

Thermodynamics: An Engineering Approach

Chapter 5 MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

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Introduction

- In Chap. 4, we applied the general energy balance relation expressed as
$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$
to closed systems.
- In this chapter, we extend the energy analysis to systems that involve mass flow across their boundaries i.e., control volumes, with particular emphasis to steady-flow systems.
- We start this chapter with the development of the general conservation of mass relation for control volumes, and we continue with a discussion of flow work and the energy of fluid streams.
- We then apply the energy balance to systems that involve steady-flow processes and **analyze the common steady-flow devices such as nozzles, diffusers, compressors, turbines, throttling devices, mixing chambers, and heat exchangers.**
- Finally, we apply the energy balance to general unsteady-flow processes such as the charging and discharging of vessels.

Objectives

The objectives of Chapter 5 are to:

- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.

- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

Mass and Volume Flow Rates

The amount of mass flowing through a cross section per unit time is called the **mass flow rate** and is denoted by \dot{m} . The dot over a symbol is used to indicate *time rate of change*, as explained in Chap. 2.

A fluid usually flows into or out of a control volume through pipes or ducts. The differential mass flow rate of fluid flowing across a small area element dA_c on a flow cross section is proportional to dA_c itself, the fluid density ρ , and the component of the flow velocity normal to dA_c , which we denote as V_n , and is expressed as (Fig. 5–2)

$$\delta\dot{m} = \rho V_n dA_c \quad (5-2)$$

Note that both δ and d are used to indicate differential quantities, but δ is typically used for quantities (such as heat, work, and mass transfer) that are *path functions* and have *inexact differentials*, while d is used for quantities (such as properties) that are *point functions* and have *exact differentials*. For flow through an annulus of inner radius r_1 and outer radius r_2 , for example,

$\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^2 - r_1^2)$ but $\int_1^2 \delta\dot{m} = \dot{m}_{\text{total}}$ (total mass flow rate through the annulus), not $\dot{m}_2 - \dot{m}_1$. For specified values of r_1 and r_2 , the value of the integral of dA_c is fixed (thus the names point function and exact differential), but this is not the case for the integral of dm . (thus the names path function and inexact differential).

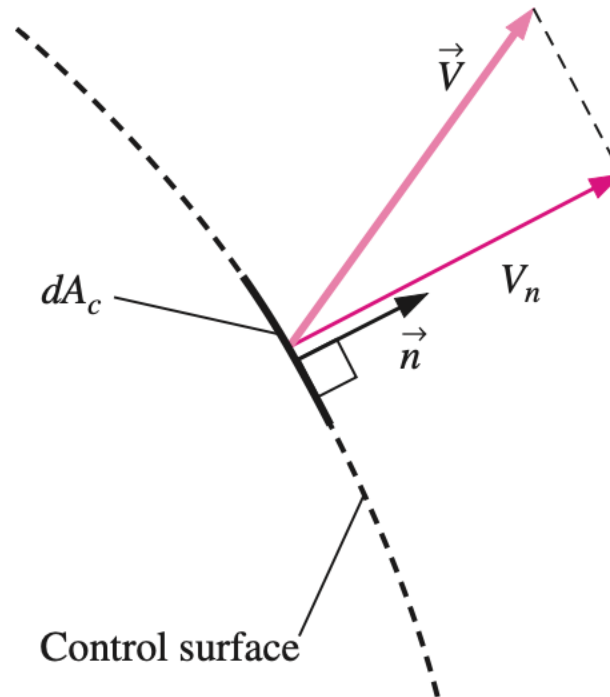


FIGURE 5–2

The normal velocity V_n for a surface is the component of velocity perpendicular to the surface.

The mass flow rate through the entire cross-sectional area of a pipe or duct is obtained by integration:

$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n dA_c \quad (\text{kg/s}) \quad (5-3)$$

- While Eq. 5–3 is always valid (in fact it is *exact*), it is not always practical for engineering analyses because of the integral.
- We would like instead to express mass flow rate in terms of average values over a cross section of the pipe.
- In a general compressible flow, both ρ and V_n vary across the pipe. In many practical applications, however, the density is essentially uniform over the pipe cross section, and we can take ρ outside the integral of Eq. 5–3. Velocity, however, is *never* uniform over a cross section of a pipe because of the fluid sticking to the surface and thus having zero velocity at the wall (the no-slip condition).
- Rather, the velocity varies from zero at the walls to some maximum value at or near the centerline of the pipe.

We define the **average velocity** V_{avg} as the average value of V_n across the entire cross section (Fig. 5–3),

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n dA_c \quad (5-4)$$

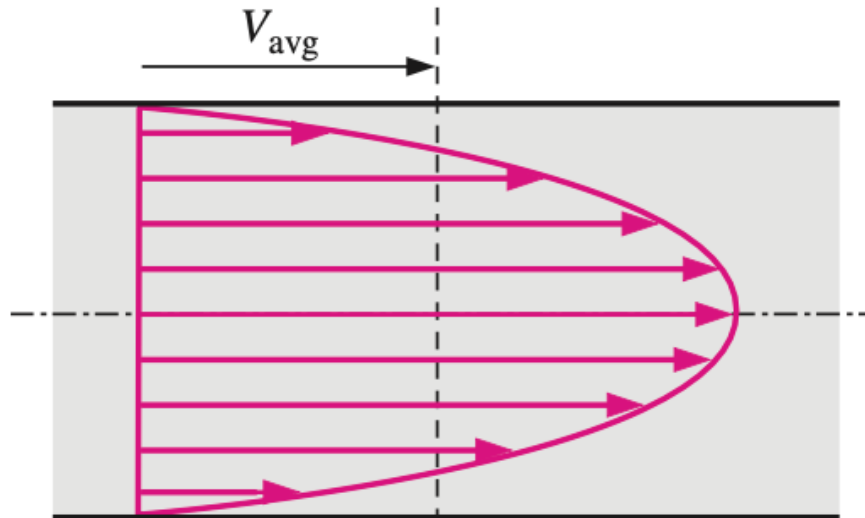


FIGURE 5–3

The average velocity V_{avg} is defined as the average speed through a cross section.

Eq. 5–3 becomes

$$\dot{m} = \rho V_{\text{avg}} A_c \quad (\text{kg/s}) \quad (5-5)$$

$$\dot{V} = \int_{A_c} V_n dA_c = V_{\text{avg}} A_c = V A_c \quad (\text{m}^3/\text{s}) \quad (5-6)$$

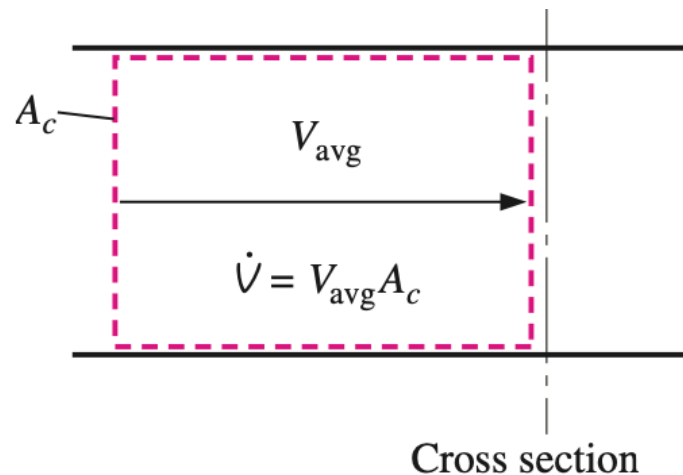


FIGURE 5–4

The volume flow rate is the volume of fluid flowing through a cross section per unit time.

The mass and volume flow rates are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{\nu} \quad (5-7)$$

where ν is the specific volume. This relation is analogous to $m = \rho V = V/\nu$, which is the relation between the mass and the volume of a fluid in a container.

Conservation of Mass Principle

The **conservation of mass principle** for a control volume can be expressed as: *The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt .* That is,

$$\left(\begin{array}{c} \text{Total mass entering} \\ \text{the CV during } \Delta t \end{array} \right) - \left(\begin{array}{c} \text{Total mass leaving} \\ \text{the CV during } \Delta t \end{array} \right) = \left(\begin{array}{c} \text{Net change in mass} \\ \text{within the CV during } \Delta t \end{array} \right)$$

or

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \quad (\text{kg}) \quad (5-8)$$

where $\Delta m_{\text{CV}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the control volume during the process (Fig. 5–5). It can also be expressed in *rate form* as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt \quad (\text{kg/s}) \quad (5-9)$$

where \dot{m}_{in} and \dot{m}_{out} are the total rates of mass flow into and out of the control volume, and dm_{CV}/dt is the time rate of change of mass within the control volume boundaries. Equations 5–8 and 5–9 are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

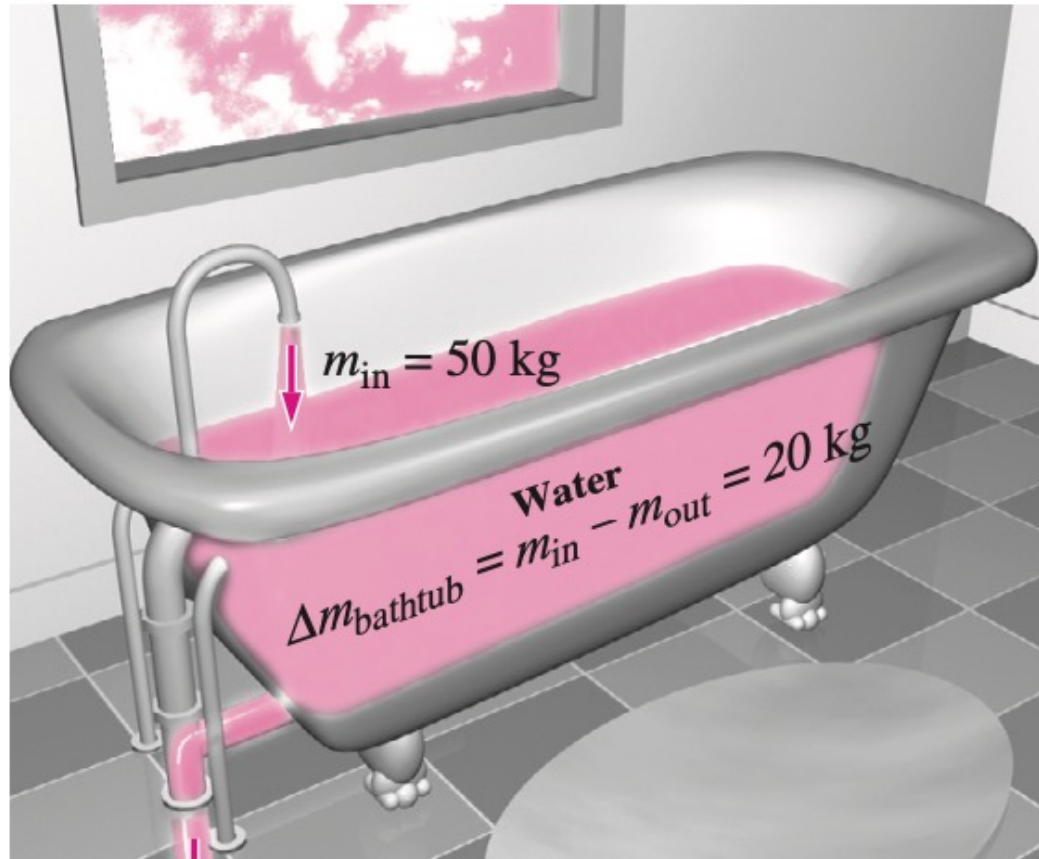


FIGURE 5–5

Conservation of mass principle for an ordinary bathtub.

Consider a control volume of arbitrary shape, as shown in Fig. 5–6. The mass of a differential volume dV within the control volume is $dm = \rho dV$. The total mass within the control volume at any instant in time t is determined by integration to be

Total mass within the CV:
$$m_{\text{CV}} = \int_{\text{CV}} \rho dV \quad (5-10)$$

Then the time rate of change of the amount of mass within the control volume can be expressed as

Rate of change of mass within the CV:
$$\frac{dm_{\text{CV}}}{dt} = \frac{d}{dt} \int_{\text{CV}} \rho dV \quad (5-11)$$

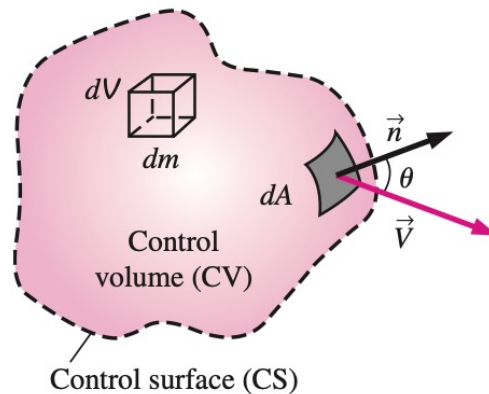


FIGURE 5–6

The differential control volume dV and the differential control surface dA used in the derivation of the conservation of mass relation.

For the special case of no mass crossing the control surface (i.e., the control volume resembles a closed system), the conservation of mass principle reduces to that of a system that can be expressed as $dm_{CV}/dt = 0$. This rela-

Normal component of velocity: $V_n = V \cos \theta = \vec{V} \cdot \vec{n}$ (5-12)

The mass flow rate through dA is proportional to the fluid density ρ , normal velocity V_n , and the flow area dA , and can be expressed as

Differential mass flow rate: $\delta \dot{m} = \rho V_n dA = \rho (V \cos \theta) dA = \rho (\vec{V} \cdot \vec{n}) dA$ (5-13)

The net flow rate into or out of the control volume through the entire control surface is obtained by integrating $\delta\dot{m}$ over the entire control surface,

Net mass flow rate:
$$\dot{m}_{\text{net}} = \int_{\text{CS}} \delta\dot{m} = \int_{\text{CS}} \rho V_n dA = \int_{\text{CS}} \rho(\vec{V} \cdot \vec{n}) dA \quad (5-14)$$

Note that $\vec{V} \cdot \vec{n} = V \cos \theta$ is positive for $\theta < 90^\circ$ (outflow) and negative for $\theta > 90^\circ$ (inflow). Therefore, the direction of flow is automatically accounted for, and the surface integral in Eq. 5-14 directly gives the *net* mass flow rate. A positive value for \dot{m}_{net} indicates net outflow, and a negative value indicates a net inflow of mass.

Rearranging Eq. 5-9 as $dm_{\text{CV}}/dt + \dot{m}_{\text{out}} - \dot{m}_{\text{in}} = 0$, the conservation of mass relation for a fixed control volume can then be expressed as

General conservation of mass:
$$\frac{d}{dt} \int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho(\vec{V} \cdot \vec{n}) dA = 0 \quad (5-15)$$

It states that *the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.*

Splitting the surface integral in Eq. 5–15 into two parts—one for the outgoing flow streams (positive) and one for the incoming streams (negative)—the general conservation of mass relation can also be expressed as

$$\frac{d}{dt} \int_{CV} \rho dV + \sum_{\text{out}} \int_A \rho V_n dA - \sum_{\text{in}} \int_A \rho V_n dA = 0 \quad (5-16)$$

where A represents the area for an inlet or outlet, and the summation signs are used to emphasize that *all* the inlets and outlets are to be considered. Using the definition of mass flow rate, Eq. 5–16 can also be expressed as

$$\frac{d}{dt} \int_{CV} \rho dV = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad \text{or} \quad \frac{dm_{CV}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m} \quad (5-17)$$

Equations 5–15 and 5–16 are also valid for moving or deforming control volumes provided that the *absolute velocity* \vec{V} is replaced by the *relative velocity* \vec{V}_r , which is the fluid velocity relative to the control surface.

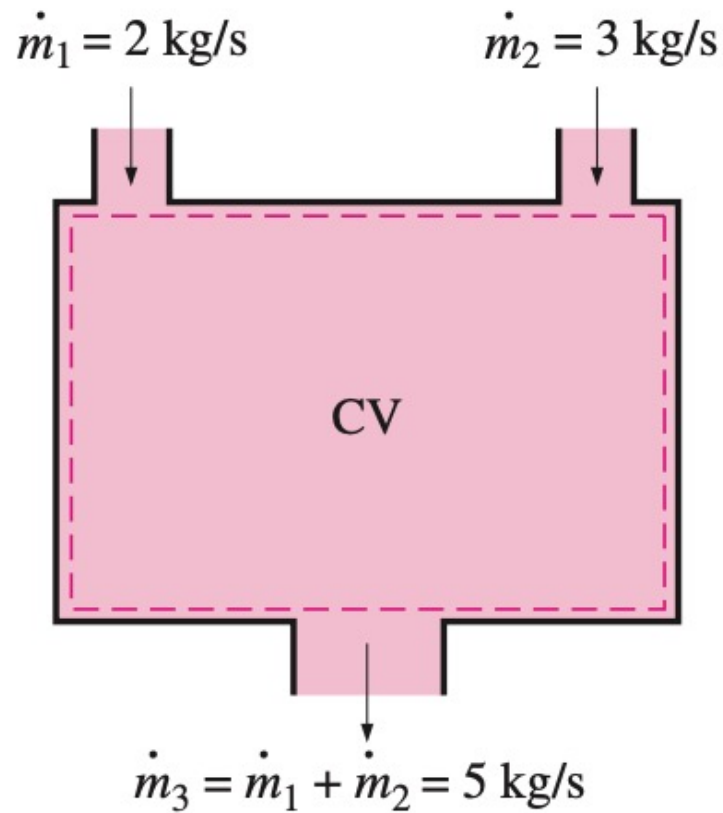


FIGURE 5–7

Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

Mass Balance for Steady-Flow Processes

- During a steady-flow process, the total amount of mass contained within a control volume does not change with time ($m_{CV} = \text{constant}$).
- Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.
- For a garden hose nozzle in steady operation, for example, the amount of water entering the nozzle per unit time is equal to the amount of water leaving it per unit time.
- When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate m* .
- *The conservation of mass principle* for a general steady-flow system with multiple inlets and outlets can be expressed in rate form as (Fig. 5–7)

Steady flow:

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \quad (\text{kg/s}) \quad (5-18)$$

- It states that *the total rate of mass entering a control volume is equal to the total rate of mass leaving it.*
- Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).
- For these cases, we denote the inlet state by the subscript 1 and the outlet state by the subscript 2, and drop the summation signs. Then Eq. 5–18 reduces, for *single-stream steady-flow systems*, to

$$\text{Steady flow (single stream):} \quad \dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad (5-19)$$

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids. Canceling the density from both sides of the general steady-flow relation gives

Steady, incompressible flow:
$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \quad (\text{m}^3/\text{s}) \quad (5-20)$$

For single-stream steady-flow systems it becomes

Steady, incompressible flow (single stream):
$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2 \quad (5-21)$$

- It should always be kept in mind that there is no such thing as a “conservation of volume” principle. Therefore, the volume flow rates into and out of a steady-flow device may be different.
- The volume flow rate at the outlet of an air compressor is much less than that at the inlet even though the mass flow rate of air through the compressor is constant (Fig. 5–8).
- This is due to the higher density of air at the compressor exit.

- For steady flow of liquids, however, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible (constant-density) substances.
- Water flow through the nozzle of a garden hose is an example of the latter case.

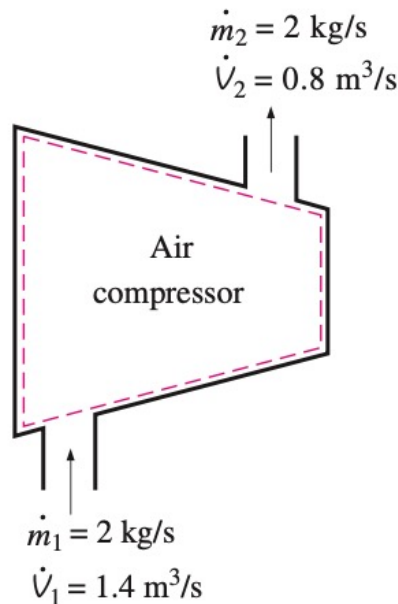


FIGURE 5–8

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

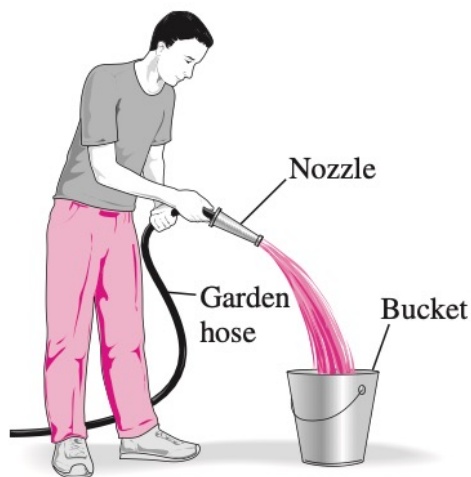


FIGURE 5–9
Schematic for Example 5–1.

EXAMPLE 5–1 Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 5–9). If it takes 50 s to fill the bucket with water, determine (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.

Solution A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be $1000 \text{ kg/m}^3 = 1 \text{ kg/L}$.

Analysis (a) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}} \left(\frac{3.7854 \text{ L}}{1 \text{ gal}} \right) = \mathbf{0.757 \text{ L/s}}$$

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757 \text{ kg/s}}$$

(b) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi (0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2} \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right) = \mathbf{15.1 \text{ m/s}}$$

Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

EXAMPLE 5–2 Discharge of Water from a Tank

A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out (Fig. 5–10). The average velocity of the jet is given by $V = \sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it will take for the water level in the tank to drop to 2 ft from the bottom.

Solution The plug near the bottom of a water tank is pulled out. The time it takes for half of the water in the tank to empty is to be determined.

Assumptions **1** Water is an incompressible substance. **2** The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height. **3** The gravitational acceleration is 32.2 ft/s^2 .

Analysis We take the volume occupied by water as the control volume. The size of the control volume decreases in this case as the water level drops, and thus this is a variable control volume. (We could also treat this as a fixed control volume that consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

The conservation of mass relation for a control volume undergoing any process is given in the rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt} \quad (1)$$

During this process no mass enters the control volume ($\dot{m}_{\text{in}} = 0$), and the mass flow rate of discharged water can be expressed as

$$\dot{m}_{\text{out}} = (\rho VA)_{\text{out}} = \rho \sqrt{2gh} A_{\text{jet}} \quad (2)$$

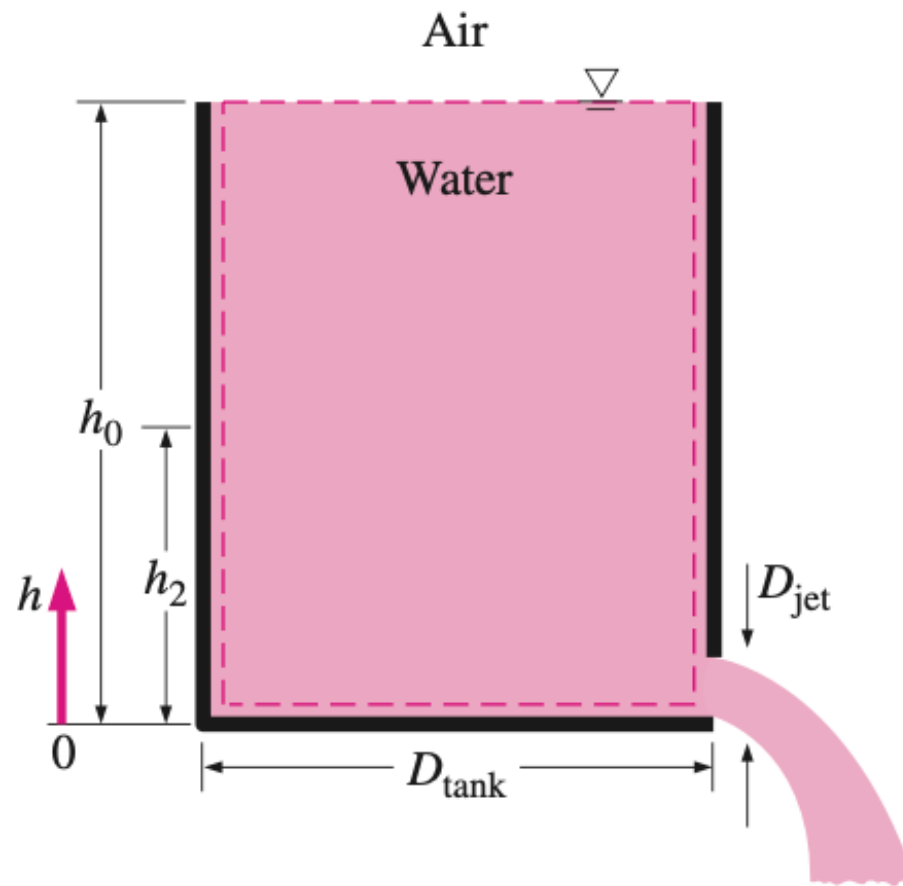


FIGURE 5–10

Schematic for Example 5–2.

where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

$$m_{\text{CV}} = \rho V = \rho A_{\text{tank}} h \quad (3)$$

where $A_{\text{tank}} = \pi D_{\text{tank}}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. 2 and 3 into the mass balance relation (Eq. 1) gives

$$-\rho \sqrt{2gh} A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \rightarrow -\rho \sqrt{2gh} (\pi D_{\text{jet}}^2/4) = \frac{\rho (\pi D_{\text{tank}}^2/4) dh}{dt}$$

Canceling the densities and other common terms and separating the variables give

$$dt = \frac{D_{\text{tank}}^2}{D_{\text{jet}}^2} \frac{dh}{\sqrt{2gh}}$$

Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = - \frac{D_{\text{tank}}^2}{D_{\text{jet}}^2 \sqrt{2g}} \int_{h_0}^{h_2} \frac{dh}{\sqrt{h}} \rightarrow t = \frac{\sqrt{h_0} - \sqrt{h_2}}{\sqrt{g/2}} \left(\frac{D_{\text{tank}}}{D_{\text{jet}}} \right)^2$$

Substituting, the time of discharge is

$$t = \frac{\sqrt{4 \text{ ft}} - \sqrt{2 \text{ ft}}}{\sqrt{32.2/2 \text{ ft/s}^2}} \left(\frac{3 \times 12 \text{ in}}{0.5 \text{ in}} \right)^2 = 757 \text{ s} = \mathbf{12.6 \text{ min}}$$

Therefore, half of the tank is emptied in 12.6 min after the discharge hole is unplugged.

Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1$ min for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h .

FLOW WORK AND THE ENERGY OF A FLOWING FLUID

- Unlike closed systems, **control volumes involve mass flow across their boundaries, and some work is required to push the mass into or out of the control volume.**
- **This work is known as the flow work, or flow energy, and is necessary for maintaining a continuous flow through a control volume.**
- To obtain a relation for flow work, consider a fluid element of volume V as shown in Fig. 5–11.
- The fluid immediately upstream forces this fluid element to enter the control volume; thus, it can be regarded as an imaginary piston.
- The fluid element can be chosen to be sufficiently small so that it has uniform properties throughout
- If the fluid pressure is P and the cross-sectional area of the fluid element is A (Fig. 5–12), the force applied on the fluid element by the imaginary piston is

$$F = PA$$

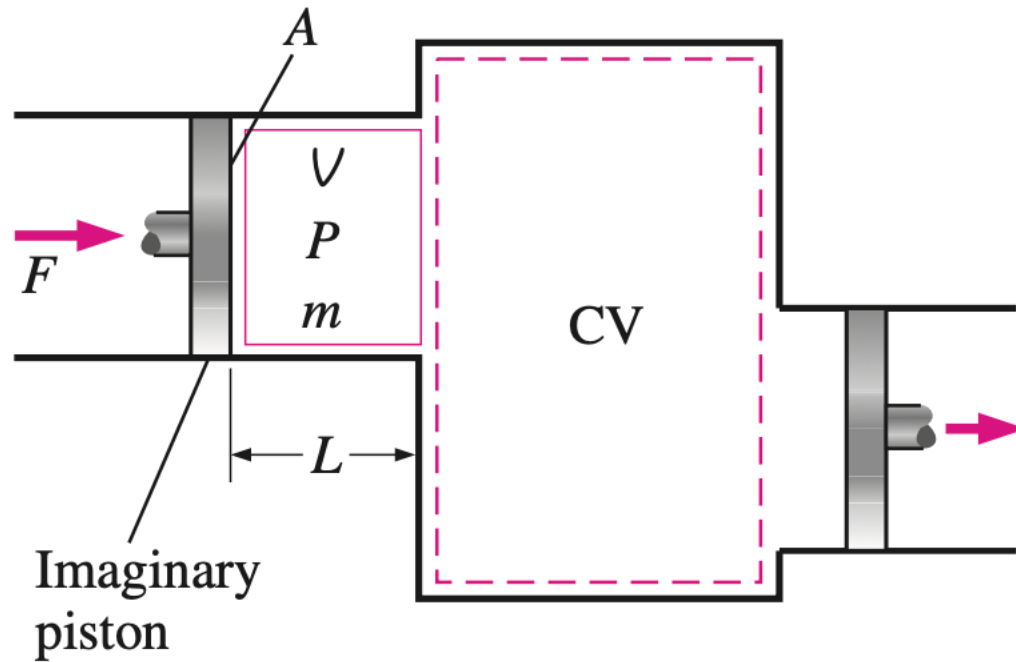


FIGURE 5–11

Schematic for flow work.

- To push the entire fluid element into the control volume, this force must act through a distance L . Thus, the work done in pushing the fluid element across the boundary (i.e., the flow work) is

$$W = FL = PAL = PV \text{ kJ (5-23)}$$

- The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

$$w_{\text{flow}} = Pv \text{ kJ/kg (5-24)}$$

- The flow work relation is the same whether the fluid is pushed into or out of the control volume (Fig. 5-13).
- The flow work relation is the same whether the fluid is pushed into or out of the control volume (Fig. 5-13).
- **It is interesting that unlike other work quantities, flow work is expressed in terms of properties.**

- **In fact, it is the product of two properties of the fluid. For that reason, some people view it as a *combination property* (like enthalpy) and refer to it as *flow energy*, *convected energy*, or *transport energy* instead of flow work.**
- In the discussions that follow, we consider the flow energy to be part of the energy of a flowing fluid, since this greatly simplifies the energy analysis of control volumes.

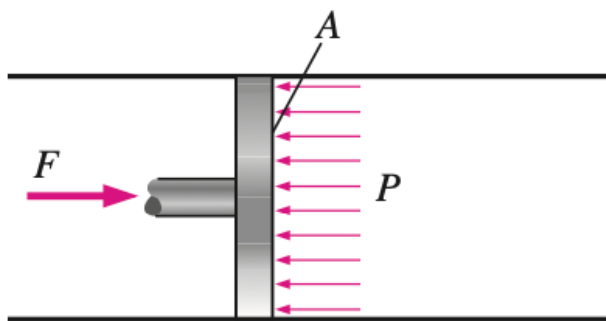
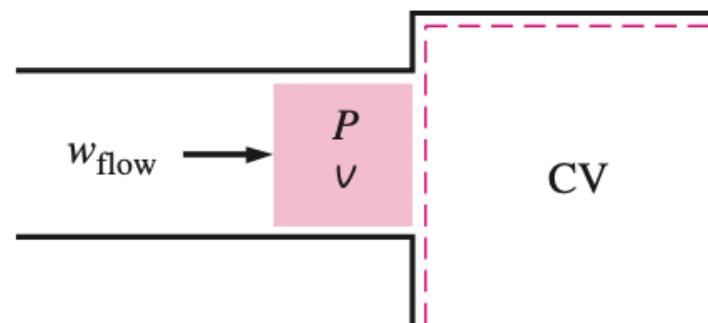
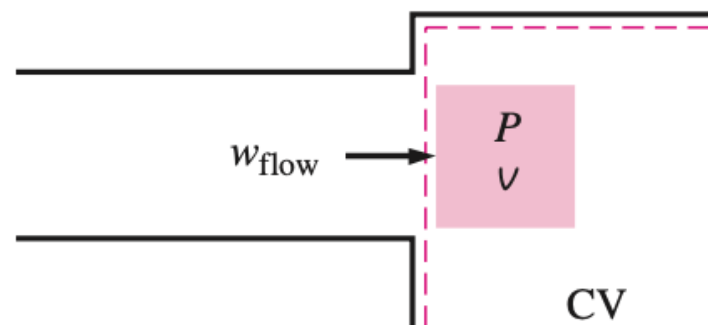


FIGURE 5–12

In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.



(a) Before entering



(b) After entering

FIGURE 5–13

Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to PV .

Total Energy of a Flowing Fluid

As we discussed in Chap. 2, the total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies (Fig. 5–14). On a unit-mass basis, it is expressed as

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \quad (\text{kJ/kg}) \quad (5-25)$$

where V is the velocity and z is the elevation of the system relative to some external reference point.

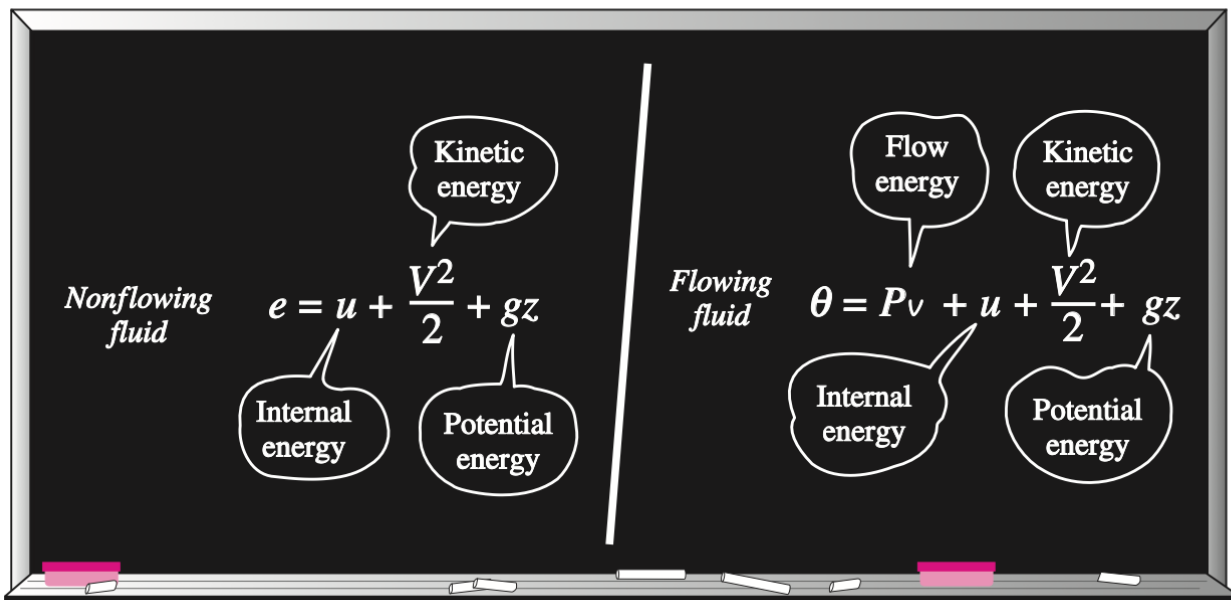


FIGURE 5–14

The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

The fluid entering or leaving a control volume possesses an additional form of energy—the *flow energy* Pv , as already discussed. Then the total energy of a **flowing fluid** on a unit-mass basis (denoted by θ) becomes

$$\theta = Pv + e = Pv + (u + ke + pe) \quad (5-26)$$

But the combination $Pv + u$ has been previously defined as the enthalpy h . So the relation in Eq. 5-26 reduces to

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz \quad (\text{kJ/kg}) \quad (5-27)$$

By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, one does not need to be concerned about the flow work. The energy associated with pushing the fluid into or out of the control volume is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy. From now on, the energy of a fluid stream flowing into or out of a control volume is represented by Eq. 5-27, and no reference will be made to flow work or flow energy.

Energy Transport by Mass

Noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass m is simply $m\theta$, provided that the properties of the mass m are uniform. Also, when a fluid stream with uniform properties is flowing at a mass flow rate of \dot{m} , the rate of energy flow with that stream is $\dot{m}\theta$ (Fig. 5–15). That is,

Amount of energy transport: $E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \quad (\text{kJ}) \quad (5-28)$

Rate of energy transport: $\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \quad (\text{kW}) \quad (5-29)$

When the kinetic and potential energies of a fluid stream are negligible, as is often the case, these relations simplify to $E_{\text{mass}} = mh$ and $\dot{E}_{\text{mass}} = \dot{m}h$.

In general, the total energy transported by mass into or out of the control volume is not easy to determine since the properties of the mass at each inlet or exit may be changing with time as well as over the cross section. Thus, the only way to determine the energy transport through an opening as a result of mass flow is to consider sufficiently small differential masses δm that have uniform properties and to add their total energies during flow.

Again noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass δm is $\theta \delta m$. Then the total energy transported by mass through an inlet or exit ($m_i\theta_i$ and $m_e\theta_e$) is obtained by integration. At an inlet, for example, it becomes

$$E_{\text{in, mass}} = \int_{m_i} \theta_i \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i \right) \delta m_i \quad (5-30)$$

Most flows encountered in practice can be approximated as being steady and one-dimensional, and thus the simple relations in Eqs. 5-28 and 5-29 can be used to represent the energy transported by a fluid stream.

EXAMPLE 5–3 Energy Transport by Mass

Steam is leaving a 4-L pressure cooker whose operating pressure is 150 kPa (Fig. 5–16). It is observed that the amount of liquid in the cooker has decreased by 0.6 L in 40 min after the steady operating conditions are established, and the cross-sectional area of the exit opening is 8 mm^2 . Determine (a) the mass flow rate of the steam and the exit velocity, (b) the total and flow energies of the steam per unit mass, and (c) the rate at which energy leaves the cooker by steam.

Solution Steam leaves a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

Assumptions 1 The flow is steady, and the initial start-up period is disregarded. 2 The kinetic and potential energies are negligible, and thus they are not considered. 3 Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at the cooker pressure.

Properties The properties of saturated liquid water and water vapor at 150 kPa are $v_f = 0.001053 \text{ m}^3/\text{kg}$, $v_g = 1.1594 \text{ m}^3/\text{kg}$, $u_g = 2519.2 \text{ kJ/kg}$, and $h_g = 2693.1 \text{ kJ/kg}$ (Table A–5).

Analysis (a) Saturation conditions exist in a pressure cooker at all times after the steady operating conditions are established. Therefore, the liquid has the properties of saturated liquid and the exiting steam has the properties of saturated vapor at the operating pressure. The amount of liquid that has evaporated, the mass flow rate of the exiting steam, and the exit velocity are

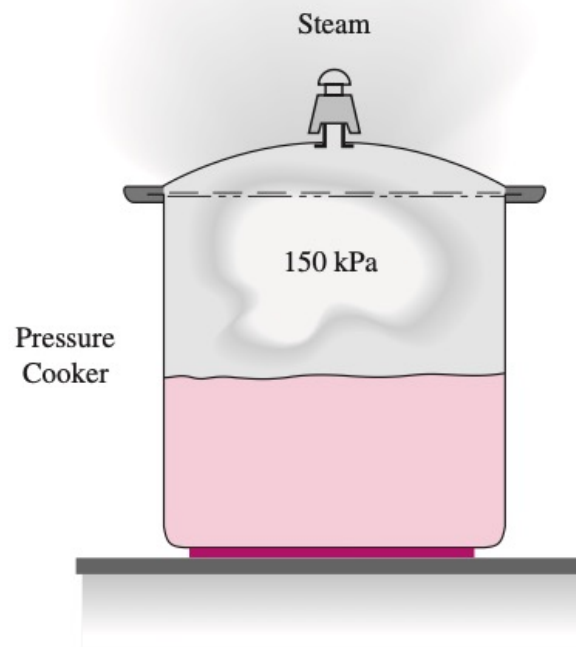


FIGURE 5–16
Schematic for Example 5–3.

$$m = \frac{\Delta V_{\text{liquid}}}{v_f} = \frac{0.6 \text{ L}}{0.001053 \text{ m}^3/\text{kg}} \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right) = 0.570 \text{ kg}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{0.570 \text{ kg}}{40 \text{ min}} = 0.0142 \text{ kg/min} = \mathbf{2.37 \times 10^{-4} \text{ kg/s}}$$

$$V = \frac{\dot{m}}{\rho_g A_c} = \frac{\dot{m} v_g}{A_c} = \frac{(2.37 \times 10^{-4} \text{ kg/s})(1.1594 \text{ m}^3/\text{kg})}{8 \times 10^{-6} \text{ m}^2} = \mathbf{34.3 \text{ m/s}}$$

(b) Noting that $h = u + Pv$ and that the kinetic and potential energies are disregarded, the flow and total energies of the exiting steam are

$$e_{\text{flow}} = Pv = h - u = 2693.1 - 2519.2 = \mathbf{173.9 \text{ kJ/kg}}$$

$$\theta = h + ke + pe \cong h = \mathbf{2693.1 \text{ kJ/kg}}$$

Note that the kinetic energy in this case is $ke = V^2/2 = (34.3 \text{ m/s})^2/2 = 588 \text{ m}^2/\text{s}^2 = 0.588 \text{ kJ/kg}$, which is small compared to enthalpy.

(c) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

$$\dot{E}_{\text{mass}} = \dot{m}\theta = (2.37 \times 10^{-4} \text{ kg/s})(2693.1 \text{ kJ/kg}) = 0.638 \text{ kJ/s} = \mathbf{0.638 \text{ kW}}$$

Discussion The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is h_{fg}) since it relates directly to the amount of energy supplied to the cooker.

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

- A large number of engineering devices such as turbines, compressors, and nozzles operate for long periods of time under the same conditions once the transient start-up period is completed and steady operation is established, and they are classified as *steady-flow devices* (Fig. 5–17).
- Processes involving such devices can be represented reasonably well by a somewhat idealized process, called the **steady-flow process**, which was defined in Chap. 1 as *a process during which a fluid flows through a control volume steadily*.
- That is, the fluid properties can change from point to point within the control volume, but at any point, they remain constant during the entire process. (Remember, *steady* means *no change with time*.)

- During a steady-flow process, no intensive or extensive properties *within the control volume* change with time. Thus, the volume V , the mass m , and the total energy content E of the control volume remain constant (Fig. 5–18).
- As a result, the boundary work is zero for steady-flow systems (since $V_{CV} = \text{constant}$), and the total mass or energy entering the control volume must be equal to the total mass or energy leaving it (since $m_{CV} = \text{constant}$ and $E_{CV} = \text{constant}$). These observations greatly simplify the analysis.
- The fluid properties at an inlet or exit remain constant during a steady-flow process. The properties may, however, be different at different inlets and exits.
- They may even vary over the cross section of an inlet or an exit. However, all properties, including the velocity and elevation, must remain constant with time at a fixed point at an inlet or exit.
- It follows that the mass flow rate of the fluid at an opening must remain constant during a steady-flow process (Fig. 5–19).

- As an added simplification, the fluid properties at an opening are usually considered to be uniform (at some average value) over the cross section. Thus, the fluid properties at an inlet or exit may be specified by the average single values.
- Also, the *heat* and *work* interactions between a steady-flow system and its surroundings do not change with time. Thus, the power delivered by a system and the rate of heat transfer to or from a system remain constant during a steady-flow process.
- The *mass balance* for a general steady-flow system was given in Sec. 5–1 as

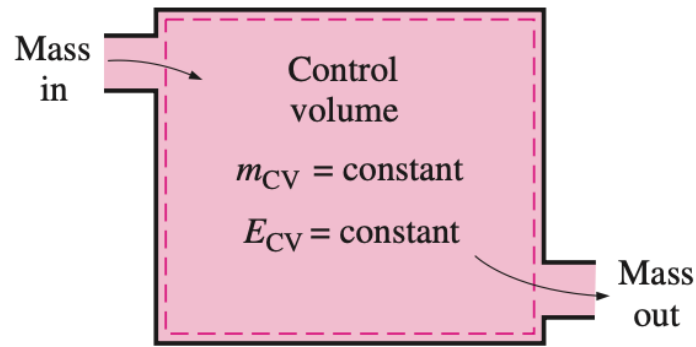


FIGURE 5–18

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

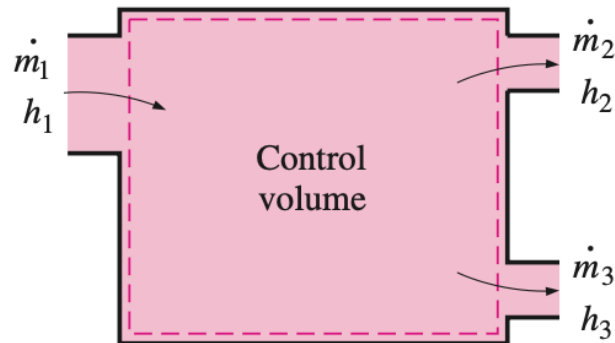


FIGURE 5–19

Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \quad (\text{kg/s}) \quad (5-31)$$

The mass balance for a single-stream (one-inlet and one-outlet) steady-flow system was given as

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad (5-32)$$

where the subscripts 1 and 2 denote the inlet and the exit states, respectively, ρ is density, V is the average flow velocity in the flow direction, and A is the cross-sectional area normal to flow direction.

During a steady-flow process, the total energy content of a control volume remains constant ($E_{\text{CV}} = \text{constant}$), and thus the change in the total energy of the control volume is zero ($\Delta E_{\text{CV}} = 0$). Therefore, the amount of energy entering a control volume in all forms (by heat, work, and mass) must be equal to the amount of energy leaving it. Then the rate form of the general energy balance reduces for a steady-flow process to

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0 \quad (5-33)$$

or

$$\text{Energy balance:} \quad \underbrace{\dot{E}_{\text{in}}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{\text{out}}}_{\text{Rate of net energy transfer out by heat, work, and mass}} \quad (\text{kW}) \quad (5-34)$$

Noting that energy can be transferred by heat, work, and mass only, the energy balance in Eq. 5–34 for a general steady-flow system can also be written more explicitly as

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m}\theta = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m}\theta \quad (5-35)$$

or

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)} = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)} \quad (5-36)$$

- Consider, for example, an ordinary electric hot-water heater under steady operation, as shown in Fig. 5–20.
- A cold-water stream with a mass flow rate \dot{m} is continuously flowing into the water heater, and a hot-water stream of the same mass flow rate is continuously flowing out of it.
- The water heater (the control volume) is losing heat to the surrounding air at a rate of \dot{Q}_{out} , and the electric heating element is supplying electrical work (heating) to the water at a rate of \dot{W}_{in} .
- On the basis of the conservation of energy principle, we can say that the water stream experiences an increase in its total energy as it flows through the water heater that is equal to the electric energy supplied to the water minus the heat losses.**

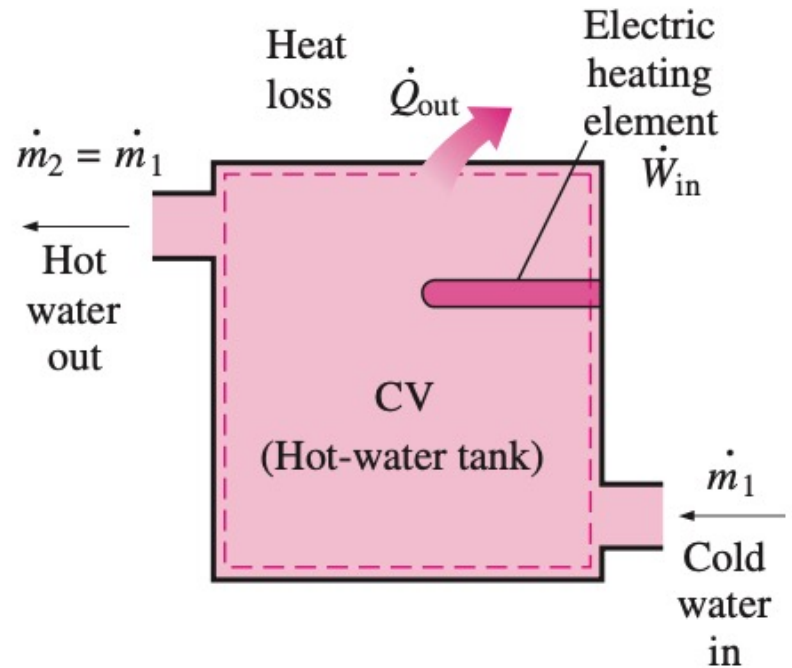


FIGURE 5–20

A water heater in steady operation.

The energy balance relation just given is intuitive in nature and is easy to use when the magnitudes and directions of heat and work transfers are known. When performing a general analytical study or solving a problem that involves an unknown heat or work interaction, however, we need to assume a direction for the heat or work interactions. In such cases, it is common practice to assume heat to be transferred *into the system* (heat input) at a rate of \dot{Q} , and work produced *by the system* (work output) at a rate of \dot{W} , and then solve the problem. The first-law or energy balance relation in that case for a general steady-flow system becomes

$$\dot{Q} - \dot{W} = \underbrace{\sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)}_{\text{for each exit}} - \underbrace{\sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right)}_{\text{for each inlet}} \quad (5-37)$$

Obtaining a negative quantity for \dot{Q} or \dot{W} simply means that the assumed direction is wrong and should be reversed. For single-stream devices, the steady-flow energy balance equation becomes

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] \quad (5-38)$$

Dividing Eq. 5–38 by \dot{m} gives the energy balance on a unit-mass basis as

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad (5-39)$$

where $q = \dot{Q}/\dot{m}$ and $w = \dot{W}/\dot{m}$ are the heat transfer and work done per unit mass of the working fluid, respectively. When the fluid experiences negligible changes in its kinetic and potential energies (that is, $\Delta ke \cong 0$, $\Delta pe \cong 0$), the energy balance equation is reduced further to

$$q - w = h_2 - h_1 \quad (5-40)$$

The various terms appearing in the above equations are as follows:

\dot{Q} = rate of heat transfer between the control volume and its surroundings. When the control volume is losing heat (as in the case of the water heater), \dot{Q} is negative. If the control volume is well insulated (i.e., adiabatic), then $\dot{Q} = 0$.

\dot{W} = **power**. For steady-flow devices, the control volume is constant; thus, there is no boundary work involved. The work required to push mass into and out of the control volume is also taken care of by using enthalpies for the energy of fluid streams instead of internal energies. Then \dot{W} represents the remaining forms of work done per unit time (Fig. 5–21). Many steady-flow devices, such as turbines, compressors, and pumps, transmit power through a shaft, and \dot{W} simply becomes the shaft power for those devices. If the control surface is crossed by electric wires (as in the case of an electric water heater), \dot{W} represents the electrical work done per unit time. If neither is present, then $\dot{W} = 0$.

$\Delta h = h_2 - h_1$. The enthalpy change of a fluid can easily be determined by reading the enthalpy values at the exit and inlet states from the tables. For ideal gases, it can be approximated by $\Delta h = c_{p,\text{avg}}(T_2 - T_1)$. Note that $(\text{kg/s})(\text{kJ/kg}) \equiv \text{kW}$.

$\Delta \text{ke} = (V_2^2 - V_1^2)/2$. The unit of kinetic energy is m^2/s^2 , which is equivalent to J/kg (Fig. 5–22). The enthalpy is usually given in kJ/kg . To add these two quantities, the kinetic energy should be expressed in kJ/kg . This is easily accomplished by dividing it by 1000. A velocity of 45 m/s corresponds to a kinetic energy of only 1 kJ/kg , which is a very small value compared with the enthalpy values encountered in practice. Thus, the kinetic energy term at low velocities can be neglected. When a fluid stream enters and leaves a steady-flow device at about the same velocity ($V_1 \cong V_2$), the change in the kinetic energy is close to zero regardless of the velocity. Caution should be exercised at high velocities, however, since small changes in velocities may cause significant changes in kinetic energy (Fig. 5–23).

$\Delta p_e = g(z_2 - z_1)$. A similar argument can be given for the potential energy term. A potential energy change of 1 kJ/kg corresponds to an elevation difference of 102 m. The elevation difference between the inlet and exit of most industrial devices such as turbines and compressors is well below this value, and the potential energy term is always neglected for these devices. The only time the potential energy term is significant is when a process involves pumping a fluid to high elevations and we are interested in the required pumping power.

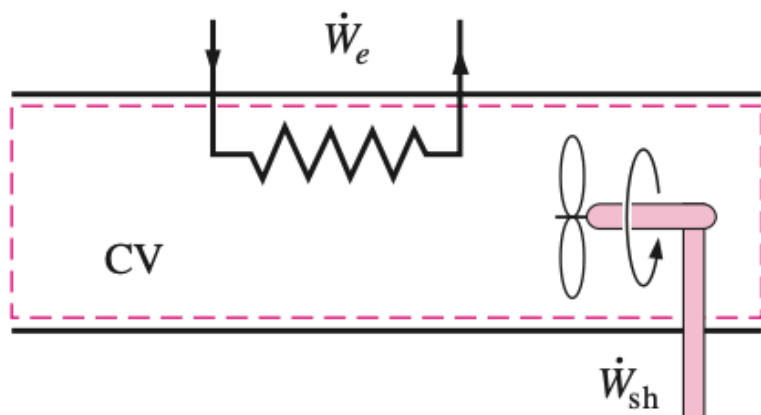


FIGURE 5–21

Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

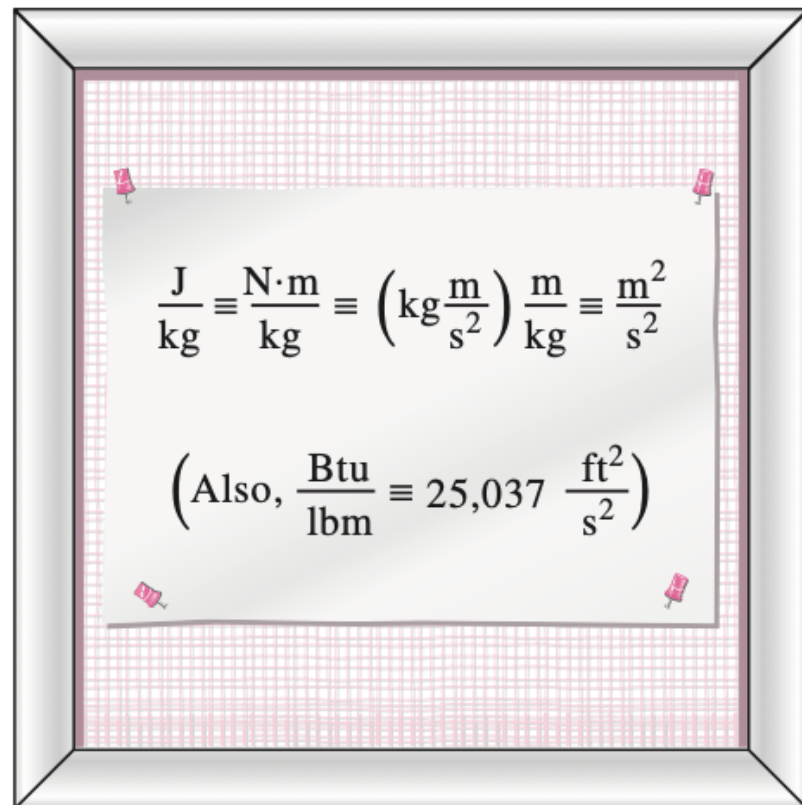


FIGURE 5–22

The units m^2/s^2 and J/kg are equivalent.

SOME STEADY-FLOW ENGINEERING DEVICES

- Many engineering devices operate essentially under the same conditions for long periods of time.
- The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance (Fig. 5–24). Therefore, these devices can be conveniently analyzed as steady-flow devices.
- In this section, some common steady-flow devices are described, and the thermodynamic aspects of the flow through them are analyzed.
- The conservation of mass and the conservation of energy principles for these devices are illustrated with examples.

Nozzles and Diffusers

- Nozzles and diffusers are commonly utilized in jet engines, rockets, space- craft, and even garden hoses.
- A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure. A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down. That is, nozzles and diffusers perform opposite tasks.
- The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.
- **The rate of heat transfer between the fluid flowing through a nozzle or a diffuser and the surroundings is usually very small ($\dot{Q} = 0$)** since the fluid has high velocities, and thus it does not spend enough time in the device for any significant heat transfer to take place.
- **Nozzles and diffusers typically involve no work ($\dot{W} = 0$) and any change in potential energy is negligible ($\Delta pe = 0$).**

- But nozzles and diffusers usually involve very high velocities, and as a fluid passes through a nozzle or diffuser, it experiences large changes in its velocity (Fig. 5–25). Therefore, the kinetic energy changes must be accounted for in analyzing the flow through these devices ($\Delta ke \neq 0$).

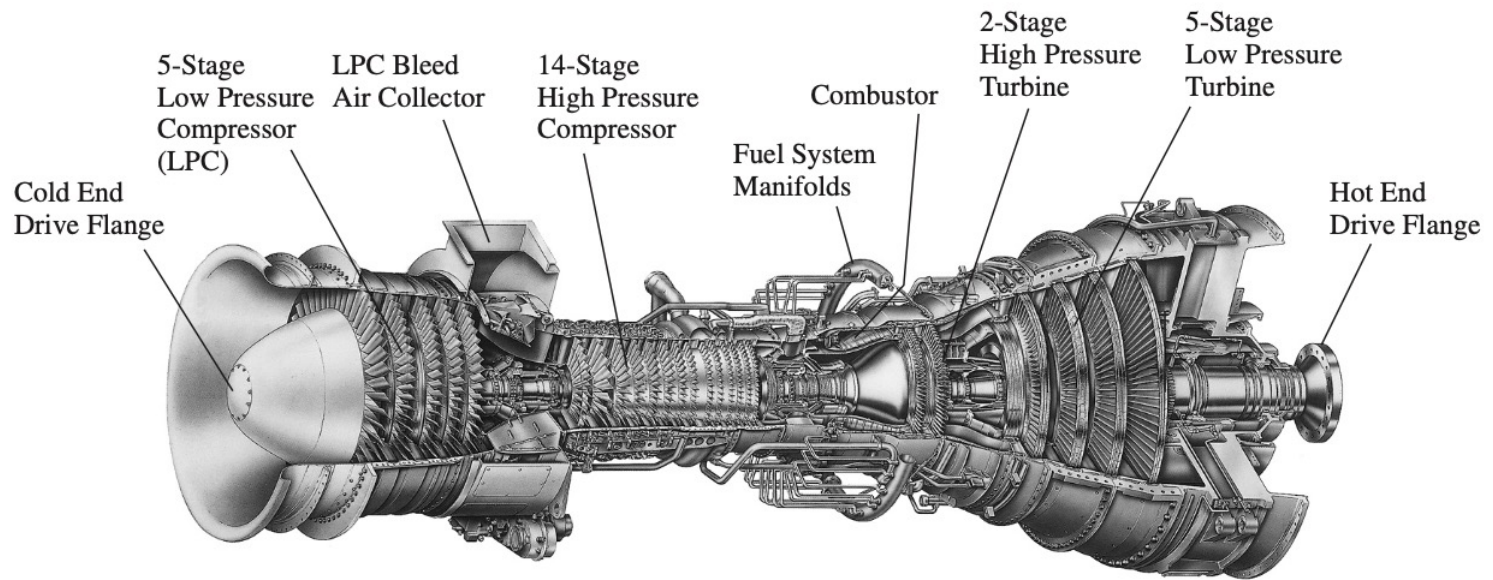


FIGURE 5–24

A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

Courtesy of GE Power Systems

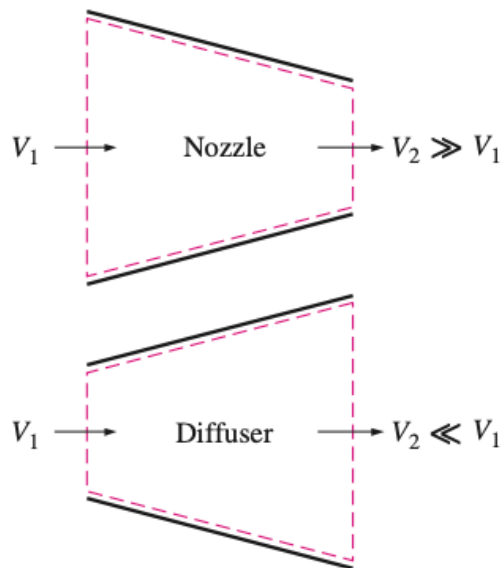


FIGURE 5–25

Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

EXAMPLE 5–4 Deceleration of Air in a Diffuser

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s . The inlet area of the diffuser is 0.4 m^2 . The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (a) the mass flow rate of the air and (b) the temperature of the air leaving the diffuser.

Solution Air enters the diffuser of a jet engine steadily at a specified velocity. The mass flow rate of air and the temperature at the diffuser exit are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{cv}} = 0$ and $\Delta E_{\text{cv}} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The potential energy change is zero, $\Delta pe = 0$. **4** Heat transfer is negligible. **5** Kinetic energy at the diffuser exit is negligible. **6** There are no work interactions.

Analysis We take the *diffuser* as the system (Fig. 5–26). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

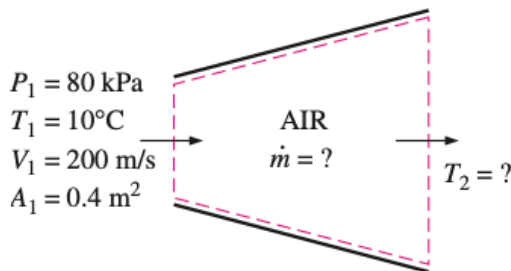


FIGURE 5–26

Schematic for Example 5–4.

(a) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K} (283 \text{ K})}{80 \text{ kPa}} = 1.015 \text{ m}^3/\text{kg}$$

Then,

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{1.015 \text{ m}^3/\text{kg}} (200 \text{ m/s}) (0.4 \text{ m}^2) = \mathbf{78.8 \text{ kg/s}}$$

Since the flow is steady, the mass flow rate through the entire diffuser remains constant at this value.

(b) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}}} \overset{0 \text{ (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ and } \Delta pe \cong 0)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ($V_2 \ll V_1$); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table (Table A–17) to be

$$h_1 = h @ 283 \text{ K} = 283.14 \text{ kJ/kg}$$

Substituting, we get

$$\begin{aligned} h_2 &= 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= 303.14 \text{ kJ/kg} \end{aligned}$$

From Table A-17, the temperature corresponding to this enthalpy value is

$$T_2 = \mathbf{303 \text{ K}}$$

Discussion This result shows that the temperature of the air increases by about 20°C as it is slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

EXAMPLE 5–5 Acceleration of Steam in a Nozzle

Steam at 250 psia and 700°F steadily enters a nozzle whose inlet area is 0.2 ft². The mass flow rate of steam through the nozzle is 10 lbm/s. Steam leaves the nozzle at 200 psia with a velocity of 900 ft/s. Heat losses from the nozzle per unit mass of the steam are estimated to be 1.2 Btu/lbm. Determine (a) the inlet velocity and (b) the exit temperature of the steam.

Solution Steam enters a nozzle steadily at a specified flow rate and velocity. The inlet velocity of steam and the exit temperature are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$ and $\Delta E_{cv} = 0$. **2** There are no work interactions. **3** The potential energy change is zero, $\Delta pe = 0$.

Analysis We take the *nozzle* as the system (Fig. 5–26A). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

(a) The specific volume and enthalpy of steam at the nozzle inlet are

$$\left. \begin{array}{l} P_1 = 250 \text{ psia} \\ T_1 = 700^\circ\text{F} \end{array} \right\} \begin{array}{l} v_1 = 2.6883 \text{ ft}^3/\text{lbm} \\ h_1 = 1371.4 \text{ Btu/lbm} \end{array} \quad (\text{Table A-6E})$$

Then,

$$\dot{m} = \frac{1}{v_1} V_1 A_1$$

$$10 \text{ lbm/s} = \frac{1}{2.6883 \text{ ft}^3/\text{lbm}} (V_1)(0.2 \text{ ft}^2)$$

$$V_1 = \mathbf{134.4 \text{ ft/s}}$$

(b) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{Q}_{\text{out}} + \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) \quad (\text{since } \dot{W} = 0, \text{ and } \Delta p_e \cong 0)$$

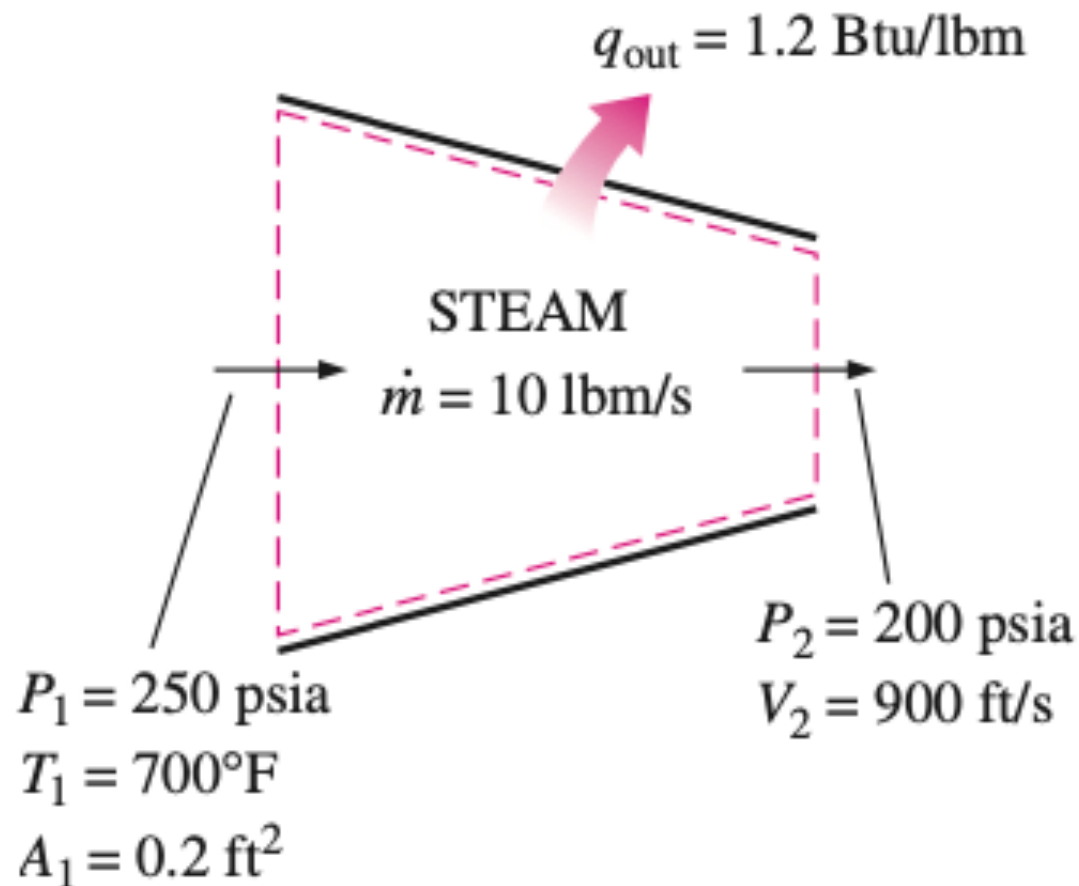


FIGURE 5–26A

Schematic for Example 5–5.

Dividing by the mass flow rate \dot{m} and substituting, h_2 is determined to be

$$\begin{aligned}h_2 &= h_1 - q_{\text{out}} - \frac{V_2^2 - V_1^2}{2} \\&= (1371.4 - 1.2) \text{ Btu/lbm} - \frac{(900 \text{ ft/s})^2 - (134.4 \text{ ft/s})^2}{2} \left(\frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2} \right) \\&= 1354.4 \text{ Btu/lbm}\end{aligned}$$

Then,

$$\left. \begin{array}{l}P_2 = 200 \text{ psia} \\h_2 = 1354.4 \text{ Btu/lbm}\end{array} \right\} T_2 = \mathbf{662.0^\circ\text{F}} \quad (\text{Table A-6E})$$

Discussion Note that the temperature of steam drops by 38.0°F as it flows through the nozzle. This drop in temperature is mainly due to the conversion of internal energy to kinetic energy. (The heat loss is too small to cause any significant effect in this case.)

Turbines and Compressors

- In steam, gas, or hydroelectric power plants, the device that drives **the electric generator is the turbine**. As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, **the shaft rotates, and the turbine produces work**.
- Compressors, as well as pumps and fans, are devices used to increase **the pressure of a fluid**. Work is supplied to these devices from an external source through a rotating shaft. Therefore, **compressors involve work inputs**.
- Even though these three devices function similarly, they do differ in the tasks they perform. A *fan* increases the pressure of a gas slightly and is mainly used to mobilize a gas. A *compressor* is capable of compressing the gas to very high pressures.
- *Pumps* work very much like compressors except that they handle liquids instead of gases. Note that turbines produce power output whereas compressors, pumps, and fans require power input.
- Heat transfer from turbines is **usually negligible ($\dot{Q} = 0$)** since they are typically well insulated. Heat transfer is also negligible for compressors unless there is intentional cooling.

- **Potential energy changes are negligible for all of these devices ($\Delta p_e = 0$). The velocities involved in these devices, with the exception of turbines and fans, are usually too low to cause any significant change in the kinetic energy ($\Delta k_e = 0$).**
- **The fluid velocities encountered in most turbines are very high, and the fluid experiences a significant change in its kinetic energy. However, this change is usually very small relative to the change in enthalpy, and thus it is often disregarded.**

EXAMPLE 5–6 Compressing Air by a Compressor

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

Solution Air is compressed steadily by a compressor to a specified temperature and pressure. The power input to the compressor is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The kinetic and potential energy changes are zero, $\Delta \text{ke} = \Delta \text{pe} = 0$.

Analysis We take the *compressor* as the system (Fig. 5–27). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, heat is lost from the system and work is supplied to the system.

Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta \text{ke} = \Delta \text{pe} \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}q_{\text{out}} + \dot{m}(h_2 - h_1)$$

The enthalpy of an ideal gas depends on temperature only, and the enthalpies of the air at the specified temperatures are determined from the air table (Table A-17) to be

$$h_1 = h_{@ 280 \text{ K}} = 280.13 \text{ kJ/kg}$$

$$h_2 = h_{@ 400 \text{ K}} = 400.98 \text{ kJ/kg}$$

Substituting, the power input to the compressor is determined to be

$$\begin{aligned} \dot{W}_{\text{in}} &= (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg} \\ &= \mathbf{2.74 \text{ kW}} \end{aligned}$$

Discussion Note that the mechanical energy input to the compressor manifests itself as a rise in enthalpy of air and heat loss from the compressor.

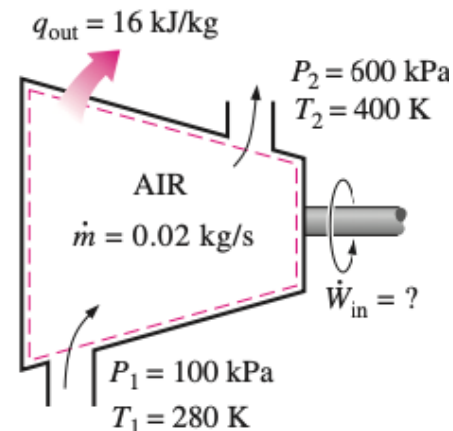


FIGURE 5-27

Schematic for Example 5-6.

EXAMPLE 5–7 Power Generation by a Steam Turbine

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in Fig. 5–28.

- (a) Compare the magnitudes of Δh , Δke , and Δpe .
- (b) Determine the work done per unit mass of the steam flowing through the turbine.
- (c) Calculate the mass flow rate of the steam.

Solution The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$ and $\Delta E_{cv} = 0$. **2** The system is adiabatic and thus there is no heat transfer.

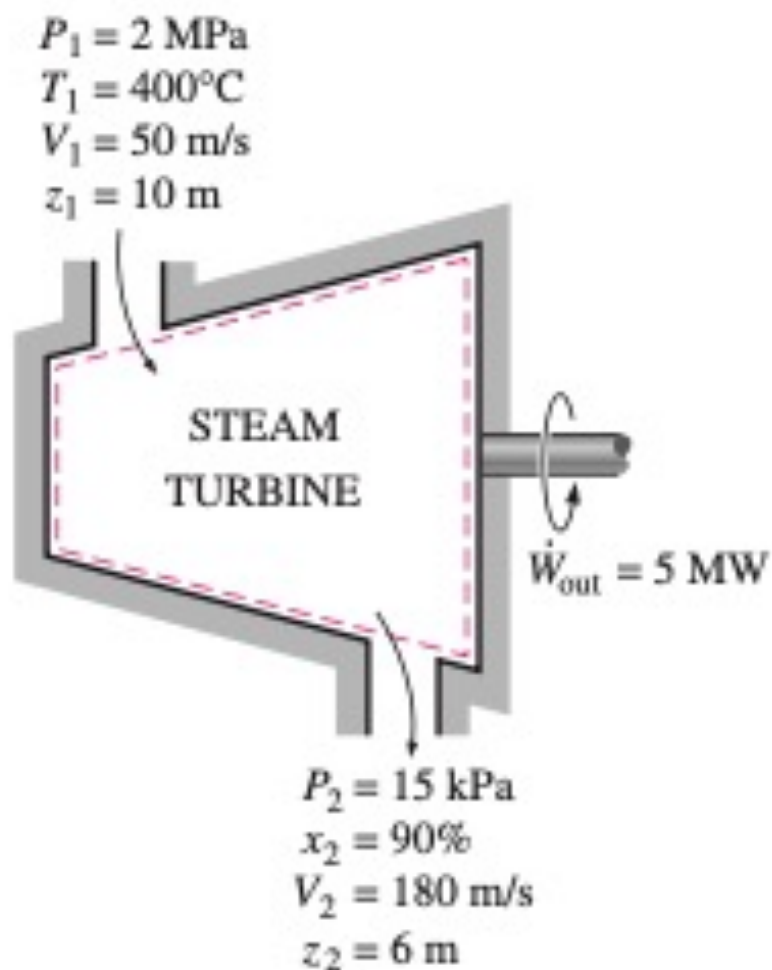


FIGURE 5-28

Schematic for Example 5-7.

Analysis We take the *turbine* as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, work is done by the system. The inlet and exit velocities and elevations are given, and thus the kinetic and potential energies are to be considered.

(a) At the inlet, steam is in a superheated vapor state, and its enthalpy is

$$\left. \begin{array}{l} P_1 = 2 \text{ MPa} \\ T_1 = 400^\circ\text{C} \end{array} \right\} h_1 = 3248.4 \text{ kJ/kg} \quad (\text{Table A-6})$$

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)] \text{ kJ/kg} = 2361.01 \text{ kJ/kg}$$

Then

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4) \text{ kJ/kg} = -887.39 \text{ kJ/kg}$$

$$\Delta \text{ke} = \frac{V_2^2 - V_1^2}{2} = \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 14.95 \text{ kJ/kg}$$

$$\Delta \text{pe} = g(z_2 - z_1) = (9.81 \text{ m/s}^2)[(6 - 10) \text{ m}] \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = -0.04 \text{ kJ/kg}$$

(b) The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{\text{out}} + \dot{m} \left(h_2 + \frac{V_2^2}{2} + gz_2 \right) \quad (\text{since } \dot{Q} = 0)$$

Dividing by the mass flow rate \dot{m} and substituting, the work done by the turbine per unit mass of the steam is determined to be

$$\begin{aligned} w_{\text{out}} &= - \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = -(\Delta h + \Delta \text{ke} + \Delta \text{pe}) \\ &= -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = \mathbf{872.48 \text{ kJ/kg}} \end{aligned}$$

(c) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{out}}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = \mathbf{5.73 \text{ kg/s}}$$

Discussion Two observations can be made from these results. First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. This is typical for most engineering devices. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high. Yet the change in kinetic energy is a small fraction of the change in enthalpy (less than 2 percent in our case) and is therefore often neglected.

Throttling Valves

- Throttling valves are *any kind of flow-restricting devices* **that cause a significant pressure drop in the fluid**. Some familiar examples are ordinary **adjustable valves, capillary tubes, and porous plugs** (Fig. 5–29).
- Unlike turbines, they produce a pressure drop without involving any work. The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.
- The magnitude of the temperature drop (or, sometimes, the temperature rise) during a throttling process is governed by a property called the *Joule-Thomson coefficient*, discussed in Chap. 12.
- Throttling valves are usually small devices, and the flow through them may be **assumed to be adiabatic ($q = 0$)** since there is neither sufficient time nor large enough area for any effective heat transfer to take place.

- Also, **there is no work done** ($w = 0$), and the change in potential energy, if any, is very **small** ($\Delta pe = 0$). Even though the exit velocity is often considerably higher than the inlet velocity, in many cases, the increase in kinetic energy is insignificant ($\Delta ke = 0$). Then the conservation of energy equation for this single-stream steady-flow device reduces to

$$h_2 \cong h_1 \text{ (kJ/kg)} \dots\dots\dots \text{Eq. 5-41}$$

- That is, enthalpy values at the inlet and exit of a throttling valve are the same. For this reason, a throttling valve is sometimes called **an isenthalpic device**. Note, however, that for throttling devices with large exposed surface areas such as capillary tubes, heat transfer may be significant.
- To gain some insight into how throttling affects fluid properties, let us express Eq. 5-41 as follows:

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$

- Thus the final outcome of a throttling process depends on which of the two quantities increases during the process. If the flow energy increases during the process ($P_2 v_2 = P_1 v_1$), it can do so at the expense of the internal energy. As a result, internal energy decreases, which is usually accompanied by a drop in temperature.
- If the product Pv decreases, the internal energy and the temperature of a fluid will increase during a throttling process. In the case of an ideal gas, $h = h(T)$, and thus the temperature has to remain constant during a throttling process (Fig. 5–30).

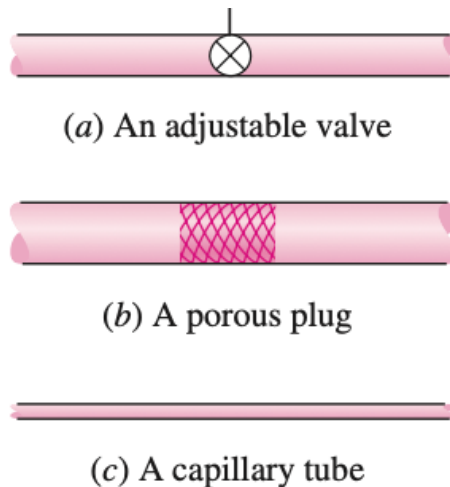


FIGURE 5–29

Throttling valves are devices that cause large pressure drops in the fluid.

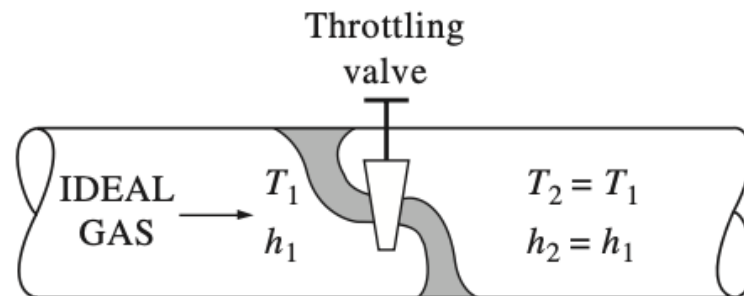


FIGURE 5–30

The temperature of an ideal gas does not change during a throttling ($h = \text{constant}$) process since $h = h(T)$.

EXAMPLE 5–8 Expansion of Refrigerant-134a in a Refrigerator

Refrigerant-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine the quality of the refrigerant at the final state and the temperature drop during this process.

Solution Refrigerant-134a that enters a capillary tube as saturated liquid is throttled to a specified pressure. The exit quality of the refrigerant and the temperature drop are to be determined.

Throttling
valve

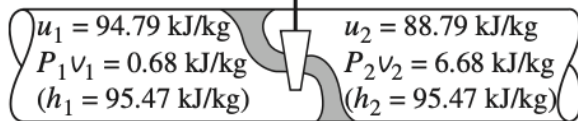


FIGURE 5–31

During a throttling process, the enthalpy (flow energy + internal energy) of a fluid remains constant. But internal and flow energies may be converted to each other.

Assumptions 1 Heat transfer from the tube is negligible. 2 Kinetic energy change of the refrigerant is negligible.
Analysis A capillary tube is a simple flow-restricting device that is commonly used in refrigeration applications to cause a large pressure drop in the refrigerant. Flow through a capillary tube is a throttling process; thus, the enthalpy of the refrigerant remains constant (Fig. 5–31).

$$\begin{aligned} \text{At inlet: } & \left. \begin{array}{l} P_1 = 0.8 \text{ MPa} \\ \text{sat. liquid} \end{array} \right\} \begin{array}{l} T_1 = T_{\text{sat}} @ 0.8 \text{ MPa} = 31.31^\circ\text{C} \\ h_1 = h_f @ 0.8 \text{ MPa} = 95.47 \text{ kJ/kg} \end{array} \quad (\text{Table A-12}) \\ \\ \text{At exit: } & \begin{array}{l} P_2 = 0.12 \text{ MPa} \\ (h_2 = h_1) \end{array} \longrightarrow \begin{array}{l} h_f = 22.49 \text{ kJ/kg} \\ h_g = 236.97 \text{ kJ/kg} \end{array} \quad T_{\text{sat}} = -22.32^\circ\text{C} \end{aligned}$$

Obviously $h_f < h_2 < h_g$; thus, the refrigerant exists as a saturated mixture at the exit state. The quality at this state is

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{95.47 - 22.49}{236.97 - 22.49} = 0.340$$

Since the exit state is a saturated mixture at 0.12 MPa, the exit temperature must be the saturation temperature at this pressure, which is -22.32°C . Then the temperature change for this process becomes

$$\Delta T = T_2 - T_1 = (-22.32 - 31.31)^\circ\text{C} = -53.63^\circ\text{C}$$

Discussion Note that the temperature of the refrigerant drops by 53.63°C during this throttling process. Also note that 34.0 percent of the refrigerant vaporizes during this throttling process, and the energy needed to vaporize this refrigerant is absorbed from the refrigerant itself.

Mixing Chambers

- In engineering applications, mixing two streams of fluids is not a rare occurrence. The section where the mixing process takes place is commonly referred to as a **mixing chamber**.
- The mixing chamber does not have to be a distinct “chamber.” An ordinary T-elbow or a Y-elbow in a shower, for example, serves as the mixing chamber for the cold- and hot-water streams (Fig. 5–32).
- The conservation of mass principle for a mixing chamber requires that the sum of the incoming mass flow rates equal the mass flow rate of the outgoing mixture.
- **Mixing chambers are usually well insulated ($q = 0$) and usually do not involve any kind of work ($w = 0$). Also, the kinetic and potential energies of the fluid streams are usually negligible ($ke = 0$, $pe = 0$).**
- Then all there is left in the energy equation is the total energies of the incoming streams and the outgoing mixture. The conservation of energy principle requires that these two equal each other. Therefore, the conservation of energy equation becomes analogous to the conservation of mass equation for this case.

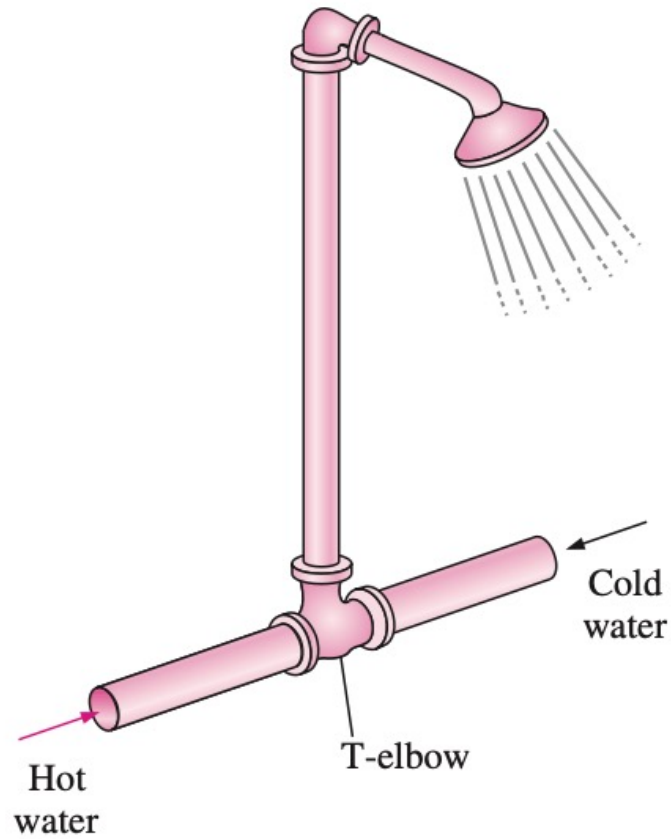


FIGURE 5-32

The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.

EXAMPLE 5–9 Mixing of Hot and Cold Waters in a Shower

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.

Solution In a shower, cold water is mixed with hot water at a specified temperature. For a specified mixture temperature, the ratio of the mass flow rates of the hot to cold water is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$ and $\Delta E_{cv} = 0$. **2** The kinetic and potential energies are negligible, $ke \cong pe \cong 0$. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There is no work interaction involved.

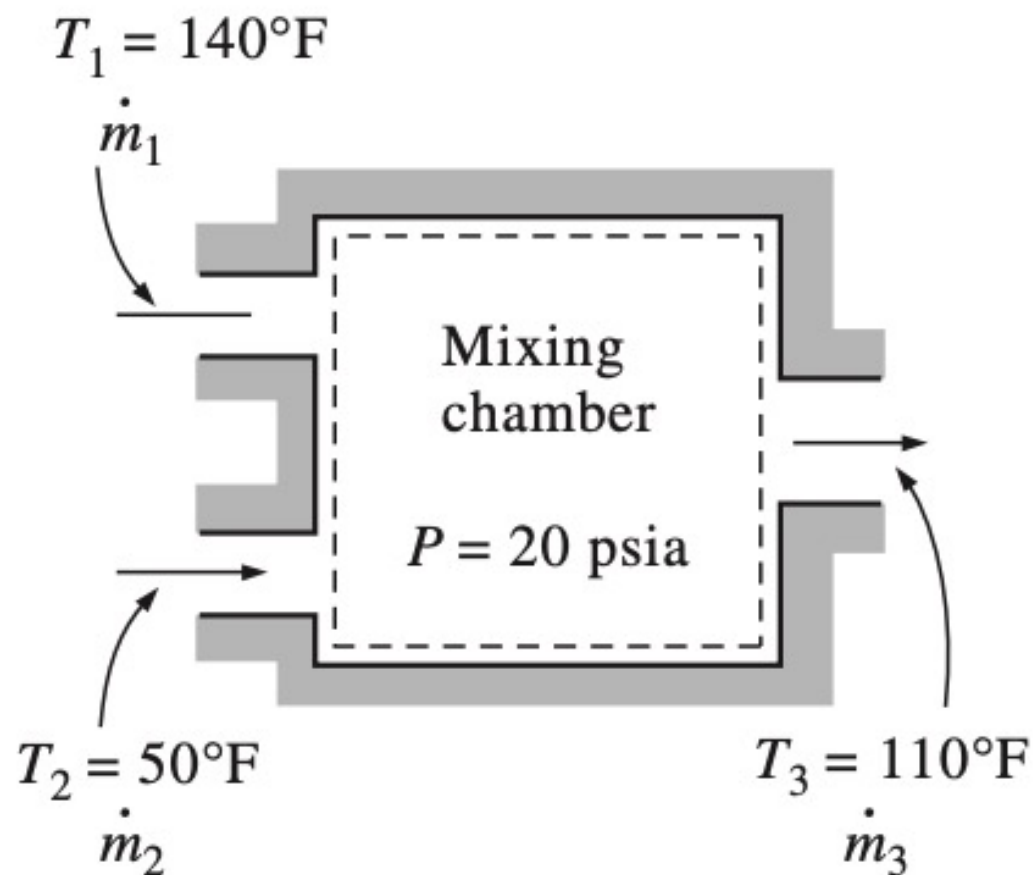


FIGURE 5–33

Schematic for Example 5–9.

Analysis We take the *mixing chamber* as the system (Fig. 5–33). This is a *control volume* since mass crosses the system boundary during the process. We observe that there are two inlets and one exit.

Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

$$\text{Mass balance:} \quad \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{Energy balance:} \quad \underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc., energies}}} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ke} \cong \text{pe} \cong 0)$$

Combining the mass and energy balances,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

Dividing this equation by \dot{m}_2 yields

$$y h_1 + h_2 = (y + 1) h_3$$

where $y = \dot{m}_1 / \dot{m}_2$ is the desired mass flow rate ratio.

where $y = \dot{m}_1/\dot{m}_2$ is the desired mass flow rate ratio.

The saturation temperature of water at 20 psia is 227.92°F. Since the temperatures of all three streams are below this value ($T < T_{\text{sat}}$), the water in all three streams exists as a compressed liquid (Fig. 5–34). A compressed liquid can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_f @ 140^\circ\text{F} = 107.99 \text{ Btu/lbm}$$

$$h_2 \cong h_f @ 50^\circ\text{F} = 18.07 \text{ Btu/lbm}$$

$$h_3 \cong h_f @ 110^\circ\text{F} = 78.02 \text{ Btu/lbm}$$

Solving for y and substituting yields

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = 2.0$$

Discussion Note that the mass flow rate of the hot water must be twice the mass flow rate of the cold water for the mixture to leave at 110°F.

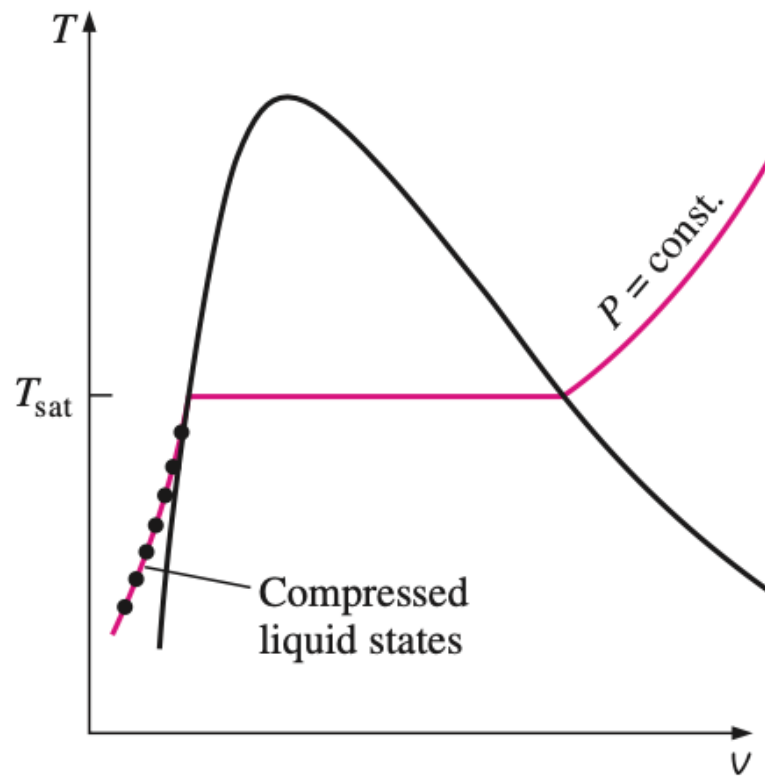


FIGURE 5–34

A substance exists as a compressed liquid at temperatures below the saturation temperatures at the given pressure.

Heat Exchangers

- As the name implies, **heat exchangers** are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.
- The simplest form of a heat exchanger is a *double-tube* (also called *tube- and-shell*) *heat exchanger*, shown in Fig. 5–35. It is composed of two concentric pipes of different diameters. One fluid flows in the inner pipe, and the other in the annular space between the two pipes.
- Heat is transferred from the hot fluid to the cold one through the wall separating them. Some- times the inner tube makes a couple of turns inside the shell to increase the heat transfer area, and thus the rate of heat transfer.
- **The mixing chambers discussed earlier are sometimes classified as *direct-contact* heat exchangers.**

- The conservation of mass principle for a heat exchanger in steady operation requires that the sum of the inbound mass flow rates equal the sum of the outbound mass flow rates. This principle can also be expressed as follows: *Under steady operation, the mass flow rate of each fluid stream flowing through a heat exchanger remains constant.*
- Heat exchangers typically involve no work interactions ($w = 0$) and negligible kinetic and potential energy changes ($\Delta ke = 0$, $\Delta pe = 0$) for each fluid stream.
- The heat transfer rate associated with heat exchangers depends on how the control volume is selected. Heat exchangers are intended for heat transfer between two fluids *within* the device, and the outer shell is usually well insulated to prevent any heat loss to the surrounding medium.
- When the entire heat exchanger is selected as the control volume, Q becomes zero, since the boundary for this case lies just beneath the insulation and little or no heat crosses the boundary (Fig. 5–36). If, however, only one of the fluids is selected as the control volume, then heat will cross this boundary as it flows from one fluid to the other and Q will not be zero. In fact, Q in this case will be the rate of heat transfer between the two fluids.

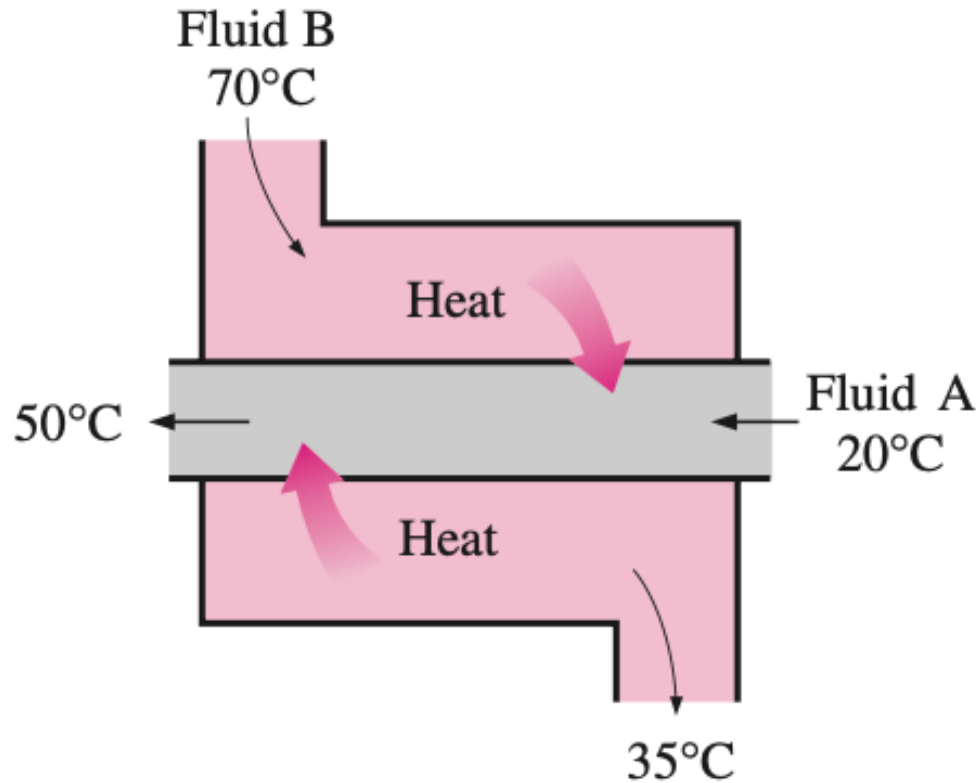


FIGURE 5–35

A heat exchanger can be as simple as two concentric pipes.

EXAMPLE 5–10 Cooling of Refrigerant-134a by Water

Refrigerant-134a is to be cooled by water in a condenser. The refrigerant enters the condenser with a mass flow rate of 6 kg/min at 1 MPa and 70°C and leaves at 35°C. The cooling water enters at 300 kPa and 15°C and leaves

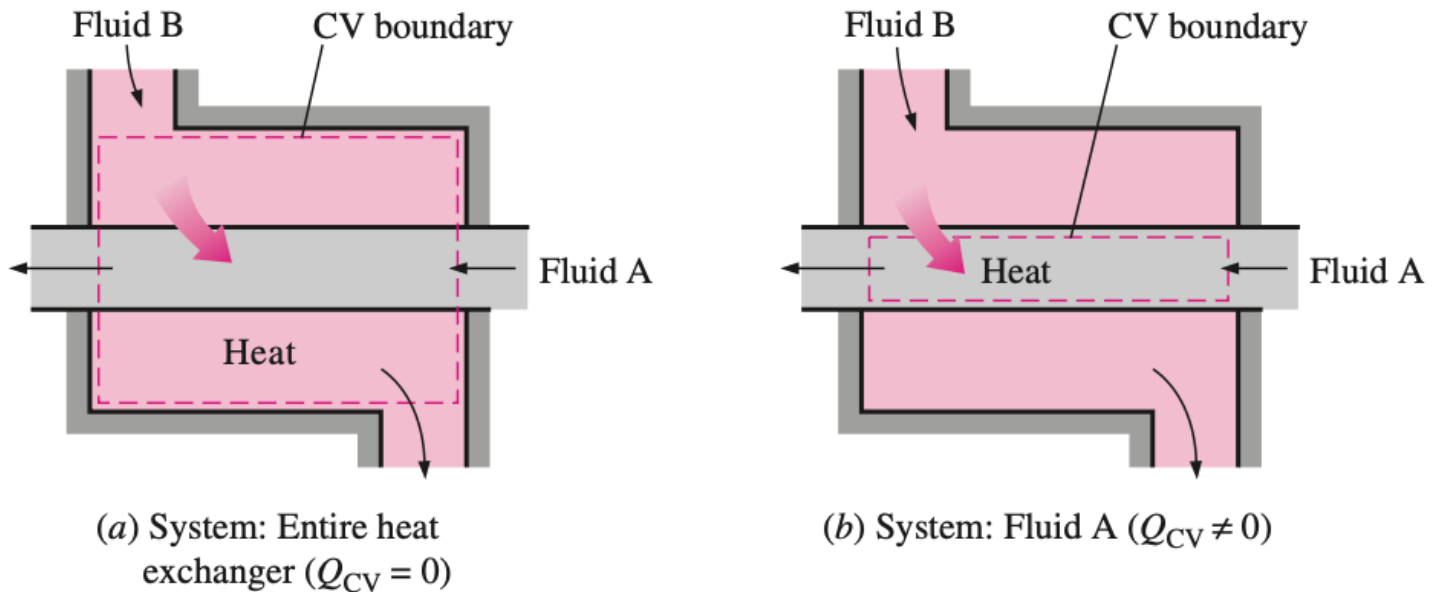


FIGURE 5–36

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

at 25°C. Neglecting any pressure drops, determine (a) the mass flow rate of the cooling water required and (b) the heat transfer rate from the refrigerant to water.

Solution Refrigerant-134a is cooled by water in a condenser. The mass flow rate of the cooling water and the rate of heat transfer from the refrigerant to the water are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$ and $\Delta E_{cv} = 0$. **2** The kinetic and potential energies are negligible, $ke \cong pe \cong 0$. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There is no work interaction.

Analysis We take the *entire heat exchanger* as the system (Fig. 5–37). This is a *control volume* since mass crosses the system boundary during the process. In general, there are several possibilities for selecting the control volume for multiple-stream steady-flow devices, and the proper choice depends on the situation at hand. We observe that there are two fluid streams (and thus two inlets and two exits) but no mixing.

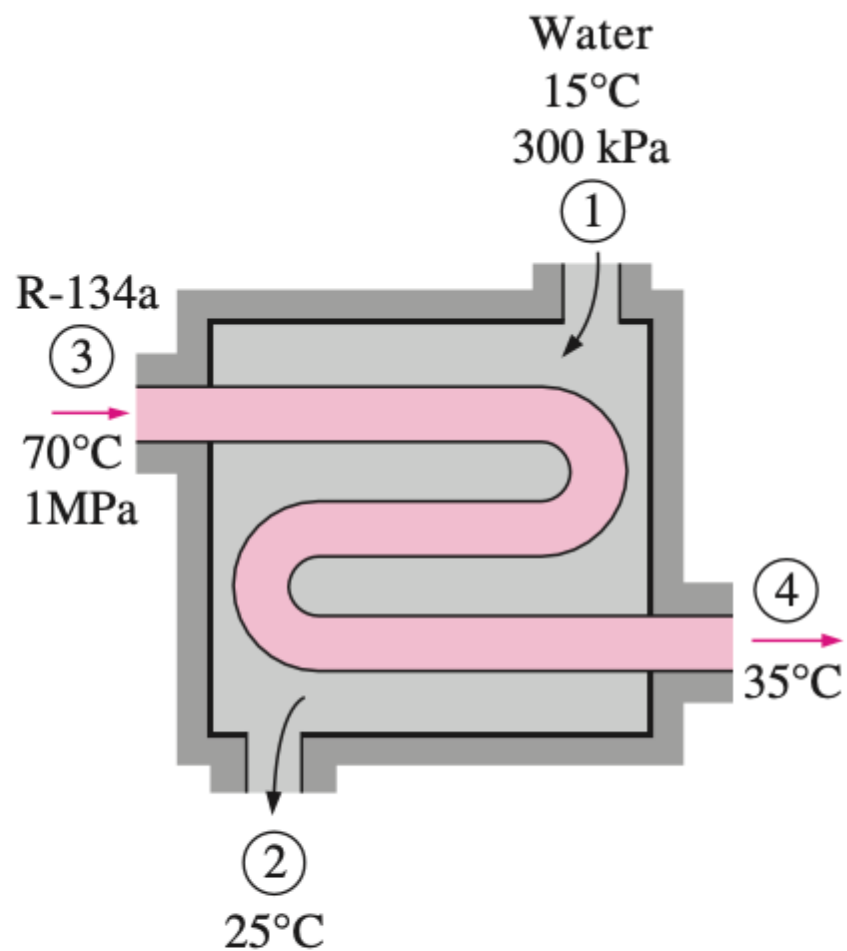


FIGURE 5–37

Schematic for Example 5–10.

(a) Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance: $\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$

for each fluid stream since there is no mixing. Thus,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance: $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \xrightarrow{0 \text{ (steady)}} 0$

Rate of net energy transfer by heat, work, and mass
Rate of change in internal, kinetic, potential, etc., energies

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ke} \cong \text{pe} \cong 0)$$

Combining the mass and energy balances and rearranging give

$$\dot{m}_w(h_1 - h_2) = \dot{m}_R(h_4 - h_3)$$

Now we need to determine the enthalpies at all four states. Water exists as a compressed liquid at both the inlet and the exit since the temperatures at both locations are below the saturation temperature of water at 300 kPa (133.52°C). Approximating the compressed liquid as a saturated liquid at the given temperatures, we have

$$\begin{aligned}h_1 &\cong h_{f@15^\circ\text{C}} = 62.982 \text{ kJ/kg} \\h_2 &\cong h_{f@25^\circ\text{C}} = 104.83 \text{ kJ/kg}\end{aligned}\quad (\text{Table A-4})$$

The refrigerant enters the condenser as a superheated vapor and leaves as a compressed liquid at 35°C. From refrigerant-134a tables,

$$\begin{aligned}\left. \begin{array}{l}P_3 = 1 \text{ MPa} \\T_3 = 70^\circ\text{C}\end{array} \right\} h_3 &= 303.85 \text{ kJ/kg} \quad (\text{Table A-13}) \\[1em]\left. \begin{array}{l}P_4 = 1 \text{ MPa} \\T_4 = 35^\circ\text{C}\end{array} \right\} h_4 &\cong h_{f@35^\circ\text{C}} = 100.87 \text{ kJ/kg} \quad (\text{Table A-11})\end{aligned}$$

Substituting, we find

$$\dot{m}_w(62.982 - 104.83) \text{ kJ/kg} = (6 \text{ kg/min})[(100.87 - 303.85) \text{ kJ/kg}]$$

$$\dot{m}_w = \mathbf{29.1 \text{ kg/min}}$$

(b) To determine the heat transfer from the refrigerant to the water, we have to choose a control volume whose boundary lies on the path of heat transfer. We can choose the volume occupied by either fluid as our control volume. For no particular reason, we choose the volume occupied by the water. All the assumptions stated earlier apply, except that the heat transfer is no longer zero. Then assuming heat to be transferred to water, the energy balance for this single-stream steady-flow system reduces to

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q}_{w, \text{in}} + \dot{m}_w h_1 = \dot{m}_w h_2$$

Rearranging and substituting,

$$\dot{Q}_{w, \text{in}} = \dot{m}_w(h_2 - h_1) = (29.1 \text{ kg/min})[(104.83 - 62.982) \text{ kJ/kg}]$$

$$= \mathbf{1218 \text{ kJ/min}}$$

Discussion Had we chosen the volume occupied by the refrigerant as the control volume (Fig. 5–38), we would have obtained the same result for $\dot{Q}_{R, \text{out}}$ since the heat gained by the water is equal to the heat lost by the refrigerant.

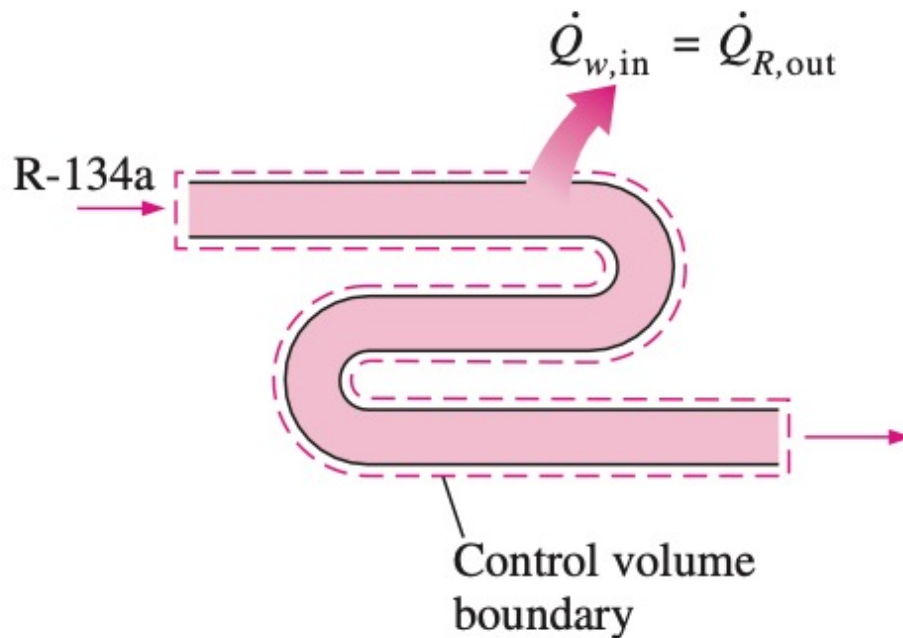


FIGURE 5–38

In a heat exchanger, the heat transfer depends on the choice of the control volume.

Pipe and Duct Flow

- The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions and thus can be analyzed as a steady-flow process. This, of course, excludes the transient start-up and shut-down periods.
- The control volume can be selected to coincide with the interior surfaces of the portion of the pipe or the duct that we are interested in analyzing
- Under normal operating conditions, the amount of heat gained or lost by the fluid may be very significant, particularly if the pipe or duct is long (Fig. 5–39). Sometimes heat transfer is desirable and is the sole purpose of the flow.
- Water flow through the pipes in the furnace of a power plant, the flow of refrigerant in a freezer, and the flow in heat exchangers are some examples of this case.

- At other times, heat transfer is undesirable, and the pipes or ducts are insulated to prevent any heat loss or gain, particularly when the temperature difference between the flowing fluid and the surroundings is large. Heat transfer in this case is negligible.
- If the control volume involves a heating section (electric wires), a fan, or a pump (shaft), the work interactions should be considered (Fig. 5–40). Of these, fan work is usually small and often neglected in energy analysis.

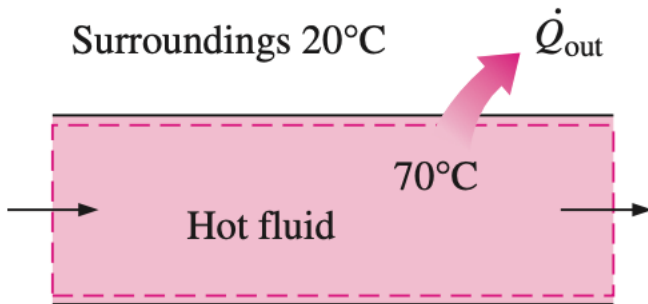


FIGURE 5–39

Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

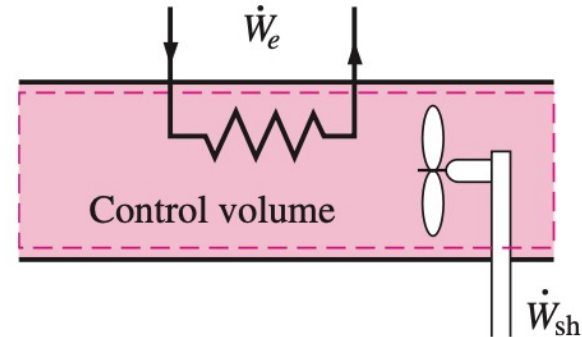


FIGURE 5–40

Pipe or duct flow may involve more than one form of work at the same time.

- The velocities involved in pipe and duct flow are relatively low, and the kinetic energy changes are usually insignificant. This is particularly true when the pipe or duct diameter is constant and the heating effects are negligible.
- Kinetic energy changes may be significant, however, for gas flow in ducts with variable cross-sectional areas especially when the compressibility effects are significant.
- The potential energy term may also be significant when the fluid undergoes a considerable elevation change as it flows in a pipe or duct.

EXAMPLE 5–11 Electric Heating of Air in a House

The electric heating systems used in many houses consist of a simple duct with resistance heaters. Air is heated as it flows over resistance wires. Consider a 15-kW electric heating system. Air enters the heating section at 100 kPa and 17°C with a volume flow rate of 150 m³/min. If heat is lost from the air in the duct to the surroundings at a rate of 200 W, determine the exit temperature of air.

Solution The electric heating system of a house is considered. For specified electric power consumption and air flow rate, the air exit temperature is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **4** Constant specific heats at room temperature can be used for air.

Analysis We take the *heating section portion of the duct* as the system (Fig. 5–41). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, heat is lost from the system and electrical work is supplied to the system.

At temperatures encountered in heating and air-conditioning applications, Δh can be replaced by $c_p \Delta T$ where $c_p = 1.005 \text{ kJ/kg} \cdot ^\circ\text{C}$ —the value at room temperature—with negligible error (Fig. 5–42). Then the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

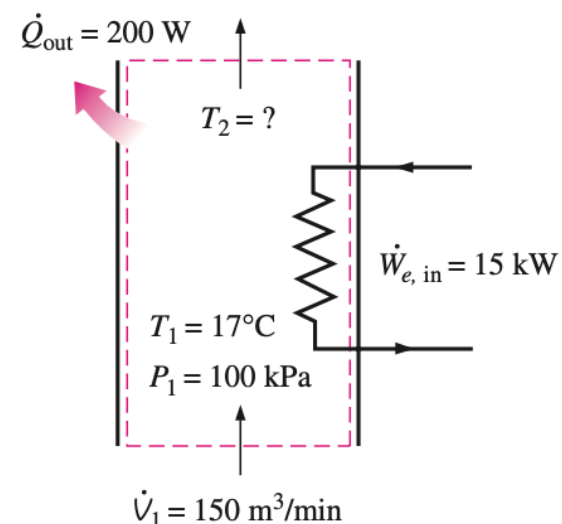


FIGURE 5–41
Schematic for Example 5–11.

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{e,\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$

$$\dot{W}_{e,\text{in}} - \dot{Q}_{\text{out}} = \dot{m}c_p(T_2 - T_1)$$

From the ideal-gas relation, the specific volume of air at the inlet of the duct is

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290 \text{ K})}{100 \text{ kPa}} = 0.832 \text{ m}^3/\text{kg}$$

The mass flow rate of the air through the duct is determined from

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{150 \text{ m}^3/\text{min}}{0.832 \text{ m}^3/\text{kg}} \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 3.0 \text{ kg/s}$$

Substituting the known quantities, the exit temperature of the air is determined to be

$$(15 \text{ kJ/s}) - (0.2 \text{ kJ/s}) = (3 \text{ kg/s})(1.005 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 17)^\circ\text{C}$$

$$T_2 = \mathbf{21.9^\circ\text{C}}$$

Discussion Note that heat loss from the duct reduces the exit temperature of air.

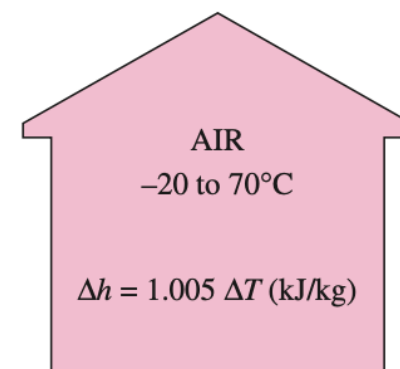


FIGURE 5–42

The error involved in $\Delta h = c_p \Delta T$, where $c_p = 1.005 \text{ kJ/kg} \cdot ^\circ\text{C}$, is less than 0.5 percent for air in the temperature range -20 to 70°C .

ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

- During a steady-flow process, no changes occur within the control volume; thus, one does not need to be concerned about what is going on within the boundaries. Not having to worry about any changes within the control volume with time greatly simplifies the analysis.
- Many processes of interest, however, involve *changes* within the control volume with time. Such processes are called *unsteady-flow*, or *transient-flow*, processes. The steady-flow relations developed earlier are obviously not applicable to these processes. When an unsteady-flow process is analyzed, it is important to keep track of the mass and energy contents of the control volume as well as the energy interactions across the boundary.
- Some familiar unsteady-flow processes are the charging of rigid vessels from supply lines (Fig. 5–43), discharging a fluid from a pressurized vessel, driving a gas turbine with pressurized air stored in a large container, inflating tires or balloons, and even cooking with an ordinary pressure cooker.

- Unlike steady-flow processes, unsteady-flow processes start and end over some finite time period instead of continuing indefinitely. Therefore in this section, we deal with changes that occur over some time interval Δt instead of with the rate of changes (changes per unit time).
- An unsteady-flow system, in some respects, is similar to a closed system, except that the mass within the system boundaries does not remain constant during a process.
- Another difference between steady- and unsteady-flow systems is that steady-flow systems are fixed in space, size, and shape. Unsteady-flow systems, however, are not (Fig. 5–44). They are usually stationary; that is, they are fixed in space, but they may involve moving boundaries and thus boundary work.

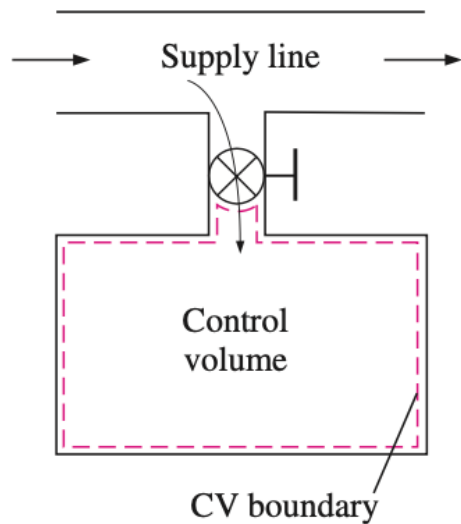


FIGURE 5-43

Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.

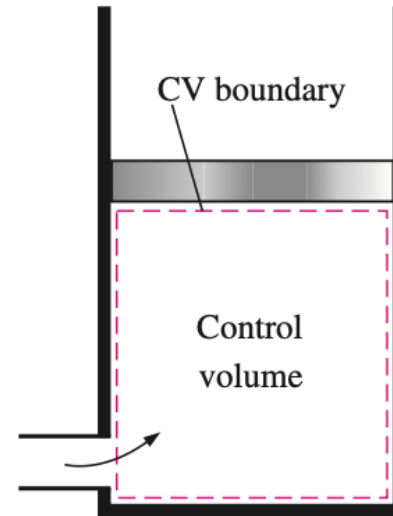


FIGURE 5-44

The shape and size of a control volume may change during an unsteady-flow process.

The *mass balance* for any system undergoing any process can be expressed as (see Sec. 5–1)

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \quad (\text{kg}) \quad (5-42)$$

where $\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the system. For control volumes, it can also be expressed more explicitly as

$$m_i - m_e = (m_2 - m_1)_{\text{CV}} \quad (5-43)$$

where i = inlet, e = exit, 1 = initial state, and 2 = final state of the control volume. Often one or more terms in the equation above are zero. For example, $m_i = 0$ if no mass enters the control volume during the process, $m_e = 0$ if no mass leaves, and $m_1 = 0$ if the control volume is initially evacuated.

The energy content of a control volume changes with time during an unsteady-flow process. The magnitude of change depends on the amount of energy transfer across the system boundaries as heat and work as well as on the amount of energy transported into and out of the control volume by mass during the process. When analyzing an unsteady-flow process, we must keep track of the energy content of the control volume as well as the energies of the incoming and outgoing flow streams.

The general energy balance was given earlier as

$$\text{Energy balance:} \quad \underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ}) \quad (5-44)$$

- The general unsteady-flow process, in general, is difficult to analyze because the properties of the mass at the inlets and exits may change during a process.
- Most unsteady-flow processes, however, can be represented reasonably well by the **uniform-flow process**, which involves the following idealization: *The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.*
- Note that unlike the steady-flow systems, the state of an unsteady-flow system may change with time, and that the state of the mass leaving the control volume at any instant is the same as the state of the mass in the control volume at that instant.
- The initial and final properties of the control volume can be determined from the knowledge of the initial and final states, which are completely specified by two independent intensive properties for simple compressible systems.

Then the energy balance for a uniform-flow system can be expressed explicitly as

$$\left(Q_{\text{in}} + W_{\text{in}} + \sum_{\text{in}} m\theta \right) - \left(Q_{\text{out}} + W_{\text{out}} + \sum_{\text{out}} m\theta \right) = (m_2 e_2 - m_1 e_1)_{\text{system}} \quad (5-45)$$

where $\theta = h + \text{ke} + \text{pe}$ is the energy of a fluid stream at any inlet or exit per unit mass, and $e = u + \text{ke} + \text{pe}$ is the energy of the nonflowing fluid within the control volume per unit mass. When the kinetic and potential energy changes associated with the control volume and fluid streams are negligible, as is usually the case, the energy balance above simplifies to

$$Q - W = \sum_{\text{out}} mh - \sum_{\text{in}} mh + (m_2 u_2 - m_1 u_1)_{\text{system}} \quad (5-46)$$

where $Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ is the net heat input and $W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ is the net work output. Note that if no mass enters or leaves the control volume during a process ($m_i = m_e = 0$, and $m_1 = m_2 = m$), this equation reduces to the energy balance relation for closed systems (Fig. 5-45). Also note that an unsteady-flow system may involve boundary work as well as electrical and shaft work (Fig. 5-46).

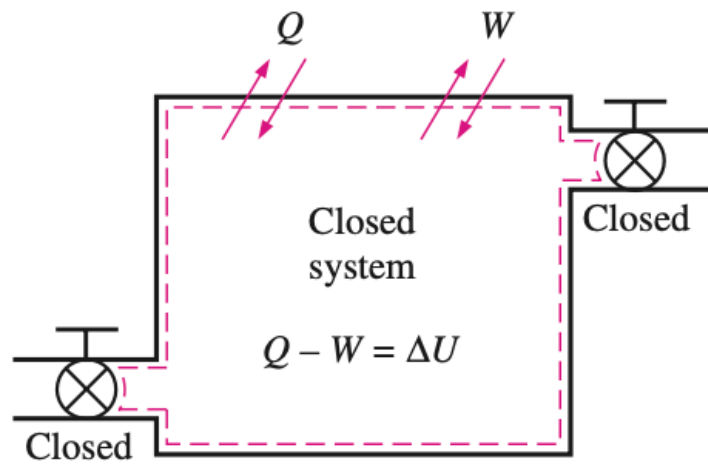


FIGURE 5-45

The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

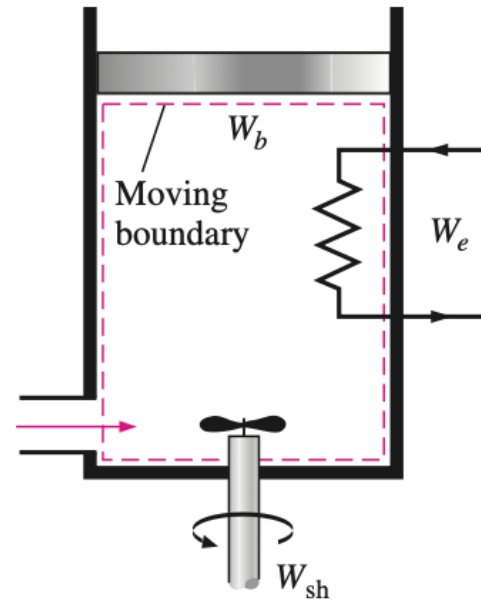


FIGURE 5-46

A uniform-flow system may involve electrical, shaft, and boundary work all at once.

- Although both the steady-flow and uniform-flow processes are somewhat idealized, many actual processes can be approximated reasonably well by one of these with satisfactory results. The degree of satisfaction depends on the desired accuracy and the degree of validity of the assumptions m

EXAMPLE 5–12 Charging of a Rigid Tank by Steam

A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 1 MPa and 300°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure reaches 1 MPa, at which point the valve is closed. Determine the final temperature of the steam in the tank.

Solution A valve connecting an initially evacuated tank to a steam line is opened, and steam flows in until the pressure inside rises to the line level. The final temperature in the tank is to be determined.

Assumptions **1** This process can be analyzed as a *uniform-flow process* since the properties of the steam entering the control volume remain constant during the entire process. **2** The kinetic and potential energies of the streams are negligible, $ke \cong pe \cong 0$. **3** The tank is stationary and thus its kinetic and potential energy changes are zero; that is, $\Delta KE = \Delta PE = 0$ and $\Delta E_{\text{system}} = \Delta U_{\text{system}}$. **4** There are no boundary, electrical, or shaft work interactions involved. **5** The tank is well insulated and thus there is no heat transfer.

Analysis We take the *tank* as the system (Fig. 5–47). This is a *control volume* since mass crosses the system boundary during the process. We observe that this is an unsteady-flow process since changes occur within the control volume. The control volume is initially evacuated and thus $m_1 = 0$ and $m_1 u_1 = 0$. Also, there is one inlet and no exits for mass flow.

Noting that microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u , respectively, the mass and energy balances for this uniform-flow system can be expressed as

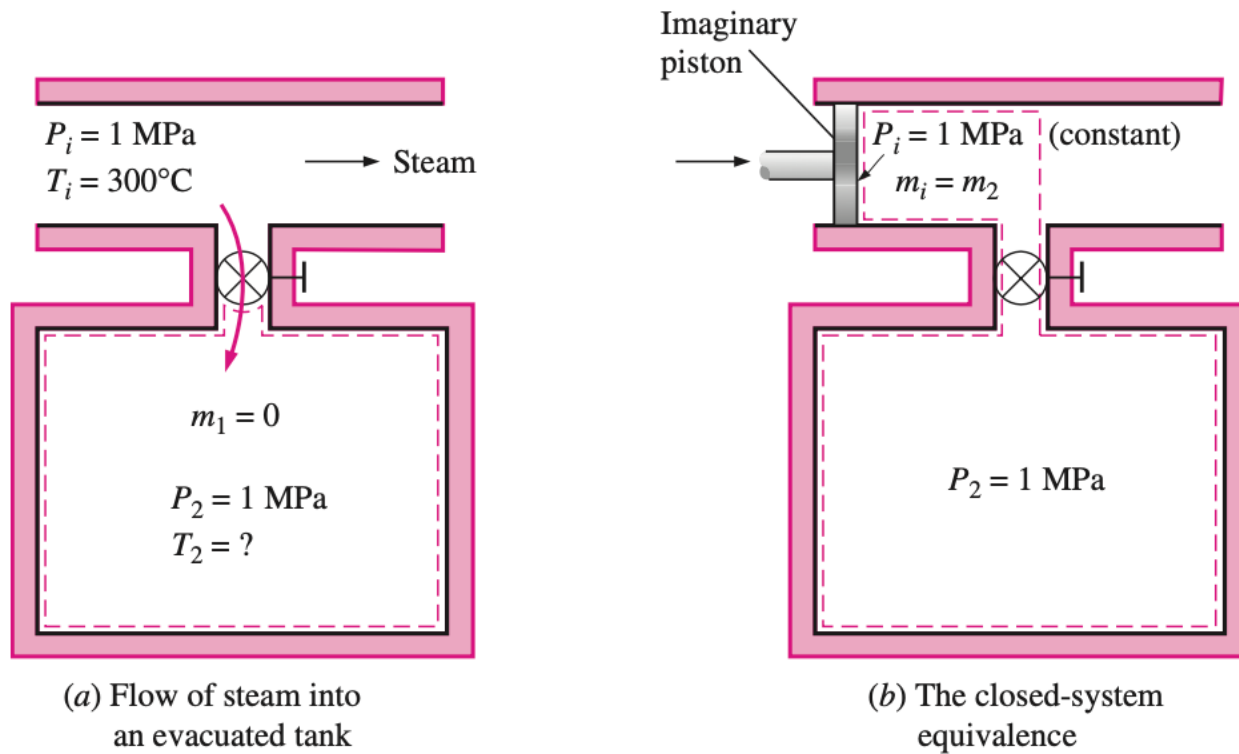


FIGURE 5-47

Schematic for Example 5-12.

$$\text{Mass balance: } m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow m_i = m_2 - m_1^0 = m_2$$

$$\text{Energy balance: } \underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$m_i h_i = m_2 u_2 \quad (\text{since } W = Q = 0, \text{ ke} \cong \text{pe} \cong 0, m_1 = 0)$$

Combining the mass and energy balances gives

$$u_2 = h_i$$

That is, the final internal energy of the steam in the tank is equal to the enthalpy of the steam entering the tank. The enthalpy of the steam at the inlet state is

$$\left. \begin{array}{l} P_i = 1 \text{ MPa} \\ T_i = 300^\circ\text{C} \end{array} \right\} h_i = 3051.6 \text{ kJ/kg} \quad (\text{Table A-6})$$

which is equal to u_2 . Since we now know two properties at the final state, it is fixed and the temperature at this state is determined from the same table to be

$$\left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ u_2 = 3051.6 \text{ kJ/kg} \end{array} \right\} T_2 = \mathbf{456.1^\circ\text{C}}$$

Discussion Note that the temperature of the steam in the tank has increased by 156.1°C. This result may be surprising at first, and you may be wondering where the energy to raise the temperature of the steam came from. The answer lies in the enthalpy term $h = u + Pv$. Part of the energy represented by enthalpy is the flow energy Pv , and this flow energy is converted to sensible internal energy once the flow ceases to exist in the control volume, and it shows up as an increase in temperature (Fig. 5–48).

Alternative solution This problem can also be solved by considering the region within the tank and the mass that is destined to enter the tank as a closed system, as shown in Fig. 5–47*b*. Since no mass crosses the boundaries, viewing this as a closed system is appropriate.

During the process, the steam upstream (the imaginary piston) will push the enclosed steam in the supply line into the tank at a constant pressure of 1 MPa. Then the boundary work done during this process is

$$W_{b,\text{in}} = - \int_1^2 P_i dV = -P_i(V_2 - V_1) = -P_i[V_{\text{tank}} - (V_{\text{tank}} + V_i)] = P_i V_i$$

where V_i is the volume occupied by the steam before it enters the tank and P_i is the pressure at the moving boundary (the imaginary piston face). The energy balance for the closed system gives

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$W_{b,\text{in}} = \Delta U$$

$$m_i P_i V_i = m_2 u_2 - m_i u_i$$

$$u_2 = u_i + P_i v_i = h_i$$

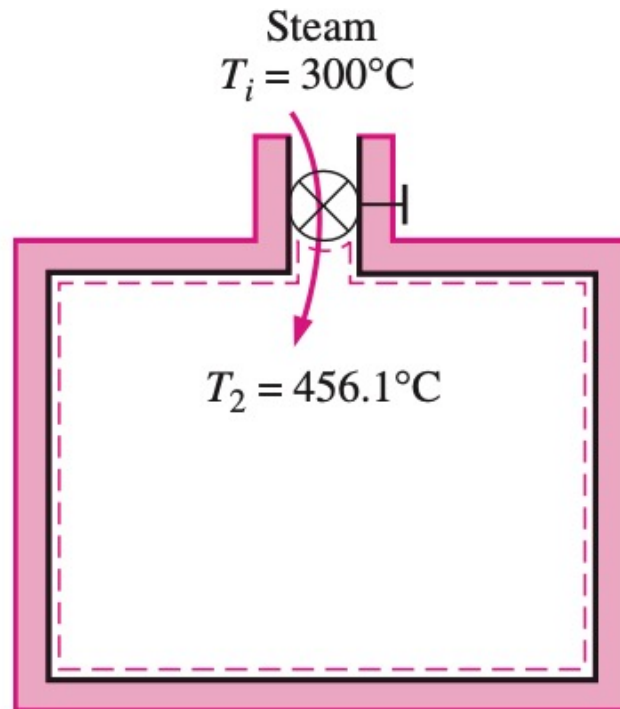


FIGURE 5–48

The temperature of steam rises from 300 to 456.1°C as it enters a tank as a result of flow energy being converted to internal energy.

EXAMPLE 5–13 Cooking with a Pressure Cooker

A pressure cooker is a pot that cooks food much faster than ordinary pots by maintaining a higher pressure and temperature during cooking. The pressure inside the pot is controlled by a pressure regulator (the petcock) that keeps the pressure at a constant level by periodically allowing some steam to escape, thus preventing any excess pressure buildup.

Pressure cookers, in general, maintain a gage pressure of 2 atm (or 3 atm absolute) inside. Therefore, pressure cookers cook at a temperature of about 133°C (or 271°F) instead of 100°C (or 212°F), cutting the cooking time by as much as 70 percent while minimizing the loss of nutrients. The newer pressure cookers use a spring valve with several pressure settings rather than a weight on the cover.

A certain pressure cooker has a volume of 6 L and an operating pressure of 75 kPa gage. Initially, it contains 1 kg of water. Heat is supplied to the pressure cooker at a rate of 500 W for 30 min after the operating pressure is reached. Assuming an atmospheric pressure of 100 kPa, determine (a) the temperature at which cooking takes place and (b) the amount of water left in the pressure cooker at the end of the process.

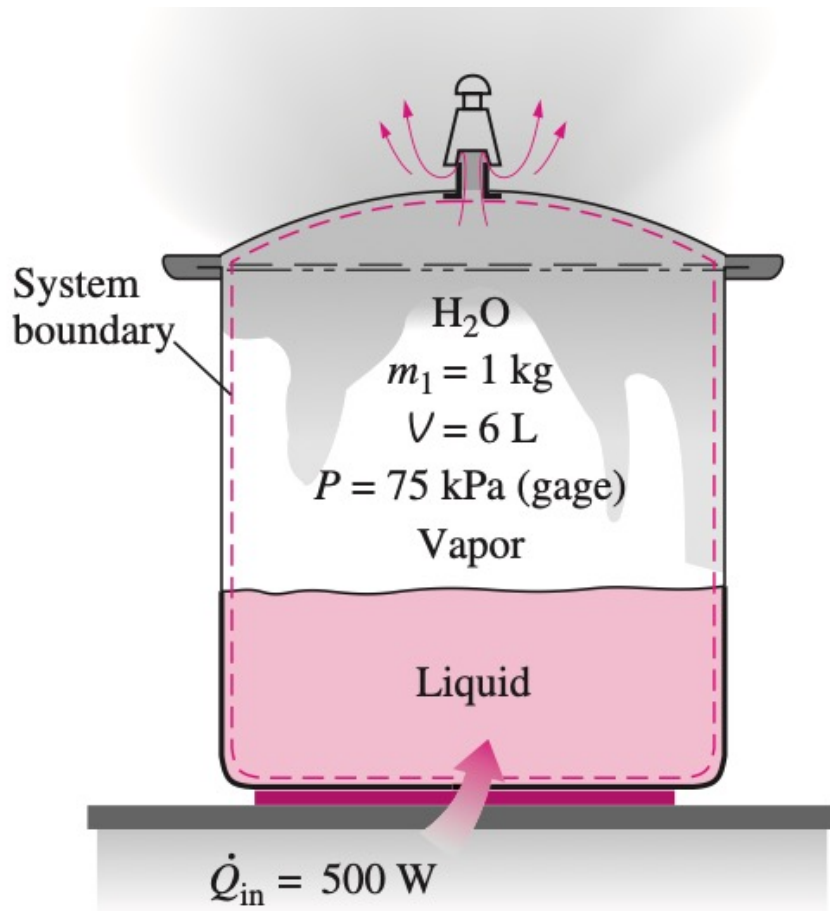


FIGURE 5–49

Schematic for Example 5–13.

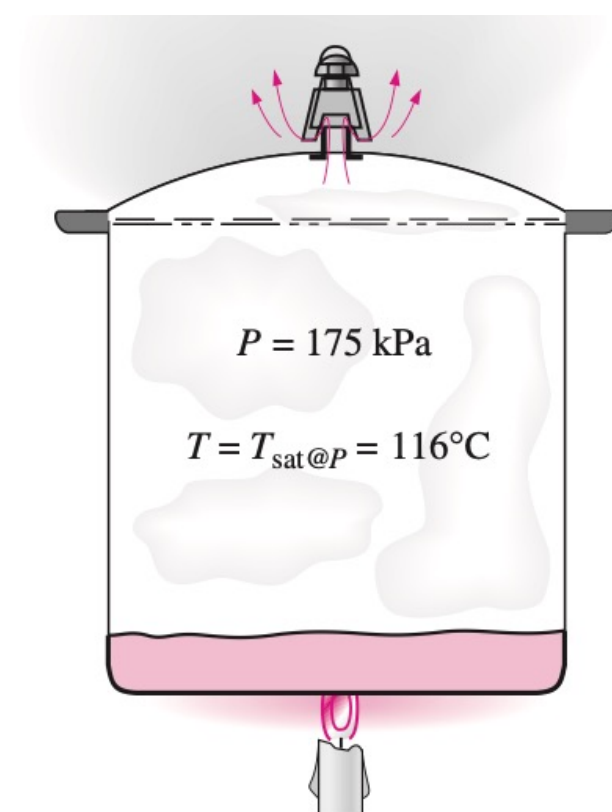


FIGURE 5–50

As long as there is liquid in a pressure cooker, the saturation conditions exist and the temperature remains constant at the saturation temperature.

Solution Heat is transferred to a pressure cooker at a specified rate for a specified time period. The cooking temperature and the water remaining in the cooker are to be determined.

Assumptions **1** This process can be analyzed as a *uniform-flow process* since the properties of the steam leaving the control volume remain constant during the entire cooking process. **2** The kinetic and potential energies of the streams are negligible, $ke \cong pe \cong 0$. **3** The pressure cooker is stationary and thus its kinetic and potential energy changes are zero; that is, $\Delta KE = \Delta PE = 0$ and $\Delta E_{\text{system}} = \Delta U_{\text{system}}$. **4** The pressure (and thus temperature) in the pressure cooker remains constant. **5** Steam leaves as a saturated vapor at the cooker pressure. **6** There are no boundary, electrical, or shaft work interactions involved. **7** Heat is transferred to the cooker at a constant rate.

Analysis We take the *pressure cooker* as the system (Fig. 5–49). This is a *control volume* since mass crosses the system boundary during the process. We observe that this is an unsteady-flow process since changes occur within the control volume. Also, there is one exit and no inlets for mass flow.

(a) The absolute pressure within the cooker is

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 75 + 100 = 175 \text{ kPa}$$

Since saturation conditions exist in the cooker at all times (Fig. 5–50), the cooking temperature must be the saturation temperature corresponding to this pressure. From Table A–5, it is

$$T = T_{\text{sat @ 175 kPa}} = \mathbf{116.04^\circ\text{C}}$$

which is about 16°C higher than the ordinary cooking temperature.

(b) Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u , respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance:

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}} \rightarrow -m_e = (m_2 - m_1)_{\text{CV}} \quad \text{or} \quad m_e = (m_1 - m_2)_{\text{CV}}$$

Energy balance:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q_{\text{in}} - m_e h_e = (m_2 u_2 - m_1 u_1)_{\text{CV}} \quad (\text{since } W = 0, \text{ke} \cong \text{pe} \cong 0)$$

Combining the mass and energy balances gives

$$Q_{\text{in}} = (m_1 - m_2)h_e + (m_2 u_2 - m_1 u_1)_{\text{CV}}$$

The amount of heat transfer during this process is found from

$$Q_{\text{in}} = \dot{Q}_{\text{in}} \Delta t = (0.5 \text{ kJ/s})(30 \times 60 \text{ s}) = 900 \text{ kJ}$$

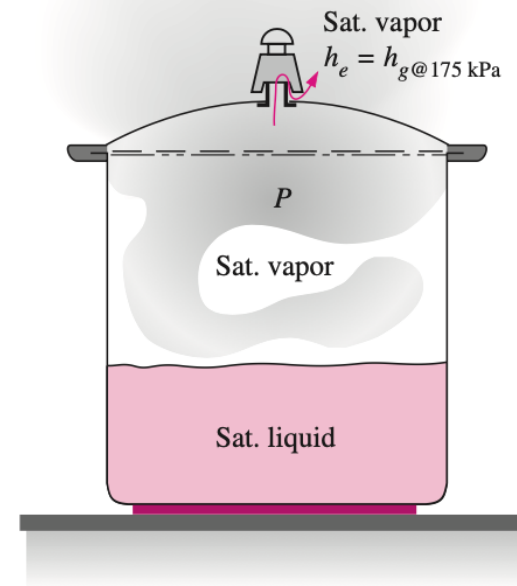


FIGURE 5–51

In a pressure cooker, the enthalpy of the exiting steam is $h_g @ 175 \text{ kPa}$ (enthalpy of the saturated vapor at the given pressure).

Steam leaves the pressure cooker as saturated vapor at 175 kPa at all times (Fig. 5–51). Thus,

$$h_e = h_g @ 175 \text{ kPa} = 2700.2 \text{ kJ/kg}$$

The initial internal energy is found after the quality is determined:

$$v_1 = \frac{V}{m_1} = \frac{0.006 \text{ m}^3}{1 \text{ kg}} = 0.006 \text{ m}^3/\text{kg}$$

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.006 - 0.001}{1.004 - 0.001} = 0.00499$$

Thus,

$$u_1 = u_f + x_1 u_{fg} = 486.82 + (0.00499)(2037.7) \text{ kJ/kg} = 497 \text{ kJ/kg}$$

and

$$U_1 = m_1 u_1 = (1 \text{ kg})(497 \text{ kJ/kg}) = 497 \text{ kJ}$$

The mass of the system at the final state is $m_2 = V/v_2$. Substituting this into the energy equation yields

$$Q_{\text{in}} = \left(m_1 - \frac{V}{v_2} \right) h_e + \left(\frac{V}{v_2} u_2 - m_1 u_1 \right)$$

There are two unknowns in this equation, u_2 and v_2 . Thus we need to relate them to a single unknown before we can determine these unknowns. Assuming there is still some liquid water left in the cooker at the final state (i.e., saturation conditions exist), v_2 and u_2 can be expressed as

$$v_2 = v_f + x_2 v_{fg} = 0.001 + x_2(1.004 - 0.001) \text{ m}^3/\text{kg}$$

$$u_2 = u_f + x_2 u_{fg} = 486.82 + x_2(2037.7) \text{ kJ/kg}$$

Recall that during a boiling process at constant pressure, the properties of each phase remain constant (only the amounts change). When these expressions are substituted into the above energy equation, x_2 becomes the only unknown, and it is determined to be

$$x_2 = 0.009$$

Thus,

$$v_2 = 0.001 + (0.009)(1.004 - 0.001) \text{ m}^3/\text{kg} = 0.010 \text{ m}^3/\text{kg}$$

and

$$m_2 = \frac{V}{v_2} = \frac{0.006 \text{ m}^3}{0.01 \text{ m}^3/\text{kg}} = \mathbf{0.6 \text{ kg}}$$

Therefore, after 30 min there is 0.6 kg water (liquid + vapor) left in the pressure cooker.

Discussion Note that almost half of the water in the pressure cooker has evaporated during cooking.