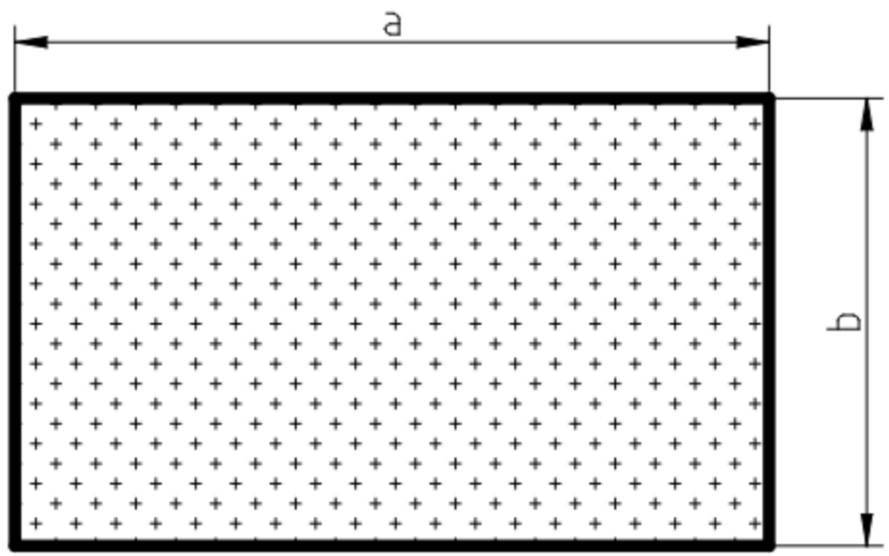
length. It is termed the *hydraulic diameter* and is defined as

$$D_h \equiv \frac{4A_c}{P} \quad (8.66)$$

where A_c and P are the *flow* cross-sectional area and the *wetted perimeter*, respectively. It is this diameter that should be used in calculating parameters such as Re_D and Nu_D.

For the flow of the fluid through the annulus, the hydraulic diameter is:

$$D_h = \frac{4(\pi/4)(D_o^2 - D_i^2)}{\pi D_o + \pi D_i} = D_o - D_i \quad (8.71)$$



Based on equation *(1)* the hydraulic diameter of a rectangular duct or pipe can be calculated as

$$d_h = 4\ a\ b\ /\ (2\ (a + b))$$

$$= 2\ a\ b\ /\ (a + b) \qquad (4)$$

where

a = width/height of the duct (m, ft)

b = height/width of the duct (m, ft)

The annular flow is therefore laminar. Assuming uniform temperature along the inner surface of the annulus and a perfectly insulated outer surface, the convection coefficient at the inner surface may be obtained from Table 8.2. With $(D_i/D_o) = 0.56$, linear interpolation provides

$$Nu_i = \frac{h_o D_h}{k} = 5.63$$

and

$$h_o = 5.63 \frac{0.138 \text{ W/m} \cdot \text{K}}{0.020 \text{ m}} = 38.8 \text{ W/m}^2 \cdot \text{K}$$

The overall convection coefficient is then

$$U = \frac{1}{(1/2250 \text{ W/m}^2 \cdot \text{K}) + (1/38.8 \text{ W/m}^2 \cdot \text{K})} = 38.1 \text{ W/m}^2 \cdot \text{K}$$

and from the rate equation it follows that

$$L = \frac{q}{U\pi D_i \Delta T_{\text{lm}}} = \frac{8524 \text{ W}}{38.1 \text{ W/m}^2 \cdot \text{K}\ \pi\,(0.025 \text{ m})\,(43.2°\text{C})} = 65.9 \text{ m} \qquad \triangleleft$$

Comments:

1. The hot side convection coefficient controls the rate of heat transfer between the two fluids, and the low value of h_o is responsible for the large value of L. Incorporation of heat transfer enhancement methods, such as described in Section 8.8, could be used to decrease the size of the heat exchanger.
2. Because $h_i \gg h_o$, the tube wall temperature will follow closely that of the coolant water. Accordingly, the assumption of uniform wall temperature, which is inherent in the use of Table 8.2 to obtain h_o, is reasonable.

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	*Comments*
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$

Used with permission from W. M. Kays and H. C. Perkins, in W. M. Rohsenow and J. P. Hartnett, Eds., *Handbook of Heat Transfer,* Chap. 7, McGraw-Hill, New York, 1972.

EXAMPLE 11.2

The counterflow, concentric tube heat exchanger of Example 11.1 is replaced with a compact, plate-type heat exchanger that consists of a stack of thin metal sheets, separated by N gaps of width a. The oil and water flows are subdivided into $N/2$ individual flow streams, with the oil and water moving in opposite directions within alternating gaps. It is desirable for the stack to be of a cubical geometry, with a characteristic exterior dimension L. Determine the exterior dimensions of the heat exchanger as a function of the number of gaps if the flow rates, inlet temperatures, and desired oil outlet temperature are the same as in Example 11.1. Compare the pressure drops of the water and oil streams within the plate-type heat exchanger to the pressure drops of the flow streams in Example 11.1, if 60 gaps are specified.

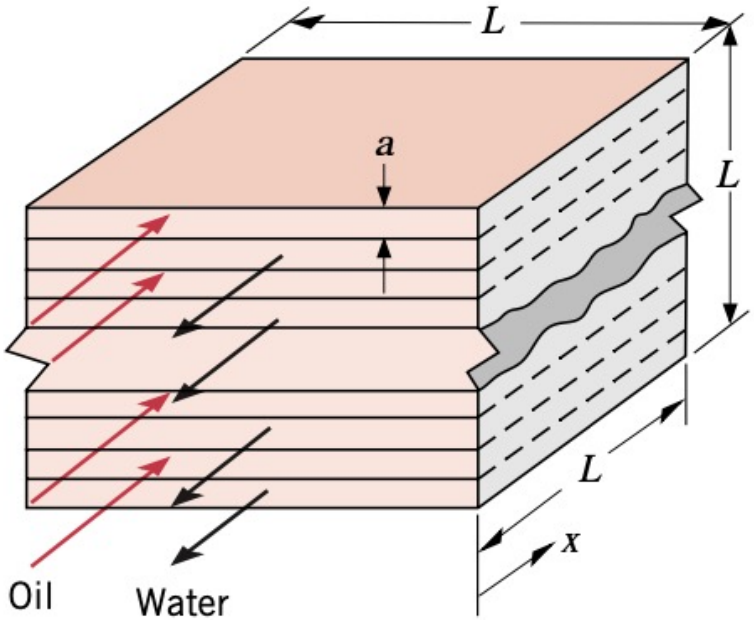
SOLUTION

Known: Configuration of a plate-type heat exchanger. Fluid flow rates, inlet temperatures, and desired oil outlet temperature.

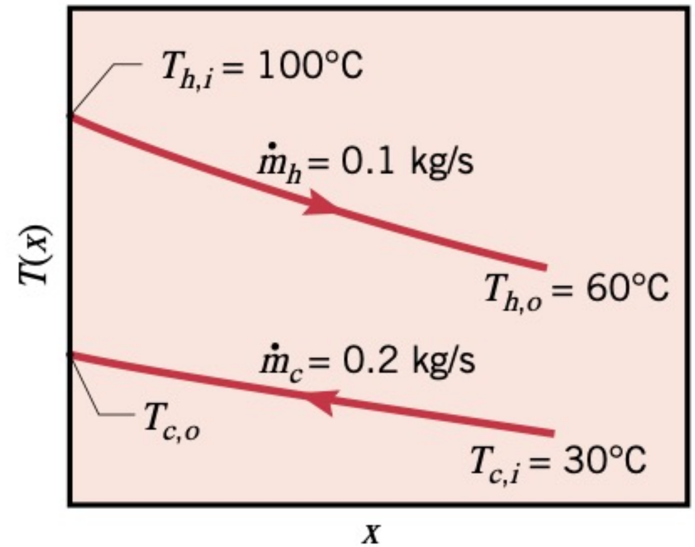
Find:

1. Exterior dimensions of the heat exchanger.
2. Pressure drops within the plate-type heat exchanger with $N = 60$ gaps, and the concentric tube heat exchanger of Example 11.1.

Schematic:



L
a
L
L
x
Oil
Water



$T_{h,i}$ = 100°C
$\dot{m}_h$ = 0.1 kg/s
$T_{h,o}$ = 60°C
$\dot{m}_c$ = 0.2 kg/s
$T_{c,o}$
$T_{c,i}$ = 30°C
T(x)
x

Assumptions:

1. Negligible heat loss to the surroundings.
2. Negligible kinetic and potential energy changes.
3. Constant properties.
4. Negligible plate thermal resistance and fouling factors.
5. Fully developed conditions for the water and oil.
6. Identical gap-to-gap heat transfer coefficients.
7. Heat exchanger exterior dimension is large compared to the gap width.

Properties: See Example 11.1. In addition, Table A.5, unused engine oil ($\bar{T}_h$ = 353 K): $\rho = 852.1\ \text{kg/m}^3$. Table A.6, water ($\bar{T}_c \approx 35°\text{C}$): $\rho = v_f^{-1} = 994\ \text{kg/m}^3$.

Analysis:

1. The gap width may be related to the overall dimension of the heat exchanger by the expression $a = L/N$, and the total heat transfer area is $A = L^2\,(N - 1)$. Assuming $a \ll L$ and the existence of laminar flow, the Nusselt number for each interior gap is provided in Table 8.1 and is

$$Nu_D = \frac{hD_h}{k} = 7.54$$

From Equation 8.66, the hydraulic diameter is $D_h = 2a$. Combining the preceding expressions yields for the water:

$$h_c = 7.54kN/2L = 7.54 \times 0.625\ \text{W/m}\cdot\text{K} \times N/2L = (2.36\ \text{W/m}\cdot\text{K})N/L$$

Likewise, for the oil:

$$h_h = 7.54kN/2L = 7.54 \times 0.138\ \text{W/m}\cdot\text{K} \times N/2L = (0.520\ \text{W/m}\cdot\text{K})N/L$$

and the overall convection coefficient is

$$U = \frac{1}{1/h_c + 1/h_h}$$

From Example 11.1, the required log mean temperature difference and heat transfer rate are $\Delta T_{\text{lm}} = 43.2°\text{C}$ and $q = 8524$ W, respectively. From Equation 11.14 it follows that

$$UA = \frac{L^2(N-1)}{1/h_c + 1/h_h} = \frac{q}{\Delta T_{\text{lm}}}$$

From Example 11.1, the required log mean temperature difference and heat transfer rate are $\Delta T_{lm} = 43.2°C$ and $q = 8524$ W, respectively. From Equation 11.14 it follows that

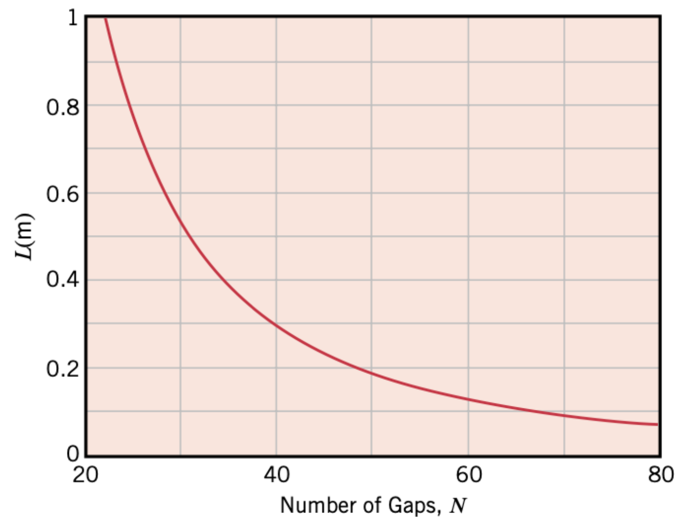
$$UA = \frac{L^2(N-1)}{1/h_c + 1/h_h} = \frac{q}{\Delta T_{lm}}$$

which may be rearranged to yield

$$L = \frac{q}{\Delta T_{lm}(N-1)}\left[\frac{1}{h_c L} + \frac{1}{h_h L}\right]$$

$$= \frac{8524 \text{ W}}{43.2°\text{C}(N-1)N}\left[\frac{1}{2.36 \text{ W/m} \cdot \text{K}} + \frac{1}{0.520 \text{ W/m} \cdot \text{K}}\right] = \frac{463 \text{ m}}{N(N-1)} \quad \triangleleft$$

The size of the compact heat exchanger decreases as the number of gaps is increased, as shown in the figure below.



2. For $N = 60$ gaps, the stack dimension is $L = 0.131$ m from the results of part 1, and the gap width is $a = L/N = 0.131\ \text{m}/60 = 0.00218$ m.

The hydraulic diameter is $D_h = 0.00436$ m, and the mean velocity in each water-filled gap is

$$u_m = \frac{\dot{m}}{\rho L^2/2} = \frac{2 \times 0.2\ \text{kg/s}}{994\ \text{kg/m}^3 \times 0.131^2\ \text{m}^2} = 0.0235\ \text{m/s}$$

providing a Reynolds number of

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{994\ \text{kg/m}^3 \times 0.0235\ \text{m/s} \times 0.00436\ \text{m}}{725 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2} = 141$$

For the oil-filled gaps

$$u_m = \frac{\dot{m}}{\rho L^2/2} = \frac{2 \times 0.1\ \text{kg/s}}{852.1\ \text{kg/m}^3 \times 0.131^2\ \text{m}^2} = 0.0137\ \text{m/s}$$

yielding a Reynolds number of

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{852.1\ \text{kg/m}^3 \times 0.0137\ \text{m/s} \times 0.00436\ \text{m}}{3.25 \times 10^{-2}\ \text{N} \cdot \text{s/m}^2} = 1.57$$

Therefore, the flow is laminar for both fluids, as assumed in part 1. Equations 8.19 and 8.22a may be used to calculate the pressure drop for the water:

$$\Delta p = \frac{64}{Re_D} \cdot \frac{\rho u_m^2}{2D_h} \cdot L = \frac{64}{141} \times \frac{994\ \text{kg/m}^3 \times 0.0235^2\ \text{m}^2/\text{s}^2}{2 \times 0.00436\ \text{m}} \times 0.131\ \text{m}$$

$$= 3.76\ \text{N/m}^2 \qquad \triangleleft$$

8.1.4 Pressure Gradient and Friction Factor in Fully Developed Flow

Substituting Equations 8.1 and 8.14 into 8.16, it follows that, for fully developed laminar flow,

$$f = \frac{64}{Re_D} \tag{8.19}$$

Note that f, hence dp/dx, is a constant in the fully developed region. From Equation 8.16 the pressure drop $\Delta p = p_1 - p_2$ associated with fully developed flow from the axial position x_1 to x_2 may then be expressed as

$$\Delta p = -\int_{p_1}^{p_2} dp = f\frac{\rho u_m^2}{2D}\int_{x_1}^{x_2} dx = f\frac{\rho u_m^2}{2D}(x_2 - x_1) \tag{8.22a}$$

Similarly, for the oil

$$\Delta p = \frac{64}{Re_D} \cdot \frac{\rho u_m^2}{2D_h} \cdot L = \frac{64}{1.57} \times \frac{852.1 \text{ kg/m}^3 \times 0.0137^2 \text{ m}^2/\text{s}^2}{2 \times 0.00436 \text{ m}} \times 0.131 \text{ m}$$

$$= 98.2 \text{ N/m}^2 \quad \triangleleft$$

For Example 11.1, the friction factor associated with the water flow may be calculated using Equation 8.21, and for a smooth surface condition is $f = (0.790 \ln(14{,}050) - 1.64)^{-2} = 0.0287$. The mean velocity is $u_m = 4\dot{m}/[\rho\pi D_i^2 = 4 \times 0.2$ kg/s/(994 kg/m$^3 \times \pi \times 0.025^2$ m^2)] = 0.410 m/s, and the pressure drop is

$$\Delta p = f \cdot \frac{\rho u_m^2}{2D_h} \cdot L = 0.0287 \times \frac{994 \text{ kg/m}^3 \times 0.410^2 \text{ m}^2/\text{s}^2}{2 \times 0.025 \text{ m}} \times 65.9 \text{ m}$$

$$= 6310 \text{ N/m}^2 \quad \triangleleft$$

For the oil flowing in the annular region, the mean velocity is $u_m = 4\dot{m}/[\rho\pi (D_o^2 - D_i^2)] = 4 \times 0.1$ kg/s/[852.1 kg/m$^3 \times \pi \times (0.045^2 - 0.025^2)$m^2] = 0.107 m/s, and the pressure drop is

$$\Delta p = \frac{64}{Re_D} \cdot \frac{\rho u_m^2}{2D_h} \cdot L = \frac{64}{56} \times \frac{852.1 \text{ kg/m}^3 \times 0.107^2 \text{ m}^2/\text{s}^2}{2 \times 0.020 \text{ m}} \times 65.9 \text{ m}$$

$$= 18{,}300 \text{ N/m}^2 \quad \triangleleft$$

Comments:

1. Increasing the number of gaps increases the UA product by simultaneously providing more surface area and increasing the heat transfer coefficients associated with the flow of the fluids through smaller passages.
2. The area-to-volume ratio of the $N = 60$ heat exchanger is $L^2(N - 1)/L^3 = (N - 1)/L = (60 - 1)/0.131 \text{ m} = 451 \text{ m}^2/\text{m}^3$.
3. The volume occupied by the concentric tube heat exchanger is $V = \pi D_o^2 L/4 = \pi \times 0.045^2 \text{ m}^2 \times 65.9 \text{ m}/4 = 0.10 \text{ m}^3$, while the volume of the compact plate-type exchanger is $V = L^3 = 0.131^3 \text{ m}^3 = 0.0022 \text{ m}^3$. Use of the plate-type heat exchanger results in a 97.8% reduction in volume relative to the conventional, concentric tube heat exchanger.
4. The pressure drops associated with use of the compact heat exchanger are significantly less than for a conventional concentric tube configuration. Pressure drops are reduced by 99.9% and 99.5% for the water and oil flows, respectively.
5. Fouling of the heat transfer surfaces may result in a decrease in the gap width, as well as an associated reduction in heat transfer rate and increase in pressure drop.
6. Because $h_c > h_h$, the temperatures of the thin metal sheets will follow closely that of the water, and, as in Example 11.1, the assumption of uniform temperature conditions to obtain h_c and h_h is reasonable.
7. One method to fabricate such a heat exchanger is presented in C. F. McDonald, *Appl. Thermal Engin.*, 20, 471, 2000.

Heat Exchanger Analysis: The Effectiveness–NTU Method

- It is a simple matter to use the log mean temperature difference (LMTD) method of heat exchanger analysis when the fluid inlet temperatures are known and the outlet temperatures are specified or readily determined from the energy balance expressions, Equations 11.6b and 11.7b.

- The value of ΔT_{lm} for the exchanger may then be determined.

- However, if only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure.

- It is therefore preferable to use an alternative approach termed the effectiveness–NTU (or NTU) method.

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) \tag{11.18}$$

where C_{min} is equal to C_c or C_h, whichever is smaller.

It is now logical to define the effectiveness, ε, as the ratio of the actual heat transfer rate for a heat exchanger to the maximum possible heat transfer rate:

$$\varepsilon \equiv \frac{q}{q_{max}} \tag{11.19}$$

From Equations 11.6b, 11.7b, and 11.18, it follows that

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} \tag{11.20}$$

or

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} \tag{11.21}$$

By definition the effectiveness, which is dimensionless, must be in the range $0 \leq \varepsilon \leq 1$. It is useful because, if ε, $T_{h,i}$, and $T_{c,i}$ are known, the actual heat transfer rate may readily be determined from the expression

$$q = \varepsilon C_{min}(T_{h,i} - T_{c,i}) \tag{11.22}$$

For any heat exchanger it can be shown that [5]

$$\varepsilon = f\left(\text{NTU}, \frac{C_{min}}{C_{max}}\right) \tag{11.23}$$

where C_{min}/C_{max} is equal to C_c/C_h or C_h/C_c, depending on the relative magnitudes of the hot and cold fluid heat capacity rates. The *number of transfer units* (NTU) is a dimensionless parameter that is widely used for heat exchanger analysis and is defined as

$$\text{NTU} \equiv \frac{UA}{C_{min}} \tag{11.24}$$

where Cr is the heat capacity ratio Cr =Cmin/Cmax

Table 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation		
Concentric tube			
Parallel flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - C_r)]}{1 - C_r \exp[-\text{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\text{NTU}}{1 + \text{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n Shell passes (2n, 4n, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\text{NTU})^{0.22}\{\exp[-C_r(\text{NTU})^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\text{NTU})]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\text{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\text{NTU})$		(11.35a)

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$	(11.28b)
Counterflow	$\text{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$	
	$\text{NTU} = \dfrac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\text{NTU})_1 = -(1 + C_r^2)^{-1/2}\ln\left(\dfrac{E - 1}{E + 1}\right)$	(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$	(11.30c)
n Shell passes (2*n*, 4*n*, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F - 1}{F - C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \text{NTU} = n(\text{NTU})_1$	(11.31b, c, d)
Cross-flow (single pass)		
C_{max} (mixed), C_{min} (unmixed)	$\text{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$	(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\text{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$	(11.34b)
All exchangers ($C_r = 0$)	$\text{NTU} = -\ln(1 - \varepsilon)$	(11.35b)

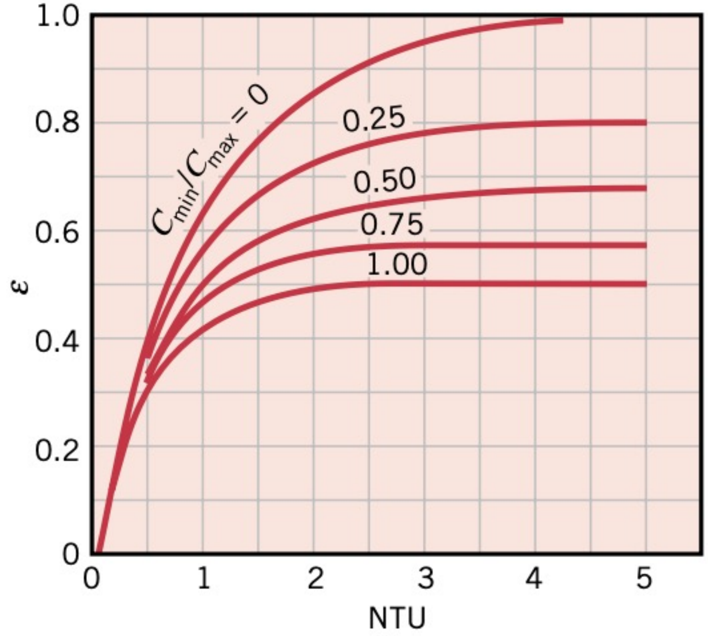


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

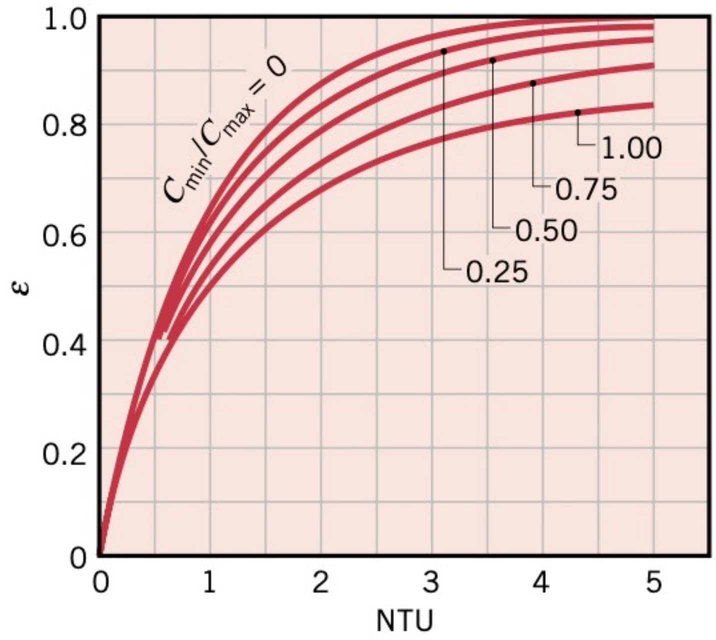


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

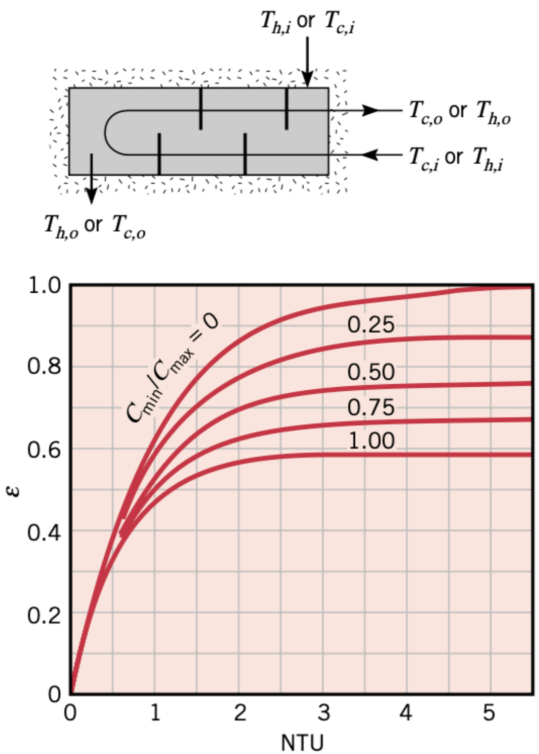


FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

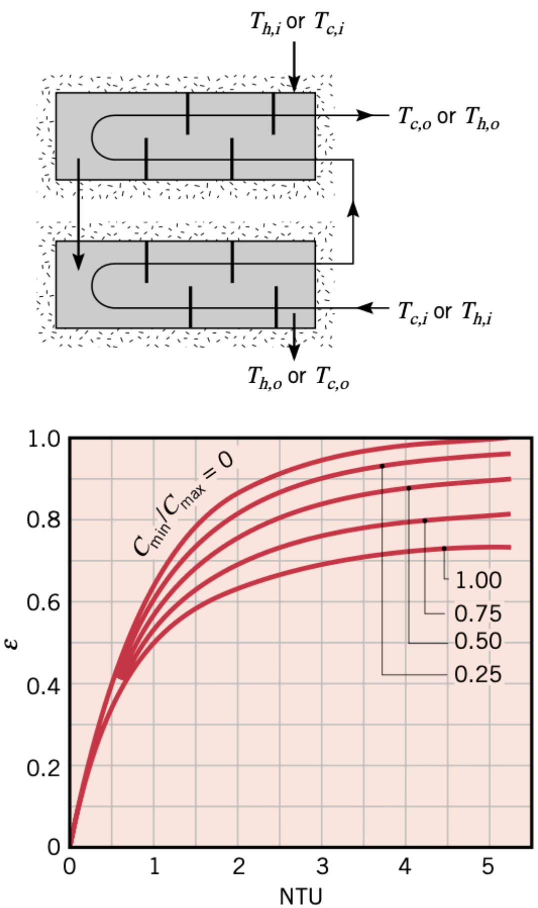


FIGURE 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with $n = 2$).

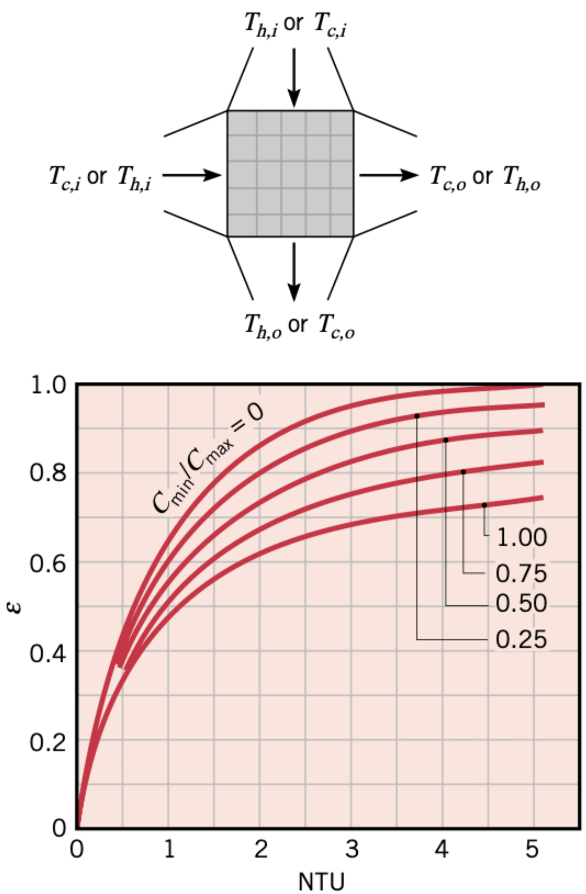


FIGURE 11.14 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).

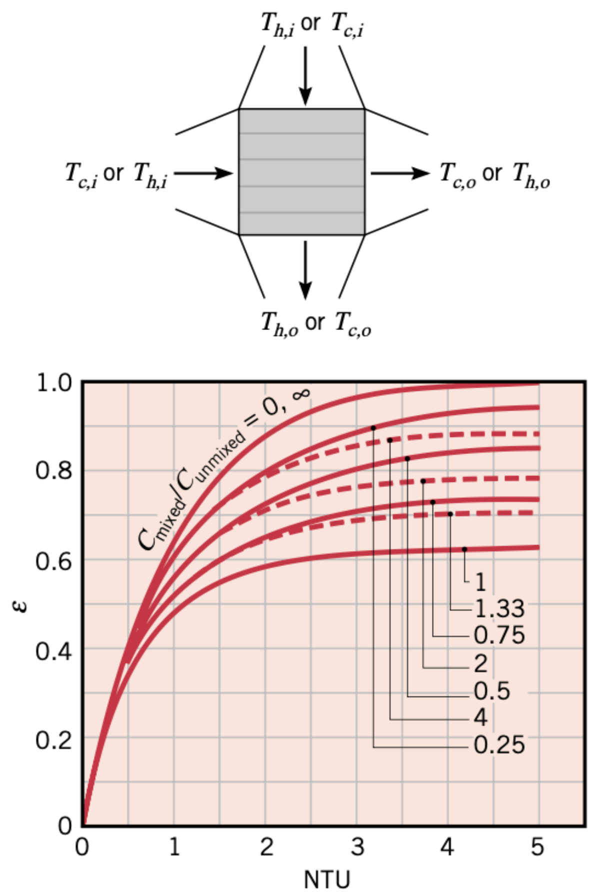


FIGURE 11.15 Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed (Equations 11.33, 11.34).

Example 11.3

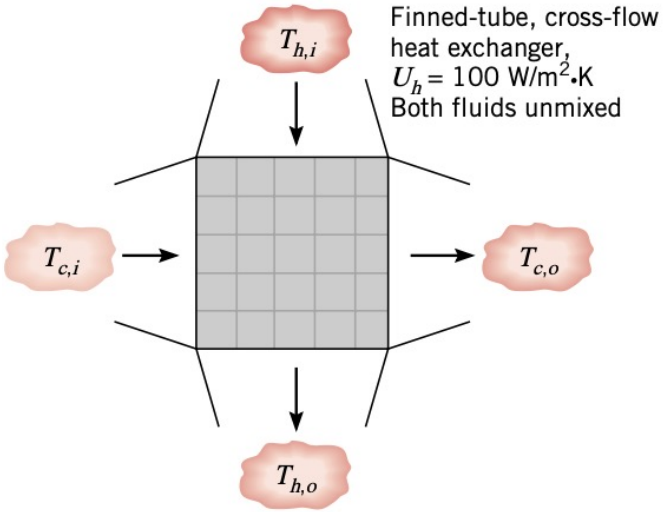
Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at 300°C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125°C. The exhaust gas specific heat is approximately 1000 J/kg · K, and the overall heat transfer coefficient based on the gas-side surface area is $U_h = 100$ W/m^2 · K. Determine the required gas-side surface area A_h using the NTU method.

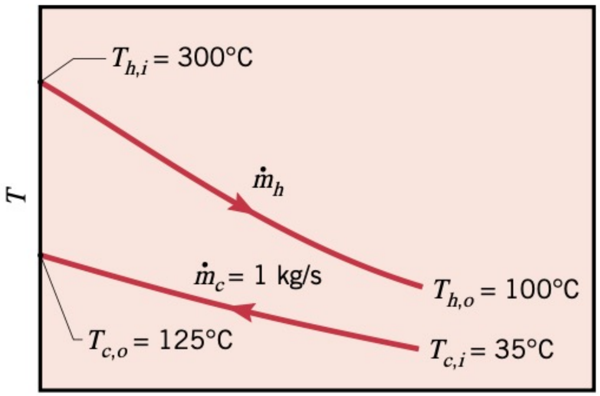
Solution

Known: Inlet and outlet temperatures of hot gases and water used in a finned-tube, cross-flow heat exchanger. Water flow rate and gas-side overall heat transfer coefficient.

Find: Required gas-side surface area.

Schematic:





Assumptions:

1. Negligible heat loss to the surroundings and kinetic and potential energy changes.
2. Constant properties.

Properties: Table A.6, water ($\overline{T}_c = 80°C$): $c_{p,c} = 4197$ J/kg · K. Exhaust gases: $c_{p,h} = 1000$ J/kg · K.

Analysis: The required surface area may be obtained from knowledge of the number of transfer units, which, in turn, may be obtained from knowledge of the ratio of heat capacity rates and the effectiveness. To determine the minimum heat capacity rate, we begin by computing

$$C_c = \dot{m}_c c_{p,c} = 1 \text{ kg/s} \times 4197 \text{ J/kg} \cdot \text{K} = 4197 \text{ W/K}$$

Since $\dot{m}_h$ is not specified, C_h is obtained by combining the overall energy balances, Equations 11.6b and 11.7b:

$$C_h = \dot{m}_h c_{p,h} = C_c \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}} = 4197 \frac{125 - 35}{300 - 100} = 1889 \text{ W/K} = C_{\min}$$

From Equation 11.18

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) = 1889 \text{ W/K} (300 - 35)^\circ\text{C} = 5.00 \times 10^5 \text{ W}$$

From Equation 11.7b the actual heat transfer rate is

$$q = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) = 1 \text{ kg/s} \times 4197 \text{ J/kg} \cdot \text{K} (125 - 35)^\circ\text{C}$$

$$q = 3.78 \times 10^5 \text{ W}$$

Hence from Equation 11.19 the effectiveness is

$$\varepsilon = \frac{q}{q_{\max}} = \frac{3.78 \times 10^5 \text{ W}}{5.00 \times 10^5 \text{ W}} = 0.75$$

With

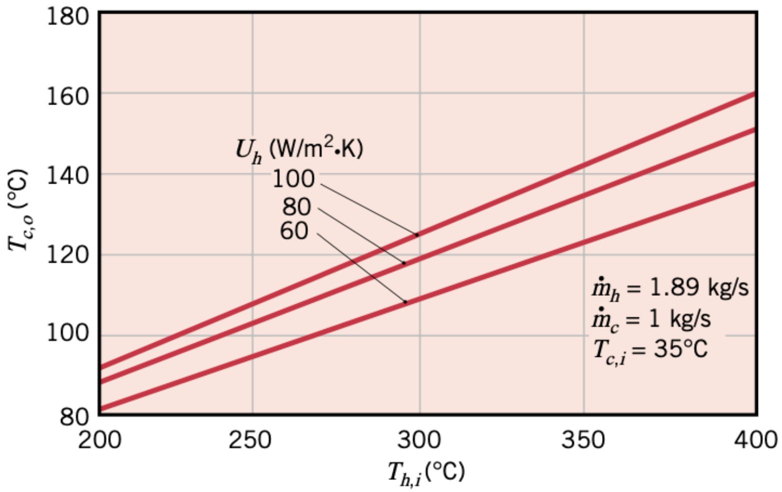
$$\frac{C_{\min}}{C_{\max}} = \frac{1889}{4197} = 0.45$$

it follows from Figure 11.14 that

$$\text{NTU} = \frac{U_h A_h}{C_{\min}} \approx 2.1$$

$$A_h = \frac{2.1(1889\ \text{W/K})}{100\ \text{W/m}^2 \cdot \text{K}} = 39.7\ \text{m}^2 \qquad \triangleleft$$

Comments: With the heat exchanger sized ($A_h = 39.7\ \text{m}^2$) and placed into operation, its actual performance is subject to uncontrolled variations in the exhaust gas inlet temperature ($200 \leq T_{h,i} \leq 400°\text{C}$) and to gradual degradation of the heat exchanger surfaces due to fouling (U_h decreasing from 100 to 60 $\text{W/m}^2 \cdot \text{K}$). For a fixed value of $C_{\min} = C_h = 1889$ W/K, the reduction in U_h corresponds to a reduction in the NTU (to NTU $\approx$ 1.26) and hence to a reduction in the heat exchanger effectiveness, which can be computed from Equation 11.32. The effect of the variations on the water outlet temperature has been computed and is plotted as follows:



If the intent is to maintain a fixed water outlet temperature of $T_{c,o} = 125°\text{C}$, adjustments in the flow rates, $\dot{m}_c$ and $\dot{m}_h$, could be made to compensate for the variations. The model equations could be used to determine the adjustments and hence as a basis for designing the requisite *controller*.

Heat Exchanger Design and Performance Calculations: Using the Effectiveness– NTU Method

Example 11.4

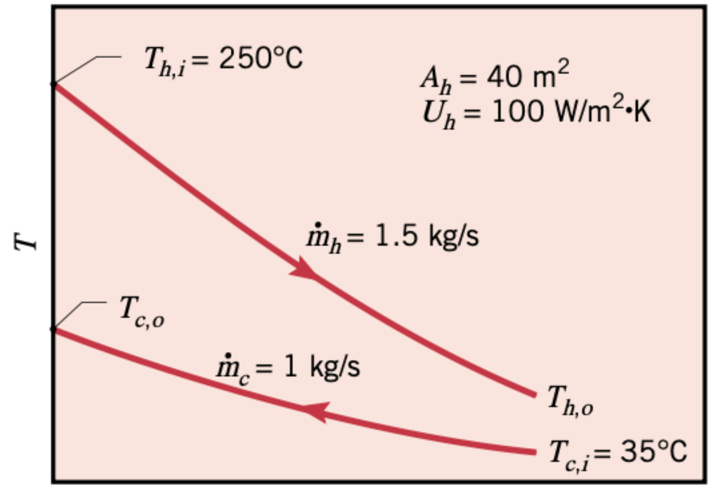
Consider the heat exchanger design of Example 11.3, that is, a finned-tube, cross-flow heat exchanger with a gas-side overall heat transfer coefficient and area of 100 $W/m^2 \cdot K$ and 40 m^2, respectively. The water flow rate and inlet temperature remain at 1 kg/s and 35°C. However, a change in operating conditions for the hot gas generator causes the gases to now enter the heat exchanger with a flow rate of 1.5 kg/s and a temperature of 250°C. What is the rate of heat transfer by the exchanger, and what are the gas and water outlet temperatures?

Solution

Known: Hot and cold fluid inlet conditions for a finned-tube, cross-flow heat exchanger of known surface area and overall heat transfer coefficient.

Find: Heat transfer rate and fluid outlet temperatures.

Schematic:



Assumptions:

1. Negligible heat loss to surroundings and kinetic and potential energy changes.
2. Constant properties (unchanged from Example 11.3).

Analysis: The problem may be classified as one requiring a heat exchanger *performance calculation.* The heat capacity rates are

$$C_c = \dot{m}_c c_{p,c} = 1\ \text{kg/s} \times 4197\ \text{J/kg} \cdot \text{K} = 4197\ \text{W/K}$$

$$C_h = \dot{m}_h c_{p,h} = 1.5\ \text{kg/s} \times 1000\ \text{J/kg} \cdot \text{K} = 1500\ \text{W/K} = C_{\min}$$

in which case

$$\frac{C_{\min}}{C_{\max}} = \frac{1500}{4197} = 0.357$$

The number of transfer units is

$$\text{NTU} = \frac{U_h A_h}{C_{\min}} = \frac{100\ \text{W/m}^2 \cdot \text{K} \times 40\ \text{m}^2}{1500\ \text{W/K}} = 2.67$$

From Figure 11.14 the heat exchanger effectiveness is then $\varepsilon \approx 0.82$, and from Equation 11.18 the maximum possible heat transfer rate is

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i}) = 1500\ \text{W/K}\ (250 - 35)°\text{C} = 3.23 \times 10^5\ \text{W}$$

Accordingly, from the definition of ε, Equation 11.19, the actual heat transfer rate is

$$q = \varepsilon q_{\max} = 0.82 \times 3.23 \times 10^5\ \text{W} = 2.65 \times 10^5\ \text{W} \qquad \triangleleft$$

It is now a simple matter to determine the outlet temperatures from the overall energy balances. From Equation 11.6b

$$T_{h,o} = T_{h,i} - \frac{q}{\dot{m}_h c_{p,h}} = 250°\text{C} - \frac{2.65 \times 10^5\ \text{W}}{1500\ \text{W/K}} = 73.3°\text{C} \qquad \triangleleft$$

and from Equation 11.7b

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 35°\text{C} + \frac{2.65 \times 10^5\ \text{W}}{4197\ \text{W/K}} = 98.1°\text{C} \qquad \triangleleft$$

Comments:

1. From Equation 11.32, $\varepsilon = 0.845$, which is in good agreement with the estimate obtained from the charts.

2. The overall heat transfer coefficient has tacitly been assumed to be unaffected by the change in $\dot{m}_h$. In fact, with an approximately 20% reduction in $\dot{m}_h$, there would be a significant, albeit smaller percentage, reduction in U_h.

3. As discussed in the Comment of Example 11.3, flow rate adjustments could be made to maintain a fixed water outlet temperature. If, for example, the outlet temperature must be maintained at $T_{c,o} = 125°\text{C}$, the water flow rate could be reduced to an amount prescribed by Equation 11.7b. That is,

$$\dot{m}_c = \frac{q}{c_{p,c}(T_{c,o} - T_{c,i})} = \frac{2.65 \times 10^5\ \text{W}}{4197\ \text{J/kg} \cdot \text{K}\ (125 - 35)°\text{C}} = 0.702\ \text{kg/s}$$

The change in flow rate has again been presumed to have a negligible effect on U_h. In this case the assumption is good, since the dominant contribution to U_h is made by the gas-side, and not the water-side, convection coefficient.

EXAMPLE 11.5

The condenser of a large steam power plant is a heat exchanger in which steam is condensed to liquid water. Assume the condenser to be a *shell-and-tube* heat exchanger consisting of a single shell and 30,000 tubes, each executing two passes. The tubes are of thin wall construction with $D = 25$ mm, and steam condenses on their outer surface with an associated convection coefficient of $h_o = 11{,}000\ \mathrm{W/m^2 \cdot K}$. The heat transfer rate that must be effected by the exchanger is $q = 2 \times 10^9$ W, and

this is accomplished by passing cooling water through the tubes at a rate of 3×10^4 kg/s (the flow rate per tube is therefore 1 kg/s). The water enters at 20°C, while the steam condenses at 50°C. What is the temperature of the cooling water emerging from the condenser? What is the required tube length L per pass?

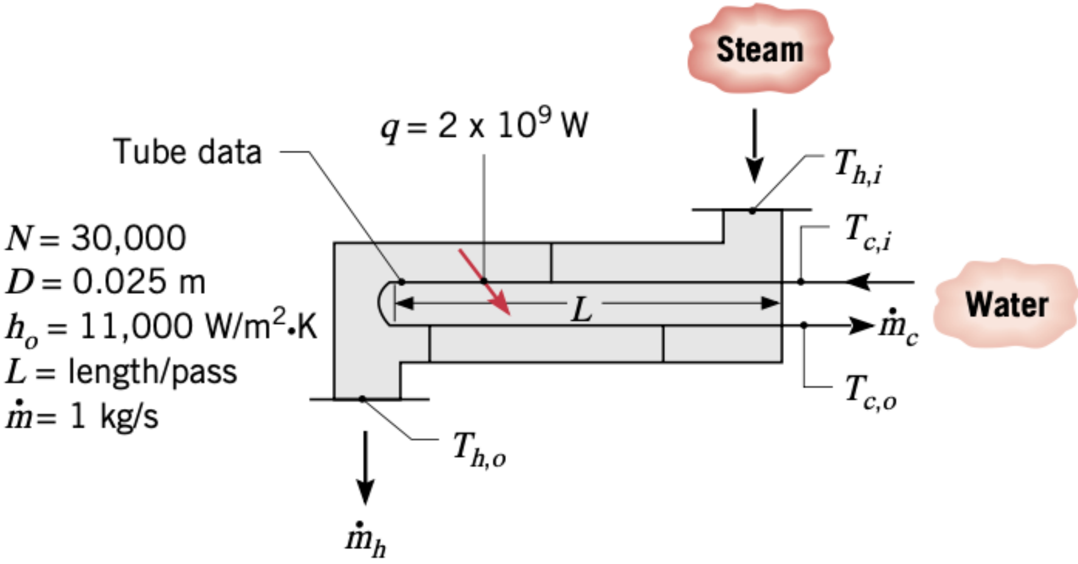
SOLUTION

Known: Heat exchanger consisting of single shell and 30,000 tubes with two passes each.

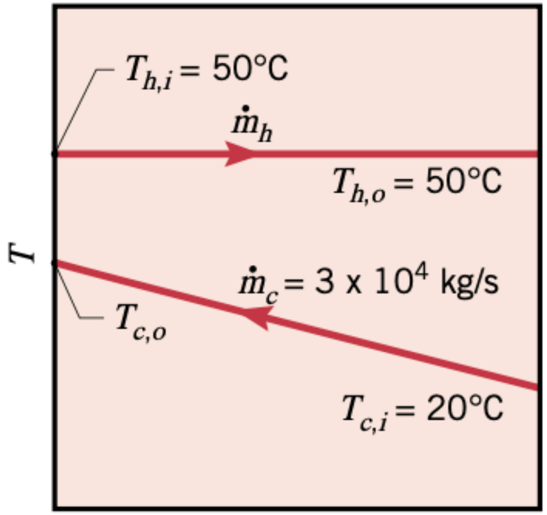
Find:

1. Outlet temperature of the cooling water.
2. Tube length per pass to achieve required heat transfer.

Schematic:



Tube data
$N = 30,000$
$D = 0.025$ m
$h_o = 11,000$ W/m²•K
L = length/pass
$\dot{m} = 1$ kg/s
$q = 2 \times 10^9$ W
Steam
$T_{h,i}$
$T_{c,i}$
L
$\dot{m}_c$
Water
$T_{c,o}$
$T_{h,o}$
$\dot{m}_h$



$T_{h,i} = 50°C$
$\dot{m}_h$
$T_{h,o} = 50°C$
T
$\dot{m}_c = 3 \times 10^4$ kg/s
$T_{c,o}$
$T_{c,i} = 20°C$

Assumptions:

1. Negligible heat transfer between exchanger and surroundings and negligible kinetic and potential energy changes.
2. Tube internal flow and thermal conditions fully developed.
3. Negligible thermal resistance of tube material and fouling effects.
4. Constant properties.

Properties: Table A.6, water (assume $\overline{T}_c \approx 27°\text{C} = 300\text{ K}$): $\rho = 997\text{ kg/m}^3$, $c_p = 4179\text{ J/kg}\cdot\text{K}$, $\mu = 855 \times 10^{-6}\text{ N}\cdot\text{s/m}^2$, $k = 0.613\text{ W/m}\cdot\text{K}$, $Pr = 5.83$.

Analysis:

1. The cooling water outlet temperature may be obtained from the overall energy balance, Equation 11.7b. Accordingly,

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 20°\text{C} + \frac{2 \times 10^9\text{ W}}{3 \times 10^4\text{ kg/s} \times 4179\text{ J/kg}\cdot\text{K}}$$

$$T_{c,o} = 36.0°\text{C} \quad \triangleleft$$

2. The problem may be classified as one requiring a *heat exchanger design calculation.* First, we determine the overall heat transfer coefficient for use in the NTU method.

   From Equation 11.5

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

where h_i may be estimated from an internal flow correlation. With

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 1 \text{ kg/s}}{\pi(0.025 \text{ m})855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 59{,}567$$

the flow is turbulent and from Equation 8.60

$$Nu_D = 0.023\, Re_D^{4/5} Pr^{0.4} = 0.023(59{,}567)^{0.8}(5.83)^{0.4} = 308$$

Hence

$$h_i = Nu_D \frac{k}{D} = 308 \frac{0.613 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} = 7543 \text{ W/m}^2 \cdot \text{K}$$

$$U = \frac{1}{[(1/7543) + (1/11{,}000)] \text{ m}^2 \cdot \text{K/W}} = 4474 \text{ W/m}^2 \cdot \text{K}$$

Using the design calculation methodology, we note that

$$C_h = C_{\max} = \infty$$

and

$$C_{\min} = \dot{m}_c c_{p,c} = 3 \times 10^4 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 1.25 \times 10^8 \text{ W/K}$$

from which

$$\frac{C_{\min}}{C_{\max}} = C_r = 0$$

The maximum possible heat transfer rate is

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1.25 \times 10^8 \text{ W/K} \times (50 - 20) \text{ K} = 3.76 \times 10^9 \text{ W}$$

from which

$$\varepsilon = \frac{q}{q_{max}} = \frac{2 \times 10^9 \text{ W}}{3.76 \times 10^9 \text{ W}} = 0.532$$

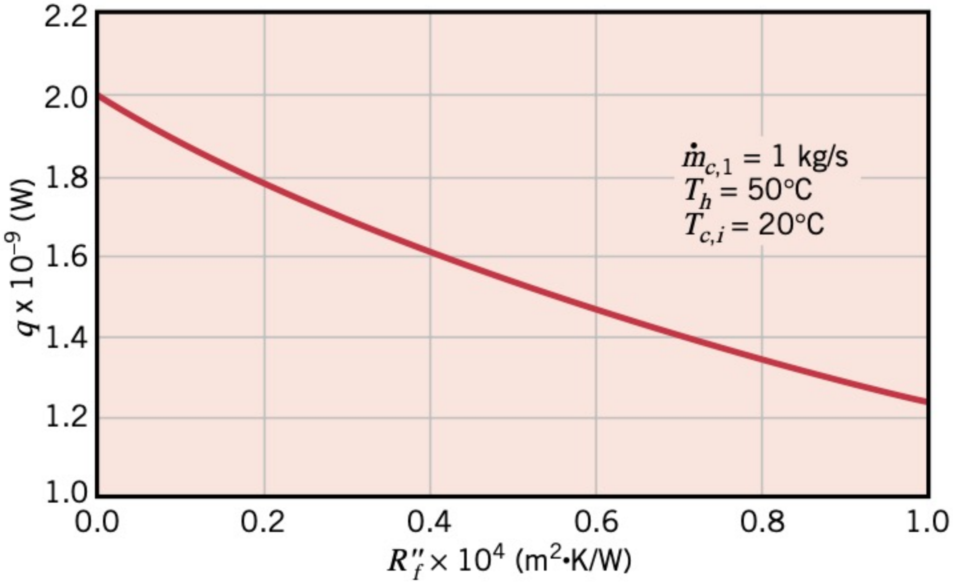
From Equation 11.35b or Figure 11.12, we find NTU = 0.759. From Equation 11.24, it follows that the tube length per pass is

$$L = \frac{\text{NTU} \cdot C_{min}}{U(N2\pi D)} = \frac{0.759 \times 1.25 \times 10^8 \text{ W/K}}{4474 \text{ W/m}^2 \cdot \text{K} (30{,}000 \times 2 \times \pi \times 0.025 \text{ m})} = 4.51 \text{ m} \quad \triangleleft$$

Comments:

1. Recognize that L is the tube length per pass, in which case the total length per tube is 9.0 m. The entire length of tubing in the condenser is $N \times L \times 2 = 30{,}000 \times 4.51 \text{ m} \times 2 = 271{,}000 \text{ m}$ or 271 km.

2. Over time, the performance of the heat exchanger would be degraded by fouling on both the inner and outer tube surfaces. A representative maintenance schedule would call for taking the heat exchanger off-line and cleaning the tubes when fouling factors reached values of $R''_{f,i} = R''_{f,o} = 10^{-4} \text{ m}^2 \cdot \text{K/W}$. To determine the effect of fouling on performance, the ε–NTU method may be used to calculate the total heat rate as a function of the fouling factor, with $R''_{f,o}$ assumed to equal $R''_{f,i}$. The following results are obtained:



To maintain the requirement of $q = 2 \times 10^9$ W with the maximum allowable fouling and the restriction of $\dot{m}_{c,1} = 1$ kg/s, the tube length or the number of tubes would have to be increased. Keeping the length per pass at $L = 4.51$ m, $N =$ 48,300 tubes would be needed to transfer 2×10^9 W for $R''_{f,i} = R''_{f,o} = 10^{-4}$ $m^2 \cdot K/W$. The corresponding increase in the total flow rate to $\dot{m}_c = N\dot{m}_{c,1} = 48{,}300$ kg/s would have the beneficial effect of reducing the water outlet temperature to $T_{c,o} = 29.9°C$, thereby ameliorating potentially harmful effects associated with discharge into the environment. The additional tube length associated with increasing the number of tubes to $N = 48{,}300$ is 165 km, which would result in a significant increase in the capital cost of the condenser.

3. The steam plant generates 1250 MW of electricity with a wholesale value of \$0.05 per kW·h. If the plant is shut down for 48 hours to clean the condenser tubes, the loss in revenue for the plant's owner is 48 h $\times$ 1250 $\times$ 10^6 W $\times$ \$0.05/(1 $\times$ 10^3 W·h) = \$3 million.
4. Assuming a smooth surface condition within each tube, the friction factor may be determined from Equation 8.20b, $f = 0.184(59{,}567)^{-0.2} = 0.020$. The pressure drop within one tube of length $L = 9$ m may be determined from Equation 8.22a, where $u_m = 4\dot{m}/(\rho\pi D^2) = (4 \times 1 \text{ kg/s})/(997 \text{ kg/m}^3 \times \pi \times 0.025^2 \text{m}^2) = 2.04$ m/s.

$$\Delta p = f\frac{\rho u_m^2}{2D}L = 0.020\,\frac{997 \text{ kg/m}^3(2.04 \text{ m/s})^2}{2(0.025 \text{ m})}\,9.0 \text{ m} = 15{,}300 \text{ N/m}^2$$

Therefore, the power required to pump the cooling water through the 48,300 tubes may be found by using Equation 8.22b and is

$$P = \frac{\Delta p \dot{m}}{\rho} = \frac{15{,}300 \text{ N/m}^2 \times 48{,}300 \text{ kg/s}}{997 \text{ kg/m}^3} = 742{,}000 \text{ W} = 0.742 \text{ MW}$$

The cooling water pump is driven by an electric motor. If the combined efficiency of the pump and motor is 87%, the annual cost to overcome friction losses in the condenser tubes is 24 h/day $\times$ 365 days/yr $\times$ 0.742 $\times$ 10^6 W $\times$ \$0.05/1 $\times$ 10^3 W·h/0.87 = \$374,000.
5. Optimal condenser designs are based on the desired thermal performance and environmental considerations as well as on the capital cost, operating cost, and maintenance cost associated with the device.