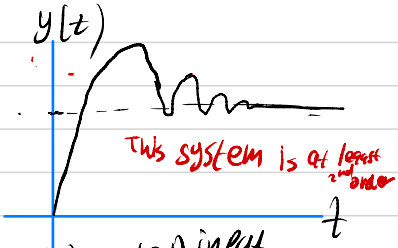
Calculus of Variation.

* Reverse engineering

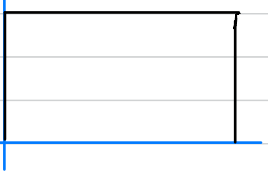
⟹ We have a machine in existence, can know what are the components of the system knowing the input and output?



2nd order system $y(t) = e^{\text{Real part}}\left[\cos(\omega_d t) + \sin(\omega_d t)\right]$ (imaginary)

$r(t) \equiv$ step input

$r(t) \equiv$ step $\equiv r$



All we need to do is find ω_d, ζ, ω_n

$\omega_d = \omega_n\left(\sqrt{1-\zeta^2}\right)$

$\zeta = \frac{c}{c_r} = 2m\omega_n = \frac{\omega_n}{\omega_d}$

↳ Find damping coefficient ratio by 3 ways

1) logarithmic decreament

1) $J = \frac{1}{n}\cdot\frac{y}{2}\sum_{i=1}^{}\left(x_i^{\text{model}} - x_i^{\text{test}}\right)^2$

---

goals of log decrement

1) Damping ratio

$x = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \theta)$

Max displacement at the crest

$\delta = \frac{1}{n}\ln\left[\frac{x_1}{x_{n+1}}\right]$

$\ln\left(\frac{8.39}{7.84}\right) = 0.0654$ $(n+1)$

$t_1 = 6.3,\ x_1 = 8.37$
$t_2 = 7.4,\ x_2 = 7.84$
$t_3 = 8.5,\ x_3 = 7.62$

$x_1 = 8.4 \quad x_3 = 7.5$
$x_2 = 7.7 \quad \ln\left(\frac{8.4}{7.5}\right)\cdot\frac{1}{2}$
$\delta = \ln\left(\frac{8.4}{7.7}\right) = 0.087$

$\zeta = \frac{\delta}{\sqrt{\delta^2+4\pi^2}} = 0.0138$

$\zeta = 0.0138$

$t_1 = 6.3,\ t_2 = 7.9$

$\zeta = \frac{1}{2}\ln\left(\frac{8.39}{7.62}\right) = 0.0481$

$\zeta_3 = \frac{\delta}{\sqrt{\delta^2+4\pi^2}} = 7.66\times10^{-3}$

(3) $T_3 = 0.9$

$\omega_{d_3} = \frac{2\pi}{T} = 6.98131$

$\omega_{n_3} = \omega_d\sqrt{1-\zeta^2} = 6.98111$

$T =$

$t_1 = 5.1 \quad X_1 = 6.23$
$t_2 = 8.2 \quad X_2 = 5.81$
$t_5 = 10.4 \quad X_5 = 5.18$

$$\delta = \frac{1}{4} \ln\left(\frac{6.23}{5.18}\right) = 0.04614$$

$$f_1 = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} = 0.00732$$

---

(2)

$X_1 = 7.45 \quad X_4 = 6.58$
$t_1 = 7 \quad t_4 = 10.4$

$$\delta = \frac{1}{3} \ln\left(\frac{7.45}{6.58}\right) = 0.04139$$

$$f_2 = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} = 0.006587$$

$5.711^2 m_{eq} = K(0.99787)$
$5.711^2 m_{eq} - 5.711^2 = K(0.9882)$
$32.615 m_{eq} - 0.99787K = 0$
$32.615 m_{eq} - 0.9982K = -32.615$

$m_{eq} = 3023.848 \text{ kg}, \quad K = 98835.33 \text{ N/m}$

$$c_1 = \frac{2\xi K_1}{\omega_{n_1}}$$

$$c_1 = \frac{2 \times 0.04614}{5.7108} = 0.01615$$

$$c_2 = \frac{2\xi f_2}{\omega_{n_2}} = 0.01449$$