<u>Suggested Problems from Chapter 3 and 5</u>

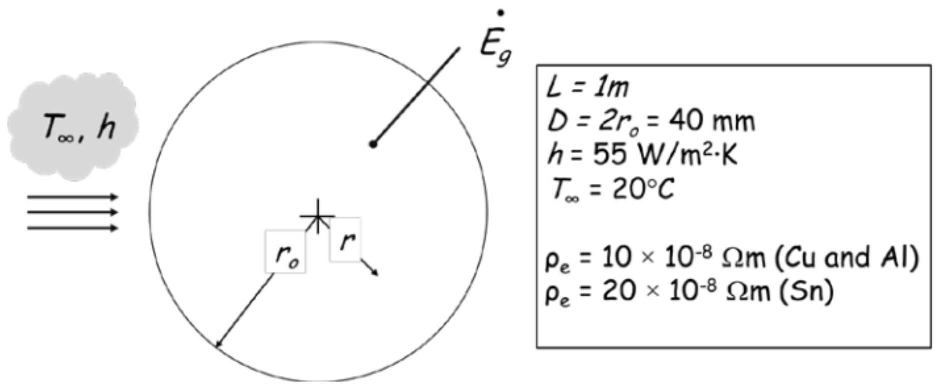
3.63 An uncoated, solid cable of length $L = 1$ m and diameter $D = 40$ mm is exposed to convection conditions characterized by $h = 55$ W/m2 · K and $T_\infty = 20°C$. Determine the maximum electric current that can be carried by the cable if it is pure copper, pure aluminum, or pure tin. Calculate the corresponding minimum wire temperatures. The electrical resistivity is $\rho_e = 10 \times 10^{-8}\ \Omega \cdot m$ for copper and aluminum at their melting points, while the electrical resistivity of tin is $\rho_e = 20 \times 10^{-8}\ \Omega \cdot m$ at its melting point.

PROBLEM 3.63

KNOWN: Dimensions and resistivity of current-carrying cable. Temperature and heat transfer coefficient of environment.

FIND: Maximum operating current and corresponding minimum cable temperature for copper, aluminum, and tin cables.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Given,* Cu: $\rho_e = 10 \times 10^{-8}$ Ω·m, Al: $\rho_e = 10 \times 10^{-8}$ Ω·m, Sn: $\rho_e = 20 \times 10^{-8}$ Ω·m. *Table A.1*, Cu: T_m = 1358 K, k = 339 W/m·K; Al: T_m = 933 K, k = 218 W/m·K; Sn: T_m = 505 K, k = 62.2 W/m·K. Thermal conductivities have been evaluated at highest available temperature.

ANALYSIS: The maximum temperature should not exceed the melting temperature of the material. The maximum temperature occurs at the centerline and can be found from Equation 3.58 evaluated at r = 0. The surface temperature can be related to the known environment temperature through Equation 3.60. Thus,

$$T_{max} = T(r=0) = \frac{\dot{q}r_o^2}{4k} + \frac{\dot{q}r_o}{2h} + T_\infty \qquad (1)$$

Or solving for $\dot{q}$,

$$\dot{q}_{max} = \frac{T_{max} - T_\infty}{r_o^2 / 4k + r_o / 2h} \qquad (2)$$

The heat generation rate depends on the current and resistivity according to:

$$\dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2 (\rho_e L / A_c)}{LA_c} = \frac{I^2 \rho_e}{A_c^2} = \frac{I^2 \rho_e}{\left(\pi r_o^2\right)^2} \qquad (3)$$

Thus the maximum allowable current is given by:

PROBLEM 3.63 (Cont.)

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2} \pi r_o^2 = \left(\frac{T_{max} - T_\infty}{\rho_e\left(r_o^2 / 4k + r_o / 2h\right)}\right)^{1/2} \pi r_o^2 \qquad (4)$$

The minimum temperature occurs at the surface of the cable, so from Equation 3.60,

$$T_{min} = \frac{\dot{q} r_o}{2h} + T_\infty = \frac{I_{max}^2 \rho_e r_o}{2h\left(\pi r_o^2\right)^2} + T_\infty \qquad (5)$$

Evaluating Eqs.(4) and (5) for the properties of copper yields,

$$I_{max} = \left(\frac{(1358 - 293)\,\text{K}}{10\times10^{-8}\,\Omega\cdot\text{m}\left(\frac{(0.02\,\text{m})^2}{4\times339\ \text{W/m}\cdot\text{K}} + \frac{0.02\,\text{m}}{2\times55\ \text{W/m}^2\cdot\text{K}}\right)}\right)^{1/2} \pi\times(0.02\,\text{m})^2 = 9610\,\text{A}$$ <

$$T_{min} = \frac{(9610\ \text{A})^2 \times 10\times10^{-8}\,\Omega\cdot\text{m}\times0.02\ \text{m}}{2\times55\ \text{W/m}^2\cdot\text{K}\times\left(\pi(0.02\ \text{m})^2\right)^2} + 293\ \text{K} = 1356\ \text{K}$$ <

Similarly for aluminum,

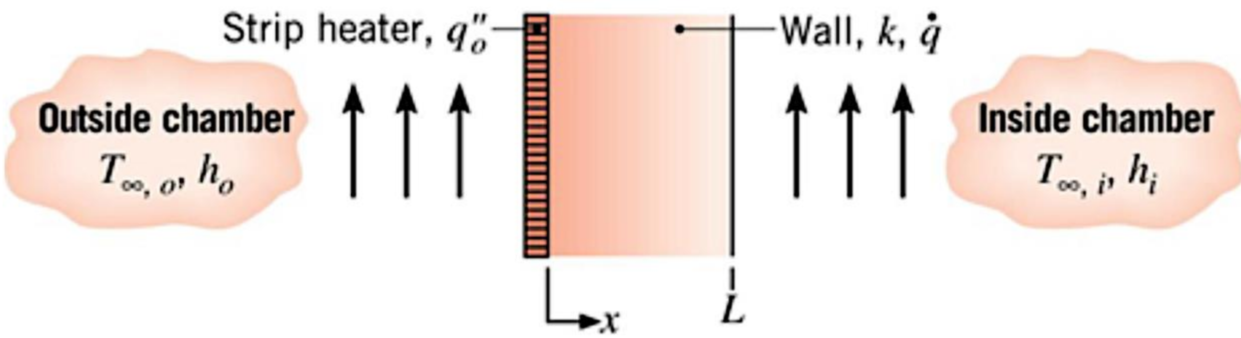
$I_{max} = 7450\,\text{A}, \quad T_{min} = 931\ \text{K}$ <

And for tin,

$I_{max} = 3020\,\text{A}, \quad T_{min} = 503\ \text{K}$ <

COMMENTS: (1) The minimum (surface) temperature is almost equal to the maximum (melting) temperature; the cable is nearly isothermal. (2) The neglect of radiation is a poor assumption for such large temperatures. Assuming the surroundings are also at 293 K and the emissivity is one, the ratio of radiation to convection heat transfer is given by $q_{rad} / q_{conv} = \sigma(T_{min}^4 - T_\infty^4) / [h(T_{min} - T_\infty)] =$ 3.3, 1.2, 0.28 for Cu, Al, and Sn, respectively. If radiation were included in the analysis, it would enhance heat transfer and increase the current-carrying capacity.

3.64 The air inside a chamber at T∞,i = 50°C is heated convectively with hi = 20 W/m2 · K by a 200-mm-thick wall having a thermal conductivity of 4 W/m · K and a uniform heat generation of 1000 W/m3. To prevent any heat generated within the wall from being lost to the outside of the chamber at T∞,o = 25°C with ho = 5 W/m2 · K, a very thin electrical strip heater is placed on the outer wall to provide a uniform heat flux, q_o''.



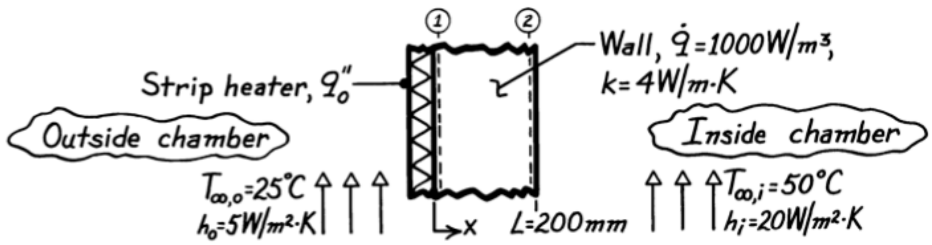
(a) Sketch the temperature distribution in the wall on T − x coordinates for the condition where no heat generated within the wall is lost to the outside of the chamber.
(b) What are the temperatures at the wall boundaries, T(0) and T(L), for the conditions of part (a)?
(c) Determine the value of q_o'' that must be supplied by the strip heater so that all heat generated within the wall is transferred to the inside of the chamber.
(d) If the heat generation in the wall were switched off while the heat flux to the strip heater remained constant, what would be the steady-state temperature, T(0), of the outer wall surface?

PROBLEM 3.64

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

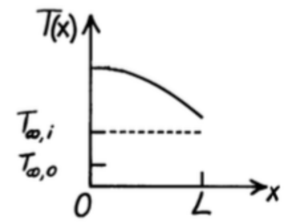
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}$=0 but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \qquad (1)$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \qquad (2)$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \qquad (3)$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \qquad (4)$$

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2\times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$ <

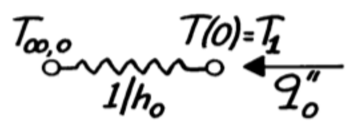
Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

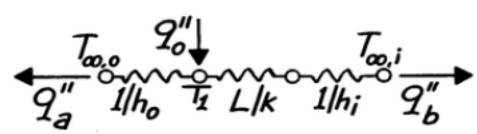
(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

$$T_1 = 55^\circ\text{C}.$$ <

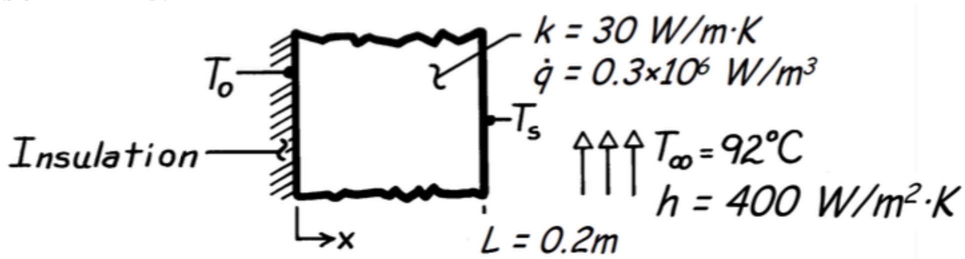
3.65 A plane wall of thickness 0.2 m and thermal conductivity 30 W/m · K having uniform volumetric heat generation of 0.4 MW/m3 is insulated on one side, while the other side is exposed to a fluid at 92°C. The convection heat transfer coefficient between the wall and the fluid is 400 W/m2 · K. Determine the maximum temperature in the wall.

PROBLEM 3.65

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: The temperature at the inner surface is given by Eq. 3.48 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.51,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ C + 0.4\times10^6 \frac{W}{m^3} \times 0.2m/400 W/m^2\cdot K = 92^\circ C + 200^\circ C = 292^\circ C.$$

It follows that

$$T_o = 0.4\times10^6 W/m^3 \times (0.2m)^2/2\times30 W/m\cdot K + 292^\circ C$$

$$T_o = 267^\circ C + 292^\circ C = 559^\circ C.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 400 W/m^2\cdot K(292-92)^\circ C = 80 kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

$$q''_{conv} = \dot{q}L = 0.4\times10^6 W/m^3 \times 0.2m = 80 kW/m^2.$$

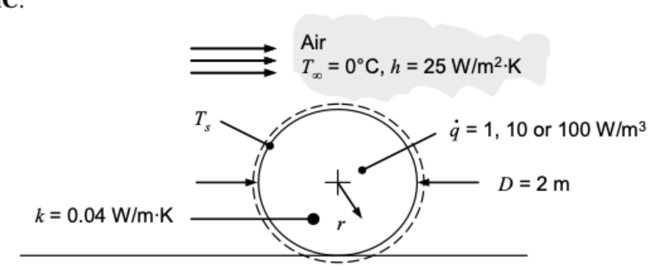
3.66 Large, cylindrical bales of hay used to feed livestock in the winter months are D = 2 m in diameter and are stored end-to-end in long rows. Microbial energy generation occurs in the hay and can be excessive if the farmer bales the hay in a too-wet condition. Assuming the thermal conductivity of baled hay to be k = 0.04 W/m · K, determine the maximum steady-state hay temperature for dry hay $\dot{q} = 1\ W/m^3$, moist hay $\dot{q} = 10\ W/m^3$, and wet hay $\dot{q} = 100\ W/m^3$. Ambient conditions are T∞ = 0°C and h = 25 W/m2 · K.

PROBLEM 3.66

KNOWN: Diameter, thermal conductivity and microbial energy generation rate in cylindrical hay bales. Ambient conditions.

FIND: The maximum hay temperature for $\dot{q}$ = 1, 10, and 100 W/m³.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional heat transfer (4) Uniform volumetric generation, (5) Negligible radiation, (6) Negligible conduction to or from the ground.

PROPERTIES: k = 0.04 W/m·K (given).

ANALYSIS: The surface temperature of the dry hay is (Eq. 3.60)

$$T_s = T_\infty + \frac{\dot{q}r_o}{2h} = 0°\text{C} + \frac{1\text{W/m}^3 \times 1\text{m}}{2 \times 25\text{W/m}^2 \cdot \text{K}} = 0.02°\text{C} \qquad <$$

whereas T_s = 0.2°C and 2.0°C for the moist and wet hay, respectively. <

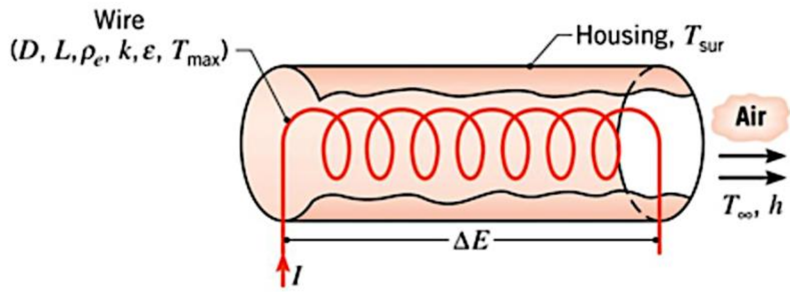
The maximum hay temperature occurs at the centerline, r = 0. From Eq. 3.58, for the dry hay,

$$T_{\max} = \frac{\dot{q}r_o^2}{4k} + T_s = \frac{1\text{W/m}^3 \times (1\text{m})^2}{4 \times 0.04\ \text{W/m} \cdot \text{K}} + 0.02°\text{C} = 6.27°\text{C} \qquad <$$

whereas $T_{\max}$ = 62.7°C and 627°C for the moist and wet hay, respectively. <

COMMENTS: (1) The hay begins to lose its nutritional value at temperatures exceeding 50°C. Therefore the center of the moist hay bale will lose some of its nutritional value. (2) The center of the wet hay bale can experience very high temperatures without combusting due to lack of oxygen internal to the hay bale. However, when the farmer breaks the bale apart for feeding, oxygen is suddenly supplied to the hot hay and combustion may occur. (3) The outer surface of the hay bale differs by only 2°C from the dry to the wet condition, while the centerline temperature differs by over 600 degrees. The farmer cannot anticipate the potential for starting a fire by touching the outer surface of the hay bale. (4) See Opuku, Tabil, Crerar and Shaw, "Thermal Conductivity and Thermal Diffusivity of Timothy Hay," *Canadian Biosystems Engineering*, Vol. 48, pp. 3.1 - 3.6, 2006 for hay property information.

3.69 An air heater may be fabricated by coiling Nichrome wire and passing air in cross flow over the wire. Consider a heater fabricated from wire of diameter D = 2 mm, electrical resistivity $\rho_e = 10^{-6}\ \Omega \cdot m$, thermal conductivity k = 25 W/m · K, and emissivity ε = 0.20. The heater is designed to deliver air at a temperature of T_∞ = 60°C under flow conditions that provide a convection coefficient of h = 250 W/m2 · K for the wire. The temperature of the housing that encloses the wire and through which the air flows is Tsur = 60°C.

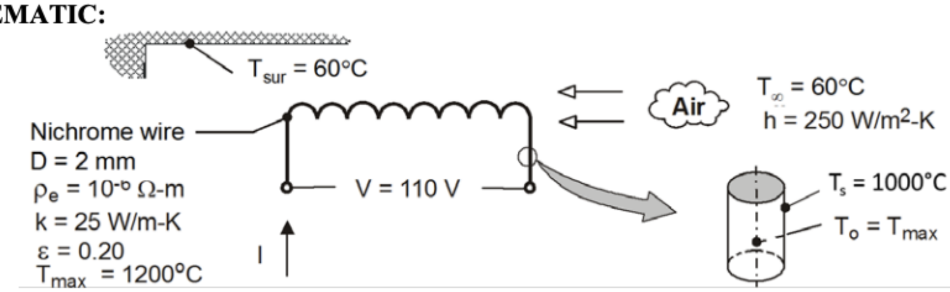


PROBLEM 3.69

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.60), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\,(1273 + 333)\,K\left(1273^2 + 333^2\right)K^2 = 31.5\,W/m^2 \cdot K$$

$$h_t = (250 + 31.5)\,W/m^2 \cdot K = 281.5\,W/m^2 \cdot K$$

$$\dot{q}_{max} = \frac{2h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(281.5\,W/m^2 \cdot K\right)}{0.001m}(940°C) = 5.29 \times 10^8\,W/m^3$$

Hence, with $\dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2\left(\rho_e L/A_c\right)}{LA_c} = \frac{I^2 \rho_e}{A_c^2} = \frac{I^2 \rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2} \frac{\pi D^2}{4} = \left(\frac{5.29 \times 10^8\,W/m^3}{10^{-6}\,\Omega \cdot m}\right)^{1/2} \frac{\pi(0.002\,m)^2}{4} = 72.3\,A \qquad <$$

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E \cdot A_c}{I_{max}\,\rho_e} = \frac{110\,V\left[\pi(0.002m)^2/4\right]}{72.3\,A\left(10^{-6}\,\Omega \cdot m\right)} = 4.78m \qquad <$$

and the power rating is

$$P_{elec} = \Delta E \cdot I_{max} = 110\,V(72.3A) = 7950\,W = 7.95\,kW \qquad <$$

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.58) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1000°C. It follows that $T_o = \frac{\dot{q}\,r_o^2}{4k} + T_s = \frac{5.29 \times 10^8\,W/m^3\,(0.001m)^2}{4(25\,W/m \cdot K)} + 1000°C = 1005°C$. With only a 5°C temperature difference between the centerline and surface of the wire, the assumption is excellent.

3.70 Consider the composite wall of Example 3.7. In the Comments section, temperature distributions in the wall were determined assuming negligible contact resistance between materials A and B. Compute and plot the temperature distributions if the thermal contact resistance is .

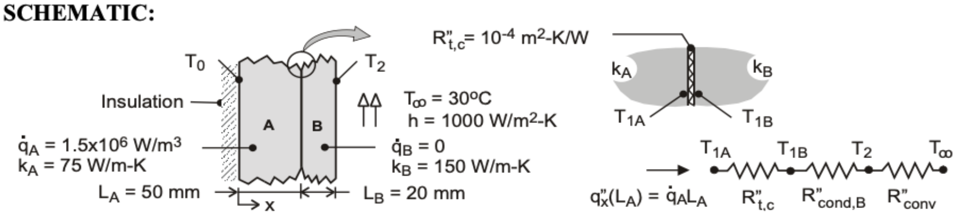
$$R''_{t,c} = 10^{-4}\ \mathrm{m^2 \cdot K/W}.$$

PROBLEM 3.70

KNOWN: Composite wall of materials A and B. Wall of material A has uniform generation, while wall B has no generation. The inner wall of material A is insulated, while the outer surface of material B experiences convection cooling. Thermal contact resistance between the materials is $R''_{t,c} = 10^{-4}\ \mathrm{m^2 \cdot K/W}$. See Example 3.7 that considers the case without contact resistance.

FIND: Compute and plot the temperature distribution in the composite wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties, and (3) Inner surface of material A is adiabatic.

ANALYSIS: From the analysis of Example 3.8, we know the temperature distribution in material A is parabolic with zero slope at the inner boundary, and that the distribution in material B is linear. At the interface between the two materials, $x = L_A$, the temperature distribution will show a discontinuity.

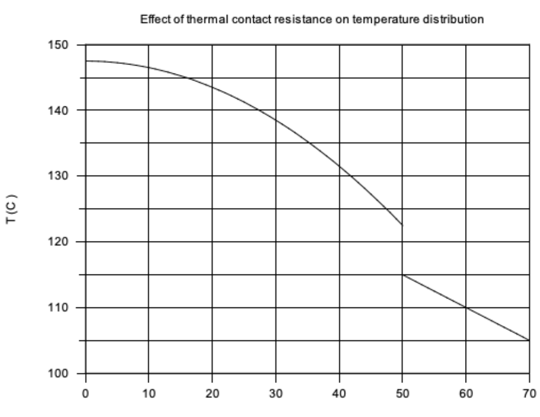
$$T_A(x) = \frac{\dot{q} L_A^2}{2k_A}\left(1 - \frac{x^2}{L_A^2}\right) + T_{1A} \qquad 0 \le x \le L_A$$

$$T_B(x) = T_{1B} - (T_{1B} - T_2)\frac{x - L_A}{L_B} \qquad L_A \le x \le L_A + L_B$$

Considering the thermal circuit above (see also Example 3.8) including the thermal contact resistance,

$$q'' = \dot{q} L_A = \frac{T_{1A} - T_\infty}{R''_{tot}} = \frac{T_{1B} - T_\infty}{R''_{cond,B} + R''_{conv}} = \frac{T_2 - T_\infty}{R''_{conv}}$$

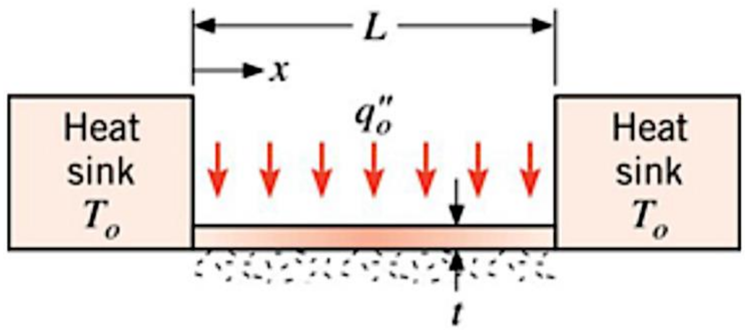
find $T_A(0) = 147.5°C$, $T_{1A} = 122.5°C$, $T_{1B} = 115°C$, and $T_2 = 105°C$. Using the foregoing equations in IHT, the temperature distributions for each of the materials can be calculated and are plotted on the graph below.



COMMENTS: (1) The effect of the thermal contact resistance between the materials is to increase the maximum temperature of the system.

(2) Can you explain why the temperature distribution in the material B is not affected by the presence of the thermal contact resistance at the materials' interface?

3.92 A thin flat plate of length L, thickness t, and width W ≫ L is thermally joined to two large heat sinks that are maintained at a temperature To. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q_o''.



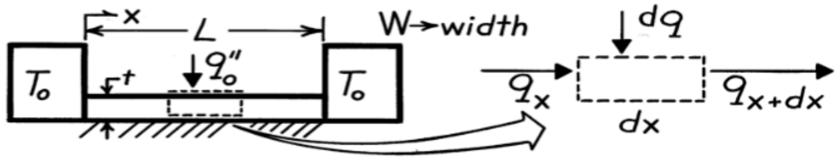
(a) Derive the differential equation that determines the steady-state temperature distribution T(x) in the plate.
(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

PROBLEM 3.92

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\,dx$ and $dq = q_o''\,(W\cdot dx)$. Hence, $(dq_x/dx) - q_o''\,W = 0$. From Fourier's law, $q_x = -k(t\cdot W)\,dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\,\frac{dT}{dx}\right] - q_o''\,W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0. \qquad <$$

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1\,x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o''L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o''}{2kt}\left(x^2 - Lx\right) + T_o. \qquad <$$

Applying Fourier's law at x = 0, and at x = L,

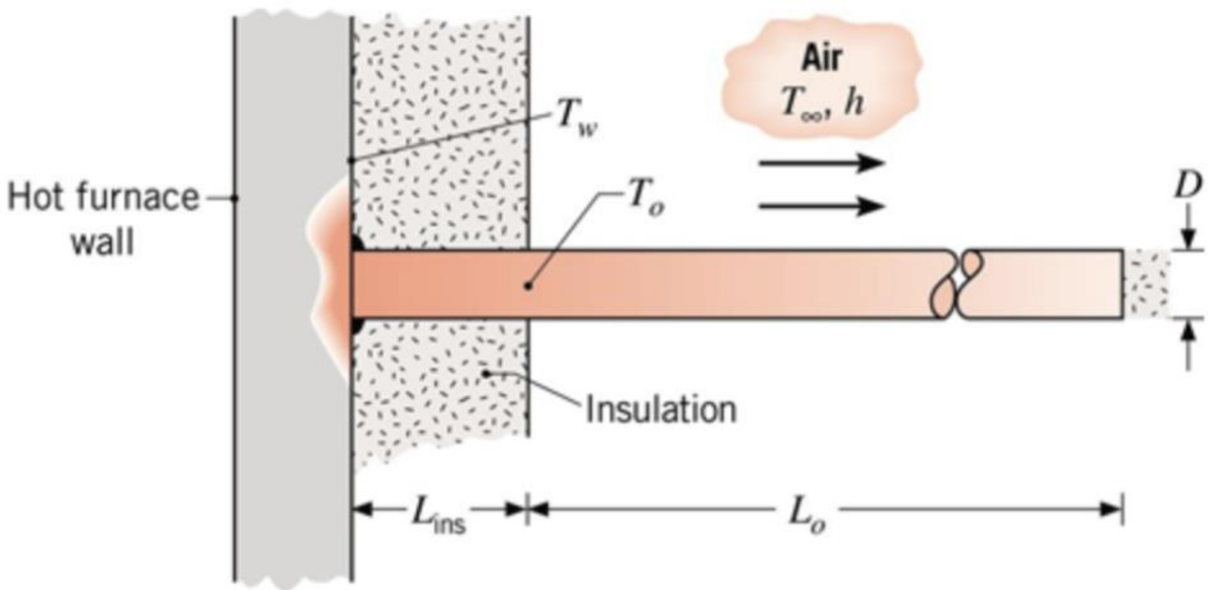
$$q(0) = -k(Wt)\,dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o''WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o''WL}{2}$$

Hence the heat loss from the plates is $q = 2\left(q_o''WL/2\right) = q_o''WL.$ <

COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

3.97 A rod of diameter D = 25 mm and thermal conductivity k = 60 W/m · K protrudes normally from a furnace wall that is at Tw = 200°C and is covered by insulation of thickness Lins = 200 mm. The rod is welded to the furnace wall and is used as a hanger for supporting instrumentation cables. To avoid damaging the cables, the temperature of the rod at its exposed surface, To, must be maintained below a specified operating limit of Tmax = 100°C. The ambient air temperature is T∞ = 25°C, and the convection coefficient is h = 15 W/m2 · K.



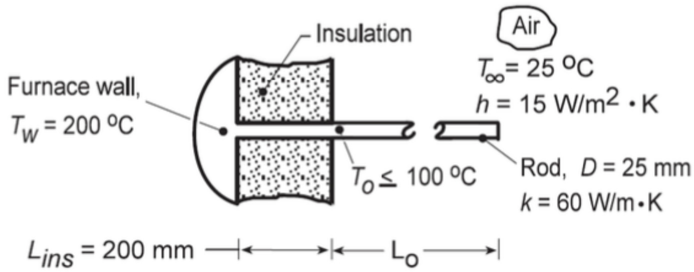
(a) Derive an expression for the exposed surface temperature To as a function of the prescribed thermal and geometrical parameters. The rod has an exposed length Lo, and its tip is well insulated.
(b)Computer Icon Will a rod with Lo = 200 mm meet the specified operating limit? If not, what design parameters would you change? Consider another material, increasing the thickness of the insulation, and increasing the rod length. Also, consider how you might attach the base of the rod to the furnace wall as a means to reduce To.

PROBLEM 3.97

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

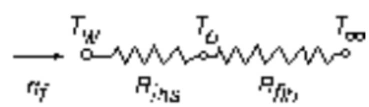
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.81,

$$R_{fin} = \theta_b/q_f = \frac{1}{(hPkA_c)^{1/2}\tanh(mL_o)} \qquad (2)$$

$$m = (hP/kA_c)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_W - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}(T_W - T_\infty) \qquad (6) <$$

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}(200 - 25)°C = 109°C \qquad <$$

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi(0.025\,m)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh(6.324\times0.200) = 6.298\,K/W$$

$$(hPkA_c) = \left(15\,W/m^2\cdot K\times\pi(0.025\,m)\times60\,W/m\cdot K\times4.909\times10^{-4}m^2\right) = 0.0347\,W^2/K^2$$

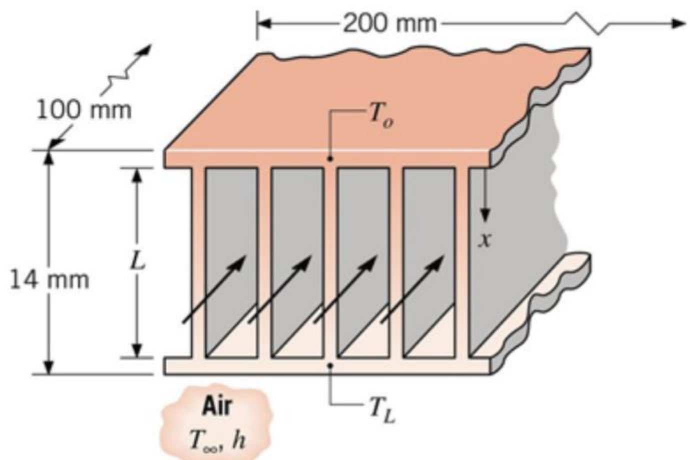
Continued...

$$m = (hP/kA_c)^{1/2} = \left(15\,W/m^2\cdot K \times \pi(0.025\,m)/60\,W/m\cdot K \times 4.909\times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with $L_o = 400$ and 600 mm, T_o is 102.8°C and 102.3°C, respectively. Hence, increasing L_o will reduce T_o only modestly. (2) Decreasing the thermal conductivity: backsolving the above equation set with $T_0 = 100°C$, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for $T_o = 100°C$, the required insulation thickness would be $L_{ins} = 211$ mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit. (6) Using a tube rather than a rod will decrease A_c. For a 3 mm tube wall and 25 mm outside diameter, $A_c = 2.07 \times 10^{-4}$ m^2, $R_{ins} = 16.103$ K/W and $R_{fin} = 8.61$ K/W, yielding $T_o = 86°C$. (conduction within the air inside the tube is neglected).

3.113 Finned passages are frequently formed between parallel plates to enhance convection heat transfer in compact heat exchanger cores. An important application is in electronic equipment cooling, where one or more air-cooled stacks are placed between heat-dissipating electrical components. Consider a single stack of rectangular fins of length L and thickness t, with convection conditions corresponding to h and T∞.

(a) Obtain expressions for the fin heat transfer rates, qf,o and qf,L, in terms of the base temperatures, To and TL.
(b) In a specific application, a stack that is 200 mm wide and 100 mm deep contains 50 fins, each of length L = 12 mm. The entire stack is made from aluminum, which is everywhere 1.0 mm thick. If temperature limitations associated with electrical components joined to opposite plates dictate maximum allowable plate temperatures of To = 400 K and TL = 350 K, what are the corresponding maximum power dissipations if h = 150 W/m2 · K and T∞ = 300 K?

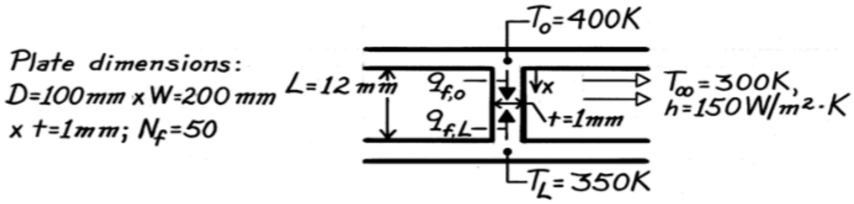


PROBLEM 3.113

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Uniform h, (6) Negligible variation in T_∞, (7) Negligible contact resistance.

PROPERTIES: *Table A.1,* Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in a fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty, \qquad \theta(L) = \theta_L = T_L - T_\infty.$

Hence $\theta_o = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence
$$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \left[e^{m(L-x)} - e^{-m(L-x)} \right] + \theta_L \left(e^{mx} - e^{-mx} \right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c \frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL} \cosh m(L-x) + \frac{\theta_L m}{\sinh mL} \cosh mx \right].$$

Hence
$$q_{f,o} = kDt\left(\frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL} \right)$$ <

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL} \right).$$ <

Continued …

(b) $$m=\left(\frac{hP}{kA_c}\right)^{1/2}=\left(\frac{150\ \text{W/m}^2\cdot\text{K}\left(2\times0.1\ \text{m}+2\times0.001\ \text{m}\right)}{240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}}\right)^{1/2}=35.5\ \text{m}^{-1}$$

$$mL=35.5\ \text{m}^{-1}\times0.012\ \text{m}=0.43$$

$\sinh\ mL=0.439$ $\tanh mL=0.401$ $\theta_o=100\ \text{K}$ $\theta_L=50\ \text{K}$

$$q_{f,o}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}\right)$$

$q_{f,o}=115.4\ \text{W}$ (*from* the top plate)

$$q_{f,L}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}\right)$$

$q_{f,L}=87.8\ \text{W.}$ (*into* the bottom plate)

Maximum power dissipations are therefore

$$q_{o,max}=N_f q_{f,o}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{o,max}=50\times115.4\ \text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times100\ \text{K}$$

$$q_{o,max}=5770\ \text{W}+225\ \text{W}=5995\ \text{W}$$ <

$$q_{L,max}=-N_f q_{f,L}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{L,max}=-50\times87.8\text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times50\ \text{K}$$

$$q_{L,max}=-4390\ \text{W}+112\text{W}=-4278\ \text{W.}$$ <

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air}=\frac{q_{tot}}{c_p\Delta T_\infty}=\frac{1717\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\times5\ \text{K}}=0.34\ \text{kg/s.}$$

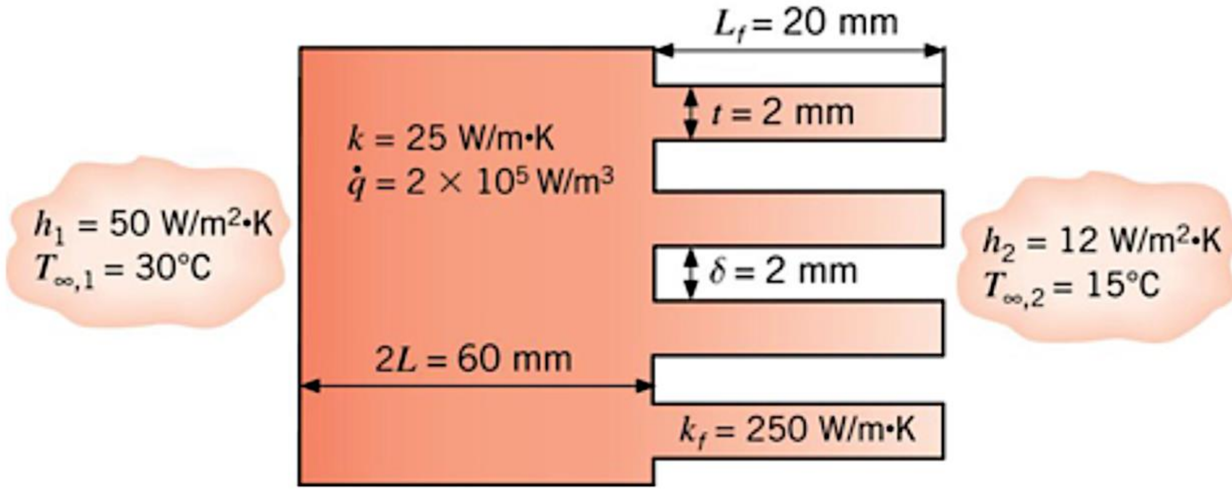
Its mean velocity is then

$$V_{air}=\frac{\dot{m}_{air}}{\rho_{air}A_c}=\frac{0.34\ \text{kg/s}}{1.16\ \text{kg/m}^3\times0.012\ \text{m}\left(0.2-50\times0.001\right)\text{m}}=163\ \text{m/s.}$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

(2) A negative value of $q_{L,max}$ implies that the bottom plate must be cooled externally to maintain the plate at 350 K.

3.122 Heat is uniformly generated at the rate of 2 × 105 W/m3 in a wall of thermal conductivity 25 W/m · K and thickness 60 mm. The wall is exposed to convection on both sides, with different heat transfer coefficients and temperatures as shown. There are straight rectangular fins on the right-hand side of the wall, with dimensions as shown and thermal conductivity of 250 W/m · K. What is the maximum temperature that will occur in the wall?

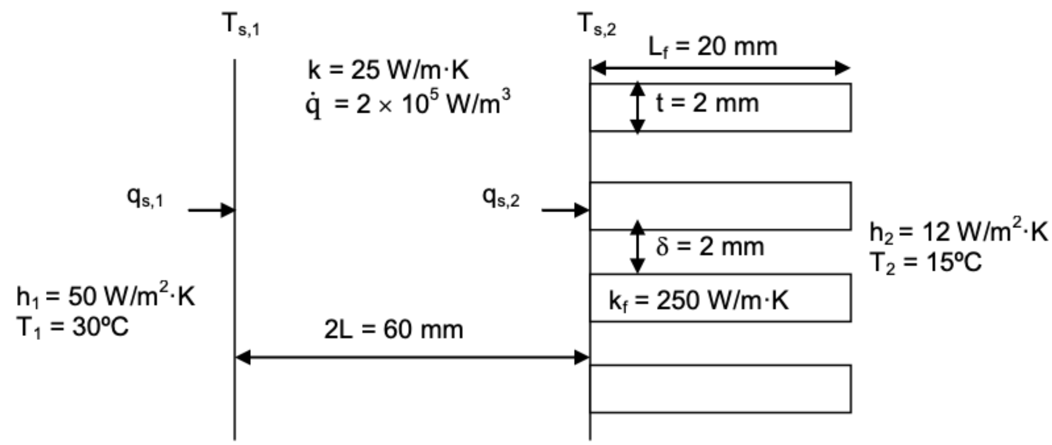


PROBLEM 3.122

KNOWN: Wall with known heat generation rate, thermal conductivity, and thickness. Dimensions and thermal conductivity of fins. Heat transfer coefficients and environment temperatures.

FIND: Maximum temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Wall surface temperatures are uniform. (3) No contact resistance between fins and wall, (4) Heat transfer from the fin tips can be neglected.

ANALYSIS: The temperature distribution in a wall with uniform volumetric heat generation and specified temperature boundary conditions is, from Equation 3.46

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \qquad (1)$$

The heat transfer rates at the two surfaces, for a wall section of area A, can be found from Fourier's law:

$$q_{s,1} = -kA\left.\frac{dT}{dx}\right|_{x=-L} = -\dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \qquad (2)$$

$$q_{s,2} = -kA\left.\frac{dT}{dx}\right|_{x=L} = \dot{q}LA - kA\frac{T_{s,2} - T_{s,1}}{2L} \qquad (3)$$

We can express these same heat transfer rates alternatively, as follows:

$$q_{s,1} = h_1 A(T_1 - T_{s,1}) \qquad (4)$$

$$q_{s,2} = h_2 A_t (T_{s,2} - T_2)\eta_o \qquad (5)$$

where η_o is given by Equation 3.107. Equating the two expressions for $q_{s,1}$, Equations (2) and (4), and equating the expressions for $q_{s,2}$, Equations (3) and (5), and solving for $T_{s,1}$ and $T_{s,2}$ yields

Continued…

PROBLEM 3.122 (Cont.)

$$T_{s,1} = \frac{\left(\frac{k}{2L} + h_2\tilde{A}\right)h_1T_1 + \frac{k}{2L}h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_2\tilde{A}\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

$$T_{s,2} = \frac{\frac{k}{2L}h_1T_1 + \left(\frac{k}{2L} + h_1\right)h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_1\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

where

$$\tilde{A} = \frac{A_t\eta_o}{A} = \frac{A_t}{A} - \frac{NA_f}{A}(1 - \eta_f)$$

Performing the calculations:

$$m = \sqrt{\frac{h_2P}{k_fA_c}} = \sqrt{\frac{2h_2}{k_ft}} = \sqrt{\frac{2 \times 12\ \text{W/m}^2\cdot\text{K}}{250\ \text{W/m}\cdot\text{K} \times 0.002\ \text{m}}} = 6.9\ \text{m}^{-1}$$

$$\eta_f = \frac{\tanh(mL_f)}{mL_f} = \frac{\tanh\left(6.9\ \text{m}^{-1} \times 0.02\ \text{m}\right)}{6.9\ \text{m}^{-1}\times 0.02\ \text{m}} = 0.994$$

$$\frac{NA_f}{A} = \frac{N2wL_f}{(\delta+t)Nw} = \frac{2L_f}{\delta+t} = \frac{2 \times 0.02\ \text{m}}{0.004\ \text{m}} = 10.0$$

$$\frac{A_t}{A} = \frac{NA_f}{A} + \frac{A_b}{A} = \frac{NA_f}{A} + \frac{\delta Nw}{(\delta+t)Nw} = \frac{NA_f}{A} + \frac{\delta}{\delta+t} = 10. + \frac{0.002\ \text{m}}{0.004\ \text{m}} = 10.5$$

$$\tilde{A} = 10.5 - 10.(1 - 0.994) = 10.4$$

$$h_2\tilde{A} = 12\ \text{W/m}^2\cdot\text{K} \times 10.4 = 125\ \text{W/m}^2\cdot\text{K}$$

$$\frac{k}{2L} = \frac{25\ \text{W/m}\cdot\text{K}}{0.06\ \text{m}} = 417\ \text{W/m}^2\cdot\text{K}$$

Thus

$$T_{s,1} = \left(\begin{array}{l}(417+125)\ \text{W/m}^2\cdot\text{K}\times 50\ \text{W/m}^2\cdot\text{K}\times 30°\text{C} \\ +\ 417\ \text{W/m}^2\cdot\text{K}\times 125\ \text{W/m}^2\cdot\text{K}\times 15°\text{C} \\ +(2\times 417+125)\ \text{W/m}^2\cdot\text{K}\times 2\times 10^5\ \text{W/m}^3\times 0.03\ \text{m}\end{array}\right) \Bigg/ \left(\begin{array}{l}(417\times 50 \\ +\ 50\times 125 \\ +\ 417\times 125)\ \left(\text{W/m}^2\cdot\text{K}\right)^2\end{array}\right)$$

Continued…

$$T_{s,1} = 92.7°C$$

Similarly,

$$T_{s,2} = 85.8°C$$

The location of the maximum temperature in the wall can be found by setting the gradient of the temperature (from Equation (1)) to zero:

$$\frac{dT}{dx} = -\frac{\dot{q}x}{k} + \frac{T_{s,2} - T_{s,1}}{2L} = 0$$

Thus, $x_{max} = k\frac{T_{s,2} - T_{s,1}}{2L\dot{q}}$. Substituting this back into the temperature distribution,

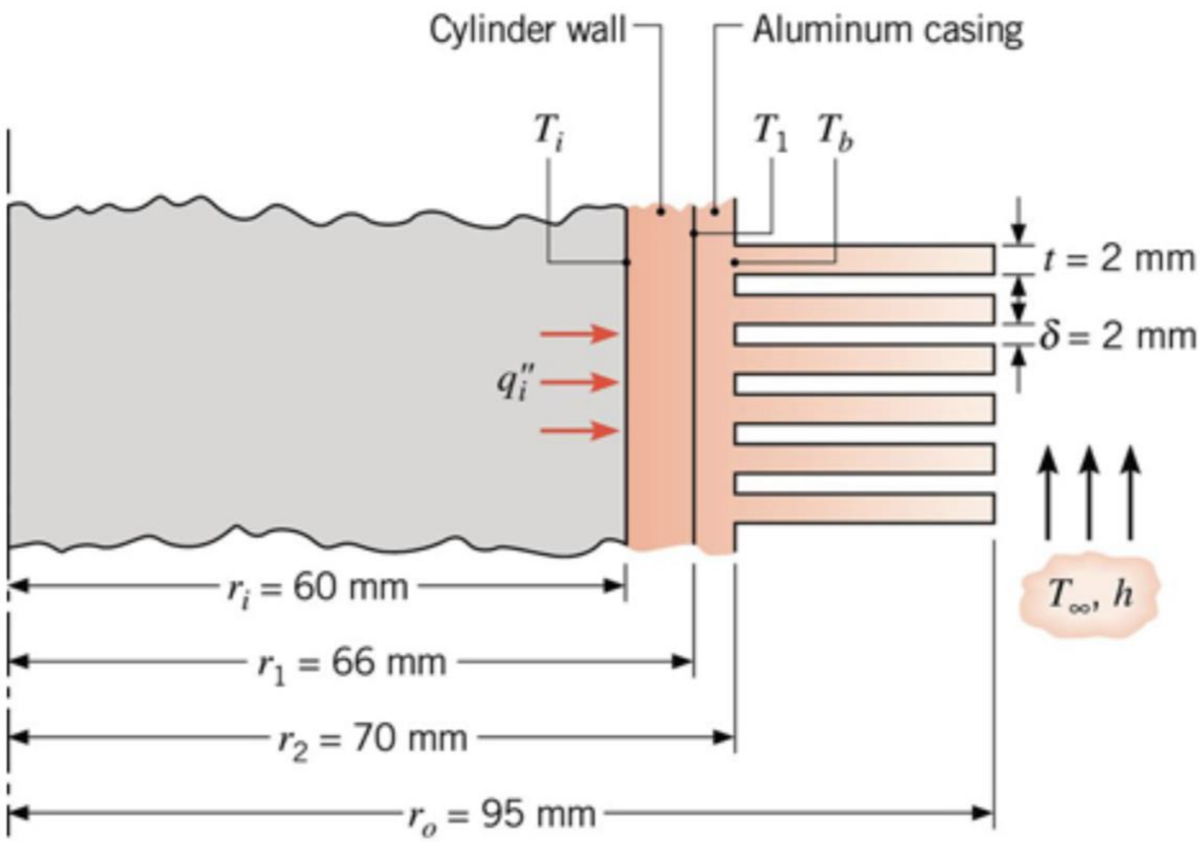
$$T_{max} = \frac{\dot{q}L^2}{2k} + \frac{k\left(T_{s,2} - T_{s,1}\right)^2}{8L^2\dot{q}} + \frac{T_{s,1} + T_{s,2}}{2}$$

$$= \frac{2 \times 10^5\ \text{W/m}^3 \times (0.03\ \text{m})^2}{2 \times 25\ \text{W/m}\cdot\text{K}} + \frac{25\ \text{W/m}\cdot\text{K}\left(85.8°C - 92.7°C\right)^2}{8 \times (0.03\ \text{m})^2 \times 2 \times 10^5\ \text{W/m}^3}$$

$$+ \frac{92.7°C + 85.8°C}{2} = 93.7°C$$

<

3.125 It is proposed to air-cool the cylinders of a combustion chamber by joining an aluminum casing with annular fins ($k = 240\ W/m \cdot K$) to the cylinder wall ($k = 50\ W/m \cdot K$).

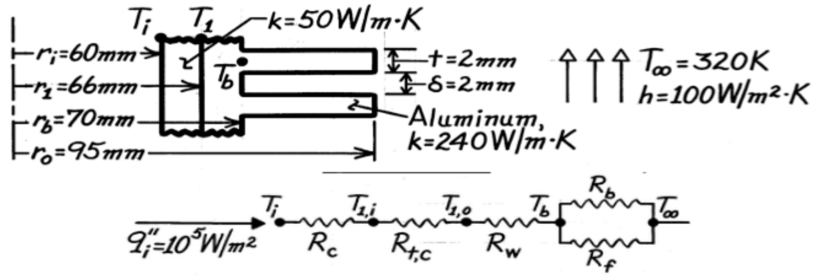


PROBLEM 3.125

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{tot}} = q''_i(2\pi\, r_i) = 10^5\ \text{W/m}^2 \times 2\pi \times 0.06\ \text{m} = 37,700\ \text{W/m}$$

where $R'_{tot} = R'_c + R'_{t,c} + R'_w + R'_{equiv}$; $\quad R'_{equiv} = \left(1/R'_f + 1/R'_b\right)^{-1}$.

R'_c, *Conduction resistance of cylinder wall:*

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi\, k} = \frac{\ln(66/60)}{2\pi(50\ \text{W/m}\cdot\text{K})} = 3.034\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$R'_{t,c}$, *Contact resistance:*

$$R'_{t,c} = R''_{t,c}/2\pi\, r_1 = 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\pi\times0.066\ \text{m} = 2.411\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_w, *Conduction resistance of aluminum base:*

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi\, k} = \frac{\ln(70/66)}{2\pi\times240\ \text{W/m}\cdot\text{K}} = 3.902\times10^{-5}\ \text{m}\cdot\text{K/W}$$

R'_b, *Resistance of prime or unfinned surface:*

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100\ \text{W/m}^2\cdot\text{K}\times0.5\times2\pi(0.07\ \text{m})} = 454.7\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_f, *Resistance of fins:* The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f h A'_f}$$

The fin efficiency may be obtained from Fig. 3.20,

$$r_{2c} = r_o + t/2 = 0.096\ \text{m} \qquad L_c = L + t/2 = 0.026\ \text{m}$$

Continued …

$$A_p = L_c t = 5.2\times10^{-5}\ m^2 \quad r_{2c}/r_1 = 1.45 \quad L_c^{3/2}\left(h/kA_p\right)^{1/2} = 0.375$$

Fig. 3.20 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A'_f = 250\left[\pi\left(r_o^2 - r_b^2\right)\times 2\right] = 250\ m^{-1}\left[2\pi\left(0.096^2 - 0.07^2\right)\right]m^2 = 6.78\ m.$$

Hence $\quad R'_f = \left[0.88\times100\ W/m^2\cdot K\times6.78\ m\right]^{-1} = 16.8\times10^{-4}\ m\cdot K/W$

$$1/R'_{equiv} = \left(1/16.8\times10^{-4} + 1/454.7\times10^{-4}\right)W/m\cdot K = 617.2\ W/m\cdot K$$

$$R'_{equiv} = 16.2\times10^{-4}\ m\cdot K/W.$$

Neglecting the *contact resistance,*

$$R'_{tot} = (3.034 + 0.390 + 16.2)10^{-4}\ m\cdot K/W = 19.6\times10^{-4}\ m\cdot K/W$$

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times19.6\times10^{-4}\ m\cdot K/W + 320\ K = 393.9\ K$$ <

$$T_1 = T_i - q'R'_w = 393.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 382.5\ K$$ <

$$T_b = T_1 - q'R'_b = 382.5\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 381.0\ K.$$ <

Including the *contact resistance,*

$$R'_{tot} = \left(19.6\times10^{-4} + 2.411\times10^{-4}\right)m\cdot K/W = 22.0\times10^{-4}\ m\cdot K/W$$

$$T_i = 37,700\ W/m\times22.0\times10^{-4}\ m\cdot K/W + 320\ K = 402.9\ K$$ <

$$T_{1,i} = 402.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 391.5\ K$$ <

$$T_{1,o} = 391.5\ K - 37,700\ W/m\times2.411\times10^{-4}\ m\cdot K/W = 382.4\ K$$ <

$$T_b = 382.4\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 380.9\ K.$$ <

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R'_{tot} = R'_c + R'_{conv}$, where

$$R'_{conv} = \frac{1}{h2\pi r_1} = \frac{1}{100\ W/m^2\cdot K\ 2\pi(0.066\ m)} = 241.1\times10^{-4}\ m\cdot K/W.$$

Hence, $R_{tot} = 244.1\times10^{-4}\ m\cdot K/W$, and

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times244.1\times10^{-4}\ m\cdot K/W + 320\ K = 1240\ K.$$

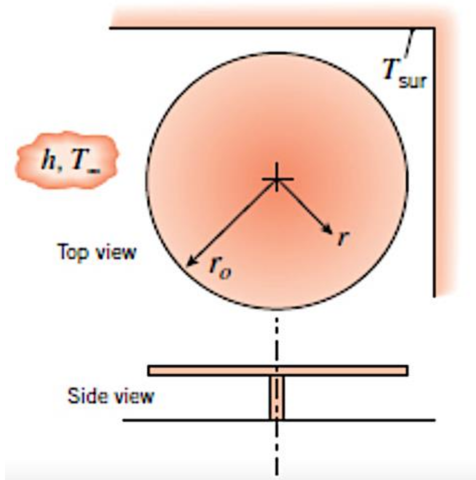
Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - \left(A'_f/A'_t\right)\left(1-\eta_f\right) = 1 - 6.78\ m/7.00\ m(1-0.88) = 0.884.$$

It follows that $q' = \eta_o h_o A'_t \theta_b = 37,700\ W/m$, which agrees with the prescribed value.

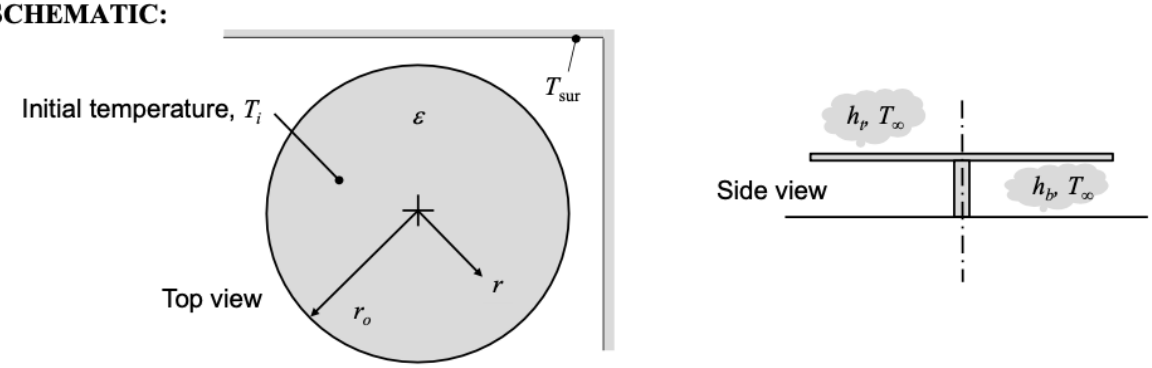
5.4 A thin stainless steel disk of thickness b and outer radius ro has been heat treated to a high, uniform initial temperature of Ti. The disk is then placed upon a small stand and allowed to cool. The ambient air and surroundings are at T∞ and Tsur, respectively. The convection coefficients on the top and bottom disk surfaces, ht and hb, are known, as is the plate emissivity, ε. Derive a differential equation that could be solved to determine the transient thermal response of the disk, T(r, t). List the appropriate initial and boundary conditions. Assume adiabatic conditions at r = ro.



KNOWN: Stainless steel disk of known thickness, radius, emissivity, and initial temperature. Environment and surroundings temperatures and heat transfer coefficients.

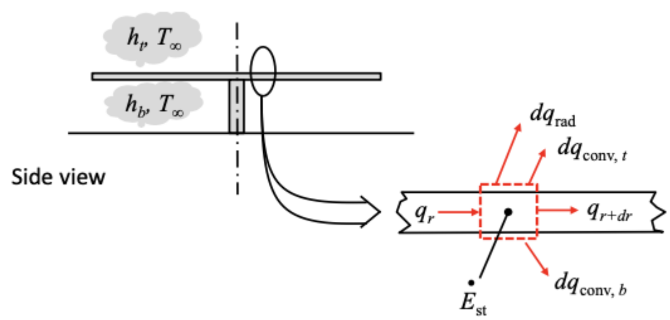
FIND: Derive differential equation governing transient temperature distribution, $T(r,t)$. List initial and boundary conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform properties. (2) Temperature is nearly uniform across the disk thickness. (3) The surroundings are large relative to the disk. (4) Convection and radiation from the rim are negligible because it is so thin.

ANALYSIS: Consider a differential control volume that is a ring of width dr and thickness b, as shown. Assuming the temperature variation through the thickness is negligible, and applying conservation of energy to the differential control volume:



$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{st}}}{dt},$$

$$q_r - q_{r+dr} - dq_{\text{conv},b} - dq_{\text{conv},t} - dq_{\text{rad}} = \rho c V \frac{\partial T}{\partial t}$$

where

$$V = 2\pi r dr b, \quad q_r = -k 2\pi r b \frac{\partial T}{\partial r}, \quad q_{r+dr} = q_r + \frac{\partial q_r}{\partial r} dr, \quad q_r - q_{r+dr} = \frac{\partial}{\partial r}\left(k 2\pi r b \frac{\partial T}{\partial r}\right) dr$$

$$dq_{\text{conv},b} = h_b 2\pi r dr (T - T_\infty), \quad dq_{\text{conv},t} = h_t 2\pi r dr (T - T_\infty), \quad dq_{\text{rad}} = \varepsilon\sigma 4\pi r dr (T^4 - T_{\text{sur}}^4)$$

Combining these terms,

$$\frac{\partial}{\partial r}\left(k 2\pi r b \frac{\partial T}{\partial r}\right) dr - \left[h_b (T - T_\infty) + h_t (T - T_\infty) + 2\varepsilon\sigma (T^4 - T_{\text{sur}}^4)\right] 2\pi r dr = \rho c 2\pi r dr b \frac{\partial T}{\partial t}$$

Dividing by $2\pi r dr b$ yields the differential equation for $T(r,t)$:

Continued…

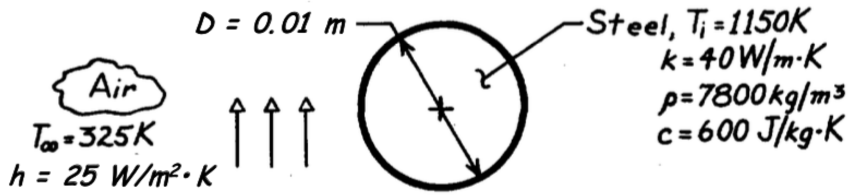
5.7 Steel balls 10 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 450 K in an air environment for which T∞ = 325 K and h = 25 W/m2 · K. Assuming the properties of the steel to be k = 40 W/m · K, ρ = 7800 kg/m3, and c = 600 J/kg · K, estimate the time required for the cooling process.

PROBLEM 5.7

KNOWN: Diameter and initial temperature of steel balls cooling in air.

FIND: Time required to cool to a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation effects, (2) Constant properties.

ANALYSIS: Applying Eq. 5.10 to a sphere ($L_c = r_o/3$),

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{25\ W/m^2 \cdot K\ (0.005\ m/3)}{40\ W/m \cdot K} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used. From Eqs. 5.4 and 5.5,

$$t = \frac{\rho V c_p}{hA_s} \ln\frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho\left(\pi D^3/6\right)c_p}{h\pi D^2} \ln\frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800\ kg/m^3\,(0.01\ m)\,600\ J/kg \cdot K}{6 \times 25\ W/m^2 \cdot K} \ln\frac{1150 - 325}{450 - 325}$$

$$t = 589\ s\ = 0.164\ h$$ <

COMMENTS: Due to the large value of T_i, radiation effects are likely to be significant during the early portion of the transient. The effect is to shorten the cooling time.

5.8 Consider the steel balls of Problem 5.7, except now the air temperature increases with time as $T_\infty(t)$ = 325 K + at, where a = 0.1875 K/s.

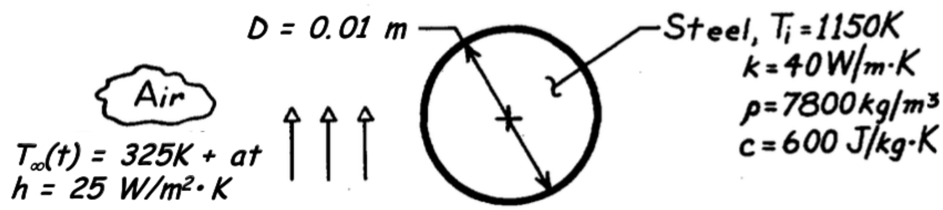
(a) Sketch the ball temperature versus time for $0 \le t \le 1$ h. Also show the ambient temperature, T_∞, in your graph. Explain special features of the ball temperature behavior.

(b) Find an expression for the ball temperature as a function of time T(t), and plot the ball temperature for $0 \le t \le 1$ h. Was your sketch correct?

KNOWN: Diameter and initial temperature of steel balls in air. Expression for the air temperature versus time.

FIND: (a) Expression for the sphere temperature, T(t), (b) Graph of T(t) and explanation of special features.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation heat transfer.

PROPERTIES: Given: k = 40 W/m·K, ρ = 7800 kg/m^3, c = 600 J/kg·K.

ANALYSIS:
(a) Applying Equation 5.10 to a sphere ($L_c = r_o/3$),

$$B_i = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{25\ \text{W/m}^2\cdot\text{K}\ (0.005\ \text{m}/3)}{40\ \text{W/m}\cdot\text{K}} = 0.001$$

Hence, the temperature of the steel sphere remains approximately uniform during the cooling process. Equation 5.2 is written, with $T_\infty = T_o + at$, as

$$-hA_s(T - T_o - at) = \rho \forall c \frac{dT}{dt}$$

Letting $\theta = T - T_o$, $dT = d\theta$ and $-hA_s(\theta - at) = \rho\forall c\frac{d\theta}{dt}$ or $\frac{d\theta}{dt} = -C(\theta - at)$ where $C = \frac{hA_s}{\rho\forall c}$

The solution may be written as the sum of the homogeneous and particular solutions,

$$\theta = \theta_h + \theta_p \quad \text{where} \quad \theta_h = c_1\exp(-Ct).$$

Assuming $\theta_p = f(t)\theta_h$, we substitute into the differential equation to find

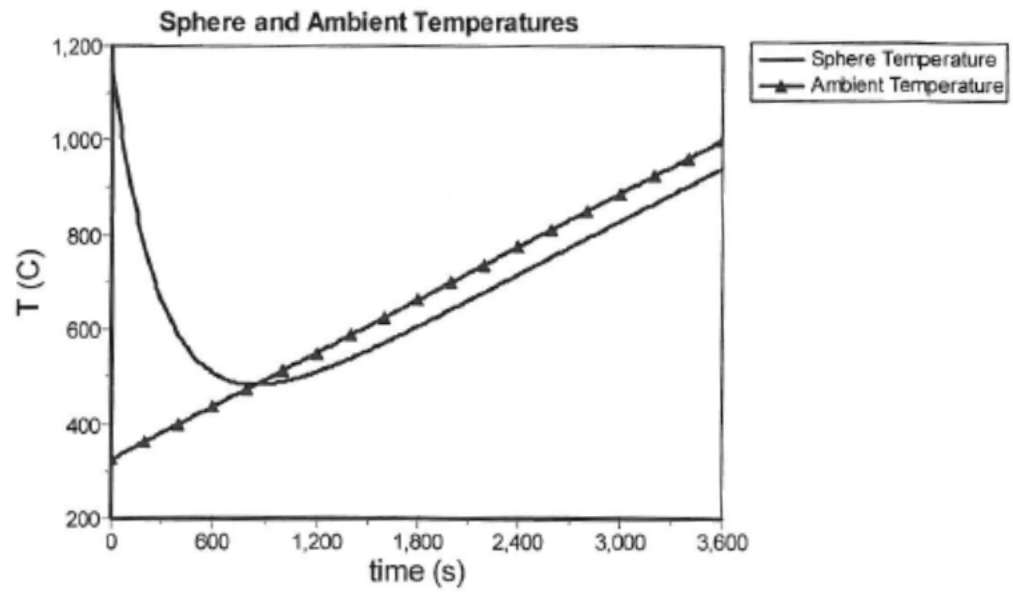
$$\frac{df}{dt} = Cat\exp(Ct)/c_1 \text{ from which } f = a(t - 1/C)\exp(Ct)/c_1.$$

Thus, the complete solution is

$$\theta = c_1\exp(-Ct) + a(t - 1/C) \text{ and applying the initial condition we find}$$

$$T = (T_i - T_o + a/C)\exp(-Ct) + a(t - 1/C) + T_o$$ <

(b) The ambient and sphere temperatures for $0 \leq t \leq 3600$ s are shown in the plot below.



Note that:

1) For small times ($t \leq 300$s) the sphere temperature decreases rapidly,

2) at $t \approx 850$ s, $T = T_\infty$ and, from Equation 5.2, $dT/dt = 0$,

3) at $t \geq 850$ s, $T < T_\infty$,

4) at large time, $T - T_\infty$ and dT/dt are constant.

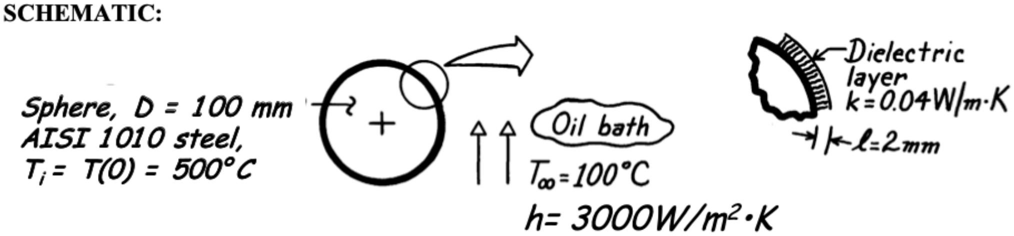
COMMENTS: Unless the air environment of Problem 5.7 is cooled, the air temperature will increase in temperature as energy is transferred from the balls. However, the actual air temperature versus time may not be linear.

5.10 A steel sphere (AISI 1010), 100 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m · K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which T∞ = 100°C and h = 3000 W/m2 · K. Estimate the time required for the coated sphere temperature to reach 150°C. Hint: Neglect the effect of energy storage in the dielectric material, since its thermal capacitance (ρcV) is small compared to that of the steel sphere.

KNOWN: Solid steel sphere (AISI 1010), coated with dielectric layer of prescribed thickness and thermal conductivity. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath.

FIND: Time required for sphere to reach 150°C.

SCHEMATIC:



PROPERTIES: *Table A-1*, AISI 1010 Steel $\left(\bar{T} = [500+150]^\circ C/2 = 325^\circ C \approx 600K\right)$:

$\rho = 7832\ kg/m^3,\ c = 559\ J/kg \cdot K,\ k = 48.8\ W/m \cdot K.$

ASSUMPTIONS: (1) Steel sphere is space-wise isothermal, (2) Dielectric layer has negligible thermal capacitance compared to steel sphere, (3) Layer is thin compared to radius of sphere, (4) Constant properties, (5) Neglect contact resistance between steel and coating.

ANALYSIS: The thermal resistance to heat transfer from the sphere is due to the dielectric layer and the convection coefficient. That is,

$$R'' = \frac{\ell}{k} + \frac{1}{h} = \frac{0.002m}{0.04\ W/m \cdot K} + \frac{1}{3000\ W/m^2 \cdot K} = (0.050 + 0.00033) = 0.0503 \frac{m^2 \cdot K}{W},$$

or in terms of an overall coefficient, $U = 1/R'' = 19.87\ W/m^2 \cdot K$. The effective Biot number is

$$Bi_e = \frac{UL_c}{k} = \frac{U(r_o/3)}{k} = \frac{19.87\ W/m^2 \cdot K \times (0.100/6)m}{48.8\ W/m \cdot K} = 0.0068$$

where the characteristic length is $L_c = r_o/3$ for the sphere. Since $Bi_e < 0.1$, the lumped capacitance approach is applicable. Hence, Eq. 5.5 is appropriate with h replaced by U,

$$t = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{\theta_i}{\theta_o} = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{T(0) - T_\infty}{T(t) - T_\infty}.$$

Substituting numerical values with $(V/A_s) = r_o/3 = D/6$,

$$t = \frac{7832\ kg/m^3 \times 559\ J/kg \cdot K}{19.87\ W/m^2 \cdot K}\left[\frac{0.100m}{6}\right] \ln\frac{(500-100)^\circ C}{(150-100)^\circ C}$$

$$t = 7623\ s = 2.12\ h.$$ <

COMMENTS: (1) Note from calculation of R'' that the resistance of the dielectric layer dominates and therefore nearly all the temperature drop occurs across the layer.