Part One-Simple pendulum:

Table-1.1 Collected data for the simple pendulum part

Trial	Steel Ball		Plastic Ball	
	L(cm)	T(second)	L(cm)	T(second)
1	18.5	9.03	16	8.5
2	25	10.4	22.5	9.87
3	34	11.96	27.2	10.76
4	39.4	12.96	34.6	11.78
5	42.5	13.3	43.4	13.21
6	45.5	14.23	51.8	14.56

Part two-Compound Pendulum:

L= 91.5 cm

Table-1.2 Collected data for the compound pendulum part

Trial	H(cm)	T(second)
1	42	15.1
2	39	14.65
3	35	14.61
4	31.5	14.4
5	27.5	14.38
6	22	14.33
7	18.5	14.81
8	14.5	15.93
9	9.5	18.16
10	8	20.20

Table-1.4 Data processing analysis for the simple pendulum part

Plastic Ball				
Trail	L (cm)	t Exper (second)	t theor (second)	(t Exper)^2 (second)^2
1	16	0.85	0.802	0.7225
2	22.5	0.987	0.9516	0.974
3	27.2	1.076	1.046	1.1578
4	34.6	1.178	1.18	1.3877
5	43.4	1.321	1.321	1.745
6	51.8	1.456	1.444	2.12
7				
8				

Table-1.6 :- Data processing analysis for the compound pendulum part

Trial	h (m)	t(second)	h^2 (m)^2	(t^2)*h (m.sec^2)
1	0.42	1.51	0.1764	0.9576
2	0.39	1.465	0.1521	0.8370
3	0.35	1.461	0.1225	0.7471
4	0.315	1.44	0.0992	0.6532
5	0.275	1.438	0.0756	0.5687
6	0.22	1.433	0.0484	0.4518
7	0.185	1.481	0.0342	0.4058
8	0.145	1.593	0.0210	0.3680
9	0.095	1.816	0.0090	0.3133
10	0.08	2.02	0.0064	0.3264

Table-1.7 :- Data processing results for the compound pendulum part

From figure-1.3		
Slope (sec^2/m)	g (m^2/sec.)	Percent Error (%)
3.9518	9.99	1.83
YInt (sec^2.m)	**K**CG (cm)	
0.2949	27.07	
XInt (m^2)	**K**CG (cm)	Percent Error (%)
- 0.0763	27.62	2.01

From Figure-1.4	**From Eqn-11**	
t min (sec)	**t** min (sec)	Percent Error (%)
1.385	1.484	6.67
h aT **t** = **t** min (cm)	**h** (cm)	Percent Error (%)
29.2	27.345	6.76

$$g = \frac{4\pi^2}{4.2718} = 9.242 \ m/s^2$$

part two (plastic Ball)

$T_1 = 8.5$ sec , $L_1 = 16$ cm

$$\tau_{exp} = \frac{T}{10} = \frac{8.5}{10} = 0.85 \ sec$$

$$\tau_{theor} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{16*10^{-2}}{9.81}}$$

$$= 0.802 \ sec$$

$$\text{Error}\% = \frac{\tau_{exp} - \tau_{theor}}{\tau_{theor}} * 100\% = \frac{0.85 - 0.802}{0.802} * 100\%$$

$$= 5.985\%$$

sample of calculation

part one (steel Ball)

$T_1 = 9.03$ sec , $L_1 = 18.5$ cm

$$\tau_{exp} = \frac{T_1}{10} = \frac{9.03}{10} = 0.903 \text{ sec}$$

$$\tau_{theor} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{18.5\times10^{-2}}{9.81}} = 0.8628 \text{ sec}$$

$$\text{error}\% = \frac{\tau_{exp} - \tau_{theor}}{\tau_{theor}} * 100\% = \frac{0.903 - 0.8628}{0.8628} * 100\%$$

$$= 4.659\%$$

$T_2 = 10.4$ sec, $L_2 = 25$ cm

$$\tau_{exp} = \frac{T}{10} = 1.04 \text{ sec}$$

$$\tau_{theor} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{25\times10^{-2}}{9.81}}$$

$$= 1.003 \text{ sec}$$

$$\text{Error}\% = \frac{\tau_{exp} - \tau_{theor}}{\tau_{theor}} * 100\% = \frac{1.04 - 1.003}{1.003} * 100\% = 3.68\%$$

$T_3 = 11.96$ sec , $L_3 = 34$ cm

$$\tau_{exp} = \frac{11.96}{10} = 1.196 \text{ sec}$$

$$\tau_{theor} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{34\times10^{-2}}{9.81}} = 1.1697 \text{ sec}$$

$$\text{error}\% = \frac{\tau_{exp} - \tau_{theor}}{\tau_{theor}} * 100\% = \frac{1.196 - 1.1697}{1.1697} * 100\% = 2.248$$

$T_3 = 10.76$ sec , $L_3 = 27.2$ cm

$$\tau_{exp} = \frac{T}{10} = \frac{10.76}{10} = 1.076 \text{ sec}$$

$$\tau_{theor} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{27.2\times10^{-2}}{9.81}}$$

$$= 1.046 \text{ sec}$$

$$\text{Error}\% = \frac{\tau_{exp} - \tau_{theor}}{\tau_{theor}} * 100\%$$

$$= \frac{1.076 - 1.046}{1.046} * 100\% = 2.87\%$$

from figure 1 : (plastic Ball)
the slope = 3.8365

$$\text{slope} = \frac{4\pi^2}{\cancel{\text{slope}}\, g}$$

$$g = \frac{4\pi^2}{\text{slope}} = \frac{4\pi^2}{3.8365} = 10.29 \text{ m/s}^2$$

Calculations :-

1. From fig. 2-1 :-

- slop = 3.9518
- Squaring eqn. 10 $\Rightarrow \tau^2 h = \frac{4\pi^2}{g}\left(K_{cG}^2 + h^2\right)$

$\therefore$ slop $= \frac{4\pi^2}{g} \longrightarrow g_{exp} = \frac{4\pi^2}{slop} = \frac{4\pi^2}{3.9518}$

$$\boxed{g_{exp} = 9.99 \ m/s^2} \ \#$$

- Error in g :- $\epsilon_g = \frac{|9.81 - 9.99|}{9.81} * 100\% = \boxed{1.83\%}$ #

2. From fig 2-1 :-

$X_{int} = -0.0763 \ (m^2)$, $Y_{int} = 0.2949 \ (s^2.m)$

$X_{int} = -K_{cG}^2$, $Y_{int} = \left(\frac{4\pi^2}{g}\right) K_{cG}^2$

$\therefore \boxed{K_{cG} = 0.2762 \ m}$, $K_{cG} = \sqrt{\frac{(9.81)}{4(\pi^2)}(0.2949)} = \boxed{0.2707 \ m}$ #

$$K_{cG_{avg}} = \frac{0.2762 + 0.2707}{2} = \boxed{0.27345 \ m} \ \#$$

3. From Fig 2-2 :-

- $\boxed{h_{at\ t = \tau_{min}} = 0.292\ m}$ #

- $\boxed{\tau_{min} = 1.385\ s}$ #

4. From eqn 10 $\Rightarrow \tau = 2\pi\left(\frac{K_{CG}^2 + h^2}{gh}\right)^{1/2}$

$$\frac{d\tau}{dh} = 0 \rightarrow \left(\frac{1}{2}\right)(2\pi)\left(\frac{K_{CG}^2 + h_o^2}{gh}\right)^{-1/2}\left(\frac{2h(gh) - g(K_{CG}^2 + h^2)}{g^2h^2}\right) =$$

Then this eqn equal zero when $h = K_{CG}$

- To check sub $h = K_{CG}$:-

$$\pi\left(\frac{K_{CG}^2 + K_{CG}^2}{gh}\right)^{-1/2}(0) \overset{?}{=} 0$$

- Then at $h = K_{CG} \rightarrow \tau = \tau_{min}$

- sub $h = K_{CG}$ in eqn 10 :-

$$\tau = \sqrt{\frac{8\pi^2 K_{CG}}{g}} \quad \text{---}(11)$$

then $\tau = \sqrt{\dfrac{(8)(\pi)^2(0.27345)}{9.81}} = \boxed{1.484 \text{ s}}$ #

$\boxed{h = K_{CG_{avg}} = 0.2735 \text{ m}}$ #

5. Error in τ_{min} and h :-

- $E_\tau = \left| \dfrac{1.484 - 1.385}{1.484} \right| * 100\% \Rightarrow \boxed{E_\tau = 6.67\ \%}$ #

- $E_h = \left| \dfrac{0.2735 - 0.272}{0.2735} \right| * 100\% \Rightarrow \boxed{E_h = 6.76\ \%}$ #

6. sample of calculation's For h^2, and $\tau^2 h$:-

- For the First Row in Table 1.6 :-

$h = 0.42$ m , $t = 1.51$ sec ;-

* $h^2 = (0.42)^2 = 0.1764 \text{ m}^2$

* $(t^2)(h) = (1.51)^2(0.42) = 0.9576 \text{ m.s}^2$

CONCLUSION

1- *Simple harmonic motion is the simplest form of periodic motion, usually sine or cosine, typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law. The motion is sinusoidal in time and demonstrates a single resonant frequency. In such a vibration, the acceleration is proportional to the displacement and directed towards the mean positio.*

2- We use two masses with identical geometries because :

- Similar geometry ensures the same air drag resistance force acting on both balls as well as any frictional forces during oscillation.
- To detect the independence of motion and period of oscillation of the simple pendulum from the acting concentrated mass.

3- The physical meaning of h being equal to zero that the center of gravity of the concentrated mass is equal and the same as the center of oscillation

4- The compound pendulum have the identity of possessing two values of *h* corresponding to the same period of oscillation because :

The mass of the compound pendulum is distributed along its length, its mass moment of inertia will have a squared part according to parallel axis theorem, which means a positive or negative distance from the center of mass will produce the same effect of inertia. So it has two points that vibrates with the same frequency and period of oscillation.

5- **Simple pendulum equation of motion:**

$$I\ddot{\theta} + mg\theta = 0$$

$$where\ I = m\ell$$

The mass moment of inertia of the concentrated mass from a distance "ℓ" to the point of oscillation

- **Compound pendulum equation of motion:**

$$I\ddot{\theta} + mgh\theta = 0$$

$$where\ I = I_{CG} + mh^2 = m\left(K_{CG}^2 + h^2\right)$$

To replace the compound pendulum with an equivalent simple pendulum we get :

$$\left(\frac{I}{h}\right)_{compound} = I_{simple}$$

$$m\left(K_{CG}^2 + h^2\right)_{compound} = m\ell_{simple}$$

$$\Rightarrow \ell_{simple} = \left(K_{CG}^2 + h^2\right)$$

So we can assemble a simple pendulum with the equivalent value of "ℓ"(thread length from center of concentrated mass to center of oscillation) and any concentrated mass connected at the free end of length "ℓ"that will vibrate at same frequency and period of oscillation compared to the corresponding compound pendulum.

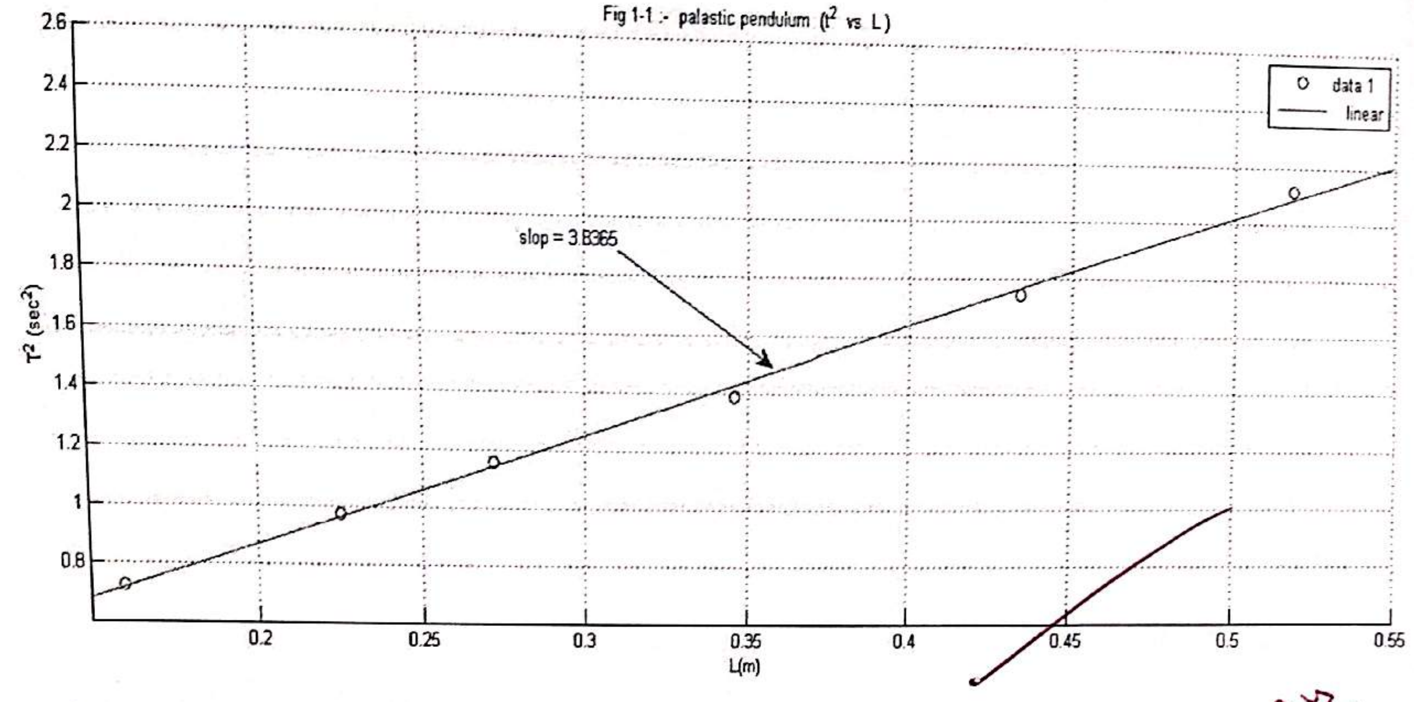

Figure 1:

$slope = \frac{\Delta y}{\Delta x}$

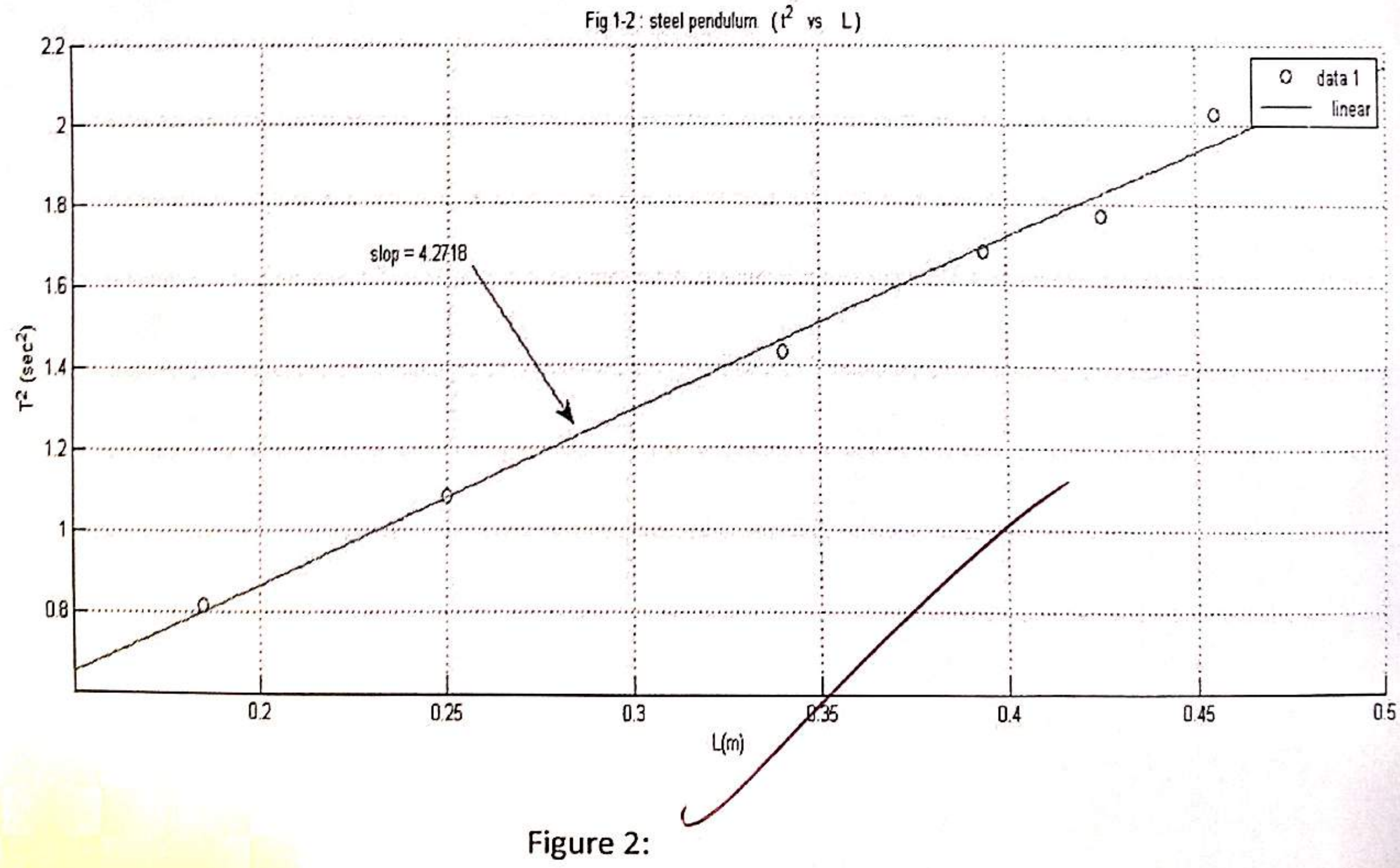

Figure 2:

Why the pendulum is Important :

it can be used to measure "g" (the acceleration due to gravity), which is important in determining the **shape** of the earth and the distribution of materials within it (the science of **geodesy**).

the pendulum helps scientists determine "**g**" force which is affected by:

1) distance from center of earth .
2) masses attracting upwards (ie. mountains) .
3) density of matter around us .

Pendulum Clock

A **pendulum clock** is a clock that uses a pendulum, a swinging weight, as its timekeeping element. The advantage of a pendulum for timekeeping is that it is a harmonic oscillator; it swings back and forth in a precise time interval dependent on its length, and resists swinging at other rates. From its invention in 1656 by Christian Huygens until the 1930s, the pendulum clock was the world's most precise timekeeper, accounting for its widespread use. Pendulum clocks must be stationary to operate; any motion or accelerations will affect the motion of the pendulum, causing inaccuracies, so other mechanisms must be used in portable timepieces. They are now kept mostly for their decorative and antique value

Error Types:-

1- Absolute Error: if we have Experimental and ~~actual~~ theoritical values, $E = \frac{\text{theoritical} - \text{Experimental}}{\text{theoritical}} * 100\%$

2- relative Error: if we have Experimental data only

$$E = \frac{exp_1 - exp_2}{exp_1} * 100\%$$