VI-Collected Data:

Basic Parameters:

Table-3.1 Dimensions to be used according to figures-3.1 & 2

Parameter	Value	Parameter	Value
L(cm)	50.8	r_c(mm)	9.5
w(mm)	25.2	h_c(mm)	51.2
h(mm)	12.5	R(mm)	38.0
b(mm)	483	h_m(mm)	51.0
r(mm)	4.5		

Part one-Bifilar Suspension Technique:

Table-3.2 Collected data for the *Bifilar Suspension Technique part*

Trial	L(cm)	T(second)
1	77.8	11.8
2	70	11.31
3	64.5	10.65
4	57	10.13
5	47	9.21
6	40.5	8.18

Part two-Auxiliary Mass Method:

L= 27 cm   m= 1.804 (kg)

Table-3.3 Collected data for the *Auxiliary Mass Method* part

Trial	L(cm)	T(second)
1	18	10.5
2	15.5	9.88
3	13	8.45
4	10.5	7.73
5	8	6.95
6	5.5	6.16

VII-Results:

Part one- Bifilar Suspension Technique:

M = ...1.1551...(kg)

Table-3.4 Data processing analysis for the *Bifilar Suspension Technique part*

Trial	L(cm)	t(second)	t^2(second^2)
1	77.8	1.180	1.392
2	70.0	1.131	1.279
3	64.5	1.065	1.134
4	57.0	1.013	1.026
5	47.0	0.921	0.848
6	40.5	0.818	0.669

fig.1:- t^2 vs l

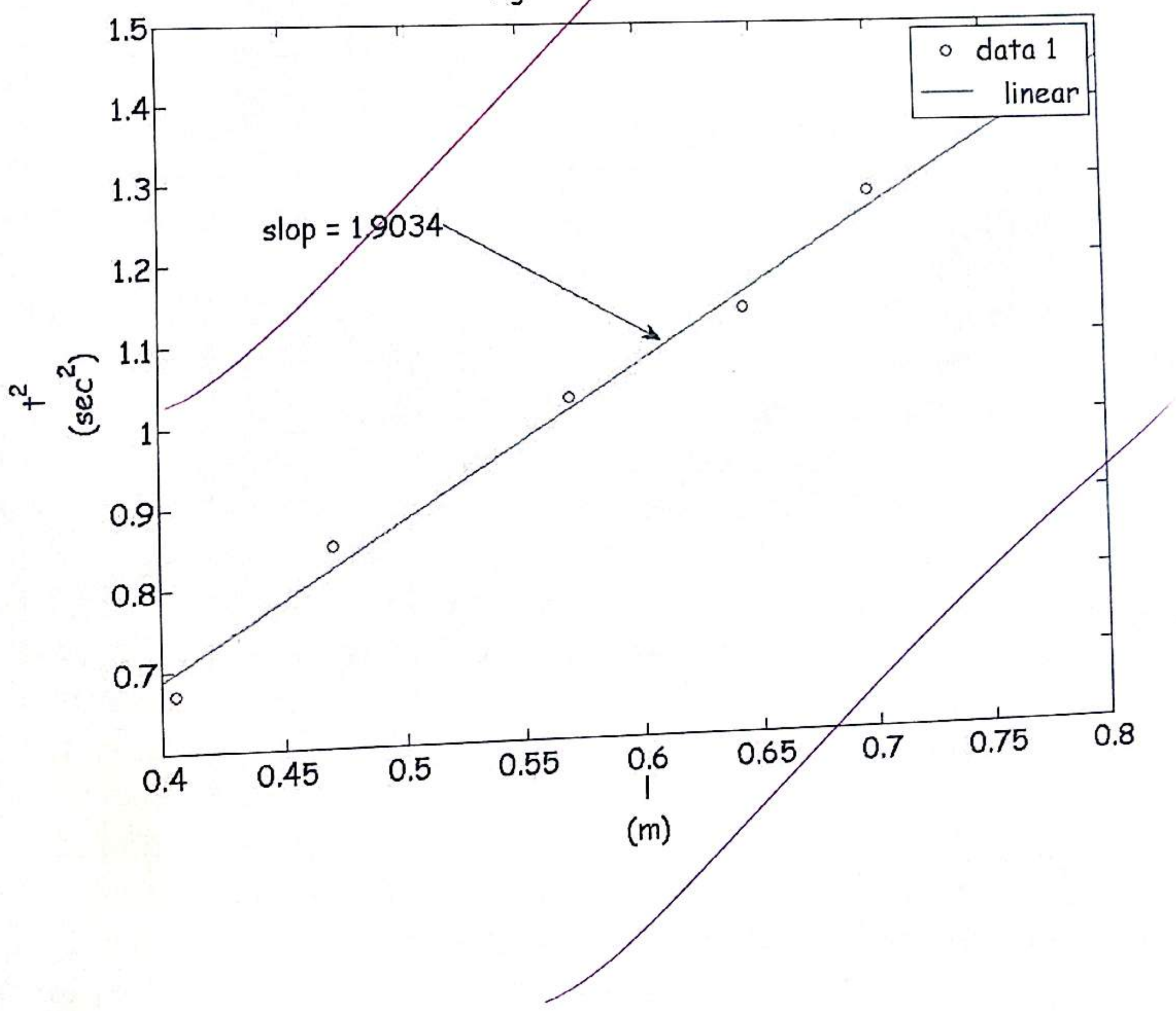

Table-3.5 Data processing results for the *Bifilar Suspension Technique part*

Quantity	Slope (sec.^{2}/m)	I(kg/m^2)
From Figure-3.3	1.9034	0.03186

Part two- Auxiliary Mass Method:

Table-3.6 Data processing analysis for the *Auxiliary Mass Method* part

Trial	Y(cm)	I_m(kg.m^2)	t^2(second^2)
1	18	0.119504176	1.3225
2	15.5	0.089287176	0.976144
3	13	0.063580176	0.714025
4	10.5	0.042383176	0.597529
5	8	0.025696176	0.483025
6	5.5	0.013519176	0.379456

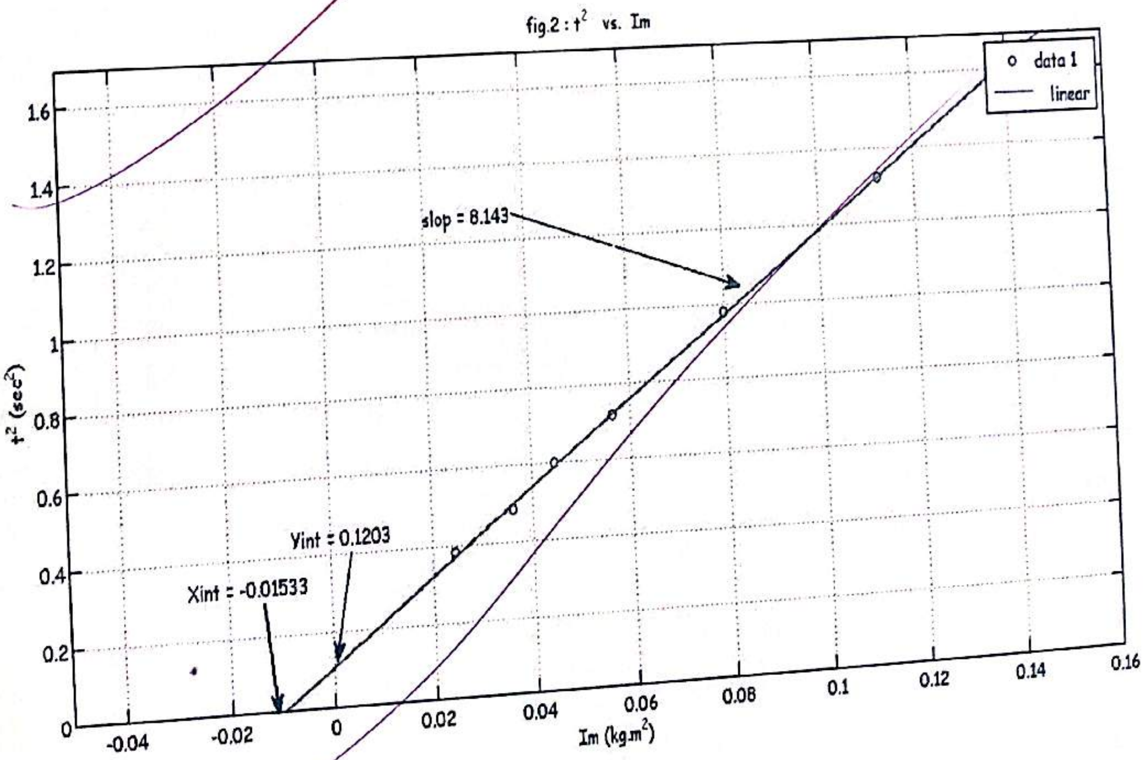

Table-3.7 Data processing results for the *Auxiliary Mass Method* part

From figure-3.4			
Slope($s^2/m^2.kg$)	8.143	**g(m/sec.^{2})**	9.424
Y_{Int}(sec.^{2})	0.1203	**I(kg.m^2)**	0.030756372
X_{Int}(kg.m^2)	-0.01533	**I(kg.m^2)**	0.3066

Part three- Analytical Solution:

Table-3.8 Analytical determination of the mass moment of inertia I

I_S(kg.m^2)	0.0268420
I_H(kg.m^2)	0.0016856
I_C(kg.m^2)	0.0013218
$I = I_S - I_H + I_C$ (kg.m^2)	0.0264782

Comparison :

Table-3.9 comparison of I obtained by the two methods with the analytical value

Method:	I(kg.m^2)	Percentage error (%)
Analytical	0.0264782	-
Bifilar Suspension	0.0318600	20.3
Auxiliary Mass	0.0307563	16.2

Sample of calcalations

Part one:-

- $M = \rho V$

$= \text{Density} * \text{Volum}$

# of holes

$7800 \frac{Kg}{m^3} \quad (\underbrace{LWh}_{\text{Volume without holes}} - 15\pi \underbrace{r^2 h}_{\text{Volume of a hole}})$

$= (7800) * ((0.508 * 0.0252 * 0.0125) - 15(\pi * 0.0045^2 * 0.025))$

$= 1.1551 \text{ Kg}$ #

- From Fig1 → slop = 1.9034 sec^2/m

- $\text{slop} = \frac{16\pi^2 I}{Mgb^2} \rightarrow I = \frac{\text{slop} * M * g b^2}{16\pi^2}$

$I = \frac{(1.9034)(1.1551)(9.81)(0.483)^2}{16\pi^2}$

$I = 0.03186 \text{ Kg.m}^2$ #

• I :

① $Y_{int} \Rightarrow \quad Y_{int} = \dfrac{16\pi^2 I L}{g b^2 (M+2m)} \rightarrow I = \dfrac{Y_{int}\, g b^2 (M+2m)}{16\pi^2 L}$

$\boxed{Y_{int} = 0.1203}$

$\therefore \quad I = \dfrac{(0.1203)(9.81)(0.483^2)(1.1551 + 2 * 1.804)}{16 * \pi^2 * 0.27}$

$= 0.030756372 \ Kg.m^2$ ⋕

② $X_{int} \Rightarrow \quad X_{int} = \dfrac{-I}{2} \rightarrow I = -2 * X_{int}$

$\boxed{X_{int} = -0.01533}$

$I = (-2)(-0.01533)$

$= 0.3066 \ Kg.m^2$ ⋕

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Part three :-

1) $I_s = \frac{M_s L^2}{12} = \frac{\rho w h L^3}{12} = \frac{7800 * 0.0252 * 0.0125 * 0.508^3}{12}$

$= 0.026842 \ Kg.m^2$

2) $I_H = \frac{15}{2} M_H r^2 + 2 M_H \sum X^2 = \rho \pi r^2 h \left(\frac{15}{2} r^2 + 2 \sum X^2 \right)$

X : distance for the holes.

$I_H = 7800 * \pi * 0.0045^2 * 0.0125 * \left[\frac{15}{2} * 0.0045^2 + 2 * (0.055^2 + 0.08^2 + 0.13^2 + 0.105^2 + 0.155^2 + 0.18^2 + 0.205^2) \right]$

$I_H = 0.0016856 \ Kg.m^2$ #

3) $I_c = M_c R_c^2 + M_c \left(\frac{b^2}{2} \right) \rightarrow I_c = \rho \pi r_c^2 h_c \left(r_c^2 + \frac{b^2}{2} \right)$

$I_c = 7800 * \pi * 0.0095^2 * 0.00512 * \left(0.0095^2 + \frac{0.483^2}{2} \right)$

$I_c = 0.0013218 \ Kg.m^2$

#

4) $I = I_s - I_H + I_c = 0.026842 - 0.0016856 + 0.0013218$

$I = 0.0264782 \ Kg.m^2$ #

Error's :-

$$E = \frac{|I_{analytical} - I_{Bifilar}|}{I_{analytical}} * 100\%$$

$$= \frac{0.0264782 - 0.03186}{0.0264782} * 100\% = 20.3\%$$

#

$$E = \frac{|I_{analytical} - I_{Aux.mass}|}{I_{analytical}} * 100\% = 16.2\%$$

#

• Note :- the error in estimating I is somehow big that would be caused by error in taking the priod time τ.

Part two :-

- $m = \rho V = \rho(\pi r^2 h) = 7800(\pi * 0.038^2 * 0.051)$

$= 1.804\ Kg$ #

- I_m For trail (1) :-

$I_m = m(R^2 + 2Y^2)$ eq. page 24

$R = 0.038\ m$, $m = 1.804\ Kg$

$Y = 18 * 10^{-2}\ cm$

$\therefore$ $I_m = 1.804(0.038^2 + 2 * 0.18^2) = 0.119504176\ Kg.m^2$ #

- g :

* From fig 2 : slop = 8.143 → eq → $slop = \frac{32\pi^2 L}{g b^2 (M+2m)}$

$\therefore\ g = \frac{32 * \pi^2 * L}{b^2(M+2m) * slop} = \frac{(32)(\pi^2)(0.27)}{(0.483)^2((1.1551) + 2*1.804)(8}$

$\therefore\ g = 9.424\ m/s^2$ #

1. **In the first part, what modifications should be done *(concerning the derivation of equation of motion)* in order to determine the mass moment of inertia about any point other than the middle point of the beam? Derive the equation of motion for this case.**

Changing the center of oscillation to a point other than the center of gravity (CG) will cause a behavior of the beam similar to that of compound pendulum. For this reason, the parallel axis theorem is applied to get the mass moment of inertia about the new pivot point:

$$I_b = I_{CG} + md^2 = m(K_{CG}^2 + d^2)$$

Where; d = The distance between the middle point (CG) and the pivot point.

$\Rightarrow$ Equation of motion becomes:

$$\left(I_{CG} + md^2\right)\ddot{\theta} + \left(Mgb/2\right)\theta = 0$$

2. **In the second part *(the Auxiliary Mass Method part)*, is it acceptable to use only one mass at either sides of the beam? Explain?**

The experiment is designed to obtain a horizontal oscillation of the beam. Using one mass on either side will disturb this condition and the beam will deflect at the heavier side hence causing a vertical oscillation in addition to horizontal oscillation.

3. **Referring to the derivation of the equation of motion for the beam, why is it important to keep the angle of oscillation of the beam small during the execution of the experiment? What is the basic assumption that is based on assuming a small angle of oscillation?**

In the derivation of equation of motion, linearity of the motion was assumed. This will make the derivation of equation of motion and using it a matter of ease. This can be achieved by assuming the following:

$$\left.\begin{array}{l} \sin\theta \approx \theta \\ \tan\theta \approx \theta \\ \cos\theta \to 1 \end{array}\right\} \text{for small angles of } \theta \,;\, \theta \in [0^\circ, 10^\circ]$$

Thus small angles of oscillation are to be obtained in performing such experiments.

4. From your results, comment on the accuracy of the two methods, mentioning the major sources of errors in each part of the experiment?

From the results obtained above, it was shown that the bifilar suspension method gave more accurate results than the auxiliary mass method. Some possible sources of error can be recorded as:

- Inaccurate time measurement.
- Errors in major length parameters measurement.
- Not well even fixing of the beam in the horizontal plane due to differences in Chord's lengths.
- The slender rod inserted in the middle point as an axis of rotation was not well fixed allowing some translational motion of the beam.
- Slight part of the oscillation was in the vertical plane.
- Introducing the two masses on the beam in a not exact equilibrium position in the horizontal plane will cause a difference moment on the beam which disturbs the required pure horizontal vibration.
- External sources of vibration like Base Vibration may affect the results.