V-Collected Data:

Part one-rotor's inertia determination:

Table-4.1 Collected data for the *rotor's inertia determination part*

Rotor	h(cm)	t(second)
Rotor-1	87.5	4.67
Rotor-2	87.5	6.25

Part two-modulus of rigidity determination:

Table-4.2 Collected data for the modulus of rigidity determination part

trial	L(cm)	T(second)
1	51	4.31
2	46.3	4.13
3	43.2	3.95
4	38	3.73
5	33.5	3.4
6	29.3	3.3
7	23.2	2.78
8	21	2.68

Part three-two rotors' system:

Table-4.3 Collected data for the *two rotors' system* part

Parameter	L(cm)	T(second)	d(mm)
Value	63.5	4.28	5

Basic Parameters And Dimensions:

Table-4.4 Dimensions of the two rotors according to figure-4.4

Dimension	Rotor-1	Rotor-2	Dimension	Rotor-1	Rotor-2
R_1(mm)	25	25	R_2(mm)	18.8	18.8
R_3(mm)	37.8	37.8	R_4(mm)	90.9	127.1
t_1(mm)	40.5	40.5	t_2(mm)	22	22
t_3(mm)	14.5	14.5	t_4(mm)	51	51

VII-Results:

Part one-rotor's inertia determination:

Table-4.6 Data processing analysis for the *rotor's inertia determination part*

Rotor	I($kg.m^2$) [eqn-5]	I($kg.m^2$) [Analytically]	Percent Error (%)
Rotor-1	0.040076	0.040868	1.938
Rotor-2	0.140849	0.154635	8.915

Part two-modulus of rigidity determination:

Table-4.7 Data processing analysis for the modulus of rigidity determination part

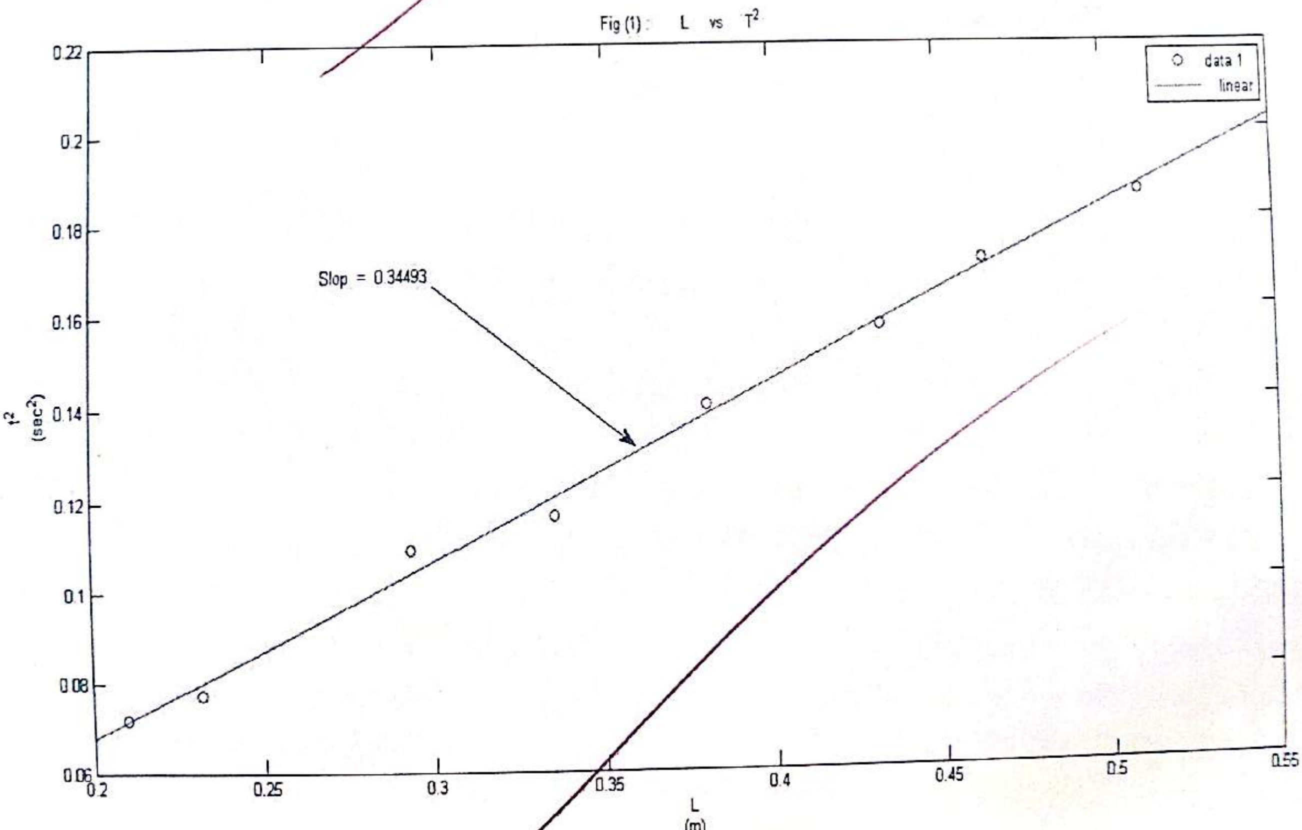
Parameter	Value
J(m^4)	6.1359e-11
K_t(N.m/rad)	7.73

Table-4.8 Data processing analysis for the modulus of rigidity determination part

Rotor-1, I_t = 0.040067 (kg.m^2)			
Trial	L(cm)	t(second)	t^2(second^2)
1	51	0.431	0.1858
2	46.3	0.413	0.1706
3	43.2	0.395	0.1560
4	38	0.373	0.1391
5	33.5	0.34	0.1156
6	29.3	0.33	0.1089
7	23.2	0.278	0.0773
8	21	0.268	0.0718

Table-4.9 Data processing results for the modulus of rigidity determination part

Rotor	Slope (kg/N)	G(Gpa)	Percent Error (%)
Rotor-1	0.34493	74.737	6.579



Part three-two rotors' system:

Table-4.10 Data processing results for the *two rotors' system* part

Parameter	Theoretically	Experimentally	Percent Error (%)
t(second)	0.4	0.428	6.54
L_1(cm)	49.43	49.5	0.142
L_2(cm)	14.07	14	0.5

Sample of calculations :-

Part $\boxed{1}$:- * Experimentally

$$I_1 = mR_1^2\left[\frac{g t_1^2}{2h} - 1\right] = (0.04)(90.9*10^{-3})^2\left[\frac{9.81*4.67^2}{2*87.5*10^{-2}} - 1\right]$$

$$= 0.040076 \ Kg\,m^2$$

$$I_2 = mR_2^2\left[\frac{g t_2^2}{2h} - 1\right] = (0.04)(127.1*10^{-3})^2\left[\frac{9.81*6.25^2}{2*87.5*10^{-2}} - 1\right]$$

$$= 0.140849 \ Kg\,m^2$$

* Analytically

$$I_1 = \frac{1}{2}\sum M_i R_i^2 \qquad \text{Where :} \quad M_i = \rho * V_i$$

$$I_2 = \frac{1}{2}\sum M_i R_i^2 \qquad V = \pi R_i^2 * t_i$$

$$M_1R_1^2 = 7370 * \pi * 0.025^4 * 0.0405 = 0.000366 \ Kg\,m^2$$

$$M_2R_2^2 = 7370 * \pi * 0.0188^4 * 0.022 = 0.000064 \ Kg\,m^2$$

$$M_3R_3^2 = 7370 * \pi * 0.0378^4 * 0.0145 = 0.000685 \ Kg\,m^2$$

$$(M_4R_4^2)_{Rotor\,1} = 7370 * \pi * 0.0909^4 * 0.051 = 0.080620 \ Kg\,m^2$$

$$I_1 = \frac{1}{2}\left(M_1 R_1^2 + M_2 R_2^2 + M_3 R_3^2 + (M_4 R_4^2)_{Rotor\ 1} \right) = 0.040868 \ Kg\,m^2$$

$$I_2 = \frac{1}{2}\left(M_1 R_1^2 + M_2 R_2^2 + M_3 R_3^2 + (M_4 R_4^2)_{Rotor\ 2} \right) = 0.154635 \ Kg.m^2$$

* Error's :-

$$\epsilon_1 = \frac{| I_{1exp} - I_{1Ana} |}{I_{1Ana}} * 100\% = \frac{|0.040076 - 0.040868|}{0.040868} * 100\%$$

$$= 1.938\%$$

$$\epsilon_2 = \frac{| I_{2exp} - I_{2Ana} |}{I_{2Ana}} * 100\% = \frac{|0.140849 - 0.154635|}{0.154635} * 100$$

$$= 8.915\%$$

• part $\boxed{2}$:-

eq 9 :- $\tau = 2\pi\sqrt{\frac{I_T}{K_t}} \rightarrow \tau^2 = \frac{4\pi^2 I_T}{K_t}$ ....①

eq 13 :- $K_t = \frac{GJ}{L}$ → sub in ①

$\therefore \quad \tau^2 = \frac{4\pi^2 I_T L}{GJ} \rightarrow \text{slop} = \frac{4\pi^2 I_T}{GJ}$

• but I_T From part $\boxed{1}$ → $I_T = 0.040076$ Kg.m²

• and $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} * (0.005)^4 = 6.1359 * 10^{-11}$ m^4

• and From Fig 1 "Results" : slop = 0.34493 Kg/N #

Then $G_{exp} = \frac{4\pi^2 I_T}{(\text{slop}) J} = \frac{(4\pi^2)(0.040067)}{0.34493(6.1359*10^{-11})}$

$= 74.737$ GPa #

$\epsilon = \frac{|80 - 74.737|}{80} * 100\% = 6.579\%$ #

$K_t = \frac{GJ}{L} = \frac{(80*10^9)(6.1359*10^{-11})}{0.635} = 7.73$ N.m/rad #

part $\boxed{3}$:-

eq 18 :- $\tau_{th} = 2\pi \sqrt{\dfrac{I_1 I_2}{K_T (I_1 + I_2)}}$

From part $\boxed{1}$: $I_1 = 0.040076 \ Kg.m^2$

$I_2 = 0.140849 \ Kg.m^2$

From part $\boxed{2}$:- $K_T = 7.73 \ N.m/rad$

Then $\tau_{th} = 2\pi \sqrt{\dfrac{(0.040076)(0.140849)}{7.73\,(0.040076 + 0.140849)}} = 0.4 \ sec$

$\tau_{exp} = 0.428 \ sec$

$\epsilon = \left| \dfrac{0.428 - 0.4}{0.428} \right| = 6.54\,\%$ #

eq. 19 :- $L_1 = \dfrac{I_2}{I_1 + I_2} L$ $L_2 = \dfrac{I_1}{I_1 + I_2} L$

but $L = 63.5 \ cm \rightarrow L_1 = 49.43 \ cm$ # , $L_2 = 14.07 \ cm$ #

$L_{1\,exp} = 49.5 \ cm$, $L_2 = 14 \ cm$ #

$\epsilon_1 = \left| \dfrac{49.5 - 49.43}{49.43} \right| * 100\% = 0.141\,\%$ # , $\epsilon_2 = \left| \dfrac{14 - 14.07}{14.07} \right| * 100\% = 0.5\%$ #

VIII- Discussion and Conclusions:

1. Where and why do we use flywheels? Give practical examples.

Flywheels are connected with rotational shafts where constant torque is desired and energy storage is needed during motion. It ensures gradual acceleration and deceleration of the rotating shaft, thus prevents large energy loss in little time. For example, in automotives the flywheel connects the crank shaft with the gear box and drive shaft.

2. In determining the value of *G*; several period readings have been taken to draw a graph, and from its slope *G* was found. Why do not we take discrete reading*(s)* and apply directly in *eqns-9 & 13* to find *G*? What are the benefits of making such a graph?

The graph connects the most précised points that approach from the true value of ***G***. This yields to minimal errors due to linear fitting which gives a general slope for the points with best accuracy. Discrete points may consider the far points obtained in the experiment due to some errors and this will yield in greater errors in determining ***G***.

3. Does the nodal point have the maximum or minimum stresses along the shaft, why?

It will have maximum stress. This because at this point the shaft section is subjected to opposed shear stress which means that the applied torque at this section will be an add up from the two rotors, thus increasing the effect of torsional shear stress and decrease the fatigue life of the shaft.

4. Discuss the factors affecting the period of oscillation of a Two-Rotor System?

$$\tau = 4\pi^2 \frac{I_T}{K_T} = 4\pi^2 \frac{I_T L}{GJ}$$

- The mass moment of inertia of the two rotors and shaft length are directly proportional to the period of oscillation.
- The modulus of rigidity and the polar moment of area of the shaft and hence the shaft diameter are inversely proportional to the period of oscillation.
- Therefore, the torsional stiffness of the shaft is inversely proportional with the period of oscillation of the system.

5. You are given a system of a similar layout as the one shown in *Figure-4.2*; in which the rotor has an unknown inertia, and fitted to the end of a shaft of an unknown material, with a number of different couples of auxiliary masses available. Describe *(with the necessary equations)* how to find both *G* and *I* with such a set-up?

1) The total mass moment of inertia of the whole assembly is found by part on of the experiment:

$$I_T = I_{Rotor} + \sum I_{auxiliary} = mR^2\left(\frac{gt^2}{2h} - 1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

2) The torsional stiffness of the shaft is given by:

$$K_T = \frac{GJ}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where; $J = \frac{\pi}{32} d^4$

3) Equation (1) is squared to get:

$$\tau^2 = 4\pi^2 \frac{I_T}{K_T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

4) Equation (2) is substituted in equation (3) to give:

$$\tau^2 = 4\pi^2 \frac{I_T L}{GJ} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

5) A linear graph is plotted between τ^2 versus L.

6) The slope of the line $= \frac{4\pi^2 I_T}{GJ} \Rightarrow G$ is found.

*Applications of flywheel

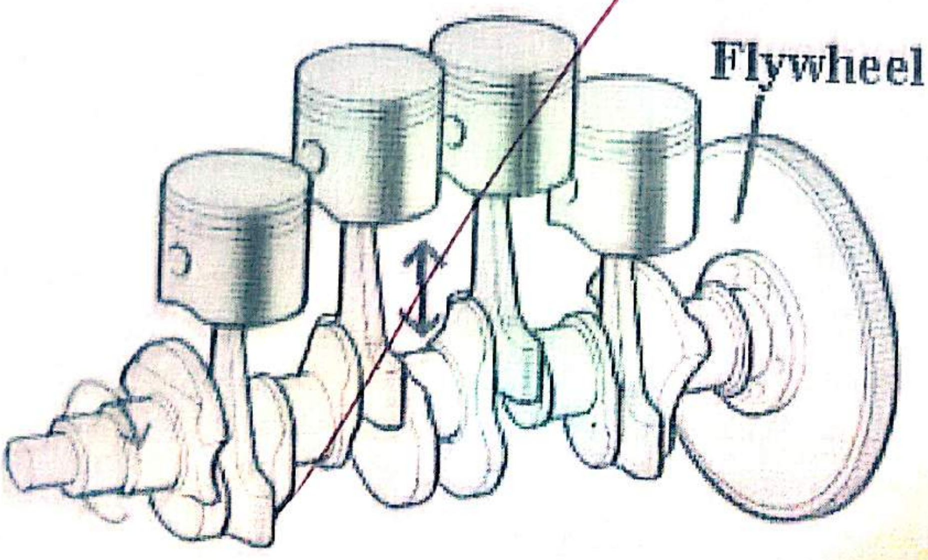
1) Transportation

Automotive

In the 1950s, flywheel-powered buses, known as gyrobuses, were used in Yverdon, Switzerland and there is ongoing research to make flywheel systems that are smaller, lighter, cheaper and have a greater capacity. It is hoped that flywheel systems can replace conventional chemical batteries for mobile applications, such as for electric vehicles. Proposed flywheel systems would eliminate many of the disadvantages of existing battery power systems, such as low capacity, long charge times, heavy weight and short usable lifetimes. Flywheels may have been used in the experimental Chrysler Patriot, though that has been disputed.

Flywheels have also been proposed for use in continuously variable transmissions. Punch Powertrain is currently working on such a device.

During the 1990s, Rosen Motors developed a gas turbine powered series hybrid automotive powertrain using a 55,000 rpm flywheel to provide bursts of acceleration which the small gas turbine engine could not provide. The flywheel also stored energy through regenerative braking. The flywheel was composed of a titanium hub with a carbon fibercylinder and was gimbal-mounted to minimize adverse gyroscopic effects on vehicle handling. The prototype vehicle was successfully road tested in 1997 but was never mass-produced.



2)Uninterruptible power supplies

Flywheel power storage systems in production as of 2001 have storage capacities comparable to batteries and faster discharge rates. They are mainly used to provide load leveling for large battery systems, such as an uninterruptible power supply for data centers as they save a considerable amount of space compared to battery systems.

Flywheel maintenance in general runs about one-half the cost of traditional battery UPS systems. The only maintenance is a basic annual preventive maintenance routine and replacing the bearings every five to ten years, which takes about four hours. Newer flywheel systems completely levitate the spinning mass using maintenance-free magnetic bearings, thus eliminating mechanical bearing maintenance and failures

Costs of a fully installed flywheel UPS are about $330 per kilowatt In combination with a diesel generator set or integrated design, it supplies continuous power as long as there is fuel.