Table-7.1 Data collected from the experiment execution

Trial	M(kg)	L(cm)	T(second)
1	0.4	20.5	2.88
2	0.8	21	3.43
3	1.2	21.5	3.49
4	1.6	21.9	3.61
5	2.0	22.3	4.01
6	2.4	22.7	4.1
7	2.8	23.1	4.13
8	3.2	23.6	4.26
9	3.6	24	4.7
10	4.4	24.8	4.98

Table-7.2 Dimensions and parameters of the spring

Parameter	Value
N(turns)	18
D(mm)	42.52
d(mm)	3.3
Lo(cm)	20.2

Table-7.3 Data processing analysis

Trial	M(kg)	delta(mm)	t(second)	t^2(second)^2
1	0.4	0.3	0.288	0.082944
2	0.8	0.8	0.343	0.117649
3	1.2	1.3	0.349	0.121801
4	1.6	1.7	0.361	0.130321
5	2	2.1	0.401	0.160801
6	2.4	2.5	0.41	0.1681
7	2.8	2.9	0.413	0.170569
8	3.2	3.4	0.426	0.181476
9	3.6	3.8	0.47	0.2209
10	4.4	4.6	0.498	0.248004

Table-7.4 Data processing results

Spring Stiffness K			
K(theoretical) = 857.045 (N/m)			
From:	Slope	K(N/m)	Percent Error (%)
Figure-3	S_2=94	922.14	7.6
Figure-4	S_1=0.039	1012.27	18.1

Spring Effective Mass m_s			
From Figure-7.2:			
Y Inter (kg.m/N)	0.073	m_s(kg)	0.4
X Inter (kg)	-1.87	m_s(kg)	0.4

Gravitational Acceleration g			
From figures-7.2&4	S_1S_2(sec^2/m)	g(m/sec^2)	Percent Error (%)
	3.666	10.77	9.79

Modulus Of Rigidity G			
From figure-7.2	Slope (m/N)	G(Gpa)	Percent Error (%)
	0.039	94.5	18.13

$\ldots, N=18, D=0.04252\ m, m_c=1.47\ kg$

$$K_{theor} = \frac{G d^4}{8 N D^3} = \frac{80\times10^9 \times (0.0033)^4}{(8)(18)(0.04252)^3} = 857.045\ N/m$$

from figure (1), $S_1 = 0.039$

$$S_1 = \frac{4\pi^2}{K} \Rightarrow K = \frac{4\pi^2}{S_1} = \frac{4\pi^2}{0.039} = 1012.27\ N/m$$

$$Error\% = \frac{1012.27 - 857.045}{857.045} * 100\% = 18.1\%$$

from figure (2), $S_2 = 94$

$$S_2 = \frac{K}{g} \Rightarrow K = S_2 g = 94(9.81) = 922.14\ N/m$$

$$Error\% = \frac{922.14 - 857.045}{857.045} * 100\% = 7.6\%$$

from figure (1)

$Y_{Inter} = 0.073$

$X_{Inter} = -1.87$

$$Y_{Inter} = \frac{4\pi^2}{K}(m_c + m_s) \Rightarrow 0.073 = \frac{4\pi^2}{1012.27}(1.47 + m_s)$$

$m_s = 0.4\ kg$

$$X_{Inter} = -(m_c + m_s) \Rightarrow -1.87 = -(1.47 + m_s)$$

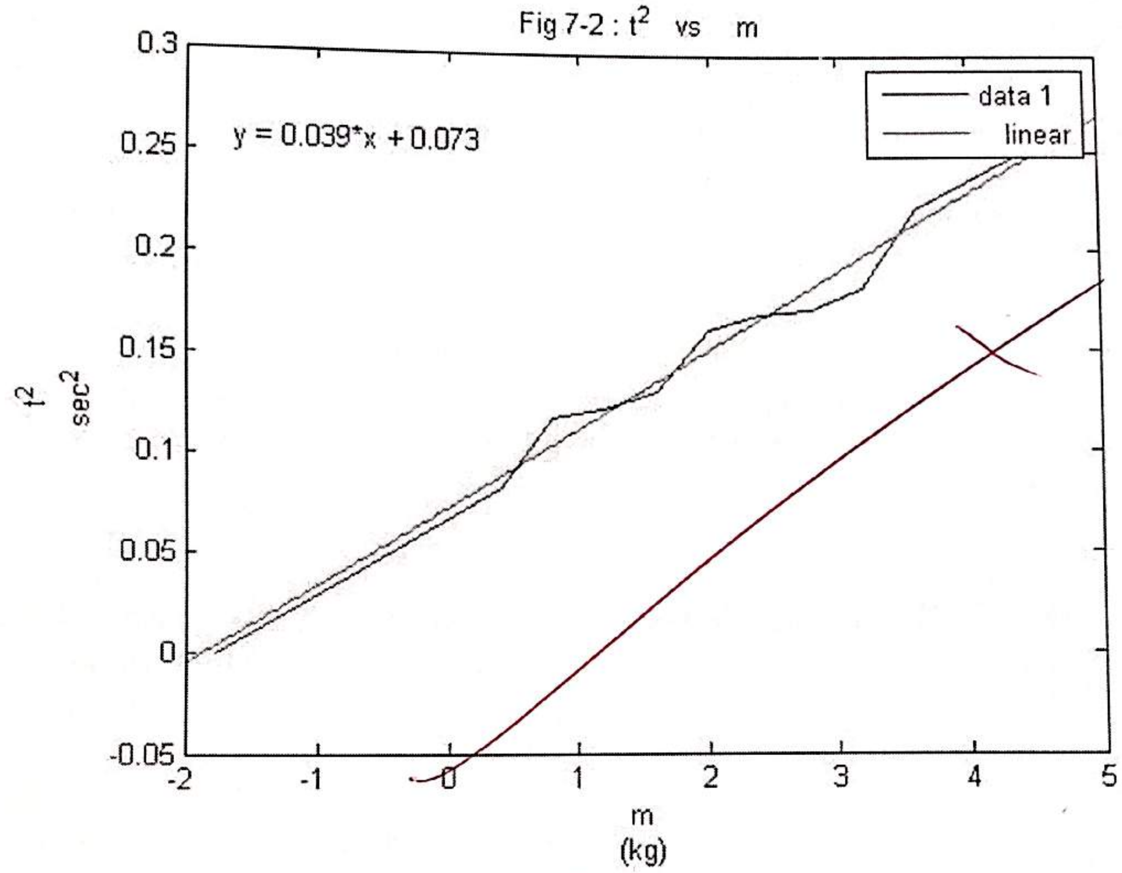
$m_s = 0.4\ kg$

$$S_1 S_2 = \frac{4\pi^2}{g_{exp}} \Rightarrow g_{exp} = \frac{4\pi^2}{3.666} = 10.77\ m/s^2$$

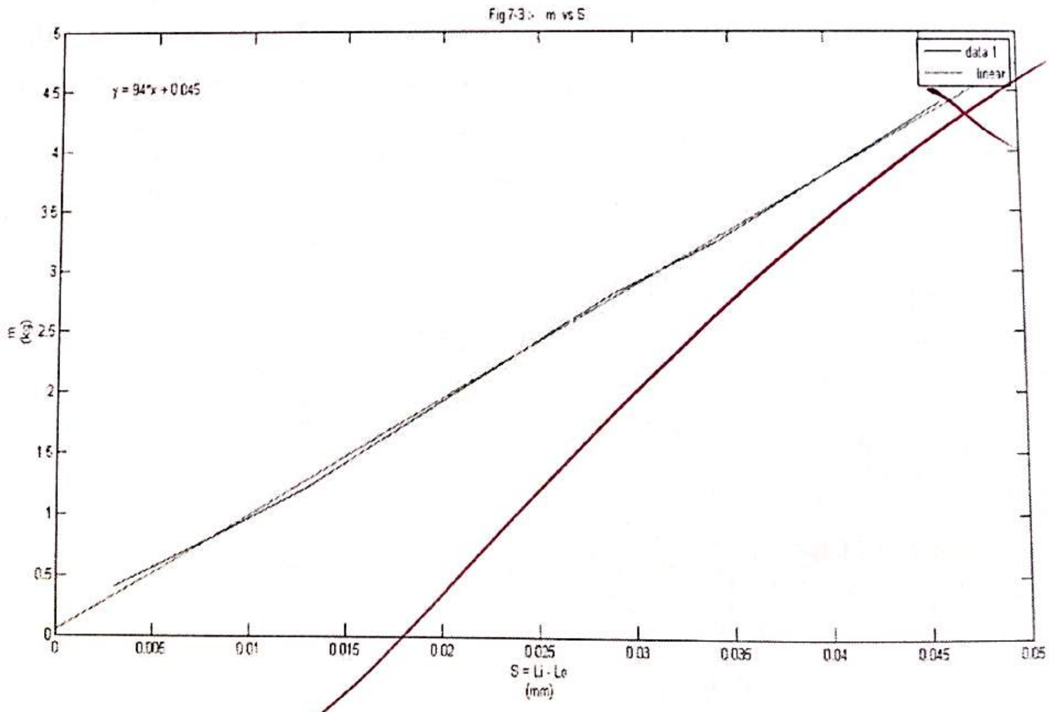
$$Error\% = \frac{10.77 - 9.81}{9.81} = 9.79\%$$

$$S_1 = \frac{32\pi^2 D^3 N}{G_{exp}(d)^4} \Rightarrow G_{exp} = \frac{32\pi^2 (42.52\times10^{-3})^3 (18)}{(0.039)(0.0033)^4}$$

Figures :-



slope = Δy/Δx

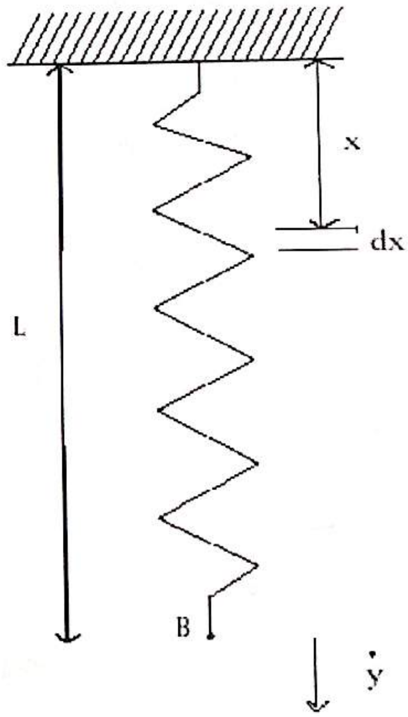


1) What is the physical meaning of the *Effective Mass* of a spring? Is there an effective mass for Torsion springs?

*In a real spring-mass system, the spring has a non-negligible mass m. Since not all of the spring's length moves at the same velocity u as the suspended mass M, its kinetic energy is not equal to $mu^2/2$. As such, m cannot be simply added to M in order to determine the frequency of oscillation, and the **effective mass** of the spring is defined as the mass that needs to be added to M in order to correctly predict the behavior of the system.*

This approximation is valid for linear springs that follow the linearization assumption. Thus it can be applied to the torsion springs, too. Instead of effected mass, effective mass moment of inertia is used.

2) Derive a formula for the effective mass of a linear helical spring m_s in terms of its total mass M_s?



Vertical spring of mass m_s = μ L, where μ is its mass per unit length

The effective mass can be calculated using Raleigh's method. Let the spring have a uniform mass per unit length $\mu = m_s/L$ and total mass m_s and length L. If the coordinate of the mass (the spring's end-point) is **y**, then its velocity is $\dot{y}$. The velocity of a point at a distance **x** below the top of the spring is then,by proportionality:

$$\frac{\dot{x}}{x} = \frac{\dot{y}}{L}$$

$$\dot{x} = x\frac{\dot{y}}{L}$$

Hence an infinitesimal piece of the spring of length **dx** located at a distance **x** below the top of the spring contributes a kinetic energy:

$$dK_s = \frac{1}{2}(\mu dx)\left(\frac{\dot{y}}{L}x\right)^2 = \frac{1}{2}\mu\frac{\dot{y}^2}{L^2}x^2\,dx$$

And the complete spring contributes a kinetic energy, **Ks**:

$$K_s = \int_0^L dK_s = \int_0^L \frac{1}{2}\mu \frac{\dot{y}^2}{L^2} x^2 \, dx$$

$$= \frac{1}{2}(\mu L)\frac{\dot{y}^2}{L^3}\int_0^L x^2 \, dx$$

$$= \frac{1}{2} m_s \frac{\dot{y}^2}{L^3}\left[\frac{L^3}{3}\right]$$

$$= \frac{1}{6} m_s \dot{y}^2$$

Which means that the effective mass of the spring = $\frac{m_s}{3}$

3) Use the dimensions of the spring to estimate its volume and total mass *(by approximate calculations)*, and apply in the formula derived above to find its effective mass. Verify your experimental results.

If the spring was compressed so its turns close to each other, then we get a hollow cylindrical tube as in the figure.

$$V = A_c \times h = \frac{\pi}{4}\left[D^2 - (D-2d)^2\right]\times(18d)$$

$$V = \frac{\pi}{4}\times\left[0.04245^2 - (0.04245 - 2\times 0.0033)^2\right]\times 18\times 0.0033$$

$$V =- 24.11\times 10^{-6}\,m^3$$

$$\rho_{steel} = 7840\,kg/m^3$$

$$\Rightarrow m_s = \rho \times V = 7840\times 24.11\times 10^{-6} = 0.189\,kg$$

$$\therefore m_{effictive} = \frac{m_s}{3} = \frac{0.189}{3} = 0.063\,kg$$

4) In *eqn-5* $F_S = mg$, why didn't we equate the spring force *Fs* with the total weight of the system *Mg*?

At the static equilibrium position: $F_s = k\delta = Mg$

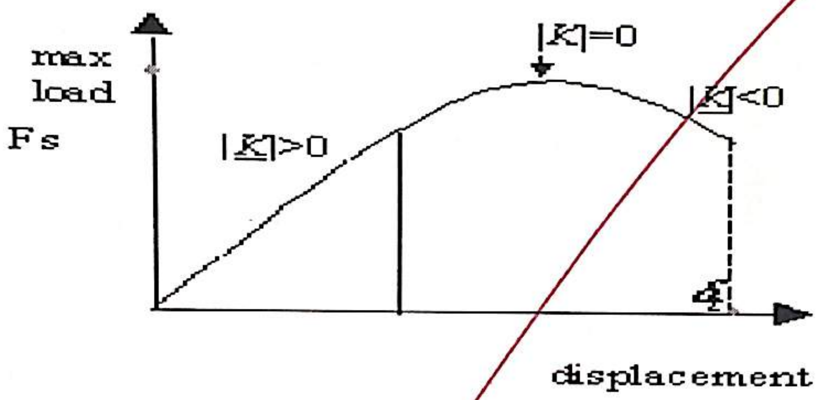
$$(M + m_d)g - k(\Delta x + \delta_{static}) = 0$$

When discs masses are added: Mg cancels out with $k\delta$

$$\Rightarrow m_d g - k\Delta x = 0$$

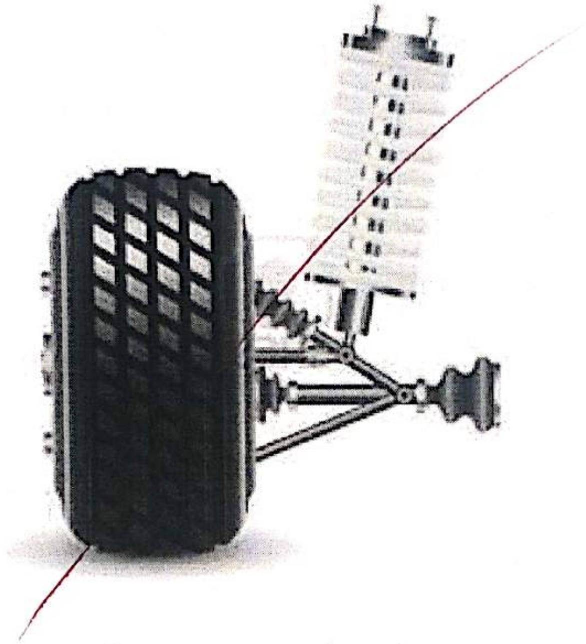
5) In determining the stiffness of the spring using the deflection curve of *Figure-7.3*, what is the essential implicit assumption that has been made? How could you ensure that you did not violate it in the experiment using your graph?

The relation between the spring force and the spring deflection was assumed ***Linear***. This means that the spring's behavior follows Hooke's Law of simple harmonic motion. This linearization was taken in consideration in the deflection curve in this experiment so it is still valid.



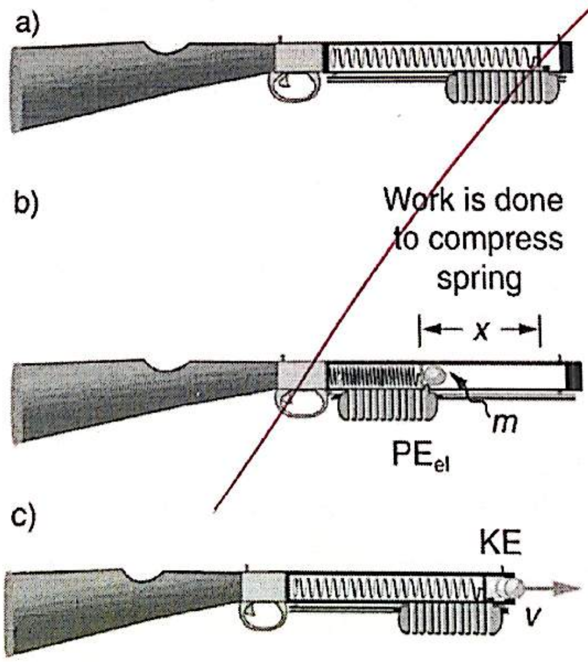
Simple examples of Mass Spring Systems that we use in real life

***Shock absorbers in a vehicle:



***Complex example of a Mass Spring System used in our profession:

designing firearms



- Experimental Procedures:

1) Hang the spring vertically with the load carrier attached to its end, and then measure the total length of the spring L_o.

(This length is not the initial free length of the spring L_i)

2) Add one disk to the carrier *($m = m_d$)*, and measure the total length of the spring after elongation L.
3) With this loading, stretch the spring downward, then leave it to oscillate freely and record the time needed to complete ten oscillations T.
4) Add another disk so that *($m = 2m_d$)*, and repeat steps-2 & 3.
5) Continue by adding a disk each time for total ten disks *($m = 10m_d$)*, and each time measure the parameters L and T.