

2014

notebook

STATICS

الدكتور ياسر الحنيطي

إعداد : عاصم الحنيطي



ech-family

* Engineering mechanics :-

may be defined as the science which describes and the conditions of rest or motion of bodies under the action of forces.

* The Parallelogram law for the addition of forces :-

Any 2 forces acting on a particle may be replaced by a single force called their resultant.

Particle → جُزْء

* The Principle of Transmissibility :-

Conditions of equilibrium will remain unchanged if the force acting at a given point of the rigid body is replaced by a force of the same magnitude and direction at a different point provided that the two forces have the same line of action.

* Newton's laws *

* Force systems:- 2D

Force is the action of one body on another.

types of force: ① actual contact ② remote.

Force is characterized by:-

* Point of application

* magnitude and

* direction.

Units → Newtons → N

+ إذا كانت الوحدة مُشتقة من اسم

نرمز لها بحرف Capital

مثل N

+ إذا كانت الوحدة ليست

مشتقة من اسم ← حرف Small مثل kg

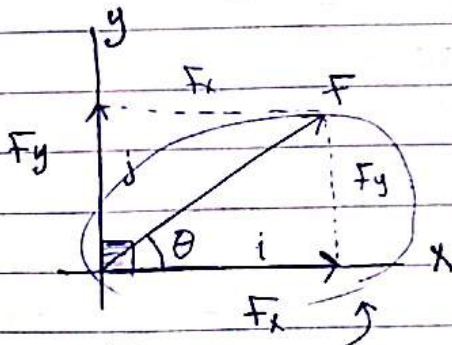
* kg

* kN

* * Components of force:-

* Rectangular Components

يجب أن يكونوا متعامدان



Force Triangle

$$F_x = F \cos \theta$$

$$F_y = F \cos(90^\circ - \theta) = F \sin \theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

* Vector expression of the force

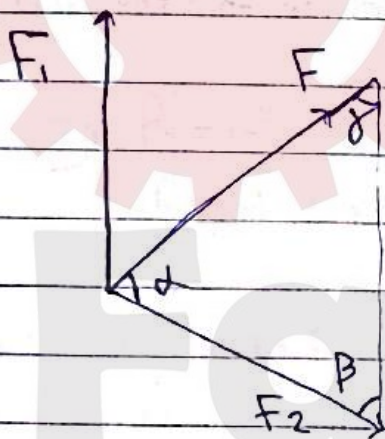
$$F = F_x + F_y$$

$$\rightarrow F_x = F_x i$$

$$\rightarrow F_y = F_y j$$

$$\rightarrow \cancel{F} F = F_x i + F_y j = F (\cos \alpha i + \sin \alpha j)$$

* Non-Rectangular Components:- ليس مثلثي



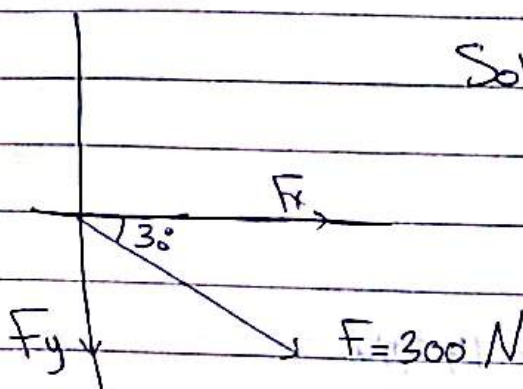
* law of Sines:-

$$F_1 \rightarrow \frac{F}{\sin \beta} = \frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \gamma}$$

* law of cosines:-

$$\rightarrow F^2 = F_1^2 + F_2^2 - 2 F_1 F_2 \cos \beta$$

* Example :- calculate F_x and F_y and express F as a vector



$$\text{Sol: } F_x = F \cos 30^\circ$$

$$= 300 \times 0.78 = 260 \text{ N}$$

$$F_y = F \sin 30^\circ$$

$$= 300 \times 0.5 = 150 \text{ N}$$

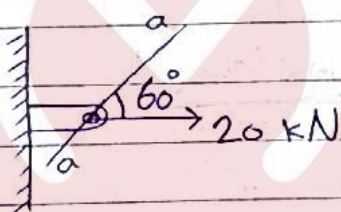
* $\sin(180-\theta) = \sin\theta$ * $\sin(90-\theta) = \cos\theta$

* F as a vector:-

$$\begin{aligned} F &= F_x + F_y \\ &= F \cos\theta i + F \sin\theta j \end{aligned}$$

$$F = (260i - 150j) \text{ N}$$

* Example:-

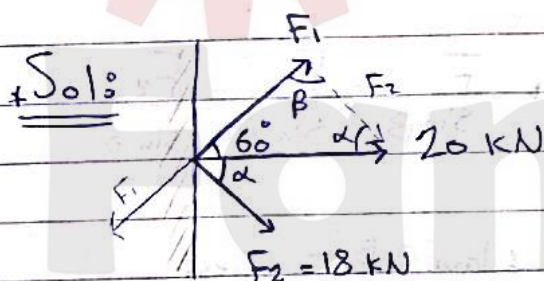


Replaced the 20 kN force by:

F_1 along axis a-a

$F_2 = 18 \text{ kN}$

Determine F_1 and α which F_2 makes with the horizontal axis



* Law of sines:-

$$\frac{20}{\sin\beta} = \frac{18}{\sin 60^\circ} = \frac{F_1}{\sin\alpha} \quad \Rightarrow \quad \beta = 74.2^\circ \text{ or } 105.8^\circ$$

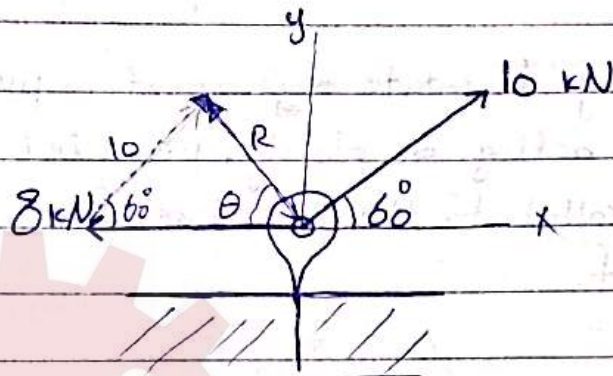
$$\rightarrow 60^\circ + \alpha + \beta = 180^\circ$$

$$\text{So } \alpha = 14.2^\circ \text{ or } 45.8^\circ$$

$$\rightarrow \text{For } \alpha = 45.8^\circ \rightarrow \frac{F_1}{\sin 45.8^\circ} = \frac{20}{\sin 74.2^\circ} \Rightarrow F_1 = 14.9 \text{ kN}$$

→ For $\alpha = 14.2^\circ$: $F_1 = 90 \rightarrow F_1 = 5.10 \text{ kN}$
 $\sin 14.2^\circ \quad \sin 105.8^\circ$

* Example:



Determine the resultant R

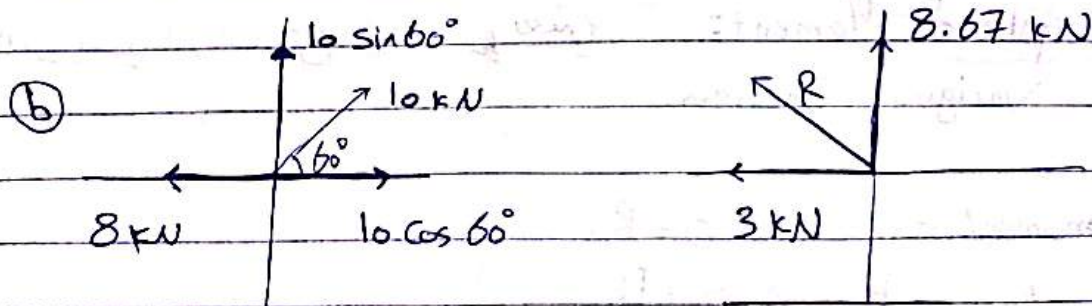
a Parallelogram rule for vector addition

b Summing scalar components

→ Sol: (a) $R^2 = 10^2 + 8^2 - 2(8)(10) \cos 60^\circ$
 $R = 9.17 \text{ kN}$

→ law of sines:

$\frac{R}{\sin 60^\circ} = \frac{10}{\sin \theta} \rightarrow \theta = 70.8^\circ$

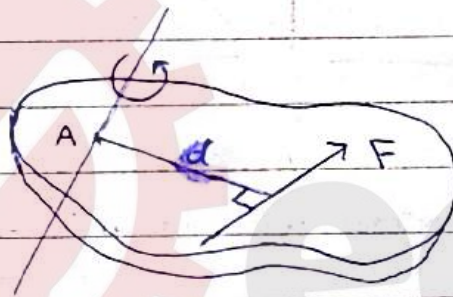


$$R = \sqrt{3^2 + 8.67^2}$$

$$R = 9.17 \text{ kN}$$

* Moment العزم

→ the tendency to rotate ~~about~~ about a given axis caused by the force acting on a body, provided that this axis is not parallel to the force line of action and doesn't intersect it.



* Magnitude:

$$M = F \cdot d$$

$$\text{Units} = \text{N} \cdot \text{m}$$

$$\text{Signs: } \begin{matrix} \curvearrowright (+) \\ \curvearrowleft (-) \end{matrix}$$

* نستخدم قاعدة اليد اليمنى

• الأصابع مع اتجاه الحركة

والإبهام اتجاه M

* Principle of Moment:

Varignon's Theorem

يعني

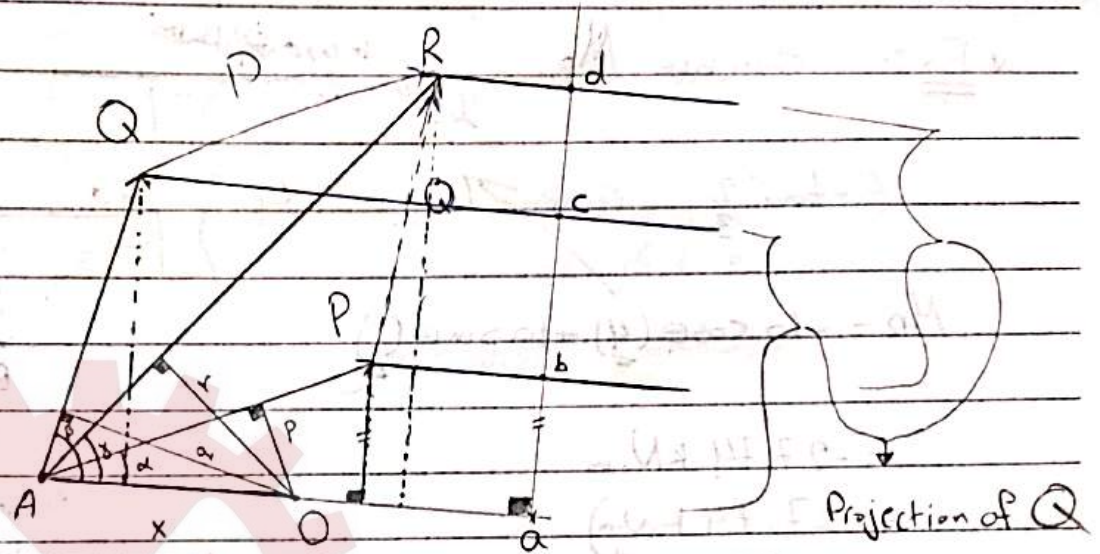
المoment للقوة يساوي moment لمركباتها

$$\text{moment of } R \text{ about } O = R \cdot r$$

$$\text{" " } P \text{ " } O = P \cdot p$$

$$\text{" " } Q \text{ " } O = Q \cdot q$$

* Projection for any force means rectangular components



* $ac = bd$ → Projection of Q

∴ في الارتفاع

* $ad = ab + bd$
 $= ab + ac$

⊗ $ab = P \sin \alpha$

⊗ $ac = Q \sin \beta$

⊗ $ad = R \sin \gamma$

$ad = ab + ac$

* $R \sin \gamma = P \sin \alpha + Q \sin \beta$

نضرب الطرفين بـ $\frac{1}{\sin \gamma}$

* $AD = X$

→ $X R \sin \gamma = X P \sin \alpha + X Q \sin \beta$

→ $\frac{X R \sin \gamma}{\sin \gamma} = \frac{X P \sin \alpha}{\sin \alpha} + \frac{X Q \sin \beta}{\sin \beta}$

→ $r.R = P.P + q.Q$

نحن الوجب إيجاد لحظة اعودية عن B خط عمل القوة
لأن نجد M لحركات القوة لأن إيجاد لحظة اعودية لمركبان

* Ex:- Calculate M_B .

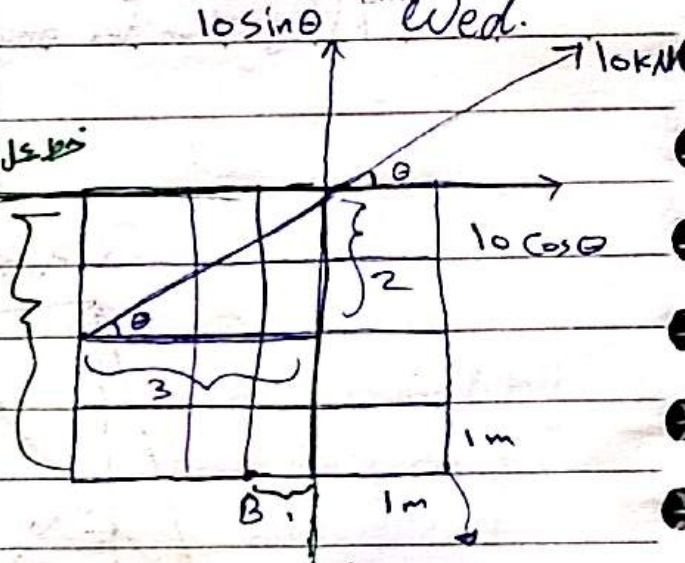
خط عمل القوة 10
يبعد عن B 4

$$\theta = \tan^{-1} \frac{2}{3}$$

$$M_B = -10 \cos \theta (4) + 10 \sin \theta (1)$$

$$= -27.74 \text{ kN.m}$$

$$= 27.74 \text{ kN.m}$$



خط عمل القوة 10
يبعد عن B 1
plate

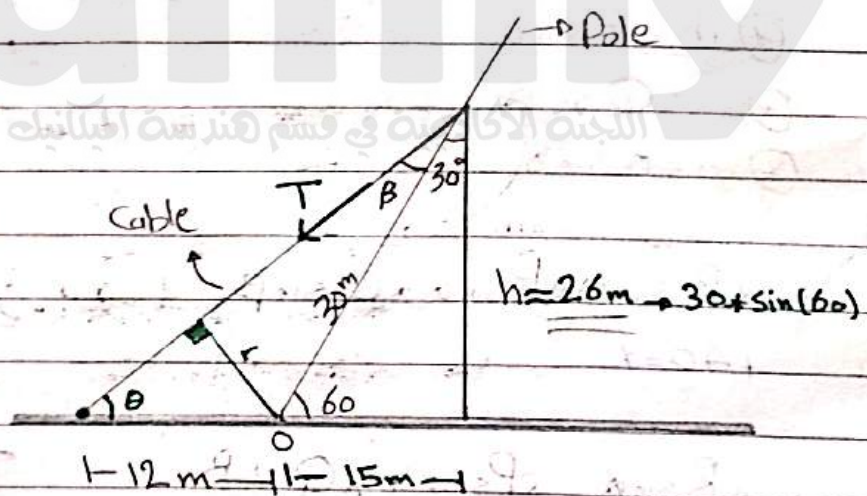
* Ex:- calculate the Tension in the cable to cause $M_o = 72 \text{ kN.m}$

$$\text{given } M_o = T \cdot r = 72 \text{ kN.m}$$

$$\theta = \tan^{-1} \frac{26}{27}$$

$$(\beta + 30^\circ) = \tan^{-1} \frac{27}{26}$$

$$\beta = 16.1^\circ$$



$$T = \frac{M_o}{r}$$

$$= \frac{72}{r}$$

$$30(\sin \beta)$$

$$T = 8.65 \text{ kN}$$



$$2.7 \text{ m}$$

$$15 = 30 \cdot \cos(60^\circ)$$

PAGE: _____
DATE: _____

PAGE: _____
DATE: _____

* 17x :-

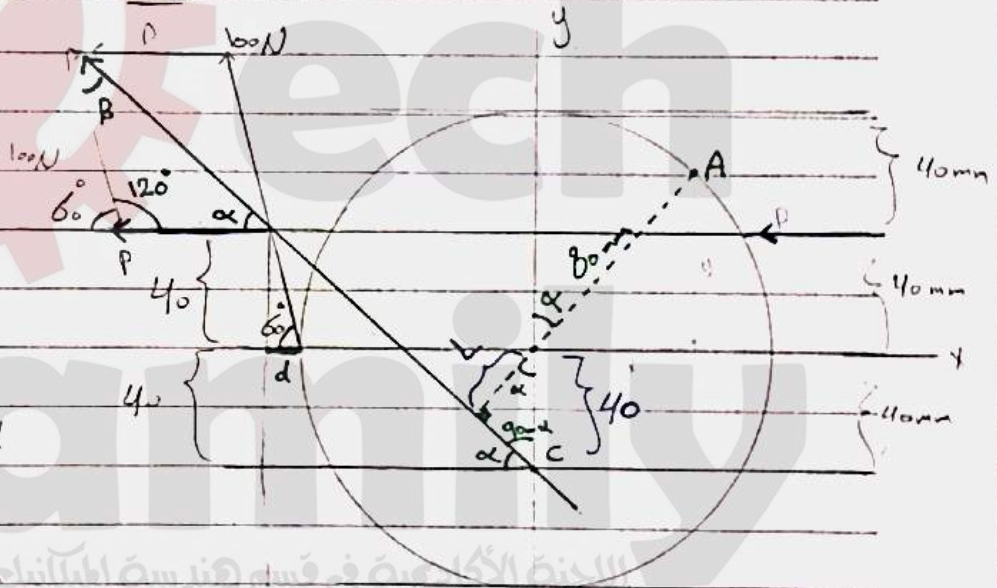
$$M_{C=O}$$

moment of the
two forces shown about c
is zero.

النقطة A أبعد نقطة عن R وذلك حتى يكون M أكبر شيء
وحتى تكون أبعد شيء يجب أن تكون
عادية على R ولتكن في المركز.

Determine:-

P, R, x and y of point A
rational about which M is max



$$\alpha = \tan^{-1} \frac{80}{80+d}$$

$$\tan 60^\circ = \frac{40}{d}$$

$$d = \frac{40}{\tan 60^\circ} = 23.1 \text{ nm}$$

$$\alpha = \tan^{-1} \frac{80}{80+23.1} = 37.8^\circ$$

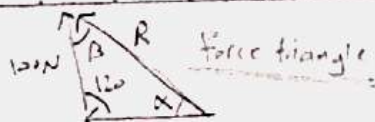
$$\beta = 22.2^\circ$$

* Law of sines:-

$$\frac{P}{\sin \beta} = \frac{100}{\sin \alpha} = \frac{R}{\sin 120}$$

$P = 61.6 \text{ N}$, $R = 141 \text{ N}$

$$x = 80 \sin \alpha = 49$$



حول من mm
بالقسمة على 1000

$$+ \text{Now } M_A = R \cdot (80 + 40 \cos \alpha)$$

$$\rightarrow L = 40 \cos \alpha$$

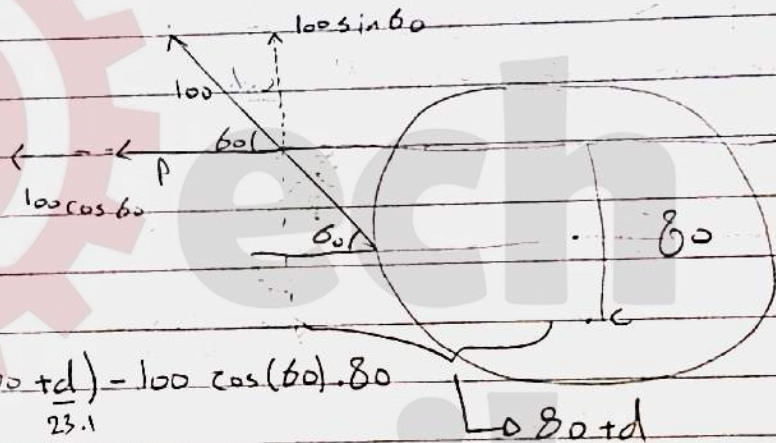
$$= 141 (80 + 40 \cos \alpha)$$

$$= 15.7 \text{ N.m}$$

$$+ 80 + 40 \cos \alpha$$

→ distance between
A and R.

طريقة أخرى لا، P, R دون استيعاب Law of sines.



$$M_C = \text{Zero}$$

$$P \cdot 80 = 100 \sin(60) \cdot (80 + \frac{d}{23.1}) - 100 \cos(60) \cdot 80$$

$$P = 61.6 \text{ N}$$

* فوتين لهما نفس المقدار + خطي عملهما متوازيان + مفاكستان بالارجاه.

* Couple :-

Two forces having the same magnitude, parallel lines of action and opposite sense are said to form a couple.

→ Characteristics:-

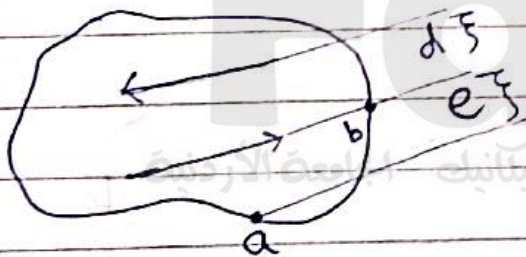
1) $\sum R = 0$

these two forces can't be combined into a single force

2) Cause rotation.

3) the moment of the couple is given by

$M = F \cdot d$ and is the same at any point

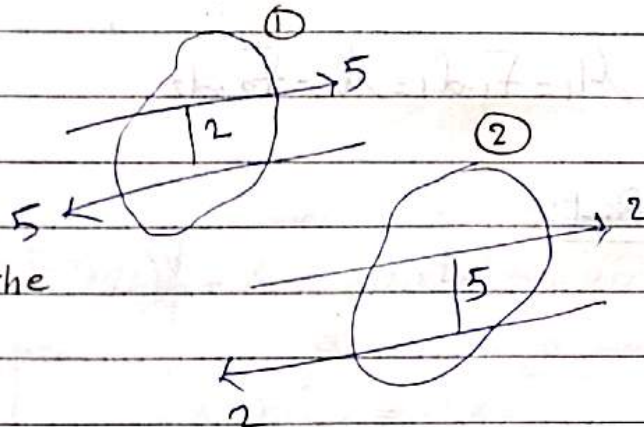


$$\begin{aligned} M_a &= +F(d+e) \\ &\quad - F(e) \\ &= +F \cdot d \end{aligned}$$

$$M_b = +F \cdot d$$

* Equivalent Couple:-

→ Couples are said to be equivalent if they produce the same moment



$$M_1 = 5 \cdot 2 = 10$$

$$M_2 = 2 \cdot 5 = 10$$

29/6/2014 Sunday

* Two Systems of forces are equivalent if we can transform one of them into the other by means of one or several of the following operations:-

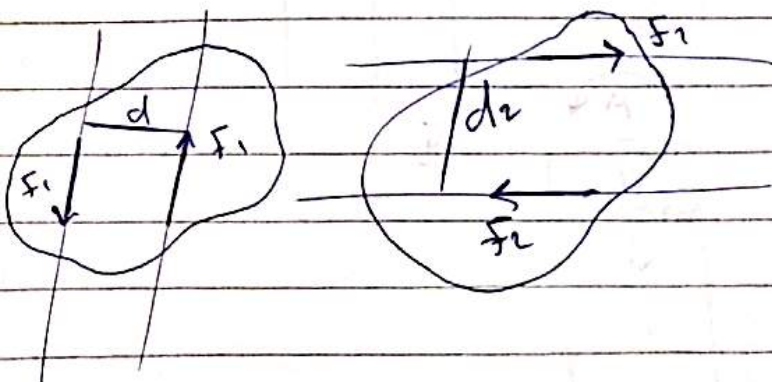
⊕ Replacing two forces by their resultant.

⊕ Resolving a force into two components

⊕ Canceling two equal and opposite forces acting on a particle.

⊕ Attaching to the same particle two equal and opposite forces

⊕ Moving a force along its line of action.



$$M_1 = F_1 \cdot d_1 = M_2 = F_2 \cdot d_2$$

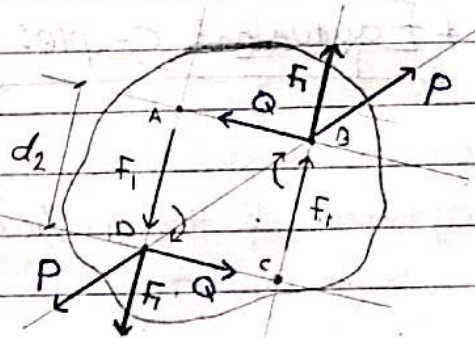
* Proof:-

$$F_1 d_1 = M_{P_s} + M_{Q_s}$$

$$= Q d_2$$

$$= F_2 d_2$$

$$\Rightarrow Q = F_2$$



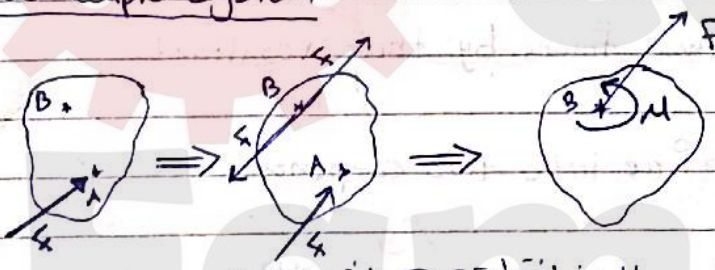
* لنقل القوتين على خط

لعمري.

* نختار القوتين P لأبها متساويتان ومعاكستان

وعلى نفس خط عمل القوي

* Force Couple System:-

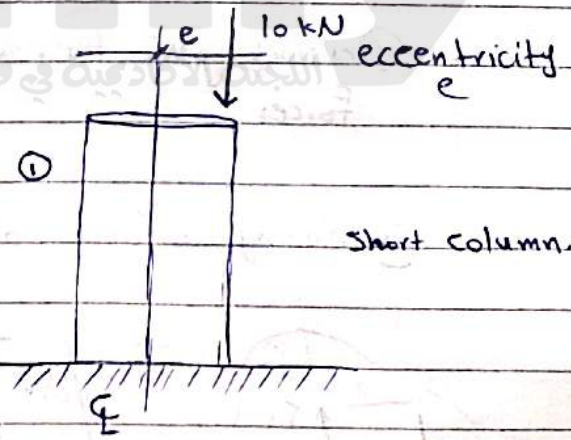
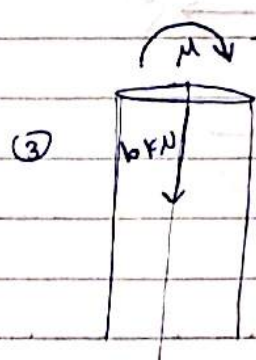
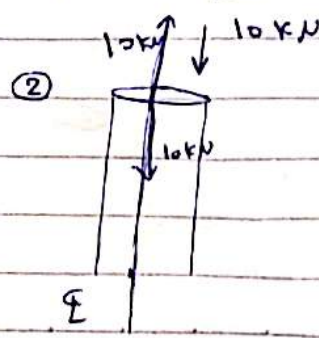


* لنقل قوة من A إلى B: (1) احسب M عند النقطة B (2) انقل القوة

* Example:

$$\text{If } M_c = 800 \text{ N.m}$$

determine e.



$$10,000 \text{ e} = 800 \text{ N.m}$$

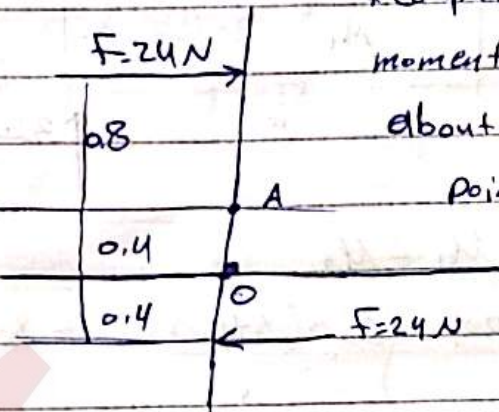
$$e = 80 \text{ mm.}$$

* Example:

$$M_A = M_O = F \cdot d$$

$$= 24(1.6)$$

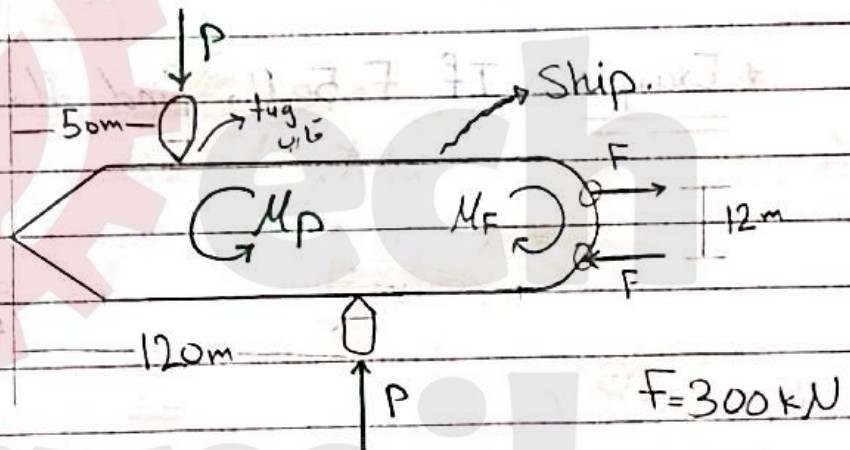
$$= 38.4 \text{ N.m}$$



* Compute the combined moment of two 24 N forces about point O, and point A

* Example:

* Sol:-



$$P(70) = F(12)$$

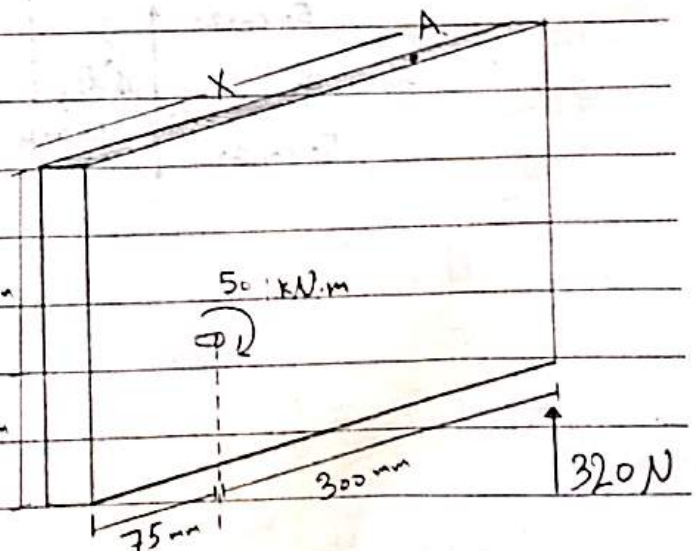
$$P = \frac{300(12)}{70}$$

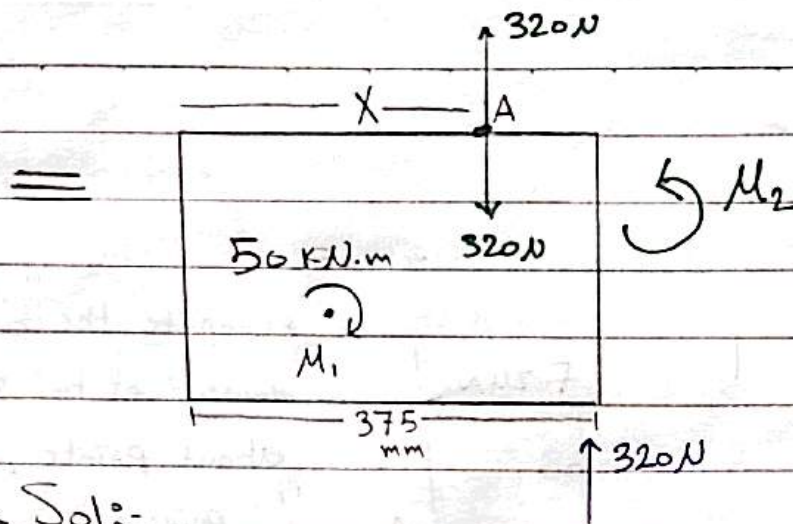
$$P = 51.4 \text{ kN}$$

* Example:-

The (50 N.m) couple and the 320 N force are replaced by a single force at A

* determine X



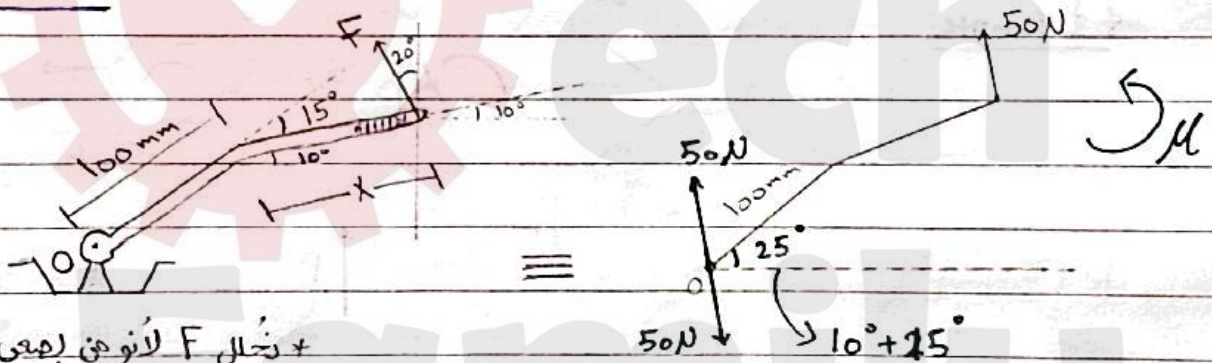


→ Sol:-

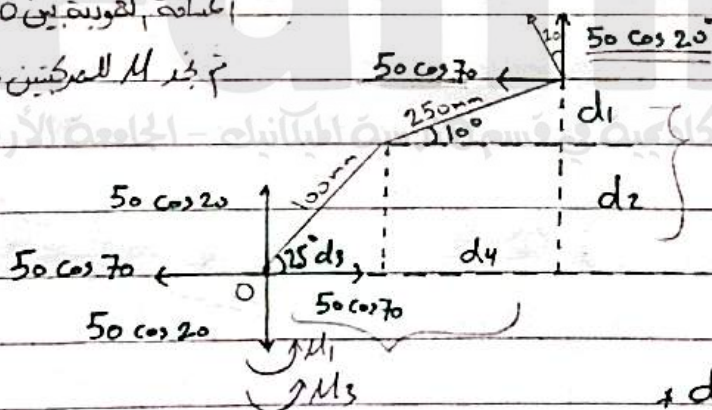
$$M_1 = M_2$$

$$50(1000) = 320(375 - x) \Rightarrow x = 218.75 \Rightarrow x = 219 \text{ mm}$$

* Example:- If $F = 50 \text{ N}$, find M_0 ($x = 250 \text{ mm}$)



* نحل F لأنوفين إصبعي إيجار
المسافة القوية بين O ونقطة عمل F
ثم نجد M للمركبتين.



M_1 - ناتج عن المركبة لأعلى
 M_2 - ناتج عن المركبة لليمين

$$M_0 = M_1 + M_2$$

$$M_1 = 50 \cos 20 (d_3 + d_4)$$

$$M_2 = 50 \cos 70 (d_1 + d_2)$$

$$* d_1 = 250 \sin 10$$

$$* d_2 = 100 \sin 25$$

$$* d_3 = 100 \cos 25$$

$$* d_4 = 250 \cos 10$$

$$M_0 = 17.3 \text{ N.m}$$

* لا تنس تحويل الأمتار إلى m

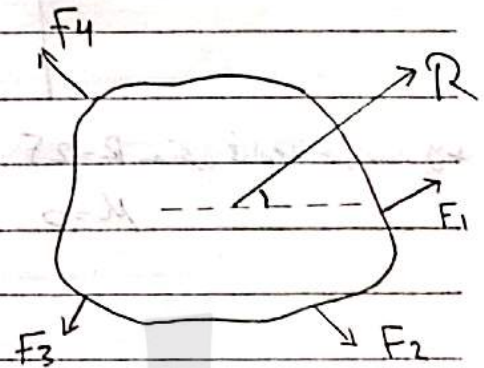
* Resultant:

The resultant may be defined as the single force or (couple) which replace the original forces with no change to the external effect.

$$* R_x = \sum F_x = F_{1x} + F_{2x} + \dots$$

$$* R_y = \sum F_y = F_{1y} + F_{2y} + \dots$$

$$\rightarrow R = \sqrt{R_x^2 + R_y^2}, \quad \theta = \tan^{-1} \frac{R_y}{R_x}$$



** The principle of moment:- (to determine point of apply)

$$Rd = F_1d_1 + F_2d_2 + F_3d_3$$

$$\rightarrow d = \frac{\sum Fd}{R}$$

* Example:- Find R!?

$$R_x = \sum F_x = 80 \cos 45 + 20 \cos 60^\circ$$

$$= 15.65 \text{ kN}$$

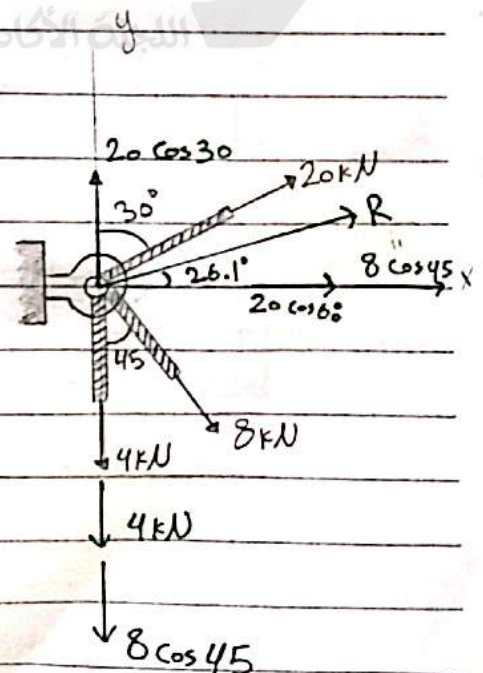
$$R_y = \sum F_y = 20 \cos 30 - 4 - 8 \cos 45$$

$$= 7.66 \text{ kN}$$

$$R = \sqrt{15.65^2 + 7.66^2}$$

$$= 17.42 \text{ kN}$$

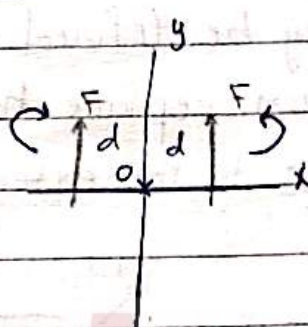
$$\theta = \tan^{-1} \frac{R_y}{R_x} = 26.1^\circ$$



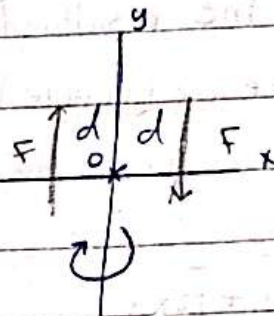
31/6/2014

DATE

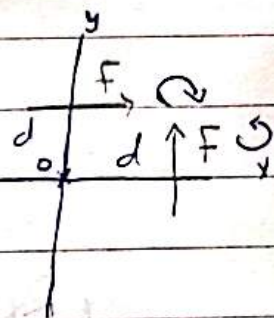
* Example:-



$R = 2F$
 $M = 0$



$R = 0$
 $M = F \cdot 2d$



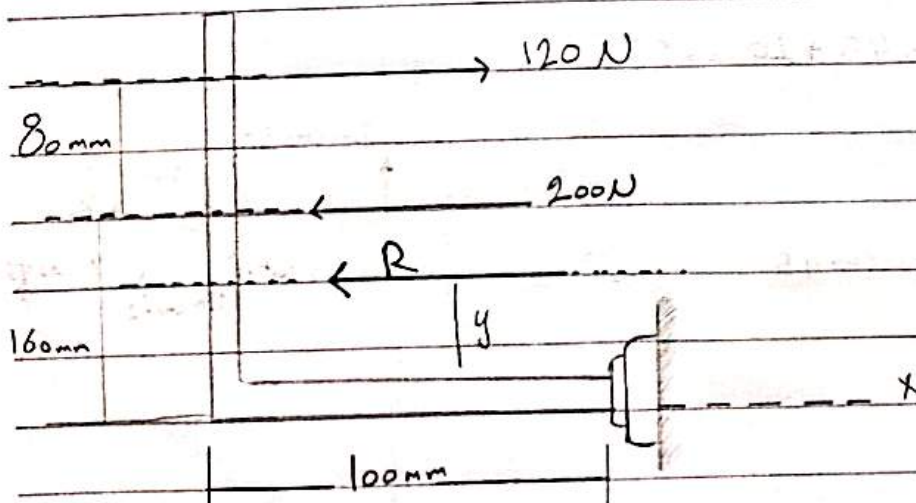
$R = F\sqrt{2}$
 $M = 0$

* Example:-

Replace the two forces by a single force R , and determine y

① $R = \sum F_i = 120 - 200 = -80 \text{ N}$

② $R \cdot y = 200(160) - 120(240) \Rightarrow y = 40 \text{ mm}$



3

* Example:-

Four Slender rods subjected to force as shown.

Determine F and θ if the resultant R is equal to 100N vertically upward.

$$* R_x = \sum F_x = 0$$

$$1000 - 750 \cos 30^\circ + F \cos \theta + 600 \cos 50^\circ = 0$$

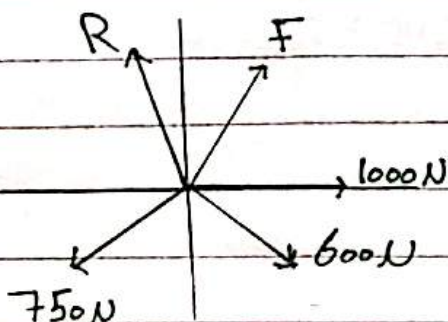
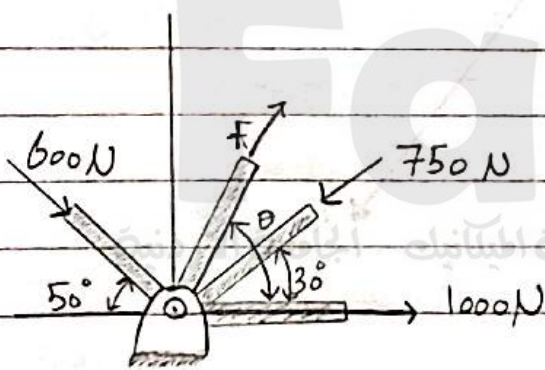
$$\Rightarrow F \cos \theta = -736\text{N}$$

$$* R_y = \sum F_y = 100$$

$$-750 \sin 30^\circ + F \sin \theta - 600 \sin 50^\circ = 100$$

$$\Rightarrow F \sin \theta = 935\text{N}$$

$$\Rightarrow \theta = 128.2^\circ \Rightarrow F = 1190\text{N}$$



* Force Systems 3D

* اليد اليمنى :-

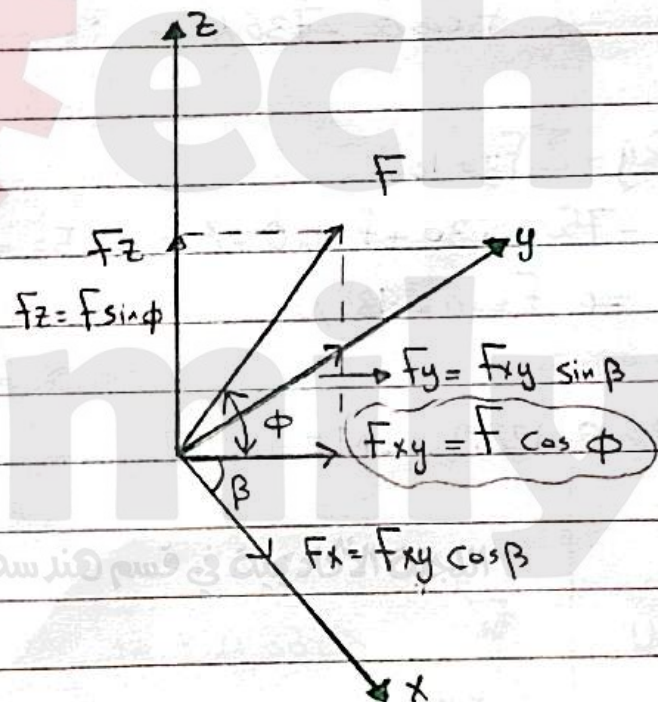
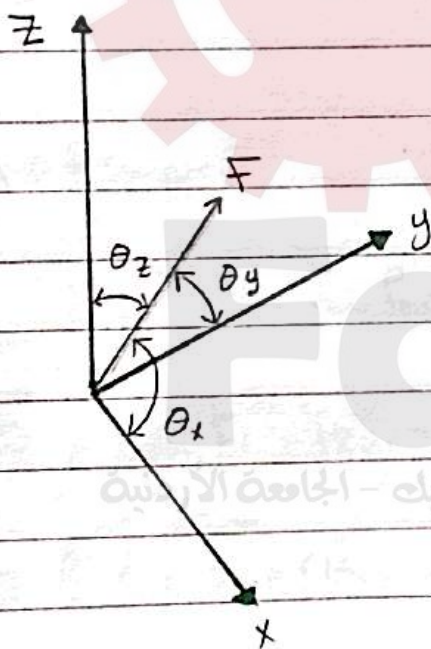
← يكون الدوران من X ← Y بأصابع اليد
والإبهام يكون +Z

The rectangular components of the force F in the 3 directions are

$$* F_x = F \cos \theta_x$$

$$* F_y = F \cos \theta_y$$

$$* F_z = F \cos \theta_z$$



$$\rightarrow F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\rightarrow F = \sqrt{F_z^2 + \underbrace{F_{xy}^2}_{= F_x^2 + F_y^2}}$$

$$\rightarrow F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

* Example:-^①

If $P_x = 60 \text{ kN}$, find P_y and P

* Sol:- $P_y = P \cos 40$

$P_{xz} = P \sin 40$

$\phi = \tan^{-1} \frac{2}{3} = 33.69^\circ$

$P_x = P_{xz} \cos \phi$
 $= P \sin 40^\circ \cos \phi$
 $= 60 \text{ kN}$

$\Rightarrow P = 112.2 \text{ kN}$

$P_y = P \cos 40^\circ$
 $= 85.94 \text{ kN}$

$AB = d$

$dx = x_2 - x_1$

$dy = y_2 - y_1$

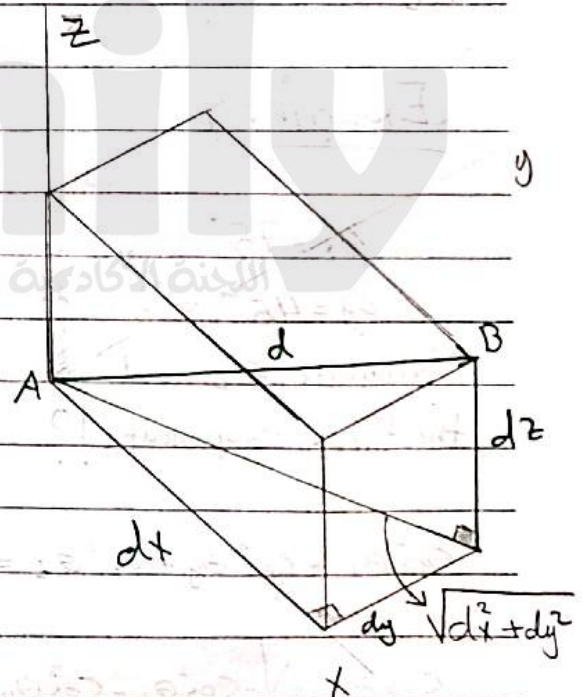
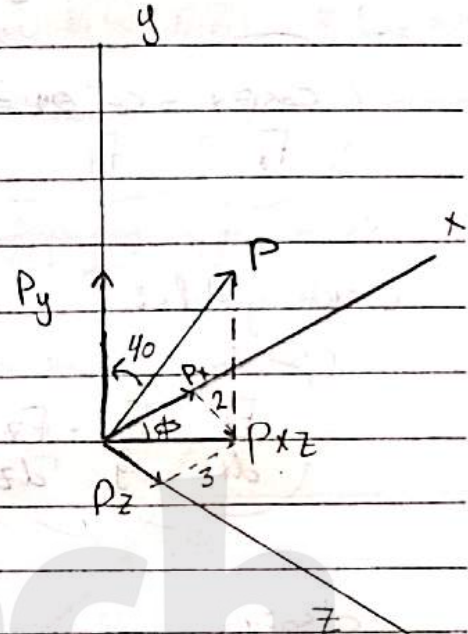
$dz = z_2 - z_1$

$d = \sqrt{dx^2 + dy^2 + dz^2}$

$\rightarrow \text{let } \cos \theta_x = L$

$\cos \theta_y = m$, we now that $L^2 + m^2 + n^2 = 1$

$\cos \theta_z = n$



← حاول تتخيلها مع Pp

also we can say that:-

$$\frac{\cos \theta_x}{F_x} = \frac{\cos \theta_y}{F_y} = \frac{\cos \theta_z}{F_z} = \frac{1}{F}$$

which yields:

$$\frac{F_x}{dx} = \frac{F_y}{dy} = \frac{F_z}{dz} = \frac{F}{d}$$

also:

$$\frac{\cos \theta_x}{dx} + \frac{\cos \theta_y}{dy} + \frac{\cos \theta_z}{dz} = \frac{1}{d}$$

Example:

$$\theta_x = 75^\circ$$

$$\theta_y = 45^\circ$$

Calculate θ_z and the force components!?

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\cos \theta_z = \sqrt{1 - \cos^2 \theta_x - \cos^2 \theta_y}$$

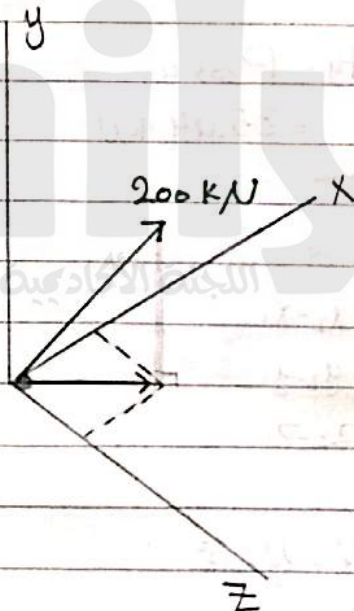
$$\theta_z = 48.85^\circ$$

$$F_x = F \cos \theta_x = 51.76 \text{ kN}$$

$$F_y = F \cos \theta_y = 141.42 \text{ kN}$$

$$F_z = F \cos \theta_z = 131.6 \text{ kN}$$

$$\mathbf{F} = (51.76\mathbf{i} + 141.42\mathbf{j} + 131.6\mathbf{k}) \text{ kN}$$



* Example: What is the projection of F onto Line OA ?

↳ Rectangular Component in the direction of Line OA .

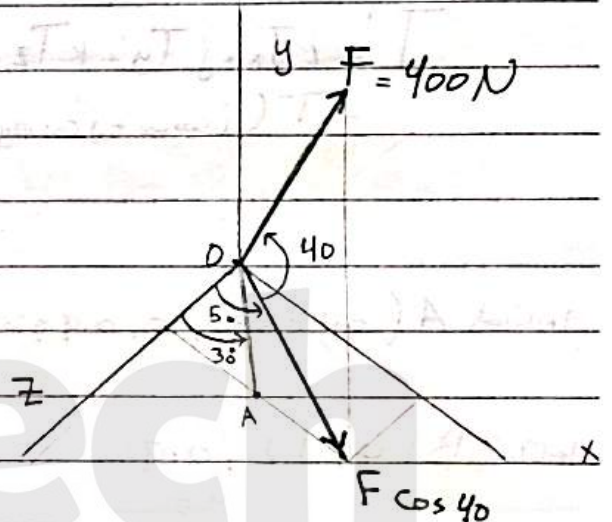
Sol: $F = i F_x + j F_y + k F_z$
 $F \cos \theta_x \quad F \cos \theta_y \quad F \cos \theta_z$

$$F_y = 400 \cos 50^\circ = 257 \text{ N.}$$

$$F_x = 400 \cos 40^\circ \cos 40^\circ = 234.7 \text{ N}$$

$$F_z = 400 \cos 40^\circ \cos 50^\circ = 197 \text{ N}$$

$$\rightarrow F = (234.7 i + 257 j + 197 k) \text{ N}$$

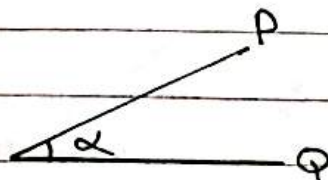


* Remark: (projection → Rectangular Comps)

$$S_o : F_{OA} = F \cos 40^\circ \cos 20^\circ = 288 \text{ N.}$$

* Dot Product :-

$$P \cdot Q = P \cdot Q \cos \alpha$$



* هو عبارة عن مركبة أحد المتجهين في اتجاه الآخر موزونة في الناتج الآخر

$$P \cdot Q \cos \alpha$$

له مركبة Q في اتجاه P



* Example:- (2/107 / Page 69)

T_{CD} ?

$$T_{CD} = T \cdot \lambda$$

$$T = i T_x + j T_y + k T_z$$

$$= T (\cos \theta_x i + \cos \theta_y j + \cos \theta_z k)$$

نقاط A (0.9 cos 30°, 0, 0.9 sin 30°)

نقاط B (0, 1.2, 0.9)

$$dx = -0.9 \cos 30^\circ = -0.779 \text{ m}$$

$$dy = 1.2 \text{ m}$$

$$dz = 0.45 \text{ m}$$

$$AB = d = \sqrt{dx^2 + dy^2 + dz^2} = 1.5 \text{ m}$$

$$\cos \theta_x = \frac{dx}{d}$$

$$\cos \theta_y = \frac{dy}{d}$$

$$\cos \theta_z = \frac{dz}{d}$$

Unit vector λ direction

$$T = \frac{100}{1.5} (-0.779 i + 1.2 j + 0.45 k)$$

$$\lambda = \frac{1}{1.5} (-0.779 i + 1.2 j + 0.45 k)$$

راجع احسبهم في البيت

واضرب المركبات في بعض

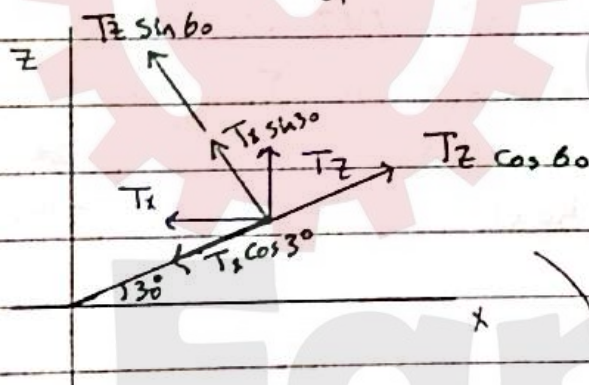
$$T_{CD} = T \cdot \lambda = 46 \text{ N}$$

$$T_{CD} = \left(\frac{100 \times -0.779}{1.5} \times \frac{0.779}{1.5} \right) + \left(\frac{1.2 \times 100}{1.5} + \frac{1.2}{1.5} \right) + \left(\frac{0.45 \times 100}{1.5} + \frac{0.45}{1.5} \right)$$

$$= -27 + 64 + 9 = 46$$

$$+ T_x = T \cos \theta_x = T \frac{dx}{d} = 52 \text{ N} \quad + T_y = T \cos \theta_y = T \frac{dy}{d} = 80 \text{ N}$$

$$+ T_z = T \cos \theta_z = T \frac{dz}{d} = 30 \text{ N}$$



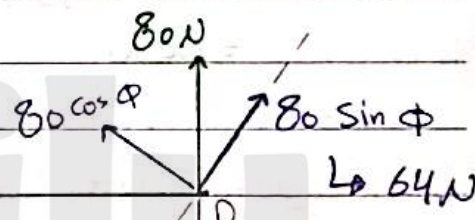
y

T_y

$T_x \cos 30^\circ$ $T_z \cos 60^\circ$

45 N $\hookrightarrow$ 15 N

resultant = 30 N



80 N

$80 \cos \phi$ $80 \sin \phi$

$\hookrightarrow$ 64 N

1.2

0.9

$\phi = \tan^{-1} \left(\frac{1.2}{0.9} \right)$

$= 53.13^\circ$

$30 \sin \phi$

30 N

$30 \cos \phi$

$\hookrightarrow$ 18 N

$$\therefore 64 - 18 = 46 \checkmark$$

الكل صحيح

* Exam : 17/7/2014 - Thursday *

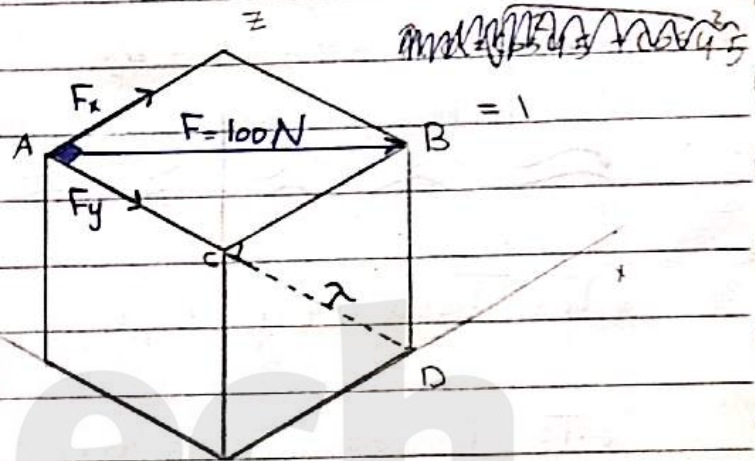
PAGE 6/7/2014
DATE Sunday.

* Example : ① Calculate F_{CD} ?!

$$F_x = F \cos 45^\circ$$

$$F_y = F \cos 45^\circ$$

$$F_z = 0$$



$$\rightarrow \mathbf{F} = F(\mathbf{i} \cos 45^\circ + \mathbf{j} \cos 45^\circ)$$

$$= F(\mathbf{i} \cos 45^\circ + \mathbf{j} \cos 45^\circ)$$

$$\rightarrow \lambda = \frac{1}{\sqrt{2}}(\mathbf{i} \cos 45^\circ + \mathbf{j} \cos 45^\circ)$$

$$F_{CD} = \mathbf{F} \cdot \lambda$$

$$= F \cos^2 45^\circ$$

$$= 50 \text{ N}$$

$$\mathbf{F} \cdot \lambda = F \cos 45^\circ (\cos 45^\circ) + F \cos 45^\circ (\cos 45^\circ) + F \cos 45^\circ (0)$$

$$\mathbf{F} \cdot \lambda = F \cos^2 45^\circ$$

$$= 50 \text{ N}$$

Problem 2/97, Page 67

* Example : 2

Tension in Cable AB is 2.4 kN

vector expression of $\mathbf{T}$ acting on A

$$\beta = \tan^{-1} \frac{1}{2} = 26.6^\circ$$

$\mathbf{T}_{AC}$

$$\phi = \tan^{-1} \frac{\sqrt{3}}{5} = 24.1^\circ$$

$$T_{xy} = T \sin \phi = 0.98 \text{ kN}$$

$$T_z = T \cos \phi = 2.19 \text{ kN} = -2.19 \text{ kN (downward)} \downarrow$$

$$T_x = T_{xy} \cos \beta = 0.876 \text{ kN}$$

$$T_y = T_{xy} \sin \beta = 0.438 \text{ kN}$$

$$T_{AC} = T, \Delta$$

$$\lambda = \frac{1}{5.74} (2i - 2j - 5k)$$

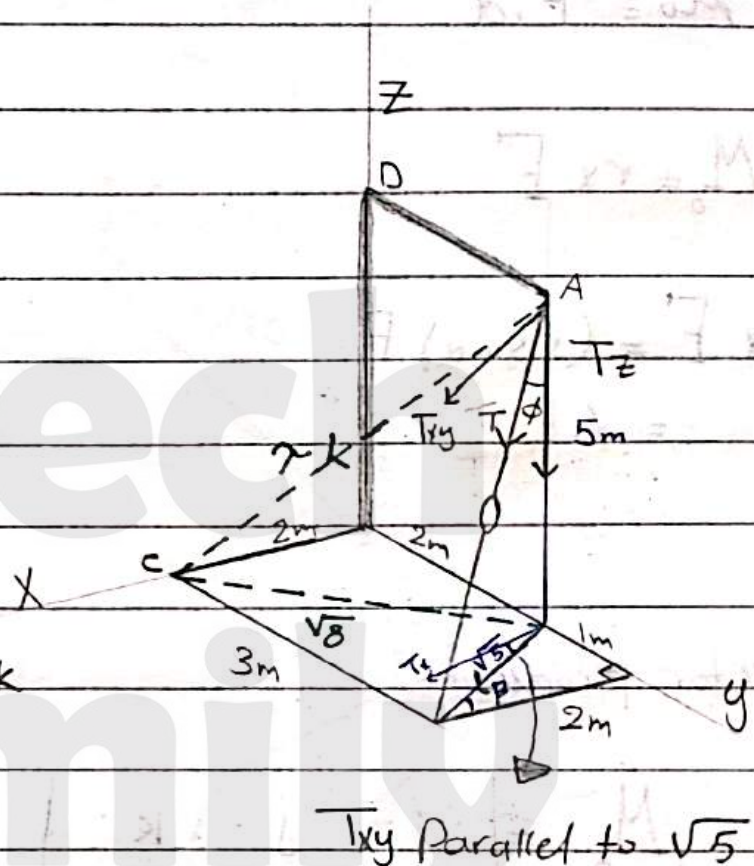
$$d = dr^2 + dy^2 = dz^2$$

$$d = \sqrt{(\sqrt{8})^2 + 5^2}$$
$$= 5.74$$

$$T = 0.876i + 0.438j - 2.19k$$

$$\vec{A} = \frac{1}{5.74} (2\hat{i} - 2\hat{j} - 5\hat{k})$$

$$T \cdot \lambda = 2.06 \text{ kN}$$

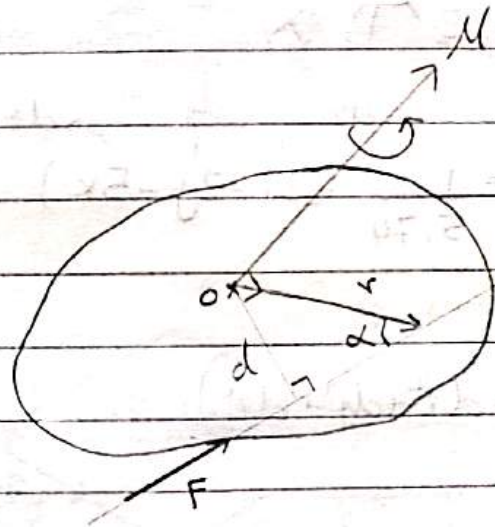


* Moment 3D

$$M_o = F \cdot d$$

$$M_o = r \times F$$

$$r \times F = (r \sin \alpha) F \\ = d \cdot F$$



* Vector product

$$M_o = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_o = \overset{M_x}{i(r_y F_z - r_z F_y)} + \overset{M_y}{j(r_z F_x - r_x F_z)} + \overset{M_z}{k(r_x F_y - r_y F_x)}$$

أو $j(r_x F_z - r_z F_x)$ + استبدال الترتيب

$$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

i	j	k	i	j
r_x	r_y	r_z	r_x	r_y
F_x	F_y	F_z	F_x	F_y

- +

$$M_x = -F_y r_z + F_z r_y$$

$$M_y = +F_x r_z - F_z r_x$$

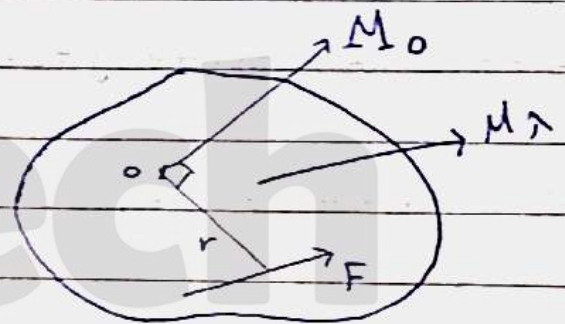
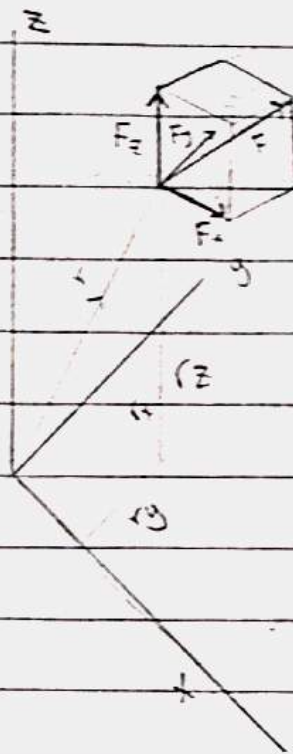
$$M_z = +F_y r_x - F_x r_y$$

$$M_\lambda = M_o \cdot \lambda$$

$$= r \times F \cdot \underline{n} \rightarrow \text{unit vector}$$

in the direction
of λ

$$M_\lambda = (r \times F \cdot n) n$$



* Example:

Calculate $M_o = ?!$

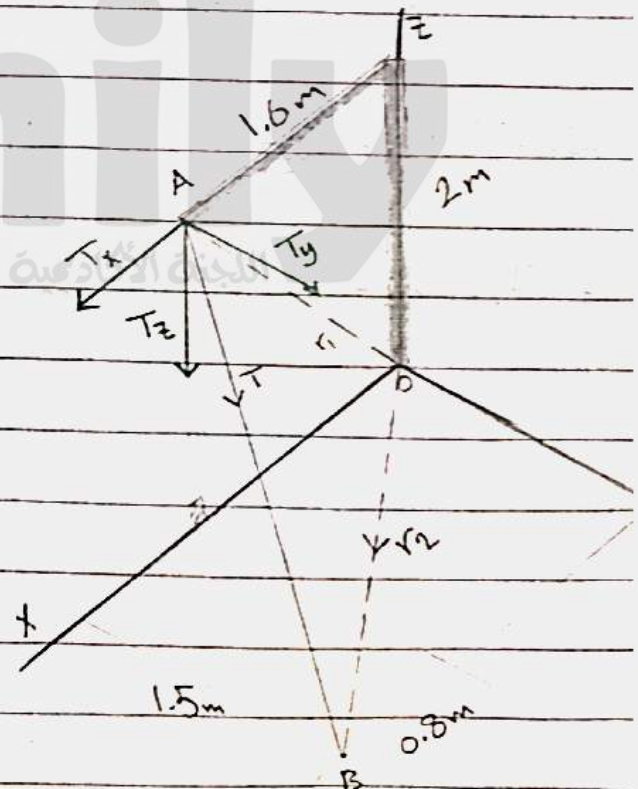
$$T = 1.2 \text{ kN}$$

$$\left. \begin{array}{l} dx = 0.8 \text{ m} \\ dy = 1.5 \text{ m} \\ dz = -2 \text{ m} \end{array} \right\} \begin{array}{l} d = 2.625 \text{ m} \end{array}$$

$$T_x = \frac{dx}{d} \cdot T$$

$$T_y = \frac{dy}{d} \cdot T$$

$$T_z = \frac{dz}{d} \cdot T$$



$$T = i T_x + j T_y + k T_z$$

$$= \frac{1.2}{2.825} (0.8i + 1.5j - 2k)$$

$$M_o = r_2 \times T$$

ر-أى متجه من 0 إلى خط عمل T
إلى r₂, r₁

$$r = 2.4i + 1.5j$$

$$\checkmark M_o = 0.457 (-3i + 4.8j + 2.4k)$$

$$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2} = 2.81 \text{ kN.m.}$$

لا نقل moment لأنها موازية $T_x \rightarrow$
لا نقل moment لأنها تقاطعها $T_z \rightarrow$

$$\checkmark M_x = -T_y (2) = 1.37 \text{ kN.m} \Rightarrow T_x \rightarrow$$

$$M_y = T_x (2) + T_z (1.6) = 2.194 \text{ kN.m}$$

$$M_z = T_y (1.6) = 1.097 \text{ kN.m} \Rightarrow T_x \rightarrow z\text{-axis} \text{ تقع}$$

$$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2} = 2.81 \text{ kN.m} \checkmark$$

* Equilibrium :-

A rigid body is said to be in equilibrium when the exerted force acting on it form a system of forces equivalent to zero:

$$R = \sum F = 0 \text{ and}$$

$$M = \sum M = 0$$

* Mechanical System Isolation :-

Free Body Diagram FBD

diagrammatic representation of the body or bodies under consideration showing all forces applied to it by other bodies that imagined to be removed.

* Construction of the F.B.D. :-

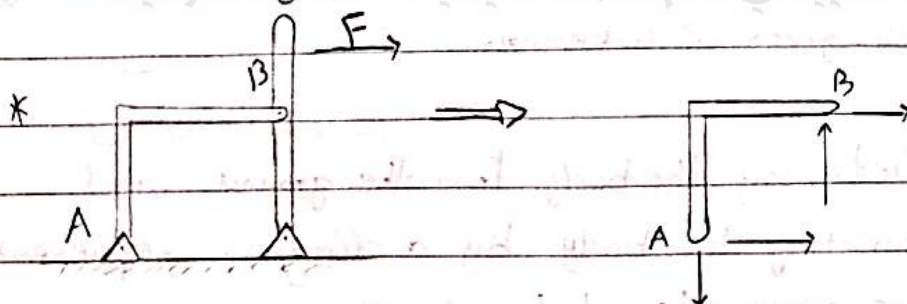
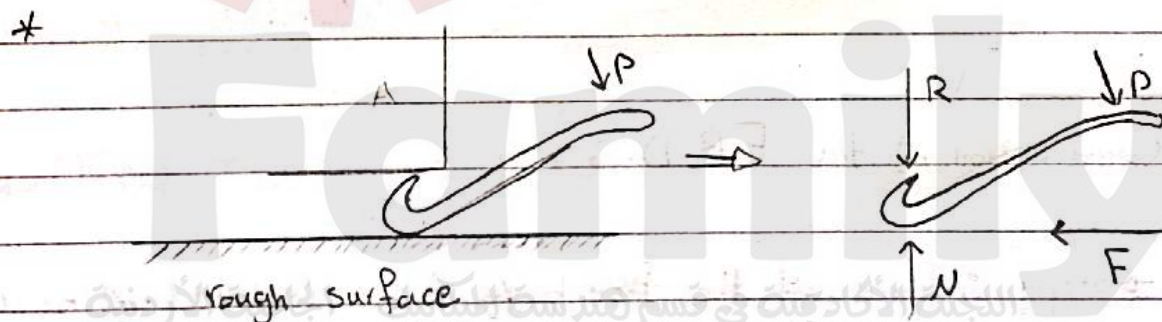
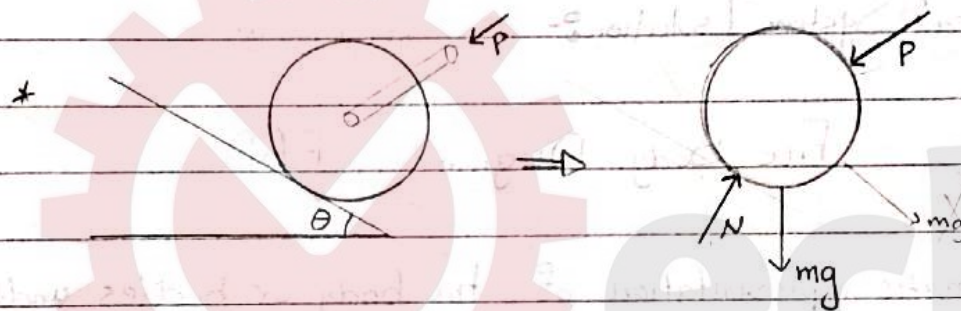
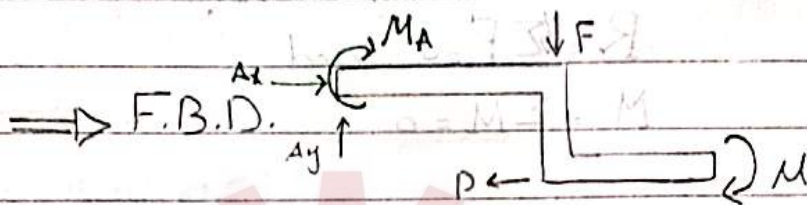
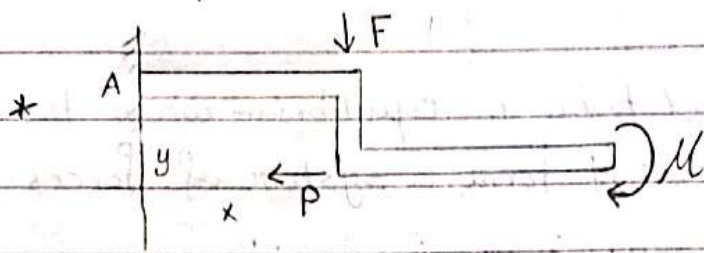
→ Step 1 : choice of the Free body to be isolated. A body which gives all unknowns

→ Step 2 : Detaching the body from the ground and separating the body by a diagram representing its complete external boundaries.

→ Step 3 : Indicating all external forces.

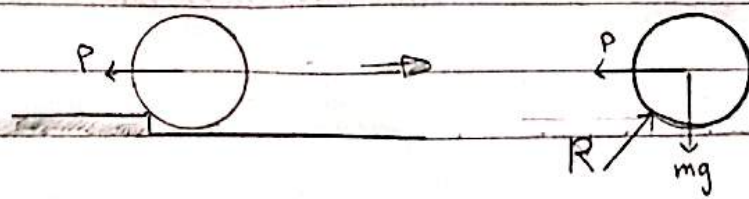
→ diagram should include axes, dimensions, directions.

* Example :-

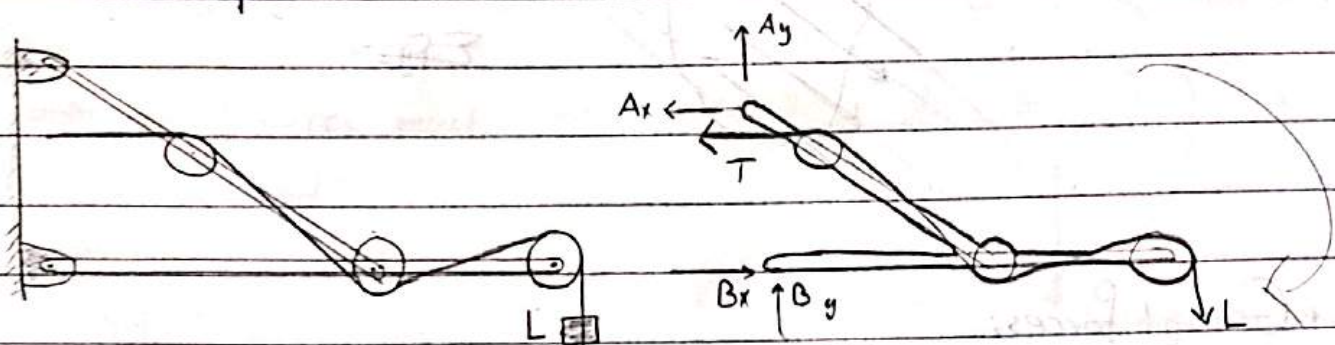


* Example :- wheel of mass m on verge of being rolled over curb by Pull P ?

* علی دینک این تاقاب



* Example :-



* Equilibrium Conditions:

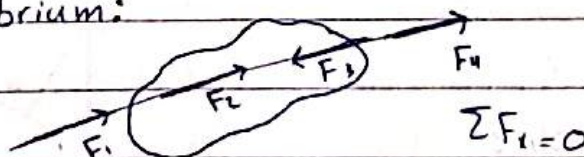
The necessary and sufficient conditions for complete equilibrium in 2D are:

$$\sum F_x = 0$$

$$\sum F_y = 0 \text{ and } \sum M = 0$$

* Categories of equilibrium:

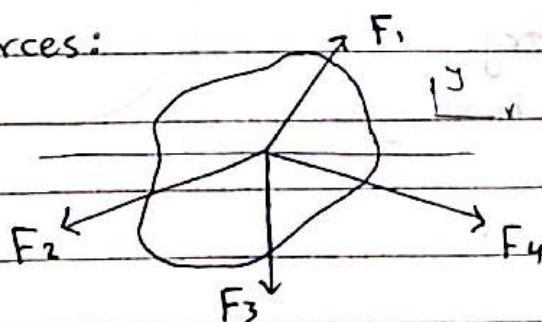
* Collinear forces:



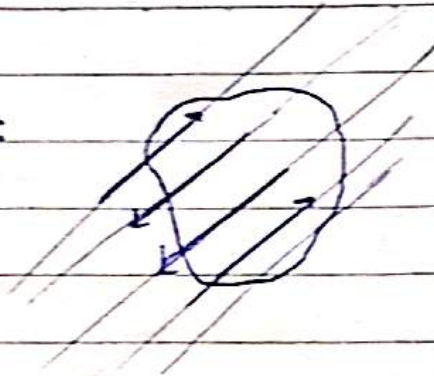
(من نفس النقطة) * Concurrent forces:

$$\sum F_x = 0$$

$$\sum F_y = 0$$



+ Parallel Forces:



$$\sum F_x = 0, \sum M = 0$$

$$\sum F_y = 0$$

+ General Forces:



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

13/7/2014: Sunday

* Special Cases of Equilibrium:-

1] Equilibrium of a two-force body.



the two forces must be equal, opposite and collinear.

2] Equilibrium of 3-force body.

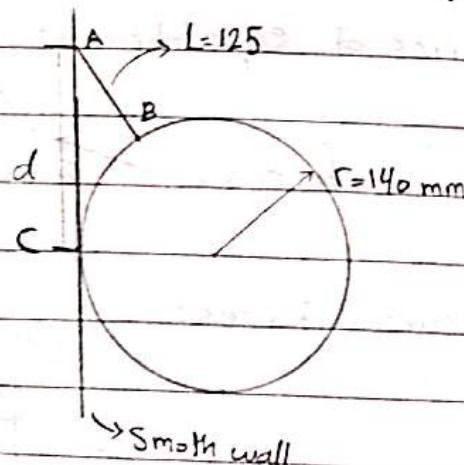
Multi اختر من ٣ the 3 forces must be either concurrent or Parallel

* Example:

Thin ring of mass 2kg.

Determine d, T in string AB

R_c



The ring is 3-Force body

⇒ the 3 Forces must be concurrent.

Sol:

$$d = \sqrt{265^2 + 140^2} = 225 \text{ mm.}$$

$$\sum F_x = 0$$

$$T_{AB} \cos \theta = R_c$$

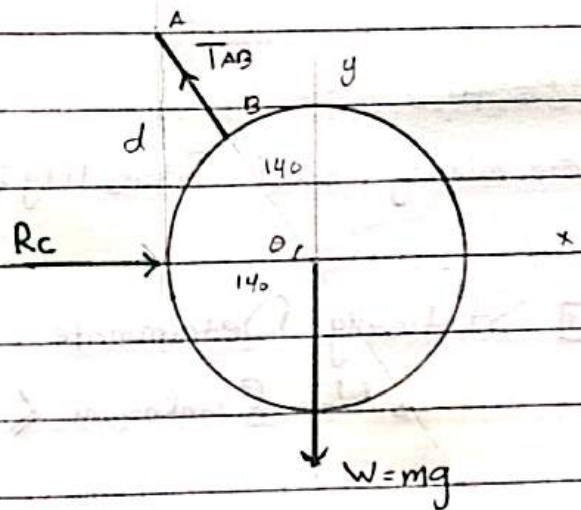
$$\sum F_y = 0$$

$$T_{AB} \sin \theta = W$$

$$T_{AB} = \frac{2(9.81)}{\sin \theta} = 23.11 \text{ N}$$

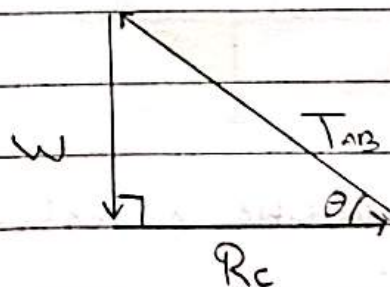
$$\Rightarrow \text{From } \sum F_x = 0$$

$$R_c = 12.21 \text{ N}$$



From the force triangle:

$$\frac{T_{AB}}{\sin \theta} = \frac{R_c}{\cos \theta} = \frac{W}{\sin \theta}$$



* ~~Determines~~

+ Determinacy and Stability:

1] Statically Determinate

→ No. of unknowns $\leq$ Equations of equi.

2] Statically Indeterminate

→ No. of unknowns $>$ Equations of equi.

* Examples:-

Unknowns 4 Equ. 3 $\Rightarrow$ 1st degree indeterminate.



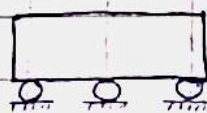
Unknowns 5 Equ. 3 $\Rightarrow$ 2nd degree indeterminate



Unknowns 6 Equ. 3 $\Rightarrow$ 3rd degree indeterminate.



* Stability:

1]  , unstable no restraint against horizontal movement

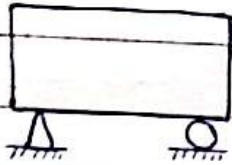
all constraint parallel
كواج

حتى يكون Stable القوى يجب أن لا تكون موازية ولا متقاطعة.
لهما نتيجة من اللحظات أو الكواج

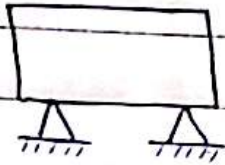
2]  , unstable no restraint against rotation.

constraint concurrent

كواج



Stable determinate adequate constraints.



Stable Indet.

↳ Redundant constraints (causes extra fixability)

① 14/7/2014

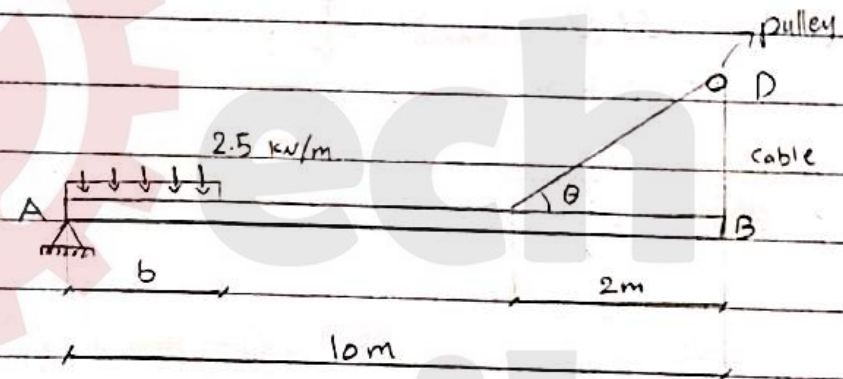
* Example:

$$\theta = 60^\circ$$

Beam AB of 100 kg

If $T_{\max}$ in the cable is 800 N

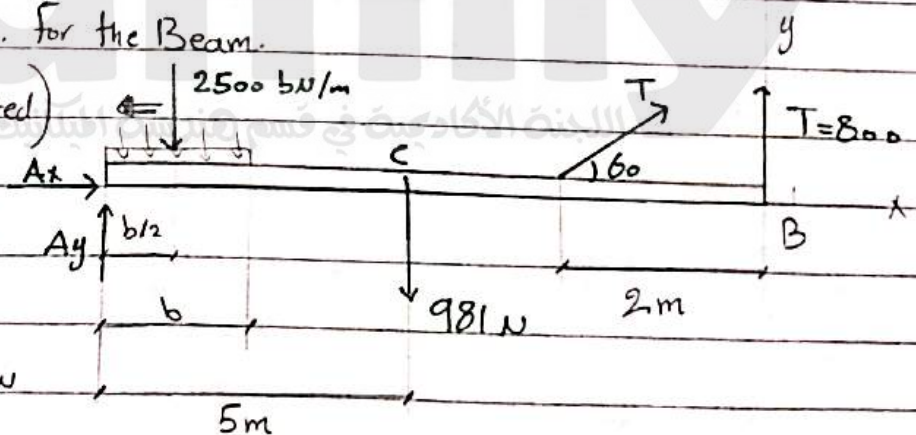
determine b , A_x and A_y



* Solⁿ:- F.B.D. for the Beam.

$2500b$: (equivalent concentrated force)

* 2500 N/m $\rightarrow$ each 1 m make
force eq^l to 2500 N
on the beam



$$\rightarrow \sum M_A = 0 \quad -2500b\left(\frac{b}{2}\right) - 981(5) + T \sin(60^\circ) + T(10) = 0$$

$$\Rightarrow b = 2.63 \text{ m}$$

$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$A_x + T \cos 60 = 0$$

$$A_x = 400 \text{ N} \leftarrow$$

$$\sum F_y = 0$$

$$A_y - 2500(2.63) - 981 + T \sin 60 + T = 0$$

$$A_y = 6663 \text{ N}$$

$$= 6.66 \text{ kN}$$

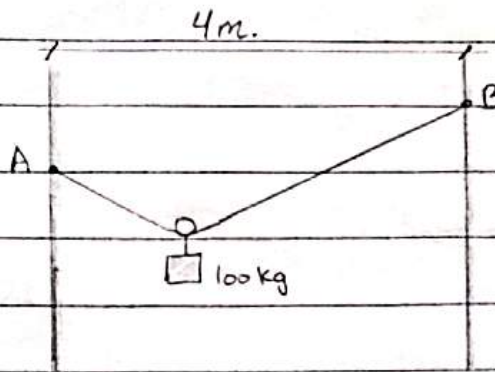
15/7/2014 Tuesday.

PAGE
DATE

①
* Example:

length of cable $AB = 6m$

Calculate the Tension in the cable.



$$\sum F_x = 0$$

$$T \cos \alpha = T \cos \beta$$

$$\Rightarrow \alpha = \beta$$

From geometry:

$$a \cos \beta + b \cos \alpha = 4$$

$$(a+b) \cos \beta = 4$$

$$a+b=6 \Rightarrow \beta = \cos^{-1} \frac{4}{6}, \beta = 48^\circ$$

$$\sum F_y = 0$$

$$2T \sin \beta = 981N$$

$$T = 658N$$

* Example:

Force in the spring

$$F = K \cdot x$$

(Elongation

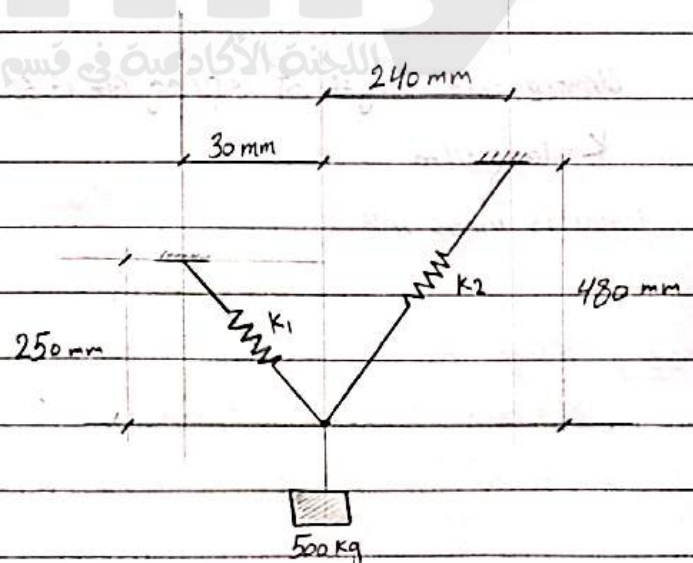
or contraction)

* Springs stiffness

$$K_1 = 20 \text{ kN/m}$$

$$K_2 = 10 \text{ kN/m}$$

Determine the unstretched length of the springs.



$$\theta = \tan^{-1} \frac{480}{240} = 63.43^\circ$$

$$\phi = \tan^{-1} \frac{250}{300} = 39.8^\circ$$

$$\sum F_x = 0$$

$$F_{s2} \cos \theta = F_{s1} \cos \phi \dots (1)$$

$$\sum F_y = 0$$

$$W = F_{s1} \sin \phi + F_{s2} \sin \theta \dots (2)$$

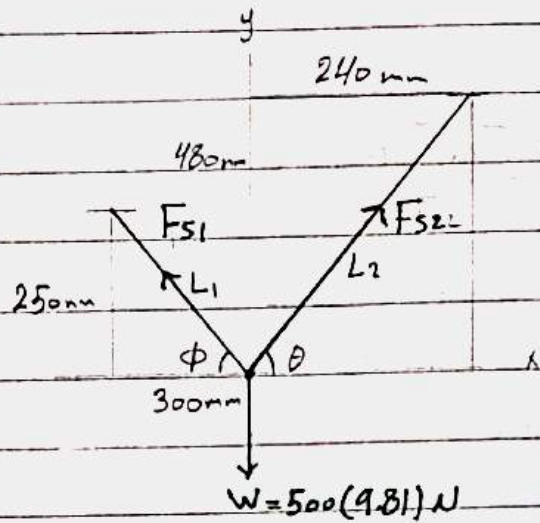
$$\Rightarrow F_{s1} = 2.253 \text{ kN}$$

$$\Rightarrow F_{s2} = 3.871 \text{ kN}$$

$$\rightarrow F_{s1} = K_1 \cdot X_1 \Rightarrow L_1 - L_{01} = \frac{F_{s1}}{K_1}$$

$$L_{01} = 278 \text{ mm}$$

الطول الأصلي غير مشدود



$$L_1 = 390.5 \text{ mm}$$

$$L_2 = 536.7 \text{ mm}$$

$$\rightarrow F_{s2} = K_2 X_2 \Rightarrow L_2 - L_{02} = \frac{F_{s2}}{K_2}$$

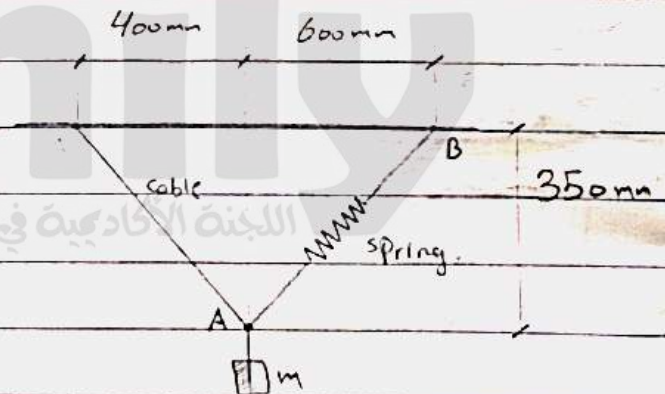
$$L_{02} = 150 \text{ mm}$$

* Example:

Unstretched length of spring AB is 600mm

$$K = 1000 \text{ N/m}$$

What is mass m?

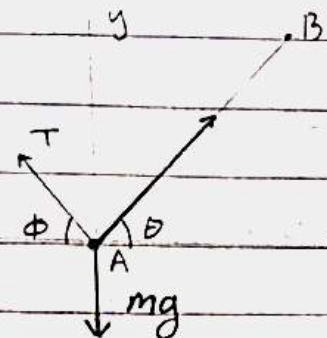


$$\text{Sol: length AB} = 694.62 \text{ mm}$$

$$e = 694.62 - 600 = 94.62 \text{ mm}$$

$$\theta = 30.26^\circ (\tan^{-1} \frac{350}{600})$$

$$\phi = 41.2^\circ (\tan^{-1} \frac{350}{400})$$



$$F_s = k \cdot e$$

$$= \frac{1000 \cdot (34.62)}{1000} \rightarrow \text{حوّل إلى m}$$

$$= 34.62 \text{ N}$$

$$\sum F_x = 0$$

$$F_s \cos \theta = T \cos \phi \rightarrow T = \frac{F_s}{\cos \phi} \cos \theta = \frac{34.62}{\cos 41.2^\circ} \cos 30.2^\circ \Rightarrow T = 39.74 \text{ N}$$

$$\sum F_y = 0$$

$$F_s \sin \theta + T \sin \phi = mg \rightarrow m = \frac{F_s \sin \theta + T \sin \phi}{g} \rightarrow m = 4.447 \text{ kg}$$

16/7/2014

* Example:

rod AB $m = 40 \text{ kg}$

wheels 80 kg each

→ Show that the 3 bodies

can not be in equi. as shown.